



The #1 Best Selling Self Study Guide

BITSAT

Prep Guide

Fully Updated

Complete Study of all
the Sections; Physics,
Chemistry, Maths etc.

Coverage of Past Years'
Question Papers

With 5 Practice Sets



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ARIHANT PRAKASHAN (Series), MEERUT



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SYLLABUS

PART I PHYSICS

1. Units & Measurement

- 1.1 Units (Different systems of units, SI units, fundamental and derived units)
- 1.2 Dimensional Analysis
- 1.3 Precision and significant figures
- 1.4 Fundamental measurements in Physics (Vernier calipers, screw gauge, Physical balance etc.)

2. Kinematics

- 2.1 Properties of vectors
- 2.2 Position, velocity and acceleration vectors
- 2.3 Motion with constant acceleration
- 2.4 Projectile motion
- 2.5 Uniform circular motion
- 2.6 Relative motion

3. Newton's Laws of Motion

- 3.1 Newton's laws (free body diagram, resolution of forces)
- 3.2 Motion on an inclined plane
- 3.3 Motion of blocks with pulley systems
- 3.4 Circular motion – centripetal force
- 3.5 Inertial and non-inertial frames

4. Impulse and Momentum

- 4.1 Definition of impulse and momentum
- 4.2 Conservation of momentum
- 4.3 Collisions
- 4.4 Momentum of a system of particles
- 4.5 Center of mass

5. Work and Energy

- 5.1 Work done by a force
- 5.2 Kinetic energy and work-energy theorem
- 5.3 Power
- 5.4 Conservative forces and potential energy
- 5.5 Conservation of mechanical energy

6. Rotational Motion

- 6.1 Description of rotation (angular displacement, angular velocity and angular acceleration)
- 6.2 Rotational motion with constant angular acceleration
- 6.3 Moment of inertia, Parallel and perpendicular axes theorems, rotational kinetic energy
- 6.4 Torque and angular momentum
- 6.5 Conservation of angular momentum
- 6.6 Rolling motion

7. Gravitation

- 7.1 Newton's law of gravitation
- 7.2 Gravitational potential energy, Escape velocity
- 7.3 Motion of planets – Kepler's laws, satellite motion

8. Mechanics of Solids and Fluids

- 8.1 Elasticity
- 8.2 Pressure, density and Archimedes' principle
- 8.3 Viscosity and Surface Tension
- 8.4 Bernoulli's theorem

9. Oscillations

- 9.1 Kinematics of simple harmonic motion
- 9.2 Spring mass system, simple and compound pendulum
- 9.3 Forced & damped oscillations, resonance

10. Waves

- 10.1 Progressive sinusoidal waves
- 10.2 Standing waves in strings and pipes
- 10.3 Superposition of waves, beats
- 10.4 Doppler Effect

11. Heat and Thermodynamics

- 11.1 Kinetic theory of gases
- 11.2 Thermal equilibrium and temperature

- 11.3 Specific heat, Heat Transfer - Conduction, convection and radiation, thermal conductivity, Newton's law of cooling
- 11.4 Work, heat and first law of thermodynamics
- 11.5 2nd law of thermodynamics, Carnot engine Efficiency and Coefficient of performance.

12. Electrostatics

- 12.1 Coulomb's law
- 12.2 Electric field (discrete and continuous charge distributions)
- 12.3 Electrostatic potential and Electrostatic potential energy
- 12.4 Gauss' law and its applications
- 12.5 Electric dipole
- 12.6 Capacitance and dielectrics (parallel plate capacitor, capacitors in series and parallel)

13. Current Electricity

- 13.1 Ohm's law, Joule heating
- 13.2 D.C circuits – Resistors and cells in series and parallel, Kirchoff's laws, potentiometer and Wheatstone bridge,
- 13.3 Electrical Resistance (Resistivity, origin and temperature dependence of resistivity).

14. Magnetic Effect of Current

- 14.1 Biot-Savart's law and its applications
- 14.2 Ampere's law and its applications
- 14.3 Lorentz force, force on current carrying conductors in a magnetic field
- 14.4 Magnetic moment of a current loop, torque on a current loop, Galvanometer and its conversion to voltmeter and ammeter

15. Electromagnetic Induction

- 15.1 Faraday's law, Lenz's law, eddy currents
- 15.2 Self and mutual inductance

- 15.3 Transformers and generators
- 15.4 Alternating current (peak and rms value)
- 15.5 AC circuits, LCR circuits

16. Optics

- 16.1 Laws of reflection and refraction
- 16.2 Lenses and mirrors
- 16.3 Optical instruments – telescope and microscope
- 16.4 Interference – Huygen's principle, Young's double slit experiment
- 16.5 Interference in thin films
- 16.6 Diffraction due to a single slit
- 16.7 Electromagnetic waves and their characteristics (only qualitative ideas), Electromagnetic spectrum
- 16.8 Polarization – states of polarization, Malus' law, Brewster's law

17. Modern Physics

- 17.1 Dual nature of light and matter – Photoelectric effect, De-Broglie wavelength
- 17.2 Atomic models – Rutherford's experiment, Bohr's atomic model
- 17.3 Hydrogen atom spectrum
- 17.4 Radioactivity
- 17.5 Nuclear reactions Fission and fusion, binding energy

18. Electronic Devices

- 18.1 Energy bands in solids (qualitative ideas only), conductors, insulators and semiconductors;
- 18.2 Semiconductor diode – I-V characteristics in forward and reverse bias, diode as a rectifier; I-V characteristics of LED, photodiode, solar cell, and Zener diode; Zener diode as a voltage regulator.
- 18.3 Junction transistor, transistor action, characteristics of a transistor; transistor as an amplifier (common emitter configuration) and oscillator
- 18.4 Logic gates (OR, AND, NOT, NAND and NOR). Transistor as a switch.

PART II CHEMISTRY

1. States of Matter

- 1.1 Measurement Physical quantities and SI units, Dimensional analysis, Precision, Significant figures.
- 1.2 Chemical Reactions Laws of chemical combination, Dalton's atomic theory; Mole concept; Atomic, molecular and molar masses; Percentage composition empirical & molecular formula; Balanced chemical equations & stoichiometry
- 1.3 Three states of matter, intermolecular interactions, types of bonding, melting and boiling points Gaseous state: Gas Laws, ideal behavior, ideal gas equation, empirical derivation of gas equation, Avogadro number, Kinetic theory – Maxwell distribution of velocities, Average, root mean square and most probable velocities and relation to temperature, Diffusion; Deviation from ideal behaviour – Critical temperature, Liquefaction of gases, van der Waals' equation.
- 1.4 Liquid State Vapour pressure, surface tension, viscosity.
- 1.5 Solid State Classification; Space lattices & crystal systems; Unit cell in two dimensional and three dimensional lattices, calculation of density of unit cell – Cubic & hexagonal systems; Close packing; Crystal Structures Simple AB and AB₂ type ionic crystals, covalent crystals – diamond & graphite, metals. Voids, number of atoms per unit cell in a cubic unit cell, Imperfections- Point defects, non-stoichiometric crystals; Electrical, magnetic and dielectric properties; Amorphous solids qualitative description. Band theory of metals, conductors, semiconductors and insulators, and *n*- and *p*- type semiconductors.

2. Atomic Structure

- 2.1 Introduction Radioactivity, Subatomic particles; Atomic number, isotopes and isobars, Thompson's model and its limitations, Rutherford's picture of atom and its limitations; Hydrogen atom spectrum and Bohr model and its limitations.
- 2.2 Quantum Mechanics Wave-particle duality de-Broglie relation, Uncertainty principle; Hydrogen atom: Quantum numbers and wavefunctions, atomic orbitals and their shapes (*s*, *p*, and *d*), Spin quantum number.

- 2.3 Many Electron Atoms Pauli exclusion principle; Aufbau principle and the electronic configuration of atoms, Hund's rule.
- 2.4 Periodicity Brief history of the development of periodic tables Periodic law and the modern periodic table; Types of elements: *s*, *p*, *d*, and *f* blocks; Periodic trends: ionization energy, atomic, and ionic radii, inter gas radii, electron affinity, electro negativity and valency. Nomenclature of elements with atomic number greater than 100.

3. Chemical Bonding & Molecular Structure

- 3.1 Valence Electrons, Ionic Bond Lattice Energy and Born-Haber cycle; Covalent character of ionic bonds and polar character of covalent bond, bond parameters
- 3.2 Molecular Structure Lewis picture & resonance structures, VSEPR model & molecular shapes
- 3.3 Covalent Bond Valence Bond Theory- Orbital overlap, Directionality of bonds & hybridization (*s*, *p* & *d* orbitals only), Resonance; Molecular orbital theory- Methodology, Orbital energy level diagram, Bond order, Magnetic properties for homonuclear diatomic species (qualitative idea only).
- 3.4 Metallic Bond Qualitative description.
- 3.5 Intermolecular Forces Polarity; Dipole moments; Hydrogen Bond.

4. Thermodynamics

- 4.1 Basic Concepts Systems and surroundings; State functions; Intensive & Extensive Properties; Zeroth Law and Temperature
- 4.2 First Law of Thermodynamics Work, internal energy, heat, enthalpy, heat capacities and specific heats, measurements of *U* and *H*, Enthalpies of formation, phase transformation, ionization, electron gain; Thermochemistry; Hess's Law, Enthalpy of bond dissociation, combustion, atomization, sublimation, solution and dilution
- 4.3 Second Law Spontaneous and reversible processes; entropy; Gibbs free energy related to spontaneity and non-spontaneity, non-mechanical work; Standard free energies of formation, free energy change and chemical equilibrium
- 4.4 Third Law Introduction

5. Physical and Chemical Equilibria

- 5.1 Concentration Units Mole Fraction, Molarity, and Molality
- 5.2 Solutions Solubility of solids and gases in liquids, Vapour Pressure, Raoult's law, Relative lowering of vapour pressure, depression in freezing point; elevation in boiling point; osmotic pressure, determination of molecular mass; solid solutions, abnormal molecular mass, van't Hoff factor. Equilibrium: Dynamic nature of equilibrium, law of mass action
- 5.3 Physical Equilibrium Equilibria involving physical changes (solid-liquid, liquid-gas, solid-gas), Surface chemistry, Adsorption, Physical and Chemical adsorption, Langmuir Isotherm, Colloids and emulsion, classification, preparation, uses.
- 5.4 Chemical Equilibria Equilibrium constants (K_p , K_c), Factors affecting equilibrium, Le-Chatelier's principle.
- 5.5 Ionic Equilibria Strong and Weak electrolytes, Acids and Bases (Arrhenius, Lewis, Lowry and Bronsted) and their dissociation; degree of ionization, Ionization of Water; ionization of polybasic acids, pH; Buffer solutions; Henderson equation, Acid-base titrations; Hydrolysis; Solubility Product of Sparingly Soluble Salts; Common Ion Effect.
- 5.6 Factors Affecting Equilibria Concentration, Temperature, Pressure, Catalysts, Significance of ΔG and ΔG° in Chemical Equilibria.

6. Electrochemistry

- 6.1 Redox Reactions Oxidation-reduction reactions (electron transfer concept); Oxidation number; Balancing of redox reactions; Electrochemical cells and cell reactions; Standard electrode potentials; EMF of Galvanic cells; Nernst equation; Factors affecting the electrode potential; Gibbs energy change and cell potential; Secondary cells; dry cells, Fuel cells; Corrosion and its prevention.
- 6.2 Electrolytic Conduction Electrolytic Conductance; Specific and molar conductivities; variations of conductivity with concentration, Kohlrausch's Law and its application, Electrolysis, Faraday's laws of electrolysis; Coulometer; Electrode potential and electrolysis, Commercial production of the chemicals, NaOH, Na, Al, Cl_2 & F_2 .

7. Chemical Kinetics

- 7.1 Aspects of Kinetics Rate and Rate expression of a reaction; Rate constant; Order and molecularity of the reaction; Integrated rate expressions and half life for zero and first order reactions.
- 7.2 Factor Affecting the Rate of the Reactions Concentration of the reactants, catalyst; size of particles, Temperature dependence of rate constant concept of collision theory (elementary idea, no mathematical treatment); Activation energy; Catalysis, Surface catalysis, enzymes, zeolites; Factors affecting rate of collisions between molecules.
- 7.3 Mechanism of Reaction Elementary reactions; Complex reactions; Reactions involving two/three steps only.
- 7.4 Surface Chemistry Adsorption Physisorption and chemisorption; factors affecting adsorption of gases on solids; catalysis: homogeneous and heterogeneous, activity and selectivity: enzyme catalysis, colloidal state: distinction between true solutions, colloids and suspensions; lyophilic, lyophobic multi molecular and macromolecular colloids; properties of colloids; Tyndall effect, Brownian movement, electrophoresis, coagulations; emulsions – types of emulsions.

8. Hydrogen and s-block Elements

- 8.1 Hydrogen Element Unique position in periodic table, occurrence, isotopes; Dihydrogen: preparation, properties, reactions, and uses; Molecular, saline, ionic, covalent, interstitial hydrides; Water: Properties; Structure and aggregation of water molecules; Heavy water; Hydrogen peroxide: preparation, reaction, structure & use, Hydrogen as a fuel.
- 8.2 s-block Elements Abundance and occurrence; Anomalous properties of the first elements in each group; diagonal relationships; trends in the variation of properties (ionization energy, atomic & ionic radii).
- 8.3 Alkali Metals Lithium, sodium and potassium: occurrence, extraction, reactivity, and electrode potentials; Biological importance; Reactions with oxygen, hydrogen, halogens water and liquid ammonia; Basic nature of oxides and hydroxides; Halides; Properties and uses of compounds such as NaCl, Na_2CO_3 , NaHCO_3 , NaOH, KCl and KOH.

- 8.4 Alkaline Earth Metals Magnesium and calcium: Occurrence, extraction, reactivity and electrode potentials; Reactions with O_2 , H_2O , H_2 and halogens; Solubility and thermal stability of oxo salts; Biological importance of Ca and Mg; Preparation, properties and uses of important compounds such as CaO , $Ca(OH)_2$, plaster of Paris, $MgSO_4$, $MgCl_2$, $CaCO_3$, and $CaSO_4$; Lime and limestone, cement.

9. *p*-, *d*- and *f*-block Elements

- 9.1 General Abundance, distribution, physical and chemical properties, isolation and uses of elements; Trends in chemical reactivity of elements of a group; electronic configuration, oxidation states; anomalous properties of first element of each group.
- 9.2 Group 13 Elements Boron, Properties and uses of borax, boric acid, boron hydrides & halides. Reaction of aluminium with acids and alkalis;
- 9.3 Group 14 Elements Carbon, carbon catenation, physical & chemical properties, uses, allotropes (graphite, diamond, fullerenes), oxides, halides and sulphides, carbides; Silicon: Silica, silicates, silicone, silicon tetrachloride, Zeolites, and their uses
- 9.4 Group 15 Elements Dinitrogen; Preparation, reactivity and uses of nitrogen; Industrial and biological nitrogen fixation; Compound of nitrogen; Ammonia: Haber's process, properties and reactions; Oxides of nitrogen and their structures; Properties and Ostwald's process of nitric acid production; Fertilizers – NPK type; Production of phosphorus; Allotropes of phosphorus; Preparation, structure and properties of hydrides, oxides, oxoacids (elementary idea only) and halides of phosphorus, phosphine.
- 9.5 Group 16 Elements Isolation and chemical reactivity of dioxygen; Acidic, basic and amphoteric oxides; Preparation, structure and properties of ozone; Allotropes of sulphur; Preparation/production properties and uses of sulphur dioxide and sulphuric acid; Structure and properties of oxides, oxoacids (structures only), hydrides and halides of sulphur.
- 9.6 Group 17 and group 18 Elements Structure and properties of hydrides, oxides, oxoacids of halogens (structures only); preparation, properties & uses of chlorine & HCl; Inter halogen compounds; Bleaching Powder; Uses of Group 18 elements, Preparation, structure and reactions of xenon fluorides, oxides, and oxoacids.
- 9.7 *d*-Block Elements General trends in the chemistry of first row transition elements; Metallic character; Oxidation state; ionization enthalpy; Ionic radii; Color; Catalytic properties; Magnetic properties; Interstitial compounds; Occurrence and extraction of iron, copper, silver, zinc, and mercury; Alloy formation; Steel and some important alloys; preparation and properties of $CuSO_4$, $K_2Cr_2O_7$, $KMnO_4$, Mercury halides; Silver nitrate and silver halides; Photography.
- 9.8 *f*-Block Elements Lanthanoids and actinoids; Oxidation states and chemical reactivity of lanthanoids compounds; Lanthanide contraction and its consequences, Comparison of actinoids and lanthanoids.
- 9.9 Coordination Compounds Coordination number; Ligands; Werner's coordination theory; IUPAC nomenclature; Application and importance of coordination compounds (in qualitative analysis, extraction of metals and biological systems e.g. chlorophyll, vitamin B_{12} , and hemoglobin); Bonding: Valence-bond approach, Crystal field theory (qualitative); Stability constants; Shapes, color and magnetic properties; Isomerism including stereoisomerisms; Organometallic compounds.

10. Principles of Organic Chemistry and Hydrocarbons

- 10.1 Classification General Introduction, classification based on functional groups, trivial and IUPAC nomenclature. Methods of purification: qualitative and quantitative.
- 10.2 Electronic Displacement in a Covalent Bond Inductive, resonance effects, and hyperconjugation; free radicals; carbocations, carbanions, nucleophiles and electrophiles; types of organic reactions, free radical halogenations.
- 10.3 Alkanes and Cycloalkanes Structural isomerism, general properties and chemical reactions, free radical halogenation, combustion and pyrolysis.

- 10.4 Alkenes and Alkynes General methods of preparation and reactions, physical properties, electrophilic and free radical additions, acidic character of alkynes and (1,2 and 1,4) addition to dienes.
- 10.5 Aromatic Hydrocarbons Sources; properties; isomerism; resonance delocalization; aromaticity; polynuclear hydrocarbons; IUPAC nomenclature; mechanism of electrophilic substitution reaction, directive influence and effect of substituents on reactivity; carcinogenicity and toxicity.
- 10.6 Haloalkanes and Haloarenes Physical properties, nomenclature, optical rotation, chemical reactions and mechanism of substitution reaction. Uses and environmental effects; di, tri, tetrachloromethanes, iodoform, freon and DDT.
- 10.7 Petroleum Composition and refining, uses of petrochemicals.

11. Stereochemistry

- 11.1 Introduction Chiral molecules; optical activity; polarimetry; R,S and D,L configurations; Fischer projections; enantiomerism; racemates; diastereomerism and meso structures.
- 11.2 Conformations Ethane conformations; Newman and Sawhorse projections.
- 11.3 Geometrical isomerism in alkenes

12. Organic Compounds with Functional Groups Containing Oxygen and Nitrogen

- 12.1 General Nomenclature, electronic structure, important methods of preparation, identification, important reactions, physical and chemical properties, uses of alcohols, phenols, ethers, aldehydes, ketones, carboxylic acids, nitro compounds, amines, diazonium salts, cyanides and isocyanides.
- 12.2 Specific Reactivity of α -hydrogen in carbonyl compounds, effect of substituents on α -carbon on acid strength, comparative reactivity of acid derivatives, mechanism of nucleophilic addition and dehydration, basic character of amines, methods of preparation, and their separation, importance of diazonium salts in synthetic organic chemistry.

13. Biological, Industrial and Environmental Chemistry

- 13.1 The Cell Concept of cell and energy cycle.
- 13.2 Carbohydrates Classification; Monosaccharides; Structures of pentoses and hexoses; Anomeric carbon; Mutarotation; Simple chemical reactions of glucose, Disaccharides: reducing and nonreducing sugars – sucrose, maltose and lactose; Polysaccharides: elementary idea of structures of starch, cellulose and glycogen.
- 13.3 Proteins Amino acids; Peptide bond; Polypeptides; Primary structure of proteins; Simple idea of secondary, tertiary and quaternary structures of proteins; Denaturation of proteins and enzymes.
- 13.4 Nucleic Acids Types of nucleic acids; Primary building blocks of nucleic acids (chemical composition of DNA & RNA); Primary structure of DNA and its double helix; Replication; Transcription and protein synthesis; Genetic code.
- 13.5 Vitamins Classification, structure, functions in biosystems; Hormones
- 13.6 Polymers Classification of polymers; General methods of polymerization; Molecular mass of polymers; Biopolymers and biodegradable polymers; methods of polymerization (free radical, cationic and anionic addition polymerizations); Copolymerization: Natural rubber; Vulcanization of rubber; Synthetic rubbers. Condensation polymers.
- 13.7 Pollution Environmental pollutants; soil, water and air pollution; Chemical reactions in atmosphere; Smog; Major atmospheric pollutants; Acid rain; Ozone and its reactions; Depletion of ozone layer and its effects; Industrial air pollution; Green house effect and global warming; Green Chemistry, study for control of environmental pollution.
- 13.8 Chemicals in medicine, health-care and food: Analgesics, Tranquilizers, antiseptics, disinfectants, anti-microbials, anti-fertility drugs, antihistamines, antibiotics, antacids; Preservatives, artificial sweetening agents, antioxidants, soaps and detergents.

14. Theoretical Principles of Experimental Chemistry

- 14.1 Volumetric Analysis Principles; Standard solutions of sodium carbonate and oxalic acid; Acidbase titrations; Redox reactions involving KI, H_2SO_4 , Na_2SO_3 , $\text{Na}_2\text{S}_2\text{O}_3$ and H_2S ; Potassium permanganate in acidic, basic and neutral media; Titrations of oxalic acid, ferrous ammonium sulphate with KMnO_4 , $\text{K}_2\text{Cr}_2\text{O}_7/\text{Na}_2\text{S}_2\text{O}_3$, $\text{Cu(II)}/\text{Na}_2\text{S}_2\text{O}_3$.
- 14.2 Qualitative Analysis of Inorganic Salts Principles in the determination of the cations Pb^{2+} , Cu^{2+} , As^{3+} , Mn^{2+} , Al^{3+} , Zn^{2+} , Co^{2+} , Ca^{2+} , Sr^{2+} , Ba^{2+} , Mg^{2+} , NH_4^+ , Fe^{3+} , Ni^{2+} and the anions CO_3^{2-} , S^{2-} , SO_4^{2-} , SO_3^{2-} , NO_2^- , NO_3^- , Cl^- , Br^- , I^- , PO_4^{3-} , CH_3COO^- , $\text{C}_2\text{O}_4^{2-}$.
- 14.3 Physical Chemistry Experiments Preparation and crystallization of alum, copper sulphate. Benzoic acid ferrous sulphate, double salt of alum and ferrous sulphate, potassium ferric sulphate; Temperature vs. solubility; Study of pH changes by common ion effect in case of weak acids and weak bases; pH measurements of some solutions obtained from fruit juices, solutions of known and varied concentrations of acids, bases and salts using pH paper or universal indicator; Lyophilic and lyophobic sols; Dialysis; Role of emulsifying agents in emulsification.
- Equilibrium studies involving ferric and thiocyanate ions (ii) $[\text{Co}(\text{H}_2\text{O})_6]^{2+}$ and chloride ions; Enthalpy determination for strong acid vs. strong base neutralization reaction
- (ii) hydrogen bonding interaction between acetone and chloroform; Rates of the reaction between (i) sodium thiosulphate and hydrochloric acid, (ii) potassium iodate and sodium sulphite (iii) iodide vs. hydrogen peroxide, concentration and temperature effects in these reactions.
- 14.4 Purification Methods Filtration, crystallization, sublimation, distillation, differential extraction, and chromatography. Principles of melting point and boiling point determination; principles of paper chromatographic separation – R_f values.
- 14.5 Qualitative Analysis of Organic Compounds Detection of nitrogen, sulphur, phosphorous and halogens; Detection of carbohydrates, fats and proteins in foodstuff; Detection of alcoholic, phenolic, aldehydic, ketonic, carboxylic, amino groups and unsaturation.
- 14.6 Quantitative Analysis of Organic Compounds Basic principles for the quantitative estimation of carbon, hydrogen, nitrogen, halogen, sulphur and phosphorous; Molecular mass determination by silver salt and chloroplatinate salt methods; Calculations of empirical and molecular formulae.
- 14.7 Principles of Organic Chemistry Experiments Preparation of iodoform, acetanilide, p-nitro acetanilide, di-benzyl acetone, aniline yellow, b-naphthol; Preparation of acetylene and study of its acidic character.
- 14.8 Basic Laboratory Technique Cutting glass tube and glass rod, bending a glass tube, drawing out a glass jet, boring of cork.

PART III

a. English Proficiency, b. Logical Reasoning

a. English Proficiency

This test is designed to assess the test takers' general proficiency in the use of English language as a means of self-expression in real life situations and specifically to test the test takers' knowledge of basic grammar, their vocabulary, their ability to read fast and comprehend, and also their ability to apply the elements of effective writing.

1. Grammar

- 1.1 Agreement, Time and Tense, Parallel construction, Relative pronouns
- 1.2 Determiners, Prepositions, Modals, Adjectives
- 1.3 Voice, Transformation
- 1.4 Question tags, Phrasal verbs

2. Vocabulary

- 2.1 Synonyms, Antonyms, Odd Word, One Word, Jumbled letters, Homophones, Spelling
- 2.2 Contextual meaning.
- 2.3 Analogy

3. Reading Comprehension

- 3.1 Content/Ideas
- 3.2 Vocabulary
- 3.3 Referents
- 3.4 Idioms/Phrases
- 3.5 Reconstruction (rewording)

4. Composition

- 4.1 Rearrangement
- 4.2 Paragraph Unity
- 4.3 Linkers/Connectives

b. Logical Reasoning

The test is given to the candidates to judge their power of reasoning spread in verbal and nonverbal areas. The candidates should be able to think logically so that they perceive the data accurately, understand the relationships correctly, figure out the missing numbers or words, and to apply rules to new and different contexts. These indicators are measured through performance on such tasks as detecting missing links, following directions, classifying words, establishing sequences, and completing analogies.

5. Verbal Reasoning

- 5.1 Analogy Analogy means correspondence.
In the questions based on analogy, a particular

relationship is given and another similar relationship has to be identified from the alternatives provided.

- 5.2 Classification Classification means to assort the items of a given group on the basis of certain common quality they possess and then spot the odd option out.
- 5.3 Series Completion Here series of numbers or letters are given and one is asked to either complete the series or find out the wrong part in the series.
- 5.4 Logical Deduction – Reading Passage Here a brief passage is given and based on the passage the candidate is required to identify the correct or incorrect logical conclusions.
- 5.5 Chart Logic Here a chart or a table is given that is partially filled in and asks to complete it in accordance with the information given either in the chart / table or in the question.

6. Non-verbal Reasoning

- 6.1 Pattern Perception Here a certain pattern is given and generally a quarter is left blank. The candidate is required to identify the correct quarter from the given four alternatives.
- 6.2 Figure Formation and Analysis The candidate is required to analyze and form a figure from various given parts.
- 6.3 Paper Cutting It involves the analysis of a pattern that is formed when a folded piece of paper is cut into a definite design.
- 6.4 Figure Matrix In this more than one set of figures is given in the form of a matrix, all of them following the same rule. The candidate is required to follow the rule and identify the missing figure.
- 6.5 Rule Detection Here a particular rule is given and it is required to select from the given sets of figures, a set of figures, which obeys the rule and forms the correct series.

PART IV MATHEMATICS

1. Algebra

- 1.1 Complex numbers, addition, multiplication, conjugation, polar representation, properties of modulus and principal argument, triangle inequality, roots of complex numbers, geometric interpretations; Fundamental theorem of algebra.
- 1.2 Theory of Quadratic equations, quadratic equations in real and complex number system and their solutions, relation between roots and coefficients, nature of roots, equations reducible to quadratic equations.
- 1.3 Arithmetic, geometric and harmonic progressions, arithmetic, geometric and harmonic means, arithmetico-geometric series, sums of finite arithmetic and geometric progressions, infinite geometric series, sums of squares and cubes of the first n natural numbers.
- 1.4 Logarithms and their properties.
- 1.5 Exponential series.
- 1.6 Permutations and combinations, Permutations as an arrangement and combination as selection, simple applications.
- 1.7 Binomial theorem for a positive integral index, properties of binomial coefficients, Pascal's triangle
- 1.8 Matrices and determinants of order two or three, properties and evaluation of determinants, addition and multiplication of matrices, adjoint and inverse of matrices, Solutions of simultaneous linear equations in two or three variables, elementary row and column operations of matrices,
- 1.9 Sets, Relations and Functions, algebra of sets applications, equivalence relations, mappings, one-one, into and onto mappings, composition of mappings, binary operation, inverse of function, functions of real variables like polynomial, modulus, signum and greatest integer.
- 1.10 Mathematical Induction
- 1.11 Linear Inequalities, solution of linear inequalities in one and two variables.

2. Trigonometry

- 2.1 Measurement of angles in radians and degrees, positive and negative angles, trigonometric ratios, functions and identities.
- 2.2 Solution of trigonometric equations.
- 2.3 Properties of triangles and solutions of triangles
- 2.4 Inverse trigonometric functions
- 2.5 Heights and distances

3. Two-dimensional Coordinate Geometry

- 3.1 Cartesian coordinates, distance between two points, section formulae, shift of origin.
- 3.2 Straight lines and pair of straight lines: Equation of straight lines in various forms, angle between two lines, distance of a point from a line, lines through the point of intersection of two given lines, equation of the bisector of the angle between two lines, concurrent lines.
- 3.3 Circles and family of circles : Equation of circle in various form, equation of tangent, normal & chords, parametric equations of a circle , intersection of a circle with a straight line or a circle, equation of circle through point of intersection of two circles, conditions for two intersecting circles to be orthogonal.
- 3.4 Conic sections : parabola, ellipse and hyperbola their eccentricity, directrices & foci, parametric forms, equations of tangent & normal, conditions for $y = mx + c$ to be a tangent and point of tangency.

4. Three Dimensional Coordinate Geometry

- 4.1 Co-ordinate axes and co-ordinate planes, distance between two points, section formula, direction cosines and direction ratios, equation of a straight line in space and skew lines.
- 4.2 Angle between two lines whose direction ratios are given, shortest distance between two lines.
- 4.3 Equation of a plane, distance of a point from a plane, condition for coplanarity of three lines, angles between two planes, angle between a line and a plane.

5. Differential Calculus

- 5.1 Domain and range of a real valued function, Limits and Continuity of the sum, difference, product and quotient of two functions, Differentiability.
- 5.2 Derivative of different types of functions (polynomial, rational, trigonometric, inverse trigonometric, exponential, logarithmic, implicit functions), derivative of the sum, difference, product and quotient of two functions, chain rule.
- 5.3 Geometric interpretation of derivative, Tangents and Normals.
- 5.4 Increasing and decreasing functions, Maxima and minima of a function.
- 5.5 Rolle's Theorem, Mean Value Theorem and Intermediate Value Theorem.

6. Integral Calculus

- 6.1 Integration as the inverse process of differentiation, indefinite integrals of standard functions.
- 6.2 Methods of integration: Integration by substitution, Integration by parts, integration by partial fractions, and integration by trigonometric identities.
- 6.3 Definite integrals and their properties, Fundamental Theorem of Integral Calculus, applications in finding areas under simple curves.
- 6.4 Application of definite integrals to the determination of areas of regions bounded by simple curves.

7. Ordinary Differential Equations

- 7.1 Order and degree of a differential equation, formulation of a differential equation whole general solution is given, variables separable method.

- 7.2 Solution of homogeneous differential equations of first order and first degree

- 7.3 Linear first order differential equations

8. Probability

- 8.1 Various terminology in probability, axiomatic and other approaches of probability, addition and multiplication rules of probability.
- 8.2 Conditional probability, total probability and Baye's theorem
- 8.3 Independent events
- 8.4 Discrete random variables and distributions with mean and variance.

9. Vectors

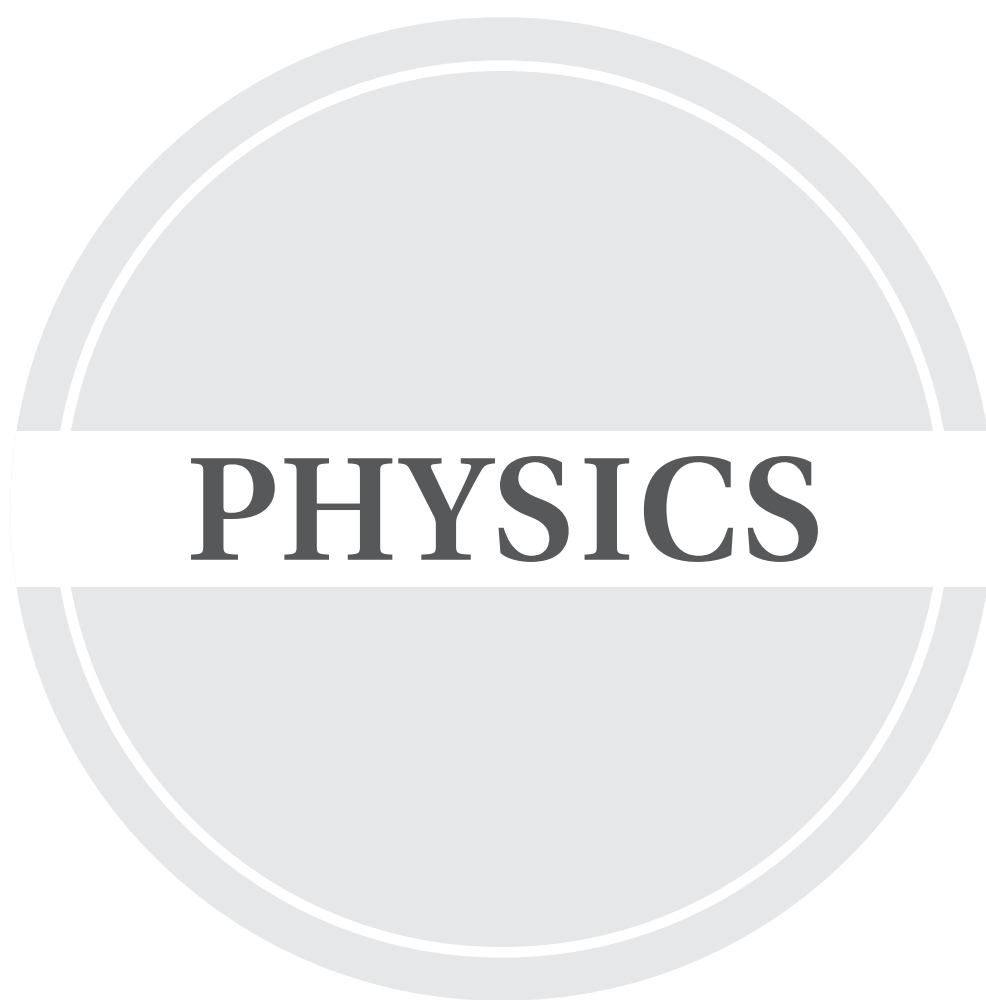
- 9.1 Direction ratio/cosines of vectors, addition of vectors, scalar multiplication, position vector of a point dividing a line segment in a given ratio.
- 9.2 Dot and cross products of two vectors, projection of a vector on a line.
- 9.3 Scalar triple products and their geometrical interpretations.

10. Statistics

- 10.1 Measures of dispersion
- 10.2 Measures of skewness and Central Tendency, Analysis of frequency distributions with equal means but different variances

11. Linear Programming

- 11.1 Various terminology and formulation of linear Programming
- 11.2 Solution of linear Programming using graphical method, feasible and infeasible regions, feasible and infeasible solutions, optimal feasible solutions (upto three nontrivial constraints)



1

Units, Measurement and Dimensions

Introduction

Science is a systematic attempt to understand natural phenomena in as much detail and depth as possible and use the knowledge, so gained to predict, modify and control the phenomena.

Every natural occurrence around us like the Sun, the wind, the planets, atmosphere, human body etc., follows some basic laws. To understand these laws, by observing natural occurrence is called **Physics**. These laws of physics are related and applicable to every aspect of life, thus understanding them leads to their applications in several fields for further development of society, which is also known as **technology**.

Physical Quantities

All those quantities which can be measured directly or indirectly and in terms of which the laws of Physics can be expressed, are called physical quantities. For example, length, mass, temperature, speed and force, electric current, etc.

Units of Physical Quantities

Unit of any physical quantity is its measurement compared to certain basic, arbitrarily chosen, internationally accepted reference standard. There are several systems of units like CGS (Centimetre, Gram and Second), FPS (Foot, Pound and Second) and MKS (Metre, Kilogram and Second).

Fundamental and Derived Units

The number of physical quantities is quite large. Thus, we may define a set of **fundamental** quantities and all other quantities may be expressed in terms of these fundamental quantities. These all other quantities are known as **derived quantities**. Units of fundamental and derived quantities are known as the **fundamental units** and **derived units**, respectively. A complete set of these units, both fundamental and derived units is known as the **system of units**.

System of Units

There are some systems used in units, can be defined as

1. **CGS System** (Centimetre, Gram, Second) is often used in scientific work. This system measures, length in centimetre (cm), mass in gram (g) and time in second (s).
2. **FPS System** (Foot, Pound, Second) It is also called the British Unit System. This unit measures, length in foot (foot), mass in gram (pound) and time in second (s).
3. **MKS System** (Metre, Kilogram, Second) This system measures length in metre(m), mass in kilogram (kg) and time in second (s).
4. **SI Units** (International System of Units) A variety of system of units (CGS, FPS and MKS) leads to the need of a unique system of units which is accepted world-wide. So, in 1971, a system of units named SI (System International in French) was developed and recommended by general conference on weights and measures. It is an extended version of the MKS system. SI system has seven fundamental units and two supplementary units, which are as follows

The two supplementary units of SI system are

- (i) **Radian for Plane Angle** Angle subtended by an arc at the centre of the circle having length equal to radius of circle has unit radians. It is denoted by **rad**.
- (ii) **Steradian for Solid Angle** It is the solid angle which has the vertex at the centre of the sphere and cut-off an area of the surface of sphere equal to that of square with sides of length equal to radius of sphere. It is expressed in unit steradian and denoted by **sr**.

Precision of Measuring Instruments

Measurement is the foundation of all experimental science and technology. The instruments used for measurement in any experiment is called measuring instruments.

Accuracy, Precision and Resolution of an Instrument

- (i) **Accuracy** An instrument is said to be the accurate, if the physical quantity measured by a measuring instrument resembles very close to its true value.
- (ii) **Precision** An instrument is said to have high degree of precision, if the value measured by it remains unchanged, however large number of times it may have been repeated.
- (iii) **Resolution** It stands for the minimum reading, which an instrument can read.

Least Count (LC)

The least count of a measuring instrument is the least value, that can be measured using the instrument. It is denoted as LC.

Least Count of Certain Measuring Instruments

- Vernier calliper, Least count = $\frac{1 \text{ mm}}{10 \text{ divisions}} = 0.1 \text{ mm}$

- Screw gauge, Least count = $\frac{\text{Value of 1 pitch scale reading}}{\text{Total number of head scale divisions}}$

$$\text{Least count} = \frac{1 \text{ mm}}{100 \text{ divisions}} = 0.01 \text{ mm}$$

- Travelling microscope,

$$\text{Least count} = \frac{\text{Value of 1 main scale division}}{\text{Total number of vernier scale divisions}} = \frac{0.5 \text{ mm}}{50 \text{ divisions}} = 0.01 \text{ mm}$$

- Spectrometer, Least count = $\frac{0.5 \text{ degree}}{30 \text{ divisions}} = \frac{30^\circ}{30 \text{ divisions}} = 1^\circ$

$$1 \text{ degree (angle)} = 60' \text{ and } 1' = 60''$$

Errors in Measurement

The uncertainty in results of every measurement by any measuring instruments, is called **error in measurement**. There can be several causes of errors like instrumental errors, imperfection in experimental techniques or procedures, error caused by random changes in temperature, pressure, humidity etc. In systematic errors, mean of many separate measurement differs significantly.

Calculation of Magnitude of Errors

(i) True Value

It is the mean of observed values.

$$\therefore a_{\text{true}} = a_{\text{mean}} = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n} = \frac{1}{n} \sum_{i=1}^n a_i$$

where, $a_1, a_2, \dots, a_n$ are observed values and n is the number of observations.

(ii) Absolute Error

Absolute error of a particular measurement is the difference between mean of observed value and true value.

Absolute error,

$$\Delta a_1 = a_{\text{mean}} - a_1,$$

$$\Delta a_2 = a_{\text{mean}} - a_2,$$

$$\vdots \quad \vdots \quad \vdots$$

and

$$\Delta a_n = a_{\text{mean}} - a_n$$

(iii) Mean Absolute Error

The arithmetic mean of the magnitudes of different values of absolute errors, is known as the mean absolute error.

∴ Mean absolute error,

$$\Delta a_{\text{mean}} = \frac{|\Delta a_1| + |\Delta a_2| + \dots + |\Delta a_n|}{n}$$

The final result of measurement can be written as $a = a_{\text{mean}} \pm \Delta a_{\text{mean}}$. This implies that value of a is likely to lie as $a_{\text{mean}} + \Delta a_{\text{mean}}$ and $a_{\text{mean}} - \Delta a_{\text{mean}}$.

(iv) Relative or Fractional Error

The ratio of the mean value of absolute error and the true value, is known as the mean relative error.

$$\begin{aligned} \text{Mean relative error} &= \frac{\text{Mean absolute error}}{\text{Mean value of measurement}} \\ &= \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \end{aligned}$$

(v) Percentage Error

When relative error is expressed in terms of percentage, then relative error is called the percentage error.

Hence,

$$\text{Percentage error} = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100\%$$

Combination of Errors

- (i) **Sum of errors** (Z) of two physical quantities A and B , where ΔA and ΔB are their absolute errors, is

$$\Delta Z = \pm (\Delta A + \Delta B)$$

- (ii) **Difference of errors** (Z) of two physical quantities A and B , where ΔA and ΔB are their absolute errors, is

$$\Delta Z = \pm (\Delta A + \Delta B)$$

- (iii) **Errors of a product** If $Z = AB$, then

$$\frac{\Delta Z}{Z} = \pm \left(\frac{\Delta A}{A} + \frac{\Delta B}{B} \right)$$

- (iv) **Errors of a quotient** If $Z = \frac{A}{B}$, then

$$\frac{\Delta Z}{Z} = \left(\frac{\Delta A}{A} \right) + \left(\frac{\Delta B}{B} \right)$$

Significant Figures

Significant figure in the measured value of a physical quantity tells the number of digits in which we have confidence. All accurately known digits in a measurement plus the first (only one uncertain digit together in a measured value form significant figures). Larger the number of significant figures obtained in a measurement, greater is the accuracy of the measurement.

Rules for Counting Significant Figures

- All the non-zero digits are significant. In 2.738, the number of significant figures is 4.
- All the zeroes between two non-zero digits are significant, no matter where the decimal point is, if at all. As examples, 209 and 3.002 have 3 and 4 significant figures respectively.
- If the measurement of number is less than 1, the zero (es) on the right of decimal point and to the left of the first non-zero digit are non-significant. In 0.00807, first three underlined zeroes are non-significant and the number of significant figures is only 3.
- The terminal or trailing zero (es) in a number without a decimal point are not significant. Thus, $12.3 = 1230\text{cm} = 12300\text{mm}$ has only 3 significant figures.
- The trailing zero (es) in number with a decimal point are significant. Thus, 3.800 kg has 4 significant figures.
- A choice of change of units does not change the number of significant digits or figures in a measurement.

Rules for Arithmetic Operations with Significant Figures

- In addition or subtraction, the final results should retain as many decimal places as there are in the number with the least decimal place. As an example sum of 423.5 g, 164.92 g and 24.381 g is 612.801 g, but it should be expressed as 612.8 g only because the least precise measurement (423.5 g) is correct to only one decimal place.
- In multiplication or division, the final result should retain as many significant figures, as are there in the original number with the least significant figures. For example, suppose an expression is performed like

$$\frac{24.3 \times 1243}{44.65} = 676.481522$$

Rounding the above result upto three significant figures, the result would become 676.

Rules for Rounding off the Uncertain Digits

Result of arithmetic computation, we get a number having more digits than the appropriate number of significant figures, then these uncertain digits are rounded off as per the rules given ahead.

- (i) The preceding digit is raised by 1, if the insignificant digit to be dropped is more than 5 and is left unchanged, if the latter is less than 5.
e.g. 18.764 will be rounded off to 18.8 and 18.74 to 18.7.
- (ii) If the insignificant figure is 5 and the preceding digit is even, then the insignificant digit is simply dropped. However, if the preceding digit is odd, then it is raised by one, so as to make it even. e.g. 17.845 will be rounded off to 17.84 and 17.875 to 17.88.

Dimensions of Physical Quantities

The dimensions of a physical quantity are the power to which the base quantities are raised to represent that quantity. The expression which shows how and which base quantities represent the dimensions of a physical quantity, is called the **dimensional formula**. e.g. for volume, dimensional formula is $[M^0L^3T^0]$. An equation, where a physical quantity is equated with its dimensional formula is called **dimensional equation**. e.g. dimensional equation for force is

$$[F] = [MLT^{-2}]$$

Principle of Homogeneity of Dimensions

According to this principle, a correct dimensional equation must be homogeneous,

i.e. dimensions of all the terms in a physical expression must be same.

$$LHS = RHS$$

Dimensional analysis can be used in conversion of units, to check the dimensional correctness of physical relation and to establish relation among various physical quantities.

Dimensional Analysis and Its Applications

Dimensional analysis help us in deducing certain relations among different quantities. Main applications of dimensional analysis are as follows:

To check the correctness of a given physical equation

If both sides of a physical relation have same dimensions, then the relation is dimensionally correct. Dimensional analysis is also used to deduce relation among the physical quantities, i.e. if the dimensions of physical quantities on both sides is known, then we can deduce relations correlating the quantities with these dimensions.

To convert a physical quantity from one system to another

Let dimensional formula of a given physical quantity be $[M^aL^bT^c]$. If a physical quantity is known in one system of unit (n_1). Then, we can relate it with another system of unit (n_2) as below

$$\frac{n_1}{n_2} = \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

NOTE Here, a system having base units $[M_1, L_1, T_1]$ the numerical value of the given quantity be n_1 , and the numerical value n_2 in another unit system having the base units M_2, L_2, T_2 .

Practice Exercise

- Which one is not a unit of time?
a. Leap year b. Year c. Shake d. Light year
- The height of the building is 50 ft. The same in millimetre is
a. 560 mm b. 285 mm
c. 1786.8 mm d. 15240 mm
- Which of the following is the most precise device for measuring length?
a. A vernier calliper with 20 divisions of the sliding scale
b. An optical instrument that can measure length within wavelength of light
c. A screw gauge of pitch 1 mm and 100 divisions on the circular scale
d. None of the above
- The radius of hydrogen atom in ground state is 5×10^{-11} m. Find the radius of hydrogen atom in fermimetre. (1 fm = 10^{-15} m).
a. 5×10^4 fm b. 2×10^4 fm
c. 5×10^2 fm d. 5×10^6 fm
- One nautical mile is 6080 ft. The same in kilometre is
a. 0.9 km b. 0.8 km
c. 1.85 km d. None of these
- The area of a room is 10 m^2 . The same in ft^2 is
a. 107.6 ft^2 b. 77 ft^2
c. 77.6 ft^2 d. None of these
- The density of iron is 7.87 g/cm^3 . If the atoms are spherical and closely packed. The mass of iron atom is 9.27×10^{-26} kg. What is the volume of an iron atom?
a. $1.18 \times 10^{-29} \text{ m}^3$ b. $2.63 \times 10^{-29} \text{ m}^3$
c. $1.73 \times 10^{-28} \text{ m}^3$ d. $0.53 \times 10^{-29} \text{ m}^3$
- In the previous question, what is the distance between the centres of adjacent atoms?
a. $2.82 \times 10^{-9} \text{ m}$ b. $0.282 \times 10^{-9} \text{ m}$
c. $0.63 \times 10^{-9} \text{ m}$ d. $6.33 \times 10^{-9} \text{ m}$
- The world's largest cut diamond is the first star of Africa (mounted in the British Royal Sceptre and kept in the tower of London). Its volume is 1.84 cubic inch. What is its volume in cubic metre?
a. $30.2 \times 10^{-6} \text{ m}^3$ b. 33.28 m^2
c. 4.8 m^3 d. None of these
- Crane is British unit of volume.
(One crane = 170.474 litre). Convert crane into SI unit.
a. 0.170474 m^3 b. 17.0474 m^3
c. 0.0017474 m^3 d. 1704.74 m^3
- The nearest star to our solar system is 4.29 light year away. How much is this distance in terms of parsecs?
a. 1.32 b. 3.21 c. 2.31 d. 3.12
- The concorde is the fastest airlines used for commercial service. It can cruise at 1450 mile per hour (about two times the speed of sound or in other words, mach 2). What is it in m/s?
a. 644.4 m/s b. 80 m/s
c. 40 m/s d. None of these
- One light year is defined as the distance travelled by light in one year. The speed of light is 3×10^8 m/s. Find the same in metre.
a. 3×10^{12} m b. 9.461×10^{15} m
c. 3×10^{15} m d. None of these
- The acceleration of a car is 10 mile per hour per second. The same in $\frac{\text{ft}}{\text{s}^2}$ is
a. $1.467 \frac{\text{ft}}{\text{s}^2}$ b. $14.67 \frac{\text{ft}}{\text{s}^2}$
c. 40 ft/s^2 d. None of these
- The speed of light in vacuum is 3×10^8 m/s. How many nanosecond does it take to travel one metre in a vacuum?
a. 8 ns b. $\frac{10}{3}$ ns
c. 3.34 ns d. None of these
- The time taken by an electron to go from ground state to excited state is one shake (one shake = 10^{-8} s). Find this time in nanosecond.
a. 10 ns b. 4 ns c. 2 ns d. 25 ns
- The time between human heart beat is 8×10^{-1} s. How many heart beats are measured in one minute?
a. 75 b. 60 c. 82 d. 64
- The age of the universe is 5×10^{17} s. Find the age of universe in year.
a. 158×10^6 year b. 158×10^9 year
c. 158×10^8 year d. 158×10^{11} year
- Assuming the length of the day uniformly increases by 0.001 second per century. Calculate the net effect on the measure of time over 20 centuries.
a. 3.2 hour b. 2.1 hour c. 2.4 hour d. 5 hour
- Find the number of molecules of H_2O in 90 g of water.
a. 35.6×10^{23} molecules b. 41.22×10^{23} molecules
c. 27.2×10^{23} molecules d. 30.11×10^{23} molecules

21. The mass of Earth is 5.98×10^{24} kg. The average atomic weight of atoms that make up Earth is 40 u. How many atoms are there in Earth?
 a. 9×10^{51} b. 9×10^{49} c. 9×10^{46} d. 9×10^{55}
22. One amu is equivalent to 931 MeV energy. The rest mass of electron is 9.1×10^{-31} kg. The mass equivalent energy is (Here, 1 amu = 1.67×10^{-27} kg)
 a. 0.5073 MeV b. 0.693 MeV
 c. 4.0093 MeV d. None of these
23. One atomic mass unit in amu = 166×10^{-27} kg. The atomic weight of oxygen is 16. Find the mass of one atom of oxygen.
 a. 26.56×10^{-27} kg b. 10.53×10^{-27} kg
 c. 74×10^{-27} kg d. 2.73×10^{-27} kg
24. One horse power is equal to
 a. 746 W b. 756 W c. 736 W d. 766 W
25. If $E = mc^2$
 where, m = mass of the body, c = speed of light
 Guess the name of physical quantity E .
 a. Energy b. Power
 c. Momentum d. None of these
26. One calorie of heat is equivalent to 4.2 J. BTU (British Thermal Unit) is equivalent to 1055 J. The value of one BTU in calorie is
 a. 251.2 cal b. 200 cal
 c. 263 cal d. None of these
27. It is claimed that the two cesium clocks, if allowed to run for 100 yr, free from any disturbance, may differ by only about 0.02s. Which of the following is the correct fractional error?
 a. 10^{-9} b. 10^{-5} c. 10^{-13} d. 10^{-11}
28. Which of the following is the average mass density of sodium atom assuming, its size to be about 2.5 Å (Use the known values of Avogadro's number and the atomic mass of sodium).
 a. 0.64×10^3 kg/m³ b. 8.0×10^2 kg/m³
 c. 8.6×10^3 kg/m³ d. 6.4×10^5 kg/m³
29. Electron volt is the unit of energy (1 eV = 1.6×10^{-19} J). In H-atom, the binding energy of electron in first orbit is 13.6 eV. The same in joule (J) is
 a. 10×10^{-19} J b. 21.76×10^{-19} J
 c. 13.6×10^{-19} J d. None of these
30. 1 mm of Hg pressure is equivalent to one torr and one torr is equivalent to 133.3 N/m². The atmospheric pressure in mm of Hg pressure is
 a. 70 mm b. 760 mm
 c. 3.76 mm d. None of these
31. One bar is equivalent to 10^5 N/m². The atmosphere pressure is 1.013×10^5 N/m². The same in bar is
 a. 1.88 bar b. 1.013 bar
 c. 2.013 bar d. None of these
32. 1 revolution is equivalent to 360°. The value of 1 revolution per minute is
 a. 2π rad/s b. 0.1047 rad/s
 c. 3.14 rad/s d. None of these
33. The height of a man is 5.87532 ft. But measurement is correct upto three significant figures. The correct height is
 a. 5.86 ft b. 5.87 ft
 c. 5.88 ft d. 5.80 ft
34. $4.32 \times 2.0 = \dots\dots\dots$
 a. 8.64 b. 8.6
 c. 8.60 d. 8.640
35. $4.338 + 4.835 \times 3.88 \div 3.0$ is equal to
 a. 10.6 b. 10.59
 c. 10.5912 d. 10.591267
36. 10×2.88 is equal to
 a. 2.88 b. 2.880
 c. 2.9 d. None of these
37. 100×2.88 is equal to
 a. 2.88 b. 2.880
 c. 2.9 d. None of these
38. If v = velocity of a body, c = speed of light. Then, the dimension of $\frac{v}{c}$ is
 a. $[M^0L^0T^0]$ b. $[MLT^{-1}]$
 c. $[ML^2T^{-2}]$ d. None of these
39. The expression for centripetal force depends upon mass of body, speed of the body and the radius of circular path. Find the expression for centripetal force.
 a. $F = \frac{mv^2}{2r^3}$ b. $F = \frac{mv^2}{r}$
 c. $F = \frac{mv^2}{r^2}$ d. $F = \frac{m^2v^2}{2r}$
40. The maximum static friction on a body is $F = \mu N$. Here, N = normal reaction force on the body, μ = coefficient of static friction. The dimensions of μ is
 a. $[MLT^{-2}]$ b. $[M^0L^0T^0\theta^{-1}]$
 c. dimensionless d. None of these
41. What are dimensions of Young's modulus of elasticity?
 a. $[ML^{-1}T^{-2}]$ b. $[MLT^{-2}]$
 c. $[MLT^{-1}]$ d. None of these
42. The surface tension is $T = \frac{F}{l}$, then the dimensions of surface tension is
 a. $[MLT^{-2}]$ b. $[MT^{-2}]$
 c. $[M^0L^0T^0]$ d. None of these
43. The dimension of heat capacity is
 a. $[L^2T^{-2}\theta^{-1}]$ b. $[ML^2T^{-2}\theta^{-1}]$
 c. $[M^{-1}L^2T^{-2}\theta^{-1}]$ d. None of these

44. If $\Delta H = mL$, where m is mass of body.
 ΔH = total thermal energy supplied to the body
 L = latent heat of fusion.
 Find the dimensions of latent heat of fusion.
 a. $[ML^2T^{-2}]$ b. $[L^2T^{-2}]$ c. $[M^0L^0T^{-2}]$ d. $[ML^0T^{-1}]$
45. Solar constant is defined as energy received by Earth per cm^2 per minute. Find the dimensions of solar constant.
 a. $[ML^2T^{-3}]$ b. $[M^2L^0T^{-1}]$ c. $[MT^{-3}]$ d. $[MLT^{-2}]$
46. The unit of electric permittivity is $\frac{C^2}{Nm^2}$. Find the dimensions of electric permittivity.
 a. $[A^2M^{-1}L^{-3}T^4]$ b. $[AM^{-1}L^{-3}T^4]$
 c. $[A^2M^{-1}L^{-3}T^0]$ d. $[A^2M^0L^{-3}T^4]$
47. A physical relation is $\varepsilon = \varepsilon_0 \varepsilon_r$
 where, ε = electric permittivity of a medium
 ε_0 = electric permittivity of vacuum
 ε_r = relative permittivity of medium
 What are dimensions of relative permittivity?
 a. $[ML^2T^{-2}]$ b. $[M^0L^2T^{-3}]$
 c. $[M^0L^0T^0]$ d. $[M^0L^0T^{-1}]$
48. The dimensions of $\frac{1}{2} \varepsilon E^2$ are same as
 a. energy density (energy per unit volume)
 b. energy
 c. power
 d. None of the above
49. The electric flux is given by scalar product of electric field strength and area. What are the dimensions of electric flux?
 a. $[A^{-2}ML^3T^{-2}]$ b. $[A^{-1}ML^3T^{-2}]$
 c. $[A^{-1}ML^3T^{-3}]$ d. $[A^0M^2LT^{-1}]$
50. Electric displacement is given by $D = \varepsilon E$
 Here,
 ε = electric permittivity
 E = electric field strength.
 Find the dimensions of electric displacement.
 a. $[AML^{-2}T]$ b. $[AL^{-2}T^{-1}]$
 c. $[AL^{-2}T]$ d. None of these
51. The energy stored in an electric device known as capacitor, is given by $U = \frac{q^2}{2C}$
 where, U = energy stored in capacitor
 C = capacity of capacitor
 q = charge on capacitor
 Find the dimensions of capacity of the capacitor
 a. $[A^2M^{-1}L^{-2}T^4]$
 b. $[AM^{-1}L^{-2}T^4]$
 c. $[A^2M^{-2}L^{-2}T^4]$
 d. $[A^0M^0L^{-2}T^4]$
52. The work done by a battery is $W = \varepsilon \Delta q$, where
 Δq = charge transferred by battery ε = emf of the battery. What are dimensions of emf of battery?
 a. $[A^{-2}M^0L^0T^{-2}]$ b. $[A^{-2}ML^2T^{-3}]$
 c. $[A^0M^2T^{-3}]$ d. $[A^{-1}ML^2T^{-3}]$
53. The expression for drift speed is $v_d = \frac{J}{ne}$.
 Here, J = current density,
 n = number of electrons per unit volume,
 $e = 1.6 \times 10^{-19} \text{unit}$
 The unit and dimensions of e are
 a. coulomb and $[AT]$
 b. ampere per second and $[AT^{-1}]$
 c. no sufficient information
 d. None of the above
54. The unit of current element is ampere-metre. Find the dimensions of current element.
 a. $[AML]$ b. $[AML^2T]$
 c. $[MLT^2]$ d. $[AL]$
55. The magnetic force on a point moving charge is $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$.
 Here, q = electric charge
 $\mathbf{v}$ = velocity of the point charge
 $\mathbf{B}$ = magnetic field
 The dimensions of $\mathbf{B}$ is
 a. $[AMLT^{-1}]$
 b. $[A^{-1}MLT^{-2}]$
 c. $[A^{-1}MT^{-2}]$
 d. None of these
56. What are dimensions of $\frac{E}{B}$?
 a. $[LT^{-1}]$ b. $[LT^{-2}]$
 c. $[MLT^{-1}]$ d. $[ML^2T^{-1}]$
57. What are the dimensions of $\mu_0 \varepsilon_0$?
 Here, μ_0 = magnetic permeability in vacuum,
 ε_0 = electric permittivity in vacuum
 a. $[ML^{-2}T^{-2}]$ b. $[L^{-2}T^{-2}]$
 c. $[L^{-2}T^2]$ d. None of these
58. In the formula, $a = 3bc^2$ 'a' and 'c' have dimensions of electric capacitance and magnetic induction, respectively. What are dimensions of 'b' in MKS system?
 a. $[M^{-3}L^{-2}T^4Q^4]$ b. $[M^{-3}T^4Q^4]$
 c. $[M^{-3}T^3Q]$ d. $[M^{-3}L^2T^4Q^{-4}]$
59. Find the dimensions of $\frac{R}{L}$.
 Here, R = electric resistance
 L = self-inductance
 a. $[T^{-2}]$ b. $[T^{-1}]$
 c. $[ML^{-1}]$ d. $[T]$

60. The magnetic energy stored in an inductor is given by $E = \frac{1}{2} L^a I^b$. Find the value of 'a' and 'b'.

Here, L = self-inductance, I = electric current.

- a. $a = 3, b = 0$ b. $a = 2, b = 1$
c. $a = 0, b = 2$ d. $a = 1, b = 2$

61. In L - R circuit, $I = I_0 [1 - e^{-t/\lambda}]$

Here, I = electric current in the circuit. Then,

- a. the dimensions of I_0 and λ are same
b. the dimensions of t and λ are same
c. the dimensions of I and I_0 are not same
d. All of the above

62. A physical quantity u is given by the

$$\text{relation } u = \frac{B^2}{2\mu_0}.$$

Here, B = magnetic field strength

μ_0 = magnetic permeability of vacuum

The name of physical quantity u is

- a. energy b. energy density
c. pressure d. None of these
63. The energy of a photon depends upon Planck's constant and frequency of light. Find the expression for photon energy.
- a. $E = h\nu$ b. $E = \frac{h}{\nu}$
c. $E = \frac{\nu}{h}$ d. $E = h\nu^2$
64. If energy of photon is $E \propto h^a c^b \lambda^d$.

Here, h = Planck's constant, c = speed of light and
 λ = wavelength of photon

Then, the value of a, b and d are

- a. 1, 1, 1 b. 1, -1, 1
c. 1, 1, -1 d. None of these
65. The radius of nucleus is $r = r_0 A^{1/3}$, where A is mass number. The dimensions of r_0 is
- a. $[MLT^{-2}]$ b. $[M^0L^0T^{-1}]$
c. $[M^0LT^0]$ d. None of these
66. The power of lens is $P = \frac{1}{f'}$ where f is focal length of

the lens. The dimensions of power of lens is

- a. $[LT^{-2}]$ b. $[M^0L^{-1}T^0]$
c. $[M^0L^0T^0]$ d. None of these

67. The dimensions of frequency is

- a. $[T^{-1}]$ b. $[M^0L^0T]$
c. $[M^0L^0T^{-2}]$ d. None of these

68. The dimensions of wavelength is

- a. $[M^0L^0T^0]$ b. $[M^0LT^0]$
c. $[M^0L^{-1}T^0]$ d. None of these

69. The optical path difference is defined as $\Delta x = \frac{2\pi}{\lambda}$.

What are dimensions of optical path difference?

- a. $[M^0L^{-1}T^0]$ b. $[M^1L^1T^0]$
c. $[ML^0T^1]$ d. $[ML^{-2}T]$

70. The unit of intensity of a wave is $\frac{W}{m^2}$? What are dimensions of intensity of wave?

- a. $[MT^{-3}]$ b. $[AML^0T^{-2}]$
c. $[M^0L^{-1}T^{-2}]$ d. None of these

71. If $x = \frac{a \sin \theta + b \cos \theta}{a + b}$, then

- a. the dimensions of x and a are same
b. the dimensions of a and b are not same
c. x is dimensionless
d. None of the above

72. $\int \frac{dv}{\sqrt{2nv - v^2}} = a^n \sin^{-1} \left[\frac{x}{a} - 1 \right]$ on the basis of

dimensional analysis, the value of n is

- a. 0 b. -2
c. 3 d. None of these

Find the value of following on the basis of significant figure rule

73. Calculate the fractional error $\left(\frac{\Delta x}{x} \right)$, if $x = a^n$.

- a. $\pm \left(\frac{\Delta a}{a} \right)^n$ b. $\pm n \left(\frac{\Delta a}{a} \right)$
c. $\pm n \log_e \frac{\Delta a}{a}$ d. $\pm n \log \frac{\Delta a}{a}$

74. The relation gives the value of 'x' $x = \frac{a^3 b^3}{c\sqrt{d}}$.

Find the percentage error in 'x', if the percentage error in a, b, c and d are 2%, 1%, 3%, and 4%, respectively.

- a. $\pm 8\%$ b. $\pm 10\%$ c. $\pm 12\%$ d. $\pm 14\%$

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- For the equation $F \propto A^a v^b d^c$, where F is the force, A is the area, v is the velocity and d is the density, the values of a, b and c are, respectively [2014]
a. 1, 2, 1 b. 2, 1, 1 c. 1, 1, 2 d. 0, 1, 1
- If edge lengths of a cuboid are measured to be 1.2 cm, 1.5 cm and 1.8 cm, then volume of the cuboid is [2014]
a. 3.240 cm^3 b. 3.24 cm^3 c. 3.2 cm^3 d. 3.0 cm^3
- If the force is given by $F = at + bt^2$ with t as time. The dimensions of a and b are [2012]
a. $[\text{MLT}^{-4}]$ and $[\text{MLT}^{-2}]$ b. $[\text{MLT}^{-3}]$ and $[\text{MLT}^{-4}]$
c. $[\text{ML}^2\text{T}^{-3}]$ and $[\text{ML}^2\text{T}^{-2}]$ d. $[\text{ML}^2\text{T}^{-3}]$ and $[\text{ML}^3\text{T}^{-4}]$
- The dimensional formula for inductance is [2012]
a. $[\text{ML}^2\text{T}^{-2}\text{A}^{-2}]$ b. $[\text{ML}^2\text{TA}^{-2}]$
c. $[\text{ML}^2\text{T}^{-1}\text{A}^{-2}]$ d. $[\text{ML}^2\text{T}^{-2}\text{A}^{-1}]$
- A cube has a side of length $1.2 \times 10^{-2} \text{ m}$. Calculate its volume. [2011]
a. $1.7 \times 10^{-6} \text{ m}^3$ b. $1.73 \times 10^{-6} \text{ m}^3$
c. $1.70 \times 10^{-6} \text{ m}^3$ d. $1.732 \times 10^{-6} \text{ m}^3$
- The dimensions of the quantity hc (where, $h = \frac{h}{2\pi}$) is [2010]
a. $[\text{ML}^2\text{T}^{-1}]$ b. $[\text{MLT}^{-1}]$
c. $[\text{ML}^3\text{T}^{-2}]$ d. $[\text{ML}^2\text{T}^{-1}]$
- A resistor of $10 \text{ k}\Omega$ has a tolerance of 10% and another resistor of $20 \text{ k}\Omega$ has a tolerance of 20%. The tolerance of the series combination is nearly [2009]
a. 10% b. 20% c. 15% d. 17%
- The energy (E), angular momentum (L) and universal gravitational constant (G) are chosen as fundamental quantities. The dimensions of universal gravitational constant in the dimensional formula of Planck's constant (h) is [2008]
a. zero
b. -1
c. $5/3$
d. 1
- In the relation $p = \frac{\alpha}{\beta} e^{-\frac{az}{k\theta}}$, where p is the pressure, z the distance, k is Boltzmann constant and θ is the temperature, the dimensional formula of β will be
a. $[\text{M}^0\text{L}^2\text{T}^0]$ b. $[\text{ML}^2\text{T}]$ [2007]
c. $[\text{ML}^0\text{T}^{-1}]$ d. $[\text{ML}^2\text{T}^{-1}]$
- A physical quantity is given by $X = [\text{M}^a\text{L}^b\text{T}^c]$. The percentage error in measurement of M, L and T are α, β and γ , respectively. Then, the maximum % error in the quantity X is [2006]
a. $\alpha\alpha + b\beta + c\gamma$
b. $\alpha\alpha + b\beta - c\gamma$
c. $\frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma}$
d. None of the above
- Which one of the following is not a unit of Young's modulus? [2006]
a. Nm^{-1}
b. Nm^{-2}
c. dyne cm^{-2}
d. mega pascal

Answer with Solutions

Practice Exercise

- (d) leap year, year and shake are units of time and light year is the unit of distance.
- (d) $n_1 u_1 = n_2 u_2$
 $\therefore n_2 = \frac{n_1 u_1}{u_2} \Rightarrow \frac{50 \text{ ft}}{\text{mm}} = \frac{50 \times 12 \times 2.54 \text{ cm}}{0.1 \text{ cm}} = 15240 \text{ mm}$
 $\therefore 50 \text{ ft} = 15240 \text{ mm}$
- (b) An optical instrument gives most precise measurement.
- (a) $\therefore 10^{-15} \text{ m} = 1 \text{ fm}$
 $\therefore 1 \text{ m} = 10^{15} \text{ fm}$
 $\therefore 5 \times 10^{-11} \text{ m} = 5 \times 10^{-11} \times 10^{15} \text{ fm} = 5 \times 10^4 \text{ fm}$
- (c) $\therefore n_1 u_1 = n_2 u_2$
or $6080 \text{ ft} = n_2 \text{ km}$

$$\text{or } 6080 \times 12 \times 2.54 \text{ cm} = n_2 \times 10^3 \times 100 \text{ cm}$$

$$\therefore n_2 = \frac{6080 \times 12 \times 2.54}{10^5} = 185 \text{ km}$$

- (a) $n_1 u_1 = n_2 u_2$
 $\therefore n_2 = \frac{n_1 u_1}{u_2} = \frac{10 \text{ m}^2}{\text{ft}^2} = \frac{10 \times (100 \text{ cm})^2}{(12 \times 2.54 \text{ cm})^2} = 107.6 \text{ ft}^2$
- (a) Volume = $\frac{\text{mass}}{\text{density}} = \frac{9.27 \times 10^{-26} \times 10^{-3} \text{ g}}{7.87 \text{ g/cm}^3} = 118 \times 10^{-29} \text{ m}^3$
- (b) Volume = $\frac{4}{3} \pi r^3 = 118 \times 10^{-29}$
 $\Rightarrow r = 1.409 \times 10^{-10}$

So, distance between centre of adjacent atoms is $0.282 \times 10^{-9} \text{ m}$.

9. (a) $n_1 u_1 = n_2 u_2$

$$\therefore n_2 = \frac{n_1 u_1}{u_2} = \frac{1.84 \text{ (inch)}^3}{\text{m}^3} = \frac{1.84 (2.54)^3 \text{ cm}^3}{(100 \text{ cm})^3}$$

$$= 30.2 \times 10^{-6} \text{ m}^3$$

10. (a) 1 crane = 170.474 litre

SI unit of volume is m^3 .

Using, 1 litre = 10^{-3} m^3

or 1 litre = 10^{-3} m^3

$$170.474 \text{ litre} = 170.474 \times 10^{-3} \text{ m}^3 = 0.170474 \text{ m}^3$$

11. (a) Distance = 4.29 light year.

$$= 4.29 \times 9.46 \times 10^{15} \text{ m}$$

$$[\because 1 \text{ light year} = 9.46 \times 10^{15} \text{ m}]$$

$$= \frac{4.29 \times 9.46 \times 10^{15}}{3.08 \times 10^{16}} \text{ parsec}$$

$$[\because 1 \text{ parsec} = 3.08 \times 10^{16} \text{ m}]$$

$$= 132 \text{ parsec}$$

12. (a) $n_1 u_1 = n_2 u_2$

$$\therefore n_2 = \frac{n_1 u_1}{u_2} = \frac{1450 \text{ mile}}{\text{m/s hour}} = \frac{1450 \text{ s mile}}{\text{m hour}}$$

$$= \frac{1450 \text{ s} \times 1.6 \text{ km}}{10^{-3} \text{ km } 60 \times 60 \text{ s}} = 644.4 \text{ m/s}$$

13. (b)

14. (a) $n_1 u_1 = n_2 u_2$

or $\frac{10 \text{ mile}}{\text{hour second}} = n_2 \frac{\text{ft}}{\text{s}^2}$

or $\frac{10 \times 1760 \times 3 \text{ ft}}{60 \times 60 \text{ s}^2} = n_2 \frac{\text{ft}}{\text{s}^2}$

$$\therefore n_2 = 1467 \text{ ft/s}^2$$

15. (b) $t = \frac{s}{v} = \frac{1}{3 \times 10^8} = \frac{1}{3} \times 10^{-8} \text{ s}$

$$= \frac{1}{3} \times \frac{10^{-8}}{10^{-9}} \text{ ns} = \frac{10}{3} \text{ ns}$$

16. (a) $\therefore n_1 u_1 = n_2 u_2$

$$1 \times \text{shake} = n_2 \text{ s}$$

or $10^{-8} \text{ s} = n_2 \text{ s}$

$$\therefore n_2 = 10 \text{ ns}$$

17. (a) $n = \frac{t}{8 \times 10^{-1} \text{ s}} = \frac{1 \text{ minute}}{8 \times 10^{-1} \text{ s}} = \frac{60 \text{ s}}{8 \times 10^{-1} \text{ s}} = \frac{600}{8} = 75$

18. (c) $n_1 u_1 = n_2 u_2$

or $5 \times 10^{17} \text{ s} = n_2 \text{ year}$

$$\therefore n_2 = \frac{5 \times 10^{17} \text{ s}}{\text{year}} = \frac{5 \times 10^{17} \text{ s}}{365 \times 24 \times 60 \times 60 \text{ s}} = 158 \times 10^8 \text{ year}$$

19. (b) The length of day after 20 century is 24 hour and 0.21 s

$$\therefore \text{The change in time, } \Delta t = \frac{0.21 \times 365 \times 100 \text{ hour}}{60 \times 60}$$

$$= 2.1 \text{ hour}$$

20. (d) $\therefore 18 \text{ g of H}_2\text{O} = 6.022 \times 10^{23} \text{ molecules}$

$$\therefore 1 \text{ g of H}_2\text{O} = \frac{6.022 \times 10^{23}}{18}$$

$$\therefore 90 \text{ g of H}_2\text{O} = \frac{6.022 \times 10^{23}}{18} \times 90$$

$$= 30.110 \times 10^{23} \text{ molecules}$$

21. (b) $n = \frac{m}{M} = \frac{5.98 \times 10^{24}}{40 \times 166 \times 10^{-27}} = 9 \times 10^{49}$

22. (a) The mass of electron = $\frac{9.1 \times 10^{-31}}{1.67 \times 10^{-27}} \text{ (amu)}$

$$\therefore E = \frac{9.1 \times 10^{-31}}{1.67 \times 10^{-27}} \times 931 \text{ MeV} = 0.5073 \text{ MeV}$$

23. (a) The mass of one atom of oxygen,

$$m = 16 \text{ amu} = 16 \times 166 \times 10^{-27} \text{ kg} = 26.56 \times 10^{-27} \text{ kg}$$

24. (a) 25. (a)

26. (a) $\therefore 4.2 \text{ J} = 1 \text{ cal}$

$$\therefore 1 \text{ J} = \frac{1}{4.2} \text{ cal} \therefore 1055 \text{ J} = \frac{1}{4.2} \times 1055 \text{ cal} = 251.2 \text{ cal}$$

27. (d) Time interval = 100 years

$$= 100 \times 365 \times 24 \times 60 \times 60 \text{ s}$$

$$= 3.155 \times 10^9 \text{ s}$$

Difference in time = 0.2 s

$$\therefore \text{Fractional error} = \frac{\text{Difference in time (s)}}{\text{Time interval (s)}}$$

$$= \frac{0.2}{3.155 \times 10^9} = 6.34 \times 10^{-12} = 10 \times 10^{-12} \approx 10^{-11}$$

28. (a) Average radius of sodium atom,

$$r = 2.5 \text{ \AA} = 2.5 \times 10^{-10} \text{ m}$$

$$\therefore \text{Volume of sodium atom} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times 3.14 \times (2.5 \times 10^{-10})^3 = 65.42 \times 10^{-30} \text{ m}^3$$

Mass of a mole of sodium = 23 g = $23 \times 10^{-3} \text{ kg}$

One mole contains 6.023×10^{23} atoms, hence the mass of sodium atom,

$$M = \frac{23 \times 10^{-3}}{6.023 \times 10^{23}} \text{ kg} = 3.82 \times 10^{-26} \text{ kg}$$

$\therefore$ Average mass density of sodium atom

$$\rho = \frac{M}{V} = \frac{3.82 \times 10^{-26}}{65.42 \times 10^{-30}} \text{ kg m}^{-3} = 0.64 \times 10^3 \text{ kg m}^{-3}$$

29. (b) $\therefore 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

$$\therefore 13.6 \text{ eV} = 13.6 \times 1.6 \times 10^{-19} \text{ J} = 21.76 \times 10^{-19} \text{ J}$$

30. (b) 1 atm = $1.013 \times 10^5 \text{ N/m}^2$

$$\therefore 133.3 \text{ N/m}^2 = 1 \text{ mm of Hg}$$

$$\therefore 1 \text{ N/m}^2 = \frac{1}{133.3} \text{ mm of Hg} = 1.013 \times 10^5 \text{ N/m}^2$$

$$= \frac{1}{133.3} \times 1.013 \times 10^5 \text{ mm of Hg} = 760 \text{ mm of Hg}$$

$$31. (b) 1 \text{ bar} = 10^5 \text{ N/m}^2 \\ = 1.013 \times 10^5 \text{ N/m}^2 = \frac{1.013 \times 10^5}{10^5} \text{ bar} = 1.013 \text{ bar}$$

$$32. (b) 1 \frac{\text{rev}}{\text{min}} = \frac{1 \times 2\pi \text{ rad}}{60 \text{ s}} = \frac{\pi}{30} \text{ rad/s} = 0.1047 \text{ rad/s}$$

$$33. (a) \quad 34. (b) \quad 35. (a) \quad 36. (c) \quad 37. (a) \quad 38. (a)$$

$$39. (b) F = m^a v^b r^c \\ [\text{MLT}^{-2}] = [\text{M}^0 \text{T}^0]^a [\text{M}^0 \text{LT}^{-1}]^b [\text{M}^0 \text{LT}^0]^c \\ [\text{MLT}^{-2}] = [\text{M}^a \text{L}^{b+c} \text{T}^{-b}] \\ \Rightarrow \quad a = 1, b + c = 1 \text{ and } b = 2 \\ 2 + c = 1 \text{ and } c = -1 \\ F = mv^2 r^{-1} \Rightarrow F = \frac{mv^2}{r}$$

$$40. (c) \because F = \mu N \quad \therefore \mu = \frac{F}{N} \\ \therefore [\mu] = \left[\frac{F}{N} \right] = \frac{[\text{MLT}^{-2}]}{[\text{MLT}^{-2}]} = \text{dimensionless}$$

$$41. (a) \text{ Young's modulus of elasticity, } Y = \frac{\text{stress}}{\text{strain}} \\ \therefore [Y] = \left[\frac{F}{A \Delta l} \right] = \left[\frac{Fl}{A \Delta l} \right] = \frac{[\text{MLT}^{-2} \text{L}]}{[\text{L}^2 \text{L}]} = [\text{ML}^{-1} \text{T}^{-2}]$$

$$42. (b) T = \frac{F}{l} = \frac{[\text{MLT}^{-2}]}{[\text{M}^0 \text{LT}^0]} = [\text{ML}^0 \text{T}^{-2}]$$

$$43. (b) \Delta H = c \Delta \theta \\ \therefore c = \frac{\Delta H}{\Delta \theta} = \frac{[\text{ML}^2 \text{T}^{-2}]}{[\theta]} = [\text{ML}^2 \text{T}^{-2} \theta^{-1}]$$

$$44. (b) \Delta H = mL \quad \therefore [L] = \frac{\Delta H}{m} = \frac{[\text{ML}^2 \text{T}^{-2}]}{[\text{M}]} = [\text{L}^2 \text{T}^{-2}]$$

$$45. (c) \text{ Unit} = \frac{\text{energy}}{\text{cm}^2 \text{ minute}} = \frac{[\text{ML}^2 \text{T}^{-2}]}{[\text{L}^2 \text{T}]} \\ \therefore \text{The dimensions of solar constant} = [\text{MT}^{-3}]$$

$$46. (a) \text{ Unit of } \epsilon = \frac{\text{C}^2}{\text{Nm}^2} \\ \text{Dimensions of } \epsilon = \frac{[(\text{AT})^2]}{[\text{MLT}^{-2} \text{L}^2]} = [\text{A}^2 \text{M}^{-1} \text{L}^{-3} \text{T}^4]$$

$$47. (c) \epsilon = \epsilon_0 \epsilon_r \Rightarrow \epsilon_r = \frac{\epsilon}{\epsilon_0} \\ \text{Relative permittivity is the ratio of } \epsilon \text{ and } \epsilon_0, \text{ hence it is dimensionless.} \\ \text{i.e.} \quad [\text{M}^0 \text{L}^0 \text{T}^0]$$

$$48. (a)$$

$$49. (c) \text{ Electric flux is } \phi = \mathbf{E} \cdot \mathbf{S} = ES \cos \theta \\ \therefore [\phi] = \frac{\text{N}}{\text{C}} \text{m}^2 = \frac{\text{MLT}^{-2} \times \text{L}^2}{\text{AT}} = [\text{A}^{-1} \text{ML}^3 \text{T}^{-3}]$$

$$50. (c) D = \epsilon E \\ \text{Unit of } D = \frac{\text{C}^2}{\text{Nm}^2} \frac{\text{N}}{\text{C}} \quad \therefore [D] = \left(\frac{\text{C}}{\text{m}^2} \right) = \frac{\text{AT}}{\text{L}^2} = [\text{AL}^{-2} \text{T}]$$

$$51. (a) U = \frac{q^2}{2C}$$

$$\therefore C = \frac{q^2}{2U} \quad \therefore [C] = \frac{[\text{AT}]^2}{[\text{ML}^2 \text{T}^{-2}]} = [\text{A}^2 \text{M}^{-1} \text{L}^{-2} \text{T}^4]$$

$$52. (d) W = \epsilon \Delta q \\ \Rightarrow \epsilon = \frac{W}{q} = \frac{[\text{ML}^2 \text{T}^{-2}]}{[\text{M}^0 \text{L}^0 \text{AT}]} \Rightarrow [\epsilon] = [\text{ML}^2 \text{A}^{-1} \text{T}^{-3}]$$

$$53. (a) \because v_d = \frac{J}{ne} \\ [e] = \left[\frac{J}{nv_d} \right] = \frac{[\text{AL}^{-2}]}{\left[\frac{1}{\text{L}^3} \right] [\text{LT}^{-1}]} = [\text{AT}] = \text{amp-second} = \text{coulomb}$$

$$54. (d) \text{ Dimensions of current element are [ampere-metre]} \\ = [\text{AL}]$$

$$55. (c) \because \mathbf{F} = q\mathbf{v} \times \mathbf{B} \text{ or } F = qvB \sin \theta \\ \therefore [B] = \left[\frac{F}{qv} \right] = \frac{[\text{MLT}^{-2}]}{[\text{ATLT}^{-1}]} = [\text{A}^{-1} \text{MT}^{-2}]$$

$$56. (a) \text{ The unit of } \frac{E}{B} = \frac{\text{N/C}}{\text{N/Am}} = \frac{\text{N Am}}{\text{C N}} = \frac{\text{Am}}{\text{C}} = \frac{\text{Am}}{\text{As}} = \frac{\text{m}}{\text{s}} \\ \therefore \frac{E}{B} = \left[\frac{\text{m}}{\text{s}} \right] = \left[\frac{\text{L}}{\text{T}} \right] = [\text{LT}^{-1}]$$

$$57. (c) \text{ As we know the speed of light, } c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \\ \text{or } c^2 = \frac{1}{\mu_0 \epsilon_0} \quad \therefore \mu_0 \epsilon_0 = \frac{1}{c^2} = \frac{1}{[\text{LT}^{-1}]^2} = [\text{L}^{-2} \text{T}^2]$$

$$58. (a)$$

$$59. (b) \because \frac{L}{R} = \text{time constant} \\ \therefore \frac{[L]}{[R]} = [\text{T}] \Rightarrow \frac{[R]}{[L]} = [\text{T}^{-1}]$$

$$60. (d) E = \frac{1}{2} L^a I^b \\ [E] = [\text{ML}^2 \text{T}^{-2}], [L] = [\text{ML}^2 \text{T}^{-2} \text{A}^{-2}], [I] = [\text{M}^0 \text{L}^0 \text{T}^0 \text{A}]$$

Using principle of Homogeneity,

$$[\text{ML}^2 \text{T}^{-2}] = [\text{ML}^2 \text{T}^{-2} \text{A}^{-2}]^a [\text{M}^0 \text{L}^0 \text{T}^0 \text{A}]^b$$

$$[\text{MLT}^{-2}] = [\text{M}^a \text{L}^{2a} \text{T}^{-2a} \text{A}^{-2a+b}]$$

$$\Rightarrow a = 1 \text{ and } -2a + b = 0 \Rightarrow b = 2$$

$$61. (b) \text{ Exponent has no dimensions.}$$

$$\therefore [e^{-t/\lambda}] = [\text{M}^0 \text{L}^0 \text{T}^0]$$

$$\text{or } \left[-\frac{t}{\lambda} \right] = [\text{M}^0 \text{L}^0 \text{T}^0] \text{ or } [t] = [\lambda] \quad \therefore [\lambda] = [\text{T}]$$

Hence, (b) is correct

$$62. (b) u = \frac{B^2}{2\mu}$$

$$\text{Unit of } u = \frac{(\text{N/Am})^2}{\text{N/A}^2} = \frac{\text{N}^2 \text{A}^2}{\text{NA}^2 \text{m}^2} = \frac{\text{N}}{\text{m}^2} = \frac{\text{Nm}}{\text{m}^3} = \frac{\text{J}}{\text{m}^3} \\ = \text{energy per unit volume} = \text{energy density}$$

$$63. (a) E = h^a \nu^b \quad \dots(i) \\ \text{where, } h = \text{Planck's constant and } \nu = \text{frequency.}$$

$$[E] = [ML^2T^{-2}], [v] = [M^0L^0T^{-1}] \text{ and } [h] = [ML^2T^{-1}]$$

$$\text{From Eq. (i), } [ML^2T^{-2}] = [ML^2T^{-1}]^a [M^0L^0T^{-1}]^b$$

$$[ML^2T^{-2}] = [M^aL^{2a}T^{-a-b}]$$

$$\Rightarrow a = 1 \text{ and } -a - b = -2 \Rightarrow a + b = 2 \Rightarrow b = 1$$

$$\Rightarrow E = hv$$

$$64. (c) \quad 65. (b) \quad 66. (b) \quad 67. (a) \quad 68. (a) \quad 69. (a)$$

$$70. (a) \text{ Unit} = \frac{W}{m^2} = \frac{J}{sm^2}$$

$$\therefore [\text{Intensity of wave}] = \frac{[ML^2T^{-2}]}{[TL^2]} = [MT^{-3}]$$

$$71. (c) x = \frac{a \sin \theta + b \cos \theta}{a + b}$$

$$\therefore [x] = \left[\frac{a \sin \theta}{a + b} \right] = \left[\frac{b \cos \theta}{a + b} \right] \text{ or } [x] = \text{dimensionless}$$

$$72. (a) [LHS] = [RHS]$$

$$\text{or } \left[\frac{dv}{\sqrt{2av - v^2}} \right] = \left[a^n \sin^{-1} \left(\frac{x}{a} - 1 \right) \right] \text{ or } \frac{[dv]}{[\sqrt{v^2}]} = [a^n]$$

$$\text{or } \frac{[dv]}{[v]} = [a^n] \text{ or } [M^0L^0T^0] = [a^n] \therefore n = 0$$

$$73. (b) \quad x = a^n \quad \therefore \frac{\Delta x}{x} = \pm n \frac{\Delta a}{a}$$

$$74. (d)$$

BITSAT Archives

$$1. (a) [MLT^{-2}] = [L^{2a}] \times [L^{bT^{-b}}] [M^c L^{-3c}] = [M^c L^{2a+b-3c} T^{-b}]$$

Comparing powers of M, L and T, on both sides, we get

$$c = 1, 2a + b - 3c = 1, -b = -2 \text{ or } b = 2$$

$$\text{Also, } 2a + 2 - 3(1) = 1 \Rightarrow 2a = 2 \text{ or } a = 1$$

$$\therefore \text{ This is } 1, 2, 1$$

$$2. (a) \text{ Volume of cuboid} = l \times b \times h = 1.8 \times 1.5 \times 1.2 \text{ cm}^3 \\ = 2.70 \times 1.2 = 3.240 \text{ cm}^3$$

Using concept of significant figures, product is reported in number of significant figures present in measurement which has least number of significant figures, here all measurement have 2 significant figures.

$$\text{So, volume} = 3.2 \text{ cm}^3$$

(Keeping 2 significant figures only)

$$3. (b) \text{ Dimension of } at = \text{Dimension of } F$$

$$[at] = [F] \Rightarrow [a] = \left[\frac{F}{t} \right]$$

$$[b] = \left[\frac{MLT^{-2}}{T} \right] \Rightarrow [a] = [MLT^{-3}]$$

$$\text{Dimension of } bt^2 = \text{Dimension of } F$$

$$[bt^2] = [F] \Rightarrow [b] = \left[\frac{F}{t^2} \right]$$

$$[b] = \left[\frac{MLT^{-2}}{T^2} \right] \Rightarrow [b] = [MLT^{-4}]$$

$$4. (a) \text{ EMF induced in an electrical circuit}$$

$$e = L \frac{dl}{dt} \quad (\text{numerically})$$

$$\text{or } L = e \frac{dt}{dl} = \frac{W}{Q} \cdot \frac{dt}{dl} \quad \left(\because e = V = \frac{W}{Q} \right)$$

$$= \frac{Wdt}{lt \cdot dl} = \frac{[ML^2T^{-2}][T]}{[A][T][A]} = [ML^2T^{-2}A^{-2}]$$

$$5. (a) \text{ Volume, } V = l^3 = (1.2 \times 10^{-2} \text{ m})^3 = 1.728 \times 10^{-6} \text{ m}^3$$

Since length (l) has two significant figure, the volume (V) will also have two significant figure.

Therefore, the correct answer is $V = 1.7 \times 10^{-6} \text{ m}^3$

$$6. (c) hc = E\lambda = [ML^2T^{-2}] [L] = [ML^3T^{-2}]$$

$$7. (d) r_1 = 10 \text{ k}\Omega, \Delta r_1 = \frac{10}{100} \times 10 = 1 \text{ k}\Omega$$

$$r_2 = 20 \text{ k}\Omega, \Delta r_2 = \frac{20}{100} \times 20 = 4 \text{ k}\Omega$$

$$\text{Maximum tolerance} = \Delta r_1 + \Delta r_2 = 1 + 4 = 5$$

$$r_1 + r_2 = 10 + 20 = 30 \text{ k}\Omega$$

$$\% \text{ age of tolerance} = \frac{5}{30} \times 100 = 16.67\% = 17\%$$

$$8. (a) h \propto G^x L^y E^z$$

Write the dimensions on both sides

$$[ML^2T^{-1}] \propto [M^{-1}L^3T^{-2}]^x [ML^2T^{-1}]^y [ML^2T^{-2}]^z$$

$$[ML^2T^{-1}] = k [M^{-1}L^3T^{-2}]^x [ML^2T^{-1}]^y [ML^2T^{-2}]^z$$

Comparing the powers, we get

$$1 = -x + y + z \quad \dots(i)$$

$$2 = 3x + 2y + 2z \quad \dots(ii)$$

$$-1 = -2x - y - 2z \quad \dots(iii)$$

On solving Eqs. (i), (ii) and (iii), we get

$$x = 0$$

$$9. (a) \text{ In given equation, } \frac{\alpha z}{k\theta} \text{ should be dimensionless.}$$

$$\therefore \alpha = \frac{k\theta}{z}$$

$$\Rightarrow [a] = \frac{[ML^2T^{-2}K^{-1} \times K]}{[L]} = [MLT^{-2}]$$

$$\text{and } p = \frac{\alpha}{\beta} \Rightarrow [p] = \left[\frac{\alpha}{\beta} \right] = \left[\frac{MLT^{-2}}{ML^{-1}T^{-2}} \right] = [M^0L^2T^0]$$

$$10. (a) X = [M^a L^b T^c]$$

$$\text{Maximum percentage error} = a\alpha + b\beta + c\gamma$$

$$11. (a) Y = \frac{\text{stress}}{\text{strain}} = \text{N/m}^2 \text{ or pascal} \quad [\text{in SI system}]$$

$$\text{and } y = \frac{\text{dyne}}{\text{cm}^2} \quad [\text{in CGS system}]$$

Thus, Nm^{-1} is not the unit of Young's modulus.

2

Scalar and Vectors

Physical Quantity

Physical quantity is that which can be measured by available apparatus.

Scalar and Vector Quantities

A **scalar quantity** is one whose specification is completed with its magnitude only. Two or more than two similar scalar quantities can be added according to the ordinary rules of algebra. e.g., mass, distance, speed, energy etc.

A **vector quantity** is a quantity that has magnitude as well as direction. Not all physical quantities have a direction. Temperature, energy, mass, and time, for example, do not “point” in the spatial sense. We call such quantities scalars, and we deal with them by the rules of ordinary algebra.

Vector quantities can be added according to the **law of parallelogram or triangle law**.

A vector quantity can be represented by an arrow. The front end (arrow head) represents the direction and length of the arrow gives its magnitude.

NOTE ***Orthogonal vectors** If two or more vectors are perpendicular to each other, then they are known as orthogonal vectors.*

***Unit vector** A vector of unit magnitude and whose direction is same as the given vector is called unit vector. Basically, unit vector represents the direction of the given vector.*

*Consider a vector **A**. This vector is represented as*

Vector = (Magnitude of the vector) $\times$ (Direction of the vector)

i.e.

$$A = |A| \hat{A}$$

So,

$$\hat{A} = \frac{A}{|A|}$$

*where, $\hat{A}$ is a **unit vector** drawn in the direction of **A**.*

Unit vector is a dimensionless physical quantity. Unit vectors along X, Y and Z-axes are $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively.

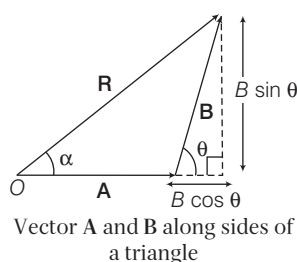
Laws of Vector Addition

There are three laws for the addition of vectors

- Triangle law of vector addition
- Parallelogram law of vector addition
- Polygon law of vector addition

Triangle Law of Vector Addition

If two vectors are represented both in magnitude and direction by the two sides of a triangle taken in the same order, then the resultant of these vectors is represented both in magnitude and direction by the third side of the triangle taken in reverse order as shown below.



i.e. $\mathbf{R} = \mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$

The resultant $\mathbf{R}$ can be calculated as

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

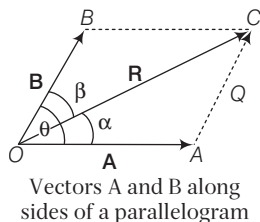
If θ is the angle between $\mathbf{A}$ and $\mathbf{B}$, then

$$|\mathbf{A} + \mathbf{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

If $\mathbf{R}$ makes an angle α with $\mathbf{A}$, then $\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$

Parallelogram Law of Vector Addition

According to parallelogram law of vector addition, if two vectors acting on a particle are represented in magnitude and direction by two adjacent sides of a parallelogram, then the diagonal of the parallelogram represents the magnitude and direction of the resultant of the two vectors acting on the particle.



i.e. $\mathbf{OA} + \mathbf{AC} = \mathbf{OC}$; $\mathbf{A} + \mathbf{B} = \mathbf{R}$

Magnitude of the resultant $\mathbf{R}$ is given by

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

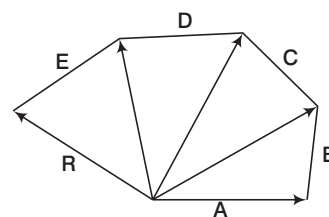
Here, θ = Angle between $\mathbf{A}$ and $\mathbf{B}$. So, the direction of $\mathbf{R}$ can be found by angle α or β of $\mathbf{R}$ with $\mathbf{A}$ and $\mathbf{B}$.

Here, $\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$

$$\Rightarrow \tan\beta = \frac{A\sin\theta}{B + A\cos\theta}$$

Polygon Law of Vector Addition

If a number of non-zero vectors are represented by the $(n - 1)$ sides of an n sided polygon taken in same order then the resultant is given by the closing side or the n th side of the polygon taken in opposite order. So,



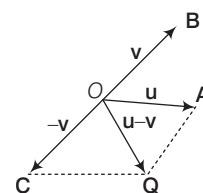
$$\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D} + \mathbf{E}$$

Subtraction of Vectors

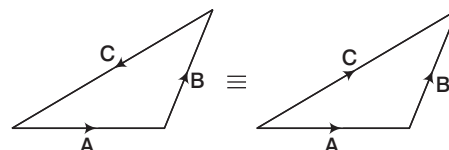
Negative of a vector $(-\mathbf{v})$ is a vector of the same magnitude as vector $\mathbf{v}$ but pointing in a direction opposite to that of $\mathbf{v}$.

Therefore,

$$\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v})$$



NOTE If the vectors form a closed n sided polygon with all the sides in the same order, then the resultant is 0.



Vector in three dimension

If $\mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$(a) |\mathbf{r}| = \sqrt{x^2 + y^2 + z^2}$$

(b) Let $\mathbf{r}$ makes α, β and γ angles with x -axis, y -axis and z -axis respectively, then

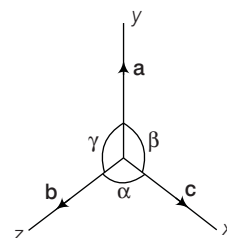
$$\cos\alpha = \frac{x}{\sqrt{x^2 + y^2 + z^2}}, \cos\beta = \frac{y}{\sqrt{x^2 + y^2 + z^2}}$$

$$\text{and } \cos\gamma = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$(c) \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$\therefore$ Lami's theorem

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

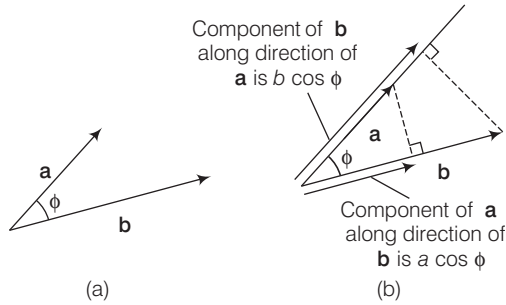


The Scalar Product or Dot Product

The scalar product of two vectors **a** and **b** in Fig. (a) is written as **a** · **b** and is defined to be

$$\mathbf{a} \cdot \mathbf{b} = ab \cos \phi \quad \dots(i)$$

where, ϕ is the angle between the vectors **a** and **b**.



Because of the notation, **a** · **b** is also known as the **dot product** and is spelled as “a dot b.”

- (i) Dot product of the vectors with itself is equal to the square of the magnitude of the vector.

$$\mathbf{a} \cdot \mathbf{a} = \mathbf{a} \cdot \mathbf{a} \cos 0^\circ \Rightarrow \mathbf{a} \cdot \mathbf{a} = a^2 \quad (\cos 0^\circ = 1)$$

If $\theta = 180^\circ$, i.e. vectors are anti-parallel.

$$\begin{aligned} \text{Then, } \mathbf{a} \cdot \mathbf{b} &= ab(-1) \quad [\because \cos 180^\circ = -1] \\ \mathbf{a} \cdot \mathbf{b} &= -ab \end{aligned}$$

i.e. If two vectors are anti-parallel then their dot product equals the negative product of the magnitudes of vectors.

If $\theta = 90^\circ$, i.e. vectors are perpendicular.

$$\mathbf{a} \cdot \mathbf{b} = ab \cos 90^\circ = ab(0) = 0$$

Vectors are perpendicular $\Leftrightarrow$ Dot product = 0

- (ii) If $\mathbf{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$ and $\mathbf{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$ and θ

is the angle between **a** and **b**, then $\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{ab}$ where,

$$\mathbf{a} \cdot \mathbf{b} = ab \cos \theta$$

The component of **a** parallel to **b** in the vector form is

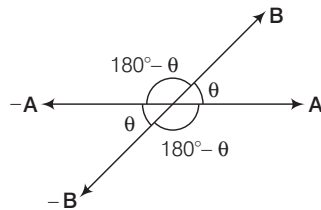
$$\mathbf{c} = \frac{(\mathbf{a} \cdot \mathbf{b}) \mathbf{b}}{|\mathbf{b}|^2}$$

The component of **a** perpendicular to **b** in vector form is

$$\mathbf{d} = \mathbf{a} - \left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|^2} \right) \mathbf{b}$$

If angle between **A** and **B** is θ , then

- (a) Angle between **-A** and **B** is $(180^\circ - \theta)$
 (b) Angle between **A** and **-B** is $(180^\circ - \theta)$
 (c) Angle between **-A** and **-B** is θ



Important Points

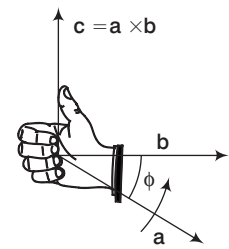
- The dot product of force **F** and displacement **s** gives work (scalar quantity), i.e. $\mathbf{F} \cdot \mathbf{s} = W$.
- The dot product of force (**F**) and velocity (**v**) is equal to power (scalar quantity), i.e. $\mathbf{F} \cdot \mathbf{v} = P$.
- The dot product of magnetic induction (**B**) and area vector (**A**) is equal to the magnetic flux (ϕ) linked with the surface (scalar quantity) $\mathbf{B} \cdot \mathbf{A} = \phi_B$

The Vector Product or Cross Product

The vector product of **a** and **b**, written as **a** × **b**, produces a third vector **c** whose magnitude is $c = ab \sin \phi$

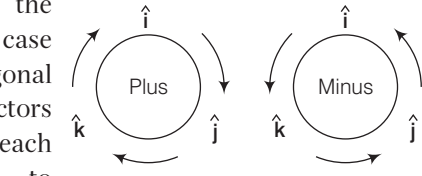
where, ϕ is the smaller of the two angles between **a** and **b**.

Because of the notation, **a** × **b** is also known as the **cross product**, and it is spelled as “a cross b”.



- (i) If two vectors are perpendicular to each other, we have $\theta = 90^\circ$ and therefore, $\sin \theta = 1$. So that, $\mathbf{a} \times \mathbf{b} = ab \hat{n}$.

- (ii) The vectors **a**, **b** and **a** × **b** thus form a right handed system of mutually perpendicular vectors. It follows at once from the above that in case of the orthogonal triad of unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ (each perpendicular to each other).



$$\hat{i} \times \hat{j} = -\hat{j} \times \hat{i} = \hat{k}$$

$$\hat{j} \times \hat{k} = -\hat{k} \times \hat{j} = \hat{i}$$

and

$$\hat{k} \times \hat{i} = -\hat{i} \times \hat{k} = \hat{j}$$

Cross Product of Two Vectors in Determinant Form

For two vectors **a** and **b**, their cross product is given by

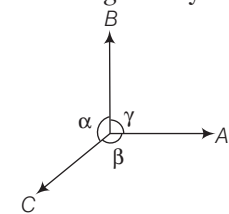
$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix},$$

where,

$$\mathbf{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

and

$$\mathbf{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}.$$



Practice Exercise

1. An insect moves on a circular path of radius 7 m. Find the maximum magnitude of displacement of the insect.

a. 7 m b. 14π m c. 7π m d. 14 m

2. In previous problem, if the insect moves with constant speed 10 m/s. Find the minimum time to achieve maximum magnitude of displacement.

a. 10 s b. 2 s c. 1.4 s d. 2.2 s

3. Two forces of magnitudes 3 N and 4 N are acted on a body. The ratio of magnitude of minimum and maximum resultant force on the body, is

a. $\frac{3}{4}$ b. $\frac{4}{3}$
c. $\frac{1}{7}$ d. None of these

4. A vector **a** makes 30° and **b** makes 120° angle with the x-axis. The magnitude of these vectors are 3 unit and 4 unit, respectively. The magnitude of resultant vector is

a. 3 unit b. 4 unit c. 5 unit d. 1 unit

5. If two forces of equal magnitude 4 units acting at a point and the angle between them is 120° , then find the magnitude and direction of the sum of the two vectors

a. 4, $\theta = \tan^{-1}(1.73)$ b. 4, $\theta = \tan^{-1}(0.73)$
c. 2, $\theta = \tan^{-1}(1.73)$ d. 6, $\theta = \tan^{-1}(0.73)$

6. If $\frac{|\mathbf{a} + \mathbf{b}|}{|\mathbf{a} - \mathbf{b}|} = 1$, then the angle between **a** and **b** is

a. 0° b. 45° c. 90° d. 60°

7. The angle between **A** and the resultant of **(A + B)** and **(A - B)** will be

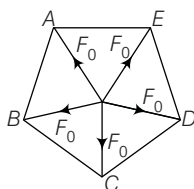
a. 0° b. $\tan^{-1}\left(\frac{A}{B}\right)$
c. $\tan^{-1}\left(\frac{B}{A}\right)$ d. $\tan^{-1}\left(\frac{A - B}{A + B}\right)$

8. Three forces are acted on a body. Their magnitudes are 3 N, 4 N and 5 N. Then,

a. the acceleration of body must be zero
b. the acceleration of body may be zero
c. the acceleration of the body must not be zero
d. None of the above

9. In the given figure, **O** is the centre of regular pentagon **ABCDE**. Five forces each of magnitude F_0 are acted as shown in figure. The resultant force is

a. $5F_0$ b. $5F_0 \cos 72^\circ$
c. $5F_0 \sin 72^\circ$ d. zero



10. **ABCD** is a parallelogram, and **a, b, c** and **d** are the position vector of vertices **A, B, C** and **D** of a parallelogram, choose the correct option.

a. $\mathbf{c} + \mathbf{b} = \mathbf{d} - \mathbf{a}$ b. $\mathbf{c} - \mathbf{b} = \mathbf{d} - \mathbf{a}$
c. $\mathbf{c} - \mathbf{c} = \mathbf{d} - \mathbf{a}$ d. None of these

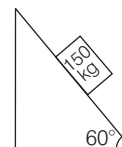
11. A man walks 4 km due West, 500 m due South finally 750 m in South-West direction. Find the distance and magnitude of displacement travelled by the man.

a. 4646.016 m and 5250 m b. 5250 m and 4646.016 m
c. 4550.016 m and 2300 m d. None of these

12. Calculate the resultant force, when four force of 30 N due East, 20 N due North, 50 N due West and 40 N due South, are acted upon a body.

a. $20\sqrt{2}$ N, 60° , South of West
b. $20\sqrt{2}$ N, 45° , South of West
c. $20\sqrt{2}$ N, 45° , South of East
d. $20\sqrt{2}$ N, 45° , South of East

13. A block of 150 kg is placed on an inclined plane with an angle of 60° . Calculate of the weight parallel to the inclined plane.



a. 1300 N b. 1400 N c. 1100 N d. 750 N

14. A cat is situated at a point **A** (0, 3, 4) and rat is situated at point **B** (5, 0, -8). The cat is free to move but the rat is always at rest. Find the minimum distance travelled by cat to catch the rat.

a. 5 unit b. 12 unit c. 13 unit d. 17 unit

15. An insect fly start from one corner of a cubical room and reaches at diagonally opposite corner. The magnitude or displacement of the insect is $40\sqrt{3}$ ft. Find the volume of cube.

a. $64\sqrt{3}$ ft³ b. 1600 ft³
c. 64000 ft³ d. None of these

16. In above problem, if the insect does not fly but crawls. What is the minimum distance travelled by the insect?

a. 89.44 ft b. 95.44 ft
c. 40 ft d. 80 ft

17. If a particle is moving on an elliptical path given by $\mathbf{r} = b \cos \omega t \hat{i} + a \sin \omega t \hat{j}$, then find its radial acceleration along **r**.

a. $\omega \mathbf{r}$ b. $\omega^2 \mathbf{r}$
c. $-\omega^2 \mathbf{r}$ d. None of these

18. Obtain the magnitude and direction cosines of vector $(\mathbf{A} - \mathbf{B})$, if $\mathbf{A} = 2\hat{i} + 3\hat{j} + \hat{k}$, $\mathbf{B} = 2\hat{i} + 2\hat{j} + 3\hat{k}$
- a. $0, \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}$ b. $0, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}$
 c. $0, 0, \frac{1}{\sqrt{5}}$ d. None of these
19. The vertices of a quadrilateral are $A(1, 2, -1)$, $B(-4, 2, -2)$, $C(4, 1, -5)$ and $D(2, -1, 3)$. Forces of magnitude 2, 3, 2 N are acting at point A along the lines AB, AC, AD , respectively. Find their resultant.
- a. $\frac{10\hat{i} - 9\hat{j} + 6\hat{k}}{\sqrt{26}}$ b. $\left(\frac{\hat{i} - 9\hat{j} - 6\hat{k}}{\sqrt{26}} \right)$
 c. $\frac{\hat{i} - 9\hat{j} + 16\hat{k}}{\sqrt{26}}$ d. $\frac{\hat{i} - 19\hat{j} + 6\hat{k}}{\sqrt{26}}$
20. A force $\mathbf{F} = a\hat{i} + b\hat{j} + c\hat{k}$ is acted upon a body of mass m . If the body starts from rest and was at the origin initially, find its new coordinate after time t .
- a. $\frac{at^2}{2m}, \frac{bt^2}{2m}, \frac{ct^2}{2m}$ b. $\frac{at^2}{2m}, \frac{2bt^2}{m}, \frac{ct^2}{2m}$
 c. $\frac{at^2}{m}, \frac{bt^2}{m}, \frac{ct^2}{2m}$ d. None of these
21. The angle between vector $\mathbf{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\mathbf{b} = 3\hat{i} - 4\hat{j}$ is equal to
- a. $\cos^{-1}\left(\frac{3}{15}\right)$ b. $\cos^{-1}\left(\frac{1}{15}\right)$
 c. zero d. $\cos^{-1}\frac{2}{15}$
22. The resultant of two vectors $\mathbf{P}$ and $\mathbf{Q}$ is $\mathbf{R}$. If the vector $\mathbf{Q}$ is reversed, then the resultant becomes $\mathbf{S}$, then choose the correct option.
- a. $R^2 + S^2 = 2(P^2 - Q^2)$ b. $R^2 + S^2 = 2(P^2 + Q^2)$
 c. $R^2 + S^2 = (P^2 - Q^2)$ d. $R^2 - S^2 = 2(P^2 + Q^2)$
23. For what value of x , will the two vectors $\mathbf{A} = 2\hat{i} + 2\hat{j} - x\hat{k}$ and $\mathbf{B} = 2\hat{i} - \hat{j} - 3\hat{k}$ are perpendicular to each other?
- a. $x = -2/3$ b. $x = 3/2$ c. $x = -4/3$ d. $x = 2/3$
24. Calculate the work done by a force $\mathbf{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N to displace a body from position A to position B . The position vector of A is $\mathbf{r}_1 = (\hat{i} + 3\hat{j} + \hat{k})$ m and the position vector of B is $\mathbf{r}_2 = (2\hat{i} + 2\hat{j} + 3\hat{k})$ m.
- a. 5 J b. 3 J c. 2 J d. 10 J
25. If $\mathbf{c} = \mathbf{a} \times \mathbf{b}$, then
- a. the direction of $\mathbf{c}$ changes, when the angle between $\mathbf{a} \times \mathbf{b}$ increases up to θ ($\theta < 180^\circ$)
 b. the direction of $\mathbf{c}$ changes, when the angle between $\mathbf{a}$ and $\mathbf{b}$ decreases up to θ ($\theta > 0^\circ$)
 c. the direction of $\mathbf{c}$ does not change, when the angle between $\mathbf{a}$ and $\mathbf{b}$ increases
 d. None of the above
26. The unit vector perpendicular to vectors $\mathbf{a} = 3\hat{i} + \hat{j}$ and $\mathbf{b} = 2\hat{i} - \hat{j} - 5\hat{k}$ is
- a. $\pm \frac{(\hat{i} - 3\hat{j} + \hat{k})}{\sqrt{11}}$ b. $\pm \frac{3\hat{i} + \hat{j}}{\sqrt{11}}$
 c. $\pm \frac{(2\hat{i} - \hat{j} - 5\hat{k})}{\sqrt{30}}$ d. None of these
27. If three vectors along coordinate axes represent the adjacent sides of a cube of length b , then the unit vector along its diagonal passing through the origin will be
- a. $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{2}}$ b. $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}b}$ c. $\hat{i} + \hat{j} + \hat{k}$ d. $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$
28. Choose the correct option $\mathbf{A} \times \mathbf{B} = \mathbf{C}$.
- (i) $\mathbf{C}$ is perpendicular to $\mathbf{A}$
 (ii) $\mathbf{C}$ is perpendicular to $\mathbf{B}$
 (iii) $\mathbf{C}$ is perpendicular to $(\mathbf{A} + \mathbf{B})$
 (iv) $\mathbf{C}$ is perpendicular to $(\mathbf{A} \times \mathbf{B})$
- a. Only (i) and (ii) are correct
 b. Only (ii) and (iv) are correct
 c. (i), (ii) and (iii) are correct
 d. All of the above
29. Find the vector area of a triangle whose vertices are a, b and c .
- a. $\frac{1}{2}(b \times c + c \times a + a \times b)$ b. $\frac{1}{3}(b \times c + c \times a + a \times b)$
 c. $\frac{1}{3}(b \times c + a \times c + b \times a)$ d. None of these
30. If three vectors $xa - 2b + 3c$, $-2a + yb - 4c$ and $-zb + 3c$ are coplanar, where a, b and c are unit (or any) vectors, then
- a. $xy + 3zx - 3z = 4$ b. $2xy - 3zx - 3z - 4 = 0$
 c. $4xy - 3zx - 3z = 4$ d. $xy - 2zx - 3z - 4 = 0$
31. A force $\mathbf{F} = (2\hat{i} + 3\hat{j} - \hat{k})$ N is acting on a body at a position $\mathbf{r} = (6\hat{i} + 3\hat{j} - 2\hat{k})$. Calculate the torque about the origin.
- a. $(3\hat{i} + 2\hat{j} + 12\hat{k})$ Nm b. $(9\hat{i} + 2\hat{j} + 7\hat{k})$ Nm
 c. $(\hat{i} + 2\hat{j} + 12\hat{k})$ Nm d. $(3\hat{i} + 12\hat{j} + \hat{k})$ Nm
32. Find the values of x and y for which vectors $\mathbf{A} = (6\hat{i} + x\hat{j} - 2\hat{k})$ and $\mathbf{B} = (5\hat{i} - 6\hat{j} - y\hat{k})$ are parallel.
- a. $x = 0, y = \frac{2}{3}$ b. $x = -\frac{36}{5}, y = \frac{5}{3}$
 c. $x = -\frac{15}{3}, y = \frac{23}{5}$ d. $x = \frac{36}{5}, y = \frac{15}{4}$
33. Find the area of the parallelogram determined by $\mathbf{A} = 2\hat{i} + \hat{j} - 3\hat{k}$ and $\mathbf{B} = 12\hat{j} - 2\hat{k}$
- a. 42 b. 56
 c. 38 d. 74

34. Choose the correct option.
 a. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = 0$
 b. $\mathbf{a} \times (\mathbf{c} \times \mathbf{b}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = 0$
 c. $\mathbf{a} \times (\mathbf{c} \times \mathbf{b}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) - \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = 0$
 d. None of the above
35. The three conterminous edges of a parallelepiped are
 $\mathbf{a} = 2\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \mathbf{b} = 5\hat{\mathbf{j}}, \mathbf{c} = -2\hat{\mathbf{i}} + \hat{\mathbf{k}}$
 Calculate the volume of parallelepiped.
 a. 36 cubic units b. 45 cubic units
 c. 40 cubic units d. 54 cubic units
36. If the three vectors are coplanar, then find x .
 $\mathbf{A} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \mathbf{B} = x\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \mathbf{C} = 7\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 11\hat{\mathbf{k}}$
 a. 36/21 b. -51/32 c. 51/32 d. -36/21
37. A particle is moving along a circular path with a constant speed 30 m/s. What is change in velocity of a particle, when it describe an angle of 90° at the centre of the circle.
 a. zero b. $30\sqrt{2}$ m/s c. $60\sqrt{2}$ m/s d. $30\sqrt{2}$ m/s
38. One day in still air, a motor-cyclist riding north at 30 m/s, suddenly the wind starts blowing Westward with a velocity 50 m/s, then calculate the apparent velocity with which the motor-cyclist will move.
 a. 58.3 m/s b. 65.4 m/s c. 73.2 m/s d. 53.8 m/s
39. Calculate the distance travelled by the car, if a car travels 4 km towards north at an angle of 45° to the east and then travels a distance of 2 km towards north at an angle of 135° to the east.
 a. 6 km b. 8 km
 c. 5 km d. 2 km
40. On one rainy day a car starts moving with a constant acceleration of 1.2 m/s^2 . If a toy monkey is suspended from the ceiling of the car by a string, then find the angle with the vertical with the string be now inclined.
 a. $\tan^{-1}(0.25)$
 b. $\tan^{-1}(0.63)$
 c. $\tan^{-1}(0.12)$
 d. $\tan^{-1}(\sqrt{3})$

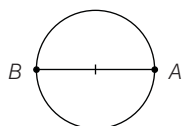
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1. If $\mathbf{A}$ and $\mathbf{B}$ denote the sides of a parallelogram and its area is $\frac{1}{2} AB$ (A and B are magnitude of $\mathbf{A}$ and $\mathbf{B}$ respectively), the angle between $\mathbf{A}$ and $\mathbf{B}$ is [2014]
 a. 30° b. 45°
 c. 60° d. 90°
2. A vector $\mathbf{F}_1$ acts along positive x-axis. If its vector product with another $\mathbf{F}_2$ is zero, then $\mathbf{F}_2$ could be [2009]
 a. $4\hat{\mathbf{j}}$ b. $(\hat{\mathbf{k}} + \hat{\mathbf{j}})$
 c. $(\hat{\mathbf{j}} + \hat{\mathbf{k}})$ d. $-4\hat{\mathbf{i}}$
3. The component of vector $\mathbf{A} = a_x \hat{\mathbf{i}} + a_y \hat{\mathbf{j}} + a_z \hat{\mathbf{k}}$ along the direction of $\hat{\mathbf{i}} - \hat{\mathbf{j}}$ is [2008]
 a. $a_x - a_y + a_z$ b. $a_x - a_y$
 c. $(a_x - a_y) / \sqrt{2}$ d. $(a_x + a_y + a_z)$
4. If $\mathbf{A} \cdot \mathbf{B} = \mathbf{A} \times \mathbf{B}$, then angle between $\mathbf{A}$ and $\mathbf{B}$ is [2006]
 a. 45° b. 30° c. 60° d. 90°
5. If $\mathbf{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\mathbf{b} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, then the angle between the vectors $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ is [2005]
 a. 60° b. 90° c. 45° d. 55°

Answer with Solutions

Practice Exercise

1. (d) The maximum magnitude of displacement



$$= |\mathbf{AB}| = 2r = 14 \text{ m}$$

2. (d) Time = $\frac{\text{Distance}}{\text{Speed}} = \frac{\pi r}{10} = \frac{22}{7} \times \frac{7}{10} = 2.2 \text{ s}$

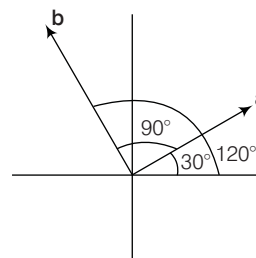
3. (c) $F_1 = 3 \text{ N}$ and $F_2 = 4 \text{ N}$

$$F_{\max} = F_1 + F_2 = (3 + 4) = 7 \text{ N}$$

$$F_{\min} = F_2 - F_1 = 4 - 3 = 1 \text{ N}$$

$$\Rightarrow \frac{F_{\min}}{F_{\max}} = \frac{1}{7}$$

4. (c) The figure shown as described in the question.



According to parallelogram law of vectors,

$$R = \sqrt{a^2 + b^2 + 2ab \cos \alpha}$$

$$= \sqrt{3^2 + 4^2 + 2 \times 3 \times 4 \cos 90^\circ} = 5 \text{ units}$$

5. (a) $|\mathbf{R}| = |\mathbf{a} + \mathbf{b}| = \sqrt{4^2 + 4^2 + 2(4)(4) \cos 120^\circ}$

$$|\mathbf{R}| = 4$$

Let θ = angle between $\mathbf{a} + \mathbf{b}$ with x -axis

$$\theta = \tan^{-1} \left(\frac{4 \sin(120^\circ)}{4 + 4 \cos(120^\circ)} \right) = \tan^{-1} \left(\frac{3.464}{2} \right) = \tan^{-1}(1.73)$$

6. (c) $\frac{|\mathbf{a} + \mathbf{b}|}{|\mathbf{a} - \mathbf{b}|} = 1$

or $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$

or $\sqrt{a^2 + b^2 + 2ab \cos \alpha} = \sqrt{a^2 + b^2 - 2ab \cos \alpha}$

or $a^2 + b^2 + 2ab \cos \alpha = a^2 + b^2 - 2ab \cos \alpha$

or $4ab \cos \alpha = 0$ or $\cos \alpha = 0$

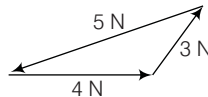
or $\alpha = 90^\circ$

7. (a) $\mathbf{R} = (\mathbf{A} + \mathbf{B}) + (\mathbf{A} - \mathbf{B})$

$$\mathbf{R} = 2\mathbf{A}$$

The angle between $\mathbf{A}$ and $2\mathbf{A}$ is zero, because they are parallel vectors.

8. (b) The magnitude of three forces 3N, 4N and 5N will be zero, if these vectors from a close polygon will all the sides in the same order as shown in figure.



Hence, option (b) is correct.

9. (d) According to polygon law, resultant force will be zero.

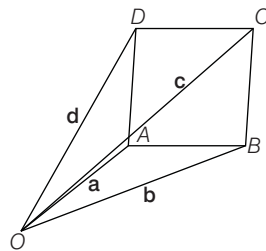
10. (b) As shown in figure, according to condition given in question.

$\mathbf{BC} = \mathbf{c} - \mathbf{b}$

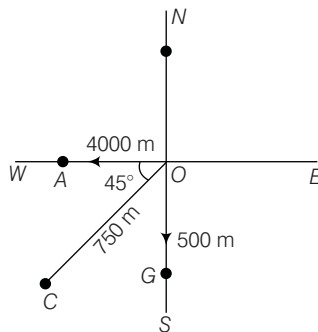
$\mathbf{AD} = \mathbf{d} - \mathbf{a}$

but $\mathbf{BC} = \mathbf{AD}$

$\therefore \mathbf{c} - \mathbf{b} = \mathbf{d} - \mathbf{a}$



11. (b) The given figure shows the direction of motion of man.



Here, distance = 4000 + 750 + 500 = 5250 m

Here, $x = -4000 - 750 \cos 45^\circ = -4530.33$

$y = -500 - 750 \sin 45^\circ = -1030.33$

$$\therefore |\mathbf{s}| = \sqrt{x^2 + y^2}$$

$$= \sqrt{20523889.9 + 1061580.08} = 4646.016 \text{ m}$$

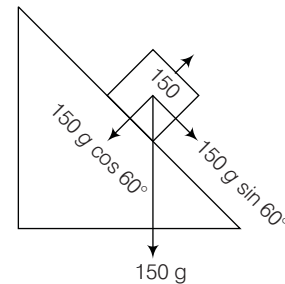
12. (b) Effective force along West (x) = (50 - 30) = 20 N

Effective force along South, (y) = 40 - 20 = 20 N

Resultant = $R = \sqrt{x^2 + y^2} = \sqrt{20^2 + 20^2} = 20\sqrt{2} \text{ N}$

$$\tan \theta = \frac{y}{x} = 1 = \tan 45^\circ \Rightarrow \theta = \frac{\pi}{4} = 45^\circ$$

13. (a) Component parallel to inclined plane, is $150g \sin 60^\circ$.



$$= \frac{1500 \sqrt{3}}{2} = 1299 \approx 1300 \text{ N}$$

14. (c) The minimum distance

= The magnitude of displacement of cat = $|\mathbf{r}_B - \mathbf{r}_A|$

Here, $\mathbf{r}_B = 5\hat{i} - 8\hat{k}$, $\mathbf{r}_A = 3\hat{j} + 4\hat{k}$

$$\therefore \mathbf{s} = \mathbf{r}_B - \mathbf{r}_A = 5\hat{i} - 3\hat{j} - 12\hat{k}$$

$$\therefore |\mathbf{s}| = \sqrt{(5)^2 + (-3)^2 + (-12)^2}$$

$$= \sqrt{25 + 9 + 144}$$

$$= 13.34 \approx 13 \text{ unit}$$

15. (c) Here, $\sqrt{3}a = 40\sqrt{3} \text{ ft}$

$$\therefore a = 40 \text{ ft}$$

$$\therefore \text{Volume} = a^3 = (40 \text{ ft})^3 = 64000 \text{ ft}^3$$

16. (a) Distance = $\sqrt{(40)^2 + (40 + 40)^2}$
- $$= \sqrt{1600 + 6400} = 89.44 \text{ ft}$$

17. (a) $\frac{d\mathbf{r}}{dt} = -b\omega \sin \omega t \hat{i} + a\omega \cos \omega t \hat{j}$

$$\frac{d^2\mathbf{r}}{dt^2} = -b\omega^2 \sin \omega t \hat{i} + a\omega^2 \cos \omega t \hat{j}$$

$$\frac{d^2\mathbf{r}}{dt^2} = -\omega^2 [b \cos \omega t \hat{i} + a \sin \omega t \hat{j}] = -\omega^2 \mathbf{r}$$

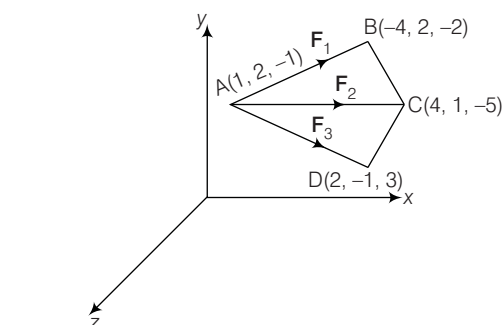
18. (a) $(\mathbf{A} - \mathbf{B}) = 2\hat{i} + 3\hat{j} + \hat{k} - 2\hat{i} - 2\hat{j} - 3\hat{k} = \hat{j} - 2\hat{k}$

$$|\mathbf{A} - \mathbf{B}| = \sqrt{1 + 4} = \sqrt{5}$$

Direction cosines = $\frac{0}{\sqrt{5}}, \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}$

19. (b) $\mathbf{F}_1 = 2 \left[\frac{(-4 - 1)\hat{i} + (2 - 2)\hat{j} + (-2 + 1)\hat{k}}{(-5)^2 + (0)^2 + (-1)^2} \right]$

$$\mathbf{F}_1 = 2 \left(\frac{-5\hat{i} - \hat{k}}{\sqrt{26}} \right)$$



$$\mathbf{F}_2 = 3 \left[\frac{(4-1)\hat{i} + (1-2)\hat{j} + (-5+1)\hat{k}}{(1)^2 + (-3)^2 + (-4)^2} \right]$$

$$\mathbf{F}_2 = 3 \left[\frac{3\hat{i} - \hat{j} - 4\hat{k}}{\sqrt{26}} \right]$$

$$\mathbf{F}_3 = 2 \left[\frac{(2-1)\hat{i} + (-1-2)\hat{j} + (3+1)\hat{k}}{(1)^2 + (-3)^2 + (4)^2} \right]$$

$$\mathbf{F}_3 = 2 \left[\frac{\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{26}} \right]$$

$$\begin{aligned} \mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ &= \frac{1}{\sqrt{26}} [-10\hat{i} - 2\hat{k} + 9\hat{i} - 3\hat{j} - 12\hat{k} + 2\hat{i} - 6\hat{j} + 8\hat{k}] \\ &= \frac{1}{\sqrt{26}} [\hat{i} - 9\hat{j} - 6\hat{k}] \end{aligned}$$

20. (a) $\mathbf{F} = m\mathbf{A} \Rightarrow \mathbf{A} = \frac{1}{m} (a\hat{i} + b\hat{j} + c\hat{k})$

Distance travelled,

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2} \mathbf{A}t^2$$

$$\mathbf{s} = 0 \times t + \frac{1}{2} \frac{1}{m} (a\hat{i} + b\hat{j} + c\hat{k}) t^2$$

$$\mathbf{s} = (a\hat{i} + b\hat{j} + c\hat{k}) \frac{t^2}{2m}$$

21. (d) $\therefore \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$

$$\begin{aligned} \therefore \cos \theta &= \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} = \frac{(2\hat{i} + \hat{j} - 2\hat{k}) \cdot (3\hat{i} - 4\hat{j})}{\sqrt{2^2 + 1^2 + (-2)^2} \sqrt{(3)^2 + (-4)^2}} \\ &= \frac{6-4}{3 \times 5} = \frac{2}{15} \Rightarrow \theta = \cos^{-1} \left(\frac{2}{15} \right) \end{aligned}$$

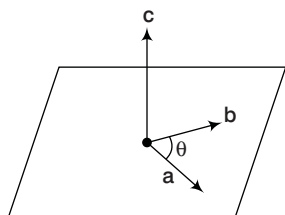
22. (b)

23. (a)

24. (a)

25. (c) $\mathbf{c} = \mathbf{a} \times \mathbf{b}$

$\mathbf{c}$ is perpendicular to $\mathbf{a}$ and $\mathbf{b}$ as show in figure. The direction of $\mathbf{c}$ does not change when the angle between $\mathbf{a}$ and $\mathbf{b}$ is increases or decreases.



26. (a) $\mathbf{a} = 3\hat{i} + \hat{j}$ and $\mathbf{b} = 2\hat{i} - \hat{j} - 5\hat{k}$

$$\begin{aligned} \mathbf{a} \times \mathbf{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 0 \\ 2 & -1 & -5 \end{vmatrix} = (-5)\hat{i} - (-15-0)\hat{j} + (-3-2)\hat{k} \\ &= -5\hat{i} + 15\hat{j} - 5\hat{k} \end{aligned}$$

$$\mathbf{R} = \mathbf{a} \times \mathbf{b} = -5(\hat{i} - 3\hat{j} + \hat{k})$$

$$\hat{\mathbf{R}} = \frac{\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{(1)^2 + (-3)^2 + (1)^2}} = \frac{\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{11}}$$

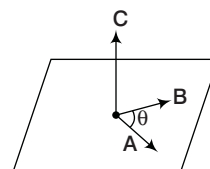
27. (d) Diagonal vector $\mathbf{A} = b\hat{i} + b\hat{j} + b\hat{k}$

or $A = \sqrt{b^2 + b^2 + b^2} = \sqrt{3}b$

$$\therefore \hat{\mathbf{A}} = \frac{\mathbf{A}}{A} = \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$$

28. (c) $\mathbf{A} \times \mathbf{B} = \mathbf{C}$

It is clear from the figure that $\mathbf{C}$ is perpendicular to $\mathbf{A}$ and $\mathbf{B}$, and $\mathbf{C}$ is parallel to $\mathbf{A} \times \mathbf{B}$.



29. (a) Vector area of triangle $= \frac{1}{2} (\mathbf{BC} \times \mathbf{BA})$

$$\begin{aligned} &= \frac{1}{2} [(c-b) \times (a-b)] = \frac{1}{2} [c \times a - b \times a + b \times b - c \times b] \\ &= \frac{1}{2} [c \times a + b \times c + a \times b] \end{aligned}$$

30. (d) Condition of coplanarity, $\begin{vmatrix} x & -2 & 3 \\ -2 & y & -4 \\ 0 & -z & 3 \end{vmatrix} = 0$

$$\Rightarrow x(2y - 4z) + 2(-4 - 0) + 3(2z - 0) = 0$$

$$\Rightarrow 2xy - 4zx - 8 + 6z = 0$$

$$\Rightarrow xy - 2zx - 4 + 3z = 0$$

31. (a) $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 3 & -2 \\ 2 & 3 & -1 \end{vmatrix}$

$$= \hat{i}(-3+6) - \hat{j}(-6+4) + \hat{k}(18-6)$$

$$\boldsymbol{\tau} = (3\hat{i} + 2\hat{j} + 12\hat{k}) \text{ Nm}$$

32. (b) Condition for parallel vectors, $\mathbf{A} \times \mathbf{B} = 0$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & x & -2 \\ 5 & -6 & -y \end{vmatrix} = 0$$

$$= \hat{i}(-xy - 12) - \hat{j}(-6y + 10) + \hat{k}(-36 - 5x) = 0$$

$$\text{or } xy = -12, +6y = 10 - 36 = 5x \Rightarrow x = -\frac{36}{5}, y = \frac{5}{3}$$

$$\begin{aligned}
 33. (a) \mathbf{A} \times \mathbf{B} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 0 & 12 & -2 \end{vmatrix} \\
 &= \hat{i}(-2 + 36) - \hat{j}(-4 - 0) + \hat{k}(24 - 0) \\
 &= 34\hat{i} + 4\hat{j} + 24\hat{k} \\
 |\mathbf{A} \times \mathbf{B}| &= \sqrt{1156 + 16 + 576} = 41.81 \approx 42
 \end{aligned}$$

34. (a) From vector triple product,

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c}) \mathbf{b} - (\mathbf{a} \cdot \mathbf{b}) \mathbf{c} \quad \dots(i)$$

$$\mathbf{b} \times (\mathbf{c} \times \mathbf{a}) = (\mathbf{b} \cdot \mathbf{a}) \mathbf{c} - (\mathbf{b} \cdot \mathbf{c}) \mathbf{a} \quad \dots(ii)$$

$$\mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = (\mathbf{c} \cdot \mathbf{b}) \mathbf{a} - (\mathbf{c} \cdot \mathbf{a}) \mathbf{b} \quad \dots(iii)$$

Now adding Eqs. (i), (ii) and (iii), we get

$$\Rightarrow \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = 0$$

$$\begin{aligned}
 35. (c) \text{ Volume, } V &= \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} 2 & -6 & 3 \\ 0 & 5 & 0 \\ -2 & 0 & 1 \end{vmatrix} \\
 &= 2(5 - 0) + 6(0) + 3(+10) = 10 + 30 = 40 \text{ cubic units}
 \end{aligned}$$

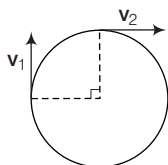
36. (b) The three vectors are coplanar, if their scalar triple product is zero.

$$\text{i.e. } \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0 = \begin{vmatrix} 1 & -2 & 3 \\ 0 & x & 3 \\ 7 & 3 & -11 \end{vmatrix} = 0$$

$$(1)[-x(11) - 9] + 2(0 - 21) + 3(0 - 7x) = 0$$

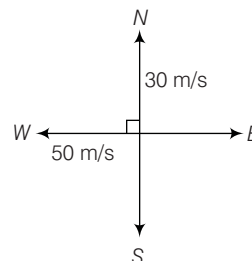
$$-11x - 9 - 42 - 21x = 0 \Rightarrow -32x - 51 = 0 \Rightarrow x = -\frac{51}{32}$$

$$37. (b) \Delta v = |\mathbf{v}_2 - \mathbf{v}_1|$$



$$\begin{aligned}
 |v_1| &= |v_2| = v \\
 |\Delta v| &= \sqrt{v^2 + v^2 - 2v \cos 90^\circ} \\
 &= \sqrt{2} v = 30\sqrt{2} \text{ m/s}
 \end{aligned}$$

38. (a) Here, $\theta = 90^\circ$

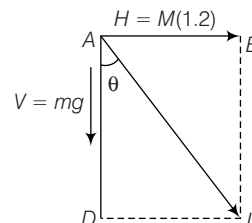


Let v' = apparent velocity

$$\begin{aligned}
 v' &= \sqrt{30^2 + 50^2 + 2(50)(30) \cos 90^\circ} \\
 &= \sqrt{900 + 2500} \\
 v' &= 58.3 \text{ m/s}
 \end{aligned}$$

39. (a) As the distance is a scalar quantity. So, total distance travelled = $4 + 2 = 6 \text{ km}$

40. (c) Let mass of toy = m



$$\begin{aligned}
 \tan \theta &= \frac{H}{V} = \frac{m(1.2)}{m(10)} \\
 \theta &= \tan^{-1}(0.12)
 \end{aligned}$$

BITSAT Archives

1. (a) Area of parallelogram = $|\mathbf{A} \times \mathbf{B}|$

$$AB \sin \theta = \frac{1}{2} AB \Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

2. (d) Let $\mathbf{F}_1 = x\hat{i}$

As, $\mathbf{F}_1 \times \mathbf{F}_2 = 0$ and only $\hat{i} \times \hat{i} = 0$

$$\therefore \mathbf{F}_2 = -4\hat{i}$$

3. (c) Let $\mathbf{B} = \hat{i} - \hat{j}$

Then, component of vector $\mathbf{A}$ along $\mathbf{B} = \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{B}|}$

$$= \frac{(a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \cdot (\hat{i} - \hat{j})}{|\hat{i} - \hat{j}|} = \frac{a_x - a_y}{\sqrt{2}}$$

4. (a) $\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \cos \theta \Rightarrow AB \cos \theta = AB \sin \theta$

$$\tan \theta = 1 \Rightarrow \tan \theta = \tan 45^\circ \Rightarrow \theta = 45^\circ$$

5. (b) Given that, $\mathbf{a} = \hat{i} + 2\hat{j} - 3\hat{k}$

and $\mathbf{b} = 3\hat{i} - \hat{j} + 2\hat{k}$

$$\text{Now, } \mathbf{a} + \mathbf{b} = \hat{i} + 2\hat{j} - 3\hat{k} + 3\hat{i} - \hat{j} + 2\hat{k} = 4\hat{i} + \hat{j} - \hat{k}$$

$$\text{and } \mathbf{a} - \mathbf{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) - (3\hat{i} - \hat{j} + 2\hat{k}) = -2\hat{i} + 3\hat{j} - 5\hat{k}$$

Let θ be the angle between $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$.

$$\begin{aligned}
 \therefore \cos \theta &= \frac{(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b})}{|\mathbf{a} + \mathbf{b}| |\mathbf{a} - \mathbf{b}|} = \frac{(4\hat{i} + \hat{j} - \hat{k}) \cdot (-2\hat{i} + 3\hat{j} - 5\hat{k})}{|4\hat{i} + \hat{j} - \hat{k}| \cdot |-2\hat{i} + 3\hat{j} - 5\hat{k}|} \\
 &= \frac{-8 + 3 + 5}{\sqrt{16 + 1 + 1} \sqrt{4 + 9 + 25}} = 0 \Rightarrow \theta = 90^\circ
 \end{aligned}$$

3

Motion in 1, 2 & 3 Dimensions and Projectile Motion

Motion

If the position of an object is continuously changing with respect to its surroundings with time, then it is said to be in the state of motion. A bird flying in air, a train moving on rails, a ship sailing on water, are some of the examples of motion.

Motion in a Straight Line

The motion of a point object in a straight line is one-dimensional motion. During such motion, the point object occupies definite position on the path at each instant of time t .

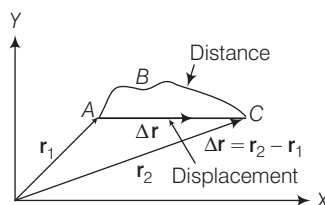
Distance and Displacement

Distance It is the total length of the path travelled by a particle in a given interval of time. It is a scalar quantity having unit metre or centimetre.

Displacement It is the shortest distance between initial and final positions with the direction from initial to final position. It is a vector quantity having unit metre or centimetre.

Mathematically, it is equal to difference of position vectors, i.e. $\Delta r = |r_2 - r_1|$, while distance is actual path length, i.e. ABC as shown in figure.

The displacement of motion may be zero or negative but path length or distance can never be negative.



NOTE Magnitude of displacement is less than or equal to distance.

Thus, $|\text{Displacement}| \leq \text{Distance}$.

Speed

It is defined as the total path length (i.e. actual distance covered) divided by total time taken by the object

$$\text{i.e. Speed} = \frac{\text{Total path length}}{\text{Total time taken}}$$

So, speed is a scalar quantity. It gives no idea about the direction of motion of the object. Hence, speed of the object can be zero or positive but never negative.

Average Speed

Average speed of an object is defined as the ratio of the total distance travelled by the object to the total time taken, i.e.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Average speed of particles in different cases is discussed below

Case I If a particle travels distances s_1, s_2, s_3 etc., with speeds v_1, v_2, v_3 etc., in same direction, then distance travelled $= s_1 + s_2 + s_3 + \dots$

$$\text{Total time taken} = \frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + \dots$$

$$\text{Average speed, } v_{av} = \frac{s_1 + s_2 + \dots}{\left(\frac{s_1}{v_1} + \frac{s_2}{v_2} + \dots\right)}$$

If $s_1 = s_2 = s$, i.e. the body covers equal distance with different speeds, then

$$\begin{aligned} v_{av} &= \frac{2s}{s\left[\frac{1}{v_1} + \frac{1}{v_2}\right]} \\ &= \frac{2v_1v_2}{v_1 + v_2} \end{aligned}$$

It means average speed is equal to harmonic mean of individual speeds.

Case II If a particle travels with speeds v_1, v_2, v_3 etc., during time intervals t_1, t_2, t_3 etc., then total distance travelled, $s = v_1t_1 + v_2t_2 + \dots$

$$\text{Total time taken} = t_1 + t_2 + t_3 + \dots$$

So, average speed,

$$v_{av} = \frac{v_1t_1 + v_2t_2 + v_3t_3 + \dots}{t_1 + t_2 + t_3 + \dots}$$

Case III If $t_1 = t_2 = t_3 = \dots = t_n$, then we have

$$v_{av} = \frac{(v_1 + v_2 + \dots + v_n)t}{nt} = \frac{v_1 + v_2 + \dots + v_n}{n}$$

It means average speed is equal to arithmetic mean of individual speeds.

Velocity

It is defined as the ratio of displacement and the corresponding time interval taken by the object, i.e.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time interval}}$$

Different types of velocity are given below

(i) **Variable Velocity** An object is said to have variable velocity, if either its speed or its direction or both change w.r.t. time.

(ii) **Average Velocity** Average velocity is defined as ratio of change in position (displacement) to the time interval (Δt).

$$\begin{aligned} \text{Average velocity, } v &= \frac{\text{Total displacement}}{\text{Total time taken}} \\ &= \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \end{aligned}$$

(iii) **Instantaneous Velocity** Instantaneous velocity is defined as the velocity at a particular instant of time. Instantaneous velocity at particular instant of time t is

$$v_i = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Acceleration

Acceleration of an object is defined as the rate of change of velocity. It is a vector quantity having unit m/s^2 or ms^{-2} . Acceleration can be positive, zero or negative. Positive acceleration means velocity increasing with time, zero acceleration means velocity is uniform, while negative acceleration (retardation) means velocity is decreasing with time.

Average and Instantaneous Acceleration

Average acceleration is defined as the change in velocity (Δv) divided by the time interval (Δt).

Let us consider the motion of a particle. Suppose that, the particle has velocity v_1 at $t = t_1$ and at a later time $t = t_2$ it has velocity v_2 . Thus, the average acceleration during the time interval $\Delta t = t_2 - t_1$, is

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

On a plot of velocity *versus* time, the average acceleration is the slope of the straight line connecting the points (v_2, t_2) and (v_1, t_1) .

If the time interval approaches zero, average acceleration is known as **instantaneous acceleration**. Mathematically,

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

When a body is moving with a constant acceleration, then acceleration time graph is a straight line.

Negative acceleration is known as retardation. It indicates that velocity is decreasing with respect to time.

Uniformly Accelerated Motion

A motion in which change in velocity in each unit of time is constant, is called as uniformly accelerated motion. For a uniformly accelerated motion ($\mathbf{a} = \text{constant}$), equations of motion are as follows

Kinematic equations of motion

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t \quad \dots(i)$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2 \quad \dots(ii)$$

$$\text{and} \quad \mathbf{v} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{u} + 2\mathbf{a} \cdot \mathbf{s} \quad \dots(iii)$$

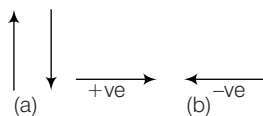
$$v^2 = u^2 + 2as$$

where, $\mathbf{u}$ = initial velocity, $\mathbf{v}$ = velocity at time t ,

$\mathbf{s}$ = displacement of particle at time t .

If motion is described in one dimension, so vector sign ($\rightarrow$) need not be used.

Sign convention for Fig. (a) motion in the vertical direction, and Fig. (b) motion in the horizontal direction is shown in figure.



Non-uniformly Accelerated Motion

When motion of a particle is not uniform, i.e. acceleration of particle is not constant or acceleration is a function of time, then following relations hold for one-dimensional motion.

$$(i) \quad v = \frac{ds}{dt}$$

$$(ii) \quad a = \frac{dv}{dt} = v \frac{dv}{ds}$$

$$(iii) \quad ds = v dt \text{ and}$$

$$(iv) \quad dv = a dt \text{ or } v dv = a ds$$

Such problems can be solved either by differentiation or integration by applying some boundary conditions.

Free Fall Acceleration

An important example of a straight line motion with constant acceleration, is that of an object rising or falling freely near the earth's surface.

The constant acceleration equation describes this motion but we make two changes in notation, (1) we refer the motion to the vertical y -axis with $+y$ vertically up, and (2) we replace a with g , where g is the magnitude of the free fall acceleration. Near earth's surface, $g = 9.8 \text{ ms}^{-2}$ (32 fts^{-2})

Some Special Cases

- If an object starts from rest and moves with a uniform acceleration, then displacement covered by it in time t is directly proportional to t^2 , i.e. $s \propto t^2$ and if a body starts from rest and moves with a uniform acceleration, then displacement covered by it in successive seconds will be in the ratio of odd natural number, i.e. $s_{1st} : s_{2nd} : s_{3rd} = 1 : 3 : 5 \dots$

When particle covers one-third distance at speed v_1 , next one-third at speed v_2 and last one-third at speed v_3 , then

$$v_{av} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$$

- If a particle is accelerated for time t_1 with acceleration a_1 and for time t_2 with acceleration a_2 , then average acceleration is

$$a_{av} = \frac{a_1t_1 + a_2t_2}{t_1 + t_2}$$

- If an object is moving with uniform acceleration, then
Instantaneous acceleration = Uniform acceleration
= Average acceleration
- If a particle is moving with constant acceleration from point A to B in a straight line. If u and v are corresponding velocities at A and B respectively, then velocity at the mid-point of A

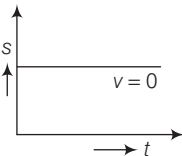
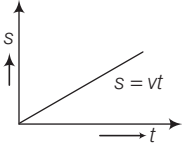
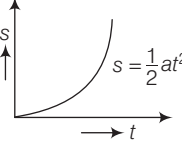
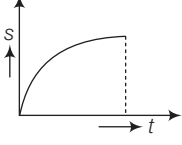
$$\text{and B is } v_0 = \sqrt{\frac{u^2 + v^2}{2}}.$$

Graphs

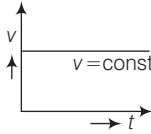
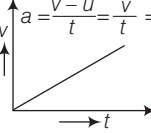
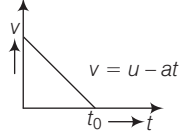
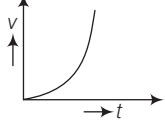
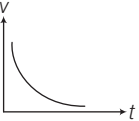
Graphs in One-Dimensional Motion

Graphs can be generalised on the basis of position-time and velocity-time in one-dimensional motion with uniform velocity and uniform acceleration.

Displacement-Time Graph

Different cases	s-t Graph	Characteristics
At rest		Slope of s-t graph $\left(\frac{ds}{dt}\right)$ is zero, i.e. zero velocity.
Uniform motion $a = 0$		Slope of s-t graph is constant, i.e. $v = \text{constant}$, $a = 0$
Uniformly accelerated motion with $u = 0, s = 0$, at $t = 0$		Slope of s-t graph is increasing, i.e. positive value.
Uniformly retarded motion		Slope of s-t graph is decreasing, i.e. negative value.

Velocity-Time Graph

Different cases	v-t Graph	Characteristics
Uniform motion		(i) $\theta = 0^\circ$ (ii) $v = \text{constant}$ Slope of v-t graph $\left(\frac{dv}{dt}\right)$ is zero, i.e. acceleration is zero.
Uniformly accelerated motion with $u = 0, s = 0$ at $t = 0$	 $a = \frac{v-u}{t} = \frac{v}{t} = \text{constant}$	Slope of v-t graph is constant.
Uniformly decelerated motion	 $v = u - at$	Slope of v-t graph is negative (retarded)
Non-uniformly accelerated motion		Slope of v-t graph increases with time. v is increasing, so acceleration is increasing.
Non-uniformly decelerating motion		v is decreasing so acceleration is decreasing.

NOTE Slope of s-t graph $= \frac{ds}{dt} = \tan\theta$

where, θ is the angle made by the tangent at any point (with time axis) on the graph taken anti-clockwise.

Relative Motion

If the velocity of two bodies are known w.r.t. a common frame of reference, then the velocity of one of the two bodies can be measured w.r.t. the second body.

Let us consider, if the velocities of the two bodies A and B w.r.t. earth are $\mathbf{v}_A$ and $\mathbf{v}_B$, then the relative velocity of A w.r.t. B is $\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$

Similarly, we see that $\mathbf{v}_{AB} = -\mathbf{v}_{BA}$

Also, acceleration of A w.r.t. B is

$$\mathbf{a}_{AB} = \mathbf{a}_A - \mathbf{a}_B$$

and

$$\mathbf{a}_{AB} = -\mathbf{a}_{BA}$$

If a car moves from west to east (the direction of motion of the earth) $v_C = v_{CE} + v_E$

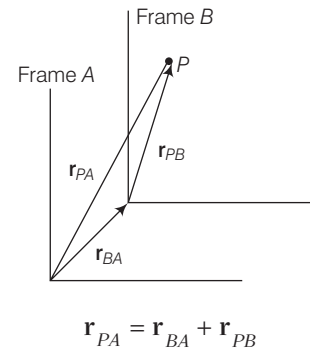
and if the car moves from east to west (opposite to the motion of the earth) $v_C = v_{CE} - v_E$

where, v_C and v_E are the velocities of car and the earth respectively with respect to an inertial frame of reference.

Also, note that the earth is assumed as the rotating frame which is a non-inertial one.

Relative Motion in Two Dimensions

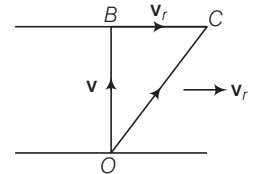
The description of the motion of an object in two or three dimensions depends on the choice of the coordinate system. Figure shows two reference frames in two dimensions. The vectors $\mathbf{r}_{PA}$ and $\mathbf{r}_{PB}$ are the position vectors of the object P in reference frame A and in reference frame B , respectively. Vector $\mathbf{r}_{BA}$ is the position of observer B (located at the origin of reference frame B) w.r.t. frame A . The position vector of the object P in reference frame B can be obtained from the position vector in reference frame A .



The velocity and acceleration of the object P are therefore,

$$\mathbf{v}_{PA} = \mathbf{v}_{BA} + \mathbf{v}_{PB} \quad \text{and} \quad \mathbf{a}_{PA} = \mathbf{a}_{BA} + \mathbf{a}_{PB}$$

For crossing the river in the shortest time, the boat should sail perpendicular to the flow. If the width of the river is d and v is the velocity of the boat in still water, then

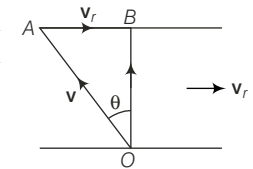


$$t = \frac{d}{v} \quad \text{and} \quad OC = \sqrt{d^2 + \left(v_r \times \frac{d}{v}\right)^2}$$

For crossing the river by the shortest distance, the boat moves such that the horizontal component of the velocity balances the speed of flow. Time of crossing,

$$t = \frac{d}{\sqrt{v^2 - v_r^2}}$$

where, v_r = velocity of river flow



Applications of the Concept of Relative Motion

(i) Rain Problems

In this case, the three velocities with which we have to deal are

$\mathbf{v}_{RG}$ → velocity of rain w.r.t. ground

$\mathbf{v}_{MG}$ → velocity of man w.r.t. ground

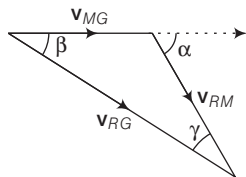
$\mathbf{v}_{RM}$ → velocity of rain w.r.t. man

From the equation of relative motion,

$$\mathbf{v}_{RM} = \mathbf{v}_{RG} - \mathbf{v}_{MG} \quad \text{OR} \quad \mathbf{v}_{RM} + \mathbf{v}_{MG} = \mathbf{v}_{RG}$$

From this equation we can construct a vector diagram, which would be very helpful in solving the questions.

The vector diagram for a particular situation is shown in figure.



From sine law,

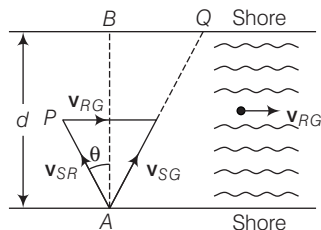
$$\frac{|\mathbf{v}_{MG}|}{\sin \gamma} = \frac{|\mathbf{v}_{RM}|}{\sin \beta} = \frac{|\mathbf{v}_{RG}|}{\sin(\pi - \alpha)}$$

Remember $|\mathbf{v}_{MG}|$, $|\mathbf{v}_{RM}|$ or $|\mathbf{v}_{RG}|$ are not representing the length of triangles.

(ii) River Boat Problems

In this case, we have to deal with the following velocities.

$\mathbf{v}_{SR}$ → velocity of swimmer/steamer/boat w.r.t. river /water/stream



$\mathbf{v}_{RG}$ → Velocity of river/water/stream w.r.t. ground.

$\mathbf{v}_{SG}$ → Velocity of swimmer steamer/boat w.r.t. ground.

From relative motion equation $\mathbf{v}_{SR} + \mathbf{v}_{RG} = \mathbf{v}_{SG}$

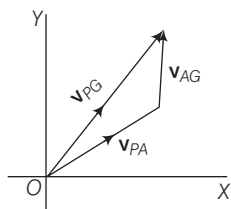
For the situation shown in the figure, a boat is crossing the river and it is heading (moving) along AP as shown in the figure then the time taken by the boat to cross the river is

$$t = \frac{d}{|\mathbf{v}_{SR} \cos \theta|}$$

and it will reach the point Q due to river flow. The distance $BQ = (|\mathbf{v}_{RG}| - |\mathbf{v}_{SR}| \sin \theta)t$ is termed as **drift**.

(iii) Aeroplane Problems

Here the following velocities are to be analysed



$\mathbf{v}_{PA}$ → velocity of plane w.r.t. air.

$\mathbf{v}_{AG}$ → velocity of air w.r.t. ground.

$\mathbf{v}_{PG}$ → velocity of plane w.r.t. ground.

The relative motion equation would be as shown in figure.

$$\mathbf{v}_{PG} = \mathbf{v}_{PA} + \mathbf{v}_{AG}$$

Projectile Motion

We considered objects moving along straight line paths, such as the x -axis. Now, let us look at some cases in which an object moves in a plane. By this we mean that the object has a motion in both the x and the y -directions simultaneously.

Which is another way of saying, it moves in two dimensions. The motion of a particle thrown in a vertical plane, making an angle with the horizontal ($\neq 90^\circ$), is an example of two-dimensional motion. This is called **projectile motion**.

Time of Flight

It is defined as the total time for which the projectile

remains in air, $T = \frac{2u \sin \theta}{g}$

Maximum Height

It is defined as the maximum vertical distance covered by

projectile, $H = \frac{u^2 \sin^2 \theta}{2g}$

Horizontal Range

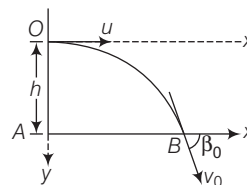
It is defined as the maximum distance covered in horizontal

distance, $R = \frac{u^2 \sin 2\theta}{g}$

- Horizontal range is maximum when it is thrown at an angle of 45° from the horizontal $R_{\max} = \frac{u^2}{g}$
- For angle of projection θ and $(90^\circ - \theta)$ the horizontal range is same.

A Projectile Fired Horizontally from a Certain Height

Let a particle be projected horizontally with a velocity u from a height h above the ground. Then, taking the point of projection O as the origin, horizontal direction as x -axis and vertically upward direction as y -axis, we find that



(i) Equation of trajectory is $y = \frac{g}{2u^2} x^2$

which represents the equation of a parabola.

(ii) Time taken by the projectile to reach the ground

$$T = \sqrt{\frac{2h}{g}}$$

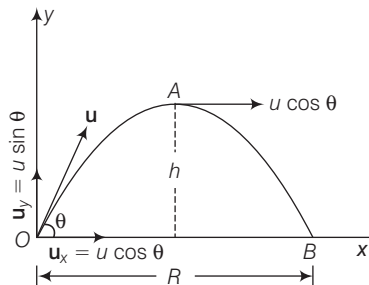
The time is the same as that taken by an object falling freely from the same height.

(iii) Horizontal range, $AB = R = uT = u \sqrt{\frac{2h}{g}}$

(iv) Velocity of projectile at any instant of time t is $\mathbf{v} = (u\hat{i} + gt\hat{j}) \Rightarrow v = \sqrt{u^2 + g^2t^2}$ and $\tan\beta = \frac{gt}{u}$

A Projectile Fired at an Angle of Elevation θ from the Ground

Let a particle be projected at an angle of elevation θ from the ground level at point O with an initial velocity $\mathbf{u}$. The velocity may be resolved into (i) a horizontal component $u_x = u \cos\theta$ and (ii) a vertical component $u_y = u \sin\theta$. Moreover, here, $a_x = 0$ and $a_y = -g$. From these values of $\mathbf{u}$ and $\mathbf{a}$, we get the following results.



(i) Equation of the trajectory, $y = x \tan\theta - \frac{g}{2u^2 \cos^2\theta} x^2$

which is the equation of a parabola, symmetric about the y -axis (i.e. vertical direction)

(ii) Vertical height covered, $h = \frac{u^2 \sin^2\theta}{2g}$

(iii) Time of flight, $T = \frac{2u \sin\theta}{g}$

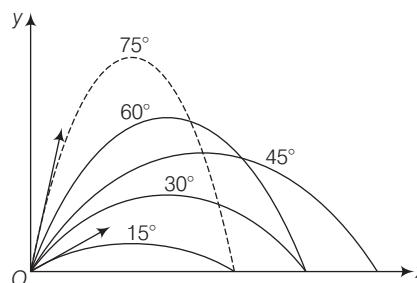
For complementary angles ϕ and $(90^\circ - \phi)$, if T_ϕ and $T_{(90^\circ - \phi)}$ are the time of flight and R is the range, then

$$T_\phi T_{90^\circ - \phi} = \frac{2R_\phi}{g} = \frac{2R_{90^\circ - \phi}}{g} = \frac{2R}{g}$$

e.g., $T_{1^\circ} T_{89^\circ} = \frac{2R_{1^\circ}}{g} = \frac{2R_{89^\circ}}{g}$

(iv) Horizontal range, $R = OB = u_x T = \frac{u^2 \sin 2\theta}{g}$

For a given range R , there are two different angles of projection, one of them being the complement of the other (15° or 75° , 30° or 60°) as shown in figure. However, the vertical height and the time of flight will be different for the two values of θ .



(v) Maximum horizontal range

$$R_{\max} = \frac{u^2}{g}, \text{ when } \theta = 45^\circ$$

For a given speed of projection, R depends on θ , the angle of projection. Range will be maximum for a given speed of projection, when $\sin 2\theta = 1$, i.e. $2\theta = 90^\circ$ or $\theta = 45^\circ$. Hence, for maximum range, the angle of projection must be 45° .

(vi) **Velocity at any instant t** Let the instantaneous velocity be $\mathbf{v}$, inclined at an angle β with the horizontal. Then,

$$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(u \cos\theta)^2 + (u \sin\theta - gt)^2}$$

$$\text{and } \tan\beta = \frac{u \sin\theta - gt}{u \cos\theta}$$

If K is the kinetic energy at the point of launch, then kinetic energy at the highest point is

$$K' = \frac{1}{2} m v_x^2 = \frac{1}{2} m u^2 \cos^2\theta$$

$$\text{or } K' = K \cos^2\theta$$

(vii) **Relation between R and h** It is found that

$$\frac{R}{h} = 4 \cot\theta$$

and if $\theta = 45^\circ$, then $\frac{R_{\max}}{h_{\max}} = 4$

or $R_{\max} = 4h_{\max}$

(viii) **Two angles of projection for the same range** It is found that the range of a projectile is the same when it is projected either at an angle of elevation θ° or at an angle of $(90^\circ - \theta)$.

Practice Exercise

1. A cat wants to catch a rat. The cat follows the path whose equation is $x + y = 0$. But rat follows the path whose equation is $x^2 + y^2 = 4$. The coordinates of possible points of catching the rat are

a. $(\sqrt{2}, \sqrt{2})$ b. $(-\sqrt{2}, \sqrt{2})$ c. $(\sqrt{2}, \sqrt{3})$ d. $(0, 0)$

2. Speed is too velocity as

a. centimetre is too metre
b. force is too torque
c. velocity is too acceleration
d. distance is too displacement

3. A person travelling on a straight line moves with a uniform velocity v_1 for some time and with uniform velocity v_2 for the next the equal time. The average velocity v is given by

a. $v = \frac{v_1 + v_2}{2}$

b. $\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$

c. $v = \sqrt{v_1 v_2}$

d. $\frac{1}{v} = \frac{1}{v_1} + \frac{1}{v_2}$

4. A car moves at 80 km in the first half of total time of motion and at 40 km in the later half. Its average speed is

a. 60 km/h b. 30 km/h
c. 120 km/h d. None of these

5. A particle moves with constant speed v along a regular hexagon $ABCDEF$ in same order. (i.e. A to B, B to C, C to D, D to E, E to F and F to A). The magnitude of average velocity for its motion from A to C is

a. v b. $\frac{v}{2}$
c. $\frac{\sqrt{3}v}{2}$ d. None of these

6. During the shooting of a superhit film 'MARD' Amitabh Bachchan was waiting for his beloved Amrita Singh with his dog. When he saw her approaching, the dog was excited and dashed to her than back to master and so on, never stopping. How far would you estimate the dog ran if his speed is 30 km/h and each of them walked at 4 km/h, starting 400 m apart?

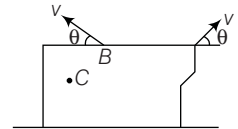
a. 400 m b. 880 m c. 1500 m d. 30 km

7. Two particles start from the same point with different speeds but one moves along $y = a \sin \omega x$ and other moves along curve $y = a \cos \omega x$

a. they must collide after some time
b. they never collide with each other
c. they may collide at the point $P\left(\frac{\pi}{4\omega}, \frac{a}{\sqrt{2}}\right)$
d. they must collide at the point P

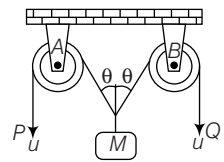
8. A sheet of wood moves over a smooth surface (shown in the figure). The magnitude of velocity of C is

a. v
b. $2v \cos \theta$
c. $2v \sin \theta$
d. $2v$

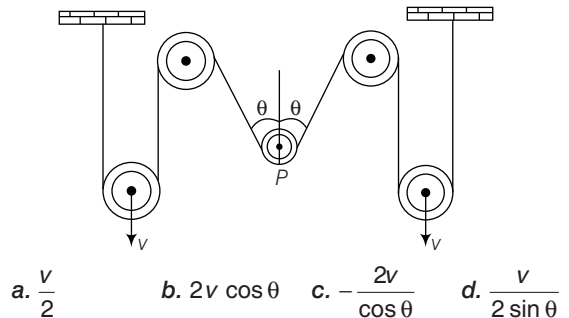


9. In the arrangement shown in figure, the ends P and Q of an inextensible string move downwards with uniform speed u . Pulleys A and B are fixed. The mass m moves upwards with a speed

a. $2u \cos \theta$ b. $\frac{u}{\cos \theta}$ c. $\frac{2u}{\cos \theta}$ d. $u \cos \theta$

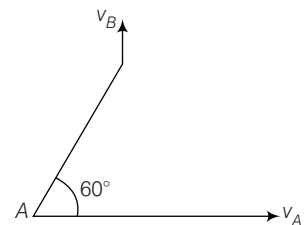


10. In the given figure, find the speed of pulley P.



11. A link AB is moving in a vertical plane. At a certain instant when the link is inclined 60° to the horizontal the point A is moving horizontally at 3 m/s, while B is moving in the vertical direction. What is the velocity of B?

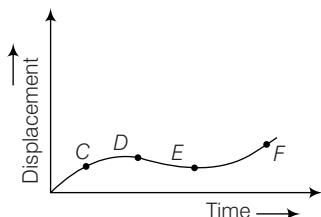
a. $\frac{1}{\sqrt{3}}$ m/s b. $2\sqrt{3}$ m/s
c. $\sqrt{3}$ m/s d. $\frac{\sqrt{3}}{2}$ m/s



12. Two intersecting straight lines move parallel to themselves with speeds 3 m/s and 4 m/s, respectively. The speed of the point of intersection of the lines, if the angle between them is 90° will be

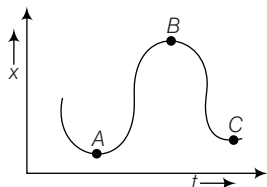
a. 5 m/s b. 3 m/s
c. 4 m/s d. None of these

13. The displacement-time graph is shown in figure. The instantaneous velocity is negative at the point



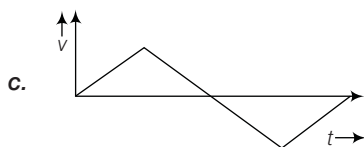
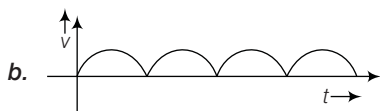
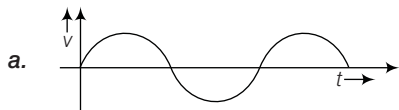
a. D b. F c. C d. E

14. In the given $x-t$ curve,



a. the velocity at A is zero but at B is non-zero
b. the velocity at A and B are zero
c. the velocity at A and B are non-zero
d. the directions of velocity at A and B are definite

15. Which of the following speed-time graph exists in the nature?

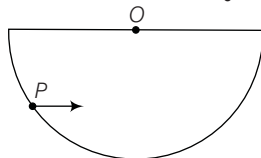


d. All of the above

16. Two particles describe the same circle of radius a in the same sense with the same speed v . What is their relative angular velocity?

a. v/a b. $2v/a$ c. $v/2a$ d. va

17. A particle of mass m is initially situated at the point P inside a hemispherical surface of radius r as shown in figure. A horizontal acceleration of magnitude a_0 is suddenly produced on the particle in the horizontal direction. If gravitational acceleration is neglected, the time taken by particle to touch the sphere again is



a. $\sqrt{\frac{4r \sin \alpha}{a_0}}$

b. $\sqrt{\frac{4r \tan \alpha}{a_0}}$

c. $\sqrt{\frac{4r \cos \alpha}{a_0}}$

d. None of these

18. A particle starts with a velocity of 2 m/s and moves in a straight line with a retardation of 0.1 m/s^2 . The time that it takes to describe 15 m is

a. 10 s in its backward journey
b. 30 s in its forward journey
c. 10 s in its forward journey
d. 30 s in its backward journey
e. Both (b) and (c) are correct

19. A particle starts from rest with acceleration 2 m/s^2 . The acceleration of the particle decreases down to zero uniformly during time-interval of 4 s. The velocity of particle after 2 s is

a. 3 m/s b. 4 m/s c. zero d. 8 m/s

20. An aeroplane moves 400 m towards north, 300 m towards west and then 1200 m vertically upwards. Then, its displacement from the initial position is

a. 1300 m b. 1400 m
c. 1500 m d. 1600 m

21. The displacement of a particle is given by $x = (t - 2)^2$ where, x is in metres and t in seconds. The distance covered by the particle in first 4 s is

a. 4 m b. 8 m
c. 12 m d. 16 m

22. A person walks up a stalled escalator in 90 s when standing on the same escalator, now moving, he is carried in 60 s. The time it would take him to walk up the moving escalator will be

a. 27 s b. 72 s c. 18 s d. 36 s

23. A body starts from rest and moves with a constant acceleration. The ratio of distance covered in the n th second to the distance covered in n second is

a. $\frac{2}{n} - \frac{1}{n^2}$ b. $\frac{1}{n^2} - \frac{1}{n}$ c. $\frac{2}{n^2} - \frac{1}{n}$ d. $\frac{2}{n} + \frac{1}{n^2}$

24. A particle moving with a uniform acceleration along a straight line covers distances a and b in successive intervals of p and q second. The acceleration of the particle is

a. $\frac{pq(p+q)}{2(bp-aq)}$ b. $\frac{2(aq-bp)}{pq(p-q)}$ c. $\frac{bp-aq}{pq(p-q)}$ d. $\frac{2(bp-aq)}{pq(p+q)}$

25. A body moves along x -axis with velocity v_x at position x . If the plot v_x-x is an ellipse with major axis $2A$ and minor axis $2v_0$, the maximum acceleration has a modulus

a. $\frac{v_0^2}{A}$

b. $\frac{A}{v_0^2}$

c. $v_0 A$

d. None of these

26. The distance time graph of a particle at time t makes angle 45° with respect to time axis. After 1s, it makes angle 60° with respect to time axis. What is the acceleration of the particle?

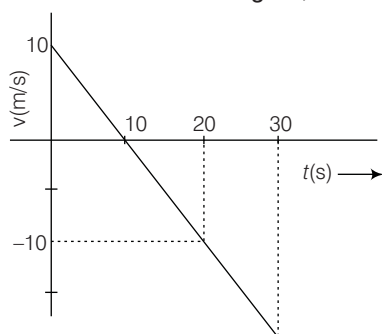
a. $(\sqrt{3} - 1)$ unit

b. $(\sqrt{3} + 1)$ unit

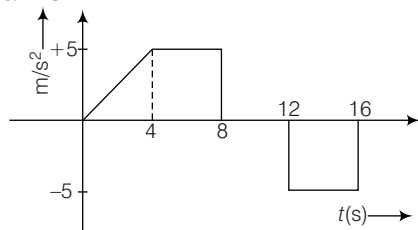
c. $\sqrt{3}$ unit

d. 1 unit

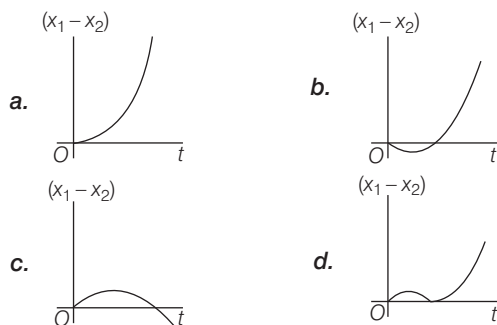
27. The velocity-time plot for a particle moving on a straight line is shown in the figure, then



- a. the particle has a constant acceleration
 b. the particle has never turned around
 c. the average speed in the interval 0 to 10 s is the same as the average speed in the interval 10 s to 20 s
 d. Both (a) and (c) are correct
28. The acceleration of a train between two stations 2 km apart is shown in the figure. The maximum speed of the train is



- a. 60 m/s b. 30 m/s c. 120 m/s d. 90 m/s
29. A body is at rest at $x = 0$, it starts moving in the positive x -direction with a constant acceleration. At the same instant another body passes through $x = 0$ moving in the positive x -direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time t ?



30. A particle moves in a straight line, so that after t second, the distance x from a fixed point O on the line is given by $x = (t - 2)^2(t - 5)$. Then,
- a. after 2 s, velocity of particle is zero
 b. after 2 s, the particle reaches at O
 c. the acceleration is negative, when $t < 3$ s
 d. All of the above

31. A bee flies in a line from a point A to another point B in 4 s with a velocity of $|t - 2|$ m/s. The distance between A and B in metre is

a. 2 b. 4 c. 6 d. 8

32. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time ($t = 1$) is

a. $v_0 + 2g + 3f$ b. $v_0 + \frac{g}{2} + \frac{f}{3}$
 c. $v_0 + g + f$ d. $v_0 + \frac{g}{2} + f$

33. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x -direction with a velocity v that varies as $v = \alpha \sqrt{x}$. The displacement of the particle varies with time as

a. t^2 b. t c. $t^{1/2}$ d. t^3

34. A particle moves as such whose acceleration is given by $a = 3 \sin 4t$, then

a. the initial velocity of the particle must be zero
 b. the acceleration of the particle becomes zero after each interval of $\frac{\pi}{4}$ second
 c. the particle does not come at its initial position after some time
 d. the particle must move on a circular path

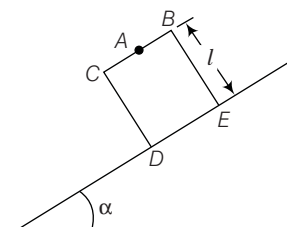
35. A particle moves along a straight line such that its position x at any time t is $x = 3t^2 - t^3$, where x is in metre and t in second, then

a. at $t = 0$ acceleration is 6 m/s^2
 b. $x-t$ curve has maximum at 8 m
 c. $x-t$ curve has maximum at 2 s
 d. Both (a) and (c) are correct

36. The motion of a body falling from rest in a resting medium is described by the equation $\frac{dv}{dt} = a - bv$, where a and b are constant. The velocity at any time t is

a. $a(1 - b^{2t})$ b. $\frac{a}{b}(1 - e^{-bt})$
 c. abe^{-t} d. $ab^2(1 - t)$

37. A rectangular box is sliding on a smooth inclined plane of inclination θ . At $t = 0$, the box starts to move on the inclined plane. A bolt starts to fall from point A . Find the time after which bolt strikes the bottom surface of the box.



a. $\sqrt{\left(\frac{2l}{g \cos \alpha}\right)}$ b. $\sqrt{\left(\frac{2l}{g \sin \alpha}\right)}$
 c. $\sqrt{\left(\frac{2l}{g}\right)}$ d. $\sqrt{\left(\frac{l}{g}\right)}$

38. A car starting from rest, accelerates at the rate f through a distance s , then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance travelled is $15s$, then
 a. $s = ft$ b. $s = \frac{1}{6}ft^2$ c. $s = \frac{1}{72}ft^2$ d. $s = \frac{1}{4}ft^2$

39. An object moves, starting from rest through a resistive medium, such that its acceleration is related to velocity as $a = 3 - 2v$. Then,

- a. the terminal velocity is 1.5 unit
 b. the terminal velocity is 3 unit
 c. the slope of $a - v$ graph is not constant
 d. initial acceleration is 2 unit

40. If the velocity of a moving particle, $v \propto x^n$, where x is the displacement, then

- a. when $x = 0$, the velocity and acceleration are zero
 b. $n > \frac{1}{2}$
 c. $n < \frac{1}{2}$
 d. Both (a) and (b) are correct

41. A particle is projected vertically upward in vacuum with a speed 40 m/s, then velocity of particle when it reaches at maximum height 2 s before, is

(Take, $g = 10 \text{ m/s}^2$)

- a. 20 m/s b. 4.2 m/s
 c. 9.8 m/s d. None of these

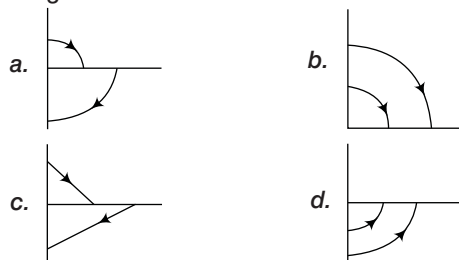
42. A juggler keeps on moving four balls in the air throws the balls in regular interval of time. When one ball leaves his hand (speed = 20 m/s), the position of other balls will be (Take, $g = 10 \text{ m/s}^2$)

- a. 10 m, 20 m, 10 m b. 15 m, 20 m, 15 m
 c. 5 m, 15 m, 20 m d. 5 m, 10 m, 20 m

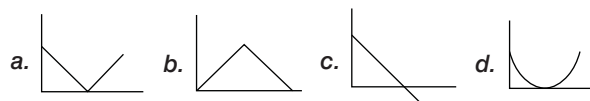
43. Balls are thrown vertically upward in such a way that the next ball is thrown when the previous one is at the maximum height. If the maximum height is 5 m, the number of balls thrown per minute will be (Take, $g = 10 \text{ m/s}^2$)

- a. 60 b. 40 c. 50 d. 120

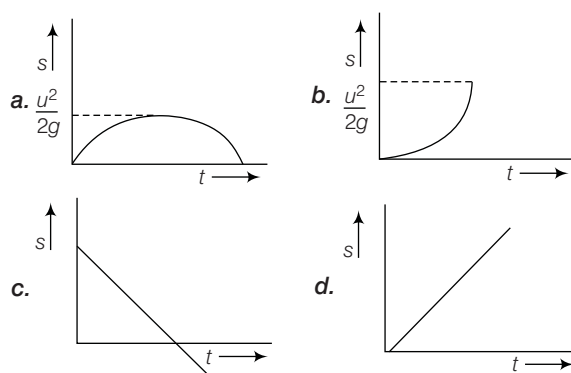
44. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its speed v varies with the height h above the ground as



45. A ball is projected vertically upwards. If resistance due to air is ignored, then which of the following graphs represent the velocity-time graph of the ball during its flight?



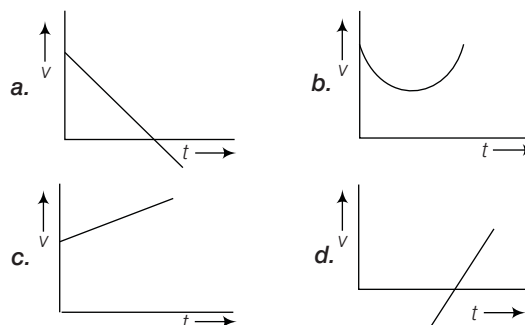
46. An object is thrown upward with a velocity u , then displacement-time graph is



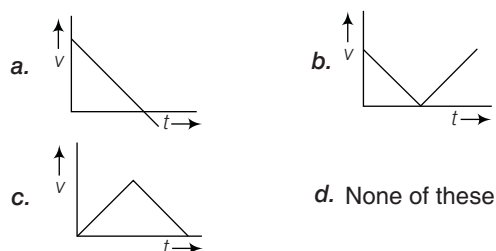
47. A balloon going upward with a velocity of 12 m/s is at a height of 65 m from the earth surface at any instant. Exactly at this instant a packet drops from it. How much time will the packet take in reaching the surface of earth? (Take $g = 10 \text{ m/s}^2$)

- a. 7.5 s b. 10 s
 c. 5 s d. None of these

48. A stone is released from a balloon moving upward with velocity v_0 at height h at $t = 0$. Which of the following graph is best representation of velocity-time graph for the motion of stone?



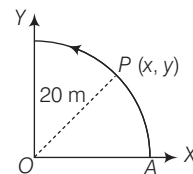
49. For Q. 48, draw speed-time graph.



50. A particle P is at the origin starts with velocity $\mathbf{u} = (2\hat{i} - 4\hat{j})$ m/s with constant acceleration $(3\hat{i} + 5\hat{j})$ m/s². After travelling for 2 s, its distance from the origin is
 a. 10 m b. 10.2 m c. 9.8 m d. 11.7 m
51. At an instant t , the coordinates of a particles are $x = at^2$, $y = bt^2$ and $z = 0$. The magnitude of velocity of particle at an instant t is
 a. $t\sqrt{a^2 + b^2}$ b. $\frac{v}{\sqrt{2}}$ c. $\frac{v}{\sqrt{3}}$ d. $2t\sqrt{a^2 + b^2}$
52. If $x = a(\cos \theta + \theta \sin \theta)$ and $y = a(\sin \theta - \theta \cos \theta)$ and θ increases at uniform rate ω . The velocity of particle is
 a. $a\omega$ b. $\frac{a^2\theta}{\omega}$ c. $\frac{a\theta}{\omega}$ d. $a\theta\omega$
53. A particle is moving with velocity $\mathbf{v} = k(Y\hat{i} + X\hat{j})$, where k is a constant. The general equation for its path is
 a. $Y = X^2 + \text{constant}$ b. $Y^2 = X + \text{constant}$
 c. $XY = \text{constant}$ d. $Y^2 = X^2 + \text{constant}$
54. A light rigid rod is placed on a smooth horizontal surface. Initially the end A begins to move vertically upward with constant velocity v_0 and centre of the rod upward with a velocity $v_0/2$ having downward acceleration $a_0/2$, the other end moves downward with
 a. zero initial velocity having zero acceleration
 b. zero initial velocity having a_0 downward acceleration
 c. non-zero initial velocity and zero acceleration
 d. None of the above
55. At the top of the trajectory of a projectile, the directions of its velocity and acceleration are
 a. parallel to each other
 b. inclined to each other at an angle of 45°
 c. anti-parallel to each other
 d. perpendicular to each other
56. A projectile is thrown at an angle of $\theta = 45^\circ$ to the horizontal, reaches a maximum height of 16 m, then
 a. its velocity at the highest point is zero
 b. its range is 64 m
 c. it is thrown at an angle of 30° , its range will decrease
 d. (b) and (c) both are correct
57. A heavy stone is thrown from a cliff of height h in a given direction. The speed with which it hits the ground (air resistance may be neglected)
 a. must depend on the speed of projection
 b. must be larger than the speed of projection
 c. must be independent of the speed of projection
 d. (a) and (b) both are correct
58. A particle of mass m is projected with a velocity v moving at an angle of 45° with the horizontal. The magnitude of angular momentum of projectile about the point of projection when the particle is at its maximum height h is

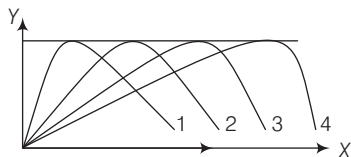
- a. zero b. $\frac{mvh}{\sqrt{2}}$
 c. $\frac{mvh^2}{\sqrt{2}}$ d. None of these

59. Two particles are projected vertically upwards with the same velocity on two different planets with accelerations due to gravities g_1 and g_2 respectively. If they fall back to their initial points of projection after lapse of time t_1 and t_2 respectively. Then,
 a. $t_1 t_2 = g_1 g_2$ b. $t_1 g_1 = t_2 g_2$
 c. $\frac{t_1 g_2}{t_2 g_1} = 2$ d. $t_1^2 + t_2^2 = g_1 + g_2$
60. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in metre and t is in second. The radius of the path is 20 m. The acceleration of P when $t = 2$ s is nearly.



- a. 13 ms^{-2} b. 12 ms^{-2} c. 7.2 ms^{-2} d. 14 ms^{-2}
61. A number of particles are projected from a given point with equal velocities in different directions in the same vertical plane. At any instant, they will lie on
 a. parabola b. circle c. hyperbola d. rectangle
62. Rain water is falling vertically downward with velocity v . When velocity of wind is u in horizontal direction, water is collected at the rate of $R \text{ m}^3/\text{s}$. When velocity of wind becomes $2u$ in horizontal direction. The rate of collection of water in vessel is
 a. R b. $\frac{R}{2}$
 c. $2R$ d. $\frac{R\sqrt{4u^2 + v^2}}{\sqrt{u^2 + v^2}}$
63. A ball projected from ground at an angle of 45° just clears a wall in front. If point of projection is 4 m from the foot of wall and ball strikes the ground at a distance of 6 m on the other side of the wall, the height of the wall is
 a. 4.4 m b. 2.4 m c. 3.6 m d. 1.6 m
64. The maximum range of a bullet fired from a toy pistol mounted on a car at rest is $R_0 = 40$ m. What will be the acute angle of inclination of the pistol for maximum range when the car is moving in the direction of firing with uniform velocity $v = 20 \text{ m/s}$, on a horizontal surface? ($g = 10 \text{ m/s}^2$)
 a. 30° b. 60°
 c. 75° d. 45°

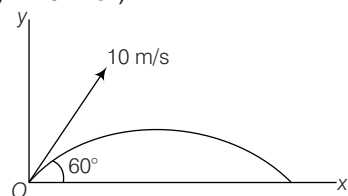
65. Figure shows four paths for a kicked football. Ignoring the effect of air on the flight, rank the paths according to initial horizontal velocity component highest first.



a. 1, 2, 3, 4 b. 2, 3, 4, 1 c. 3, 4, 1, 2 d. 4, 3, 2, 1

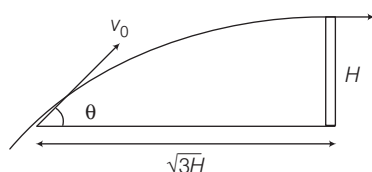
66. A body is projected at an angle θ with the horizontal. When it is at the highest point, the ratio of the potential and kinetic energies of the body is
- a. $\tan \theta$ b. $\tan^2 \theta$
c. $\cot \theta$ d. $\cot^2 \theta$

67. A particle is projected at an angle 60° with the horizontal with a speed 10 m/s. Then, latus rectum is (Take, $g = 10 \text{ m/s}^2$)



a. 5 m b. 15 m
c. 10 m d. 0

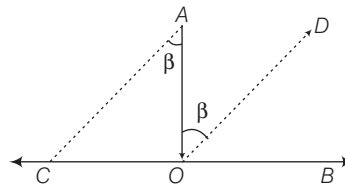
68. A projectile is thrown at an angle θ such that it is just able to cross a vertical wall at its highest point of journey as shown in the figure. The angle θ at which the projectile is thrown is given by



a. $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ b. $\tan^{-1}(\sqrt{3})$
c. $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ d. $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

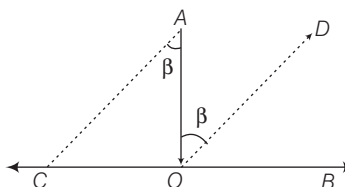
69. Two cars move in the same direction along parallel roads. One of them is a 200 m long travelling with a velocity of 20 m/s. The second one is 800 m long travelling with a velocity of 7.5 m/s. How long will it take for the first car to overtake the second car?
- a. 20 s b. 40 s
c. 60 s d. 80 s
70. A motorboat covers the distance between two spots on the river banks in $t_1 = 8 \text{ h}$ and $t_2 = 12 \text{ h}$ in down stream and upstream respectively. The time required for the boat to cover this distance in still water will be
- a. 6.9 h b. 9.6 h c. 69 s d. 96 s

71. For a man who wants to cross the river with the shortest path AB , find the angle θ (see figure).



a. $\sin^{-1}\left(\frac{3}{4}\right)$
b. $\sin^{-1}\left(\frac{4}{3}\right)$
c. 30°
d. Given situation is not possible

72. To a person going towards East in a car with a velocity of 25 km/h, a train appears to move towards North with a velocity of $25\sqrt{3}$ km/h. The actual velocity of the train will be



a. 25 km/h b. 50 km/h
c. 5 km/h d. 53 km/h

73. A man can swim with a speed of 4 km/h in still water. How long does he take to cross a river 1 km wide, if the river flows steadily 3 km/h and he makes his strokes normal to the river current. How far down the river does he go, when he reaches the other bank?
- a. 800 m b. 900 m
c. 400 m d. 750 m

74. Rain drops fall vertically at a speed of 20 m/s. At what angle do they fall on the wind screen of a car moving with a velocity of 15 m/s, if the wind screen velocity inclined at an angle of 23° to the vertical?

$$\left(\cot^{-1} \frac{4}{3} \approx 37^\circ\right)$$

a. 60° b. 30°
c. 45° d. 90°

75. A bus moves over a straight level road with a constant acceleration a . A boy in the bus drops a ball out side. The acceleration of the ball with respect to the bus and the earth are respectively
- a. a and g
b. $a + g$ and $g - a$
c. $\sqrt{a^2 + g^2}$ and g
d. $\sqrt{a^2 + g^2}$ and a

BITSAT Archives

1. A train accelerating uniformly from rest attains a maximum speed of 40 ms^{-1} in 20 s. It travels at this speed for 20 s and is brought to rest with uniform retardation in further 40 s. What is the average velocity during this period? [2014]

a. 80 m/s b. 25 m/s c. 40 m/s d. 30 m/s

2. A ball is projected upwards from the top of tower with a velocity 50 m/s making an angle 30° with the horizontal. The height of tower is 70 m. After how many seconds from the instant of throwing will the ball reach the ground? [2014]

a. 2 s b. 5 s c. 7 s d. 9 s

3. A ball thrown upward from the top of a tower with speed v reaches the ground in t_1 sec. If this ball is thrown downward from the top of the same tower with speed v it reaches the ground in t_2 sec. In what time, the ball shall reach the ground, if it is allowed to fall freely under gravity from the top of the tower? [2014]

a. $\frac{t_1 + t_2}{2}$ b. $\frac{t_1 - t_2}{2}$ c. $\sqrt{t_1 t_2}$ d. $t_1 + t_2$

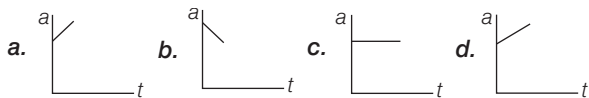
4. A man runs at a speed of 4 m/s to overtake a standing bus. When he is 6 m behind the door at $t = 0$, the bus moves forward and continuous with a constant acceleration of 1.2 m/s^2 . The man reaches the door in time t . Then, [2013]

a. $4t = 6 + 0.6 t^2$ b. $1.2 t^2 = 4t$
c. $4t^2 = 1.2 t$ d. $6 + 4t = 0.2 t^2$

5. For a given velocity, a projectile has the same range R for two angles of projection if t_1 and t_2 are the time of flight in the two cases, then [2012]

a. $t_1 t_2 \propto R$ b. $t_1 t_2 \propto R^2$ c. $t_1 t_2 \propto \frac{1}{R^2}$ d. $t_1 t_2 \propto \frac{1}{R}$

6. A body moves with uniform acceleration, then which of the following graph is correct? [2012]



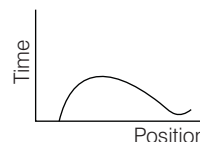
7. One second after the projection, a stone moves at an angle of 45° with the horizontal. Two seconds from the start, it is travelling horizontally, its angle of projection with the horizon is (given, $g = 10 \text{ ms}^{-2}$) [2010]

a. 60° b. $\tan^{-1}(4)$ c. $\tan^{-1}(3)$ d. $\tan^{-1}(2)$

8. A body covers a total distance of $3s$. The first s is covered with a velocity u the second s with v and the last s with ω . Then, the average velocity during the whole journey is [2010]

a. $\frac{u + v + \omega}{3}$ b. $\frac{3uv\omega}{u + v + \omega}$
c. $\frac{3uv\omega}{uv + v\omega + u\omega}$ d. zero

9. Do we observe the time variation of position is nature as shown in the graph? [2009]

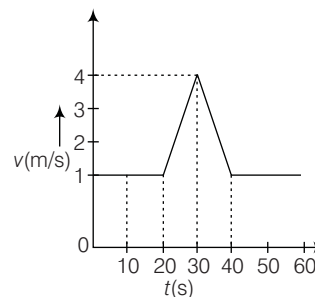


a. Yes b. No c. Often d. Rarely

10. If a body is projected with an angle θ to the horizontal, then [2008]

a. its velocity is always perpendicular to its acceleration
b. its velocity becomes zero at its maximum height
c. its velocity makes zero angle with the horizontal at its maximum height
d. the body just before hitting the ground, the direction of velocity coincides with the acceleration

11. Velocity-time ($v-t$) graph for a moving object is shown in the figure. Total displacement of the object during the time interval when there is non-zero acceleration and retardation is [2007]



a. 60 m b. 50 m c. 30 m d. 40 m

12. At the top of the trajectory of a projectile, the direction of its velocity and acceleration are [2007]

a. perpendicular to each other
b. parallel to each other
c. inclined to each other at an angle of 45°
d. anti-parallel to each other

13. At what point of a projectile motion, acceleration and velocity are perpendicular to each other? [2006]

a. At the point of projection
b. At the point of drop
c. At the top most point
d. Anywhere in between the point of projection and top most point

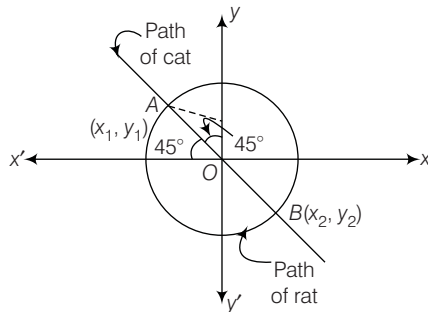
14. From the top of a tower of two stones, whose masses are in the ratio 1 : 2 are thrown on straight up with an initial speed u and the second straight down with the same speed u . Then, neglecting air resistance [2005]

a. the heavier stone hits the ground with a higher speed
b. the lighter stone hits the ground with a higher speed
c. both the stone will have the same speed when they hit the ground
d. the speed cannot be determined with the given data

Answer with Solutions

Practice Exercise

1. (b) The paths of rat and cat are shown in figure. The possible points of catching the rat by the cat are $A(x_1, y_1)$ and $B(x_2, y_2)$. From figure,



$OA = OB = \text{radius of circle} = 2 \text{ unit}$

$$\therefore x_1 = -OA \sin 45^\circ = -\frac{2}{\sqrt{2}} = -\sqrt{2}$$

$$\text{and } y_1 = OA \sin 45^\circ = \sqrt{2}$$

Similarly for point B , $x_2 = \sqrt{2}$, $y_2 = -\sqrt{2}$

Alternate

Let the catching point of rat by cat is $P(x_1, y_1)$.

The coordinates of point P satisfy the both equations.

$$\therefore x_1 + y_1 = 0$$

$$\therefore y_1 = -x_1 \quad \dots(i)$$

$$\text{For rate, } x_1^2 + y_1^2 = 4$$

$$\text{or } x_1^2 + (-x_1)^2 = 4$$

$$\therefore 2x_1^2 = 4$$

$$\therefore x_1 = \pm \sqrt{2}$$

The corresponding values of $y_1 = \mp \sqrt{2}$

$\therefore$ The possible coordinates of point P are $(\sqrt{2}, -\sqrt{2})$ and $(-\sqrt{2}, \sqrt{2})$.

2. (d) Speed is too velocity = $\frac{\text{m/s}}{\text{m/s}} = \text{No unit}$

Similarly, distance is too displacement = $\frac{\text{m}}{\text{m}} = \text{No unit}$

3. (a)

$$4. (a) \text{ Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$\text{Here, } s_1 = 80 \times \frac{t}{2} \text{ and } s_2 = 40 \times \frac{t}{2}$$

$$s_1 = 40t \text{ and } s_2 = 20t$$

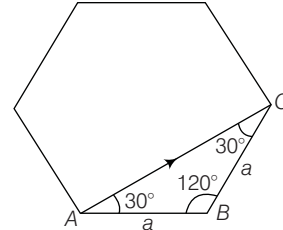
$$\text{Total distance} = s_1 + s_2 = 40t + 20t$$

$$\therefore \text{Total time} = \frac{t}{2} + \frac{t}{2} = t$$

$$\therefore \text{Average speed} = \frac{40t + 20t}{t} = 60 \text{ km/h}$$

5. (c) Average velocity = $\frac{\text{Total displacement}}{\text{Total time}}$

$$\mathbf{v}_{av} = \frac{\mathbf{AC}}{t} \therefore |\mathbf{v}_{av}| = \frac{AC}{t}$$



The time taken by particle to reach from A to C is

$$t = \frac{AB + BC}{v} = \frac{2a}{v}$$

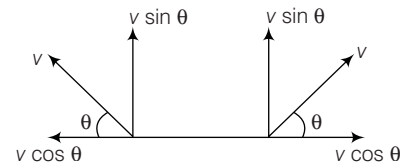
$$\text{In } \triangle ABC, \cos 120^\circ = \frac{a^2 + a^2 - AC^2}{2a^2} \text{ or } -\frac{1}{2} = \frac{2a^2 - AC^2}{2a^2}$$

$$\therefore AC = \sqrt{3}a \therefore |\mathbf{v}_{av}| = \frac{\sqrt{3}a}{\frac{2a}{v}} = \frac{\sqrt{3}}{2}v$$

6. (c) 7. (c)

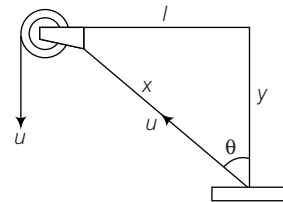
8. (c) From figure of problem, horizontal components of velocities vanish.

Hence, net velocity is $v' = 2v \sin \theta$



9. (b) Here, $\frac{dx}{dt} = u$

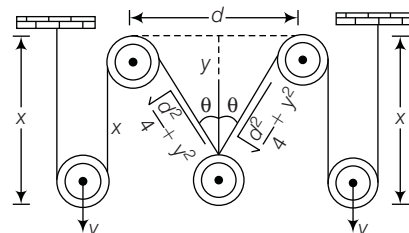
From figure, $l^2 + y^2 = x^2$



$$2y \frac{dy}{dt} = 2x \frac{dx}{dt} \text{ or } \frac{dy}{dt} = \frac{x}{y} u = \frac{u}{\cos \theta} \left(\because \frac{y}{x} = \cos \theta \right)$$

10. (c) The total length of string is

$$l = 2x + 2\sqrt{\left(\frac{d^2}{4} + y^2\right)} + 2x \quad \dots(i)$$



Differentiating of Eq. (i) on both sides with respect to t ,

$$0 = 4 \frac{dx}{dt} + \frac{2y}{\sqrt{\frac{d^2}{4} + y^2}} \frac{dy}{dt}$$

$$\text{or } 4v + 2 \cos \theta \frac{dy}{dt} = 0 \quad \text{or } \frac{dy}{dt} = -\frac{2v}{\cos \theta}$$

11. (c) The situation of problem is shown in figure. Let the length of link AB is l . From figure,

$$x_A^2 + x_B^2 = l^2$$

$$\text{or } 2x_A \frac{dx_A}{dt} + 2x_B \frac{dx_B}{dt} = 0$$

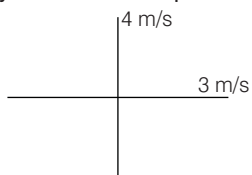
$$\text{or } -x_A v_A + x_B v_B = 0 \quad \left(\because \frac{dx_A}{dt} = -v_A \right)$$

because x_A decreases with respect to time.

$$\therefore v_B = \frac{x_A v_A}{x_B} = v_A \cot 60^\circ$$

$$= \frac{v_A}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3} \text{ m/s}$$

12. (a) The velocity of intersection point is



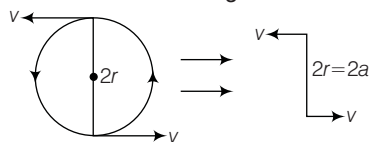
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{3^2 + 4^2} = 5 \text{ m/s}$$

13. (d) The slope of displacement-time graph gives velocity. At the point E in the given graph slope is negative. Hence, instantaneous velocity will be negative corresponding to point E .

14. (b) The slope of $x-t$ graph gives velocity. The tangents at points A and B on the given graph are parallel to X -axis. So, the slopes of the graph at points A and B are zero. Hence, velocities at A and B are zero.

15. (b) Speed never be negative. Hence, (b) is correct.

16. (a) The situation is shown in figure.



The relative angular velocity is

$$\omega_{\text{rel}} = \frac{\left[\begin{array}{c} \text{The relative velocity of component} \\ \text{of velocity in perpendicular direction} \\ \text{of line joining the particles} \end{array} \right]}{\text{The shortest distance between particles}} = \frac{2v}{2a} = \frac{v}{a}$$

17. (c) Let the particle touches the sphere at the point A .

Let $PA = l$

$$\therefore PB = \frac{l}{2}$$

$$\text{In } \triangle OPB, \cos \alpha = \frac{PB}{r}$$

$$\therefore PB = r \cos \alpha$$

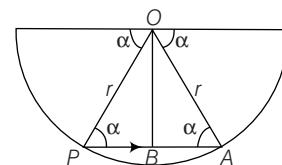
$$\text{or } \frac{l}{2} = r \cos \alpha$$

$$\therefore l = 2r \cos \alpha$$

$$\text{But, } l = \frac{1}{2} a_0 t^2$$

$$\therefore t = \sqrt{\left(\frac{2l}{a_0} \right)} = \sqrt{\left(\frac{2 \times 2r \cos \alpha}{a_0} \right)}$$

$$= \sqrt{\left(\frac{4r \cos \alpha}{a_0} \right)}$$



18. (e) Let the particle returns back after time $t = t_0$.

For this, $v = 0$, at $t = t_0$

$$\therefore v = u + at \quad \text{or } 0 = 2 - 0.1t_0$$

$$\therefore t_0 = \frac{2}{0.1} = 20 \text{ s} \quad \left(\because s = ut + \frac{1}{2} at^2 \right)$$

$$\text{or } 15 = 2t - \frac{1}{2} 0.1t^2 \quad \text{or } 30 = 4t - 0.1t^2$$

$$\text{or } 300 = 40t - t^2 \quad \text{or } t^2 - 40t + 300 = 0$$

$$\therefore t = \frac{40 \pm \sqrt{1600 - 1200}}{2 \times 1} = \frac{40 \pm 20}{2}$$

$$\therefore t = 30 \text{ s or } 10 \text{ s}$$

19. (a)

20. (a) Given, $x = 400 \text{ m}$, $y = 300 \text{ m}$, $z = 1200 \text{ m}$

$$\therefore r = \sqrt{x^2 + y^2 + z^2} = \text{magnitude of displacement}$$

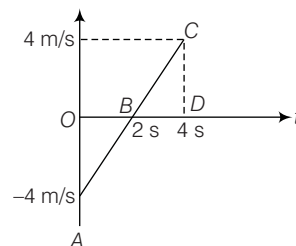
$$= \sqrt{(400)^2 + (300)^2 + (1200)^2}$$

$$= 10^2 \sqrt{16 + 9 + 144} = 1300 \text{ m}$$

21. (b) Here, $x = (t - 2)^2$

$$\text{Velocity } v = \frac{dx}{dt} = 2(t - 2) \text{ m/s}$$

$$\text{Acceleration, } a = \frac{dv}{dt} = 2 \text{ ms}^{-2} \quad (\text{i.e. uniform})$$



When $t = 0$, $v = -4 \text{ m/s}$

$$t = 2 \text{ s}, v = 0$$

$$t = 4 \text{ s}, v = 4 \text{ m/s}$$

Velocity (v) -time (t) graph of this motion is as shown in figure.

Distance travelled = area AOB + area BCD

$$= \frac{4 \times 2}{2} + \frac{4 \times 2}{2} = 8 \text{ m}$$

22. (d) Let the walking distance is l .

$$\therefore v_1 = \text{velocity of person} = \frac{l}{90}$$

$$v_2 = \text{the velocity of escalator} = \frac{l}{60}$$

$\therefore$ The resulting velocity of person on moving escalator is

$$\begin{aligned} v &= v_1 + v_2 \\ \therefore t &= \frac{l}{v_1 + v_2} = \frac{l}{\frac{l}{60} + \frac{l}{90}} \\ t &= \frac{1}{\frac{3}{60} + \frac{2}{90}} = \frac{180}{5} = 36 \text{ s} \end{aligned}$$

23. (a) Here, $s_n = \frac{1}{2}an^2$

s_{nth} = distance travelled in n second – distance travelled in $(n-1)$ second

$$s_{nth} = \left(\frac{2n-1}{2}\right)a \quad \therefore \frac{s_{nth}}{s_n} = \frac{2n-1}{n^2} = \frac{2}{n} - \frac{1}{n^2}$$

24. (b) According to problem, when $s = a$, $t = p$

$$\therefore s = ut + \frac{1}{2}ft^2 \quad [\text{Here, } f = \text{acceleration}]$$

$$\text{or} \quad a = up + fp^2$$

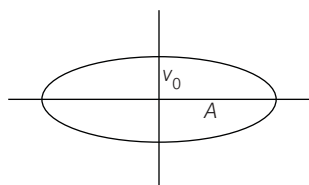
$$\text{or} \quad a = up + \frac{fp^2}{2} \quad \dots(i)$$

For $s = b$, $t = q$

$$b = uq + \frac{fq^2}{2} \quad \dots(ii)$$

After solving Eqs. (i) and (ii), we get $f = \frac{2(aq - bp)}{pq(p - q)}$

25. (a) The corresponding equation is $\frac{x^2}{A^2} + \frac{v^2}{v_0^2} = 1$



$$\text{or} \quad \frac{v^2}{v_0^2} = 1 - \frac{x^2}{A^2} \quad \dots(i)$$

Differentiating of Eq. (i) on both sides with respect to t ,

$$\frac{2v}{v_0^2} \frac{dv}{dt} = -\frac{2x}{A^2} \frac{dx}{dt}$$

$$\text{or} \quad \frac{2v}{v_0^2} \frac{dv}{dt} = -\frac{2x}{A^2} v \quad \text{or} \quad a = -\frac{v_0^2}{A^2} x$$

$$\therefore a_{\max} = -\frac{v_0^2}{A^2} x_{\max}$$

$$\begin{aligned} \therefore a_{\max} &= -\frac{v_0^2}{A^2} A \quad [\because x_{\max} = A] \\ &= -\frac{v_0^2}{A} \quad [\text{In the sense of magnitude}] \end{aligned}$$

26. (a) The slope of distance-time graph gives speed.

The change in velocity in one second

$$= \tan 60^\circ - \tan 45^\circ = \sqrt{3} - 1$$

$$\therefore \text{Acceleration} = \frac{\Delta v}{\Delta t} = \frac{\sqrt{3} - 1}{1} = (\sqrt{3} - 1) \text{ unit}$$

27. (d) The slope of velocity-time graph gives acceleration. Since, the given graph is straight line slope of graph is constant. Hence, acceleration is constant.

The area of given v - t graph between 0 to 10 s is same as between 10 s to 20 s.

28. (b) The area of a - t graph gives change in velocity. The area of the graph from 0 to 8 s

$$= v - u = \frac{1}{2} \times 4 \times 5 + 4 \times 5 = 30$$

But, $u = 0 \therefore v = 30 \text{ m/s}$

29. (b) As, $x_1(t) = \frac{1}{2}at^2$ and $x_2(t) = vt$

$$\therefore x_1 - x_2 = \frac{1}{2}at^2 - vt \quad [\text{parabola}]$$

Clearly, graph (b) represents it correctly.

30. (d) $\because x = (t-2)^2(t-5)$

$$v = \frac{dx}{dt} = (t-5) \frac{d}{dt}(t-2)^2 + (t-2)^2 \frac{d}{dt}(t-5)$$

$$\text{or} \quad v = (t-5)2(t-2) + (t-2)^2 = 2(t-5)(t-2) + (t-2)^2$$

$$\text{At } t = 2 \text{ s, } v = 0$$

$$\text{At } t = 2 \text{ s, } x = 0$$

$$a = \frac{dv}{dt} = \frac{d}{dt} \{2(t-5)(t-2) + (t-2)^2\}$$

$$= 2(t-5) + 2(t-2) + 2(t-2)$$

$$= 2t - 10 + 2t - 4 + 2t - 4 = 6t - 18$$

$$\text{When } t < 3, \quad \text{then } \frac{a+18}{6} < 3 \quad \text{or } a < 0$$

Hence, at $t < 3$, acceleration a will be negative.

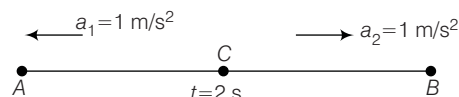
31. (b) $\because v = |t-2| \text{ m/s}$

$$v = t - 2 \quad \text{when } t > 2 \text{ s}$$

$$v = 2 - t, \quad \text{when } t < 2 \text{ s}$$

$$\therefore a = \frac{dv}{dt} = 1 \text{ m/s}^2 \quad t > 2 \text{ s}$$

$$a = -1 \text{ m/s}^2 \quad t < 2$$



In the direction of motion from A to C, bee decelerates but from C to B, bee accelerates.

Let $AC = s_1$ $CB = s_2$

$$u_A = 2 \text{ m/s at } t = 0, u_C = 0 \text{ at } t = 2 \text{ s, } u_B = 2 \text{ m/s at } t = 4 \text{ s}$$

$$\therefore s_1 = \left(\frac{u_A + u_C}{2}\right)t_1, s_2 = \left(\frac{u_C + u_B}{2}\right)t_2$$

$$\therefore s = s_1 + s_2 = \left(\frac{2+0}{2}\right)2 + \left(\frac{0+2}{2}\right)2 = 4 \text{ m}$$

32. (b) As, $v = v_0 + gt + ft^2$ or $\frac{dx}{dt} = v_0 + gt + gt^2$ $\left(\because v = \frac{dx}{dt}\right)$

$$\Rightarrow dx = (v_0 + gt + ft^2)dt$$

$$\text{So, } \int_0^x dx = \int_0^1 (v_0 + gt + ft^2)dt \Rightarrow x = v_0 + \frac{g}{2} + \frac{f}{3}$$

33. (a) Given, $v = \alpha \sqrt{x}$

$$\text{or } \frac{dx}{dt} = \alpha \sqrt{x} \quad \text{or } \frac{dx}{\sqrt{x}} = \alpha dt$$

$$\text{On integrating } \int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha dt$$

[$\therefore$ at $t = 0$, $x = 0$ and let at any time t , particle is at x]

$$\Rightarrow \left. \frac{x^{1/2}}{1/2} \right|_0^x = \alpha t \text{ or } x^{1/2} = \frac{\alpha}{2} t \text{ or } x = \frac{\alpha^2}{4} \times t^2 \text{ or } x \propto t^2$$

34. (b) $a = 3 \sin 4t$

As the acceleration is a sin function, hence it is a periodic function and the acceleration of the particle becomes zero after each interval of $\frac{\pi}{4}$ seconds $\left[a = 3 \sin \left(4 \times \frac{\pi}{4}\right) = 0\right]$

35. (d) The given expression is $x = 3t^2 - t^3$

$$v = \frac{dx}{dt} = 6t - 3t^2, a = \frac{dv}{dt} = 6 - 6t$$

$$\text{At } t = 0, a = 6 \text{ m/s}^2$$

(c) For maximum of $x-t$ curve,

$$\frac{dx}{dt} = 0$$

$$\therefore 6t - 3t^2 = 0 \Rightarrow t = 2 \text{ s}$$

Hence, (c) is correct.

Hence, finally (d) is correct.

36. (b) $\because \frac{dv}{dt} = a - bv$

$$\text{or } \int_0^v \frac{dv}{a - bv} = \int_0^t dt \quad \text{or } -\frac{1}{b} [\log(a - bv)]_0^v = t$$

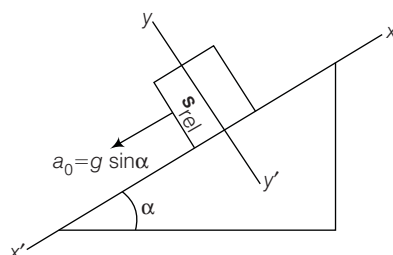
$$\text{or } \left(\frac{a - bv}{a}\right) = e^{-bt} \quad \text{or } 1 - \frac{b}{a}v = e^{-bt}$$

$$\therefore v = \frac{a}{b}(1 - e^{-bt})$$

Tricky approach According to homogeneity principle, the dimensions of left hand side should be equal to dimensions of right hand side. Options (a), (c) and (d) are dimensionally wrong. But, (b) is dimensionally correct.

37. (a) The actual acceleration of bolt

$$= \mathbf{a}_1 = -g \sin \hat{\alpha} \hat{i} - g \cos \hat{\alpha} \hat{j}$$



$$\mathbf{a}_2 = \text{The actual acceleration of box} = -g \sin \hat{\alpha} \hat{i}$$

$$\therefore \mathbf{s}_{\text{rel}} = -l \hat{j}, \mathbf{u}_{\text{rel}} = 0$$

$$\mathbf{s}_{\text{rel}} = \mathbf{u}_{\text{rel}} t + \frac{1}{2} \mathbf{a}_{\text{rel}} t^2$$

$$\text{or } -l \hat{j} = 0 + \frac{1}{2} (\mathbf{a}_1 - \mathbf{a}_2) t^2 \quad \therefore l = \frac{gt^2 \cos \alpha}{2}$$

$$\therefore t = \sqrt{\left(\frac{2l}{g \cos \alpha}\right)}$$

38. (c)

39. (a) The meaning of terminal velocity is constant velocity.

For constant velocity, acceleration should be zero

(i.e. $a = 0$)

$$\therefore a = 3 - 2v \quad \text{or } 0 = 3 - 2v$$

$$\therefore v = \frac{3}{2} = 1.5 \text{ unit}$$

40. (d) Here, $v \propto x^n$

$$\text{or } \frac{dx}{dt} = kx^n$$

$$\therefore \frac{d^2x}{dt^2} = knx^{n-1} \frac{dx}{dt}$$

$$\text{or } \frac{d^2x}{dt^2} = knx^{n-1} kx^n \quad \text{or } \frac{d^2x}{dt^2} = k^2 nx^{2n-1}$$

Since, at $a = 0$, $v = 0$

It means for moving the body, acceleration should be positive. For this

$$\therefore 2n - 1 > 0 \Rightarrow n > \frac{1}{2}$$

If $n < \frac{1}{2}$, particle remains in rest.

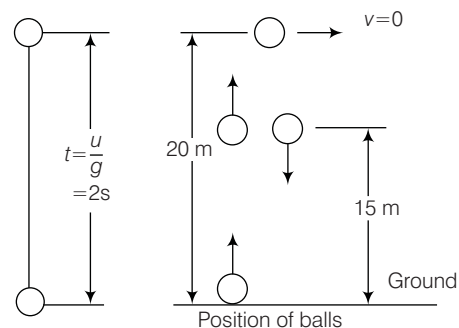
41. (a) At the maximum height, $v = 0$

$$\therefore v = u - gt \Rightarrow 0 = u - gt_0$$

$$\therefore t_0 = \frac{u}{g} = \frac{40}{10} = 4 \text{ s}$$

$$\therefore v = u - gt = 40 - 10(4 - 2) = 20 \text{ m/s}$$

42. (b)



$$h_1 = \frac{1}{2} gt^2 = \frac{1}{2} \times 10 \times 1^2 = 5 \text{ m}$$

$$h_2 = \frac{1}{2} gt^2 = \frac{1}{2} \times 10 \times 2^2 = 20 \text{ m}$$

From ground, 15 m, 20 m, 15 m (shown in figure).

43. (a) Let the time taken by ball to reach at maximum height is t_0 .

At maximum height, $v = 0$

$$\text{Here, } h = \frac{(u+v)t}{2} \quad \text{or } 5 = \frac{u}{2} t_0$$

$$\therefore t_0 = \frac{10}{u} \quad \dots(i)$$

But for upward motion, $v^2 = u^2 - 2gh$

$$\text{or } (0)^2 = u^2 - 2 \times 10 \times 5$$

$$\therefore u = \sqrt{2 \times 10 \times 5} = 10 \text{ m/s}$$

$$\therefore t_0 = \frac{10}{10} = 1 \text{ s}$$

$$\therefore \text{Number of balls per minute} = \frac{t}{t_0} = \frac{1 \text{ min}}{1 \text{ s}} = \frac{60 \text{ s}}{1 \text{ s}} = 60$$

44. (b)

45. (a) For the given condition, initial height $h = d$ and velocity of the ball is zero. When the ball moves downwards its velocity increases and it will be maximum when the ball hits the ground and just after the collision it becomes half and in opposite direction. As the ball moves upward, its velocity again decreases and becomes zero at height $\frac{d}{2}$.

This explanation match with graph (a).

46. (a)

47. (c) In this case, the motion of balloon is undergravity. So, acceleration is constant. Hence, kinematic equation

$$s = ut + \frac{1}{2} at^2$$

is applicable.

$$\text{Here, } s = -65 \hat{j}$$

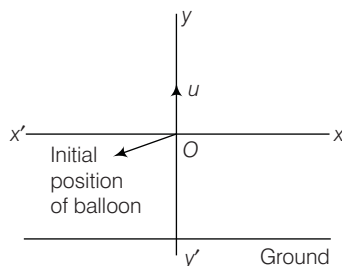
$$a = g = -10 \hat{j}$$

$$u = 12 \hat{j}$$

$$\therefore -65 \hat{j} = 12t - \frac{1}{2} 10t^2 \hat{j}$$

$$\text{or } -65 = 12t - 5t^2$$

$$\therefore t = 5 \text{ s}$$



Remarks In this case, magnitude of displacement is not equal to distance.

48. (a) According to kinematic equation,

$$v = v_0 - gt$$

Upward direction is taken as positive and downward direction is taken as negative. Hence, $v-t$ graph is straight line having negative slope.

49. (b) 50. (b) 51. (d) 52. (d)

53. (d) Given, velocity $v = kY \hat{i} + kX \hat{j}$

$$\frac{dX}{dt} = kY, \frac{dY}{dt} = kX$$

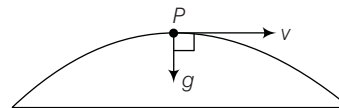
$$\frac{dY}{dX} = \frac{dY}{dt} \times \frac{dt}{dX} = \frac{kX}{kY}$$

$$Y dY = X dX$$

$$\Rightarrow Y^2 = X^2 + C \quad (\text{where, } C = \text{constant})$$

54. (b)

55. (d) The trajectory is shown in figure. From figure, at the top, velocity and gravitational acceleration are mutually perpendicular to each other.



56. (d) The maximum height is $H = \frac{u^2 \sin^2 \theta}{2g}$

$$\text{or } 16 = \frac{u^2 \sin^2 45^\circ}{19.6}$$

$$\therefore u = \sqrt{16 \times 2 \times 19.6}$$

$$\begin{aligned} \text{But, } R &= \frac{u^2 \sin 2\theta}{g} \\ &= \frac{16 \times 2 \times 19.6 \sin 90^\circ}{9.8} = 64 \text{ m} \end{aligned}$$

When $\theta = 45^\circ$, $\sin 2\theta = 1$

But, when $\theta = 30^\circ$,

$$\sin 2\theta = \frac{\sqrt{3}}{2} = 0.866$$

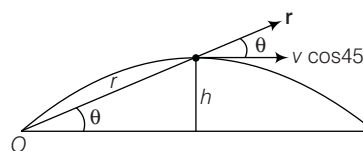
So, at $\theta = 30^\circ$, range is lesser than that of at $\theta = 45^\circ$

57. (d) When the stone reaches at ground. The potential energy of stone decreases. This decrease in potential energy of stone. Hence, speed increases. Hence, (d) is correct.

58. (b) The angular momentum of a particle is given by

$$L = r \times mv$$

$$\therefore L = mvr \sin \theta$$



From figure, $L = rm(v \cos 45^\circ) \sin \theta = \frac{mv}{\sqrt{2}} (r \sin \theta)$

$$= \frac{mvh}{\sqrt{2}} \quad \left(\because \sin \theta = \frac{h}{r} \right)$$

59. (b) In the absence of air,

$$t = \text{time of flight} = 2 \times \text{time of ascent} = 2 \times \frac{u}{g} = \frac{2u}{g}$$

$$\therefore t_1 = \frac{2u}{g_1} \quad \dots(i)$$

$$\text{and } t_2 = \frac{2u}{g_2} \quad \dots(ii)$$

Now, from Eqs. (i) and (ii), we get

$$\therefore t_1 g_1 = t_2 g_2$$

60. (d) Given, $s = t^3 + 5$

$$\therefore \text{Speed, } v = \frac{ds}{dt} = 3t^2$$

and rate of change of speed $a_t = \frac{dv}{dt} = 6t$

$\therefore$ Tangential acceleration at $t = 2$ s

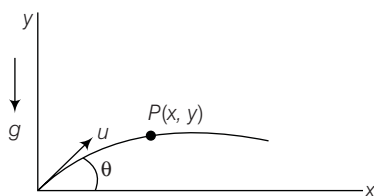
$$a_t = 6 \times 2 = 12 \text{ ms}^{-2}$$

and at $t = 2$ s, $v = 3(2)^2 = 12 \text{ ms}^{-1}$

$\therefore$ Centripetal acceleration, $a_c = \frac{v^2}{R} = \frac{144}{20} \text{ ms}^{-2}$

Net acceleration $= \sqrt{a_c^2 + a_t^2} = 14 \text{ ms}^{-2}$

61. (b) Let a particle is projected at an angle θ with horizontal (shown in figure)



At an instant t , the particle reaches at point $P(x, y)$.

$$\therefore x = (u \cos \theta)t$$

$$\therefore ut \cos \theta = x \quad \dots(i)$$

and $y = (u \sin \theta)t - \frac{1}{2}gt^2$

$$\therefore y = (u \sin \theta)t - \frac{1}{2}gt^2$$

$$\therefore ut \sin \theta = y + \frac{1}{2}gt^2 \quad \dots(ii)$$

Squaring Eqs. (i) and (ii) and then adding, we get

$$(ut \sin \theta)^2 + (ut \cos \theta)^2 = x^2 + \left(y + \frac{1}{2}gt^2\right)^2$$

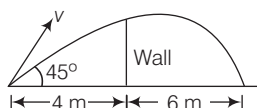
$$\therefore u^2 t^2 = x^2 + y^2 + \frac{g^2 t^4}{4} + gt^2 y$$

$$x^2 + y^2 + \frac{g^2 t^4}{4} + gt^2 y - u^2 t^2 = 0$$

This is an equation of circle.

62. (a) The rate of storing water in vessel depends upon vertical component of velocity. In this problem, vertical component of velocity in both cases are same.

63. (c) As range $= 10 = \frac{u^2 \sin 2\theta}{g} \Rightarrow u^2 = 10g$



$\therefore u = 10 \text{ m/s}$ (as $g = 10 \text{ m/s}^2$)

$$Y = x \tan \theta - \frac{1}{2} \frac{gx^2}{v_0^2 \cos^2 \theta} = 4 \tan 45^\circ - \frac{1}{2} \frac{g \times 16}{2 \times 10^2 \cos^2 45^\circ}$$

$$= 4 \times 1 - \frac{1}{2} \frac{10 \times 16}{2 \times 10 \times 10 \times \frac{1}{2}} = 4 - 0.8 = 3.2 \text{ m}$$

64. (b)

65. (d)

66. (b) Potential energy PE

$$= mgh_{\max} = \frac{mg(u^2 \sin^2 \theta)}{2g} = \frac{mu^2 \sin^2 \theta}{2}$$

$$\text{KE} = \frac{1}{2} m(u \cos \theta)^2$$

$$\text{Hence, } \frac{\text{PE}}{\text{KE}} = \tan^2 \theta$$

67. (a) From review of concepts of the chapter

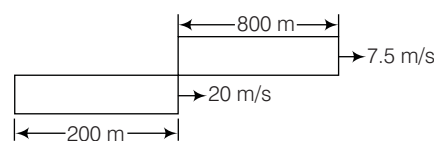
$$\text{Latusrectum} = \frac{2a^2 \cos^2 \alpha}{g} = \frac{2 \times 10^2 \times \cos^2 60^\circ}{10} = 5 \text{ m}$$

68. (c) We can write $\frac{R/2}{H} = \frac{\sqrt{3}H}{H} = \sqrt{3}$

$$\text{or } \frac{(v_0^2 \sin \theta \cos \theta)/g}{(v_0^2 \sin^2 \theta)/2g} = \sqrt{3}$$

$$2 \cot \theta = \sqrt{3} \Rightarrow \tan \theta = \frac{2}{\sqrt{3}}, \theta = \tan^{-1} \left(\frac{2}{\sqrt{3}} \right)$$

69. (d) From the figure, the relative displacement is



$$s_{\text{rel}} = (800 + 200) \text{ m} = 1000 \text{ m}$$

$$v_{\text{rel}} = v_1 - v_2 = (20 - 7.5) \text{ m/s} = 12.5 \text{ m/s}$$

$$\therefore t = \frac{s_{\text{rel}}}{v_{\text{rel}}} = \frac{1000}{12.5} = 80 \text{ s}$$

70. (b) Let the distance between spots A and B is l .

u = speed of river stream

v = speed of motorboat in still water

During downstream,

$$t_1 = \frac{l}{v + u} \quad \dots(i)$$

Similarly in upstream

$$t_2 = \frac{l}{v - u} \quad \dots(ii)$$

$$\text{In still water, } t = \frac{l}{v} \quad \dots(iii)$$

After solving Eqs. (i), (ii) and (iii), we get $t = 9.6 \text{ h}$

71. (d) For the man to cross the river from the shortest path,

$$V_m \sin \theta = v_r$$

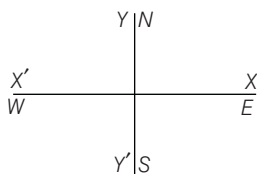
$$\sin \theta = \frac{v_r}{v_m}$$

$$\sin \theta = \frac{4}{3}$$

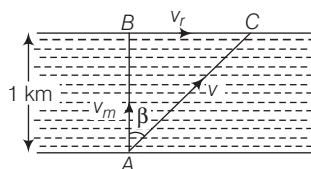
$$\theta = \sin^{-1} \left(\frac{4}{3} \right)$$

But it is not possible, because $\sin \theta$ could not be greater than 1. Hence, the man could not cross the river from the shortest path AB.

72. (b) Let
- $\mathbf{v}_t$
- = actual velocity of train

 $\mathbf{v}_c$ = actual velocity of car $\therefore \mathbf{v}_{tc}$ = relative velocity of train with respect to car $\therefore \mathbf{v}_c = 25\hat{i}$ and $\mathbf{v}_{tc} = 25\sqrt{3}\hat{j}$ $\therefore \mathbf{v}_{tc} = \mathbf{v}_t - \mathbf{v}_c$ $\therefore \mathbf{v}_t = \mathbf{v}_{tc} + \mathbf{v}_c = 25\sqrt{3}\hat{j} + 25\hat{i}$ $\therefore v_t = \sqrt{(25\sqrt{3})^2 + (25)^2} = 25\sqrt{4} = 50 \text{ km/h}$

73. (d) Given, speed of man
- $v_m = 4 \text{ km/h}$

Speed of river $v_r = 3 \text{ km/h}$ Width of the river, $d = 1 \text{ km}$

Time taken by the man to cross the river,

$$t = \frac{\text{Width of the river}}{\text{Speed of the man}} = \frac{1 \text{ km}}{4 \text{ km/h}} = \frac{1}{4} \text{ h} = \frac{1}{4} \times 60 = 15 \text{ min}$$

Distance travelled along the river $= v_r \times t$

$$= 3 \times \frac{1}{4} = \frac{3}{4} \text{ km} = \frac{3000}{4} = 750 \text{ m}$$

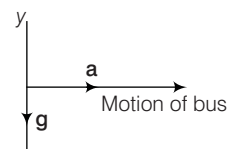
74. (a) $\tan(90^\circ - \theta) = \frac{20}{15}$

$$\therefore \cot \theta = \frac{20}{15} = \frac{4}{3}$$

$$\Rightarrow \theta = 37^\circ$$

$$\therefore \alpha = 37^\circ + 23^\circ = 60^\circ$$

75. (c) Let
- $\mathbf{a}_{\text{rel}}$
- = acceleration of ball with respect to ground – acceleration of bus with respect to ground



$$= -g\hat{j} - a\hat{i}$$

$$\therefore |\mathbf{a}_{\text{rel}}| = \sqrt{g^2 + a^2}$$

BITSAT Archives

1. (b) (i)
- $v = u + at_1$

$$40 = 0 + a \times 20$$

$$a = 2 \text{ m/s}^2$$

$$v^2 - u^2 = 2as$$

$$40^2 - 0 = 2 \times 2s_1$$

$$\therefore s_1 = 400 \text{ m}$$

$$\text{(ii) } s_2 = v \times t_2 = 40 \times 20 = 800 \text{ m}$$

$$\text{(iii) } v = u + at$$

$$0 = 40 + a \times 40$$

$$\therefore a = -1 \text{ m/s}^2$$

$$0^2 - 40^2 = 2(-1)s_3$$

$$\therefore s_3 = 800 \text{ m}$$

$$\text{Total distance travelled} = s_1 + s_2 + s_3$$

$$= 400 + 800 + 800 = 2000 \text{ m}$$

$$\text{Total time taken} = 20 + 20 + 40 = 80 \text{ s}$$

$$\therefore \text{Average velocity} = \frac{2000}{80}$$

$$= 25 \text{ m/s}$$

2. (c) Taking vertical downward motion of projectile from point of projection to ground, we have

$$u = -50 \sin 30^\circ = -25 \text{ m/s}$$

$$a = +10 \text{ m/s}^2, s = 70 \text{ m}, t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$\text{So, } 70 = -25 \times t + \frac{1}{2} \times 10 \times t^2$$

$$\text{or } 5t^2 - 25t - 70 = 0 \text{ or } t^2 - 5t - 14 = 0$$

$$\text{On solving } = 7 \text{ s}$$

3. (c) $h = -vt_1 + \frac{1}{2}gt_1^2$ or $\frac{h}{t_1} = -v + \frac{1}{2}gt_1$... (i)

$$h = vt_2 + \frac{1}{2}gt_2^2 \text{ or } \frac{-h}{t_2} = -v + \frac{1}{2}gt_2 \text{ ... (ii)}$$

$$\therefore \frac{h}{t_1} + \frac{h}{t_2} = \frac{1}{2}g(t_1 + t_2)$$

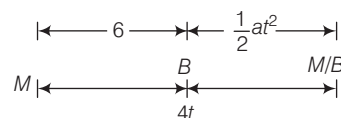
$$\text{or } h = \frac{1}{2}gt_1t_2$$

For falling under gravity from the top of the tower

$$h = \frac{1}{2}gt^2$$

$$\therefore \frac{1}{2}gt_1t_2 = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{t_1t_2}$$

4. (a) Let us draw the figure for given situation,



$$\Rightarrow 4t = 6 + \frac{1}{2} \times 12 \times t^2$$

$$\Rightarrow 4t = 6 + 0.6t^2$$

$$5. (a) \quad t_1 = \frac{2u \sin \alpha}{g}$$

$$t_2 = \frac{2u \sin(90^\circ - \alpha)}{g}$$

$$\text{So,} \quad t_1 \times t_2 = 2 \frac{u^2}{g^2} \sin 2\alpha$$

$$\text{or} \quad t_1 \times t_2 = \frac{2R}{g} \quad \left(\because R = \frac{u^2 \sin 2\alpha}{g} \right)$$

$$t_1 \times t_2 \propto R$$

6. (c) An object is said to be moving with a uniform acceleration, if its velocity changes by equal amount in equal intervals of time. The velocity-time graph of uniformly accelerated motion is a straight line inclined to time axis. Acceleration of an object in a uniformly accelerated motion in one dimension is equal to the slope of the velocity-time graph with time axis.

7. (d) When stone travels horizontally, then it must be at the maximum height.

$$\text{So,} \quad 2 = \frac{u \sin \theta}{g}$$

$$\Rightarrow \quad u \sin \theta = 20 \quad \dots(i)$$

$$\text{Also,} \quad \tan 45^\circ = \frac{u \sin \theta - g(1)}{u \cos \theta}$$

$$\Rightarrow \quad u \cos \theta = 20 - 10$$

$$\Rightarrow \quad u \cos \theta = 10 \quad \dots(ii)$$

Squaring of Eqs. (i) and (ii) and adding, we get

$$u = 10\sqrt{5} \text{ ms}^{-1} \text{ and } \tan \theta = 2$$

$$\Rightarrow \quad \theta = \tan^{-1}(2)$$

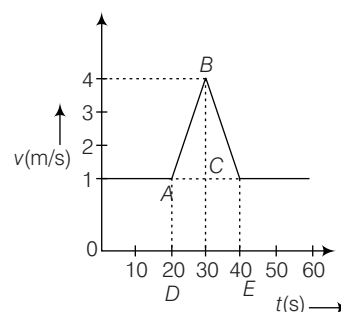
8. (c) Average velocity $v_{av} = \frac{\text{Total distance}}{\text{Total time}}$

$$\Rightarrow \quad v_{av} = \frac{s + s + s}{\frac{s}{u} + \frac{s}{v} + \frac{s}{\omega}} \Rightarrow v_{av} = \frac{3uv\omega}{u\omega + v\omega + u\omega}$$

9. (b) As x increases, time first increases and then decreases. This is not possible.

10. (c) Direction of velocity is always tangent to the path so at the top of trajectory, it is in horizontal direction.

11. (b) Total displacement of the object = Area (DABCE)
= Area of $\triangle ABC$ + Area of rectangle (ACED)



$$= \left[\frac{1}{2} \times (40 - 20) \times (4 - 1) \right] + [(40 - 20) \times 1]$$

$$= \frac{1}{2} \times 20 \times 3 + 20$$

$$= 30 + 20$$

$$= 50 \text{ m}$$

12. (a) Direction of velocity is always tangent to the path, so at the top of trajectory it is in horizontal direction and acceleration due to gravity is always in vertically downward direction.

Hence, $\mathbf{v}$ and $\mathbf{g}$ are perpendicular to each other.

13. (c) At the top most point of the projectile there is only horizontal component of velocity and acceleration due to gravity is vertically downward, so velocity and acceleration are perpendicular to each other.

14. (c)

4

Newton's Laws of Motion and Friction

Force

Force is an external effort in the form of push or pull which

- (i) generates or tends to generate motion in a body at rest,
- (ii) stops or tends to stop a body in motion,
- (iii) increases or decreases the magnitude of velocity of the moving body,
- (iv) changes the direction of motion of the body.

Classification of Forces

Based on the nature of interaction between two bodies, forces may be broadly classified as under

- (i) **Field forces** are those forces that act between two bodies separated by a distance without any actual contact. Gravitational force between two bodies and electrostatic force between two charges are the examples of field forces.
- (ii) **Contact forces** are those forces that act between two bodies in contact.
e.g. tension, normal reaction, friction, etc.

Inertia

It is an inherent property of all bodies, by virtue of which they cannot change by themselves their state of rest or of uniform motion along a straight line.

As inertia of a body is measured by the mass of the body. So, heavier the body, greater is the force required to change its state.

Newton's Laws of Motion

There are three laws of motion stated by Newton. These laws describe the relationship between a body and the forces acting upon it and its motion in response to said forces. Newton's laws are not valid in the non-inertial frames, they have to be modified by introducing the concept of pseudo force.

First Law or Law of Inertia

If the net force ΣF exerted on an object is zero, then the object continues in its original state of motion (or rest). That is, if $\Sigma F = 0$, an object at rest remains at rest and or object moving with constant velocity. This is **Newton's first law**.

Linear Momentum

The linear momentum of a body is defined as a product of mass and velocity of the body, i.e.

$$\mathbf{p} = m\mathbf{v}$$

Momentum is a vector quantity.

A body at rest cannot possess linear momentum and a moving body always possesses linear momentum.

Second Law

The acceleration of an object is directly proportional to the net force acting on it and is inversely proportional to its mass.

$$a = \frac{\Sigma F}{m}$$

This is the **Newton's second law**.

The above equation in vector form is $\Sigma \mathbf{F} = m\mathbf{a}$,

where $\mathbf{a}$ is the acceleration of the object, m is its mass and $\Sigma \mathbf{F}$ represents the vector sum of all the forces acting on the object.

It also states that the time rate of change of momentum of body is equal to the net external force exerted on that body

i.e.
$$\mathbf{F}_{\text{net}} = \frac{d\mathbf{p}}{dt}$$

If $F_1, F_2, F_3 \dots$ are the concurrent forces acting at the same point, then the point will be in equilibrium, if

$$F_1 + F_2 + F_3 \dots = 0$$

Impulse

- When a force of large magnitude acts on an object for a small time interval, the force is called impulsive force. In such cases, we measure the total effect of force.

- Impulse of a force is a measure of the total effect of force. Mathematically, impulse $\mathbf{J} = \mathbf{F}_{\text{av}} dt = \int_0^{\Delta t} \mathbf{F} dt$

- For a force-time ($F-t$) graph, the area under the graph gives the value of the impulse. As, $\mathbf{F} = \frac{d\mathbf{p}}{dt}$.

- Hence, impulse $\mathbf{J} = \int_0^{\Delta t} \mathbf{F} dt$
 $= \int_0^{\Delta t} \frac{d\mathbf{p}}{dt} dt = \int_{p_i}^{p_f} d\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i$. This relation is known as the **impulse-momentum theorem**.

Third Law

If two objects interact, the force F_{12} which the object 1 exerts on object 2 is equal in magnitude but opposite in direction to the force F_{21} which the object 2 exerts on object 1. This is **Newton's third law**, i.e. $F_{12} = -F_{21}$

Conservation of Linear Momentum

According to the principle of conservation of linear momentum, if there is no external force acting on a system, then total momentum of the system remains constant.

According to second law of motion,

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

If no force is acting, then $\mathbf{F} = 0$.

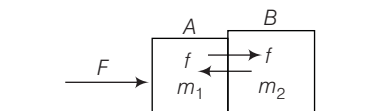
So,
$$\frac{d\mathbf{p}}{dt} = 0 \Rightarrow \mathbf{p} = \text{constant}$$

$$m_1 v_1 = m_2 v_2 = \text{constant}$$

Connected Motion

The connected motion describes the relative motion between two objects whether it is between two blocks and a plane or it is between a block and an inclined plane.

- If two blocks of masses m_1 and m_2 are placed on a perfectly smooth surface and are in contact, then

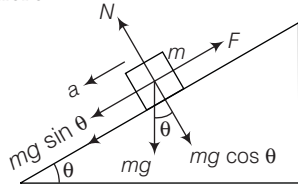


Acceleration of the blocks,
$$a = \frac{F}{m_1 + m_2}$$

and the contact force (acting normally) between the two blocks is
$$f = m_2 a = \frac{F m_2}{(m_1 + m_2)}$$

- For a block of mass m placed on a fixed, perfectly smooth inclined plane of angle θ , the forces acting on the block are shown in the figure.

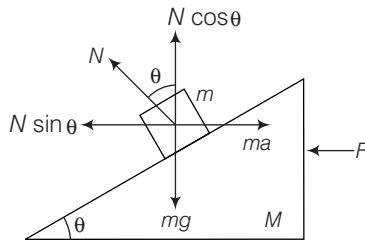
Obviously, here



$$\Rightarrow a = g \sin \theta$$

3. If a block of mass m is placed on a smooth movable wedge of mass M , which in turn is placed on smooth surface, then a force F is applied on the wedge, horizontally.

The acceleration of the wedge and the block is



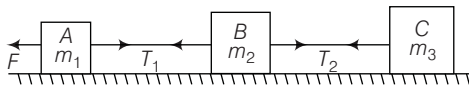
$$a = \frac{F}{(M + m)}$$

Force on the block, $F = (M + m)a$

$$= (M + m)g \tan \theta$$

4. For a block system shown in the figure, acceleration of the system

$$a = \frac{F}{m_1 + m_2 + m_3}$$



Tension in the string

$$T_1 = (m_2 + m_3)a = \frac{F(m_2 + m_3)}{(m_1 + m_2 + m_3)}$$

and tension $T_2 = m_3a = \frac{Fm_3}{m_1 + m_2 + m_3}$

5. For a block system suspended freely from a rigid support as shown in the figure, the acceleration of the system $a = 0$.

Tension in the string,

$$T_1 = (m_1 + m_2 + m_3)g$$

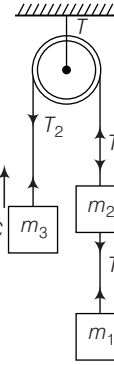
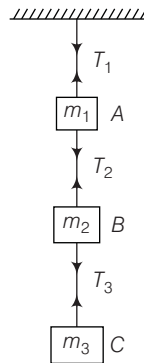
$$T_2 = (m_2 + m_3)g$$

and

$$T_3 = m_3g$$

6. For a block system and a pulley shown in the figure, value of the acceleration of the system

$$a = \frac{(m_1 + m_2 - m_3)g}{(m_1 + m_2 + m_3)}$$

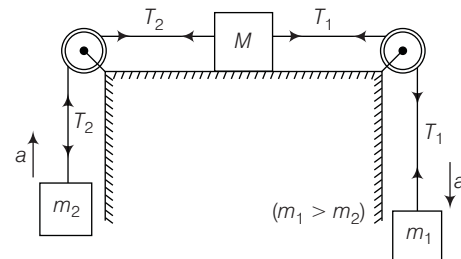


$$\text{Tension, } T_1 = \frac{2m_1m_3g}{(m_1 + m_2 + m_3)}$$

$$\text{Tension, } T_2 = \frac{2m_3(m_1 + m_2)g}{(m_1 + m_2 + m_3)} \text{ and}$$

$$\text{Tension, } T = 2T_2 = \frac{4m_3(m_1 + m_2)g}{(m_1 + m_2 + m_3)}$$

7. For the pulley and block arrangement shown in the figure, we have



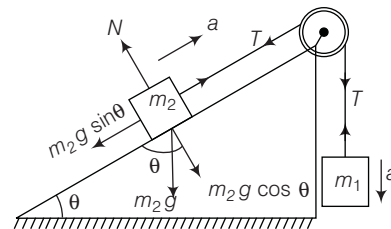
Net acceleration

$$a = \frac{\text{Net accelerating force}}{\text{Total mass}} = \frac{(m_1 - m_2)g}{(m_1 + m_2 + M)}$$

$$\text{Tension, } T_1 = m_1(g - a) = \frac{(M + 2m_2)m_1g}{(M + m_1 + m_2)}$$

$$\text{and Tension, } T_2 = m_2(g + a) = \frac{(M + 2m_1)m_2g}{(M + m_1 + m_2)}$$

8. For the system of block and pulley, with a smooth inclined plane shown in the figure, we have



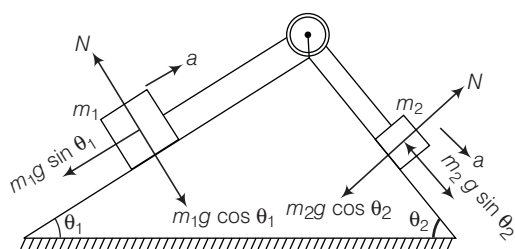
$$\text{Net acceleration, } a = \frac{(m_1 - m_2 \sin \theta)g}{(m_1 + m_2)}$$

If $m_1 g > m_2 g \sin \theta$
 and $a = \frac{(m_2 \sin \theta - m_1)g}{m_1 + m_2},$

If $m_1 g < m_2 g \sin \theta$
 and tension in the string

$$T = m_1(g - a) = \frac{m_1 m_2 (1 + \sin \theta)}{(m_1 + m_2)}$$

9. For a pulley and block system on a smooth double inclined plane as shown in the figure, we have



Net acceleration

$$a = \frac{(m_1 \sin \theta_1 - m_2 \sin \theta_2)g}{(m_1 + m_2)},$$

for $\theta_1 > \theta_2, m_1 > m_2$
 and tension in the string

$$T = m_1(g \sin \theta_1 - a) = \frac{m_1 m_2 (\sin \theta_1 + \sin \theta_2)g}{(m_1 + m_2)}$$

Lami's Theorem

For three concurrent forces in equilibrium position. If three forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other. Thus, if the forces are P, Q and R ; α, β, γ be the angles between Q and R, R and P, P and Q respectively, also the forces are in equilibrium, we have

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$

Friction

The opposing force which comes into play when a body moves or tries to move over the surface of another body, is known as friction. Force of friction acts tangentially to the surface in contact. e.g. A ball rolling over the floor stops after sometime.

Types of Frictions

Force of friction can be classified into following types

Laws of Friction

1. Limiting friction is directly proportional to the normal reaction (R), i.e. $f_s \propto R$.
2. It acts opposite to the direction of body which is at the verge of moving over the other.
3. It depends upon the nature of the material and nature of the surface in contact.

Coefficient of Friction (μ)

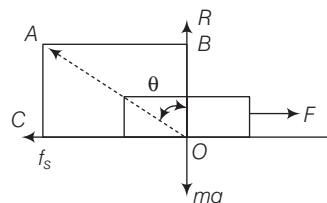
It is defined as the division of the force of limiting friction (f_s) and normal reaction (R) between them.

i.e.
$$\mu = \frac{f_s}{R}$$

So, value of μ depends on (a) nature of the surface in contact i.e. dry or wet; rough or smooth, (b) material of the surface in contact.

Angle of Friction (θ)

It is defined as the angle between the normal reaction (R) and the resultant of the frictional force (f_s) and reaction R , as shown in figure.



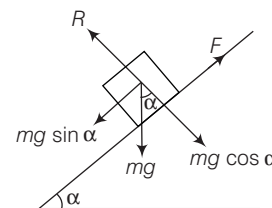
In $\triangle AOB$,

$$\tan \theta = \frac{AB}{OB} = \frac{f_s}{R} = \frac{\mu_s R}{R} \text{ or } \tan \theta = \mu_s \Rightarrow \theta = \tan^{-1}(\mu_s)$$

i.e. coefficient of limiting friction between any two surfaces in contact is equal to tangent of the angle of friction between them.

Angle of Repose (α)

The minimum angle of inclination of a plane with the horizontal, such that a body placed on the plane just begins to slide down the inclined plane is known as the angle of repose or angle of sliding. From figure, we have



$$\mu mg \sin \alpha = F = \text{Force of limiting friction}$$

$$mg \cos \alpha = R = \text{Force of normal reaction}$$

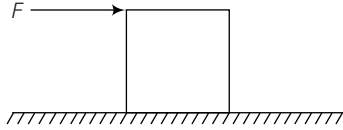
So,
$$\frac{\mu mg \sin \alpha}{mg \cos \alpha} = \frac{F}{R}$$

$$\Rightarrow \tan \alpha = \frac{F}{R} = \frac{\mu R}{R} = \mu \Rightarrow \alpha = \tan^{-1}(\mu) \text{ or } \alpha = \theta$$

i.e. Angle of repose = Angle of friction

Some Important Points

- (i) When F increases, normal reaction shifts from centre of mass to right. At the time of toppling, normal reaction acts through point A . Also, the net force on the body gives acceleration of the centre of mass.



- (ii) If the body is in rest with respect to the surface, then $f_r < \mu_s N$.
 (iii) If the body is just in motion, then $f_r = \mu_s N$
 (iv) If the body is in motion, then $f_r = \mu_k N$
 (v) If some bodies have same accelerations, then they are taken as a system. If they do not move together, bodies are not taken as a system.
 (vi) The force of friction during pushing is greater than that of pulling in the same manner.
 (vii) It is misconception to say that friction always opposes the motion of the body. It only opposes the relative motion between surfaces.

$$mg - T = ma$$

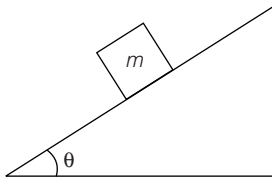
- (viii) Gravitational force : Electromagnetic force : Strong force : Weak force = $1 : 10^{36} : 10^{38} : 10^{25}$

- (ix) For the equilibrium of a body on an inclined plane,

$$mg \sin \theta = \mu_s mg \cos \theta$$

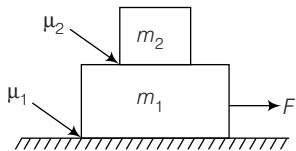
$$\Rightarrow \mu_s = \tan \theta$$

In this case, $0 < \mu < 1$



- (x) The force acting on m_2 is $f_2 = \mu_2 m_2 g$.

If the system moves with the common acceleration, then



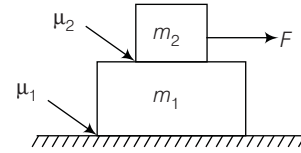
$$F - \mu_1(m_1 + m_2)g = (m_1 + m_2)a$$

$$f_2 = m_2 a$$

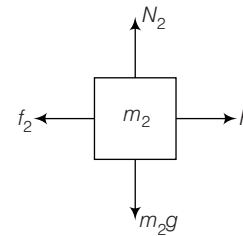
$$\mu_2 m_2 g = m_2 a$$

$$\therefore a = \mu_2 g$$

- (xi) If force is applied on upper block, $f_2 =$ limiting friction between m_1 and $m_2 = \mu_2 m_2 g$
 $f_1 =$ limiting friction between the surface and $m_1 = \mu_1(m_1 + m_2)g$



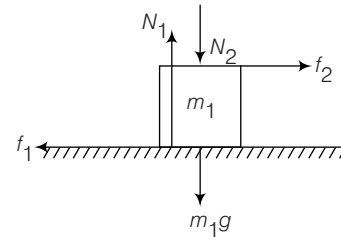
If $F > f_1$, then both blocks move with different acceleration and the maximum friction acts between the blocks. $F - f_2 = m_2 a_2$



$$F - \mu_2 m_2 g = m_2 a_2$$

$$N_2 = m_2 g$$

$$N_1 = N_2 + m_1 g = (m_1 + m_2)g$$



$$\therefore f_1 = \mu_1 N_1 = \mu_1(m_1 + m_2)g$$

$$f_2 = \mu_2 m_2 g$$

If $f_2 < f_1$, then m_1 remains in rest.

If $f_2 > f_1$, then m_1 moves in the direction of f_2 .

$$f_2 - f_1 = m_1 a_1$$

If $F < f_2$, then no relation is found between m_1 and m_2 . i.e. m_1 and m_2 move together.

If $F < f_1$, then the system is in rest.

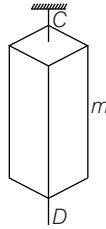
If $F > f_1$, the system moves with the common acceleration a . In this case,

$$F - f_1 = (m_1 + m_2)a$$

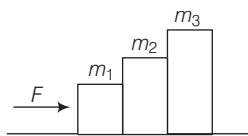
$$\text{or } F - \mu_1(m_1 + m_2)g = (m_1 + m_2)a$$

Practice Exercise

- Two bodies have same mass and speed, then
 - their momentums are same
 - the ratio of momentums is not determined
 - the ratio of their magnitudes of momentum is one
 - Both a and b are correct
- In the superhit film 'Raja Hindustani' Amir khan greets his beloved by shaking hand, What kind of force do they exert?
 - Nuclear
 - Gravitational
 - Weak
 - Electromagnetic
- A heavy block of mass m is supported by a cord C attached to the ceiling, and another cord D is attached to the bottom of the block. If a sudden jerk is given to D , then
 - cord C breaks
 - cord D breaks
 - cord C and D both break
 - None of the cords breaks

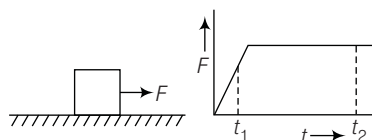


- At time t second, a particle of mass 3 kg has position vector $\mathbf{r}$ metre, where $\mathbf{r} = 3t\hat{i} - 4\cos t\hat{j}$. Find the impulse of the force during the time interval $0 \leq t \leq \frac{\pi}{2}$
 - $12\hat{j}$ N-s
 - $9\hat{j}$ N-s
 - $4\hat{j}$ N-s
 - $14\hat{j}$ N-s
- Three blocks of masses $m_1 = 1\text{ kg}$, $m_2 = 2\text{ kg}$ and $m_3 = 3\text{ kg}$ are placed in contact on a horizontal frictionless plane as shown in figure. A force of 12 N is applied on m_1 . Acceleration of the system is
 - 12 m/s^2
 - 2 m/s^2
 - 6 m/s^2
 - 4 m/s^2
- A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that
 - its velocity is constant
 - its kinetic energy is constant
 - it moves in a circular path
 - Both b and c are correct
- A particle of mass m moves on the x -axis as follows. It starts from rest at $t = 0$ from the point $x = 0$ and comes to rest at $t = 1$ at the point $x = 1$. No other information is available about its motion at intermediate time ($0 < t < 1$). If α denotes the instantaneous acceleration of the particle, then
 - α cannot remain positive for all t in the interval $0 \leq t \leq 1$
 - $|\alpha|$ cannot exceed 2 at any point in its path
 - $|\alpha|$ be ≥ 4 at point or some points in its path
 - Both a and c are correct

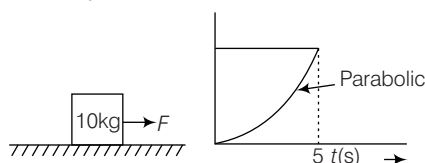


- A 0.1 kg body moves at a constant speed of 10 m/s. It is pushed by applying a constant force for 2 s. Due to this force, it starts moving exactly in the opposite direction with a speed of 4 m/s. Then,
 - the deceleration of the body is 7 m/s^2
 - the magnitude of change in momentum is 1.4 kg-m/s
 - impulse of the force is 1.4 N-s
 - the force which acts on the ball is 0.7 N
 - All of the above
- Water jet issues water from a nozzle of 2 cm^2 cross-section with velocity 30 cm/s and strikes a plane surface placed at right angles to the jet. The force exerted on the plane is
 - 200 dyne
 - 400 dyne
 - 1800 dyne
 - None of the above
- The action and reaction forces referred to Newton's third law of motion
 - must act upon the same bodies
 - must act upon different bodies
 - need not to be equal in magnitude but must have the same line of action
 - must be equal in magnitude but need not have the same line of action
- A man is pulling a rope attached to a block on a smooth horizontal table. The tension in the rope will be the same at all points
 - if and only if the rope is not accelerated
 - if and only if the rope is massless
 - if either the rope is not accelerated or is massless
 - always
- A particle of mass m moves on the x -axis under the influence of a force of attraction towards the origin O given by $F = -\frac{k}{x^2}\hat{i}$. If the particle starts from rest at $x = a$. The speed of it will attain to reach at distance x from the origin O will be
 - $\sqrt{\frac{2k}{m}} \left[\frac{x-a}{ax} \right]^{1/2}$
 - $\sqrt{\frac{2k}{m}} \left[\frac{a+x}{ax} \right]^{1/2}$
 - $\sqrt{\frac{k}{m}} \left[\frac{ax}{x-a} \right]$
 - $\sqrt{\frac{m}{2k}} \left[\frac{a-x}{ax} \right]^{1/2}$

13. A particle is on a smooth horizontal plane. A force F is applied whose F - t graph is given. Then,

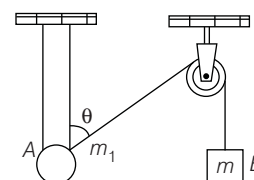


- a. at t_1 , acceleration is constant
 b. initially body must be in rest
 c. at t_2 , acceleration is constant
 d. initially acceleration is zero
 e. both c and d are correct
14. A force F is applied to the initially, stationary cart. The variation of force with time is shown in the figure. The speed of cart at $t = 5$ s is

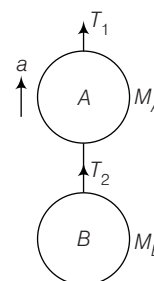


- a. 10 m/s b. 8.33 m/s c. 2 m/s d. zero
15. The mass m is placed on a body of mass M . There is no friction. The force F is applied on M and it moves with acceleration a . Then, the force on the top body is
- a. F b. ma
 c. $F - ma$ d. None of these
16. Three identical blocks each of mass M are along a frictionless table and a force F is acting as shown. Which of the following statements is false?
- a. The net vertical force on block A is zero
 b. The net force on block A is $\frac{F}{3}$
 c. The acceleration of block C is $\frac{F}{3M}$
 d. The force of interaction between A and B is $\frac{2F}{3}$
17. A 40 N block is supported by two ropes. One rope is horizontal and the other makes an angle of 30° with the ceiling. The tension in the rope attached to the ceiling is approximately
- a. 80 N b. 40 N c. 34.6 N d. 46.2 N
18. A weight w is suspended from the midpoint of a rope, whose ends are at the same level. In order to make the rope perfectly horizontal, the force applied to each of its ends must be
- a. less than w b. equal to w
 c. equal to $2w$ d. infinitely large
19. A ring of mass 5 kg sliding on a frictionless vertical rod connected by a block B of mass 10 kg by the help of a massless string.

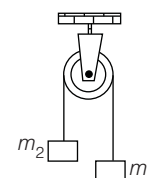
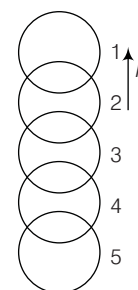
Then, at the equilibrium of the system, the value of θ is



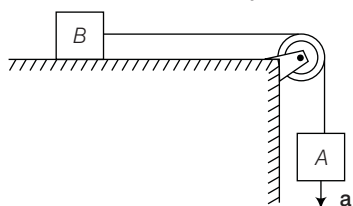
- a. 30° b. 60°
 c. 90° d. 0°
20. A body of mass 10 kg is to be raised by a massless string from rest to rest, through a height 9.8 m. The greatest tension which the string can safely bear is 20 kg-wt. The least time of ascent is
- a. 1 s
 b. 3 s
 c. 4 s
 d. None of the above
21. Which of the following expression correctly represents T_1 and T_2 if the system is given an upward acceleration by a pulling up mass A?



- a. $T_1 = M_A(a - g) + M_B(a - g)$, $T_2 = M_B(a - g)$
 b. $T_1 = M_A(g - a) + M_B(g - a)$, $T_2 = M_B(g - a)$
 c. $T_1 = M_A(g + a) + M_B(g + a)$, $T_2 = M_B(g + a)$
 d. $T_1 = M_A(g + a)$, $T_2 = M_B(g + a)$
22. A chain consisting of 5 links each of mass 0.1 kg is lifted vertically with a constant acceleration of 2.5 m/s^2 as shown in the figure. The force of interaction between the top link and the link immediately below it, will be
- a. 6.15 N
 b. 4.92 N
 c. 3.69 N
 d. 2046 N
23. In the given figure,
- a. acceleration of m_1 and m_2 are same
 b. the magnitude of relative acceleration of m_1 with respect to m_2 is twice the magnitude of acceleration of m_1
 c. the velocities of m_1 and m_2 are same
 d. the speed of m_1 and m_2 are not same

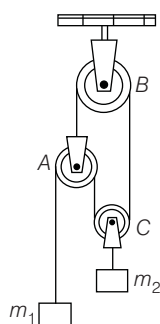


24. The actual acceleration of body A is a . Then,



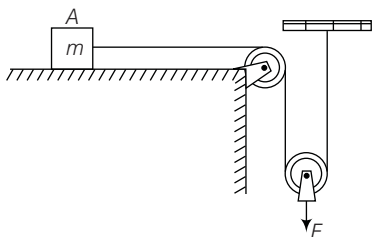
- the acceleration of B is a
- the acceleration of B is $2a$
- the magnitude of relative acceleration of B with respect to A is $\sqrt{2}a$
- the momentum of A may be equal to that of B

25. In the given ideal pulley system,



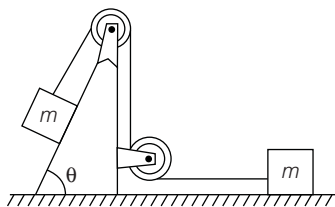
- tension in string is zero
- pulleys B and C rotate counter anti-clockwise and the pulley A clockwise
- A and B are same and is equal to g
- All of the above

26. Pulleys and strings are massless. The horizontal surface is smooth. The acceleration of the block A is



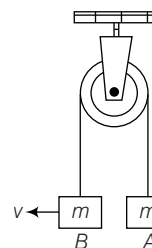
- $\frac{F}{m}$
- $\frac{F}{2m}$
- $\frac{F}{4m}$
- zero

27. For the system shown in the figure, the pulleys are light and frictionless. The tension in the string will be



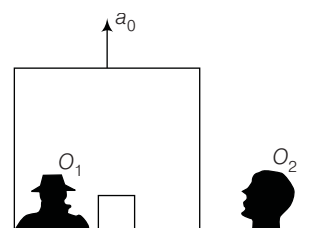
- $\frac{2}{3}mg \sin \theta$
- $\frac{3}{2}mg \sin \theta$
- $\frac{1}{2}mg \sin \theta$
- $2mg \sin \theta$

28. In the given figure,



- both masses always remain in same level
- after sometime, A is lower than B
- after sometime, B is lower than A
- no sufficient information

29. Observer O_1 is in a lift going upwards and O_2 is on the ground. Both apply Newton's law and measure normal reaction on the body



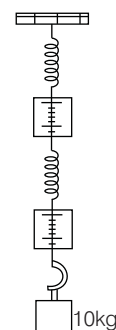
- both measure the same value
- both measure zero
- both measure different value
- no sufficient data

30. A particle is found to be at rest when seen from frame S_1 and moving with a constant velocity when seen from another frame S_2 .

Mark the possible points from the following.

- Both the frames are inertial
- Both the frames are non-inertial
- S_1 is non-inertial and S_2 is inertial
- Both a and b are correct

31. A block of mass 10 kg is suspended through two light springs which are balanced as shown in the figure. Then,



- both the scales will read 10 kg
- both the scales will read 5 kg
- the upper scale will read 10 kg and the lower zero
- the readings may be of any value but their sum will be 10 kg

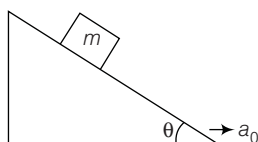
32. The normal reaction on a body placed in a lift moving up with constant acceleration 2 m/s^2 is 120 N . Mass of body is (Take, $g = 10\text{ m/s}^2$)

- 10 kg
- 15 kg
- 12 kg
- 5 kg

33. A block of mass m is moving on a wedge with the acceleration a_0 . The wedge is moving with the acceleration a_1 . The magnitude of pseudo force on the block is

a. ma_0 b. ma_1 c. $m\sqrt{a_0^2 + a_1^2}$ d. $m\left(\frac{a_1 + a_0}{2}\right)$

34. A point mass m is moving along inclined plane with acceleration a with respect to smooth triangular block. The triangular block is moving horizontally with acceleration a_0 . The value of a is



a. $g \sin \theta + a_0 \cos \theta$ b. $g \sin \theta - a_0 \cos \theta$
c. $g \cos \theta - a_0 \sin \theta$ d. $g \cos \theta - a_0 \tan \theta$

35. A uniform fine chain of length l is suspended with lower end just touching a horizontal table. Find the pressure on the table, when a length x has reached the table.

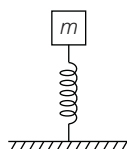
a. mgx b. $2mgx$ c. $3mgx$ d. $\frac{mgx}{2}$

36. A uniform chain is coiled up on a horizontal plane and one end passes over a small light pulley at a height 'a' above the plane. Initially, a length 'b' hangs freely on the other side. If $b = 2a$, then

a. the end descends with a constant acceleration $g/3$
b. the end descends with acceleration depends upon hanging position
c. acceleration cannot be determined
d. acceleration is variable

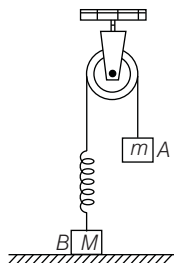
37. A mass m is placed over a spring of spring constant k , the acceleration of mass at the lowest position is

a. g
b. zero
c. $\left(\frac{kx}{m} - g\right)$, where x is compression in spring
d. None of the above



38. In the figure, the ball A is released from rest when the spring is at its natural length. For the block B of mass M to leave contact with the ground at some stage, the minimum mass of A must be

a. $2M$
b. M
c. $\frac{M}{2}$
d. a function of M and the force constant of the spring



39. A bicycle is in motion. When it is not pedaled, the force of friction exerted by the ground on the two wheels is such that it acts

a. in the backward direction on the front wheel and in the forward direction on the rear wheel

b. in the forward direction on the front wheel and in the backward direction on rear wheel
c. in the backward direction on both the front and the rear wheels
d. in the forward direction on both the front and the rear wheels

40. If a body of mass m is moving on a rough horizontal surface of coefficient of kinetic friction μ , the net electromagnetic force exerted by surface on the body is

a. $mg\sqrt{1+\mu^2}$ b. μmg c. mg d. $mg\sqrt{1-\mu^2}$

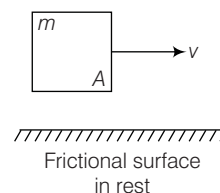
41. A block is placed on a rough floor and a horizontal force F is applied on it. The force of friction by the floor on the block is measured for different values of F and a graph is plotted between them

a. the graph is a straight line of slope 45°
b. the graph is a straight line parallel to the F -axis
c. the graph is a straight line of slope 45° for small F and a straight line parallel to the F -axis for large F
d. there is a small kink on the graph

42. When a body is in rest in the condition of a horizontal applied force. Then, the slope of force and friction from graph is

a. 1 b. μ c. 0 d. -1

43. Look at the situation, when the body is in air and is moving with pure translation. This situation is shown in the figure. What happens, when the body hits the surface?



a. Sliding friction will act in the backward direction
b. The velocity of the point of contact gradually decreases
c. The sliding friction acts in such a way so as to try to make the point of contact velocity of the body same as that of the surface
d. Both a and b are correct

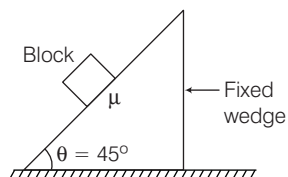
44. Let F , F_N and f denote the magnitudes of the contact force, normal force and the frictional force exerted by one surface on the other kept in contact. If one of these is zero, then

a. $F > F_N$ b. $F > f$
c. $F_N - f < F_N + f$ d. All of these

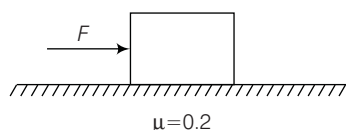
45. A car starts from rest to cover a distance s . The coefficient of friction between the road and the tyres is μ . The minimum time in which the car can cover the distance is proportional to

a. μ b. $\sqrt{\mu}$ c. $\frac{1}{\mu}$ d. $\frac{1}{\sqrt{\mu}}$

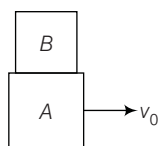
46. A block of mass 1 kg is placed on a wedge shown in figure. Find out minimum coefficient of friction between wedge and block to stop the block on it.



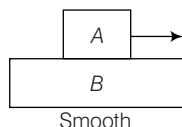
- a. 0.6 b. 0.9 c. 1 d. 0.2
47. A body of mass 2 kg is placed on rough horizontal plane. The coefficient of friction between body and plane is 0.2. Then,



- a. body will move in forward direction if $F = 5 \text{ N}$
 b. body will move in backward direction with acceleration 0.5 m/s^2 if force $F = 3 \text{ N}$
 c. if $F = 3 \text{ N}$, then body will be in rest condition
 d. Both a and c are correct
48. Two blocks of masses $M = 3 \text{ kg}$ and $m = 2 \text{ kg}$ are in contact on a horizontal table. A constant horizontal force $F = 5 \text{ N}$ is applied to block M as shown. There is a constant frictional force of 2 N between the table and the block m but no frictional force between the table and the first block M , then acceleration of the two blocks is
- a. 0.4 ms^{-2} b. 0.6 ms^{-2} c. 0.8 ms^{-2} d. 1 ms^{-2}
49. The coefficient of static friction between the bodies A and B is 0.30. Determine minimum stopping distance that the body A can have from a speed of 70 km/h with constant deceleration if the body B is not to slip forward



- a. 3 m b. 30.3 m c. 70 km d. 63 m
50. In the given figure, force of friction on body is

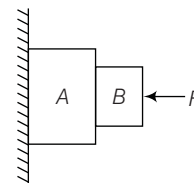


- a. towards left b. towards right
 c. either left or right d. no sufficient data

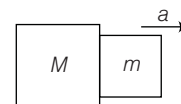
51. A piece of ice slides down a rough inclined plane at 45° inclination in twice the time that it takes to slide down an identical but frictionless inclined plane. What is the coefficient of friction between ice and incline?

a. $\frac{3}{7 \cot \theta}$ b. $\frac{4}{7 \cot \theta}$ c. $\frac{3}{4 \cot \theta}$ d. $\frac{7}{9 \cot \theta}$

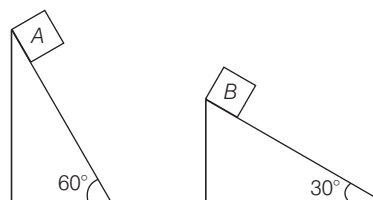
52. Consider the situation shown in the figure. The wall is smooth but the surfaces of A and B in contact are rough. Then,



- a. system may remain in equilibrium
 b. both bodies must move together
 c. the system cannot remain in equilibrium
 d. None of the above
53. The coefficient of static friction between the two blocks is 0.363. What is the acceleration of block 1 so that block 2 does not fall?



- a. 6 ms^{-2} b. 12 ms^{-2}
 c. 18 ms^{-2} d. 27 ms^{-2}
54. Two fixed frictionless inclined plane making angles 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B ?



- a. 4.9 ms^{-2} in horizontal direction
 b. 9.8 ms^{-2} in vertical direction
 c. Zero
 d. 4.9 ms^{-2} in vertical direction
55. A body is in equilibrium on a rough inclined plane under its own weight. If the angle of inclination of the inclined plane is α and the angle of friction is λ , then
- a. $\alpha > \lambda$ b. $\alpha > \frac{\lambda}{2}$ c. $\alpha = \lambda$ d. $\alpha \geq \lambda$
56. For the equilibrium of a body on an inclined plane of inclination 45° , the coefficient of static friction will be
- a. greater than one b. less than one
 c. zero d. less than zero

BITSAT Archives

1. A block of mass 0.18 kg is attached to a spring of force constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially, the block is at rest and the spring is unstretched. An impulse is given to the block.

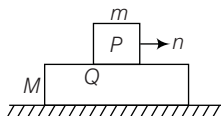
The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is $v = N/10$. Then, N is [2014]

- a. 2 b. 3 c. 4 d. 6

2. The sum of the magnitudes of two forces acting at a point is 16 N. The resultant of these forces is perpendicular to the smaller force which has a magnitude of 8 N. If the magnitude of smaller force is x , then the value of x is [2014]

- a. 2 N b. 4 N
c. 6 N d. 7 N

3. A block of mass $m = 1$ kg is placed over a plank Q of mass $M = 6$ kg, placed over a smooth horizontal surface as shown in figure. Block P is given a velocity $v = 2$ m/s² to the right. If the coefficient of friction between P and Q is $\mu = 0.3$. Find the acceleration of Q relative to P . [2013]

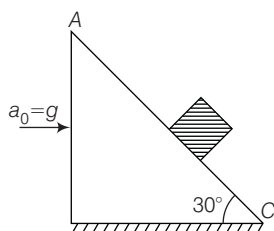


- a. 4 m/s² b. 3.5 m/s²
c. 2 m/s² d. 10.0 m/s²

4. A bomb at rest explodes into three parts of the same mass. The linear momentum of two parts are $-2p \hat{i}$ and $p \hat{j}$. The magnitude of momentum of third part is $p\sqrt{x}$. Find x . [2013]

- a. p b. $\sqrt{5}p$
c. $2p$ d. $10p$

5. A block is placed on an inclined plane. The block is moving towards right horizontally with an acceleration $a_0 = g$. The length of the inclined plane (AC) is equal to 1 m. Whole the situation are shown in the figure. Assume that all the surfaces are frictionless. The time taken by the block to reach from C to A is (Take, $g = 10$ m/s²) [2013]



- a. 0.74 s b. 0.9 s
c. 0.52 s d. 1.24 s

6. Pseudo force is [2013]

- a. electromagnetic in nature
b. a nuclear force
c. a gravitational force
d. None of the above

7. A body is moved in straight line by constant power of machine. What will be the relation between the travelling distance and time? [2012]

- a. $s^2 \propto t^3$ b. $s^4 \propto t^3$
c. $s^3 \propto t^2$ d. $s \propto t^3$

8. A 10 kg stone is suspended with a rope of breaking strength 30 kg-wt. The minimum time in which the stone can be raised through a height 10 m starting from rest is (Take, $g = 10$ N kg⁻¹) [2012]

- a. 0.5 s b. 1.0 s
c. $\sqrt{\frac{2}{3}}$ s d. 2.0 s

9. The minimum force required to move a body up an inclined plane is three times the minimum force required to prevent it from sliding down the plane. If the coefficient of friction between the body and the inclined plane is $\frac{1}{2\sqrt{3}}$, the angle of the inclined plane is [2012]

- a. 60° b. 45°
c. 30° d. 15°

10. Natural length of a spring is 60 cm and its spring constant is 4000 N/m. A mass of 20 kg is hung from it. The extension produced in the spring is (Take, $g = 9.8$ m/s²) [2011]

- a. 4.9 cm b. 0.49 cm
c. 9.4 cm d. 0.94 cm

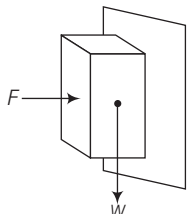
11. The linear momentum p of a body moving in one dimension varies with time t according to the equation $p = a + bt^2$, where a and b are positive constant. The net force acting on the body is [2011]

- a. a constant
b. proportional to t^2
c. inversely proportional to t
d. proportional to t

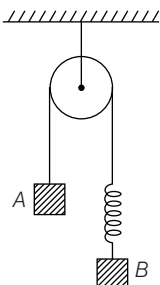
12. A block of mass 5 kg is placed on a rough inclined plane. The inclination of the plane is gradually increased till the block just begins to slide down. The inclination of the plane is then 3 in 5. The coefficient of friction between the block and the plane is (Take, $g = 10$ m/s²) [2010]

- a. $\frac{3}{5}$ b. $\frac{3}{4}$
c. $\frac{4}{5}$ d. $\frac{2}{3}$

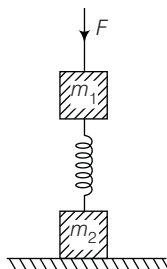
13. A block weighing w is held against a vertical wall by pressing horizontally with a force F . Then, F needed to hold the block is [2010]



- a. equal to w b. equal to $\frac{w}{\mu}$ only
c. greater than $\frac{w}{\mu}$ only d. greater than or equal to $\frac{w}{\mu}$
14. A body of mass 2 kg is placed on a horizontal surface having coefficient of kinetic friction 0.4 and limiting coefficient of static friction 0.5. If a horizontal force 2.5 N is applied on the body, the frictional force acting on the body will be (Take, $g = 10 \text{ ms}^{-2}$) [2010]
a. 8 N b. 10 N c. 20 N d. 2.5 N
15. In figure, block A is released from rest, when spring is at its natural unstretched length. For block B of mass M to leave contact with the ground at some stage, the minimum mass of A must be [2009]



- a. $3M$ b. $\frac{M}{2}$ c. M d. cannot say
16. A 600 kg rocket is set for a vertical firing. If the exhaust speed is 1000 m/s, the mass of the gas ejected per second to supply the thrust needed to overcome the weight of rocket is [2009]
a. 117.6 kgs^{-1} b. 58.6 kgs^{-1} c. 6 kgs^{-1} d. 46.4 kgs^{-1}
17. A system consists of two cubes of masses m_1 and m_2 respectively connected by a spring of force constant k . force (F) that should be applied to the upper cube for which the lower one just lifts after the force is removed, is [2009]

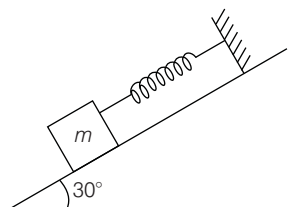


- a. $m_1 g$ b. $m_2 g$
c. $(m_1 + m_2)g$ d. $\frac{m_1 m_2}{m_1 + m_2} g$

18. Starting from rest, the time taken by a body sliding down on a rough inclined plane at 45° with the horizontal is twice the time taken to travel on a smooth plane of same inclination and same distance. Then, the coefficient of kinetic friction is [2008]
a. 0.25 b. 0.33 c. 0.50 d. 0.75
19. Three weights w , $2w$ and $3w$ are connected to identical spring suspended from a rigid horizontal rod. The assembly of the rod and the weights fall freely. The positions of the weight from the rod are such that [2007]
a. $3w$ will be farthest
b. w will be farthest
c. all will be at the same distance
d. $2w$ will be farthest
20. Consider the following statement. When jumping from some height, you should bend your knees as you come to rest instead of keeping your legs stiff. Which of the following relations can be useful in explaining the statement? [2007]
a. $\Delta p_1 = -\Delta p_2$
b. $\Delta E = -\Delta(\text{PE} + \text{KE}) = 0$
c. $F\Delta t = m\Delta v$
d. $\Delta x \propto \Delta F$

where symbols have their usual meaning.

21. A body of mass 5 kg is suspended by a spring balance on an inclined plane as shown in figure. The spring balance measure [2006]



- a. 50 N b. 25 N
c. 500 N d. 10 N
22. Under the action of a force $F = Cx$, the position of a body changes from 0 to x . The work done is [2006]
a. $\frac{1}{2} Cx^2$ b. Cx^2
c. Cx d. $\frac{1}{2} Cx$
23. A student unable to answer a question on Newton's laws of motion attempts to pull himself up by tugging on his hair. He will not succeed [2005]
a. as the force exerted small
b. the frictional force while gripping is small
c. Newton's law of inertia is not applicable to living beings
d. as the force applied is internal to the system

Answer with Solutions

Practice Exercise

- (d) (a) The magnitudes of momentum are same to each other.
(b) Momentum is a vector quantity and vector does not obey division law. So, the ratio of momentum is not determined.
- (d) Contact force is an electromagnetic force.
- (b) Cord D will break due to inertia offered by the block of mass m .
- (a) $\therefore \mathbf{r} = 3t\hat{i} - 4\cos t\hat{j}$
 $\therefore \mathbf{v} = \frac{d\mathbf{r}}{dt} = 3\hat{i} + 4\sin t\hat{j}$
 $\therefore \mathbf{a} = \frac{d^2\mathbf{r}}{dt^2} = 4\cos t\hat{j}$
 $\therefore \mathbf{F} = m\mathbf{a} = 12\cos t\hat{j}$
 $\therefore \text{Impulse} = \int_0^{\pi/2} F dt$
 $= 12 \int_0^{\pi/2} \cos t dt$
 $= 12[\sin t]_0^{\pi/2} = 12 \text{ N-s}$
The direction of impulse will be in the direction of force.
 $\therefore \text{Impulse} = 12 \hat{j} \text{ N-s}$
- (b) The acceleration of the system is
$$a = \frac{\text{Net force}}{\text{Total mass}} = \frac{12}{1+2+3} = \frac{12}{6} = 2 \text{ m/s}^2$$
- (d) A force of constant magnitude which is always perpendicular to the velocity of the particle does not do work. Hence, KE of the particle is constant. As force is perpendicular to the velocity, the particle moves in a circular path.
- (d) Since, at $t = 0$ and $t = 1\text{s}$, particle is in rest. This is possible only when the particle accelerates for sometime and then decelerates to come in rest.
- (e) $a = \frac{v-u}{t} = \frac{-4-10}{2} = -7 \text{ m/s}^2$
The magnitude of change in momentum
 $= |m(v-u)| = 14 \text{ kg-m/s} = \text{Impulse}$
Now, $F = ma = 0.7 \text{ N}$
- (c) $\therefore F = \frac{\Delta p}{\Delta t} = v \frac{\Delta m}{\Delta t} = \rho A v \frac{\Delta x}{\Delta t}$
 $= \rho A v \left(\frac{v \Delta t}{\Delta t} \right) = 1 \times 2 \times (30)^2 = 1800 \text{ dyne}$
- (b) The action and reaction forces must act upon different bodies.
- (c) The tension in the rope will be same at all points if it is massless or if the string has mass, the tension will be the same in the string when it is not accelerating.

$$12. (a) \therefore F = -\frac{k}{x^2}$$

$$\therefore \text{Acceleration, } f = -\frac{k}{mx^2}$$

When x decreases, v increases.

$$\therefore f = -v \frac{dv}{dx}$$

$$\therefore -v \frac{dv}{dx} = -\frac{k}{mx^2}$$

$$\text{or } \int_0^v v dv = \frac{k}{m} \int_a^x \frac{1}{x^2} dx$$

$$\therefore v = \sqrt{\frac{2k}{m} \left(\frac{x-a}{ax} \right)^{1/2}}$$

- (e) At t_2 , force is constant.

So, acceleration is constant.

At $t = 0$, force is zero. Hence, acceleration is zero.

- (b) The equation of parabola is

$$x^2 = 4ay$$

$$\text{Here, } t^2 = 4aF$$

When $t = 5 \text{ s}$, $F = 50 \text{ N}$

(see graph)

$$\therefore 5^2 = 4a \times 50$$

$$\therefore a = \frac{25}{200} = \frac{1}{8}$$

$$\therefore t^2 = 4 \times \frac{1}{8} F = \frac{F}{2} \Rightarrow F = 2t^2$$

$$\text{Acceleration} = \frac{F}{m} = \frac{2t^2}{m} = \frac{2t^2}{10} = \frac{t^2}{5}$$

$$\therefore \frac{dv}{dt} = \frac{t^2}{5} \quad \text{or} \quad \int_0^v dv = \int_0^{5t^2} \frac{t^2}{5} dt$$

$$\therefore v = \frac{1}{5} \left[\frac{t^3}{3} \right]_0^{5t^2} = \frac{125}{15} = 8.33 \text{ m/s}$$

- (d) Since, no force of friction is present. So, no horizontal force is present on body of mass m . In vertical direction normal force balances weight of the body. Hence, net force of top body must be zero. Lesser contact force corresponds more comfortableness.

- (b,c,d) The acceleration of system in rightward direction is

$$a = \frac{F}{m_A + m_B + m_C} = \frac{F}{3M}$$

For A,

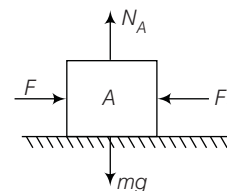
From force diagram of A,

Net force on A = $F - F_1$

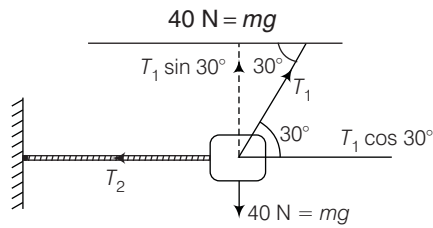
$$= m_A a = \frac{MF}{3M} = \frac{F}{3}$$

The force interaction between A and B is

$$F_1 = F - \frac{F}{3} = \frac{2F}{3}$$



17. (a) From force diagram shown in figure,



$$\Rightarrow T_1 \sin 30^\circ = 40$$

$$\therefore T_1 = 80 \text{ N}$$

18. (d) For equilibrium of body,

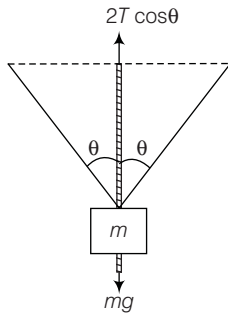
$$mg = 2T \cos \theta$$

$$\therefore T = \frac{mg}{2 \cos \theta}$$

For the string to be horizontal,
 $\theta \rightarrow 90^\circ$

$$\therefore T = \frac{mg}{2 \cos 90^\circ}$$

$$T \rightarrow \infty$$



19. (b) For ring,

From force diagram of ring,

$$T \cos \theta = m_1 g \quad \dots(i)$$

For block B,

$$mg = T$$

$$T \cos \theta = m_1 g$$

$$mg \cos \theta = m_1 g$$

$$\therefore \cos \theta = \frac{m_1}{m} = \frac{5}{10} = \frac{1}{2}$$

$$\Rightarrow \theta = 60^\circ$$

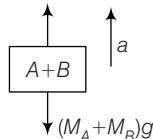
20. (a) $a_{\max} = \frac{T - mg}{m} = \frac{(20 - 10)}{10} = g$

During half time, body is accelerated from rest and during next half time decelerates with same magnitude to come to rest.

$$\therefore \frac{h}{2} = \frac{1}{2} a_{\max} t_1^2$$

$$\therefore t_1 = \sqrt{\frac{h}{a_{\max}}} = 1 \text{ s}$$

21. (c) The force diagram of system (A + B) is shown in figure.



$$\text{Here, } T_1 - (M_A + M_B)g = (M_A + M_B)a$$

$$\therefore T_1 = (M_A + M_B)g + (M_A + M_B)a$$

$$= (M_A + M_B)(g + a)$$

The force diagram of body B,

$$T_2 - M_B g = M_B a$$

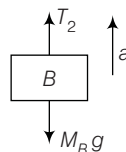
$$\therefore T_2 = M_B(g + a)$$

22. (b) $\therefore$

$$F = 5mg = 5ma$$

$$F = 5mg + 5ma$$

$$= 5m(g + a) \quad \dots(i)$$



If the force of interaction between top (first) link and second is T. Then,

$$ma = F - mg - T$$

$$T = F - mg - ma$$

$$= 5mg + 5ma - mg - ma$$

$$= 4mg + 4ma = 4m(g + a)$$

$$T = 4 \times 0.1(9.8 + 2.5) = 4.92 \text{ N}$$

23. (b) $x_1 + x_2 = l$ (length of string)

$$\frac{dx_1}{dt} + \frac{dx_2}{dt} = 0$$

$$v_1 + v_2 = 0$$

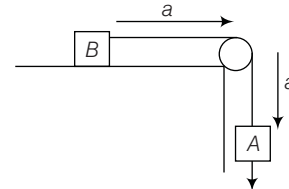
$$\Rightarrow \text{and } v_1 = -v_2$$

$$a_1 = -a_2$$

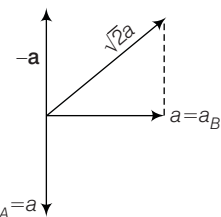
$$a_{m_1 m_2} = a_{m_1} - a_{m_2}$$

$$= a - (-a) = 2a$$

24. (c) $a_{BA} = a_B - a_A$



$$= a_B + (-a_A)$$



25. (d) Since, the mass of the pulleys and the string are negligible, so tension is same everywhere.

Considering free body diagrams of m_1 , m_2 and the pulley A, we have

$$m_1 g - T = m_1 a_1$$

$$2T - m_2 g = m_2 a_2$$

$$2T - T = 0$$

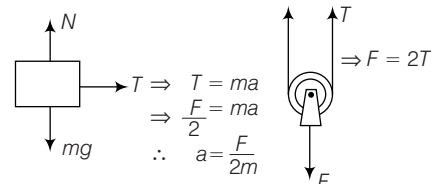
$$2T = T$$

But this is possible only when $T = 0$

$$\therefore a_1 = a_2 = g$$

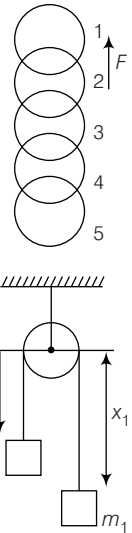
Thus, weights fall freely and pulleys B and C rotate anti-clockwise but pulley A clockwise.

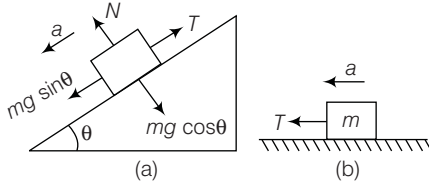
26. (b)



27. (c) From force diagram in Fig. (a),

$$mg \sin \theta - T = ma \quad \dots(i)$$





In force diagram (b), $T = ma$

...(ii)

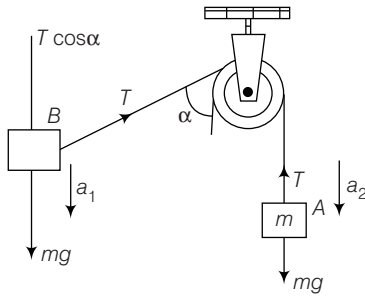
From Eqs. (i) and (ii), we get

$$a = \frac{g}{2} \sin \theta$$

$\therefore$

$$T = \frac{mg \sin \theta}{2}$$

28. (c) After sometime, right load moves on straight line ($mg - T$), but left load will oscillate in addition to the motion in straight line ($mg - T \cos \alpha$).



But, $mg - T \cos \alpha > mg - T$

$\therefore a_1 > a_2$ [in downward direction]

So, at any instant, the left load is lower than the right one.

29. (c) The value measured by O_1 is $N_1 = mg$ because acceleration of body with respect to O_1 is zero.

The value measured by O_2 is

$$N_2 - mg = ma_0$$

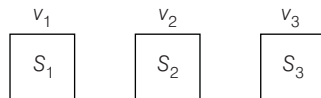
$\therefore N_2 = m(g + a_0)$

So,

$$N_1 \neq N_2$$

30. (d) Let the velocities of S_1 ,

S_2 and the particle in the frame of ground are v_1 , v_2 and v_3 , respectively.



The velocity of the particle in the frame of S_1 is

$$\mathbf{v}_{31} = \mathbf{v}_3 - \mathbf{v}_1 = 0 \quad \dots(i)$$

From this expression, it is obvious that at every instant, velocity of S_1 and the particle are same with respect to ground. The velocity of the particle in the frame of S_2 is

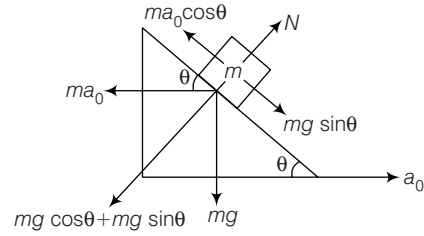
$$\mathbf{v}_{32} = \mathbf{v}_3 - \mathbf{v}_2 = \text{constant} \quad \dots(ii)$$

If S_1 and S_2 are inertial, then accelerations of S_1 and S_2 in the frame of ground are zero. If the velocity of the particle is constant in the frame of ground, then the statements of Eqs. (i) and (ii) are easily satisfied. If S_1 and S_2 are non-inertial frames of acceleration, then the statements of Eqs. (i) and (ii) are also satisfied.

31. (a) 32. (a)

33. (b)

34. (b)



From force diagram,

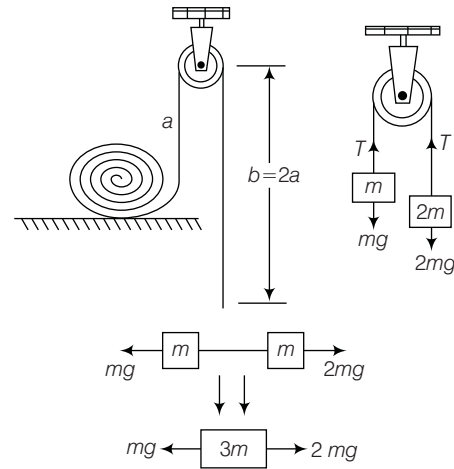
$$mg \sin \theta - ma_0 \cos \theta = ma$$

$\therefore$

$$a = g \sin \theta - a_0 \cos \theta$$

35. (c)

36. (a) Let the mass of hanging part of chain in left is m .



Since, $b = 2a$, so the mass of hanging part of chain in right will be $2m$.

$$\therefore 2mg - mg = 3mf$$

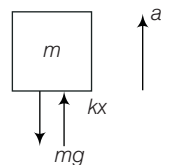
$$\therefore f = \frac{g}{3}$$

37. (c) The force diagram of the body at lowest position is shown in the figure force diagram of body.

$$kx - mg = ma$$

$\therefore$

$$a = \frac{kx}{m} - g$$



38. (c) For minimum mass of m , mass M breaks off contact when elongation in spring is maximum. At the time of break off, block A is at lowest position and its speed is zero. At an instant t_1

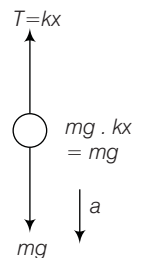
$$mg - kx = ma$$

$$v \frac{dv}{dx} = \frac{mg - kx}{m}$$

$$\int_0^0 v dv = \int_0^{x_0} \left(g - \frac{k}{m} x \right) dx$$

where, x_0 is maximum elongation in spring.

$$0 = gx_0 - \frac{kx_0^2}{2m}$$



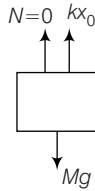
$$x_0 = \frac{2mg}{k}$$

At the time of break off of block B,

$$Mg = kx_0$$

$$Mg = 2mg$$

$$\therefore m = \frac{M}{2}$$



39. (c) When the bicycle is being pedaled then during pedaling, the force is acting in the backward direction on the rear wheel and thus frictional force will act in the forward direction. The choice (c) is correct, when the bicycle is not pedaled.

40. (a) The net electromagnetic force = $\sqrt{N^2 + f^2}$

$$\text{But, } N = mg, f = \mu mg$$

$$\Rightarrow \text{Force} = mg\sqrt{1 + \mu^2}$$

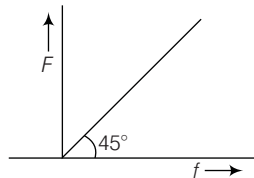
41. (d)

42. (a) When body remains in rest. For equilibrium, static friction f is equal to applied force F .

$$\therefore F = f$$

The corresponding graph is shown in figure.

From graph, slope = $\tan 45^\circ = 1$



43. (d) Since, motion of body is in forward direction. So, friction acts in backward direction. Due to this, velocity of body starts to decrease.

44. (d)

45. (d) $F_{\max} = \mu mg$

Maximum acceleration to the car is provided by the limiting frictional force.

$$\Rightarrow a = \frac{F}{m} = \mu g$$

$$\text{Using, } s = ut + \frac{1}{2}at^2 \Rightarrow s = 0 + \frac{1}{2}a_{\max}t_{\min}^2$$

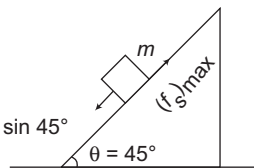
$$\Rightarrow \sqrt{\frac{2s}{\mu g}} = t \Rightarrow t \propto \frac{1}{\sqrt{\mu}}$$

46. (c) For minimum value of μ , we can write

$$mg \sin(45^\circ) = (f_s)_{\max}$$

$$mg \sin 45^\circ = (\mu)_{\min} mg \cos 45^\circ$$

$$(\mu)_{\min} = 1$$



47. (d) For moving the body, applied force should be greater or equal to limiting friction (i.e. μN).

$$\text{The limiting friction is } f_2 = \mu N = \mu mg$$

$$= 0.2 \times 2 \times 9.8 = 3.92 \text{ N}$$

- (a) Since, applied force $F = 5 \text{ N}$, so body starts to move in forward direction.

- (b) Here, applied force is lesser than limiting friction. So, body does not move.

- (c) The reason is same as option (b).

48. (b) If both blocks move together

$$F - f = (m + M)a$$

$$\therefore a = \frac{F - f}{m + M}$$

$$= \frac{5 - 2}{5} = \frac{3}{5} = 0.6 \text{ m/s}^2$$

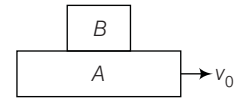
49. (d) The maximum deceleration is

$$a = \mu g = 3 \text{ m/s}^2$$

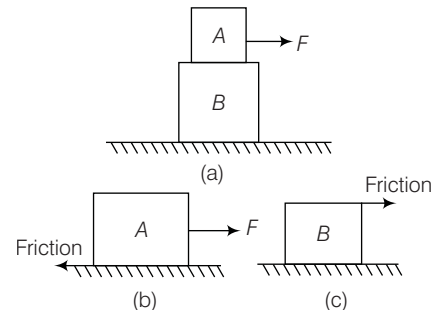
$$\therefore v^2 = u^2 - 2as$$

$$0 = v_0^2 - 2\mu g s_{\min}$$

$$\therefore s_{\min} = \frac{v_0^2}{2\mu g} = \frac{(70 \times \frac{5}{18})^2}{6} = 63 \text{ m}$$

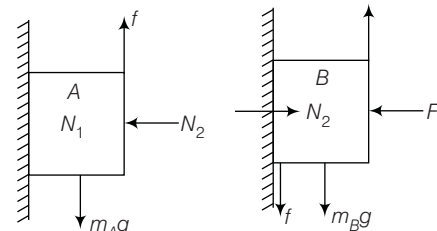


50. (b) From force diagram shown in Fig. (b) and (c), the direction of force of friction on B is rightward.



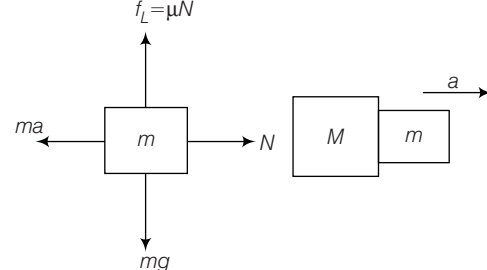
51. (c)

52. (c) From the diagram shown in figure, the resultant force of B in vertical direction is not zero. Hence, the system never be in equilibrium.



53. (d) In the frame of block M, m is in rest.

$$\therefore N = ma$$



For not falling, $mg < f_L$

$$\text{or } mg < \mu N$$

$$\text{or } mg < \mu ma$$

$$\text{or } a > \frac{g}{\mu}$$

$$\therefore a_{\min} = \frac{g}{\mu} = \frac{9.8}{0.368} = 27 \text{ m/s}^2$$

54. (d) We can write, $mg \sin \theta = ma$

$$\therefore a = g \sin \theta$$

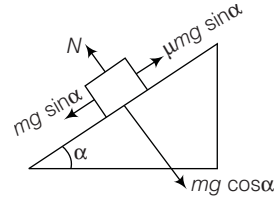
where, a is along the inclined plane.

$\therefore$ Vertical component of acceleration is $g \sin^2 \theta$.

$\therefore$ Relative vertical acceleration of A with respect to B is

$$g(\sin^2 60^\circ - \sin^2 30^\circ) = \frac{g}{2} = 4.9 \text{ ms}^{-2} \text{ [in vertical direction]}$$

55. (c) For equilibrium,



$$mg \sin \alpha = \mu mg \cos \alpha$$

$$\tan \alpha = \mu = \tan \lambda \Rightarrow \alpha = \lambda$$

56. (a)

BITSAT Archives

1. (d) Here, $m = 0.18 \text{ kg}$, $k = 2 \text{ N/m}$, $\mu = 0.1$, $x = 0.06 \text{ m}$.

According to conservation of mechanical energy principle, we know

Decrease in mechanical energy

= Work done against friction

$$\frac{1}{2}mv^2 - \frac{1}{2}kx^2 = \mu mgx$$

$\Rightarrow$

$$v = \sqrt{\frac{2\mu mgx + kx^2}{m}}$$

Substituting the values of m, μ, g, x and k , we get

$$v = \sqrt{\frac{2 \times 0.1 \times 0.18 \times 9.8 \times 0.06 + 2 \times 0.06^2}{0.18}}$$

$$v = (4/10) \text{ m/s}$$

So, $N = 4$

2. (c) $x + y = 16$,

$$\text{Also, } y^2 = 8^2 + x^2$$

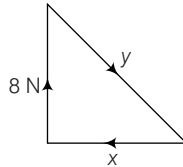
$$\text{or } y^2 = 64 + (16 - y)^2 \quad [\because x = 16 - y]$$

$$\text{or } y^2 = 64 + 256 + y^2 - 32y$$

$$\text{or } 32y = 320$$

$$y = 10 \text{ N}$$

$$\therefore x + 10 = 16 \Rightarrow x = 6 \text{ N}$$



3. (b) Frictional force between P and Q is $f = \mu mg$ which will retard P and accelerate Q .

$$\text{Retardation of } P \text{ is } a_P = -\frac{f}{m} = -\frac{\mu mg}{m} = -\mu g$$

$$\text{Acceleration of } Q \text{ is } a_Q = \frac{f}{M} = \frac{\mu mg}{M}$$

Acceleration of Q relative to P is

$$a_{QP} = a_Q - a_P = \frac{\mu mg}{M} - (-\mu g)$$

$$= \mu g \left[1 + \frac{m}{M} \right] = 0.3 \times 10 \left[1 + \frac{1}{6} \right] = 3.5 \text{ m/s}^2$$

4. (b) Given, $p_1 = -2p \hat{i} = 2p$ along negative x -axis.

$$p_2 = p \hat{i} = p \text{ along } y\text{-axis.}$$

The resultant momentum of two parts

$$p' = \sqrt{p_1^2 + p_2^2} = \sqrt{(2p)^2 + p^2} = p\sqrt{5}$$

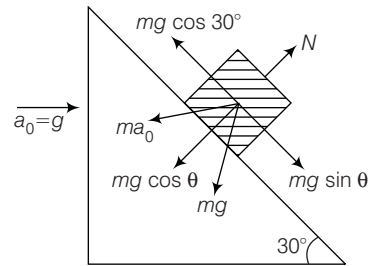
As the bomb was initially at rest final momentum of all the three parts must be zero

$$p_3 + p' = 0$$

$$p_3 = -p' = -p\sqrt{5} = \sqrt{5} p$$

$$p_3 = \sqrt{5} p$$

5. (a) The forces on smaller block is given as



For the motion of the block along the incline plane in upward direction.

Net force on the block = mass $\times$ acceleration of the block

$$\Rightarrow mg \cos 30^\circ - mg \sin 30^\circ = ma \quad [\because a_0 = g]$$

$$\Rightarrow a = \left(\frac{\sqrt{3} - 1}{2} \right) g = 3.66 \text{ m/s}^2$$

$$\text{Now, from equation of motion } s = \frac{1}{2}at^2$$

$$\Rightarrow t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 1}{3.66}} = 0.74 \text{ s}$$

6. (d) Pseudo force is not a real force.

7. (a) Power = $[ML^2T^{-3}]$ = constant

$$\therefore \left[\frac{ML^2}{T^3} \right] = \text{constant}$$

$$\therefore [L^2] \propto [T^3] \quad \text{or } s^2 \propto t^3$$

8. (b) Tension in the string $T = mg$

$$= 30 \times 10 = 300 \text{ N}$$

$$T - Mg = Ma$$

$$\text{From the figure, } 300 - 10 \times 10 = 10a$$

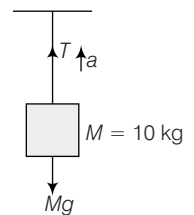
$$\therefore a = 20 \text{ ms}^{-2}$$

Thus, the maximum acceleration with which the stone can be raised is 20 ms^{-2} .

$$\text{Given, } s = 10 \text{ m} \quad \text{and } u = 0$$

$$\therefore 10 = \frac{1}{2}(20)t^2$$

$$\Rightarrow t = 1 \text{ s}$$



9. (c) Minimum force required to move a body up a rough inclined plane is

$$F_1 = mg(\sin \theta + \mu \cos \theta)$$

Minimum force required to prevent the body from sliding down the rough inclined plane, is

$$F_2 = \mu mg \cos \theta$$

According to the question,

$$F_1 = 3F_2$$

$$\therefore mg(\sin \theta + \mu \cos \theta) = 3(\mu mg \cos \theta)$$

$$\Rightarrow \sin \theta + \mu \cos \theta = 3\mu \cos \theta$$

$$\Rightarrow \sin \theta = 2\mu \cos \theta$$

$$\Rightarrow \tan \theta = 2\mu = 2 \times \frac{1}{2\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$$\theta = 30^\circ$$

10. (a) Given, $l = 60 \text{ cm} = 60 \times 10^{-2} \text{ m}$

$$m = 20 \text{ kg}, k = 400 \text{ N/m}$$

The weight hung from the spring

$$= mg = 20 \times 9.8 = 196 \text{ N}$$

Suppose x is the extension produced in spring

Now, force applied by the spring

= downward force on the spring

$$\therefore kx = mg \Rightarrow x = \frac{mg}{k}$$

$$\Rightarrow x = \frac{20 \times 9.8}{4000} = 0.049 \text{ m} = 4.9 \text{ cm}$$

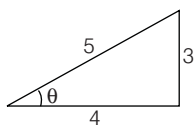
11. (d) Given, $p = a + bt^2$

$$\frac{dp}{dt} = 2bt$$

$$\therefore F = \frac{dp}{dt}$$

$$\therefore F = 2bt \quad \text{or} \quad F \propto t$$

12. (b) $\sin \theta = \frac{3}{5}$



$$\mu = \tan \theta \Rightarrow \mu = \frac{3}{4}$$

13. (b) $F - N + 0$

$$\Rightarrow F = N$$

$$\text{Further, } f = \mu N = w$$

$$\Rightarrow \mu F = w \Rightarrow F = \frac{w}{\mu}$$

14. (d) $f_{\text{static}} = (\mu_{\text{static}}) mg$

$$\Rightarrow f_{\text{static}} = (0.5)(2)(10) \Rightarrow f_{\text{static}} = 10 \text{ N}$$

$\therefore$ Applied force is just $2.5 \text{ N} < 10 \text{ N}$. So, the block will not move. As, we know that in static region, friction is a self adjusting force and has a value equal to that of applied force.

$$\text{So, } f = F_{\text{applied}} = 2.5 \text{ N}$$

15. (b) The spring will exert maximum force when A is at lowest position, say at distance x from the present position.

$$\therefore mgx = \frac{1}{2} kx^2$$

where, k is force constant of the spring, for the block B to leave contact, $kx = Mg$

$$\therefore mgx = \frac{1}{2} Mg$$

$$\therefore m = \frac{M}{2}$$

16. (c) Thrust $= u \frac{dm}{dt} = Mg$

$$\text{or } \frac{dm}{dt} = \frac{Mg}{u} = \frac{600 \times 9.8}{1000} = 6 \text{ kgs}^{-1}$$

17. (c) Let x_0 be the maximum compression, then

$$m_1 g + F = kx_0 \quad \dots(i)$$

$$\text{and } \frac{1}{2} kx_0^2 = m_1 g x_0 \quad \dots(ii)$$

If x is the maximum elongation, then

$$kx = m_2 g \quad \dots(iii)$$

$$\text{and } -\frac{1}{2} kx^2 = m_1 g(x + x_0) \quad \dots(iv)$$

$$\text{or } k(x_0 - x) = 2m_1 g$$

From Eqs. (i) and (iii), we get

$$m_1 g + F - m_2 g = k(x_0 - x) = 2m_1 g$$

$$\text{or } F = 2m_1 g + m_2 g - m_1 g = (m_1 + m_2)g$$

18. (d) $\mu = \tan \theta \left[1 - \frac{1}{n^2} \right]$

Here, $\theta = 45^\circ$ and $n = 2$

$$\therefore \mu = \tan 45^\circ \left[1 - \frac{1}{2^2} \right] = 1 - \frac{1}{4} = \frac{3}{4} = 0.75$$

19. (c) For $w, 2w, 3w$, apparent weight will be zero because the system is falling freely. So, the distances of the weights from the rod will be same.

20. (c) $F \Delta t = m \Delta v$

$$\Rightarrow F = \frac{m \Delta v}{t}$$

By doing so time of change in momentum increases and impulsive force on knees decreases.

21. (b) Acceleration of the body down the rough inclined plane. $= g \sin \theta$

Force applied on spring balance $= mg \sin \theta$

$$= 5 \times 10 \times \sin 30^\circ = 5 \times 10 \times \frac{1}{2} = 25 \text{ N}$$

22. (a) Work done, $W = \int_0^x F \cdot dx = \int_0^x Cx \, dx = C \left(\frac{x^2}{2} \right)_0^x = \frac{1}{2} Cx^2$

23. (d) In this case, the internal force is applied on the system, so he will not succeed. According to Newton's law, the state of a body can only be changed if some external force is applied on it.

5

Circular Motion

Concept of Circular Motion

The circular motion differs from the linear motion in one very important aspect that in a circular motion particles move along circular track such that direction of motion changes continuously unlike in a linear motion. Therefore, circular motion is described in terms of angular displacement i.e. angle turned by the rotating body in an unit time.

Terms Related to Circular Motion

Some important terms used in circular motion are given as

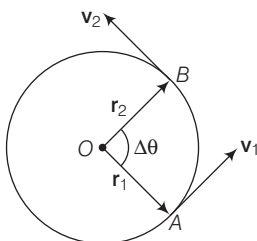
1. Angular Displacement

It is defined as the angle turned by the particle from some reference line. Angular displacement $\Delta\theta$ is usually measured in radian.

Finite angular displacement $\Delta\theta$ is a scalar but an infinitesimally small displacement is a vector.

2. Angular Velocity

It is defined as the rate of change of the angular displacement of the body.



$$\therefore \text{Angular velocity, } \omega = \lim_{\Delta t \rightarrow 0} \left(\frac{\Delta\theta}{\Delta t} \right) = \frac{d\theta}{dt}$$
$$\omega = \frac{2\pi}{T}$$

It is an axial vector whose direction is given by the right hand rule. Its unit is rad/s.

3. Angular Acceleration

It is the rate of change of angular velocity.

Thus, $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

Its unit is rad/s^2 .

4. Velocity

A particle in a circular motion has two types of velocities and corresponding two speeds.

(i) **Linear velocity** (v) or **speed** (v)

$$\mathbf{v} = \frac{ds}{dt} \text{ and } v = |\mathbf{v}| = \left| \frac{ds}{dt} \right|$$

(ii) **Angular velocity** (ω) or **speed** (ω)

$$\omega = \frac{d\theta}{dt} \text{ and } \omega = |\omega| = \left| \frac{d\theta}{dt} \right| = \frac{d\theta}{dt}$$

Relation between linear speed (v) and angular speed (ω) is

$$v = r\omega$$

In vector form, $\mathbf{v} = \omega \times \mathbf{r}$, which is a relation between linear velocity ($\mathbf{v}$) and angular velocity (ω).

Here, $\mathbf{r}$ is the position vector of particle with respect to the centre of the circle.

5. Acceleration

Acceleration of a particle in circular motion has two components

(i) **Tangential acceleration** (a_t), which is the component of $\mathbf{a}$ in the direction of velocity.

a_t = component of $\mathbf{a}$ along $\mathbf{v}$

$$= \frac{dv}{dt} = \frac{d|\mathbf{v}|}{dt}$$

(ii) **Radial acceleration** (a_r), which is the component of $\mathbf{a}$ towards the centre of the circular motion. This is responsible for a change in the direction of velocity.

$$a_r = \frac{v^2}{r} = r\omega^2$$

(iii) **Tangential acceleration** If we differentiate above equation, we get,

$$\frac{d\mathbf{v}}{dt} = \frac{d\omega}{dt} \times \mathbf{r}$$

i.e.

$$\mathbf{a}_t = \alpha \times \mathbf{r}$$

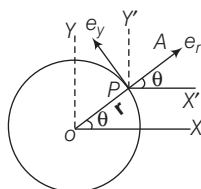
where, $\mathbf{a}_t$ is the tangential component of acceleration which is responsible for changing the magnitude of velocity.

when speed of the particle increases a_t is positive or in the direction of the velocity.

Thus, net acceleration in this case will be

$$a = \sqrt{a_t^2 + a_r^2}$$

$$\text{where, } a_t = \frac{dv}{dt} \text{ or } = \frac{d|\mathbf{v}|}{dt}$$



Dynamics of Uniform Circular Motion

If a particle in circular motion moves with a uniform speed, then motion of the particle is called **uniform circular motion**. In such a case,

$$\frac{dv}{dt} = 0 \text{ and } a = \omega^2 r$$

or

$$a = \frac{v^2}{r^2} r$$

$$a = \frac{v^2}{r}$$

Thus, if a particle moves in a circle of radius r with a uniform speed v , then its acceleration is $\frac{v^2}{r}$ towards the centre. This acceleration is termed as **centripetal acceleration**.

Forces in Circular Motion

In circular motion of an object, two kinds of forces occur which are described below

(i) Centripetal Force

A particle performing circular motion is acted upon by a force directed along the radius towards the centre of the circle. This force is called the centripetal force.

If m is the mass of the particle, then the magnitude of centripetal force is given by
Centripetal force = Mass $\times$ Centripetal acceleration

$$\Rightarrow F = m \left(\frac{v^2}{r} \right)$$

or

$$F = mr\omega^2$$

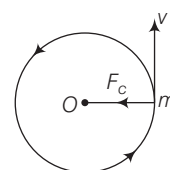
In vector form, centripetal force is given by

$$\mathbf{F} = -\frac{mv^2}{r} \hat{\mathbf{r}}$$

or

$$\mathbf{F} = -(m\omega^2 r) \hat{\mathbf{r}}$$

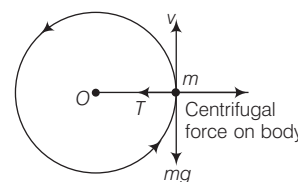
where, $\hat{\mathbf{r}}$ is a unit vector directed away from the centre along the radius.



A particle performing uniform circular motion

(ii) Centrifugal Force

It can be defined as the radially directed outward force acting on a body in circular motion, as observed by a person moving with the body.



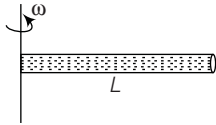
Centrifugal force = Mass $\times$ Centrifugal acceleration

$$\Rightarrow F = \frac{mv^2}{r} = mr\omega^2 \quad \text{or} \quad F = mv\omega \quad [\because v = \omega r]$$

This can be written in vector form as $\mathbf{F} = \frac{mv^2}{r} \hat{\mathbf{r}}$

Some Important Points

If a tube filled an incompressible fluid of mass m and closed at both ends is rotated with constant angular velocity ω about an axis passing through one end. Then, the force exerted by liquid at the other end is $\frac{1}{2}mL\omega^2$.



Applications of Centripetal and Centrifugal Forces

1. Motion of a Vehicle on a Level Circular Road

When a vehicle negotiates a circular path, it requires a centripetal force. In such cases the lateral force of friction may provide the required centripetal force. Thus, for maintaining its circular path required centripetal force

$$\left(\frac{mv^2}{r} \right) \leq \text{frictional force } (\mu mg)$$

$$\Rightarrow v_{\max} = \sqrt{\mu rg}$$

where, μ = coefficient of friction between road and vehicle tyres and r = radius of circular path.

2. Banking of a Curved Road

For the safe journey of a vehicle on a curved (circular) road, without any risk of skidding, the road is slightly raised towards its outer end. Let the road be banked at an angle θ from the horizontal, as shown in the figure.

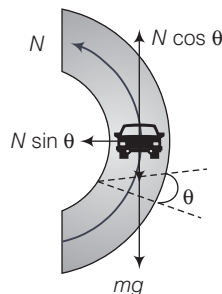
$$v_{\max} = \sqrt{rg \cdot \tan \theta}$$

If b = width of the road and h = height of the outer edge of the road as compared to the inner edge, then $\tan \theta = \frac{v^2}{rg} = \frac{h}{b}$

In case of friction is present between road and tyre

$$\text{then} \quad v_{\max} = \sqrt{\frac{rg(\mu_s + \tan \theta)}{1 - \mu_s \tan \theta}}$$

where, μ_s = coefficient of static friction.

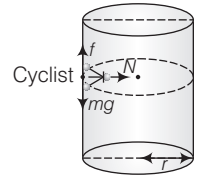


3. Bending of a Cyclist

If a curved road is not banked, then a cyclist/scooterist bends away from the vertical, while negotiating a turn on a curved road. The angle θ at which the cyclist bends from the vertical, is given by $\tan \theta = \frac{v^2}{rg}$

4. Motion of a Cyclist in a Death Well

For equilibrium of cyclist in a death well, as shown in the figure, the normal reaction N provides the centripetal force needed and the force of friction balances his weight mg .

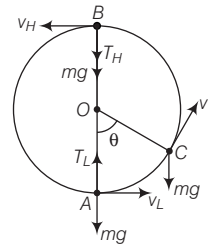


$$\text{Thus, } N = \frac{mv^2}{r}$$

$$\text{and } f = \mu N = mg \Rightarrow v_{\max} = \sqrt{\frac{rg}{\mu}}$$

Vertical Circular Motion

It is an example of non-uniform circular motion in which speed of object decreases due to effect of gravity as the object goes from its lowest position A to highest position B.



- (i) At the lowest point A, the tension T_L and the weight mg are in mutually opposite directions and their resultant provides the necessary centripetal force,

$$\text{i.e.} \quad T_L - mg = \frac{mv_L^2}{r}$$

$$\text{or} \quad T_L = mg + \frac{mv_L^2}{r}$$

- (ii) At the highest point B, tension T_H and the weight mg are in the same direction and hence,

$$T_H + mg = \frac{mv_H^2}{r} \quad \text{or} \quad T_H = \frac{mv_H^2}{r} - mg$$

Moreover, v_L and v_H are correlated as $v_H^2 = v_L^2 - 2gr$

- (iii) In general, if the revolving particle, at any instant of time, is at position C, inclined at an angle θ from the vertical, then

$$v^2 = v_L^2 - 2gr(1 - \cos \theta)$$

$$\text{and} \quad T = mg \cos \theta + \frac{mv^2}{r}$$

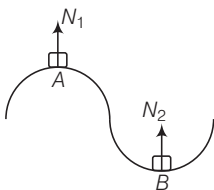
- (iv) In the critical condition of just looping the vertical loop, (i.e. when the tension just becomes zero at the highest point B), we obtain the following results

$$T_H = 0, T_L = 6mg, v_L = \sqrt{5rg} \text{ and } v_H = \sqrt{rg}$$

In general, $T_L - T_H = 6mg$

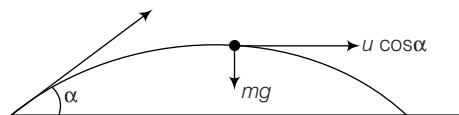
Some Important Points

- When a vehicle is moving over a convex bridge, the maximum velocity $v = \sqrt{rg}$, where r is the radius of the road.
- When the vehicle is at the maximum height, the reaction of the road, is $N_1 = mg - \frac{mv^2}{r}$
- When the vehicle is moving in a dip B, then $N_2 = mg + \frac{mv^2}{r}$
- The weight that we feel is the normal force and not the actual weight.
- When body loses the contact, normal force reduces to zero.
- The normal on tangent at a point on the curve gives the direction of radius.



i.e.
$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$$

- where, ρ is radius of curvature.

Expression for the Radius of Curvature for a Particle at the Highest Point in the Case of Projectile Motion

$$mg = \frac{mv^2}{r} \quad (\text{But, } v = u \cos \alpha)$$

$$r = \frac{v^2}{g}$$

$$= \frac{u^2 \cos^2 \alpha}{g}$$

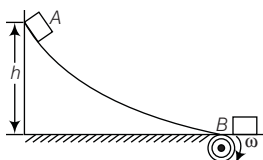
or

where, r is the radius of curvature.

Practice Exercise

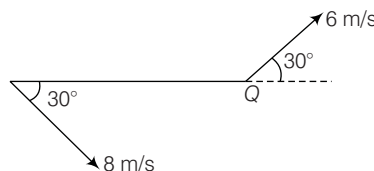
- In circular motion
 - radial acceleration is non-zero
 - radial velocity is zero
 - body is in equilibrium
 - All of the above
- A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that
 - its velocity is constant
 - its acceleration is constant
 - its kinetic energy is constant
 - it does not move on a circular path
- A stone of mass m tied to a string of length l is rotated in a circle with the other end of the string as the centre. The speed of the stone is v . If the string breaks, the stone will
 - move towards the centre
 - move away from the centre
 - move along a tangent
 - stop

- Particles are released from rest at A and slide down the smooth surface of height h to a conveyor B. The correct angular velocity ω of the conveyor pulley of radius r to prevent any sliding on the belt as the particles transfer to the conveyor is



- a. $\sqrt{2gh}$ b. $\frac{2gh}{r}$ c. $\frac{\sqrt{2gh}}{r}$ d. $\frac{2gh^2}{r^2}$

- Two moving particles P and Q are 10 m apart at a certain instant. The velocity of P is 8 m/s making an angle of 30° with the line joining P and Q and that of Q is 6 m/s making an angle 30° with PQ as shown in figure.



Then, angular velocity of P with respect to Q is

- a. zero b. 0.1 rad/s c. 0.4 rad/s d. 0.7 rad/s

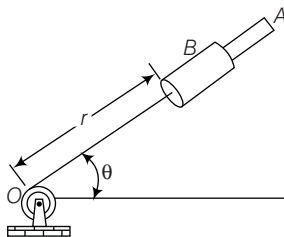
- A solid body rotates about a stationary axis, so that its angular velocity depends on the rotational angle ϕ as $\omega = \omega_0 - k\phi$, where ω_0 and k are positive constants. At the moment $t = 0$, $\phi = 0$. Find the time dependence of rotation angle.

- a. $k\omega_0 e^{-kt}$ b. $\frac{\omega_0}{k} e^{-kt}$
 c. $\frac{\omega_0}{k} (1 - e^{-kt})$ d. $\frac{k}{\omega_0} (e^{-kt} - 1)$

- The position of a point P is $\mathbf{r} = a \cos \theta \hat{i} + b \sin \theta \hat{j}$, where a and b are constants and θ is angle between $\mathbf{r}$ and x -axis. If the rate of increasing of θ is ω . Find the equation of path of particle.

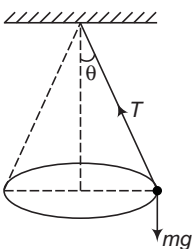
- a. Circle b. Parabola
 c. Ellipse d. Straight line

8. The angular displacement of the rod is defined as $\theta = \frac{3}{20}t^2$, where θ is in radian and t is in second. The collar B slides along the rod in such a way that its distance from O is $r = 0.9 - 0.12t^2$, where r is in metre and t is in second. The velocity of collar at $\theta = 30^\circ$ is
 a. 0.45 m/s b. 0.48 m/s c. 0.52 m/s d. 0.27 m/s
9. Two buses A and B are moving around concentric circular paths of radii r_A and r_B . If the two buses complete the circular paths in the same time. The ratio of their linear speeds is
 a. 1 b. r_A / r_B
 c. r_B / r_A d. None of these
10. A stone of mass 0.3 kg attached to a 1.5 m long string is whirled around in a horizontal circle at a speed of 6 m/s. The tension in the string is
 a. 10 N b. 20 N c. 7.2 N d. None
11. A cyclist goes round a circular path of length 400 m in 20 second. Calculate the angle through which he bends from vertical in order to maintain the balance
 a. $\sin^{-1}(0.64)$ b. $\tan^{-1}(0.64)$
 c. $\cos^{-1}(0.64)$ d. None of these
12. Find the maximum speed with which an automobile can round a curve of radius 8 m without slipping if the road is unbanked and the coefficient of friction between the road and the tyres is 0.8. ($g = 10 \text{ m/s}^2$)
 a. 8 m/s b. 10 m/s
 c. 20 m/s d. None of these
13. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω . The force exerted by the liquid at the other end is
 a. $\frac{ML\omega^2}{2}$ b. $ML\omega^2$ c. $\frac{ML\omega^2}{4}$ d. $\frac{ML^2\omega^2}{2}$
14. A point on the periphery of a rotating disc has its acceleration vector making an angle of 30° with the velocity vector. The ratio a_c / a_t (a_c is centripetal acceleration and a_t is tangential acceleration) equals
 a. $\sin 30^\circ$ b. $\cos 30^\circ$ c. $\tan 30^\circ$ d. None
15. A car of 1400 kg is moving on a circular path of radius 30 m with a speed of 40 km/h. When the driver applies the brakes and the car continues to move along the circular path, what is the maximum deceleration possible if the tyres are limited to a total horizontal friction of 10.6 kN?
 a. 10 m/s^2 b. 6.36 m/s^2 c. 4 m/s^2 d. None



16. A cyclist is travelling on a circular section of highway of radius 2500 ft at the speed of 60 mile/h. The cyclist suddenly applies the brakes causing the bicycle to slow down at constant rate. Knowing that after 8 second, the speed has been reduced to 45 mile/h. The acceleration of the bicycle immediately after the brakes have been applied is
 a. 2 ft/s^2 b. 4.14 ft/s^2 c. 3.10 ft/s^2 d. 2.75 ft/s^2
17. A road of width 20 m forms an arc of radius 15 m, its outer edge is 2 m higher than its inner edge. Calculate for what velocity the road is banked?
 a. $\sqrt{10} \text{ m/s}$ b. $\sqrt{14.7} \text{ m/s}$
 c. $\sqrt{9.8} \text{ m/s}$ d. None of these
18. Three identical cars A, B and C are moving at the same speed on three bridges. The car A goes on a plane bridge, B on a bridge convex upwards and C goes on a bridge concave upwards. Let F_A, F_B and F_C be the normal forces exerted by the cars on the bridges when they are at the middle of the bridges. Then,
 a. F_A is maximum of the three forces
 b. F_B is maximum of the three forces
 c. F_C is maximum of the three forces
 d. $F_A = F_B = F_C$
19. A car runs from east to west and another car B of the same mass runs from west to east at the same path along the equator. A presses the track with a force N_1 and B presses the track with a force N_2 . Then,
 a. $N_1 > N_2$ b. $N_1 < N_2$
 c. $N_1 = N_2$ d. None of these
20. A small body of mass m slides down from the top of a hemisphere of radius R . The surface of block and hemisphere are frictionless. The height at which the body loss contact with the surface of the sphere is
 a. $\frac{3}{2}R$ b. $\frac{2}{3}R$ c. $\frac{1}{2}R$ d. $\frac{1}{3}R$
21. A person wants to drive on the vertical surface of a large cylindrical wooden well commonly known as death well in a circus. The radius of well is R and the coefficient of friction between the tyres of the motorcycle and the wall of the well is μ_s . The minimum speed, the motorcyclist must have in order to prevent slipping should be
 a. $\sqrt{\left(\frac{Rg}{\mu_s}\right)}$ b. $\sqrt{\left(\frac{\mu_s}{Rg}\right)}$
 c. $\sqrt{\left(\frac{\mu_s g}{R}\right)}$ d. $\sqrt{\left(\frac{R}{\mu_s g}\right)}$
22. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A plumb bob is suspended from the roof of the car by a light rigid rod of length 1 m. The angle made by the rod with the track is
 a. zero b. 30° c. 45° d. 60°

23. A heavy sphere of mass m is suspended by a string of length l . The sphere is made to revolve about a vertical line passing through the point of suspension, in a horizontal circle such that the string always remains inclined to the vertical making an angle θ . What is the period of revolution?



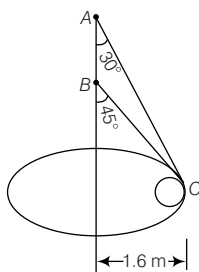
- a. $T = 2\pi\sqrt{\frac{l}{g}}$ b. $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$
 c. $T = 2\pi\sqrt{\frac{l\sin\theta}{g}}$ d. $T = 2\pi\sqrt{\frac{l\tan\theta}{g}}$

24. A rod of length L is hinged from one end. It is brought to a horizontal position and released. The angular velocity of the rod when it is in vertical position is

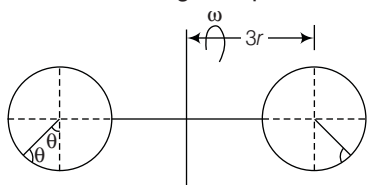
- a. $\sqrt{\frac{2g}{L}}$ b. $\sqrt{\frac{3g}{L}}$ c. $\sqrt{\frac{g}{2L}}$ d. $\sqrt{\frac{g}{L}}$

25. Two wires AC and BC are tied at C of small sphere of mass 5 kg , which revolves at a constant speed v in the horizontal circle of radius 1.6 m . The minimum value of v is

- a. 3.01 m/s
 b. 4.01 m/s
 c. 8.2 m/s
 d. 3.96 m/s



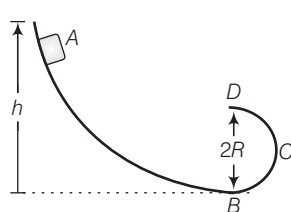
26. The small spherical balls are free to move on the inner surface of the rotating spherical chamber of radius $R = 0.2\text{ m}$. If the balls reach a steady state at angular position $\theta = 45^\circ$, the angular speed ω of device is



- a. 8 rad/s b. 2 rad/s c. 3.64 rad/s d. 9.34 rad/s

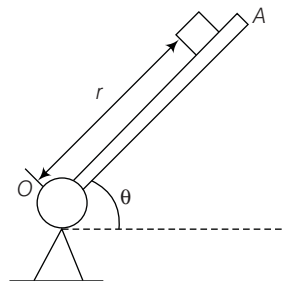
27. A frictionless track $ABCD$ ends in a semicircular loop of radius R . A body slides down the track from point A which is at a height $h = 5\text{ cm}$. Maximum value of R for the body to successfully complete the loop is

- a. 5 cm b. $\frac{15}{4}\text{ cm}$ c. $\frac{10}{3}\text{ cm}$ d. 2 cm



28. A rod OA rotates about a horizontal axis through O with a constant anti-clockwise velocity $\omega = 3\text{ rad/s}$. As it passes the position $\theta = 0^\circ$ a small block of mass m is placed on it at a radial distance $r = 450\text{ mm}$. If the

block is observed to slip at $\theta = 50^\circ$, the coefficient of static friction between the block and the rod is (Given that, $\sin 50^\circ = 0.766$, $\cos 50^\circ = 0.64$)



- a. 0.2 b. 0.55 c. 0.8 d. 1

29. A particle of mass m is moving in a horizontal circle of radius r under a centripetal force given by $\left(\frac{-k}{r^2}\right)$,

where k is a constant, then

- a. the total energy of the particle is $\left(\frac{-k}{2r}\right)$
 b. the kinetic energy of the particle is $\left(\frac{k}{r}\right)$
 c. the potential energy of the particle is $\left(\frac{k}{2r}\right)$
 d. the kinetic energy of the particle is $\left(-\frac{k}{r}\right)$

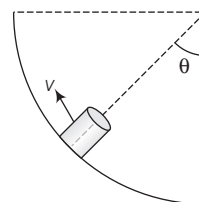
30. Kinetic energy of a particle moving along a circle of radius R depends on the distance covered as $K = as^2$, where a is a constant. Find the force acting on the particle as a function of s .

- a. $\frac{2a}{s}\sqrt{1+\left(\frac{s}{R}\right)^2}$ b. $2as\sqrt{1+\left(\frac{R}{s}\right)^2}$
 c. $2as\sqrt{1+\left(\frac{s}{R}\right)^2}$ d. $\frac{2s}{a}\sqrt{1+\left(\frac{R}{s}\right)^2}$

31. A projectile is projected at a angle 60° with horizontal with speed 10 m/s . The minimum radius of curvature of the trajectory described by the projectile is

- a. 2.55 m b. 2 m
 c. 10 m d. None of these

32. The skate board negotiates the circular surface of radius 4.5 m . At $\theta = 45^\circ$, its speed of centre of mass is 6 m/s . The combined mass of skate board and the person is 70 kg and his centre of mass is 0.75 m from the surface. The normal reaction between the surface and the skate board wheel is



- a. 500 N b. 2040 N c. 1045 N d. zero

BITSAT Archives

- A body of mass $m = 20$ g is attached to an elastic spring of length $L = 50$ cm and spring constant $k = 2 \text{ Nm}^{-1}$. The system is revolved in a horizontal plane with a frequency $\nu = 30$ rev/min. Find the radius of the circular motion and the tension in the spring.
 a. 0.25 m, 0.1 N [2014]
 b. 0.5 m, 0.52 N
 c. 0.55 m, 0.1 N
 d. 0.9 m, 0.2 N
- A car is moving on a circular road of diameter 50 m with a speed of 5 m/s. It is suddenly accelerated at a rate of 1 m/s^2 . If the mass of the car is 500 kg, then the net force acting on the car is [2011]
 a. 5 N
 b. 1000 N
 c. $500\sqrt{2}$ N
 d. $\frac{500}{\sqrt{2}}$ N
- An inclined track ends in a circular loop of radius r . From what height on the track a particle should be released so that it completes the loop, assuming there is no friction? [2010]
 a. $\frac{r}{2}$
 b. $\frac{3r}{2}$
 c. $2r$
 d. $\frac{5r}{2}$
- A small ball describes a horizontal circle on the smooth inner surface of a conical funnel. If the height of the plane of the circle above the vertex be 10 cm, what is the speed of the particle? [2009]
 a. 2 m/s
 b. 4 m/s
 c. 16 m/s
 d. 1 m/s
- The angle turned by a body undergoing circular motion depends on time as $\theta = \theta_0 + \theta_1 t + \theta_2 t^2$. Then, the angular acceleration of the body is [2006]
 a. θ_1
 b. θ_2
 c. $2\theta_1$
 d. $2\theta_2$

Answer with Solutions

Practice Exercise

- (d) In circular motion radial acceleration $a_r = \frac{v^2}{r}$ and the particle does not move along the radius of the circular path hence radial velocity is zero, radial acceleration only changes the direction of motion of the particle.
- (c) Perpendicular force does not change the magnitude of velocity hence the kinetic energy remain constant.
- (c) When the string break, there is no tension in the string, hence there is no radial acceleration to change the direction of the stone the required (tension provide centripetal force), hence the stone will move along the tangent.
- (c) First of all, we have to calculate the velocity of particles at the point B.

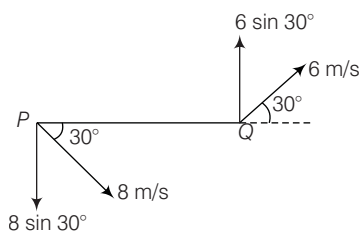
$$\therefore \text{Loss in PE} = \text{gain in KE} \quad \text{or} \quad mgh = \frac{1}{2}mv^2$$

$$\therefore v = \sqrt{2gh}$$

To prevent sliding, $v = r\omega$

$$\therefore \omega = \frac{v}{r} = \frac{\sqrt{2gh}}{r}$$

- (d)



$$\therefore \omega_{\text{rel}} = \frac{8 \sin 30^\circ + 6 \sin 30^\circ}{10} = 0.7 \text{ rad/s}$$

- (c) $\therefore \omega = \omega_0 - k\phi$

$$\text{or} \quad \frac{d\phi}{dt} = \omega_0 - k\phi$$

$$\text{or} \quad \int_0^\phi \frac{d\phi}{\omega_0 - k\phi} = \int_0^t dt$$

$$\text{or} \quad -\frac{1}{k} [\ln(\omega_0 - k\phi)]_0^\phi = t$$

$$\text{or} \quad \ln\left(\frac{\omega_0 - k\phi}{\omega_0}\right) = -kt$$

$$\therefore \phi = \frac{\omega_0}{k} (1 - e^{-kt})$$

- (c) $\therefore \mathbf{r} = x\hat{i} + y\hat{j}$

$$\text{But, } \mathbf{r} = a \cos \theta \hat{i} + b \sin \theta \hat{j}$$

$$\therefore x = a \cos \theta$$

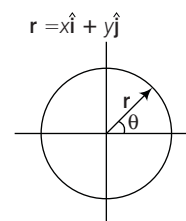
$$y = b \sin \theta$$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = \cos^2 \theta + \sin^2 \theta = 1$$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

This is an equation of ellipse.

- (c) $\therefore v_r = \frac{dr}{dt} = -0.24t$



$$v_t = r\omega = r \frac{d\theta}{dt} = r \left(\frac{6}{20} t \right) = (0.9 - 0.12 t^2) \left(\frac{6t}{20} \right)$$

$$\therefore v = \sqrt{v_r^2 + v_t^2}$$

$$\therefore \theta = \frac{3}{20} t^2 \quad \text{or} \quad \frac{\pi}{6} = \frac{3}{20} t^2$$

$$\therefore t = \sqrt{\left(\frac{20\pi}{18} \right)}$$

Putting the value of θ , $v = 0.52 \text{ m/s}$

9. (b)

10. (c) The force of tension in the string provides required centripetal force to keep the stone in circular motion.

$$\therefore T = \frac{mv^2}{R} = \frac{0.3 \times (6)^2}{1.5} = 7.2 \text{ N}$$

11. (b) $400 = 2\pi r$

$$\therefore \tan \theta = \frac{v^2}{rg} = \frac{\left(\frac{400}{20} \right)^2}{\left(\frac{400}{2\pi} \right)g}$$

$$\tan \theta = \frac{400 \times 2\pi}{400g} = \frac{6.28}{9.8} = 0.64$$

$$\therefore \theta = \tan^{-1}(0.64)$$

12. (a) Here centripetal force is provided by force of friction.

$$\therefore f = \mu_s N$$

$$\therefore f = \frac{mv^2}{r}$$

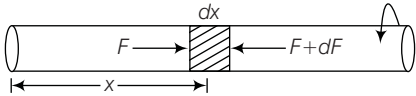
$$\text{or} \quad \mu_s N = \frac{mv^2}{r}$$

$$\text{or} \quad \mu_s mg = \frac{mv^2}{r}$$

$$\therefore v = \sqrt{\mu_s gr} = \sqrt{0.8 \times 10 \times 8} = 8 \text{ m/s}$$

13. (a)

$$dm = \frac{M}{L} dx$$

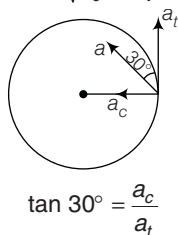
$$dF = dm x \omega^2$$


$$\int_0^L dF = \frac{M}{L} \omega^2 \int_0^L x dx$$

$$F = \frac{M}{L} \omega^2 \frac{L^2}{2} = \frac{ML\omega^2}{2}$$

14.(c) The tangential acceleration is along the tangent and a_c acts radially inwards. The resultant acceleration

$$a = \sqrt{a_c^2 + a_t^2} \quad (\text{from figure})$$



15. (b) The net acceleration to car is provided by force of friction.

$$\therefore a = \sqrt{a_n^2 + a_t^2}$$

$$\therefore F = ma = m \sqrt{\left(\frac{v^2}{r} \right)^2 + a_t^2}$$

$$\text{or} \quad 10.6 \times 10^3 = m \sqrt{\frac{v^4}{r^2} + a_t^2}$$

After solving, $a_t = 6.36 \text{ m/s}^2$

16. (b) Tangential acceleration

$$= a_t = \frac{v_f - v_i}{t} = \left(\frac{45 - 60}{8} \right) \left(\frac{22}{15} \right) = -\frac{11}{4} \text{ ft/s}^2$$

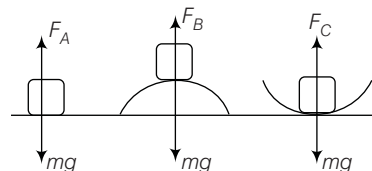
The radial acceleration is

$$a_n = \frac{v^2}{r} = \frac{\left(60 \times \frac{22}{15} \right)^2}{2500}$$

$$\therefore a = \sqrt{a_n^2 + a_t^2} = 4.14 \text{ ft/s}^2$$

17. (b)

18. (c)



For car A, $F_A = mg$... (i)

For car B, $mg - F_B = \frac{mv^2}{r}$... (ii)

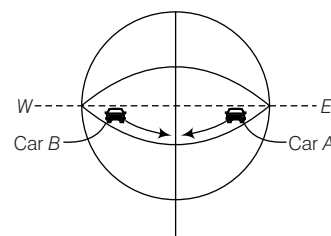
For car C, $F_C - mg = \frac{mv^2}{r}$... (iii)

$\therefore F_C > F_A > F_B$

19. (a) Earth rotates from west to east about its axis

$\Rightarrow \omega_A > \omega_B$... (i)

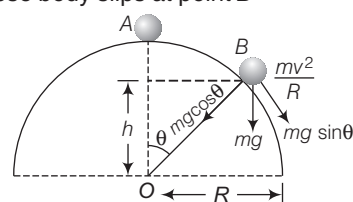
Using, $N - mg = m\omega^2 R$... (ii)



From Eqs. (i) and (ii), we get $N_A > N_B$

or $N_1 > N_2$

20. (b) Suppose body slips at point B



$$mg \cos \theta = \frac{mv^2}{R} \quad [v = \sqrt{2g(R-h)}]$$

$$g \cos \theta = \frac{2g(R-h)}{R}$$

$$\cos \theta = \frac{2(R-h)}{R}$$

$$\frac{h}{R} = \frac{2(R-h)}{R}$$

$$\left[\cos \theta = \frac{h}{R} \right]$$

$\therefore$

$$h = \frac{2}{3}R$$

21. (a) $N = \frac{mv^2}{R}$

$$\mu_s N > mg$$

$$\frac{\mu_s mv^2}{R} < mg \quad \text{or} \quad v < \sqrt{\left(\frac{Rg}{\mu_s}\right)}$$

$\therefore$

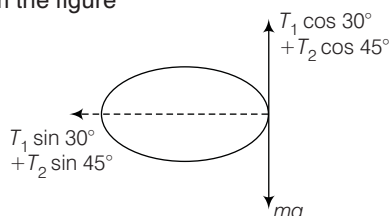
$$v_{\min} = \sqrt{\left(\frac{Rg}{\mu_s}\right)}$$

22. (c)

23. (b)

24. (b)

25. (d) From the figure



$$T_1 \cos 30^\circ + T_2 \cos 45^\circ = mg$$

$$\text{and } T_1 \sin 30^\circ + T_2 \sin 45^\circ = \frac{mv^2}{r}$$

...(i)

...(ii)

$$\text{From Eq. (i) and Eq. (ii), we get } T_1 = \frac{mg - \frac{mv^2}{r}}{\left(\frac{\sqrt{3}-1}{2}\right)}$$

But,

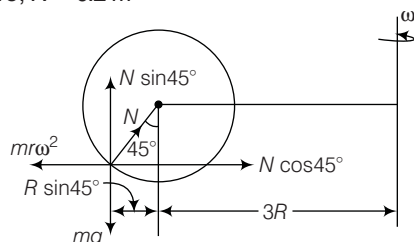
$$T_1 \geq 0$$

$$\Rightarrow T_1 = \frac{mg - \frac{mv^2}{r}}{\left(\frac{\sqrt{3}-1}{2}\right)} \geq 0$$

$$mg \geq \frac{mv^2}{r} \Rightarrow v \leq \sqrt{rg}$$

$$v_{\max} = \sqrt{rg} = \sqrt{1.6 \times 9.8} = 3.96 \text{ m/s}$$

26. (c) Here, $R = 0.2 \text{ m}$



From figure, $r = 3R + R \sin 45^\circ$

In the frame of rotating spherical chamber (non-inertial)

$$N \cos 45^\circ = mr \omega^2 \quad \dots (i)$$

$$N \sin 45^\circ = mg \quad \dots (ii)$$

Dividing Eq. (ii) by Eq. (i), we get

$$\therefore \tan 45^\circ = \frac{mg}{mr \omega^2} = \frac{g}{r \omega^2}$$

$$\therefore \omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{g}{3R + \frac{R}{\sqrt{2}}}} = 3.64 \text{ rad/s}$$

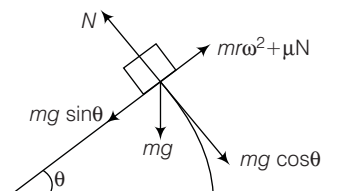
27. (d) Velocity at the bottom is $\sqrt{2gh}$

For completing the loop,

$$\sqrt{2gh} = \sqrt{5gR}$$

Hence, $R = 2h/5 = (2 \times 5)/5 = 2 \text{ cm}$

28. (b) At the time of slipping, maximum friction acts on the body.



In the frame of rod,

$$mg \sin \theta = \mu N + mr \omega^2$$

Also,

$$N = mg \cos \theta$$

$$\therefore mg \sin \theta = \mu mg \cos \theta + mr \omega^2$$

Here,

$$r = 0.45 \text{ m}, \theta = 50^\circ,$$

$$\sin 50^\circ = 0.766 \text{ and } \cos 50^\circ = 0.64$$

$$\therefore \mu = \frac{mg \sin \theta - mr \omega^2}{mg \cos \theta} = \frac{g \sin \theta - r \omega^2}{g \cos \theta} = 0.55$$

29. (a)

30. (c) According to given problem,

$$\frac{1}{2} Mv^2 = as^2 \quad [\text{Here, } M = \text{mass}]$$

$$\Rightarrow v = s \sqrt{\frac{2a}{M}} \quad \dots (i)$$

So,

$$a_g = \frac{v^2}{R} = \frac{2as^2}{MR}$$

Further more as $a_t = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = v \frac{dv}{ds}$ (By chain rule)

from Eq. (i), we get

$$\text{i.e. } v = s \sqrt{\frac{2a}{M}} \text{ yields}$$

$$\therefore a_t = \left[s \sqrt{\frac{2a}{M}} \right] \left[\frac{2a}{M} \right] = \frac{2as}{M} \quad \left(\because \frac{v}{s} = \sqrt{\frac{2a}{M}} \right)$$

So that,

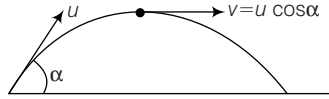
$$a_{\text{net}} = \sqrt{a_R^2 + a_t^2} = \sqrt{\left(\frac{2as^2}{MR}\right)^2 + \left(\frac{2as}{M}\right)^2} = \frac{2as}{M} \left(\sqrt{1 + \left(\frac{s}{R}\right)^2} \right)$$

$$\therefore F = Ma_{\text{net}} = 2as \sqrt{1 + (s/R)^2}$$

31. (a) The radius of curvature is given by

$$\rho = \frac{mv^3}{|\mathbf{F} \times \mathbf{v}|} = \frac{mv^3}{Fv \sin \theta} = \frac{mv^3}{mgv \sin \theta} \quad [\because F = mg]$$

or
$$\rho = \frac{v^2}{g \sin \theta}$$



$$\therefore \rho_{\min} = \frac{v^2}{g(\sin \theta)_{\max}}$$

$$\therefore \rho_{\min} = \frac{v^2}{g} \quad [\because \theta_{\max} = 90^\circ]$$

$\theta = 90^\circ$ is only possible when the projectile is at the maximum height. At the maximum height,

$$v = u \cos \alpha = 10 \cos 60^\circ = 5 \text{ m/s}$$

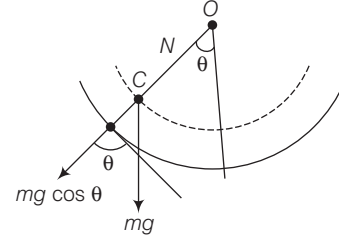
$$\therefore \rho_{\min} = \frac{5 \times 5}{9.8} = 2.55 \text{ m}$$

32. (c) From the diagram,

$$N - mg \cos \theta = \frac{mv^2}{r}$$

where, $r = CO = 4.5 - 0.75 = 3.75 \text{ m}$

$$\therefore N = \frac{mv^2}{r} + mg \cos \theta$$



From the figure, $\theta = 45^\circ$, $m = 70 \text{ kg}$, $g = 9.8 \text{ m/s}^2$,

$$v' = 6 \text{ m/s}, r = 4.5 \text{ m}$$

Putting these values in above equation, we get

$$N = 1045 \text{ N}$$

BITSAT Archives

1. (c) Angular velocity $\omega = 2\pi r = 2\pi \times \frac{30}{60} = \pi \text{ rad/s}$

For an elastic spring force $F = kx$, where x is the extension

Radius of circular motion $r = L + x$

Centripetal force $= m\omega^2 r = F$

$$\Rightarrow m(L + x)\omega^2 = kx$$

$$\Rightarrow x = \frac{mL\omega^2}{k - m\omega^2} = \frac{0.02 \times 0.5 \times (3.14)^2}{2 - 0.02 \times (3.14)^2}$$

Radius of the circular motion (r)

$$= L + x = 0.5 + 0.05 = 0.55 \text{ m}$$

Tension in the spring $T = kx$

$$= 2 \times 0.05 = 0.1 \text{ N}$$

2. (c) Given, $r = 25 \text{ m}$, $v = 5 \text{ m/s}$, $m = 500 \text{ kg}$

$$a_t = 1 \text{ m/s}^2, a_r = \frac{v^2}{r} = \frac{5 \times 5}{25} = 1 \text{ m/s}^2$$

$$a_{\text{net}} = \sqrt{a_t^2 + a_r^2}$$

$$= \sqrt{1^2 + 1^2} = \sqrt{2} \text{ m/s}^2$$

$$F = ma_{\text{net}} = 500\sqrt{2} \text{ N}$$

3. (d) For looping the loop, $v = \sqrt{5gr}$

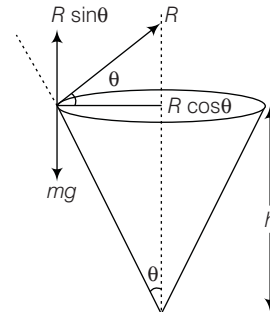
By law of conservation of energy.

Loss in potential energy = Gain in kinetic energy

$$\Rightarrow mgh = \frac{1}{2}mv^2$$

$$\Rightarrow h = \frac{5r}{2}$$

4. (d) As it is clear from figure



$$R \sin \theta = mg \quad \dots(i)$$

$$R \cos \theta = \frac{mv^2}{r} \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\therefore \tan \theta = \frac{rg}{v^2}$$

$$\text{or } \frac{r}{h} = \frac{rg}{v^2} \text{ or } v = \sqrt{gh}$$

$$v = \sqrt{10 \times \frac{1}{10}} = 1 \text{ m/s}$$

5. (d) Angle turned by the body $\theta = \theta_0 + \theta_1 t + \theta_2 t^2$

$$\text{Angular velocity, } \omega = \frac{d\theta}{dt} = \frac{d}{dt}(\theta_0 + \theta_1 t + \theta_2 t^2)$$

$$\omega = \theta_1 + 2\theta_2 t$$

$$\text{Angular acceleration, } \alpha = \frac{d\omega}{dt}$$

$$\alpha = \frac{d}{dt}(\theta_1 + 2\theta_2 t)$$

$$\alpha = 2\theta_2$$

6

Work, Energy and Power

Work

Work is said to be done when a force applied on a body displaces the body through a certain distance, in the direction of force.

The work done by the force F in displacing the body through a distance s is given by

$$W = (F \cos \theta)s = Fs \cos \theta = F \cdot s$$

where, $F \cos \theta$ is the component of the force acting along the direction of the displacement produced.

The SI unit of work is joule (J).

$$1 \text{ J} = 1 \text{ N-m}$$

Work is a scalar quantity. Work can be of three types

- (i) **Positive work** Work is said to be positive, if value of the angle θ between the directions of F and s is either zero or an acute angle.
- (ii) **Negative work** Work is said to be negative, if value of angle θ between the directions of F and s is either 180° or an obtuse angle.
- (iii) **Zero work** As work done $W = F \cdot s = Fs \cos \theta$, hence work done can be zero if both F and s are finite but the angle θ between the directions of force and displacement is 90° . In such a case,

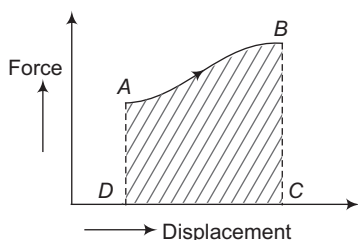
$$\begin{aligned} W &= F \cdot s \\ &= Fs \cos \theta \\ &= Fs \cos 90^\circ = 0 \end{aligned}$$

Work Done in Different Conditions

- Work done by a variable force is given by

$$W = \int F \cdot ds$$

It is equal to the area under the force-displacement graph, along with proper sign.



Work done = Area ABCDA

- The force (or field) is said to be **conservative**, if the work done by the force, also referred to as the line integral of the force, i.e. $\int \mathbf{F} \cdot d\mathbf{l}$ is independent of the path followed between any two points. In such cases, the work done simply depends upon the initial and the final positions. Moreover, work done by a conservative force for a closed path is always zero. Gravitational force, force of gravity, electrostatic force, elastic force are some examples of conservative forces (fields).
- The force is said to be **non-conservative**, if the work done by the force depends on the actual path followed by the body for a displacement between any two points. Friction, viscous force, damping force, etc., are some examples of non-conservative forces.

Energy

Energy is defined as the capacity or ability of a body to do work. Energy is scalar quantity and its units and dimensions are the same as that of work. Thus, SI unit of energy is joule. There are so many types of energy e.g. kinetic, potential, electrostatic, magnetic, geothermal, elastic, solar etc. Some of them are described below.

Some other commonly used units of energy are

$$1 \text{ erg} = 10^{-7} \text{ J}$$

$$1 \text{ cal} = 4.186 \text{ J} \cong 4.2 \text{ J}$$

$$1 \text{ kcal} = 4186 \text{ J}, 1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$$

and 1 electron volt = 1 eV = $1.60 \times 10^{-19} \text{ J}$

Kinetic Energy

Kinetic Energy (KE) is the capacity of a body to do work by virtue of its motion. Motion may be either translational or rotational.

A body of mass m , moving with a velocity v , has a kinetic energy

$$K = \frac{1}{2}mv^2$$

Thus,

$$K \propto m$$

$$K \propto v^2$$

Kinetic energy of a body is always positive irrespective of the sign of velocity v . Negative kinetic energy is impossible.

Kinetic energy is correlated with momentum as

$$K = \frac{p^2}{2m} \text{ or } p = \sqrt{2mK}$$

Kinetic energy for a system of particle will be

$$K = \frac{1}{2} \sum_i m_i v_i^2$$

Kinetic energy depends on the frame of reference. Kinetic energy of a passenger sitting in a running train is zero in the frame of reference of the train but is finite in the frame of reference of the earth.

Potential Energy

Potential Energy (PE) is energy of the body by virtue of its position, configuration or state of strain. The relation between potential energy and work done is

$$W = -\Delta U$$

where, ΔU is change in potential energy.

Change in potential energy of a body between any two points is equal to the negative of work done by the conservative force in displacing the body between these two points, without there being any change in kinetic energy. Thus,

$$dU = -dW = -\mathbf{F} \cdot d\mathbf{r}$$

and

$$\begin{aligned} U_2 - U_1 &= -W \\ &= -\int_{r_1}^{r_2} \mathbf{F} \cdot d\mathbf{r} \end{aligned}$$

Value of the potential energy in a given position can be defined only by assigning some arbitrary value to the reference point. Generally, reference point is taken at infinity and potential energy at infinity is taken as zero. In that case,

$$U = -W = -\int_{\infty}^r \mathbf{F} \cdot d\mathbf{r}$$

Potential energy is a scalar quantity but has a sign. It may be positive as well as negative.

Generally potential energy is of two types

Gravitational Potential Energy

It is the energy associated with the state of separation between two bodies which interact *via* the gravitational force. The gravitational potential energy of two particles of masses m_1 and m_2 separated by a distance r is

$$U = \frac{-Gm_1m_2}{r}$$

Generally, one of the two bodies is our earth of mass M and radius R . If m is the mass of the other body, situated at a distance r ($r \geq R$) from the centre of earth, the potential energy of the body

$$U(r) = -\frac{GMm}{r}$$

Some Important Points

- If a body of mass m is raised to a height h from the surface of earth, the change in potential energy of the system (earth+body) comes out to be

$$\Delta U = \frac{mgh}{\left(1 + \frac{h}{R}\right)} \text{ or } \Delta U \approx mgh, \text{ if } h \ll R$$

Thus, the potential energy of a body at height h , i.e. mgh is really the change in potential energy of the system for $h \ll R$.

- For the gravitational potential energy, the zero of the potential energy is chosen to be the ground.

Elastic Potential Energy

Whenever an elastic body (say a spring) is either stretched or compressed, work is being done against the elastic spring force. The work done is $W = \frac{1}{2}kx^2$

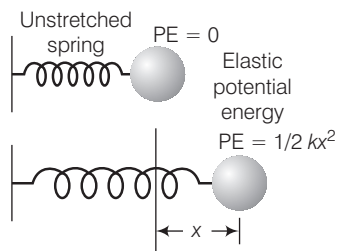
where, k is spring constant and x is the displacement and elastic potential energy

$$U = \frac{1}{2}kx^2$$

Elastic potential energy is always positive.

Potential Energy of a Spring

Elastic potential energy is equal to work done to stretch the spring, which depends upon the spring constant k as well as the distance stretched. From Hooke's law, the force required to stretch the spring will be directly proportional to the amount of stretch.



$$F = -kx \quad [\text{force by spring}]$$

Then, the work done to stretch the spring by a distance x is

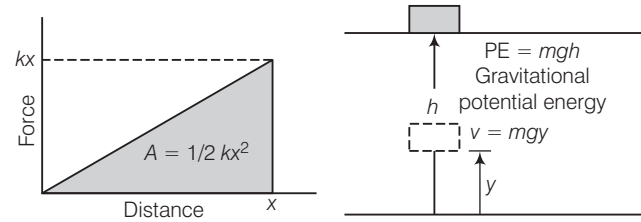
$$W = PE = \frac{1}{2}kx^2 \quad [\text{work done by stretcher}]$$

Since, the change in potential energy of an object between two positions is equal to the work that must be done to move the object from one point to the other, the calculation of potential energy is equivalent to calculating work. Since, the force required to stretch a spring changes with distance, the calculation of the work involves an integral.

$$W = \int_0^x kx \, dx = k \frac{x^2}{2}$$

Work can also be visualized as the area under the force curve. Here, object is displaced slowly.

If the potential energy function U is known, the force at any point can be obtained by taking the derivative of the potential.



$$F_y = -\frac{dU}{dy} = -\frac{d}{dy}(mgy)$$

$$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right)$$

$$F_y = -mg, F_x = -kx$$

Electric Potential Energy

The electric potential energy of two point charges q_1 and q_2 separated by a distance r in vacuum is given by

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

where, $\frac{1}{4\pi\epsilon_0} = 9.1 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2} = \text{Constant}$

Work-Energy Theorem

According to this theorem, work done by all the forces (conservative or non-conservative, external or internal) acting on a particle or an object is equal to the change in kinetic energy of it.

$$\text{i.e. } W_{\text{net}} = W_c + W_{nc} + W_{\text{ext}} = \Delta KE = K_f - K_i$$

If work is being done by the force on the body, then its kinetic energy increases, i.e. $\Delta K = +ve$. On the other hand, if work is being done by the body on the force, then its kinetic energy decreases i.e. $\Delta K = -ve$.

This theorem is true for all type of forces

(i) The work-energy theorem is not independent of the Newton's second law. It may be viewed as scalar form of second law.

(ii) When a body moves along a circular path with uniform speed, there is no change in its kinetic energy. By the work-energy theorem, the work done by the centripetal force is zero.

Law of Conservation of Energy

Total mechanical energy of a system remains constant if the internal forces are conservative and external forces do not work. We define the change in potential energy of a system corresponding to a conservative internal force as

$$U_f - U_i = -W = -\int_i^f \mathbf{F} \cdot d\mathbf{r}$$

where, W is work done by internal force on the system as system passes from the position i to f .

$$U_f - U_i = -W = -(K_f - K_i) \Rightarrow U_i + K_i = U_f + K_f$$

If non-conservative internal forces operate within the system or external forces do work on the system, then apply work-energy theorem.

$$\Rightarrow W_c + W_{nc} + W_{ext} = K_f - K_i \quad [\because W_c = -(U_f - U_i)]$$

$$\Rightarrow W_{nc} + W_{ext} = E_f - E_i$$

Power

Power is defined as the rate of doing work. If an agent does work W in time t , then its average power is given by

$$P_{av} = \frac{W}{t}$$

Power is a scalar quantity and its SI unit is watt, where

$$1W = 1J/s$$

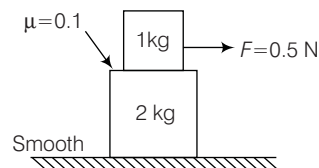
$$\text{Instantaneous power, } P_{inst} = \frac{dW}{dt} = \frac{\mathbf{F} \cdot d\mathbf{s}}{dt} = \mathbf{F} \cdot \mathbf{v}$$

The power of a system is defined as the rate of change of kinetic energy per unit time. Mathematically,

$$\text{Power, } P = \frac{d}{dt}(\text{KE}) = \frac{dK}{dt}$$

Practice Exercise

- A body of mass 10 kg is moving on a horizontal surface by applying a force of 10 N in forward direction. Find the work done by force of friction.
 - 20 J
 - 10 J
 - 20 J
 - 5 J
- In Q. (1), the work done by normal reaction is
 - 20 J
 - 196 J
 - zero
 - None of these
- A body of mass 10 kg is moving on an inclined plane of inclination 30° with an acceleration 2 m/s^2 . The body starts from rest. Find the work done by force of gravity in 2 s.
 - 10 J
 - zero
 - 98 J
 - 196 J
- In Q. (3), find the work done by force of friction.
 - 58 J
 - 58 J
 - 98 J
 - 116 J
- A body of mass 1 kg moves from point A(2m, 3m, 4m) B(3 m, 2 m, 5 m). During motion of body, a force $\mathbf{F} = (2\text{N})\hat{i} - (4\text{N})\hat{j}$ acts on it. The work done by the force on the particle during displacement is
 - $2\hat{i} - 4\hat{j}$
 - 2 J
 - 2 J
 - None of these
- During swinging of simple pendulum,
 - the work done by gravitational force is zero
 - the work done by tension force is always zero
 - the mechanical energy of bob does not remain constant in the absence of air
 - the mechanical energy remains constant in the presence of air resistance
- A force of 0.5 N is applied on upper block as shown in figure. Find the work done by lower block on upper block for a displacement 3 m of the upper block. (Take, $g = 10 \text{ m/s}^2$)
 - 1 J
 - 1 J
 - 2 J
 - 2 J
- A force $\mathbf{F} = -k(y\hat{i} + x\hat{j})$, acts on a particle moving in the xy -plane. Starting from the origin, the particle is taken along the positive x -axis to the point $(a, 0)$ and is then taken parallel to the y -axis to the point (a, a) . The total work done by the force is
 - $-2ka^2$
 - $2ka^2$
 - $-ka^2$
 - ka^2
- A particle moves along a curve of unknown shape but magnitude of force F is constant and always acts along tangent to the curve. Then,
 - $\mathbf{F}$ may be conservative
 - $\mathbf{F}$ must be conservative
 - $\mathbf{F}$ may be non-conservative
 - $\mathbf{F}$ must be non-conservative
- If a man of mass M jumps to the ground from a height h and his centre of mass moves a distance x in the time taken by him to hit the ground, the average force acting on him is
 - $\frac{Mgh}{x}$
 - $\frac{Mgx}{h}$
 - $Mg\left(\frac{h}{x}\right)^2$
 - $Mg\left(\frac{x}{h}\right)^2$



11. A particle is moving in a region where the potential U is given by $U = k(x^2 + y^2 + z^2)$. The force acting on the particle is

a. $k(\hat{i}x + \hat{j}y + \hat{k}z)$ b. $-k(\hat{i}x + \hat{j}y + \hat{k}z)$
 c. $-2k(\hat{i}x + \hat{j}y + \hat{k}z)$ d. zero

12. The potential energy is a function for the force between two atoms in a diatomic molecule is given by

$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}, \text{ where } a \text{ and } b \text{ are positive constants}$$

and x is the distance between the atoms. The position of stable equilibrium for the system of the two atoms is given by

a. $x = \frac{a}{b}$ b. $x = \frac{\sqrt{a}}{b}$ c. $x = \frac{\sqrt{3a}}{b}$ d. $x = \sqrt[6]{\left(\frac{2a}{b}\right)}$

13. The potential energy of a particle of mass 5 kg moving in the xy -plane is given by $U = (-7x + 24y)$ J. x and y being in metre. If the particle starts from rest from origin then speed of particle at $t = 2$ s is

a. 5 m/s b. 14 m/s c. 17.5 m/s d. 10 m/s

14. The potential energy of a particle of mass 5 kg moving in the xy -plane is given by $U = -7x + 24y$ J, x and y being in metre. Initially at $t = 0$, the particle is at the origin $(0, 0)$ moving with a velocity of $6[2.4\hat{i} + 0.7\hat{j}]$ m/s. The magnitude of force on the particle is

a. 25 units b. 24 units
 c. 7 units d. None of these

15. If a compressed spring is dissolved in acid, then

a. the energy of the spring increases
 b. the energy of acid decreases
 c. the potential energy and kinetic energy of molecules of acid increases
 d. the temperature of acid decreases

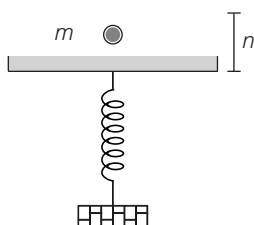
16. A long spring, when stretched by x cm has a potential energy U . On increasing the length of spring by stretching to nx cm, the potential energy stored in the spring will be

a. $\frac{U}{n^2}$ b. nU c. n^2U d. $\frac{U}{n^2}$

17. Mark the correct option.

a. The negative change in potential energy is equal to work done
 b. Mechanical energy of a system remains constant
 c. If internal forces are non-conservative, then the net work done by internal forces must be zero
 d. None of the above

18. A ball of mass m is dropped from a height h on a massless platform fixed at the top of a vertical spring as shown below. The platform is depressed by a distance x . What will be the value of the spring constant?



a. $2mg/x$ b. $2mg h/x$
 c. $2mg(h+x)/x^2$ d. $2mg(h) + 2mg hx/x^2$

19. The kinetic energy of a particle moving on a curved path continuously increases with time. Then,

a. resultant force on the particle must be parallel to the velocity at all instants
 b. the resultant force on the particle must be at an angle less than 90° all the time
 c. its height above the ground level must continuously decrease
 d. the magnitude of its linear momentum is increasing continuously
 e. both b and d are correct

20. Force F acts on a body by mass 1 kg moving with an initial velocity v_0 for 1 s. Then,

a. distance covered by the body is $v_0 + \frac{F}{2}$
 b. final velocity of body is $(v_0 + F)$
 c. momentum of body increased by F
 d. All of the above

21. When a bullet of mass 10 g and speed 100 m/s penetrates upto distance 1 cm in a human body in rest. The resistance offered by human body is

a. 2000 N b. 1500 N c. 5000 N d. 1000 N

22. A 60 g bullet is fired through a stack of fibre board sheet, 200 mm thick. If the bullet approaches the stack with a velocity of 600 m/s. The average retarding force offered to the bullet is

a. 54 kN b. 2 kN c. 20.25 kN d. 10 kN

23. A bucket tied to a string is lowered at a constant acceleration of $\frac{g}{4}$. If the mass of the bucket is m and is

lowered by a distance d , the work done by the string will be

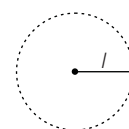
a. $\frac{mgd}{4}$ b. $-\frac{3}{4}mgd$
 c. $-\frac{4}{3}mgd$ d. $\frac{4}{3}mgd$

24. A uniform chain of length l and weight w is hanging from its ends A and B which are close together. At a given instant end B is released. The tension at A when B has fallen a distance $x < \frac{l}{2}$ is

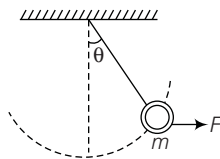
a. $\frac{w}{2} \left[\frac{3x}{l} - 2 \right]$ b. $\frac{w}{2} \left[3 - \frac{3x}{4} \right]$
 c. $\frac{w}{2} \left[1 + \frac{3x}{l} \right]$ d. $\frac{w}{2} \left[\frac{3x}{l} + 4 \right]$

25. A small sphere of mass m is suspended by a thread of length l . It is raised upto the height of suspension with thread fully stretched and released. Then, the maximum tension in thread will be

a. mg b. $2mg$ c. $3mg$ d. $6mg$



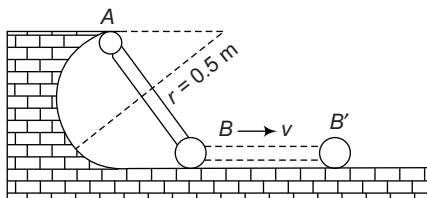
26. An object of mass m is tied to a string of length L and a variable horizontal force is applied on it which starts at zero and gradually increases until the string makes an angle θ with the vertical. Work done by the force F is
- a. $mgL(1 - \sin \theta)$ b. mgL
c. $mgL(1 - \cos \theta)$ d. $mgL(1 + \cos \theta)$



27. An elastic string of unstretched length L and force constant k is stretched by a small length x . It is further stretched by another small length y . The work done in the second stretching is
- a. $\frac{1}{2}ky^2$ b. $\frac{1}{2}k(x^2 + y^2)$
c. $\frac{1}{2}k(x + y)^2$ d. $\frac{1}{2}ky(2x + y)$

28. A body of mass m is accelerated uniformly from rest to a speed v in a time interval T . The instantaneous power delivered to the body as a function of time, is given by
- a. $\frac{mv^2}{T^2}t$ b. $\frac{mv^2}{T^2}t^2$ c. $\frac{1}{2}\frac{mv^2}{T^2}t$ d. $\frac{1}{2}\frac{mv^2}{T^2}t^2$

29. Two small balls of equal masses are joined by a light rigid rod. If they are released from rest in the position shown and slide on the smooth track in the vertical plane. The speed of balls when A reaches B 's position and B is at B' is



- a. 4 m/s
b. 4.21 m/s
c. 2.21 m/s
d. None of the above

30. A man weighing 60 kg climbs a staircase carrying a 20 kg load on his hand. The staircase has 20 steps and each step has a height of 20 cm. If he takes 20 s to climb, his power is

a. 160 W b. 230 W c. 320 W d. 80 W

31. The average human heart forces 4 L of blood per minute through arteries a pressure of 125 mm. If the density of blood is $1.03 \times 10^3 \text{ kg/m}^3$, then the power of heart is

a. $112.76 \times 10^{-6} \text{ HP}$ b. 112.76 HP
c. $1.03 \times 10^3 \text{ HP}$ d. $1.03 \times 10^{-6} \text{ HP}$

32. A bullet fired into a fixed target loses half of its velocity after penetrating distance of 3 cm. How much further it will penetrate before coming to rest, assuming that it faces constant resistance to its motion?

a. 3.0 cm b. 2.0 cm c. 1.5 cm d. 1.0 cm

33. The power supplied by a force acting on a particle moving in a straight line is constant. The velocity of the particle varies with the displacement x as

a. $\sqrt{x}$ b. x c. x^2 d. $x^{1/3}$

34. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2 r t^2$. The power is

a. $2\pi m k^2 r^2 t$ b. $m k^2 r^2 t$ c. $\frac{m k^4 r^2 t^5}{3}$ d. zero

35. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table?

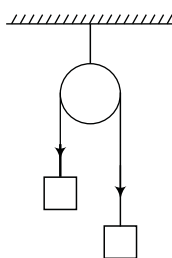
a. 7.2 J b. 3.6 J c. 120 J d. 1200 J

36. A particle move with a velocity $(5\hat{i} - 3\hat{j} + 6\hat{k}) \text{ m/s}$ under the influence of a constant force $\mathbf{F} = 10\hat{i} + 10\hat{j} + 20\hat{k}$. The instantaneous power applied to the particle is

a. 200 J/s b. 40 J/s c. 140 J/s d. 170 J/s

BITSAT Archives

1. A light inextensible string that goes over a smooth fixed pulley as shown in the figure, connect two blocks of masses 0.36 kg and 0.72 kg. Taking $g = 10 \text{ m/s}^2$. Find the work done by string on the block of mass 0.36 kg during the first second after the system is released from rest. [2013]



- a. 4 J
b. 2 J
c. 8 J
d. 10 J

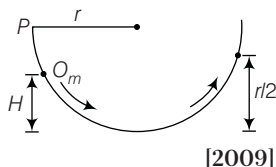
2. How much work must be done by a force on 50 kg body in order to accelerate it from rest to 20 m/s in 10 s? [2012]

a. 10^3 J b. 10^4 J c. $2 \times 10^3 \text{ J}$ d. $4 \times 10^4 \text{ J}$

3. An object placed on a ground is in stable equilibrium. If the object is given a slight push, then initially the position of centre of gravity [2012]

a. moves nearer to ground
b. rises higher above the ground
c. remains as such
d. may remain at same level

4. A shell of mass 20 kg at rest explodes into two fragments whose masses are in the ratio 2 : 3. The smaller fragment moves with a velocity of 6 ms^{-1} . The kinetic energy of the larger fragment is [2012]
 a. 96 J b. 216 J c. 144 J d. 360 J
5. The force constant of a spring gun is 50 N/m. If a ball of 20 g be shoot by the gun, so that its spring is compressed by 10 cm, the velocity of the ball is [2011]
 a. 5 m/s b. 15 m/s
 c. 25 m/s d. 20 m/s
6. A stone of mass m is pushed down a frictionless hemispherical bowl of radius r from a point O . The stone just rises up to the half of other side of the bowl as shown in the figure, what is the initial speed of stone with which it was pushed? [2009]



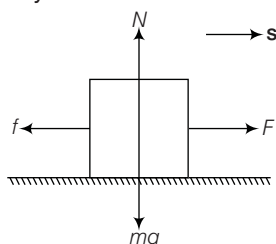
- a. $\sqrt{g(r-H)}$ b. $\sqrt{2gH}$
 c. $\sqrt{g(r-2H)}$ d. Zero

7. A river of salty water is flowing with a velocity 2 m/s. If the density of the water is 1.2 g/cc, then the kinetic energy of each cubic metre of water is [2008]
 a. 2.4 J b. 24 J c. 2.4 kJ d. 4.8 kJ
8. A ball is released from the top of a tower. The ratio of work done by force of gravity in first, second and third seconds of the motion of the ball is [2007]
 a. 1 : 2 : 3 b. 1 : 4 : 9 c. 1 : 3 : 5 d. 1 : 5 : 3
9. Two springs have their force constant as k_1 , and k_2 ($k_1 > k_2$). When they are stretched by the same force [2007]
 a. no work is done in case of both the springs
 b. equal work is done in case of both the springs
 c. more work is done in case of second spring
 d. more work is done in case of first spring

Answer with Solutions

Practice Exercise

1. (a) Since, body moves with constant velocity. So, net force on the body is zero.



Here, $N = mg$
 $F = f$
 $\therefore W = \mathbf{f} \cdot \mathbf{s} = fs \cos 180^\circ$
 $= -fs = -10 \times 2 = -20 \text{ J}$

2. (c) $\therefore W = \mathbf{F} \cdot \mathbf{s} = N \cdot \mathbf{s}$
 $= Ns \cos 90^\circ = 0$

3. (d) $s = \frac{1}{2} at^2 = \frac{1}{2} \times 2 \times 4 = 4 \text{ m}$

$\therefore$ Work done by force of gravity

$$= W = m\mathbf{g} \cdot \mathbf{s}$$

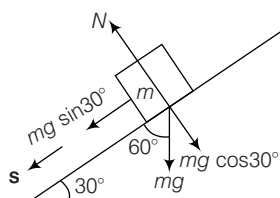
$$= mgs \cos 60^\circ$$

$$= 10 \times 9.8 \times 4 \times \frac{1}{2} = 196 \text{ J}$$

4. (d) From force diagram,

$$mg \sin 30^\circ - f = ma$$

$$\therefore f = mg \sin 30^\circ - ma$$



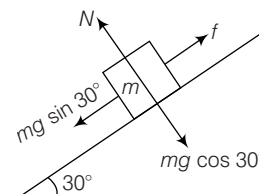
$$= 10 \times 9.8 \times \frac{1}{2} - 10 \times 2$$

$$= 49 - 20 = 29 \text{ N}$$

$$\therefore W = \mathbf{f} \cdot \mathbf{s} = fs \cos 180^\circ$$

$$= -fs = -29 \times 4$$

$$= -116 \text{ J}$$



5. (c) The displacement of body is

$$\mathbf{AB} = \mathbf{r}_B - \mathbf{r}_A$$

$$= (3\hat{i} + 2\hat{j} + 5\hat{k}) - (2\hat{i} + 3\hat{j} + 4\hat{k})$$

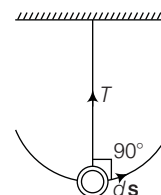
$$= \hat{i} - \hat{j} + \hat{k}$$

$$\therefore W = \mathbf{F} \cdot \mathbf{AB}$$

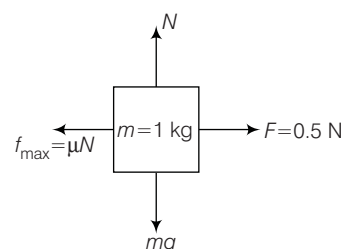
$$= (2\hat{i} - 4\hat{j}) \cdot (\hat{i} - \hat{j} + \hat{k})$$

$$= 2 - 4 = -2 \text{ J}$$

6. (b) Since, the tension force is in perpendicular direction of instantaneous displacement of bob. Hence, work done is zero. [$\therefore W = \int F ds \cos 90^\circ = 0$]



7. (b) From force diagram shown in figure.



$$\therefore f_{\max} = \mu N = \mu mg = 0.1 \times 1 \times 10 = 1 \text{ N}$$

Since, $F < f$

So, both blocks move together. The acceleration of system is

$$a = \frac{F}{1+2} = \frac{0.5}{3} = \frac{5}{30} = \frac{1}{6} \text{ m/s}^2$$

For lower block,

$$f = 2a = 2 \times \frac{1}{6} = \frac{1}{3} \text{ N}$$

So, the force of friction due to lower block on upper block is $\frac{1}{3}$ N in the opposite direction of displacement of the block.

$\therefore$ Work done by lower block on upper block is

$$W = -fs = -\frac{1}{3} \times 3 = -1 \text{ J}$$

$$8. (c) W_1 = \int_0^a \mathbf{F} \cdot d\mathbf{x} = \int_0^a -k(y\hat{i} + x\hat{j}) \cdot \hat{i} dx$$

$$= \int_0^a -k(0\hat{i} + x\hat{j}) \cdot \hat{i} dx = \text{zero}$$

$$W_2 = \int_0^a \mathbf{F} \cdot d\mathbf{y} = \int_0^a -k(y\hat{i} + x\hat{j}) \cdot \hat{j} dy$$

$$= \int_0^a -k(a\hat{i} + a\hat{j}) \cdot \hat{j} dy = -ka \int_0^a dy = -ka^2$$

Total work done,

$$W = W_1 + W_2 = 0 - ka^2 = -ka^2$$

9. (d) Since, work done by this force depends upon path followed by the particle between two points. Hence, the force must be non-conservative.

$$10. (a) \text{ Here, } F = -\frac{\Delta U}{\Delta x} = -\frac{(-Mgh)}{x} = \frac{Mgh}{x}$$

$$11. (c) U = k(x^2 + y^2 + z^2) = kr^2$$

$$F = -\frac{dU}{dr} = -\frac{d}{dr}(kr^2)$$

$$-2kr = -kr = -2k(\hat{i}x + \hat{j}y + \hat{k}z)$$

$$12. (d) \therefore U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$$

At the stable equilibrium,

$$\frac{dU}{dx} = 0 \Rightarrow -\frac{12a}{x^{13}} + \frac{6b}{x^7} = 0 \Rightarrow x = \left(\frac{2a}{b}\right)^{1/6}$$

$$13. (d) \mathbf{F} = -\frac{\partial U}{\partial x} \hat{i} - \frac{\partial U}{\partial y} \hat{j} = 7\hat{i} - 24\hat{j}$$

$$\therefore a_x = \frac{F_x}{m} = \frac{7}{5} = 1.4 \text{ m/s}^2$$

$$a_y = \frac{F_y}{m} = -\frac{24}{5}$$

$$= 4.8 \text{ m/s}^2 \text{ along negative } y\text{-axis}$$

$$\therefore v_x = a_x t = 1.4 \times 2$$

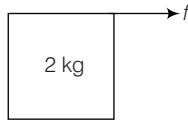
$$= 2.8 \text{ m/s along negative } y\text{-axis}$$

$$\text{and } v_y = 4.8 \times 2 = 9.6 \text{ m/s}$$

$$\therefore v = \sqrt{v_x^2 + v_y^2} = 10 \text{ m/s}$$

$$14. (a) \mathbf{F} = -\frac{\partial U}{\partial x} \hat{i} - \frac{\partial U}{\partial y} \hat{j} = 7\hat{i} - 24\hat{j}$$

$$\therefore |\mathbf{F}| = \sqrt{(7)^2 + (-24)^2} = 25 \text{ units}$$



15. (c) The spring loses elastic potential energy which appears as energy of molecules of acid.

$$16. (c) \text{ Initially potential energy} = \frac{1}{2} kx^2$$

$$\Rightarrow U = \frac{1}{2} kx^2 \text{ or } 2U = kx^2 \Rightarrow k = \frac{2U}{x^2}$$

When it is stretched to nx cm.

$$\therefore \text{PE} = \frac{1}{2} kx_1^2 = \frac{1}{2} \times \frac{2U}{x^2} \times n^2 x^2 = n^2 U$$

$\therefore$ Potential energy stored in the spring $= n^2 U$

17. (d) (a) This statement is correct only when forces are of conservative nature.

(b) In the absence of non-conservative force, mechanical energy and non-mechanical energy are separately constant.

(c) The work done by internal forces may not be zero.

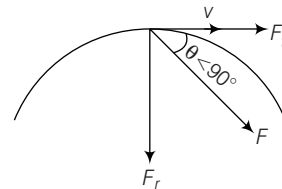
$$18. (c) \text{ Here, } mg(h+x) = \frac{1}{2} kx^2$$

$$\Rightarrow kx^2 = 2mg(h+x) \Rightarrow k = \frac{2mg(h+x)}{x^2}$$

19. (e) If particle moves on a curved path, then there are two components of force

(i) **Tangential Component** This component provides tangential acceleration. Due to this, speed increases.

(ii) **Radial Force** This component provides centripetal acceleration.



The resultant force is

$$F = \sqrt{F_r^2 + F_t^2}$$

$$\tan \theta = \frac{F_r}{F_t}$$

But F_t is not zero.

$$\therefore \tan \theta < \infty \Rightarrow \theta < 90^\circ$$

When speed increases, $p = mv$ increases.

$$20. (d) a = \frac{F}{m} = \frac{F}{1} = F \text{ m/s}^2$$

$$v = u + at$$

$$v = v_0 + (F \times 1) \Rightarrow v = v_0 + F$$

where, v = final velocity

$$s = ut + \frac{1}{2} at^2$$

$$s = v_0 + \frac{F}{2}$$

$$p_i \text{ (initial momentum)} = mv_0 - v_0$$

$$p_f \text{ (final momentum)} = mv = v_0 + F$$

$$\text{Change in momentum, } \Delta p = p_f - p_i = F$$

21. (c) Here, $u = 100 \text{ m/s}$, $v = 0$, $s = 1 \text{ cm} = 10^{-2} \text{ m}$

$$\therefore v^2 = u^2 - 2as$$

$$\therefore a = \frac{u^2}{2s} = \frac{100 \times 100}{2 \times 1 \times 10^{-2}} = 5 \times 10^5 \text{ m/s}^2$$

$$\therefore F = ma = 10 \times 10^{-3} \times 5 \times 10^5 = 5000 \text{ N}$$

Alternate method

$$W = \Delta K \quad \text{or} \quad -Fs = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$= 0 - \frac{1}{2} \times 10 \times 10^{-3} \times 10000$$

$$\text{or} \quad -F \times 10^{-2} = -\frac{1}{2} \times 10 \times 10^{-3} \times 10^4$$

$$\therefore F = 5000 \text{ N}$$

22. (a) $m = 60 \text{ g} = \frac{60}{1000} = 0.06 \text{ kg}$

$$s = 200 \text{ mm} = 0.2 \text{ m}$$

$$u = 600 \text{ m/s}$$

$$v = 0$$

$$\text{Using} \quad v^2 = u^2 + 2as$$

$$(0)^2 = (600)^2 + 2 \times a \times 0.2$$

$$a = \frac{-600 \times 600 \times 10}{0.4}$$

Average retarding force

$$F = ma = \frac{0.06 \times 600 \times 600 \times 10}{4}$$

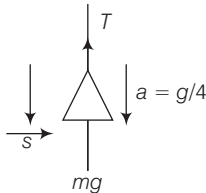
$$F = 54 \text{ kN}$$

23. (b) From force diagram as shown in figure, $mg - T = ma$

$$\therefore T = mg - ma = mg - \frac{mg}{4} = \frac{3mg}{4}$$

$$\therefore W_T = \text{work done by tension}$$

$$= \mathbf{T} \cdot \mathbf{s} = Ts \cos 180^\circ = -\frac{3mgd}{4}$$



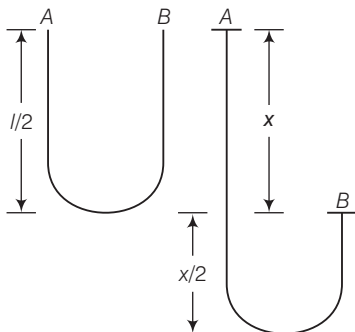
24. (c) Mass per unit length, $\lambda = \frac{m}{l} = \frac{w}{lg}$

$$\text{Velocity, } v^2 = 2g\left(\frac{x}{2}\right) \text{ or } v = \sqrt{gx}$$

Change in momentum when an element dx falls is

$$\frac{w}{lg} \sqrt{gx} \, dx$$

$$F = \frac{dp}{dt} = \frac{w}{lg} \sqrt{gx} \frac{dx}{dt} = \frac{w}{l} x \quad \left[\because \frac{dx}{dt} = \sqrt{gx} \right]$$



Tension at A = Weight of half the chain

$$+ \text{Weight of } \frac{x}{2} \text{ length} + F$$

$$= \frac{w}{2} + \frac{wx}{2l} + \frac{wx}{l} = \frac{w}{2} \left[1 + \frac{3x}{l} \right]$$

- 25 (c) The tension in string is maximum at the lowest position of the ball.

$$\therefore T_{\max} - mg = \frac{mv^2}{l}$$

$$\therefore T_{\max} = mg + \frac{mv^2}{l} \dots (i)$$

The loss in potential energy = gain in kinetic energy

$$\text{or} \quad mgl = \frac{1}{2}mv^2$$

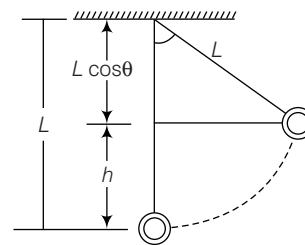
$$\therefore v = \sqrt{2gl} \dots (ii)$$

$$\therefore \frac{mv^2}{l} = \frac{m}{l} 2gl = 2mg \dots (iii)$$

From Eqs. (i), (ii) and (iii),

$$T_{\max} = mg + 2mg = 3mg$$

26. (c)



$$W = \Delta K \quad \text{or} \quad W_T + W_g + W_F = 0$$

[Since, change in kinetic energy is zero]

Here, $W_T = \text{work done by tension} = 0$

$W_g = \text{work done by force of gravity}$

$$= -mgh = -mgL(1 - \cos \theta)$$

$$\therefore W_F = -W_g = mgL(1 - \cos \theta)$$

27. (d) Elastic force in string is conservative in nature.

$$\therefore W = -\Delta U$$

where, $W = \text{work done by elastic force of string}$

$\Delta U = \text{change in elastic potential energy}$

$$\therefore W = -(U_f - U_i) = U_i - U_f$$

$$\text{or} \quad W = \frac{1}{2}kx^2 - \frac{1}{2}k(x+y)^2$$

$$\text{or} \quad W = \frac{1}{2}kx^2 - \frac{1}{2}k(x^2 + y^2 + 2xy)$$

$$= \frac{1}{2}kx^2 - \frac{1}{2}kx^2 - \frac{1}{2}ky^2 - \frac{1}{2}k(2xy)$$

$$= -kxy - \frac{1}{2}ky^2 = \frac{1}{2}ky(-2x - y)$$

The work done against elastic force is

$$W_{\text{external}} = -W = \frac{ky}{2}(2x + y)$$

28. (a) $F = ma = \frac{mv}{T}$ $\left[\because a = \frac{v-0}{T} \right]$

Instantaneous power $= Fv = mav = \frac{mv}{T} at$
 $= \frac{mv}{T} \frac{v}{T} t = \frac{mv^2}{T^2} t$

29. (c) The final speeds of each ball are same.
 $\therefore$ Loss in PE by ball A = gain in KE by the system

or $mgr = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$

or $mgr = mv^2$

$\therefore v = \sqrt{gr} = \sqrt{9.8 \times 0.5} = \sqrt{4.9}$
 $= \sqrt{\left(\frac{49}{10}\right)} = 2.21 \text{ m/s}$

30. (a) Power $= \frac{(60 + 20) \times (10) \times (20 \times 0.2)}{20} = 160 \text{ W}$

31. (a) Height of liquid column, $h = 125 \text{ mm} = 0.125 \text{ m}$

Density of blood, $\rho = 1.03 \times 10^3 \text{ kg/m}^3$.

Acceleration due to gravity, $g = 9.8 \text{ m/s}^2$

Volume of blood, $V = 4 \text{ litre} = 4 \times 10^{-3} \text{ m}^3$

Time, $t = 1 \text{ minute} = 60 \text{ second}$.

Work done, $W = \text{force} \times \text{distance}$
 $= \text{pressure} \times \text{area} \times \text{distance}$
 $= \text{pressure} \times \text{volume}$
 $= h\rho gV$

Power, $P = \frac{W}{t} = \frac{h\rho gV}{t}$
 $= \frac{0.125 \times 1.03 \times 10^3 \times 9.8 \times 4 \times 10^{-3}}{60}$
 $= 112.76 \times 10^{-6} \text{ HP}$

32. (d) According to the work-energy theorem,

$$W = \Delta K$$

Case I $-F \times 3 = \frac{1}{2}m\left(\frac{v_0}{2}\right)^2 - \frac{1}{2}mv_0^2$

where, F is the resistive force and v_0 is the initial speed.

Case II Let the further distance travelled by the bullet before coming to rest is s .

$\therefore -F(3+s) = K_f - K_i = -\frac{1}{2}mv_0^2$
 $\Rightarrow -\frac{1}{8}mv_0^2(3+s) = -\frac{1}{2}mv_0^2$

or $\frac{1}{4}(3+s) = 1$

or $\frac{3}{4} + \frac{s}{4} = 1$

or $s = 1 \text{ cm}$

33. (d) $\because P = Fv = (ma)v = m\left(\frac{d^2x}{dt^2}\right)\left(\frac{dx}{dt}\right)$

Since, power is constant.

$\therefore \left(\frac{d^2x}{dt^2}\right)\left(\frac{dx}{dt}\right) = k$

or $\frac{d}{dt}\left(\frac{dx}{dt}\right)^2 = k$ or $\left(\frac{dx}{dt}\right)^2 = k_1t$

or $\frac{dx}{dt} = \sqrt{k_1t} = k_2t^{1/2}$ or $x = k_3t^{3/2}$

Hence, $\frac{dx}{dt} \propto t^{1/2} \propto x^{1/3}$

34. (b) $a_c = \frac{v^2}{r} = k^2rt^2$

$\therefore v = krt$

The tangential acceleration is

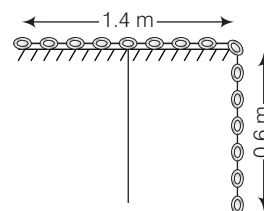
$$a_t = \frac{dv}{dt} = \frac{d(krt)}{dt} = kr$$

The work done by centripetal force will be zero.

So, power is delivered to the particle by only tangential force which acts in the same direction of instantaneous velocity.

$\therefore \text{Power} = F_t v$
 $= ma_t krt$
 $= m(kr)(krt) = mk^2r^2t$

35. (b) Mass per unit length $= \frac{M}{L} = \frac{4}{2} = 2 \text{ kg m}^{-1}$



The mass of 0.6 m of chain $= 0.6 \times 2 = 1.2 \text{ kg}$

The height of the centre of mass of the hanging part

$$h = \frac{0.6 + 0}{2} = 0.3 \text{ m}$$

Hence, work done in pulling the chain on the table =

work done against gravity the force of gravity

i.e.

$$W = mgh$$

$$= 1.2 \times 10 \times 0.3 = 3.6 \text{ J}$$

36. (c) $P_{\text{inst}} = \mathbf{F} \cdot \mathbf{v}$

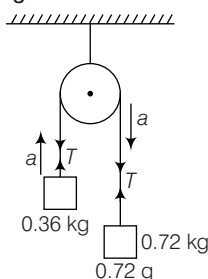
$$= (10\hat{i} + 10\hat{j} + 20\hat{k}) \cdot (5\hat{i} - 3\hat{j} + 6\hat{k})$$

$$= 50 - 30 + 120$$

$$= 140 \text{ J/s}$$

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1. (c) According to figure



$$\text{So, acceleration} = \frac{\text{Net pulling force}}{\text{Total mass}} = \frac{0.72g - 0.36g}{0.72 + 0.36} = \frac{g}{3}$$

$$\text{Distance, } s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times \frac{g}{3} \times (1)^2 = \frac{g}{6}$$

$$\begin{aligned} \text{So, } T - mg &= ma \\ T - 0.36g &= 0.36a \\ T &= 0.48g \end{aligned}$$

Now, work done by string on the block

$$\begin{aligned} W_T &= Ts \cos 0^\circ \text{ (on 0.36 kg of mass)} \\ &= (0.48g) \left(\frac{g}{6} \right) (1) = 0.08g^2 \\ &= 0.08 \times (10)^2 = 8 \text{ J} \end{aligned}$$

2. (b) Now,
- $s = ut + \frac{1}{2}at^2$

$$s = 0 + \frac{1}{2} \times 2 \times 10 \times 10$$

$$s = 100 \text{ m}$$

Hence, work done

$$W = F \times s$$

$$W = ma \times s$$

$$\therefore W = 50 \times 2 \times 100$$

$$W = 10000 = 10^4 \text{ J}$$

3. (b) In stable equilibrium, the centre of gravity of object, lies at minimum height from ground. As the object is given a slight push, its centre of gravity rises because it comes in unstable equilibrium.

4. (a) Total mass of the shell = 20 kg

Ratio of the masses of the fragments = 2 : 3

 $\therefore$ Masses of the fragments are 8 kg and 12 kg. Now, according to the conservation of momentum

$$m_1 v_1 = m_2 v_2$$

$$\therefore 8 \times 6 = 12 \times v$$

 v (velocity of the larger fragment) = 4 m/s

$$\begin{aligned} \text{Kinetic energy} &= \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 12 \times (4)^2 = 96 \text{ J} \end{aligned}$$

5. (a)
- $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$$\begin{aligned} v &= \sqrt{\frac{R}{m}x} \\ &= \sqrt{\frac{50}{20 \times 10^{-3}}} (10 \times 10^{-2}) \\ &= 50 \times 10^{-1} \\ &= 5 \text{ m/s} \end{aligned}$$

6. (c) Let the velocity with which stone is pushed =
- u
- .

We have,

$$\begin{aligned} \frac{1}{2}mu^2 + mgH &= mg \frac{r}{2} \\ u &= \sqrt{2g \left(\frac{r}{2} - H \right)} \end{aligned}$$

or

$$u = \sqrt{g(r - 2H)}$$

7. (c) Given, velocity of river,
- $(v) = 2 \text{ m/s}$

Density of water $\rho = 1.2 \text{ g/cc}$

Mass of each cubic metre

$$\begin{aligned} m &= \frac{1.2 \times 10^{-3}}{(10^{-2})^3} \\ &= 1.2 \times 10^3 \text{ kg} \end{aligned}$$

$$\begin{aligned} \therefore \text{Kinetic energy} &= \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 1.2 \times 10^3 \times (2)^2 = 2.4 \text{ kJ} \end{aligned}$$

8. (c) When the ball is released from the top of tower, then ratio of distance covered by the ball in first, second and third seconds is

$$h_I : h_{II} : h_{III} = 1 : 3 : 5 \quad [\text{because } h_n \propto (2n - 1)]$$

 $\therefore$ Ratio of work done

$$mgh_I : mgh_{II} : mgh_{III} = 1 : 3 : 5$$

9. (c)
- $W = \frac{F^2}{2k}$

If both springs are stretched by same force, then $W \propto \frac{1}{K}$

As

$$K_1 > K_2$$

Therefore, $W_1 > W_2$

i.e. more work is done in case of second spring.

7

Centre of Mass, Momentum and Collision

Centre of Mass

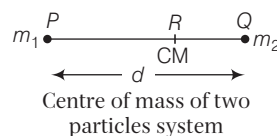
The **centre of mass** of a body is a point where the whole mass of the body is supposed to be concentrated for describing its translatory motion. In a uniform gravitational field, the centre of mass and the centre of gravity of a system are coincident.

Centre of Mass of Two Particles System

Consider the two particles system in which two masses m_1 and m_2 are separated by a distance d .

The Centre of Mass (CM) divides internally the line joining two particles (or masses) in inverse ratio of their masses.

$$PR = \frac{m_2 d}{m_1 + m_2} \text{ and } QR = \frac{m_1 d}{m_1 + m_2}$$



Centre of Mass of N-Particles System

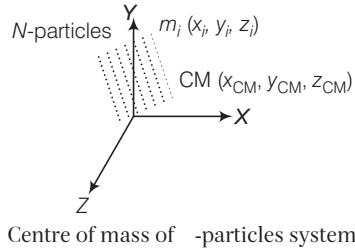
If there are N -particles distributed in space whose masses are $m_1, m_2, m_3, \dots, m_N$ and position vectors $r_1, r_2, r_3, \dots, r_N$ respectively, then the coordinates of the centre of mass are given as below :

$$X_{\text{CM}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots + m_N x_N}{m_1 + m_2 + m_3 + \dots + m_N}, \quad Y_{\text{CM}} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \dots + m_N y_N}{m_1 + m_2 + m_3 + \dots + m_N}$$

$$Z_{\text{CM}} = \frac{m_1 z_1 + m_2 z_2 + m_3 z_3 + \dots + m_N z_N}{m_1 + m_2 + m_3 + \dots + m_N}$$

We can also write as,

$$X_{CM} = \frac{1}{M} \sum m_i x_i; \quad Y_{CM} = \frac{1}{M} \sum m_i y_i; \quad Z_{CM} = \frac{1}{M} \sum m_i z_i$$



where, $M = m_1 + m_2 + m_3 + \dots + m_N$

The position vector of centre of mass of N -particles system is given by

$$\mathbf{r}_{CM} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2 + m_3 \mathbf{r}_3 + \dots + m_N \mathbf{r}_N}{m_1 + m_2 + m_3 + \dots + m_N}$$

Important Formulae of Centre of Mass

- Velocity of centre of mass of N -particles system is given by

$$\mathbf{v}_{CM} = \frac{\sum_{i=1}^n m_i \mathbf{v}_i}{M}$$

- Similarly, acceleration of centre of mass is given by

$$\mathbf{a}_{CM} = \frac{\sum_{i=1}^n m_i \mathbf{a}_i}{M}$$

- In accordance with Newton's second law of motion

$$\mathbf{F}_{CM} = \mathbf{F}_1 + \mathbf{F}_2 + \dots + \mathbf{F}_n$$

or

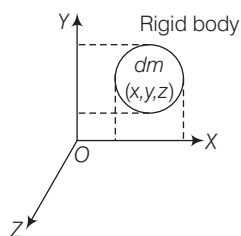
$$\mathbf{F}_{CM} = \sum_{i=1}^n \mathbf{F}_i$$

- This is called as equation of motion of centre of mass. In pure translation, every particle of the body moves with the same velocity at any instant of time.

Rigid Body

It is a continuous solid body of finite size in which deformation is not possible, i.e. the relative distance between the two given points within the rigid body is always constant whatever the force is applied on it. Practically, there is no existence of it. In order to assume a body to be completely rigid, deformation would be neglected.

For a rigid body which is not under any external force, centre of mass is always a fixed point while in case of body which is not rigid, the centre of mass can vary.



Coordinates of centre of mass of a rigid body are

$$X_{CM} = \frac{1}{M} \int x \, dm, \quad Y_{CM} = \frac{1}{M} \int y \, dm$$

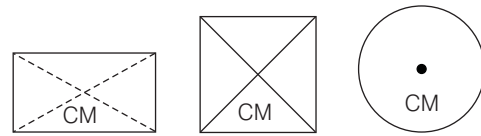
and

$$Z_{CM} = \frac{1}{M} \int z \, dm$$

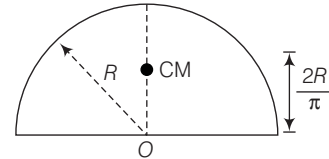
where, M is total mass of the rigid body and x, y and z are the coordinates of elementary mass dm .

Centre of Mass of Some Rigid Bodies

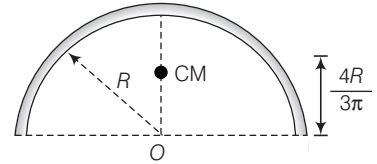
- Centre of mass of a uniform rectangular, square or circular plate lies at its centre.



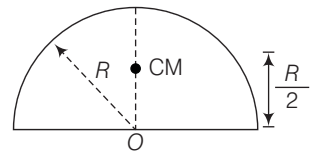
- Centre of mass of a uniform semi-circular ring lies at a distance of $h = \frac{2R}{\pi}$ from its centre on the axis of symmetry where R is the radius of the ring.



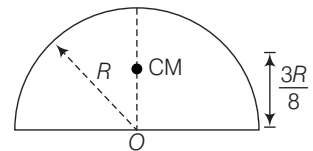
- Centre of mass of a uniform semi-circular disc of radius R lies at a distance of $h = \frac{4R}{3\pi}$ from the centre on the axis of symmetry as shown in figure.



- Centre of mass of a hemispherical shell of radius R lies at a distance of $h = \frac{R}{2}$ from its centre on the axis of symmetry as shown in figure.



- Centre of mass of a solid hemisphere of radius R lies at a distance of $h = \frac{3R}{8}$ from its centre on the axis of symmetry. If some mass or area is removed from a rigid body, then the position of centre of mass of the remaining portion is obtained from the following formula



$$\mathbf{r}_{CM} = \frac{m_1 \mathbf{r}_1 - m_2 \mathbf{r}_2}{m_1 - m_2} \quad \text{or} \quad \mathbf{r}_{CM} = \frac{A_1 \mathbf{r}_1 - A_2 \mathbf{r}_2}{A_1 - A_2}$$

In rotation about a fixed axis, every particle of the rigid body moves in a circle which is in a plane perpendicular to the axis and has its centre on the axis.

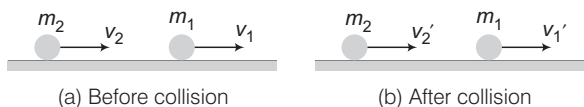
Collision

It is defined as an isolated event in which two or more colliding bodies exert relatively strong forces on each other for a relatively short time, as a result of which the kinetic energy and momentum of the colliding particles change.

Types of Collision

There are two basic types of collision in one dimension as given below :

- (i) **Elastic collision** The collision is said to be elastic, if both the bodies regain their original shape and size completely, after they are separated. Also, the total linear momentum as well as kinetic energy of bodies remain conserved.



Let the two balls of mass m_1 and m_2 collide each other elastically with their respective velocities v_1 and v_2 in the same direction as shown in figure. Their velocities become v_1' and v_2' after the collision along the same line. Applying conservation of linear momentum, we get

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \quad \dots(i)$$

In an elastic collision, kinetic energy before and after collision is also conserved. Hence,

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 \quad \dots(ii)$$

Solving Eqs. (i) and (ii) for v_1' and v_2' , we get

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_1 + \left(\frac{2m_2}{m_1 + m_2} \right) v_2 \quad \dots(iii)$$

$$\text{and } v_2' = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) v_2 + \left(\frac{2m_1}{m_1 + m_2} \right) v_1 \quad \dots(iv)$$

- (ii) **Inelastic collision** The collision is said to be inelastic, if the bodies do not return to their original form after the collision. In this type of collision, the total linear momentum of the system of bodies will remain conserved but kinetic energy of the system is not conserved.

$$\text{i.e. } \mathbf{p}_{\text{initial}} = \mathbf{p}_{\text{final}}$$

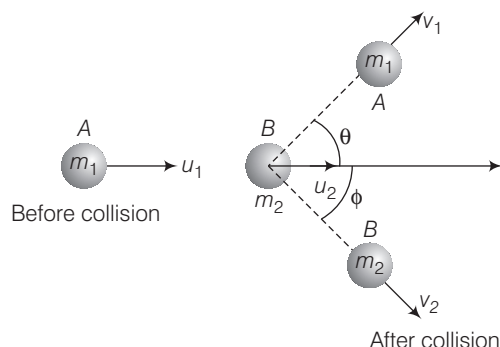
$$\Rightarrow \Delta \mathbf{p} = 0$$

$$\text{But, } K_{\text{initial}} \neq K_{\text{final}}$$

$$\Rightarrow \Delta K \neq 0$$

Perfectly Elastic Collision in a Plane

In a two-dimensional (or oblique) collision between two bodies, momentum remains conserved.



$\therefore$ Along the x -axis

$$m_1 u_1 + m_2 u_2 = m_1 v_1 \cos \theta + m_2 v_2 \cos \phi \quad \dots(i)$$

and along the y -axis

$$0 = m_1 v_1 \sin \theta - m_2 v_2 \sin \phi \quad \dots(ii)$$

As the total kinetic energy remains unchanged.

$$\text{Hence, } \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad \dots(iii)$$

We can solve these equations provided that either the value of θ or ϕ is known to us.

Rebounding of a Ball on Collision with the Floor

1. Speed of the ball after the n th rebound

$$v_n = e^n v_0 = e^n \sqrt{2gh_0}$$

2. Height covered by the ball after the n th rebound

$$h_n = e^{2n} h_0$$

3. Total distance (vertical) covered by the ball before it stops bouncing

$$H = h_0 + 2h_1 + 2h_2 + 2h_3 + \dots = h_0 \left(\frac{1+e^2}{1-e^2} \right)$$

4. Total time taken by the ball before it stops bouncing

$$T = t_0 + t_1 + t_2 + t_3 + \dots$$

$$= \sqrt{\frac{2h_0}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + \dots = \sqrt{\frac{2h_0}{g}} \left(\frac{1+e}{1-e} \right)$$

Coefficient of Restitution (e)

For a collision, it is defined as the ratio of relative velocity of separation to the relative velocity of approach.

$$\text{Thus, coefficient of restitution, } e = \frac{v_2 - v_1}{u_1 - u_2}$$

For a perfectly elastic collision, $e = 1$.

If $0 < e < 1$, the collision is said to be partially elastic. But if, $e = 0$, the collision is said to be perfectly inelastic.

- In a perfectly inelastic collision, $e = 0$ which means that $v_2 - v_1 = 0$ or $v_2 = v_1$
- It can be shown that for an inelastic collision, the final velocities of the colliding bodies are given by

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2} \right) u_1 + \frac{(1+e)m_2}{(m_1 + m_2)} u_2$$

$$\text{and } v_2 = \frac{(1+e)m_1}{(m_1 + m_2)} u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2} \right) u_2$$

- If a particle of mass m , moving with velocity u , hits an identical stationary target inelastically, then final velocities of projectile and target are correlated as

$$\text{i.e. } m_1 = m_2 = m$$

$$\text{and } u_2 = 0; \frac{v_1}{v_2} = \frac{1-e}{1+e}$$

Important Concepts Regarding the Collision

- Two bodies of mass m_1 (heavy) and mass m_2 (light) are moving with same kinetic energy. If they are stopped by the same retarding force, then
 - The bodies cover the same distance before coming to rest.
 - The time taken to come to rest is lesser for m_2 as it has less momentum, i.e. $t = \frac{p}{F}$.
 - The time taken to come to rest is more for m_1 as it has greater momentum.

- Two bodies A and B having masses m_1 and m_2 have equal kinetic energies. If they have velocities v_1 and v_2 , then

$$\frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}} \text{ and } \frac{p_1}{p_2} = \sqrt{\left(\frac{m_1}{m_2} \right)}$$

where, p_1 and p_2 are their momenta.

- If a bullet of mass m_1 moving with a velocity u , strikes a mass m_2 which is free to move in the direction of the bullet and is embedded in it, then the loss of kinetic energy is $\frac{1}{2} \left(\frac{m_1 m_2 u^2}{m_1 + m_2} \right)$.
- A shell of mass m_1 is ejected from a gun of mass m_2 by an explosion which generates kinetic energy equals to E . Then, the initial velocity of the shell is

$$\sqrt{\left[\frac{2m_2 E}{m_1 (m_1 + m_2)} \right]}$$

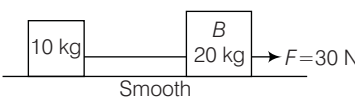
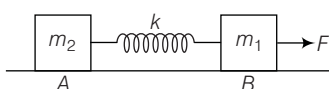
- A gun of mass m_2 fires a shell of mass m_1 horizontally and the energy of explosion is such as would be sufficient to project the shell vertically to a height h . Then, the velocity or recoil of the gun is

$$\sqrt{\left[\frac{2m_1^2 gh}{m_2 (m_1 + m_2)} \right]}$$

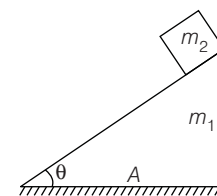
- A bullet of mass m_1 penetrates a thickness of a fixed plate of mass m_2 . If m_2 is free to move and the resistance is supposed to be uniform, then the thickness penetrated is $\frac{m_2 s}{m_1 + m_2}$.

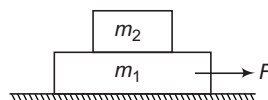
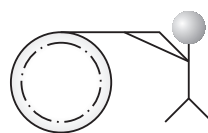
Practice Exercise

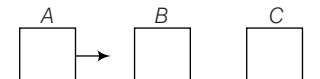
- Two homogeneous spheres A and B of masses m and $2m$ having radii $2a$ and a respectively, are placed in touch. The distance of centre of mass from first sphere is
 - a
 - $2a$
 - $3a$
 - None of these
- A circular hole of radius 1 cm is cut-off from a disc of radius 6 cm. The centre of hole is 3 m from the centre of the disc. Find the position of centre of mass of the remaining disc from the centre of disc.
 - $-\frac{3}{35}$ cm
 - $\frac{1}{35}$ cm
 - $\frac{3}{10}$ cm
 - None of these
- A non-uniform thin rod of length L is placed along X-axis as such its one end is at the origin. The linear mass density of rod is $\lambda = \lambda_0 x$. Find the distance of centre of mass of rod from the origin.
 - $L/2$
 - $2L/3$
 - $L/4$
 - $L/5$
- Which of the following has centre of mass not situated in the material of body?
 - A rod bent in the form of a circle
 - Football
 - Hand ring
 - All of the above

5. A ball kept in a closed box moves in the box making collisions with the walls. The box is kept on a smooth surface. The centre of mass
- of the box remains constant
 - of the box plus the ball system remains constant
 - of the ball remains constant
 - of the ball relative to the box remains constant
6. A man of mass M stands at one end of a plank of length L which lies at rest on a frictionless surface. The man walks to the other end of the plank. If the mass of plank is $M/3$, then the distance that the mass moves relative to the ground is
- $3L/4$
 - $L/4$
 - $4L/5$
 - $L/3$
7. The centre of mass of a system cannot change its state of motion, unless there is an external force acting on it. Yet the internal force of the brakes can bring a car to rest. Then
- the brakes stop the wheels
 - the friction between the brake pads and the wheel stops the car
 - the car is stopped by the road
 - the car is stopped by the driver pressing the pedal
8. Two blocks A and B are connected by a massless string (shown in figure) A force of 30 N is applied on block B . Find the distance travelled by centre of mass in 2 s starting from rest.
- 
- 1 m
 - 2 m
 - 3 m
 - None of these
9. A loaded spring gun of mass M fires a shot of mass m with a velocity v at an angle of elevation θ . The gun is initially at rest on a horizontal frictionless surface. After firing, the centre of mass of the gun-shot system
- moves with a velocity $v \frac{m}{M}$
 - moves with a velocity $\frac{vm}{M} \cos \theta$ in the horizontal direction
 - remains at rest
 - moves with a velocity $\frac{v(M-m)}{(M+m)}$ in the horizontal direction
10. Two bodies A and B of masses m_1 and m_2 respectively are connected by a massless spring of force constant k . A constant force F starts acting on the body A at $t = 0$. Then,
- 
- at every instant, the acceleration of centre of mass is $\frac{F}{m_1 + m_2}$
 - at $t = 0$, acceleration of B is zero but that of A is maximum
 - the acceleration of A decreases continuously and finally becomes equal
 - All of the above

11. In the given figure, the mass m_2 starts with velocity v_0 and moves with constant velocity on the surface. During motion, the normal reaction between the horizontal surface and fixed triangular block m_1 is N . Then, during motion



- $N = (m_1 + m_2)g$
 - $N = m_1g$
 - $N < (m_1 + m_2)g$
 - $N > (m_1 + m_2)g$
12. Four particles of masses 1 kg , 2 kg , 3 kg and 4 kg are placed at the corners A, B, C and D respectively of a square $ABCD$ of edge 1 m . If point A is taken as origin, edge AB is taken along X -axis and edge AD is taken along Y -axis. Find the coordinates of centre of mass in SI
- $(1, 1)$
 - $(5, 7)$
 - $(0.5, 0.7)$
 - None of these
13. Three bodies having masses 5 kg , 4 kg and 2 kg are moving at the speed of 5 m/s , 4 m/s and 2 m/s , respectively along X -axis. The magnitude of velocity of CM is
- 1.0 m/s
 - 4 m/s
 - 0.9 m/s
 - 1.3 m/s
14. Figure shows the situation involving two blocks of masses m_1 and m_2 . The heavier block of mass m_1 is pulled by a force F . The acceleration of CM is (all the surfaces are smooth)
- 
- $\frac{F}{m_2}$
 - $\frac{F}{m_1}$
 - $\frac{F}{m_1 + m_2}$
 - $\frac{2F}{m_1 m_2}$
15. A particle is projected such that its horizontal range would be R . At the highest point, the particle breaks into two identical parts P and Q . If P comes to rest, the horizontal distance of point from the point of projection (i.e. origin) where the particle Q lands on the ground is
- $1.5 R$
 - R
 - $2.5 R$
 - $2 R$
16. A string of negligible thickness is wrapped several times around a cylinder kept on a rough horizontal surface. A boy standing at a distance l from the cylinder holds one end of the string and pulls the cylinder towards him. Assuming no slipping, the length of the thread passed through the hands of the man is
- 
- $\frac{l}{2}$
 - l
 - $2l$
 - $\frac{3l}{2}$
17. If momentum of a body remains constant. Then mass-speed graph of body is
- circle
 - straight line
 - rectangular hyperbola
 - parabola

18. Two bodies of masses m and $4m$ are moving with equal linear momentum. The ratio of their kinetic energies is
 a. 1 : 4 b. 4 : 1
 c. 1 : 1 d. 1 : 2
19. A man is sitting in a moving train, then
 a. his momentum must not be zero
 b. his kinetic energy is zero
 c. his kinetic energy is not zero
 d. his kinetic energy may be zero
20. When a meteorite burns in the atmosphere. Then,
 a. the momentum conservation principle is applicable to the meteorite system
 b. the energy of meteorite remains constant
 c. the conservation principle of momentum is applicable to a system consists of meteorites, earth and air molecules
 d. the meteorite momentum remains constant
21. If a bullet is fired from a gun, then
 a. the mechanical energy of bullet-gun system remains constant
 b. the mechanical energy is converted into non-mechanical energy
 c. the mechanical energy may be conserved
 d. the non-mechanical energy is converted into mechanical energy
22. A nucleus moving with a velocity $\mathbf{n}$ emits an α -particle. Let the velocities of the α -particle and the remaining nucleus be $\mathbf{n}_1$ and $\mathbf{n}_2$ and their masses be m_1 and m_2 , then
 a. $\mathbf{n}$, $\mathbf{n}_1$ and $\mathbf{n}_2$ must be parallel to each other
 b. none of the two of $\mathbf{n}$, $\mathbf{n}_1$, $\mathbf{n}_2$ should be parallel to each other.
 c. $\mathbf{n}_1 + \mathbf{n}_2$ must be parallel to $\mathbf{n}$
 d. $m_1 \mathbf{n}_1 + m_2 \mathbf{n}_2$ must be parallel to $\mathbf{n}$
23. Two blocks of mass m_1 and m_2 are connected by a massless spring and placed at smooth surface. The spring initially stretched and released. Then,
 a. the momentum of each particle remains constant separately
 b. the momentum of both bodies are same to each other
 c. the magnitude of momentum of both bodies are same to each other
 d. the mechanical energy of system remains constant
 e. Both (c) and (d) are correct
24. When two blocks A and B coupled by a spring on a frictionless table are stretched and then released, then
 a. kinetic energy of body at any instant after releasing is inversely proportional to their masses
 b. kinetic energy of body at any instant may or may not be inversely proportional to their masses
 c. $\frac{\text{KE of } A}{\text{KE of } B} = \frac{\text{mass of } B}{\text{mass of } A}$, when spring is massless
 d. Both (b) and (c) are correct
25. Two bodies are projected from roof with same speed in different directions. If air resistance is not taken into account. Then,
 a. they reach at ground with same magnitude of momenta if bodies have same masses
 b. they reach at ground with same kinetic energy
 c. they reach at ground with same speed
 d. both (a) and (c) are correct
26. If a ball is dropped from rest. It bounces from the floor respectively. The coefficient of restitution is 0.5 and the speed just before the first bounce is 5 m/s. The total time taken by the ball to come to the rest is
 a. 2 s b. 1 s c. 0.5 s d. 0.25 s
27. Three identical blocks A , B and C are placed on horizontal frictionless surface. The blocks B and C are in rest. But A is approaching towards B with a speed of 10 m/s. The coefficient of restitution for all collision is 0.5. The speed of the block C just after collision is
 a. 5.6 m/s b. 6 m/s c. 8 m/s d. 10 m/s
- 
28. A thin uniform bar lies on a frictionless horizontal surface and is free to move in any way on the surface. Its mass is 0.16 kg and length is 1.7 m. Two particles each of mass 0.08 kg are moving on the same surface and towards the bar in the direction perpendicular to the bar, one with a velocity of 10 m/s and other with velocity of 6 m/s. If collision between particles and bar is completely inelastic. Both particles strike with the bar simultaneously. The velocity of centre of mass after collision is
 a. 2 m/s b. 4 m/s c. 10 m/s d. 16 m/s
29. When two bodies collide elastically. The force of interaction between them is
 a. conservative
 b. non-conservative
 c. either conservative or non-conservative
 d. zero
30. A body of mass M moving with a speed u has a head-on collision with a body of mass m originally at rest. If $M \gg m$, then the speed of the body of mass m after collision will be nearly
 a. $\frac{um}{M}$ b. $\frac{uM}{m}$ c. $\frac{u}{2}$ d. $2u$
31. A ball moving with a certain velocity hits another identical ball at rest. If the plane is frictionless and collision is elastic, then the angle between the directions in which the balls move after collision, will be
 a. 30° b. 60° c. 90° d. 120°
32. A shell is fired from a cannon with a velocity v at an angle θ with the horizontal direction. At the highest point in its path, it explodes into two pieces, one retraces its path to the cannon and the speed of the other pieces immediately after the explosion is
 a. $3v \cos \theta$ b. $2v \cos \theta$
 c. $\left(\frac{3}{2}\right)v \cos \theta$ d. $\frac{\sqrt{3}}{2}v \cos \theta$

7. If gas molecules undergo inelastic collision with the walls of the container, [2009]
 a. temperature of gas will increase
 b. temperature of gas will fall
 c. pressure of the gas will increase
 d. neither the temperature nor the pressure will change
8. A ball is dropped from a height h on a floor of coefficient of restitution e . The total distance covered by the ball just before second hit is [2008]
 a. $h(1 - 2e^2)$ b. $h(1 + 2e^2)$ c. $h(1 + e^2)$ d. he^2
9. Two particles A and B initially at rest, move towards each other, under mutual force of attraction. At an

instance when the speed of A is v and speed of B is $2v$, the speed of centre of mass (CM) is [2008]

- a. zero b. v c. $2.5v$ d. $4v$

10. Consider the following statement. When jumping from some height, you should bend your knees as you come to rest instead of keeping your legs stiff. Which of the following relations can be useful in explaining the statement? [2007]

- a. $\Delta \mathbf{p}_1 = -\Delta \mathbf{p}_2$ b. $\Delta E = -\Delta(\text{PE} + \text{KE}) = 0$
 c. $\mathbf{F} \Delta t = m \Delta \mathbf{v}$ d. $\Delta \mathbf{x} \propto \Delta \mathbf{F}$

where, symbols have their usual meanings.

Answer with Solutions

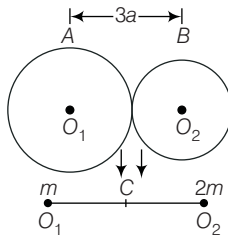
Practice Exercise

1. (b) We assume that origin is situated at the O_1 .

$$\therefore x_1 = 0, m_1 = m$$

$$x_2 = 3a, m_2 = 2m$$

$$\begin{aligned} \therefore x_{\text{CM}} &= \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \\ &= \frac{m \times 0 + 2m \times 3a}{m + 2m} \\ &= \frac{6ma}{3m} = 2a \end{aligned}$$



2. (a) For the calculation of the position of centre of mass, cut-off mass is taken as negative. The mass of disc is

$$m_1 = \pi r_1^2 \sigma = \pi 6^2 \sigma = 36\pi\sigma$$

where, σ = Surface mass density

The mass of cutting portion is

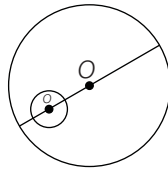
$$m_2 = \pi (1)^2 \sigma = \pi\sigma$$

$$x_{\text{CM}} = \frac{m_1 x_1 - m_2 x_2}{m_1 + m_2}$$

Taking origin at the centre of disc,

$$x_1 = 0, x_2 = 3 \text{ cm}$$

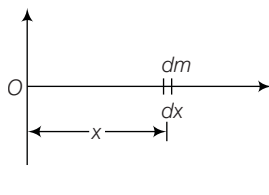
$$\begin{aligned} x_{\text{CM}} &= \frac{36\pi\sigma \times 0 - \pi\sigma \times 3}{36\pi\sigma - \pi\sigma} \\ &= \frac{-3\pi\sigma}{35\pi\sigma} = -\frac{3}{35} \text{ cm} \end{aligned}$$



3. (b) The mass of considered element is

$$dm = \lambda dx = \lambda_0 x dx$$

$$\therefore x_{\text{CM}} = \frac{\int_0^L x dm}{\int_0^L dm} = \frac{\int_0^L x (\lambda_0 x dx)}{\int_0^L \lambda_0 x dx}$$



$$\begin{aligned} &= \frac{\lambda_0 \left[\frac{x^3}{3} \right]_0^L}{\lambda_0 \left[\frac{x^2}{2} \right]_0^L} = \frac{\lambda_0 \frac{L^3}{3}}{\lambda_0 \frac{L^2}{2}} = \frac{2}{3} L \end{aligned}$$

4. (d) (a) If a rod is bent in the form of a circle. Its centre of mass is at the centre of curvature.

(b) Football is in the shape of hollow sphere. So, its centre of mass is at the centre of sphere.

(c) The reason is same as (a).

In all the above geometrical shapes, no material is present at the centre of mass.

5. (b) Since, no external force is present on the system. So, the centre of mass of system will not be changed.
6. (b) Since, external force on system is zero. So, no change takes place in the centre of mass.

$$x_{\text{CM}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

or

$$\Delta x_{\text{CM}} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$

Here,

$$\Delta x_{\text{CM}} = 0$$

$$\therefore m_1 \Delta x_1 + m_2 \Delta x_2 = 0 \text{ or } M \Delta x_1 + \frac{M}{3} \Delta x_2 = 0$$

$$\therefore \Delta x_1 = -\frac{M}{3M} \Delta x_2 = -\frac{\Delta x_2}{3} \quad \dots (i)$$

$$\text{But, } x_{\text{rel}} = \Delta x_1 - \Delta x_2 \text{ or } L = \Delta x_1 - \Delta x_2$$

$$\text{or } L = -\frac{\Delta x_2}{3} - \Delta x_2 = -\frac{4}{3} \Delta x_2 \quad [\text{from Eq. (i)}]$$

$$\therefore \Delta x_2 = -\frac{3}{4} L$$

$$\therefore \Delta x_1 = -\frac{\Delta x_2}{3} = -\frac{-3L}{4 \times 3} = -\frac{L}{4}$$

Negative sign indicates that both move in opposite direction.

7. (c) The car is stopped by the contact force exerted due to road.

8. (b) The acceleration of centre of mass is

$$a_{CM} = \frac{F}{m_A + m_B} = \frac{30}{10 + 20} = 1 \text{ m/s}^2$$

$$\therefore s = \frac{1}{2} a_{CM} t^2 = \frac{1}{2} \times 1 \times 2^2 = 2 \text{ m}$$

9. (c) If we consider a system consists of gun plus mass m . Then, no external force is present on the system. So, the acceleration of centre of mass of system will be zero. Since, initially the system is in rest. So, centre of mass will always remains at rest.

10. (d) (a) Since, external force on the system is F .

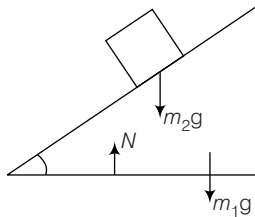
$$\therefore a_{CM} = \frac{F}{m_1 + m_2}$$

(b) At $t = 0$, elongation in spring is zero. So, force on m_2 is zero but force on m_1 is only in forward direction, i.e. F .

$$\therefore a_1 = \frac{F}{m_1}, \text{ at } t = 0$$

This is maximum acceleration of m_1 .

11. (c) When mass m_2 moves downwards, the centre of mass of system ($m_1 + m_2$) moves downwards. It means the acceleration is found in centre of mass in downward direction during motion of m_2 . This is possible only when net downward force is greater than that of upward force.



Mathematically,

$$m_1g + m_2g > N$$

$$\therefore N < m_1g + m_2g$$

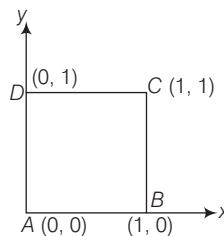
12. (c) $x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C + m_D x_D}{m_A + m_B + m_C + m_D}$
- $$= \frac{1 \times 0 + 2 \times 1 + 3 \times 1 + 4 \times 0}{1 + 2 + 3 + 4}$$
- $$= \frac{2 + 3}{10} = \frac{1}{2} = 0.5 \text{ m}$$

Similarly,

$$y_{CM} = \frac{m_A y_A + m_B y_B + m_C y_C + m_D y_D}{m_A + m_B + m_C + m_D}$$

$$= \frac{1 \times 0 + 2 \times 0 + 3 \times 1 + 4 \times 1}{1 + 2 + 3 + 4}$$

$$= \frac{7}{10} = 0.7 \text{ m}$$



13. (b) As, $v_{CM} = \frac{m_1 v_1 + m_2 v_2 + m_3 v_3}{m_1 + m_2 + m_3}$
- $$= \frac{5 \times 5 + 4 \times 4 + 2 \times 2}{5 + 4 + 2}$$
- $$= \frac{25 + 16 + 4}{11}$$
- $$\therefore v_{CM} = \frac{45}{11} \approx 4.09 \approx 4 \text{ m/s}$$

14. (c) FBD of two blocks

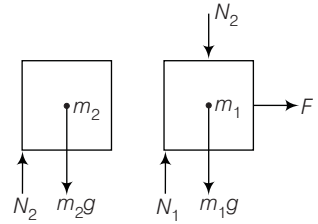
$$\text{For } m_1, a_1 = \frac{F}{m_1}$$

$$\text{and } N_2 + m_1g = N_1$$

$$\text{For } m_2, a_2 = 0, N_2 = m_2g$$

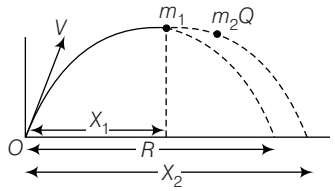
$$\therefore a_{CM} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$$

$$= \frac{m_1 \times \frac{F}{m_1} + 0}{m_1 + m_2} = \frac{F}{m_1 + m_2}$$



15. (a) Here, $m_1 = m_2 = m$ (suppose), when P comes to rest, it falls vertically downward.

$$X_1 = \frac{R}{2}$$



Centre of mass,

$$X_{CM} = \frac{m_1 X_1 + m_2 X_2}{m_1 + m_2}$$

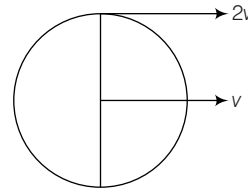
$$R = \frac{m \frac{R}{2} + m X_2}{2m}$$

$$[\because X_{CM} = R]$$

$\Rightarrow$

$$X_2 = 1.5 R$$

16. (c) If velocity of centre of mass is v , then velocity of contact is 0 and that of the top is $2v$, hence when centre of mass covers a distance l , thread covers a distance $2l$.



17. (c)

$$p = mv$$

$\therefore$

$$m = \frac{p}{v}$$

Hence, m - v graph will be rectangular hyperbola.

18. (b) $\frac{E_{k1}}{E_{k2}} = \frac{\frac{p_1^2}{2m_1}}{\frac{p_2^2}{2m_2}} = \frac{m_2}{m_1} = \frac{4m}{m} = 4:1$ $[\because p_1 = p_2]$

19. (d) The momentum and kinetic energy depend upon reference frame. In the frame of train, the man is in rest. So, the momentum and kinetic energy of the man in the frame of train are zero. But in the frame of ground, velocity of the man is non-zero. So, in the frame of ground, momentum and kinetic energy are not zero.

20. (c) Since, gravitational force is present. So, conservation principle is only applicable to a system consists of air molecules + earth + meteorite.

21. (d) (a) During firing of the bullet from gun, the chemical energy of powder of bullet is converted into the kinetic energy of the system. So, final mechanical energy of system is greater than initial kinetic energy of the system.

(b) In this case, chemical energy (non-mechanical) is converted into mechanical energy of the system.

(c) The same reason as the above.

(d) The same reason as (a) and (b).

22. (d) Since, no external force is present. So, momentum conservation principle is completely applicable.

$$\therefore m \mathbf{n} = m_1 \mathbf{n}_1 + m_2 \mathbf{n}_2$$

$$\text{or } (m_1 + m_2) \mathbf{n} = m_1 \mathbf{n}_1 + m_2 \mathbf{n}_2$$

23. (e) Since, no external force is present on the system. So, conservation principle of momentum is applicable.

$$\therefore \mathbf{p}_i = \mathbf{p}_f = \mathbf{p}_1 + \mathbf{p}_2$$

$$\therefore \mathbf{p}_1 = -\mathbf{p}_2 \quad [\because \mathbf{p}_i = 0]$$

$$\therefore |\mathbf{p}_1| = |\mathbf{p}_2|$$

$$\therefore \mathbf{p}_1 = \mathbf{p}_2$$

From this point of view, it is clear that momenta of both particles are equal in magnitude but opposite in direction.

Also, friction is absent. So, total mechanical energy of system remains conserved.

24. (d) When spring is massless. Then according to momentum conservation principle,

$$\mathbf{p}_i = \mathbf{p}_f \text{ or } 0 = m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2$$

$$\therefore m_1 \mathbf{v}_1 = -m_2 \mathbf{v}_2$$

$$\therefore m_1 v_1 = m_2 v_2 \text{ or } p_1 = p_2$$

$$\therefore K_1 = \frac{p_1^2}{2m_1} \Rightarrow K_2 = \frac{p_2^2}{2m_2}$$

$$\therefore \frac{K_1}{K_2} = \frac{m_2}{m_1} \quad [\because p_1 = p_2]$$

25. (d) In the case of projectile motion, if bodies are projected with same speed, they reach at ground with same speeds. So, if bodies have same masses, then momentum of body will be different for different directions of projection but magnitude of momenta must be same.

26. (c) $\therefore$ Acceleration, $a = \frac{v - u}{t}$ or $a = \frac{v - v_0}{t}$

$$\text{or } g = \frac{v - v_0}{t} \quad [\text{considered average force}]$$

$$\therefore v = 0$$

Speed before first bounce

$$v_0 = 5 \text{ m/s } (\uparrow)$$

$$\therefore t = \frac{v - v_0}{g} = \frac{0 - (-5)}{10} = \frac{5}{10} = 0.5 \text{ s}$$

27. (a) For collision between blocks A and B,

$$e = \frac{v_B - v_A}{u_A - u_B} = \frac{v_B - v_A}{10 - 0} = \frac{v_B - v_A}{10}$$

$$\therefore v_B - v_A = 10e = 10 \times 0.5 = 5 \quad \dots (i)$$

From momentum conservation principle,

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$\text{or } m \times 10 + 0 = m v_A + m v_B$$

$$\therefore v_A + v_B = 10 \quad \dots (ii)$$

Adding Eqs. (i) and (ii), we get

$$v_B = 7.5 \text{ m/s}$$

Similarly for collision between B and C,

$$v_C - v_B = 7.5e = 7.5 \times 0.5 = 3.75$$

$$\therefore v_C - v_B = 3.75 \text{ m/s} \quad \dots (iii)$$

$$\text{and } v_C + v_B = 7.5 \quad \dots (iv)$$

Adding Eqs. (iii) and (iv), we get

$$2v_C = 11.25$$

$$\therefore v_C = \frac{11.25}{2} = 5.6 \text{ m/s}$$

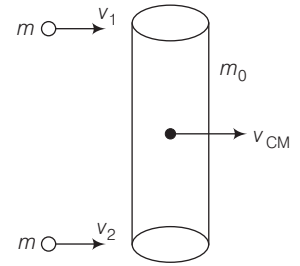
28. (b) Here, $m = 0.08 \text{ kg}$

$$m_0 = 0.16 \text{ kg}$$

According to conservation principle of momentum

$$mv_1 + mv_2 = (2m + m_0) v_{CM}$$

$$\begin{aligned} \therefore v_{CM} &= \frac{mv_1 + mv_2}{2m + m_0} \\ &= \frac{0.08 \times 16}{0.16 + 0.16} \\ &= \frac{1.28}{0.32} = \frac{128}{32} = 4 \text{ m/s} \end{aligned}$$



29. (a)

30. (d)

31. (c)

32. (a) As we know that at the highest point, the shell has only the horizontal component of velocity which is $v \cos \theta$. If u be the velocity of second exploded piece, then applying conservation of linear momentum along X-axis.

$$\therefore 2mv \cos \theta = -mv \cos \theta + m$$

$$\text{or } u = 3v \cos \theta$$

33. (b) Since, no force is present along the surface of plate. So, momentum conservation principle for ball is applicable along the surface of plate.

$$mv \sin \theta_1 = mv_1 \sin \theta_2$$

$$\text{or } v \sin \theta_1 = v_1 \sin \theta_2 \quad \dots (i)$$

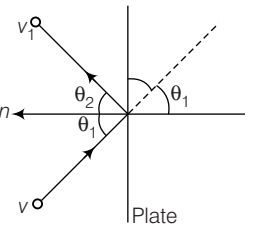
$$e = \frac{v_1 \cos \theta_2}{v \cos \theta_1} = \frac{v_1 \cos \theta_2}{v \cos \theta} \quad [\text{Let } \theta_1 = \theta]$$

$$\therefore v_1 \cos \theta_2 = ev \cos \theta \quad \dots (ii)$$

$$\therefore \frac{v_1 \sin \theta_2}{v_1 \cos \theta_2} = \frac{v \sin \theta}{ev \cos \theta} = \frac{\tan \theta}{e}$$

$$\therefore \tan \theta_2 = \frac{\tan \theta}{e}$$

$$\therefore \theta_2 = \tan^{-1} \left(\frac{\tan \theta}{e} \right)$$



34. (c) $F = v \frac{\Delta m}{\Delta t}$

$$\text{Here, } \frac{\Delta m}{\Delta t} = \frac{nm}{t} = \frac{120 \times 10 \times 10^{-3}}{60}$$

$$= 20 \times 10^{-3} \text{ kg/s}$$

$$\therefore = 800 \times 20 \times 10^{-3} \text{ N} = 16 \text{ N}$$

35. (c) $F = \frac{\Delta p}{\Delta t} = v \frac{\Delta m}{\Delta t} = v \frac{nm}{t}$

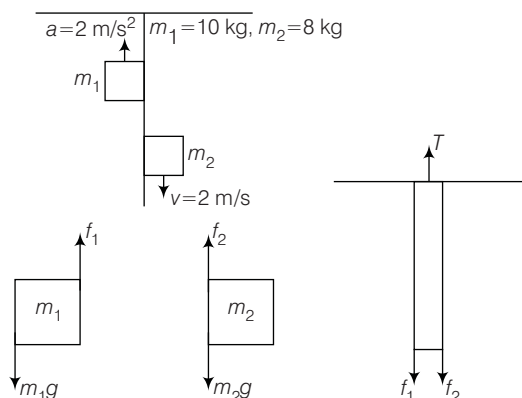
$$F = vm \frac{n}{60} = \frac{mvn}{60}$$

36. (a) Since, m_2 moves with constant velocity.

$$\therefore \quad \begin{aligned} f_2 &= m_2 g \\ f_2 &= 8 \times 10 = 80 \text{ N} \end{aligned}$$

Since, block m_1 moves with acceleration $a = 2 \text{ m/s}^2$ in upward direction

$$\begin{aligned} \therefore \quad f_1 - m_1 g &= m_1 a \\ \therefore \quad f_1 &= m_1 g + m_1 a = 10 \times 10 + 10 \times 2 = 120 \text{ N} \end{aligned}$$



For equilibrium of rope,

$$T = f_1 + f_2 = (120 + 80) \text{ N} = 200 \text{ N}$$

37. (a) 38. (a)

39. (b) Initially from conservation of momentum theorem, for bullet and block system

$$mv' = (m + M)v \Rightarrow v' = \frac{mv}{(m + M)}$$

Let the bullet-block system rises to height h , so from conservation of mechanical energy,

$$\frac{1}{2}(m + M)v'^2 = (m + M)gh$$

$$h = \frac{v'^2}{2g}$$

$$\Rightarrow \quad h = \frac{m^2 v^2}{(m + M)^2} \times \frac{1}{2g}$$

40. (a) From definition, we can write coefficient of restitutions,

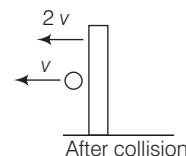
$$e = \frac{v' - 2v}{v + 2v}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{v' - 2v}{3v}$$

$$\Rightarrow \quad 3v' = 9v$$

$$\Rightarrow \quad v' = 3v$$

As wood sheet is heavier than ball, so its speed will not change.



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1. (b) In any type of collision, the linear momentum of the system remain conserved even during collision.

2. (a) Total mass of the shell = 20 kg

Ratio of the masses of the fragments = 2 : 3

$\therefore$ Masses of the fragments are 8 kg and 12 kg. Now, according to the conservation of momentum

$$m_1 v_1 = m_2 v_2 \quad \therefore 8 \times 6 = 12 \times v$$

v (velocity of the larger fragment) = 4 m/s

$$\text{Kinetic energy} = \frac{1}{2} m v^2 = \frac{1}{2} \times 12 \times (4)^2 = 96 \text{ J}$$

3. (a) Momentum is conserved in non-elastic collision but kinetic energy is not conserved.

4. (d) A ball bearing striking another ball bearing is not an example of perfectly inelastic collision.

5. (b) Since, if momentum is conserved, then

$$mu = mv_1 + mv_2$$

$$\Rightarrow \quad u = v_1 + v_2 \dots (i)$$

As we know that coefficient of restitution can be given as

$$e = \frac{v_1 - v_2}{u} \dots (ii)$$

From Eqs. (i) and (ii), we get

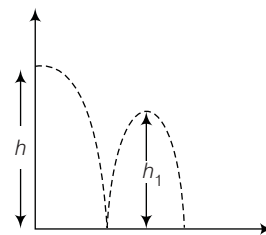
$$e = \frac{v_1 - v_2}{v_1 + v_2} \dots (iii)$$

On solving Eq. (iii), we get $\frac{v_1}{v_2} = \frac{1+e}{1-e}$

6. (c) Thrust = $u \frac{dm}{dt} = Mg$ or $\frac{dm}{dt} = \frac{Mg}{u} = \frac{600 \times 9.8}{1000} = 6 \text{ kgs}^{-1}$

7. (d) Usually, temperature of the gas in a container is same as the temperature of the walls of the container. Therefore, during collision, there is no exchange of energy. Hence, whether collision is elastic or inelastic, molecules rebound with same average speed. Therefore, neither the temperature nor the pressure will change.

8. (b) Total distance travelled by the ball before its second hit is



$$H = h + 2h_1 = h[1 + 2e^2] \quad [\because h_1 = he^2]$$

9. (a) As initially both the particles were at rest, therefore velocity of centre of mass was zero and there is no external force on the system. So, speed of centre of mass remains constant, i.e. it should be equal to zero.

10. (c) $F \Delta t = m \Delta v \Rightarrow F = \frac{m \Delta v}{\Delta t}$

By doing so, time of change in momentum increases and impulsive force on knees decreases.

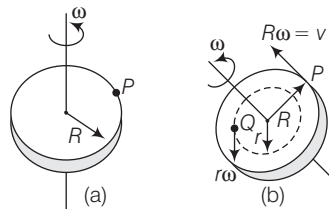
8

Rotational Motion

Concept of Rotational Motion

If a force is exerted on some body that does not pass through its centre of mass, then it can produce a turning effect in the body about its centre of mass. If the body is fixed, then there will not be its linear motion. Thus, it will go under pure rotation. But if the body is not fixed, then there will be a linear motion along with the rotational motion.

Terms Related to Rotational Motion of the Body



(R and r are two radii)

(i) Linear speed

$$v = \omega R$$

[Its SI unit is m/s]

(ii) Angular speed

$$\omega = \frac{2\pi}{T}$$

[Its SI unit is rad/s]

Also,

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$

[where, $\Delta \theta$ = angular displacement]

(iii) Angular acceleration

$$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt} = \frac{\text{Change in angular speed}}{\text{Time taken}}$$

α is same for all the points of the rotational body. Its SI unit is rad/s^2 .

(iv) Linear acceleration

$$a = R\alpha$$

[Its SI unit is m/s^2]

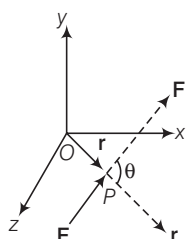
Equations of Rotational Motion

If angular acceleration α is uniform, then equations of rotational motion may be written as

$$\begin{aligned} \text{(i)} \quad \omega &= \omega_0 + \alpha t & \text{(ii)} \quad \theta &= \omega_0 t + \frac{1}{2} \alpha t^2 \\ \text{(iii)} \quad \omega^2 - \omega_0^2 &= 2\alpha\theta & \text{(iv)} \quad \theta_{n\text{th}} &= \omega_0 + \frac{\alpha}{2}(2n-1) \end{aligned}$$

Torque or Moment of a Force

Torque is a quantity which measures the capability of a force to rotate a body. It is a vector quantity which is directly perpendicular to both position vector (r) and force (F). Its SI unit is Nm.



Representation of torque on axes

Torque, $\tau = \mathbf{r} \times \mathbf{F} = rF \sin \theta \hat{n}$

where, θ = angle between $\mathbf{r}$ and $\mathbf{F}$.

and $\hat{n}$ is unit vector which shows the direction of τ .

Torque due to a force is also known as the **moment of a force**. It is defined as the cross product of the position vector of point (P) from the origin (O) on which the force is acting, with the force (see figure).

Angular Momentum

The moment of linear momentum of a given body about an axis of rotation is called as its angular momentum. If $\mathbf{p}$ is the linear momentum of a particle and $\mathbf{r}$ is its position vector from the point of rotation, then

Angular momentum, $\mathbf{L} = \mathbf{r} \times \mathbf{p} = rpsin\theta \hat{n}$

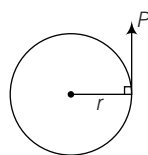
where, $\hat{n}$ is a unit vector in the direction of rotation.

Angular momentum is an axial vector and its SI unit is $\text{kgm}^2\text{s}^{-1}$ or J-s.

- (i) For rotational motion of a rigid body, angular momentum is equal to the product of angular velocity and moment of inertia of the body about the axis of rotation. Mathematically, $\mathbf{L} = I\boldsymbol{\omega}$

- (ii) According to the second law of rotational motion, the rate of change of angular momentum of a body is equal to the external torque applied on it and takes place in the direction of torque. Thus,

$$\tau = \frac{d\mathbf{L}}{dt} = \frac{d}{dt}(I\boldsymbol{\omega}) = I \frac{d\boldsymbol{\omega}}{dt} = I\alpha$$



- (iii) Total effect of a torque applied on a rotating body in a given time is called angular impulse. Angular impulse is equal to total change in angular momentum of the system in given time. Thus, angular impulse

$$J = \int_0^{\Delta t} \tau dt = \Delta L = L_f - L_i$$

- (iv) The angular momentum of a system of particles about the origin is

$$L = \sum r_i \times p_i$$

Law of Conservation of Angular Momentum

According to the law of conservation of angular momentum, if no external torque is acting on a system, then total vector sum of angular momentum of different particles of the system remains constant.

We know that, $\frac{d\mathbf{L}}{dt} = \tau_{\text{ext}}$

Hence, if $\tau_{\text{ext}} = 0$, then $\frac{d\mathbf{L}}{dt} = 0 \Rightarrow L = \text{constant}$

Therefore, in the absence of any external torque, total angular momentum of a system must remain conserved.

Relation between Angular Momentum and Torque

If the external force exerted on a body rotates it, then there is an external torque on the body. This external torque will be equal to the time rate of change of angular momentum.

$$\tau_{\text{ext}} = \frac{d\mathbf{L}}{dt}$$

Moment of Inertia of Different Bodies

Body	Axis	Moment of Inertia
1. Rectangular Lamina	Passing through its CG and perpendicular to its plane of length of breadth.	$\frac{M(l^2 + b^2)}{12}$
2. Uniform thin bar	(i) Through centre of gravity and perpendicular to length.	$\frac{Ml^2}{12}$
	(ii) Through one end and perpendicular to length.	$\frac{Ml^2}{3}$
3. Disc	(i) Passing through its centre and perpendicular to its plane.	$\frac{MR^2}{2}$
	(ii) About a diameter.	$\frac{MR^2}{4}$
4. Ring or Hoop	(i) Passing through its centre and perpendicular to its plane.	MR^2
	(ii) About a diameter.	$\frac{MR^2}{2}$

Body	Axis	Moment of Inertia
5. Solid cylinder	(i) About its own geometric axis.	$\frac{MR^2}{2}$
	(ii) Passing through CG and perpendicular to its geometric axis.	$M\left(\frac{l^2}{12} + \frac{R^2}{4}\right)$
6. Hollow disc of radii R_1 and R_2	Passing through its centre and perpendicular to its plane.	$\frac{M(R_1^2 + R_2^2)}{2}$
7. Hollow cylinder	(i) About its own geometric axis.	MR^2
	(ii) Passing through CG and perpendicular to length.	$M\left(\frac{l^2}{12} + \frac{R^2}{2}\right)$
8. Thin spherical shell	(i) About a diameter.	$\frac{2}{3}MR^2$
	(ii) About a tangent.	$\frac{5}{3}MR^2$
9. Diatomic molecule	Passing through centre of gravity and perpendicular to bond length.	$\frac{m_1 m_2}{m_1 + m_2} r^2$
10. Solid sphere	(i) About a diameter.	$\frac{2}{5}MR^2$
	(ii) About a tangent.	$\frac{7}{5}MR^2$

Moment of Inertia

The property by virtue of which a body opposes its change in its state of motion is called its inertia.

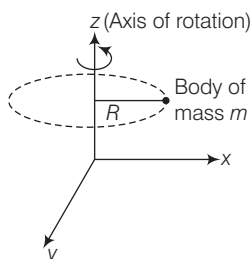
In similar manner, a body have the tendency to oppose its rotational motion or change in rotational motion about its axis is known as moment of inertia.

Mathematically, it is given as the product of mass and square of the perpendicular distance of a particle from the axis of rotation.

$$I = mR^2$$

where, m = mass of the particle

and R = perpendicular distance from axis of rotation.

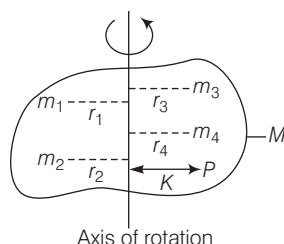


A particle is rotating about z-axis in a circle of radius R .

Radius of Gyration

Radius of gyration of a given body about a given axis of rotation is the normal distance of a point from the axis, where if whole mass of the body is placed, then its moment of inertia will be exactly same as it has with its actual distribution of mass.

Thus, radius of gyration



$$K = \sqrt{\frac{I}{M}} \quad \text{or} \quad K = \left[\frac{r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2}{n} \right]^{-1/2}$$

SI unit of radius of gyration is metre.

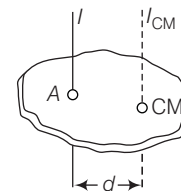
NOTE Radius of gyration depends upon shape and size of the body, position and configuration of the axis of rotation and also on distribution of mass of body w.r.t. axis of rotation.

Theorems on Moment of Inertia

There are two theorems of moment of inertia, i.e. theorem of parallel axes and theorem of perpendicular axes. Both the theorems are described below

1. Theorem of Parallel Axes

Moment of inertia of a body about a given axis I is equal to the sum of moment of inertia of the body about a parallel axis passing through its centre of mass I_{CM} and the product of mass of body (M) and square of normal distance d between the two axes.

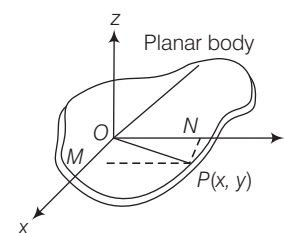


Mathematically, $I = I_{CM} + Md^2$

2. Theorem of Perpendicular Axes

The sum of moment of inertia of a plane lamina body about two mutually perpendicular axes lying in its plane is equal to its moment of inertia about an axis passing through the point of intersection of these two axes and perpendicular to the plane of lamina body. If I_x and I_y be moment of inertia of the body about two perpendicular axes in its own plane and I_z be the moment of inertia about an axis passing through point O and perpendicular to the plane of lamina, then

$$I_z = I_x + I_y$$



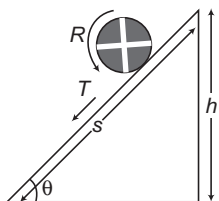
Rigid Body Rotation

If a rigid body of moment of inertia I about a given axis is rotating with an angular speed ω , then its rotational kinetic energy is given by, $K_R = \frac{1}{2} I \omega^2$. Rotational kinetic energy is a scalar having SI unit joule (J). Rotational kinetic energy is related to angular momentum as per relation,

$$K_R = \frac{L^2}{2I} \quad \text{or} \quad L = \sqrt{2IK_R}$$

Rolling on an Inclined Plane

For a body of mass M , rolling on an inclined plane having inclination θ , slant height s and height h .



(i) Height of an inclined plane, $h = \frac{v^2}{2g} \left(1 + \frac{K^2}{R^2} \right)$

(ii) Acceleration in motion $a = g \sin \theta / \left(1 + \frac{K^2}{R^2} \right)$

(iii) Time of descent,

$$t = \frac{1}{\sin \theta} \sqrt{\frac{2h \left(1 + \frac{K^2}{R^2} \right)}{g}}$$

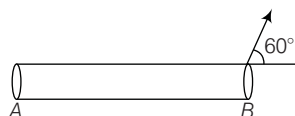
(iv) The velocity of CM at the bottom of the incline,

$$v = \sqrt{\frac{2gh}{1 + K^2/R^2}}$$

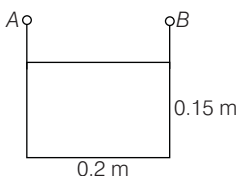
Here, K = Radius of gyration.

Practice Exercise

- The angular displacement at any time t is given by $\theta(t) = 2t^3 - 6t^2$. The torque on the wheel will be zero at
 a. 1 s b. 0.1 s c. 2 s d. 0.2 s
- Four 2 kg masses are connected by 1/4 m spokes to an axle. A force of 24 N acts on a lever 1/2 m long to produce angular acceleration α . The magnitude of α (in rad s^{-2}) is (the angle between $\mathbf{r}$ and $\mathbf{F}$ is 30°)
 a. 24 b. 12 c. 6 d. 3
- A flywheel of moment of inertia 0.4 kg m^2 and radius 0.2 m is free to rotate about a central axis. If a string is wrapped around it and it is pulled with a force of 10 N, then its angular velocity after 4 s will be
 a. 10 rad s^{-1} b. 5 rad s^{-1} c. 20 rad s^{-1} d. None
- The centre of a wheel rolling on a plane surface moves with a speed v_0 . A particle on the rim of the wheel at the same level as the centre will be moving at speed
 a. zero b. v_0 c. $\sqrt{2}v_0$ d. $2v_0$
- A meter stick is held vertically with one end on the floor and then is allowed to fall. Assuming that the end on the floor of the sticks does not move. The velocity of the other end when it hits the floor, will be
 a. 9.8 m/s b. 6.2 m/s c. 8.9 m/s d. 5.4 m/s
- The instantaneous velocity of point B of the given rod of length 0.5 is 3 m/s in the represented direction.
 The angular velocity of the rod for minimum velocity of end A is
 a. 1.5 rad/s b. 5.2 rad/s c. 2.5 rad/s d. 3.6 rad/s
- A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A plumb bob is suspended from the roof of the car by a light rigid rod of length 1 m. The angle made by the rod with the track is (Take $g = 10 \text{ m/s}^2$)
 a. zero b. 30°
 c. 45° d. 60°
- If a raw egg and a boiled egg are spinned on the table by applying same torque. Then
 a. boiled egg will spin faster
 b. raw egg will spin faster
 c. moment of inertia of boiled egg will be lesser than that of the raw egg
 d. Both a and c are correct
- The torque τ on a body about a given point is found to be equal to $\mathbf{A} \times \mathbf{L}$, where $\mathbf{A}$ is constant vector and $\mathbf{L}$ is the angular momentum of the body about that point. From this, it follows that
 a. $\frac{d\mathbf{L}}{dt}$ is perpendicular to $\mathbf{L}$ at all instant of time
 b. the component of $\mathbf{L}$ in the direction of $\mathbf{A}$ does not change with time
 c. the magnitude of $\mathbf{L}$ does not change with time
 d. All of the above
- The tricycle weighing 20 kg has a small wheel symmetrically placed 1 m behind the two large wheels, which are also 1 m apart. If the centre of gravity of machine is at a horizontal distance of 25 cm behind the front wheels and the rider whose weight is 40 kg is 10 cm behind the front wheels. Then, the thrust on each front wheel is
 a. 255 N b. 90 N
 c. 200 N d. 400 N

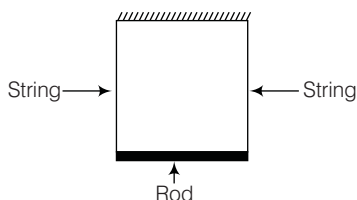


11. A rectangular plate of mass 20 kg is suspended from points A and B as shown. If the pin B is suddenly removed, then determine the angular acceleration (in rad/sec^2) of the plate.



a. 48 b. 19.6 c. 29.4 d. 23.6

12. A uniform rod of length l and mass m is suspended by two vertical inextensible strings as shown in figure. Then, tension in the left string when right string snaps, is

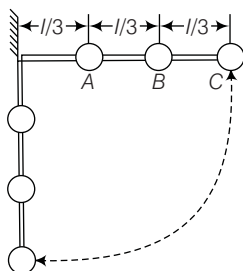


a. $\frac{3mg}{4}$ b. $\frac{mg}{4}$
c. $\frac{mg}{2}$ d. $\frac{mg}{8}$

13. A rod of length L is hinged from one end. It is brought to a horizontal position and released. The angular velocity of the rod, when it is in vertical position, is

a. $\sqrt{\frac{2g}{L}}$ b. $\sqrt{\frac{3g}{L}}$ c. $\sqrt{\frac{g}{2L}}$ d. $\sqrt{\frac{g}{L}}$

14. A light rod carries three equal masses A, B and C as shown in figure. Find the velocity of B in vertical position of rod, if it is released from horizontal position as shown in figure.



a. $\sqrt{2gl}$ b. $\sqrt{\frac{18gl}{7}}$ c. $\sqrt{\frac{4gl}{3}}$ d. $\sqrt{\frac{8gl}{7}}$

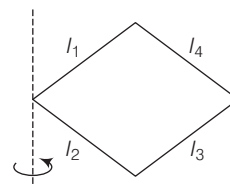
15. The kinetic energy of a lamina moving in its plane is

a. $M(v_{CM}^2 + K^2\omega^2)$ b. $\frac{1}{2}M(v_{CM}^2 + K^2\omega^2)$
c. $\frac{1}{2}I\omega^2$ d. None of these

16. A wheel has mass of the rim 1 kg, having 50 spokes each of mass 5 g. The radius of the wheel is 40 cm. The moment of inertia is

a. 0.273 kg m^2 b. 1.73 kg m^2
c. 0.173 kg m^2 d. 2.73 kg m^2

17. The moment of inertia of a system of four rods each of length l and mass m about the axis shown is



a. $\frac{2}{3}ml^2$ b. $2ml^2$ c. $3ml^2$ d. $\frac{8}{3}ml^2$

18. The surface density of a circular disc of radius a depends on the distance as $\rho(r) = A + Br$. The moment of inertia about the line perpendicular to the plane of the disc is

a. $\pi a^4 \left(\frac{A}{2} + \frac{2a}{5} B \right)$ b. $\pi a^4 \left(\frac{A}{2} + \frac{2B}{5} \right)$
c. $2\pi a^3 \left(\frac{A}{2} + \frac{Ba}{5} \right)$ d. None of these

19. A hoop of radius 2 m, weight 100 kg. It rolls along horizontal floor so that its centre of mass has a speed of 20 cm/s. How much work has to be done to stop it?

a. 4.8 J b. 4.0 J c. 6.2 J d. 7.2 J

20. Mass of bigger disc having radius $2R$ is M . A disc of radius R is cut from bigger disc. Moment of inertia of disc about an axis passing through periphery and perpendicular to plane is

a. $\frac{27}{8}MR^2$ b. $\frac{29}{8}MR^2$ c. $3.5MR$ d. $2MR^2$

21. Three thin rods each of length L and mass M are placed along x, y and z-axes such that one of each rod is at origin. The moment of inertia of this system about z-axis is

a. $\frac{2}{3}ML^2$ b. $\frac{4ML^2}{3}$ c. $\frac{5}{3}ML^2$ d. $\frac{ML^2}{3}$

22. If r is the distance between the Earth and the Sun. Then, angular momentum of the Earth around the Sun is proportional to

a. $r^{-1/2}$ b. $r^{1/2}$ c. r^2 d. r^{-2}

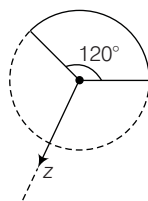
23. Two wheels A and B are mounted on 6 kg m^2 the same shaft. One of them having their moment of inertia 8 kg m^2 with angular speed 600 rpm and other is at rest. The value of moment of inertia of another wheel in order to achieve the combined angular speed 300 rpm is

a. 4 kg m^2 b. 3 kg m^2 c. 6 kg m^2 d. 9 kg m^2

24. Choose the correct option.

a. Friction is necessary for rolling motion
b. Friction is necessary for pure accelerated rolling motion
c. Friction is necessary for pure accelerated rolling on an inclined plane
d. None of the above

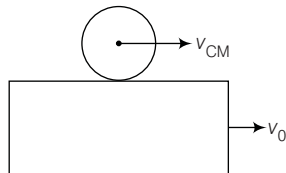
25. An arm making an angle of 120° at the centre of ring of mass m and radius r is cut from the ring. The arc is made to rotate about z-axis perpendicular to its plane and passing through the centre of the ring. The moment of inertia of the arc about the z-axis is



- a. mr^2 b. $\frac{mr^2}{3}$ c. $\frac{mr^2}{2}$ d. $\frac{mr^2}{4}$
26. A particle of mass m rotates in a circle of radius a with a uniform angular speed ω_0 . It is viewed from a frame rotating about the z-axis with a uniform angular speed ω . The centrifugal force on the particles is
- a. $m\omega^2 a$ b. $m\omega_0^2 a$
 c. $m\left(\frac{\omega + \omega_0}{2}\right)^2 a$ d. $m\omega\omega_0$

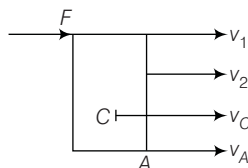
27. At any instant, a rolling body may be considered to be in pure rotation about an axis through the point of contact. This axis is translating forward with speed
- a. equal to centre of mass b. zero
 c. twice of centre of mass d. No sufficient data

28. In the given figure, the sphere rolls without slipping on the plank which is moving with constant velocity v_0 . The radius and angular velocity of the sphere is r and ω , respectively. The velocity of centre of mass of the sphere is



- a. $v_0 + r\omega$ b. $v_0 - r\omega$ c. $r\omega$ d. v_0
29. A uniform cube of mass m and edge a moves on a horizontal surface along the positive x-axis, with initial velocity v_0 . Then,
- a. during motion, $N > mg$
 b. during motion, normal reaction acts on the centre of mass
 c. during motion, the normal reaction shifts towards positive x-axis from the centre of mass
 d. during motion, normal reaction shifts in the direction of the forces of friction

30. In the case of toppling of the body about the point A (shown in the figure),



- a. $v_C > v_2 > v_1 > v_A$
 b. $v_1 > v_2 > v_C > v_A$
 c. $v_A \geq 0$
 d. $v_C < v_1 < v_2 < v_A$
31. In Q. 30, acceleration of the point A is
- a. ≥ 0 b. > 0
 c. < 0 d. $= 0$

32. Two cubes A and B of same shape, size and mass are placed on a rough surface in the same manner. Equal forces are applied on the both cubes. But at the cube A, the force is applied at the top in horizontal direction. But at the cube B just above the centre of mass of the cube in the same manner. Then,
- a. A will topple first
 b. B will topple first
 c. Both will topple at the same time
 d. None of the above

33. A regular polygon of n sides is placed on a rough surface vertically as such one of the side of regular polygon touches the surface. A force is applied horizontally at the top. The chosen value of n are 3, 5 and 8. For which value of n , the polygon first is likely to topple?

- a. 3 b. 5
 c. 8 d. All of these

34. A particle performs uniform circular motion with an angular momentum L . If the frequency of particles motion is doubled and its KE is halved, then the angular momentum becomes

- a. $2L$ b. $4L$ c. $\frac{L}{2}$ d. $\frac{L}{4}$

35. A mass m is moving with a constant velocity along a line parallel to the x-axis, away from the origin. Its angular momentum with respect to the origin
- a. is zero
 b. remains constant
 c. goes on increasing
 d. goes on decreasing

36. A particle of mass m is projected with velocity v moving at an angle of 45° with horizontal. The magnitude of angular momentum of projectile about point of projection when particle is at maximum height, is

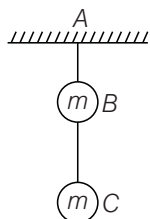
- a. zero b. $\frac{mv^3}{4\sqrt{2}g}$
 c. $\frac{mv^3}{\sqrt{2}g}$ d. $m\sqrt{2}gh^3$

37. A uniform rod of length $2a$ is held with one end resulting on a smooth horizontal table making an angle α with the vertical. When the rod is released,
- a. its centre of mass moves vertically downwards on a straight line
 b. its centre of mass remains in rest
 c. the rod rotates about a vertical axis
 d. Both a and c are correct

38. A 70 kg man standing on ice throws a 3 kg body horizontally at 8 m/s. The friction coefficient between the ice and his feet is 0.02. The distance, the man slips is

- a. 0.3 m b. 2 m
 c. 1 m d. ∞

39. A weightless rod of length l carries two equal masses m one fixed at the end and other in the middle of the rod. The rod can revolve in a vertical plane about A . Then, horizontal velocity which must be imparted to end C of rod to deflect it to horizontal position is



a. $\sqrt{\frac{12}{5}gl}$ b. $\sqrt{3gl}$ c. $\sqrt{\frac{16}{5}gl}$ d. $\sqrt{2gl}$

40. Two balls A and B of angular velocities ω_A and ω_B collide with each other. Then, after collision
- both have same angular velocities
 - $\omega_A > \omega_B$
 - $\omega_A = \omega_B$, when balls are smooth
 - $\omega_A > \omega_B$, when balls are smooth

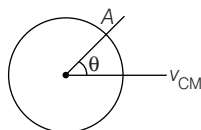
41. A uniform rod OA of mass M and length $2a$ rests on a smooth table and is free to turn about a smooth pivot at its end O , in contact with it at a distance b from O is an inelastic particle of mass m , a horizontal blow of impulse p is given to rod at a distance x from O in a direction perpendicular to the rod. The resultant instantaneous angular velocity of the rod is

a. $\frac{px}{\frac{4Ma^2}{3} + mb^2}$ b. $\frac{px}{M}$
c. $\frac{px}{ma^2 + mb^2}$ d. None of these

42. A uniform rod AB of mass m and length l is at rest on a smooth horizontal surface. An impulse p is applied to the end B . The time taken by the rod to turn through a right angle is

a. $2\pi \frac{ml}{p}$ b. $2\pi \frac{p}{ml}$ c. $\frac{\pi ml}{12p}$ d. $\frac{\pi p}{ml}$

43. A sphere of radius R is rolling on a rough horizontal surface. The magnitude of velocity of A with respect to ground will be



a. $\sqrt{2}v_{CM}$
b. $2v_{CM} \sin \theta$
c. $\sqrt{2}v_{CM}\sqrt{1 + \sin \theta}$
d. No sufficient information

44. When a wheel moves a distance shorter than $2\pi R$ while making one rotation. Then,

a. $v_{CM} < R\omega$ b. $v_{CM} \leq R\omega$
c. $v_{CM} > R\omega$ d. $v_{CM} \geq R\omega$

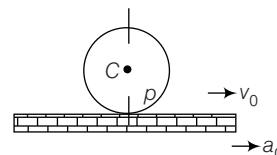
45. If a body moves through a distance greater than $2\pi R$ in one full rotation. Then,

a. $v_{CM} > R\omega$
b. $v_{CM} < R\omega$
c. $v_{CM} \geq R\omega$
d. $v_{CM} \leq R\omega$

46. A uniform sphere of radius a rotating with an angular velocity ω about an axis perpendicular to the plane of motion and its centre impinges on a horizontal plane, let u and v are horizontal and vertical component of velocity before impact. Then

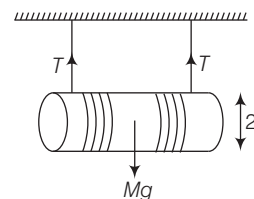
a. if $u = a\omega$, then u and ω are unaltered
b. if $u = a\omega$, then surface is frictionless
c. if $u > a\omega$, then angular velocity increases
d. All of the above

47. In the given figure, a solid sphere is placed on a plank having acceleration a_0 (shown in the figure). Then,



a. if $a_p = a_0$, then pure rolling takes place
b. if $v_p = v_0$, then pure rolling takes place
c. if $a_p = a_0$, $v_p \neq v_0$, then pure rolling takes place
d. if $a_p = a_0$, $v_p = v_0$, then pure rolling takes place

48. The cylinder of mass M is suspended through two strings as shown in figure. The speed of cylinder after descending through a depth h , is



a. $\sqrt{\frac{3gh}{2}}$ b. $\sqrt{\frac{4gh}{3}}$
c. $\sqrt{\frac{2gh}{3}}$ d. $\sqrt{\frac{8gh}{3}}$

BITSAT Archives

- Two rings of radius R and nR made of same material have the ratio of moment of inertia about an axis passing through centre is 1:8. The value of n is [2013, 07]
 a. 2 b. $2\sqrt{2}$ c. 4 d. $1/2$
- A mass m is moving with a constant velocity along a line parallel to x -axis. Its angular momentum with respect to origin on z -axis is [2013, 07]
 a. zero b. remain constant
 c. goes on increasing d. goes on decreasing
- The moment of inertia of the body about an axis is 1.2 kg m^2 . Initially, the body is at rest. In order to produce a rotational kinetic energy of 1500J , an angular acceleration of 25 rad/s^2 must be applied about the axis for the duration of [2011]
 a. 2 s b. 4 s c. 8 s d. 10 s
- When the mass is rotated in a plane about a fixed point, then angular momentum is directed along the [2010]
 a. radius
 b. tangent to orbit
 c. axis of rotation
 d. line at an angle of 45° to plane of rotation
- A solid cylinder of mass 2 kg rolls down (pure rolling) an inclined plane from a height of 4 m. Its rotational kinetic energy, when it reaches the foot of the plane is (Take $g = 10 \text{ ms}^{-2}$) [2010]
 a. 20 J b. 40 J c. $\frac{80}{3} \text{ J}$ d. 80 J
- The radius of a wheel is R and its radius of gyration about its axis passing through its centre and perpendicular to its plane is K . If the wheel is rolling without slipping, then the ratio of its rotational kinetic energy to its translational kinetic energy is [2010]
 a. $\frac{K^2}{R^2}$ b. $\frac{R^2}{K^2}$
 c. $\frac{R^2}{R^2 + K^2}$ d. $\frac{K^2}{R^2 + K^2}$
- The moment of inertia of a thin circular disc about an axis passing through its centre and perpendicular to its plane, is I . Then, the moment of inertia of the disc about an axis parallel to its diameter and touching the edge of the rim is [2008]
 a. I b. $2I$ c. $\frac{3}{2}I$ d. $\frac{5}{2}I$
- The moment of inertia of a circular disc about an axis passing through the circumference perpendicular to the plane of the disc is [2006]
 a. MR^2 b. $\frac{3}{2}MR^2$
 c. $\frac{MR^2}{2}$ d. $\frac{5}{4}MR^2$
- Angular momentum is conserved [2006]
 a. always
 b. never
 c. when external force is absent
 d. when external torque is absent

Answer with Solutions

Practice Exercise

1. (a) Given, $\theta(t) = 2t^3 - 6t^2 \Rightarrow \frac{d\theta}{dt} = 6t^2 - 12t$

$$\frac{d^2\theta}{dt^2} = 12t - 12$$

For torque, $\tau = I\alpha = 0$

$$\frac{d^2\theta}{dt^2} = 0 \Rightarrow 12t^2 - 12 = 0$$

$$\therefore t = 1 \text{ s}$$

2. (b) As, force $\times$ distance $= \tau = I\alpha$

$$\Rightarrow F \sin 30^\circ \times \frac{1}{2} = 4 \left[2 \times \left(\frac{1}{4} \right)^2 \right] \alpha$$

$$\text{or } 24 \times \frac{1}{2} \times \frac{1}{2} = \frac{\alpha}{2}$$

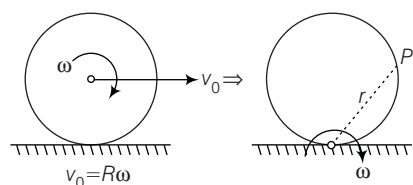
$$\therefore \alpha = 12 \text{ rad s}^{-2}$$

3. (c) As, $\tau = F \times r = I\alpha$

$$= I \frac{(\omega_2 - \omega_1)}{t}$$

$$\therefore \omega_2 - \omega_1 = \frac{F \times r \times t}{I} = \frac{10 \times 0.2 \times 4}{0.4} = 20 \text{ rad s}^{-1}$$

4. (c) $v_p = r\omega = (\sqrt{2}R)\omega = \sqrt{2}v_0$



5. (d) From energy conservation,

$$\frac{M}{2}gl = \frac{1}{2}I\omega^2 \quad \dots(i)$$

Moment of inertia of the rod,

$$I = \frac{1}{3}Ml^2 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{M}{2}gl = \frac{1}{2} \frac{Ml^2}{3} \omega^2 \Rightarrow \omega = \sqrt{\frac{3g}{l}}$$

Given, $l = 1\text{m}$, $\omega = \sqrt{3g}$

Also, $v = r\omega = 1 \times \sqrt{3 \times 9.8} = 5.4\text{m/s}$

6. (b) If rod is rotated about end A, then vertical component of velocity, $v_{\perp}$ of end A will be zero.

$$\therefore \omega = \frac{v \sin 60^\circ}{l} = \frac{\sqrt{3}v}{2l} = \frac{\sqrt{3} \times 3}{2 \times 0.5} = 5.2 \text{ rad/s}$$



7. (c) From figure,

$$T \cos \theta = mg \quad \dots(i)$$

$$\text{or } T \sin \theta = \frac{mv^2}{r} \quad \dots(ii)$$

Dividing Eq. (ii) by Eq. (i)

$$\therefore \tan \theta = \frac{v^2}{rg} = \frac{10^2}{10 \times 10} = 1$$

$$\Rightarrow \theta = 45^\circ$$

8. (d) $\tau = I\alpha$, $\alpha = \frac{\tau}{I}$, $\alpha \propto \frac{1}{I}$

It means, greater moment of inertia, lesser the spin. The MI of boiled egg is less than that of the raw egg. So, options (a) and (c) are correct.

9. (c) $\therefore \tau = \mathbf{A} \times \mathbf{L}$ or $\frac{d\mathbf{L}}{dt} = \mathbf{A} \times \mathbf{L}$... (i)

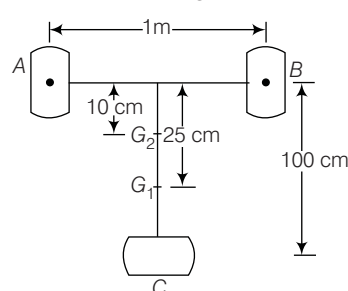
i.e. $\left(\frac{d\mathbf{L}}{dt}\right)$ is always perpendicular to $\mathbf{L}$

(and also $\mathbf{A}$)

$$\text{From Eq. (i), } \left(\frac{dL}{dt}\right) = AL \sin \theta \quad \dots(ii)$$

The component of $\mathbf{L}$ in the direction of $\mathbf{A}$ ($L \cos \theta$) will not change with time. And also torque is perpendicular to $\mathbf{L}$, the magnitude of $\mathbf{L}$ will not change with time.

10. (a)



In the given figure, the tricycle with two front wheels A and B with one rear wheel C with centre of gravity of machine G_1 and that of rider G_2 .

Now, force at $G_1 = 20 \times 10 = 200\text{N}$

Force at $G_2 = 400\text{N}$

Let F_C = Force on ground under rear wheel C

Force at $D = F_D = 2F_A$ acts at mid-point of AB.

Taking torque about D, for equilibrium,

$$F_C \times 100 = 200 \times 25 + 400 \times 10 = 9000$$

$$\therefore F_C = 90\text{N}$$

$\therefore$ Thrust on each front wheel is N.

$\therefore 2N + F_C = \text{Total weight} = 600\text{N}$

$$\therefore N = \frac{600 - F_C}{2} = \frac{600 - 90}{2} = 255\text{N}$$

11. (a) When plate starts to rotate about point A, then

$$\tau = mg \frac{l}{2} \quad \left(\because \tau = mg \frac{l}{2} \right)$$

$$\text{or } I\alpha = mg \frac{l}{2}$$

Here, $l = 0.2\text{m}$, $b = 0.15\text{m}$

$$\therefore d^2 = \frac{l^2}{4} + \frac{b^2}{4} = \frac{(0.2)^2}{4} + \frac{(0.15)^2}{4} = \frac{0.0625}{4}$$

$\therefore$ The moment of inertia about vertical axis passing through point A is

$$I = I_C + md^2 = \frac{m}{12}(l^2 + b^2) + md^2$$

$$= \frac{20}{12}\{(0.2)^2 + (0.15)^2\} + 20 \times \frac{0.0625}{4}$$

On putting the values,

$$\alpha = 48 \text{ rad/s}^2$$

12. (b) The torque on rod about point O is

$$\tau = mg \frac{l}{2} \text{ or } I\alpha = mg \frac{l}{2}$$

$$\text{or } \frac{ml^2}{3} \alpha = mg \frac{l}{2} \text{ or } \alpha = \frac{3}{2}g$$

For centre of rod,

$$mg - T = ma_{CM}$$

$$\therefore T = mg - ma_{CM} = mg - m \frac{l}{2} \alpha$$

$$= mg - \frac{3}{4}mg = \frac{mg}{4}$$

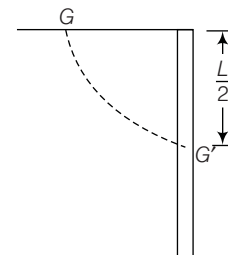
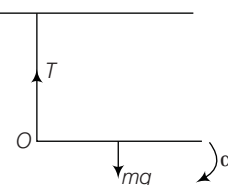
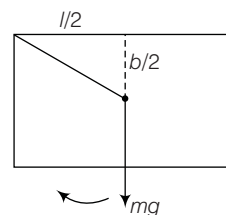
13. (b) The loss in potential energy = Gain in angular kinetic energy

$$\therefore mg \frac{L}{2} = \frac{1}{2}I\omega^2$$

$$\text{or } mg \frac{L}{2} = \frac{1}{2} \frac{mL^2}{3} \omega^2$$

$$\text{or } g = \frac{L\omega^2}{3}$$

$$\therefore \omega = \sqrt{\left(\frac{3g}{L}\right)}$$



14. (d) Loss in PE = Gain in angular kinetic energy

$$\text{or } mg \frac{l}{3} + mg \left(\frac{2l}{3} \right) + mgl = \frac{1}{2} I \omega^2 \quad \dots(i)$$

Here, I = Moment of inertia about fixed point

$$= m \left(\frac{l}{3} \right)^2 + m \left(\frac{2l}{3} \right)^2 + ml^2 = \frac{14}{9} ml^2$$

From Eq. (i),

$$2mgl = \frac{1}{2} \left(\frac{14}{9} ml^2 \right) \omega^2 \therefore \omega = \sqrt{\left(\frac{36g}{14l} \right)}$$

$$\therefore v_B = r\omega = \frac{2l}{3} \sqrt{\left(\frac{36g}{14l} \right)} = \sqrt{\left(\frac{8gl}{7} \right)}$$

15. (b) If a lamina is moving with
- v_{CM}
- having angular velocity
- ω
- .

Then,

KE = Angular kinetic energy + Translatory kinetic energy

$$= \frac{1}{2} I_{CM} \omega^2 + \frac{1}{2} m v_{CM}^2$$

$$= \frac{1}{2} M K^2 \omega^2 + \frac{1}{2} M v_{CM}^2 = \frac{1}{2} M (K^2 \omega^2 + v_{CM}^2)$$

16. (c)
- $I = mr^2 + 50 \frac{ml^2}{3}$

$$= 1 \times (0.4)^2 + \frac{50 (5 \times 10^{-3}) (0.4)^2}{3}$$

$$= 0.16 (1.083) = 0.173 \text{ kg m}^2$$

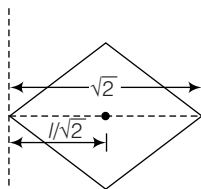
17. (d) Consider a square lamina, then

$$4m \left(\frac{l^2}{12} + \frac{l^2}{12} \right) \text{ about}$$

$$\text{Centre of mass} = \frac{2ml^2}{3}$$

Apply perpendicular axes theorem

$$= \frac{2ml^2}{3} + 4m \left(\frac{l}{\sqrt{2}} \right)^2 = \frac{8}{3} ml^2$$



18. (a)
- $dm = 2\pi r dr$
- (
- $\rho = (A + Br) (2\pi r dr)$
-)

$$I = \int_0^a dm r^2 = \frac{\pi A a^4}{2} + \frac{2\pi B a^5}{5} = \pi a^4 \left(\frac{A}{2} + \frac{2a}{5} B \right)$$

19. (b) Given,
- $R = 2 \text{ m}$
- ,
- $M = 100 \text{ kg}$

Speed of centre of mass,

$$v = 20 \text{ cm/s} = 0.20 \text{ m/s}$$

Work done to stop the hoop = Total kinetic energy of the hoop

$$W = \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2$$

But moment of inertia, $I = Mr^2$ and angular velocity, $\omega = \frac{v}{R}$

$$W = \frac{1}{2} M v^2 + \frac{1}{2} (Mr^2) \times \left(\frac{v^2}{R^2} \right)$$

$$= \frac{1}{2} M v^2 + \frac{1}{2} M v^2 = M v^2$$

$$= 100 \times (0.20)^2 = (100 \times 0.04) \text{ J} = 4.0 \text{ J}$$

20. (b) Surface density of motional disc is

$$\sigma = \frac{M}{\pi(2R)^2} = \frac{M}{4\pi R^2}$$

Mass of cutting portion is

$$m_1 = \sigma \times \pi R^2 = \frac{M}{4}$$

$$I = I_1 - I_2$$

where,

 I_1 = Moment of inertia of disc about given axis without cutting portion I_2 = Moment of inertia due to cutting portion

$$I = \frac{M(2R)^2}{2} + M(2R)^2 - \left[\frac{m_1 R^2}{2} + m_1 (3R)^2 \right]$$

$$= 6MR^2 - \frac{19MR^2}{8} = \frac{29MR^2}{8}$$

21. (a) Moment of inertia of the rod lying along z-axis will be

zero. MI of the rods along x and y-axes will be $\frac{ML^2}{3}$ each.Hence, total moment of inertia is $\frac{2}{3} ML^2$.

22. (b) By the Kepler's rule,

$$L = mvr = m \sqrt{\frac{GM}{r}} r$$

 $\Rightarrow$

$$L \propto \sqrt{r}$$

23. (c) From the conservation of angular momentum,

$$(I_1 + I_2) \omega = I_1 \omega_1 + I_2 \omega_2$$

$$\Rightarrow (6 + I_2) \times \frac{300}{60} \times 2\pi = 6 \times \frac{600}{60} \times 2\pi + I_2 \times 0$$

$$\Rightarrow I_2 = 6 \text{ kg m}^2$$

24. (c) For pure rolling, friction is not necessary.

For pure accelerated rolling on a horizontal floor, friction is not necessary in every case.

For pure accelerated rolling on an inclined plane, friction is necessary.

25. (b) The mass of ring per unit angle is
- $\lambda = \frac{m}{2\pi}$

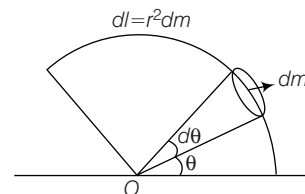
$$\therefore dm = \frac{m}{2\pi} d\theta$$

 $\therefore$ The moment of inertia of considered element about z-axis passing through point O is

$$dl = r^2 dm$$

 $\therefore$ The moment of inertia of the arc is

$$I = r^2 = \frac{mr^2}{3} \int_{\theta=0}^{\theta=2\pi/3} d\theta = \frac{mr^2}{3}$$



26. (b) Centrifugal force does not depend upon angular velocity of particle. It depends upon angular velocity of frame of reference.

 $\therefore$ Centrifugal force is $F = m\omega_0^2 a$

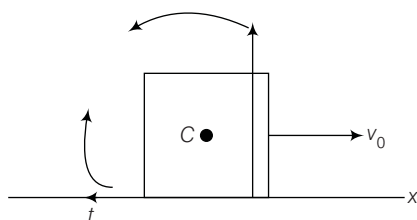
27. (a) Since, in this case, instantaneous axis of rotation is always below the centre of mass. This is possible only when point of contact moves with a velocity equal to centre of mass.

28. (a) For pure rolling motion of sphere, relative motion of point of contact of sphere and plank should be zero. For this, the point of contact has a velocity equal to the velocity of plank.

$$\therefore v_0 = v_{CM} - r\omega$$

$$\therefore v_{CM} = v_0 + r\omega$$

29. (c) According to figure,



In this case, the force of friction has a tendency to rotate the cube in clockwise direction. So, normal reaction will shift in forward direction to balance the torque due to force of friction.

30. (b)

31. (d)

32. (a) The greater height of the point of applying force from ground, more chance of toppling.

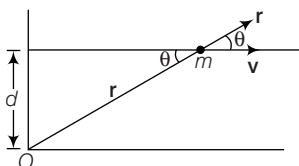
33. (c) The body has smaller base, greater chance of toppling. When the value of n increases, base size decreases.

34. (d)

35. (b) From figure, the angular momentum is

$$\mathbf{L} = \mathbf{r} \times m\mathbf{v}$$

$$\therefore L = rmv \sin \theta = mv(r \sin \theta) = mvd = \text{constant}$$



36. (b) The angular momentum is

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

$$L = rmv \cos \theta \sin \theta$$

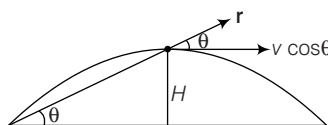
$$\text{But, } H = r \sin \theta$$

$$L = mvH \cos \theta$$

$$= mv \times \frac{v^2 \sin^2 \theta}{2\sqrt{2}g}$$

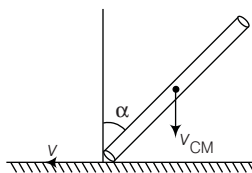
$$\text{But, } \theta = 45^\circ$$

$$\therefore L = \frac{mv^3}{4\sqrt{2}g}$$

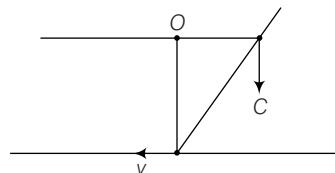


37. (d) Since, no force is present in horizontal direction.

So, acceleration of centre of mass in horizontal direction is zero. Hence, the centre of mass of rod moves in vertically downward direction.



If perpendiculars are drawn at the velocities of centre of mass and lower end. They are passing through point O. So, rod is rotating about instantaneous axis passing through point O.



38. (a)

39. (a) Loss in kinetic energy = Gain in potential energy

$$\therefore \frac{1}{2} I \omega^2 = mg \frac{l}{2} + mgl \quad \dots (i)$$

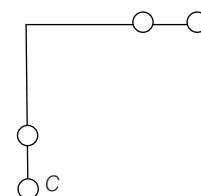
$$\text{Here, } I = \frac{ml^2}{4} + ml^2 = \frac{5}{4} ml^2$$

$$v_C = l\omega$$

From Eq. (i),

$$\frac{1}{2} \left(\frac{5}{4} ml^2 \right) \omega^2 = \frac{mgl}{2} + mgl = \frac{3}{2} mgl$$

$$ml^2 \omega^2 = \frac{12}{5} mgl \therefore v_C = \sqrt{\left(\frac{12gl}{5} \right)}$$



40. (c) (a) This is possible, when balls are smooth.

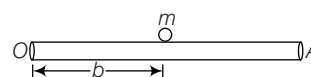
- (b) $\omega_A = \omega_B$ is not always possible. It depends upon conditions. But no conditions are mentioned.

41. (a) Angular impulse,

$$J = \Delta L = I \Delta \omega = I(\omega - 0) = I\omega$$

$$\text{Here, } px = l\omega$$

$$\therefore \omega = \frac{px}{I} + \frac{px}{mb^2 + \frac{M(2a)^2}{3}} = \frac{px}{\frac{4Ma^2}{3} + mb^2}$$



42. (c) The angular impulse about centre of mass is

$$J = L_f - L_i = I\omega - 0$$

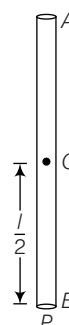
$$\therefore J = l\omega$$

$$\text{or } p \frac{l}{2} = \frac{ml^2}{12} \omega \text{ or } p = \frac{ml}{6} \omega$$

$$\therefore \omega = \frac{6p}{ml}$$

$$\therefore \theta = \omega t$$

$$\therefore t = \frac{\theta}{\omega} = \frac{\pi}{2\omega} = \frac{\pi}{2 \times \frac{6p}{ml}} = \frac{\pi ml}{12p}$$



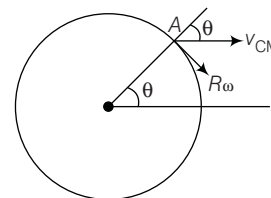
43. (c) Here, $v_{CM} = R\omega$

The resultant velocity of point A is

$$v_A = \sqrt{v_{CM}^2 + (R\omega)^2 + 2v_{CM}(R\omega) \cos(90^\circ - \theta)}$$

$$= \sqrt{v_{CM}^2 + v_{CM}^2 + 2v_{CM}^2 \sin \theta}$$

$$= \sqrt{2} v_{CM} \sqrt{1 + \sin \theta}$$



44. (a) $\because v_{CM} T < 2\pi R$ or $v_{CM} < \frac{2\pi R}{T}$

$\therefore v_{CM} < R\omega$

45. (a) $\because v_{CM} T > 2\pi R$

or $v_{CM} > \frac{2\pi R}{T} \therefore v_{CM} > R\omega$

46. (d)

47. (d) For pure rolling motion, the relative motion between point of contact and surface should be zero.

This is possible only when,

$$a_p = a_0, v_p = v_0$$

48. (b) From the equation of centre of mass of the cylinder,
 $Mg - 2T = Ma \quad \dots(i)$

For rotational motion about CM

$$2Tr = \frac{1}{2} Mr^2 \alpha$$

$$4T = Mr\alpha$$

where, r = radius of cylinder

$$\Rightarrow 2T = \frac{1}{2} Ma \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

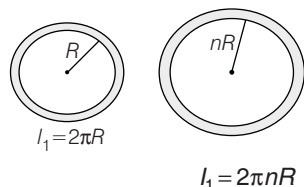
$$a = \frac{2}{3} g \text{ and } T = \frac{mg}{6}$$

Now, from equation of motion, $v^2 = u^2 + 2gh$, we get

$$v^2 = 2 \left(\frac{2}{3} g \right) h \Rightarrow v = \sqrt{\frac{4gh}{3}}$$

BITSAT Archives

1. (a)



Ratio of moment of inertia of the rings

$$\frac{I_1}{I_2} = \left(\frac{M_1}{M_2} \right) \left(\frac{R_1}{R_2} \right)^2 = \left(\frac{\lambda L_1}{\lambda L_2} \right) \left(\frac{R_1}{R_2} \right)^2$$

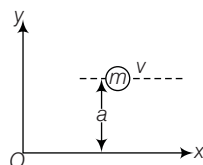
$$= \left(\frac{2\pi R}{2\pi nR} \right) \left(\frac{R}{nR} \right)^2 \quad [\lambda = \text{linear density of wire} = \text{constant}]$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{1}{n^3} = \frac{1}{8} \quad [\text{given}]$$

$$\therefore n^3 = 8$$

$$\Rightarrow n = 2$$

2. (b) Angular momentum of particle w.r.t. origin = linear momentum $\times$ perpendicular distance on line of action of linear momentum from origin
 $[= mv \times a = mva = \text{constant}]$



3. (a) Rotational kinetic energy = $\frac{1}{2} I\omega^2$

According to question, $\frac{1}{2} I\omega^2 = 1500$

$$\frac{1}{2} I(\alpha t)^2 = 1500$$

$$(1.2) \times (25)^2 \times t^2 = 3000$$

$$1.2 \times 625 \times t^2 = 3000$$

$$t^2 = \frac{3000}{1.2 \times 625} = 4$$

$$t = 2 \text{ s}$$

4. (c) The angular momentum will be directed along the axis of rotation.

5. (c) At foot of the plane,

$$mgh = \frac{1}{2} I\omega^2 + \frac{1}{2} mu^2$$

$$2 \times 10 \times 4 = \frac{1}{2} \times \left(\frac{1}{2} mR^2 \right) \times \left(\frac{u^2}{R^2} \right) + \frac{1}{2} mu^2$$

$$80 = \frac{3}{4} mu^2 \quad \dots(i)$$

$$\therefore \text{Rotational KE} = \frac{1}{2} I\omega^2$$

$$= \frac{1}{4} mu^2 = \frac{1}{4} \times \frac{80 \times 4}{3} = \frac{80}{3} \text{ J}$$

6. (a) $\frac{\text{Rotational KE}}{\text{Translational KE}} = \frac{\frac{1}{2} I\omega^2}{\frac{1}{2} Mv^2} = \frac{K^2}{R^2} \quad [\because I = MK^2, v = R\omega]$

7. (d) Moment of inertia of a circular disc about an axis passing through centre of gravity and perpendicular to its plane

$$I = \frac{1}{2} MR^2 \quad \dots(i)$$

From Eq. (i),

$$MR^2 = 2I$$

Then, moment of inertia of disc about tangent in a plane

$$= \frac{5}{4} MR^2 = \frac{5}{4} (2I) = \frac{5}{2} I$$

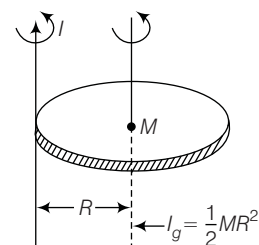
8. (b) $I_{CM} = \frac{1}{2} MR^2$

Now, moment of inertia about an axis is given as

$$I = I_{CM} + Mh^2 = \frac{1}{2} MR^2 + MR^2$$

$$[as h = R]$$

$$I = \frac{3}{2} MR^2$$



9. (d) According to law of conservation of angular momentum, if there is no torque on the system, then the angular momentum remains constant.

9

Gravitation

Introduction

Gravitation is the name given to the force of attraction between any two massive bodies of the universe.

Universal Law of Gravitation

Every body in the universe attracts every other body. Force of gravitational attraction between two bodies of mass m_1 and m_2 is given as

$$F = G \frac{m_1 m_2}{r^2}$$

The proportionality constant G is called universal gravitational constant. In SI system, the value of gravitational constant G is $6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$. Dimensional formula of G is $[\text{M}^{-1}\text{L}^3\text{T}^{-2}]$.

Gravity

In Newton's law of gravitation, we consider the gravitation as the force. But in case of two gravitationally interacting masses (or bodies), if one is the earth, then the gravitational force is called as the **gravity**.

Acceleration due to Gravity (g)

At the earth's surface, the value of g is found to be 9.8 ms^{-2} or 32 ft s^{-2} .

Mathematically,
$$g = \frac{F}{m} = \frac{GMm}{R^2m}$$

where, F = force exerted by the earth on a mass m .

$$\Rightarrow g = \frac{GM}{R^2}$$

Here, M = mass of earth, R = radius of earth.

Variation in Acceleration due to Gravity (g)

The value of g is variable and can vary in some cases as mentioned below (R is treated to be radius of the earth, i.e. 6400 km)

- (i) **Variation in g with the Height from the Earth's Surface** The new value of g at height h from the earth's surface, is

$$g' = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$$

and if $h \ll R$, then

$$g' = g\left(1 - \frac{2h}{R}\right)$$

- (ii) **Variation in g with Depth Inside the Earth** The new value of g at a place inside the earth with depth h from the earth's surface, is

$$g' = g\left(1 - \frac{h}{R}\right)$$

- (iii) **Variation in g Due to Daily Rotation of the Earth** The new value of g at point P with latitude θ , is

$$g' = g - \omega^2 R \cos^2 \theta$$

- (a) At the pole,

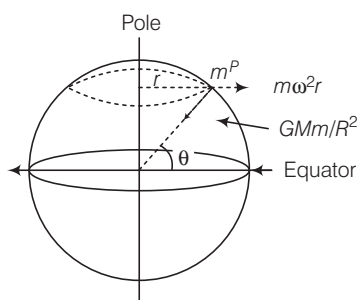
$$\cos \theta = 0 \quad [\because \theta = 90^\circ]$$

$$g' = g$$

- (b) At the equator,

$$\cos \theta = 1 \quad [\because \theta = 0^\circ]$$

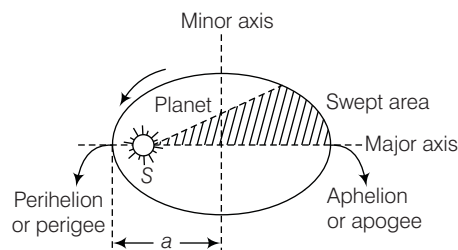
$$g' = g - \omega^2 R$$



Kepler's Laws of Planetary Motion

Kepler discovered three empirical laws which accurately describe the motion of planets.

These laws are



1. Law of Orbits

Every planet moves in an elliptical orbit around the sun, with sun situated at one of the foci of the ellipse.

2. Law of Areas

The line joining a planet to the sun sweeps equal areas in equal intervals of time, howsoever small these time intervals may be. Mathematically,

$$\frac{dA}{dt} = \frac{1}{2} r v = \text{constant} = \frac{L}{2m}$$

where, L = angular momentum of planet and
 m = mass of planet.

3. Law of Periods

The square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse traced out by the planet. Thus,

$$T^2 \propto r^3 \quad \text{or} \quad \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

Gravitational Field

Gravitational field of a given body is the space around it, within which gravitational effect due to that body may be experienced.

Gravitational Field Intensity

Gravitational field intensity at any point in the field of given body is defined as the force experienced by a unit mass placed at that point. Mathematically,

$$\text{Gravitational field intensity, } \mathbf{I} = \lim_{m_0 \rightarrow 0} \frac{\mathbf{F}}{m_0}$$

where, m_0 is a small test mass. The SI unit of gravitational intensity is N kg^{-1} .

Gravitational intensity at a point situated at a distance r from a point mass M is given by

$$I = \frac{GM}{r^2}$$

Gravitational Field in Different Cases

The calculation of gravitational field in some situations are mentioned below

- **Field due to point mass** The field at point P , at a distance r from the point mass (M) is

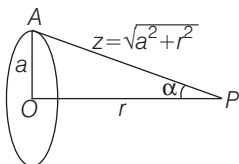
$$E_P = \frac{-GM}{r^2} \mathbf{e}_r$$

where, $\mathbf{e}_r$ is the unit vector along r .

- **Field at an axial point of a circular ring**

The field at an axial point P is

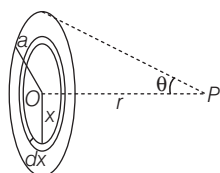
$$E_P = \frac{GMr}{(a^2 + r^2)^{3/2}}$$



- **Field at an axial point of a disc**

The field at an axial point P is

$$E_P = \frac{2GM}{a^2} (1 - \cos\theta)$$

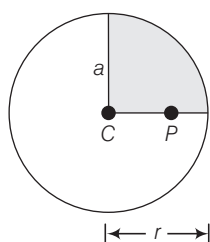


- **Field due to a uniform spherical shell**

There may be two conditions as

- When point P is an internal point of the shell (i.e. $r < a$), then

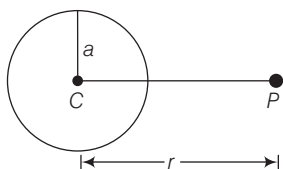
$$E_P = 0$$



Field at any point inside the shell is zero, which implies that the potential inside the shell is constant everywhere.

- When point P is an external point of the shell (i.e. $r \geq a$), then

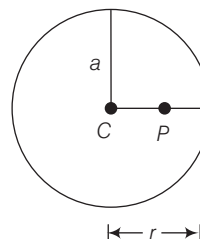
$$E_P = \frac{GM}{r^2}$$



- **Field due to a uniform solid sphere** There may be two conditions as

- When point P is an internal point, then

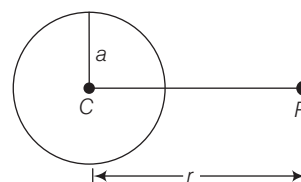
$$E_P = \frac{GM}{a^3} r$$



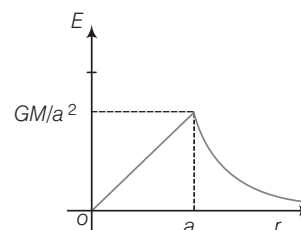
At the centre, $r = 0$, $E_P = 0$

- When point P is an external point, then

$$E_P = \frac{GM}{r^2}$$



In the case of uniform solid sphere, the graph of E versus r is shown below

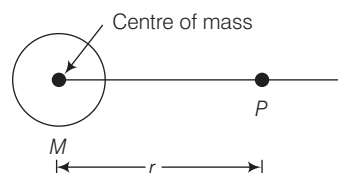


Gravitational Potential

The amount of work done by the applied force required to bring a body of unit mass from reference position (i.e. infinity) to a given position within the field is known as gravitational potential of the field (created by mass).

Gravitational Potential in Different Cases

- Potential due to a point mass



Gravitational potential at P ,

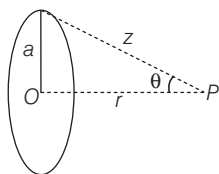
$$V_P = -\frac{GM}{r}$$

2. Potential at an axial point of a uniform ring

Gravitational potential at point P ,

$$V_P = \frac{-GM}{\sqrt{a^2 + r^2}}$$

where, a = radius of the ring
 M = mass of the ring



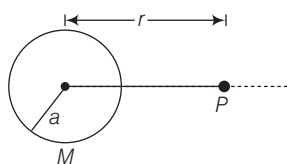
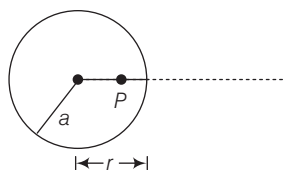
3. Potential due to a uniform thin spherical shell

There may be two conditions as

(i) When point P is an internal point (i.e. $r < a$)

$$V_P = \frac{-GM}{a}$$

(ii) When point P is an external point (i.e. $r \geq a$)

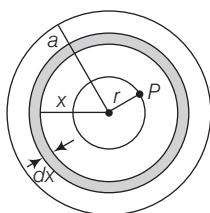


$$V_P = \frac{-GM}{r}$$

4. Potential due to solid sphere

There may be two conditions

(i) Potential at an internal point P

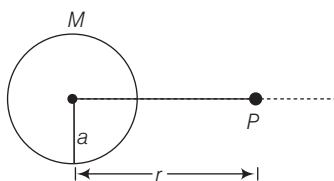


$$V_P = \frac{-GM}{2a^3} (3a^2 - r^2), \quad (r \leq a)$$

When point is at the centre of sphere, then

$$V = \frac{-3GM}{2a}$$

(ii) Potential at an external point P



$$V_P = \frac{-GM}{r}, \quad (r \geq a)$$

Gravitational Potential Energy

Gravitational potential energy of a body or system is negative of work done by the conservative gravitational forces, F in bringing it from infinity to the present position. Mathematically, gravitational potential energy

$$U = -W = -\int_{\infty}^r \mathbf{F} \cdot d\mathbf{r}$$

The gravitational potential energy of two particles of masses m_1 and m_2 separated by a distance r is given by

$$U = -\frac{Gm_1m_2}{r}$$

The gravitational potential energy of mass m at the surface of the earth is

$$U = -\frac{GMm}{R}$$

Difference in potential energy of mass m at a height h from the earth's surface and at the earth's surface is

$$U_{(R+h)} - U_R = \frac{mgh}{1 + \frac{h}{R}} \approx mgh, \text{ if } h \ll R$$

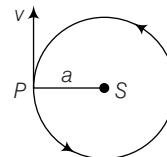
• For three particles system,

$$U = -\left[\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}} \right]$$

• For n -particles system, $\frac{n(n-1)}{2}$ pairs form and total potential energy of the system is sum of potential energies of all such pairs.

The Orbital Speed, Time Period and Energy of the Satellite

Let the mass of the satellite be m and that of the planet under study be M and a is radius of the orbit.



(i) Orbital Speed

The speed with which a satellite revolves in its orbit is known as its orbital speed (v). The gravitational force between earth-satellite provides required centripetal force. i.e.

$$\frac{GMm}{a^2} = m\left(\frac{v^2}{a}\right) \Rightarrow v = \sqrt{GM/a}$$

(ii) Time Period

The time taken by satellite to just complete a revolution around the earth is known as its time period (T).

$$T = \frac{2\pi a}{v} = \sqrt{\frac{4\pi^2}{GM} a^3}$$

(iii) Height of Satellite in Terms of Period

The height of the satellite (from the earth's planet) can be determined by its time period and *vice-versa*.

As the height of the satellite in terms of time period,

$$h = a - R = \left[\frac{gR^2 T^2}{4\pi^2} \right]^{1/3} - R$$

(iv) Angular Momentum of Satellite

Angular momentum of a satellite, $L = mv_0 a = \sqrt{m^2 G M a}$.

During the motion of planet and satellite, the angular momentum will remain conserved.

Energy

The total mechanical energy (E) associated with sun-planet (earth-satellite) system (i.e. sum of kinetic energy and potential energy) is treated to be energy of the system.

E = Potential energy + Kinetic energy

$$= U + K = \frac{GMm}{2a} - \frac{GMm}{a}$$

$$\Rightarrow E = -\frac{GMm}{2a}$$

Binding Energy of Satellite

It is the energy required to remove the satellite from its orbit and take it to infinity.

$$\text{Binding energy} = -E = +\frac{GMm}{2a}$$

Escape Velocity

The minimum velocity with which a particle or body should be launched from the earth's surface to just cross the earth's gravitational field (i.e. it never comes back to the earth), is known as escape velocity.

It is given by the equation,

$$v_e \geq \sqrt{\frac{2GM}{R}}$$

where, R = radius of the earth and M = Mass of the earth.

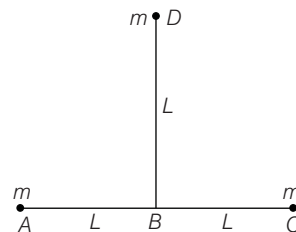
At the earth's surface, the value of escape velocity is 11.6 kms^{-1} .

Practice Exercise

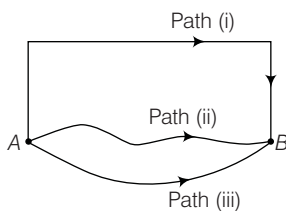
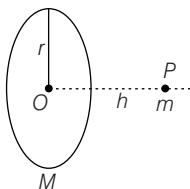
- Calculate the gravitational force of attraction between two spherical bodies, each of mass 1 kg placed at 10 m apart ($G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$).
 a. $6.67 \times 10^{-13} \text{ N}$ b. $6.67 \times 10^{-11} \text{ N}$
 c. $6.67 \times 10^{-7} \text{ N}$ d. None of these
- If the distance between the two particles is increased by 2%, then the force of attraction between them will
 a. decrease 6% b. decrease 4%
 c. increase 4% d. increase 6%
- How the gravitational constant will change if a brass plate is introduced between two bodies?
 a. No change b. Decreases
 c. Increases d. Insufficient data
- Six particles each of mass m are placed at the corners of a regular hexagon of edge length a . If a point mass m_0 is placed at the centre of the hexagon. Then, the net gravitational force on the point mass m_0 is

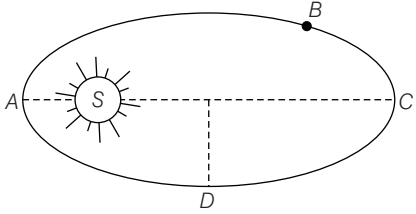
- a. $\frac{6Gm^2}{a^2}$ b. $\frac{6Gmm_0}{a^2}$ c. zero d. None

5. Two particles each of mass m are placed at A and C as such $AB = BC = L$. Calculate the gravitational force on the third particle placed at D at a distance L metre on the perpendicular bisector of the line AC .



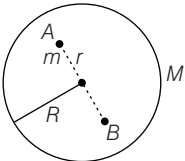
- a. $\frac{Gm^2}{\sqrt{2}L^2}$ along BD b. $\frac{Gm^2}{\sqrt{2}L^2}$ along DB
 c. $\frac{Gm^2}{L^2}$ along AC d. None of these

6. In a hypothetical concept, electron of mass m_e revolves around nucleus due to gravitational force of attraction between electron and proton of mass m_p . If the radius of circular path of electron is r , then the speed of electron is
- a. $\sqrt{\left(\frac{Gm_p m_e}{4r}\right)}$ b. $\sqrt{\left(\frac{Gm_p m_e}{r}\right)}$
 c. $\sqrt{\left(\frac{Gm_p}{r}\right)}$ d. None of these
7. Three point masses each of mass m rotate in a circle of radius r with constant angular velocity ω due to their mutual gravitational attraction. If at any instant, the masses are on the vertex of an equilateral triangle of side a , then the value of ω is
- a. $\sqrt{\left(\frac{Gm}{a^3}\right)}$ b. $\sqrt{\left(\frac{3Gm}{a^3}\right)}$ c. $\sqrt{\left(\frac{Gm}{3a^3}\right)}$ d. None
8. A gravitational field is present in a region. Point A mass is shifted from A to B, from different paths shown in the figure. If W_1 , W_2 and W_3 represent work done by gravitational force for respective paths. Then,
- a. $W_1 = W_2 = W_3$
 b. $W_1 > W_2 > W_3$
 c. $W_1 > W_3 > W_2$
 d. None of the above
- 
9. A point mass m_0 is placed at distance $R/3$ from the centre of a spherical shell of mass m and radius R . The gravitational force on the point mass m_0 is
- a. $\frac{4Gmm_0}{R^2}$ b. zero
 c. $\frac{9Gmm_0}{R^2}$ d. None of these
10. A uniform ring of mass M and radius R is placed directly above a uniform sphere of mass $8M$ and of same radius R . The centre of the ring is at a distance of $d = \sqrt{3}R$ from the centre of the sphere. The gravitational attraction between the sphere and the ring is
- a. $\frac{GM^2}{R^2}$ b. $\frac{3GM^2}{2R^3}$ c. $\frac{2GM^2}{\sqrt{2}R^2}$ d. $\frac{\sqrt{3}GM^2}{R^2}$
11. A mass m is placed at P at a distance h along the normal through the centre O of a thin circular ring of mass M and radius r as shown in figure.
- 
- If the mass is removed further away such that OP becomes $2h$, by what factor the force of gravitation will decrease, if $h = r$?
- a. $\frac{3\sqrt{2}}{4\sqrt{3}}$ b. $\frac{5\sqrt{2}}{\sqrt{3}}$ c. $\frac{4\sqrt{3}}{5}$ d. $\frac{4\sqrt{2}}{5\sqrt{5}}$
12. A point mass of 10 kg is placed at the centre of earth. The weight of the point mass is
- a. zero b. 98 N c. 49 N d. 38 N
13. How will you weight the sun, i.e. estimate its mass? You will need to know the period of one of its planets and the radius of the planetary orbit. The mean orbital radius of the earth around the sun is 1.5×10^8 km, then the mass of the sun would be calculated as
- a. 2×10^{15} kg
 b. 2×10^{20} kg
 c. 2×10^{27} kg
 d. 2×10^{30} kg
14. A particle hanging from a massless spring stretches it by 2 cm at earth's surface. How much will the same particle stretch the spring at height 2624 km from the surface of earth? (radius of earth = 6400 km)
- a. 1 cm b. 2 cm
 c. 3 cm d. 4 cm
15. There is a mine of depth 2 km. The conditions as compared to those at the surface of the earth are
- a. lower value of g b. higher value of g
 c. Both (a) and (b) d. None of these
16. In order to make the weight of a 5 kg body to zero at the equator. The angular speed of the earth would must be (take, $g = 10 \text{ m/s}^2$ and radius of the earth, $R = 6400$ km)
- a. $0.00125 \text{ rad s}^{-1}$ b. $0.0125 \text{ rad s}^{-1}$
 c. 0.125 rad s^{-1} d. $0.0325 \text{ rad s}^{-1}$
17. At what height, the weight of the body is same as that at same depth from the earth's surface? (take, earth's radius = R)
- a. $R/2$ b. $\frac{(\sqrt{5}-1)R}{2}$
 c. $\frac{(\sqrt{3}R-1)}{2}$ d. $\frac{\sqrt{5}}{3}R$
18. At the surface of a certain planet acceleration due to gravity is one-quarter of that on the earth. If a brass ball is transported on this planet, then which one of the following statements is not correct?
- a. The brass ball has the same mass on the other planet as on the earth
 b. The mass of the brass ball on this planet is a quarter of its mass as measured on the earth
 c. The weight of the brass ball on this planet is a quarter of the weight as measured on the earth
 d. The brass ball has the same volume on the other planet as on the earth
19. If both the mass and radius of the earth, each decreases by 50%, the acceleration due to gravity would
- a. remain same
 b. decrease by 50%
 c. decrease by 100%
 d. increase by 100%

20. A body is suspended on a spring balance in a ship sailing along the equator with a speed v' . If ω is the angular speed of the earth and ω_0 is the scale reading when the ship is at rest, the scale reading when the ship is sailing, is
- ω_0
 - zero
 - $\omega_0 \left(1 \pm \frac{2\omega v'}{g}\right)$
 - $\omega_0 \left(1 - \frac{g}{2\omega}\right)$
21. The maximum vertical distance through which a full dressed astronaut can jump on the earth is 0.5 m. Estimate the maximum vertical distance through which he can jump on the moon, which has a mean density $2/3$ rd that of the earth and radius one-quarter that of the earth.
- 1.5 m
 - 3 m
 - 6 m
 - 7.5 m
22. Two satellites A and B revolve round the same planet in coplanar circular orbits lying in the same plane. Their periods of revolutions are 1 h and 8 h, respectively. The radius of the orbit of A is 10^4 km. The speed of B relative to A when they are closed, in kmh^{-1} is
- $3\pi \times 10^4$
 - zero
 - $2\pi \times 10^4$
 - $\pi \times 10^4$
23. A satellite is moving on a circular path of radius r around earth has a time period T . If its radius slightly increases by Δr , determine the change in its time period.
- $\frac{3}{2} \left(\frac{T}{r}\right) \Delta r$
 - $\left(\frac{T}{r}\right) \Delta r$
 - $\frac{3}{2} \left(\frac{T^2}{r^2}\right) \Delta r$
 - None of these
24. The gravitational field in a region is $10 \text{ N/kg } (\hat{i} - \hat{j})$. Find the work done by gravitational force to shift slowly a particle of mass 1 kg from point (1m, 1m) to a point (2 m, - 2 m).
- 10 J
 - 10 J
 - 40 J
 - + 40 J
25. In previous problem, find the work done by external agent.
- 40 J
 - 40 J
 - zero
 - + 10 J
26. The gravitational force in a region is given by
- $$\mathbf{E} = ay \hat{i} + ax \hat{j}$$
- Find the work done by gravitational force to shift a point mass m from $(0, 0, 0)$ to (x_0, y_0, z_0) .
- $ma x_0 y_0 z_0$
 - $ma x_0 y_0$
 - $-ma x_0 y_0$
 - Zero
27. The work done by an external agent to shift a point mass from infinity to the centre of earth is
- = 0
 - > 0
 - < 0
 - ≤ 0
28. The work done in shifting a particle of mass m from centre of earth to the surface of earth is
- $-mgR$
 - $\frac{mgR}{2}$
 - zero
 - None of these
29. The change in potential energy of a body of mass m when it elevated upto height of nR from the earth's surface, is
- $nmgR$
 - $mgR = \left(\frac{n^2 + 1}{n^2}\right)$
 - $mgR \left(\frac{n-1}{n}\right)$
 - $mgR \left(\frac{n+1}{n}\right)$
30. Suppose the gravitational force varies inversely as the n th power of the distance. Then, the time period of a planet in circular orbit of radius R around the sun will be proportional to
- R^n
 - $R^{(n+1)/2}$
 - $R^{(n-1)/2}$
 - R^{-n}
31. The time period of a simple pendulum at the centre of earth is
- zero
 - infinity
 - less than zero
 - None of these
32. If a rocket is fired with a speed $v = 2\sqrt{gR}$ near the earth's surface and coasts upwards, its speed in the inter-stellar space is
- $4\sqrt{gR}$
 - $\sqrt{2gR}$
 - $\sqrt{gR}$
 - $4\sqrt{gR}$
33. In Q. 32. if the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, the speed with which it hits the surface of the earth is (Take, $g = 9.8 \text{ m/s}^2$ and $R = 6400 \text{ km}$)
- 4 km/s
 - 8 km/s
 - 2 km/s
 - 6 km/s
34. A projectile is fired vertically upward from the surface of the earth with a velocity kv_e , where v_e is the escape velocity and $k < 1$. If R is the radius of the earth, the maximum height to which it will rise measured from the centre of earth will be (neglect air resistance)
- $\frac{1-k^2}{R}$
 - $\frac{R}{1-k^2}$
 - $R(1-k^2)$
 - $\frac{R}{1+k^2}$
35. A satellite of mass M revolving in a circular orbit of radius r_s around the earth of mass M has a total energy E . Then, its angular momentum will be
- $\sqrt{\frac{E}{m_s r_s^2}}$
 - $\frac{E}{2m_s r_s^2}$
 - $(2Em_s r_s^2)^{1/2}$
 - $\sqrt{2Em_s r_s^2}$
36. A planet revolves in elliptical orbit around the sun. (see figure). The linear speed of the planet will be maximum at
- 
- A
 - B
 - C
 - D
37. Two bodies each of mass 1 kg are at a distance of 1 m. The escape velocity of a body of mass 1 kg which is midway between them is
- $8 \times 10^{-5} \text{ m/s}$
 - $2.31 \times 10^{-5} \text{ m/s}$
 - $4.2 \times 10^{-5} \text{ m/s}$
 - zero

38. If an artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of the escape velocity from the earth, the height of the satellite above the surface of the earth is
 a. $2R$ b. $R/2$
 c. R d. $R/4$
39. A particle of mass m is projected from the surface of earth with a speed v_0 ($v_0 < \text{escape velocity}$). Find the speed of particle at height $h=R$ (radius of earth). (Take, $R=6400\text{km}$ and $g=9.8\text{ m/s}^2$)
 a. $\sqrt{gR}$
 b. $\sqrt{v_0^2 - 2gR}$
 c. $\sqrt{v_0^2 - gR}$
 d. None of these
40. The binding energy of earth-sun system is (neglecting the other planets)
 a. $2.60 \times 10^{33}\text{ J}$ b. $6.33 \times 10^{16}\text{ J}$
 c. $3.40 \times 10^{33}\text{ J}$ d. $8.60 \times 10^{24}\text{ J}$
41. If one of the satellites of jupiter has an orbital period of 1.769 days and the radius of the orbit is $4.22 \times 10^8\text{ m}$. The mass of jupiter is about
 a. one thousandth that of the sun
 b. one hundredth that of the sun
 c. one tenth that of the sun
 d. half of that of the sun
42. A satellite is orbiting closely to earth and having kinetic energy K . The kinetic energy required by it to just overcome the gravitational pull of the earth, is
 a. $2K$ b. $\sqrt{2}K$ c. $\sqrt{3}K$ d. $2\sqrt{2}K$

BITSAT Archives

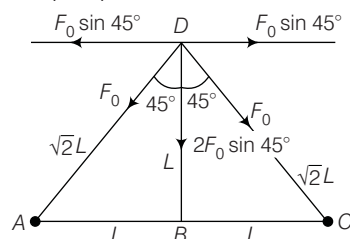
1. The total energy of a revolving satellite around the earth is $-K\text{ J}$. The minimum energy required to throw it out of earth's gravitational fields is [2013]
 a. $K\text{ J}$ b. $\frac{K}{2}\text{ J}$
 c. $2K\text{ J}$ d. None of these
2. There is a shell of mass M and density of shell is uniform. The work done to take a point mass from point A to B is ($AB=r$) [2013]
- 
- a. $\frac{GmM}{r}$ b. $\frac{GmM}{R}$ c. $-\frac{GmM}{r}$ d. zero
3. A satellite is in a circular orbit round the earth at an altitude R above the earth's surface, where R is the radius of the earth. If g is the acceleration due to gravity on the surface of the earth, the speed of the satellite is [2012]
 a. $\sqrt{2Rg}$ b. $\sqrt{Rg}$
 c. $\sqrt{\frac{Rg}{2}}$ d. $\frac{\sqrt{Rg}}{4}$
4. Which is constant, the earth revolving around the sun? [2012]
 a. Angular momentum b. Linear momentum
 c. Rotational kinetic energy d. Kinetic energy
5. Suppose the gravitational force varies inversely as the n th power of distance. Then, the time period of a planet in circular orbit of radius R around the sun will be proportional to [2011]
 a. $R^{(n+1)/2}$ b. $R^{(n-1)/2}$
 c. R^n d. $R^{(n-2)/2}$
6. The satellite of mass m revolving in a circular orbit of radius r around the earth has kinetic energy E . Then, its angular momentum will be [2011]
 a. $\sqrt{\frac{E}{mr^2}}$ b. $\frac{E}{2mr^2}$
 c. $\sqrt{2Emr^2}$ d. $\sqrt{2Emr}$
7. If a new planet is discovered rotating around sun with the orbital radius double that of the earth, then what will be its time period? (in earth's days) [2011]
 a. 1032 b. 1023
 c. 1024 d. 1043
8. If the radius of the earth were to shrink by one percent, its mass remaining the same, the value of g on the earth's surface would [2010]
 a. increase by 0.5% b. increase by 2%
 c. decrease by 0.5% d. decrease by 2%
9. The mean radius of the earth's orbit around the sun is $15 \times 10^{11}\text{ m}$ and that of the orbit of mercury is $6 \times 10^{10}\text{ m}$. The mercury will revolve around the sun is nearly [2010]
 a. $\sqrt{\frac{2}{5}}\text{ yr}$ b. $\frac{2}{5}\text{ yr}$ c. $\left(\frac{2}{5}\right)^2\text{ yr}$ d. $\left(\frac{2}{5}\right)^{3/2}\text{ yr}$
10. A satellite of mass m is orbiting around the earth at a height equal to twice the radius of the earth (R). Its potential energy is given by [2009]
 a. $-2mgR$ b. $-mg\frac{R}{2}$ c. $-\frac{2}{3}mgR$ d. $-mg\frac{R}{2}$
11. A small mass m is moved slowly from the surface of earth to a height h above the earth. The work done in doing this is [2009]
 a. $-mgR$ for $h \ll R$ b. mgh for all values of h
 c. $-\frac{1}{2}mgR$ for $h=R$ d. $\frac{1}{2}mgR$ for $h=R$

14. A satellite of mass m is placed at a distance r from the centre of earth (mass M). The mechanical energy of the satellite is [2006]
- a. $-\frac{GMm}{r}$ b. $\frac{GMm}{r}$ c. $\frac{GMm}{2r}$ d. $-\frac{GMm}{2r}$
15. If M is the mass of the earth and R its radius, the ratio of the gravitational acceleration and the gravitational constants is [2005]
- a. $\frac{R^2}{M}$ b. $\frac{M}{R^2}$ c. MR^2 d. $\frac{M}{R}$

Practice Exercise

1. (a) 2. (b) 3. (a)
4. (c) If equal masses are placed at respective corners of a regular polygon. Then, the gravitational force on a point mass placed at centre of the regular hexagon is zero.

5. (b) Here, $F_0 = \frac{Gm^2}{(\sqrt{2}L)^2} = \frac{Gm^2}{2L^2}$



$\therefore$ Resultant force,

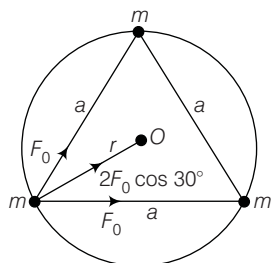
$$F = 2F_0 \cos 45^\circ \quad (\text{along } DB)$$

$$= 2F_0 \times \frac{1}{\sqrt{2}} = 2 \frac{Gm^2}{2l^2} \times \frac{1}{\sqrt{2}} = \frac{Gm^2}{\sqrt{2}l^2} \quad (\text{along } DB)$$

- 6. (c)**

7. (b) Here, $F_0 = \frac{Gm^2}{a^2}$

$$\therefore F = 2F_0 \cos 30^\circ = \frac{2Gm^2}{a^2} \frac{\sqrt{3}}{2}$$



$$= \frac{\sqrt{3} Gm^2}{a^2} \text{ or } mr\omega^2 = \frac{\sqrt{3} Gm^2}{a^2}$$

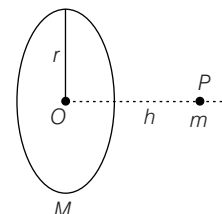
$$\text{or } m \frac{a}{\sqrt{3}} \omega^2 = \frac{\sqrt{3} Gm^2}{a^2} \therefore \omega = \sqrt{\left(\frac{3Gm}{a^3}\right)}$$

8. (a)
9. (b)
10. (d) Gravitational intensity due to the ring at a distance $d = \sqrt{3} R$ on its axis is

$$I = \frac{GMd}{(d^2 + R^2)^{3/2}} = \frac{GM \times \sqrt{3} R}{(3R^2 + R^2)^{3/2}} = \frac{\sqrt{3} GM}{8R^2}$$

$$\text{Force on sphere} = (8M)I = (8M) \times \frac{\sqrt{3} GM}{8R^2} = \frac{\sqrt{3} GM^2}{R^2}$$

11. (d) Gravitational force acting on an object of mass m , placed at point P at a distance h along the normal through the centre of a circular ring of mass M and radius r is given by



$$F = \frac{GMmh}{(r^2 + h^2)^{3/2}}$$

When mass is displaced upto distance $2h$, then

$$F' = \frac{GMm \times 2h}{(r^2 + (2h)^2)^{3/2}} = \frac{2GMmh}{(r^2 + 4h^2)^{3/2}}$$

When $h = r$, then

$$F = \frac{GMm \times r}{(r^2 \times r^2)^{3/2}} = \frac{GMm}{2\sqrt{2}r^2}$$

and

$$F' = \frac{2GMmr}{(r^2 + 4r^2)^{3/2}} = \frac{2GMm}{5\sqrt{5}r^2}$$

$$\therefore \frac{F'}{F} = \frac{4\sqrt{2}}{5\sqrt{5}} \text{ or } F' = \frac{4\sqrt{2}}{5\sqrt{5}} F$$

12. (a)
13. (d) Mean orbital radius of the earth around the sun,

$$r = 1.5 \times 10^8 \text{ km} = 1.5 \times 10^{11} \text{ m}$$

Time period of the earth around the sun

$$= 1 \text{ yr} = 365 \text{ days}$$
$$= 365 \times 24 \times 60 \times 60 \text{ s}$$

As, required centripetal force is obtained from the gravitational force, therefore

Centripetal force = Gravitational force

$$\frac{mv^2}{r} = \frac{GM_s m}{r^2}, v^2 = \frac{GM_s}{r}$$

$$(r\omega)^2 = \frac{GM_s}{r} \text{ or } \omega^2 = \frac{GM_s}{r^3} \quad [\because v = r\omega]$$

But, $\omega = \frac{2\pi}{T}$

$$\therefore \left(\frac{2\pi}{T}\right)^2 = \frac{GM_s}{r^3} \text{ or } M_s = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{4 \times (3.14)^2 \times (1.5 \times 10^{11})^3}{6.67 \times 10^{-11} (365 \times 24 \times 60 \times 60)^2}$$

$$\approx 2 \times 10^{30} \text{ kg}$$

14. (a) $g_1 = \frac{GM}{(R+h)^2}$

$$\therefore \frac{g_1}{g} = \left(\frac{R}{R+h}\right)^2 \therefore g_1 = \left(\frac{R}{R+h}\right)^2 g$$

$$g_1 = \left(\frac{6400}{6400 + 2624}\right)^2 \times g = \frac{g}{2}$$

 $\therefore$ At surface of earth, $mg = kx_0$ At the height $h = 2624$ km, $mg_1 = kx$

$$\therefore \frac{x}{x_0} = \frac{g_1}{g}$$

$$\therefore x = \frac{g_1}{g} \times x_0 = \frac{1}{2} \times 2 = 1 \text{ cm}$$

15. (a)

16. (a) As $g' = g - \omega^2 R \cos^2 \phi$ At the equator, $\phi = 0$, i.e. $\cos \phi = 1$ For weight to be zero $g' = 0$

$$\therefore \omega = \sqrt{\frac{g}{R}} = 0.00125 \text{ rad s}^{-1}$$

17. (b) The value of g at two places will must be same

$$\Rightarrow \frac{gR^2}{(R+h)^2} = g \left(1 - \frac{h}{R}\right)$$

$$\Rightarrow \left(1 - \frac{h}{R}\right) \left(1 + \frac{h^2}{R^2} + \frac{2h}{R}\right) = 1$$

$$\Rightarrow \frac{h^3}{R^3} + \frac{h^2}{R^2} - \frac{h}{R} = 0$$

$$\Rightarrow \frac{h}{R} \left(\frac{h^2}{R^2} + \frac{h}{R} - 1\right) = 0$$

$$\Rightarrow h = \frac{\sqrt{5}R - R}{2} = \frac{R(\sqrt{5} - 1)}{2}$$

18. (a)

19. (d) Here, $g = GM/R^2$ and

$$g' = \frac{G(M/2)}{(R/2)^2} = \frac{2GM}{R^2} = 2g$$

$$\therefore \% \text{ increase in } g = \left(\frac{g' - g}{g}\right) \times 100$$

$$= \left(\frac{2g - g}{g}\right) \times 100 = 100\%$$

20. (c)

21. (b) On the moon, $g_m = \frac{4}{3} \pi G (R/4) (2\rho/3)$

$$= \frac{1}{6} \left(\frac{4}{3} \pi G R \rho\right) = \frac{1}{6} g$$

Work done in jumping $= m \times g \times 0.5 = m \times (g/6) h_1$

$$h_1 = 0.5 \times 6 = 3.0 \text{ m}$$

22. (d) From the Kepler's law,

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

$$\Rightarrow \frac{1}{64} = \frac{r_1^3}{r_2^3} \Rightarrow r_2 = 4 \times 10^4 \text{ km}$$

$$\therefore v_1 = \frac{2\pi r_1}{T_1} \text{ and } v_2 = \frac{2\pi r_2}{T_2}$$

$$\Rightarrow v_1 = 2 \times 10^4 \pi \text{ kmh}^{-1}$$

and $v_2 = \pi \times 10^4 \text{ kmh}^{-1}$

Thus, speed of B relative to A is v_{21} .

i.e. $v_{21} = \pi \times 10^4 \text{ kmh}^{-1}$

23. (a) 24. (d)

25. (b) Since, particle is shifted slowly. So, change in kinetic energy is zero.

According to work-energy theorem

$$W + W_{\text{ext}} = 0$$

$$W_{\text{ext}} = -W = -40 \text{ J}$$

26. (b) Here, $\mathbf{F} = m \mathbf{E} = m a (\mathbf{y}\hat{\mathbf{i}} + \mathbf{x}\hat{\mathbf{j}})$

$$\therefore \mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

$$\therefore d\mathbf{r} = dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}$$

$$\therefore W = \int \mathbf{F} \cdot d\mathbf{r} = ma \int_{0,0,0}^{x_0,y_0,z_0} (y\hat{\mathbf{i}} + x\hat{\mathbf{j}}) \cdot (dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}})$$

$$= ma \int d(xy) = ma [xy]_{0,0,0}^{x_0,y_0,z_0} = ma x_0 y_0$$

27. (c)

28. (b)

29. (c) Change in PE $= \Delta U = U_f - U_i = GmM \left(\frac{1}{R} - \frac{1}{nR}\right)$

$$= \frac{GMm}{R} \left(\frac{n-1}{n}\right) = mgR \left(\frac{n-1}{n}\right) \quad \left(\because g = \frac{GM}{R^2}\right)$$

30. (b) $T = \frac{2\pi R}{v}$

$$E = \frac{1}{2} mv^2 = \frac{GMm}{R^{n-1}} \text{ or } v = \left[\frac{2GM}{R^{n-1}}\right]^{1/2}$$

$$\therefore T = \frac{2\pi R}{\sqrt{\frac{2GM}{R^{n-1}}}} = \frac{2\pi}{\sqrt{2GM}} \times R^{(n+1)/2}$$

$$\therefore T \propto R^{(n+1)/2}$$

31. (b)

32. (b) Applying law of conservation of energy,

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}mv_1^2 + 0$$

$$\text{or} \quad \frac{1}{2}mv_1^2 = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$\text{or} \quad v_1^2 = v^2 - \frac{2GM}{R} = v^2 - v_e^2$$

$$\therefore v_1 = \sqrt{v^2 - v_e^2} = \sqrt{4gR - 2gR} = \sqrt{2gR}$$

33. (b)

 34. (b) If a body is projected from the surface of the earth with a velocity v and reaches a height h , then according to the law of conservation of energy,

$$\frac{1}{2}mv^2 = \frac{mgh}{1 + \frac{h}{R}}$$

$$\text{Here, } v = kv_e = k\sqrt{2gR}$$

$$\therefore \frac{1}{2}mk^2 \cdot 2gR = \frac{mgh}{1 + \frac{h}{R}}$$

$$k^2R \left[1 + \frac{r-R}{R} \right] = r - R$$

$$\text{or} \quad k^2r = r - R$$

$$\text{or} \quad r = \frac{R}{1 - k^2}$$

$$35. (c) \therefore \frac{GMm_s}{r_s^2} = \frac{m_s v_s^2}{r_s}$$

$$v_s = \left(\frac{GM}{r_s} \right)^{1/2}$$

$$\text{Angular momentum} = m_s v_s r_s = J$$

$$\therefore J = \left(\frac{GM}{r_s} \right)^{1/2} m_s r_s = (GMm_s^2 r_s)^{1/2}$$

$$\therefore \text{KE} = \frac{1}{2}m_s v_s^2 = \frac{1}{2}m_s \frac{GM}{r_s}$$

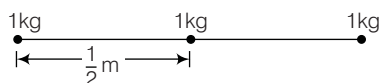
$$\text{PE} = -\frac{GMm_s}{r_s}$$

$$\text{Total energy, } E = -\frac{GMm_s}{2r_s}$$

$$\therefore J = (2Em_s r_s^2)^{1/2}$$

36. (a)

$$37. (b) U_i = -\frac{6.67 \times 10^{-11} \times 1^2}{\left(\frac{1}{2}\right)} - \frac{6.67 \times 10^{-11} \times 1^2}{\left(\frac{1}{2}\right)}$$



$$= -6.67 \times 10^{-11} (2 + 2)$$

$$= -4 \times 6.67 \times 10^{-11}$$

$$U_f = 0 \text{ at infinity}$$

$$\therefore \frac{1}{2}mv_e^2 = \Delta U_f - U_i = 4 \times 6.67 \times 10^{-11}$$

$$\therefore v_e = \sqrt{2 \times 4 \times 6.67 \times 10^{-11}} = 2.31 \times 10^{-5} \text{ m/s}$$

$$38. (c) v_e = \text{escape velocity} = \sqrt{\frac{2GM}{R+h}}$$

$$v_o = \text{orbital velocity} = \sqrt{\frac{GM}{R+h}}$$

$$\text{Given, } v_o = \frac{v_e}{2}$$

$$\therefore \sqrt{\frac{GM}{R+h}} = \frac{1}{2} \sqrt{\frac{2GM}{R}}$$

$$\text{or} \quad \frac{GM}{R+h} = \frac{1}{4} \times \frac{2GM}{R}$$

 On solving, we get $h = R$

39. (c) During motion of the particle, total mechanical energy remains constant.

At the surface of earth, total mechanical energy is

$$\begin{aligned} E_i &= -\frac{GmM}{R} + \frac{1}{2}mv_0^2 \\ &= -\frac{GM}{R^2}mR + \frac{1}{2}mv_0^2 \\ &= -gmR + \frac{1}{2}mv_0^2 \end{aligned}$$

 Total mechanical energy at height $h = R$ is

$$\begin{aligned} E_f &= -\frac{GmM}{2R} + \frac{1}{2}mv^2 \\ &= -\frac{gmR}{2} + \frac{1}{2}mv^2 \end{aligned}$$

According to conservation principle of energy,

$$\begin{aligned} E_i &= E_f \\ \therefore -gmR + \frac{1}{2}mv_0^2 &= -\frac{gmR}{2} + \frac{1}{2}mv^2 \end{aligned}$$

$$\text{or} \quad -2gR + v_0^2 = -gR + v^2$$

$$\therefore v = \sqrt{v_0^2 - gR}$$

40. (a) 41. (a)

 42. (a) Orbital speed closer to the earth is $\sqrt{gR}$.

$$K = \text{KE} = \frac{1}{2}m(\sqrt{gR})^2 = \frac{mgR}{2}$$

 Now, if the velocity is changed to $\sqrt{2gR}$ (escape speed), then the kinetic energy K' will be

$$K' = \frac{1}{2}m(\sqrt{2gR})^2 = mgR$$

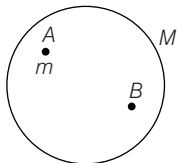
$$\Rightarrow K' = 2K$$

BITSAT Archives

1. (a) If the satellite could be through outside the earth's gravitational field, its minimum total energy = 0.

So, $-K + K' = 0$
 $K' = K \text{ J}$

2. (d)



The gravitational field at A and B are zero. So, no work required to make change between the points A and B.

3. (c) Orbital velocity (v_o) at a height h above the earth's surface is given by

$$v_o = R_e \sqrt{\frac{g}{R_e + h}}$$

Given,

$$h = R_e$$

$\therefore$

$$v_o = R \sqrt{\frac{g}{2R}}$$

$\Rightarrow$

$$v_o = \sqrt{\frac{Rg}{2}}$$

4. (a) Kepler's second law

$$\frac{dA}{dt} = \frac{L}{2m}$$

$$\frac{dA}{dt} = \text{constant}$$

$$L = \text{constant}$$

5. (a) The necessary centripetal force required for a planet to move round the sun

= gravitational force exerted on it

$$\frac{mv^2}{R} = \frac{GM_e m}{R^n} \text{ or } v = \left(\frac{GM}{R^{n-1}} \right)^{1/2}$$

As,

$$T = \frac{2\pi R}{v} = 2\pi R \times \left(\frac{R^{n-1}}{GM} \right)^{1/2}$$

$$T = 2\pi \left[\frac{R^{\frac{(n+1)}{2}}}{(GM_e)^{1/2}} \right]$$

$$T \propto R^{(n+1)/2}$$

6. (c) KE of a satellite

$$E = \frac{1}{2} mv^2$$

or

$$mv = \sqrt{2Em}$$

Angular momentum,

$$L = mvr = (\sqrt{2Em}) \times r = \sqrt{2mEr^2}$$

7. (a) By Kepler's third law, $T^2 \propto R^3$

$$\therefore \left(\frac{T_2}{365 \text{ days}} \right)^2 = \left(\frac{2r}{r} \right)^3$$

$$T_2 = 365 \times 2\sqrt{2} = 1032 \text{ days}$$

8. (b) $g = \frac{GM}{R^2}$

$$\frac{\Delta g}{g} \times 100 = 2 \left(\frac{\Delta R}{R} \times 100 \right) = 2 \times 1\% = 2\%$$

Therefore, the value of g will increase by 2%.

9. (d) According to Kepler's law,

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

$$\frac{1}{T_2^2} = \left(\frac{15 \times 10^{11}}{6 \times 10^{10}} \right)^3 = \left(\frac{5}{2} \right)^3$$

$$T_2 = \left(\frac{2}{5} \right)^{3/2} \text{ yr}$$

10. (c) $PE = \int_R^{3R} -\frac{GMm}{r^2} dr$

$$= -GMm \left[\frac{r^{-1}}{-1} \right]_R^{3R} = GMm \left[\frac{1}{3R} - \frac{1}{R} \right]$$

$$= -\frac{2}{3} \frac{GMm}{R} = -\frac{2}{3} \frac{gR^2 m}{R} = -\frac{2}{3} mgR$$

11. (d) When $h = R$

$$\text{Work done} = -\frac{GMm}{(R+R)} - \left(-\frac{GMm}{R} \right) = \frac{GMm}{2R} = \frac{1}{2} mgR$$

12. (a) Time period of satellite,

$$T \propto \frac{1}{M^{1/2}}, \text{ where } M \text{ is mass of earth.}$$

$T \propto (R+h)^{3/2}$, where R is radius of the orbit, h is the height of satellite from the earth's surface.

13. (a) $v = \sqrt{2gR}$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{g_1}{g_2} \times \frac{R_1}{R_2}} = \sqrt{g \times K} = (Kg)^{1/2}$$

14. (d) If a satellite of mass m is placed at a distance r from the centre of the earth (of mass M), the mechanical energy of the satellite is given by $\frac{-GMm}{2r}$.

15. (b) Gravitational acceleration is given by $g = \frac{GM}{R^2}$

where, G = gravitational constant

$$\therefore \frac{g}{G} = \frac{M}{R^2}$$

10

Simple Harmonic Motion

Periodic Motion

A motion which repeats itself over and over again after a regular interval of time is called a **periodic motion**. A periodic motion in which a body moves back and forth repeatedly about a fixed point (called mean position) is called **oscillatory** or **vibratory motion**.

Period The regular interval of time after which periodic motion repeats itself is called period of the motion.

Frequency The number of times of motion repeated in one second is called frequency of the periodic motion. Every oscillatory motion is periodic but every periodic motion is not an oscillatory motion.

Displacement as a Function of Time

In a periodic motion each displacement value is repeated after a regular interval of time, displacement can be represented as a function of time $y = f(t)$.

Periodic Function

A function which repeats its value after a fix interval of time is called a **periodic function**.

$$y(t) = y(t + T)$$

where, T is the period of the function.

Trigonometric functions $\sin\theta$ and $\cos\theta$ are simplest periodic functions having period of 2π .

Simple Harmonic Motion (SHM)

It is a special kind of oscillatory motion in which particle moves to and fro about a mean position (on a straight line) under a restoring force which is directed towards mean position and its magnitude (of restoring force) is directly proportional to the displacement of particle (at all instants). For simple harmonic motion, displacement should be very small.

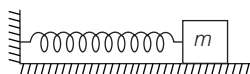
Simple harmonic motion is a special form of oscillatory or vibratory motion.

It is of two types

1. Linear simple harmonic motion
2. Angular simple harmonic motion

Linear SHM

In linear SHM, a particle moves to and fro (on a straight line) about fixed point under a restoring force (always directed towards mean position) whose magnitude is directly proportional to displacement of the particle (at all instants).



e.g. Motion of a block connected to a spring on a smooth surface.

Restoring force $\propto$ Displacement

$$F \propto -x \quad (\text{if displacement is along } X\text{-axis})$$

$\therefore$ Acceleration, $a \propto -x$

$$\text{or} \quad \frac{d^2x}{dt^2} \propto -x$$

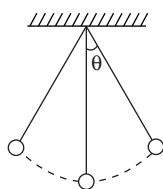
$$\Rightarrow \quad a = -\omega^2 x$$

where, a is acceleration and ω is angular frequency.

Negative sign indicates direction of restoring force and acceleration is towards equilibrium position, but in opposite direction of displacement.

Angular SHM

In angular SHM, the restoring torque (or angular acceleration) acting on the particle is proportional to the angular displacement of the particle and directed towards equilibrium position.



Restoring torque $\propto$ Angular displacement

$$\tau \propto -\theta$$

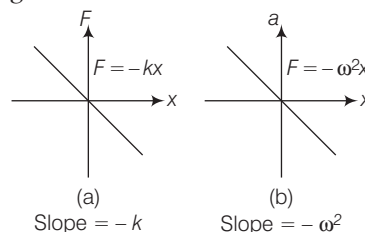
$$\Rightarrow \quad \alpha \propto -\theta$$

$$\text{or} \quad \frac{d^2\theta}{dt^2} \propto -\theta$$

Some Terms Related to SHM

- Mean position** It is a point at which restoring force on the particle is zero.
- Restoring force** The force which is acting on the particle tends to bring the particle towards its mean position is known as restoring force.

Restoring force, $F = -kx$, where x is displacement, k is spring constant.



(iii) Force constant (Spring constant)

We know that, defining equation of SHM is

$$a = -\omega^2 x, \text{ also } a = \frac{F}{m}$$

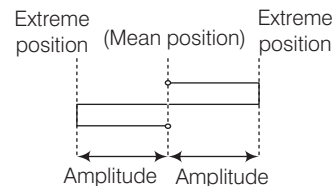
$$\therefore \quad \frac{F}{m} = -\omega^2 x$$

$$F = -m\omega^2 x \Rightarrow F = -kx$$

k is called spring constant or force constant.

$$\text{Also, } k = m\omega^2.$$

- Amplitude** Maximum displacement of particle from mean position on either side is defined as amplitude. It is a scalar quantity.



$$\text{Amplitude} = \frac{1}{2} \times (\text{Distance between extreme points})$$

or position)

- Instantaneous displacement** Displacement of particle from mean position in a particular direction (at any instant of time) is defined as instantaneous displacement.

$$\text{It is given by } x = A \sin(\omega t + \phi)$$

where, A = amplitude

ϕ = initial phase or phase constant

Velocity

The velocity of a particle executing SHM at an instant is defined as the time rate of change of its displacement at that instant.

$$\text{Velocity, } v = \omega \sqrt{A^2 - y^2}$$

At the mean position ($y=0$), during its motion $v = A\omega = v_{\max}$ and at the extreme positions ($y = \pm A$) $v = 0$.

$$\text{Velocity amplitude} = v_{\max} = A\omega$$

Acceleration

The acceleration of a particle executing SHM at an instant is defined as the time rate of change of velocity at that instant.

Acceleration, $a = -\omega^2 y$

The acceleration is also a variable. At the mean position ($y = 0$), acceleration $a = 0$ and at the extreme position ($y = \pm A$), the acceleration is $a_{\max} = -A\omega^2$.

$\therefore$ Acceleration amplitude, $a_{\max} = A\omega^2$

Phase and Phase Relationship

Phase is that physical quantity which tells about the position and direction of motion of any particle at any moment. It is denoted by ϕ . In SHM, the velocity is ahead of the displacement by a phase $\frac{\pi}{2}$ and the acceleration is further ahead of the velocity by a phase of $\frac{\pi}{2}$.

Phase Difference

The difference of total phase angles of two particles executing SHM with respect to their mean positions is known as **phase difference**. If phase angles of two particles executing SHM are $(\omega t + \phi_1)$ and $(\omega t + \phi_2)$ respectively, then

$$\Delta\phi = (\omega t + \phi_2) - (\omega t + \phi_1)$$

$$\Delta\phi = \phi_2 - \phi_1$$

Time Period

The time taken by a particle to complete one oscillation is called **time period**. It is denoted by T .

Acceleration in SHM, $a = -\omega^2 y$

Hence, $|a| = \omega^2 |y| \Rightarrow \omega = \sqrt{\frac{|a|}{|y|}}$

$\therefore$ Time period of SHM,

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{|y|}{|a|}} = 2\pi \sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

Equation of Simple Harmonic Motion

A simple harmonic motion may be mathematically expressed by a single sinusoidal (sine or cosine) function of time. The equation of simple harmonic motion is given by

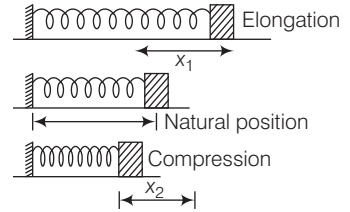
$$y = A \sin \omega t$$

$$\text{and } x = A \cos \omega t$$

- Differential equations of SHM for linear SHM, $\frac{d^2 y}{dt^2} + \omega^2 y = 0$, for angular SHM, $\frac{d^2 \theta}{dt^2} + \omega^2 \theta = 0$
- All sine and cosine functions of t are simple harmonic in nature, i.e. for the function
- $y = A \sin(\omega t \pm \phi)$ or $y = A \cos(\omega t \pm \phi)$
- $\frac{d^2 y}{dt^2}$ is directly proportional to $-y$. Hence, they are simple harmonic in nature.

Oscillations of a Spring Block System

A spring pendulum consists of a point (small sized) mass m either suspended from a massless (or light) spring or placed on a smooth horizontal plane attached with a spring.



Here, $x_1 = x_2 = l$

If the mass is once pulled so as to stretch the spring and is then released, then a restoring force acts on it which continuously tries to restore its mean position, restoring force $F = -kl$, where k is force constant and l is the change in length of the spring under the restoring force the spring pendulum oscillates simple harmonically having time period and frequency given by

$$T = 2\pi \sqrt{\frac{m}{k}} \text{ and } \nu = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

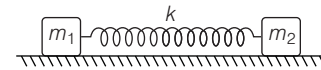
where, k is the force constant of the spring and it is numerically equal to the force required to increase the length of the spring by unity.

If the spring is not light but has a mass m_s , then

$$T = 2\pi \sqrt{\frac{m + 1/3 m_s}{k}}$$

NOTE If two masses m_1 and m_2 , connected by a spring are made to oscillate on a horizontal surface, then its period will be

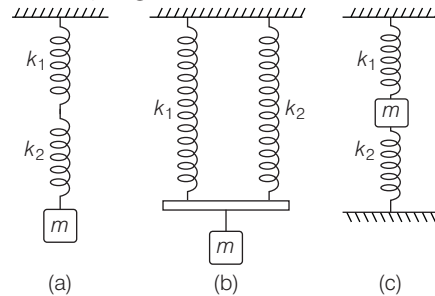
$$T = 2\pi \sqrt{\frac{\mu}{k}}$$



where, $\mu = \frac{m_1 m_2}{m_1 + m_2}$ = reduced mass of the system.

Series Combination of Springs

If two springs of spring constants k_1 and k_2 are joined in series (horizontally and vertically), then their equivalent spring constant k_s is given by



$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$\Rightarrow k_s = \frac{k_1 k_2}{k_1 + k_2}$$

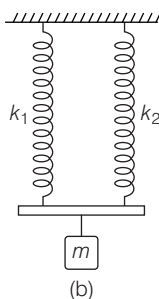
$$\therefore T = 2\pi \sqrt{\frac{m}{k_s}} = 2\pi \sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$

Parallel Combination of Springs

If the two springs of spring constants k_1 and k_2 are joined in parallel as shown in the figure then their equivalent spring constant $k_p = k_1 + k_2$, hence

$$T = 2\pi \sqrt{\frac{m}{k_p}}$$

$$= 2\pi \sqrt{\frac{m}{(k_1 + k_2)}}$$



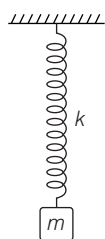
Spring Pendulum

A point mass suspended from a massless (or light) spring constitutes a spring pendulum. If the mass is once pulled downwards so as to stretch the spring and then released, the system oscillates up and down about its mean position simple harmonically. Time period (T) and frequency (ν) of oscillations are given by

$$T = 2\pi \sqrt{\frac{m}{k}} \quad \text{or} \quad \nu = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

If the spring is not light but has a definite mass m_s , then it can be easily shown that period of oscillation will be

$$T = 2\pi \sqrt{\frac{m + \frac{m_s}{3}}{k}}$$



Energy in Simple Harmonic Motion

If a particle executes SHM, its kinetic energy changes into potential energy and *vice-versa* keeping total energy constant (if friction of air is neglected).

A particle executing SHM possesses two types of energy

1. **Kinetic energy** This is an account of the velocity of the particle.

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2 \cos^2(\omega t + \phi) = \frac{1}{2}m\omega^2(A^2 - x^2)$$

From this expression, we conclude that kinetic energy is maximum at the centre ($x = 0$) and zero at the extremes of oscillation ($x = \pm A$).

2. **Potential energy** This is an account of the displacement of the particle from its mean position.

$$U = \frac{1}{2}m\omega^2 x^2$$

$$= \frac{1}{2}mA^2\omega^2 \sin^2(\omega t + \phi)$$

Thus, potential energy has its minimum at the centre ($x = 0$) and increases as the particle approaches either extreme of oscillation ($x = \pm A$).

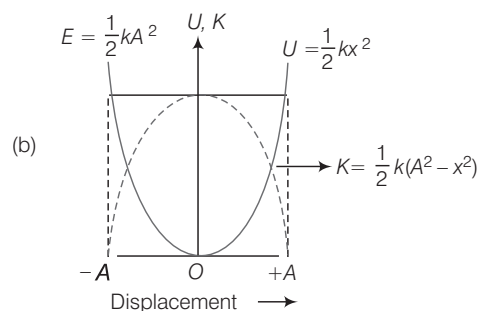
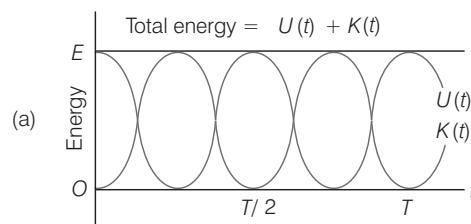
3. **Mechanical energy** This is an account of kinetic and potential energy.

Thus, total mechanical energy = kinetic energy + potential energy

$$\text{or} \quad E = \frac{1}{2}m\omega^2 A^2$$

Obviously, the total energy is constant and is proportional to the square of amplitude (A) of motion.

Figures show the variations of total energy (E), potential energy (U) and kinetic energy (K) with displacement (x).



Some Important Points

- KE and PE also varies periodically.
- Frequency of PE or KE is double than that of SHM.
- The frequency of total energy in SHM is zero.
- The average values of KE and PE in one complete revolution is $\frac{1}{4}m\omega^2 A^2$.
- The average value of total energy in one complete revolution is $\frac{1}{2}m\omega^2 A^2$.

Simple Pendulum

A simple pendulum, in practice, consists of a heavy but small sized metallic bob suspended by a light, inextensible and flexible string. The motion of a simple pendulum is simple harmonic for very small angular displacement (α) whose time period and frequency are given by

$$T = 2\pi\sqrt{\frac{l}{g}} \text{ and } \nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

where, l is the effective length of the string and g is acceleration due to gravity.

- (i) If a pendulum of length l at temperature $\theta^\circ\text{C}$ has a time period T , then on increasing the temperature by $\Delta\theta^\circ\text{C}$ its time period changes to $T \times \Delta T$,

$$\text{where, } \frac{\Delta T}{T} = \frac{1}{2}\alpha\Delta\theta$$

where, α is the temperature coefficient of expansion of the string.

- (ii) A second's pendulum is a pendulum whose time period is 2s. At a place where $g = 9.8\text{m/s}^2$, the length of a second's pendulum is 0.9929 m (or 1 m approx).

- (iii) If the bob of a pendulum (having density ρ) is made to oscillate in a non-viscous fluid of density σ , then it can be shown that the new period is

$$T = 2\pi\sqrt{\frac{l}{g\left(1 - \frac{\sigma}{\rho}\right)}}$$

- (iv) If a pendulum is in a lift or in some other carriage moving vertically with an acceleration a , then the effective value of the acceleration due to gravity becomes $(g \pm a)$ and hence,

$$T = 2\pi\sqrt{\frac{l}{(g \pm a)}}$$

Here, positive sign is taken for an upward accelerated motion and negative sign for a downward accelerated motion.

- (v) If a pendulum is made to oscillate in a freely falling lift or an orbiting satellite then the effective value of g is zero and hence, the time period of the pendulum will be infinity and therefore pendulum will not oscillate at all.

Free, Forced, Damped and Resonant Vibrations

Free Vibrations

If a given body is once set into vibrations and then let free to vibrate with its own natural frequency, the vibrations are

said to be free vibrations. The natural frequency of free vibrations depends on the nature and structure of the body and in ideal situation, the amplitude, frequency and the energy of the vibrating body remain constant.

Forced Vibrations

The vibrations in which a body oscillates under the effect of an external periodic force, whose frequency is different from the natural frequency of oscillating body are called forced vibrations. In forced vibrations, the oscillating body vibrates with the frequency of external force and amplitude of oscillations is generally small.

If an external driving force is represented by

$$F(t) = F_0 \cos \omega_d t$$

The motion of particle is under combined action of restoring force ($-kx$), damping force ($-bv$) and driving force $F(t)$.

Now,

$$ma = -kx - bv + F_0 \cos \omega_d t$$

or

$$\frac{d^2x}{dt^2} = -\frac{kx}{m} - \frac{b}{m} \frac{dx}{dt} + \frac{F_0 \cos \omega_d t}{m}$$

The solution of this equation gives $x = x_0 \sin(\omega_d t + \phi)$ with amplitude

$$x_0 = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega_d^2)^2 + \left(\frac{b\omega_d}{m}\right)^2}}, \tan \theta = \frac{\omega_0^2 - \omega_d^2}{b\omega_d/m}$$

and $\omega_0 = \sqrt{\frac{k}{m}}$ = natural frequency

Damped Vibrations

When a body is set in free vibrations, generally there is a dissipation of energy due to dissipative causes like viscous drag of a fluid, frictional force, hysteresis, electromagnetic damping force, etc., and as a result amplitude of vibration regularly decreases with time. Such vibrations of continuously falling amplitudes are called damped vibrations. In these oscillations, the amplitude of oscillation decreases exponentially and hence, energy also decreases exponentially. If the velocity of an oscillator is v , the damping force

$$F_d = -bv$$

where, b = damping constant.

Resultant force on a damped oscillator is given by

$$F = F_R + F_D = -kx - bv \text{ or } \frac{md^2x}{dt^2} + \frac{bdx}{dt} + kx = 0$$

Displacement of a damped oscillator is given by

$$x = x_m e^{-bt/2m} \sin(\omega' t + \phi)$$

$$\omega' = \sqrt{\omega_0^2 - (b/2m)^2}$$

where, ω' = angular frequency of the damped oscillator

For a damped oscillator, if the damping is small then the mechanical energy decreases exponentially with time

as $E = \frac{1}{2} kx_m^2 e^{-bt/m}$

Resonant Vibrations

It is a special case of forced vibrations in which frequency of external force is exactly same as the natural frequency of oscillator. As a result the oscillating body begins to vibrate with a large amplitude leading to the resonance phenomenon to occur. Resonant vibrations play a very important role in music and tuning of station/channel in a radio/TV.

Some Important Points

(i) Physical or compound pendulum, $T = 2\pi \sqrt{\left(\frac{l + \frac{K^2}{l}}{g} \right)}$

where, l = distance of centre of gravity from point of suspension and

K = radius of gyration about an axis passing through centre of gravity.

(ii) In the case of water oscillating in a U-tube,

$$T = 2\pi \sqrt{\left(\frac{h}{g} \right)}$$

where, h is the height of liquid column in each limb.

(iii) When a ball of mass m is made to oscillate in the neck of an air chamber having volume V and neck area A , then

$$T = 2\pi \sqrt{\left(\frac{mV}{pA^2} \right)}$$

(iv) If a person sitting on an oscillating swings stands up, the time period of the swing decreases.

(v) When a pendulum is kept in a car which is sliding down, then

$$T = 2\pi \sqrt{\left(\frac{1}{g \cos \theta} \right)}$$

where, θ is an angle of inclination.

(vi) If time period of one spring is T_1 and that of second spring is T_2 and if they are connected in series, then $T_{\text{series}} = \sqrt{T_1^2 + T_2^2}$. If they are connected in parallel, then

$$T_{\text{parallel}} = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}}$$

(vii) The time period of a simple pendulum having long

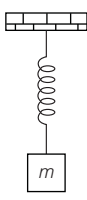
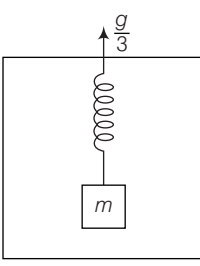
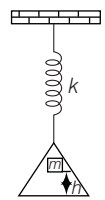
length is $T = 2\pi \sqrt{\left(\frac{IR}{(l+R)g} \right)}$

where, R = radius of earth

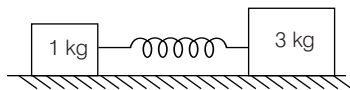
If length is infinite, then $T = 2\pi \sqrt{\left(\frac{R}{g} \right)}$

Practice Exercise

- A particle executing simple harmonic motion has amplitude of 1 m and time period 2 s. At $t = 0$, net force on the particle is zero. The equation of displacement of particle is
a. $x = \sin \pi t$ b. $x = \cos \pi t$ c. $x = \sin 2\pi t$ d. $x = \cos 2\pi t$
- In Q. 1, find maximum velocity and maximum acceleration.
a. 1 m/s, π m/s² b. π m/s and π^2 m/s²
c. π m/s and π m/s² d. None of these
- A particle executes simple harmonic motion. The amplitude of vibration of particle is 2 cm. The displacement of particle in one time period is
a. 1 cm b. 2 cm c. 4 cm d. zero
- In Q. 3, the distance travelled by the particle is
a. 8 cm b. 2 cm
c. 4 cm d. zero
- A particle moves along Y-axis according to equation $y = 3 + 4 \cos \omega t$. The motion of particle is
a. not SHM b. oscillatory but not SHM
c. SHM d. None of these
- In Q. 5, the amplitude of vibration is
a. 3 units b. 4 units
c. 5 units d. None of these
- If $\mathbf{s} = a \sin \omega t \hat{i} + b \cos \omega t \hat{j}$, the equation of path of particle is
a. $x^2 + y^2 = \sqrt{a^2 + b^2}$ b. $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$
c. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ d. None of these
- In previous question, the amplitude of vibration is
a. 4 units b. 8 units
c. 10.58 units d. None of these

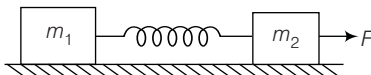
9. The motion of a particle varies with time according to the relation
 $y = a \sin \omega t + a \cos \omega t$. Then,
 a. the motion is oscillatory but not SHM
 b. the motion is SHM with amplitude a
 c. the motion is SHM with amplitude $\sqrt{2}a$
 d. None of the above
10. A particle executes SHM along a straight line so that its period is 12 s. The time it takes in traversing a distance equal to half its amplitude from its equilibrium position is
 a. 6 s
 b. 4 s
 c. 2 s
 d. 1 s
11. A particle executes SHM with an amplitude of 10 cm and frequency 2 Hz. At $t = 0$, the particle is at a point where, potential energy and kinetic energy are same. The equation of displacement of particle is
 a. $0.1 \sin \left(4\pi t + \frac{\pi}{4} \right)$
 b. $0.1 \sin 4\pi t$
 c. $0.1 \cos \left(4\pi t + \frac{\pi}{4} \right)$
 d. None of the above
12. A particle executes simple harmonic motion with a frequency f . The frequency with which the potential energy oscillates is
 a. f
 b. $f/2$
 c. $2f$
 d. zero
13. A particle of mass m is executing oscillation about the origin on the X -axis. Its potential energy is $u(x) = k|x|^3$, where k is a positive constant. If the amplitude of oscillation is a , then its time period T is
 a. proportional to $\frac{1}{\sqrt{a}}$
 b. independent of a
 c. proportional to $\sqrt{a}$
 d. proportional to $a^{3/2}$
14. A particle of mass m is free to move along the x -axis and has potential energy given by $u(x) = k[1 - e^{-x^2}]$ for $-\infty \leq x \leq \infty$, where k is positive constant of appropriate dimensions. Then,
 a. at points away from the origin, the particle is in unstable equilibrium
 b. for any finite non-zero value of x , there is a force directed away from the origin
 c. its total mechanical energy is $\frac{k}{2}$, it has its minimum kinetic energy at origin
 d. for small displacement from $x = 0$, motion is SHM
15. A simple harmonic oscillator has amplitude A , angular velocity ω , and mass m . Then, average energy in one time period will be
 a. $\frac{1}{4} m \omega^2 A^2$
 b. $\frac{1}{2} m \omega^2 A^2$
 c. $m \omega^2 A^2$
 d. zero
16. A point mass $m = 20$ kg, is suspended by a massless spring of constant 2000 N/m. The point mass is released when elongation in the spring is 15 cm. The equation of displacement of particle as function of time is (take, $g = 10 \text{ m/s}^2$)
 a. $y = 10 \sin 10t$
 b. $Y = 10 \cos 10t$
 c. $y = 10 \sin \left(10t + \frac{\pi}{6} \right)$
 d. None of these
- 
17. A spring of spring constant 200 N/m has a block of mass 1 kg hanging at its one end and from other end spring is attached to a ceiling of an elevator. The elevator rising upwards with an acceleration of $g/3$. When acceleration is suddenly cease, then what should be the angular frequency and elongation during the time when the elevator is accelerating?
 a. 14.14 rad/s, 0.07 m
 b. 14 rad/s, 0.1 m
 c. 14.14 rad/s, 0.05 m
 d. 10 rad/s, 0.07 m
- 
18. A spring of force constant k is cut into two pieces such that one piece is double the length of the other. Then, the long piece will have a force constant of
 a. $\frac{2}{3}k$
 b. $\frac{3}{2}k$
 c. $3k$
 d. $6k$
19. A solid copper sphere is suspended from a massless spring. The time period of oscillation of the system is 4 s. The sphere is now completely immersed in a liquid whose density is $1/8$ th that of brass. The sphere remains in liquid during oscillation. Now, the time period is
 a. 4 s
 b. 2 s
 c. 3 s
 d. None of these
20. A load of mass m falls from a height h on to the scale pan hung from a spring as shown in the adjoining figure. If the spring constant is k and mass of the scale pan is zero and the mass m does not bounce relative to the pan, then the amplitude of vibration is
 a. $\frac{mg}{k}$
 b. $\frac{mg}{k} \sqrt{\left(\frac{1+2hk}{mg} \right)}$
 c. $\frac{mg}{k} + \frac{mg}{k} \sqrt{\left(\frac{1+2hk}{mg} \right)}$
 d. $\frac{mg}{k} \sqrt{\left(\frac{1+2hk}{mg} - \frac{mg}{k} \right)}$
- 
21. In Q. 20, find the amplitude of vibration.
 a. $\sqrt{\left(\frac{mv^2}{k} \right)}$
 b. $\sqrt{\left(\frac{mv^2}{2k} \right)}$
 c. $\sqrt{\left(\frac{mv^2}{4k} \right)}$
 d. None of these

22. Two point masses of 3 kg and 1 kg are attached to opposite ends of a horizontal springs whose spring constant is 300 N/m as shown in figure. The natural vibration frequency of the system is of order



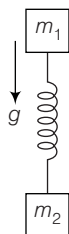
a. 4 Hz b. 3 Hz c. 2 Hz d. 1 Hz

23. Two blocks connected by a spring rest on a smooth horizontal plane as shown in figure. If a constant force F starts acting on the block m_2



- a. length of spring increases continuously, if $m_1 > m_2$
 b. blocks start performing SHM about centre of mass of the system with increasing amplitude
 c. blocks start performing SHM about centre of mass of the system which moves rectilinearly with constant acceleration
 d. acceleration of m_2 is maximum at initial moment of time only

24. There is a spring with nature length L_0 . Two masses m_1 and m_2 are connected to both of its ends as shown in figure. The whole system is held at rest. At any time $t = 0$, m_2 is released and system starts free fall. Initial stretched length of spring before fall is L_0 . What is the displacement of centre of mass as a function of time?



- a. gt^2 b. $\frac{1}{2}gt^2$ c. $\frac{g}{k}t^2$ d. $\frac{m_1 + m_2}{m_1 m_2} \times t$

25. A second's pendulum has time period 2s. The spherical bob which is empty has mass of 50 g. This is replaced by another solid bob of same radius but having mass of 100 g. The new time period will be

- a. 4 s b. 1 s
 c. 2 s d. 8 s

26. A clock pendulum is adjusted for giving correct time in Patna. This clock pendulum also gives correct time in

- a. Delhi b. Kota
 c. Hyderabad d. None of these

27. A simple pendulum of length L and mass M is oscillating in a plane about a vertical line between angular limits $-\phi$ and $+\phi$. For an angular displacement, the tension in the string and the velocity of the bob are T and v respectively. The following relation holds good under the above conditions

- a. $T = Mg \cos \theta$ b. $T \cos \theta = Mg$
 c. $T - Mg \cos \theta = \frac{Mv^2}{L}$ d. None of these

28. From the ceiling of a train, a pendulum of length ' l ' is suspended. The train is moving with an acceleration a_0 on horizontal surface. What must be the period of oscillation of pendulum?

a. $T = 2\pi \sqrt{\left(\frac{l}{g}\right)}$ b. $T = 2\pi \sqrt{\left(\frac{l}{a_0^2 + g^2}\right)}$
 c. $T = \pi \sqrt{\left(\frac{l}{\sqrt{a_0^2 + g^2}}\right)}$ d. $T = 2\pi \sqrt{\left(\frac{l}{\sqrt{a_0^2 - g^2}}\right)}$

29. There is a clock which give correct time at 20°C is subjected to 40°C . The coefficient of linear expansion of the pendulum is 12×10^{-6} per $^\circ\text{C}$, how much is gain or loss in time?

- a. 10.3 s/day b. 19 s/day c. 5.5 s/day d. 6.8 s/day

30. There are two pendulums of length l_1 and l_2 start vibrating. At some instant, the two are in mean position in the same phase. Calculate after how many vibrations of shorter pendulum, the two will be in phase in the mean position?

$[(l_1 > l_2), l_1 = 121\text{cm}, l_2 = 100\text{cm}]$

- a. 11 b. 10 c. 9 d. 8

31. A clock is performing SHM along a vertical line with amplitude of 40 cm on a horizontal plank. The block just lose the contact with plank when plank is momentarily at rest. Then, (Take, $g = 10\text{ m/s}^2$)

- a. the period of its oscillation is $\frac{2\pi}{5}\text{ s}$
 b. the period of its oscillation is $\frac{2\pi}{6}\text{ s}$
 c. the period of its oscillation is $\frac{\pi}{5}\text{ s}$
 d. None of the above

32. There is a ring or mass m and radius R is pivoted at a point O on its periphery. It is free to rotate about an axis perpendicular to its plane. What is the period of ring?

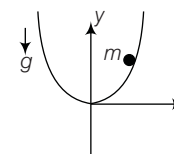
a. $T = 2\pi \sqrt{\left(\frac{R}{g}\right)}$ b. $T = 2\pi \sqrt{\left(\frac{2R}{g}\right)}$
 c. $T = \pi \sqrt{\left(\frac{2R}{g}\right)}$ d. $T = 2\pi \sqrt{\left(\frac{3R}{g}\right)}$

33. There is a rod of length l and mass m . It is hinged at one end to the ceiling. Find the period of small oscillation.

a. $T = 2\pi \sqrt{\left(\frac{2l}{3g}\right)}$ b. $T = \pi \sqrt{\left(\frac{l}{3g}\right)}$
 c. $T = 2\pi \sqrt{\left(\frac{l}{3g}\right)}$ d. $T = 2\pi \sqrt{\left(\frac{l}{g}\right)}$

34. A particle of mass m is allowed to oscillate near the minimum of a vertical parabolic path having the equation $x^2 = 4ay$. The angular frequency of small oscillations is given by

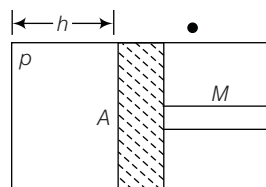
a. $\sqrt{gh}$ b. $\sqrt{2gh}$ c. $\sqrt{\left(\frac{g}{2a}\right)}$ d. $\sqrt{\left(\frac{g}{a}\right)}$



35. A highly rigid cubical block of mass m and side L is fixed rigidly on to another cubical block B of the same dimensions and of low modulus of rigidity η such that the lower face of A completely covers the upper face of B . The lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the side faces of A . After the force is withdrawn, block A executes small oscillation, the period is

a. $2\pi\sqrt{\eta mL}$ b. $2\pi\sqrt{\left(m\frac{\eta}{L}\right)}$ c. $2\pi\sqrt{\left(\frac{mL}{\eta}\right)}$ d. $2\pi\sqrt{\left(\frac{m}{\eta L}\right)}$

36. A cylindrical piston of mass M slide smoothly inside a long cylinder closed at one end, enclosing a certain mass of gas.

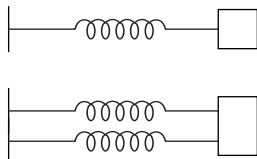


The cylinder is kept with its axis horizontal. If the piston is disturbed from its equilibrium position, it oscillates simple harmonically. Find its period.

a. $T = 2\pi\sqrt{\left(\frac{Mh}{pA}\right)}$ b. $T = 2\pi\sqrt{\left(\frac{MA}{ph}\right)}$
 c. $T = 2\pi\sqrt{\left(\frac{M}{pah}\right)}$ d. $T = 2\pi\sqrt{MphA}$

BITSAT Archives

1. A particle of mass $m = 5\text{ g}$ is executing simple harmonic motion with an amplitude 0.3 m and time period $\pi/5$ second. The maximum value of force acting on the particle is [2013]
 a. 5 N b. 4 N c. 0.5 N d. 0.15 N
2. Pulse rate of a normal person is 75 per minute. The time period of heart is [2013]
 a. 0.8 s b. 0.75 s c. 1.25 s d. 1.75 s
3. A simple wave motion represented by $y = 5(\sin 4\pi t + \sqrt{3}\cos 4\pi t)$. Its amplitude is [2012]
 a. 5 b. $5\sqrt{3}$ c. $10\sqrt{3}$ d. 10
4. If the displacement of simple pendulum at any time is 0.02 m and acceleration is 2 m/s^2 , then in this time angular velocity will be [2012]
 a. 100 rad/s b. 10 rad/s c. 1 rad/s d. 0.1 rad/s
5. If k_s and k_p respectively are effective spring constant in series and parallel combination of springs as shown in figure, find $\frac{k_s}{k_p}$. [2012]



- a. $\frac{9}{2}$ b. $\frac{3}{7}$ c. $\frac{2}{9}$ d. $\frac{7}{3}$
6. What is the maximum acceleration of the particle doing the SHM?

$y = 2\sin\left[\frac{\pi t}{2} + \phi\right]$, where 2 is in cm [2011]

a. $\frac{\pi}{2}\text{ cm/s}^2$ b. $\frac{\pi^2}{2}\text{ cm/s}^2$ c. $\frac{\pi}{4}\text{ cm/s}^2$ d. $\frac{\pi}{4}\text{ cm/s}^2$

7. The equation $\frac{d^2y}{dt^2} + \frac{b}{m}\frac{dy}{dt} + \omega^2y = 0$ represents the equation of motion for a [2010]
 a. free vibration b. damped vibration
 c. forced vibration d. resonant vibration

8. A pole is floating in a liquid with 80 cm of its length immersed. It is pushed down a certain distance and then released. Time period of vertical oscillation is [2010]

a. $\frac{4\pi}{7}\text{ s}$ b. $\frac{3\pi}{7}\text{ s}$ c. $\frac{2\pi}{7}\text{ s}$ d. $\frac{\pi}{7}\text{ s}$

9. A pendulum has a ball of mass m attached to the string and is suspended from the roof of a trolley. If the trolley rolls upwards with acceleration a , then what is the angle made by the string with the inclined plane? (Given that, $a = 5\text{ m/s}^2$, $g = 10\text{ m/s}^2$ and the angle of inclination of the plane is 30°) [2009]

a. $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$ b. $\cot^{-1}\left(\frac{2}{\sqrt{3}}\right)$ c. $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ d. $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

10. A particle is executing simple harmonic motion with an amplitude A and time period T . The displacement of the particles after $2T$ period from its initial position is [2008]

a. A b. $4A$
 c. $8A$ d. zero

11. In a second's pendulum, mass of bob is 30 g . If it is replaced by 90 g mass, then its time period will be [2007]

a. 1 s b. 2 s
 c. 4 s d. 3 s

12. A simple pendulum hanging from the ceiling of a stationary lift has time period t_1 . When the lift moves downward with constant velocity, the time period is t_2 , then [2006]

a. t_2 is infinity b. $t_2 > t_1$
 c. $t_2 < t_1$ d. $t_2 = t_1$

Answer with Solutions

Practice Exercise

1. (a) $\therefore$ In the case of SHM,

$$x = a \sin(\omega t + \phi)$$

Here, $a = 1\text{m}, \omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \text{ rad/s}$

At $t = 0$, net force on the particle is zero.

Hence, particle is at mean position at $t = 0$

$$\therefore x = 0 \text{ at } t = 0$$

$$\therefore 0 = a \sin(\omega \times 0 + \phi) \Rightarrow \phi = 0$$

$$\therefore x = a \sin \omega t$$

$$\therefore x = 1 \times \sin \pi t$$

$$\therefore x = \sin \pi t$$

2. (b) $v_{\max} = a\omega = 1 \times \pi = \pi \text{ m/s}^2$

$$a_{\max} = -\omega^2 a = -\pi^2 \text{ m/s}^2$$

$$\therefore |a_{\max}| = \pi^2 \text{ m/s}^2$$

3. (d) $\therefore x = A \sin \omega t$

$$\therefore x = A \sin\left(\frac{2\pi}{T} \times T\right)$$

$$\therefore x = A \sin 2\pi = 0$$

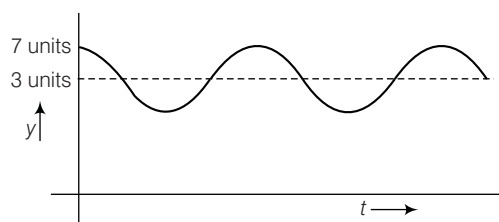
4. (a) Since, the distance travelled in $T/4$ time is $S_1 = A$

$\therefore$ The distance travelled in one time.

$$s = 4S_1 = 4A$$

$$\therefore s = 8 \text{ cm} \quad (\because A = 2 \text{ cm})$$

5. (c) $y = 3 + 4 \cos \omega t$



The y - t graph is shown in the figure. Since, graph is sinusoidal, hence motion is SHM.

6. (b) $y - 3 = 4 \cos \omega t$

Comparing with $x = A \cos \omega t$

$\Rightarrow$ Amplitude of SHM, $A = 4$ units

7. (c) Here, $x = a \sin \omega t$

$$\therefore \sin \omega t = \frac{x}{a}$$

Similarly, $y = b \cos \omega t$

$$\therefore \cos \omega t = \frac{y}{b}$$

$$\therefore \sin^2 \omega t + \cos^2 \omega t = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (\text{Ellipse})$$

8. (c)

9. (c) $y = a \sin \omega t + a \cos \omega t = a \sin \omega t + a \sin\left(\omega t + \frac{\pi}{2}\right)$

The resultant motion is SHM but resultant amplitude is

$$A = \sqrt{a^2 + a^2 + 2a^2 \cos \frac{\pi}{2}} = \sqrt{2}a$$

10. (d) Since, particle starts from mean position.

$$\therefore x = A \sin \omega t$$

$$\therefore \frac{A}{2} = A \sin \omega t_0 \text{ or } \frac{1}{2} = \sin \omega t_0$$

$$\text{or } \sin \frac{2\pi}{T} t_0 = \sin \frac{\pi}{6}$$

$$\text{or } \frac{2\pi}{T} t_0 = \frac{\pi}{6}$$

$$\therefore t_0 = \frac{T}{12} = \frac{12}{12} = 1\text{s}$$

11. (a) Let $x = A \sin(\omega t + \phi)$

$$\therefore v = \frac{dx}{dt} = A \omega \cos(\omega t + \phi)$$

$$\therefore \text{KE} = \frac{1}{2} m v^2 = \frac{1}{2} m A^2 \omega^2 \cos^2(\omega t + \phi)$$

$$\therefore (\text{KE})_{\max} = \frac{1}{2} m A^2 \omega^2$$

$$\therefore \text{PE} = \frac{1}{2} m A^2 \omega^2 - \text{KE}$$

$$= \frac{1}{2} m A^2 \omega^2 - \frac{1}{2} m A^2 \omega^2 \cos^2(\omega t + \phi)$$

$$= \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t + \phi)$$

According to problem, $\text{KE} = \text{PE}$ (at $t = 0$)

$$\therefore \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t + \phi)$$

$$= \frac{1}{2} m A^2 \omega^2 \cos^2(\omega t + \phi)$$

$$\therefore \tan^2(\omega t + \phi) = 1$$

$$\Rightarrow \tan^2(\omega t + \phi) = \tan^2 \frac{\pi}{4}$$

$$\Rightarrow \omega t + \phi = \frac{\pi}{4}$$

$$\Rightarrow \phi = \frac{\pi}{4}$$

($\because t = 0$)

$$\therefore x = A \sin(\omega t + \phi)$$

Here, $A = 10 \text{ cm} = 0.1 \text{ m}$

$$\omega = 2\pi f = 2\pi \times 2 = 4\pi \text{ rad/s}$$

$$\phi = \frac{\pi}{4}$$

$$\therefore x = 0.1 \sin\left(4\pi t + \frac{\pi}{4}\right)$$

12. (c) If
- $x = A \sin \omega t$

Then, $PE = \frac{1}{2} mA^2 \omega^2 \sin^2 \omega t$

$$\therefore PE = \frac{1}{2} mA^2 \omega^2 \left(\frac{1 - \cos 2\omega t}{2} \right)$$

$$\therefore \omega' = 2\omega$$

$$\text{or } 2\pi f' = 2 \times 2\pi f$$

$$\therefore f' = 2f$$

13. (a)
- $u = k|x|^3$

$$u_{\max} = ka^3$$

The oscillation energy is given by $\frac{1}{2} ma^2 \omega^2$.

$$\therefore \frac{1}{2} ma^2 \omega^2 = ka^2 a$$

$$\text{or } \omega^2 = \frac{2k}{m} a$$

$$\text{or } \omega = \sqrt{\left[\left(\frac{2k}{m} \right) a \right]}$$

$$\therefore T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{2ka}}$$

$$\therefore T \propto \frac{1}{\sqrt{a}}$$

14. (d)

$$15. (a) \text{ Average energy} = \frac{\int_0^T u dt}{\int_0^T dt} = \frac{1}{T} \int_0^T u dt$$

$$= \frac{1}{2T} \int_0^T m\omega^2 A^2 \cos^2(\omega t + \phi) dt = \frac{1}{4} m\omega^2 A^2$$

16. (c)

17. (a) The angular frequency under all circumstances is

$$\omega = \sqrt{\left(\frac{k}{m} \right)} = \sqrt{\left(\frac{200}{1} \right)} = 14.14 \text{ rad/s}$$

When elevator is moving up, the equation of motion is

$$T - mg = \frac{mg}{3} \Rightarrow T = \frac{4mg}{3}$$

This tension elongates the spring by x

$$T = kx$$

$$\Rightarrow x = \frac{4mg}{3k} = 0.07 \text{ m}$$

$$18. (b) \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \text{ or } \frac{1}{k} = \frac{2}{k_2} + \frac{1}{k_2} = \frac{3}{k_2}$$

$$\therefore k_2 = 3k$$

$$\therefore k_1 = \frac{k_2}{2} = \frac{3}{2} k$$

$$19. (a) \text{ Since, } T = 2\pi \sqrt{\frac{m}{k}}$$

Here, m and k remain constant.

Hence, T remains constant.

20. (b) According to conservation principle

$$mgh = \frac{1}{2} kx_0^2 - mgx_0$$

where, x_0 is maximum elongation in spring (when particle is in its lowest extreme position).

$$\text{or } \frac{1}{2} kx_0^2 - mgx_0 - mgh = 0$$

$$\text{or } x_0^2 - \frac{2mg}{k} x_0 - \frac{2mg}{k} h = 0$$

$$\therefore x_0 = \frac{\frac{2mg}{k} \pm \sqrt{\left[\left(\frac{2mg}{k} \right)^2 + 4 \times \frac{2mg}{k} h \right]}}{2}$$

$\therefore$ Amplitude = elongation in spring for lowest extreme position – elongation in spring for equilibrium position

$$= x_0 - x_1 = \frac{mg}{k} \sqrt{\left(1 + \frac{2hk}{mg} \right)}$$

$$21. (b) \text{ Here, } \omega = \sqrt{\left(\frac{k}{2m} \right)}$$

$$\therefore E_0 = \frac{1}{2} (2m) A^2 \omega^2$$

$$\text{or } \frac{mv^2}{4} = mA^2 \omega^2 \text{ or } \frac{mv^2}{4} = mA^2 \frac{k}{2m}$$

$$\therefore A = \sqrt{\left(\frac{mv^2}{2k} \right)}$$

22. (b) The reduced mass of the system is
- $\mu = \frac{m_1 m_2}{m_1 + m_2}$

$$T = 2\pi \sqrt{\left(\frac{\mu}{k} \right)} = 2\pi \sqrt{\left[\frac{m_1 m_2}{(m_1 + m_2) k} \right]}$$

$$\therefore f = \frac{1}{T} = 3 \text{ Hz}$$

23. (c) In this case, a constant force
- F
- acts on the system. So, the centre of mass of system moves with constant acceleration.

$$a_{\text{CM}} = \frac{F}{m_1 + m_2}$$

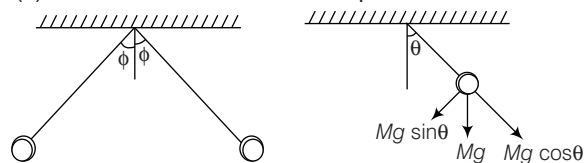
In the frame of centre of mass, particles execute SHM because force on each particles varies linearly.

24. (b)

25. (c) Time period of second's pendulum does not depend on the mass of bob.

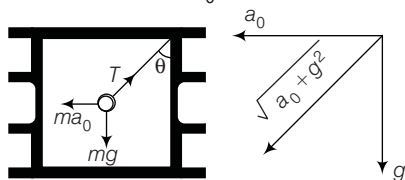
26. (d) The time period of clock pendulum at different places are different due to variation of
- g
- .

27. (c) The bob moves on a circular path



$$\therefore T - Mg \cos \theta = \frac{Mv^2}{L}$$

28. (b) Let train is moving towards right. The pseudo force is acting in left direction $= ma_0$



Effective acceleration, $a_{\text{eff}} = \sqrt{a_0^2 + g^2}$

$$\therefore \text{Time period} = 2\pi \sqrt{\left(\frac{l}{a_{\text{eff}}}\right)}$$

$$T = 2\pi \sqrt{\left(\frac{l}{\sqrt{a_0^2 + g^2}}\right)}$$

29. (a) $\frac{\Delta t}{t} = \frac{1}{2} \frac{\Delta L}{L} = \frac{1}{2} \alpha \Delta \theta$
 $= \frac{1}{2} \times 12 \times 10^{-6} \times (40^\circ - 20^\circ) = 12 \times 10^{-5}$
 $\Delta t = t \times 12 \times 10^{-5} = 86400 \times 12 \times 10^{-5} = 10.3 \text{ s/day}$

30. (a) $T_1 = 2\pi \sqrt{\left(\frac{l_1}{g}\right)}, T_2 = 2\pi \sqrt{\left(\frac{l_2}{g}\right)} (l_1 > l_2)$

Let shorter pendulum makes n vibrations, then the longer will make one less than n to come in phase again.

$$\frac{nT_2}{2\pi} = (n-1) \frac{T_1}{2\pi}$$

$$2\pi \sqrt{\left(\frac{l_2}{g}\right)} n = (n-1) 2\pi \sqrt{\left(\frac{l_1}{g}\right)}$$

$$\therefore n = 11$$

31. (a) The block just loose the contact from the plank if acceleration of plank is just equal to g . (Vertically downward)

Since, block just loses contact, when the plank is momentarily at rest. Therefore, at that instant the plank is at its upper extreme position.

But at extreme position acceleration $= a\omega^2$

where, $a = 40 \text{ cm} = 0.4 \text{ m}$

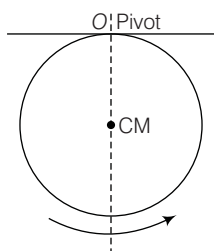
Hence, $0.4\omega^2 = g = 10 \text{ m/s}^2$

$$\omega = 5 \text{ rad/s} \Rightarrow T = \frac{2\pi}{5} \text{ s}$$

32. (b) $d = R$ and $I = 2mR^2$

$$\therefore T = 2\pi \sqrt{\left(\frac{I}{mgd}\right)} = 2\pi \sqrt{\left(\frac{2mR^2}{mgR}\right)}$$

$$\Rightarrow T = 2\pi \sqrt{\left(\frac{2R}{g}\right)}$$



33. (a)

34. (c) At the vertex, tangential acceleration is zero. When particle is at mean position, it is at vertex. When it is displaced for small angle θ ,

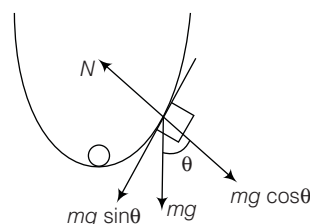
$$-mf = mg \sin \theta$$

$$\text{or } f = -g \sin \theta = -g \theta \tan \theta$$

$$\text{or } f = -g \left(\frac{dy}{dx}\right)$$

$$\therefore x^2 = 4ay \text{ or } 2x = 4a \frac{dy}{dx}$$

$$\text{or } \frac{dy}{dx} = \frac{x}{2a} \therefore \tan \theta = \frac{x}{2a}$$



$$\therefore f = -g \left(\frac{x}{2a}\right)$$

$$\text{or } -\omega^2 x = -g \left(\frac{x}{2a}\right)$$

$$\therefore \omega = \sqrt{\left(\frac{g}{2a}\right)}$$

35. (d) Stress $= \frac{F}{L^2}, \eta = \frac{F}{L^2 \phi}$

$$F = L^2 \eta \phi$$

$$F = L^2 \eta \left(\frac{-x}{L}\right) \quad \left[\because \text{Strain} = \phi = -\frac{x}{L} \right]$$

$$F = (L\eta)(-x)$$

$$\Rightarrow ma = (L\eta)(-x)$$

$$a = \left(\frac{L\eta}{m}\right)(-x)$$

$$\therefore a \propto -\omega^2 x$$

$$\frac{L\eta}{m}(-x) = -\omega^2 x$$

$$\text{or } \omega = \sqrt{\left(\frac{L\eta}{m}\right)} \therefore T = 2\pi \sqrt{\left(\frac{m}{\eta L}\right)}$$

36. (a) $f = \frac{1}{2\pi} \sqrt{\left(\frac{BA^2}{MV_0}\right)}$

where, B = Bulk modulus of elasticity

$$T = 2\pi \sqrt{\left(\frac{MV_0}{BA^2}\right)}$$

$$= 2\pi \sqrt{\left(\frac{M(hA)}{pA^2}\right)}$$

$$T = 2\pi \sqrt{\left(\frac{Mh}{pA}\right)}$$

BITSAT Archives

1. (d) We know maximum acceleration, $a_{\max} = \omega^2 A = \frac{4\pi^2}{T^2} A$

$$= \frac{4\pi^2}{\left(\frac{\pi}{5}\right)^2} \times 0.3 = 30 \text{ m/s}^2$$

Maximum force, $F_{\max} = ma_{\max} = \frac{5}{1000} \times 30 = 0.15 \text{ N}$

2. (a) The beat frequency of heart is

$$v = \frac{75}{(1 \text{ min})} = \frac{75}{60 \text{ s}} = 1.25 = 1.25 \text{ Hz}$$

The time period of heart is

$$T = \frac{1}{v} = \frac{1}{1.25 \text{ s}^{-1}} = 0.8 \text{ s}$$

3. (d) $y = 5 (\sin 4\pi t + \sqrt{3} \cos 4\pi t)$

$$y = 5 (\sin 4\pi t + 5\sqrt{3} \cos 4\pi t)$$

$$A = \sqrt{A_1^2 + A_2^2}$$

$$A = \sqrt{(5)^2 + (5\sqrt{3})^2}$$

$$= \sqrt{25 + 75} = \sqrt{100} = 10$$

4. (b) Acceleration $|\alpha| = \omega^2 x$

or

$$\omega = \sqrt{\frac{\alpha}{x}}$$

$$= \sqrt{\frac{2}{0.002}} = 10 \text{ rad/s}$$

5. (c) The effective spring constant k_s of this arrangement is

$$\frac{1}{k_s} = \frac{1}{k} + \frac{1}{2k} \Rightarrow \frac{1}{k_s} = \frac{2+1}{2k} = \frac{3}{2k}$$

$$k_s = \frac{2k}{3}$$

The effective spring constant k_p of this arrangement is

$$k_p = k_1 + k_2 = k + 2k = 3k$$

$$\therefore \frac{k_s}{k_p} = \frac{2k/3}{3k} = \frac{2}{9}$$

6. (b) $y = 2 \sin\left(\frac{\pi t}{2} + \phi\right)$

Comparing the equation with the standard equation

$$y = A \sin(\omega t + \phi)$$

So $A = 2 \text{ cm}, \omega = \frac{\pi}{2}$

Acceleration of particle is

$$a = \omega^2 x$$

[numerically]

At $x = +A, a = a_{\max}$

$$\therefore a_{\max} = \omega^2 A = \left(\frac{\pi}{2}\right)^2 \times 2$$

$$= 2 \times \frac{\pi^2}{4} = \frac{\pi^2}{2} \text{ cm/s}^2$$

7. (b) This equation represented damped vibration.

8. (a) $T = 2\pi \sqrt{\frac{l_{\text{immersed}}}{g}}$

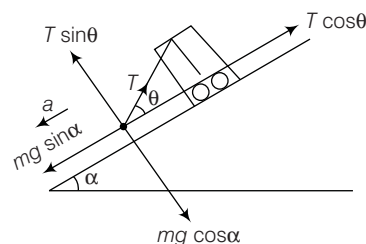
$$\Rightarrow T = 2\pi \sqrt{\frac{0.80}{9.8}} = \frac{4\pi}{7}$$

9. (b) In equilibrium,

$$T \cos \theta = ma + mg \sin \alpha$$

and

$$T \sin \theta = mg \cos \alpha$$



This gives, $\cot \theta = \frac{a + g \sin \alpha}{g \cos \alpha}$

$$= \frac{5 + 10 \times \left(\frac{1}{2}\right)}{10 \times \frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

$$\therefore \theta = \cot^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

10. (d) It is the least interval of time after which the periodic motion of a body repeats itself.

Therefore, displacement will be zero.

11. (a) Time period is independent of mass of bob of pendulum.

12. (d) The lift is moving with constant velocity, so there will be no change in the acceleration. Hence, time period will remain same.

11

Fluid Mechanics

Fluid Statics

The substances which flow are called fluids. Fluids include both liquids and gases. The science of fluids at rest is called **fluid statics**.

Pressure in a Fluid

Consider a point A inside the fluid (see figure alongside). Imagine a small area ΔS containing the point A . We define pressure of fluid at point A as

$$p = \lim_{\Delta S \rightarrow 0} \frac{F}{\Delta S}$$

where, p = pressure of fluid, F = force exerted by fluid

The SI unit of pressure is Nm^{-2} called pascal which is abbreviated as Pa. Common unit of pressure is 1 atmosphere (atm), i.e. the pressure exerted by the atmosphere at sea level.

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

Other unit of pressure is torricelli's, 1 torr = 133 Pa

Another important property of fluid is its **density**, which is defined as the mass of fluid per unit volume occupied by it, i.e.

$$\text{Density of fluid} = \frac{\text{Mass of fluid}}{\text{Volume occupied by the fluid}}$$

It is denoted by ρ and its SI unit is kg/m^3 .

Relative density is defined as the ratio of the density of a substance to the density of water at 4°C , i.e.

$$\text{Relative density} = \frac{\text{Density of substance}}{\text{Density of water at } 4^\circ\text{C}}$$

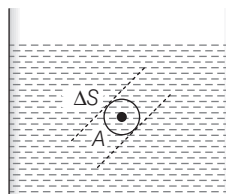
Relative density has no unit and no dimension.

Pressure due to Fluid Column

Hydrostatic pressure at a point A depth h below the fluid surface is given by

$$p = h\rho g$$

where, ρ = density of fluid.



Gauge Pressure

The pressure difference between the real hydrostatic pressure and the atmospheric pressure is known as the gauge pressure.

∴ Gauge pressure

$$= \text{real pressure } (p) - \text{atmospheric pressure } (p_0)$$

Pascal's Law

This law can be defined as “if the pressure in a liquid is changed at a particular point, then this change is transmitted to the entire liquid without being diminished in magnitude”. Assuming liquid to be incompressible and non-viscous. This law has various interesting applications in the form of hydraulic machines such as hydraulic lift, hydraulic brakes etc.

Archimedes' Principle

Whenever a body is partly or wholly immersed in a fluid at rest, there is a decrease in its weight and this decrease in weight is equal to the weight of the fluid displaced by the immersed part of the body. In fact, when a body is immersed in a fluid, it experiences an upthrust due to the fluid and as a result the apparent weight of the body is reduced.

∴ Apparent weight of the body

= weight of the body – upthrust due to fluid

= weight of the body – weight of the fluid displaced

e.g. for a floating body, the volume of a body ($V - V_s$) remaining outside the liquid will be given by

$$V_0 = V - V_s = V - V \frac{\rho}{\sigma} = V \left(1 - \frac{\rho}{\sigma} \right)$$

Laws of Floatation

When a body of density ρ_B and volume V is immersed in a liquid of density σ , the forces acting on the body are

(i) The weight of body $w = mg = V\rho_B g$ acting vertically downwards through the centre of gravity of the body.

(ii) The upthrust $F = V\sigma g$ acting vertically upwards through the centre of gravity of the displaced liquid i.e. centre of buoyancy.

So, the following three cases are possible.

Case I The density of body is greater than that of liquid (i.e. $\rho_B > \sigma$). In this case, as weight will be more than upthrust, the body will sink.

Case II The density of body is equal to the density of liquid (i.e. $\rho_B = \sigma$). In this case, $w = F$, so the body will float fully submerged in neutral equilibrium anywhere in the liquid.

Case III The density of body is lesser than that of liquid (i.e. $\rho_B < \sigma$). In this case, $w < F$, so the body will move upwards and in equilibrium will float partially immersed in the liquid, such that

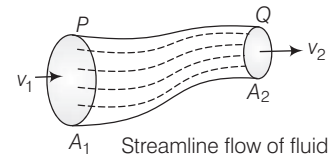
$$w = V_{in}\sigma g \quad [V_{in} \text{ is the volume of body in the liquid}]$$

$$\text{or} \quad V\rho_B g = V_{in}\sigma g \quad [\text{as } w = mg = \rho_B V g]$$

$$\text{or} \quad V\rho_B = V_{in}\sigma \quad \dots(i)$$

Equation of Continuity

Consider a streamline flow of a fluid through a tube of varying cross-section (see figure alongside).



Now, according to equation of continuity,

$$A_1 v_1 = A_2 v_2 \quad (\text{or } Av = \text{constant})$$

where, A_1 and A_2 are the cross-section areas and v_1 and v_2 are the velocities of the fluids at two section respectively.

Energy of a Flowing Fluid

There are three types of energies in a flowing liquid, which are defined below

Pressure Energy

If p is the pressure at the cross-section area A of a fluid and the fluid moves through a distance l against this pressure. Then, the pressure energy stored in fluid = $(pA) \times l = pAl$
Volume of liquid moved = Al

∴ Pressure energy (stored in fluid) per unit volume = p

Kinetic Energy

If any fluid element of mass m and volume V is flowing with velocity v , then

$$\therefore \text{KE per unit volume} = \frac{1}{2} \left(\frac{m}{V} \right) v^2 = \frac{1}{2} \rho v^2 \quad \left\{ \because \rho = \frac{m}{V} \right\}$$

Here, ρ is the density of the fluid.

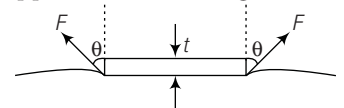
Potential Energy

If a fluid element of mass m and volume V is at a height of h above the ground level (reference level), then the potential energy stored in fluid is mgh . So, potential energy per unit

$$\text{volume is given by} = \frac{mgh}{V} = \rho gh$$

Bernoulli's Theorem

Bernoulli's theorem is an outcome of the principle of conservation of energy, when applied to the flowing fluid. It states that “for a streamline flow of an ideal fluid (incompressible, non-viscous), the sum of



pressure energy per unit volume, kinetic energy per unit volume and potential energy per unit volume remains constant at every cross-section of the flow".

$$\text{i.e. } p + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

Applications of Bernoulli's Theorem

- The action of carburetor, paintgun, scent sprayer, atomiser is based on Bernoulli's equation.
- Action of Bunsen's burner, gas burner, oil stove is also based on it.
- Motion of spinning ball (Magnus effect) is based on Bernoulli's equation.
- Aerofoil or lift of aircraft wing is also based on it. When aerofoil moves against the wind, the flow of speed of air on top is more as compared to the speed of air below it, which results into an upward force providing lift to the aircraft.
- Torricelli's law and venturimeter are also based on this equation.

Limitations of Bernoulli's Theorem

- When a fluid is at rest, i.e. its velocity is zero everywhere, Bernoulli's equation becomes $p_1 + \rho gh_1 = p_2 + \rho gh_2 \Rightarrow (p_1 - p_2) = \rho g(h_2 - h_1)$
- Bernoulli's equation ideally applies to fluids with zero viscosity or non-viscous fluids.
- Bernoulli's equation applies to fluids which must be incompressible, as the elastic energy of the fluid is also not taken into consideration.
- Bernoulli's equation does not hold for steady or turbulent flows, because in that situation velocity and pressure are constantly fluctuating in time.

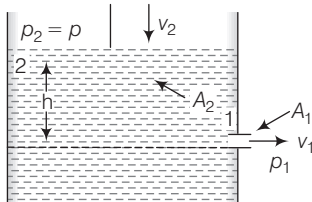
Torricelli's Law

This law states that "speed of liquid coming out through a hole at a depth h below the free surface of a tank is same as that of a particle fallen freely through height h under gravity". The speed of the liquid coming out is called the **speed of efflux**.

In the above figure, p_1, v_1, A_1 are the pressure, velocity and area of orifice at 1 and p_2, v_2, A_2 be pressure, velocity and area at 2 respectively.

Now, speed of efflux, $v_1 = \sqrt{2gh}$

$$\therefore \text{Volume of liquid coming out of orifice per second} \\ = A_1 v_1 = A_1 \sqrt{2gh}$$



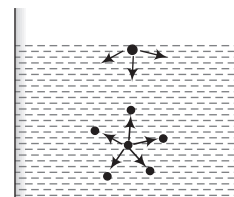
The time of making empty a hemispherical tank through an orifice at its bottom. Initially, tank is full of water and finally tank is completely made empty.

$$T = 14\pi \frac{R^{5/2}}{15a\sqrt{2g}}$$

where, R = the radius of the hemispherical tank
 a = cross-sectional area of the orifice

Surface Tension

Each molecule (or atom) of liquid attracts the neighbouring molecule with the same force (called cohesive force). However, as every molecule is gathered by a number of molecules in every direction, the net effect on the molecule is zero (figure).



Now, if we consider a molecule on the surface of the liquid, as clear from the figure, the molecule would feel a net downward force due to which the free surface behaves like a stretched membrane under tension.

If we draw an imaginary line of a length l on the surface of the liquid, the net value of force acting on this line per unit length is called surface tension.

$$\text{Surface tension, } S = \frac{\text{Force}}{\text{Length}} \Rightarrow S = \frac{F}{L}$$

SI unit of surface tension (S or T) is Nm^{-1} and its dimensional formula is $[\text{MT}^{-2}]$. It is the characteristic of the two media in contact.

Applications of Surface Tension (Drop, Bubble)

As every system tends to have minimum potential energy, the surface of a liquid too tends to have minimum surface area. As, sphere has the minimum surface area for a given volume, drops and bubbles tend to have spherical shape.

If any other force such as gravitation, air resistance is absent, then the drops and bubbles would be perfect spheres.

Another interesting consequence of surface tension is that pressure inside a spherical drop or bubble is more than the pressure outside.

$$\text{Excess pressure for liquid drop } (\Delta p)_d = \frac{2S}{r}$$

$$\text{Excess pressure for soap bubble } (\Delta p)_b = \frac{4S}{r}$$

where, S is the surface tension of the liquid and r is the radius of drop and bubble.

Surface Energy

Surface energy of a liquid is the potential energy of the molecules of a surface film of the liquid by virtue of its position.

When the surface area of a liquid is increased, work is done against the cohesive force of molecules and this work is stored in the form of additional surface energy.

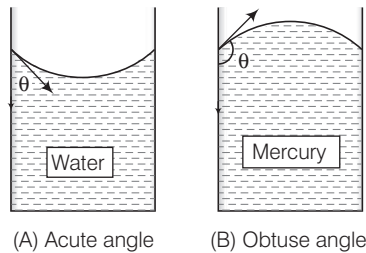
Increase in surface potential energy

$$\Delta U = \text{Work done } (\Delta W) = S \Delta A$$

where, ΔA is the increase in surface area of the liquid.

Angle of Contact

The angle of contact between a liquid and a solid is defined as the angle enclosed between the tangent to the liquid surface and the solid surface which is in contact with the liquid as shown in the figure given below.



(A) Acute angle

(B) Obtuse angle

The angle of contact varies between 0° to 180° .

For the liquids which wet the surface of solid (such as water and glass), the angle of contact is acute, while those for liquids which do not wet the surface of solid (such as mercury and glass), the angle of contact is obtuse.

Angle of contact decreases with the addition of impurities whereas it increases if temperature is increased.

Capillary Rise or Capillarity

Capillarity is the phenomenon of rise or fall of a liquid in a capillary tube as compared to that in a surrounding liquid.

The height h upto which a liquid will rise in a capillary tube

$$\text{is given by } h = \frac{2S \cos \theta}{r \rho g} = \frac{2S}{r \rho g}$$

where, r = radius of the capillary tube and

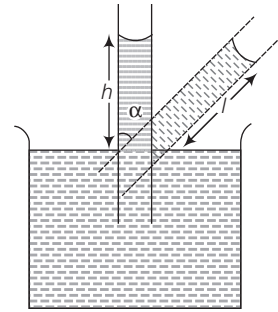
$$R = \frac{r}{\cos \theta} = \text{radius of liquid meniscus.}$$

(i) The rise in capillary tube $h \propto \frac{1}{r}$.

(ii) If a capillary tube, dipped in a liquid is tilted at an angle α from the vertical, the vertical height h of the liquid column remains the same. However, the length of the liquid column (l) in the capillary tube increases to

$$l = \frac{h}{\cos \alpha}$$

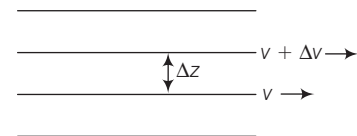
(iii) If the capillary tube is of insufficient length, the liquid rises upto the upper end of the tube and then the radius of its meniscus changes from R to R' such that $hR = h'R'$, where h' = insufficient length of the tube.



(iv) After connection due to the weight of liquid contained in the meniscus, the formula for the height is given by $h = \frac{2S}{\rho r g} - \frac{r}{3}$

Viscosity

Viscosity is the property of a fluid in motion, by virtue of which a force of resistance exists between the different layers of the fluid whenever there is relative motion between them.



When a fluid is flowing in a pipe or a tube, then the velocity of the liquid layer along the axis of the tube is maximum and decreases gradually as we move towards the walls, where it becomes zero.

The relative speed of the liquid layer (closest to boundary) is assumed to be zero.

Hence, due to viscosity, liquid flows in layers. This type of flow is called **laminar flow**.

Poiseuille's Formula

In case of steady flow of liquid of viscosity η in a capillary tube of length L and radius R under a pressure difference p across it, the volume of liquid flowing per second is given by

$$\frac{dQ}{dt} = \frac{\pi p R^4}{8 \eta L}$$

This is called poiseuille's formula.

Stokes' Law

Stokes proved that for a small spherical body of radius r moving with a constant speed v called terminal velocity through a fluid having coefficient of viscosity η , the viscous force F is given by $F = 6\pi\eta r v$. It is known as the **Stokes' law**.

Terminal Velocity

If a small spherical body is dropped in a fluid, then initially it is accelerated under the action of gravity. However, with an increase in speed, the viscous force increases and soon it balances the weight of the body. Now, the body moves with

a constant velocity, called the **terminal velocity**. Terminal velocity v_t is given by

$$v_t = \frac{2}{9} \frac{r^2(\rho - \sigma)g}{\eta}$$

where, r = radius of the falling body,
 ρ = density of the falling body and
 σ = density of the fluid.

Reynold's Number

Reynold defined a dimensionless number called Reynold's number (R_e), whose value gives an approximate idea whether the flow would be laminar or turbulent.

Flow is streamline, if $R_e < 2000$

Flow is turbulent, if $R_e > 3000$

Between 2000 to 3000, the flow is transient.

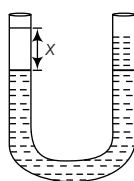
Practice Exercise

- The pressure at the bottom of a tank of liquid is not proportional to
 - the density of the liquid
 - the area of the liquid surface
 - the height of the liquid
 - the acceleration

- If a vessel containing a fluid of density ρ upto height h is accelerated vertically downwards with acceleration a_0 . Then, the pressure by fluid at the bottom of vessel is
 - $p = p_0 + \rho gh + \rho ha_0$
 - $p = p_0 + \rho gh$
 - $p = p_0 + \rho h(g - a_0)$
 - $p = p_0 - \rho gh$

- In each heart beat, a heart pumps 80 ml of blood at an average pressure of 100 mm of Hg. What will be the power output of the heart? (Assume 60 heart beat per minute)
 - 1 W
 - 2.75 W
 - 1.06 W
 - 0.5 W

- One end of a U-tube of uniform bore (area A) containing mercury is connected to a suction pump. Because of it the level of liquid of density ρ falls in one limb. When the pump is removed, the restoring force in the other limb is



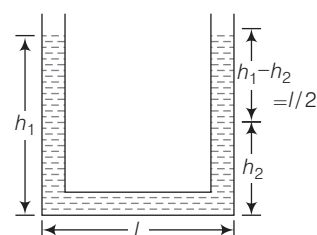
- $2\rho Ag$
 - $x\rho g$
 - $A\rho g$
 - $x\rho Ag$
- A cylindrical vessel of radius r containing a liquid is rotating about a vertical axis through the centre of circular base. If the vessel is rotating with angular velocity ω , then what is the difference of the heights of liquid at centre of vessel and edge?
 - $\frac{r\omega}{2g}$
 - $\frac{r^2\omega^2}{2g}$
 - $\sqrt{2g r\omega}$
 - $\frac{\omega^2}{2gr^2}$

- If the weight of a body in vacuum is w and w_1 and w_2 are weights when it is immersed in a liquid of specific gravity ρ_1 and ρ_2 respectively, then find the relation among w , w_1 and w_2 .

$$\begin{aligned} \text{a. } w &= \frac{w_1\rho_2 + w_2\rho_1}{w_1 + w_2} & \text{b. } w &= \frac{w_1\rho_2 - w_2\rho_1}{\rho_2 - \rho_1} \\ \text{c. } w &= \frac{w_1\rho_1 + w_2\rho_2}{\rho_1 + \rho_2} & \text{d. } w &= \frac{w_1\rho_2 + w_2\rho_1}{\rho_1 + \rho_2} \end{aligned}$$

- A closed rectangular tank 10 m long, 5 m wide and 3 m deep is completely filled with an oil of specific gravity 0.92. Find the pressure difference between the rear and front corners of the tank, if its moving with an acceleration of 3 m/s^2 in the horizontal direction
 - 27.6 kPa
 - 50 kPa
 - 60 kPa
 - 70 kPa

- A U-tube having a liquid of density ρ is accelerated at $a \text{ m/s}^2$, so as to create be the height difference between two columns of $l/2$ (as shown in figure). If l is the length of the base of U-tube, the value of acceleration given to the system is

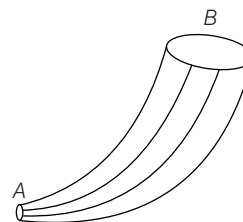


- 4.9 m/s^2
 - 9.8 m/s^2
 - 5.6 m/s^2
 - 6.4 m/s^2
- A body weighs 5 N in air and 2 N when immersed in a liquid. The buoyant force is
 - 2 N
 - 3 N
 - 5 N
 - 7 N
 - A neckless weighing 50 g in air, but it weight 46 g in water. Assume copper is mixed with gold to prepare the neckless. Find how much copper is present in it. (Specific gravity of gold is 20 and that of copper is 10)
 - $m = 25 \text{ g}$
 - $m = 30 \text{ g}$
 - $m = 35 \text{ g}$
 - $m = 20 \text{ g}$

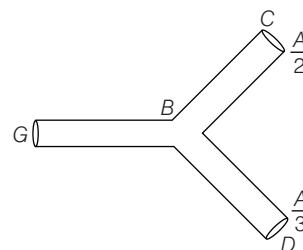
11. If air of weight w is filled in a empty balloon which weights w_1 the weight of balloon will become w_2 . Suppose the density of air inside and out side the balloon is same, then
 - a. $w_2 = w_1 + w$
 - b. $w_2 = \sqrt{w_1 w}$
 - c. $w_2 = w_1$
 - d. $w_2 = w_1 - w$
12. A soft plastic bag of weight w_0 is filled with air at STP. Now, weight of the bag is w in air. Then,
 - a. $w > w_0$
 - b. $w = w_0$
 - c. $w \geq w_0$
 - d. $w < w_0$
13. A block of ice of area A and thickness 0.5 m is floating in the fresh water. In order to just support a man of 100 kg, the area A should be (specific gravity of ice 0.917 and density of water = 1000 kg/m³)
 - a. 124 m²
 - b. 4.21 m²
 - c. 2.41 m²
 - d. 7.23 m²
14. A piece of ice is floating in water. Find the fraction of volume of the piece of ice outside the water
(Given, density of ice = 900 kg/m³ and density of water = 1000 kg/m³)
 - a. 0.21
 - b. 0.01
 - c. 0.1
 - d. 0.9
15. A block of wood floats with 1/4 of its volume under water. What is the density of the wood? (Density of water = 1000 kg/m³)
 - a. 750 kg/m³
 - b. 250 kg/m³
 - c. 300 kg/m³
 - d. 260 kg/m³
16. A block weighs 15 N and 12 N in air and water respectively. When it is immersed in another liquid, it weights 13 N, then find the relative density of the block.
 - a. 5
 - b. 6
 - c. 10
 - d. 2
17. In English, the phrase 'tip of the iceberg' is used to mean a small visible fraction of something that is mostly hidden. For a real iceberg, what is this fraction, if the density of sea water is 1.03 g/cc and that of ice is 0.92 g/cc?
 - a. 0.106
 - b. 10.6
 - c. 0.901
 - d. 0.801
18. A vessel contains oil (density 0.8 g/cc) over mercury (density 13.6 g/cc). A homogeneous sphere floats with half its volume immersed in mercury and the other half in oil. The density of the material of the sphere in g/cc is
 - a. 3.3
 - b. 6.4
 - c. 7.2
 - d. 12.8
19. In a steady incompressible flow of a liquid
 - a. the speed does not change, if the area of cross-section changes
 - b. the speed increases, if the area of cross- section increases
 - c. the speed decreases, if the area of cross- section increases
 - d. bubbles are produced when the area of the cross-section increases

20. Water from a tap emerges vertically downward with an initial speed of 1 m/s. The cross-sectional area of the tap is 10⁻⁴ m². Assume that the pressure is constant throughout the stream of water and that the flow is steady. The cross-sectional area of the steam 0.15 m below the tap is
 - a. 5×10^{-4} m²
 - b. 1×10^{-5} m²
 - c. 5.83×10^{-5} m²
 - d. 2×10^{-5} m²

21. A tube of flow is shown in the figure.

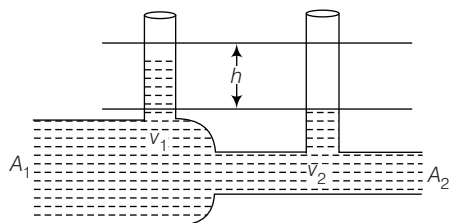


- a. The fluid particles must be accelerated from A to B
 - b. Fluid particles may accelerate from A to B
 - c. The fluid particles must be decelerated from A to B
 - d. The fluid particles may be decelerated from B to A
22. A pipe GB is fitted with two pipes C and D as shown in the figure. The pipe has area $A = 24$ m² at G and velocity of water at G is 10 m/s, and at C is 6 m/s. The velocity of water at D is
 - a. 21 m/s
 - b. 3.3 m/s
 - c. 30 m/s
 - d. None of these



23. Bernoulli's equation is applicable to points
 - a. in a steadily flowing liquid
 - b. in a stream line
 - c. in a straight line perpendicular to stream line
 - d. in any non- viscous liquid
24. The horizontal flow of fluid depends upon
 - a. pressure difference
 - b. amount of fluid
 - c. density of fluid
 - d. All of the above
25. In steady horizontal flow,
 - a. the pressure is greatest where the speed is least
 - b. the pressure is independent of speed
 - c. the pressure is least where the speed is least
 - d. Both a and c are correct

26. From a horizontal tube with area of cross-section A_1 and A_2 as shown in figure, liquid is flowing with velocities v_1 and v_2 respectively. The difference in the level of the liquid in the two vertical tubes is h .

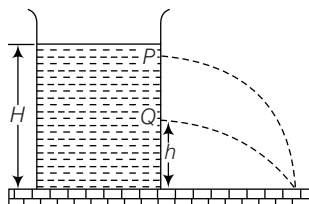


- a. The volume of the liquid flowing through the tube in time is $A_1 v_1$
 b. $v_2 - v_1 = \sqrt{2gh}$
 c. $v_2^2 - v_1^2 = 2gh$
 d. The energy per unit mass of the liquid is the same in both sections of the tube
27. A vessel is filled with water and kerosene oil. The vessel has a small hole in the bottom. Neglecting viscosity if the thickness of water layer is h_1 and kerosene layer is h_2 , then the velocity v of flow of water will be (density of water is ρ_1 g/cc and that of kerosene is ρ_2 g/cc)

a. $v = \sqrt{2g(h_1 + h_2)}$ b. $v = \sqrt{2g\left(h_1 + h_2 \frac{\rho_2}{\rho_1}\right)}$
 c. $v = \sqrt{2g(h_1 \rho_1 + h_2 \rho_2)}$ d. $v = \sqrt{2g\left(h_1 \frac{\rho_1}{\rho_2} + h_2\right)}$

28. Mark the correct option(s).
 a. two stream lines may cross each other
 b. two stream lines must cross each other
 c. two stream lines never cross each other
 d. None of the above
29. Water flows along a horizontal pipe whose cross-section is not constant. The pressure is 1 cm of Hg, where the velocity is 35 cm/s. At a point where the velocity is 65 cm/s, then pressure will be
 a. 0.89 cm of Hg b. 8.9 cm of Hg
 c. 0.5 cm of Hg d. 1 cm of Hg
30. A pilot tube was inserted in a pipe to measure the velocity of water in it. If the water rises in the tube is 200 mm. Find the velocity of water.
 a. 9.8 m/s b. 1.98 m/s c. 19.6 m/s d. 196 m/s

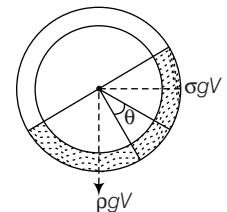
31. A cylindrical vessel filled with water to a height H . A vessel has two small holes in the side, from which water is rushing out horizontally and the two streams strike the ground at the same point. If the lower hole Q



Q is h height above the ground, then the height of hole P above the ground will be

- a. $2h$ b. $\frac{H}{h}$
 c. $H - h$ d. $\frac{H}{2}$
32. A liquid having area of free surface A and has an orifice at a depth of h with an area a , below the liquid surface, then find the velocity v of flow through the orifice.
 a. $v = \sqrt{2gh}$ b. $v = \sqrt{2gh} \sqrt{\frac{A^2}{A^2 - a^2}}$
 c. $v = \sqrt{2gh} \sqrt{\frac{A}{A - a}}$ d. $v = \sqrt{2gh} \sqrt{\frac{A^2 - a^2}{A^2}}$
33. A capillary tube of area of cross-section A is dipped in water vertically. Calculate the amount of heat evolved as the water rises in the capillary tube upto height h . The density of water is ρ
 a. $\frac{A\rho gh^2}{2}$ b. $Agh^2\rho$
 c. $2Agh^2\rho$ d. None of these
34. The radius of the biggest metal coin of thickness t and density ρ , which would be able to float on water surface of surface tension S , is
 a. $\frac{S}{2\rho gt}$ b. $\frac{S}{\rho gt}$
 c. $\frac{2S}{\rho gt}$ d. $\frac{4S}{3\rho gt}$
35. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg)
 a. 38 cm b. 6 cm
 c. 16 cm d. 22 cm
36. While measuring surface tension of water using capillary rise method, height of the lower meniscus from free surface of water is 3 cm, while inner radius of capillary tube is found to be 0.5 cm. Then, compute tension of water using this data. [Take, contact angle between glass and water as 0 and $g = 9.81 \text{ m/s}^2$]
 a. 0.72 N/m
 b. 0.77 N/m
 c. 1.67 N/m
 d. None of the above
37. To what depth must a rubber ball be taken in deep sea so that its volume is decreased by 0.1%. (The bulk modulus of rubber is $9.8 \times 10^8 \text{ N/m}^2$, and the density of sea water is 10^3 kg/m^3)
 a. 100 m b. 60 m c. 75 m d. 65 m

38. A wooden block of mass m and density ρ is tied to a string, the other end of the string is fixed to the bottom of a tank. The tank is filled with a liquid of density σ with $\sigma > \rho$. The tension in the string will be
- $\left(\frac{\sigma - \rho}{\sigma}\right) mg$
 - $\left(\frac{\sigma - \rho}{\rho}\right) mg$
 - $\frac{\rho mg}{\sigma}$
 - $\frac{\sigma mg}{\rho}$
39. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T , density of liquid is ρ and L is its latent heat of vaporisation.
- $\rho L/T$
 - $\sqrt{T/\rho L}$
 - $T/\rho L$
 - $2T/\rho L$
40. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1} . The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to
- 7.5×10^{-3} m
 - 9.6×10^{-3} m
 - 3.6×10^{-3} m
 - 5.0×10^{-3} m
41. In an experiment a capillary tube is kept vertical, then water rises up in the tube upto 3 mm height. When the tube is tilted at an angle of 60° with vertical, what should be the height of water rise
- 6 mm
 - 4 mm
 - 3 mm
 - None of these
42. Calculate for the rise of water in a capillary tube when kept vertical in water whose radii is $1/4$ th of that capillary tube which when kept vertical water rise in it upto a height of 3 mm.
- 12 mm
 - 10 mm
 - 4 mm
 - 3 mm
43. Calculate the heat evolved for the rise of water when one end of the capillary tube of radius r is immersed vertically into water. Assume surface tension = T and density of water to be ρ .
- $\frac{2\pi T}{\rho g}$
 - $\frac{\pi T^2}{\rho g}$
 - $\frac{2\pi T^2}{\rho g}$
 - None of these
44. In a liquid there is air bubble of radius 1 mm at a depth 10 cm below the free space. The surface tension of liquid 0.075 N/m and density is 1000 kg/m^3 . By what amount is the pressure inside the bubble greater than the atmospheric pressure?
- 1130 pascal
 - 1200 pascal
 - 1100 pascal
 - 1000 pascal
45. Calculate the work done by a boy in making a soap bubble of diameter 1.4 cm by blowing, if the surface tension of soap solution is 0.03 N/m .
- $3 \times 10^{-5} \text{ J}$
 - $3.696 \times 10^{-5} \text{ J}$
 - $2 \times 10^{-5} \text{ J}$
 - $4.2 \times 10^{-5} \text{ J}$
46. A drop of radius r is broken into n equal drops. Calculate the work done if surface tension of water is T .
- $4\pi R^2 n T$
 - $4\pi R^2 T (n^{2/3} - 1)$
 - $4\pi R^2 T (n^{1/3} - 1)$
 - None of the above
47. What will happen if n drops of a liquid each has surface energy E , combine to form a single drop.
- No energy will be released in the process
 - Some energy will be absorbed in the process
 - Energy released or absorbed will be $E(n - n^{2/3})$
 - Energy released or absorbed will be $nE(2^{2/3} - 1)$
48. If a bigger drop of liquid at temperature t , breaks up into number of small droplets, then what is temperature of the droplets? (Assume, bigger drop is isolated from its surroundings)
- Equal to t
 - Greater than t
 - Less than t
 - Either (a), (b) and (c) depending on surface tension of liquid
49. The excess pressure inside a soap bubble of radius 4 cm is 30 dyne/cm^2 . The surface tension is
- 30 dyne/cm
 - 20 dyne/cm
 - 40 dyne/cm
 - 80 dyne/cm
50. Calculate the work done against surface tension in formation of a drop of mercury of radius 4 cm (surface tension for mercury = 465 dyne/cm)
- $9.34 \times 10^{-3} \text{ J}$
 - $10 \times 10^{-2} \text{ J}$
 - $4 \times 10^{-3} \text{ J}$
 - 466 J
51. Calculate the energy required to increase the radius of a soap bubble from 1 cm to 2 cm (The surface tension is 30 dyne/cm).
- $240 \pi \text{ erg}$
 - $720 \pi \text{ erg}$
 - $480 \pi \text{ erg}$
 - None of these
52. A film of a liquid is held on a circular ring of radius r , if the surface tension of the liquid is T . The surface tension of the liquid is
- $\pi r^2 T$
 - $2\pi r^2 T$
 - $4\pi r^2 T$
 - None of these
53. A small uniform tube is bent into a circle of radius r whose plane is vertical. The equal volumes of two fluids whose densities are ρ and σ ($\rho > \sigma$), fill half the circle. Find the angle that the radius passing through the interface makes with the vertical.
- $\cot \theta = \frac{\rho - \sigma}{\rho + \sigma}$
 - $\tan \theta = \frac{\rho - \sigma}{\rho + \sigma}$
 - $\sin \theta = \frac{\rho + \sigma}{\rho - \sigma}$
 - $\sin \theta = \frac{\rho}{\sigma}$



54. A liquid drop of radius R breaks into N smaller droplets of radii r . If liquid has density ρ , specific heat s and surface tension T , then the drop in temperature is given by
- $\frac{NT}{\rho s} \left(\frac{1}{R} - \frac{1}{r} \right)$
 - $\frac{NT}{\rho s} \left(\frac{R}{r} - 1 \right)$
 - $\frac{3T}{4\rho s} \left(\frac{1}{R} - \frac{1}{r} \right)$
 - $\frac{3T}{\rho s} \left(\frac{1}{R} - \frac{1}{r} \right)$
55. A body of density ρ is dropped from height h into a liquid having density σ ($\sigma > \rho$). If the body just touches the base of the container, then the distance of fallen would be proportional to (Neglect viscous forces)
- $\frac{h}{\sigma - \rho}$
 - $\frac{h}{\sigma + \rho}$
 - $h \times (\sigma - \rho)$
 - $\frac{h\rho}{\sigma - \rho}$
56. A block of mass m and density ρ is hanging from a string. If it is lowered into a vessel of cross-sectional area A containing a liquid of density σ ($\sigma < \rho$) and gets fully immersed, the increase in pressure at the bottom of vessel would be
- $\frac{m\rho g}{\sigma A}$
 - $\frac{m\sigma g}{\rho A}$
 - $\frac{mg}{A}$
 - zero
57. A solid sphere falls with a terminal velocity of 32 m/s in air. If it is allowed to fall in vacuum, then
- the terminal velocity will be 32 m/s
 - the terminal velocity will be less than 32 m/s
 - the terminal velocity will be greater than 32 m/s
 - there will be no terminal velocity
58. Find the common radius of curvature r when two soap bubbles with radii r_1 and r_2 ($r_1 > r_2$) come in contact
- $r = \frac{r_1 + r_2}{2}$
 - $r = \frac{r_1 r_2}{r_1 - r_2}$
 - $r = \frac{r_1 r_2}{r_1 + r_2}$
 - $r = \sqrt{r_1 r_2}$
59. Water is flowing in a river. If the velocity of a layer at a distance 10 cm from the bottom is 20 cm/s. Find the velocity of layer at a height of 40 cm from the bottom.
- 10 m/s
 - 20 cm/s
 - 30 cm/s
 - 80 cm/s
60. A horizontal plate (10 cm $\times$ 10 cm) moves on a layer of oil of thickness 4 mm with a constant speed of 10 cm/s. The coefficient of viscosity of oil is 4 poise. The tangential force applied on the plate to maintain the constant speed of the plate is
- 10^3 dyne
 - 10^4 dyne
 - 10^5 dyne
 - None of these
61. A liquid is flowing through a narrow tube. The coefficient of viscosity of liquid is 0.1308 poise. The length and inner radius of tube are 50 cm and 1 mm respectively. The rate of flow of liquid is 360 cm³/min. Find the pressure difference between ends of tube.
- 10^6 dyne/cm²
 - 10^4 dyne/cm²
 - 10 dyne/cm²
 - None of the above
62. Find the terminal velocity of solid sphere of radius 0.1 m moving in air in vertically downward direction. ($\eta = 18 \times 10^{-5}$ Ns/m², density of sphere = 1000 kg/m³ and $g = 10$ m/s²)
- 2 m/s
 - 1.2 cm/s
 - 4 cm/s
 - None of these
63. Eight equal drops of water each of radius $r = 2$ mm are falling through air with a terminal velocity of 16 cm/s. The eight drops combine to form a big drop. Calculate the terminal velocity of big drop.
- 16 cm/s
 - 32 cm/s
 - 64 cm/s
 - None of these
64. At 20°C, to attain the terminal velocity how fast will an aluminium sphere of radii 1 mm fall through water. Assume flow to be laminar flow and specific gravity (Al) = 2.7, $\eta_{\text{water}} = 8 \times 10^{-4}$ Pa.
- 5 m/s
 - 4.6 m/s
 - 4 m/s
 - 2 m/s

BITSAT Archives

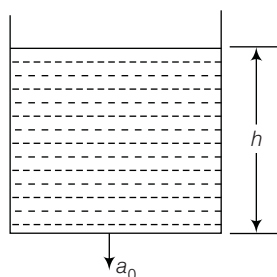
- The work done in blowing a soap bubble of surface tension 0.06 Nm^{-1} from 2 cm radius to 5 cm radius is
 - 0.004168 J
 - 0.003168 J
 - 0.003158 J
 - 0.004568 J
 [2014]
- A ball whose density is $0.4 \times 10^3 \text{ kg/m}^3$ falls into water from a height of 9 cm. To what depth does the ball sink?
 - 9 cm
 - 6 cm
 - 4.5 cm
 - 2.25 cm
 [2013]
- The relative humidity on a day when partial pressure of water vapour is $0.012 \times 10^6 \text{ Pa}$ at 12°C is (Take, vapour pressure of water at this temperature as $0.016 \times 10^5 \text{ Pa}$)
 - 70%
 - 40%
 - 75%
 - 25%
 [2013]
- In the absence of intermolecular forces of attraction, the observed pressure p will be
 - remain same
 - decrease
 - increase
 - zero
 [2013]

5. 10000 small balls, each weighting 1g, strike one square centimetre of area per second with a velocity 100 m/s in a normal directions and rebound with the same velocity. The value of pressure on the surface will be [2013]
 a. $2 \times 10^{10} \text{ N/m}^2$ b. $2 \times 10^5 \text{ N/m}^2$
 c. 10^7 N/m^2 d. $2 \times 10^7 \text{ N/m}^2$
6. At a given place where acceleration due to gravity is $g \text{ m/s}^2$, a sphere of lead of density $d \text{ kg/m}^3$ is gently released in a column of liquid of density $\rho \text{ kg/m}^3$. If $d > \rho$, the sphere will [2013]
 a. fall vertically with an acceleration $g \text{ m/s}^2$
 b. fall vertically with no acceleration
 c. fall vertically with an acceleration $g \left(\frac{d - \rho}{d} \right)$
 d. fall vertically with an acceleration $g \left(\frac{\rho}{d} \right)$
7. Motion of a fluid in a tube is best described by [2011]
 a. Bernoulli's theorem b. Poiseuille's principle
 c. Archimedes' principle d. Stokes' law
8. The velocity of efflux of a liquid through an orific in the bottom of the tank does not depend upon [2011]
 a. size of orifice
 b. height of liquid
 c. acceleration due to gravity
 d. density of liquid
9. Density of sea water is 1.03 g/cc^{-1} . A ship passes from fresh water into sea water. It will [2010]
 a. rise
 b. sink
 c. remain at the same depth
 d. rise or sink depending on its shape and size
10. Water is flowing through a tube of non-uniform cross-section. If the radii of the tube at the entrance and the exit are in the ratio 3 : 2, then the ratio of the velocities of flow of water at the entrance and the exit is [2010]
 a. 9 : 4 b. 4 : 9
 c. 8 : 27 d. 27 : 8
11. A liquid X of density 3.36 g/cm^3 is poured in a U-tube, which contains Hg. Another liquid Y is poured in left arm with height 8 cm, upper levels of X and Y are same. What is density of Y? [2010]
 a. 0.8 g/cc b. 1.2 g/cc
 c. 1.4 g/cc d. 1.6 g/cc
12. The surface tension of soap solution is 0.03 N/m . The work done in blowing to form a soap bubble of surface area 40 cm^2 , (in J), is [2008]
 a. 12×10^{-4} b. 2.4×10^{-4}
 c. 12×10^{-4} d. 24×10^{-4}
13. Two rain drops reach the earth with different terminal velocities having ratio 9 : 4. Then, the ratio of their volumes is [2008]
 a. 3 : 2 b. 4 : 9
 c. 9 : 4 d. 27 : 8
14. Water falls from a tap down the streamline, [2006]
 a. area decreases
 b. area increases
 c. velocity remains same
 d. area remains same
15. If a liquid does not wet glass, its angle of contact is [2005]
 a. zero b. acute
 c. obtuse d. right angle

Answer with Solutions

Practice Exercise

1. (b)
 2. (c) Here, effected gravitational acceleration is



$$\therefore \begin{aligned} g' &= g - a_0 \\ p &= p_0 + \rho g' h = p_0 + \rho(g - a_0)h \end{aligned}$$

3. (c) Work done by the heart = $p dV$

$$dV = (60) \times (80 \times 10^{-6} \text{ m}^3)$$

$$p = (100 \text{ mm of Hg}) \times \frac{101 \times 10^5 \text{ Pa}}{760 \text{ mm of Hg}}$$

$$= 1.33 \times 10^4 \text{ Pa}$$

$$\text{Power} = \frac{\text{Work done}}{\text{Time}}$$

$$= \frac{1.33 \times 10^4 \text{ Pa} \times 60 \times 80 \times 10^{-6} \text{ m}^3}{60 \text{ s}}$$

$$\text{Power} = 1.06 \text{ W}$$

4. (a) $\therefore$ The force due to excess pressure = restoring force
 or $pghA = \text{restoring force}$
 or $pg(2x)A = \text{restoring force}$
 $\therefore$ Restoring force = $2pg \times A$

27. (b) Net pressure = $h_1 \rho_1 g + h_2 \rho_2 g$

According to Bernoulli's theorem, this pressure energy will be converted into KE, while flowing through the hole is

$$h_1 \rho_1 g + h_2 \rho_2 g = \frac{1}{2} \rho v^2$$

$$v = \sqrt{2g \left(h_1 + h_2 \frac{\rho_2}{\rho_1} \right)}$$

28. (c) If two stream lines cross each other, then at a given point, there are two directions of motion of fluid particles. This is physically not possible.

Hence, (c) is correct,

29. (a) In horizontal pipe,

$$p_1 + \frac{1}{2} \rho v_1^2 = p_2 + \frac{1}{2} \rho v_2^2 \quad \dots(i)$$

Here,

$$p_1 = \rho_m g h_1 = 13600 \times 9.8 \times 10^{-2}$$

$$p_2 = 13600 \times 9.8 \times h$$

$$\rho = 1000 \text{ kg/m}^3$$

$$v_1 = 35 \times 10^{-2} \text{ m/s}$$

$$v_2 = 65 \times 10^{-2} \text{ m/s}$$

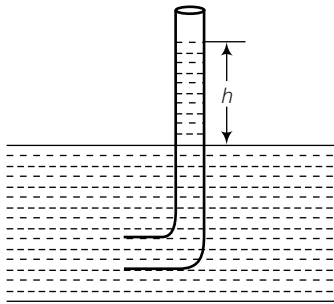
$\therefore$ From Eq. (i), we get

$$13600 \times 9.8 \times 10^{-2} + \frac{1}{2} \times 1000 \times (0.35)^2$$

$$= 13600 \times 9.8 \times h + \frac{1}{2} \times 1000 \times (0.65)^2$$

After solving, 0.89 cm of Hg.

30. (b) $\frac{1}{2} \rho v^2 = \rho g h$



$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 200 \times 10^{-3}} = 1.98 \text{ m/s}$$

31. (c)

32. (a) Applying Bernoulli's theorem,

$$\frac{p}{\rho} + \frac{1}{2} (v')^2 + gh = \frac{p}{\rho} + \frac{1}{2} v^2 + 0$$

where, v' is velocity of all surfaces of liquid and v is velocity of efflux.

From equation of continuity

$$Av' = av \Rightarrow v' = \frac{av}{A}$$

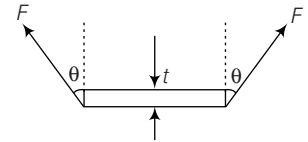
$$\frac{1}{2} \left(\frac{av}{A} \right)^2 + gh = \frac{1}{2} v^2$$

$$v = \sqrt{2gh} \sqrt{\left(\frac{A^2 - a^2}{A^2} \right)}$$

33. (a) Heat evolved = Potential energy of water

$$= \frac{mgh}{2} = \frac{\rho Ah gh}{2} = \frac{\rho Ah^2 g}{2}$$

34. (c) For the biggest coin, the force due to surface tension shall be vertical (i.e. $\theta = 0$)
 $\therefore$ Upward force exerted on the coin at the verge of its sinking



$$F = (S) \times (2\pi r) \quad (r = \text{radius of coin})$$

Now,

$$F = \text{weight of coin}$$

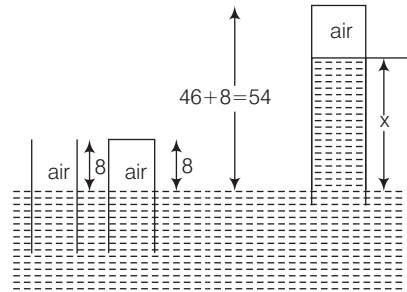
$$\therefore S \times 2\pi r = \pi t^2 \tau \times \rho g \Rightarrow r = \frac{2S}{\rho g t}$$

35. (c) Assuming, temperature remains same.

Hence, we can write $p_1 V_1 = p_2 V_2$

$$\Rightarrow p_1 l_1 = p_2 l_2 \Rightarrow (76)(8) = (54 - x)(76 - x)$$

$$\Rightarrow x = 38 \text{ cm}$$



Length of air column = $54 - 38 = 16 \text{ cm}$

$$r \left(h + \frac{r}{3} \right) \rho g$$

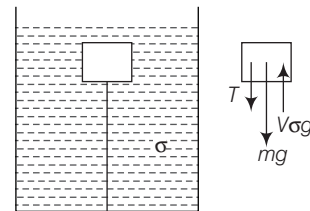
36. (b) As, $T = \frac{r \left(h + \frac{r}{3} \right) \rho g}{2 \cos \theta}$

$$= \frac{0.5 \times 10^{-2} \left[3 + \frac{0.5}{3} \right] \times 10^{-2} \times 10^3 \times 9.81}{2} = 0.77 \text{ N/m}$$

37. (a)

38. (b) From free body diagram of the wooden block,

$$V\sigma g = mg + T \quad [V \text{ is the volume of block}]$$



$$T = V\sigma g - mg$$

$$T = \frac{m}{\rho} \sigma g - mg = mg \left(\frac{\sigma - \rho}{\rho} \right)$$

39. (a) When radius is decreased by
- dr
- .

Decrease in surface energy = Heat required for vaporisation $(4\pi r dr) \times T \times 2 = 4\pi r^2 dp$

$$\Rightarrow r = \frac{2T}{\rho L}$$

40. (c) From Bernoulli's theorem,
- $\rho gh = \frac{1}{2} \rho (v_2^2 - v_1^2)$

$$\Rightarrow gh = \frac{1}{2} v_1^2 \left[\left(\frac{v_2}{v_1} \right)^2 - 1 \right] = \frac{1}{2} v_1^2 \left[\left(\frac{A_1}{A_2} \right)^2 - 1 \right]$$

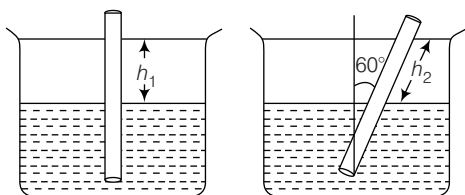
($\because A_1 v_1 = A_2 v_2$)

$$\Rightarrow \left(\frac{A_1}{A_2} \right)^2 = 1 + \frac{2gh}{v_1^2}$$

$$\Rightarrow \left(\frac{D_1^2}{D_2^2} \right)^2 = 1 + \frac{2gh}{v_1^2}$$

$$\Rightarrow D_2 = \frac{D_1}{\left(1 + \frac{2gh}{v_1^2} \right)^{1/4}} = \frac{8 \times 10^{-3}}{\left(1 + \frac{2 \times 10 \times 0.2}{(0.4)^2} \right)^{1/4}} = 3.6 \times 10^{-3} \text{ m}$$

41. (a) Since, the atmospheric pressure and temperature are same.



$$\therefore h_2 = \frac{h_1}{\cos 60^\circ} = \frac{3}{1/2} = 6 \text{ mm}$$

42. (a)
- $h = \frac{2S \cos \theta}{\rho g}$

$$\Rightarrow hr = \frac{2S \cos \theta}{\rho g}$$

For both experiment S, θ, ρ, g are same

$$hr = \frac{2S \cos \theta}{\rho g} = \text{constant}$$

$$h_1 r_1 = h_2 r_2$$

$$h_2 = \left(\frac{r_1}{r_2} \right) 3 = 12 \text{ mm}$$

43. (c) Water rise to height
- $h = \frac{2T}{\rho g r}$

$$\text{Potential energy of water column } U = \frac{mgh}{2} = \frac{2\pi T^2}{\rho g}$$

The work performed by force of surface tension is

$$W = 2\pi r T h = \frac{4\pi T^2}{\rho g}$$

From conservation of energy, the heat evolved

$$Q = W - U = \frac{2\pi T^2}{\rho g}$$

44. (a) Let atmospheric pressure is
- p_0
- .

Pressure of liquid just outside the bubble is

$$p = p_0 + h \rho g$$

Pressure of liquid inside the bubble

$$p' = p + \frac{2T}{r} = p_0 + h \rho g + \frac{2T}{r}$$

$$p' - p_0 = \frac{1}{10} \times 1000 \times 9.8 + \frac{2 \times 0.075}{1 \times 10^{-3}}$$

$$p' - p_0 = 1130 \text{ Pa}$$

45. (b) Initial area of soap bubble = 0

Soap bubble has two surfaces internal and external.

Increase in surface area = $\Delta A = 2 \times 4\pi r^2$

$$= 2 \times 4 \times \frac{22}{7} \times (0.7 \times 10^{-2})^2$$

$$= 12.32 \times 10^{-4} \text{ m}^2$$

Work done = $W = S \Delta A$

$$= 0.03 \times 12.32 \times 10^{-4}$$

$$= 3.696 \times 10^{-5} \text{ J}$$

46. (c) The volume of
- n
- smaller drop = volume of bigger drop

$$\therefore n \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3$$

$$\therefore R = n^{1/3} r$$

$$\therefore r = \frac{R}{n^{1/3}}$$

Here, R = radius of bigger drop r = radius of smaller drop

$$\therefore W = 4\pi(nr^2 - R^2)T$$

$$= 4\pi \left[n \left(\frac{R}{n^{1/3}} \right)^2 - R^2 \right] T = 4\pi \left[\frac{nR^2}{n^{2/3}} - R^2 \right] T$$

$$= 4\pi R^2 (n^{1-2/3} - 1) T = 4\pi R^2 T (n^{1/3} - 1)$$

47. (c) Let radius of small drop =
- r

Radius of single drop = R Surface tension S = surface energy per unit area

$$n \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3$$

$$R = n^{1/3} r$$

Initial surface energy = $E_i = n \times 4\pi r^2 \times S = nE$ Final surface energy = $E_f = 4\pi R^2 S$

$$= 4\pi r^2 n^{2/3} S = n^{2/3} E$$

$$\text{Energy released} = E_i - E_f = E(n - n^{2/3})$$

48. (c) 49. (a) 50. (a) 51. (b)

52. (b)

53. (b) Taking torque about the point
- O
- .

$$\therefore \rho V r \sin(45^\circ - \theta) g = \sigma V r \sin(45^\circ + \theta) g$$

$$\tan \theta = \frac{\rho - \sigma}{\rho + \sigma}$$

54. (d) Volume will remain same, hence we can write

$$\frac{4}{3}\pi R^3 = N \times \frac{4}{3}\pi r^3 \Rightarrow N = \frac{R^3}{r^3}$$

Increase in surface energy due to breaking of drop into N droplets,

$$\Delta U = 4\pi T(R^2 - Nr^2)$$

As this energy is provided at the cost of lowering of temperature by $\Delta\theta$

$$ms\Delta\theta = \Delta U$$

$$\Delta\theta = \frac{\Delta U}{ms} = \frac{4\pi T(R^2 - Nr^2)}{\left(\frac{4}{3}\pi R^3 \rho\right)s}$$

$$\Rightarrow \Delta\theta = \frac{3T}{\rho s} \left(\frac{1}{R} - \frac{r^2}{R^3} N \right) = \frac{3T}{\rho s} \left(\frac{1}{R} - \frac{1}{r} \right) \quad \left(\because N = \frac{R^3}{r^3} \right)$$

55. (a) Buoyant force, $F \propto (\sigma - \rho)$

$$\therefore \text{Deceleration} \propto (\sigma - \rho) \Rightarrow a = -k(\sigma - \rho)$$

Now, as initial velocity, $u = \sqrt{2gh}$

and final velocity, $v = 0$

Distance fallen by the body,

$$s = \frac{2gh}{2 \times k(\sigma - \rho)} \quad \left[\because s = \frac{u^2}{2a} \right]$$

$$\Rightarrow s \propto \frac{h}{\sigma - \rho}$$

56. (b) Volume of block = $\frac{m}{\rho}$

$$\therefore \text{Buoyant force on block} = \text{Volume} \times \text{Density of liquid} \times g$$

$$= \frac{m}{\rho} \times \sigma \times g$$

By Newton's third law of motion, force exerted by block on liquid surface = $\frac{m\sigma g}{\rho A}$

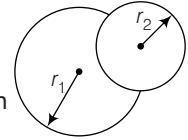
$$\therefore \text{Increase in pressure} = \frac{m\sigma g}{\rho A}$$

57. (d)

58. (b) Let p_0 = atmosphere pressure

p_1 and p_2 are pressure difference across the common surface.

Let r = radius of curvature of the common surface.



$$p_2 - p_1 = \frac{4S}{r}$$

$$\frac{4S}{r} = \frac{4S}{r_2} - \frac{4S}{r_1} \quad (S = \text{surface tension})$$

$$\frac{1}{r} = \frac{1}{r_2} - \frac{1}{r_1} \Rightarrow r = \frac{r_1 r_2}{r_1 - r_2}$$

59. (d) $\therefore F = -\eta A \frac{dv}{dx}$

$$\therefore F \propto \frac{dv}{dx} \text{ or } F \propto \frac{\Delta v}{\Delta x}$$

$$F \propto \frac{v}{x} \text{ or } x \propto \frac{v}{F}$$

But F is constant,

$$\therefore x \propto v$$

$$\frac{v_2}{v_1} = \frac{x_2}{x_1} \text{ or } \frac{v_2}{20} = \frac{40}{10}$$

$$\therefore v_2 = 80 \text{ cm/s}$$

60. (b)

61. (a)

62. (b)

63. (c)

64. (b)

BITSAT Archives

1. (b) As given, $s = 0.06 \text{ Nm}^{-1}$,

$$r_1 = 2 \text{ cm} = 0.02 \text{ m}, r_2 = 5 \text{ cm} = 0.05 \text{ m}$$

Since, bubble has two surface

$$\text{Initial surface area of the bubble} = 2 \times 4\pi r_1^2$$

$$= 2 \times 4\pi \times (0.02)^2$$

$$= 32\pi \times 10^{-4} \text{ m}^2$$

$$\text{Final surface area of the bubble} = 2 \times 4\pi r_2^2$$

$$= 2 \times 4 \times \pi \times (0.05)^2$$

$$= 200 \times \pi \times 10^{-4} \text{ m}^2$$

So, work done = $s \times \text{increase in surface}$

$$= 0.06 \times (200\pi \times 10^{-4} - 32\pi \times 10^{-4})$$

$$= 0.06 \times 168\pi \times 10^{-4}$$

$$= 0.003168 \text{ J}$$

2. (b) The velocity of ball before entering the water surface

$$v = \sqrt{2gh} = \sqrt{2g \times 9}$$

When ball enters into water, due to upthrust of water, the velocity of ball decreases (or retarded)

The retardation,

$$a = \frac{\text{apparent weight}}{\text{mass of ball}}$$

$$= \frac{V(\rho - \sigma)g}{V\rho} = \frac{(\rho - \sigma)g}{\rho}$$

$$= \left(\frac{0.4 - 1}{0.4} \right) g = -\frac{3}{2}g$$

If h be the depth upto which ball sink, then

$$0 - v^2 = 2 \times \left(-\frac{3}{2}g \right) \times h$$

$$\Rightarrow 2g \times 9 = 3gh$$

$$\therefore h = 6 \text{ cm}$$

3. (c) Relative humidity at a given temperature (R)

$$= \frac{\text{partial pressure of water pressure}}{\text{vapour pressure of water}}$$

$$= \frac{0.012 \times 10^5}{0.016 \times 10^5}$$

$$= 0.75 = 75\%$$

4. (c) In the absence of intermolecular forces, there will be no stickiness of molecules. Hence, pressure will increase.

$$5. (d) p = \frac{F}{A} = \frac{n\{mv - (-mv)\}}{A} = \frac{2mnv}{A}$$

$$= \frac{2 \times 10^{-3} \times 10^4 \times 10^2}{10^{-4}} = 2 \times 10^7 \text{ N/m}^2$$

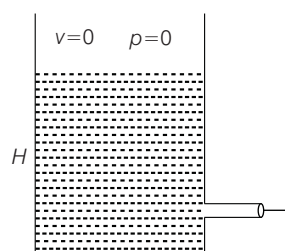
6. (c) Apparent weight = actual weight – upthrust

$$Vdg' = Vdg - V\rho g$$

$$\Rightarrow g' = \left(\frac{d - \rho}{d} \right) g$$

7. (a) Bernoulli's theorem is applicable only for tube flow of non-uniform cross-section.

8. (a) v = velocity of efflux through an orifice

$$= \sqrt{2gH}$$


It is independent of the size of orifice.

9. (a) Fresh water has lesser density as compared to sea water. So, upthrust in sea water will be more as compared to fresh water.

10. (b) From equation of continuity

$$A_1 v_1 = A_2 v_2$$

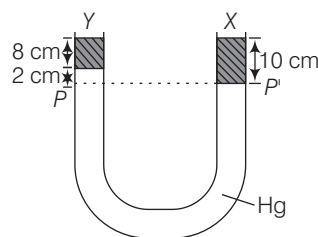
$$\Rightarrow \pi r_1^2 v_1 = \pi r_2^2 v_2$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{r_2^2}{r_1^2} = \frac{4}{9}$$

11. (a) As shown in figure, in the two arms of a tube pressure remains same on surface PP' . Hence,

$$8 \times \rho_y \times g + 2\rho_{\text{Hg}} \times g = 10 \times \rho_x \times g$$

$$8\rho_y = 2 \times 13.6 = 10 \times 3.36$$



or

$$\rho_y = \frac{39.6 - 27.2}{8}$$

$$= 0.8 \text{ g/cc}$$

12. (b) In case of soap bubble,

$$W = T \times 2 \times \Delta A$$

$$= 0.03 \times 2 \times 40 \times 10^{-4}$$

$$= 2.4 \times 10^{-4} \text{ J}$$

13. (d) Terminal velocity, $v_T \propto r^2$

or

$$\frac{v_{T1}}{v_{T2}} = \frac{r_1^2}{r_2^2}$$

$$\therefore \sqrt{\frac{9}{4}} = \frac{r_1}{r_2} \text{ or } \frac{r_1}{r_2} = \frac{3}{2}$$

$$\therefore V = \frac{4}{3} \pi r^3 \text{ or } \frac{V_1}{V_2} = \frac{r_1^3}{r_2^3} = \frac{27}{8}$$

14. (b) If water falls from a tap down the streamline, then its area increases.

15. (c) For the liquids, which do not wet the glass, the liquid meniscus is convex upward, so angle of contact is obtuse.

12

Elasticity

Introduction

Elasticity is the property of body by virtue of which a body regains or tends to regain its original configuration (shape as well as size), when the external deforming forces acting on it, is removed.

Cause of Elasticity

When a solid is deformed by a force, atoms or molecules get displaced from their equilibrium position causing a change in interatomic/intermolecular distances. When deforming force is removed, the interatomic forces tend to drive back the molecules to their original positions. This gives rise to the property elasticity.

Elastic Behaviour of Solids

If a body completely regains its original configuration on removal of external deforming forces, it is called a **perfectly elastic body**.

If a body has no tendency to regain its original configuration and tends to maintain its deformed state even after the removal of the deforming force, the body is called a **plastic** (or **non-elastic**) **body**.

Stress

The internal restoring force per unit area of cross-section of the deformed body is called **stress**. Thus,

$$\text{Stress, } \sigma = \frac{\text{Restoring force}}{\text{Area}} = \frac{F}{A}$$

Stress is numerically equal to the deforming force per unit area of cross-section.

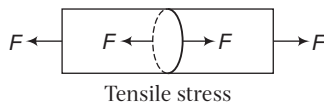
The SI unit of stress is Nm^{-2} or pascal (Pa).

Types of Stress

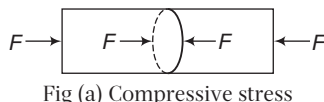
Stress are of two types normal stress or longitudinal stress and tangential stress or shearing stress.

Also the longitudinal stress can be of two types

- (i) If a solid in the form of a wire/rod is stretched by applying a force normally, then the stress is called **tensile stress** (first type of longitudinal stress).



- (ii) If the solid wire/rod is compressed, then the stress is called **compressive stress**. (Other type of longitudinal stress)



- (iii) For a bulk solid or liquid or gas if the deforming force is applied normal to the surface such that magnitude of force on any small area is proportional to the area, the stress is called **volumetric stress**.

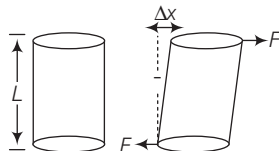


Fig (b) Volumetric stress

The given Fig. (b). Here, force is applied tangentially to the cross-section of the body. So, the restoring force developed due to applied tangential force is called **tangential or shearing stress**.

Strain

When the size or shape of a body is changed under an external force, the body is said to be strained. The change occurred in the unit size of the body is called **strain**.

$$\text{i.e. Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}} = \frac{\Delta x}{x}$$

where, Δx = Change (may be in length, in volume etc)

x = Original value of quantity in which change has occurred

It is a dimensionless quantity.

Types of Strain

- (i) For a wire or rod, **linear strain** is defined as the ratio of change in length to the original length.

$$\therefore \text{Longitudinal strain} = \frac{\text{Change in length } (\Delta L)}{\text{Original length } (L)}$$

- (ii) When the deforming force causes a change in volume, the strain is called **volumetric strain**.

$$\text{Volumetric strain} = \frac{\text{Change in volume } (\Delta V)}{\text{Original volume } (V)}$$

- (iii) When the deforming force, applied tangentially to a surface, produces a change in shape of the body, the strain developed is called **shearing strain or shear**.

$$\text{Shearing strain} = \phi$$

Hooke's Law

According to the Hooke's law, for any body within the elastic limit, stress developed is directly proportional to the strain produced.

$$\text{Stress} \propto \text{strain}$$

$$\text{Stress} = E \times \text{strain}$$

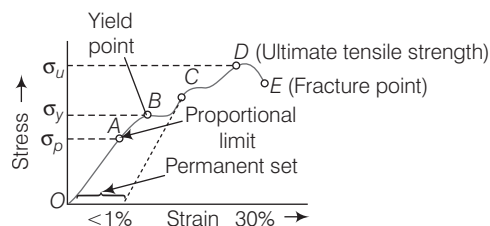
The ratio of stress to strain, within the elastic limit, is called the coefficient (or modulus) of elasticity for the given material.

Depending on the type of stress applied and resulting strain, we have the following three of elasticity given as

$$E = \frac{\text{Stress}}{\text{Strain}}$$

Stress-Strain Relationship

When a wire is stretched by an applied force, then a typical graph is obtained (especially in case of metals) as shown below



Modulus of Elasticity

Within the proportional region (region OA in stress-strain graph), the ratio of stress and strain remains constant. This ratio is called **modulus of elasticity**.

It has same units and dimensions as that of stress.

As there are three types of stresses and corresponding strains. So, there are three different modulus of elasticity which are given below.

1. Young's Modulus

Young's modulus of elasticity (Y) is defined as the ratio of normal stress (either tensile or compressive) to the longitudinal strain within a elastic limit.

$$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}} = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

2. Bulk Modulus

It is defined as the ratio of the normal stress to the volumetric strain.

Coefficient of volume elasticity

$$B = \frac{F/A}{\Delta V/V} = -\frac{pV}{\Delta V}$$

where, $p = \frac{F}{A}$ = the pressure or stress negative sign signifies that for an increase in pressure, the volume will decrease. Reciprocal of bulk modulus is called **compressibility**.

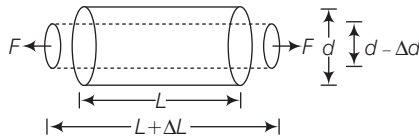
3. Modulus of Rigidity (Shear Modulus)

It is defined as the ratio of tangential stress to shearing stress.

$$\eta = \frac{\text{Tangential stress}}{\text{Shearing strain}} = \frac{F/A}{\phi} = \frac{F}{A\phi} = \frac{FL}{Ax}$$

Poisson's Ratio (σ)

In general, a deforming force in one direction can produce strains in other directions such as when wire is stretched, then force will produce longitudinal strain (obviously) but it will also change its lateral dimensions, (i.e. radius of cross-section) as described in figure given below



The fractional change in transverse length is proportional to the fractional change in longitudinal length and is constant for a given material. This constant of proportionality is called **Poisson's ratio** and is given by

$$\sigma = -\frac{\text{Transverse strain}}{\text{Longitudinal strain}} \Rightarrow \sigma = -\frac{\frac{\Delta d}{d}}{\frac{\Delta L}{L}}$$

Theoretical value of σ varies from -1 to 0.5 .

Elastic Potential Energy of a Strained Body

When force is applied on a wire, it gets under tension and work is done against the internal forces which is stored in the form of potential energy in the body.

$$\therefore \text{Young's modulus, } Y = \frac{\text{Stress}}{\text{Strain}}$$

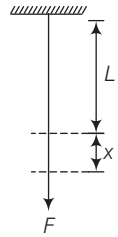
$$\text{or } F = \frac{AY}{L} \cdot x$$

$\therefore$ Work done is equal to potential energy

$$W = \int_0^l \frac{AY}{L} \cdot x \, dx = \frac{AY}{2L} \cdot l^2 \quad (\because dW = F \cdot dx)$$

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

$$\therefore \text{Energy per unit volume} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$



Inter-relations between Elastic Constants

Y = Young's modulus, η = Rigidity modulus, K = Bulk modulus, σ = Poisson's ratio

$$Y = 2\eta(1 + \sigma)$$

$$Y = 3K(1 - 2\sigma)$$

$$\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K}$$

or

$$Y = \frac{9K\eta}{\eta + 3K}$$

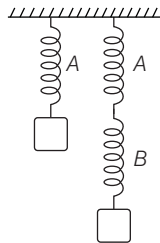
$$\sigma = \frac{3K - 2\eta}{6K + 2\eta}$$

Volume elasticity of a gas under an isothermal condition is equal to the pressure exerted by the gas $B_{\text{iso}} = p$

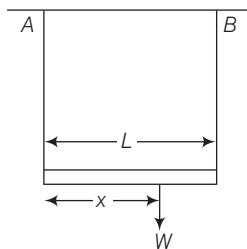
Adiabatic elasticity of a gas $B_{\text{adia}} = \gamma p$, where γ is the ratio of the two principal specific heats of the gas

Practice Exercise

- Calculate the stress for one litre of a perfect gas, at a pressure of 72 cm of Hg, when it is compressed isothermally to a volume of 900 cc.
 a. $9.88 \times 10^3 \text{ N/m}^2$ b. $10.88 \times 10^3 \text{ N/m}^2$
 c. $1.088 \times 10^3 \text{ N/m}^2$ d. $2 \times 10^3 \text{ N/m}^2$
- If a metal wire of length L , having area of cross-section A and Young's modulus Y , behaves as a spring of spring constant k . The value of k is
 a. $\frac{YA}{L}$ b. $\frac{YA}{2L}$ c. $\frac{2YA}{L}$ d. $\frac{YL}{A}$
- In the figure three identical springs are shown. From spring A , a mass of 4 kg is hung and spring shows elongation of 1 cm. But when a weight of 6 kg is hung on B , the Hooke's descends



- a. 1 cm b. 2 cm c. 3 cm d. 4 cm
- A system consists of two springs and a mass $m = 1 \text{ kg}$ as shown in figure. If mass m is displaced slightly along vertical and released. The system oscillates with a period of 2 s. Then, the spring constant k is
 a. $\frac{\pi^2}{4}$ b. $\frac{\pi^2}{6}$
 c. $\frac{\pi^2}{8}$ d. $\frac{\pi^2}{2}$
 - A spring of constant k is cut into two parts of length in the ratio 2 : 3. The spring constant of larger spring is
 a. $\frac{5}{3}k$ b. $\frac{2}{3}k$
 c. k d. $\frac{3}{5}k$
 - A light rod of length L is suspended from a support horizontally by means of two vertical wires A and B of equal lengths as shown in the figure. Cross-section area of A is half that of B and Young's modulus of A is double than that of B . A weight W is hung



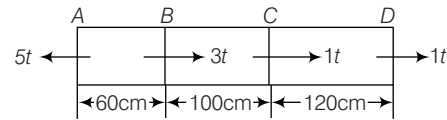
on the rod as shown. The value of x , so that the stress in A is same as that in B , is

- a. $\frac{L}{3}$ b. $\frac{L}{2}$ c. $\frac{2L}{3}$ d. $\frac{3L}{4}$

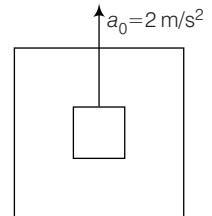
- Two wires, one made of copper and other of steel are joined end to end. (as shown in figure). The area of cross-section of copper wire is twice that of steel wire. They are placed under compressive force of magnitudes F . Find the ratio of their lengths such that change in lengths of both wires are same ($Y_s = 2 \times 10^{11} \text{ N/m}^2$ and $Y_c = 11 \times 10^{11} \text{ N/m}^2$).



- a. 2.1 b. 1.1 c. 1.2 d. 2
- Find the total elongation of the bar, if the bar is subjected to axial forces as shown in figure. The cross-sectional area of bar is 10 cm^2 . [Take $E = 8 \times 10^2 \text{ dyne/cm}^2$]



- a. 0.01 cm b. 0.5 cm c. 0.0675 cm d. 0.775 cm
- When tension in a metal wire is T_1 , its length was l_1 , and when tension is T_2 , the length is l_2 . Find its unstretched length.
 a. $\sqrt{l_1 l_2}$ b. $\frac{l_1 + l_2}{2}$ c. $\frac{(l_1 T_2 - l_2 T_1)}{T_2 - T_1}$ d. $\frac{l_1 T_2 + T_1 l_2}{T_1 + T_2}$
 - One end of a steel wire is fixed to ceiling of an elevator moving up with an acceleration 2 m/s^2 and a load of 10 kg hangs from other end. Area of cross-section of the wire is 2 cm^2 . Find the longitudinal strain in the wire (Take $g = 10 \text{ m/s}^2$ and $Y = 2 \times 10^{11} \text{ N/m}^2$)
 a. 4×10^{11} b. 3×10^{-6} c. 8×10^{-6} d. 2×10^{-6}
 - Two wires one of copper and other of steel having same cross-sectional area and lengths 1.0 m and 0.5 m respectively, are fastened end to end and stretched by a load M . If copper wire is stretched by 1 mm, the total extension of the combined wire is (Given, Young's moduli are $Y_{\text{copper}} = 1 \times 10^{11} \text{ N/m}^2$, and $Y_{\text{steel}} = 2 \times 10^{11} \text{ N/m}^2$)
 a. 1.125 cm b. 0.2 cm c. 0.120 cm d. 0.25 cm



12. A body of mass 1 kg is fastened to one end of a steel wire of cross-sectional area $3 \times 10^{-6} \text{ m}^2$ and is rotated in horizontal circle of radius 20 cm with a constant speed 2 m/s. Find the elongation of the wire ($Y = 2 \times 10^{11} \text{ N/m}^2$)

a. $0.33 \times 10^{-5} \text{ m}$ b. $0.67 \times 10^{-5} \text{ m}$
c. $2 \times 10^{-5} \text{ m}$ d. $4 \times 10^{-5} \text{ m}$

13. If a conical wire is stretched by two forces F applied parallel to its length and in opposite direction. Normal to end faces. The length of wire is L and its end radius are r_1 and r_2 . Find out the extension produced. (Given, Y = Young's modulus of wire)

a. $\frac{FL}{\pi r_1^2 Y}$ b. $\frac{FL}{\pi r_1 Y}$ c. $\frac{FL}{\pi r_1 r_2 Y}$ d. $\frac{FLY}{\pi r_1 r_2}$

14. If for a material, Y and B are Young's modulus and Bulk's modulus, then

a. $Y < 3B$ b. $Y = 3B$ c. $Y > 3B$ d. $3Y = B$

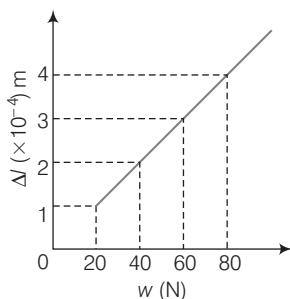
15. When a sphere is taken to bottom of sea 1 km deep, it contracts by 0.01%. Find the bulk modulus of elasticity of the material of sphere. (Given, density of water = 1 g/cm^3)

a. $9.8 \times 10^{10} \text{ N/m}^2$ b. $10.2 \times 10^{10} \text{ N/m}^2$
c. $0.98 \times 10^{10} \text{ N/m}^2$ d. $8.4 \times 10^{10} \text{ N/m}^2$

16. The Young's modulus of brass and steel are $1.0 \times 10^{11} \text{ Nm}^{-2}$ and $2.0 \times 10^{11} \text{ Nm}^{-2}$ respectively. A brass wire and a steel wire of the same length extend by 1 mm, each under the same force. If radii of brass and steel wires are R_B and R_S respectively, then

a. $R_S = \sqrt{2} R_B$ b. $R_S = \frac{R_B}{\sqrt{2}}$
c. $R_S = 4R_B$ d. $R_S = \frac{R_B}{2}$

17. The adjacent graph shows the extension Δl of a wire of length 1 m, suspended from the top of a roof at one end and with a load w connected to the other end. If the cross-sectional area of the wire is 10^{-6} m^2 , calculate the Young's modulus of the material of the wire.



a. $2 \times 10^{11} \text{ Nm}^{-2}$ b. $2 \times 10^{-11} \text{ Nm}^{-2}$
c. $3 \times 10^{-12} \text{ Nm}^{-2}$ d. $2 \times 10^{-13} \text{ Nm}^{-2}$

18. A wire of mass m and length l is suspended from a ceiling. Due to its own weight it elongates, consider cross-section area of wire as A and Young's modulus of material of wire as Y . The elongation in the wire is

a. $\frac{2mg}{3YA}$ b. $\frac{mgl}{YA}$
c. $\frac{mgl}{2YA}$ d. Cannot be calculated

19. A body of mass $m = 10 \text{ kg}$ is attached to a wire of length 0.3 m. Calculate the maximum angular velocity with which it can be rotated in a horizontal circle (Breaking stress of wire = $4.8 \times 10^7 \text{ N/m}^2$ and area of cross-section of a wire = 10^{-6} m^2)

a. 4 rad/s b. 8 rad/s c. 1 rad/s d. 2 rad/s

20. Find the velocity of projection of a missile of mass 5 g. When a rubber cord is stretched to 12 cm and then released to project the missile (Given, area of cross-section of cord = 1 mm^2 , total unstretched length = 10 cm, Young's modulus of rubber = $5 \times 10^8 \text{ N/m}^2$)

a. 20 m/s b. 25 m/s
c. 22 m/s d. 18 m/s

21. A rubber cord catapult has a cross-sectional area of 25 mm^2 and the initial length of rubber cord is 10 cm. It is stretched by 5 cm and then released to project a missile of mass 5 g. Taking, $Y_{\text{rubber}} = 5 \times 10^8 \text{ Nm}^{-2}$, velocity of the projected missile is

a. 20 ms^{-1} b. 100 ms^{-1} c. 250 ms^{-1} d. 200 ms^{-1}

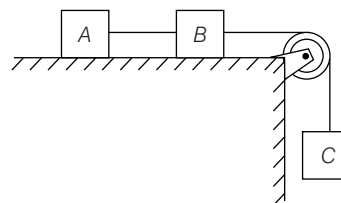
22. One end of a wire 2 m long and diameter 2 mm is fixed in a ceiling. A naughty boy of mass 10 kg jumps to catch the free end and stays there. The change in length of wire is (Take $g = 10 \text{ m/s}^2$, $Y = 2 \times 10^{11} \text{ N/m}^2$)

a. $3185 \times 10^{-5} \text{ m}$ b. 2 mm
c. 3 mm d. 4 mm

23. In above problem, if Poisson's ratio is $\sigma = 0.1$ Find the change in diameter.

a. $3.184 \times 10^{-5} \text{ m}$ b. $31.84 \times 10^{-5} \text{ m}$
c. $3.184 \times 10^{-8} \text{ m}$ d. $31.84 \times 10^{-8} \text{ m}$

24. Each of three blocks shown in figure has a mass 3 kg. The wire connecting blocks A and B has area of cross-section 0.005 cm^2 and Young's modulus of elasticity $Y = 2 \times 10^{11} \text{ N/m}^2$. Neglect friction. Find the elastic potential energy stored per unit volume in wire connecting blocks A and B in steady state. (Take $g = 10 \text{ m/s}^2$)



a. 500 J/m^3 b. 1000 J/m^3
c. 2000 J/m^3 d. 3000 J/m^3

25. A steel rod of Young's modulus $2 \times 10^{11} \text{ N/m}^2$ undergoes an elastic strain of 0.05%. The energy per unit volume stored in J/m^3 is

a. 12500 b. 5000
c. 10000 d. 25000

26. If F is the force applied to an elastic bar to produce an extension of Δl . Then, the energy lost in the process is

a. $F \cdot \Delta l$ b. $\frac{F \cdot \Delta l}{2}$
 c. zero d. $\frac{3F \cdot \Delta l}{2}$

27. If the work done in stretching a wire by 1 mm is W . Then, the work required to stretch another wire of same material but with half the radius of cross-section and double the length by 2 mm is

a. $4W$ b. W c. $\frac{1}{4}W$ d. $\frac{1}{2}W$

28. Which of the following correctly gives the elastic energy stored in the metal bar?

(σ = stress, ϵ = strain, Y = Young's modulus, L = length, Δl = extension, F = load, A = cross-sectional area).

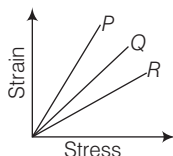
a. $\frac{1}{2} \sigma^2 Y$
 b. $\frac{1}{2} \sigma^2 Y$
 c. $\frac{1}{2} \epsilon^2 Y \cdot (AL)$
 d. $\frac{1}{2} \sigma^2 Y \cdot (AL)$

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1. The strain-stress curves of three wires of different material are shown in the figure.

P, Q, R are the elastic limits of the wires, the figure shows that [2014]

a. elasticity of wire P is maximum
 b. elasticity of wire R is maximum
 c. tensile strength of R is maximum
 d. None of the above



2. A steel wire of length 4.7 m and cross-section $3.0 \times 10^{-5} \text{ m}^2$ stretches by the same amount as a copper wire of length 3.5 m and cross-section $4.0 \times 10^{-5} \text{ m}^2$ under a given load. What is the ratio of the Young's modulus of steel so that of copper? [2014]

a. 1.5 : 2 b. 1.8 : 2
 c. 1.5 : 1 d. 1.8 : 1

3. The average depth of Indian Ocean is about 3000 m. Bulk modulus of water is $2.2 \times 10^4 \text{ N/m}^2$,

$g = 10 \text{ m/s}^2$, then fractional compression $\frac{\Delta V}{V}$ of water

at the bottom of the Indian Ocean will be [2014]

a. 1.36% b. 20.6%
 c. 13.9% d. 0.52%

4. One end of steel wire is fixed to ceiling of an elevator moving up with an acceleration 2 m/s^2 and a load of 10 kg hangs from other end. Area of cross-section of the wire is 2 cm^2 . The longitudinal strain in the wire is ($g = 10 \text{ m/s}^2$ and $Y = 2 \times 10^{11} \text{ Nm}^{-2}$) [2014]

a. 4×10^{11} b. 3×10^{-6}
 c. 8×10^{-6} d. 2×10^{-6}

5. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If length of wire 1 increased by Δx on applying force F ,

how much force is needed to stretch wire 2 by the same amount? [2011]

a. $4F$ b. $6F$
 c. $9F$ d. F

6. The Poisson's ratio of a material is 0.5. If a force is applied to a wire of this material, there is a decrease in the cross-sectional area by 4%. The percentage increase in the length is [2011]

a. 1% b. 2%
 c. 2.5% d. 4%

7. A load of 1 kg weight is attached to one end of a steel wire of area of cross-section 3 mm^2 and Young's modulus 10^{11} N/m^2 . The other end is suspended vertically from a hook on a wall, then the load is pulled horizontally and released. When the load passes through its lowest position the fractional change in length is (Take $g = 10 \text{ m/s}^2$) [2008]

a. 0.3×10^{-4} b. 0.3×10^{-3}
 c. 0.3×10^3 d. 0.3×10^4

8. There is some change in length when a 33000 N tensile force is applied on a steel rod of area of cross-section 10^{-3} m^2 . The change of temperature required to produce the same elongation, if the steel rod is heated, is (The modulus of elasticity is $3 \times 10^{11} \text{ N/m}^2$ and the coefficient of linear expansion of steel is $11 \times 10^{-5} / ^\circ \text{C}$). [2008]

a. 20°C b. 15°C
 c. 10°C d. 0°C

9. Which one of the following is not a unit of Young's modulus? [2005]

a. Nm^{-1} b. Nm^{-2}
 c. dyne cm^{-2} d. mega pascal

Answer with Solutions

Practice Exercise

1. (b) Let p = pressure of the gas in compressed state.

Condition for isothermal process is

$$p \times 900 = 72 \times 1000 \Rightarrow p = 80 \text{ cm}$$

Stress = increase in pressure = $(80 - 72) = 8 \text{ cm of Hg}$

$$= 0.08 \times 13.6 \times 10^3 \times 10 = 10.88 \times 10^3 \text{ N/m}^2$$

2. (a) Let the extension of wire be x for an external force F .

$$\text{Stress} = \frac{F}{A}$$

$\Rightarrow$

$$\text{Strain} = \frac{x}{L}$$

$$Y = \frac{F/A}{x/L} = \frac{FL}{xA}$$

$$\text{Equivalent force constant} = k = \frac{F}{x} = \frac{YA}{L}$$

3. (c) $x = \frac{F}{k}$

If spring constant is k for the first case,

it is $\frac{k}{2}$ for second case.

$$\text{For first case, } 1 = \frac{4}{k} \quad \dots(i)$$

$$\text{For second case, } x' = \frac{6}{k/2} = \frac{12}{k} \quad \dots(ii)$$

Dividing Eqs. (i) and (ii), we get $x' = \frac{12/k}{4/k} = 3 \text{ cm}$

4. (d) $T = 2\pi\sqrt{\left(\frac{m}{2k}\right)} \Rightarrow k = \frac{2\pi^2 m}{T^2} = \frac{\pi^2}{2} \quad \left(\because T = 2 \text{ s}\right)$
 $(m = 1 \text{ kg})$

5. (a) As, $Y = \frac{Fl}{A \cdot \Delta l}$
 $\Rightarrow F = \left(\frac{AY}{l}\right) \cdot \Delta l$

$$\text{As, } |F| = kl$$

$$\text{So, } k \propto \frac{AY}{l} \therefore k \propto \frac{1}{l}$$

$$\text{Length of bigger spring, } l' = \frac{3}{5} l$$

$$\text{So, } \frac{k'}{k} = \frac{l}{\frac{3}{5}l} = \frac{5}{3}$$

6. (c) Stress in $A = \frac{T_A}{A_A}$, where A_A is cross-sectional area of wire A .

Stress in $B = \frac{T_B}{A_B}$, where A_B is cross-sectional area of wire B .

$$\text{It is given, } A_A = \frac{A_B}{2}, \frac{T_A}{A_A} = \frac{T_B}{A_B} \text{ which gives } x = \frac{2L}{3}$$

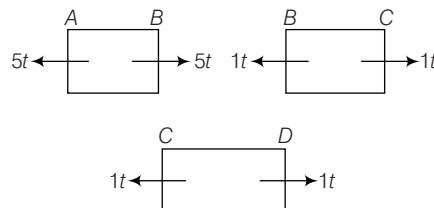
7. (b) $Y_S = \frac{FL_S}{A_S \Delta l_S}$ and $Y_C = \frac{FL_C}{A_C \Delta l_C}$

$$\therefore \frac{L_C}{L_S} = \frac{\frac{Y_C A_C \Delta l_C}{F}}{\frac{Y_S A_S \Delta l_S}{F}} = \left(\frac{Y_C}{Y_S}\right) \left(\frac{A_C}{A_S}\right) \left(\frac{\Delta l_C}{\Delta l_S}\right)$$

$$\text{Here } \frac{A_C}{A_S} = 2, \frac{\Delta l_C}{\Delta l_S} = 1, \frac{Y_C}{Y_S} = \frac{11}{2}$$

$$\therefore \frac{L_C}{L_S} = \left(\frac{11}{2}\right) (2) (1) = 11$$

8. (c) Let δl = total elongation of the bar, A = area,
 $E = 8 \times 10^2$. Let make the free body diagram.



$$\text{Using equation } \delta l = \frac{1}{AE} (F_1 l_1 + F_2 l_2 + F_3 l_3)$$

$$\delta l = \frac{1}{10 \times 8 \times 10^2} (5 \times 60 + 2 \times 60 + 1 \times 120)$$

$$\delta l = 0.0675 \text{ cm}$$

9. (c) Let unstretched length = l

$$T_1 = k(l_1 - l) \Rightarrow T_2 = k(l_2 - l)$$

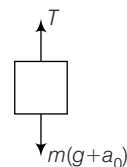
$$\frac{T_1}{T_2} = \frac{l_1 - l}{l_2 - l} \Rightarrow l = \left(\frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}\right)$$

10. (b) Here, $T = m(g + a_0) = 10(10 + 2) = 120 \text{ N}$

$$\therefore \text{Stress} = \frac{T}{A} = \frac{120}{2 \times 10^{-4}} = 60 \times 10^4 \text{ N/m}^2$$

$$\therefore Y = \frac{\text{stress}}{\text{strain}}$$

$$\therefore \text{Strain} = \frac{\text{stress}}{Y} = \frac{60 \times 10^4}{2 \times 10^{11}} = 30 \times 10^{-7} = 3 \times 10^{-6}$$



11. (a) $\Delta l = \frac{F}{A} \frac{l}{Y}$

$\frac{F}{A}$ is identical for both the wire.

$$\frac{\Delta l_1 (\text{copper})}{\Delta l_2 (\text{steel})} = \frac{l_1}{l_2} \cdot \frac{Y_2}{Y_1}$$

$$\Delta l_2 = \Delta l_1 \frac{Y_1}{Y_2} \cdot \frac{l_2}{l_1} = (1) \left(\frac{0.5}{1}\right) \left(\frac{1 \times 10^{11}}{2 \times 10^{11}}\right) = 0.25 \text{ mm}$$

$$\text{Total extension} = 1 + 0.25 = 1.125 \text{ mm}$$

12. (b)

$$13. (c) \Delta L = \frac{FL}{YA} = \frac{FL}{Y\sqrt{A_1 A_2}} = \frac{FL}{Y\sqrt{\pi r_1^2 \cdot \pi r_2^2}}, \Delta L = \frac{FL}{Y\pi r_1 r_2}$$

$$14. (a) \frac{Y}{3B} = 1 - 2\sigma$$

$$\text{and } 0 < \sigma < \frac{1}{2}$$

$$\Rightarrow 0 < 1 - 2\sigma < 1$$

$$\text{Hence, } \frac{Y}{3B} < 1 \therefore Y < 3B$$

$$15. (a) \text{ Bulk modulus, } B = \frac{\Delta p}{-\Delta V/V} \Rightarrow -\frac{\Delta V}{V} = \frac{0.01}{100} = 10^{-4}$$

$$\Delta p = \text{pressure of water} = H\rho g = (10)^3 \times (1 \times 10^3) \times (9.8)$$

$$\Delta p = 9.8 \times 10^6 \Rightarrow B = \frac{9.8 \times 10^6}{10^{-4}} = 9.8 \times 10^{10} \text{ N/m}^2$$

16. (b)

17. (a) $(\Delta l - w)$ graph is a straight line, where

$$\frac{w}{\Delta l} = \frac{(80 - 20)}{(4 - 1) \times 10^{-4}} = 2 \times 10^5 \text{ Nm}^{-2}$$

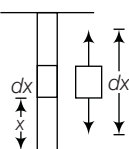
$$\text{Moreover, } L = 1 \text{ m and } A = 10^{-6} \text{ m}^2$$

$$\text{Hence, } Y = \frac{FL}{A\Delta l} = \frac{wL}{A\Delta l} = \frac{2 \times 10^5 \times 1}{10^{-6}} = 2 \times 10^{11} \text{ Nm}^{-2}$$

18. (c) Consider an element of wire of width dx at a distance x from bottom end of wire.

The force experienced by this element is due to the gravitational force of portion of wire lower to it.

$$\text{So, } Y = \frac{T/A}{\Delta(dx)/dx}, \text{ where } \Delta(dx) \text{ is the}$$



elongation in

this element.

$$\text{Now, } \Delta(dx) = \frac{T}{YA} dx = \frac{mg}{YA} \times x dx$$

$$\text{Total elongation, } \Delta l = \int_0^l \Delta(dx) = \int_0^l \frac{mg \times x dx}{YA} = \frac{mgl}{2YA}$$

19. (a) Breaking strength = tension in the wire = $mr\omega^2$

$$4.8 \times 10^7 \times 10^{-6} = 10 \times 0.3 \times \omega^2$$

$$\omega^2 = \frac{48}{0.3 \times 10} = 16 \Rightarrow \omega = 4 \text{ rad/s}$$

20. (a) Equivalent rubber constant of rubber cord

$$k = \frac{YA}{l} = \frac{(5 \times 10^8)(1 \times 10^{-6})}{(0.1)} = 5 \times 10^3 \text{ N/m}$$

From conservation of energy

$$\text{Elastic PE of cord} = \text{KE of missile } \frac{1}{2}k(\Delta l)^2 = \frac{1}{2}mv^2$$

$$v = \sqrt{\left(\frac{k}{m}\right) \cdot \Delta l} = \sqrt{\left(\frac{5 \times 10^3}{5 \times 10^{-3}}\right) \times (12 - 10) \times 10^{-2}}$$

$$v = 20 \text{ m/s}$$

21. (c) Elastic potential energy stored in the catapult

$$\left(U = \frac{1}{2}F\Delta L = \frac{1}{2}\frac{YA}{L}\Delta L^2\right) \text{ is converted into the kinetic}$$

$$\text{energy of the projectile } \left(K = \frac{1}{2}mv^2\right) \Rightarrow v = \sqrt{\frac{YA\Delta L^2}{Lm}}$$

$$= \left[\frac{(5 \times 10^8) \times (25 \times 10^{-6}) \times (5 \times 10^{-2})^2}{(10 \times 10^{-2}) \times (5 \times 10^{-3})}\right]^{1/2} = 250 \text{ ms}^{-1}$$

22. (a) For equilibrium of the boy,

$$\text{Tension in wire, } T = \text{weight of boy} = mg = 100 \text{ N}$$

$$= \frac{100 \times 10^6}{3.14} \text{ N/m}^2 \Rightarrow \text{Strain} = \frac{\Delta L}{L} = \frac{\Delta L}{2}$$

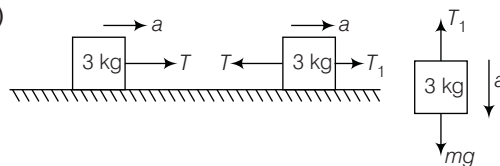
$$\therefore Y = \frac{\text{stress}}{\text{strain}} = \frac{100 \times 10^6 \times 2}{3.14 \times \Delta L}$$

$$\therefore \Delta L = \frac{100 \times 10^6 \times 2}{3.14 \times 2 \times 10^{11}} = 31.85 \times 10^{-5} \text{ m}$$

$$23. (c) \therefore \sigma = \frac{\Delta dL}{d\Delta L}$$

$$\therefore \Delta d = \frac{\sigma d\Delta L}{L} = \frac{0.1 \times 2 \times 10^{-3} \times 31.84 \times 10^{-5}}{2} = 3.184 \times 10^{-8} \text{ m}$$

24. (b)



From force diagram,

$$T = 3a \quad \dots (i)$$

$$T_1 - T = 3a \quad \dots (ii)$$

$$3g - T_1 = 3a \quad \dots (iii)$$

After solving Eqs. (i) and (ii), we get $T = 10 \text{ N}$

$$\therefore \text{Stress} = \frac{T}{A} = \frac{10}{0.005 \times 10^{-4}}$$

 $\therefore$ The elastic energy stored per unit volume

$$= \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times \text{stress} \times \frac{\text{stress}}{Y} \quad \left(\because Y = \frac{\text{stress}}{\text{strain}}\right)$$

$$= \frac{\text{stress}^2}{2Y} = \frac{\left(\frac{10}{0.005 \times 10^{-4}}\right)^2}{2 \times 2 \times 10^{11}}$$

$$= \frac{100}{25 \times 10^{-6} \times 10^{-8} \times 4 \times 10^{11}} = \frac{100}{100 \times 10^{-14} \times 10^{11}}$$

$$= 10^3 \text{ J/m}^3 = 1000 \text{ J/m}^3$$

25. (d)

26. (b)

27. (c)

$$28. (c) \text{ Energy stored} = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times \sigma \times \epsilon \times (AL)$$

$$\text{As, } Y = \frac{\sigma}{\epsilon} \Rightarrow \sigma = \epsilon Y$$

$$\text{So, } E = \frac{1}{2} \cdot \epsilon^2 Y \cdot (AL)$$

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1. (b) As stress is shown on x-axis and strain on y-axis.

So, we can say that $y = \cot \theta = \frac{1}{\tan \theta} = \frac{1}{\text{slope}}$

So, elasticity of wire P is minimum and R is maximum.

2. (d) As given for steel wire

$$A_1 = 3 \times 10^{-5} \text{ m}^2, l_1 = 4.7 \text{ m}, \Delta l_1 = \Delta l; F_1 = F$$

For copper wire,

$$A_2 = 4 \times 10^{-5} \text{ m}^2,$$

$$l_2 = 3.5 \text{ m}, \Delta l_2 = \Delta l, F_2 = F$$

Let Y_1 and Y_2 be the Young's modulus of steel wire and copper wire, respectively.

$$\text{So, } Y_1 = \frac{F_1}{A_1} \times \frac{l_1}{\Delta l_1} = \frac{F}{3 \times 10^{-5}} \times \frac{4.7}{\Delta l}$$

$$\text{and } Y_2 = \frac{F_2}{A_2} \times \frac{l_2}{\Delta l_2} = \frac{F \times 3.5}{4 \times 10^{-5} \times \Delta l}$$

$$\frac{Y_1}{Y_2} = \frac{4.7 \times 4 \times 10^{-5}}{3.5 \times 3 \times 10^{-5}} = 1.8$$

So, $Y_1 : Y_2 = 1.8 : 1$

3. (a) The pressure exerted by a 3000 m column of water on the bottom layer.

$$p = h\rho g = 3000 \times 1000 \times 10 \\ = 3 \times 10^7 \text{ kg m}^{-1}\text{s}^{-2} = 3 \times 10^7 \text{ N/m}^2$$

Fractional compression $\left(\frac{\Delta V}{V}\right)$

$$= \frac{\text{Stress}}{B} = \frac{3 \times 10^7}{2.2 \times 10^9} = 1.36 \times 10^{-2}$$

$$\frac{\Delta V}{V} \times 100 = 1.36\%$$

4. (b) As, $T = m(g + a_0) = 10(10 + 2) = 120 \text{ N}$

$$\text{Stress} = \frac{T}{A} = \frac{120}{2 \times 10^{-4}} = 60 \times 10^4 \text{ Nm}^{-2}$$

$$\text{and } Y = \frac{\text{Stress}}{\text{Strain}}, \text{ Strain} = \frac{\text{Stress}}{Y}$$

$$= \frac{60 \times 10^4}{2 \times 10^{11}}$$

$$= 30 \times 10^{-7} = 3 \times 10^{-6}$$

5. (c) $Y = \frac{FL}{A\Delta L}$

$$\text{or } F = \frac{YA\Delta L}{L} = \frac{YA^2\Delta L}{AL} = \frac{YA^2\Delta L}{V} = \frac{YA^2\Delta x}{V}$$

where, $AL = V = \text{volume of wire}$, Young modulus in the same as both the wires are made of same material. It is given that both the wires have same volume and same extension in length.

$$\therefore \frac{F'}{F} = \frac{A'^2}{A^2} = \frac{(3A)^2}{A^2} = 9 \\ F' = 9F$$

6. (d) Poisson's ratio = 0.5

Since, density is constant therefore change in volume is zero, we have

$$V = A \times l = \text{constant}$$

$$\log V = \log A + \log l$$

$$\text{or } \frac{dA}{A} + \frac{dl}{l} = 0$$

$$\frac{dl}{l} = -\frac{dA}{A}$$

$\therefore$ Percentage increase in length = 4%

7. (a) $Y = \frac{mgl}{A\Delta l}$

$$\Rightarrow \frac{\Delta l}{l} = \frac{mg}{AY}$$

$$\therefore \frac{\Delta l}{l} = \frac{1 \times 10}{3 \times 10^{-6} \times 10^{11}}$$

$$= 0.3 \times 10^{-4}$$

8. (c) Modulus of elasticity = $\frac{\text{Force}}{\text{Area}} \times \frac{l}{\Delta l}$

$$3 \times 10^{11} = \frac{33000}{10^{-3}} \times \frac{l}{\Delta l}$$

$$\frac{\Delta l}{l} = \frac{33000}{10^{-3}} \times \frac{1}{3 \times 10^{11}}$$

$$= 11 \times 10^{-5}$$

$$\text{Change in length, } \frac{\Delta l}{l} = \alpha \Delta T$$

$$11 \times 10^{-5} = 11 \times 10^{-5} \times \Delta T$$

$$\Rightarrow \Delta T = 10 \text{ K or } 10^\circ \text{ C}$$

9. (a) $Y = \frac{\text{stress}}{\text{strain}} = \text{N/m}^2$ or pascal (in SI system)

$$\text{and } Y = \frac{\text{dyne}}{\text{cm}^2} \quad (\text{in CGS System})$$

Thus, Nm^{-1} is not the unit of Young's modulus.

13

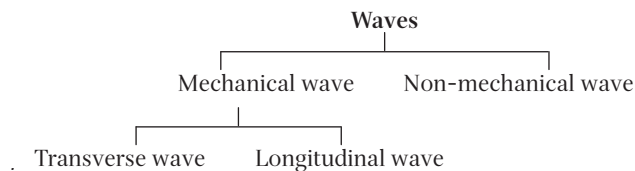
Wave Motion

Wave

In physics, a **wave** is a disturbance or oscillation that travels through space and matter, accompanied by a transfer of energy. In simple terms, we can say that wave motion involves transfer of disturbance (energy) from one point to the another with particles of medium oscillating about their mean positions, i.e. particles of the medium do not travel themselves along with the wave.

Types of Waves

Types of waves are shown as below



Mechanical Waves Waves which can be produced or propagated only in material medium are defined as mechanical waves, e.g. waves on water surface, waves on strings, sound waves, etc.

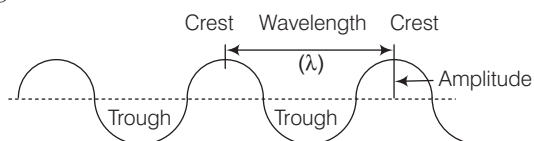
Non-mechanical Waves Waves which can be independent of propagation medium are defined as non-mechanical waves, e.g. visible light, ultraviolet light, X-rays, etc.

Transverse Waves

A wave motion in which the particles of the medium oscillate in simple harmonic motion in a direction perpendicular to its direction of propagation, are defined as **transverse waves**.

e.g. Movement of string of a sitar or violin. On propagation of transverse waves, medium is divided into alternate crests and troughs.

The distance between two consecutive crests or troughs is called wavelength of the transverse wave. It is denoted by λ .



Speed of Transverse Wave

The expression for speed of transverse waves in a solid and in case of a stretched string can be obtained theoretically given as

(i) In solids, $v = \sqrt{\frac{\eta}{d}}$

where, η is the modulus of rigidity and d is the density of the medium.

(ii) In a stretched string, $v = \sqrt{\frac{T}{m}} = \sqrt{\frac{Mg}{\pi r^2 d}}$

where, T = the tension in the string,

m = the mass per unit length of the string,

M = mass suspended from the string,

r = radius of the string and

d = density of the material of the string.

Progressive Waves

A wave which travels continuously in the same direction without any change in its amplitude is called a progressive wave or a travelling wave. A progressive wave may be transverse or longitudinal.

Displacement Relation for a Progressive Wave

- (i) If a plane wave travels in a medium along the positive x -direction, then the displacement y of a particle located at x at time t is given by

$$y(x, t) = A \sin(\omega t - kx)$$

where, A = amplitude of the wave

$$\omega = \frac{2\pi}{T} \text{ or } 2\pi\nu \text{ is angular frequency}$$

$$k = \left(\frac{2\pi}{\lambda}\right), \text{ i.e. angular wave number}$$

- (ii) If a wave is travelling along the negative x -direction, then

$$y(x, t) = A \sin(\omega t + kx)$$

- (iii) **Phase change with position** At any instant t , if ϕ_1 and ϕ_2 are the phase of two particles whose distance from origin are x_1 and x_2 .

$$\text{Then, } \phi_1 = (\omega t - kx_1) \text{ and } \phi_2 = (\omega t - kx_2)$$

$$\phi_1 - \phi_2 = k(x_2 - x_1)$$

$$\text{So, phase difference, } \Delta\phi = \frac{2\pi}{\lambda} \times \text{path difference } (\Delta x)$$

- (iv) **Phase change with time** If a particle at a distance x from origin has phase ϕ_1 at t_1 and ϕ_2 at t_2 .

$$\text{Then, } \phi_1 = (\omega t_1 - kx) \text{ and } \phi_2 = (\omega t_2 - kx)$$

$$\phi_1 - \phi_2 = \omega(t_1 - t_2)$$

$$\text{So, phase difference, } \Delta\phi = \frac{2\pi}{T} \times \text{time difference } (\Delta t)$$

- (v) A particle velocity at a given position at a given time is equal to product of wave velocity and negative of slope of the wave curve at the given position and time.

$$u(x, t) = \frac{d}{dt}[y(x, t)]$$

- (vi) Acceleration of a particle at (x, t) , $a = \frac{d^2 y}{dt^2}$

$$\text{Also, } |a_{\max}| = -\omega^2 A$$

- (vii) **Intensity** A flow of energy per unit area of cross-section of the string in unit time is defined as intensity.

$$\text{Thus, } I = \frac{\text{Power}}{\text{Area of cross-section}}$$

$$= \frac{P}{S} = \frac{1}{2} \rho \omega^2 A^2 v$$

where, ρ is density of string and v is speed of the wave.

NOTE From the formula for intensity, we can write $I \propto A^2$, where A is amplitude.

Principle of Superposition of Waves

When two or more than two waves of similar type propagate in a medium simultaneously, then resultant displacement of any particle of the medium is equal to the vector sum of displacements produced by individual waves separately. This principle is called principle of superposition of waves.

$$y = y_1 + y_2 + y_3 + \dots$$

Interference of Waves

When two waves of same frequency (or same wavelength) travelling along same path superimpose each other, there occurs redistribution of energy in the medium. If at a given position (x being constant) displacement due to two waves be

$$y_1 = A_1 \sin \omega t \text{ and } y_2 = A_2 \sin(\omega t + \phi)$$

Then, resultant displacement

$$y = y_1 + y_2 = A \sin(\omega t + \phi)$$

where,

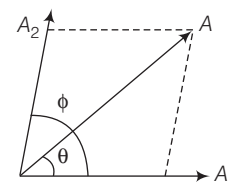
$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \phi}$$

and

$$\tan \theta = \frac{A_2 \sin \phi}{A_1 + A_2 \cos \phi}$$

At those points where phase difference $\phi = 0^\circ$ or $2n\pi$, i.e. an integer multiple of 2π ($n = 1, 2, 3, \dots$),

$$A = A_1 + A_2 = A_{\max}$$



This is known as **constructive interference**.

At those points where $\phi = (2n - 1)\pi$, i.e. phase difference is an odd multiple of π and hence,

$$A = A_1 \sim A_2 = A_{\min}$$

This is known as **destructive interference**.

If I_1 and I_2 are intensities of the interfering waves and ϕ is the phase difference, then resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2 \text{ for } \phi = 2\pi n$$

and $I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

$$I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2 \text{ for } \phi = (2n + 1)\pi$$

Stationary or Standing Waves in String

When two sets of progressive waves of the same type having same amplitude, same frequency and same speed along the same straight line in opposite directions superimposed, then a new wave is formed. This is called stationary wave or standing wave. Consider two waves of the same frequency, speed and amplitude, which are travelling in opposite directions along a string. Two such waves may be represented by the equations

$$y_1 = A \sin(kx - \omega t) \text{ and } y_2 = A \sin(kx + \omega t)$$

Hence, the resultant may be written as

$$y = y_1 + y_2 = A \sin(kx - \omega t) + A \sin(kx + \omega t)$$

$$y = 2A \sin kx \cos \omega t$$

This is the equation of a standing wave.

Characteristics of Stationary Waves

- (i) There are certain points in the medium in a standing wave, which are permanently at rest. These are called as nodes. Distance between two consecutive nodes is $\frac{\lambda}{2}$.
- (ii) Some points in the medium in a standing wave, have maximum amplitude. These are called as ant-nodes. Distance between two consecutive ant-nodes is $\frac{\lambda}{2}$.

Laws of Vibrations of Stretched Strings

- (i) **Law of length** If T and m are constants, then fundamental frequency of vibration of stretched string (f) is inversely proportional to the length (l) of the string.

i.e. $f \propto \frac{1}{l}$

- (ii) **Law of tension** If l and m are constants, then frequency of vibration of stretched string (f) is directly proportional to the square root of tension (T) of the string.

i.e. $f \propto \sqrt{T}$

- (iii) **Law of mass** If l and T are constants, then frequency of vibration of stretched string (f) is inversely proportional to the square root of mass per unit length of wire.

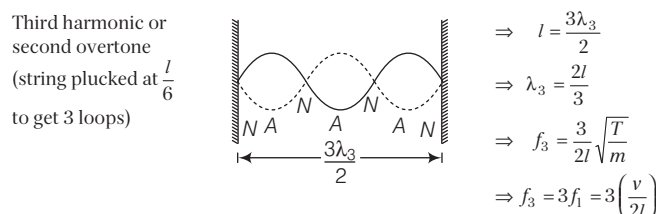
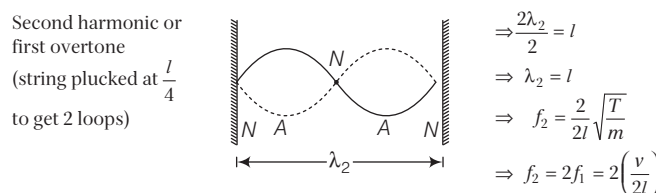
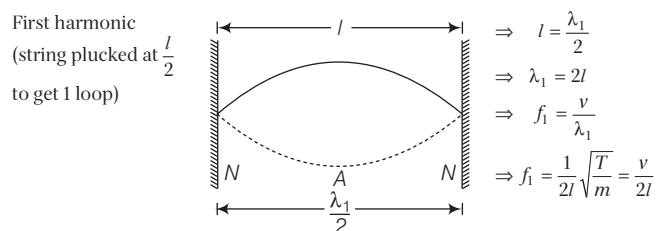
i.e. $f \propto \frac{1}{\sqrt{m}}$

- (iv) **Law of density** If l, T and D are constants, then frequency of vibration of stretched string (f) is inversely proportional to the square root of the density of wire.

$$\left[\because f = \frac{n}{2l} \sqrt{\frac{T}{m}} = \frac{n}{2l} \sqrt{\frac{TA}{MA}} = \frac{n}{2l} \sqrt{\frac{4T}{\rho D^2 \pi}} \right], \text{ i.e. } f \propto \frac{1}{\sqrt{\rho}}$$

Fundamental Mode and Harmonics

Normal modes of vibration in a string fixed at both ends are discussed as below



Note (i) For string fixed at both ends, the waves produced having all harmonics. (ii) For a string fixed at only one end antinode will be formed at free end and node at fixed end, only odd harmonics are present.

n th harmonic
 $f_n = n f_1 \Rightarrow \lambda_n = \frac{2l}{n}$

Practice Exercise

1. The equation of a wave travelling on a stretched string along the x -axis is $y = ae^{-(bx + ct)}$. The direction of propagation of wave is

a. along negative y -axis b. along positive y -axis
c. along negative x -axis d. along positive x -axis

2. In (Q. 1.) the maximum displacement of particle of string is

a. a b. b c. c d. c/b

3. In (Q. 1.) the speed of wave is

a. c/b b. b/c c. a d. c

4. The wave travels along a string whose equation is

$$y = \frac{p^3}{p^2 + (px - qt)^2}, \text{ where } p = 2 \text{ unit and } q = 0.5 \text{ unit.}$$

Find the direction of propagation of wave.

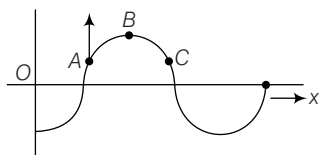
a. along $+y$ -axis b. along $-x$ -axis
c. along $+x$ -axis d. None of these

5. If wave $y = A \cos(\omega t + kx)$ is moving along x -axis.

The shape of pulse at $t = 0$ and $t = 2$ s

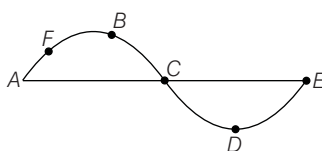
a. are different b. are same
c. may not be same d. None of these

6. At any instant a wave travelling along the string shown in figure. Here, point A is moving upward. Which of the following statement is true?



a. The wave is travelling to the right
b. The displacement amplitude of wave is equal to displacement of B at this instant
c. At this instant C also directed upward.
d. None of the above

7. In the given figure,



a. the velocity of particles B and C are same
b. the velocity of particles A, C and E are maximum
c. the particle F moves upward
d. all particles have same velocity

8. If a wave propagates through a medium. Then, the velocity of particle of medium is given by

a. wave velocity $\times$ strain b. $\frac{\text{wave velocity}}{\text{strain}}$
c. wave velocity d. $\frac{\text{angular frequency}}{\text{propagation constant}}$

9. In a wave motion $y = a \sin(kx - \omega t)$, y can represent

a. electric field b. magnetic field
c. displacement, pressure d. All of these

10. The equation of a wave travelling on a stretched string

$$\text{is } y = 4 \sin 2\pi \left(\frac{t}{0.02} - \frac{x}{100} \right)$$

Here, x and y are in cm and t is in second. The speed of wave is

a. 50 m/s b. 40 m/s c. 50 cm/s d. 40 cm/s

11. In (Q. 10.) the ratio of particle velocity amplitude and wavelength is

a. π b. 2π c. 3π d. 4π

12. In (Q. 10.) the relative deformation amplitude of medium is

a. 0.02π b. 0.08π
c. 0.06π d. None of these

13. Along a stretched string equation of transverse wave

$$\text{is } y = 3 \sin \left[2\pi \left(\frac{x}{20} - \frac{t}{0.01} \right) \right]$$

where, x, y are in cm and t is in second. Calculate wave velocity.

a. 20 m/s b. 30 m/s c. 15 m/s d. 25 m/s

14. A transverse wave along a string is given by

$$y = 2 \sin \left(2\pi (3t - x) + \frac{\pi}{4} \right)$$

where x and y are in cm and t in second. Find acceleration of a particle located at $x = 4$ cm at $t = 1$ s.

a. $36\sqrt{2}\pi^2 \text{ cm/s}^2$ b. $36\pi^2 \text{ cm/s}^2$
c. $-36\sqrt{2}\pi^2 \text{ cm/s}^2$ d. $-36\pi^2 \text{ cm/s}^2$

15. If $y = y_0 \sin 2\pi \left(ft - \frac{x}{\lambda} \right)$ is the equation of transverse

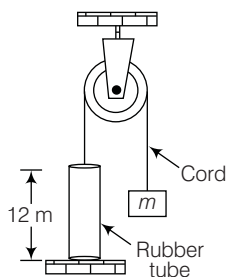
wave, then for what value of λ the maximum particle velocity is equal to four times the wave velocity?

a. $y_0\pi$ b. $\frac{y_0\pi}{2}$ c. $2y_0\pi$ d. $1.5y_0\pi$

16. Along a stretched wire a transverse wave passes with speed 3000 m/s. If the tension in the wire increased four times, then the velocity of the wave is

a. 1500 m/s b. 3000 m/s
c. 6000 m/s d. 9000 m/s

17. A long rubber tube having mass 0.9 kg is fastened to a fixed support and the free end of the tube is attached to a cord which passes over a pulley and supports an object, with a mass of 5 kg as shown in figure. If the tube is struck by a transverse blow at one end, find the time required for the pulse to reach the other end.



- a. 5 s b. 0.47 s c. 4.7 s d. 3.2 s
18. The time taken by a transverse wave going on a wire having mass 5 g, from one end to another end of wire is 0.5 s. The area of cross-section of wire is 1 mm^2 and Young's modulus of elasticity is $16 \times 10^{11} \text{ N/m}^2$. The speed of wave is 80 m/s. The strain in wire is
- a. 5×10^{-7} b. 2×10^{-7} c. 3×10^{-6} d. 4×10^{-6}
19. Find the time taken by a transverse wave to travel the full length of a uniform rope of mass 0.1 kg and length 2.45 m hangs from the ceiling
- a. 1 s b. 0.5 s c. 2 s d. 1.5 s
20. A transverse wave of equation $y = 2 \sin(0.01x + 30t)$ moves on a stretched string from one end to another end. In the equation of wave, x and y are in cm and t is in second. The time taken by wave to reach from one end to another end of string is 5 s. The length of string is
- a. 10 m b. 100 m c. 150 m d. 160 m
21. A sinusoidal wave travelling in the same direction have amplitudes of 3 cm and 4 cm and difference in phase by $\pi/2$. The resultant amplitude of the superimposed wave is
- a. 7 cm b. 5 cm
c. 2 cm d. 0.5 cm
22. Two simple harmonic motions are represented by the equations $y_1 = 10 \sin\left(3\pi t + \frac{\pi}{4}\right)$ and $y_2 = 5(3 \sin 3\pi t + \sqrt{3} \cos 3\pi t)$. Their amplitudes are in the ratio of
- a. $\sqrt{3}$ b. $1/\sqrt{3}$ c. 2 d. 1/6
23. Predict for the wave $y = A \cos \frac{2\pi x}{\lambda} \sin\left(\frac{2\pi vt}{\lambda}\right)$
- a. It is a progressive wave
b. It is a transverse progressive wave
c. It is a longitudinal progressive wave
d. It is a stationary wave
24. A string of length l is fixed at both ends and is vibrating in second harmonic. The amplitude at anti-node is 2 mm. The amplitude of a particle at distance $l/8$ from the fixed end is
- a. $5\sqrt{2} \text{ mm}$ b. $\frac{5}{\sqrt{2}} \text{ mm}$ c. 5 mm d. $\frac{10}{\sqrt{2}} \text{ mm}$

25. In (Q. 24.) the tension in string is T and the linear mass density of string is μ . The ratio of magnitude of maximum velocity of particle and the magnitude of maximum acceleration is

a. $\frac{1}{2\pi} \sqrt{\frac{\mu l^2}{T}}$ b. $2\pi \sqrt{\frac{\mu l^2}{T}}$ c. $\frac{1}{2\pi} \sqrt{\frac{T}{\mu l^2}}$ d. $\frac{1}{4\pi} \sqrt{\frac{\mu l^2}{T}}$

26. In (Q. 24.) if at $t = 0$, $y = 2.5 \text{ mm}$, the equation of standing wave is

a. $(2.5 \text{ mm}) \sin \frac{2\pi}{l} x \cos \left(2\pi \sqrt{\frac{T}{\mu l^2}} t\right)$

b. $5 \text{ mm} \sin \frac{\pi}{l} x \cos 2\pi t$

c. $5 \text{ mm} \sin \frac{2\pi}{l} x \cos \left(2\pi \sqrt{\frac{T}{\mu l^2}} t\right)$

d. $5 \text{ mm} \cos \frac{2\pi}{l} x \cos \left(2\pi \sqrt{\frac{T}{\mu l^2}} t\right)$

27. If a string fixed at both ends vibrates in four loops. The wavelength is 10 cm. The length of string is

a. 5 cm b. 15 cm c. 30 cm d. None

28. In (Q. 27.) the distance of plucking point from the fixed end is

a. 5 cm b. 10 cm c. 2.5 cm d. 7.5 cm

29. A stretched wire carries a body of density $\sigma = 8000 \text{ kg/m}^3$ at its end. The fundamental frequency of vibration of wire is 280 Hz. The body is dipped completely in a vessel of water. Find the new frequency of fundamental mode of vibrations. (Density of water is $\rho = 1000 \text{ kg/m}^3$)

a. 262 Hz b. 260 Hz c. 243.2 Hz d. 255.5 Hz

30. An elastic string of length 2 m is fixed at its end. The string starts to vibrate in third overtone with a frequency 1200 Hz. The ratio of frequency of lower overtone and fundamental is

a. 1 b. 2 c. 3 d. 4

31. If a string is stretched with a weight 4 kg then the fundamental frequency is equal to 256 Hz. What weight is needed to produce its octave?

a. 4 kg-wt b. 12 kg-wt c. 16 kg-wt d. 24 kg-wt

32. Calculate the minimum possible length of the string when both ends of string are fixed and has consecutive standing wave mode for which distance between adjacent nodes are 18 cm and 16 cm respectively.

a. 150 cm b. 144 cm c. 140 cm d. 142 cm

33. A sonometer wire of length 114 cm is fixed at both the ends. Where should the two bridges be placed so as to divide the wire into three segments whose fundamental frequencies are in the ratio 1:3:4?

a. At 36 cm and 84 cm from one end
b. At 24 cm and 72 cm from one end
c. At 48 cm and 96 cm from one end
d. At 72 cm and 96 cm from one end

34. A wave of angular frequency ω propagates so that a certain phase of oscillation moves along x -axis, y -axis, z -axis with speeds c_1 , c_2 and c_3 respectively. Find the propagation constant k .

a. $\frac{\omega}{\sqrt{c_1^2 + c_2^2 + c_3^2}} (\hat{i} + \hat{j} + \hat{k})$ b. $\frac{\omega}{c_1} \hat{i} + \frac{\omega}{c_2} \hat{j} + \frac{\omega}{c_3} \hat{k}$
 c. $(\omega \hat{i} + \omega \hat{j} + \omega \hat{k}) \frac{1}{c}$ d. None of these

35. Wave of frequency 500 Hz has a phase velocity 360 m/s. The phase difference between two displacement at a certain point at time 10^{-3} s apart will be

a. π rad b. $\frac{\pi}{2}$ rad c. $\frac{\pi}{4}$ rad d. 2π rad

36. Equation of a plane wave is given by $4 \sin \frac{\pi}{4} \left[2t + \frac{x}{8} \right]$.

The phase difference at any given instant of two particles 16 cm apart is

a. 60° b. 90° c. 30° d. 120°

37. The transverse displacement $y(x, t)$ of a wave on a string is given by $y(x, t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}xt)}$. This represents a

a. wave moving in $-x$ -direction with speed $\sqrt{\frac{b}{a}}$
 b. standing wave of frequency $\sqrt{b}$
 c. standing wave of frequency $\frac{1}{\sqrt{b}}$
 d. wave moving in $+x$ -direction with speed $\sqrt{a/b}$

38. For a wave $y = 0.0002 \sin \left[2\pi \left(110t - \frac{x}{3} \right) + \frac{\pi}{3} \right]$

is travelling in a medium. Find energy per unit volume being transferred by wave if density of medium is 1.5 kg/m^3 .

a. $14 \times 10^{-4} \text{ J/m}^3$
 b. $143.2 \times 10^{-4} \text{ J/m}^3$
 c. $14.3 \times 10^{-4} \text{ J/m}^3$
 d. $143 \times 10^{-4} \text{ J/m}^3$

BITSAT Archives

1. The equation of a simple harmonic motion is given by $y = 3 \sin \frac{\pi}{2} (50t - x)$, where x and y are in metres and

t is in seconds, the ratio of maximum particle velocity to the wave velocity is [2014]

a. 25 m/s b. 30 m/s
 c. 50 m/s d. None of these

2. A stretched string of length 2 m vibrates in 4 segments. The distance between consecutive nodes is [2010]

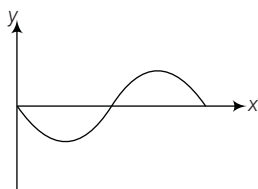
a. 0.5 m b. 0.25 m c. 1.0 m d. 0.75 m

3. At $t = 0$, a transverse wave pulse in a wire is described by the function $y = \frac{6}{x^2 - 3}$, where x and y are in

metre. The function $y(x, t)$ that describes this wave equation, if it is travelling in the positive x -direction with a speed of 4.5 ms^{-1} is [2010]

a. $y = \frac{6}{(x + 4.5t)^3 - 3}$ b. $y = \frac{6}{(x - 4.5t^2) + 3}$
 c. $y = \frac{6}{(x + 4.5t)^2 - 3}$ d. $y = \frac{6}{(x - 4.5t)^2 - 3}$

4. In a sine wave, position of different particles at time $t = 0$ is shown in figure. The equation for this wave travelling along positive x -axis can be [2009]



a. $y = A \sin(\omega t - kx)$ b. $y = A \cos(kx - \omega t)$
 c. $y = A \cos(\omega t - kx)$ d. $y = A \sin(kx - \omega t)$

5. A wave has velocity v in medium P and velocity $2v$ in medium Q . If the wave is incident in medium P at an angle of 30° , then the angle of refraction will be [2007]

a. 30°
 b. 45°
 c. 60°
 d. 90°

6. The equation of progressive wave is $y = 0.2 \sin 2\pi \left[\frac{t}{0.01} - \frac{x}{0.3} \right]$, where x and y are in metre and t is in second. The velocity of propagation of the wave is [2007]

a. 30 m/s
 b. 40 m/s
 c. 300 m/s
 d. 400 m/s

7. Two progressive waves having equation $x_1 = 3 \sin \omega t$ and $x_2 = 4 \sin(\omega t + 90^\circ)$ are superimposed. The amplitude of the resultant wave is [2006]

a. 5 unit
 b. 1 unit
 c. 3 unit
 d. 4 unit

8. A particle on the trough of a wave at any instant will come to the mean position after a time (T = time period) [2005]

a. $T/2$ b. $T/4$ c. T d. $2T$

Answer with Solutions

Practice Exercise

1. (c)

2. (a) $\therefore y = ae^{-(bx + ct)}$

At $t = 0, y = ae^{-bx}$

The value of y is $y_{\max} = a$.

3. (a) $\therefore bx + ct = \text{constant}$

$$\therefore b \frac{dx}{dt} + c \frac{dt}{dt} = 0$$

$$\therefore \text{Wave speed} = \left| \frac{dx}{dt} \right| = \frac{c}{b}$$

4. (c) At $t = 0, y = \frac{p^3}{(p^2 + p^2 x^2)} = \frac{p}{1 + x^2}$

$$y = f(px - qt)$$

$$y = f(\omega t - kx)$$

Here, sign of f and x are opposite, so direction should be + x -axis.

5. (b)

6. (b)

7. (b)

8. (a) Strain = $\frac{\partial y}{\partial x}$,

$$\text{As we know, } v_{\text{particle}} = -c \frac{\partial y}{\partial x}$$

$$= \text{wave velocity} \times \text{strain}$$

9. (d)

10. (a)

11. (d) We know that,

$$k = \frac{2\pi}{100}$$

$$\text{or } \frac{2\pi}{\lambda} = \frac{2\pi}{100}$$

$$\therefore \lambda = 100 \text{ cm}$$

$$\text{But } v_{\text{Particle}} = \frac{\partial y}{\partial t} = a\omega \cos(\omega t - kx)$$

$$(v_{\text{Particle}})_{\max} = a\omega = 4 \times \frac{2\pi}{0.02} = 400\pi \text{ cm/s}$$

$$\therefore \frac{(v_{\text{Particle}})_{\max}}{\lambda} = \frac{400\pi}{100} = 4\pi$$

12. (b) The relative deformation is

$$\frac{\partial y}{\partial x} = \frac{\partial}{\partial x} [4 \sin(\omega t - kx)]$$

$$= -4k \cos(\omega t - kx)$$

The relative deformation amplitude

$$\left(\frac{\partial y}{\partial x} \right)_{\max} = ak = 4 \times \frac{2\pi}{100} = 0.08\pi$$

13. (a) Let wave velocity = v

$$v = \lambda f$$

The given equation of wave

$$y = 3 \sin \left[2\pi \left(\frac{x}{20} - \frac{t}{0.01} \right) \right]$$

Compare the given equation with standard equation

$$y = A \sin \left[2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) \right]$$

We get $A = 3 \text{ cm}$

$$\lambda = 20 \text{ cm}$$

$$f = \frac{1}{T} = 100 \text{ Hz}$$

$$v = \lambda f = \frac{20 \times 100}{100} = 20 \text{ m/s}$$

14. (c)

15. (b) $\omega = 2\pi f$ and $v_{\max} = \omega r = 2\pi f y_0$

$$\text{Wave velocity} = f\lambda$$

$$2\pi f y_0 = 4f\lambda$$

(Given, velocity of particle is four times of wave velocity)

$$\lambda = \frac{y_0 \pi}{2}$$

16. (c) From relation, $v = \sqrt{\left(\frac{T}{m} \right)} = 3000 \text{ m/s}$

$$v' = \sqrt{\left(\frac{4T}{m} \right)} = 2 \times \sqrt{\left(\frac{T}{m} \right)}$$

$$v' = 2 \times 3000 = 6000 \text{ m/s}$$

17. (b) Let μ = mass per unit length of rubber tube

$$\mu = \frac{0.9}{12} = 0.075 \text{ kg/m}$$

$$\text{Tension in tube, } T = mg = 5 \times 9.8 = 49 \text{ N}$$

Speed of wave on the tube

$$v = \sqrt{\left(\frac{T}{\mu} \right)} = \sqrt{\frac{49}{0.075}} = 25.56 \text{ m/s}$$

$$\text{Time required} = t = \frac{AB}{v} = \frac{12}{25.56} = 0.47 \text{ s}$$

18. (a) $c = \sqrt{\left(\frac{T}{\mu} \right)}$

$$\therefore T = \mu c^2, y = \frac{T}{A \text{ strain}}$$

$$\therefore \text{Strain} = \frac{T}{AY} = \frac{\mu c^2}{AY} = \frac{mc^2}{lAY}$$

$$= \frac{mc^2}{(ct)AY} = \frac{mc}{AYt} = \frac{5 \times 10^{-3} \times 80}{10^{-6} \times 16 \times 10^{11} \times 0.5} = 5 \times 10^{-7}$$

19. (a) Speed of transverse wave at lower end = 0.

$$\text{Speed of transverse wave at the upper end} = \sqrt{lg}$$

Let, acceleration = a , time to travel = t

$$a = \left(\frac{\sqrt{gl}}{t} \right) \therefore s = \frac{1}{2} at^2$$

$$l = \frac{1}{2} \left(\frac{\sqrt{gl}}{t} \right) t^2$$

$$t = 2 \sqrt{\left(\frac{l}{g} \right)} = 2 \sqrt{\left(\frac{2.45}{9.8} \right)}$$

$$t = 1 \text{ s}$$

20. (c)

21. (b)

22. (b)

23. (d)

24. (b) The equation of stationary wave on a string fixed at both ends is

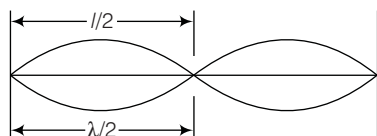
$$y = 2a \sin kx \cos \omega t$$

 The amplitude of a particle at distance x from fixed end is

$$A = 2a \sin kx.$$

For second harmonic, string vibrates in two loops (shown in figure).

$$\therefore \frac{\lambda}{2} = \frac{l}{2}$$



$$\therefore \lambda = l$$

$$\therefore k = \frac{2\pi}{\lambda} = \frac{2\pi}{l}$$

$$\therefore A = 2a \sin kx = 2a \sin \frac{2\pi}{l} \times \frac{l}{8} = 2a \sin \frac{\pi}{4}$$

 The amplitude at antinode is $2a = 5 \text{ mm}$

$$\therefore A = 5 \sin \frac{\pi}{4} = \frac{5}{\sqrt{2}} \text{ mm}$$

 25. (a) Here, $v_{\max} = A\omega = 2a\omega$ (at antinode $A = 2a$) and acceleration is

$$f_{\max} = \omega^2 A = 2a\omega^2$$

$$\therefore \text{Ratio} = \frac{2a\omega}{2a\omega^2} = \frac{1}{\omega}$$

$$\therefore \text{Frequency, } \nu = \frac{n}{2l} \sqrt{\left(\frac{T}{\mu} \right)} = \frac{2}{2l} \sqrt{\left(\frac{T}{\mu} \right)}$$

 (for second harmonic, $n = 2$)

$$= \sqrt{\left(\frac{T}{\mu l^2} \right)}$$

$$\therefore \omega = 2\pi\nu$$

$$\therefore \text{Ratio} = \frac{1}{2\pi\nu}$$

$$= \frac{1}{2\pi} \sqrt{\left(\frac{\mu l^2}{T} \right)}$$

26. (c) The equation of standing wave on a string fixed both ends is

$$y = 2a \sin kx \cos \omega t$$

Here,

$$2a = 5 \text{ mm}$$

$$k = \frac{2\pi}{l}$$

$$\omega = 2\pi\nu = 2\pi \sqrt{\left(\frac{T}{\mu l^2} \right)}$$

$$\therefore y = (5 \text{ mm}) \sin \frac{2\pi}{l} x \cos 2\pi \sqrt{\left(\frac{T}{\mu l^2} \right)} t$$

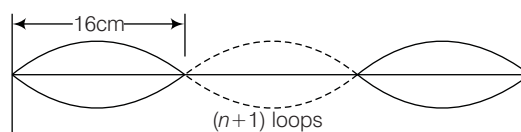
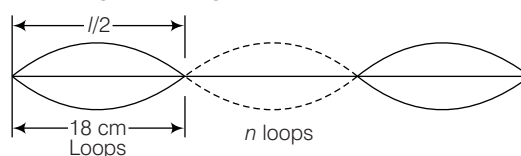
27. (b)

28. (c)

29. (c)

30. (b)

31. (c) To produce an octave, force should be 4 times the initial value as octave has double frequency.

 32. (b) Let length of string = l


$$18n = l$$

$$16(n+1) = l$$

$$n = 8$$

$$l = 144 \text{ cm}$$

33. (c)

34. (b) Propagation constant is a vector quantity.

$$\therefore \mathbf{k} = k_x \hat{i} + k_y \hat{j} + k_z \hat{k}$$

Here,

$$k_x = \frac{\omega}{v_x} = \frac{\omega}{c_1}$$

$$k_y = \frac{\omega}{v_y} = \frac{\omega}{c_2}$$

and

$$k_z = \frac{\omega}{v_z} = \frac{\omega}{c_3}$$

$$\therefore \mathbf{k} = \frac{\omega}{c_1} \hat{i} + \frac{\omega}{c_2} \hat{j} + \frac{\omega}{c_3} \hat{k}$$

 35. (a) $\Delta x = ct$

... (i)

$$\therefore \Delta\phi = \frac{2\pi}{\lambda} \Delta x = \frac{2\pi}{c/f} \Delta x$$

$$= \frac{2\pi ct}{c/f} = 2\pi ft$$

[from Eq. (i)]

$$= 2\pi \times 500 \times 10^{-3}$$

$$= \pi \text{ rad}$$

$$36. (b) y = 4 \sin \left(2t + \frac{x}{8} \right)$$

... (i)

$$\text{or } y = 4 \sin \left(\frac{\pi}{2} t + \frac{\pi x}{32} \right)$$

... (ii)

The standard equation is

$$y = 4 \sin (\omega t \pm kx)$$

Comparing the Eq. (i) with Eq. (ii), we get

$$\begin{aligned}
 k &= \frac{\pi}{32} \\
 \therefore \frac{2\pi}{\lambda} &= \frac{\pi}{32} \\
 \therefore \frac{\lambda}{2} &= 32 \\
 \therefore \lambda &= 64 \\
 \therefore \Delta\phi &= \frac{2\pi}{\lambda} \Delta x \\
 &= \frac{2\pi}{64} \times 16 = \frac{\pi}{2} = 90^\circ
 \end{aligned}$$

BITSAT Archives

1. (c) The given wave equation is

$$\begin{aligned}
 y &= 3 \sin \frac{\pi}{2} (50t - x) \\
 \Rightarrow y &= 3 \sin \left(25\pi t - \frac{\pi}{2} x \right) \quad \dots(i)
 \end{aligned}$$

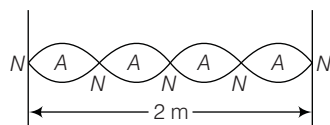
Comparing with standard equation

$$\begin{aligned}
 y &= a \sin(\omega t - kx) \quad \dots(ii) \\
 \omega &= 25\pi, k = \pi/2
 \end{aligned}$$

Wave velocity,

$$v = \frac{\omega}{k} = \frac{25\pi}{\pi/2} = 50 \text{ m/s}$$

2. (a) Since, it vibrates in 4 segments, so we must get 5 nodes and 4 antinodes.



$$\Rightarrow 2 = 2\lambda \Rightarrow \lambda = 1 \text{ m}$$

Distance between two consecutive nodes is

$$\frac{\lambda}{2} = \frac{1}{2} = 0.5 \text{ m}$$

3. (c) $y(x, t) = \frac{a}{(x \pm vt)^2 + b}$ is another form of progressive

wave equation propagating with a speed v . Negative sign to be taken for propagation along $+x$ -axis and positive sign to be taken for propagation along $-x$ -axis.

4. (d) It is clear from figure that at $t = 0$, $x = 0$ displacement $y = 0$. Therefore, option (a) or (b) may be correct.

In case of (d), $y = A \sin(kx - \omega t)$

$$\frac{dy}{dt} = A \cos(kx - \omega t)(-\omega)$$

$$\frac{dy}{dx} = A \cos(kx - \omega t)(k)$$

$$37. (a) y(x, t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}xt)} = e^{-(\sqrt{a}x + \sqrt{b}t)^2}$$

It is a function of type

$$y = f(\omega t + kx)$$

$\therefore y(x, t)$ represents wave travelling along $-x$ -direction.

$$\text{Speed of wave} = \frac{\omega}{k} = \frac{\sqrt{b}}{\sqrt{a}} = \sqrt{\frac{b}{a}}$$

$$\begin{aligned}
 38. (b) \text{ We know } E_0 &= 2\pi^2 \rho A^2 v^2 = \frac{1}{2} \rho A^2 \omega^2 \\
 &= \frac{1}{2} \times 15 \times (0.02)^2 (220\pi)^2 \times 10^{-4} \\
 E_0 &= 143.2 \times 10^{-4} \text{ J/m}^3
 \end{aligned}$$

$$\therefore \frac{dy/dt}{dy/dx} = \frac{-\omega A \cos(kx - \omega t)}{kA \cos(kx - \omega t)}$$

$$\frac{dx}{dt} = -\frac{\omega}{k} = -v$$

$$\therefore \frac{dy}{dt} = -v \left(\frac{dy}{dx} \right)$$

i.e. particle velocity = $-(\text{wave speed}) \times \text{slope}$.

And slope at $x = 0$ and $t = 0$ is positive in figure. Therefore, particle velocity is in negative y -direction. Therefore, (d) is correct option.

$$5. (a) v = \frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

$$\Rightarrow \sin r = \sin 30^\circ \times \frac{2v}{v}$$

$$\Rightarrow \sin r = \frac{1}{2} \times 2 \times 1$$

$$\Rightarrow r = 90^\circ$$

$$6. (d) v = \frac{\text{Coefficient of } t}{\text{Coefficient of } x} = \frac{2\pi / 0.01}{2\pi / 0.3} = 30 \text{ m/s}$$

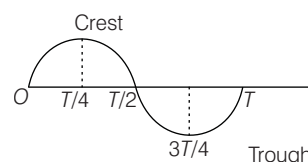
$$7. (d) x_1 = 2 \sin \omega t \text{ and } x_2 = 4 \sin(\omega t + 90^\circ)$$

The phase difference between the two waves is 90° .

So, resultant amplitude

$$\begin{aligned}
 a &= \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} \\
 &= 5 \text{ unit}
 \end{aligned}$$

8. (b) The time taken by the particle to come to mean position from the trough = $\frac{T}{4}$



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Sound Wave

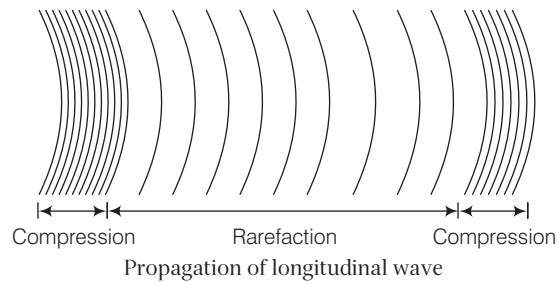
Introduction

Sound may be defined as the physical cause which enables us to have the sensation of hearing. Sound wave is a longitudinal wave.

Longitudinal Waves

In this wave motion, the particles of the medium oscillate about their mean or equilibrium position along the direction of propagation of the wave motion itself. Longitudinal wave travels in the form of compression and rarefactions.

- **Compression** It is a region of the medium in which particles are compressed (i.e. come closer), so distance between particles become less than the normal distance between them.
- **Rarefaction** It is a region of the medium in which particles of the medium are rarefied, i.e. particles get farther apart than the normal distance between them.



Speed of Longitudinal Wave (or Sound Wave)

According to Newton's formula, speed of sound in a gas is

(i) Newton's Formula

$v = \sqrt{\frac{B}{D}}$, where B denotes bulk modulus of elasticity and D denotes density of medium.

(ii) Laplace's Correction

For gases, E = coefficient of adiabatic elasticity.

$$E = \gamma p$$

Here, p is the pressure of the gas.

$$v = \sqrt{\frac{\gamma p}{D}}, \text{ where } \gamma \text{ denotes adiabatic constant} = \frac{C_p}{C_v}$$

(iii) Effect of Temperature on Velocity

With rise in temperature then velocity of sound increases

$$\text{as } v = \sqrt{\frac{\gamma RT}{M}}; \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\text{i.e. } v \propto \sqrt{T}$$

(iv) Effect of Pressure for Gases Medium

$\frac{P}{D}$ remains constant. Pressure has no effect on the velocity of sound, provide temperature remains constant.

(v) Effect of Humidity

When humidity in air increases, its density decreases and so velocity of sound increases.

$$\text{For solids, } v = \sqrt{\frac{Y}{D}}. \text{ For liquids, } v = \sqrt{\frac{K}{D}}$$

where, Y = Young's modulus of elasticity

K = bulk modulus of elasticity.

(vi) Effect of Wind Velocity

The velocity of sound in a particular direction is the algebraic sum of the velocity of sound and the component of wind velocity.

NOTE The density of a gas mixture can be calculated as

$$\rho_m = \frac{\rho_1 V_1 + \rho_2 V_2 + \dots + \rho_n V_n}{V_1 + V_2 + \dots + V_n}$$

where, ρ_m = Density of the gas mixture

$\rho_1 \dots \rho_n$ = Density of each of the components

$V_1, V_2 \dots V_n$ = Volume share of each of the components

Sound Waves

Mechanical waves in air having a frequency ranging from 20 Hz to 20 kHz are known as **audio waves** or sound waves. Waves having frequencies less than 20 Hz are known as the infrasonic waves. Waves in air having frequencies greater than 20 kHz are known as **ultrasonic waves**.

Sound waves do not undergo polarisation. Transverse waves only get polarised.

Sound exhibits reflection, refraction, interference and diffraction but not polarisation. A person can emit 5 syllables in 1s. Each syllable is produced in 0.2 s.

Relation between Phase Difference, Path Difference and Time Difference

$$\text{Phase difference } (\phi) = \frac{2\pi}{\lambda} \times \text{path difference } (x)$$

$$\Rightarrow \phi = \frac{2\pi x}{\lambda} \Rightarrow x = \frac{\phi \lambda}{2\pi}$$

$$\text{Phase difference } (\phi) = \frac{2\pi}{T} \times \text{time difference } (t)$$

$$\Rightarrow \phi = \frac{2\pi t}{T}$$

$$\Rightarrow t = \frac{T\phi}{2\pi}$$

$$\text{Time difference } (t) = \frac{T}{\lambda} \times \text{path difference } (x)$$

$$\Rightarrow t = \frac{Tx}{\lambda} \Rightarrow x = \frac{\lambda t}{T}$$

(i) For a wave, velocity

$$v = \text{frequency } (n) \times \text{wavelength } (\lambda)$$

$$\Rightarrow v = n\lambda$$

$$(ii) \text{ Angular speed, } \omega = 2\pi n = \frac{2\pi}{T} = \frac{2\pi v}{\lambda}$$

Reflection and Transmission of Waves

When sound waves are incident on a boundary separating two media, a part of it is reflected back into the initial medium while the remaining is partly absorbed and partly transmitted into the second medium.

Characteristics

(i) In case of reflection and transmission of sound, the frequency of the wave remains unchanged, i.e. $\omega_i = \omega_r = \omega_t = \omega$.

(ii) The incident ray, the reflected ray, normal and the refracted ray are always in the same plane.

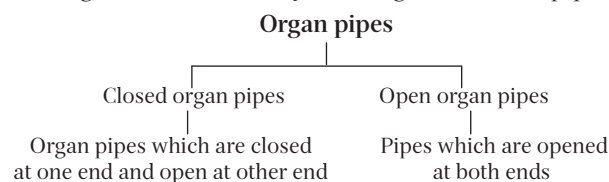
(iii) In case of reflection of sound, angle of incidence = angle of reflection

(iv) In case of refraction of sound, $\frac{\sin i}{\sin r} = \frac{v_i}{v_t}$

(v) When a longitudinal wave is reflected from a denser medium, compression and as rarefaction do not invert after reflection.

Standing Wave in Organ Pipes

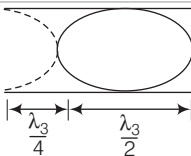
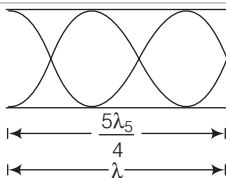
Organ pipes are the musical instruments which are used for producing musical sound by blowing air into the pipe.



Closed Organ Pipes

If one end of a pipe is closed, the reflected wave is 180° out of phase with the incoming wave. The displacement of the small volume elements at the closed end must always be zero. Hence, the closed end must be a displacement node.

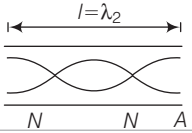
Various modes of vibrations of air column in a closed organ pipe are shown below

First harmonic (fundamental mode)		$\Rightarrow l = \frac{\lambda_1}{4}$ $\Rightarrow \lambda_1 = 4l$ $\Rightarrow f_1 = \frac{v}{4l}$
Third harmonic (first overtone)		$\Rightarrow l = \frac{3\lambda_3}{4}$ $\Rightarrow \lambda_3 = \frac{4l}{3}$ $\Rightarrow f_3 = \frac{3v}{4l}$ $\Rightarrow f_3 = 3f_1$
Fifth harmonic (second overtone)		$\Rightarrow l = \frac{5\lambda_5}{4}$ $\Rightarrow \lambda_5 = \frac{4l}{5}$ $\Rightarrow f_5 = \frac{5v}{4l}$ $\Rightarrow f_5 = 5f_1$
Note In open organ pipe, only odd harmonics are present nth harmonic $\lambda_n = \frac{4l}{(2n-1)}$ $v_n = (2n-1)v_1$		

Open Organ Pipes

If both ends of a pipe are open and a system of air is directed against an edge, standing longitudinal waves can be set up in the tube. The open end has a displacement antinode.

Various modes of vibration of air column in an open organ pipe are shown below

First harmonic (fundamental mode)		$\Rightarrow l = \frac{\lambda_1}{2}$ $\Rightarrow \lambda_1 = 2l$ $\Rightarrow f_1 = \frac{v}{2l}$
Second harmonic or first overtone		$\Rightarrow l = \lambda_2$ $\Rightarrow f_2 = \frac{2v}{2l}$ $\Rightarrow f_2 = 2f_1$
Third harmonic or second overtone		$\Rightarrow l = \frac{3\lambda_3}{2}$ $\Rightarrow f_3 = \frac{3v}{2l}$ $\Rightarrow f_3 = 3f_1$
nth harmonic $f_n = nf_1$ $\lambda_n = 2l/n$		

Beats

- When two sound waves of nearly equal (but never equal) or slightly different frequencies and equal or nearly equal amplitudes travelling along the same direction superimpose at a given point, the resultant sound intensity alternately rises and falls. This alternate rise and fall of sound at a given position is called **beats**.
- One beat is said to be formed when starting from minimum sound the intensity once rises to its maximum value and then falls back to the minimum level.
- Number of beats formed per second is called the frequency of beats. If two sound waves of frequencies v_1 and v_2 superimpose, then frequency of beats $= (v_1 - v_2)$, i.e. either $(v_1 - v_2)$ or $(v_2 - v_1)$.
- For formation of distinct beats then difference between the frequencies of two superposing notes should be less than 10 Hz.
- If source of sound is linear, then at a distance d from the line source, $I = \frac{P}{2\pi / d}$
- Our perception of loudness is better correlated with the second level measured in decibel (dB) and defined as follows $\beta = 10 \log_{10} \left(\frac{I}{I_0} \right)$, where $I_0 = 10^{-12} \text{ Wm}^2$ at 1 kHz.

Doppler's Effect in Sound

When apparent change in the frequency of sound caused by the relative motion between source of sound and the observer, it is called Doppler's effect. Doppler's effect is a wave phenomena, it holds not only for sound waves but also for electromagnetic waves. So, change in frequency can be analysed under three different situations

- Observer is stationary but the source is moving.
- Observer is moving but the source is stationary.
- Both the observer and source are moving.

Expression for Apparent Frequency

Consider a source of sound and an observer are moving with velocity v_s and v_o , respectively. The source is provided a sound of frequency f and the frequency of sound heard by the observer be f' .

Velocity of sound w.r.t. source $= v - v_s$

Apparent wavelength, $\lambda' = \frac{v - v_s}{f}$

Velocity of sound wave w.r.t. listener $= v - v_o$

Wavelength of wave reacting by the listeners $= \lambda'$

Hence, apparent frequency of sound wave heard by listener is

$$f' = \frac{v - v_o}{\lambda'} = \frac{v - v_o}{v - v_s} f$$

Change in frequency can be analysed under different situations, which are given below

- (i) If source is moving towards _____
listener but listener is at rest,
then v_s is positive and
 $v_o = 0$

$$\therefore f' = \frac{v}{v - v_s} \times f, \text{ i.e. } f' > f$$

- (ii) If source is moving away _____
from listener but listener is
at rest, then v_s is negative
and $v_o = 0$.

$$\therefore f' = \left(\frac{v}{v - (-v_s)} \right) \times f = \left(\frac{v}{v + v_s} \right) \times f$$

$$\text{i.e. } f' < f$$

- (iii) If source is at rest and
listener is moving away _____
from source, then $v_s = 0$ and
 v_o is positive.

$$\therefore f' = \left(\frac{v - v_o}{v} \right) \times f$$

$$\text{i.e. } f' < f$$

- (iv) If source is at rest and
listener is moving towards _____
the source, then $v_s = 0$ and
 v_o is negative.

$$\therefore f' = \left[\frac{v - (-v_o)}{v} \right] \times f = \left(\frac{v + v_o}{v} \right) \times f$$

$$\text{i.e. } f' > f$$

- (v) If source and listener are approaching each other,
then v_s is positive and v_o is negative.

$$\therefore f' = \left[\frac{v - (-v_o)}{v - v_s} \right] \times f = \left(\frac{v + v_o}{v - v_s} \right) \times f$$

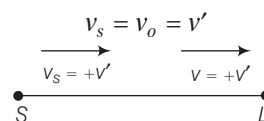
$$\text{i.e. } f' > f$$

- (vi) If source and listener are moving away from each
other, then v_s is negative and v_o is positive.

$$\therefore f' = \left[\frac{v - v_o}{v - (-v_s)} \right] \times f = \left(\frac{v - v_o}{v + v_s} \right) \times f$$

$$\text{i.e. } f' < f$$

- (vii) If source and listener both are in motion in the same
direction and with same velocity, then



$$\therefore f' = \left[\frac{(v - v')}{(v - v')} \right] \times f$$

$$\text{i.e. } f' = f$$

- (viii) If source and listener moves at right angles to the
direction of wave propagation, then velocity of
source and listener along the direction of wave
propagation is $v_s \cos 90^\circ = 0$ and $v_o \cos 90^\circ = 0$

$$\therefore f' = \frac{(v - 0)}{(v - 0)} \times f \text{ or } f' = f$$

NOTE • Velocity of source and observer measured in above expressions are with respect to the medium in which the sound wave travels.

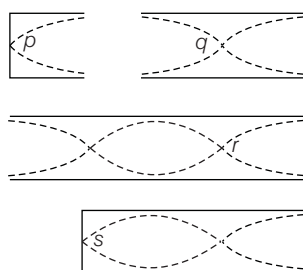
• If the medium carrying the sound wave is itself moving then we have to consider speed of the wave relative to the speed of the medium.

Practice Exercise

- A physicist points out that glass is rarer than water.
 - This statement is correct in the case of sound
 - This statement is always wrong
 - This statement is correct in the case of light
 - This statement is always correct
- When height increases, velocity of sound decreases
 - this is due to decrease of pressure
 - this is due to decrease in temperature
 - this is due to both decrease in temperature and pressure
 - statement is wrong
- The velocity of sound is not affected by change in
 - temperature
 - medium
 - pressure
 - wavelength
- If copper has modulus of rigidity $4 \times 10^{10} \text{ N/m}^2$ and Bulk modulus $1.2 \times 10^{11} \text{ N/m}^2$ and density 9 g/cm^3 , then find the velocity of longitudinal wave, when set-up in solid copper.
 - 4389 m/s
 - 5000 m/s
 - 4000 m/s
 - 4300 m/s
- A piezo electric quartz plate of Young's modulus of elasticity $8 \times 10^{10} \text{ N/m}^2$ and density $2.65 \times 10^3 \text{ kg/m}^3$ is vibrating in resonant condition. The fundamental frequency of vibrating is 550 kHz. What is thickness of the plate?
 - 0.05 cm
 - 0.5 cm
 - 1.25 cm
 - 0.55 cm

6. The value of adiabatic constant γ for oxygen and nitrogen is same. The speed of sound in oxygen is 470 m/s at STP. The speed of sound in nitrogen at STP is
 a. 340 m/s b. 580 m/s
 c. 502 m/s d. None of these
7. At STP, the speed of sound in hydrogen is 1324 m/s then the speed of sound in air
 a. 331 m/s b. 220 m/s c. 340 m/s d. 230 m/s
8. If the speed of sound is changed by 1 per cent, how much must the temperature of air near 0°C be changed
 a. 5°C b. 6°C c. 5.5°C d. 6.5°C
9. If the speed of sound wave in a stretched string is v and Hooke's law is obeyed, then the extension in string is x . Find the extension in the string if the speed of sound wave will become $1.22v$.
 a. $1.5x$ b. $1x$ c. $0.5x$ d. $2x$
10. A boy watches a jet plane flying from north to south. When the jet is just seen above his head, the sound of jet appears to reach him making some angle with horizontal from north. If the velocity of sound is v and velocity of jet is $v/2$, then find the angle.
 a. 60° b. 30° c. 45° d. 15°
11. A stone is dropped into a lake from a tower of 500 m high. The sound of the splash will be heard by the man approximately after (Take $g = 10\text{ ms}^{-2}$)
 a. 11.5 s b. 21 s c. 10 s d. 14 s
12. A light pointer fixed to one prong of a tuning fork touches a vertical plate. The fork is set vibrating at a frequency of 56 Hz and allowed to free fall. Calculate how many complete oscillation are counted when plate falls at 10 cm.
 a. 10 b. 9 c. 8 d. 7
13. The equation of a sound wave in air is $P = 0.01 \cos(1000t - 3x)$ where, P , x and t are in SI. The bulk modulus of elasticity is $1.4 \times 10^5 \text{ N/m}^2$. The displacement amplitude is
 a. 0.24 m b. $0.24 \times 10^{-7} \text{ m}$
 c. $8 \times 10^{-7} \text{ m}$ d. 10 m
14. A sound wave of pressure amplitude 14 pascal propagates through the air medium. The normal pressure of air is $1.0 \times 10^5 \text{ N/m}^2$. The difference between maximum and minimum pressure in the medium is
 a. $5 \times 10^5 \text{ N/m}^2$ b. $10 \times 10^5 \text{ N/m}^2$
 c. 10 N/m^2 d. None of these
15. A sound wave having a frequency of 100 Hz and pressure amplitude of 10 Pa, then calculate the displacement amplitude (Given speed of sound in air = 340 m/s and density of air = 1.29 kg/m^3)
 a. $3.63 \times 10^{-5} \text{ m}$ b. $3 \times 10^{-5} \text{ m}$
 c. $4.2 \times 10^{-5} \text{ m}$ d. $6.4 \times 10^{-5} \text{ m}$
16. When a wave is propagated from rarer medium to denser medium. Which of the following will remain unchanged?
 a. Wave speed b. Propagation constant
 c. Frequency d. None of these
17. A sonar system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the sonar with a speed of 360 km/h. What is the frequency of sound reflected by the submarine? (Take the speed of sound in water to be 1450 m/s)
 a. 52.3 kHz b. 45.93 kHz c. 62.49 Hz d. 54.34 kHz
18. A small speaker has a capacity of power 3 W. A microphone is placed at distance 2 m from the speaker. Find the displacement amplitude of particles of air near the microphone. The frequency of sound emitted by speaker is 1.0 kHz.
 (Density of air = 1.2 kg/m^3) and speed of sound in air = 330 m/s)
 a. $2.76 \times 10^{-4} \text{ cm}$ b. $4 \times 10^{-4} \text{ cm}$
 c. $10 \times 10^{-4} \text{ cm}$ d. $3.8 \times 10^{-3} \text{ cm}$
19. From a height of 2 m, a drop of water of radius $2 \times 10^{-3} \text{ m}$ fall and produces a sound. The sound produced can be heard upto a distance of 20 m. If the gravitational energy is converted into sound energy in 0.5 s, then calculate the intensity at a distance of 20 m.
 a. $2 \times 10^{-7} \text{ W/m}^2$ b. $2.6 \times 10^{-6} \text{ W/m}^2$
 c. $2.6 \times 10^{-7} \text{ W/m}^2$ d. $3 \times 10^{-7} \text{ W/m}^2$
20. Velocity of sound wave in air is 330 m/s for a particular sound in air. A path difference of 40 cm is equivalent to a phase difference of 1.6π . The frequency of this wave is
 a. 165 Hz b. 150 Hz c. 660 Hz d. 330 Hz
21. A tuning fork A of frequency 512 Hz produces 4 beats per second when sounded with a tuning fork B. Due to filing of the prongs of the tuning fork B, the number of the beats per second becomes 6. The actual frequency of B is
 a. 516 Hz b. 508 Hz
 c. 512 Hz d. None of these
22. A tuning fork A of frequency 260 c/s produces 4 beats per second with tuning fork B. When the tuning fork A is loaded with wax, then the number of beats produced per second becomes 3. Then, what is the frequency of tuning fork B?
 a. 264 b. 263 c. 256 d. 260
23. When temperature of air is 20°C , a tuning fork sounded over the open end of an air column produces 4 beats per second, the tuning fork given a lower note. If the frequency of tuning fork is 34 Hz, then find how many beats will be produced by the tuning fork if temperature falls to 5°C ?
 a. 2 beat/s b. 4 beat/s c. 1 beat/s d. 3 beat/s

24. Two wires A and B are of same length, radius and same material are in unison. If tension in A is increased by 4%, 4 beats are heard, then the frequency of the note produced when they were in unison, will be
a. 50 Hz b. 100 Hz c. 150 Hz d. 200 Hz
25. The velocity of sound in air at NTP is 331 m/s. Find its velocity when temperature rises to 91°C and its pressure is doubled.
a. 372 m/s b. 382.1 m/s c. 423 m/s d. 392.5 m/s
26. The fundamental frequency of a closed organ pipe is equal to second overtone of an open organ pipe. If the length of closed organ pipe is 15 cm. The length of open organ pipe is
a. 90 cm b. 30 cm c. 15 cm d. 20 cm
27. An organ pipe closed at one end resonates with a tuning fork of frequencies 180 Hz and 300 Hz. It will also resonate with tuning fork of frequencies
a. 360 Hz b. 420 Hz c. 480 Hz d. 540 Hz
28. Figures shows the vibrations of four air columns. The ratio of frequencies $n_p : n_q : n_r : n_s$ is



- a. 12 : 6 : 3 : 4 b. 1 : 2 : 4 : 3
c. 4 : 2 : 3 : 1 d. 4 : 3 : 2 : 1
29. A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod are given to be 2.53 kHz. What is the speed of sound in steel?
a. 6.2 km/s b. 5.06 km/s c. 7.23 km/s d. 7.45 km/s
30. Two organ pipes are emitting their fundamental notes, when each closed at end, give 5 beat/s. If their fundamental frequencies are 250 Hz and 255 Hz, then find the ratio of their lengths.
a. $\frac{49}{50}$ b. $\frac{49}{51}$ c. $\frac{50}{51}$ d. $\frac{51}{50}$
31. In the case of vibration of closed end organ pipe in fundamental mode of vibration, the pressure is maximum at
a. open end b. closed end
c. at middle d. None of these
32. An air column in a pipe which is closed at one end will be in resonance with a vibrating tuning fork of frequency 264 Hz if the length of the air column in cm is (Speed of sound in air = 340 m/s)
a. 32.19 cm b. 64.39 cm c. 100 cm d. 140 cm

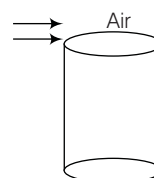
33. When two waves of almost equal frequencies n_1 and n_2 are produced simultaneously, then the time interval between successive maxima is

a. $\frac{1}{n_1 - n_2}$ b. $\frac{1}{n_1} - \frac{1}{n_2}$ c. $\frac{1}{n_1} + \frac{1}{n_2}$ d. $\frac{1}{n_1 + n_2}$

34. A metal rod of length 1.5 m is clamped at the centre. When it is set with longitudinal vibrations it emits a note of 1000 Hz. Determine the Young's modulus if the density of material = $8 \times 10^3 \text{ kg/m}^3$

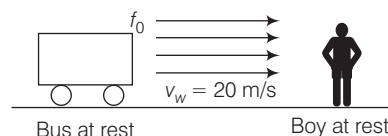
a. $7 \times 10^{10} \text{ N/m}^2$ b. $7.2 \times 10^{10} \text{ N/m}^2$
c. $0.7 \times 10^{10} \text{ N/m}^2$ d. $6.8 \times 10^{10} \text{ N/m}^2$

35. A long tube of length $l = 25 \text{ cm}$ and diameter equal to 2 cm is taken and at its mouth air is blown as shown in figure. The sound emitted by tube will have all the frequencies of the group (velocity of sound = 330 m/s)



a. 660, 1320, 2640 Hz b. 660, 1000, 3300 Hz
c. 302, 684, 1320 Hz d. 330, 990, 1690 Hz

36. In the above situation, bus is at rest blowing horn of frequency f_0 . A boy is at rest at some distance. What will be apparent frequency of sound, if the air start moving with the speed of 20 m/s from bus towards boy?



a. $< f_0$ b. $> f_0$ c. $= f_0$ d. None of these

37. The first overtone of an open organ pipe beats with the first overtone of a closed organ pipe with a beat frequency of 2.2 Hz. The fundamental frequency of closed organ pipe is 110 Hz. Find length of the open pipe. (Given, sound in air = 330 m/s)

a. 2.43 m b. 0.73 m c. 0.993 m d. 2.93 m

38. The apparent frequency is f_1 when a source of sound approaches a stationary observer with a speed u and f_2 when the observer approaches the stationary source with same speed. If v is the velocity of sound, then

a. $f_1 = f_2$ b. $f_1 > f_2$ if $u < v$
c. $f_2 > f_1$ if $u < v$ d. $f_2 > f_1$ if $u < v$

39. A locomotive engine approaches a railway station and whistles at a frequency of 400 Hz. A stationary observer on the platform observes a changes of 40 Hz as the engine passes across him. If the velocity of sound is 330 m/s. The speed of the engine is

a. 33 m/s b. 18 m/s c. 16.5 m/s d. 24 m/s

40. Two trains, one moving at a speed of 30 mile/h and other at 60 mile/h, approaching each other. When a faster train blows a whistle, the apparent frequency of the note heard by an observer at rest behind the faster train is 1852 Hz. Calculate the frequency of note produced by faster train (Assume speed of sound to be 1100 ft/s)
- a. 2000 vib/s b. 1500 vib/s c. 1000 vib/s d. 2500 vib/s
41. A boy with a radio, playing a music at a frequency ' f ', is moving towards a wall with velocity v_b . A motorist is following the boy with a speed v_m . Find the expression for the beat frequency heard by the motorist., if the speed of sound is v

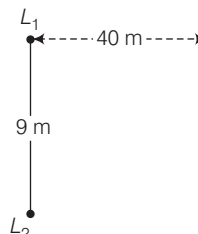
$$\begin{array}{ll} \text{a. } \left(\frac{v + v_m}{v + v_b} \right) f & \text{b. } \frac{v + v_m}{v - v_b} f \\ \text{c. } \frac{2v_b(v + v_m)}{(v^2 - v_b^2)} f & \text{d. } \frac{2v_m(v + v_b)}{(v^2 - v_m^2)} f \end{array}$$

42. There is a road between two parallel rows of building and distance between the rows of building is 106 m. Find the velocity of car if a car blows a horn whose echo is heard by the driver after 1 s. (Given, speed of sound = 340 m/s)
- a. 180 m/s b. 165 m/s
c. 323 m/s d. 150 m/s

BITSAT Archives

1. A sound source is moving towards stationary listener with $\frac{1}{10}$ th of the speed of sound. The ratio of apparent to real frequency is [2012]
- a. $\left(\frac{9}{10}\right)^2$ b. $\frac{10}{9}$
c. $\frac{11}{10}$ d. $\left(\frac{11}{10}\right)^2$
2. The produced rays in sonography are [2012]
- a. microwaves
b. infrared waves
c. sound waves
d. ultra sound
3. The source of sound generating a frequency of 3 kHz reaches an observer with a speed of 0.5 times the velocity of sound in air. The frequency heard by the observer is [2011]
- a. 1 kHz b. 2 kHz
c. 4 kHz d. 6 kHz
4. If v_m is the speed of sound in moist air and v_d is the speed of sound in dry air under identical conditions of pressure and temperature, then [2011]
- a. $v_m > v_d$
b. $v_m < v_d$
c. $v_m = v_d$
d. $v_m \cdot v_d = 1$
5. Two loudspeakers L_1 and L_2 , driven by a common oscillator and amplifier, are arranged as shown. The frequency of the oscillator is gradually increased from zero and the detector at D records a series of maxima

and minima. If the speed of sound is 330 ms^{-1} , then the frequency at which the first maximum is observed is [2010]



- a. 165 Hz b. 330 Hz c. 495 Hz d. 660 Hz
6. When a sound wave of wavelength λ is propagating in a medium, the maximum velocity of the particle is equal to the wave velocity. The amplitude of wave is [2008]
- a. λ b. $\frac{\lambda}{2}$ c. $\frac{\lambda}{2\lambda}$ d. $\frac{\lambda}{4\lambda}$
7. The disc of a siren containing 60 holes rotates at a constant speed of 360 rpm. The emitted sound is in unison with a tuning fork of frequency [2005]
- a. 10 Hz b. 360 Hz c. 216 Hz d. 60 Hz
8. The ratio of velocity of sound in hydrogen and oxygen at STP is [2005]
- a. 16 : 1 b. 8 : 1 c. 4 : 1 d. 2 : 1
9. In an experiment with sonometer a tuning fork of frequency 256 Hz resonates with a length of 25 cm and another tuning fork resonates with a length of 16 cm. Tension of the string remaining constant the frequency of the second tuning fork is [2005]
- a. 163.84 Hz b. 400 Hz
c. 320 Hz d. 204.8 Hz

Answer with Solutions

Practice Exercise

1. (a) 2. (b) 3. (c) 4. (a)

$$5. (b) \text{ Here, } c = \sqrt{\left(\frac{Y}{\rho}\right)} = \sqrt{\left(\frac{8 \times 10^{10}}{2.65 \times 10^3}\right)}$$

$$= \sqrt{\left(\frac{80 \times 10^6}{2.65}\right)} = 5494.4 \text{ m/s}$$

$$\therefore \lambda = \frac{c}{f} = \frac{5494.4}{550 \times 10^3}$$

$\therefore$ For fundamental mode of vibration, thickness

$$t = \frac{\lambda}{2} = 0.4995 = 0.5 \text{ cm}$$

$$6. (c) c = \sqrt{\left(\frac{\gamma RT}{M}\right)}$$

$$\therefore \frac{c_2}{c_1} = \sqrt{\left(\frac{M_1}{M_2}\right)} \text{ or } \frac{c_{\text{nitrogen}}}{c_{\text{oxygen}}} = \sqrt{\left(\frac{32}{28}\right)} = \sqrt{\left(\frac{8}{7}\right)}$$

$$\therefore c_{\text{nitrogen}} = \sqrt{\left(\frac{8}{7}\right)} c_{\text{oxygen}} = \sqrt{\left(\frac{8}{7}\right)} \times 470$$

$$= 502.45 \text{ m/s} \approx 502 \text{ m/s}$$

7. (a) Molecular weight of hydrogen = 2

Molecular weight of oxygen = 32

$$\text{Both are diatomic } v_{\text{air}} = \sqrt{\left(\frac{\gamma_{\text{air}} P_{\text{air}}}{\rho_{\text{air}}}\right)}, v_{\text{H}} = \sqrt{\frac{\gamma_{\text{H}} P_{\text{H}}}{\rho_{\text{H}}}}$$

$$\frac{\rho_{\text{air}}}{\rho_{\text{H}}} = \frac{M_{\text{air}}}{M_{\text{H}}} = \frac{32}{2} = \frac{16}{1}$$

$$\frac{v_{\text{air}}}{v_{\text{H}}} = \sqrt{\left(\frac{\gamma_{\text{air}} P_{\text{H}}}{\gamma_{\text{H}} P_{\text{air}}}\right)} \Rightarrow \frac{v_{\text{air}}}{1324} = \sqrt{\left(\frac{\rho_{\text{H}}}{\rho_{\text{air}}}\right)}$$

$$v_{\text{air}} = \frac{1324}{4} = 331 \text{ m/s}$$

8. (c)

9. (a) $T \propto x$

So, $T' \propto x'$

$$v = \sqrt{\left(\frac{T}{m}\right)}; v' = \sqrt{\left(\frac{T'}{m}\right)}$$

$$\frac{v'}{v} = \sqrt{\left(\frac{T'}{T}\right)} \Rightarrow \sqrt{\left(\frac{x'}{x}\right)} = \frac{1.22}{v} \Rightarrow x' \approx 1.5x$$

10. (a) Distance covered by sound to reach the boy
 $= \frac{\text{Distance covered by jet}}{\cos \theta}$

Velocity of jet = (velocity of sound) $\times \cos \theta$

$$\Rightarrow \frac{v}{2} = v \cos \theta \Rightarrow \theta = 60^\circ$$

11. (a)

12. (c) Let n = frequency

and N = number of oscillations

Time taken to fall freely to a distance 10 cm

$$s = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2 \times 10}{g}}$$

The frequency of oscillation during this time

$$n = \frac{N}{t} \Rightarrow N = nt \Rightarrow N = 56 \times \sqrt{\frac{2 \times 10}{980}}$$

$$N = 7.999 \approx 8$$

13. (b) The standard equation is

$$p = p_0 \cos(\omega t - x) \quad \dots(i)$$

The given equation is

$$P = 0.01 \cos(1000t - 3x) \quad \dots(ii)$$

Comparing the Eq. (i) and Eq. (ii), we get

$$p_0 = 0.01 \text{ N/m}^2$$

$$\omega = 1000 \text{ rad/s}$$

$$k = 3 \text{ per metre}$$

But

$$p_0 = B \Delta k$$

$$\therefore A = \frac{p_0}{Bk} \quad [B = \text{Bulk modulus of elasticity}]$$

$$= \frac{0.01}{14 \times 10^5 \times 3} = 0.2381 \times 10^{-7} \text{ m} \approx 0.24 \times 10^{-7} \text{ m}$$

14. (c) The minimum pressure is $p_{\text{min}} = (10^5 - 5) \text{ N/m}^2$

The maximum pressure is $p_{\text{max}} = (10^5 + 5) \text{ N/m}^2$

$$\therefore \Delta p = p_{\text{max}} - p_{\text{min}} = 10 \text{ N/m}^2$$

15. (a) 16. (c)

17. (b) Sonar frequency,

$$v_s = 40 \text{ kHz} = 40 \times 10^3 \text{ Hz}$$

Speed of enemy submarine,

$$v_s = 360 \text{ km/h} = 360 \times \frac{5}{18} \text{ m/s} = 100 \text{ m/s} \left(\because 1 \text{ km/h} = \frac{5}{18} \text{ m/s} \right)$$

Speed of sound in water = 1450 m/s

Apparent frequency received by the enemy submarine,

$$v' = \left(\frac{v + v_0}{v}\right) v = \left(\frac{1450 + 100}{1450}\right) \times 40 \times 10^3$$

$$= 42.76 \times 10^3 \text{ Hz}$$

Now, the reflected waves received by sonar having a different frequency

$$v'' = \left(\frac{v}{v - v_s}\right) v'$$

Here, $v_s = 100 \text{ m/s}$ is velocity of enemy submarine.

$$\therefore v'' = \left(\frac{1450}{1450 - 100}\right) \times 42.76 \times 10^3$$

$$= 45.93 \times 10^3 \text{ Hz}$$

$$= 45.93 \text{ kHz}$$

$$18. (a) \because l = \frac{P}{4\pi r^2} = \frac{3}{4\pi \times 2^2} = \frac{3}{16\pi}$$

$$\text{But } l = 2\pi^2 A^2 v^2 \rho_0 c \text{ or } A^2 = \frac{l}{2\pi^2 v^2 \rho_0 c}$$

$$\therefore A = \sqrt{\left(\frac{l}{2\pi^2 v^2 \rho_0 c}\right)} = \sqrt{\left(\frac{3}{16\pi \times 2\pi^2 \times 10^6 \times 12 \times 330}\right)}$$

$$= 0.002764 \times 10^{-3} \text{ m} = 2.76 \times 10^{-4} \text{ cm}$$

19. (c) Change in gravitational energy = mgh

$$= \left(\frac{4}{3}\pi r^3\right) \rho gh \text{ joule}$$

$$\text{Hence, power} = \frac{\frac{4}{3}\pi r^3 \rho gh}{t} \text{ watt}$$

$$\text{and intensity} = \frac{\frac{4}{3}\pi r^3 \rho gh}{(4\pi R^2)t} = 2.6 \times 10^{-7} \text{ Wm}^2$$

20. (c) As, phase difference = $\frac{2\pi}{\lambda} \times \text{Path difference}$

$$\Rightarrow 1.6\pi = \frac{2\pi}{\lambda} \times 40 \Rightarrow \lambda = 50 \text{ cm} = 0.5 \text{ m}$$

$$\text{Now, as } v = \lambda f \Rightarrow f = \frac{v}{\lambda} = \frac{330}{0.5} = 660 \text{ Hz}$$

21. (a) Due to filing, frequency of tuning fork B increases. Since, after filing beats frequency increases.

$$\therefore f_B > f_A \quad \therefore f_B - f_A = 4$$

$$\therefore f_B = 4 + f_A = 4 + 512 = 516 \text{ Hz}$$

22. (c) When tuning fork A is loaded with wax, then its frequency decreases

$$\therefore f_A - f_B = 4 \quad \therefore 260 - f_B = 4$$

$$\therefore f_B = 256 \text{ Hz}$$

23. (d) Let beat produced at $5^\circ\text{C} = b$

$$\text{Frequency of air column at } 20^\circ\text{C} = 34 + 4$$

$$\text{Frequency of air column at } 5^\circ\text{C} = 34 + b$$

$$\text{Ratio of velocity of sound at temperature } 20^\circ\text{C and } 5^\circ\text{C}$$

$$\frac{v_{20^\circ}}{v_{5^\circ}} = \sqrt{\left(\frac{273 + 20}{273 + 5}\right)} \Rightarrow \frac{34 + 4}{34 + b} = \sqrt{\left(\frac{293}{278}\right)}$$

$$b = 3 \text{ beat/s}$$

$$24. (d) \frac{104}{100} = \left(\frac{n+4}{n}\right)^2 \text{ or } \sqrt{\frac{104}{100}} = \frac{n+4}{n} \Rightarrow n = 200 \text{ Hz}$$

25. (b) Given velocity of sound in air $v_0 = 331 \text{ m/s}$

$$t = 91^\circ\text{C}$$

$$v_t = v_0 \sqrt{\frac{273 + t}{273}} = 331 \sqrt{\frac{273 + 91}{273}} = 331 \sqrt{1 + \frac{91}{273}}$$

$$v_t = 331 \sqrt{1 + \frac{1}{3}} = 331 \times \frac{2}{\sqrt{3}}$$

$$= 382.1 \text{ m/s}$$

26. (a) For closed organ pipe $f_1 = \frac{v}{4l_1}$ [for fundamental]

$$\text{For open organ pipe, } f_2 = \frac{3v}{2l_2} \quad [\text{second overtone}]$$

$$\therefore f_1 = f_2 \quad \therefore \frac{v}{4l_1} = \frac{3v}{2l_2} \Rightarrow \frac{l_2}{l_1} = 6$$

$$\therefore l_2 = 6l_1 = 6 \times 15 = 90 \text{ cm}$$

27. (b) Resonate frequencies are $n_1 f$ and $n_2 f$, where n_1 and n_2 are integers and f is fundamental frequency.

$$\text{Here, } n_1 f = 180 \text{ and } n_2 f = 300$$

$$\therefore \frac{n_1}{n_2} = \frac{180}{300} = \frac{18}{30} = \frac{3}{5} \quad \therefore n_1 = 3 \text{ and } n_2 = 5$$

$$\therefore 3f = 180 \quad \therefore f = \frac{180}{3} = 60 \text{ Hz}$$

Possible harmonics in the case of closed end organ pipe is 1, 3, 5, 7, 9, 11, 13, ...

The corresponding frequencies are 60, 180, 300, 420, ...

28. (b)

29. (b) In fundamental mode,

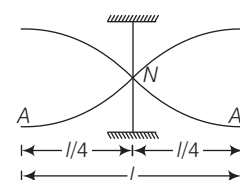
$$l = 2\left(\frac{\lambda}{4}\right) = \frac{\lambda}{2} \Rightarrow \lambda = 2l$$

$$\text{Given, } l = 100 \text{ cm, } v = 253 \text{ kHz}$$

$$\text{Using } v = \lambda f$$

$$\Rightarrow v = 2.53 \times 10^3 \times 2 \times 1000 \times 10^{-2}$$

$$= 5.06 \times 10^3 \text{ m/s} = 5.06 \text{ km/s}$$



30. (c)

31. (b)

32. (a) For fundamental mode of vibration,

$$\therefore \frac{\lambda}{4} = l \quad \therefore \lambda = 4l$$

$$\therefore v = f\lambda \text{ or } 340 = 264 \times 4l$$

$$\therefore l = 32.19 \text{ cm}$$

33. (a) Number of beats per second = $n_1 - n_2$

$$\therefore \text{Time interval between two successive maxima} = \frac{1}{n_1 - n_2}$$

34. (b)

$$35. (a) n = \frac{v}{2l} = \frac{330}{2 \times 25} \times 100 = 660 \text{ Hz}$$

$$n_1 = 2 \times 660 = 1320 \text{ Hz}$$

$$n_2 = 3 \times 660 = 1980 \text{ Hz}$$

36. (c)

37. (c) Fundamental frequency of closed organ pipe,

$$v_c = \frac{v}{4L_c} \Rightarrow L_c = \frac{v}{4 \times 110} = 0.75 \text{ m}$$

Frequency of first overtone of open organ pipe,

$$v_1 = 2 \times \frac{v}{2L_0} = \frac{v}{L_0}$$

Frequency of first overtone of closed organ pipe, $v_2 = 3v_c$

$$= 3 \times \frac{v}{4L_c} = \frac{3 \times 330}{4 \times 0.75} = 330 \text{ Hz}$$

$$\therefore v_1 - v_2 = 2.2; \frac{v}{L_0} - 330 = 2.2$$

$$\Rightarrow \frac{v}{L_0} = 332.2$$

$$\Rightarrow L_0 = \frac{330}{332.2} = 0.993 \text{ m}$$

38. (d) When source approaches the observer,

$$f_1 = \left(\frac{v}{v - v_s} \right) f = \left(\frac{v}{v - u} \right) f = \left\{ \frac{1}{\left(1 - \frac{u}{v} \right)} \right\} f$$

When observer approaches towards the source

$$f_2 = \left(\frac{v + v_o}{v} \right) f = \left(\frac{v + u}{v} \right) f = \left(1 + \frac{u}{v} \right) f$$

$$\therefore f_2 > f_1$$

39. (c) Here,
- $n' = \left(\frac{v}{v - v_s} \right) n$
- and
- $n'' = \left(\frac{v}{v + v_s} \right) n$

According to problem,

$$40 = n' - n'' = \left(\frac{v}{v - v_s} \right) n - \left(\frac{v}{v + v_s} \right) n$$

$$\text{or } 40 = \left(\frac{330 \times 400}{330 - v_s} \right) - \left(\frac{330 \times 400}{330 + v_s} \right)$$

After solving, $v_s = 16.5 \text{ m/s}$

40. (a)
- $\therefore 60 \text{ mile/h} = 88 \text{ ft/s}$

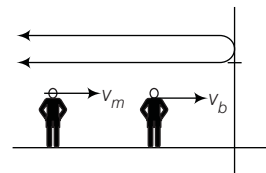
$$n' = \left(\frac{v}{v + v_s} \right) n \text{ or } \left(\frac{1100}{1100 + 88} \right) \times n = 1852$$

$$n = \frac{1852 \times 1188}{1100} = 2000.16 = 2000 \text{ vib/s}$$

41. (c) Motorist will receive two sounds.

$$\text{For direct sound waves, } f' = \left(\frac{v + v_m}{v + v_b} \right) f$$

$$\text{For reflected sound waves } f'' = \left(\frac{v}{v - v_b} \right) \times f$$

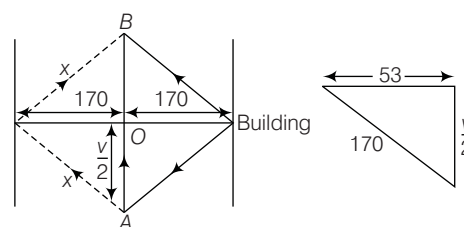
 $\Rightarrow$ Sound reflected from wall

$$f''' = \left(\frac{v + v_m}{v} \right) \times f'' = \left(\frac{v + v_m}{v + v_b} \right) f$$

Beat frequency $f''' - f'$

$$= \left(\frac{v + v_m}{v - v_b} \right) f - \left(\frac{v + v_m}{v + v_b} \right) f = \frac{2v_b(v + v_m)}{(v^2 - v_b^2)} f$$

42. (c) Signal goes from A to B. Let the velocity of car =
- v
- .

Distance travelled $AB = v$, $OA = \frac{v}{2}$

$$\frac{2x}{1} = 340 \Rightarrow x = 170 \text{ m}$$

$$(170)^2 = (53)^2 + \frac{v^2}{4}$$

$$\sqrt{4(28900 - 2809)} = v \Rightarrow v = 323 \text{ m/s}$$

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1. (b) Given,
- $n_s = \frac{n}{10}$

$$\text{Apparent frequency } n' = n \left(\frac{v}{v - v_s} \right)$$

where, n = real frequency of source v = velocity of sound, v_s = velocity of source

$$\text{So, } \frac{n'}{n} = \frac{v}{v - \frac{v}{10}} = \frac{10}{9}$$

2. (d) The produced rays in sonography are ultrasound.

3. (d) Apparent frequency heard will be
- $n' = n \left(\frac{v}{v - v_s} \right)$

 v = velocity of sound, v_s = velocity of source of sound n = frequency = 3 kHz

$$\therefore n' = 3 \times \frac{v}{v - 0.5v} = 3 \times \frac{v}{0.5v} = 6 \text{ kHz}$$

4. (a) Under identical pressure and temperature condition, speed of sound in moist air is more than that in dry air, i.e.
- $v_m > v_d$

5. (b) Let
- Δx
- = path difference

$$\Rightarrow \Delta x = L_2 D - L_1 D \Rightarrow \Delta x = \sqrt{40^2 + 9^2} - 40$$

$$\Rightarrow \Delta x = 41 - 40 \Rightarrow \Delta x = 1 \text{ m}$$

$$\text{For first maximum, } \Delta x = (2n) \frac{\lambda}{2} \quad (\text{where } n = 1)$$

$$\Rightarrow 1 = 2(1) \frac{\lambda}{2} \Rightarrow \lambda = 1 \text{ m} \Rightarrow f = \frac{v}{\lambda} = 330 \text{ Hz}$$

6. (c)
- $v_{\max} = v \Rightarrow A\omega = v \Rightarrow A \times 2\pi v = v\lambda$
- or
- $A = \frac{\lambda}{2\pi}$

7. (b) Speed =
- $360 \text{ rev/min} = \frac{360}{60} \text{ rev/s} = 6$

$$\therefore \text{Frequency} = 6 \times 60 = 360$$

8. (c) Velocity of sound,
- $v = \sqrt{\frac{\gamma RT}{M}}$

$$\frac{v_H}{v_O} = \sqrt{\frac{M_O}{M_H}} = \sqrt{\frac{16}{1}} = 4:1$$

9. (b) For sonometer,
- $n \propto \frac{1}{l}$

$$\therefore \frac{n_1}{n_2} = \frac{l_2}{l_1} \Rightarrow \frac{256}{n_2} = \frac{16}{25} \Rightarrow n_2 = \frac{256 \times 25}{16} = 400 \text{ Hz}$$

15

Heat, Temperature and Calorimetry

Heat

Heat is the energy, which is transferred between system and surroundings due to the temperature difference. In other words, heat is a form of energy that flows from one body to another because of temperature difference between them.

The conventional unit of heat is called the calorie. It is defined as the amount of heat required to raise the temperature of 1 g of water through 1°C. $1 \text{ cal} = 4.186 \text{ J}$.

Temperature

Temperature is the measure of degree of hotness or coldness of a body. So, a body at higher temperature would be hotter relative to a body at lower temperature.

Whenever, there is a difference of temperature between a body and its surroundings, then exchange of energy takes place. This energy is called **heat energy**. So, heat is a form of energy transferred between two (or more) systems or a system and its surroundings by virtue of its temperature difference.

Measurement of Temperature

Temperature is measured by using a thermometer. Various temperature scales such as celsius scale, fahrenheit scale and kelvin scale is used for measuring the temperature.

Properties of matter such as volume expansion or contraction with the change of temperature is the generally used principle of thermometers. In commonly used thermometers (liquid in glass), mercury or alcohol are the generally used liquids. Whereas, the Kelvin scale was derived by using the gas thermometers which is based on ideal gas equation.

The minimum temperature for an ideal gas at which its pressure is reduced to zero is called **absolute zero** (0 K), which is found to be equal to -273.15°C .

Relation between the three mentioned temperature scales is given below

$$\frac{T - 273.15}{100} = \frac{t_C - 0}{100} = \frac{t_F - 32}{180}$$

where, T = Absolute temperature (in kelvin),

t_C = Celsius temperature

t_F = Fahrenheit temperature

Since, at melting point of water $t_C = 0^\circ\text{C}$

So, $T_{\text{melting}} = 273.15\text{ K}$ and $t_{F, \text{melting}} = 32^\circ\text{F}$

And at boiling point of water $t_C = 100^\circ\text{C}$

So, $T_{\text{boiling}} = 373.15\text{ K}$ and $t_{F, \text{boiling}} = 212^\circ\text{F}$

Thermometry

The branch dealing with measurement of temperature is called thermometry and the devices used to measure temperature are called thermometers.

At changing temperature, change in pressure of a gas at constant volume, change in electric resistance of a metallic wire etc. Such property of a substance is called thermometric property. Let thermometric properties at temperatures 0°C (ice point), 100°C (steam point) and $t^\circ\text{C}$ (unknown temperature) are X_0 , X_{100} , and X_t respectively. Then,

$$\frac{X_t - X_0}{t} = \frac{X_{100} - X_0}{100}$$

$$\text{or } \frac{X_t - X_0}{X_{100} - X_0} = \frac{t}{100}$$

$$\text{Thus, } t = \left(\frac{X_t - X_0}{X_{100} - X_0} \right) \times 100^\circ\text{C}$$

Triple Point

Triple point is a state in which ice, water and water vapour can stay together in equilibrium. It refers to temperature at the equilibrium.

$\therefore$ The temperature scale by the equation

$$T = \lim_{p_{\text{tr}} \rightarrow 0} \frac{p}{p_{\text{tr}}} \times 273.16\text{ K}$$

where, p = pressure

p_{tr} = pressure at equilibrium and K stands for Kelvin scale of temperature.

If values of thermometric properties at 0 K , 273.16 K and $T\text{ K}$ are 0 , X_{tr} and X respectively, then

$$\frac{T}{T_{\text{tr}}} = \frac{X}{X_{\text{tr}}} \text{ or } T = \frac{X}{X_{\text{tr}}} T_{\text{tr}} = \left[\frac{X}{X_{\text{tr}}} \times 273.16 \right] \text{ K}$$

Different Thermometers

Constant Volume Gas Thermometer

If p_0 , p_{100} , p_{tr} and p_t are the pressures of gas at temperatures 0°C , 100°C , triple point of water and unknown temperature ($t^\circ\text{C}$) respectively keeping the volume constant, then

$$t = \left(\frac{p - p_0}{p_{100} - p_0} \times 100 \right)^\circ\text{C} \text{ or } T = \left(273.16 \frac{p}{p_{\text{tr}}} \right) \text{ K}$$

Platinum Resistance Thermometer

If R_0 , R_{100} , R_{tr} and R_t are the resistances of a platinum wire at temperatures 0°C , 100°C , triple point of water and unknown temperature ($t^\circ\text{C}$) respectively, then

$$t = \left(\frac{R_t - R_0}{R_{100} - R_0} \times 100 \right)^\circ\text{C}$$

$$\text{or } T = \left(\frac{R_t}{R_{\text{tr}}} \times T_{\text{tr}} \right) \text{ K} = \left(\frac{R_t}{R_{\text{tr}}} \times 273.16 \right) \text{ K}$$

Mercury Thermometer

In this thermometer, the length of a mercury column from some fixed point is taken as thermometric property. Thus,

$$t = \left(\frac{l_t - l_0}{l_{100} - l_0} \right) \times 100^\circ\text{C} \text{ or } T = \left(\frac{l_t}{l_{\text{tr}}} \times 273.16 \right) \text{ K}$$

Thermal Expansion

A change in temperature of a body causes change in its dimensions. When the body's temperature is increased, body expands in dimensions. It is called **thermal expansion**.

Linear Expansion

(Expansion in Length of a Solid)

Consider a rod of length l_1 at a temperature θ_1 . Let it be heated to a temperature θ_2 and the increased length of the rod be l_2 , then

$$l_2 = l_1 (1 + \alpha \Delta\theta)$$

where, α = coefficient of linear expansion and $\theta = \theta_2 - \theta_1$

Areal Expansion

(Expansion in Surface Area)

If A_1 is the area of solid at $\theta_1^\circ\text{C}$ and A_2 is the area at $\theta_2^\circ\text{C}$, then

$$A_2 = A_1 (1 + \beta \Delta\theta)$$

where, β = coefficient of areal (superficial) expansion and $\Delta\theta = \theta_2 - \theta_1$

Volume Expansion (Expansion in Volume)

If V_1 is the volume of solid at $\theta_1^\circ\text{C}$ and V_2 is the volume at $\theta_2^\circ\text{C}$, then

$$V_2 = V_1(1 + \gamma \Delta\theta)$$

where, γ = coefficient of cubical (volume) expansion and $\Delta\theta = \theta_2 - \theta_1$

Apparent Expansion of Liquid

If in a beaker (container) a liquid is fully filled and if the temperature of the system increases, then because of the fact that $\gamma_{\text{liquid}} > \gamma_{\text{solid}}$, the expansion in liquid is more than the expansion in solid and thus the liquid overflows from the container. This is termed as apparent expansion of liquid.

Consider a vessel of volume V_0 fully filled with a liquid of coefficient of cubical expansion γ_l . If temperature of the system is increased by ΔT , then

$\Delta V_l = V_0 \gamma_l \Delta T$, $\Delta V_c = V_0 \gamma_c \Delta T$, where subscript c denotes the container.

The volume of overflowing liquid is

$$\Delta V = V_0(\gamma_l - \gamma_c) \Delta T = V_0 \gamma_{lc} \Delta T$$

where, $\gamma_{lc} = \gamma_l - \gamma_c$ is termed as the apparent coefficient of cubical expansion.

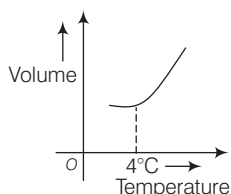
Dulong and Petit's method In this method, a column of experimental liquid at $t^\circ\text{C}$ is balanced against other column of the experimental liquid at 0°C is by taking them in U-tube.

Here,

$$\gamma = \frac{h_t - h_0}{h_0 t}$$

Anomalous Behaviour of Water

Water exhibits an anomalous behaviour. It contracts on heating between 0°C and 4°C . But it shows normal behaviour above 4°C i.e. it expands above 4°C . For this reason, water has maximum density at 4°C (see graph).



Since, $pV = nRT$

So, at constant pressure,

$$p \cdot \Delta V = nR \cdot \Delta T$$

So,

$$\frac{\Delta V}{V} = \frac{\Delta T}{T}$$

or

$$\gamma = \frac{1}{\Delta T} \cdot \left(\frac{\Delta V}{V} \right) = \frac{1}{T}$$

Specific Heat Capacity

The specific heat of a substance is the quantity of heat in calorie required to raise the temperature of 1 g of that substance by 1°C . Its unit is $\text{cal g}^{-1} \text{C}^{-1}$.

The quantity of heat ΔQ required to change the temperature of a body of mass m by ΔT , is approximately proportional to the product of m and ΔT i.e. $\Delta Q \propto m \Delta T$ or $\Delta Q = ms \Delta T$, where s is the specific heat capacity of the material. The product of mass of the body and specific heat capacity is termed as heat capacity.

$s = \frac{\Delta Q}{\Delta T}$ i.e. heat capacity is defined as the amount of heat required to raise the temperature of a body by 1°C .

Principle of Calorimetry

Let two bodies of masses m_1 and m_2 , specific heats s_1 and s_2 and at temperatures θ_1 and θ_2 are brought in contact with each other. Assuming $\theta_1 > \theta_2$, heat will flow from body 1 to body 2. If θ is the common temperature of the two bodies at the state of thermal equilibrium, then (assuming no heat is gained or lost from or to the surroundings)

Heat lost by body 1 = Heat gained by body 2

$$m_1 s_1 (\theta_1 - \theta) = m_2 s_2 (\theta - \theta_2) \because (\theta_2 < \theta < \theta_1)$$

Latent Heat

The heat required to change the state of a system is proportional to the mass of the system, i.e. $Q \propto m \Rightarrow Q = mL$, where L is the **latent heat**.

Water Equivalent of a Substance

Water equivalent of certain amount of substance is defined as the amount of water, which when replaced by the substance requires the same amount of heat for the same rise in temperature.

$$m_w = \frac{mS}{S_w},$$

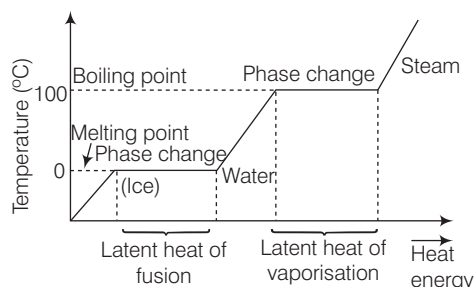
where, m_w = water equivalent of substance whose mass is m ,

S = specific heat capacity of substance and

S_w = specific heat capacity of water.

Heating Curve

A plot of temperature *versus* heat energy for 1 kg of water is shown below



Debyer T^3 law: Specific heat of a solid varies with temperature. It is $3R$ at higher temperature and near absolute zero $C_V \propto T^3$. Clausius Clapeyron's equation (or latent heat equation) representing change of MP or BP with pressure is

$$\frac{dp}{dt} = \frac{Jl}{T(V_2 - V_1)}$$

Hygrometry It is a branch in which we study and measure of water vapours in atmosphere.

$$RH = \frac{m}{M} \times 100$$

Here, m = mass of water, M = mass of water vapour to saturate the same volume of air.

Let p and P are the actual and maximum vapour pressures respectively at the temperature, then

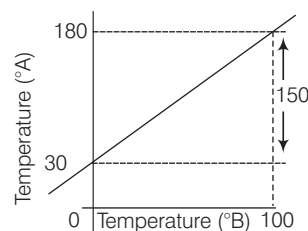
$$RH = \left(\frac{\text{Maximum vapour pressure at dew point}}{\text{Maximum vapour pressure at } t^\circ \text{C}} \right) \times 100\% \\ = \frac{p}{P} \times 100\%$$

Practice Exercise

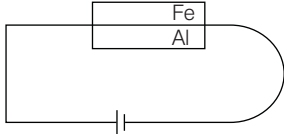
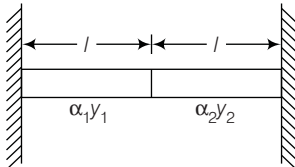
- The temperature at which Centigrade thermometer and Kelvin thermometer gives the same reading, is
a. 4° b. 273°
c. not possible d. 0
- What is the change in the temperature on Fahrenheit scale and on Kelvin scale, if a iron piece is heated from 30 to 90°C ?
a. 108°F , 60 K b. 100°F , 55 K
c. 100°F , 65 K d. 60°F , 108 K
- At 30°C , the hole in a steel plate has diameter of 0.99970 cm . A cylinder of diameter exactly 1 cm at 30°C is to be slide into the hole. To what temperature the plate must be heated?
(Given $\alpha_{\text{steel}} = 11 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$)
a. 58°C b. 55°C c. 57.3°C d. 60°C
- A monoatomic ideal gas, initially at temperature T_1 , is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 be the lengths of the gas column before and after expansion respectively, then $\frac{T_1}{T_2}$ is given by

- $\left(\frac{L_1}{L_2}\right)^{2/3}$
- $\left(\frac{L_1}{L_2}\right)$
- $\left(\frac{L_2}{L_1}\right)$
- $\left(\frac{L_2}{L_1}\right)^{2/3}$

- The graph between two temperature scales A and B is shown in figure given below. Between upper fixed point and lower fixed point, there are 150 equal divisions on scale A and 100 on scale B . The relationship for conversion between two scales is given by



- $\frac{t_A - 180}{100} = \frac{t_B}{150}$
 - $\frac{t_A - 30}{150} = \frac{t_B}{100}$
 - $\frac{t_B - 180}{150} = \frac{t_A}{100}$
 - $\frac{t_B - 40}{100} = \frac{t_A}{180}$
- Heat given to a system can be associated with
a. kinetic energy of random motion of molecules
b. kinetic energy of orderly motion of molecules
c. total kinetic energy of random and orderly motion of molecules
d. kinetic energy of random motion in some cases and kinetic energy of orderly motion in other
 - At what temperature (in $^\circ\text{C}$), the fahrenheit and celsius scale gives same reading?
a. 40 b. -40 c. 8 d. -8

8. Which one of the following would raise the temperature of 40 g of water at 20°C most when mixed with?
- 20 g of water at 40°C
 - 30 g of water at 30°C
 - 10 g of water at 60°C
 - 4 g of water at 100°C
9. If same amount of heat is supplied to two identical spheres (one is hollow and other is solid), then
- the expansion in hollow is greater than the solid
 - the expansion in hollow is same as that in solid
 - the expansion in hollow is lesser than the solid
 - the temperature of both must be same to each other
10. At 30°C, a lead bullet of 50 g, is fired vertically upwards with a speed of 840 m/s. The specific heat of lead is 0.02 cal/g°C. On returning to the starting level, it strikes to a cake of ice at 0°C. Calculate the amount of ice melted (Assume all the energy is spent in melting only)
- 62.7 g
 - 55 g
 - 52.875 g
 - 52.875 kg
11. A second's pendulum clock having steel wire is calibrated at 20°C. When temperature is increased to 30°C, then calculate how much time does the clock lose or gain in one week?
- $[\alpha_{\text{steel}} = 12 \times 10^{-5} (^{\circ}\text{C}^{-1})]$
- 0.3628 s
 - 3.626 s
 - 362.8 s
 - 36.23 s
12. A plate composed of welded sheets of aluminium and iron is connected to an electrical circuit as shown in figure. What will happen if a fairly strong current be passed through the circuit?
- 
- Strip bends upward
 - Strip bends downward
 - Strip remains in its initial condition
 - None of the above
13. A copper rod of length l_0 at 0°C is placed on smooth surface. Now, the rod is heated upto 100°C. Find the longitudinal strain developed.
- (α = coefficient of linear expansion)
- $\frac{100l_0\alpha}{l_0 + 100l_0\alpha}$
 - 100α
 - zero
 - None of these
14. Two metal rods are fixed end to end between two rigid supports, as shown in figure. Each rod is of length ' l ' and area of cross-section is A . When the system is heated up, determine the condition when the junction between rods does not shift.
- 

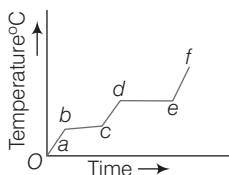
[Given, Y_1 and Y_2 are Young's modulus of materials of the rods, α_1 and α_2 are coefficient at linear expansion.]

- $\alpha_1 Y_2 = \alpha_2 Y_1$
 - $\alpha_1 Y_1 = \alpha_2 Y_2$
 - $\alpha_1 = \alpha_2$
 - $Y_1 = Y_2$
15. What will be the stress at -20°C , if a steel rod with a cross-sectional area of 150 mm^2 is stretched between two fixed points? The tensile load at 20°C is 5000 N (Assume, $\alpha = 11.7 \times 10^{-6} /^{\circ}\text{C}$ and $Y = 200 \times 10^{11}\text{ N/m}^2$)
- $12.7 \times 10^6\text{ N/m}^2$
 - $127 \times 10^6\text{ N/m}^2$
 - $127 \times 10^6\text{ N/m}^2$
 - $0.127 \times 10^6\text{ N/m}^2$
16. In an anisotropic medium, the coefficients of linear expansion of a solid are α_1 , α_2 and α_3 in three mutually perpendicular directions. The coefficient of volume expansion for the solid is
- $\alpha_1 - \alpha_2 + \alpha_3$
 - $\frac{\alpha_1 + \alpha_2 + \alpha_3}{3}$
 - $\alpha_1 + \alpha_2 + \alpha_3$
 - None of these
17. A liquid when heated in a copper vessel and when heated in a silver vessel, the apparent coefficient of expansion is C and S , respectively. If coefficients of linear expansion of copper is A , then coefficient of linear expansion of silver is
- $\frac{C + S - 3A}{3}$
 - $\frac{C + 3A - S}{3}$
 - $\frac{3A - S - C}{3}$
 - $\frac{C + S + 3A}{3}$
18. Using the following data, calculate at what temperature will the wood just sink in benzene
- Density of wood at $0^{\circ}\text{C} = 8.8 \times 10^2\text{ kg/m}^3$
 Density of benzene at $0^{\circ}\text{C} = 9 \times 10^2\text{ kg/m}^3$
 Cubical expansivity of wood = $1.5 \times 10^{-4}\text{ K}^{-1}$
 Cubical expansivity of benzene = $1.2 \times 10^{-3}\text{ K}^{-1}$
- 27°C
 - 21.7°C
 - 31°C
 - 31.7°C
19. The radius of metal sphere at room temperature T is R and the coefficient of linear expansion of the metal is α . The sphere is heated a little by a temperature T , so that new temperature is $T + \Delta T$. The increase in volume of sphere is approximately
- $2\pi R \alpha \Delta T$
 - $\pi R^2 \alpha \Delta T$
 - $4\pi R^3 \alpha \Delta T / 3$
 - $4\pi R^3 \alpha \Delta T$
20. A copper rod and steel rod having length L_c and L_s respectively at certain temperature. It is observed that difference between their length remains constant at all temperatures. If α_c and α_s are their respective coefficient of linear expansions. Then, ratio of $\frac{L_s}{L_c}$ is
- $\frac{\alpha_c}{\alpha_s}$
 - $\frac{\alpha_s}{\alpha_c}$
 - $\left(1 + \frac{\alpha_s}{\alpha_c}\right)$
 - $\left(1 + \frac{\alpha_c}{\alpha_s}\right)$

21. One mole of an ideal monoatomic gas is heated at a constant pressure of 1 atmosphere from 0°C to 100°C . Work done by the gas is

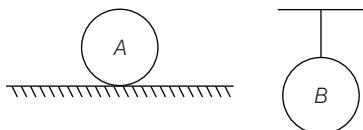
a. $8.31 \times 10^3 \text{ J}$ b. $8.31 \times 10^{-3} \text{ J}$
c. $8.31 \times 10^{-2} \text{ J}$ d. $8.31 \times 10^2 \text{ J}$

22. The following figure represents the temperature versus time plot for a given amount of a substance when heat energy is supplied to it at a fixed rate and at a constant pressure. Which part of plot represent a phase change?



a. a to b and e to f
b. b to c and c to d
c. a to e and e to f
d. b to c and d to e

23. A sphere A is placed at smooth table. Another sphere B is suspended as shown in figure. Both spheres are identical in all respects. Equal quantity of heat is supplied to both spheres. All kinds of heat loss are neglected. The final temperatures of A and B are T_A and T_B respectively, then



a. $T_A = T_B$ b. $T_A > T_B$
c. $T_A < T_B$ d. None of these

24. Calculate the resulting temperature when 20 g of boiling water is poured into an ice-cold brass vessel (specific heat = $0.1 \text{ cal/g}^\circ\text{C}$) of mass 100 g.

a. 66.66°C b. 6.66°C
c. 0.66°C d. 50°C

25. In similar calorimeters, equal volumes of water and alcohol, when poured, take 100 s and 74 s respectively to cool from 50°C to 40°C . If the thermal capacity of each calorimeter is numerically equal to volume of either liquid, then calculate the specific heat capacity of alcohol.

(Given, the relative density of alcohol as 0.8 and specific heat capacity of water as $1 \text{ cal/g}^\circ\text{C}$)

a. $0.8 \text{ cal/g}^\circ\text{C}$
b. $0.6 \text{ cal/g}^\circ\text{C}$
c. $0.9 \text{ cal/g}^\circ\text{C}$
d. $1 \text{ cal/g}^\circ\text{C}$

26. The ratio of thermal capacities of two spheres A and B, if their diameters are in the ratio 1 : 2, densities in the ratio 2 : 1, and the specific heat in the ratio of 1 : 3, will be

a. 1 : 6 b. 1 : 12
c. 1 : 3 d. 1 : 4

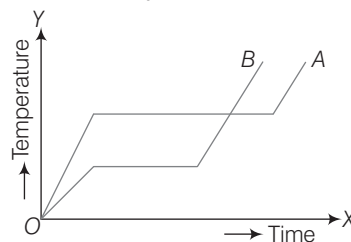
27. The molar heat capacity of rock salt at low temperatures varies with temperature according to Debye's T^3 law.

Thus, $C = k \frac{T^3}{\theta^3}$, where $k = 1940 \text{ Jmol}^{-1}\text{K}^{-1}$, $\theta = 281 \text{ K}$

Calculate how much heat is required to raise the temperature of 2 moles of rock salt from 10 K to 50 K?

a. 800 J b. 373 J
c. 273 J d. None of these

28. Equal masses of two liquids A and B contained in vessels of negligible heat capacity are supplied heat at the same rate. The temperature-time graphs for the two liquids are shown in the figure. If S represents specific heat and L represents latent heat of liquid, then



a. $S_A > S_B$, $L_A < L_B$ b. $S_A > S_B$, $L_A > L_B$
c. $S_A < S_B$, $L_A < L_B$ d. $S_A < S_B$, $L_A > L_B$

29. The specific heat of a substance at temperature $t^\circ\text{C}$ is $s = at^2 + bt + c$. Calculate the amount of heat required to raise the temperature of $m \text{ g}$ of the substance from 0°C to $t_0^\circ\text{C}$

a. $\frac{mt_0^3a}{3} + \frac{bt_0^2}{2} + ct_0$ b. $\frac{mt_0^3a}{3} + \frac{mbt_0^2}{2} + mct_0$
c. $\frac{mt_0^3a}{3} + \frac{mbt_0^2}{2}$ d. None of these

30. In Q. No. 29 find the average value of specific heat.

a. $\frac{at_0^3}{3} + \frac{bt_0^2}{2} + ct_0$ b. $\frac{at_0}{3} + \frac{bt_0^2}{2} + c$
c. $at_0^2 + bt_0 + c$ d. zero

31. A drilling machine of 10 kW power is used to drill a bore in a small aluminium block of mass 8 kg. If 50% of power is used up in heating the machine itself or lost to the surroundings then how much is the rise in temperature of the block in 2.5 min?

(Given, specific heat of aluminium = $0.91 \text{ J/g}^\circ\text{C}$)

a. 103°C b. 130°C c. 105°C d. 30°C

32. A thermally insulated piece of metal is heated under atmosphere by an electric current so that it receives electric energy at a constant power P . This leads to an increase of the absolute temperature T of the metal with time t as follows $T = a^{1/4}$

Then, the heat capacity C_p is

a. $\frac{4PT^3}{a^4}$ b. $\frac{4PT^2}{a^3}$
c. $4PT^2$ d. None of these

33. If in 1.1 kg of water which is contained in a calorimeter of water equivalent 0.02 kg at 15°C, steam at 100°C is passed, till the temperature of calorimeter and its contents rises to 80°C. The mass of steam condensed in kilogram is
a. 0.131 b. 0.065 c. 0.260 d. 0.135
34. 5 g of water at 30°C and 5 g of ice at – 20°C are mixed together in a calorimeter. The water equivalent of calorimeter is negligible and specific heat and latent heat of ice are 0.5 cal/g°C and 80 cal/g respectively. The final temperature of the mixture is
a. 0°C b. – 8°C c. – 4°C d. 2°C
35. Water at – 10°C is present in a thermally insulated container. Calculate the ratio of mass of ice formed and initial mass of water, if a small crystal of ice is thrown into it
- a. 1/15 b. 1/17
c. 2/15 d. 1/8
36. It takes 20 minutes to melt 10 g of ice, when ray from the sun are focussed by a lens of diameter 5 cm on to a block of ice. Calculate the heat received from the sun on 1 cm² per minute.
(Given, $L = 80$ kcal/kg)
a. $R = 2.04$ cal/cm²-min b. $R = 3.04$ cal/cm²-min
c. $R = 0.204$ cal/cm²-min d. $R = 204$ cal/cm²-min
37. In an energy recycling process, X g of steam at 100°C becomes water at 100°C which converts Y g of ice at 0°C into water at 100°C. The ratio of X / Y will be
a. 1/3 b. 2/3
c. 3 d. 2

BITSAT Archives

1. In an experiment on the specific heat of a metal a 0.20 kg block of the metal at 150°C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cc of water at 27°C. The final temperature is 40°C. Calculate the specific heat of the metal. If heat losses to the surroundings are not negligible, is our answer greater or smaller than the actual value of specific heat of the metal? [2014]
a. 0.02 b. 0.2 c. 0.01 d. 0.1
2. A partition wall has two layers of different materials A and B in contact with each other. They have the same thickness but the thermal conductivity of layer A is twice that of B . At steady state, if the temperature difference across the layer B is 50 K, then the corresponding difference across the layer A is [2013]
a. 50 K b. 12.5 K c. 25 K d. 60 K
3. A monoatomic gas is suddenly compressed to $(1/8)$ th of its initial volume adiabatically. The ratio of its final pressure to the initial pressure is (Given, the ratio of the specific heats of the given gas to be 5/3) [2012]
a. 32 b. 40/3 c. 24/5 d. 8
4. 1g of water (volume 1 cm³) becomes 1671 cm³ of steam when boiled at a pressure of 1 atm. The latent heat of vaporisation is 540 cal/g, then the external work done is ($1 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$) [2011]
a. 499.7 J b. 40.3 J
c. 169.2 J d. 128.57 J
5. The end A of rod AB of length 1 m is maintained at 80°C and the end B at 0°C. The temperature at a distance of 60 cm from the end A is [2010]
a. 16°C b. 32°C c. 48°C d. 64°C
6. One junction of a thermocouple is at a particular temperature T_r , and another is at T . Its thermoemf is expressed as $E = k(T - T_r) \left\{ T_0 - \frac{1}{2}(T + T_r) \right\}$.
At temperature $T = \frac{T_0}{2}$, the value of thermo electric power will be [2010]
a. $\frac{1}{2} kT_0$ b. kT_0
c. $\frac{1}{2} kT_0^2$ d. $\frac{1}{2} k(T_0 - T_r)^2$
7. In a 10 m deep lake, the bottom is at a constant temperature of 4°C. The air temperature is constant at – 4°C. The thermal conductivity of ice is 3 times that of water. Neglecting the expansion of water on freezing, the maximum thickness of ice will be [2010]
a. 7.5 m b. 6 m c. 6 m d. 2.5 m
8. A metal string is fixed between rigid supports. It is initially at negligible tension. Its Young's modulus is Y , density is ρ and coefficient of thermal expansion is α . If it is now cooled through a temperature $= t$, transverse wave will move along it with speed. [2009]
a. $Y \sqrt{\frac{\alpha t}{\rho}}$ b. $\alpha t \sqrt{\frac{Y}{\rho}}$ c. $\sqrt{\frac{Y \alpha t}{\rho}}$ d. $t \sqrt{\frac{Y \alpha}{\rho}}$
9. There is some change in length when a 33000 N tensile force is applied on a steel rod of area of cross-section 10^{-3} m^2 . The change of temperature required to produce the same elongation, if the steel rod is heated is (The modulus of elasticity is $3 \times 10^{11} \text{ N/m}^2$ and the coefficient of linear expansion of steel is $11 \times 10^{-5} / ^\circ \text{C}$) [2008]
a. 20 °C b. 15 °C c. 10 °C d. 0 °C

Answer with Solutions

Practice Exercise

1. (c)

2. (a) $\Delta T_C = 90^\circ - 30^\circ\text{C} = 60^\circ\text{C}$

$$\Delta T_F = \frac{9}{5} \Delta T_C = \frac{9}{5} (60^\circ) = 108^\circ\text{F}$$

$$\Delta T = \Delta T_C = 60\text{ K}$$

3. (c) $\Delta l = 1 - 0.99970 = 0.00030\text{ cm}$

$$\Delta l = \alpha l \Delta t$$

$$\Delta t = \frac{\Delta l}{\alpha l} = \frac{0.0003}{1.1 \times 10^{-5} \times 0.9997} = 27.3^\circ\text{C}$$

So, plate temperature must be raised to $30^\circ + 27.36 = 57.3^\circ\text{C}$

4. (d) For an adiabatic process,

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

or $T_1 L_1^{\gamma-1} = T_2 L_2^{\gamma-1}$ ($\because A = \text{constant}$)

or $\frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{\gamma-1}$

$\therefore \gamma = \frac{5}{3}$ for a monoatomic gas,

$\therefore \gamma - 1 = \frac{2}{3}$

Thus, $\frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{2/3}$

5. (b)

6. (a)

7. (b)

8. (c) If m is the mass of water added at $\theta^\circ\text{C}$, then resultant temperature T is given as

Heat lost = Heat gained

$$\Rightarrow m(\theta - T) = 40(T - 20) \Rightarrow T = \frac{800 + m \cdot \theta}{40 + m}$$

So, for option (a), $T = \frac{800 + 20 \times 40}{40 + 20} = 26.6^\circ\text{C}$

For option b, $T = \frac{800 + 30 \times 30}{40 + 30} = 24.28^\circ\text{C}$

For option (c), $T = \frac{800 + 10 \times 60}{40 + 10} = 28^\circ\text{C}$

For option (d), $T = \frac{800 + 400}{40 + 4} = 27.27^\circ\text{C}$

9. (a)

10. (d)

11. (d) Time period of second's pendulum = 2 s

Change in time period,

$$\Delta T = \frac{1}{2} T \alpha \Delta \theta$$

$$\Delta T = \left(\frac{1}{2}\right)(2)(1.2 \times 10^{-5})(30^\circ - 206) = 1.2 \times 10^{-4}\text{ s}$$

New time period, $T' = T + \Delta T \Rightarrow T' = 2.00012\text{ s}$

Time lost in a week

$$\Delta t = \frac{\Delta T}{T'} \times t = \frac{1.2 \times 10^{-4}}{2.00012} \times (7 \times 24 \times 3600) = 36.23\text{ s}$$

12. (a)

13. (c)

14. (b) Since, each rod is prevented from expansion so they are under compression and mechanical strain. The net strain in each rod

$$\epsilon_1 = \alpha_1 \Delta T - \frac{F l}{A Y_1} \Rightarrow \epsilon_2 = \alpha_2 \Delta T - \frac{F l}{A Y_2}$$

$$\epsilon_1 = \epsilon_2 = 0$$

$$\alpha_1 \Delta T - \frac{F l}{A Y_1} = 0 \text{ and } \alpha_2 \Delta T - \frac{F l}{A Y_2}$$

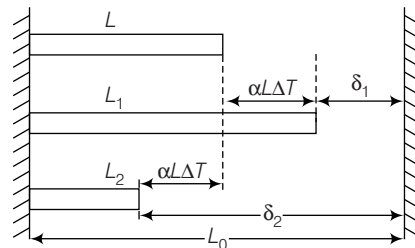
Solving, we get

$$\alpha_1 Y_1 = \alpha_2 Y_2$$

15. (c) Let L = free length at 0°C

L_0 = final stretched length in each case.

L_1 and L_2 are free lengths at $+20^\circ\text{C}$ and -20°C , respectively.



We know,

$$s_2 = s_1 + 2\alpha L \Delta T$$

Here, s_1 and s_2 are load deformation

$$\frac{F_2 L}{A Y} = \frac{F_1 L}{A Y} + 2\alpha L \Delta T$$

$$\Rightarrow \sigma_2 = \frac{F_2}{A} = \frac{F_1}{A} + 2\alpha \Delta T Y$$

$$= \frac{5000}{150 \times 10^{-6}} + (2)(11.7 \times 10^{-6})(20)(2 \times 10^{11})$$

$$= 127 \times 10^6 \text{ N/m}^2$$

16. (c)

17. (b) According to relation,

Coefficient of apparent expansion = coefficient of real expansion – coefficient of volume expansion

$$C = \gamma_L - 3A \quad \dots(i)$$

$$S = \gamma_L - 3A_0 \quad \dots(ii)$$

where, A_0 = linear expansion of coefficient of silver.

Subtracting Eq. (i) from Eq. (ii), we get

$$S - C = -3A_0 + 3A \therefore A_0 = \left(\frac{3A + C - S}{3}\right)$$

18. (b) $\rho_1 + \rho_1 \gamma_2 \Delta T = \rho_2 + \rho_2 \gamma_1 \Delta T$

$$\Delta T = \frac{\rho_1 - \rho_2}{\rho_2 \gamma_1 - \rho_1 \gamma_2}$$

$$\Delta T_2 = \frac{900 - 800}{880 \times 1.2 \times 10^{-3} - 900 \times 1.5 \times 10^{-4}} \Rightarrow \Delta T = 21.7^\circ\text{C}$$

19. (d) Since, coefficient of volume expansion,
 $\gamma = 3 \times \text{coefficient of linear expansion}$

$$\Rightarrow \gamma = 3\alpha$$

Now, $\frac{1}{\Delta T} \cdot \left(\frac{\Delta V}{V} \right) = 3\alpha \Rightarrow \Delta V = 3V \cdot \alpha \cdot \Delta T$

$$\Rightarrow \Delta V = \frac{4}{3} \pi R^3 \times 3\alpha \times \Delta T = 4\pi R^3 \alpha \Delta T$$

20. (a) If rods are heated to $\Delta t^\circ\text{C}$, the increases in length of steel and copper rods are

$$\Delta L_s = L_s \alpha_s \Delta t, \quad \text{and} \quad \Delta L_c = L_c \alpha_c \Delta t$$

Difference between their lengths will remain constant.

$$\Rightarrow \Delta L_s = \Delta L_c$$

$$\Rightarrow L_s \alpha_s \Delta t = L_c \alpha_c \Delta t \text{ or } \frac{L_s}{L_c} = \frac{\alpha_c}{\alpha_s}$$

21. (a) $dW = dQ - dU$

$$= C_p (T_2 - T_1) - C_v (T_2 - T_1) = R [T_2 - T_1]$$

$$= 8.31 \times 100 = 8.31 \times 10^2 \text{ J}$$

22. (d) 23. (c) 24. (a)

25. (b) Let $V \text{ cm}^3$ be volume of either liquid

Mass of water = $V \times 1 \text{ g}$

Mass of alcohol = $V \times 0.8 = 0.8V \text{ g}$

Rate of cooling of the water calorimeter

$$= \frac{1}{100} [V \times (50^\circ - 40^\circ) + V \times 1 (50^\circ - 40^\circ)] = \frac{1}{5} V \text{ cal/s}$$

Rate of cooling of alcohol calorimeter

$$= \frac{1}{74} [V \times (50^\circ - 40^\circ) + 0.8 V \times s (50^\circ - 40^\circ)]$$

$$= \frac{1}{74} (10V + 8Vs) \text{ cal/s}$$

As, rate of cooling of both is same

$$5V = \frac{1}{74} (10V + 8Vs) \Rightarrow s = 0.6 \text{ cal/g}^\circ\text{C}$$

26. (b) $\frac{m_A c_A}{m_B c_B} = \frac{\left(\frac{4}{3}\right) \pi r_A^3 \rho_A c_A}{\left(\frac{4}{3}\right) \pi r_B^3 \rho_B c_B} = \left(\frac{r_A}{r_B}\right)^3 \frac{\rho_A c_A}{\rho_B c_B}$

$$\frac{m_A c_A}{m_B c_B} = \left(\frac{1}{2}\right)^3 \times \frac{2}{1} \times \left(\frac{1}{3}\right) = \frac{1}{12}$$

27. (c) $dQ = nCdT, dQ = nk \frac{T^3}{\theta^3} dT$

$$Q = \frac{nk}{\theta^3} \int_{T_1}^{T_2} T^3 dT = \frac{nk}{\theta^3} \left(\frac{T_2^4 - T_1^4}{4} \right)$$

$$= \frac{2 \times 1940 (50^4 - 10^4)}{(281)^3 \times 4} = 272.79 \text{ J} \Rightarrow Q = 273 \text{ J}$$

28. (d)

29. (b) $\Delta H = \int ms dt$

$$= m \int_0^{t_0} (at^2 + bt + c) dt = m \left[\frac{at_0^3}{3} + \frac{bt_0^2}{2} + ct_0 \right]$$

30. (a) $\bar{s} = \int_0^{t_0} s dt = \int_0^{t_0} (at^2 + bt + c) dt = \frac{at_0^3}{3} + \frac{bt_0^2}{2} + ct_0$

31. (a) $P = \frac{W}{t}$

Total work done by drill machine in $2.5 \times 60 \text{ s}$
 $= (10 \times 10^3)(2.5 \times 60) = 15 \times 10^5 \text{ J}$

$\therefore$ Energy lost = 50% of $15 \times 10^5 \text{ J} = 7.5 \times 10^5 \text{ J}$

Energy taken by its surroundings, i.e. aluminium block.

$$\Delta Q = mc\Delta t = 8 \times 10^3 \times 0.91 \times \Delta T \text{ J}$$

Energy given = Energy taken

$$7.5 \times 10^5 = 8 \times 10^3 \times 0.91 \times \Delta T$$

$$\Delta T = 103^\circ\text{C}$$

32. (a) $\therefore dH = Pdt \therefore C_p = \frac{Pdt}{dT} = \frac{P}{\frac{dT}{dt}}$

Here, $T = at^{1/4}$ or $\frac{T}{a} = t^{1/4} \therefore t = \frac{T^4}{a^4}$

$$\therefore \frac{dT}{dt} = \frac{a}{4} \left(\frac{T^4}{a^4} \right)^{-3/4} = \frac{a}{4} \left(\frac{T}{a} \right)^{-3} = \frac{a^4}{4T^3} \Rightarrow C_p = \frac{P}{\frac{dT}{dt}} = \frac{4PT^3}{a^4}$$

33. (a)

34. (a) Total heat required to convert the temperature of ice from -20°C to 0°C water is

$$\Delta H = mL + ms \Delta \theta = 5 \times 80 + 5 \times 0.5 \times [0^\circ - (-20^\circ)]$$

$$= 400 + 2.5 \times 20 = 400 + 50 = 450 \text{ cal}$$

The total heat released to loss the temperature of water from 30°C to water at 0°C is

$$\Delta H = 5 \times 1 \times (30 - 0) = 150$$

Since, heat released is lesser than heat required. Hence, total ice is not converted into water. Hence, final temperature of mixture is 0°C .

35. (d) Let initial mass of water in the container be m .

m_1 = the mass of water converted into ice.

At final thermal equilibrium state, temperature of mixture should be zero.

$\therefore$ Heat loss by water = heat of fusion of water

or $ms [0 - (-10)] = m_1 L$

or $m \times 1 \times 10 = m_1 \times 80$ or $\frac{m_1}{m} = \frac{10}{80} = \frac{1}{8}$

36. (a) Let R = rate at which heat is received

$$Q_1 = R \times A \times t = R \times \pi (2.5)^2 \times (20 \text{ min}) = 392.5 R \text{ cal}$$

Heat required to melt the 10 g of ice

$$Q_2 = mL = 80 \times 10 = 800 \text{ cal}$$

$$Q_1 = Q_2 \Rightarrow 392.5 R = 800$$

$$R = \frac{800}{392.5} = 2.04 \text{ cal/cm}^2 \cdot \text{min}$$

37. (a) Specific heat of water = 4200 J/kg·K

Specific latent heat of fusion = 3.36×10^5 J/kg

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1. (a) Mass of metal, $m = 0.2 \text{ kg} = 200 \text{ g}$

Fall in temperature of metal $\Delta T = 150 - 40 = 110^\circ \text{C}$

If s is specific heat of metal, the heat lost by the metal

$$\Delta Q = ms \Delta T = 200 \times 110 \times s$$

Volume of water, = 150 cc

Mass of water $m' = 150 \text{ g}$

Water equivalent of calorimeter

$$w = 0.025 \text{ kg} = 25 \text{ g}$$

Rise in temperature of water in calorimeter

$$\Delta T' = 40 - 27 = 13^\circ \text{C}$$

Heat gained by water and calorimeter

$$\Delta Q' = (m' + w) \Delta T' = (150 + 25) \times 13$$

$$\Delta Q' = 175 \times 13 = \Delta Q$$

$$\text{So, } 200 \times s \times 110 = 175 \times 13$$

$$\Rightarrow s = \frac{175 \times 13}{200 \times 110} \approx 0.1$$

2. (c) Let T be the junction temperature

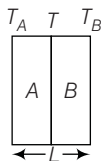
Here, $K_A = 2K_B$, $T - T_B = 50 \text{ K}$

At the steady state $H_A = H_B$

$$\Rightarrow \frac{K_A A (T_A - T)}{L} = \frac{K_B A (T - T_B)}{L}$$

$$\Rightarrow 2K_B (T_A - T) = K_B (T - T_B)$$

$$\Rightarrow T_A - T = \frac{T - T_B}{2} = \frac{50}{2} = 25 \text{ K}$$



3. (a) In an adiabatic process,

$$pV^\gamma = \text{constant}$$

$$\Rightarrow \frac{p_1}{p_2} = \left(\frac{V_2}{V_1} \right)^\gamma \Rightarrow \frac{p_1}{p_2} = \left(\frac{1}{8} \right)^{5/3}$$

$$\Rightarrow \frac{p_1}{p_2} = \left(\frac{1}{2^3} \right)^{5/3} = \frac{1}{32} \therefore \frac{p_2}{p_1} = 32$$

4. (c) Work done, $W = p \Delta V = 1.013 \times 10^5 \times (1671 - 1) \times 10^{-6}$
 $= 1.013 \times 10^5 \times 1670 \times 10^{-6} = 169.2 \text{ J}$

$$5. (b) \frac{\Delta T}{l} = \frac{80 - 0}{1} = 80 \text{ cm}^{-1}$$

$$\Delta T_1 = (0.60)(80) = 48^\circ \text{C}$$

Decrease in temperature after a distance of 60 cm is 48°C and hence actual temperature is $(80 - 48)^\circ \text{C} = 32^\circ \text{C}$.

6. (a) Thermoelectric power,

$$S = \frac{dE}{dT} = \frac{d}{dT} \left[k(T - T_r) \cdot \left\{ T_0 - \frac{1}{2}(T + T_r) \right\} \right]$$

Specific latent heat of vaporisation

$$= 22.68 \times 10^5 \text{ J/kg} = X \times 10^{-3} \times 22.68 \times 10^5$$

$$= Y \times 10^{-3} \times 3.36 \times 10^5 + Y \times 10^{-3} \times 4200 \times 100$$

$$\therefore \frac{X}{Y} = \frac{7.56}{22.68} = \frac{1}{3}$$

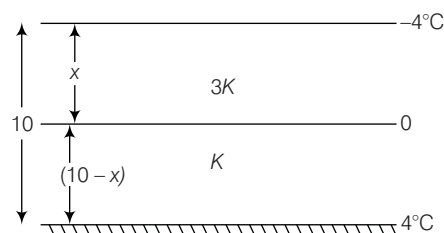
$$S = k(T - T_r) \times \left(-\frac{1}{2} \right) + \left[T_0 - \frac{1}{2}(T + T_r) \right] \times k$$

$$\text{When } T = \frac{T_0}{2}$$

$$S = -\frac{k}{2} \left(\frac{T_0}{2} - T_r \right) + \left\{ T_0 - \frac{1}{2} \left(\frac{T_0}{2} + T_r \right) \right\} \times k$$

$$= -\frac{kT_0}{4} + \frac{kT_r}{2} + kT_0 - \frac{kT_0}{4} - \frac{kT_r}{2} = \frac{1}{2} kT_0$$

$$7. (a) KA \left(\frac{4 - 0}{10 - x} \right) = 3KA \left(\frac{0 - (-4)}{x} \right)$$



$$\Rightarrow x = 7.5 \text{ m}$$

$$8. (c) \text{ From, } Y = \frac{F/A}{\Delta L/L} = \frac{T \cdot L}{A \Delta L}$$

where, $F = T = \text{tension}$.

On cooling, $\Delta L = \alpha L (\Delta \theta) = \alpha L t$

$$\therefore Y = \frac{TL}{A \alpha L t}$$

$$T = Y A \alpha t$$

Also, mass per unit length of string

$$m = A \rho$$

$$\text{As wave velocity} = \sqrt{\left(\frac{T}{m} \right)} \therefore v = \sqrt{\frac{Y A \alpha t}{A \rho}} = \sqrt{\left(\frac{Y \alpha t}{\rho} \right)}$$

$$9. (c) \text{ Modulus of elasticity} = \frac{\text{Force}}{\text{Area}} \times \frac{l}{\Delta l}$$

$$3 \times 10^{11} = \frac{33000}{10^{-3}} \times \frac{l}{\Delta l}$$

$$\frac{\Delta l}{l} = \frac{33000}{10^{-3}} \times \frac{1}{3 \times 10^{11}} = 11 \times 10^{-5}$$

$$\text{Change in length, } \frac{\Delta l}{l} = \alpha \Delta T$$

$$11 \times 10^{-5} = 11 \times 10^{-5} \times \Delta T \Rightarrow \Delta T = 10 \text{ K or } 10^\circ \text{C}$$

16

Physics for Gaseous State

Ideal Gas or Perfect Gas

An ideal gas or perfect gas is that gas which strictly obeys the gas laws such as Boyle's law, Charles' law, Gay-Lussac's law, etc., at all values of temperature and pressure.

Gas Laws

There are some laws given below

Boyle's Law

It states that for a given mass of an ideal gas at constant temperature (called isothermal process), the volume of a gas is inversely proportional to its pressure, i.e.

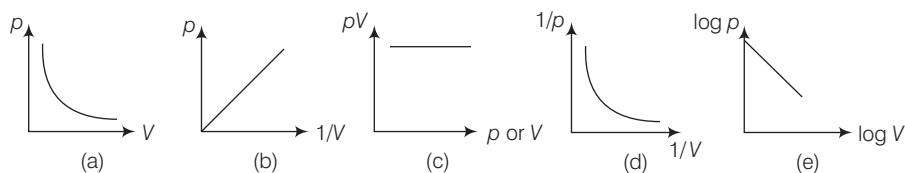
$$V \propto \frac{1}{p}$$

(where, V = volume, p = pressure, m = mass and T = temperature or $pV = \text{constant}$)

or

$$p_1V_1 = p_2V_2 = p_3V_3 = \dots$$

This law can also be shown graphically as



(m and T are constants in all cases)

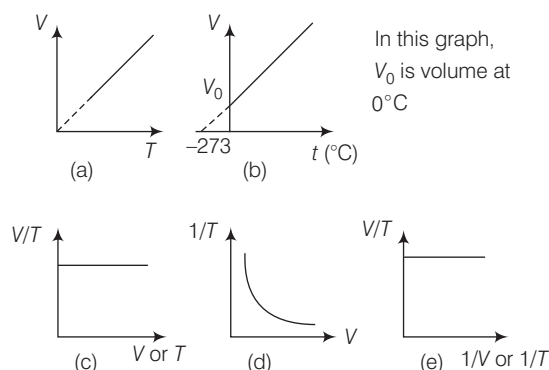
Charles' Law

It states that for a given mass of an ideal gas at constant pressure, (called isobaric process) volume of a gas is directly proportional to its absolute temperature

i.e. $V \propto T$ [if m and p are constants]

or $\frac{V}{T} = \text{constant}$ or $\frac{V_1}{T_1} = \frac{V_2}{T_2} = \dots$

This law can also be shown graphically as



(m and p are constants in all cases)

Gay-Lussac's Law or Pressure Law

It states that for a given mass of an ideal gas at constant volume (called isochoric process), pressure of a gas is directly proportional to its absolute temperature

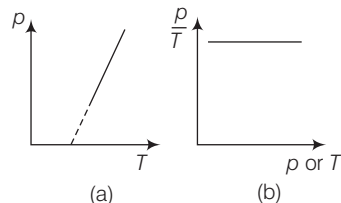
i.e. $p \propto T$ [if m and V are constants]

or $\frac{p}{T} = \text{constant}$

or $\frac{p_1}{T_1} = \frac{p_2}{T_2} = \dots$

Here, temperature is in kelvin.

This law can also be shown graphically as



(m and V are constants in all cases)

Ideal Gas Equation

An ideal gas is one whose molecules are free from intermolecular attraction and obeys gas laws at all values of temperature and pressure.

Ideal gas equation is a form of combined effect of above four laws, thus its equation is given by

$$pV = nRT = \frac{m}{M} RT$$

Kinetic Theory of Gases

The kinetic theory of gases is based on the following assumptions

1. Molecules of a gas are small hard spheres and these molecules are very far apart in comparison to their sizes.
2. The total volume of the molecules is negligible as compared to the size of the gas.
3. The molecules collide elastically with each other.
4. The molecules exert no force on each other except during collisions.

Concept of Pressure

The pressure of the gas is equal to the force exerted per unit area of the walls by the gas.

Pressure of an ideal gas

$$p = \frac{1}{3} \frac{mn}{V} v_{\text{rms}}^2 = \frac{1}{3} \rho v_{\text{rms}}^2$$

Kinetic Interpretation of Temperature

According to kinetic theory of gases, for 1 mole of an ideal gas

$$p = \frac{1}{3} \frac{M}{V} (v_{\text{rms}})^2$$

$$\Rightarrow pV = \frac{1}{3} M (v_{\text{rms}})^2 \quad \dots(i)$$

(here, M = total mass)

So, $(v_{\text{rms}})^2 = \frac{3RT}{M}$ (for 1 mole, i.e. $n = 1$) ... (ii)

Also, $pV = RT$
 $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$$\Rightarrow v_{\text{rms}} \propto \sqrt{T}$$

The absolute temperature of an ideal gas is directly proportional to mean square velocity of its molecule.

Also, $\frac{1}{3} M v_{\text{rms}}^2 = RT$ [From Eqs. (i) and (ii)]

$$\Rightarrow \frac{1}{3} \frac{M}{N_0} v_{\text{rms}}^2 = \frac{R}{N_0} T \Rightarrow \frac{1}{2} m v_{\text{rms}}^2 = \frac{3}{2} k_B T$$

$$\left(k_B = \frac{R}{N_0} = \text{Boltzmann constant} \right)$$

$$\Rightarrow (\text{KE})_{\text{trans}} = \frac{3}{2} k_B T$$

where, $\frac{1}{2} m v_{\text{rms}}^2$ is average translational energy per molecule

$$(\text{KE})_{\text{trans}} \propto T$$

Relation between Pressure and Kinetic Energy of the Gas

By using the expression for pressure (p) in terms of mean square speed ($\bar{v}^2$), we can find the relation between pressure and kinetic energy (E) as shown below.

$$\text{Since, } p = \frac{1}{3} \rho (\bar{v}^2) \quad \dots(i)$$

Mass of unit volume of gas = $1 \times \rho = \rho$

Mean kinetic energy of translation per unit volume of the gas is

$$E = \frac{1}{2} \rho (\bar{v}^2) \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{p}{E} = \frac{2}{3} \text{ or } p = \frac{2}{3} E \quad (\text{for unit volume})$$

The pressure exerted by an ideal gas is numerically equal to two-third of the mean kinetic energy of translation per unit volume of the gas.

Various Speeds of Gas Molecules

RMS Speed of Gas Molecules

The square root of the mean of the squares of the speed of gas molecules is called their root mean square speed (v_{rms}).

$$v_{\text{rms}} = \sqrt{\frac{3RT}{m}} \quad [v_{\text{rms}} \propto \sqrt{T}]$$

where, m = molecular weight.

Mean Speed or Average Speed

It is the arithmetic mean of the speeds of molecules in a gas. If $v_1, v_2, \dots, v_n$ are individual speeds of n molecules, then

$$v_{\text{av}} = \frac{v_1 + v_2 + \dots + v_n}{n}$$

According to kinetic theory of gases, it can be shown that

$$v_{\text{av}} = \sqrt{\frac{8k_B T}{\pi m}}$$

Most Probable Speed

It is defined as speed which is possessed by maximum fraction of total number of molecules of gas.

According to kinetic theory of gases, it can be shown that

$$v_{\text{mp}} = \sqrt{\frac{2k_B T}{m}}$$

Degree of Freedom

Degree of freedom represents the number of independent possible ways in which the system can have energy (due to its motion or configuration). A system can possess energy due to translational, rotational or vibrational motion or due to vibrational configuration or any combination of these.

- (i) For an ideal monoatomic gas, $f = 3$ due to translational motion in three directions.
- (ii) For an ideal diatomic gas at room temperature $f = 5$ due to 3 translational and 2 rotational. At high temperature, $f = 7$ because of 2 additional ways due to vibration (one kinetic energy and other potential energy).
- (iii) For an ideal polyatomic gas at room temperature, $f = 6$ due to 3 translational and 3 rotational. At high temperature, the vibrational mode will also come into existence. For a polyatomic gas, number of vibrational terms can be greater than two.
- (iv) For solids, $f = 6$ due to vibrational motion in all three directions.

Law of Equipartition of Energy

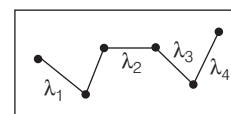
Total internal energy of an ideal gas distributes equally in all degrees of freedom and energy per degree of freedom of one mole of gas is $\frac{1}{2} RT$, where T is the absolute temperature of the gas.

If f is the degree of freedom and n is number of moles of the gas, then the internal energy of the gas is given by

$$U = \frac{n}{2} f RT$$

Mean Free Path

Every gas consists of a large number of molecules undergoing frequent collisions. These molecules are in a state of continuous random motion. They undergo perfectly elastic collision against one another. The zig-zag path of different lengths is called free path and their mean is called **mean free path**. If $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$ are successive free paths, then



$$\text{Mean free path, } \lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 + \dots + \lambda_n}{n}$$

So, mean free path of gas molecules is average distance travelled by a molecule between two successive collisions.

Mathematically, it can be expressed as

$$\lambda = \frac{1}{\sqrt{2} \pi d^2 n}$$

where, n = number of molecules per unit volume
 d = diameter of molecules

Specific Heat Capacity

It is the amount of heat required to raise the temperature of unit mass of a substance through 1°C . Consider a container containing m gram of gas of molecular mass M . If n is number of moles of gas in a container, ΔQ is the heat supplied and rise in temperature is ΔT , then

Specific heat, $c = \frac{1}{m} \frac{\Delta Q}{\Delta T}$ and SI unit = $\text{J kg}^{-1} \text{K}^{-1}$

Molar Specific Heat of Gases

Molar specific heat (C) of a substance is defined as the amount of heat required to raise the temperature of 1 mole of substance through one degree celsius

$$C = \frac{1}{n} \frac{\Delta Q}{\Delta T}$$

Also, relation between specific heat and molar specific heat

$$C = Mc$$

Here, M = molar mass of gas, n = number of moles,

C = molar specific heat and c = specific heat

(or gram specific heat)

It is of two types

Specific Heat at Constant Volume (C_V)

When heat is supplied to gas at constant volume, the entire heat supplied just increases the internal energy of gas, so specific heat at constant volume is

$$C_V = \frac{1}{n} \left(\frac{\Delta Q}{\Delta T} \right)_V$$

$\Rightarrow$

$$C_V = \frac{1}{n} \left(\frac{\Delta U}{\Delta T} \right)$$

Specific Heat at Constant Pressure (C_p)

When heat is supplied to the gas at constant pressure, a part of it increases the internal energy of the gas and remaining does an external work, so specific heat at constant pressure is

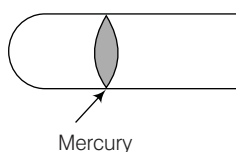
$$C_p = \frac{1}{n} \left(\frac{\Delta Q}{\Delta T} \right)_p$$

At constant pressure, to increase internal energy of gas by the same amount (as in case of heat supplied at constant volume), more amount of heat has to be supplied, so we conclude $C_p > C_V$.

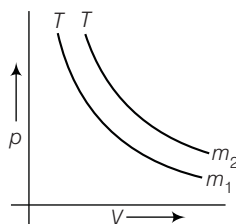
Practice Exercise

- In an ideal gas without preferred direction of motion of molecules,
 - $v_x = v_y = v_z$
 - $v_x^2 = v_y^2 = v_z^2$
 - $\bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2$
 - None of the above
- Two perfect gases at absolute temperatures T_1 and T_2 are mixed. The absolute temperature of the mixture is T . There is no loss of energy. If m_1 and m_2 are masses of molecules and n_1 and n_2 are number of molecules, then
 - $T = \frac{T_1 + T_2}{2}$
 - $T = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}$
 - $T = \frac{n_1 T_1 + n_2 T_2}{T_1 + T_2}$
 - None of the above
- In troposphere, temperature varies linearly with elevation as $T = T_0 - ay$, where T_0 is the temperature at the earth's surface, then
 - the pressure does not change with elevation in troposphere
 - the variation of pressure with elevation is linear
 - the dimension of a is $[\text{M}^0 \text{L}^{-1} \theta]$
 - the pressure is independent of variation with temperature in the given situation
- A vessel contains a mixture of nitrogen of mass 7 g and carbon dioxide of mass 11 g at temperature 290 K and pressure 1 atm. Find the density of the mixture.
 - 1.1 g/L
 - 1.2 g/L
 - 1.515 g/L
 - 1.6 g/L
- 12 g of gas occupy a volume of $4 \times 10^{-3} \text{ m}^3$ at a temperature of 7°C . After the gas is heated at constant pressure, its density becomes $6 \times 10^{-4} \text{ g/cm}^3$. What is the temperature to which the gas was heated?
 - 1000 K
 - 1400 K
 - 1200 K
 - 800 K

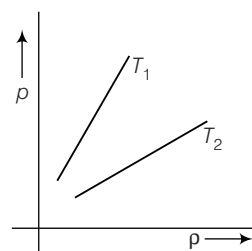
6. A closed vessel with a capacity of 1 m^3 contains 0.9 kg of water and 1.6 kg of O_2 . Find the pressure in the vessel at a temperature of 500°C at which all the water will be converted into steam.
- a. $3.2 \times 10^5 \text{ N/m}^2$ b. $6.4 \times 10^5 \text{ N/m}^2$
c. $1.6 \times 10^5 \text{ N/m}^2$ d. $9.6 \times 10^5 \text{ N/m}^2$
7. The pressure of a gas kept in an isothermal container is 200 kPa . If half the gas is removed from it, the pressure will be
a. 100 kPa b. 200 kPa c. 400 kPa d. 800 kPa
8. The pressure inside a tyre is 4 atm at 27°C . If the tyre bursts suddenly, its final temperature will be
a. $300(4)^{7/2}$ b. $300(4)^{2/7}$ c. $300(2)^{7/2}$ d. $300(4)^{-2/7}$
9. A uniform tube is shown in figure. Which is open at one end and closed at the other. To enclose a column of air inside the tube, a pellet of mercury is introduced. If the length of air column at 27°C is 18 cm , at what temperature its length will be 21.6 cm ?



- a. 87°C b. 91°C c. 85°C d. 97°C
10. Pressure of an ideal gas is increased by keeping temperature constant. What is its effect on kinetic energy of molecules?
a. Increase b. Decrease
c. No change d. Cannot be determined
11. How many cylinders of hydrogen at atmospheric pressure are required to fill a balloon whose volume is 500 m^3 , if hydrogen is stored in cylinders of volume 0.05 m^3 at an absolute pressure of $15 \times 10^5 \text{ Pa}$?
a. 700 b. 675 c. 605 d. 710
12. Two identical containers A and B have frictionless pistons. Both contain same volume of ideal gas at same temperature. The gas in each cylinder is allowed to expand isothermally to double the initial volume. The mass of the gas in A is m_A and the mass of the gas in B is m_B . The changes in the pressure in A and B are Δp and $1.5 \Delta p$, respectively, then
a. $4m_A = 9m_B$ b. $2m_A = 3m_B$ c. $3m_A = 2m_B$ d. $9m_A = 4m_B$
13. The figure shows, the p - V diagram of two different masses m_1 and m_2 drawn at constant temperature T , then
a. $m_1 > m_2$
b. $m_2 > m_1$
c. $m_1 = m_2$
d. insufficient data



14. Figure shows graph of pressure *versus* density for an ideal gas at two temperatures T_1 and T_2 , then



- a. $T_1 > T_2$
b. $T_1 = T_2$
c. $T_1 < T_2$
d. any three is possible
15. A gas is contained in a closed vessel at 250 K , then calculate the percentage increase in pressure, if the gas is heated through 1°C .
a. 0.4% b. 0.6%
c. 0.8% d. 1.0%
16. What volume will be occupied by the molecules contained in 4.5 kg water at STP, if the intermolecular forces vanish away?
a. 5.6 m^3 b. 4.5 m^3
c. 11.2 m^3 d. 5.6 L
17. Find the pressure exerted by 6×10^{23} hydrogen molecules which will strike per second a wall of area 10^{-4} m^2 at 60° with normal. The mass of hydrogen molecules and speed are $3.32 \times 10^{-27} \text{ kg}$ and 10^3 m/s respectively.
a. $19.92 \times 10^3 \text{ N/m}^2$
b. $18.2 \times 10^3 \text{ N/m}^2$
c. $1.992 \times 10^3 \text{ N/m}^2$
d. $0.1992 \times 10^3 \text{ N/m}^2$
18. Four molecules of a gas have speeds $1, 2, 3$ and 4 km/s . The value of the root mean square speed of the gas molecules is
a. $\frac{1}{2} \sqrt{15} \text{ km/s}$ b. $\frac{1}{2} \sqrt{10} \text{ km/s}$
c. 2.5 km/s d. $\sqrt{\left(\frac{15}{2}\right)} \text{ km/s}$
19. The temperature of H_2 at which the rms velocity of its molecules is seven times the rms velocity of the molecules of nitrogen at 300 K , is
a. 2100 K b. 1700 K c. 1350 K d. 1050 K
20. Choose the correct order of the root mean square velocity (v_{rms}), the average velocity (v_{av}) and the most probable velocity (v_{mp}).
a. $v_{\text{mp}} > v_{\text{av}} > v_{\text{rms}}$ b. $v_{\text{rms}} > v_{\text{av}} > v_{\text{mp}}$
c. $v_{\text{av}} > v_{\text{mp}} > v_{\text{rms}}$ d. $v_{\text{mp}} > v_{\text{rms}} > v_{\text{av}}$
21. The average kinetic energy of a gas molecule at 27°C is $6.21 \times 10^{-21} \text{ J}$. Its average kinetic energy at 127°C will be
a. $12.2 \times 10^{-21} \text{ J}$
b. $8.28 \times 10^{-21} \text{ J}$
c. $10.35 \times 10^{-21} \text{ J}$
d. $11.35 \times 10^{-21} \text{ J}$

22. Ratio of v_{mp} (most probable speed), v_{av} (average speed), v_{rms} (root mean square speed) of gas molecules are related as

a. $\sqrt{3} : \sqrt{2} : \sqrt{\frac{8}{\pi}}$ b. $2 : \sqrt{3} : \sqrt{\frac{\pi}{8}}$
 c. $\sqrt{2} : \sqrt{3} : \sqrt{\frac{8}{\pi}}$ d. $\sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3}$

23. Five gas molecules chosen at random are found to have speeds of 500, 600, 700, 800 and 900 m/s. Then,

- a. the rms speed and the average speed are the same
 b. the rms speed is 14 m/s higher than the average speed
 c. the rms speed is 14 m/s lower than the average speed
 d. the rms speed is $\sqrt{14}$ m/s higher than the average speed

24. In case of molecules of an ideal gas, which of the following, average velocities cannot be zero?

- a. $\langle \vec{v} \rangle$ b. $\langle \vec{v}^3 \rangle$
 c. $\langle \vec{v}^4 \rangle$ d. $\langle \vec{v}^5 \rangle$

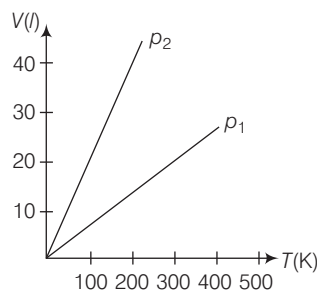
25. Choose the correct relation between the rms speed (v_{rms}) of the gas molecules and the velocity of sound in that gas (v_s) in identical situations of pressure and temperature.

a. $v_{rms} = v_s$ b. $v_{rms} = \sqrt{\left(\frac{3}{\gamma}\right)} v_s$
 c. $v_{rms} = \sqrt{\left(\frac{\gamma}{3}\right)} v_s$ d. $\gamma v_{rms} = 3 v_s$

26. At what temperature is the effective speed of gaseous hydrogen molecules equal to that of oxygen molecules at 47°C ?

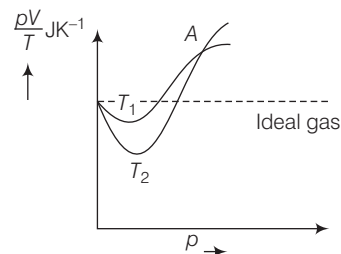
- a. 50 K b. 20 K
 c. 40 K d. 100 K

27. Volume *versus* temperature graphs for a given mass of an ideal gas are shown in figure at two different values of constant pressure. What can be inferred about relation between p_1 and p_2 ?

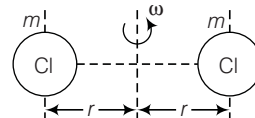


- a. $p_1 > p_2$ b. $p_1 < p_2$
 c. $p_1 = p_2$ d. Data is insufficient

28. Given the graph between $\frac{pV}{T}$ and p for 1 g of oxygen gas at two different temperatures T_1 and T_2 (in figure). Given, density of oxygen = 1.427 kg/m^3 . The value of $\frac{pV}{T}$ at point A and the relation between T_1 and T_2 are respectively



- a. 0.256 JK^{-1} and $T_1 < T_2$
 b. $8.314 \text{ J mol}^{-1} \text{ K}^{-1}$ and $T_1 > T_2$
 c. 0.256 JK^{-1} and $T_1 > T_2$
 d. 4.28 JK^{-1} and $T_1 < T_2$
29. Which of the following quantities is zero on an average for the molecules of an ideal gas in equilibrium?
- a. Kinetic energy b. Momentum
 c. Density d. Speed
30. On a fast moving train, a container is placed enclosing some gas at 300 K, while the train is in motion, the temperature of the gas
- a. rises above 300 K b. falls below 300 K
 c. remains unchanged d. becomes unsteady
31. If at a pressure of 10^6 dyne/cm^2 , one gram mole of nitrogen occupies $2 \times 10^4 \text{ cc}$ volume, then calculate the average energy of a nitrogen molecules in erg. (Given, Avogadro's number = 6×10^{23})
- a. 14×10^{-13} b. 10×10^{-12} c. 10^6 d. 2×10^6
32. Calculate the temperature at which average translational KE of a molecule is equal to the KE of an electron accelerated from rest through a potential difference of 1 V.
- a. $T = 7729 \text{ K}$ b. $T = 8879 \text{ K}$
 c. $T = 7.72 \text{ K}$ d. $T = 772.9 \text{ K}$
33. In a model of chlorine (Cl_2), two Cl-atoms are rotated about their centre of mass as shown. Here the two Cl-atoms are $2 \times 10^{-10} \text{ m}$ apart and angular speed $\omega = 2 \times 10^{12} \text{ rad/s}$. If the molar mass of chlorine is 70 g/mol, then what is the rotational kinetic energy of one Cl_2 molecule?
- a. $2.32 \times 10^{-20} \text{ J}$ b. $2.32 \times 10^{-21} \text{ J}$
 c. $2.32 \times 10^{-19} \text{ J}$ d. $2.32 \times 10^{-22} \text{ J}$



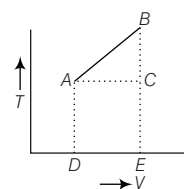
34. At 20 °C temperature, an argon gas at atmospheric pressure is confined in a vessel with a volume of 1 m³. The effective hard sphere diameter of argon atom is 3.10×10^{-10} m. Determine mean free path.
- 100 nm
 - 90 nm
 - 93.6 nm
 - 95 nm

BITSAT Archives

- Two balloons are filled one with pure He gas and other with air respectively. If the pressure and temperature of these balloons are same, then the number of molecules per unit volume is [2014]
 - more in He filled balloon
 - same in both balloons
 - more in air filled balloon
 - in the ratio 1 : 4
- A vessel containing 1 mole of O₂ gas (molar mass 32) at temperature T . The pressure of the gas is p . An identical vessel containing one mole of He gas (molar mass 4) at temperature $2T$ has a pressure of [2013]
 - $p/8$
 - p
 - $2p$
 - $8p$
- The temperature of an ideal gas is increased from 27 °C to 127 °C, then percentage increase in v_{rms} is [2013]
 - 37 %
 - 11%
 - 33%
 - 15.5%
- The ratio of the adiabatic bulk modulus to the isothermal bulk modulus of a perfect gas with f degrees of freedom is [2010]
 - $\frac{2}{f}$
 - $1 + \frac{1}{f}$
 - $1 + \frac{2}{f}$
 - $1 + \frac{(f-1)}{4}$
- If v is the molecular speed and l is the mean free path of molecule of a gas, then the collision frequency is [2010]
 - vl
 - $\frac{v}{l}$
 - $\frac{l}{v}$
 - $\sqrt{vl}$

35. In the case of saturated vapour,
- pressure depends upon volume at constant temperature
 - pressure varies non-linearly with temperature at constant volume
 - pressure becomes less than one atmosphere at boiling point
 - pressure varies linearly with temperature at constant volume

6. From the following V - T diagram, what is true about pressure? [2009]



- $p_1 < p_2$
 - $p_1 > p_2$
 - $p_1 = p_2$
 - Cannot predict
7. One litre of oxygen at a pressure of 1 atm and two litres of nitrogen at a pressure of 0.5 atm, are introduced into a vessel of volume 1 L. If there is no change in temperature, the final pressure of the mixture of gas (in atm) is [2008]
- 1.5
 - 2.5
 - 2
 - 4
8. The ratio of velocity of sound in hydrogen and oxygen at STP is [2005]
- 16 : 1
 - 8 : 1
 - 4 : 1
 - 2 : 1

Answer with Solutions

Practice Exercise

- (c) An isotropic gas is one which have same properties throughout and their molecules are all moving in random directions, so only average values of square of their velocity components are equal.
- (b) Average KE per molecule of a gas = $\frac{3}{2} kT$
Before mixing two gases, the average KE of all molecules of 2 gases

$$= \frac{3}{2} kn_1T_1 + \frac{3}{2} kn_2T_2$$

After mixing, the average KE of both gases

$$= \frac{3}{2} k (n_1 + n_2) T$$

where, T is temperature of mixture.

∴ No loss of energy.

$$\therefore \frac{3}{2} k (n_1 + n_2) T = \frac{3}{2} kn_1T_1 + \frac{3}{2} kn_2T_2$$

$$T = \frac{n_1T_1 + n_2T_2}{(n_1 + n_2)}$$

3. (c)

4. (c) $p_1 V = \frac{7}{28} \times R \times 290$

$$p_2 V = \frac{11}{44} \times R \times 290$$

$$\therefore (p_1 + p_2) V = \left(\frac{7}{28} + \frac{11}{44} \right) R \times 290$$

$$\therefore p_1 + p_2 = 1 \text{ atm}$$

$$V = \frac{\left(\frac{7}{28} + \frac{11}{44} \right) \times 8.3 \times 10^7 \times 290}{1.013 \times 10^6}$$

$$\therefore \text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{18}{V} = 1.515 \text{ g/L}$$

5. (b) Volume at $7^\circ\text{C} = 4 \times 10^{-3} \text{ m}^3$

Let it be heated to x K.

$$\begin{aligned} \text{Volume} &= \frac{12}{6 \times 10^{-4}} \text{ cm}^3 = 2 \times 10^4 \text{ cm}^3 \\ &= 2 \times 10^{-2} \text{ m}^3 \end{aligned}$$

$$\text{Since, } \frac{V}{T} = \text{constant}$$

$$\therefore \frac{4 \times 10^{-3}}{280} = \frac{2 \times 10^{-2}}{x}$$

$$\therefore x = \frac{2 \times 10^{-2} \times 280}{4 \times 10^{-3}} = 1400 \text{ K}$$

6. (b)

7. (a) For isothermal process, temperature remains constant. According to ideal gas equation,

$$pV = nRT$$

For a given container, V remains constant.

$$\therefore p \propto n$$

When half of gas is removed, number of moles becomes half. Hence, pressure becomes half.

8. (d) In an adiabatic process,

$$p_2^{(1-\gamma)} T_2^\gamma = p_1^{(1-\gamma)} T_1^\gamma$$

$$T_2 = T_1 \left(\frac{p_1}{p_2} \right)^{(1-\gamma)/\gamma} = 300 \left(\frac{4}{1} \right)^{(1-7/5)/7/5} = 300 (4)^{-2/7}$$

9. (a) From Charles' law, $L \propto T$

$$\frac{L_1}{T_1} = \frac{L_2}{T_2}$$

$$\Rightarrow T_2 = \frac{L_2}{L_1} \times T_1$$

$$T_2 = \frac{21.6}{18} \times 300$$

$$T_2 = 87^\circ\text{C}$$

10. (c) As kinetic energy depends on temperature, so there will be no change.

11. (b) $15 \times 10^5 \times V_2 = 500 \times 1.013 \times 10^5$

$$V_2 = 33.77 \text{ m}^3$$

$$\therefore \text{Number of cylinders} = \frac{33.77}{0.05} = 675$$

12. (c)

13. (b) We know that,

$$pV = nRT = \frac{m}{M} RT$$

$$m = (pV) \left(\frac{M}{RT} \right)$$

$$m \propto pV$$

$$p_2 V_2 > p_1 V_1$$

$$\text{From the curve, } m_2 > m_1$$

14. (a) According to ideal gas equation,

$$pV = nRT$$

$$pV = \frac{m}{M} RT$$

or

$$p = \frac{\rho RT}{M}$$

or

$$p = \frac{RT}{M} \rho$$

Hence, more temperature means more slope.

15. (a) $p_t = p_0 (1 + \gamma d\theta) = p_0 \left(1 + \frac{d\theta}{237} \right)$

$$\frac{p_t - p_0}{p_0} \times 100 = \frac{1}{273} \times 100$$

$$= 0.36\% \approx 0.4\%$$

16. (a) If intermolecular forces vanish away, the liquid will convert into vapours.

Molecular mass of water vapours

$$= 18 \text{ g} = 18 \times 10^{-3} \text{ kg}$$

$$\text{So, } 18 \times 10^{-3} \text{ kg of water vapour will occupy } 22400 \text{ m}^3$$

then, 4.5 kg of water vapour will occupy

$$\frac{22.4 \times 10^{-3}}{18 \times 10^{-3}} \times 4.5 = 5.6 \text{ m}^3$$

17. (a) Change in momentum $= 2mv \cos \theta$

$$\text{Force, } F = 2mv \cos \theta \times n$$

$$p = \frac{F}{A} = \frac{2mv n \cos \theta}{A}$$

$$= \frac{2 \times 3.32 \times 10^{-27} \times 10^3 \times \frac{1}{2} \times 6 \times 10^{23}}{10^{-4}}$$

$$= 19.92 \times 10^3 \text{ N/m}^2$$

18. (d) $\therefore v_{\text{rms}} = \sqrt{\left(\frac{v_1^2 + v_2^2 + \dots + v_n^2}{n} \right)}$

$$v_{\text{rms}} = \sqrt{\left(\frac{1^2 + 2^2 + 3^2 + 4^2}{4} \right)} = \sqrt{\left(\frac{1+4+9+16}{4} \right)}$$

$$= \sqrt{\left(\frac{30}{4} \right)} = \sqrt{\left(\frac{15}{2} \right)} \text{ km/s}$$

$$\therefore v_{\text{rms}} = \sqrt{\left(\frac{15}{2} \right)} \text{ km/s}$$

19. (d)

$$20. (b) v_{rms} = \sqrt{\left(\frac{3kT}{m}\right)}, v_{av} = \sqrt{\left(\frac{8kT}{\pi m}\right)},$$

$$v_{mp} = \sqrt{\left(\frac{2kT}{m}\right)}$$

$$\therefore v_{rms} > v_{av} > v_{mp}$$

$$21. (b) \text{ As, } KE \propto T \Rightarrow \frac{E_{127}}{E_{27}} = \frac{(127 + 273)}{(27 + 273)}$$

$$E_{127} = 6.21 \times 10^{-21} \times \frac{400}{300}$$

$$= 8.28 \times 10^{-21} \text{ J}$$

$$22. (d) v_{mp} : v_{av} : v_{rms} = \sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3}$$

$$23. (b) \therefore v_{av} = \frac{500 + 600 + 700 + 800 + 900}{5}$$

$$= \frac{3500}{5} = 700 \text{ m/s}$$

$$\text{and } v_{rms} = \sqrt{\frac{(500)^2 + (600)^2 + (700)^2 + (800)^2 + (900)^2}{5}}$$

$$= 714 \text{ m/s}$$

Hence, the rms speed is 14 m/s higher than the average speed.

24. (c) The average velocity of even power quantity cannot be zero.

$$25. (b) v_{rms} = \sqrt{\left(\frac{3RT}{m}\right)} \text{ and } v_s = \sqrt{\left(\frac{\gamma RT}{m}\right)}$$

$$\therefore v_{rms} = \sqrt{\left(\frac{3}{\gamma}\right)} v_s$$

26. (b)

27. (a) When pressure is constant, Charles' law is obeyed
i.e. $V \propto T$ or $\frac{V}{T} = \text{constant} = \frac{1}{p}$

From slope of curves, $p_1 > p_2$.

$$28. (c) pV = nRT = \frac{m}{M} RT$$

where, m = mass of gas

and $\frac{m}{M} = n$ = number of moles

$\Rightarrow \frac{pV}{T} = nR = \text{constant}$ for all values of p (ideally, it is a straight line).

$$\therefore \frac{pV}{T} = \frac{1 \text{ g}}{32 \text{ g}} \times 8.31 \text{ J mol}^{-1} \text{ K}^{-1} = 0.256 \text{ JK}^{-1}$$

Also, $T_1 > T_2$

29. (b) The average of speed, density and kinetic energy can never be zero. Because these quantities have only positive values. But momentum is a vector quantity. So, average value of momentum may be zero.

30. (a) Random motion of molecules is not ordered motion, causes rise of temperature.

$$31. (a) \text{ Average KE} = \frac{1}{2} m v_{rms}^2 = \frac{1}{2} m \frac{3p}{\rho} = \frac{1}{2} \frac{M}{N} \times \frac{3p}{\rho}$$

$$= \frac{1}{2} \times \frac{28}{6 \times 10^{23}} \times \frac{3 \times 10^6}{\left(\frac{1}{2 \times 10^4}\right)}$$

$$= 14 \times 10^{-13} \text{ erg}$$

32. (a)

33. (b) Let I = moment of inertia

$$I = 2 (mr^2)$$

$$m = \frac{70 \times 10^{-3}}{2 \times 6.02 \times 10^{23}} = 5.81 \times 10^{-26} \text{ kg}$$

$$r = \frac{2 \times 10^{-10}}{2} = 1 \times 10^{-10} \text{ m}$$

$$I = 1.16 \times 10^{-45} \text{ kg/m}^2$$

$$\therefore k = \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} (1.16 \times 10^{-45}) (2 \times 10^{12})^2$$

$$= 2.32 \times 10^{-21} \text{ J}$$

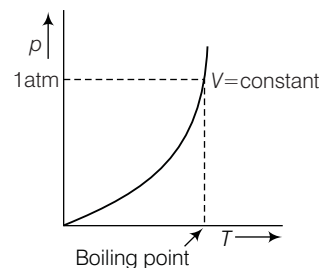
34. (c) Mean free path,

$$\lambda = \frac{kT}{\sqrt{2} \pi \rho d^2}$$

$$\lambda = \frac{(1.381 \times 10^{-23}) (293)}{\sqrt{2} \pi (1.013 \times 10^5) (3.10 \times 10^{-10})^2}$$

$$\lambda = 93.6 \text{ nm}$$

35. (b) The p - T graph for saturated vapour is shown in figure.



From graph, it is clear that pressure varies non-linearly with temperature at constant volume.

BITSAT Archives

1. (b) Assuming the balloons have the same volume, as $pV = nRT$. If p , V and T are the same, n the number of moles present will be the same, whether it is He or air. Hence, number of molecules per unit volume will be same in both the balloons.

2. (c) Applying gas equation, $pV = nRT$

We can write, $p_1V = n_1RT_1$

and

$$p_2V = n_2RT_2$$

$\Rightarrow$

$$\begin{aligned} \frac{p_2}{p_1} &= \frac{n_2}{n_1} \times \frac{T_2}{T_1} \\ &= \frac{1}{1} \times \frac{2T}{T} = 2 \end{aligned}$$

$\Rightarrow$

$$p_2 = 2p$$

3. (d) We know, $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$\Rightarrow$ % increase in

$$\begin{aligned} v_{\text{rms}} &= \frac{\sqrt{\frac{3RT_2}{M}} - \sqrt{\frac{3RT_1}{M}}}{\sqrt{\frac{3RT_1}{M}}} \times 100 \\ &= \frac{\sqrt{T_2} - \sqrt{T_1}}{\sqrt{T_1}} \times 100 \\ &= \frac{\sqrt{400} - \sqrt{300}}{\sqrt{300}} \times 100 \\ &= \frac{20 - 17.32}{17.32} \times 100 \\ &= 15.5\% \end{aligned}$$

4. (c) $\frac{E_s}{E_T} = \frac{C_p}{C_V} = \gamma = 1 + \frac{2}{f}$

5. (b) Collision frequency $= \frac{v}{l}$

6. (a) As V - T graph is a straight line with positive slope, therefore its equation, may be written as

$$V = aT + b$$

From standard gas equation,

$$p = \frac{RT}{V} = \frac{RT}{aT + b} = \frac{R}{a + b/T}$$

As,

$$T_2 > T_1$$

$\therefore$

$$p_2 > p_1 \text{ or } p_1 < p_2$$

7. (c) Ideal gas equation is given by

$$pV = nRT$$

...(i)

For oxygen, $p = 1 \text{ atm}$, $V = 1 \text{ L}$, $n = n_0$

Therefore, Eq. (i) becomes

$$\therefore 1 \times 1 = n_{\text{O}_2} RT \Rightarrow n_{\text{O}_2} = \frac{1}{RT}$$

$$\text{and } 0.5 \times 2 = n_{\text{N}_2} RT \Rightarrow n_{\text{N}_2} = \frac{1}{RT}$$

For mixture of gas,

$$p_{\text{mix}} V_{\text{mix}} = n_{\text{mix}} RT$$

Here,

$$n_{\text{mix}} = n_{\text{O}_2} + n_{\text{N}_2}$$

$$\therefore \frac{p_{\text{mix}} V_{\text{mix}}}{RT} = \frac{1}{RT} + \frac{1}{RT}$$

$\Rightarrow$

$$p_{\text{mix}} V_{\text{mix}} = 2$$

8. (c) Velocity of sound, $v = \sqrt{\frac{\gamma RT}{M}}$

$$\frac{v_{\text{H}}}{v_{\text{O}}} = \sqrt{\frac{M_{\text{O}}}{M_{\text{H}}}} = \sqrt{\frac{16}{1}} = 4 : 1$$

17

Laws of Thermodynamics

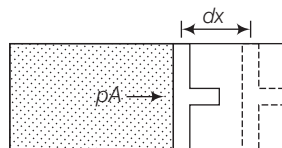
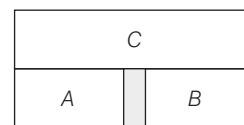
Thermal Equilibrium

When there is no exchange of heat between two objects placed in contact, then both are said to be in thermal equilibrium. Thus, the temperature remains constant throughout all the portions. Thermodynamics is a branch of science which deals with transformation of heat energy into other forms of energy and *vice-versa*.

Zeroth Law of Thermodynamics

If two objects A and B are separately in thermal equilibrium with another object C , then objects A and B will also be in thermal equilibrium.

Cylinder has a cross-sectional area A and pressure exerted by system on the piston face is p . The work done by the system on the surroundings for small displacement dx is $dW = pAdx$ as shown in figure.



$$W = \int dW = \int_{V_i}^{V_f} p dV$$

Work

Work done can be defined as ability to do work. In thermodynamics, work is actually done due to pressure of the gas.

If a gas placed in a cylinder with frictionless piston is allowed to expand from the state A to the state B , when pressure and change in volume are p and ΔV . If during expansion, piston moves by small distance dx , then work done is

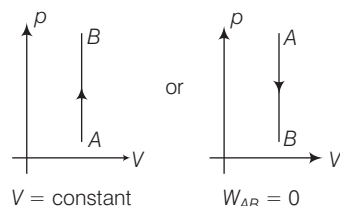
$$dW = pA(dx) = p dV \Rightarrow W = \int_{V_1}^{V_2} p dV$$

If V increases, then work done by the gas is positive. If V decreases, then work done by the gas is negative.

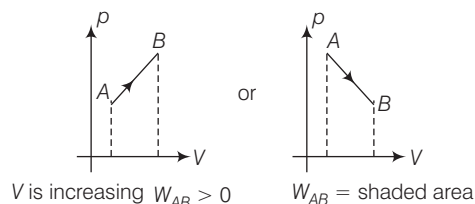
Work done = area bounded by p - V curve.

Work done in Different Cases

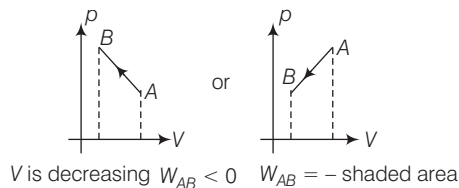
Case I When volume is constant.



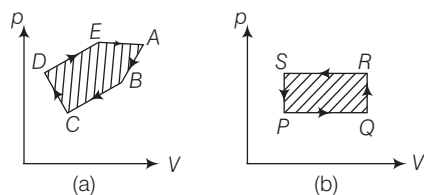
Case II When volume is increasing.



Case III When volume is decreasing.



Case IV Work done in cyclic process.



(i) Work done by clockwise cycle

$ABCDE = + \text{shaded area}$

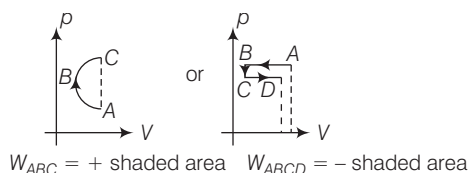
[In Fig. (a)]

(ii) Work done by anti-clockwise cycle

$PQRS = - \text{shaded area}$

[In Fig. (b)]

Case V Work done in incomplete cycle.



First Law of Thermodynamics

This law is based on law of conservation of energy. If δQ is heat supplied to a thermodynamical system and δW is work done by the thermodynamical system and resulting δU is the change in internal energy, then

$$\delta Q = \delta W + \delta U$$

This equation is the **first law of thermodynamics**.

Sign Conventions for First Law of Thermodynamics

δQ , δW and δU must be in the same units.

Heat supplied to the system = Positive

Heat rejected by the system = Negative

Work done by the system = Positive

Work done on the system = Negative

If temperature increases, δU = Positive

If temperature decreases, δU = Negative

Different Thermodynamical Processes

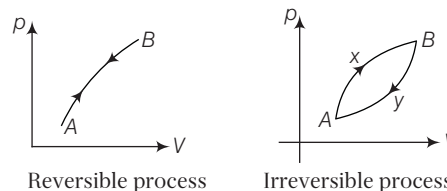
We are going to discuss different types of thermodynamics processes.

1. Reversible and Irreversible Processes

A reversible process means, if a process takes up the path AB (as shown in figure), then on reversing the conditions it comes back by BA .

A thermal process could be reversible, if the change is extremely small (infinitesimally small).

In irreversible process, one will not reach back to A , if the process AB has occurred.



Reversible process

Irreversible process

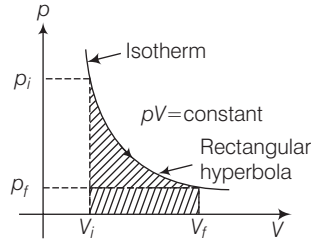
2. Isothermal Process

- In this process, temperature of the system is kept constant during the change of state.
- As $Q = nC_{iso}dT \Rightarrow C_{iso} = \frac{Q}{ndT} = \infty$, i.e. molar heat capacity for an isothermal process is infinity.
- From $dU = nC_V dT$ as $dT = 0$, so $dU = 0$, i.e. internal energy is constant.
- Gas equation is $pV = \text{constant}$.

- From first law of thermodynamics, $dQ = dW$ i.e. heat given to the system is equal to the work done by system on the surroundings.

$$W = nRT \ln\left(\frac{V_f}{V_i}\right) = nRT \ln\left(\frac{p_i}{p_f}\right)$$

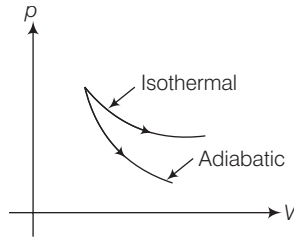
- After differentiating $pV = \text{constant}$, we have $\frac{dp}{dV} = -\frac{p}{V}$ (slope of p - V curve) and $\frac{-dp}{dV/V} = p$, i.e. bulk modulus of a gas in isothermal process, $B = p$



- p - V curve is a rectangular hyperbola.

3. Adiabatic Process

- In this process, no heat exchange takes place between the system and the surroundings, i.e. $dQ = 0$ or $Q = \text{constant}$
- From $dQ = nCdT$, $C_{ad} = 0$ as $dQ = 0$, i.e. molar heat capacity for an adiabatic process is zero.
- From first law, $dU = -dW$ i.e. work done by the system is equal to decrease in internal energy. When a system expands adiabatically, work done is positive and hence internal energy decreases i.e. the system cools down and *vice-versa*.
- Gas equation is $pV^\gamma = \text{constant}$. Gas equation can be written in many other ways as $TV^{\gamma-1} = \text{constant}$ or $p^{1-\gamma}T^\gamma = \text{constant}$, where $\gamma = \frac{C_p}{C_v}$.



- Work done in an adiabatic process is

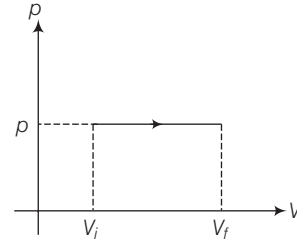
$$W = \frac{nR(T_1 - T_2)}{\gamma - 1} = \frac{p_1V_1 - p_2V_2}{\gamma - 1}$$

- Using $pV = nRT$ and $pV^\gamma = \text{constant}$, we can have $\frac{dp}{dV} = -\frac{\gamma p}{V}$ i.e. slope of p - V curve in an adiabatic process is γ times the slope of p - V curve in isothermal process.

4. Isobaric Process

- This is the process in which pressure is kept constant.
- Molar heat capacity of the process is C_p and $dQ = nC_p dT$

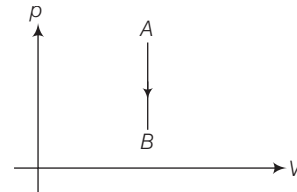
- $dU = nC_v dT$



- From the first law of thermodynamics, $dQ = dU + dW$; $dW = pdV = nRdT$
 $\Rightarrow W = p(V_f - V_i) = nR(T_f - T_i)$
- Gas equation is $\frac{V}{T} = \text{constant}$.
- p - V curve is a straight line parallel to the volume axis.

5. Isochoric Process

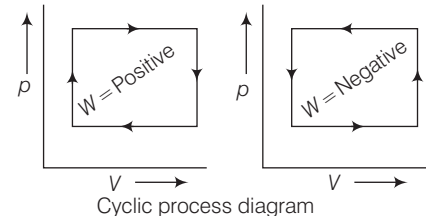
- This is the process in which volume is kept constant.
- $dQ = nC_v dT$, molar heat capacity for isochoric process is C_v .



- Volume is constant, so $dW = 0$
- From the first law of thermodynamics, $dQ = dU$. As heat is supplied to the system, internal energy increases and hence the temperature increases.
- Gas equation is $\frac{p}{T} = \text{constant}$.
- p - V curve is a straight line parallel to the pressure axis.

6. Cyclic Process

In cyclic process, final state of gas is same as the initial state of gas. For one complete cycle process, $dU = 0$.



Work done by the gas

is equal to the area enclosed by the p - V curve.

Here, net heat in the process is given by

Net heat = Total heat supplied + Total heat rejected

Efficiency of cyclic process

$$\eta = \frac{\text{Total work done in cycle}}{\text{Total heat supplied}} \times 100\%$$

Heat Engine

It is a device which is used to convert heat energy into mechanical energy, in a cyclic process.

Components of a heat engine are

- (i) A body at higher temperature T_1 from which heat is extracted is called the source.
- (ii) Body of the engine contains a working substance which performs mechanical work, when heat is supplied to it.
- (iii) A body at lower temperature T_2 to which heat can be rejected is called the sink.

Efficiency of Heat Engine

Efficiency of heat engine (η) is defined as the fraction of total heat supplied (Q_1) to the engine which is converted into work (W).

Mathematically,

$$\eta = \frac{W}{Q_1} \text{ or } \eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} \left[\because \frac{Q_2}{Q_1} = \frac{T_2}{T_1} \right]$$

Refrigerator

Refrigerator is a device which takes heat from a cold body, work is done on it and the work done together with the heat absorbed is rejected to the source. An ideal refrigerator can be regarded as an ideal heat engine working in the reverse direction.

Coefficient of Performance of a Refrigerator (β)

It is defined as the ratio of quantity of heat removed per cycle (Q_2) to the work done on the working substance per cycle to remove this heat.

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} \text{ or } \beta = \frac{T_2}{T_1 - T_2} = \frac{1 - \eta}{\eta}$$

Second Law of Thermodynamics

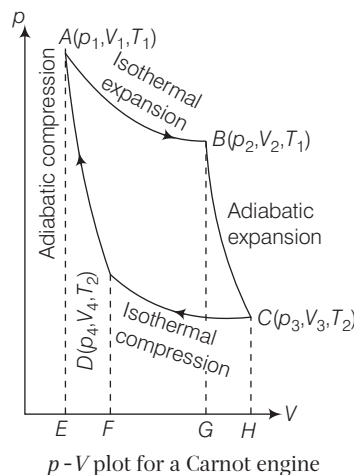
Second law of thermodynamics tells us whether in a given process conservation of energy will actually take place or not. This law gives the direction of heat flow. It can be stated as follows

- (i) **Clausius statement** It is impossible to make any such machine that can transfer heat from an object with low temperature to an object with high temperature without any external source.
- (ii) **Kelvin statement** It is impossible to obtain work continuously by cooling an object below the temperature of its surroundings.
- (iii) **Kelvin-Planck statement** It is impossible to construct any such machine that works on a cyclic process and absorbs heat from a source, converts all that heat into work and rejects no heat to sink.

Carnot Cycle

Carnot engine is an ideal reversible heat engine that operates between two temperatures T_1 (source) and T_2 (sink).

As the engine works, the working substance of the engine undergoes a cycle, known as **Carnot cycle**.



Net Work done by the Gas per Cycle

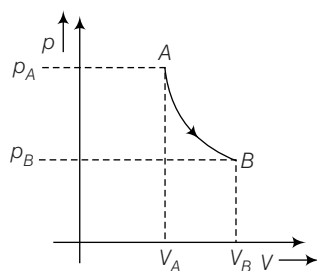
During the four strokes, W_1, W_2 are the work done by the gas and W_3, W_4 are the work done on the gas. Therefore, the net work performed by the engine,

$$\begin{aligned} W &= W_1 + W_2 - W_3 - W_4 \\ &= \text{Area } ABGE + \text{Area } BCHG - \text{Area } CDFH - \text{Area } ADEF \\ &= \text{Area } ABCD \end{aligned}$$

Thus, net work done by the engine during one cycle is equal to the area enclosed by the indicator diagram of the cycle.

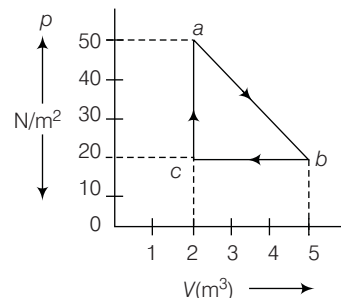
Practice Exercise

- A boy weighing 50 kg eats bananas. The energy content of banana is 1000 cal, if this energy is used to lift the boy from ground, then the height through which he is lifted, is
a. 8.57 m b. 10.57 m c. 6.57 m d. 5.57 m
- 1 kg mass of water, boils at standard atmosphere pressure, turns completely into saturated vapour. Assume saturated vapour to be an ideal gas. Find the increment of internal energy of the system and internal work done.
(Given, specific latent heat of vaporisation of water = 2250 kJ/kg)
a. 3×10^6 J b. 2.1×10^6 J c. 3.2×10^6 J d. 4.2×10^6 J
- An ideal gas is heated at constant pressure and absorbs amount of heat Q . If the adiabatic exponent is γ , then find the fraction of heat absorbed in raising the internal energy and performing the work is
a. $1 - \frac{1}{\gamma}$ b. $1 + \frac{1}{\gamma}$ c. $1 - \frac{2}{\gamma}$ d. $1 + \frac{2}{\gamma}$
- The latent heat of vaporisation of water is 2240 J. If the work done in the process of vaporisation of 1 g is 168 J, then increase in internal energy is
a. 2408 J b. 2240 J c. 2072 J d. 1904 J
- The molar heat capacity in a process of a diatomic gas if it does a work of $Q/4$, when Q amount of heat is supplied to it is
a. $\frac{2}{5}R$ b. $\frac{5}{2}R$ c. $\frac{10}{3}R$ d. $\frac{6}{7}R$
- Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal, W_2 if purely isobaric and W_3 if purely adiabatic. Then,
a. $W_2 > W_1 > W_3$ b. $W_2 > W_3 > W_1$
c. $W_1 > W_2 > W_3$ d. $W_1 > W_3 > W_2$
- Calculate the work done (W_{AB}) by the gas, if 5 moles of an ideal gas is carried by a quasi state isothermal process at 500 K to twice its volume.



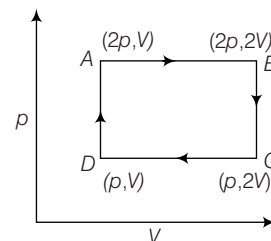
- a. 1500 J b. 14407 J c. 13380 J d. 14890 J

- The work done for the cycle shown in given figure, will be



- a. 45 J b. 54 J c. 22.5 J d. 32.5 J

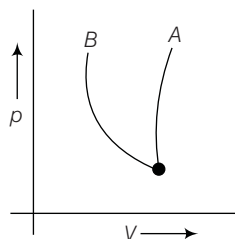
- An ideal monoatomic gas is taken around the cycle ABCDA as shown in the p - V diagram. The work done during cycle is given by



- a. $\frac{1}{2}pV$ b. pV
c. $2pV$ d. $4pV$

- A balloon that is initially flat, is inflated by filling it from a tank of compressed air. The final volume of the balloon is 5 m^3 . The barometer reads 95 kPa. The work done in this process is
a. 475×10^5 J b. 4.75×10^7 J
c. 4.75×10^3 J d. 4.75×10^5 J
- What work will be done, when 3 moles of an ideal gas are compressed to half the initial volume at a constant temperature of 300 K?
a. -5188 J b. 5000 J c. 5188 J d. -5000 J
- A gas is contained in a cylinder and expands according to the relation $pV^{1.3} = \text{Constant}$. The initial pressure and initial volume of the gas are 30 atm and 30 L respectively. If the final pressure is 15 atm, then calculate the work done on the face of piston by the pressure force of the gas.
a. 5×10^4 J b. 4.36×10^4 J
c. 3×10^4 J d. 4×10^4 J
- A vertical cylinder divided into two parts by a frictionless piston in the ratio of 5 : 4. The piston is free to slide along the length of the vessel and length of the vessel is 90 cm. Each of the two parts of the vessel contains 0.1 mole of an ideal gas and the temperature of gas is 300 K. Calculate the mass of the piston.
a. 14 kg b. 12.7 kg c. 16 kg d. 15 kg

14. During an isothermal expansion of an ideal gas,
 a. its internal energy decreases
 b. its internal energy does not change
 c. the work done by the gas is equal to the quantity of heat absorbed by it
 d. both (b) and (c) are correct
15. In the given graph, adiabatic and isothermal curves are shown. Then,



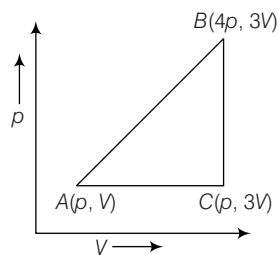
- a. the curve A is isothermal b. the curve B is isothermal
 c. the curve A is adiabatic d. the curve B is adiabatic
16. 0.2 moles of an ideal gas, is taken around the cycle *abc* as shown in the figure. The path *b-c* is adiabatic process, *a-b* is isovolumic process and *c-a* is isobaric process. The temperature at *a* and *b* are $T_a = 300\text{ K}$ and $T_b = 500\text{ K}$ and pressure at *a* is 1 atmosphere. Find the volume at *c*.
 (Given, $\gamma = \frac{C_p}{C_v} = \frac{5}{3}$, $R = 8.205 \times 10^{-2}\text{ L/atm/mol-K}$)
- a. 6.9 L b. 6.68 L c. 5.52 L d. 5.82 L
17. At 27°C a motor car tyre has pressure of 2 atmosphere. Find the temperature, if the tyre suddenly bursts.
 (Given, $\gamma_{\text{air}} = 1.4$)
- a. 24.1 K b. 250 K c. 246.1 K d. 248 K
18. In an adiabatic expansion, a gas does 25 J of work while in an adiabatic compression 100 J of work is done on a gas. The change of internal energy in the two processes respectively are
- a. 25 J and -100 J b. -25 J and 100 J
 c. -25 J and -100 J d. 25 J and 100 J
19. The molar heat capacity of oxygen gas at STP is nearly $2.5R$. As the temperature is increased, it gradually increases and approaches $3.5R$. The most appropriate reason for this behaviour is that at high temperature,
- a. oxygen does not behave as an ideal gas
 b. oxygen molecules dissociate in atoms
 c. the molecules collide more frequently
 d. molecular vibrations gradually become effective
20. *p-V* diagram of a diatomic gas is a straight line passing through origin. The molar heat capacity of the gas in the process will be
- a. $4R$ b. $3R$ c. $4R/3$ d. $2.5R$

21. An ideal gas is taken from the state *A* (pressure *p*, volume *V*) to the state *B* (pressure $p/2$, volume $2V$), a long straight line path in the *p-V* diagram. Select the correct statement from the following.
- a. The work done by the gas in the process *A* to *B*, exceeds the work that would be done by it, if system were taken along the isotherm
 b. In the *T-V* diagram, the path *AB* becomes a part of a hyperbola
 c. In the *p-T* diagram, the path *AB* becomes a part of a hyperbola
 d. In going from *A* to *B*, the temperature *T* of the gas first decreases to a minimum value and then increases
22. During adiabatic change, specific heat is
- a. zero b. greater than zero
 c. less than zero d. infinity
23. Molar heat capacity is directly related to
- a. temperature b. heat energy
 c. molecular structure d. mass
24. If at NTP, velocity of sound in a gas is 1150 m/s, then find out the rms velocity of gas molecules at NTP.
 (Given, $R = 8.3\text{ J/mol-K}$, $C_p = 4.8\text{ cal/mol-K}$)
- a. 1600 m/s b. 1532.19 m/s
 c. 160 m/s d. 16 m/s
25. What is the molar heat capacity for the process, when 10 J of heat added to a monoatomic ideal gas in a process in which the gas performs a work of 5 J on its surrounding?
- a. $2R$ b. $3R$ c. $4R$ d. $5R$
26. A gaseous mixture consists of $\mu_1 = 2$ moles of oxygen and $\mu_2 = 3$ moles of carbon dioxide. Assume gases to be ideal. Calculate, $\gamma = \frac{C_p}{C_v}$ for the gaseous mixture.
- a. 2.33 b. 1.33 c. 0.33 d. 3.33
27. Find the molar specific heat of mixture at constant volume, if one mole of a monoatomic gas is mixed with three moles of a diatomic gas.
- a. $3.33R$ b. $2.25R$ c. $1.15R$ d. $6.72R$
28. If a gas heated at constant pressure, then what percentage of total heat supplied is used up for external work? (Given, γ for gas = $\frac{4}{3}$)
- a. 25% b. 50% c. 75% d. 80%
29. One mole of a gas isobarically heated by 40 K receives an amount of heat 1.162 kJ. What is the ratio of specific heats of the gas?
- a. 1.7 b. 1.4 c. 1.3 d. 1.5
30. In a process $PT = \text{constant}$, if molar heat capacity of a gas is $C = 37.35\text{ J/mol-K}$, then find the number of degrees of freedom of molecules in the gas.
- a. $n = 10$ b. $n = 5$
 c. $n = 6$ d. $n = 7$

31. If in an adiabatic process, the pressure is increased by $\frac{2}{3}\%$, then volume decreases by $\left(\text{Assume } \frac{C_p}{C_v} = \frac{3}{2}\right)$
- a. $-\frac{4}{9}\%$ b. $\frac{2}{3}\%$ c. 4% d. $\frac{9}{4}\%$
32. A monoatomic ideal gas is expanded adiabatically to n times of its initial volume. The ratio of final rate of collision of molecules with unit area of container walls to the initial rate will be
- a. $n^{-4/3}$ b. $n^{4/3}$ c. $n^{2/3}$ d. $n^{-5/3}$
33. In the case of solid, number of degrees of freedom is
- a. 3 b. 5 c. 6 d. 7
34. A given quantity of an ideal gas is at the pressure p and the absolute temperature T . The isothermal bulk modulus of the gas is
- a. $\frac{2}{3}p$ b. p c. $\frac{3}{2}p$ d. $2p$
35. A non-conducting cylindrical vessel of cross-sectional area $29.15 \times 10^{-5} \text{ m}^2$ and volume 10^3 m^3 contains 10^{-4} kg oxygen in it. The cylinder is closed by a frictionless non-conducting piston which is free to move in vertical direction. If the piston is slightly depressed and released, it will execute simple harmonic motion, then find its frequency. (Assume $\frac{C_p}{C_v} = 1.4$ for oxygen and ignore atmosphere pressure)
- a. 0.13 s b. 0.318 s c. 0.23 s d. 0.36 s
36. At a temperature t , the moment of inertia of a body is I . When the temperature of the body is increased from $t + \Delta t$, its moment of inertia also increases from I to $(I + \Delta I)$. If coefficient of linear expansion of the body is α , then find the ratio $\frac{\Delta I}{I}$.
- a. $\frac{\Delta t}{t}$ b. $\frac{2\Delta t}{t}$ c. $\alpha \Delta t$ d. $2\alpha \Delta t$
37. A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be
- a. efficiency of Carnot engine cannot be made larger than 50%
b. 1200 K c. 750 K d. 600 K
38. The inside and outside temperatures of a refrigerator are 273 K and 303 K respectively. Assuming that refrigerator cycle is reversible, for every joule of work done, the heat delivered to the surrounding will be
- a. 10 J b. 20 J c. 30 J d. 50 J
39. Find coefficient of performance, if in a mechanical refrigerator, the lower temperature coils of a evaporator are at -23°C and compressed gas in the condenser has a temperature of 77°C .
- a. 70% b. 20% c. 0.23% d. 2.5%
40. A refrigerator works between the temperature of melting ice and room temperature (17°C). The amount of energy (in kWh) that must be supplied to freeze 1 kg of water at 0°C is
- a. 1.4 b. 1.8
c. 0.058 d. 2.5

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1. The heat energy [2014]
- a. is a state variable
b. does not depend on the state of the system
c. is equal to internal energy of the system
d. None of the above
2. In a thermodynamic process, the pressure of a fixed mass of a gas is changed in such a manner that the gas releases 20 J of heat and 8 J of work is done on the gas. If the initial internal energy of the gas was 30 J, then the final internal energy will be [2014]
- a. 2 J b. 42 J c. 18 J d. 58 J
3. A thermodynamical system is changed from state (p_1, V_1) to (p_2, V_2) by two different process, the quantity which will remain same will be [2013]
- a. ΔQ b. ΔW c. $\Delta Q + \Delta W$ d. $\Delta Q - \Delta W$
4. A sample of ideal monoatomic gas is taken round the cycle ABCA as shown in the figure. The work done during the cycle is [2012]



- a. $3 pV$ b. zero
c. $9 pV$ d. $6 pV$

5. An ideal refrigerator has a freezer at a temperature of -13°C . The coefficient of performance of the engine is 5. The temperature of the air (to which heat is rejected) will be [2011]
 a. 325°C b. 325 K c. 39°C d. 320°C
6. In the adiabatic compression, the decrease in volume is associated with [2008]
 a. increase in temperature and decrease in pressure
 b. decrease in temperature and increase in pressure
 c. decrease in temperature and decrease in pressure
 d. increase in temperature and increase in pressure
7. Which of the following is true in the case of an adiabatic process, where $\gamma = \frac{C_p}{C_v}$? [2008]
 a. $p^{1-\gamma}T^{\gamma} = \text{constant}$ b. $p^{\gamma}T^{1-\gamma} = \text{constant}$
 c. $pT^{\gamma} = \text{constant}$ d. $p^{\gamma}T = \text{constant}$
8. Initially two gas samples 1 and 2 are at the same condition. The volume of the two are halved, one isothermally and the other adiabatically. What is the relation between the final pressures p_1 and p_2 ? [2006]
 a. $p_1 = p_2$
 b. $p_1 > p_2$
 c. $p_2 > p_1$
 d. Cannot be determined
9. A can is taken out from a refrigerator at 0°C . The atmospheric temperature is 25°C . If t_1 is the time taken to heat from 0°C to 5°C and t_2 is the time taken from 10°C to 15°C , then [2006]
 a. $t_1 > t_2$
 b. $t_1 < t_2$
 c. $t_1 = t_2$
 d. there is no relation

Answer with Solutions

Practice Exercise

1. (a)
2. (b) We know, $\Delta Q = \Delta U + \Delta W_{\text{ex}} + \Delta W_{\text{in}}$
 $\Delta U = 0$, $\Delta Q = mL$
 $\Delta W_{\text{ex}} = p_0(V_{\text{vapour}} - V_{\text{water}}) = p_0 V_{\text{vapour}}$
 $PV = \frac{m}{M}RT$
 $\Rightarrow \Delta W_{\text{ex}} = p_0 V_{\text{vapour}} = \frac{m}{M}RT$
 $\Delta W_{\text{in}} = \Delta Q - \Delta W_{\text{ex}} = mL - \frac{m}{M}RT$
 $= m \left[L - \frac{RT}{M} \right] = 2.1 \times 10^6 \text{ J}$
3. (a) Heat absorbed by the system at constant pressure $Q = nC_p \Delta T$ and change in internal energy

$$\begin{aligned} \Delta U &= nC_v \Delta T \\ W &= Q - \Delta U \\ \text{Fraction} &= \frac{W}{Q} = \frac{Q - \Delta U}{Q} = 1 - \frac{\Delta U}{Q} \\ &= 1 - \frac{\Delta U}{Q} = \left(1 - \frac{1}{\gamma} \right) \end{aligned}$$

4. (c) Given, $L = 2240 \text{ J}$, $m = 1 \text{ g}$
 $dW = 168 \text{ J}$
 From first law of thermodynamics,
 $dU = 2072 \text{ J}$

5. (c) $dU = C_v dT = \left(\frac{5}{2} R \right) dT$
 $\Rightarrow dT = \frac{2(dU)}{5R}$

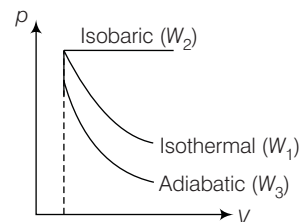
From first law of thermodynamics, $dU = dQ - dW$

$$\text{or } dU = Q - \frac{Q}{4} = \frac{3Q}{4}$$

Now, molar heat capacity,

$$C = \frac{dQ}{dT} = \frac{Q}{2 \left(\frac{dU}{5R} \right)} = \frac{5QR}{2 \left(\frac{3Q}{4} \right)} = \frac{10}{3} R$$

6. (a)



It is clear from the graph that on the basis of area under p - V curve for different processes $W_2 > W_1 > W_3$.

7. (b)
8. (a) The area of p - V diagram gives work.
 $= 45 \text{ J} = \text{work done}$
9. (b) $W = (2V - V)(2p - p) = pV$
10. (d)
11. (a) As process is isothermal
 $W = 2.303 nRT \log \left(\frac{V_2}{V_1} \right)$
 $W = 2.303 \times 3 \times 8.315 \times 300 \times \log \left(\frac{1}{2} \right)$
 $W = -5188 \text{ J}$

12. (b) $W = \int_{V_1}^{V_2} p dV$ and $pV^{1.3} = C, p = CV^{-1.3}$

$$W = \int_{V_1}^{V_2} CV^{-1.3} dV = \frac{-C}{0.3} [V^{0.3}]_{V_1}^{V_2}$$

$$C = p_1 V_1^{1.3} = p_2 V_2^{1.3}$$

$$W = \frac{p_2 V_2 - p_1 V_1}{-0.3}$$

Final volume, $V_2 = V_1 \left(\frac{p_1}{p_2} \right)^{1/1.3}$

$$= 30 \times 10^{-3} (2)^{0.77} = 30 \times 10^{-3} \times 1.71$$

$$V_2 = 51.3 \times 10^{-3} \text{ m}^3$$

$$W = \frac{15 \times 10^5 \times 51.3 \times 10^{-3} - 30 \times 10^5 \times 30 \times 10^{-3}}{0.3}$$

$$W = 4.36 \times 10^4 \text{ J}$$

13. (b)

14. (d) In isothermal process, temperature remains constant

$$\therefore \Delta U = nC_V \Delta T = 0 \quad (\because \Delta T = 0)$$

According to the first law of thermodynamics,

$$\Delta H = \Delta U + \Delta W$$

$$\therefore \Delta H = \Delta W \quad (\because \Delta U = 0)$$

Hence, (d) is correct.

15. (b, c)

16. (b) From a - b, volume is constant

$$\frac{p_a}{T_a} = \frac{p_b}{T_b}$$

$$p_b = \frac{500}{300} \times 1 = \frac{5}{3} \text{ atm}$$

For b - c, adiabatic process

$$\frac{T_c}{T_b} = \left(\frac{p_c}{p_b} \right)^{\frac{\gamma-1}{\gamma}} = \left[\frac{1}{5/3} \right]^{\frac{5/3-1}{5/3}} = \left(\frac{3}{5} \right)^{2/5}$$

Also,

$$p_b V_b^\gamma = p_c V_c^\gamma$$

$$V_c = (V_b) \left(\frac{p_b}{p_c} \right)^{1/\gamma}$$

$$= (4.928) \left(\frac{5/3}{1} \right)^{3/5} = 6.68 \text{ litre}$$

17. (c)

18. (b) In adiabatic process, $\Delta H = 0$

$\therefore$ For expansion,

$$\Delta H = \Delta U + \Delta W$$

$$0 = \Delta U + 25$$

$$U = -25 \text{ J}$$

For adiabatic compression,

$$\Delta H' = \Delta U' + \Delta W'$$

$$0 = \Delta U' - 100$$

$$\Delta U = 100 \text{ J}$$

Hence, (b) is correct.

19. (a)

20. (b) As p-V diagram is a straight line passing through origin, therefore, $p \propto V$ or $pV^{-1} = \text{constant}$

In the process, $pV^x = \text{constant}$, molar heat capacity is

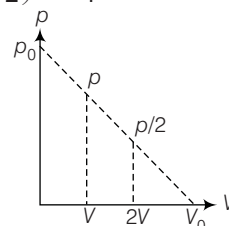
given by $C = \frac{R}{\gamma-1} + \frac{R}{1-x}$

where, $x = -1$ and $\gamma = 1.4$ for diatomic gas

$$C = \frac{R}{1.4-1} + \frac{R}{1-(-1)} = \frac{5}{2}R + \frac{R}{2}$$

$$C = 3R$$

21. (a) $W = \frac{1}{2} \left(p + \frac{p}{2} \right) V = \frac{3}{4} pV = 0.75 pV$



$$W_{\text{iso}} = RT \times 2.3026 \log_{10} \left(\frac{2V}{V} \right) = 0.693 pV$$

Thus, statement (a) is correct.

22. (a)

23. (c) Since, $C_p - C_V = R$

$$\frac{C_p}{C_V} = \gamma \text{ or } C_p = \gamma C_V$$

$$\therefore C_p - C_V = R$$

$$C_V(\gamma - 1) = R$$

$$\therefore C_V = \frac{R}{(\gamma - 1)}$$

Also,

$$C_p = \frac{\gamma R}{(\gamma - 1)}$$

But R is universal constant. The value of γ depends upon molecular structure.

24. (b) $R = \frac{8.3}{4.2} \text{ cal/mol/K}$

$$C_V = C_p - R = \left(4.8 - \frac{8.3}{4.2} \right) = 2.824$$

$$\gamma = \frac{C_p}{C_V} = \frac{4.8}{2.824} = 1.69$$

Since, $V = \sqrt{\left(\frac{3}{\gamma} \right)} \cdot V_s = 1532.19 \text{ m/s}$

25. (b)

26. (b) Let $n = n_1 + n_2$

$$nC_p = n_1 C_{p1} + n_2 C_{p2}$$

$$C_p = \frac{n_1 C_{p1} + n_2 C_{p2}}{n}$$

Similarly,

$$C_V = \frac{n_1 C_{V1} + n_2 C_{V2}}{n}$$

$$\gamma = \frac{C_p}{C_V} = \frac{n_1 C_{p1} + n_2 C_{p2}}{n_1 C_{V1} + n_2 C_{V2}}$$

$$\text{But } C_{p1} = \frac{\gamma_1 R}{(\gamma_1 - 1)} \text{ and } C_{p2} = \frac{\gamma_2 R}{(\gamma_2 - 1)}$$

$$\text{also } C_{V1} = \frac{R}{\gamma_1 - 1} \text{ and } C_{V2} = \frac{R}{\gamma_2 - 1}$$

$$\gamma = \frac{n_1 \left[\frac{\gamma_1 R}{(\gamma_1 - 1)} \right] + n_2 \left[\frac{\gamma_2 R}{(\gamma_2 - 1)} \right]}{\frac{n_1 R}{(\gamma_1 - 1)} + \frac{n_2 R}{(\gamma_2 - 1)}}$$

After solving, $\gamma = 1.33$

27. (b) Let C_{vm} = molar specific heat of mixture at constant volume.

$$\text{For mixture, } (n_1 + n_2)C_{vm} = n_1 C_{V1} + n_2 C_{V2}$$

$$\text{Here, } C_{V1} = \frac{3R}{2}, n_1 = 1 \text{ mole}$$

$$C_{V2} = \frac{5R}{2}, n_2 = 3 \text{ moles}$$

$$(1 + 3)C_{vm} = 1 \times \frac{3}{2}R + 3 \times \frac{5}{2}R$$

$$C_{vm} = \frac{9}{4}R = 2.25R$$

28. (a) Heat supplied, $dQ = nC_p dT$

$$\text{Heat used for work} = dW = nRdT$$

$$\begin{aligned} \frac{dW}{dQ} &= \frac{R}{C_p} = \frac{C_p - C_v}{C_p} = 1 - \frac{C_v}{C_p} \\ &= \left(1 - \frac{1}{\gamma}\right) = \left(1 - \frac{3}{4}\right) \\ &= \frac{1}{4} \times 100 = 25\% \end{aligned}$$

29. (b)

30. (b) $pV = nRT$ and $pT = k = \text{constant}$

$$\begin{aligned} \Rightarrow p &= \frac{k}{T} \\ V &= \frac{nR}{k} T^2, \\ dV &= \frac{2nRTdT}{k} \end{aligned}$$

For

$$\begin{aligned} n = 1, dU &= \int_0^T PdV \\ &= \int_0^T \frac{k}{T} \frac{2R}{k} TdT = 2RdT \end{aligned}$$

$$dW = nC_v dT = n \frac{R}{\gamma - 1} dT = \frac{R}{\gamma - 1} dT$$

According to law, $\Delta Q = \Delta U + \Delta W$

$$= \frac{R}{\gamma - 1} dT + 2RdT = R \left[\frac{2\gamma - 1}{\gamma - 1} \right] dT$$

$$C = \frac{dQ}{dT} = R \left[\frac{2\gamma - 1}{\gamma - 1} \right]$$

$$\gamma = \frac{7}{5} \text{ and } 1 + \frac{2}{n} = \frac{7}{5}$$

$$n = 5$$

31. (a) $\frac{C_p}{C_v} = \gamma = \frac{3}{2}$

Let $k = \text{constant}$

We know for an adiabatic process, $pV^\gamma = k$

$$pV^{3/2} = k$$

$$\log p + \frac{3}{2} \log V = \log k$$

$$\frac{\Delta p}{p} + \frac{3}{2} \frac{\Delta V}{V} = 0 \Rightarrow \frac{\Delta V}{V} = -\frac{2}{3} \frac{\Delta p}{p}$$

$$\begin{aligned} \frac{\Delta V}{V} \times 100 &= \left(-\frac{2}{3}\right) \left(\frac{\Delta p}{p} \times 100\right) \\ &= -\frac{2}{3} \times \frac{2}{3} = -\frac{4}{9} \% \end{aligned}$$

32. (d) The rate of collision of molecules with unit area of walls of container is proportional to pressure.

$$p_1 V_1^\gamma = p_2 V_2^\gamma$$

$$V_2 = n V_1$$

$$\therefore \frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^\gamma = \left(\frac{V_1}{n V_1}\right)^\gamma = \frac{1}{n^\gamma}$$

$$p_2 = p_1 n^{-\gamma}$$

For monoatomic gas, $\gamma = 1 + \frac{2}{3} = \frac{5}{3}$

$$p_2 = p_1 n^{-5/3}$$

33. (c)

$$34. (b) \beta = \frac{\Delta p}{\frac{\Delta V}{V}}$$

$\therefore$ In isothermal process $(p + \Delta p)(V - \Delta V) = pV$

or $pV - p\Delta V + V\Delta p - \Delta p\Delta V = pV$

But $\Delta p\Delta V$ may be neglected.

$$\therefore V\Delta p = p\Delta V \text{ or } \frac{\Delta p}{\frac{\Delta V}{V}} = p$$

35. (b) In equilibrium state, $p = \frac{mg}{A}$

Also Poisson's law, $pV^\gamma = \text{constant}$ as process is adiabatic.

When it is depressed, volume will decrease by ΔV and pressure increase by Δp

$$pV^\gamma = (p + \Delta p)(V - \Delta V)^\gamma$$

$$\Delta p = \gamma p \left(\frac{\Delta V}{V} \right)$$

When piston is released, restoring force will act on it.

$$F = -\Delta p(A) = -\gamma pA \left(\frac{\Delta V}{V} \right)$$

$$\Delta V = An$$

$$F = \frac{-\gamma pA^2}{V} \cdot n$$

$$\text{Acceleration of piston, } \alpha = \frac{F}{m} = \left(\frac{-\gamma p A^2}{mV} \right) x = -\omega^2 x$$

$$n = \frac{\omega}{2\pi} \Rightarrow n = \frac{1}{2\pi} \sqrt{\frac{\gamma g A}{V}} \quad \left[\because p = \frac{mg}{A} \right]$$

$$n = 0.318 \text{ s}$$

$$36. (d) I = \Sigma mr^2$$

$$\therefore I + \Delta I = \Sigma m [r(1 + \alpha \Delta t)]^2$$

$$I + \Delta I = \Sigma [mr^2(1 + 2\alpha \Delta t)] = I(1 + 2\alpha \Delta t)$$

$$\frac{\Delta I}{I} = 2\alpha \Delta t$$

$$37. (c)$$

$$38. (a)$$

$$39. (c)$$

$$40. (c)$$

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1. (b) Heat energy is not a state variable, it is just energy in transition.

2. (c) Given, $Q = -20 \text{ J}$, $W = -8 \text{ J}$

$$\text{Using 1st law, } Q = \Delta U + W \Rightarrow -20 = \Delta U + (-8)$$

$$\Rightarrow \Delta U = -12 \text{ J} \Rightarrow U_f - U_i = -12 \text{ J}$$

$$\Rightarrow U_f - 30 = -12 \text{ J} \Rightarrow U_f = 18 \text{ J}$$

3. (d) For all processes, change in internal energy $\Delta U = (\Delta Q - \Delta W)$ does not change. It depends only on initial and final states.

4. (a) The work done = area of p - V graph
= area of triangle ABC
 $= \frac{1}{2} \times 3p \times 2V = 3pV$

5. (c) Given, $T_2 = -13^\circ\text{C} = 260 \text{ K}$ and $\beta = 5$

$$\beta = \frac{T_2}{T_1 - T_2}$$

$$5 = \frac{260}{T_1 - 260}$$

$$T_1 = 312 \text{ K or } T_1 = 312 \text{ K} = 39^\circ\text{C}$$

6. (d) In adiabatic compression, temperature and hence internal energy of the gas increases. In compression pressure will increase.

7. (a) For adiabatic change, equation of state is
 $pV^\gamma = \text{constant}$

It can also be written as

$$TV^{\gamma-1} = \text{constant} \left[\text{as } p = \frac{nRT}{V} \right]$$

$$\text{and } p^{1-\gamma} T^\gamma = \text{constant} \left[\text{as } V = \frac{nRT}{p} \right]$$

8. (c) For isothermal process

$$pV = K$$

$$\Rightarrow p_1 = \frac{K}{V} \quad \dots(i)$$

$$\frac{K}{\frac{V}{2}} = 2K$$

For adiabatic process

$$pV^\gamma = K \quad (\text{constant})$$

$$\therefore p_2 = \frac{K}{V^\gamma} \quad \dots(ii)$$

$$= \frac{K}{\left(\frac{V}{2}\right)^\gamma} = K(2^\gamma)$$

From Eqs. (i) and (ii), we have

$$p_2 > p_1$$

9. (b) According to Newton's law of cooling, $t_1 < t_2$

18

Transmission

Modes of Heat Transfer

The heat can be transferred within the body and from one body to the other body through the following modes

- (i) Conduction (ii) Convection (iii) Radiation

Conduction

This process of heat transfer from one end of rod to other end, which does not involve any mass movement of molecules is called **conduction**.

Thermal Conductivity

The amount of heat transmitted through a conductor is given by

$$\Delta Q = \frac{KA\Delta T\Delta t}{l}$$

or in differential form, we can write

$$\frac{dQ}{dt} = -KA \frac{dT}{dx}$$

(where, $\frac{dT}{dx}$ = temperature gradient, A = area of cross-section, ΔT = temperature difference,

Δt = time elapsed, K = thermal conductivity and l = length)

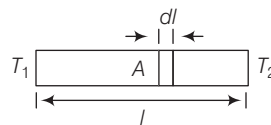
The rate of transmission of heat by conduction is given by

$$H = \frac{\Delta Q}{\Delta t} = \frac{KA\Delta T}{l}$$

The unit of thermal conductivity is $\text{Wm}^{-1}\text{K}^{-1}$.

We can write

$$\frac{dQ}{dt} = mc \frac{dT}{dt} = \pm L \frac{dm}{dt}$$



Convection

Convection is the mode of heat transfer by actual motion of the matter (or particles of matter). Convection is not possible in solids and can take place in fluids only.

Radiation

Conduction and convection require some material as a transport medium. However, the third mechanism for heat transfer needs no medium. It is called **radiation**. The electromagnetic waves which require no medium for its propagation is the carrier of this radiant energy.

Some Common Terms and Points

The thermal radiation emitted by a body comprises of all the wavelengths; intensities of radiation corresponding to different wavelengths are different.

- **Absorptive power (a)** It is defined as the ratio of the radiant energy absorbed by it in a given time to the total radiant energy incident on it in the same interval of time

$$a = \frac{\text{energy absorbed}}{\text{energy incident}}$$

As a perfectly black body absorbs all radiations incident on it, the absorptive power of a perfectly black body is maximum and unity.

- **Spectral absorptive power (a_λ)** The spectral absorptive power is the ratio of radiant energy absorbed by a surface to the radiant energy incident on it for a particular wavelength λ . It may have different values for different wavelengths for a given surface. The spectral absorptive power a_λ is related to absorptive power through the relation $a = \int_0^\infty a_\lambda d\lambda$.

- **Emissive power (e)** For a given surface it is defined as the radiant energy emitted per second per unit area of the surface. It is the total amount of energy radiated by a body per second per unit area of surface

i.e.
$$e = \frac{1}{A} \frac{\Delta Q}{\Delta t}$$

- **Spectral emissive power (e_λ)** It is emissive power for a particular wavelength λ . Thus, $e = \int_0^\infty e_\lambda d\lambda$
- **Emissivity (ϵ)** Emissivity of a body at a given temperature is defined as the ratio of the total emissive power of the body (e) to the total emissive power of a perfect black body (E) at that temperature.

i.e.
$$\epsilon = \frac{e}{E}$$

Absorption, Reflection and Transmission

When radiations are incident on a surface, then three things happen—A part of the radiation is absorbed, some is reflected back, and remaining is transmitted.

$$Q_{\text{incident}} = Q_{\text{absorbed}} + Q_{\text{reflected}} + Q_{\text{transmitted}}$$

where, Q represents the energy of thermal radiation.

Absorptivity or absorptive power, $a = \frac{Q_{\text{absorbed}}}{Q_{\text{incident}}}$

Reflectivity, $r = \frac{Q_{\text{reflected}}}{Q_{\text{incident}}}$

Transmissivity, $t = \frac{Q_{\text{transmitted}}}{Q_{\text{incident}}}$

$$\frac{Q_s}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q} = a + r + t = 1$$

For a perfect black body, $a = 1, r = t = 0$

For a perfect reflector, $a = t = 0, r = 1$

For a perfect transmitter, $a = r = 0, t = 1$

Perfectly Black Body

A perfectly black body is the one which completely absorbs the radiations of all the wavelengths that are incident on it. Thus, absorbing power of a perfectly black body is 1 (i.e. $a = 1$).

No material body is a perfectly black body. However, lamp black and platinum black are nearly perfectly black bodies.

Kirchhoff's Law

It states that the ratio of emissive power to absorptive power is the same for all the bodies at a given temperature and is equal to the emissive power of a black body at that temperature.

Thus, $\frac{e(\text{body})}{a(\text{body})} = E(\text{black body}) = \text{constant}$

So, a body with high emissive power, should also have high absorptive power and *vice-versa*.

Absorptive power is dimensionless quantity but emissive power is not.

Stefan-Boltzmann Law

It states that "energy emitted per unit area per unit time of perfectly black body is proportional to T^4 ," where T is temperature on absolute scale.

i.e.
$$U(\text{J/sm}^2) = \sigma T^4$$

Here, σ is Stefan's constant.

Measured value of σ is $5.67 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}$.

If A is the area, then radiant energy per unit time of perfectly black body is $U(\text{J/s}) = \sigma AT^4$.

For a body which is not perfectly black, energy radiated per second is given by $U = \epsilon \sigma AT^4$.

where, e is emissivity which is equal to absorptive power a of the body. (i.e. $e = a$)

Also, if a body is placed where surrounding temperature is T_0 , then energy absorbed by the body will be given by

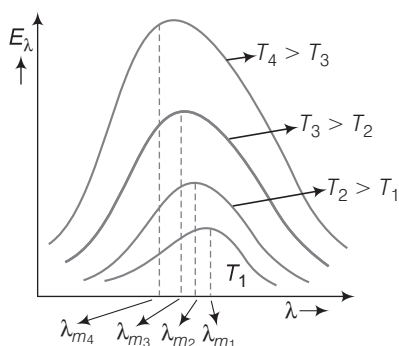
$$U = e\sigma AT_0^4$$

Thus, net loss of thermal power is given by

$$\Delta U = U - U_0 = e\sigma A(T^4 - T_0^4)$$

Wien's Displacement Law

The black body spectrum is a continuous spectrum as shown in the figure. At a given temperature, initially the intensity of thermal radiation increases with an increase in wavelength and reaches a maximum value at a particular wavelength λ_m . On increasing the wavelength beyond λ_m , the intensity of radiation E_λ starts decreasing.



The total area under E_λ - λ curve gives the total intensity of radiation at that temperature.

The area, in accordance with the Stefan's law of radiation, is directly proportional to the fourth power of the temperature.

$$\text{Emissive power, } E = \int_0^\infty E_\lambda d\lambda = \text{Area under } E_\lambda\text{-}\lambda \text{ graph} = \sigma T^4$$

From E_λ - λ graph, we find that as the temperature T of a black body increases, the wavelength λ_m corresponding to the maximum emission decreases such that

$$\lambda_m \propto \frac{1}{T} \text{ or } \lambda_m T = \text{constant} = b$$

where, b is known as the Wien's constant and its value is 2.89×10^{-3} mK.

Newton's Law of Cooling

According to the Newton's law of cooling, the rate of cooling of a body is directly proportional to the temperature difference between the body and the surroundings, provided the temperature difference is small.

$$\text{Mathematically, } \frac{dT}{dt} \propto (T - T_0) \text{ or } -\frac{dT}{dt} = k(T - T_0)$$

where, k is a constant.

Newton's law of cooling is a special case of Stefan's law under the condition that the temperature difference is small enough.

If a body cools by radiation through a small temperature difference from T_1 to T_2 in a short time t when the surrounding temperature is T_0 , then

$$\frac{dT}{dt} = \frac{T_1 - T_2}{t} = k \left[\frac{T_1 + T_2}{2} - T_0 \right]$$

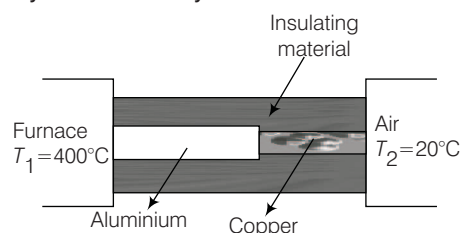
Practice Exercise

- One end of a metal rod is kept in steam. In steady state, the temperature gradient $\left(\frac{d\theta}{dx}\right)$
 - may be variable
 - must be constant
 - must be variable
 - None of the above
- Calculate the daily loss of energy by the earth, if the temperature gradient in the earth's crust is 32°C per km and mean conductivity of the rock is 0.008 of CGS unit. (Given, radius of earth = 6400 km)
 - 10^{40} cal
 - 10^{30} cal
 - 10^{18} cal
 - 10^{10} cal
- Cylindrical rod of copper of length 2 m and cross-sectional area 2 cm^2 is insulated at its curved surface. The one end of rod is maintained in steam chamber and other is maintained in ice at 0°C (The thermal conductivity of copper is $386 \text{ J/m}\cdot\text{s}\cdot^\circ\text{C}$). Find the temperature at a point which is at a distance of 120 cm from the colder end.
 - 80°C
 - 60°C
 - 50°C
 - None of the above
- In previous problem, the amount of ice melts per second is

a. 48.25 g/s	b. 80 g/s
c. 20.8 g/s	d. None of these

5. A cylindrical copper rod of length 1 m and a cylindrical steel rod of length 1.5 m are joined together end to end. The cross-sectional area of each rod is 3.14 cm^2 . The free ends of steel rod and copper rods are maintained at 0°C and 100°C , respectively. The surfaces of rods are thermally insulated. Find the temperature of copper-steel junction. (Given, thermal conductivity of steel = $46 \text{ J/m-s } ^\circ\text{C}$ and the thermal conductivity of copper = $386 \text{ J/m-s } ^\circ\text{C}$)
 - a. 40°C
 - b. 60°C
 - c. 93°C
 - d. 80.64°C
6. In previous problem, the rate of loss of heat at free end of copper is
 - a. 2 J/s
 - b. 0.89 J/s
 - c. 1.07 J/s
 - d. None of the above
7. A block of ice at 0°C rest on the upper surface of the slab of stone of area 3600 cm^2 and thickness of 10 cm. The slab is exposed on the lower surface to steam at 100°C . If 4800 g of ice is melted in one hour, then calculate the thermal conductivity of stone.
(Given, the latent heat of fusion of ice = 80 cal/g)
 - a. $K = 2.96 \times 10^{-3} \text{ cal/cm s}^\circ\text{C}$
 - b. $K = 1.96 \times 10^3 \text{ cal/cm s}^\circ\text{C}$
 - c. $K = 0.96 \times 10^3 \text{ cal/cm s}^\circ\text{C}$
 - d. None of the above
8. The ice is filled in a hollow glass sphere of thickness 2 mm and external radius 10 cm. This hollow glass sphere with ice now placed in a bath containing boiling water at 100°C . Calculate the rate at which ice melts. Neglect volume change in ice.
(Given, thermal conductivity of glass 1.1 W/m/K , latent heat of ice = $336 \times 10^3 \text{ J/kg}$)
 - a. $\frac{m}{t} = 0.01 \text{ kg/s}$
 - b. $\frac{m}{t} = 0.002 \text{ kg/s}$
 - c. $\frac{m}{t} = 0.02 \text{ kg/s}$
 - d. $\frac{m}{t} = 0.001 \text{ kg/s}$
9. 5 cm thick walls of a box like cooler is made of plastic foam. Its total surface area is 1.5 m^2 . If outside temperature is 30°C , then how much ice melts each hour inside the cooler to hold its temperature at 0°C .
(Given, K for plastic = 0.04 W/mK , $L_0 = 80 \text{ cal/g}$ and $1 \text{ kcal} = 4.184 \text{ kJ/kcal}$)
 - a. 4 kg
 - b. 0.39 kg
 - c. 3.9 kg
 - d. 0.2 kg
10. If in two identical containers, equal quantities of ice melts completely in 30 and 20 minutes respectively, then find the ratio of the thermal conductivities of the material of two containers.
 - a. 1 : 1
 - b. 1 : 2
 - c. 3 : 2
 - d. 2 : 3

11. A compound rod is formed of a steel core of diameter 1cm and outer casing is of copper, whose outer diameter is 2 cm. The length of this compound rod is 2 m and one end is maintained as 100°C , and the end is at 0°C . If the outer surface of the rod is thermally insulated, then heat current in the rod will be (Given, thermal conductivity of steel = 12 cal/m/K/s , thermal conductivity of copper = 92 cal/m/K/s)
- a. 2 cal/s b. 1.13 cal/s c. 1.42 cal/s d. 2.68 cal/s
12. An aluminium rod of length L and cross-sectional area $2A$ is joined with a copper rod of length $2L$ and area of cross-section is A , as shown in figure. Find the temperature of aluminium-copper junction in the steady state of the system.



- (Given, thermal conductivity $K_{Al} = 240 \text{ J/m/s/}^\circ\text{C}$, $K_{Cu} = 400 \text{ J/m/s/}^\circ\text{C}$)
- a. 300°C b. 400°C c. 288.24°C d. 275.4°C
- 13.** A uniform metal ring with centre C have two points A and B , such that angle ACB is θ . A and B are maintained at two different constant temperature.
- If the angle between A and B , i.e. $\theta = 180^\circ$, the rate of heat flow from A to B is 1.2 W , then what will be the rate, when $\theta = 90^\circ$?
- a. 0.6 W b. 0.9 W c. 1.6 W d. 1.8 W
- 14.** What amount of ice at -14°C required to cool 200 g of water from 25°C to 10°C ?
- (Given, $C_{ice} = 0.5 \text{ cal/g/}^\circ\text{C}$, L_f for ice $= 80 \text{ cal/g}$)
- a. 31 g b. 41 g c. 51 g d. 21 g
- 15.** A metallic sphere having inner and outer radii a and b respectively has thermal conductivity

$$K = \frac{K_0}{r} \quad (a \leq r \leq b)$$

Find the thermal resistance between inner surface and outer surface.

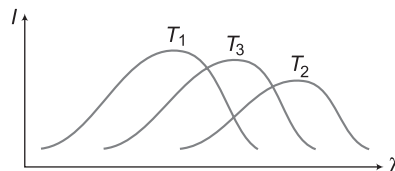
16. In above problem, if the temperature of inner and outer surfaces are $\theta_1^\circ\text{C}$ and $\theta_2^\circ\text{C}$ ($\theta_2 > \theta_1$). The rate of flow of heat in steady state is
- $\frac{4\pi K_0(\theta_1 - \theta_2)}{(b - a)}$
 - $\frac{4\pi K_0(b - a)}{(\theta_1 - \theta_2)}$
 - $\frac{4\pi K_0 ab (\theta_1 - \theta_2)}{(b^2 - a^2)}$
 - None of these

17. A body initially at 60°C cools to 50°C in 10 minutes. What will be its temperature at the end of next 10 minutes, if the room temperature is 25°C ? Assume Newton's law of cooling.
 a. 42.85°C b. 45°C c. 40.46°C d. 44.23°C
18. Find the time in which a layer of ice of thickness h will grow on the surface of the pond of surface area A , when the surrounding temperature falls to $-T^{\circ}\text{C}$.
 (Assume, K = thermal conductivity of ice, ρ = density of water, L = latent heat of fusion)
 a. $t = \frac{\rho L}{2KT} h^2$ b. $t = \frac{\rho L}{KT} h^2$
 c. $t = \frac{\rho L h^2}{3KT}$ d. $t = \frac{\rho L h^2}{4KT}$
19. A black body maintained at a certain temperature radiates heat energy at the rate Q watt. If its surface is smoothened so as to lower its emissivity by 10%, what will be the increase in its rate of radiation at double the initial temperature?
 a. $(0.9 \times 2^4 - 1) Q$ watt b. $0.9 \times 2^4 Q$ watt
 c. $(0.9 \times 2)^4 Q$ watt d. $(0.9)^4 \times 2Q$ watt
20. The thermal radiation emitted by a body per second per unit area is $\frac{\Delta H}{A \Delta t} = kT^4$. If σ is Stefan's constant, then body
 a. may be polished b. may be black body
 c. must be black body d. must not be black body
21. If temperature of black body increases from 300 K to 900 K, then the rate of energy radiation increases by
 a. 81 b. 3 c. 9 d. 2
22. Three objects coloured black, grey and white can withstand hostile conditions at 2800°C . These objects are thrown into furnace where each of them attains a temperature of 2000°C . Which object will have the brightest glow?
 a. The white object
 b. The black object
 c. All glow with equal brightness
 d. Grey object
23. The power P is received by a surface at temperature T_0 K from a small sphere at temperature T ($T \gg T_0$) and at a distance ' d '. If both ' T ' and ' d ' are doubled, then power received by surface will become
 a. P b. $2P$ c. $4P$ d. $16P$
24. A cylindrical rod is having temperatures T_1 and T_2 at its ends. The rate of flow of heat is Q_1 . If all the linear dimensions are doubled keeping the temperature constant, then rate of flow of heat Q_2 will be
 a. $4Q_1$ b. $2Q_1$
 c. $\frac{Q_1}{4}$ d. $\frac{Q_1}{2}$
25. Assuming the sun to be a spherical body of radius R at a temperature of T K, evaluate the total radiant power, incident on earth, at a distance r from the sun. Given, r_0 is the radius of earth and σ is the Stefan's constant.
 a. $4\pi r_0^2 R^2 \sigma T^4 / r^2$ b. $\pi r_0^2 R^2 \sigma T^4 / r^2$
 c. $r_0^2 R^2 \sigma T^4 / 4\pi r^2$ d. $R^2 \sigma T^4 / r^2$
26. What will be the increment in heat energy radiated when the temperature of hot body is raised by 5%?
 a. 5% b. 6% c. 11.65% d. 21.55%
27. Two spheres of the same material having radii r and $4r$ and temperatures $2T_0$ and T_0 respectively. The ratio of rate of radiation of energy by the spheres is
 a. 1 : 1 b. 1 : 2 c. 2 : 1 d. 3 : 1
28. A sphere, a cube and a thin circular plate all of same material having same mass are initially heated to 200°C . Which of these will cool fastest?
 a. Circular plate b. Sphere
 c. Cube d. All of these
29. A body at a temperature of 727°C and has surface area 5 cm^2 , radiates 300 J of energy each minute. Calculate the emissivity.
 (Given, Boltzmann constant = $5.67 \times 10^{-8} \text{ Wm}^2\text{K}^4$)
 a. $e = 0.18$ b. $e = 0.02$ c. $e = 0.2$ d. $e = 0.15$
30. Choose the correct relation, when the temperature of an isolated black body falls from T_1 to T_2 in time ' t ', and assume ' c ' to be a constant.
 a. $t = c \left(\frac{1}{T_2} - \frac{1}{T_1} \right)$ b. $t = c \left(\frac{1}{T_2^2} - \frac{1}{T_1^2} \right)$
 c. $t = c \left(\frac{1}{T_2^3} - \frac{1}{T_1^3} \right)$ d. $t = c \left(\frac{1}{T_2^4} - \frac{1}{T_1^4} \right)$
31. The temperature and the surface area of the body are 227°C and 0.15 m^2 , respectively. If its transmitting power is negligible and reflecting power is 0.5, then calculate the thermal power of the body.
 (Given, $\sigma = 5.67 \times 10^{-8} \text{ J/m}^2\text{s/K}$)
 a. 300 W b. 265.78 W c. 201 W d. 320.89 W
32. The surface temperature of the sun is ' T ' K and the solar constant for a plate is ' s '. The sun subtends an angle θ at the planet. Then,
 a. $s \propto T^4$ b. $s \propto T^2$
 c. $s \propto \theta^2$ d. $s \propto \theta$
33. When a blackened platinum wire is heated gradually, it appears
 a. first blue, then red and finally white
 b. first red, then blue and finally white
 c. first white, then blue and finally red
 d. first red, then white and finally blue

34. Surface temperature of the sun as estimated is 6032.25 K. Find the wavelength at which sun radiates maximum energy (Given, Wien's constant = 0.2898 cm-K)
- $\lambda_m = 5000 \text{ \AA}$
 - $\lambda_m = 4804.2 \text{ \AA}$
 - $\lambda_m = 3809.5 \text{ \AA}$
 - $\lambda_m = 2891.6 \text{ \AA}$

35. The plots of intensity of radiation *versus* wavelength of 3 black bodies of temperatures T_1, T_2 and T_3 as shown in the figure, then

- $T_1 > T_2 > T_3$
- $T_3 > T_2 > T_1$
- $T_1 > T_3 > T_2$
- $T_1 < T_3 < T_2$



BITSAT Archives

- A partition wall has two layers of different materials *A* and *B* in contact with each other. They have the same thickness but the thermal conductivity of layer *A* is twice that of *B*. At steady state, if the temperature difference across the layer *B* is 50 K, then the corresponding difference across the layer *A* is [2013]
 - 50 K
 - 12.5 K
 - 25 K
 - 60 K
- The rate at which a black body emits radiation at a temperature *T* is proportional to [2012]
 - $\frac{1}{T}$
 - T
 - T^3
 - T^4
- A hot and a cold body are kept in vacuum separated from each other. Which of the following cause decrease in temperature of the hot body? [2011]
 - Radiation
 - Convection
 - Conduction
 - Temperature remains unchanged
- In a 10 m deep lake, the bottom is at a constant temperature of 4°C. The air temperature is constant at -4°C. The thermal conductivity of ice is 3 times that of water. Neglecting the expansion of water on freezing, the maximum thickness of ice will be [2010]
 - 7.5 m
 - 6 m
 - 6 m
 - 2.5 m
- Two slabs *A* and *B* of equal surface area are placed one over the other such that their surfaces are completely in contact. The thickness of slab *A* is twice that of *B*. The coefficient of thermal conductivity of slab *A* is twice that of *B*. The first surface of slab *A* is maintained at 100°C, while the second surface of slab *B* is maintained at 25°C. The temperature at the contact of their surfaces is [2008]
 - 62.5°C
 - 45°C
 - 55°C
 - 85°C
- The wavelength of the radiation emitted by a body depends upon [2005]
 - the nature of the surface
 - the area of the surface
 - the temperature of the surface
 - All of the above factors

Answer with Solutions

Practice Exercise

1. (b) In steady state,

$$\frac{\Delta H}{\Delta t} = -KA \frac{d\theta}{dx}$$

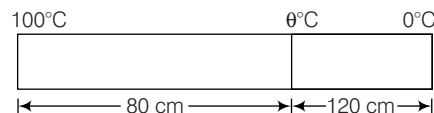
Here, $\frac{\Delta H}{\Delta t} = \text{constant} \therefore \frac{\Delta\theta}{dx} = \text{constant}$

2. (c) Temperature gradient $\frac{d\theta}{dx} = \frac{32}{1000} \text{ } ^\circ\text{C/m} = \frac{32}{10^5} \text{ } ^\circ\text{C/cm}$

Loss of energy by earth is given by

$$\begin{aligned} Q &= KA \frac{d\theta}{dx} \times 86400 \\ &= 0.008 \times \frac{4 \times 22}{7} \times (64 \times 10^8)^2 \times \frac{32}{10^5} \times 86400 \\ &= 1.1 \times 10^{18} \text{ cal} = 10^{18} \text{ cal} \end{aligned}$$

3. (b) In steady state, temperature gradient of rod remains constant.



$$\begin{aligned} \therefore \frac{100^\circ - \theta}{80} &= \frac{\theta - 0}{120} \\ \text{or } 100^\circ - \theta &= \left(\frac{80}{120}\right)\theta \\ \text{or } 100^\circ - \theta &= \frac{2}{3}\theta \quad \text{or } 100^\circ = \frac{5}{3}\theta \\ \therefore \theta &= \frac{100 \times 3}{5} = 60^\circ\text{C} \end{aligned}$$

4. (a)
- $\therefore$
- Rate of transmission of heat

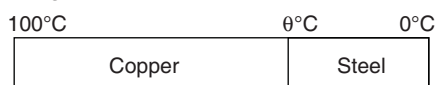
$$\frac{\Delta H}{\Delta t} = \frac{KA(100 - 0)}{l} = \frac{386 \times 2 \times 10^{-4} \times 100}{2}$$

or $\frac{\Delta mL}{\Delta t} = \frac{386 \times 2 \times 10^{-4} \times 100}{2}$

$$\therefore \frac{\Delta m}{\Delta t} = \frac{386 \times 2 \times 10^{-4} \times 100}{2 \times 80} \quad (\because L = 80)$$

$$= 4.825 \times 10^{-2} \text{ kg/s} = 48.25 \text{ g/s}$$

5. (c) Since, both rods are in series. So, the rate of flow of heat through both wires will be same.



- $\therefore$
- Rate of heat transfer

$$\frac{\Delta H}{\Delta t} = \frac{K_1 A (100 - \theta)}{l_1} = \frac{K_2 A (\theta - 0)}{l_2}$$

or $100 - \theta = \left(\frac{l_1}{l_2}\right) \left(\frac{K_2}{K_1}\right) \theta$ or $100 - \theta = \left(\frac{1}{1.5}\right) \left(\frac{46}{386}\right) \theta$

$$\therefore \theta = 92.64 \approx 93^\circ \text{C}$$

6. (b) The rate of flow of heat is equal to rate of loss of heat of free end of copper

$$\therefore \frac{\Delta H}{\Delta t} = \frac{K_1 A (100 - \theta)}{l_1}$$

$$= \frac{386 \times 3.14 \times 10^{-4} (100 - 92.64)}{1} = 0.89 \text{ J/s}$$

7. (a) We know,
- $\theta = \frac{KA \Delta T}{\Delta x} t$

Quantity of heat flowing through the stone in one hour

$$= 4800 \times 80 = 384 \times 10^3 \text{ cal}$$

$$K = \frac{\theta}{t} \times \frac{\Delta x}{\Delta T} \times \frac{1}{A} = \frac{384 \times 10^3}{3600} \times \frac{10}{100} \times \frac{1}{3600}$$

$$K = 2.96 \times 10^{-3} \text{ cal/cm s } ^\circ \text{C}$$

8. (c) Rate of transmission of heat
- $\frac{dQ}{dt} = \frac{4\pi K r_1 r_2 \Delta \theta}{(r_2 - r_1)}$

$$\therefore \frac{dQ}{dt} = \frac{4 \times \pi \times 1.1 \times 0.098 \times 0.1 \times 100}{0.002} = 6769.8 \approx 6770 \text{ W}$$

Rate of ice melt = $\frac{m}{t} \Rightarrow \left(\frac{m}{t}\right) L = \frac{d\theta}{dt}$

$$\left(\frac{m}{t}\right) \times 336 \times 10^3 = 6770 \Rightarrow \frac{m}{t} = 0.02 \text{ kg/s}$$

9. (b) Heat flow into the box =
- $\frac{\Delta Q}{\Delta t}$

$$\frac{\Delta Q}{\Delta t} = KA \left(\frac{\Delta T}{\Delta x}\right) = (0.04) (1.5) \left(\frac{30}{0.05}\right) = 36 \text{ W}$$

$\Delta Q =$ In an hour total energy

$$= 36 \times \text{one hour} = 36 \times 60 \times 60 \text{ J} = 129.6 \text{ kJ}$$

$$\therefore \Delta Q = mL = m (80 \text{ kcal/kg}) (4.184 \text{ kJ/kcal})$$

$$\therefore m = \frac{\Delta Q}{80 \times 4.184} = \frac{129.6}{80 \times 4.184} \text{ kg} = 0.39 \text{ kg ice will melt}$$

10. (d)

11. (b)
- $Q = Q_{\text{steel}} + Q_{\text{copper}}$

$$= \frac{12\pi (0.5)^2 \times 10^{-4} \times 100}{2} + \frac{92\pi (1^2 - 0.5^2) \times 10^{-4} \times 100}{2}$$

$$= 150\pi \times 10^{-4} + 3450\pi \times 10^{-4} = 3600\pi \times 10^{-4} = 1.13 \text{ cal/s}$$

12. (c) In steady state condition rate of heat flow in both rods must be equal. Let junction temperature =
- T

$$H = \frac{Q}{T} = \frac{K_{\text{Al}} A_{\text{Al}} (400^\circ - T)}{L_{\text{Al}}} = \frac{K_{\text{Cu}} A_{\text{Cu}} (T - 20^\circ)}{L_{\text{Cu}}}$$

$$= \frac{240 (2A) (400^\circ - T)}{L} = \frac{400 \times A \times (T - 20^\circ)}{2L}$$

$$T = 288.24^\circ \text{C}$$

13. (c) Let
- ΔT
- = temperature difference between the rings

R = total thermal resistance of the ring when $\theta = 180^\circ$.

The resistance will be $R/2$, $R/2$ and both are in parallel, so, equivalent resistance = $R/4$

$$\text{Rate of total heat flow} = \frac{Q_1}{t} = 1.2 = \frac{\Delta T}{(R/4)} \Rightarrow$$

When $\theta = 90^\circ$ there are two sections with resistances, $R/4$ and $3R/4$ in parallel.

So, equivalent resistance = $\frac{3R}{16}$

$$\text{Rate of heat flow} = \frac{Q_2}{t} = \frac{\Delta T}{\left(\frac{3R}{16}\right)}$$

$$\Rightarrow I_2 = \frac{16}{3} \left(\frac{\Delta T}{R}\right)$$

$$I_2 = 1.6 \text{ W}$$

$$\Rightarrow I_2 = 1.6 \text{ W}$$

14. (a) Heat given by water in cooling from
- 25°
- to
- 10°C
- .

$$Q_1 = (cm \Delta T)_w = 200 \times 1 \times (25 - 10) = 3000 \text{ cal}$$

Heat absorbed by m g of ice at -14°C

$$Q_2 = (mc \Delta T)_{\text{ice}} + mL_f + (mc \Delta T)_w$$

$$= m[(0.5)[0 - \{-14\}] + 80 + 1(10 - 0)]$$

$$= 97 \text{ m cal}$$

$$Q_2 = Q_1$$

$$97m = 3000 = 30.93 \text{ g}$$

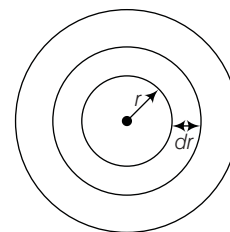
$$m = 31 \text{ g}$$

15. (a) We consider a spherical element of radius
- r
- and thickness
- dr
- . The resistance of considered element is

$$dR = \frac{dr}{KA} = \frac{dr}{K 4\pi r^2} = \frac{dr}{\frac{K_0}{r^2} 4\pi r^2}$$

$$\therefore dR = \frac{1}{4\pi K_0} dr$$

The equivalent resistance, $R = \frac{1}{4\pi K_0} \int_a^b dr = \frac{(b-a)}{4\pi K_0}$



16. (a) Rate of transmission of heat flow

$$\frac{\Delta H}{\Delta t} = \frac{\theta_1 - \theta_2}{R} = \frac{4\pi K_0 (\theta_1 - \theta_2)}{b - a}$$

17. (a) According to Newton's law of cooling,

$$\frac{d\theta}{dt} = -K(\theta - \theta_0)$$

where, θ = temperature of body θ_0 = temperature of surroundings

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta - \theta_0} = -K \int_0^1 dt$$

$$\log \left(\frac{\theta_2 - \theta_0}{\theta_1 - \theta_0} \right) = -Kt$$

$$\log \left(\frac{60 - 25}{50 - 25} \right) = -K \times 10 \times 60$$

Let α = temperature at end of 10 minutes.

$$\log \left(\frac{50 - 25}{\alpha - 25} \right) = -K \times 10 \times 60$$

$$\alpha = 42.85^\circ \text{C}$$

18. (a)

19. (a) For a black body,

Rate of radiation, $Q = \sigma T^4$ After smoothing and doubling the temperature = Rate Q

$$= 0.9 \sigma (2T)^4 = 0.9 \times 10^4 Q$$

Charge = $(0.9 \times 2^4 - 1) Q$ watt

20. (c) 21. (a) 22. (b)

23. (c) Radiation
- $\propto \frac{T^4}{d^2}$

$$P \propto \frac{T_0^2}{d_0^2}$$

When T and d are doubled

$$p' \propto \frac{(2)^4 T_0^4}{2^2 d_0^2} \propto \frac{4T_0^4}{d_0^2} \propto 4P$$

Therefore, power received by surface will become $4P$.

24. (b) Initially,
- $Q_1 = \frac{KA_1(T_1 - T_2)}{l_1}$
- but on doubling all

dimensions $l_2 = 2l_1$ and $A_2 = 4A_1$.

$$\text{Hence, } Q_2 = \frac{KA_2(T_1 - T_2)}{l_2} = \frac{K4A_1(T_1 - T_2)}{2l_1}$$

$$= 2 \frac{KA_1(T_1 - T_2)}{l_1} = 2Q_1$$

25. (b)

26. (d)
- $\frac{E + \Delta E}{E} = \frac{[T + (5/100)T]^4}{T^4}$

$$\left(1 + \frac{\Delta E}{E} \right) = (1 + 0.05)^4 = (1.05)^4$$

$$\frac{\Delta E}{E} = 1.2155 - 1 = 0.2155$$

$$\frac{\Delta E}{E} \times 100 = 0.2155 \times 100 = 21.55\%$$

27. (a) Energy radiated per second
- $P = e \sigma AT^4$

$$\therefore \frac{P_1}{P_2} = \frac{A_1 T_1^4}{A_2 T_2^4} = \left(\frac{4\pi r^2}{4\pi (4r)^2} \right) \frac{16T_0^4}{T_0^4} = \frac{r^2}{16r^2} \times \frac{16T_0^4}{T_0^4} = 1:1$$

28. (a) Radiation is directly proportional to the radiating area cooling fast.

For a given mass, area of circular plate is maximum. So, it cools first.

29. (a)
- $Q = EAt = e\sigma (T^4 - T_0^4) At$

where, t = time T_0 = temperature of surrounding when, $T > T_0$

$$Q = e\sigma T^4 At$$

$$300 = e \times (5.67 \times 10^{-8})(1000)^4 (5.00 \times 10^{-4})(60)$$

$$e = 0.18$$

30. (c) We know,
- $P = 4\pi r^2 \sigma T^4 = mS \left(-\frac{dT}{dt} \right)$

$$= -\frac{4}{3} \pi r^3 \rho \frac{dT}{dt} \text{ and } R = \frac{dT}{dt}$$

$$P \propto r^2 \text{ and } R \propto \frac{1}{r}$$

$$\int_{T_1}^{T_2} \frac{dT}{T^4} = (\text{constant}) \int_0^1 dt$$

$$t = c \left[\frac{1}{T_2^3} - \frac{1}{T_1^3} \right]$$

31. (b) Absorbing power of a body is equal to emissivity of the body.

$$\text{And also } a + r + t = 1 \Rightarrow 0 + 0.5 + a = 1$$

$$a = 0.5, \quad \varepsilon = a = 0.5$$

$$\frac{Q}{t} = \varepsilon \sigma AT^4 = (0.5)(5.67) \times 10^{-8} (0.15) \times (273 + 227)^4$$

$$\frac{Q}{t} = 265.78 \text{ W}$$

32. (a) Let radius of sun =
- R

Distance of earth from the sun = d

$$\text{Power radiated from the sun} = (4\pi R^2) \sigma T^4 = P$$

$$\text{Energy received/area/s} = S = \frac{P}{4\pi d^2}$$

$$= 4\pi R^2 \sigma \frac{T^4}{4\pi d^2} = \sigma T^4 \frac{R^2}{d^2} = \frac{1}{4} \sigma T^4 \left(\frac{2R}{d} \right)^2$$

$$\text{Angle subtended by sun at earth, } \alpha = \frac{2R}{d}$$

$$S = \text{constant} \times T^4 \times \alpha^2$$

$$\therefore S \propto T^4$$

33. (b) According to Wien's law

$$\lambda_m T = \text{constant}$$

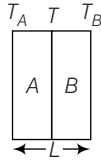
When temperature increases, wavelength decreases.

Due to this the colour of object appears first red (of longer wavelength) then blue (shorter wavelength) and finally white (all wavelengths).

34. (b) 35. (c)

BITSAT Archives

1. (c) Let
- T
- be the junction temperature

Here, $K_A = 2K_B$, $T - T_B = 50$ KAt the steady state, $H_A = H_B$

$$\Rightarrow \frac{K_A A (T_A - T)}{L} = \frac{K_B A (T - T_B)}{L}$$

$$\Rightarrow 2K_B (T_A - T) = K_B (T - T_B)$$

$$\Rightarrow T_A - T = \frac{T - T_B}{2}$$

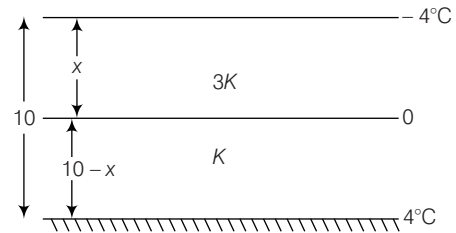
$$= \frac{50}{2} = 25 \text{ K}$$

2. (d) From Stefan's law, the rate of emission of energy per unit surface area of a black body is inversely proportional to the fourth power of absolute temperature (
- T
-) of the body.

$$E = \sigma T^4 (\sigma = \text{Stefan's constant})$$

3. (a) Heat flow through vacuum is possible in radiation mode due to which temperature of hot body falls.

$$4. (a) KA \left(\frac{4 - 0}{10 - x} \right) = 3KA \left(\frac{0 - (-4)}{x} \right)$$



$$\Rightarrow x = 7.5 \text{ m}$$

5. (a) The temperature at the contact of the surface

$$= \frac{k_2 d_2 \theta_1 + k_1 d_1 \theta_2}{k_1 d_2 + k_2 d_1}$$

$$= \frac{2k_2 d_2 \times 100 + 2d_2 \times k_2 \times 35}{2k_2 d_2 + k_2 2d_2}$$

$$= \frac{200 + 50}{4} = 62.5^\circ \text{C}$$

6. (c) According to Wien's law,

$$\lambda \propto \frac{1}{T}$$

i.e. it depends on the temperature of the surface.

19

Ray Optics

Reflection of Light

The phenomenon of bouncing back of light rays in the same medium on striking a smooth surface is called reflection of light.

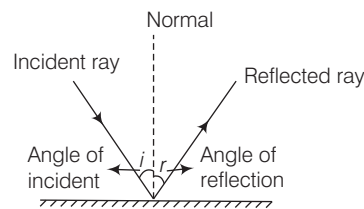
Laws of Reflection

It follows two laws, i.e.

(i) Angle of incidence, ($\angle i$) is equal to angle of reflection, ($\angle r$)

i.e. $\angle i = \angle r$

(ii) Incident ray, normal ray and reflected ray lie in the same plane.



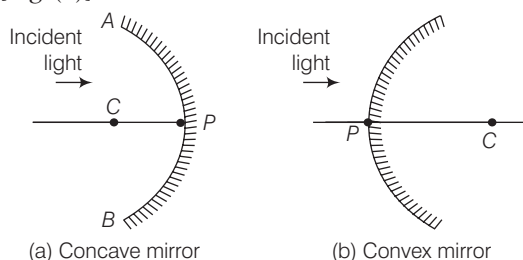
Reflection from a Plane Mirror

1. If a ray is incident on a plane mirror at an angle of incidence i , then it suffers a deviation of $(\pi - 2i)$.
2. While keeping an object fixed, a plane mirror is rotated in its plane by an angle θ , then the reflected ray rotates in the same direction by an angle 2θ .
3. Focal length as well as the radius of curvature of a plane mirror is infinity. Power of a plane mirror is zero.
4. If two plane mirrors are inclined to each other at an angle θ , the total number of images formed of an object kept between them, is $n = \frac{2\pi}{\theta}$ or $\left(\frac{2\pi}{\theta} - 1\right)$ whichever is odd.
5. The minimum size of a plane mirror fixed on a wall of a room, so that a person at the centre of the room may see the full image of the wall behind him, should be $\frac{1}{3}$ rd the size of the wall.

Reflection of Light at a Spherical Surface or Spherical Mirror

Mirrors having their reflecting surface spherical are called spherical mirrors. Spherical mirrors are of two types

- (i) **Concave mirror** If reflection takes place from the inner surface, then the mirror is called concave [Fig. (a)].
- (ii) **Convex mirror** If reflection takes place from the outer surface, then the mirror is called convex [Fig. (b)].

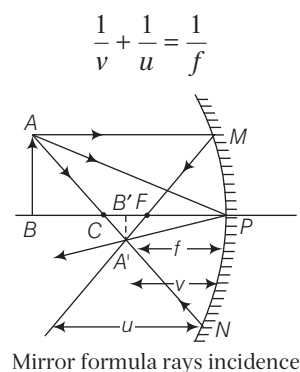


- (iii) **Focal length** It is the distance between focus F and pole P of the mirror. It is represented by f .

i.e. $f = \frac{R}{2}$, where R = radius of curvature

Mirror Formula

Mirror formula is the relation between object distance (u), image distance (v) and focal length (f) for a spherical mirror and it is given by



$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Magnification of Image in a Spherical Mirror

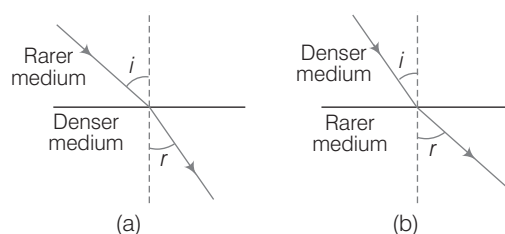
The size of an image reflected through a spherical mirror varies with the original size of object. This is called **linear magnification** of the image, which is the ratio of height of image to the height of object given by

$$m = -\frac{v}{u} = \frac{h'}{h}$$

where, m = magnification, v = image distance, u = object distance, h' = height of image and h = height of object.

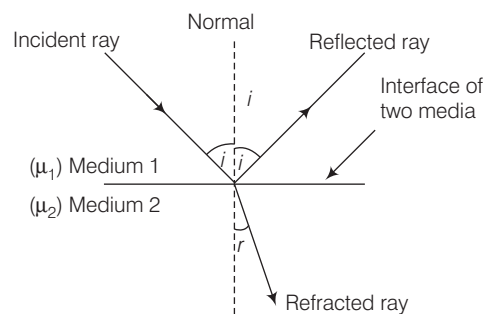
Refraction of Light

When light passes from one medium, say air, to another medium, say glass, a part is reflected back into the first medium and the rest passes into the second medium. When it passes into the second medium, it either bends towards the normal or away from the normal. This phenomenon is known as refraction.



Laws of Refraction

- (i) When light goes from one medium to another, the frequency and phase of light do not change, however velocity and wavelength of light changes.
- (ii) The incident ray, the refracted ray and normal to the interface at point of incidence, all lie in the same plane.
- (iii) The product of refractive index and sine of angle of incidence/refraction at a point in a medium is constant



i.e. $\mu \times \sin i = \text{constant}$

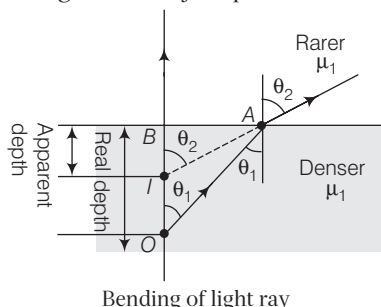
So, from figure $\frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r} = {}_1\mu_2$

where, ${}_1\mu_2$ is refractive index of medium 2 with respect to medium 1.

This law is also known as **Snell's law**.

Apparent Shift of an Object due to Refraction

Due to bending of light at the interface of two different media, the image formation due to refraction creates an illusion of shifting of the object position.



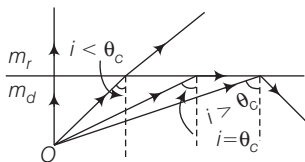
$$\frac{\text{Real depth}}{\text{Apparent depth}} = \frac{1}{\mu_2}$$

Total Internal Reflection

When light travelling in a denser medium incident on the interface between denser medium (medium with higher refractive index) and rarer medium (medium with lesser refractive index), then refracted ray bends away from the normal forming a greater angle of refraction than angle of incidence.

In this situation, when the angle of incidence exceeds a particular angle, the incident ray comes back into first medium (denser medium). This angle is called **critical angle** (θ_c).

At critical angle, there is no refracted ray found because $\angle r = 90^\circ$.



$$\frac{\sin \theta_c}{\sin 90^\circ} = \frac{\mu_{\text{rarer}}}{\mu_{\text{denser}}} = \frac{\mu_r}{\mu_d}$$

$$\Rightarrow \sin \theta_c = \frac{\mu_r}{\mu_d} \Rightarrow \theta_c = \sin^{-1} \left(\frac{\mu_r}{\mu_d} \right)$$

where, μ_r = refractive index of rarer medium
and μ_d = refractive index of denser medium

Refraction from a Spherical Surface

Let an object be placed in a medium of refractive index n_1 at a distance u from the pole of a spherical surface of radius of curvature R and after refraction, its image is formed in a medium of refractive index n_2 at a distance v , then

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

The relation is true for all surfaces, whether the image formed is real or virtual.

Lens

The lens formula is given by $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

For a thin object of height h placed perpendicular to the principal axis at a distance u , if the height of image formed is h' , then **lateral** or **transverse magnification** m is given by

$$m = \frac{h'}{h} = \frac{v}{u} = \frac{f}{f+u} = \frac{f-v}{f}$$

For a small sized object placed linearly along the principal axis, its **axial** or **longitudinal magnification** is given by

$$\text{Axial magnification} = -\frac{dv}{du} = \left(\frac{v}{u} \right)^2 = \left(\frac{f}{f+u} \right)^2 = \left(\frac{f-v}{f} \right)^2$$

The **power of a lens** is mathematically given by the reciprocal of its focal length,

$$\text{i.e. power, } P = \frac{1}{f(\text{in m})}$$

SI unit of power is dioptre (D). Power of a converging lens is positive and that of a diverging lens is negative.

Lens Maker's Formula

Focal length of the lens is given by

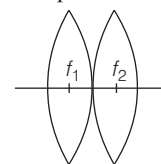
$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

If $\mu_2 = \mu$ and $\mu_1 = 1$, then

$$\Rightarrow \frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Combination of Thin Lenses in Contact

If two or more lenses are placed in contact, then



Combination of convex lenses

Total focal length of the combination,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \dots = \sum_{i=1}^n \frac{1}{f_i}$$

Total power of combination,

$$P = P_1 + P_2 + P_3 + \dots = \sum_{i=1}^n P_i$$

Total magnification of combination,

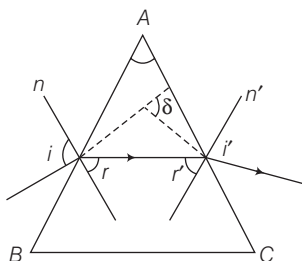
$$M = m_1 \times m_2 \times m_3 \times \dots \text{ or } M = m_1 m_2 m_3 \dots$$

NOTE If the lenses are separated by a distance d , then

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \quad \text{and} \quad P = P_1 + P_2 - P_1 P_2 d$$

Deviation by a Prism

A prism is a homogeneous, transparent medium bounded by two plane surfaces inclined at an angle A with each other. These surfaces are called as refracting surfaces and the angle between them is called angle of prism A . Deviation produced by a prism is



$$\delta = i + i' - A \Rightarrow r + r' = A$$

For grazing incidence $i = 90^\circ$, and grazing emergence $i' = 90^\circ$.
For minimum deviation,

$$(i) \ i = i' \text{ and } r = r'$$

$$(ii) \ \mu = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\frac{A}{2}}$$

(iii) $S = i + e - A$, where i is the angle of incidence, e is the angle of emergence and A is the angle of prism. In case of minimum deviation, ray is passing through prism symmetrically.

For maximum deviation ($\delta_{\max}$), $i = 90^\circ$ or $i' = 90^\circ$

For thin prism, $\delta = (\mu - 1) A$

Dispersion by a Prism

Dispersion of light is the phenomenon of splitting of white light into its constituent colours on passing light through a prism. This is because different colours have different wavelength, and hence different refractive indices

Angular dispersion $= \delta_v - \delta_r = (n_v - n_r) A$, where n_v and n_r represent refractive index for violet and red lights.

$$\text{Dispersive power, } \omega = \frac{n_v - n_r}{n - 1}$$

where $n = \frac{n_v + n_r}{2}$ is the mean refractive index.

By combining two prisms with angles A and A' and RI n and n' respectively we can create conditions of

$$(a) \text{ Dispersion without deviation when, } A' = -\frac{(n-1)A}{(n'-1)}$$

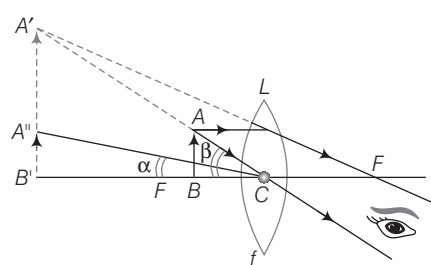
$$(b) \text{ Deviation without dispersion when } A' = -\left[\frac{n_v - n_r}{n'_v - n'_r}\right] A$$

Microscope

It is an optical instrument which forms a magnified image of a small nearby object and thus, increases the visual angle subtended by the image at the eye so that the object is seen to be bigger and distinct.

Simple Microscope (Magnifying Glass)

It consists of a single convex lens of small focal length and forms a magnified image of an object placed between the optical centre and the principal focus of the lens. If the image is formed at the near point of eye.

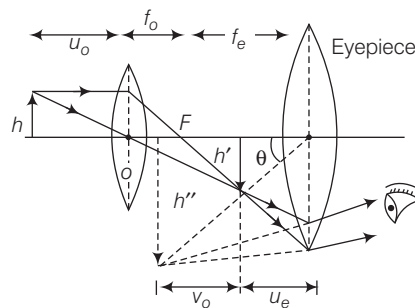


Then $m = \left(1 + \frac{D}{f}\right)$ but if the image is formed at infinity, then

$m = \frac{D}{f}$, where D = normal distance (25 cm), f = focal length of magnifying lens.

Compound Microscope

It has two converging (convex) lenses i.e. one near object, called objective lens and other near eye, called eyepiece. Objective has smaller focal length and aperture than eyepiece.



Set up of compound microscope

Magnifying Power of Compound Microscope

Magnification produced by the compound microscope is given by,

$$m = \frac{v}{a} \left(\frac{D}{f_e} \right)$$

$$\text{For relaxed eye, } M_\infty = -\frac{v_o}{u_o} \left(\frac{D}{f_e} \right)$$

and length of microscope, $L_\infty = v_o + f_e$. When final image is formed at least distance of distinct vision, then

$$M_D = -\frac{v_o}{u_o} \left(1 + \frac{D}{f_e} \right)$$

and length of microscope, $L_D = v_o + u_e$

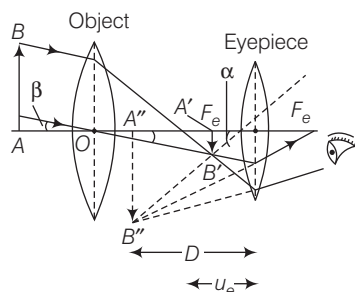
where, v_o = distance of first image from objective lens

u_o = distance of object from objective lens

f_e = focal length of eyepiece.

Astronomical Telescope

It is an optical device which increases the angle at the eye by forming the image of a distant object at the least distance of distinct vision, so that the object is seen distinct and bigger.



Set up of astronomical telescope

Magnifying Power of Astronomical Telescope

For relaxed eye, $M_{\infty} = -\frac{f_o}{f_e}$

Length of telescope, $L_{\infty} = f_o + f_e$

When a final image is formed at least distance of distinct vision, then,

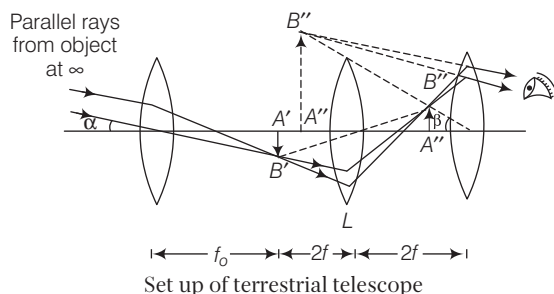
$$M_D = -\frac{f_o}{f_e} \left(1 + \frac{f_e}{D}\right)$$

Length of telescope, $L_D = f_o + u_e$

where, f_o = Focal length of objective
 f_e = Focal length of eyepiece.

Terrestrial Telescope

In an astronomical telescope, the final image is inverted with respect to the object. To remove this difficulty, a convex lens of focal length f is included between the objective and the eyepiece in such a way that the focal plane of the objective is a distance $2f$ away from this lens.



Set up of terrestrial telescope

Magnification of Terrestrial Telescope

(a) For relaxed eye, $M_{\infty} = f_o / f_e$

In this position, length of telescope,

$$L_{\infty} = f_o + 4f + f_e$$

(b) When a final image is formed at least distance of distinct vision,

$$M_D = \frac{f_o}{f_e} \left(1 + \frac{f_e}{D}\right)$$

Length of telescope,

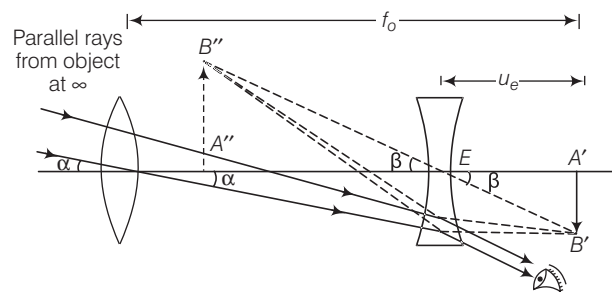
$$L_{\infty} = f_o + 4f + u_e$$

where, f_o = Focal length of objective lens

f_e = Focal length of eyepiece.

Galilean Telescope

A simple model of Galilean telescope is shown in figure. A convergent lens is used as the objective and a divergent lens as the eyepiece.



Set up of Galilean telescope

Magnification of Galilean Telescope

(a) For relaxed eye, $M_{\infty} = \frac{f_o}{f_e}$

In this position, length of telescope, $L_{\infty} = f_o - f_e$

(b) When final image is formed at least distance of distinct vision,

$$M_D = \frac{f_o}{f_e} \left(1 + \frac{f_e}{D}\right)$$

Length of telescope, $L_D = f_o - u_e$

Resolving Power of an Optical Instrument

Resolving power of an optical instrument is its ability to produce distinct images of two points of an object (or two nearby objects) very close together.

The minimum distance (or angular distance) between two points of an object whose images can be formed distinctly by the lens of an optical instrument, is called its **limit of resolution**.

Resolving Power of a Telescope

If the aperture (diameter) of the telescope objective be the D , then the minimum angular separation ($d\theta$) between two distant objects, whose images are just resolved by the telescope, is

$$d\theta = \frac{1.22\lambda}{D}$$

and resolving power of the telescope,

$$RP = \frac{1}{d\theta} = \frac{D}{1.22\lambda}$$

Resolving Power of a Microscope

The least distance (d) between two points, whose images are just seen distinctly by a microscope, is given by

$$d = \frac{1.22\lambda}{2n_m \sin\theta}$$

where, λ = wavelength of light used to illuminate the object,

n_m = refractive index of the medium between the object and the objective lens

θ = semi angle of the cone of light from the point object

The term $n_m \sin\theta$ is generally called the **numerical aperture** of the microscope the objective.

$\therefore$ Resolving power of the microscope,

$$RP = \frac{1}{d} = \frac{2n_m \sin\theta}{1.22\lambda} = \frac{(NA)}{0.61\lambda}$$

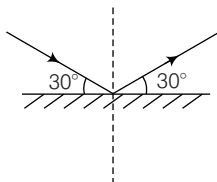
Practice Exercise

1. A hair dresser stand with her nose 20 cm in front of a plane mirror. For what distance must she focus her eyes in order to see her nose in the mirror?

a. 40 cm b. 50 cm
c. 30 cm d. 60 cm

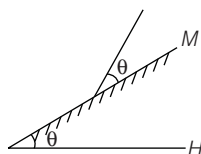
2. On a plane mirror, a ray of light is incident at an angle of 30° with horizontal. To make the reflected ray vertical, at what angle with horizontal must a plane mirror be placed?

a. 30° b. 60°
c. 45° d. 54°



3. A mirror is inclined at an angle of θ with the horizontal. If a ray of light is incident at an angle of incidence θ , then the reflected ray makes the following angle with horizontal

a. θ b. 2θ
c. $\frac{\theta}{2}$ d. None of these

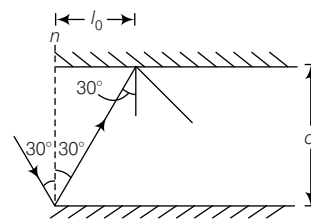


4. Two mirrors each 1.6 m long are facing each other. The distance between the mirrors is 20 cm. A light ray is incident on one end of the mirror at an angle of incidence of 30° . How many times in the ray reflected before it reaches the other end?

a. There are 15 reflections counting the first one
b. There are 13 reflections counting the first one
c. There are 12 reflections counting the first one
d. None of the above

5. A pole 5 m high is situated on a horizontal surface. Sun rays are incident at an angle 30° with the vertical. The size of shadow on horizontal surface is

a. 5 m
b. $\frac{5}{\sqrt{3}}$ m
c. $\frac{10}{\sqrt{3}}$ m
d. None of these



6. A beautiful girl with two normal eyes wants to see full width of her face by a plane mirror. The eye to eye and ear to ear distances of her face are 4 inch and 6 inch respectively. Find the minimum width of the required mirror.

a. 1 inch b. 2 inch c. 3 inch d. 4 inch

7. A ray of light falls on a plane mirror. When the mirror is turned, about an axis at right angle to the plane of the mirror through 20° , the angle between the incident ray and new reflected ray is 45° . Find the angle between the incident ray and original reflected ray.

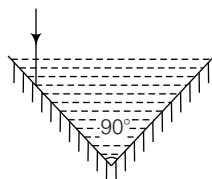
a. 15° b. 30° c. 45° d. 60°

8. The shortest height of a vertical mirror required to see the entire image of a man, will be

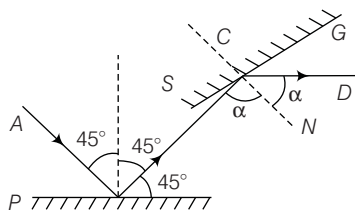
a. one-third the man's height
b. half the man's height
c. two-third the man's height
d. data insufficient

9. Two plane mirrors are perpendicular to each other. A ray after suffering reflection from the two mirrors will be
- perpendicular to the original ray
 - parallel to the original ray
 - parallel to the first mirror
 - at 45° to the original ray

10. A vessel consists of two plane mirrors at right angles (as shown in figure). The vessel is filled with water. The total deviation in incident ray is

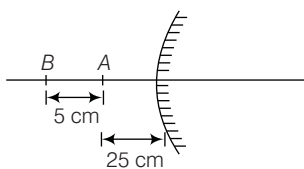


- 0°
 - 90°
 - 180°
 - None of the above
11. A light ray is incident on a horizontal plane mirror at an angle of 45° . At what angle should a second plane mirror be placed in order that the reflected ray finally be reflected horizontally from the second mirror, as shown in figure?



- $\theta = 30^\circ$
 - $\theta = 24^\circ$
 - $\theta = 22.5^\circ$
 - $\theta = 67.5^\circ$
12. If two mirrors are inclined at some angle and an object is placed between the mirrors and there are 7 images formed for an object. Then, what is angle between the mirrors?
- 54°
 - 50°
 - 60°
 - 64°
13. Two plane mirrors are placed at some angle. There are five images formed when an object is placed symmetrically between them. Find the angle between the mirrors.
- 60°
 - 65°
 - 30°
 - 45°
14. If two adjacent walls and the ceiling of a rectangular room are mirror surfaced, then how many images of himself, a man can see?
- 3
 - 5
 - 6
 - 9

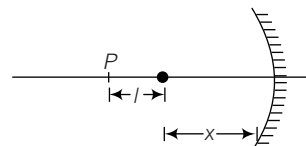
15. A convex mirror of focal length 10 cm is shown in figure. A linear object $AB = 5$ cm is placed along the optical axis. Point B is at distance 25 cm from the pole of mirror. The size of image of AB is



- 2.5 cm
- 0.64 cm
- 0.36 cm
- None of these

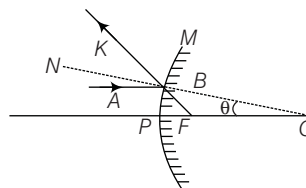
16. If u represents object distance from pole of spherical mirror and v represents image distance from pole of mirror and f is the focal length of the mirror, then a straight line $u = v$ will cut u versus v graph at
- (f, f)
 - $(2f, 2f)$
 - $(f, 2f)$
 - $(0, 0)$

17. A short linear object of length l lies on the axis of a spherical mirror of focal length f , at a distance x from the mirror. Then, the length of the image (P) so obtained will be



- $\frac{lf}{(x-f)}$
 - $\frac{lf^2}{(x-f)^2}$
 - $\frac{lf}{x}$
 - $\frac{l(x-f)}{x}$
18. A rear view mirror of a vehicle is cylindrical having radius of curvature 10 cm and length of arc of curved surface is 10 cm. Find the field of view in radian, if it is assumed that the eye of the driver is at a large distance from the mirror.
- 0.5
 - 1
 - 2
 - 4

19. In the figure, AB and BK represents incident and reflected rays. If angle $BCF = \theta$, then $\angle BFP$ will be equal to



- θ
 - 2θ
 - 3θ
 - 3.5θ
20. Find the position of 1 cm tall object which is placed 8 cm in front of a concave mirror of radius of curvature 24 cm.
- 24 cm
 - 25 cm
 - 26 cm
 - 27 cm
21. A convex driving mirror of focal length 20 cm, is fitted in a motor car. If the second car 2 m broad and 1.6 m high is 6 m away from first car and overtake the first car at a relative speed of 15 m/s, then how fast will the image be moving?
- 0.016 m/s
 - 0.0257 m/s
 - 0.162 m/s
 - 0.0073 m/s

22. When an object is placed at a distance of 25 cm from a mirror, the magnification is m_1 . But the magnification becomes m_2 , when the object is moved 15 cm farther away with respect to the earlier position. If $\frac{m_1}{m_2} = 4$,

then find the focal length of the mirror and what type of mirror it is?

- 20 cm, convex
 - 20 cm, concave
 - 10 cm, convex
 - 10 cm, concave
23. Two objects A and B when placed in turns in front of a concave mirror, give images of equal size. The focal length of the concave mirror is 7.5 cm and size of object A is three times the size of object B . Find the distance of B from the mirror, if A is placed 30 cm from the mirror.
- 18 cm
 - 15 cm
 - 20 cm
 - 25 cm

24. An object of height 5 cm is placed in midway between a concave mirror of radius of curvature 30 cm and a convex mirror of placed opposite to each other and are 60 cm apart. Find the position of the image formed by reflection at convex mirror.

a. 10 cm b. 20 cm c. 15 cm d. 13 cm

25. The focal length of plano-convex lens, the convex surface of which is silvered is 0.3 m, if μ of the lens is $7/4$, the radius of curvature of the convex surface is

a. 0.45 m b. 1.05 m c. 3 m d. 0.9 m

26. The magnification of a compound microscope is 30 and the focal length of its eye piece is 5 cm. Calculate the magnification produced by the objective, when the image is to be formed at least distance of distinct vision (25 cm).

a. 5 b. 6 c. 8 d. 10

27. A convergent doublet of separated lens, corrected for spherical aberration, are separated by 2 m and has an equivalent focal length of 10 cm. Calculate the focal length of its component lenses.

a. $f_1 = 18$ cm, $f_2 = 10$ cm

b. $f_1 = 20$ cm, $f_2 = 28$ cm

c. $f_1 = 20$ cm, $f_2 = 18$ cm

d. $f_1 = 24$ cm, $f_2 = 18$ cm

28. A ray of light falls on a transparent glass slab of refractive index 1.62. What is the angle of incidence, if the reflected ray and refracted ray are mutually perpendicular?

a. $\tan^{-1}(1.62)$ b. $\tan^{-1}\left(\frac{1}{1.62}\right)$

c. $\frac{1}{\tan^{-1}(1.62)}$ d. None of these

29. A ray of light travelling in glass having refractive index $\mu_g = 3/2$, is incident at a critical angle C on the glass-air interface. If a thin layer of water is poured on glass air interface, then what will be the angle of emergence of this ray in air when it emerges from water-air interface?

a. 180° b. 0°

c. 90° d. 45°

30. Calculate the time taken by the light to travel a distance of 500 m in water of refractive index of $4/3$. (Given, velocity of light in vacuum = 3×10^{10} cm/s)

a. 3×10^{-10} s b. 2.22×10^{-6} s

c. 4.3×10^{-5} s d. 3×10^{-6} s

31. In a medium of refractive index n_1 , a monochromatic light of wavelength λ_1 is travelling. When it enters in a denser medium of refractive index n_2 , the wavelength of the light in the second medium is

a. $\lambda_1 \left(\frac{n_1}{n_2} \right)$ b. $\lambda_1 \left(\frac{n_2}{n_1} \right)$ c. $\frac{\lambda_1(n_2 - n_1)}{n_2}$ d. $\frac{\lambda_1(n_2 - n_1)}{n_1}$

32. A ray of light is incident on the surface of separation of a medium at an angle 45° and is refracted in the medium at an angle 30° . What will be the speed of light in the medium?

a. 1.96×10^8 ms $^{-1}$ b. 2.12×10^8 ms $^{-1}$

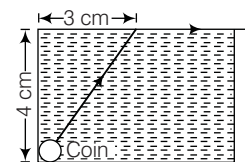
c. 3.18×10^8 ms $^{-1}$ d. 3.33×10^8 ms $^{-1}$

33. The optical path of a monochromatic light is same if, it goes through 4.0 cm of glass or 4.5 cm of water. If the refractive index of glass is 1.53, the refractive index of the water is

a. 1.30 b. 1.36

c. 1.42 d. 1.46

34. A small coin is resting on the bottom of a beaker filled with a liquid. A ray of light from the coin travels upto the surface of the liquid and moves along its surface (see figure).



How fast is the light travelling in the liquid?

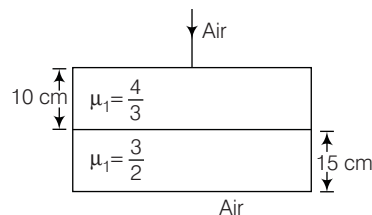
a. 1.8×10^8 ms $^{-1}$ b. 2.4×10^8 ms $^{-1}$

c. 3.0×10^8 ms $^{-1}$ d. 1.2×10^8 ms $^{-1}$

35. If c is the velocity of light in vacuum, then find the time taken by the light to travel through a glass plane of thickness t and having refractive index μ .

a. $\left(\frac{t}{\mu c} \right)$ b. $t \mu c$ c. $\frac{\mu t}{c}$ d. $\frac{tc}{\mu}$

36. Considering normal incidence of ray, find equivalent refractive index of combination of two slabs shown in figure.



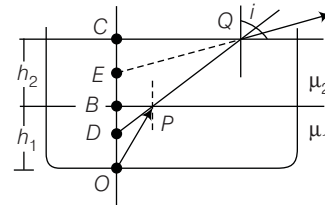
a. 1.8

b. 1.43

c. 2

d. None of these

37. A tank contains two different liquids which do not mix with each other. The lower and upper liquid are at depth h_1 and h_2 respectively and of refractive indices μ_1 and μ_2 . An object O is located at the bottom, when seen vertically from above. Locate the position of image of the object O as seen from above.



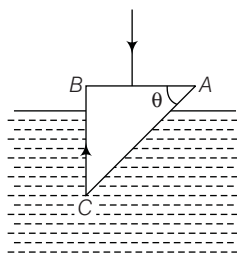
a. $\frac{h_1}{\mu_1} - \frac{h_2}{\mu_2}$

b. $\frac{h_1}{\mu_1} + \frac{h_2}{\mu_2}$

c. $\frac{h_1}{\mu_1} + \frac{h_2}{\mu_1}$

d. $\frac{h_1}{\mu_2} - \frac{h_2}{\mu_1}$

38. In a lake, a fish rising vertically to the surface of water uniformly at the rate of 3 m/s, observe a bird diving vertically towards the water at a rate of 9 m/s vertically above it. Calculate the actual velocity of the dive of the bird. (Given, refractive index of water = $4/3$)
 a. 9.2 m/s b. 4.5 m/s c. 9.0 m/s d. 3.2 m/s
39. A compound microscope has an eyepiece of focal length 10 m and an objective of focal length 4 cm. Calculate the magnification if an object is kept at a distance of 5 cm from the objective then final image is formed at the least distance of distinct vision.
 a. 10 b. 11 c. 12 d. 13
40. A simple microscope consists of a concave lens of power -10 D and a convex lens of power $+20$ D in contact. If the image formed at infinity, then calculate the magnifying power ($D = 25$ cm).
 a. 2.5 b. 3.5 c. 2.0 d. 3.0
41. A telescope consists of two lenses of focal lengths 10 cm and 1 cm. Calculate the length of the telescope, when an object is kept at a distance of 60 cm from the objective, and the final image is formed at least distance of distinct vision.
 a. 15.05 cm b. 12.96 cm c. 13.63 cm d. 14.44 cm
42. Calculate the limit of resolution of microscope, if the numerical aperture of microscope is 0.12 and the wavelength of light used is 600 nm.
 a. $0.3 \mu\text{m}$ b. $1.2 \mu\text{m}$ c. $2.3 \mu\text{m}$ d. $3 \mu\text{m}$
43. An equilateral prism deviates a ray through 45° for the two angle of incidence differing by 20° . The angle of incidence is
 a. 60° b. 40°
 c. 120° d. None of these
44. Find the refractive index of the material of prism, if a thin prism of angle $A = 6^\circ$, produces a deviation $\delta = 3^\circ$.
 a. 1.5 b. 1.2
 c. 1.1 d. 1.25
45. Figure shows, a glass prism ABC (refractive index 1.5), immersed in water (refractive index $4/3$). A ray of light incident normally on face AB . If it is totally reflected at face AC , then



- a. $\sin \theta \geq \frac{8}{9}$ b. $\sin \theta \geq \frac{2}{3}$
 c. $\sin \theta = \frac{\sqrt{3}}{2}$ d. $\frac{2}{3} < \sin \theta < \frac{8}{9}$

46. Find the refractive index of the material, if a prism having an angle $A = 60^\circ$ which produces a minimum deviation of 30° .

a. $\sqrt{3}$ b. $\sqrt{2}$ c. $\sqrt{5}$ d. $\frac{1}{\sqrt{2}}$

47. Find the power and type of the lens by which a person can see clearly the distant objects, if a person cannot see objects beyond 40 cm.

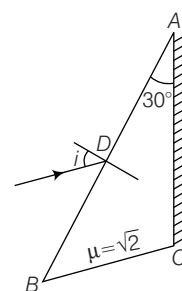
a. -2.5 D and concave lens b. -2.5 D and convex lens
 c. -3.5 D and concave lens d. -3.5 D and convex lens

48. If the resolution limit of the eye is 1 minute and at a distance x km from the eye, two persons stand with a lateral separation of 3 m, then calculate x for which the two persons can be just resolved by the naked eye.

a. 10 km b. 15 km c. 20 km d. 30 km

49. The prism shown in the figure has one side silvered. The angle of the prism is 30° and $\mu = \sqrt{2}$. What should be the angle of incidence, if the incident ray retraces its initial path?

a. 50°
 b. 45°
 c. 60°
 d. 75°



50. If a crown glass prism of refracting angle 10° have refractive indices for red and violet rays 1.514 and 1.523 respectively, then find the dispersion caused by a crown glass prism.

a. 0.07° b. 0.08° c. 0.09° d. 0.10°

51. A thin prism of angle 6° made of glass of refractive index 1.5 is combined with another prism made of glass of $\mu = 1.75$ to produce dispersion without deviation. The angle of second prism is

a. 7° b. 4.67° c. 9° d. 5°

52. A small object is enclosed in a sphere of solid glass 8 cm in radius. It is situated 2 cm from the centre and is viewed from the side to which it is nearer. Where will it appear to be if μ of glass = 1.5?

a. 6 cm from the centre
 b. 4 cm from the nearer surface
 c. $3\frac{1}{5}$ cm from the nearer surface
 d. $3\frac{2}{3}$ cm from the centre

53. In a glass sphere, there is a small bubble 2×10^{-2} m from its centre, if the bubble is viewed along a diameter of the sphere, from the side on which it lies, how far from the surface will it appear, the radius of glass sphere is 5×10^{-2} m and refractive index of glass is 1.5?

a. 2.5×10^{-2} m b. 3.2×10^{-2} m
 c. 6.5×10^{-2} m d. 0.2×10^{-2} m

54. A ray incident at a point at an angle of incidence 60° enters a glass sphere of $\mu = \sqrt{3}$ and is reflected and refracted at the further surface of the sphere. The angle between the reflected and the refracted rays at this surface is
 a. 50° b. 90°
 c. 60° d. 40°
55. The focal length of a thin convex lens for red and violet colour are 44.6 cm and 42.5 cm. Calculate the focal length for the mean colour and dispersive power of the lens.
 a. Focal length = 43.53 cm, dispersive power = 0.048
 b. Focal length = 28.53 cm dispersive power = 0.048
 c. Focal length = 63.53 cm, dispersive power = 8.48
 d. Focal length = 30.43 cm, dispersive power = 4.8
56. A wire mesh consisting of very small squares is viewed at a distance of 8 cm through a magnifying lens of focal length 10 cm, kept close to the eye. The magnification produced by the lens is
 a. 5 b. 8
 c. 10 d. 20
57. A double convex lens made of glass (refractive index $n = 1.5$) has the radii of curvature of both the surfaces as 20 cm. Incident light rays parallel to the axis of the lens will converge at a distance L such that
 a. $L = 20$ cm b. $L = 10$ cm
 c. $L = 40$ cm d. $L = \frac{20}{3}$ cm
58. An object approaches a convergent lens from the left of the lens with a uniform speed 5 m/s and stops at the focus. The image
 a. moves away from the lens with a uniform speed 5 m/s
 b. moves away from the lens with a uniform acceleration
 c. moves away from the lens with a non-uniform acceleration
 d. moves towards the lens with a non-uniform acceleration
59. A concave lens and a convex lens have the same focal length of 20 cm and both are kept in contact. The combination is used to view an object 5 cm long kept at a distance of 20 cm from the lens combination. As compared to the object, the image will be
 a. magnified and inverted
 b. diminished and erect
 c. of the same size and erect
 d. of the same size and inverted
60. A source of light lies on the angle bisector of two plane mirrors inclined at angle θ . The values of θ , so that the light reflected from one mirror does not reach the other mirror will be
 a. $\theta \geq 120^\circ$ b. $\theta \geq 90^\circ$
 c. $\theta \leq 120^\circ$ d. $\theta < 30^\circ$
61. On the axis of a spherical mirror of focal length f , a short linear object of length L lies on the axis at a distance μ from the mirror. Its image has an axial length L' equal to
 a. $L \left[\frac{f}{(\mu - f)} \right]^{1/2}$ b. $L \left[\frac{(\mu + f)}{f} \right]^{1/2}$ c. $L \left[\frac{(\mu - f)}{f} \right]^2$ d. $L \left[\frac{f}{(\mu - f)} \right]^2$
62. The radius of curvature of the face of plano-convex lens is 12 cm and its refractive index is 1.5. If the plane surface of the lens is now silvered, then find the focal length of the lens.
 a. 26 cm b. 22 cm c. 24 cm d. 20 cm
63. The speed at which the image of the luminous point object is moving, if the luminous point object is moving at speed v_o towards a spherical mirror, along its axis is
 (Given, R = radius of curvature, u = object distance)
 a. $v_i = -v_o$ b. $v_i = -v_o \left[\frac{R}{2u - R} \right]^2$
 c. $v_i = -v_o \left(\frac{2u - R}{R} \right)$ d. $v_i = -v_o \left(\frac{R}{2u - R} \right)$
64. When a thin convergent glass lens ($\mu_g = 1.5$) and has power of +5.0 D, is immersed in a liquid of refractive index μ_l , it acts as a divergent lens of focal length 100 cm. find the value of μ_l .
 a. $4/3$ b. $5/3$ c. $5/4$ d. $6/5$
65. Find the change in the focal length of the lens, if a convex lens of focal length 20 cm and refractive index 1.5, is immersed in water having refractive index 1.33.
 a. 62.2 cm b. 5.82 cm c. 58.2 cm d. 6.22 cm
66. The reflective surface is given by $y = 2 \sin x$. The reflective surface is facing positive axis. What is the least value of coordinate of the point where a ray parallel to positive x-axis becomes parallel to positive y-axis after reflection?
 a. $\left(\frac{\pi}{3}, \sqrt{3} \right)$ b. $\left(\frac{\pi}{2}, \sqrt{2} \right)$ c. $\left(\frac{\pi}{3}, \sqrt{2} \right)$ d. $\left(\frac{\pi}{4}, \sqrt{3} \right)$
67. An object is placed at the focus of convex mirror. The image will be at
 a. c b. f
 c. infinity d. None of these
68. A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification.
 a. 6.7 cm, 5/9 b. 7.5 cm, 5/9
 c. 6.7 cm, 9/5 d. 7.5 cm, 9/5
69. With a concave mirror, an object is placed at a distance x_1 from the principal focus, on the principal axis. The image is formed at a distance x_2 from the principal focus. The focal length of the mirror is
 a. $x_1 x_2$ b. $\frac{x_1 + x_2}{2}$ c. $\sqrt{\frac{x_1}{x_2}}$ d. $\sqrt{x_1 x_2}$

70. Electromagnetic radiation of frequency n , wavelength λ , travelling with velocity v in air, enters a glass slab of refractive index μ . The frequency, wavelength and velocity of light in the glass slab will be respectively

a. $\frac{n}{\mu}, \frac{\lambda}{\mu}, \frac{v}{\mu}$ b. $n, \frac{\lambda}{\mu}, \frac{v}{\mu}$ c. $n, \lambda, \frac{v}{\mu}$ d. $\frac{n}{\mu}, \frac{\lambda}{\mu}, v$

71. When a glass slab is placed on a cross made on a sheet, the cross appears to be raised by 1 cm. The thickness of the glass is 3 cm. The critical angle for glass is

a. $\sin^{-1}(0.33)$ b. $\sin^{-1}(0.5)$ c. $\sin^{-1}(0.67)$ d. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

72. A layered lens is made of materials indicated by shades in the figure. The number of images formed is



a. 1 b. 2
c. 3 d. 4

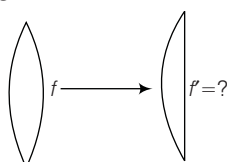
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1. Resolving power of a telescope will be more, if the diameter (a) of the objective is [2014]
a. larger
b. smaller
c. resolving power does not depend on a
d. None of the above

2. A glass piece is dipped in a liquid of refractive index $4/3$, it gets disappeared in the liquid. The refractive index of the glass piece is [2013]

a. $\frac{3}{4}$ b. $\frac{5}{3}$ c. $\frac{4}{5}$ d. $\frac{4}{3}$

3. If the bio-convex lens is cut as shown in the figure, the new focal length f' is [2013]



a. $2f$ b. f c. $f/2$ d. infinite

4. Refractive index of a medium depends [2013]

a. on the medium only
b. on the incident light only
c. on both the conditions given in options a. and b.
d. None of the above

5. A point object is placed at the focus of a convex mirror, the image will be formed at [2013]

a. infinity b. centre of curvature
c. at focus itself d. None of these

6. [2013]

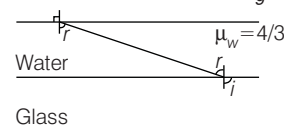
A point object is placed at the focus of the bio-convex lens. What should be the value of X , so the final image forms at infinity? [2013]

a. 10 cm b. 20 cm
c. 15 cm d. None of these

7. The image formed by a concave spherical mirror [2013]

a. is always virtual b. is always real
c. is always inverted d. may be erect

8. A ray of light is incident on the interface between water and glass at an angle i and refracted parallel to the water surface, then value of μ_g will be [2012]



a. $(4/3) \sin i$ b. $\frac{1}{\sin i}$ c. $\frac{4}{3}$ d. 1

9. A mica slit of thickness t and refractive index μ is introduced in the ray from the first source S_1 . By how much distance of fringes pattern will be displaced? [2012]

a. $\frac{d}{D}(\mu - 1)t$ b. $\frac{D}{d}(\mu - 1)t$
c. $\frac{d}{(\mu - 1)D}$ d. $\frac{D}{d}(\mu - 1)$

10. The refractive index of water is $4/3$ and that of glass is $5/3$. What will be the critical angle for the ray of light entering water from the glass? [2012]

a. $\sin^{-1}\left(\frac{4}{5}\right)$ b. $\sin^{-1}\left(\frac{5}{4}\right)$ c. $\sin^{-1}\left(\frac{1}{2}\right)$ d. $\sin^{-1}\left(\frac{2}{1}\right)$

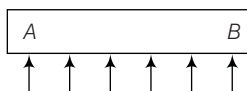
11. A film projector magnifies a 100 cm^2 film strip on a screen. If the linear magnification is 4, the area of the magnified film on the screen is [2011]

a. 1600 cm^2 b. 400 cm^2
c. 800 cm^2 d. 6400 cm^2

12. A convex lens makes a real image 4 cm long on a screen. When the lens is shifted to new position without disturbing the object, we again get a real image on the screen which is 16 cm tall. The length of the object must be [2010]

a. $\frac{1}{4}$ cm b. 8 cm c. 12 cm d. 20 cm

13. Rays of light fall on a glass slab ($\mu > 1$) as shown in figure. If μ at A is maximum and at B it is minimum, then what will happen to these rays? [2009]



a. They will tilt towards A
b. They will tilt towards B
c. They will not deviate
d. There will be total internal reflection

14. The maximum value of index of refraction of a material of a prism which allows the passage of light through it when the refracting angle of the prism is A , is [2009]

a. $\sqrt{1 + \sin\left(\frac{A}{2}\right)}$ b. $\sqrt{1 + \cos\left(\frac{A}{2}\right)}$
c. $\sqrt{1 + \tan^2\left(\frac{A}{2}\right)}$ d. $\sqrt{1 + \cot^2\left(\frac{A}{2}\right)}$

15. The focal lengths of objective and the eyepiece of a compound microscope are f_o and f_e respectively. Then, [2009]

a. $f_o > f_e$ b. $f_o < f_e$ c. $f_o = f_e$ d. None

16. The refractive index of a material of a plano-concave lens is $5/3$, the radius of curvature is 0.3 m. The focal length of the lens in air is [2008]

a. -0.45 m b. -0.6 m c. -0.75 m d. -1.0 m

17. The optical path of a monochromatic light is same if it goes through 4.0 cm of glass of 4.5 cm of water. If the refractive index of glass is 1.53, the refractive index of the water is [2007]

a. 1.30 b. 1.36 c. 1.42 d. 1.46

18. What is the refractive index of a prism whose angle $A = 60^\circ$ and angle of minimum deviation $d_m = 30^\circ$? [2006]

a. $\sqrt{2}$ b. $\frac{1}{\sqrt{2}}$ c. 1 d. $\frac{1}{\sqrt{3}}$

19. An object is placed at a distance 20 cm from the pole of a convex mirror of focal length 20 cm. The image is produced at [2006]

a. 13.3 cm b. 20 cm c. 25 cm d. 10 cm

20. The plano-convex lens of focal length 20 cm and 30 cm are placed together to form a double convex lens. The final focal length will be [2006]

a. 12 cm b. 60 cm
c. 20 cm d. 30 cm

21. Which mirror is to be used to obtain a parallel beam of light from a small lamp? [2005]

a. Plane mirror b. Convex mirror
c. Concave mirror d. Any one of these

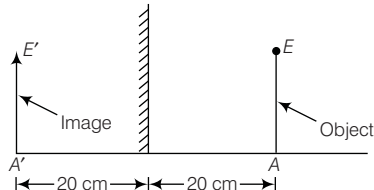
22. Which of the following is a wrong statement? [2005]

a. $D = 1/f$, where f is the focal length and D is called the refractive power of a lens
b. Power is expressed in a diopter when f is in metres
c. Power is expressed in diopter and does not depend on the system of unit used to measure f
d. D is positive for convergent lens and negative for divergent lens

Answer with Solutions

Practice Exercise

1. (a) From the figure, image is situated at 40 cm from eye.



2. (a) 3. (d)

4. (a) From the figure given in question,

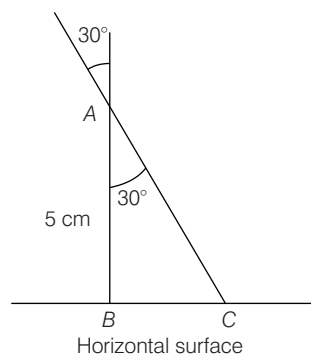
$$\tan 30^\circ = \frac{l_0}{d} \Rightarrow l_0 = d \tan 30^\circ = \frac{20}{\sqrt{3}}$$

$$\therefore \text{Number of reflections} = \left[\frac{L}{l_0} \right] = \left[\frac{16 \times 100 \text{ cm}}{\frac{20}{\sqrt{3}} \text{ cm}} \right] = 14$$

If first reflection is considered, then $n = 14 + 1 = 15$

5. (b) From figure,

$$\tan 30^\circ = \frac{BC}{5}$$



$\therefore$

$$BC = 5 \tan 30^\circ = \frac{5}{\sqrt{3}} \text{ m}$$

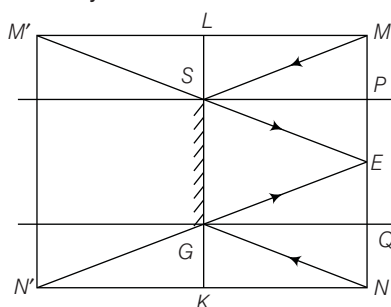
6. (a) Here the minimum size of mirror

$$= \frac{b-a}{2} = \frac{6-4}{2} = 1 \text{ inch}$$

7. (c) When mirror is rotated through an angle θ about an axis in the plane of mirror, then reflected ray is rotated through an angle 2θ .

When mirror is rotated about an axis perpendicular to the plane of mirror. The angle of incidence and angle of reflection does not change.

8. (b) Let MN = height of a man
 E = position of eye



As, $\triangle NGQ$ and $\triangle QGE$ are similar,
 $\triangle ESP$ and $\triangle PSM$ are congruent.

Now,

$$MP = PE = EQ = QN$$

$$SG = PQ = PE + EQ$$

$$SG = \frac{ME}{2} + \frac{EN}{2}$$

$$SG = \frac{1}{2} MN$$

9. (b) Here angular deviation is $\delta = 2\pi - 2\theta$

$$= 2\pi - 2 \times \frac{\pi}{2} = \pi$$

Hence, final emergent ray is parallel to incident ray.

10. (c) The phenomenon of reflection is independent of medium.

The deviation produced by combination of two plane mirrors is

$$\delta = \pi - 2\theta$$

or
$$\delta = 2\pi - 2 \times \frac{\pi}{2} \quad (\because \theta = 90^\circ)$$

$$\therefore \delta = \pi = 180^\circ$$

11. (c) From figure, CD = emergent ray and CD is parallel to PQ and BC is a line intersecting these parallel lines.

So,
$$\angle DCB + \angle CBQ = 180^\circ$$

$$\angle DCN + \angle NCB + \angle CBQ = 180^\circ$$

$$\alpha + \alpha + 45^\circ = 180^\circ$$

$$\alpha = 67.5^\circ$$

But

$$\angle NCS = 90^\circ$$

So, second mirror is at an angle of 22.5° with horizontal.

12. (b) 13. (a)

14. (c) Total number of images formed = 6, because with an object placed between two plane mirrors inclined at an angle of 90° , one image is produced due to each mirror

and one due to their combination. With three images right angles to each other, one image is produced by each plane mirror and one by each of the three pairs of combination of two mirrors each.

15. (c) For point A, $u = -25 \text{ cm}$,
 $f = +10 \text{ cm}$

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

or
$$\frac{1}{v} - \frac{1}{25} = \frac{1}{10}$$

or
$$\frac{1}{v} = \frac{1}{10} + \frac{1}{25} \quad \text{or} \quad \frac{1}{v} = \frac{5+2}{50}$$

$$\therefore v = \frac{50}{7} = 7.14 \text{ cm}$$

For point B, $u' = -30 \text{ cm}$

$$f' = 10 \text{ cm}$$

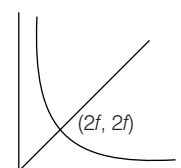
$$\frac{1}{v'} + \frac{1}{u'} = \frac{1}{f'}$$

or
$$\frac{1}{v'} - \frac{1}{30} = \frac{1}{10} \quad \text{or} \quad \frac{1}{v'} = \frac{1}{30} + \frac{1}{10} = \frac{1+3}{30} = \frac{4}{30}$$

$$\therefore v' = \frac{30}{4} = 7.5 \text{ cm}$$

$$\therefore A'B' = |v' - v| = |7.5 - 7.14| = 0.36$$

16. (b) From $u-v$ graph, (b) is correct.



17. (b) Here, $u = -x$, $F = -f$

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{F}$$

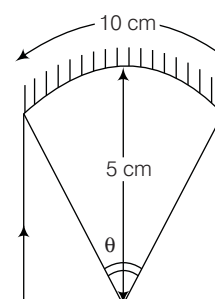
or
$$\frac{1}{v} - \frac{1}{x} = -\frac{1}{f} \quad \text{or} \quad \frac{1}{v} = \frac{1}{x} - \frac{1}{f} = \frac{f-x}{xf}$$

$$\therefore v = \frac{xf}{f-x}$$

$$\therefore \frac{\Delta v}{\Delta u} = \frac{v^2}{u^2} = \left(\frac{xf}{f-x} \right)^2 \times \frac{1}{x^2} = \frac{f^2}{(f-x)^2}$$

$$\therefore \Delta v = \frac{f^2 l}{(x-f)^2} \quad [\because \Delta u = l]$$

18. (c) $\theta = \frac{l}{r} \Rightarrow \theta = \frac{10}{5} = 2 \text{ rad}$



19. (b) Here, incident ray is parallel to principal axis, so

$$\angle AMN = \angle BCF = \theta$$

also

$$\angle CBF = \angle KMN = \angle AMN$$

$\therefore$

$$\angle BFP = \theta + \angle CBF = \theta + \theta = 2\theta$$

20. (a) $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} - \frac{1}{8} = -\frac{1}{12} \Rightarrow v = 24 \text{ cm}$$

21. (a) Mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Differentiating w.r.t. t , we get

$$0 = -\frac{1}{v^2} \frac{dv}{dt} - \frac{1}{u^2} \frac{du}{dt}$$

$$\frac{du}{dt} = -\frac{v^2}{u^2} \frac{dv}{dt}$$

But $\frac{du}{dt} = 15 \text{ m/s}$

Now $\frac{1}{v} = \left(\frac{1}{20} + \frac{1}{600} \right)$

$$\therefore v = \frac{600}{31} \text{ cm and } \frac{v}{u} = \frac{\frac{600}{31}}{600} = \frac{1}{31}$$

$$\therefore \text{Speed of image} = \frac{dv}{dt} = -\left(\frac{1}{31}\right)^2 \times 15 = 0.016 \text{ m/s}$$

22. (b)

23. (b) Let $m_A = \frac{I_A}{O_A}$ and $m_B = \frac{I_B}{O_B}$

Here, $I_A = O_A$ and $O_A = 3O_B$

$$\frac{m_A}{m_B} = \frac{I_A}{O_A} \times \frac{O_B}{I_B} = \frac{I_A}{I_B} \times \frac{O_B}{O_A} = \frac{1}{3}$$

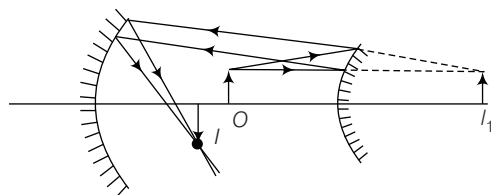
also, $m_A = \frac{f}{f - u_A}$ and $m_B = \frac{f}{f - u_B}$

$$\frac{m_A}{m_B} = \left(\frac{f - u_B}{f - u_A} \right)$$

$$\frac{1}{3} = \frac{-7.5 - u_B}{-7.5 - (-30)}$$

$$u_B = -15 \text{ cm}$$

24. (a) $u_1 = 30 \text{ cm}$, $f_1 = -\frac{30}{2} = -15 \text{ cm}$, $u_3 = v_1 = ?$



$$-\frac{1}{15} = \frac{1}{30} + \frac{1}{v_1}$$

$$v_1 = -10 \text{ cm}$$

Image formed is virtual which is formed behind the convex mirror.

25. (b) Here, $P = 2P_L + P_m = \frac{2}{f_L} - \frac{1}{f_m}$

$$P = \frac{2}{f_L} + \frac{2}{R}$$

Here, $\frac{1}{f_L} = (\mu - 1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right) = \left(\frac{7}{4} - 1 \right) \left(\frac{1}{\infty} + \frac{1}{R} \right)$

$$\frac{1}{f_L} = \frac{3}{4R}$$

$$\therefore P = -\frac{1}{F} = \frac{6}{4R} + \frac{2}{R} = \frac{6+8}{4R} = \frac{14}{4R}$$

or $-\frac{1}{0.3} = \frac{14}{4R}$

$$\therefore R = -\frac{14 \times 0.3}{4} = -\frac{4.2}{4} = -1.05$$

26. (a) When final image is formed at least distance of distinct vision, then the magnifying power

$$= M_0 \times \left(1 + \frac{D}{f_e} \right)$$

$$30 = M_0 \left(1 + \frac{25}{5} \right)$$

$$M_0 = 5$$

27. (c) Since, the doublet is corrected for spherical aberration, it satisfies the following condition

$$f_1 - f_2 = d = 2 \text{ cm}$$

$$f_1 = f_2 + 2 \text{ cm}$$

Let the equivalent focal length = F

$$F = \frac{f_1 f_2}{f_1 + f_2 - d} = 10 \text{ cm}$$

Solving it, $f_1 = 20 \text{ cm}$, $f_2 = 18 \text{ cm}$

28. (a) Here, $\angle i + 90^\circ + \angle r = 180^\circ$

$$i + r = 90^\circ$$

$$r = (90^\circ - i)$$

From Snell's law

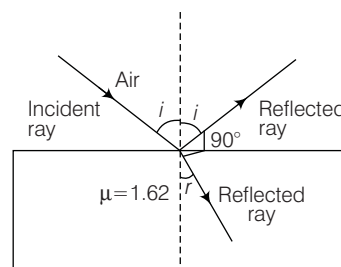
$$\mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin (90^\circ - i)}$$

$$= \frac{\sin i}{\cos i} = \tan i$$

$$\mu = \tan i$$

$$i = \tan^{-1}(\mu)$$

$$i = \tan^{-1}(1.62)$$

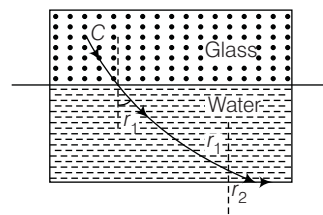


29. (c) $\sin C = \frac{1}{a\mu_g} = \frac{2}{3}$

Here, $\frac{\sin C}{\sin r_1} = \frac{a\mu_w}{a\mu_g} = \frac{\frac{3}{4}}{\frac{3}{2}} = \frac{8}{9}$

$$\sin r_1 = \frac{9}{8} \sin C = \frac{9}{8} \times \frac{2}{3} = \frac{3}{4}$$

Let r_2 = angle of refraction for water-air interface



$$\frac{\sin r_1}{\sin r_2} = \frac{1}{a\mu_w} = \frac{3}{4}$$

$$\sin r_2 = \frac{4}{3} \sin r_1 = \frac{4}{3} \times \frac{3}{4} = 1 = \sin 90^\circ$$

$$r_2 = 90^\circ$$

30. (b)

$$31. (a) \lambda_{\text{medium}} = \frac{\lambda_{\text{air}}}{n_m} = \frac{\left(\frac{c}{v}\right)}{n_m}$$

$$\lambda_1 = \frac{c}{vn_1}, \lambda_2 = \frac{c}{vn_2} \text{ or } \lambda_1 n_1 = \lambda_2 n_2$$

$$\lambda = \lambda_1 \left(\frac{n_1}{n_2} \right)$$

$$32. (b) n = \frac{\sin i}{\sin r} = \frac{c}{v}$$

Hence,

$$v = \frac{c \sin r}{\sin i} = \frac{3 \times 10^8 \times \sin 30^\circ}{\sin 45^\circ}$$

$$= \frac{3 \times 10^8}{\sqrt{2}} = 2.12 \times 10^8 \text{ ms}^{-1}$$

33. (b) As optical paths are equal, hence

$$n_g \cdot x_g = n_w \cdot x_w \Rightarrow n_w = n_g \cdot \frac{x_g}{x_w} = 1.53 \times \frac{4.0}{4.5} = 1.36$$

34. (a) As shown in figure, a light ray from the coin will not emerge out of liquid, if $i > C$.Therefore, minimum radius R corresponds to $i = C$. In $\triangle SAB$,

$$\frac{R}{h} = \tan C \text{ or } R = h \tan C$$

or

$$R = \frac{h}{\sqrt{n^2 - 1}}$$

$$\left[\because \sin C = \frac{1}{n} \text{ and } \tan C = \frac{\sin C}{\sqrt{1 - \sin^2 C}} \right]$$

Given, $R = 3 \text{ cm}$, $h = 4 \text{ cm}$

Hence,

$$\frac{3}{4} = \frac{1}{\sqrt{n^2 - 1}}$$

or

$$n^2 = \frac{25}{9} \text{ or } n = \frac{5}{3}$$

But $n = \frac{c}{v}$ or $v = \frac{c}{n} = \frac{3 \times 10^8}{5/3} = 1.8 \times 10^8 \text{ ms}^{-1}$

$$35. (c) \mu = \frac{\text{Velocity of light in vacuum}}{\text{Velocity of light in glass plate}} = \frac{c}{c'}$$

or

$$c' = \frac{c}{\mu}$$

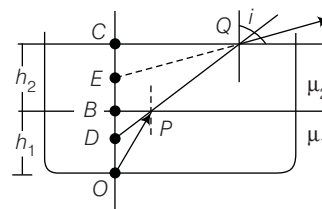
$$\text{Time taken} = \frac{\text{distance}}{\text{velocity}} = \frac{t}{\left(\frac{c}{\mu}\right)} = \frac{\mu t}{c}$$

36. (b) The equivalent refractive index of combination of slab for normal incidence is

$$\mu = \frac{\sum t_i}{\sum \frac{t_i}{\mu_i}}$$

In this case, $\mu = \frac{t_1 + t_2}{\frac{t_1}{\mu_1} + \frac{t_2}{\mu_2}} = \frac{10 + 15}{\frac{10}{3} + \frac{15}{2}}$

$$= \frac{25}{\frac{7.5 + 10}{2}} = \frac{25}{17.5} = 1.43$$

37. (b) Let $\angle BOP = \angle i$, $\angle BDP = \angle r$, $\angle CEQ = \angle i'$ 

Refraction at P

$$\mu_1 \sin i = \mu_2 \sin r$$

$$\frac{\mu_1}{\mu_2} = \frac{\sin r}{\sin i} = \frac{\tan r}{\tan i} = \frac{h_1}{BD}$$

$$\Rightarrow BD = \frac{h_1 \mu_2}{\mu_1}$$

At Q, $\mu_2 \sin r = \sin i'$

$$\mu_1 = \frac{\sin i'}{\sin r} = \frac{\tan i'}{\tan r} = \frac{BD + BC}{CE}$$

$$\Rightarrow CE = \frac{BD + BC}{\mu_2} = \frac{h_1}{\mu_1} + \frac{h_2}{\mu_2}$$

38. (b) Let at any time, fish at depth of the x and y height at which bird was above the surface of water

$$\frac{d_{AC}}{d_{AP}} = \frac{\mu_1}{\mu_2} \Rightarrow \frac{y}{d_{AP}} = \frac{1}{\mu}$$

or

$$d_{AP} = \mu y$$

$\therefore$ Total apparent distance of the bird which was seen by a fish x distance in water.

$$h = x + \mu y$$

$$\frac{dh}{dt} = \frac{dx}{dt} + \mu \frac{dy}{dt}$$

$$9 = 3 + \mu \frac{dy}{dt}$$

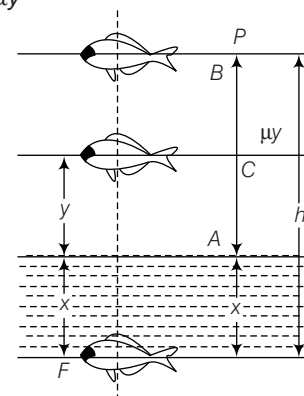
$$\frac{dy}{dt} = \frac{6}{\left(\frac{4}{3}\right)} = 4.5 \text{ m/s}$$

$$39. (c) \text{ We know, } \frac{1}{v_0} + \frac{1}{5} = \frac{1}{4}$$

$$v_0 = 20 \text{ cm}$$

and magnification (M) = $\frac{v_0}{u_0} \left[1 + \frac{D}{f_e} \right]$

$$M = \frac{20}{5} \left[1 + \frac{20}{10} \right] = 12$$



40. (a) Power of combination

$$P = P_1 + P_2$$

$$P = +20 - 10 = +10 \text{ D}$$

$$F = \frac{1}{P} = \frac{1}{10} \text{ m} = \frac{100}{10} \text{ cm} = 10 \text{ cm}$$

For final image at infinity

$$M = \frac{D}{F} = \frac{25}{10} = 2.5$$

41. (b) Two lenses used are eyepiece and objective.

For eyepiece,

$$f_e = 1 \text{ cm}, D = v_e = 25 \text{ cm}$$

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e} \text{ or } -\frac{1}{25} - \frac{1}{u_e} = 1$$

$$u_e = -\frac{25}{26} \text{ cm}$$

and for objective, $u_o = -60 \text{ cm}, f_o = 10 \text{ cm}$

$$\therefore \frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

$$\frac{1}{v_o} + \frac{1}{60} = \frac{1}{10}$$

$$\frac{1}{v_o} = \frac{1}{10} - \frac{1}{60}$$

$$\frac{1}{v_o} = \frac{5}{60}$$

$$v_o = \frac{60}{5}$$

$$v_o = 12 \text{ cm}$$

$$\therefore \text{Length of telescope} = L = v_o - u_e = \left(12 + \frac{25}{26}\right) = 12.96 \text{ cm}$$

42. (d) Let
- x
- = limit of resolution of a microscope

$$x = \frac{0.61\lambda}{\mu \sin \theta}$$

$$x = \frac{(0.61)(6 \times 10^{-7})}{(0.12)}$$

$$x = 3 \mu\text{m}$$

43. (a) Since, prism is equilateral

 $\therefore$ Angle of prism = 60° Also, $i - i' = 20$... (i)
 $\therefore \delta = i + i' - A \text{ or } \delta + A = i + i'$
 $\text{or } 40^\circ + 60^\circ = i + i' \text{ or } i + i' = 100$... (ii)

Adding Eqs. (i) and (ii), we get

$$2i = 120$$

 $\therefore i = 60^\circ$

44. (a) Condition for angle of deviation for prism

$$\delta = A(\mu - 1) \Rightarrow \mu = 1 + \frac{\delta}{A}$$

$$\mu = 1 + \frac{3}{6} = 1.5$$

$$\mu = 1.5$$

45. (a)

46. (b)

47. (a) Here, in this case, the lens used by the person should form the image of distant object of 40 cm

$$\therefore u = \infty, v = -40 \text{ cm and } \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{-40} - \frac{1}{-\infty}$$

$$\frac{1}{f} = -\frac{1}{40}$$

$$\text{Power} = \frac{1}{f} = -\frac{1}{40} = -2.5 \text{ D}$$

Negative sign shows that lens used is concave lens.

48. (a)
- $\theta = \frac{l}{r}$
- , where
- $\theta = 1$
- minute

$$\theta = \frac{1}{60^\circ} = \left(\frac{\pi}{180} \times \frac{1}{60}\right) \text{ rad and } l = 3 \text{ m}$$

$$x = r = \frac{l}{\theta} = \frac{3 \text{ m}}{\left(\frac{\pi}{180} \times \frac{1}{60}\right)} = 10318 \approx 10 \text{ km}$$

49. (b) To retrace the path ray must be incident normally on the silvered face of the prism.

$$\text{In } \triangle AED,$$

$$30^\circ + 90^\circ + \angle D = 180^\circ$$

$$\angle D = 60^\circ$$

$$\text{also } \angle D + \angle r = 90^\circ$$

$$\angle r = 90^\circ - 60^\circ = 30^\circ$$

From Snell's law

$$1 \sin i = (\sqrt{2}) \sin 30^\circ$$

$$i = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$i = 45^\circ$$

50. (c) We know, angular dispersion

$$= \delta_V - \delta_R = (\mu_V - \mu_R) A = (1.523 - 1.514) \times 10^\circ$$

$$= 0.009 \times 10^\circ = 0.09^\circ$$

51. (b)
- $\delta = 0$

$$\therefore \delta_1 - \delta_2 = 0$$

$$\text{or } (\mu_1 - 1) A_1 - (\mu_2 - 1) A_2 = 0$$

$$\text{or } (1.5 - 1) 7^\circ = (1.75 - 1) A_2$$

$$\text{or } \frac{7}{2} = \frac{3}{4} A_2$$

$$\therefore A_2 = \frac{7}{2} \times \frac{4}{3}$$

$$= \frac{14}{3} = 4.67^\circ$$

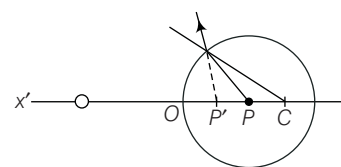
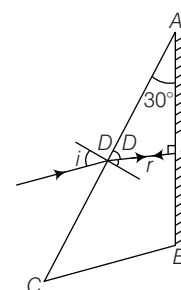
52. (c) Here,
- $\mu_1 = 1.5, \mu_2 = 1, u = +6 \text{ cm},$

$$r = +8 \text{ cm}$$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r}$$

$$\frac{1}{v} - \frac{1.5}{6} = \frac{1.5 - 1}{8}$$

$$\therefore v = 3\frac{1}{5} \text{ cm}$$



53. (a) For spherical surface

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Here, $\mu_1 = 1.5, \mu_2 = 1$

$$u = (5 \times 10^{-2} - 2 \times 10^{-2}) \\ = 3 \times 10^{-2} \text{ cm}$$

and $R = 5 \times 10^{-2} \text{ cm}$

$$\therefore \frac{1}{v} - \frac{1.5}{3 \times 10^{-2}} = \frac{1 - 1.5}{5 \times 10^{-2}}$$

$$v = 2.5 \times 10^{-2} \text{ m}$$

Image will form at a distance of $2.5 \times 10^{-2} \text{ m}$ to the right of nearer surface.

54. (b) The angle between the reflected ray and refracted ray is
- $(120^\circ - \theta)$
- .

For calculation of θ ,

$$\mu_1 \sin i = \mu_2 \sin r \\ 1 \times \sin 60^\circ = \sqrt{3} \sin \theta$$

$$\frac{\sqrt{3}}{2} = \sqrt{3} \sin \theta$$

$$\sin \theta = \frac{1}{2} = \sin 30^\circ$$

$$\theta = 30^\circ$$

Hence, $(120^\circ - 30^\circ) = 90^\circ$

55. (a) Mean focal length
- $= f = \sqrt{f_r f_v}$

$$f = \sqrt{44.6 \times 42.5} = 43.53 \text{ cm}$$

$$\therefore \text{Dispersive power} = \frac{f_r - f_v}{f} = \frac{44.6 - 42.5}{43.33} = 0.048$$

56. (a)
- $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \dots (i)$

Given, $f = 10 \text{ cm}$ (as lens is converging) $u = -8 \text{ cm}$ (as object is placed on left side of the lens)

Substituting these values in Eq. (i), we get

$$\frac{1}{10} = \frac{1}{v} - \frac{1}{-8} \Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{8} \Rightarrow \frac{1}{v} = \frac{8-10}{80}$$

$$\therefore v = \frac{80}{-2} = -40 \text{ cm}$$

Hence, magnification produced by the lens

$$m = \frac{v}{u} = \frac{-40}{-8} = 5$$

57. (a) Here,
- $n = 1.5$
- , as per sign convention followed

 $R_1 = +20 \text{ cm}$ and $R_2 = -20 \text{ cm}$

$$\therefore \frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = (1.5-1) \left[\frac{1}{(+20)} - \frac{1}{(-20)} \right] \\ = 0.5 \times \frac{2}{20} = \frac{1}{20} \Rightarrow f = 20 \text{ cm}$$

Incident rays travelling parallel to the axis of lens will converge at its second principal focus. Hence, $L = +20 \text{ cm}$

58. (c) 59. (c)

60. (a)
- $n = \frac{360}{\theta}$

Number of images, $N = n$, which is odd $= n - 1$ For the given condition, no successive reflection takes place. So, the number of images will be $N \leq 2$

$$n - 1 \leq 2$$

$$n \leq 3$$

$$\frac{360}{\theta} \leq 3$$

$$120 \leq \theta \Rightarrow \theta \geq 120^\circ$$

61. (d) For spherical mirror
- $v = \frac{fu}{(u-f)}$

The value of u for two ends of the objects of length L are

$$u_1 = \mu - \frac{L}{2} \text{ and } u_2 = \mu + \frac{L}{2}$$

Then, the axial length of the image

$$L' = v_1 - v_2 = \frac{f \left(\mu - \frac{L}{2} \right)}{\left(\mu - \frac{L}{2} - f \right)} - \frac{f \left(\mu + \frac{L}{2} \right)}{\left(\mu + \frac{L}{2} - f \right)} \\ = f \left[\frac{Lf}{(\mu - f)^2 - \frac{L^2}{4}} \right] = L \left[\frac{f}{\mu - f} \right]^2$$

62. (c)
- $\frac{1}{f} = (\mu - 1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$

$$\frac{1}{f} = (1.5 - 1) \left(\frac{1}{12} - \frac{1}{\infty} \right)$$

$$f = +24 \text{ cm}$$

63. (b)
- $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Differentiating both sides $-\frac{1}{v^2} \frac{dv}{dt} = \frac{1}{u^2} \frac{du}{dt}$

$$\frac{dv}{dt} = v_i = - \left(\frac{v}{u} \right)^2 \frac{du}{dt} = - \left(\frac{v}{u} \right)^2 v_o$$

Again,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{2}{R} - \frac{1}{u} = \frac{2u - R}{Ru}$$

$$v = \frac{uR}{2u - R}$$

$$v_i = - \left(\frac{v}{u} \right)^2 v_o = - v_o \left(\frac{R}{2u - R} \right)^2$$

64. (b)
- $P_a = \frac{1}{f_a} = \left(\frac{\mu_g - \mu_a}{\mu_a} \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

$$P_l = \frac{1}{f_l} = \left(\frac{\mu_g - \mu_l}{\mu_l} \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$= 5 \left(\frac{1.5 - 1}{1} \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \dots (i)$$

$$-1 = \left(\frac{1.5 - \mu_l}{\mu_l} \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \dots (ii)$$

From Eqs. (i) and (ii), we get $\mu_l = \frac{5}{3}$

65. (c) When the lens is in air, $\frac{1}{f_a} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

$$\frac{1}{20} = (15 - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \text{ or } \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{10}$$

When lens is in water,

$$\frac{1}{f_w} = \left(\frac{\mu_g - \mu_w}{\mu_w} \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \left(\frac{15 - 1.33}{1.33} \right) \times \frac{1}{10}$$

$$f_w = 78.2 \text{ cm}$$

The change in focal length = $78.2 - 20 = 58.2 \text{ cm}$

66. (a) Let the incidence point is $P(x, y)$

$$m = \tan 45^\circ = 1$$

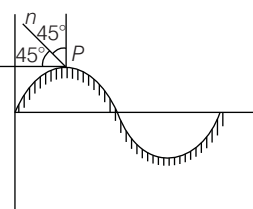
(from law of reflection)

$$y = 2 \sin x$$

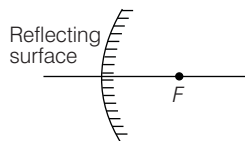
$$\therefore m = \frac{dy}{dx} = 2 \cos x = 1$$

$$\cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}$$

The corresponding value of y is $2 \sin \frac{\pi}{3} = \sqrt{3}$



67. (d) If the object is placed on the right of the convex mirror, no image will be formed.



68. (a) Given, focal length of convex mirror $f = +15 \text{ cm}$
(focal length of convex mirror is taken as positive)

Distance of object, $u = -12 \text{ cm}$

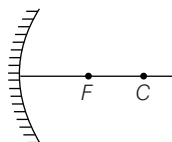
Size of object, $O = 4.5 \text{ cm}$

Using the mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{15} = \frac{1}{v} - \frac{1}{12}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{15} + \frac{1}{12} = \frac{4+5}{60} = \frac{9}{60}$$



Distance of image from the mirror, $v = 6.7 \text{ cm}$

The positive sign shows that the image is formed behind the mirror.

Using the formula of magnification,

$$m = -\frac{v}{u} = \frac{l}{O}$$

$$\frac{-6.7}{-12} = \frac{l}{4.5}$$

Size of image, $l = 2.5 \text{ cm}$

As l is positive, so image is erect and virtual.

Magnification m is given by,

$$m = \frac{l}{O} = \frac{2.5}{4.5} = \frac{25}{45} = \frac{5}{9}$$

69. (d) As, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$$\therefore \frac{1}{(f-x_1)} + \frac{1}{(f-x_2)} = \frac{1}{f}$$

$$\Rightarrow \frac{f-x_2+f-x_1}{(f-x_1)(f-x_2)} = \frac{1}{f}$$

$$\Rightarrow f^2 - fx_2 - fx_1 + x_1x_2 = 2f^2 - f(x_1+x_2)$$

$$\Rightarrow f^2 = x_1x_2 \Rightarrow f = \sqrt{x_1x_2}$$

70. (b) In going from one medium to another, frequency remains same but wavelength and velocity decreases with the increase in refractive index i.e. wavelength becomes $\lambda' = \frac{\lambda}{\mu}$ and velocity becomes, $v' = \frac{v}{\mu}$.

71. (c) We can write shift, $1 = 3 \left(1 - \frac{1}{\mu} \right)$

$$\Rightarrow 1 - \frac{1}{\mu} = \frac{1}{3}$$

$$\text{or } \frac{1}{\mu} = 1 - \frac{1}{3} = \frac{2}{3} \Rightarrow \mu = \frac{3}{2}$$

$$\text{Now, } \frac{1}{\sin i_c} = \frac{3}{2} \Rightarrow \sin i_c = \frac{2}{3}$$

$$\text{or } i_c = \sin^{-1} \left(\frac{2}{3} \right) \Rightarrow i_c = \sin^{-1} (0.67)$$

72. (d)

BITSAT Archives

1. (a)
2. (d) The glass piece will disappear only, if the refractive index of the glass and liquid is same.

So, refractive index of glass piece must be $\frac{4}{3}$.

3. (a) $f = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

$$f = (\mu - 1) \left(\frac{1}{R} + \frac{1}{R} \right)$$

$$f = (\mu - 1) \frac{2}{R} \quad \dots(i)$$

$$\text{and } f' = (\mu - 1) \left(\frac{1}{R} - \frac{1}{\infty} \right)$$

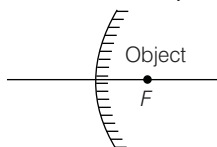
$$f' = \frac{(\mu - 1)}{R} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

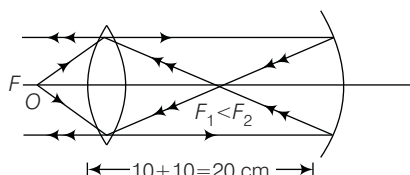
$$f' = 2f$$

4. (c) Refractive index of a medium depends on the medium as well as on the wavelength of the incident light.

5. (d) Image will not form, because object is placed on the side from where reflection is not possible.



6. (b)



The final image will be at infinity only, if the foci of lens and mirror coincide. The situation could be understood on the basis of given diagram.

7. (d) The image formed by a concave mirror could be real, virtual, erect and inverted.

8. (b) ${}_g\mu_w = \frac{\sin i}{\sin r}$

$${}_g\mu_a = \frac{\sin r}{\sin 90^\circ}$$

$$\Rightarrow {}_g\mu_w \times {}_w\mu_a = \frac{\sin i}{\sin r} \times \frac{\sin r}{\sin 90^\circ} = \sin i$$

$$\text{or } \frac{\mu_w}{\mu_g} \times \frac{\mu_a}{\mu_w} = \sin i \Rightarrow \mu_g = \frac{1}{\sin i}$$

9. (b) Fringe displacement, $x_0 = \frac{D(\mu - 1)t}{d}$

10. (a) $\mu = \frac{\mu_1}{\mu_2} = \frac{5/3}{4/3} = \frac{5}{4}$

$$\sin C = \frac{1}{\mu} = \frac{4}{5} \Rightarrow C = \sin^{-1}\left(\frac{4}{5}\right)$$

11. (a) As linear magnification $m = 4$

Hence, a real magnification $m_a = m^2 = (4)^2 = 16$

$$\therefore \text{Surface area of film image on the screen} = 16 \times 100 = 1600 \text{ cm}^2$$

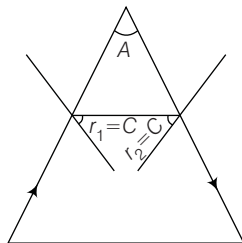
12. (b) $0 = \sqrt{l_1 l_2} = \sqrt{4 \times 16} = 8 \text{ cm}$

13. (c) For normal incidence, whatever be refractive index.

$i = 0, r = 0$, so there is no deviation of ray.

14. (d) Situation is shown in figure.

$$\begin{aligned} r_1 &= C, r_2 = C \\ r_1 + r_2 &= A \\ \therefore 2C &= A, C = \frac{A}{2} \\ \therefore \mu &= \frac{1}{\sin C} = \frac{1}{\sin\left(\frac{A}{2}\right)} \\ &= \frac{\sqrt{\sin^2\left(\frac{A}{2}\right) + \cos^2\left(\frac{A}{2}\right)}}{\sin\left(\frac{A}{2}\right)} = \sqrt{1 + \cot^2\left(\frac{A}{2}\right)} \end{aligned}$$



15. (b) For a microscope $f_o < f_e$.

16. (a) Lens maker's formula $\frac{1}{f} = (\mu - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$

Where, $R_2 = \infty, R_1 = 0.3 \text{ m}$

$$\therefore \frac{1}{f} = \left(\frac{5}{3} - 1 \right) \left(\frac{1}{0.3} - \frac{1}{\infty} \right)$$

$$\Rightarrow \frac{1}{f} = \frac{2}{3} \times \frac{1}{0.3}$$

$$\text{or } f = 0.45 \text{ m}$$

17. (b) Optical path, $\mu x = \text{constant}$

$$\text{i.e. } \mu_1 x_1 = \mu_2 x_2 \Rightarrow 1.53 \times 4 = \mu_2 \times 4.5$$

$$\Rightarrow \mu_2 = \frac{1.53 \times 4}{4.5} = 1.36$$

18. (a) Refractive index of prism

$$\begin{aligned} \mu &= \frac{\sin \frac{A + \delta_m}{2}}{\sin \frac{A}{2}} = \frac{\sin \frac{60^\circ + 30^\circ}{2}}{\sin \frac{60^\circ}{2}} \\ &= \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1/\sqrt{2}}{1/2} = \frac{1}{\sqrt{2}} \times \frac{2}{1} = \sqrt{2} \end{aligned}$$

19. (d) $u = -20 \text{ cm}, f = 20 \text{ cm}$

$$\text{From mirror formula, } \frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{20} = \frac{1}{v} + \frac{1}{-20}$$

$$\frac{1}{v} = \frac{1}{20} + \frac{1}{20}$$

$$\frac{1}{v} = \frac{1}{20} + \frac{1}{20}$$

$$\frac{1}{v} = \frac{2}{20}$$

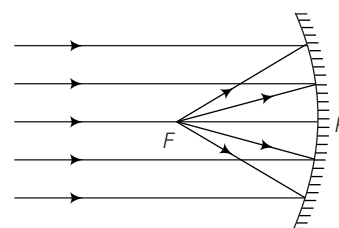
$$\Rightarrow v = 10 \text{ cm}$$

20. (a) Equivalent focal length

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{20} + \frac{1}{30}$$

$$F = \frac{20 \times 30}{20 + 30} = \frac{600}{50} = 12 \text{ cm}$$

21. (c) If lamp is placed at the focus of concave mirror, then we get parallel beam of light.



22. (c) Power is expressed in dioptre and does not depend on the system of unit used to measure F . So, among the given options, option (c) is wrong.

20

Wave Optics

Wave Optics

Wave optics describes the connection between waves and rays of light. According to wave theory of light, the light is a form of energy which travels through a medium in the form of transverse wave motion. The speed of light in a medium depends upon the nature of medium.

Huygens' Principle

Huygens' principle is used to determine the shape of a wavefront at a time t , if the shape of that wavefront is known at time $t = 0$.

It states that "each point of the wavefront is the source of a secondary disturbance and the wavelets emanating from these points spread out in all the directions with the speed of the wave known as **secondary wavelets**. If we draw a common tangent to all these spheres, then we obtain the new position of the wavefront at a later time."

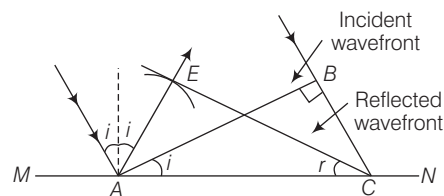
Laws of Reflection and Refraction using Huygens' Principle

Law of Reflection of a Plane Wave by a Plane Surface

Consider the diagram given below showing reflection of a plane wave by a plane surface constructed using Huygens' principle.

Here, AB is a plane wave. MN is the reflecting surface, v is the speed of wave, t is the time taken by the wavefront to reach from B to C . So, $BC = vt = AE$.

The **law of reflection** states that angle of incidence is equal to the angle of reflection, which can be proved by the diagram constructed using **Huygens' principle**.

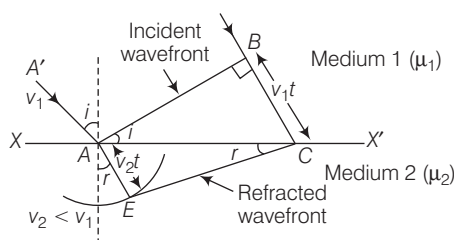


In the above diagram, we can see that $\triangle AEC$ and $\triangle ABC$ are congruent and hence their corresponding angles are equal, i.e. $\angle i = \angle r$.

Law of Refraction for a Denser Medium

Consider the given diagram showing refraction of a plane wave, constructed using Huygens' principle.

Here, XX' is the boundary between two mediums i.e. medium 1 (μ_1) and medium 2 (μ_2), where v_1, v_2 are the respective speed of light ($v_2 < v_1$), t is the time taken by the wavefront in travelling distance BC . So, CE represents the refracted wavefront.



From the figure, we have

$$\sin i = \frac{BC}{AC} \text{ and } \sin r = \frac{AE}{AC}$$

So that,

$$\frac{\sin i}{\sin r} = \frac{BC}{AC} \times \frac{AC}{AE} = \frac{BC}{AE}$$

$$\Rightarrow \frac{\sin i}{\sin r} = \frac{v_1 t}{v_2 t} = \frac{v_1}{v_2} = \frac{c/\mu_1}{c/\mu_2} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \mu_1 \sin i = \mu_2 \sin r$$

This figure proves the **Snell's law of refraction**.

i.e. $\mu_1 \sin i = \mu_2 \sin r$. Here, μ_1 and μ_2 are refractive indices of both the media, respectively.

When a ray passes from an optically denser medium to an optically rarer medium, the angle of refraction r is greater than the angle of incidence i . For a certain angle of incidence $i = i_c$, angle of refraction $r = 90^\circ$, then this angle of incidence i_c is called **critical angle**.

NOTE From law of refraction (Snell's law),

$$\mu_1 \sin i = \mu_2 \sin r \Rightarrow \mu_1 \sin i_c = \mu_2 \sin 90^\circ$$

$$\Rightarrow \sin i_c = \mu_2 / \mu_1 \Rightarrow i_c = \sin^{-1}(\mu_2 / \mu_1)$$

Interference of Light

Interference of light is the phenomenon of redistribution of light energy in space when two or more light waves of same frequency (or same wavelength) emitted by two coherent sources, travelling in a given direction, superimpose on each other.

If a_1 and a_2 be the amplitudes of two light waves of same frequency and ϕ be the phase difference between them, then the amplitude of resultant wave is given by

$$A_R = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos \phi}$$

and in terms of intensity of light

$$I_R = I_1 + I_2 + 2\sqrt{I_1I_2} \cos \phi$$

Condition for Constructive Interference

If at some point in space, the phase difference between two waves, $\phi = 0^\circ$ or $2n\pi$ (or path difference between two waves, $\Delta x = 0$ or $n\lambda$),

where, n is an integer, then

$$A_R = a_1 + a_2$$

or

$$I_R = I_1 + I_2 + 2\sqrt{I_1I_2} \text{ is maximum.}$$

Such an interference is called **constructive interference**.

Condition for Destructive Interference

If at some point in space, the phase difference between two waves, $\phi = (2n - 1)\pi$ [or path difference, $\Delta x = (2n - 1)\frac{\lambda}{2}$,

then at such points $A_R = (a_1 - a_2)$ and $I_R = I_1 + I_2 - 2\sqrt{I_1I_2}$ is minimum leading to a **destructive interference**.

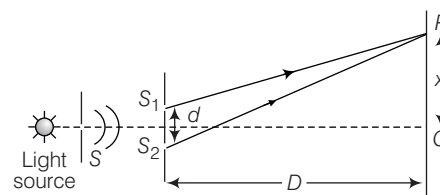
$$\frac{I_{\max}}{I_{\min}} = \frac{I_1 + I_2 + 2\sqrt{I_1I_2}}{I_1 + I_2 - 2\sqrt{I_1I_2}} = \left[\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} \right]^2 = \left[\frac{a_1 + a_2}{a_1 - a_2} \right]^2 = \left[\frac{r + 1}{r - 1} \right]^2$$

where, $r = \frac{a_1}{a_2}$ = amplitude ratio.

Young's Double Slit Experiment

This experiment is a demonstration that matter and energy can display characteristics of both waves and particles and demonstrates the fundamentally probabilistic nature of quantum mechanical phenomena.

The arrangement is shown in figure. Monochromatic light of same wavelength is used.



For light waves reaching a point P , situated at a distance x from central point O , the path difference

$$\Delta x = S_2P - S_1P = \frac{xd}{D}$$

- (i) If $\frac{xd}{D} = n\lambda$, then we get the **n th bright fringe**. Hence, position of bright fringes on the screen are given by the relation $x = \frac{nD\lambda}{d}$.
- (ii) If $\frac{xd}{D} = (2n-1)\frac{\lambda}{2}$, then we get the **n th dark fringe**. Hence, for n th dark fringe $x = \frac{(2n-1)D\lambda}{2d}$.

Fringe Width

It is the distance between two consecutive bright or two consecutive dark fringes. It is denoted by β . Fringe width between two consecutive bright or dark fringes is same.

Expression for fringe width, $\beta = \frac{\lambda D}{d}$

where, β = fringe width

λ = wavelength of light

D = distance between the screen and plane of two slits

d = distance between the two slits

Path difference between the waves coming from S_1 and S_2 , meets on the screen at a distance Y_n from central maxima

$$(O) = \Delta x \approx \frac{dY_n}{D}$$

- (i) Path difference for bright fringes,

$$\Delta x = \frac{dY_n}{D} = n\lambda$$

Distance of the n th bright fringe from the centre of the fringe system is

$$Y_n = \frac{n\lambda D}{d}; n = 0, 1, 2, \dots, \text{etc.}$$

- (ii) Path difference for dark fringes,

$$\Delta x = \frac{dY_n}{D} = \left(n + \frac{1}{2}\right)\lambda$$

Distance of the n th dark fringe from the centre of the fringe system is

$$Y_n = \left(n + \frac{1}{2}\right) \frac{\lambda D}{d}; n = 0, 1, 2, 3, \dots, \text{etc.}$$

- (iii) Angular separation between n th bright fringe and the central fringe is

$$\theta_n = \frac{Y_n}{D} = \frac{n\lambda}{d}; \theta_n \text{ is in radian.}$$

- (iv) Angular separation between n th dark fringe and the central fringe is

$$\theta_n = \left(n + \frac{1}{2}\right) \frac{\lambda}{d}$$

where, $n = 0, 1, 2, 3, \dots$ etc.

Fringe Shift

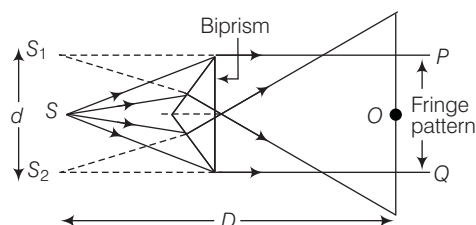
- (i) If a refracting slab of refractive index μ and thickness t is placed in front of one of the two slits of Young's double slit experiment, then the whole fringe pattern gets shifted towards the slab by distance

$$\Delta y = (\mu - 1)t$$

- (ii) If both slits are covered by refracting surfaces of thickness t_1 and t_2 and refractive indices μ_1 and μ_2 respectively, then fringe pattern shifted by $\Delta y = (\mu_2 - \mu_1)t$. Net shift will be towards the slit having greater refractive index.

Fresnel's Biprism

Fresnel's biprism is a device for obtaining **coherent sources**. The biprism is a glass prism of very small base angles.



Comparing the situation geometrically with the double slit experiment fringe pattern width $G = W = \frac{D\lambda}{d}$

where, D = distance of screen from the line joining $S_1 S_2$.

λ = wavelength of light used,

d = distance between S_1 and S_2 .

Calculation of Deviation

Let a = distance of source S from biprism

Deviation = $(\mu - 1)\alpha$ (for small angle prism)

$$\Rightarrow d = 2a\delta = 2a(\mu - 1)\alpha$$

Diffraction of Light

Diffraction of light is the deviation of light waves when it encounter any obstacle in its way.

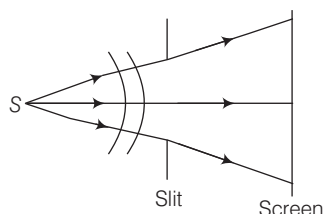
Diffraction happens due to bending of light around the edges of obstacles.

The phenomenon of diffraction is divided mainly in the following two classes

- Fresnel diffraction
- Fraunhofer diffraction

Diffraction due to a Single Slit

To study the diffraction phenomenon, Young's double slit experiment was modified to single slit.



From this experiment, it was observed that when a parallel beam of monochromatic light falls normally on a narrow slit, the diffraction pattern on a screen has bright **central maxima** bordered on both sides by secondary maxima of rapidly decreasing intensity. Suppose, λ is the wavelength of light and a is width of slit.

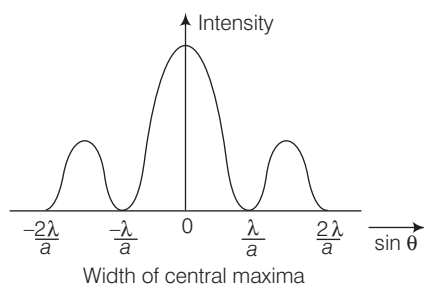
(i) Thus, for bright fringes, $\sin\theta = 0, \frac{3\lambda}{2a}, \frac{5\lambda}{2a}, \dots$, etc.

(ii) For dark fringes, $\sin\theta = \frac{\lambda}{a}, \frac{2\lambda}{a}, \frac{3\lambda}{a}, \dots$, etc.

Width of Central Maxima

Angular width of central maxima = $\frac{2\lambda}{a}$

Linear width of central maxima = $\frac{2f\lambda}{a}$



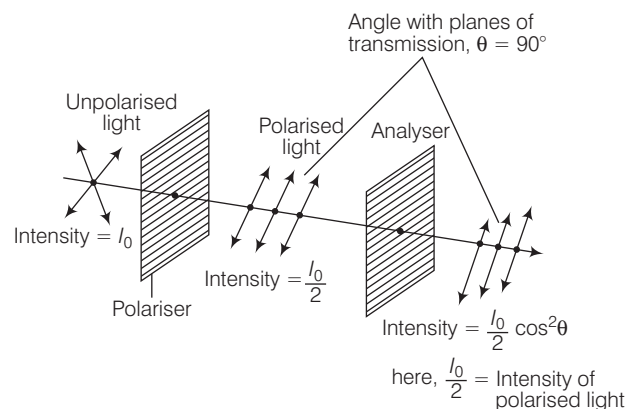
where, λ = wavelength of light, a = width of slit
 f = focal length of the convex lens placed close to the slit

Malus Law

When a beam of completely plane polarised light is incident on an analyser, the resultant intensity of light (I) transmitted from the analyser varies directly as the square of cosine of angle (θ) between plane of transmission of analyser and polariser.

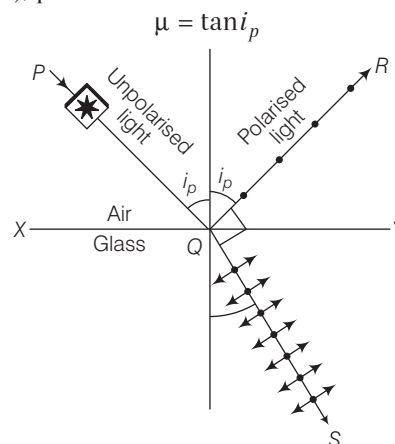
i.e.

$$I \propto \cos^2\theta$$



Brewster's Law

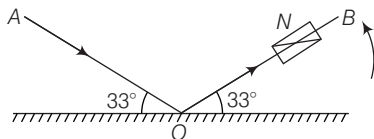
According to this law, when unpolarised light is incident at an angle called polarising angle, i_p , on an interface separating air from a medium of refractive index μ , then the reflected light is fully polarised (perpendicular to the plane of incidence), provided



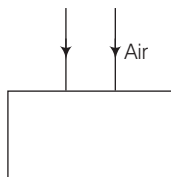
This relation represents Brewster's law.

Practice Exercise

- Light propagates 2 cm distance in glass of refractive index 1.5 in time t_0 . In the same time t_0 , light propagates a distance of 2.25 cm in a medium. The refractive index of the medium is
 a. $4/3$
 b. $3/2$
 c. $8/3$
 d. None of the above
- Two non-coherent sources emit light beam of intensities I and $4I$. The maximum and minimum intensities in the resulting beam are
 a. $9I$ and I
 b. $9I$ and $3I$
 c. $5I$ and I
 d. $5I$ and $3I$
- In an interference pattern, the position of zeroth order maxima is 4.8 mm from a certain point P on the screen. The fringe width is 0.2 mm. The position of second minima from point P is
 a. 5.1 mm
 b. 5 mm
 c. 4 mm
 d. 5.2 mm
- A beam of light AO is incident on a glass slab ($\mu = 1.54$) in a direction as shown in the figure. The reflected ray OB is passed through a nicol prism. On viewing through a nicol prism, we find on rotating the prism that



- the intensity is reduced down to zero and remains zero
 - the intensity reduces down somewhat and rises again
 - there is no change in intensity
 - the intensity gradually reduces to zero and then again increases
- A parallel beam of light of intensity I_0 is incident on a glass plate, 25% of light is reflected by upper surface and 50% of light is reflected from lower surface. The ratio of maximum to minimum intensity in interference region of reflected ray is

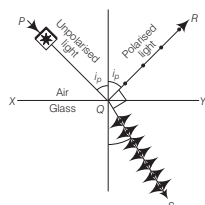


- $\left(\frac{1 + \sqrt{\frac{3}{8}}}{2 - \sqrt{\frac{3}{8}}} \right)^2$
- $\left(\frac{1 + \sqrt{\frac{3}{8}}}{2 - \sqrt{\frac{3}{8}}} \right)^2$
- $\frac{5}{8}$
- $\frac{8}{5}$

- Newton's rings are observed by keeping a spherical surface of 100 cm radius on a plane glass plate. The wavelength of light used is 5880 Å. If the diameter of the 15th bright ring is 0.590 cm, then calculate diameter of the 5th ring.
 a. 0.226 cm b. 0.446 cm c. 0.336 cm d. 0.556 cm
- If n coherent sources of intensity I_0 are superimposed at a point, the intensity of the point is
 a. nI_0 b. n^2I_0 c. n^3I_0 d. $\frac{I_0}{n}$
- In Young's double slit experiment with monochromatic light of wavelength 600 nm, the distance between slits is 10^{-3} m. For changing fringe width by 3×10^{-5} m,
 a. the screen is moved away from the slits by 5 cm
 b. the screen is moved by 5 cm towards the slits
 c. the screen is moved by 3 cm towards the slits
 d. Both (a) and (b) are correct
- If Young's double slit experiment, is performed in water
 a. the fringe width will decrease
 b. the fringe width will increase
 c. the fringe width will remain unchanged
 d. there will be no fringe
- In Young's double slit experiment, the spacing between the slits is ' d ' and wavelength of light used is 6000 Å. If the angular width of a fringe formed on a distant screen is 1° , then calculate ' d '.
 a. 1 mm b. 0.05 mm c. 0.03 mm d. 0.01 mm
- In Young's double slit experiment, when violet light of wavelength 4358 Å used, then 84 fringes are seen in the field of view, but when sodium light of certain wavelength is used, then 62 fringes are seen in the field of view, calculate wavelength of sodium light.
 a. 6893 Å b. 5904 Å c. 5523 Å d. 6429 Å
- In a Young's double slit experiment, 5th dark fringe is formed opposite to one of the slits. The wavelength of light is
 a. $\frac{d^2}{6D}$ b. $\frac{d^2}{5D}$ c. $\frac{d^2}{15D}$ d. $\frac{d^2}{9D}$
- In the ideal double slit experiment, when a glass-plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remain unchanged. The minimum thickness of the glass plate is
 a. 2λ b. $\frac{2\lambda}{3}$ c. $\frac{\lambda}{3}$ d. λ

14. For the same objective, find the ratio of the least separation between two points to be distinguished by a microscope for light of $5000 \text{ \AA}$ and electrons accelerated through 100 V used as the illuminating substance.
 a. 1×10^{-4} b. 0.2×10^{-3} c. 5×10^{-2} d. 7×10^{-4}
15. One slit of a double slit experiment is covered by a thin glass plate of refractive index 1.4 and the other by a thin glass plate of the refractive index 1.7 . The point on the screen, where the central maximum fall before the glass plate was inserted, is now occupied by what had been the fifth bright fringe was seen before? Assume the plate have the same thickness t and wavelength of light 480 nm . Then, the value of t is
 a. 2.4 mm b. 4.8 mm c. $8 \mu\text{m}$ d. 16 mm
16. In a Young's experiment, one of the slit is covered with a transparent sheet of thickness $3.6 \times 10^{-3} \text{ cm}$ due to which position of central fringe shifts to a position originally occupied by 30th bright fringe. Find the refractive index of the sheet, if $\lambda = 6000 \text{ \AA}$.
 a. 1.5 b. 1.2 c. 1.3 d. 1.7

17. A beam of light parallel to central line AB is incident on the plane of slits. The number of minima obtained on the large screen is n_1 . Now, if the beam is tilted by same angle ($\neq 90^\circ$) as shown in figure, then the number of minima obtained is n_2 . Then,

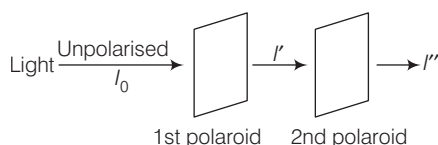


- a. $n_1 = n_2$ b. $n_1 > n_2$
 c. $n_2 > n_1$ d. n_2 will be zero

18. In Fresnel's biprism experiment, the distance between biprism and screen is 4 m . The angle of prism is $2 \times 10^{-3} \text{ rad}$, the refractive index of glass of biprism is 1.5 . The fringe width observed on the screen is $15 \times 10^{-4} \text{ m}$. Find the number of fringes on the screen.
 a. 3 b. 2 c. 6 d. 8
19. The first diffraction minimum due to single slit diffraction is θ , for a light of wavelength $5000 \text{ \AA}$. If the width of the slit is $1 \times 10^{-4} \text{ cm}$, then the value of θ is
 a. 30° b. 45°
 c. 60° d. 15°
20. A screen is at a distance of 1 m away from the aperture. If light of wavelength 500 nm falls on an aperture, then calculate the area of first HPZ and radius of third HPZ.
 a. $1.57 \text{ mm}^2, 1.22 \text{ mm}$
 b. $1.22 \text{ mm}^2, 1.57 \text{ mm}$
 c. $1.65 \text{ mm}^2, 2.79 \text{ mm}$
 d. $2.63 \text{ mm}^2, 0.22 \text{ mm}$
21. The sodium yellow doublet has wavelengths $5890 \text{ \AA}$ and $\lambda \text{ \AA}$ and resolving power of a grating to resolve these lines is 982 , then value of λ is
 a. $5896 \text{ \AA}$ b. $5880 \text{ \AA}$
 c. $5869 \text{ \AA}$ d. $5876 \text{ \AA}$
22. An unpolarised beam of light is incident on a group of four polarising sheets which are arranged in such a way that the characteristic direction of each polarising sheet makes an angle of 30° with that of the preceding sheet. The percentage of incident light transmitted by the first polariser will be
 a. 100%
 b. 50%
 c. 25%
 d. 125%

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1. The angular size of the central maxima due to a single slit diffraction is ($a \rightarrow$ slit width) [2014]
 a. $\frac{\lambda}{a}$ b. $\frac{2\lambda}{a}$ c. $\frac{3\lambda}{2a}$ d. $\frac{\lambda}{2a}$
2. Find the final intensity of light (I''), if the angle between the axes of two polaroids is 60° . [2014]



- a. $\frac{3I_0}{2}$ b. $\frac{I_0}{2}$
 c. $\frac{I_0}{4}$ d. $\frac{I_0}{8}$

3. A mica slit of thickness t and refractive index μ is introduced in the ray from the first source S_1 . By how much distance of fringes pattern will be displaced? [2012]
 a. $\frac{d}{D}(\mu - 1)t$ b. $\frac{D}{d}(\mu - 1)t$
 c. $\frac{d}{(\mu - 1)D}$ d. $\frac{D}{d}(\mu - 1)$
4. A single slit diffraction is obtained using a beam of red light. If red light is replaced by blue light, what happens? [2009]
 a. The diffraction pattern disappears
 b. There is no change in the diffraction pattern
 c. Diffraction fringes become narrower and crowded together
 d. Diffraction fringes become broader and farther apart

5. **Statement** Using Huygens' eyepiece measurements can be taken but are not correct.

Reason The cross wires, scale and final image are not magnified proportionately because the image of the object is magnified by two lenses, whereas the cross wire scale is magnified by one lens only.

Identify the correct one of the following. [2008]

- a. Both (S) and (R) are true, (R) explains (S)
 b. Both (S) and (R) are true, but (R) cannot explain (S)
 c. (S) is false, (R) is false
 d. (S) is false, (R) is true
6. In Fraunhofer diffraction experiment, L is the distance between screen and the obstacle, b is the size of obstacle and λ is wavelength of incident light. The general condition for the applicability of Fraunhofer diffraction is [2008]
- a. $\frac{b^2}{L\lambda} \gg 1$ b. $\frac{b^2}{L\lambda} = 1$
 c. $\frac{b^2}{L\lambda} \ll 1$ d. $\frac{b^2}{L\lambda} \neq 1$

7. The optical path of a monochromatic light is same if it goes through 4.0 cm of glass of 4.5 cm of water. If the refractive index of glass is 1.53, the refractive index of the water is [2007]

a. 1.30 b. 1.36 c. 1.42 d. 1.46

8. In Young's double slit experiment a minima is observed when path difference between the interfering beam is [2006]

a. λ b. 1.5λ c. 2λ d. 2.25λ

9. Two coherent light beams of intensity I and $4I$ are superposed. The maximum and minimum possible intensities in the resulting beam are [2005]

a. $9I$ and I b. $9I$ and $3I$ c. $5I$ and I d. $5I$ and $3I$

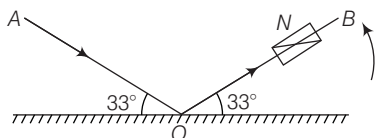
10. When unpolarised light beam is incident from air onto glass ($n = 1.5$) at the polarising angle [2005]

a. reflected beam is polarised 100%
 b. reflected and refracted beams are partially polarised
 c. the reason for (a) is that almost all the light is reflected
 d. All of the above

Answer with Solutions

Practice Exercise

1. (a) For a given time, optical path remains constant
 $\therefore \mu_1 x_1 = \mu_2 x_2$
 or $1.5 \times 2 = \mu_2 \times 2.25$
 $\therefore \mu_2 = \frac{1.5 \times 2}{2.25} = \frac{2}{1.5} = \frac{20}{15} = \frac{4}{3}$
2. (d) When two coherent light beams of intensity I_1 and I_2 superimpose, then maximum intensity is $(\sqrt{I_1} + \sqrt{I_2})^2$ and minimum intensity is $(\sqrt{I_1} - \sqrt{I_2})^2$. But when two incoherent light beams of intensities I_1 and I_2 superimposed, then maximum intensity is $(I_1 + I_2)$ and minimum intensity is $(I_1 - I_2)$.
3. (a) The distance between zeroth order maxima and second order minima is
 $y_1 = \frac{\beta}{2} + \beta = \frac{3}{2}\beta = \frac{3}{2} \times 0.2 \text{ mm} = 0.3 \text{ mm}$
 $\therefore$ The distance of second maxima from point P is
 $y = (4.8 + 0.3) \text{ mm} = 5.1 \text{ mm}$
4. (d) As $i_p = \tan^{-1}(1.54) = 57^\circ$ and in the figure given
 $i = 90^\circ - 33^\circ = 57^\circ = i_p$



$\therefore$ Reflected light along OB is plane polarised. On rotating the nicol prism, intensity gradually reduces to zero and then increases again.

5. (a) The intensity of light reflected from upper surface is

$$I_1 = I_0 \times 25\% = I_0 \times \frac{25}{100} = \frac{I_0}{4}$$

Intensity of transmitted light from upper surface is

$$I = I_0 - \frac{I_0}{4} = \frac{3I_0}{4}$$

$\therefore$ Intensity of reflected light from lower surface is

$$I_2 = \frac{3I_0}{4} \times \frac{50}{100} = \frac{3I_0}{8} \Rightarrow \frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

$$\frac{I_{\max}}{I_{\min}} = \frac{\left(\sqrt{\frac{I_0}{4}} + \sqrt{\frac{3I_0}{8}}\right)^2}{\left(\sqrt{\frac{I_0}{4}} - \sqrt{\frac{3I_0}{8}}\right)^2} = \frac{\left(\frac{1}{2} + \sqrt{\frac{3}{8}}\right)^2}{\left(\frac{1}{2} - \sqrt{\frac{3}{8}}\right)^2}$$

6. (c) Let D_{n+p} = diameter of $(n + p)$ th ring

D_n = diameter of n th ring

$D_{15} = 0.590 \text{ cm}$, $D_s = ?$

$\lambda = 5880 \text{ \AA}$, $R = 100 \text{ cm}$

$P = 10$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4PR}$$

$$5880 \times 10^{-5} = \frac{(0.59 + D_n)(0.59 - D_n)}{4 \times 10 \times 100}$$

$$D_n = 0.336 \text{ cm}$$

7. (b) The resultant amplitude is $A = na_0$

$$\therefore I \propto A^2$$

$$\therefore I = kA^2 = k n^2 a_0^2 = n^2 I_0$$

$$8. (a) \beta = \frac{D}{d} \lambda \Rightarrow \Delta \beta = \frac{\Delta D}{d} \lambda$$

$$\Delta D = \frac{d \Delta \beta}{\lambda} = \frac{3 \times 10^{-5} \times 10^{-3}}{600 \times 10^{-9}}$$

= 5 cm away or towards the slits

9. (a) As we know,

$$\beta = \frac{D}{d} \lambda \quad \dots(i)$$

$$\text{and} \quad \lambda \propto \frac{1}{\mu} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\beta \propto \lambda \propto \frac{1}{\mu} \therefore \beta \propto \frac{1}{\mu}$$

$$10. (c) \text{ Here, } \sin \theta \approx \theta = \left(\frac{y}{D} \right)$$

$$\text{So, } \Delta \theta = \frac{\Delta y}{D}$$

Angular fringe width $\theta_0 = \Delta \theta$ (width $\Delta y = \beta$)

$$\theta_0 = \frac{\beta}{D} = \frac{D \lambda}{d} \times \frac{1}{D} = \frac{\lambda}{d}$$

$$\theta_0 = 1^\circ = \frac{\pi}{180} \text{ rad and } \lambda = 60 \times 10^{-7} \text{ m}$$

$$d = \frac{\lambda}{\theta_0} = \frac{180}{\pi} \times 6 \times 10^{-7}$$

$$= 3.44 \times 10^{-5} \text{ m} = 0.03 \text{ mm}$$

11. (b)

12. (d) For dark fringe,

$$\frac{Yd}{D} = (2m-1) \frac{\lambda}{2}$$

$$\text{Here, } m=5, Y = \frac{d}{2}$$

$$\therefore \frac{d}{2} \frac{d}{D} = (2 \times 5 - 1) \frac{\lambda}{2}$$

$$\text{or } \frac{d^2}{D} = 9\lambda$$

$$\therefore \lambda = \frac{d^2}{9D}$$

$$13. (a) \text{ We can write, shift} = \frac{(\mu-1)tD}{d} = \text{fringe width, } \beta = \frac{\lambda D}{d}$$

$\therefore$ Thickness, $t = 2\lambda$

$$14. (b) d_{\min} = \frac{1.22 \lambda}{2 \sin \beta}$$

where, λ is the wavelength of light and β is the angle subtended by the objective at the object.

For the light of wavelength $5500 \text{ \AA}$

$$d_{\min} = \frac{1.22 \times 5500 \times 10^{-10}}{2 \sin \beta} \quad \dots(i)$$

For electrons accelerated through 100 V, the de-Broglie wavelength

$$\lambda = \frac{12.27}{\sqrt{V}} = \frac{12.27}{\sqrt{100}} = 0.12 \times 10^{-9} \text{ m}$$

$$d_{\min} = \frac{1.22 \times 0.12 \times 10^{-9}}{2 \sin \beta}$$

Ratio of the least separation

$$\therefore \frac{d'_{\min}}{d_{\min}} = \frac{0.12 \times 10^{-9}}{5500 \times 10^{-10}} = 0.2 \times 10^{-3}$$

15. (c) Due to the introduction of the glass plate, the change in path difference is $(\mu_2 - \mu_1)t$.

$\therefore$ Before inserting the glass plate, the path difference for central maxima is zero.

After introducing glass plate, the change in path difference is equal to 5λ .

$$(\mu_2 - \mu_1)t = 5\lambda$$

$$t = \frac{5\lambda}{\mu_2 - \mu_1} = \frac{5 \times 480 \times 10^{-9}}{1.7 - 1.4} = 8 \mu\text{m}$$

16. (a) The position of 30th bright fringe

$$y_{30} = \frac{30\lambda D}{d}$$

Now, position shift of central fringe is

$$y_0 = \frac{30\lambda D}{d}$$

$$\text{But we know, } y_0 = \frac{D}{d} (\mu - 1)t$$

$$\frac{30\lambda D}{d} = \frac{D}{d} (\mu - 1)t \Rightarrow (\mu - 1) = \frac{30\lambda}{t}$$

$$= \frac{30 \times (6000 \times 10^{-10})}{(3.6 \times 10^{-5})} = 0.5 \Rightarrow \mu = 1.5$$

17. (a) Fringe size will not change due to tilting

$$\therefore n_1 = n_2$$

18. (c) The deviation,

$$S = (\mu - 1) A = 0.5 \times 2 \times 10^{-3} = 10^{-3} \text{ rad}$$

The length of band on either side of centre of screen

$$= l = b\delta = 4 \times 10^{-3} \text{ m}$$

$\therefore$ The number of fringe on either side is

$$n_1 = \frac{l}{\beta} = \frac{4 \times 10^{-3}}{15 \times 10^{-4}} = \frac{40}{15} = 2.67$$

$\therefore$ Total number of fringes

$$= 2[n_1] + 1 = 2[2.67] + 1 = 6$$

19. (a)

20. (a)

$$21. (a) \text{ Mean wavelength, } \lambda_m = \frac{5890 + \lambda}{2}$$

Wavelength difference $d\lambda = \lambda - 5890$

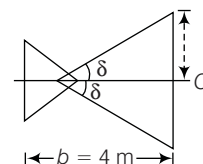
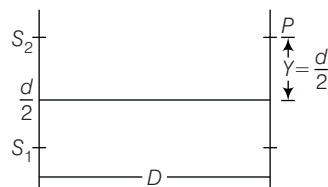
Resolving power R of a grating

$$R = \frac{\lambda_m}{d\lambda} = \frac{5890 + \lambda}{\lambda - 5890} = 982 \Rightarrow \lambda = 5896 \text{ \AA}$$

22. (b) First polariser just polarises the unpolarised light.

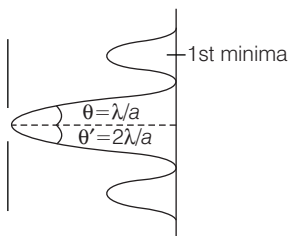
Therefore, intensity of polarised light transmitted from first polariser is

$$\frac{1}{2} I_0 = 50\% I_0$$



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1. (b)



So, angular size of central maxima is $= 2 \left(\frac{\lambda}{a} \right) = \frac{2\lambda}{a}$

2. (d) From first polaroid the unpolarised light will become polarised with half the intensity.

So, $I' = \frac{I_0}{2}$

From second polaroid

$$I'' = I' \cos^2 \theta = \frac{I_0}{2} \cos^2(60) = \frac{I_0}{2} \cdot \frac{1}{4} = \frac{I_0}{8}$$

3. (b) Fringe displacement, $x_0 = \frac{D(\mu - 1)t}{d}$

4. (c) For red light, $\lambda = \lambda_r$

$$\therefore \beta_{\text{red}} = \frac{D}{a} \lambda_r \quad \dots(i)$$

For blue light, $\lambda = \lambda_b$

$$\therefore \beta_{\text{blue}} = \frac{D}{a} \lambda_b \quad \dots(ii)$$

$$\therefore \frac{\beta_{\text{blue}}}{\beta_{\text{red}}} = \frac{\frac{D}{a} \lambda_b}{\frac{D}{a} \lambda_r} = \frac{\lambda_b}{\lambda_r}$$

$$\Rightarrow \beta_{\text{blue}} = \frac{\lambda_b}{\lambda_r} \times \beta_{\text{red}}$$

$$\because \lambda_b < \lambda_r$$

$$\therefore \beta_{\text{blue}} < \beta_{\text{red}}$$

$\therefore$ Diffraction fringes become narrower and crowded together.

5. (a) Using Huygens' eyepiece, measurements can be taken but not accurately due to the reason given.

6. (c) The general condition for Fraunhofer diffraction is

$$\frac{b^2}{L\lambda} \ll 1$$

7. (b) Optical path $\mu x = \text{constant}$

$$\text{i.e.} \quad \mu_1 x_1 = \mu_2 x_2$$

$$\Rightarrow 1.53 \times 4 = \mu_2 \times 4.5$$

$$\Rightarrow \mu_2 = \frac{1.53 \times 4}{4.5} = 1.36$$

8. (b) In Young's double slit experiment, a minima is observed when path difference between the interfering beam is 1.5λ .

9. (a) As $I \propto a^2$ or $a \propto \sqrt{I}$

$$\therefore \frac{a_1}{a_2} = \sqrt{\frac{I}{4I}} \Rightarrow \frac{a_1}{a_2} = \frac{1}{2}$$

$$\frac{I_{\text{max}}}{I_{\text{min}}} = \left(\frac{a_1 + a_2}{a_1 - a_2} \right)^2 = \left(\frac{1+2}{1-2} \right)^2 = \frac{9}{1}$$

$$\therefore I_{\text{max}} = 9I, I_{\text{min}} = I$$

10. (a) If unpolarised light is incident at polarising angle, then reflected light is completely, i.e. 100% polarised.

21

Electric Charge

Electric Charge

Charge is the property associated with matter due to which it produces and experiences electric and magnetic effects. Electric charge is of two types—positive and negative. Loss of electrons gives **positive charge** and gain of electrons gives **negative charge** to a body. The charge on any body will be an integral multiple of e , i.e. $Q = \pm ne$ where $n = 1, 2, 3, \dots$

The SI unit of charge is ampere $\times$ second = coulomb (C). Charge can also be described in mC, μC , nC.

i.e. $1 \text{ mC} = 10^{-3} \text{ C}$, $1 \mu\text{C} = 10^{-6} \text{ C}$, $1 \text{ nC} = 10^{-9} \text{ C}$

Its CGS unit is coulomb or esu. Its dimensional formula is $[AT]$.

Continuous Charge Distribution

- When charge is distributed along a line, straight or curved, then it is called linear charge distribution and its charge per unit length is called linear charge density (λ).

i.e.
$$\lambda = \frac{\Delta q}{\Delta l}$$

Its unit is coulomb/metre (Cm^{-1}).

- When charge is distributed over a surface area, then it is called surface charge distribution and charge over its per unit area is called surface charge density (σ).

i.e.
$$\sigma = \frac{\Delta q}{\Delta S}$$

Its unit is coulomb-metre⁻² (Cm^{-2}).

- When charge is distributed over entire volume of a body, then it is called volume charge distribution and charge per unit volume is called volume charge density (ρ).

i.e.
$$\rho = \frac{\Delta q}{\Delta V}$$

Its unit is coulomb-metre⁻³ (Cm^{-3}).

Coulomb's Law

If q_1 and q_2 be two stationary point charges in free space separated by a distance r , then the force of attraction or repulsion between them is given by

$$\begin{aligned} F &= \frac{k|q_1||q_2|}{r^2} \\ &= \frac{1}{4\pi\epsilon_0} \cdot \frac{|q_1||q_2|}{r^2} \quad \left[\because k = \frac{1}{4\pi\epsilon_0} \right] \\ &= \frac{9 \times 10^9 \times |q_1||q_2|}{r^2} \quad \left[\because \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \right] \end{aligned}$$

The term ϵ_0 is called the electric permittivity of free space having a value of $8.85 \times 10^{-12} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$. Its dimensional formula is $[\text{M}^{-1}\text{L}^{-3}\text{T}^4\text{A}^2]$.

If some dielectric medium is completely filled between the given charges, then the Coulomb's force between them becomes

$$\begin{aligned} F_m &= \frac{1}{4\pi\epsilon} \cdot \frac{q_1q_2}{r^2} \\ &= \frac{1}{4\pi\epsilon_0\epsilon_r} \cdot \frac{q_1q_2}{r^2} \\ &= \frac{1}{4\pi K\epsilon_0} \cdot \frac{q_1q_2}{r^2} \\ \frac{\epsilon}{\epsilon_0} &= \epsilon_r \text{ or } K \\ &= \frac{F_{\text{Free space}}}{K} \quad \left[\because F_{\text{Free space}} = \frac{q_1q_2}{4\pi r^2\epsilon_0} \right] \end{aligned}$$

Here, ϵ_0 = absolute electric permittivity of the given medium, K dielectric constant and ϵ_r is the relative permittivity of the given medium.

NOTE • Force of two charges exerts on each other is not changed by the presence of a third charge.

- Coulomb's force between two protons is 10^{36} times the gravitational force between them.

Electric Field

The space surrounding an electric charge q in which another charge q_0 experiences a force of attraction or repulsion is called the electric field of charge q .

Electric field vector E (also known as the electric field intensity) at any point is given by

$$E = \lim_{q_0 \rightarrow 0} \frac{F}{q_0}$$

where, q_0 is a small positive test charge which experiences a force F at a given point.

The SI unit of electric field is NC^{-1} and it is also known as Vm^{-1} . The dimensional formula for electric field is $[\text{MLT}^{-3}\text{A}^{-1}]$.

Motion of a Charged Particle in an Electric Field

In this condition, some cases arise as below

Case I A charged particle is released from rest in an electric field E .

Then force on charged particle is given by $F = qE$

The acceleration produced by this force is given by

$$a = \frac{F}{m} = \frac{qE}{m}$$

Since, E is constant, the acceleration a is also constant. Hence, the particle is uniformly accelerated. Let the particle starts from rest, then velocity of charged particle after time t is given by $v = u + at$

$$v = \left(\frac{qE}{m} \right) t \quad \left(u = 0 \text{ and } a = \frac{qE}{m} \right)$$

The distance travelled by the particle is given by

$$s = ut + \frac{1}{2}at^2 \Rightarrow s = \frac{1}{2} \frac{qE}{m} t^2$$

The kinetic energy gained by the particle

$$= \frac{1}{2}mv^2 = \frac{q^2E^2t^2}{2m}$$

Case II A charged particle enters the field in perpendicular direction.

Let a charged particle of mass m and charge q , enters the electric field along x -axis with speed u . The electric field E is along y -axis is given by

$$F_y = qE$$

and force along x -axis remains zero, i.e. $F_x = 0$

$\therefore$ Acceleration of the particle along y -axis is given by

$$a_y = \frac{F_y}{m} = \frac{qE}{m}$$

The initial velocity is zero along y -axis ($u_y = 0$).

$\therefore$ The deflection of charged particle along y -axis after time

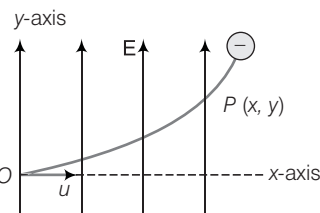
$$t \text{ is given by } y = u_y t + \frac{1}{2} a_y t^2 = \frac{qE}{2m} t^2$$

Along x -axis there is no acceleration, so the distance covered by particle in time t along x -axis is given by $x = ut$

Eliminating t , we have

$$\begin{aligned} y &= \left(\frac{qE}{2mu^2} x^2 \right) \\ y &\propto x^2 \end{aligned}$$

This shows that the path of charged particle in perpendicular field is a parabola.



Electric Field due to a Point Charge

1. Electric Field at a Distance r from a Point Charge q is

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

If q_1 and q_2 are two like point charges, separated by a distance r , a neutral point between them is obtained at a point distant r_1 from q_1 , such that

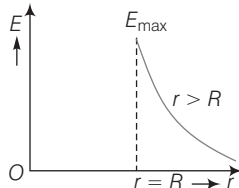
$$r_1 = \frac{r}{1 + \sqrt{\frac{q_2}{q_1}}}$$

If q_1 and q_2 are two charges of opposite nature separated by a distance r , a neutral point is obtained in the extended line joining them, at a distance r_1 from q_1 , such that

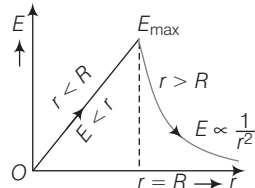
$$r_1 = \frac{r}{\sqrt{\frac{q_2}{q_1}} - 1}$$

2. Electric Field due to a Charged Cylinder

(i) For a conducting charged cylinder of linear charge density λ and radius R , the electric field is given by



(a) Conducting cylinder



(b) Non-conducting cylinder

$$E = \frac{\lambda}{2\pi\epsilon_0 r}, \text{ for } r > R,$$

$$E = \frac{\lambda}{2\pi\epsilon_0 R}, \text{ for } r = R$$

and $E = 0$, for $r < R$

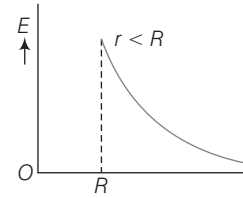
(ii) For a non-conducting charged cylinder, for $r \leq R$,

$$E = \frac{\lambda r}{2\pi\epsilon_0 R^2}$$

$$\text{and } E = \frac{\lambda}{2\pi\epsilon_0 r} \text{ for } r > R$$

3. Electric Field due to a Uniformly Charged Thin Spherical Shell

For a charged conducting sphere/ shell of radius R and total charge Q , the electric field is given by



Case I $E = 0$, for $r < R$

Case II $E = \frac{Q}{4\pi\epsilon_0 R^2}$, for $r = R$

Case III and $E = \frac{Q}{4\pi\epsilon_0 r^2}$, for $r > R$

Practice Exercise

1. One brass plate is inserted between two charges. The force between two charges will be
 a. remain the same b. increase
 c. decrease d. fluctuate

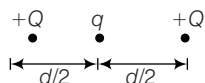
2. A sure test of electrification is
 a. attraction b. repulsion c. friction d. induction

3. In relativistic mechanics $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ the equivalent

relation in electricity for electric charge is

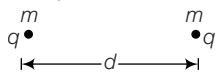
a. $q = q_0$ b. $q = \frac{q_0}{\sqrt{1 - \frac{v^2}{c^2}}}$
 c. $q_0 = \frac{q}{\sqrt{1 - \frac{v^2}{c^2}}}$ d. $q = \frac{q_0 v}{c}$

4. Two positively charged particles each having charge Q and are d distance apart. A third charge is introduced in midway on the line joining the two charges. Find nature and magnitude of third charge, so that the system is in equilibrium.



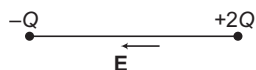
a. $q = \frac{-Q}{4}$ b. $q = \frac{Q}{4}$ c. $q = \frac{3Q}{4}$ d. $q = -\frac{3Q}{4}$

5. As shown in the adjoining figure two charge particles each having charge q and mass m are d distance apart from each other. If two particle are in equilibrium under the gravitational and electric force, then determine the ratio q / m .



a. 10^{-8} b. 10^{-10} c. 10^{10} d. None

6. Charges $+2Q$ and $-Q$ are placed as shown in figure. The point at which electric field intensity is zero will be



- a. somewhere between $-Q$ and $+2Q$
 b. somewhere on the left of $-Q$
 c. somewhere on the right of $+2Q$
 d. somewhere on the right bisector of line joining $-Q$ and $+2Q$

7. Two equal and oppositely charged particles are kept at some distance apart from each other. A spherical surface having radius equal to separation between the particles and concentric with their mid-point is considered. Then,

- a. electric field is normal to the surface at two points only
 b. electric field is zero at no point
 c. electric potential is zero at every point of one circle only
 d. All of the above

8. Two bodies A and B of definite shape are placed near one another. Electrostatic attraction is found between the bodies, then

- a. both bodies must be positively charged
 b. both bodies must be negatively charged
 c. both bodies must be oppositely charged
 d. body A may be neutral

9. If σ = surface charge density, ϵ = electric permittivity, the dimension of $\frac{\sigma}{\epsilon}$ are same as

- a. electric force b. electric field intensity
 c. pressure d. electric charge

10. What is the magnitude of a point charge due to which the electric field 30 cm away has the magnitude 2?

$[1/4 \pi \epsilon_0 = 9 \times 10^9 \text{ Nm}^2/\text{C}^2]$

- a. $2 \times 10^{-11} \text{ C}$ b. $3 \times 10^{-11} \text{ C}$
 c. $5 \times 10^{-11} \text{ C}$ d. $9 \times 10^{-11} \text{ C}$

11. A charged particle of mass m and charge q is released from rest from position $(x, 0, 0)$ in a uniform electric field $E_0 \hat{j}$. The angular momentum of the particle about origin

- a. is zero b. is constant
 c. increases with time d. decreases with time

12. An infinite plane sheet of positive charge has surface charge density σ . A metallic ball of mass m and charge $+Q$ is attached to a thread and is tied to a point A on the sheet PQ . The angle θ made by the string with the plane sheet is in equilibrium is

- a. zero b. $\tan^{-1} \left(\frac{Q\sigma}{4\epsilon_0 mg} \right)$
 c. $\tan^{-1} \left(\frac{Q\sigma}{2\epsilon_0 mg} \right)$ d. $\tan^{-1} \left(\frac{Q\sigma}{\epsilon_0 mg} \right)$

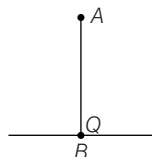
13. A dimensionless body having a physical quantity varies as $1/r^2$, where r is distance from the body. This physical quantity may be

- a. gravitational potential b. electric field
 c. gravitational field d. None of these

14. The electric field inside a conductor

a. must be zero
b. may be non-zero
c. must be non-zero
d. Both (a) and (c) are correct

15. A point charge Q is situated at point B on the ground. A point charge q of mass m is vertically dropped along line AB from a multi-storey building of height h . Find the position of the point charge q when it is in equilibrium

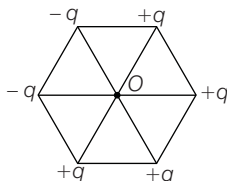


a. $\sqrt{\frac{qQ}{4\pi\epsilon_0 mg}}$
b. $\frac{Q}{h^2}$
c. $\frac{qh}{m}$
d. None of these

16. Two point charges q_1 and q_2 are released from rest in a gravity free hall when distance between them is a . Find the maximum speeds of charged particle. The mass of each charged particle is m .

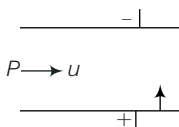
a. $\frac{q_1 q_2}{4\pi\epsilon_0 a}$
b. $\sqrt{\frac{q_1 q_2}{4\pi\epsilon_0 ma}}$
c. $\sqrt{\frac{2q_1 q_2}{4\pi\epsilon_0 ma}}$
d. None of these

17. Six point charges are arranged at the vertices of a regular hexagon of side length a (shown in figure). Find the magnitude of electric field at the centre of regular hexagon.



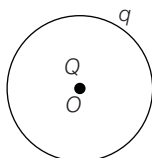
a. $\frac{q}{4\pi\epsilon_0 a^2}$
b. zero
c. $\frac{q}{2\pi\epsilon_0 a^2}$
d. None of these

18. A positively charged particle P enters the region between two parallel plates with a velocity u , in a direction parallel to the plates. There is a uniform electric field in the region. P emerges from this region with a velocity v . Taking C as a constant, v will depend on u as



a. $v = Cu$
b. $v = \sqrt{u^2 + Cu}$
c. $v = \sqrt{u^2 + \frac{C}{u}}$
d. $v = \sqrt{u^2 + \frac{C}{u^2}}$

19. A point charge Q is placed at the centre of a circular wire of radius R having charge q . Find the force of electrostatic interaction between point charge and the wire.

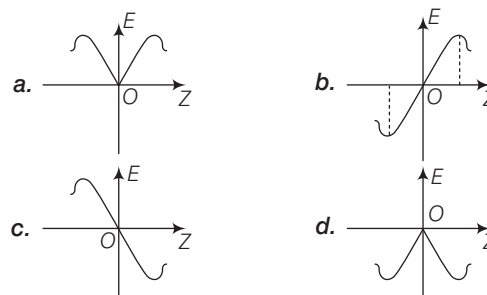


a. $\frac{qQ}{4\pi\epsilon_0 R^2}$
b. zero
c. $\frac{q^2}{4\pi\epsilon_0 R}$
d. None of these

20. A small element l is cut from a circular ring of radius a and λ charge per unit length. The net electric field at the centre of ring is

a. zero
b. $\frac{\lambda l}{4\pi\epsilon_0 a^2}$
c. ∞
d. $\frac{\lambda}{4\pi\epsilon_0 l}$

21. A circular ring carries a uniformly distributed positive charge and lies in XY -plane with the centre at origin of coordinate system. If at a point $(0, 0, Z)$ electric field is E , which of the following graphs is correct?



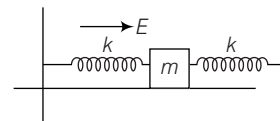
22. In the Q.21 find the separation of the point mass and wall at the equilibrium position of the mass.

a. $L + \frac{qE}{k}$
b. $L - \frac{qE}{k}$
c. $\frac{qE}{k} - L$
d. None of these

23. In the Q.21 find energy stored in spring at the equilibrium position of the point mass.

a. $\frac{q^2 E^2}{2k}$
b. $\frac{1}{2} k E^2$
c. $\frac{q^2 E^2}{k}$
d. None of these

24. A particle of mass m and having a charge q is placed on a smooth horizontal table and is connected to walls through unstressed springs of constant k (shown in figure). A horizontal electric field E parallel to spring is switched On. The maximum speed of the particle is

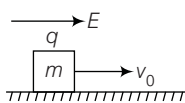


a. $\frac{qE}{\sqrt{2mk}}$
b. $\frac{qE}{mk}$
c. $\frac{qE}{m}$
d. None of these

25. A point charge is projected along the axis of circular ring of charge Q and radius $10\sqrt{2}$ cm. The distance of the point charge from centre of ring, where acceleration of charged particle is maximum, will be

a. 10 cm
b. 20 cm
c. at infinity
d. None of the above

26. If a charged particle is projected on a rough horizontal surface with speed v_0 . Find the value of dynamic coefficient of friction, if the kinetic energy of the system is constant.
- a. $\frac{qE}{mg}$ b. $\frac{qE}{m}$
 c. $\frac{q}{g}$ d. None of these
27. Two charges of value $2\mu\text{C}$ and $-50\mu\text{C}$ are placed at distance 80 cm apart. Calculate the distance of the point from the smaller charge, where the intensity will be zero.
 a. 20 cm b. 35 cm c. 30 cm d. 25 cm
28. Two charged particles of charge $+2q$ and $+q$ have masses m and $2m$, respectively. They are kept in uniform electric field and allowed to move for the same time. Find the ratio of their kinetic energies.
 a. 1 : 8 b. 16 : 1 c. 2 : 1 d. 3 : 1
29. An oil drop of charge of 2 electrons fall freely with a terminal speed. Calculate the mass of oil drop so, it can move upward with same terminal speed, if electric field of $2 \times 10^3 \text{ V/m}$ is applied.
 a. $3.0 \times 10^{-17} \text{ kg}$ b. $3.2 \times 10^{-17} \text{ kg}$
 c. $2.5 \times 10^{-17} \text{ kg}$ d. $3.3 \times 10^{-17} \text{ kg}$
30. An electron is projected with velocity 10^7 m/s at an angle $\theta (= 30^\circ)$ with horizontal, in a region of uniform electric field of 500 N/C vertically upwards. Find the

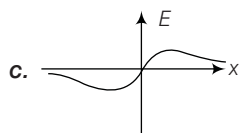
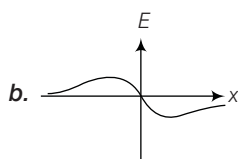
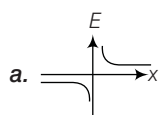
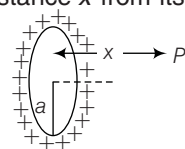


maximum distance covered by an electron in vertical direction above its initial level.

- a. 14.2 mm
 b. 15 mm
 c. 12.6 mm
 d. 14.2 cm
31. A pendulum bob of mass m and charge q is suspended by a thread of length l . The pendulum is placed in a region of a uniform electric field E directed vertically upward. If the electrostatic force acting on the sphere is less than that of gravitational force, calculate the period with which the pendulum oscillates (Assume small oscillation).
- a. $T = 2\pi \sqrt{\frac{l}{g + \frac{qE}{m}}}$ b. $T = 2\pi \sqrt{\frac{l}{g - \frac{qE}{m}}}$
 c. $T = \pi \sqrt{\frac{l}{g - \frac{qE}{m}}}$ d. $T = \pi \sqrt{\frac{l}{g + \frac{qE}{m}}}$
32. Identical charges of magnitude Q are placed at $(n-1)$ corners of a regular polygon of n sides each corner of the polygon is at a distance r from the centre. The field at the centre is
- a. $\frac{kQ}{r^2}$ b. $(n-1) \frac{kQ}{r^2}$
 c. $\frac{n}{(n-1)} \cdot \frac{kQ}{r^2}$ d. $\frac{(n-1)}{n} k \frac{Q}{r^2}$

BITSAT Archives

1. A ring shaped conductor with radius a carries a net positive charge q uniformly distributed on it as shown in figure. A point P is situated at a distance x from its centre. Which of following graph shows the correct variation of electric field (E) with distance (x)? [2014]



d. None of these

2. The electrostatic force of repulsion between two positively charged ions carrying equal charge is $3.7 \times 10^{-9} \text{ N}$, when they are separated by a distance of 5 Å. How much electrons are missing from each ion?
 a. 10 b. 8 c. 2 d. 1 [2014]
3. Two equal charges q are kept fixed at a and $+a$ along the x -axis. A particle of mass m and charge $\frac{q}{2}$ is brought to the origin and given a small displacement along the x -axis, then [2012]
 a. the particle executes oscillatory motion
 b. the particle remains stationary
 c. the particle executes, SHM along x -axis
 d. the particle executes SHM along y -axis
4. The magnitude of electric field intensity E , such that an electron placed in it would experience an electrical force equal to its weight is given by [2011]
 a. mge b. $\frac{mg}{e}$ c. $\frac{e}{mg}$ d. $\frac{e^2}{m^2} g$

5. Which one of the following is correct statement? [2009]

- Electric field is always conservative
- Electric field due to varying magnetic field is conservative
- Electric field is conservative due to electrostatic charges while non-conservative due to time varying magnetic field
- Electric field lines are always closed loops

6. A charge of $1 \mu\text{C}$ is divided into two parts such that their charges are in the ratio of 2 : 3. These two charges are kept at a distance 1 m apart in vacuum. Then, the electric force between them (in N) is [2008]

- 0.216
- 0.00216
- 0.0216
- 2.16

7. An electric line of force in the xy -plane is given by equation $x^2 + y^2 = 1$. A particle with unit positive charge, initially at rest at the point $x = 1, y = 0$ in the xy -plane [2007]

- not move at all
- will move along straight line
- will move along the circular line of force
- information is insufficient to draw any conclusion

8. Identify the wrong statement in the following. Coulomb's law correctly described the electric force that [2005]

- binds the electrons of an atom to its nucleus
- binds the protons and neutrons in the nucleus of an atom
- binds atoms together to form molecules
- binds atoms and molecules to form solids

Answer with Solutions

Practice Exercise

1. (b) 2. (b) 3. (a)

4. (a) Condition for equilibrium

$$\frac{kQ^2}{d^2} + \frac{kqQ}{(d/2)^2} = 0 \Rightarrow \frac{kQ}{d^2} (Q + 4q) = 0$$

$$q = \frac{-Q}{4}$$

5. (b) Condition for equilibrium $F_e = F_g$

$$\frac{kq^2}{d^2} = \frac{Gm^2}{d^2}$$

$$\Rightarrow \frac{1}{4\pi\epsilon_0} \frac{q^2}{d^2} = \frac{Gm^2}{d^2}$$

or

$$\frac{q^2}{m^2} = G(4\pi\epsilon_0)$$

$$\frac{q}{m} = \frac{\sqrt{G \times 4\pi\epsilon_0}}{1}$$

$$= \sqrt{\frac{6.67 \times 10^{-17}}{9 \times 10^9}} \approx 10^{-10}$$

6. (b)

7. (d)

8. (d) Attraction may take place between like charged bodies, unlike charged bodies and charged body and neutral body. Hence, (d) is correct.

9. (b)

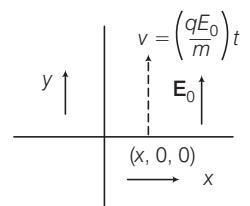
10. (a) Electric field due to a point charge, $E = \frac{q}{4\pi\epsilon_0 r^2}$

$$\therefore q = E \times 4\pi\epsilon_0 r^2 = 2 \times \frac{1}{9 \times 10^9} \times \left(\frac{30}{100}\right)^2$$

$$\Rightarrow q = 2 \times 10^{-11} \text{ C}$$

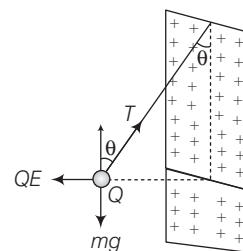
11. (b) Angular momentum, $\mathbf{L} = \mathbf{r} \times \mathbf{p} = rp \sin 90^\circ$

$$= rp = rmv = \text{constant}$$



12. (c) By using Lami's theorem,

$$\Rightarrow \tan \theta = \frac{QE}{mg} = \frac{Q\sigma}{2\epsilon_0 mg}$$



$$\theta = \tan^{-1} \left(\frac{Q\sigma}{2\epsilon_0 mg} \right) \quad (\because E = \text{field of plate} = \frac{\sigma}{2\epsilon_0})$$

13. (b)

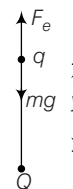
14. (b)

15. (a) For equilibrium,

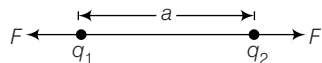
$$mg = F_e \text{ or } mg = \frac{qQ}{4\pi\epsilon_0 y^2}$$

$\therefore$

$$y = \sqrt{\left(\frac{qQ}{4\pi\epsilon_0 mg} \right)}$$



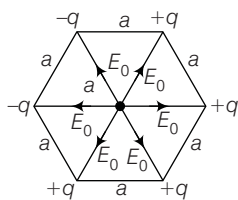
16. (b) When separation between charges is infinitely large, potential energy is zero and kinetic energy of system is maximum. According to conservation principle of energy,



$$\frac{1}{2}mv_{\max}^2 + \frac{1}{2}mv_{\max}^2 = \frac{q_1q_2}{4\pi\epsilon_0 a} \text{ or } mv_{\max}^2 = \frac{q_1q_2}{4\pi\epsilon_0 a}$$

$$\therefore v_{\max} = \sqrt{\left(\frac{q_1q_2}{4\pi\epsilon_0 ma}\right)}$$

17. (c) From figure, the resultant electric field is



$$E = 2E_0 = \frac{2q}{4\pi\epsilon_0 a^2} = \frac{q}{2\pi\epsilon_0 a^2}$$

18. (d) $u_x = u, t = \frac{l}{u}$
- $$\Rightarrow a_y = \frac{Eq}{m} = a \Rightarrow v_y = at = \frac{al}{u}$$
- $$\Rightarrow v^2 = u^2 + \frac{a^2 l^2}{u^2} = u^2 + \frac{C}{u^2} \Rightarrow v = \sqrt{u^2 + \frac{C}{u^2}}$$

19. (b)

20. (b) E_{net} = electric field due to ring at centre without cutting portion electric field due to cutting element is

$$E_{\text{net}} = 0 - \frac{q_0}{4\pi\epsilon_0 a^2} = -\frac{\lambda l}{4\pi\epsilon_0 a^2}$$

$$\therefore |E_{\text{net}}| = \frac{\lambda l}{4\pi\epsilon_0 a^2}$$

21. (b) At $Z=0$, $E=0$ and between $Z=0$ and $Z=\infty$ and E grows to a maximum value, which can be found by $\frac{dE}{dt} = 0$. Also, graph (b) should be symmetric, which is (b).

22. (a) At equilibrium position,

$$qE = kx_0 \therefore x_0 = \frac{qE}{k}$$

$$\therefore \text{Separation} = L + \frac{qE}{k}$$

23. (a)

24. (a) The compression in spring at equilibrium = x_0

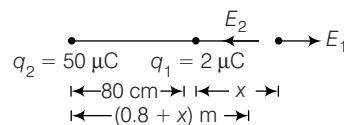
$$\therefore 2kx_0 = qE \therefore x_0 = \frac{qE}{2k} = A = \text{amplitude}$$

$$\text{Also, } \omega = \sqrt{\frac{k_1 + k_2}{m}} = \sqrt{\frac{2k}{m}}$$

$$\frac{1}{2}mv_{\max}^2 = \frac{1}{2}mA^2\omega^2 \Rightarrow v_{\max} = \frac{qE}{\sqrt{2mk}}$$

25. (a) 26. (a)

27. (a)



$$\therefore E = E_1 - E_2 \text{ or } 0 = E_1 - E_2 \therefore E_1 = E_2$$

$$\text{or } \frac{kq_1}{x^2} = \frac{kq_2}{(0.8 + x)^2}$$

$$\text{or } \frac{2 \times 10^{-6}}{x^2} = \frac{50 \times 10^{-6}}{(0.8 + x)^2}$$

$$\therefore x = 0.2 \text{ m} = 20 \text{ cm}$$

28. (b) Let E = uniform electric field

$$\text{Let } m_1 = m, m_2 = 2m$$

Force on particles,

$$F_1 = ma_1 = 2qE$$

$$\Rightarrow F_2 = 2ma_2 = qE$$

$$a_1 = \frac{2qE}{m} \text{ and } a_2 = \frac{qE}{2m}$$

Final velocities of the particle

$$v_1 = u_1 + a_1 t$$

$$\Rightarrow v_1 = \frac{2qE}{m} t \Rightarrow v_2 = \frac{qE}{2m} t$$

Let K = kinetic energy

$$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}m\left(\frac{2qE}{m}t\right)^2 = (2m)\left(\frac{qE}{m}t\right)^2$$

$$\text{Similarly, } K_2 = \frac{1}{2}m\left(\frac{qE}{2m}t\right)^2 \Rightarrow \frac{K_1}{K_2} = \frac{16}{1}$$

$$29. (b) \quad mg = F = 6\pi\eta r v \quad \dots(i)$$

$$qE - mg = 6\pi\eta r v = F \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$qE = 6\pi\eta r v + mg = 2mg$$

$$E = \frac{2mg}{q} = \frac{2mg}{2e} = \frac{mg}{e}$$

$$m = \frac{2 \times 10^3 \times 1.6 \times 10^{-19}}{10} = 3.2 \times 10^{-17} \text{ kg}$$

30. (a)

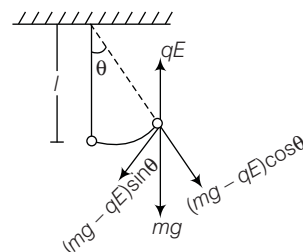
31. (b) Resultant force vertically downwards ($mg - qE$)

Net acceleration

$$g' = \left(g - \frac{qE}{m}\right)$$

$$T = 2\pi \sqrt{\left(\frac{l}{g'}\right)}$$

$$T = 2\pi \sqrt{\frac{l}{\left(g - \frac{qE}{m}\right)}}$$



32. (a)

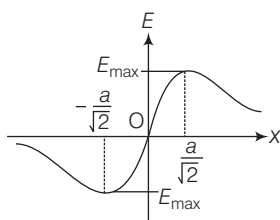
BITSAT Archives

1. (c) The net electric field at point P is given by

$$\mathbf{E} = \frac{qx}{4\pi\epsilon_0(x^2 + a^2)^{3/2}} \hat{i}$$

$\therefore$ At the centre of ring $x = 0 \Rightarrow E_0$

If $x \gg a$, $\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{x^2}$



and E will be maximum where,

$$\frac{dE}{dx} = 0 \text{ or } x = \pm \frac{a}{\sqrt{2}}$$

and

$$E_{\max} = \frac{1}{4\pi\epsilon_0} \frac{2q}{3\sqrt{3}a^2}$$

2. (c) Here, $F = 3.7 \times 10^{-9} \text{ N}$

Let, $q_1 = q_2 = q$

$$\Rightarrow r = 5 \text{ \AA} = 5 \times 10^{-10} \text{ m}$$

$$\therefore F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$\Rightarrow 3.7 \times 10^{-9} = 9 \times 10^9 \frac{q \times q}{(5 \times 10^{-10})^2}$$

$$q^2 = \frac{3.7 \times 10^{-9} \times 25 \times 10^{-20}}{9 \times 10^9}$$

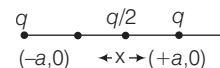
$$= 10.28 \times 10^{-38}$$

or $q = 3.2 \times 10^{-19} \text{ C}$

Now, $q = ne \therefore n = \frac{q}{e} = \frac{3.2 \times 10^{-19}}{1.6 \times 10^{-19}} = 2$

3. (c) From Coulomb's law,

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$



$$\begin{aligned} F &= \frac{1}{4\pi\epsilon_0} \frac{q \times \frac{q}{2}}{(a+x)^2} - \frac{1}{4\pi\epsilon_0} \frac{q \times \frac{q}{2}}{(a-x)^2} \\ &= \frac{1}{4\pi\epsilon_0} \frac{q^2}{2} \left[\frac{1}{(a+x)^2} - \frac{1}{(a-x)^2} \right] \\ &= \frac{1}{4\pi\epsilon_0} \frac{q^2}{2} \left[-\frac{4ax}{(a^2 - x^2)^2} \right] \end{aligned}$$

When $x \ll a$, then

$$F = -\frac{2q^2}{4\pi\epsilon_0 a^3} x \Rightarrow F \propto -x$$

Hence, SHM is along x -axis.

4. (b) Force on electron

$$|F| = qE = eE = mg \Rightarrow E = \frac{mg}{e}$$

5. (c) Electric field is conservative due to electrostatic charges, while non-conservative due to time varying magnetic field.

6. (b) Ratio of charges = 2 : 3

$$\therefore q_1 = \frac{2}{5} \times 5 \times 1 \mu\text{C} \text{ and } q_2 = \frac{3}{5} \times 1 \mu\text{C}$$

Electrostatic force between the two charges

$$\begin{aligned} F &= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \\ &= \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 3 \times 10^{-6}}{5 \times 5 \times (1)^2} \\ &= 2.16 \times 10^{-3} \text{ N} \\ &\approx 0.00216 \text{ N} \end{aligned}$$

7. (c) Charge will move along the circular line of force because $x^2 + y^2 = 1$ is the equation of circle in xy -plane.

8. (b) Coulomb's law is applicable for charged particles, it is not responsible to bind the protons and neutrons in the nucleus of an atom.

22

Gauss's Law and Electric Potential Theory

Electric Flux

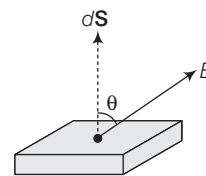
The electric flux linked with any surface in an electric field is basically a measure of total number of lines of forces passing normally through the surface. Flux is an imaginary scalar physical quantity.

Electric flux through an elementary area dS is defined as the scalar product of area of field.

i.e. $d\phi = \mathbf{E} \cdot d\mathbf{S} = E dS \cos\theta$

Hence, flux from complete area (S),

$$\phi = \int E dS \cos\theta$$



Outward flux is taken to be positive, while inward flux is taken to be negative.

SI unit of electric flux is (volt $\times$ metre) or $\frac{\text{N-C}}{\text{m}^2}$ and its dimensional formula is $[\text{ML}^3\text{T}^{-3}\text{A}^{-1}]$

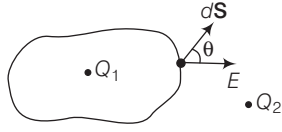
Gauss's Law

Gauss's law states that flux is produced by charge. So, total electric flux through a closed surface enclosing a charge $= \frac{Q_{\text{en}}}{\epsilon_0}$, $\phi_{\text{net}} = \frac{1}{\epsilon_0} Q_{\text{en}}$

Gauss's law is the consequence of Coulomb's law. According to Gauss's law,

$$\int \mathbf{E} \cdot d\mathbf{S} = \frac{Q_{\text{en}}}{\epsilon_0}$$

NOTE In figure shown below, E is the net electric field due to both the charges Q_1 and Q_2 but $Q_{en} = Q_1$.

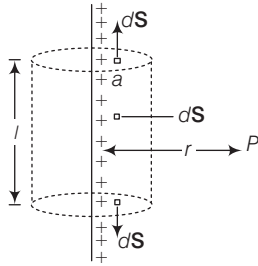


Applications of Gauss's Law

Electric Field due to Infinitely Long Line of Charge

Let us consider a uniformly charged wire of an infinite length having a constant linear charge density, i.e. $\left(\lambda = \frac{\text{Charge}}{\text{Length}}\right)$. We have to calculate electric field at point P

which is at a distance r as shown in figure.



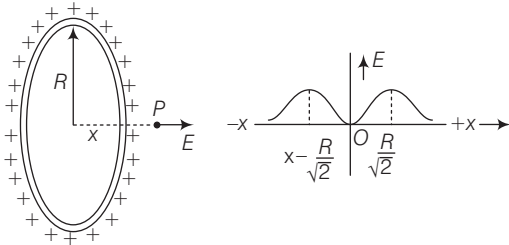
$$\int E dS = \frac{Q}{\epsilon_0} \Rightarrow E \int dS = \frac{Q}{\epsilon_0}$$

$$\Rightarrow E \times 2\pi r l = \frac{Q}{\epsilon_0}$$

$$\Rightarrow E = \frac{Q}{2\pi\epsilon_0 r l} = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k\lambda}{r} \quad \left(\because k = \frac{1}{4\pi\epsilon_0}\right)$$

Electric Field due to Charged Circular Ring

Suppose we have a charged circular ring of radius R and charge Q . On its axis, electric field is to be determined at a point x distance away from the centre of the ring.



$$E = \frac{kQx}{(x^2 + R^2)^{3/2}}$$

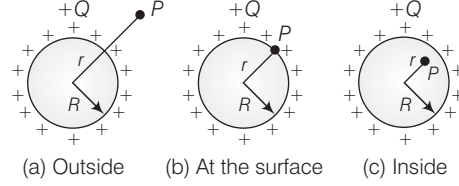
At centre, $x = 0$, so $E_{\text{centre}} = 0$.

At a point on the axis such that $x \gg R$, $E = \frac{kQ}{x^2}$

If $x = \pm \frac{R}{\sqrt{2}}$, then $E_{\text{max}} = \frac{kQ}{3\sqrt{3} R^2}$

Electric Field due to Charged Conducting Sphere

If a charge on a conducting sphere of radius R is Q and σ is the surface charge density as shown in figure, then electric field in different situations are



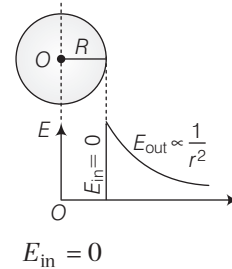
(i) **Outside the sphere** If point P lies outside the sphere, then

$$E_{\text{out}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2} = \frac{\sigma R^2}{\epsilon_0 r^2} \quad (Q = \sigma \times A)$$

(ii) **At the surface of sphere** At the surface, $r = R$

$$\text{So, } E_S = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R^2} = \frac{\sigma}{\epsilon_0}$$

(iii) **Inside the sphere** Inside the conducting charge sphere, electric field is zero.

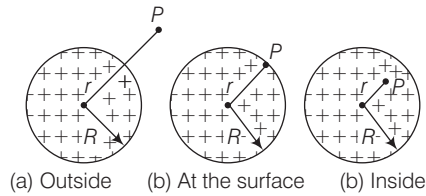


$\therefore$

$$E_{\text{in}} = 0$$

Uniformly Charged Non-conducting Sphere

Suppose a charge Q is uniformly distributed in the volume of a non-conducting sphere of radius R as shown in figure below.



(i) **Outside the sphere** If point P lies outside the sphere, then

$$E_{\text{out}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2}$$

If the sphere has a uniform volume charge density,

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}, \text{ then } E_{\text{out}} = \frac{\rho R^3}{3\epsilon_0 r^2}$$

(ii) **At the surface of sphere**

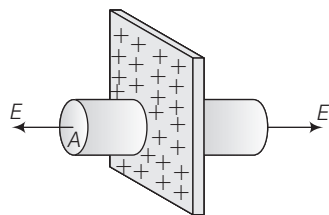
$$\text{At the surface, } r = R, \text{ so } E_S = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R^2} = \frac{\rho R}{3\epsilon_0}$$

(iii) **Inside the sphere** At a distance r from the centre,

$$E_{\text{in}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Qr}{R^3} = \frac{\rho r}{3\epsilon_0} \quad (\text{i.e. } E_{\text{in}} \propto r)$$

Infinite Thin Plane Sheet of Charge

In this case, field is uniform and does not depend on the distance from the plate.

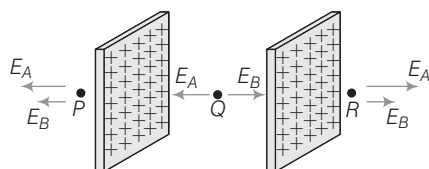


So,
$$E = \frac{\sigma}{2\epsilon_0} \quad (\text{i.e. } E \propto r^0)$$

Electric Field due to Two Thin Infinite Plane Parallel Sheets of Charge

Consider two large uniformly charged parallel sheets A and B having surface charge densities σ_A and σ_B respectively.

At P , $E_P = -(E_A + E_B) = -\frac{1}{2\epsilon_0}(\sigma_A + \sigma_B)$



At Q ,
$$E_Q = (E_A - E_B) = \frac{1}{2\epsilon_0}(\sigma_A - \sigma_B)$$

At R , $E_R = (E_A + E_B) = \frac{1}{2\epsilon_0}(\sigma_A + \sigma_B)$

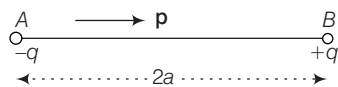
NOTE Gauss's law is easily applicable to symmetric charge distribution.

Electric Dipole

An electric dipole consists of two equal and opposite charges separated by a small distance.

The dipole moment of a dipole is defined as the product of the magnitude of either charges and the distance between them. Therefore, dipole moment $\mathbf{p} = q(2a)$.

Dipole moment is a vector whose direction is from negative to positive charge. Its SI unit is coulomb-metre, i.e. C-m and has the dimensional formula [LTA].



Electric Field due to a Dipole

- At a point distant r from the centre of a dipole, along its axial line

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2pr}{(r^2 - a^2)^2}$$

(direction of E is the same as that of $\mathbf{p}$)

For a short dipole, i.e. $r \gg a$,

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{r^3}$$

- At a point distant r from the centre of a dipole along its equatorial line

$$E = -\frac{1}{4\pi\epsilon_0} \cdot \frac{p}{(r^2 + a^2)^{3/2}}$$

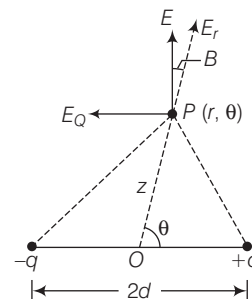
(direction of E is opposite to that of $\mathbf{p}$)

For a short dipole,

$$E = -\frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3} \quad [r \gg a]$$

- At a point distant r from the centre of a short dipole along a line inclined at an angle θ with the dipole axis

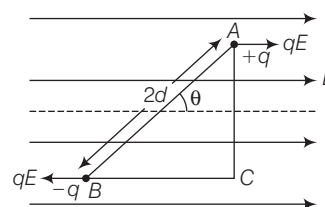
$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3} \sqrt{3 \cos^2 \theta + 1}$$



- E subtends an angle β from $\mathbf{r}$ such that, $\tan \beta = \frac{1}{2} \tan \theta$

Torque on a Dipole in a Uniform Electric Field

When a dipole is placed in an external electric field making an angle θ with the direction of the uniform electric field E , it experiences a torque given by $\tau = qE \times AC$



$$\tau = \mathbf{p} \times \mathbf{E} = pE \sin \theta$$

or

$$qE \times 2d \sin \theta = (q \times 2d) E \sin \theta$$

Work Done

If an electric dipole initially kept in an uniform electric field E making an angle θ_1 is rotated so as to finally subtends an angle θ_2 , then the work done for rotating the dipole is $W = pE(\cos \theta_2 - \cos \theta_1)$

Potential Energy of a Dipole

It is the amount of work done in rotating an electric dipole from a direction perpendicular to electric field to a particular direction. Hence, $U = -pE \cos \theta$ or $U = -\mathbf{p} \cdot \mathbf{E}$.

Obviously, potential energy of an electric dipole is a scalar quantity. It is measured in joule.

Electric Potential

Electric potential at a point in an electric field is defined as the amount of work done in bringing a unit positive charge, without any acceleration from infinity to that point, along any arbitrary path.

Mathematically, if W work is to be done to bring a test charge q_0 from infinity to a point, then the potential of that point is

$$V = \frac{W}{q_0}$$

SI unit of potential is volt, where $1 \text{ V} = \frac{1 \text{ J}}{1 \text{ C}}$. Its dimensional formula is $[\text{ML}^2\text{T}^{-3}\text{A}^{-1}]$.

Electric potential is a state function and does not depend on the path followed.

1. Electric Potential due to a Point Charge

Potential due to a point charge Q at a distance r is given by

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r}$$

- At the centre of the line joining two equal and opposite charges, $V = 0$ but $E \neq 0$.
- At the centre of the line joining two equal and like charges, $E = 0$ but $V \neq 0$.
- If four identical charges q each are placed at the four vertices of a square then the net electric field at the centre of the square is zero, but $V = \frac{\sqrt{2}q}{\pi\epsilon_0 a}$

2. Electric Potential due to a System of Charges

If a number of charges $q_1, q_2, q_3 \dots$ are present in space, then the electric potential at any point will be

$$V = V_1 + V_2 + V_3 + \dots$$

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \dots \right] = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \left(\frac{q_i}{r_i} \right)$$

3. Electric Potential due to an Electric Dipole

At any general point,

$$V = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2}$$

On the dipole axis, $\theta = 0^\circ$

and
$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^2}$$

On the equatorial axis, $\theta = 90^\circ$ and $V = 0$

4. Electric Potential due to Some Common Charge Distributions

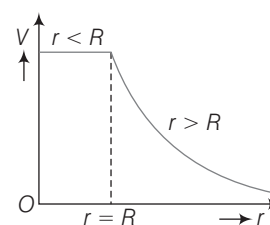
Potential at a point distant r from an infinitely long wire having linear charge density λ is $V = \frac{\lambda}{2\pi\epsilon_0} \cdot \ln r$

For a charged conducting sphere/shell having total charge Q and radius R , the potential at a point distant r from the centre of the sphere/shell is

(i) $V = \frac{Q}{4\pi\epsilon_0 r}$, for $r > R$

(ii) $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{R}$, for $r = R$

(iii) $V = \frac{Q}{4\pi\epsilon_0 R}$, for $r \leq R$



For a charged non-conducting (dielectric) sphere of radius R , the charge Q is uniformly distributed over the entire volume.

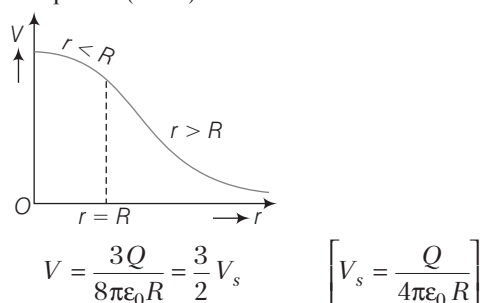
Hence,

(i) $V = \frac{Q}{4\pi\epsilon_0 r}$, for $r > R$

(ii) $V = \frac{Q}{4\pi\epsilon_0 R}$, for $r = R$

(iii) $V = \frac{Q}{4\pi\epsilon_0} \left[\frac{3R^2 - r^2}{2R^3} \right]$, for $r < R$

At the centre of the sphere ($r = 0$)



Electric Potential Energy

The electric potential energy of a system of charges is the work that has been done in bringing those charges from infinity to near each other to form the system. For two point charges q_1 and q_2 separated by distance r_{12} , the potential energy is given by

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}}$$

Electric potential energy is a scalar quantity.

Relation between E and V

Because E is force per unit charge and V is work per unit charge. E and V are related in the same way as work and force. If $\Delta V = (V_B - V_A)$ is the increase in potential over a short displacement Δs , $\Delta V = -E\Delta s$

where the negative sign indicates that the work is done against the field. If α is the angle between E and Δs , we have

$$\begin{aligned} \Delta V &= -E(\Delta s)\cos\alpha \\ &= -E\Delta x \end{aligned} \quad (\text{along } E)$$

where, $\Delta x = \Delta s\cos\alpha$ is the component of Δs along E .

Therefore,
$$E = -\frac{\Delta V}{\Delta x}$$

For small changes, above expression may be written as

$$E = -\frac{dV}{dx}$$

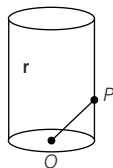
Thus, the electric field intensity E is the gradient of potential. This means that the decrease in potential is along the direction of E . The SI unit of E is volt per metre (Vm^{-1}).

Practice Exercise

1. A surface $S = 10\hat{j}$ is kept in an electric field. $E = 2\hat{i} + 4\hat{j} + 7\hat{k}$. How much electric flux will come out through the surface?

- a. 40 units
- b. 50 units
- c. 30 units
- d. 20 units

2. Electric field at point P is given by $\mathbf{E} = r E_0$. The total flux through the given cylinder of radius R and height h is



- a. $E_0\pi R^2 h$
- b. $2E_0\pi R^2 h$
- c. $3E_0\pi R^2 h$
- d. $4E_0\pi R^2 h$

3. A point charge Q is placed at the centre of a hemisphere. Find the electric flux passing through flat surface of hemisphere.

- a. $\frac{Q}{\epsilon_0}$
- b. Zero
- c. $\frac{Q}{2\epsilon_0}$
- d. None of these

4. A point charge Q is placed at the centre of a hemisphere. Find the ratio of electric flux passing through curved surface and plane surface of the hemisphere.

- a. 1 : 1
- b. 1 : 2
- c. 2π : 1
- d. 4π : 1

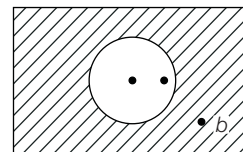
5. What should be the flux linked with the cube, if a point charge q is placed at one corner of a cube?

- a. $\frac{q}{\epsilon_0}$
- b. $\frac{q}{2\epsilon_0}$
- c. $\frac{q}{3\epsilon_0}$
- d. $\frac{q}{8\epsilon_0}$

6. Find the electric flux passing through the sphere, if an electric dipole is placed at the centre of a sphere.

- a. $\frac{1}{\epsilon_0}$
- b. $\frac{2}{\epsilon_0}$
- c. Zero
- d. None of these

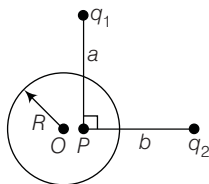
7. A point charge $q = 2 \times 10^{-7} \text{ C}$ is placed at the centre of a spherical cavity of radius 3 cm in a metal piece. Points a and b are situated at distances 1.5 cm and 4.5 cm respectively from the centre of cavity.



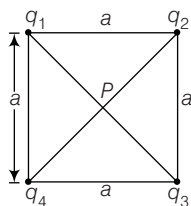
The electric field intensities at a and b are

- a. $8 \times 10^6 \text{ N/C}$ and zero
- b. zero and zero
- c. zero and $8.9 \times 10^5 \text{ N/C}$
- d. None of these

8. In the given figure, two point charges q_1 and q_2 are placed at distances a and b from centre of a metallic sphere having charge Q . Find electric field due to the metallic sphere at the point P .



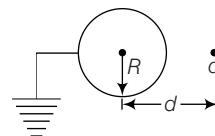
- a. $\frac{1}{4\pi\epsilon_0} \sqrt{\left(\frac{q_1}{a^2}\right)^2 + \left(\frac{q_2}{b^2}\right)^2}$
 b. $\frac{1}{4\pi\epsilon_0} \frac{Q}{R^2}$
 c. $\frac{1}{4\pi\epsilon_0} \sqrt{\left(\frac{Q}{R^2}\right)^2 + \left(\frac{q_1}{a^2} + \frac{q_2}{b^2}\right)^2}$
 d. None of the above
9. Find the minimum surface density of charge on the plate, so that a body of mass 2 kg/m^2 may just be lifted.
- a. $2.84 \times 10^{-5} \text{ C/m}^2$ b. $2.25 \times 10^{-5} \text{ C/m}^2$
 c. $1.86 \times 10^{-5} \text{ C/m}^2$ d. None of these
10. Find the surface density of electric charge at a place on the earth's surface, where the rate of fall of potential is 2.5 V .
- a. $2.0 \times 10^{-9} \text{ C/m}^2$ b. $2.21 \times 10^{-9} \text{ C/m}^2$
 c. $3.36 \times 10^{-9} \text{ C/m}^2$ d. $3.5 \times 10^{-9} \text{ C/m}^2$
11. Four point charges q_1, q_2, q_3 and q_4 are placed at the corners of the squares of side a , as shown in figure. Calculate the potential at the centre of the square.



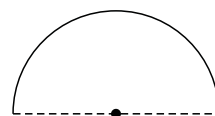
(Given, $q_1 = 1 \times 10^{-8} \text{ C}$, $q_2 = -2 \times 10^{-8} \text{ C}$,
 $q_3 = 3 \times 10^{-8} \text{ C}$, $q_4 = 2 \times 10^{-8} \text{ C}$ and $a = 1 \text{ m}$)

- a. 507 V b. 607 V c. 550 V d. 650 V
12. Over a thin ring of radius R a charge q is distributed non-uniformly. Calculate the work done of the force field in displacing a point charge q' from centre of the ring to infinity.
- a. $\frac{qq'}{4\pi\epsilon_0 R}$ b. $\frac{qq'}{2\pi\epsilon_0 R}$
 c. $\frac{qq'}{\pi\epsilon_0 R}$ d. None of these
13. Two drops of water each with a charge of $3 \times 10^{-9} \text{ C}$ having surface potential 500 V form a single drop. What is the surface potential of the new drop?
- a. 794 V b. 1000 V
 c. 250 V d. 750 V

14. Calculate the earth's potential. Assume earth has a surface charge density of 1 electron/metre². (Given, the electronic charge $= -1.6 \times 10^{-19} \text{ C}$, earth's radius $= 6.4 \times 10^6 \text{ m}$, $\epsilon_0 = 8.9 \times 10^{-12} \text{ C}^2/\text{Nm}^2$)
- a. -0.115 V b. 0.215 V c. -0.225 V d. 0.185 V
15. An electron is released from rest at one point in uniform field and moves a distance of 10 cm in 10^{-7} s . What is the voltage between the points?
- a. 10 V b. 7 V c. 11.4 V d. 8 V
16. In the figure, the charge appears on the sphere is



- a. q b. $\frac{qd}{r}$ c. $-\frac{qr}{d}$ d. 0
17. At the eight corners of a cube of side 10 cm , equal charges each of value 10 C are placed. Calculate the potential at the centre of the cube.
- a. $83.14 \times 10^{11} \text{ V}$ b. $16.62 \times 10^{11} \text{ V}$
 c. $1.66 \times 10^{11} \text{ V}$ d. $1662.7 \times 10^{11} \text{ V}$
18. Calculate the work required to bring a unit positive charge from infinity to a mid-point between two charges $20 \mu\text{C}$ and $10 \mu\text{C}$ separated by a distance of 50 m .
- a. $10.8 \times 10^4 \text{ J}$ b. $10.8 \times 10^3 \text{ J}$
 c. $108 \times 10^6 \text{ J}$ d. $0.54 \times 10^5 \text{ J}$
19. Two charges $+q$ and $-3q$ are placed at a distance of 1 m apart. Find out the points on the line joining two charges, where electric potential is zero.
- a. 0.25 m , 0.5 m b. 1 cm , 0.50 m
 c. 0.35 cm , 24 cm d. None of these
20. Two points are at distance r_1 and r_2 ($r_1 < r_2$) from a long string having charge per unit length σ . The potential difference between the points is proportional to
- a. σ b. $\log\left(\frac{r_2}{r_1}\right)$ c. $\frac{1}{\sigma}$ d. $\frac{r_2}{r_1}$
21. A semicircular wire of radius a having λ as charge per unit length is shown in the figure. Find the electric potential at the centre of the semicircular wire.



- a. $\frac{\lambda}{\epsilon_0}$ b. $\frac{\lambda}{4\pi\epsilon_0 R}$
 c. $\frac{\lambda}{4\epsilon_0}$ d. None of these

22. If a charged particle starts from rest from one conductor and reaches the other conductor with a velocity of 10^9 cm/s, then calculate the potential difference between the two conductor. The mass of the charge particle is 9×10^{-28} g and the charge is 4.8×10^{-10} esu.

a. 0.94 stat volt b. 1 stat volt
c. 1.2 stat volt d. 0.2 stat volt

23. A charge Q is uniformly distributed over the surface of two conducting concentric spheres of radii R and r ($R > r$). Then, potential at common centre of these spheres is

a. $\frac{kQ(R+r)}{Rr}$ b. $\frac{kQ(R+r)}{(R^2+r^2)}$
c. $\frac{kQ}{\sqrt{R^2+r^2}}$ d. $kQ\left(\frac{1}{R} - \frac{1}{r}\right)$

24. If a charged particle starts from rest from one conductor and reaches the other conductor with a velocity 10^9 cm/s, if the potential difference between the two conductors is 0.94 stat volt, then calculate the charge of the charged particle.

(Given, mass of charged particle = 9×10^{-28} g)

a. 5.8×10^{-10} esu b. 4.8×10^{-10} esu
c. 3.8×10^{-10} esu d. 2.75×10^{-10} esu

25. Calculate the potential of the big drop, if eight charged water drops with a radius of 1 mm and a charge of 10^{-10} C merge into a single drop.

a. 3200 V b. 4000 V c. 3600 V d. 4200 V

26. A charge $q = 2 \mu\text{C}$ is moved by some external force from infinity to a point where electric potential is 10^4 V. Calculate the work done by external force.

a. 1×10^{-2} J b. 2×10^{-2} J
c. 0.2×10^{-2} J d. 12×10^{-2} J

27. Find the electric potential energy of electron-proton system of hydrogen atom. (Given, the radius of electron orbit = $0.53 \text{ \AA}$, electronic charge = 1.6×10^{-19} C)

a. -27.17 eV b. -20.18 eV
c. 36.55 eV d. None of these

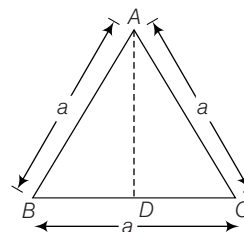
28. Three charges Q , $2Q$, $8Q$ are to be placed on a line whose length is R metre. Locate the position where these charges should be placed such that the potential energy of the system is minimum.

a. $\frac{R}{3}$ b. $\frac{2R}{3}$ c. $\frac{3R}{2}$ d. $\frac{4R}{3}$

29. A particle of mass 2 g and charge $1 \mu\text{C}$ is held at rest on a frictionless horizontal surface at a distance of 1 m from a fixed charge 1 mC. If the particle is released, it will be repelled. The speed of the particle when it is at a distance of 10 m from the fixed charge is

a. 100 m/s b. 90 m/s c. 60 m/s d. 45 m/s

30. At the corners of an equilateral triangle of side $a = 1$ m, three point charges are placed (each of 0.1 C). If this system is supplied energy at the rate of 1 kW, then calculate the time required to move one of charges to the mid-point of the line joining the other two.



a. 50 h b. 60 h c. 48 h d. 54 h

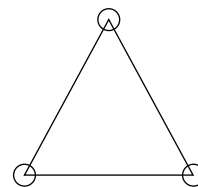
31. If Q charge is given to a sphere of radius R , the energy of the system is

a. $\frac{Q^2}{8\pi\epsilon_0 R}$ b. $\frac{Q}{4\pi\epsilon_0 R}$
c. $\frac{Q^2}{15\pi\epsilon_0 R}$ d. None of these

32. Two balls with charges $5 \mu\text{C}$ and $10 \mu\text{C}$ are at a distance of 1 m from each other. In order to reduce the distance between them to 0.5 m, the amount of work to be performed is

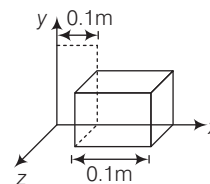
a. 45 J b. 0.45×10^{-6} J
c. 1.2×10^{-4} J d. 0.45 J

33. Three small conducting spheres each of radius a and charge q is placed at the corners of an equilateral triangle of side length l . The side length l is considerably larger than dimensions of the spheres. Find the electrical potential energy of system.



a. $\frac{3q^2}{4\pi\epsilon_0} \left(\frac{1}{2a} + \frac{1}{l} \right)$ b. $\frac{3q^2}{4\pi\epsilon_0 l}$
c. $\frac{3q^2}{8\pi\epsilon_0 a}$ d. None of these

34. Due to a charge inside a cube, the electric field is $E_x = 600 x^{1/2}$, $E_y = 0$, $E_z = 0$. The charge inside the cube is



a. $600 \mu\text{C}$ b. $60 \mu\text{C}$
c. $7 \times 10^{-6} \mu\text{C}$ d. $6 \mu\text{C}$

35. What is the electric field intensity at a point at a distance 20 cm on line making an angle of 45° with the axis of the dipole of moment 10 C-m ?

a. $1.77 \times 10^{13} \text{ V/m}$ b. $0.177 \times 10^{13} \text{ V/m}$
c. $17.7 \times 10^{13} \text{ V/m}$ d. $177 \times 10^{13} \text{ V/m}$

36. What is the electric potential at a point distance 100 cm from the centre of an electric dipole of moment $2 \times 10^{-4} \text{ C-m}$ on a line making an angle of 60° ?

a. $7 \times 10^5 \text{ V}$ b. $8 \times 10^5 \text{ V}$
c. $9 \times 10^5 \text{ V}$ d. $10 \times 10^5 \text{ V}$

37. Two point charges $q_1 = -10 \times 10^{-6} \text{ C}$ and $q_2 = 15 \times 10^{-6} \text{ C}$ are 40 cm apart as shown in figure. Find the potential difference between the points P and Q .



a. $-945 \times 10^3 \text{ V}$ b. $-1000 \times 10^3 \text{ V}$
c. $-880 \times 10^3 \text{ V}$ d. None of these

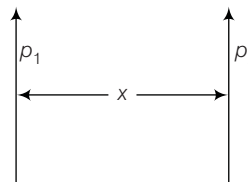
38. An electric dipole made up of a positive and negative charge, each of 1 mC and placed at a distance 2 cm apart. If the dipole is placed in an electric field of 10^5 N/C , then calculate the maximum torque which the field can exert on the dipole, if it is turn from a position $\theta = 0^\circ$ to $\theta = 180^\circ$.

a. $2 \times 10^{-3} \text{ N-m}$ b. $3 \times 10^{-3} \text{ N-m}$
c. $4 \times 10^{-3} \text{ N-m}$ d. $2.8 \times 10^{-3} \text{ N-m}$

39. What work must be done to rotate an electric dipole through an angle θ with the electric field, if an electric dipole of moment p is placed in an uniform electric field E with p parallel to E ?

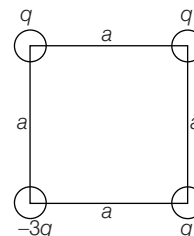
a. $W = pE(1 - \cos\theta)$ b. $W = pE(1 + \cos\theta)$
c. $E = 2pE(1 - \cos\theta)$ d. None of these

40. Find the force of interaction of two dipoles, if the two dipole moments are parallel to each other and placed at a distance x apart.



a. $\frac{3p_1p_2}{4\pi\epsilon_0x^4}$ b. $\frac{p_1p_2}{4\pi\epsilon_0x^4}$ c. $\frac{p_1p_2}{4\pi\epsilon_0x^4}$ d. $\frac{p_1p_2}{3\pi\epsilon_0x^4}$

41. Electric dipole moment of combination shown in the figure is



a. $qa + qa\sqrt{2} + qa$ b. $2\sqrt{2}qa$
c. $\sqrt{2}qa$ d. $(\sqrt{2} + 1)qa$

BITSAT Archives

1. Electric field in a region is given by $\mathbf{E} = \left(\frac{M}{x^3}\right)\hat{i}$, then

the correct expression for the potential in the region is (assume potential at infinity is zero) [2014]

a. $\frac{M}{2x^2}$ b. Mx^2 c. $\frac{M}{3x^4}$ d. None

2. A charge $(-q)$ and another charge $(+Q)$ are kept at two points A and B respectively, keeping the charge $(+Q)$ fixed at B , the charge $(-q)$ at A is moved to another points C , such that ABC forms an equilateral triangle of side l . The net work done in moving the charge $(-q)$ is [2009]

a. $\frac{1}{4\pi\epsilon_0} \frac{Qq}{l}$ b. $\frac{1}{4\pi\epsilon_0} \frac{Qq}{l^2}$
c. $\frac{1}{4\pi\epsilon_0} Qql$ d. zero

3. Two charges $+q$ and $-q$ are kept apart. Then, at any point on the right bisector of line joining the two charges [2008]

a. the electric field strength is zero
b. the electric potential is zero
c. both electric potential and electric field strength are zero
d. None of the above

4. The work done in carrying a charge q once round a circle of radius r with a charge Q at the centre is [2005]

a. $\frac{qQ}{4\pi\epsilon_0r}$
b. $\frac{qQ}{4\pi\epsilon_0^2r}$
c. $\frac{qQ}{4\pi\epsilon_0r^2}$
d. None of the above

Answer with Solutions

Practice Exercise

1. (a) The electric flux,

$$\begin{aligned}\phi &= \Sigma \mathbf{E} \cdot d\mathbf{S} = \mathbf{E} \cdot \mathbf{S} \\ &= (2\hat{i} + 4\hat{j} + 7\hat{k}) \cdot (10\hat{j}) = 40 \text{ units}\end{aligned}$$

2. (c) Here, $\mathbf{E} = E_0 \mathbf{r}$

$$\mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\therefore \mathbf{E} = E_0 x\hat{i} + E_0 y\hat{j} + E_0 z\hat{k}$$

ϕ_0 = the electric flux per unit volume

$$= \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

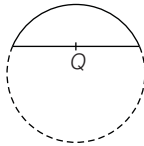
$$\phi_0 = E_0 + E_0 + E_0 = 3E_0$$

$$\therefore \text{Net flux} = \phi_0 \times \text{volume} = \phi_0 \times \pi R^2 h = 3E_0 \pi R^2 h$$

3. (c) Total flux passing through whole sphere

$$= \frac{Q}{\epsilon_0} = \text{half flux from curved surface}$$

+ half flux from plane surface



$$\therefore \text{The flux passing through plane surface} = \frac{Q}{2\epsilon_0}$$

4. (a) Net electric flux = $\frac{Q}{\epsilon_0}$

= half flux through curved surface

+ half flux through plane surface

Ratio = 1 : 1

5. (d) The flux linked with each face will be one-eighth of the flux $\frac{q}{\epsilon_0}$.

$$\text{Flux associated with given cube} = \frac{q}{8\epsilon_0}$$

6. (c)

7. (a)

8. (a) The net electric field at P is zero.

$$\therefore \mathbf{E}_P = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_{\text{sphere}} \text{ or } 0 = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_{\text{sphere}}$$

$$\therefore \mathbf{E}_{\text{sphere}} = -(\mathbf{E}_1 + \mathbf{E}_2) \therefore E_{\text{sphere}} = \sqrt{E_1^2 + E_2^2}$$

$$= \sqrt{\left(\frac{q_1}{4\pi\epsilon_0 a^2}\right)^2 + \left(\frac{q_2}{4\pi\epsilon_0 b^2}\right)^2} = \frac{1}{4\pi\epsilon_0} \sqrt{\left(\frac{q_1}{a^2}\right)^2 + \left(\frac{q_2}{b^2}\right)^2}$$

9. (c)

10. (b)

11. (a)

12. (a) Work done = qV

where, V = change in potential

V_0 = potential at the centre of the ring

$$V_0 = \frac{1}{4\pi\epsilon_0 R} \int dq = \frac{q}{4\pi\epsilon_0 R}, V_\infty = 0$$

$$W = q'(V_0 - V_\infty) = \frac{qq'}{4\pi\epsilon_0 R}$$

13. (a) Here, $q_1 = 3 \times 10^{-9} \text{ C}$, $V_1 = 500 \text{ V}$

$$V_1 = \frac{q_1}{4\pi\epsilon_0 r_1} \Rightarrow r_1 = \frac{q_1}{4\pi\epsilon_0 V_1} = \frac{3 \times 10^{-9} \times 9 \times 10^9}{500} = \frac{27}{500} \text{ m}$$

Let radius of bigger drop is R .

$$\therefore \frac{4}{3}\pi R^3 = 2 \times \frac{4}{3}\pi r_1^3 \therefore R = r_1 2^{1/3} \therefore V = \frac{2q_1}{4\pi\epsilon_0 R} = 794 \text{ V}$$

14. (a) Let $R = 6.4 \times 10^6 \text{ m}$

σ = surface charge density

Total charge on the surface of earth = q

$$q = 4\pi R^2 \sigma$$

Then, the potential V at a point on the earth's surface is same as if the entire charge q were concentrated at its centre. Thus,

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{R} = \frac{1}{4\pi\epsilon_0} \frac{4\pi R^2 \sigma}{R} = \frac{R\sigma}{\epsilon_0}$$

$$V = \frac{(6.4 \times 10^6)(-16 \times 10^{-19})}{(8.9 \times 10^{-12})} = -0.115 \text{ V}$$

15. (c) $\therefore s = \frac{1}{2}at^2$

$$\text{or } 10 \times 10^{-2} = \frac{1}{2}a \times (10^{-7})^2$$

$$\therefore a = \frac{10 \times 10^{-2} \times 2}{10^{-14}} = 2 \times 10^{13} \text{ m/s}^2$$

$$\therefore F = ma \text{ or } eE = ma$$

$$\therefore E = \frac{ma}{e} = \frac{9.1 \times 10^{-31} \times 2 \times 10^{13}}{1.6 \times 10^{-19}}$$

But uniform electric field,

$$E = \frac{\text{potential difference}}{\text{distance between points}}$$

$\therefore$ Potential difference, $V = Es$

$$= \frac{9.1 \times 10^{-31} \times 2 \times 10^{13} \times 10 \times 10^{-2}}{1.6 \times 10^{-19}} = 11.4 \text{ V}$$

16. (c)

17. (a) Let V_1 = potential at the centre of the cube due to one charge

$$V_1 = \frac{1}{4\pi\epsilon_0} \frac{q}{x}$$

and

$$x = L \frac{\sqrt{3}}{2}$$

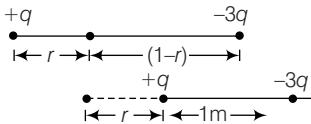
Potential due to 8 charges,

$$\Rightarrow V = 8V_1 \Rightarrow V = 8 \left[\frac{1}{4\pi\epsilon_0} \frac{q}{\frac{\sqrt{3}}{2} \times 10 \times 10^{-2}} \right]$$

$$V = \frac{16q}{4\pi\epsilon_0 \sqrt{3} \times 10 \times 10^{-2}} = 83.14 \times 10^{11} \text{ V}$$

18. (b)

19. (a) Let P is the point on the axis where potential is zero.



$$V_P = \frac{q}{4\pi\epsilon_0 r} - \frac{3q}{4\pi\epsilon_0 (1-r)} = 0 \Rightarrow r = 0.25 \text{ m}$$

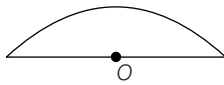
$$V_{P'} = \frac{q}{4\pi\epsilon_0 r} - \frac{3q}{4\pi\epsilon_0 (1+r)} = 0 \Rightarrow r = 0.5 \text{ m}$$

20. (b) Here, $-\int_{V_1}^{V_2} dV = \int_{r_1}^{r_2} E dr$

$$\therefore V_1 - V_2 = \int_{r_1}^{r_2} \frac{\sigma}{2\pi\epsilon_0 r} dr \therefore V_1 - V_2 = \frac{\sigma}{2\pi\epsilon_0} \ln \frac{r_2}{r_1}$$

Hence, potential difference is proportional to $\ln \log \frac{r_2}{r_1}$.

21. (c) In the case of circular arc (uniformly or non-uniformly charge), the potential at centre is



$$V = \frac{\Sigma q}{4\pi\epsilon_0 R}$$

Here,

$$\Sigma q = \lambda \pi R \therefore V = \frac{\lambda \pi R}{4\pi\epsilon_0 R} = \frac{\lambda}{4\epsilon_0}$$

22. (a) $QV = \frac{1}{2}mv^2 \therefore V = \frac{1}{2} \frac{mv^2}{Q}$

$$V = \frac{9 \times 10^{-28} \times (10^9)^2}{2 \times 4.8 \times 10^{-10}} = 0.94 \text{ stat volt}$$

23. (b) Let the surface charge density is σ .

$$\therefore Q = q_A + q_B$$

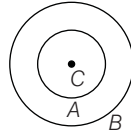
$$\text{or } Q = 4\pi r^2 \sigma + 4\pi R^2 \sigma \therefore \sigma = \frac{Q}{4\pi(r^2 + R^2)}$$

$$\therefore V_C = V_A + V_B \therefore V_C = \frac{q_A}{4\pi\epsilon_0 r} + \frac{q_B}{4\pi\epsilon_0 R}$$

$$\frac{1}{4\pi\epsilon_0} \left[\frac{q_A}{r} + \frac{q_B}{R} \right] = k \left(\frac{4\pi r^2 \sigma}{r} + \frac{4\pi R^2 \sigma}{R} \right) = k(4\pi r \sigma + 4\pi R \sigma)$$

$$= 4\pi k \left[\frac{Qr}{4\pi(R^2 + r^2)} + \frac{QR}{4\pi(r^2 + R^2)} \right] = \frac{kQ(R+r)}{(R^2 + r^2)}$$

(Here, $k = \frac{1}{4\pi\epsilon_0}$)



24. (b)

25. (c) Assume a drop to be perfect sphere of radius r .

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

The charge of n drops,

$$Q_0 = nQ = 8 \times 10^{-10} \text{ C}$$

Let radius of bigger drop = R

$$n \left(\frac{4}{3} \pi r^3 \rho \right) = \frac{4}{3} \pi R^3 \rho \Rightarrow R = r(n)^{1/3} \quad (\because n = 8)$$

$$R = 2 \times 10^{-3} \text{ m}$$

$$\text{Hence, } V = \left(\frac{9 \times 10^9 \times 8 \times 10^{-10}}{2 \times 10^{-3}} \right) = 3600 \text{ V}$$

26. (b) 27. (a)

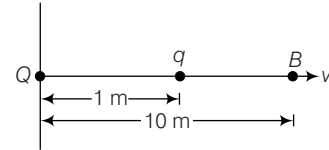
28. (a) For minimum potential energy, the largest charge should be farthest. Thus, the potential energy,

$$4\pi\epsilon_0 U = \frac{2Q^2}{x} + \frac{8Q^2}{(R-x)} + \frac{16Q^2}{R}$$

For minimum, $\frac{dU}{dx} = 0$

$$4\pi\epsilon_0 \frac{dU}{dx} = -\frac{2Q^2}{x^2} + \frac{8Q^2}{(R-x)^2} + 0 = 0 \Rightarrow x = \frac{R}{3}$$

29. (b) According to conservation principle of energy.



$$U_i + K_i = U_f + K_f$$

$$\frac{Qq}{4\pi\epsilon_0 r_1} + 0 = \frac{Qq}{4\pi\epsilon_0 r_2} + \frac{1}{2}mv^2 \quad \dots(i)$$

Here, $r_1 = 1 \text{ m}$, $r_2 = 10 \text{ m}$,

$$q = 10^{-6} \text{ C}, m = 2 \times 10^{-3} \text{ kg}, Q = 10^{-3} \text{ C}$$

Putting the values in Eq. (i), we get

$$\frac{10^{-3} \times 10^{-6} \times 9 \times 10^9}{1} - \frac{10^{-3} \times 10^{-6} \times 9 \times 10^9}{10} = \frac{1}{2} \times 2 \times 10^{-3} \times v^2$$

$$\Rightarrow v = 90 \text{ m/s}$$

30. (a) U_i = initial potential energy of the system

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{q^2}{a} + \frac{q^2}{a} + \frac{q^2}{a} \right] = \frac{1}{4\pi\epsilon_0} \left(\frac{3q^2}{a} \right)$$

$$= 9 \times 10^9 \left(3 \times \frac{(0.1)^2}{1} \right) = 27 \times 10^7 \text{ J}$$

Let charge at A is moved to mid-point D.

$$U_f = \frac{1}{4\pi\epsilon_0} \left[\frac{2q^2}{(a/2)} + \frac{q^2}{a} \right] = 5 \times \frac{1}{4\pi\epsilon_0} \left(\frac{q^2}{a} \right) = 45 \times 10^7 \text{ J}$$

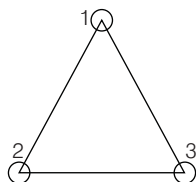
$$\text{Work done} = U_f - U_i = 18 \times 10^7 \text{ J}$$

$$\text{Also, energy supplied per second} = 1000 \text{ J}$$

$$\text{Time taken} = \frac{18 \times 10^7}{1000} = 18 \times 10^4 \text{ s} = 50 \text{ h}$$

31. (a) 32. (d)

33. (a) $U = U_1 + U_2 + U_3 + U_{12} + U_{13} + U_{23}$



$$U_1 = \text{self potential energy of first sphere} = \frac{q^2}{8\pi\epsilon_0 a} = U_2 = U_3$$

$$U_{12} = \text{interaction energy of first sphere and second sphere}$$

$$= \frac{q^2}{4\pi\epsilon_0 l} = U_{13} = U_{23}$$

$$\therefore U = \frac{3q^2}{4\pi\epsilon_0} \left(\frac{1}{2a} + \frac{1}{l} \right)$$

34. (c) $\phi_{\text{net}} = \frac{Q_{\text{en}}}{\epsilon_0}$, $Q_{\text{en}} = \phi_{\text{net}} \times \epsilon_0$

$$= 600 [(0.2)^{1/2} - (0.1)^{1/2}] (0.1)^2 \times 8.85 \times 10^{-12}$$

$$= 7 \times 10^{-12} \text{ C}$$

35. (a) 36. (c)

37. (a) Potential at P

$$V_P = \frac{1}{4\pi\epsilon_0} \left[\frac{q_2}{r_2} - \frac{q_1}{r_1} \right]$$

$$= 9 \times 10^9 \left(\frac{15 \times 10^{-6}}{0.2} - \frac{10 \times 10^{-6}}{0.2} \right) = 225 \times 10^3 \text{ V}$$

Potential at Q

$$V_Q = \frac{1}{4\pi\epsilon_0} \left[\frac{q_2}{r_2} - \frac{q_1}{r_1} \right] = 9 \times 10^9 \left[\frac{15 \times 10^{-6}}{0.1} - \frac{10 \times 10^{-6}}{0.5} \right]$$

$$= 117 \times 10^4 \text{ V} \therefore V_P - V_Q = -945 \times 10^3 \text{ V}$$

38. (a) $\tau = pE \sin \theta$ and τ is a maximum, when $\theta = 90^\circ$

$$\therefore \tau_{\text{max}} = pE \text{ and } P = (q)(2l) = (1 \times 10^{-6})(0.02)$$

$$E = 10^5 \text{ N/C}$$

$$\tau_{\text{max}} = (1 \times 10^{-6} \times 0.02) \times 10^5 = 2 \times 10^{-3} \text{ N-m}$$

39. (a) The initial potential energy $= U_i = -pE$

$$\text{The final potential energy} = U_f = -pE \cos \theta$$

But $W_{\text{ext}} = \text{change in potential energy.}$

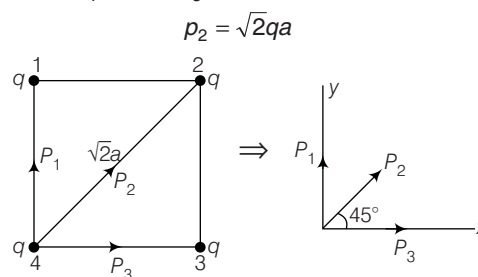
$$W_{\text{ext}} = \Delta U = U_f - U_i$$

$$W_{\text{ext}} = -pE \cos \theta - (-pE)$$

$$W_{\text{ext}} = pE(1 - \cos \theta)$$

40. (a)

41. (b) Here, $p_1 = qa = p_3$



From vector diagram shown in figure,

$$p_x = p_3 + p_2 \cos 45^\circ$$

$$= qa + \sqrt{2}qa \frac{1}{\sqrt{2}} = 2qa$$

$$p_y = p_1 + p_2 \cos 45^\circ = 2qa$$

$$\therefore p = \sqrt{p_x^2 + p_y^2} = 2\sqrt{2}qa$$

$$\Sigma \mathbf{p} = 0$$

BITSAT Archives

1. (a) $V(x, y, z) = - \int_{\infty}^{(x, y, z)} \mathbf{E} \cdot d\mathbf{r}$

$$= - \int_{\infty}^{(x, y, z)} \frac{Mdx}{x^3} = \frac{M}{2x^2}$$

2. (d) Net work done = Final potential energy – Initial potential energy

$$= \frac{Qq}{4\pi\epsilon_0 l} - \frac{Qq}{4\pi\epsilon_0 l} = 0$$

3. (b) At equatorial point,

$$E_e = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$$

(directed from + q to – q) and $V_e = 0$.

4. (d) The potential at each point on the circular path will be equal.

So, work done = q × potential difference

$$= q \times 0 = 0$$

23

Electric Capacitor

Capacitor

Capacitor is a device that stores electrical energy. It consists of conductors of any shape and size carrying charges of equal magnitudes and opposite signs and separated by an insulating medium.

Capacity of a Capacitor

Ability to store charge by a conductor is called capacity of the capacitor.

The amount of charge required to raise the potential of the capacitor by one unit (in SI one volt) is called capacitance of the capacitor.

If q = amount of charge given and V = potential of capacitor, then capacitance of the capacitor is given by

$$C = \frac{q}{V}$$

SI unit of capacitance is farad or F, 1 farad = 1 coulomb/volt

Dimensions of capacitance = $[M^{-1}L^{-2}T^4A^2]$

Capacitance is a scalar quantity.

Parallel Plate Capacitor (without Dielectric)

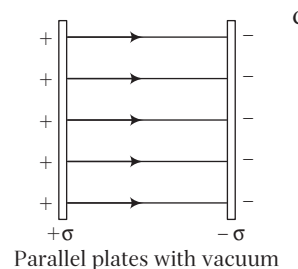
Most commonly used capacitor is parallel plate capacitor, which consists of two charged parallel plates separated by a small distance. Capacity of a parallel plate capacitor with vacuum between the plates is given by

$$C = \frac{\epsilon A}{d} = \frac{\epsilon_0 K A}{d} \text{ farad}$$

or
$$C_{\text{vacuum}} = \frac{\epsilon_0 A}{d} \quad (\because K = 1, \text{ for vacuum})$$

where, K = dielectric constant of the medium between the plates

A = area of plates and d = distance between the plates



- (i) If a dielectric slab fills completely the gap between the plates, then capacitance increases by K times.

i.e. $C' = \frac{K\epsilon_0 A}{d}$

$\Rightarrow C' = KC$

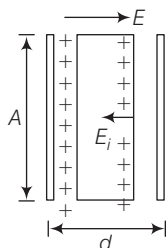
where, K = dielectric constant.

- (ii) If a dielectric slab of thickness t ($t < d$) is inserted between the plates as shown, then E = electric field between the plates, E_i = induced electric field in dielectric. $E' = (E - E_i)$, the reduced value of electric field in the dielectric.

Now, capacitance of the capacitor

$$C' = \frac{Q}{V'}$$

$$\Rightarrow C' = \frac{\epsilon_0 A}{(d - t) + \frac{t}{K}}$$



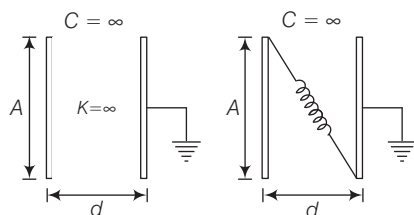
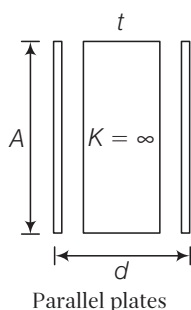
NOTE When dielectric is inserted between the plates, voltage and electric field reduces by a factor of K between the plates.

- (iii) As dielectric constant of metal is infinite.

Hence, field inside the slab does not exist. Capacitance,

$$C' = \frac{\epsilon_0 A}{(d - t)}, \text{ in case of partial}$$

filling and in case of complete filling, with metallic slab $C' = \infty$, capacitor is said to be short circuited as shown in the figures below.



NOTE (i) The capacity of a spherical capacitor is given below

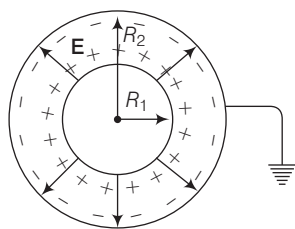
$$C = \frac{4\pi\epsilon_0 R_1 R_2}{R_2 - R_1}$$

where, R_1 is radius of inner spherical conductor and R_2 is radius of outer spherical conductor.

- (ii) Capacity of an isolated spherical capacitor.

When a smaller conductor is shielded by a conducting shell, then capacity of the combination depends upon radius of shell and is given by $C = 4\pi\epsilon_0 R$

where, R is radius of the shell.



Combination of Capacitors

- (i) Capacitors connected in series

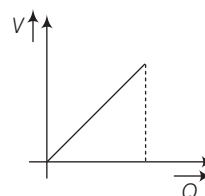
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

- (ii) Capacitors connected in parallel

$$C = C_1 + C_2$$

Energy of a Charged Conductor/Capacitor

When a conductor/capacitor is charged, its potential increases from 0 to V and work is done against repulsion, between charge stored in the conductor and charge coming from the source (battery). This work is stored as electrostatic potential energy. The graph of rise of potential of the capacitor with charge is shown as ($Q \propto V$).



Work done = Area of (Q - V) graph

$$= \frac{1}{2} QV$$

Hence, potential energy, $U = \frac{1}{2} QV$

By using $Q = CV$, we can write

$$U = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{Q^2}{2C}$$

The energy is stored in electric field between plates of capacitor, which depends on capacitance, charge and potential difference. This energy is obtained at cost of chemical energy of battery.

Energy Density

The energy stored in a capacitor is not localised on the charges or the plates but is distributed in the field. In case of a parallel plate capacitor, electric field is only between the

plates in a volume ($A \times d$), which is so called **energy density**.

$$\begin{aligned} \text{Hence, energy density} &= \frac{\text{energy}}{\text{volume}} = \frac{\frac{1}{2}CV^2}{Ad} \\ &= \frac{1}{2} \left[\frac{\epsilon_0 A}{d} \right] \frac{V^2}{Ad} \quad \left[\because C = \frac{\epsilon_0 A}{d} \right] \\ &= \frac{1}{2} \epsilon_0 \left(\frac{V}{d} \right)^2 = \frac{1}{2} \epsilon_0 E^2 \end{aligned}$$

Energy density is the property of electric field and above formula is valid in all cases. The unit of energy density is J/m^3 .

Particular Cases of Capacitors

Force between Plates of Capacitor

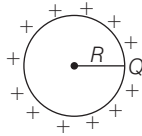
The two plates of capacitor attract each other, because they are oppositely charged.

$$\text{Electric field due to positive plate, } E = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2\epsilon_0 A}$$

$$\text{Force on negative charge } -Q, F = -QE = -\frac{Q^2}{2\epsilon_0 A}$$

$$\text{Magnitude of force, } F = \frac{Q^2}{2\epsilon_0 A}$$

Capacity of an Isolated Sphere

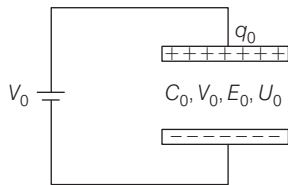


$$V_{\text{surface}} = \frac{Q}{4\pi\epsilon_0 R}; Q = 4\pi\epsilon_0 R(V)$$

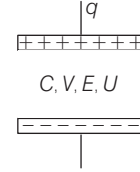
$$\text{So, } C = 4\pi\epsilon_0 R$$

Effect of Dielectric on Charge, Potential Difference, Field and Energy of Capacitor

When a dielectric is introduced between plates, it polarizes and reduces potential difference between plates and so increases its capacity.



If Charge is Held Constant, i.e. Battery is Disconnected After Charging



Charge remains unchanged, $q = q_0$

Capacity increases as $C = KC_0$

PD between the plates decreases as

$$V = \frac{q}{C} = \frac{q_0}{KC_0} = \frac{V_0}{K}$$

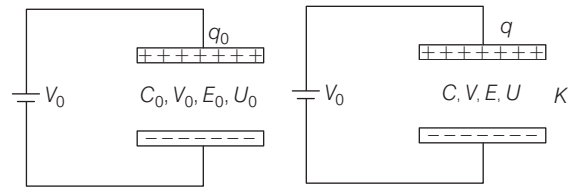
Field between the plates decreases as

$$E = \frac{V}{d} = \frac{V_0}{Kd} = \frac{E_0}{K}$$

Energy stored in the capacitor decreases as

$$U = \frac{q^2}{2C} = \frac{q_0^2}{2KC_0} = \frac{U_0}{K}$$

If Potential is Held Constant, i.e. Battery Remains Attached



PD remains constant, i.e. $V = V_0$, as battery is a source of constant potential difference

Capacity increases as,

$$C = KC_0$$

Charge on capacitor increases as

$$q = CV = (KC_0)V = Kq_0$$

Electric field remains unchanged as

$$E = \left[\frac{V}{d} \right] = \frac{V_0}{d} = E_0$$

Energy stored in the capacitor increases as

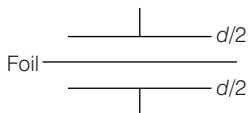
$$\begin{aligned} U &= \frac{1}{2} CV^2 \\ &= \frac{1}{2} KC_0 V_0^2 \\ &= \frac{1}{2} KU_0 \end{aligned}$$

Practice Exercise

1. In which form of the following, the energy is stored in the capacitor?

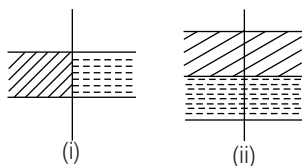
a. Charge b. Magnetic field
c. Capacitance d. Electric potential energy

2. A sheet of aluminium foil of negligible thickness is placed between the plates of a capacitor of capacitance C as shown in the figure, the capacitance of capacitor becomes.



a. $2C$ b. C c. $\frac{C}{2}$ d. zero

3. Calculate the ratio of capacitance of two capacitors filled with dielectrics of same dimensions but of different values K and $\frac{K}{4}$



arranged in two ways as shown in Fig. (i) and (ii).

a. 5 : 2 b. 25 : 16
c. 5 : 4 d. 2 : 5

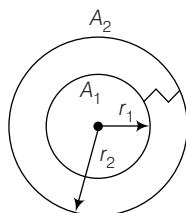
4. A conducting sphere of radius 10 cm is given a charge of $+2 \times 10^{-8} \text{ C}$. What will be its potential?

a. 0.03 kV b. 0.9 kV
c. 1.8 kV d. 3.6 kV

5. If n identical drops of mercury are combined to form a bigger drop then find the capacity of bigger drop, if capacity of each drop of mercury is C .

a. $n^{1/3} C$ b. $n^{2/3} C$
c. $n^{1/4} C$ d. nC

6. Two spherical conductors A_1 and A_2 of radii r_1 and r_2 are placed concentrically in air. The two are connected by a copper wire as shown in figure. Then, the equivalent capacitance of the system is

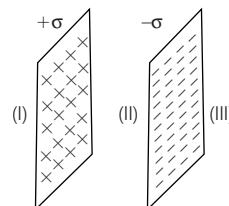


a. $\frac{4\pi \epsilon_0 K r_1 r_2}{r_2 - r_1}$ b. $4\pi \epsilon_0 (r_2 + r_1)$
c. $4\pi \epsilon_0 r_2$ d. $4\pi \epsilon_0 r_1$

7. Calculate amount of charge flow, when a conducting sphere of radius R and carrying a charge Q , is joined to an uncharged conducting sphere of radius $2R$.

a. $\frac{Q}{4}$ b. $\frac{Q}{3}$ c. $\frac{Q}{2}$ d. $\frac{2Q}{3}$

8. Find the electric field in region II as shown in figure.



a. zero b. $\frac{\sigma}{4\pi \epsilon_0}$
c. $\frac{\sigma}{\epsilon_0}$ d. infinite

9. Two capacitors A and B having capacitances $10 \mu\text{F}$ and $20 \mu\text{F}$ are connected in series with a 12 V battery. The ratio of the charge on A and B is

a. 0.5 : 1 b. 1 : 1
c. 2 : 1 d. 2 : 4

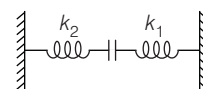
10. A $6 \times 10^{-4} \text{ F}$ parallel plate air capacitor is connected to a 500 V battery. When air is replaced by another dielectric material, $7.5 \times 10^{-4} \text{ C}$ charge flows into the capacitor. The value of the dielectric constant of the material is

a. 1.5 b. 2.0
c. 1.0025 d. 3.5

11. The 90 pF capacitor is connected to a 12 V battery. How many electrons are transferred from one plate to another?

a. 11×10^9 b. 6.7×10^9
c. 4×10^{19} d. 5×10^{19}

12. In the given figure, the capacitor of plate area A is charged upto charge q . Find the ratio of elongation (neglect force of gravity) in spring C and D at equilibrium position.

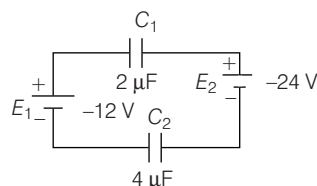


a. $\frac{k_1}{k_2}$ b. $\frac{k_2}{k_1}$
c. $k_1 k_2$ d. None of these

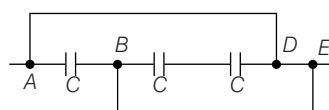
13. A potential difference of 500 V is applied to a parallel plate condenser. The separation between plates is $2 \times 10^{-3} \text{ m}$. The plates of the condenser are vertical. An electron is projected vertically upwards between the plates with a velocity of 10^5 m/s and it moves undeflected between the plates. The magnetic field acting perpendicular to the electric field has a magnitude of

a. 1.5 Wb/m^2 b. 2.0 Wb/m^2
c. 2.5 Wb/m^2 d. 3.0 Wb/m^2

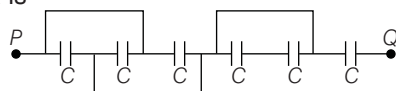
14. Two capacitors C_1 and C_2 are connected in a circuit as shown in figure. The potential difference ($V_A - V_B$) is



- a. 8 V
b. -8 V
c. 12 V
d. 12 V
15. In the given figure, the equivalent capacity between A and B is



- a. $3C$
b. $\frac{C}{3}$
c. $\frac{3}{2C}$
d. infinity
16. For the circuit, the equivalent capacitance between P and Q is



- a. $6C$
b. $4C$
c. $\frac{3C}{2}$
d. $\frac{6C}{11}$
17. A parallel plate capacitor is connected to a battery of constant emf. Let the electric field at a given point between the plate be E_0 , when there is no medium between the plates. The new electric field at the point, if a medium of dielectric constant A is introduced between them, is

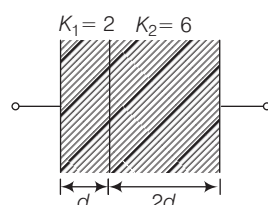
- a. $\frac{E_0}{4}$
b. $\frac{E_0}{2}$
c. E_0
d. $4E_0$
18. Force acting upon a charged particle, kept between the plates of a charged condenser is F . If one of the plates of the condenser is removed, force acting on the same particle will become

- a. 0
b. $\frac{F}{2}$
c. F
d. $2F$

19. A parallel plate capacitor has two layers of dielectrics as shown in figure.

This capacitor is connected across a battery, then the ratio of potential difference across the dielectric layers is

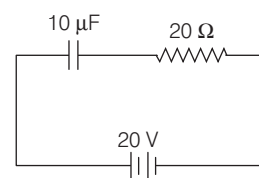
- a. $\frac{4}{3}$
b. $\frac{1}{2}$
c. $\frac{1}{3}$
d. $\frac{3}{2}$



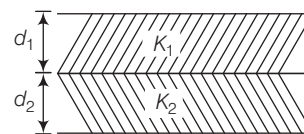
20. A parallel plate capacitor of plate separation d and plate area A is charged to a potential difference V and then the battery is disconnected. To fulfill the space between the plates of the capacitor, a slab of dielectric constant K is inserted. If the magnitude of the charge on each plate, electric field between the plate (after the slab is inserted) and work done on the system in the process of insertion of a slab are Q , E , W respectively, then

- a. $Q = \epsilon_0 \frac{AV}{d}$
b. $W = \epsilon_0 \frac{AV^2}{2d} \left(1 - \frac{1}{K}\right)$
c. $C = \frac{V}{Kd}$
d. All of these

21. A capacitor of capacitance $10 \mu\text{F}$ is charged by connecting through a resistance of 20Ω and battery of 20 V . What is the energy supplied by the battery?



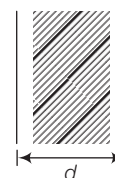
- a. Less than 2 mJ
b. Equal to 2 mJ
c. More than 2 mJ
d. Cannot be predicted
22. A capacitor of capacitance C is charged to a potential difference V_0 . The charged battery is disconnected and the capacitor is connected to a capacitor of unknown capacitance C_x . The potential difference across the combination is V . The value of C_x should be
- a. $\frac{C(V_0 - V)}{V}$
b. $\frac{C(V - V_0)}{V}$
c. $\frac{CV}{V_0}$
d. $\frac{CV_0}{V}$
23. A parallel plate capacitor is made of two dielectric blocks in series. One of the blocks has thickness d_1 and dielectric constant K_1 and the other has thickness d_2 and dielectric constant K_2 as shown in figure. This arrangement can be thought as a dielectric slab of thickness $d (= d_1 + d_2)$ and effective dielectric constant K . Constant K is equal to



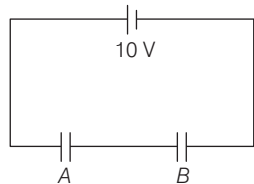
- a. $\frac{K_1 d_1 + K_2 d_2}{d_1 + d_2}$
b. $\frac{K_1 d_1 + K_2 d_2}{K_1 + K_2}$
c. $\frac{K_1 K_2 (d_1 + d_2)}{(K_1 d_1 + K_2 d_2)}$
d. $\frac{2K_1 K_2}{K_1 + K_2}$

24. A copper plate of thickness b is placed inside a parallel plate capacitor of plate distance d and area A as shown in figure. The capacitance of capacitor is

- a. $\frac{A\epsilon_0}{d}$
b. $\frac{A\epsilon_0}{b}$
c. $\frac{A\epsilon_0}{d - b}$
d. ∞

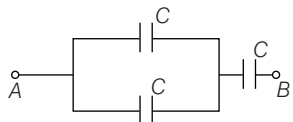


25. For the circuit shown below, capacitors A and B have identical geometry, but a material of dielectric constant 3 is present between the plates of B . The potential difference across A and B are respectively



- a. 2.5 V, 7.5 V b. 2 V, 8 V
c. 7.5 V, 2.5 V d. 8 V, 2 V

26. For the network shown we have three identical capacitors. Each of them can withstand a maximum 100 V potential difference. What maximum voltage can be applied across A and B that no capacitor gets spoiled?

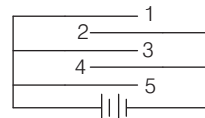


- a. 150 V b. 120 V
c. 180 V d. 0.200 V

27. Calculate the work done against the electric force, if the separation of the capacitor of area S is increased from x_1 to x_2 . Assume charge q on the capacitor is constant.

$$\begin{aligned} \text{a. } W &= \frac{q^2}{\epsilon_0 S} (x_2 - x_1) & \text{b. } W &= \frac{q}{\epsilon_0 S} (x_2 - x_1) \\ \text{c. } W &= \frac{q^2}{2\epsilon_0 S} (x_2 - x_1) & \text{d. } W &= \frac{q^2}{4\epsilon_0 S} (x_2 - x_1) \end{aligned}$$

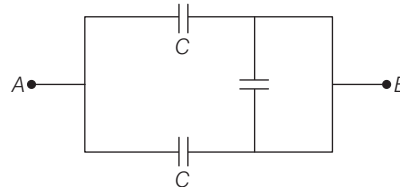
28. Five identical plates are connected across a battery as follows.



If the charge on plate 1 is $+q$, then the charges on the plates 2, 3, 4 and 5 are

- a. $-q, +q, -q, +q$ b. $-2q, +2q, -2q, +q$
c. $-q, +2q, -2q, +q$ d. None of these

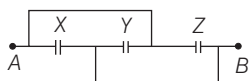
29. The equivalent capacitance of the combination of three capacitors each of capacitance C between A and B as shown in figure is



- a. C b. $2C$ c. $C/2$ d. $3C$

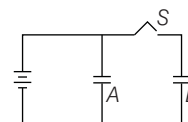
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1. Three capacitors $X = 1\mu\text{F}$, $Y = 2\mu\text{F}$ and $Z = 3\mu\text{F}$ are connected as shown in figure, then the equivalent capacitance between points A and B is [2014]



- a. $6\mu\text{F}$ b. $12\mu\text{F}$
c. $3\mu\text{F}$ d. None of these
2. A capacitor of capacity $0.1\mu\text{F}$ connected in series to a resistor of $10\text{M}\Omega$ is charged to a certain potential and then made to discharge through resistor. The time in which the potential will take to fall to half its original value is (Given, $\log_{10} 2 = 0.3010$) [2012]
- a. 2 s b. 0.693 s
c. 0.5 s d. 1.0 s
3. The work done in placing a charge of $8 \times 10^{-18}\text{C}$ on a capacitor of capacity $100\mu\text{F}$ is [2011]
- a. $32 \times 10^{-32}\text{J}$ b. $16 \times 10^{-32}\text{J}$
c. $3.1 \times 10^{-26}\text{J}$ d. $4 \times 10^{-10}\text{J}$
4. Two identical air filled parallel plate capacitors are charged to the same potential in the manner shown by closing the switch S . If now the switch S is opened and

the space between the plates is filled with a dielectric of relative permittivity ϵ_r , then [2009]



- a. the potential difference as well as charge on each capacitor goes up by a factor ϵ_r
b. the potential difference as well as the charge on each capacitor goes down by a factor ϵ_r
c. the potential difference across A remains constant and the charge on B remains unchanged
d. the potential difference across B remains constant, while the charge on A remains unchanged
5. Which of the following is discontinuous across a charged conducting surface? [2009]
- a. Electric potential
b. Electric intensity
c. Both electric potential and intensity
d. None of the above
6. Capacitance of a capacitor made by a thin metal foil is $2\mu\text{F}$. If the foil is folded with paper of thickness 0.15 mm, dielectric constant of paper is 2.5 and width of paper is 400 mm, the length of foil will be [2007]
- a. 0.34 m b. 1.33 m c. 13.4 m d. 33.9 m

Answer with Solutions

Practice Exercise

1. (d)

$$2. (b) \frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_1 = \frac{\epsilon_0 A}{d/2}, C_2 = \frac{\epsilon_0 A}{d/2}$$

$$\therefore \frac{1}{C'} = \frac{d}{\epsilon_0 A} \therefore C' = \frac{\epsilon_0 A}{d} = C$$

$$3. (b) \text{ Here, } C_1 = \frac{\epsilon_0 K_1 A}{2d} + \frac{\epsilon_0 K_2 A}{2d}$$

$$= \frac{\epsilon_0 K A}{2d} + \frac{\epsilon_0 K A}{8d} \quad \left(\begin{array}{l} \text{Given, } K_1 = K \\ \text{and } K_2 = \frac{K}{4} \end{array} \right)$$

$$= \frac{\epsilon_0 K A}{d} \left(\frac{1}{2} + \frac{1}{8} \right) = \frac{\epsilon_0 K A}{d} \left(\frac{4+1}{8} \right)$$

$$\Rightarrow C_1 = \frac{5\epsilon_0 K A}{8d} \quad \dots(i)$$

$$\text{and } C_2 = \frac{\epsilon_0 A}{\frac{d}{2K_1} + \frac{d}{2K_2}} = \frac{\epsilon_0 A}{\frac{d}{2K} + \frac{d}{2\frac{K}{4}}} = \frac{(2K)\epsilon_0 A}{5d} \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{C_1}{C_2} = \frac{\frac{5\epsilon_0 K A}{8d}}{\frac{(2K)\epsilon_0 A}{5d}} = \frac{5 \times 5}{2 \times 8} = \frac{25}{16}$$

4. (c) 5. (a) 6. (c)

7. (d) Capacitance of first sphere, $C_1 = 4\pi \epsilon_0 R$

Capacitance of second sphere

$$C_2 = 4\pi \epsilon_0 (2R) = 2C_1$$

Then, when they are in contact with the common potential between them

$$= V = \frac{Q}{C_1 + C_2}$$

$$\text{Charge on } C_2 = C_2 V = \frac{Q C_2}{C_1 + C_2} = \frac{2}{3} Q$$

8. (c)

9. (b)

10. (c) $q_0 = C_0 V_0$

$$q = C V_0$$

$$\therefore \Delta q = q - q_0 = (C - C_0) V_0$$

$$\text{or } \Delta q = (\epsilon_r C_0 - C_0) V_0 \text{ or } \Delta q = (\epsilon_r - 1) C_0 V_0 \quad \dots(i)$$

$$\text{Here, } \Delta q = 7.5 \times 10^{-4} \text{ C}$$

$$C_0 = 6 \times 10^{-4} \text{ F}$$

$$V_0 = 500 \text{ V}$$

Putting the value in Eq. (i), we get $\epsilon_r = 1.0025$

11. (b)

12. (b)

13. (c) In this case, electric force is balanced by magnetic force.

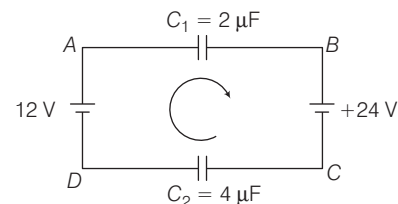
$$\therefore qE = qvB \therefore E = vB \therefore B = \frac{E}{v} = \frac{V_0}{vd} \quad \left(\because E = \frac{V_0}{d} \right)$$

$$= \frac{500}{2 \times 10^{-3} \times 10^5} = 2.5 \text{ Wb/m}^2$$

14. (b) Consider the diagram below going through the path shown

$$12 - V_A + V_B - 24 + V_D - V_C = 0$$

$$\Rightarrow V_A - V_B = 12 + V_D - V_C - 24 = -12 + \left(\frac{Q}{4\mu\text{F}} \right)$$



$$\Rightarrow \frac{-Q}{2\mu\text{F}} = -12 + \frac{Q}{4\mu\text{F}} \Rightarrow Q \left[\frac{1}{2} + \frac{1}{4} \right] = 12\mu\text{C}$$

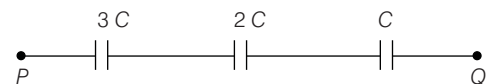
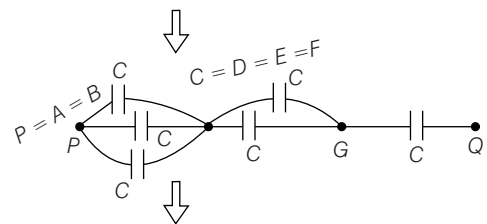
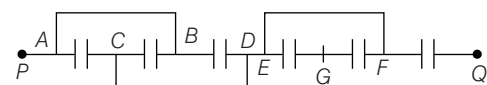
$$\Rightarrow Q = \frac{12 \times 8}{6} \mu\text{C} = 16\mu\text{C}$$

$$\Rightarrow V_A - V_B = \frac{-Q}{C_1} = \frac{-16\mu\text{C}}{2\mu\text{F}} = -8 \text{ V}$$

15. (d) From figure, $V_A = V_D = V_E = V_B$

$$V_A - V_B = 0 \quad C = \frac{q}{V_A - V_B} = \frac{q}{0} = \infty$$

16. (d)

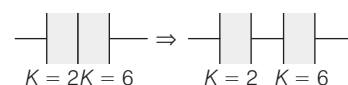


$$\therefore \frac{1}{C_{\text{eq}}} = \frac{1}{3C} + \frac{1}{2C} + \frac{1}{C} = \frac{2+3+6}{6C} = \frac{11}{6C} \therefore C_{\text{eq}} = \frac{6C}{11}$$

17. (c)

18. (b)

19. (d) As it is a series combination, so charge on each capacitor is same.



So, $V_1 = \frac{Q}{C_1}$ and $V_2 = \frac{Q}{C_2}$

$$\Rightarrow \frac{V_1}{V_2} = \frac{C_2}{C_1} = \frac{\epsilon_0 K_2 \frac{A}{2d}}{\epsilon_0 K_1 \frac{A}{d}} = \frac{6}{2 \times 2} = \frac{3}{2}$$

20. (b) When battery is disconnected, then charge remains the same.

$$Q = CV = \frac{\epsilon_0 AV}{d}$$

Potential is reduced by factor of K and electric field also.

$$E = \frac{V}{Kd}$$

Energy before dielectric material is introduced

$$U_1 = \frac{1}{2} \frac{Q^2}{C} \Rightarrow U_1 = \frac{1}{2} \frac{\epsilon_0^2 A^2 V^2}{d^2} \cdot \frac{d}{\epsilon_0 KA} = \frac{\epsilon_0 AV^2}{2d} \quad \dots(i)$$

Energy after dielectric introduced

$$U_2 = \frac{1}{2} \frac{Q^2}{C'}$$

$$\text{Here, } C' = \frac{\epsilon_0 KA}{d} \Rightarrow U_2 = \frac{1}{2} \frac{\epsilon_0^2 A^2 V^2}{2d^2} \cdot \frac{d}{\epsilon_0 KA} = \frac{\epsilon_0 AV^2}{2dK} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$U_1 - U_2 = \frac{\epsilon_0 AV^2}{2d} \left(1 - \frac{1}{K} \right)$$

21. (c)

22. (a) The initial charge on first capacitor is $Q = CV_0$

After connection, voltages of both capacitors become same, i.e. V

$$\therefore Q = q_1 + q_2 \text{ or } CV_0 = CV + C_x V$$

$$V = \frac{CV_0}{C + C_x} \therefore C_x = \frac{C(V_0 - V)}{V}$$

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1. (a) The equivalent circuit of the following figure is as follow.

$$\therefore C_{eq} = X + Y + Z = 1 + 2 + 3 = 6 \mu F$$

2. (b) By equation of charging,

$$q = q_0(1 - e^{-t/CR})$$

According to question,

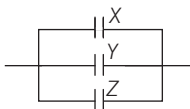
$$\frac{q}{q_0} = \frac{1}{2} = 0.50 \therefore 0.50 = 1 - e^{-t/CR}$$

$$e^{-t/CR} = 1 - 0.50 = 0.50 \Rightarrow e^{t/CR} = 2$$

$$\text{or } \frac{t}{CR} = \log_e 2 \text{ or } \frac{t}{CR} = 2.3026 \log_{10} 2$$

$$\text{or } t = CR \times 2.3026 \log_{10} 2$$

$$\text{or } t = 0.1 \times 10^{-6} \times 10 \times 10^6 \times 2.3026 \log_{10} 2$$



23. (c) 24. (d)

25. (c) If C_0 = capacitance with air between plates. Then, $3C_0$ = capacitance with dielectric. As capacitors are in series, equivalent capacitance is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_0} + \frac{1}{3C_0} \Rightarrow C_{eq} = \frac{3}{4} C_0$$

$$\text{Now, } C_{eq} = \frac{Q}{V} \Rightarrow Q = 10 \times \frac{3}{4} C_0$$

Also, charge on each capacitor is same. Charge on each capacitor is $\frac{15}{2} C_0 = Q$

$\therefore$ Potential drop across capacitors is

$$V_A = \frac{Q}{C_A} = \frac{\frac{15}{2} C_0}{C_0} = \frac{15}{2} = 7.5 \text{ V}$$

$$V_B = \frac{Q}{C_B} = \frac{\frac{15}{2} C_0}{3C_0} = \frac{5}{2} = 2.5 \text{ V}$$

26. (a) 27. (c)

28. (b) As plates are identical.

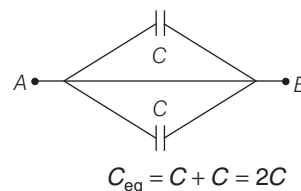
Charge on plate 5 is $+q$.

Charge on plate 3 is $+2q = +q + q$.

Charge on plate 2 is $-2q = -q - q$.

Charge on plate 4 is $-2q = -q - q$.

29. (b) Third capacitor is short-circuited as its both ends are connected to B . Equivalent circuit is



$$\text{or } t = 2.3026 \times 0.3010 \text{ or } t = 0.693 \text{ s}$$

$$3. (a) \text{ Work done, } W = \frac{q^2}{2C} = \frac{(8 \times 10^{-18})^2}{2 \times 100 \times 10^{-6}} = 32 \times 10^{-32} \text{ J}$$

4. (c) When switch S is opened, capacitor A is still connected with battery. Therefore, potential difference across A remains constant. As B is disconnected from battery, charge on B remains fixed.

5. (b) Electric intensity inside the conductor is zero.

$$6. (d) \text{ If length of the foil is } l, \text{ then } C = \frac{K\epsilon_0 (l \times b)}{d}$$

$$\Rightarrow 2 \times 10^{-6} = \frac{2.5 \times 8.85 \times 10^{-12} (l \times 400 \times 10^{-3})}{0.01 \times 10^{-3}}$$

$$\Rightarrow l = 339 \text{ m}$$

24

Current Electricity

Electric Current

The time rate of flow of charge through any cross-section of wire per unit time is called current. So, if Δq charge passes through a cross-section in time Δt , then $i_{av} = \frac{\Delta q}{\Delta t}$ and instantaneous current

$i = \lim_{\Delta t \rightarrow 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$. If rate of flow is uniform, then $i = \frac{q}{t}$. Thus, current is a scalar quantity. Its SI unit is ampere (A).

Current Density

In case of flow of charge through a cross-section, current density is defined as a vector having magnitude equal to current per unit area surrounding that point. Remember area is held normal to the direction of charge flow (or current passes) through that point. Current density at point P is given by

$$\mathbf{J} = \frac{di}{dA} \hat{\mathbf{n}}$$

Current density $\mathbf{J}$ is a vector quantity having SI unit A/m^2 and dimension $[\text{L}^{-2}\text{A}]$.

In case of uniform flow of charge through a cross-section normal to it as $i = nqvA$ so, $\mathbf{J} = \frac{i}{A} \hat{\mathbf{n}} = (nqv) \hat{\mathbf{n}}$.

Current density related with electric field is $\mathbf{J} = \sigma \mathbf{E} = \frac{\mathbf{E}}{\rho}$, where σ = conductivity and ρ = resistivity or specific resistance of substance. The direction of current density $\mathbf{J}$ is same as that of electric field $\mathbf{E}$.

Drift Velocity

Drift velocity is the average uniform velocity acquired by free electrons inside a metal by the application of an electric field which is responsible for current through it. Drift velocity is very small, it is of the order of 10^{-4} m/s as compared to thermal speed ($\approx 10^5$ m/s) of electrons at room temperature.

The current related with drift velocity is $i = neAv_d$.

We can also write $v_d = \frac{i}{neA} = \frac{J}{ne} = \frac{\sigma E}{ne} = \frac{E}{\rho ne} = \frac{V}{\rho lne}$.

where, n = number of electrons per unit volume of the conductor

A = area of cross-section

V = potential difference across the conductor

E = electric field inside the conductor

i = current, J = current density, ρ = specific resistance

σ = conductivity $\left(\sigma = \frac{1}{\rho}\right)$

Ohm's Law

According to Ohm's law, physical conditions (temperature, mechanical strain, etc.) remaining unchanged, the current flowing through a conductor is directly proportional to the potential difference across its ends.

Thus, $I \propto V$ or $V = IR$

where, R is a constant known as the **electrical resistance** of given conductor.

Electrical Resistance

The obstruction offered by any conductor in the path of flow of current is called **electrical resistance** (R). It is equal to the ratio of the potential difference (V) applied across the ends of the conductor and the electric current (I) flowing through it.

$$R = \frac{V}{I}$$

The resistance R_t of a metal conductor is given by

$$R_t = R_0(1 + \alpha t + \beta t^2)$$

where, $\alpha = \frac{R_t - R_0}{R_0 \times t}$

Its unit is ohm (Ω).

In terms of free electron, density n and relaxation time τ , the resistance of a conductor is given by

$$R = \frac{m}{ne^2\tau} \frac{l}{A}$$

where, $\rho = \frac{m}{ne^2\tau}$ is the resistivity of material.

Resistivity is a characteristic of material, i.e. it does not depend upon dimensions of the material. Although, it depends on the temperature.

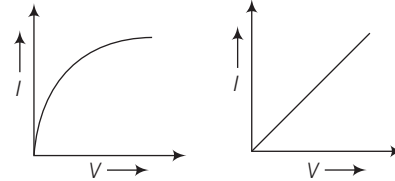
The SI unit of resistivity is $\Omega\text{-m}$ and its dimensional formula is $[\text{ML}^3\text{T}^{-3}\text{A}^{-2}]$.

V-I Characteristics of Ohmic and Non-ohmic Conductors

Substances obeying Ohm's law are called **ohmic resistors**, e.g. metals and their alloys. V - I graph for an ohmic resistor is an inclined straight line.

Substances which do not obey Ohm's law are called **non-ohmic resistors**. e.g. electrolytes, gases, thermionic tubes, transistors, rectifiers, etc.

V - I graph for them is non-linear.



Colour Code for Resistors

The value of resistance used in electric and electronic circuit vary over a wide range.

Such high resistances used are usually carbon resistances and the values of such resistances are marked on them according to a **colour code**.

Colour	Figure
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Grey	8
White	9



Colour	Tolerance
Gold	5%
Silver	10%
No colour	20%

Series and Parallel Combinations of Resistors

- **Series grouping** In series grouping of resistances, same current I flows through all the resistances.

The potential difference applied across the combination is distributed across various resistors in the direct ratio of their resistances,

i.e. $V = V_1 + V_2 + V_3 + \dots$

and $V_1 : V_2 : V_3 \dots = R_1 : R_2 : R_3 \dots$

Total equivalent resistance in series grouping is equal to the sum of individual resistances. Thus,

$$R_s = R_1 + R_2 + R_3 + \dots$$

- **Parallel grouping** In parallel grouping same potential difference V appears across each resistance.

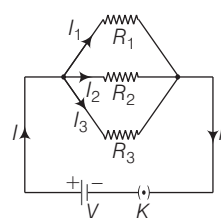
The current is distributed among various resistors in the inverse ratio of their resistances. Thus,

$$I = I_1 + I_2 + I_3 + \dots$$

and $I_1 : I_2 : I_3 \dots = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3}$

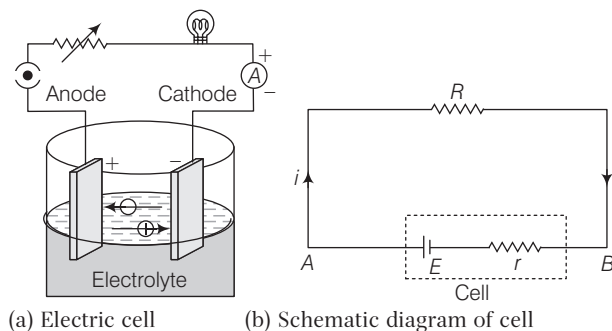
Equivalent resistance in parallel grouping R_p is given by

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$



Electric Cell

The device which converts chemical energy into electrical energy is known as electric cell.



(a) Electric cell

(b) Schematic diagram of cell

Emf, Terminal Voltage and Internal Resistance of a Cell

The potential difference between the terminals of a cell when it is on an open circuit, i.e. when no current is drawn from it is called its **emf** (E).

The potential difference between the terminals of a cell when it is in a closed circuit, i.e. when a current is drawn from it is called its **terminal voltage** (V).

Every cell has a resistance of its own called its internal resistance (r). The value of r depends upon the nature of electrodes, the nature of the electrolyte, size of electrodes and the distance between them.

Fig. (b) shows a cell of emf E , internal resistance r connected to an external resistance R . Total resistance of the circuit $= R + r$.

The current in the circuit is given by $I = \frac{E}{(R+r)}$

Potential difference across r is $V = Ir$

Potential difference across R is $V = IR$. V is called the terminal voltage and V' is the potential drop across the internal resistance.

Thus,

$$E = V + V' = I(R+r)$$

Terminal voltage is $V = E - Ir$.

Discharging of a Cell

When an external resistance is connected across a cell, a current flows in external circuit, this is also called discharging of the cell then current given by the cell

$$i = \frac{E}{R+r}$$

Potential difference across the resistance R , $V = iR$

Potential drop inside the cell $= ir$

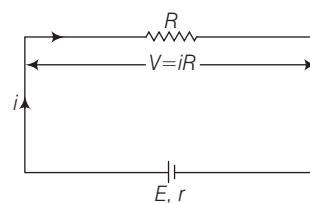
Equation of cell, $E = V + ir$ ($E > V$)

Internal resistance of the cell, $r = \left(\frac{E}{V} - 1 \right) \cdot R$

Power dissipated in external resistance (load)

$$P = Vi = i^2 R = \frac{V^2}{R} = \left(\frac{E}{R+r} \right)^2 \cdot R$$

Power delivered will be maximum when $R = r$.



So,

$$P_{\max} = \frac{E^2}{4r}$$

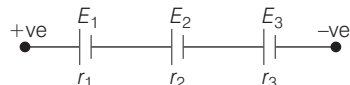
NOTE During charging of a cell, current flows into positive terminal of the cell and terminal voltage $V = E + ir$

Combination of Cells in Series and in Parallel

A group of cells is called a battery. Two common grouping of cells are

1. Series Grouping

In series grouping, if all the cells are joined so as to supply current in the same direction, then resultant emf

$$E_{\text{eq}} = E_1 + E_2 + E_3 + \dots$$


However, if one or more cells are joined so as to supply current in reverse direction, then emf of that/those cells is taken as negative while calculating the equivalent emf.

The equivalent internal resistance of cell

$$r_{\text{eq}} = r_1 + r_2 + r_3 + \dots$$

If n cells, each of emf E and internal resistance r , are joined in series, then

$$E_{\text{eq}} = nE \text{ and } r_{\text{eq}} = nr$$

2. Parallel Grouping

In parallel grouping, if positive terminals of all cells have been joined at one point and all negative terminals at another point, then

$$\frac{1}{r_{\text{eq}}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \dots$$

The equivalent emf of the parallel grouping is given by

$$\frac{E_{\text{eq}}}{r_{\text{eq}}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} + \dots$$

If n cells, each of emf E and internal resistance r , all joined in parallel, then $r_{\text{eq}} = \frac{r}{n}$

But, $E_{\text{eq}} = E$

Electric Energy and Power

The total work done (or energy supplied) by the source of emf in maintaining the electric current in the circuit for a given time is called **electric energy** consumed in the circuit.

The SI unit of electric energy is joule but another unit is watt-hour.

The bigger unit of electric energy is kilowatt-hour (kWh). It is known as **Board of Trade Unit** (BTU).

The electric energy consumed in kWh is given by

$$W = \frac{V \text{ (in volt)} \times I \text{ (in amp)} \times t \text{ (in hour)}}{1000}$$

Whenever the electric current is passed through a conductor, it becomes hot after short time. This indicates that the electric energy is being converted into heat energy. This effect is known as **heating effect** of current or **Joule heating effect**.

$$H = W = I^2 R t \text{ joule} = \frac{I^2 R t}{4.18} \text{ cal}$$

The rate at which work is done by the source of emf in maintaining the effect of current in a circuit is called electric power of the circuit

$$P = VI$$

where, V is the potential difference across the conductor, I is the current flowing through the conductor.

Other expressions for power are given as

$$P = I^2 R \Rightarrow P = \frac{V^2}{R}$$

The SI unit of electric power is watt.

Another important unit is horse power (hp).

1 hp = 746 watt.

Kirchhoff's Laws and their Applications

Sometimes complex electric circuits cannot be reduced to simple series parallel combination.

Kirchhoff's First Law

The algebraic sum of currents meeting at a junction is zero. i.e.

$$\Sigma i = 0$$

$$i_1 + i_2 = i_3 + i_4$$

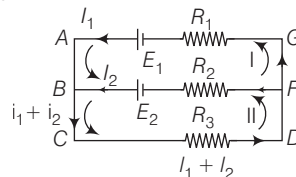
This law is simply a statement of conservation of charge as if current reaching a junction is equal to the current leaving the junction, charge will be conserved.

Kirchhoff's Second Law

The algebraic sum of the changes in potential in complete traversal of a mesh (closed loop) is zero, i.e. $\Sigma V = 0$
e.g. In the following closed loop

Applying Kirchhoff's second rule to closed mesh $ABFGA$ we have $I_1 R_1 - I_2 R_2 = E_1 - E_2$

Similarly, for the closed mesh $BCDFB$ we have $I_2 R_2 + (I_1 + I_2) R_3 = E_2$.

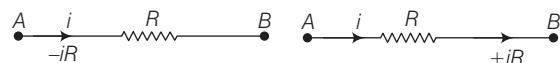


This law represents **conservation of energy** as if the sum of potential changes around a closed loop is not zero, unlimited energy could be gained by repeatedly carrying a charge around a loop.

Sign Convention for the Applications of Kirchhoff's Law

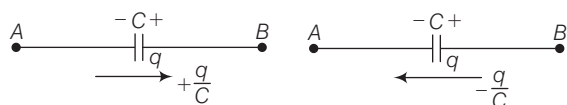
For the application of Kirchhoff's laws, the following sign conventions are to be considered

- (i) The change in potential in traversing a resistance in the direction of current is $-iR$ while in the opposite direction is $+iR$.



- (ii) The change in potential in traversing an emf source from negative to positive terminal is $+E$ while in the opposite direction is $-E$ irrespective of the direction of current in the circuit.

- (iii) The change in potential in traversing a capacitor from the negative terminal to the positive terminal is $+\frac{q}{C}$ while in opposite direction is $-\frac{q}{C}$.



- (iv) The change in voltage in traversing an inductor in the direction of current is $-L \frac{di}{dt}$ while in opposite direction it is $+L \frac{di}{dt}$.

Wheatstone's Bridge

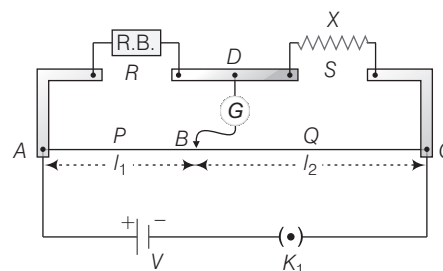
It is a sensitive arrangement to determine the value of an unknown resistance. The bridge is said to be balanced if on switching the keys K_1 and K_2 there is no deflection in galvanometer. It is possible when $V_B = V_D$. In balanced condition,

$$\frac{P}{Q} = \frac{R}{S}$$

In balanced bridge, Q and R can be interchanged without affecting the balance condition. Similarly, P and S can be interchanged. Moreover, cell and galvanometer may also be **interchanged**. If an electric circuit resembles a balanced Wheatstone's bridge, then resistance of branch BD may be ignored (or removed from the circuit) as no current is flowing through it.

Meter Bridge

A meter bridge is a practical arrangement to realise Wheatstone's bridge.

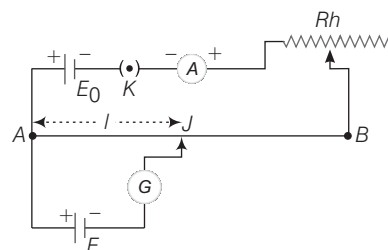


If by sliding the tapping point a null point is obtained on bridge wire at point B , then in balanced condition

$$\frac{R}{X} = \frac{P}{Q} = \frac{l_1}{l_2} \Rightarrow X = R \frac{l_2}{l_1}$$

Potentiometer

Potentiometer is an instrument which can be used for different electric measurements. It is commonly used to find emf of a given cell and to find internal resistance of a cell.



Here, AB is a long uniform resistance wire (length AB may be ranging from 1 m to 10 m). E_0 is a battery whose emf is known supplying a constant current I for flow through the potentiometer wire. If R be the total resistance of potentiometer wire and L its total length, then potential gradient, i.e. fall in potential per unit length along the potentiometer will be

$$k = \frac{V}{L} = \frac{IR}{L} = \frac{E_0 R}{(R_0 + R) L}$$

where, E_0 = emf of battery,

R_0 = resistance inserted by means of rheostat Rh .

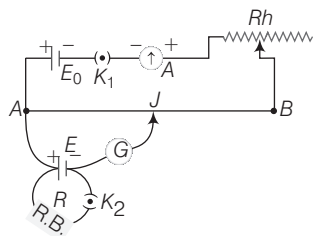
Applications of Potentiometer

The several applications of potentiometer are given below

- **Determination of emf of a cell** If with a cell of emf E on sliding the contact point we obtain zero deflection in galvanometer G when contact point is at J at a length l from the end where positive terminal of cells have been joined, then fall in potential along length l is just balancing the emf of cell. Thus, we have $E = kl$
- **Comparison of emfs of two cells** If with a given potentiometer arrangement we obtain balancing lengths l_1 and l_2 for cells of emfs E_1 and E_2 , then

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

- **Determination of internal resistance of a cell** The arrangement is shown in figure. If the cell E is in open circuit and balancing length is l_1 , then



But if by inserting key K_2 circuit of cell is closed, then potential difference V is balanced by a length l_2 of potentiometer, where $V = kl_2$

⇒ Internal resistance of a cell is given as

$$r = \frac{E - V}{V} R$$

$$= \frac{l_1 - l_2}{l_2} R$$

Variation of Thermo-emf with Temperature

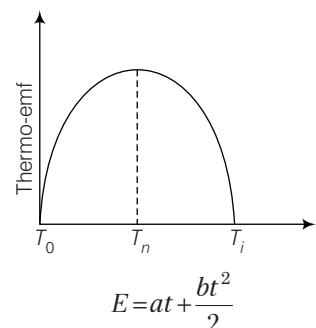
When temperature of hot junction increases by keeping the temperature of cold junction constant, thermo-emf increases and graph is parabolic. At certain temperature emf becomes maximum but thermo electric power is zero.

This temperature is known as neutral temperature (T_n). The neutral temperature is independent of temperature of cold junction. On further increasing, the temperature of hot junction, the thermo-emf decreases and becomes zero. Again further increase in temperature it reverses sign, this temperature is known as temperature of inversion T_i . Seebeck effect is reversible.

$$\therefore T_n - T_c = T_i - T_n$$

$$\Rightarrow T_n = \frac{T_i + T_c}{2}$$

Graph between emf and T can be represented as



where, a, b are constants and E is emf.

Thermoelectric power, P

$$\frac{dE}{dt} = a + 2bt$$

At neutral temperature,

$$\frac{dE}{dt} = 0$$

$$a + 2bT_n = 0$$

$$T_n = -\frac{a}{2b}$$

Inversion temperature,

$$T_i = 2T_n = -\frac{a}{b}$$

Peltier effect The absorption or evolution of heat at a junction of two dissimilar metals, when current is passed through it is known as Peltier effect. It is a reversible phenomenon.

H = Heat energy evolved or absorbed when one coulomb of charge flows across junction

$$\pi = \text{Peltier coefficient} = T \left(\frac{dE}{dt} \right)$$

If Peltier emf at junction under consideration is E volt, then energy absorbed or evolved is Eit

so,

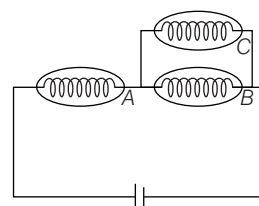
$$\pi it = Eit \text{ or } \pi = E$$

Thomson's effect The absorption or evolution of heat energy along the length of a wire, when current is passed through a wire whose ends are kept at different temperatures.

$$\text{Thomson coefficient, } \sigma = -T \left(\frac{d^2E}{dT^2} \right)$$

Practice Exercise

- Two uniform wires A and B of same metal and have equal masses, the radius of wire A is twice that of wire B . The total resistance of A and B when connected in parallel is
 - $4\ \Omega$, when resistance of wire A is $4.25\ \Omega$
 - $5\ \Omega$, when the resistance of wire A is $4\ \Omega$
 - $4\ \Omega$, when the resistance of wire B is $4.25\ \Omega$
 - $5\ \Omega$, when the resistance of wire B is $4\ \Omega$
- A battery of emf 10 V and internal resistance $0.5\ \Omega$ is connected across a variable resistance R . The value of R for which the power delivered in it is maximum, is given by
 - $2\ \Omega$
 - $0.25\ \Omega$
 - $1\ \Omega$
 - $0.5\ \Omega$
- If R_1 and R_2 are respectively the filament resistances of a 400 W bulb and a 200 W bulb, designed to operate on the same voltage, then
 - $R_1 = 2R_2$
 - $R_2 = 2R_1$
 - $R_2 = 4R_1$
 - $R_1 = 4R_2$
- Two electric bulbs rated P_1 watt V volt and P_2 watt V volt are connected in parallel across V volts mains, then the total power is
 - $P_1 + P_2$
 - $\sqrt{P_1 P_2}$
 - $P_1 P_2 / (P_1 + P_2)$
 - $\frac{P_1 + P_2}{P_1 P_2}$
- In Bohr's model of H_2 atom, the electrons move around the nucleus in a circular orbit of radius $5 \times 10^{-11}\text{ m}$. Its time period is $15 \times 10^{-16}\text{ s}$, the current associated with electron motion is
 - zero
 - $16 \times 10^{-19}\text{ A}$
 - 0.17 A
 - $107 \times 10^{-3}\text{ A}$
- A 5°C rise in temperature is observed in a conductor by passing a current. When the current is doubled, then rise in temperature will be approximately
 - 20°C
 - 16°C
 - 12°C
 - 10°C
- A uniform wire when connected directly across a 220 V line produces heat H per second. If wire is divided into n parts and all parts are connected in parallel across a 200 V line, then the heat produced per second will be
 - H_s
 - nH_s
 - $n^2 H_s$
 - H_s / n^2
- A wire of 50 cm long, 1 mm^2 in cross-section carries a current of 4 A , when connected to a 2 V battery, the resistivity of wire is
 - $2 \times 10^{-7}\ \Omega\text{-m}$
 - $5 \times 10^{-7}\ \Omega\text{-m}$
 - $4 \times 10^{-6}\ \Omega\text{-m}$
 - $1 \times 10^{-6}\ \Omega\text{-m}$
- Three equal resistors connected in series across a source of emf, together dissipated 10 watts of power. What will be the power dissipated if the same resistors are connected in parallel across the same source of emf?
 - 10 W
 - 30 W
 - 90 W
 - $(10/3)\text{ W}$
- Reciprocal of specific resistance is
 - conductive resistance
 - specific conductance
 - conductive reactance
 - plate resistance
- A heating coil is labelled 100 W , 220 V , the coil is cut in half and two pieces are joined in parallel to the same source. The energy now liberated per second is
 - 200 J
 - 400 J
 - 25 J
 - 50 J
- Two resistors of $6\ \Omega$ and $9\ \Omega$ are connected in series to a 120 V source. The power consumed by the $6\ \Omega$ resistor is
 - 384 W
 - 576 W
 - 1500 W
 - 1800 W
- A 100 W , 200 V bulb is connected to a 160 V supply. The power consumption would be
 - 64 W
 - 80 W
 - 100 W
 - 150 W
- The thermistors are usually made of
 - metals with low temperature coefficient of resistivity
 - metals with high temperature coefficient of resistivity
 - metal oxides with high temperature coefficient of resistivity
 - semiconducting materials having low temperature coefficient of resistivity
- Two wires of same material have lengths, L and $2L$ and cross-sectional area $4A$ and A , respectively. The ratio of their resistances would be
 - $1:1$
 - $1:8$
 - $8:1$
 - $1:2$
- Three bulbs A , B and C are connected as shown in figure, B and C are identical, if bulb C is fused, then
 - both A and B will glow more brightly
 - both A and B will glow less brightly than before
 - A will glow less brightly and B more brightly
 - None of the bulbs will glow



18. Three copper wires of length and cross-sectional areas are (L, A) , $(2L, \frac{A}{2})$, $(L/2, 2A)$, resistance is

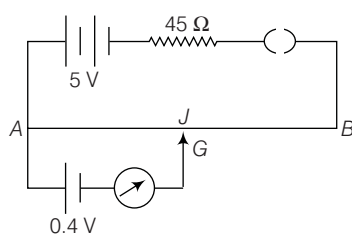
a. minimum in wire of cross-sectional area $\frac{A}{2}$
 b. minimum in wire of cross-sectional area A
 c. minimum in wire of cross-sectional area $2A$
 d. same in all three cases

19. There is a current of 1.344 A in a copper wire whose area of cross-sectional normal to the length of wire is 1 mm^2 . If the number of free electrons per cm^2 is $8.4 \times 10^{28} \text{ m}^{-3}$, then the drift velocity would be

a. 1 mm/s b. 1 mm/s c. 0.1 mm/s d. 0.01 mm/s

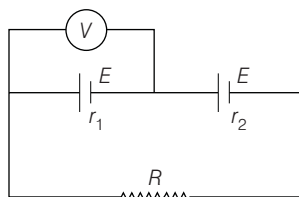
20. In the given figure, the potentiometer wire AB has a resistance of 5Ω and length 10 m. The balancing length AJ for the emf 0.4 V is

a. 8 m
 b. 0.8 m
 c. 4 m
 d. 0.4 m



21. In the following figure, the reading of an ideal voltmeter V is zero. Then, the relation between R , r_1 and r_2 is

a. $R = r_2 - r_1$
 b. $R = r_1 - r_2$
 c. $R = r_1 + r_2$
 d. $R = \frac{r_1 r_2}{r_1 + r_2}$



22. Three equal resistors, each equals to r are connected as shown in figure. Then, the equivalent resistance between points A and B is

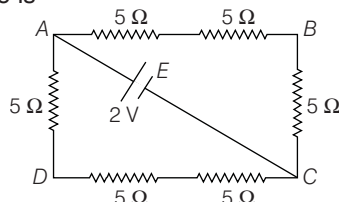
a. r b. $3r$ c. $\frac{r}{3}$ d. $\frac{2r}{3}$

23. When a resistance of 2Ω is connected across the terminals of a cell, the current is 0.5 A. When resistance is increased to 5Ω , the current is 0.25 A. The emf of the cell is

a. 1 V b. 1.5 V c. 2 V d. 2.5 V

24. The potential difference between points A and B from the figure is

a. $2/3 \text{ V}$ b. $8/9 \text{ V}$ c. $4/3 \text{ V}$ d. 2 V

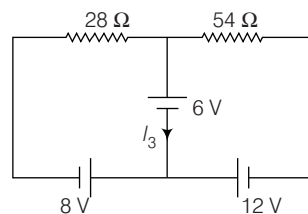


25. The 80Ω galvanometer deflects full scale for a potential of 20 mV. A voltmeter deflecting full scale of 5V is to be made using this galvanometer. We must connect

a. a resistance of $19.92 \text{ k}\Omega$ parallel to the galvanometer
 b. a resistance of $19.92 \text{ k}\Omega$ in series with the galvanometer
 c. a resistance of $20 \text{ k}\Omega$ parallel to the galvanometer
 d. a resistance of $20 \text{ k}\Omega$ in series with galvanometer

26. Consider the circuit shown in figure, the current I_3 is equal to

a. 5 A b. 3 A c. -3 A d. $-\frac{5}{6} \text{ A}$



27. Two cells having an internal resistance of 0.2Ω and 0.4Ω are connected in parallel, the voltage across the battery is 1.5 V. If the emf of one cell is 1.2 V, then the emf of second cell is

a. 2.1 V b. 2.7 V c. 3 V d. 4.2 V

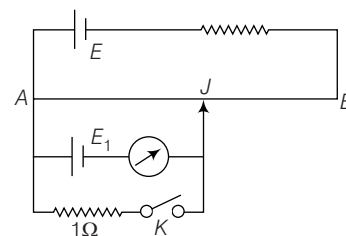
28. On a six fold increase in external resistance of a circuit the voltage across the terminals of the battery has increased from 5 V to 10 V. The emf of battery is

a. 15 V b. 18 V c. 12.5 V d. 11 V

29. In figure, AB is a potentiometer wire; length 10 m and resistance 2Ω with open the balancing length is 5.5 m. However, on closing key K the balancing length reduces to 5 m.

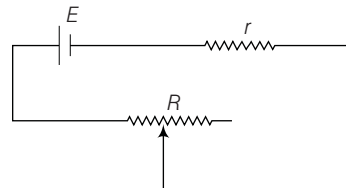
The initial resistance of the cell E_1 is

a. 0.01Ω b. 0.1Ω c. 0.2Ω d. 1Ω

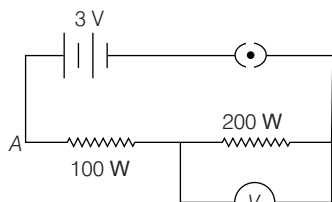


30. A battery of emf E and internal resistance r is connected to a variable resistor R as shown in figure. Which one of the following is true?

a. Potential difference across the terminals of the battery is maximum, when $R = r$
 b. Power delivered to the resistor is maximum when $R = r$
 c. Current in the circuit is maximum when $R = r$
 d. Current in the circuit is maximum when $R \gg r$

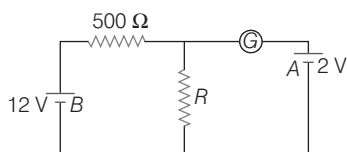


31. The voltmeter in figure has a resistance of $200\ \Omega$. The reading of voltmeter is



- a. 2 V b. 1 V c. 1.5 V d. 3 V

32. For the circuit, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be



- a. $200\ \Omega$ b. $100\ \Omega$ c. $500\ \Omega$ d. $1000\ \Omega$

33. For a potentiometer experiment, the emf of a battery in the primary circuit is 20 V and its internal resistance is $5\ \Omega$. There is a resistance box in series with the battery and the potentiometer wire, whose resistance can be varied from $120\ \Omega$ to $170\ \Omega$. Resistance of the potentiometer wire is $75\ \Omega$. The following potential differences can't be measured using this potentiometer

- a. 5 V b. 6 V c. 7 V d. 8 V

34. A 6 V battery is connected to the terminals of a 3 m long wire of uniform thickness and resistance of $100\ \Omega$. The difference of potential between two points on the wire separated by a distance of 50 cm will be

- a. 2 V b. 3 V c. 1 V d. 15 V

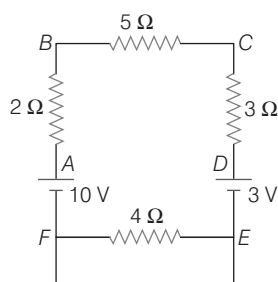
35. A cell has emf of 2.2 V, when connected to a resistance of $5\ \Omega$, the potential difference between the terminals of the cell becomes 2.1 V, the internal resistance for the cell is

- a. $0.12\ \Omega$ b. $0.48\ \Omega$ c. $0.24\ \Omega$ d. $0.50\ \Omega$

36. A uniform wire of $16\ \Omega$ resistance is made into the form of a square. Two opposite corners of square are connected by a wire of resistance $16\ \Omega$. The effective resistance between the other two opposite corners is

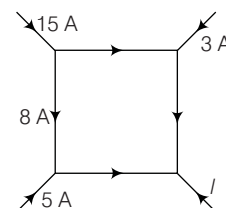
- a. $32\ \Omega$ b. $16\ \Omega$ c. $8\ \Omega$ d. $4\ \Omega$

37. For the circuit shown in figure. The point F is grounded. Which of the following is wrong statement?



- a. D is at 5 V
b. E is at zero potential
c. The current in the circuit will be 0.5 A
d. None of the above

38. The figure shows a network of currents. The magnitude of currents is shown here. The magnitude of current I will be

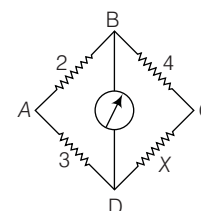


- a. 3 A
b. 13 A
c. 23 A
d. -3 A

39. Two wires having resistance R and $2R$ are connected in parallel, the ratio of heat generated in $2R$ and R is

- a. 1:2 b. 2:1 c. 1:4 d. 4:1

40. If the deflection of galvanometer in Wheatstone circuit is zero, the value of resistance will be



- a. $2\ \Omega$
b. $4\ \Omega$
c. $6\ \Omega$
d. $8\ \Omega$

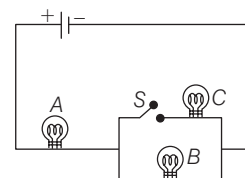
41. In which property of free electrons causes increase in the resistance of a conductor with rise in temperature?

- a. Number density b. Relaxation time
c. Mass d. None of these

42. In an electroplating experiment, mg of silver is deposited when 4 A of current flows for 2 min. The amount of silver (in g) deposited by 6 A of current for 40 s will be

- a. 4 m b. $m/2$ c. $-m/4$ d. $2m$

43. A, B, C are identical bulbs. How does the brightness of A and B change when the switch S is closed?

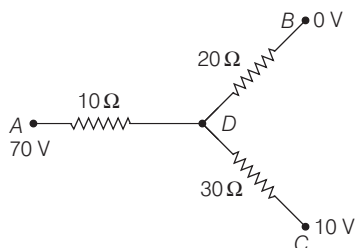


- a. The brightness of A increases and that of B decreases
b. The brightness of A does not change and that of B decreases
c. The brightness of both A and B decreases
d. The brightness of A increases and that of B does not change

44. For a thermocouple, T_c , T_n and T_i denote the temperatures of cold junction, the neutral temperature and the temperatures of inversion respectively. Which one of the following relation is correct ?

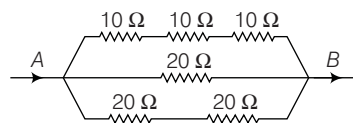
- a. $T_i = \frac{T_c + T_n}{2}$ b. $T_i = 2T_n - T_c$
c. $T_i = \frac{T_n - T_c}{2}$ d. $T_i = T_n - T_c$

45. If nearly 10^5 C liberate 1g equivalent of aluminium, then the amount of aluminium (equivalent wt. 9) deposited through electrolysis in 20 min current of 50 A will be
 a. 0.6 g b. 10.8 g c. 0.09 g d. 5.4 g
46. For the network shown in figure, points A, B and C are at potentials of 70 V, zero and 10 V respectively

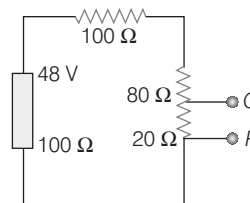


- a. point D is at a potential of 40 V
 b. the currents in the sections AD, DB and DC are in the ratio 4 : 3 : 2
 c. the currents in the sections AD, DB and DC are in the ratio 1 : 2 : 3
 d. the network draws a total power of 100 W
47. When copper voltmeter is connected with a battery of emf 12 V, 2 g of copper is deposited in 30 min. If the same voltmeter is connected across a 6 V battery, then the mass of copper deposited in 45 min would be
 a. 1 g b. 1.5 g c. 2 g d. 2.5 g
48. The emf in a thermoelectric circuit with one junction at 0°C and other at $t^\circ\text{C}$ is given by $E = At - Bt^2$, the neutral temperature is, then
 a. A/B b. $-A/2B$ c. $-B/2A$ d. $A/2B$
49. For the adjoining circuit diagram, the readings of ammeter and voltmeter are 2A and 120 V respectively. If the value of R is $75\ \Omega$, then the voltmeter resistance will be
 a. $100\ \Omega$ b. $150\ \Omega$ c. $300\ \Omega$ d. $75\ \Omega$
50. An electric heater rated as (500 W and 200 V) raises the temperature of 1 kg water from 15°C to its boiling point in 15 min. The heat efficiency of the heater is
 a. 79% b. 97% c. 69% d. 96%
51. The same mass of copper is drawn into two wires 2 mm and 3 mm thick. The two wires are connected in series and current is passed through them. The ratio of heat produced in the two wires is
 a. $\frac{9}{4}$ b. $\frac{3}{2}$ c. $\frac{2}{3}$ d. $\frac{81}{16}$
52. An electric bulb rated 500 W at 100 V is used in a circuit having a 200 V supply. What resistance R must be put in series with the bulbs so that the bulb delivers 500 W?
 a. $20\ \Omega$ b. $40\ \Omega$ c. $10\ \Omega$ d. $5\ \Omega$

53. Two heater coils made of the same material are connected in parallel across the mains, the length and the diameter of one coil is double of the other. Which will produce more heat?
 a. Thinner coil b. Thicker coil
 c. Both produce same heat d. None of these
54. Cell of emf 1 V is connected across a potentiometer, balancing length is 600 cm. What will be the balancing length for 25 V?
 a. 400 cm b. 600 cm c. 1500 cm d. 1200 cm
55. A moving coil galvanometer has a resistance of $9.8\ \Omega$ and gives a full scale deflection when a current of 10mA is passed through it. The value of the shunt required to convert it into a millimeter to measure currents upto 500mA is
 a. $0.02\ \Omega$ b. $0.2\ \Omega$ c. $2\ \Omega$ d. $0.4\ \Omega$
56. The total electrical resistance between the points A and B of the circuit shown, is



- a. $9.23\ \Omega$ b. $15\ \Omega$ c. $30\ \Omega$ d. $100\ \Omega$
57. A wire when connected to 220 V mains supply has power dissipation P_1 . Now, the wire is cut into two equal pieces which are connected in parallel to same supply. Power dissipation in this case is P_2 . Then, $P_1:P_2$ is
 a. 1 b. 2 c. 3 d. 4
58. The resistance of a 50 cm long wire is $10\ \Omega$. The wire is stretched of uniform wire of length 100 cm. The resistance now will be
 a. $15\ \Omega$ b. $30\ \Omega$ c. $20\ \Omega$ d. $40\ \Omega$
59. For the given circuit, the potential difference across P and Q will be nearest to



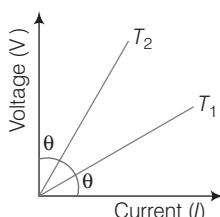
- a. 9.6 V b. 6.6 V c. 4 V d. 3.2 V
60. A voltmeter having resistance of $1800\ \Omega$ is employed to measure the potential difference across $200\ \Omega$ resistance which is connected to DC power supply of 50V and internal resistance $20\ \Omega$. What is the percentage change in potential difference across $200\ \Omega$ resistance as a result of connecting voltmeter across it?
 a. 1% b. 5% c. 10% d. 20%

61. In an experiment to measure the internal resistance of a cell by potentiometer, it is found that the balance point is at a length of 2 m when the cell is shunted by a $4\ \Omega$ resistance and is at a length of 3 m when the cell is shunted by a $8\ \Omega$ resistance. Then the internal resistance of the cell is, then

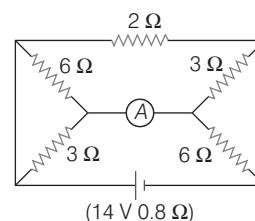
a. $12\ \Omega$ b. $8\ \Omega$ c. $16\ \Omega$ d. $1\ \Omega$

62. The V - I graph for a conductor at temperatures T_1 and T_2 are as shown in the figure. The term $T_2 - T_1$ is proportional to

a. $\cos 2\theta$ b. $\sin 2\theta$
c. $\cot 2\theta$ d. $\tan 2\theta$



63. The reading of ammeter shown in figure is



a. 6.56 A
b. 3.28 A
c. 2.18 A
d. 1.09 A

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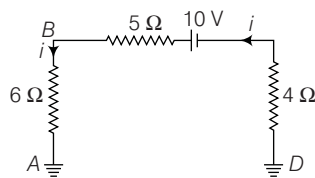
1. Ohm's law says [2014]

a. $V = IR$
b. $V/I = \text{constant}$
c. Both a and b are correct
d. Both a and b are incorrect

2. A current 4.0 A exist in a wire of cross-sectional area $2.0\ \text{mm}^2$. If each cubic metre of the wire contains 12.0×10^{28} free electrons, then the drift speed is [2014]

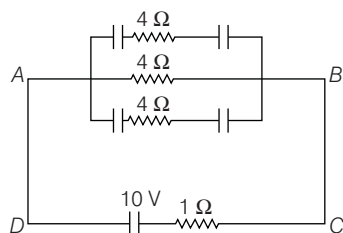
a. $2 \times 10^{-8}\ \text{m/s}$ b. $0.5 \times 10^{-3}\ \text{m/s}$
c. $1.04 \times 10^{-4}\ \text{m/s}$ d. None of these

3. What will be the value of current i in the circuit shown? [2014]



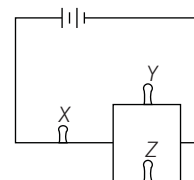
a. 0.67 A b. 1 A
c. 0.32 A d. None of these

4. In the given circuit (as shown in figure), each capacitor has a capacity of $3\ \mu\text{F}$. What will be the net charge on each capacitor? [2013]



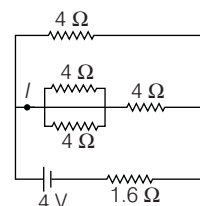
a. $48\ \mu\text{C}$ b. $24\ \mu\text{C}$
c. $12\ \mu\text{C}$ d. None of these

5. Three bulbs X, Y and Z are connected as shown in figure. The bulbs Y and Z are identical. If bulb Z gets fused, then [2013]



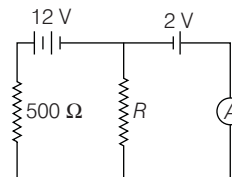
a. Both X and Y will glow more brightly
b. Both X and Y will glow less brightly
c. X will glow less brightly and Y will glow more brightly
d. X will glow more brightly and Y will glow less brightly

6. In the circuit shown, the value of i in ampere is [2012]



a. 1 b. 0.60 c. 0.4 d. 1.5

7. In the circuit shown below, the ammeter reading is zero. Then, the value of the resistance R is [2012]



a. $50\ \Omega$ b. $100\ \Omega$ c. $200\ \Omega$ d. $400\ \Omega$

8. A steady current flow in a metallic conductor of non-uniform cross-section. The quantity/quantities remaining constant along the whole length of the conductor is/are [2011]

a. current, electric field and drift speed
b. drift speed only
c. current and drift speed
d. current only

9. Two bulbs which consume powers P_1 and P_2 are connected in series. The power consumed by the combination is [2010]

a. $P_1 + P_2$ b. $\sqrt{P_1 P_2}$ c. $\frac{P_1 P_2}{P_1 + P_2}$ d. $\frac{2P_1 P_2}{P_1 + P_2}$

10. Three conductors draw respectively currents of 1 A, 2 A and 4 A when connected in turn across a battery. If they are connected in series across the same battery, the current drawn will be [2010]

a. $\frac{2}{7}$ A b. $\frac{3}{7}$ A c. $\frac{4}{7}$ A d. $\frac{5}{7}$ A

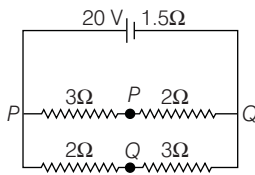
11. 24 identical cells, each of internal resistance 0.5Ω , are arranged in a parallel combination of n rows, each row containing m cells in series. The combination is connected across a resistor of 3Ω . In order to send maximum current through the resistor, we should have [2010]

a. $m = 12, n = 2$ b. $m = 8, n = 3$
c. $m = 2, n = 12$ d. $m = 3, n = 8$

12. A wire is stretched as to change its diameter by 0.25%. The percentage change in resistance is [2009]

a. 4.0% b. 2.0% c. 1.0% d. 0.5%

13. If in the circuit in figure, the internal resistance of the battery is 15Ω and V_P and V_Q are potential at P and Q respectively. What is the potential difference between the points P and Q ? [2009]



a. Zero b. 4 V ($V_P > V_Q$)
c. 4 V ($V_Q > V_P$) d. 2.5 V ($V_Q > V_P$)

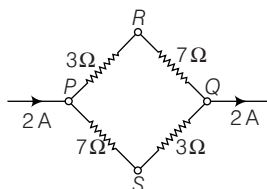
14. When the potential difference applied across a solid conductor is increased, the rate of flow of electrons [2009]

a. remains same b. decreases
c. increases d. decreases sharply

15. A box with two terminals is connected in series with a 2V battery, an ammeter and a switch. When the switch is closed the needle of the ammeter moves quickly across the scale and drops back to zero. The box contains [2009]

a. 20Ω resistor b. a strip of copper
c. a diode d. a short length of fuse wire

16. A current of 2 A flows in an electric circuit as shown in figure. The potential difference ($V_R - V_S$), in volts (V_R and V_S are potentials at R and S respectively) is [2008]



a. -4 b. +2 c. +4 d. -2

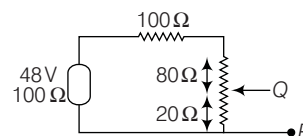
17. When a battery connected across a resistor of 16Ω , the voltage across the resistor is 12 V. When the same battery is connected across a resistor of 10Ω , voltage across it is 11 V. The internal resistance of the battery (in ohm) is [2008]

a. $\frac{10}{7}$ b. $\frac{20}{7}$ c. $\frac{25}{7}$ d. $\frac{30}{7}$

18. If a rod has resistance 4Ω and if rod is turned as half circle, then the resistance along diameter is [2007]

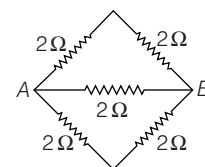
a. 156Ω b. 2.44Ω c. 4Ω d. 2Ω

19. In the circuit, the potential difference across PQ will be nearest to [2007]



a. 9.6 V b. 6.6 V c. 4.8 V d. 3.2 V

20. Each resistance shown in figure is 2Ω . The equivalent resistance between A and B is [2006]



a. 2Ω b. 4Ω
c. 8Ω d. 1Ω

21. A cell of constant emf first connected to a resistance R_1 and then connected to resistance R_2 . If power delivered in both cases is same, then the internal resistance of the cell is [2006]

a. $\sqrt{R_1 R_2}$ b. $\sqrt{\frac{R_1}{R_2}}$ c. $\frac{R_1 - R_2}{2}$ d. $\frac{R_1 + R_2}{2}$

22. Ampere - hour is the unit of [2006]

a. quantity of charge b. potential
c. energy d. current

23. A 5.0 A current is set up in an external circuit by a 6.0 V storage battery for 6.0 min. The chemical energy of the battery is reduced by [2005]

a. 1.08×10^4 J b. 1.08×10^{-4} J
c. 1.8×10^4 J d. 1.8×10^{-4} J

24. The current in a simple series circuit is 5.0 A. When an additional resistance of 2.0Ω is inserted, the current drops to 4.0 A. The original resistance of the circuit in ohms was [2005]

a. 1.25 b. 8 c. 10 d. 20

25. Two resistances are connected in two gaps of a Meter bridge. The balance point is 20 cm from the zero end. A resistance of 15Ω is connected in series with the smaller of the two. The null point shifts to 40 cm. The value of the smaller resistance in ohms is [2005]

a. 3 b. 6 c. 9 d. 12

26. By using only two resistance coils - singly, in series or in parallel one should be able to obtain resistance of 3, 4, 12, and 16 Ω . The separate resistances of the coil are [2005]

a. 3 and 4 b. 4 and 12 c. 12 and 16 d. 16 and 3

Answer with Solutions

Practice Exercise

1. (a) $r_A = 2r_B$

$$\rho_A = \rho_B \quad (\text{specific resistance})$$

In parallel, $\frac{1}{R} = \frac{1}{R_A} + \frac{1}{R_B} \quad \dots(i)$

$$A_A l_A = A_B l_B \Rightarrow \pi r_A^2 l_A = \pi r_B^2 l_B$$

$$\frac{l_A}{l_B} = \frac{1}{4} \text{ and } \frac{A_A}{A_B} = 4 \Rightarrow \frac{R_A}{R_B} = \frac{\rho \frac{l_A}{A_A}}{\rho \frac{l_B}{A_B}}$$

$$\frac{R_A}{R_B} = \frac{l_A}{l_B} \times \frac{A_B}{A_A} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

$$16R_A = R_B \quad \dots(ii)$$

$$\frac{1}{R} = \frac{1}{R_A} + \frac{1}{R_B}$$

$$= \frac{1}{R_A} + \frac{1}{16R_A} = \frac{16+1}{16R_A} = \frac{17}{16 \times R_A} \Rightarrow R = \frac{16R_A}{17}$$

If $R_A = 4.25 \Omega \Rightarrow R = \frac{16 \times 4.25}{17} = \frac{68}{17}$

$\therefore R = 4 \Omega$

2. (d) For maximum power,

$\therefore R = 0.5 \Omega$

3. (b) 4. (a)

5. (d) $r = 5 \times 10^{-11} \text{ m}$, $t = 15 \times 10^{-16} \text{ s}$

$$i = \frac{q}{t} = \frac{16 \times 10^{-19}}{15 \times 10^{-16}} \Rightarrow i = 1.07 \times 10^{-3} \text{ A}$$

6. (a)

7. (c) $V = 200 \text{ V}$

$$H = i^2 R t \quad (\because i = V / r)$$

$$H_s = \frac{V^2}{n R_{\text{eq}}} t$$

In parallel, equivalent resistance of n parts is

$$R_{\text{eq}} = \frac{r}{n}$$

where, r is the resistance of each part.

$$H = \frac{V^2}{(r/n)} t = \frac{n V^2}{r} t$$

$\therefore H = n^2 H_s$

8. (d) 9. (c) 10. (b) 11. (b)

12. (a) $V = 120 \text{ V}$

$$V = V_1 + V_2 \quad \dots(i)$$

$$V_1 = i R_1$$

$$V_2 = i R_2 \Rightarrow \frac{V_1}{R_1} = \frac{V_2}{R_2}$$

From Eq. (i), $V = V_1 + V_2 = V_1 + \frac{R_2}{R_1} V_1$

$$V = V_1 \left[\frac{R_1 + R_2}{R_1} \right] \Rightarrow V_1 = \frac{120 \times 6}{15} = 48 \text{ V}$$

Power in 6Ω , $P_1 = \frac{V_1^2}{R_1} = \frac{48 \times 48}{6} \Rightarrow P_1 = 384 \text{ W}$

13. (a) $P = V^2 / R = 160 \times \frac{160}{400} = 64 \text{ W}$

14. (c) 15. (b)

16. (c) Bulbs B and C are connected in parallel. Suppose, voltage across A and B is same. If bulb C is fused, bulb B will work

$$P_B = \frac{V^2}{R_B} \Rightarrow P_A = \frac{V^2}{R_A}$$

$$R = \frac{R_B R_C}{R_B + R_C}$$

$\therefore R_B < R_A \therefore P_B > P_A$

17. (a)

18. (c) $l_1 = L$, $A_1 = A$

$$l_2 = 2L$$

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{A_2}{A_1} \Rightarrow R_3 < R_1 < R_2$$

19. (c)

20. (a)

21. (b) $I = \frac{2E}{r_1 + r_2 + R}$

$$V_1 = E - I r_1 = 0$$

$$\Rightarrow I = E / r_1$$

$$E = \frac{2E r_1}{r_1 + r_2 + R}$$

$$\Rightarrow r_1 + r_2 + R = 2r_1$$

22. (c)

23. (b) $E = i(R + r)$

$$E = 15 \text{ V}$$

24. (a) By Kirchhoff's voltage law, for branch $ABCEA$

$$15 I_1 = 2 \Rightarrow I_1 = \frac{2}{15} \text{ A}$$

For branch $ADCEA$,

$$15 I_2 = 2 \Rightarrow I_2 = \frac{2}{15} \text{ A}$$

$$V_{AD} = I_2 R = \frac{2}{15} \times 5 = \frac{2}{3} \text{ V}$$

25. (b)

26. (d) For I_1 , by Kirchhoff's voltage law

$$28 I_1 = -6 - 8 \Rightarrow I_1 = \frac{-14}{28} = -\frac{1}{2}$$

Here, I_1 is -ve means opposite in direction as assumed.

$$\therefore I_3 = -(I_1 + I_2) = -\left(\frac{1}{2} + \frac{1}{3}\right)$$

$$I_3 = -\left(\frac{5}{6}\right) \text{ A}$$

27. (a) $r_1 = 0.2 \Omega$, $r_2 = 0.4 \Omega$

$$V = IR$$

$$R = \frac{r_1 r_2}{r_1 + r_2}, I = I_1 + I_2 \Rightarrow V = \frac{r_1 r_2}{r_1 + r_2} (I_1 + I_2)$$

$$I_1 = E_1 / r_1 \text{ and } I_2 = E_2 / r_2$$

$$V = \left(\frac{E_1}{r_1} + \frac{E_2}{r_2}\right) \frac{r_1 r_2}{r_1 + r_2} = \frac{(E_2 r_1 + E_1 r_2)}{r_1 r_2} \left(\frac{r_1 r_2}{r_1 + r_2}\right)$$

$$V = \frac{E_2 r_1 + E_1 r_2}{r_1 + r_2} \Rightarrow 15 = \frac{E_2 \times 0.2 + 1.2 \times 0.4}{0.6}$$

$$0.9 = 0.2 E_2 + 0.48 \Rightarrow \frac{0.42}{0.2} = E_2 \Rightarrow E_2 = 2.1 \text{ V}$$

28. (c) $\frac{E-V}{V} R = r \Rightarrow \frac{E-5}{5} R = r \dots(i)$

$$\frac{E-10}{10} 6R = r \dots(ii)$$

Equating Eqs. (i) and (ii), we get

$$\frac{E-5}{5} R = \frac{E-10}{10} 6R \Rightarrow E-5 = 3E-30$$

$$2E = 25 \Rightarrow E = 12.5 \text{ V}$$

29. (b) $I_{AB} = 10 \text{ m}$, $R_{AB} = 2 \Omega$, $R_{AJ} = 1 \Omega$

$$E_1 = \rho L \quad (\text{When key is open})$$

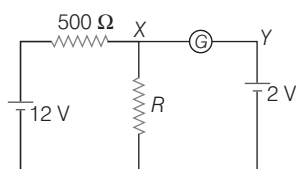
$$E_1 = \rho \times 5.5 \Rightarrow V_1 = 5 \text{ p}$$

$$\frac{E-V}{V} R = r \Rightarrow \frac{5.5 \text{ p} - 5 \text{ p}}{5 \text{ p}} \times 1 = r \Rightarrow \frac{0.5}{5} \times 1 = r$$

$$r = 0.1 \Omega$$

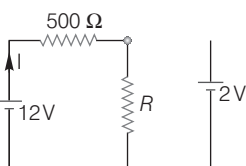
30. (c) 31. (c)

32. (b) The galvanometer shows zero deflection, i.e. current through XY is zero



As a result potential drop across R is 2 V. Circuit can be redrawn as

$$I = \frac{12}{500 + R}$$



Voltage across R ,

$$V = IR \Rightarrow 2 = \frac{12}{500 + R} \times R$$

$$\text{or } 1000 + 2R = 12R \text{ or } R = 100 \Omega$$

33. (d) Maximum external resistance put across the potentiometer battery is $(170 + 75) \Omega$.

As external resistance $= r = 5 \Omega$

So, current in potentiometer wire

$$= I = \frac{E}{(R + r)} = \frac{20}{250} = 0.08 \text{ A}$$

$$\text{Potential drop occurs across wire of potentiometer} = IR \\ = 0.08 \times 75 = 6 \text{ V}$$

Also, minimum resistance $= 120 + 75 \Omega$

$$\therefore I = \frac{20}{200} = 0.1 \text{ A}$$

$$\text{Potential drop} = IR = 0.1 \times 75 = 7.5 \text{ V}$$

So, upto 7 V it can be measured with the help of potentiometer.

34. (c)

35. (c) $E = 2.2 \text{ V}$, $R = 5 \Omega$, $V = 2.1 \text{ V}$,

$$E = iR + ir \Rightarrow \frac{E-V}{V} R = r$$

$$\frac{2.2-2.1}{2.1} \times 5 = r \Rightarrow \frac{0.1}{2.1} \times 5 = r$$

$$r = 0.24 \Omega$$

36. (d)

37. (a) Effective emf of circuit $= 10 - 3 = 7 \text{ V}$

Total resistance of circuit $= 2 + 5 + 3 + 4 = 14 \Omega$

Current $I = 7/14 = 0.5 \text{ A}$

Potential difference between A and $D = 0.5 \times 10 = 5 \text{ A}$

Potential at $D = 10 - 5 = 5 \text{ V}$

Potential at $E = 5 - 3 = 2 \text{ V}$

Hence, E cannot be at zero potential, as there is a potential drop at E .

38. (c) $\sum I = 0$ (at a junction)

$$(7 + 3) + (8 + 5) + I = 0$$

$$I = -23 \text{ A} \Rightarrow |I| = 23 \text{ A}$$

39. (b) In parallel circuit potential across each resistor is same.

$$\therefore P_1 = \text{power across } R = \frac{V^2}{R}$$

$$P_2 = \text{power across } 2R = \frac{V^2}{2R}$$

$$\frac{P_1}{P_2} = 2:1$$

40. (c)

41. (b)

42. (b) $m = z \times 4 \times 2$

$$m' = \frac{z \times 6 \times 40}{60}$$

$$\frac{m}{m'} = \frac{8z \times 60}{240z} = 2 \Rightarrow m' = \frac{m}{2}$$

43. (a) $\therefore$ Illumination of A is more than previous.

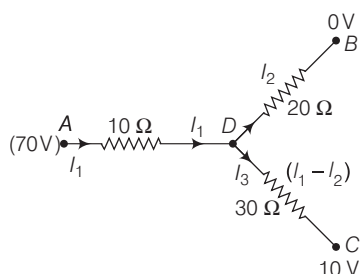
44. (b) $T_i - T_n = T_n - T_c$

$$2T_n = T_i + T_c \Rightarrow T_i = 2T_n - T_c$$

45. (d) $m = zit$

$$m = \frac{9}{10^5} \times 50 \times 20 \times 60 = \frac{54}{10} \Rightarrow m = 5.4 \text{ g}$$

46. (a) Consider the current distributions in the circuit as shown below



$$\text{By KVL, } V_A - I_1 10 - I_2 20 - V_B = 0$$

$$\Rightarrow I_1 10 + I_2 20 = 70 \quad \dots(i)$$

$$V_A - I_1 10 - I_3 30 - V_C = 0$$

$$\Rightarrow I_1 10 + I_3 30 = 60 \quad \dots(ii)$$

$$\text{Also, } I_3 = I_1 - I_2$$

So, Eq. (ii) becomes

$$I_1 10 + 30(I_1 - I_2) = 60$$

$$I_1 40 - 30 I_2 = 60 \quad \dots(iii)$$

Solving Eqs. (i) and (iii), we get

$$I_2 = 2 \text{ A, } I_1 = 3 \text{ A, } I_3 = 1 \text{ A}$$

Potential drop across branch AD,

$$V_A - V_D = I_1 \times 10 \Rightarrow 70 - V_D = 30 \Rightarrow V_D = 40 \text{ V}$$

47. (b) $m = zit$

$$m = 15 \text{ g}$$

48. (d)

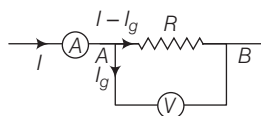
49. (c) $V_{AB} = (I - I_g) R = I_g \cdot G$

where, $G = \text{voltmeter}$
(resistance)

$$V_{AB} = 120, I = 2, R = 75$$

$$\Rightarrow 120 = (2 - I_g) 75 \Rightarrow I_g = 0.4 \text{ A}$$

$$\text{Now, } V_{AB} = I_g G \Rightarrow G = \frac{120}{0.4} = 300 \Omega$$



50. (a) Heat absorbed by water $= ms\Delta T$

$$\text{Heat supplied by heater} = VIt = Pt$$

$$\therefore \text{Efficiency} = \frac{ms\Delta T}{Pt} \times 100$$

By putting given value, efficiency $\approx 79\%$

51. (d) $m = A \cdot l \cdot x = \pi \frac{d^2}{4} l \cdot x$, $x = \text{Density}$

$$\Rightarrow d_1^2 l_1 = d_2^2 l_2 \quad (\text{as mass is same})$$

In series, current will be same.

$$\text{So, } \frac{H_1}{H_2} = \frac{i^2 R_1}{i^2 R_2} = \frac{\rho \left(\frac{l_1}{A_1} \right)}{\rho \left(\frac{l_2}{A_2} \right)} = \frac{l_1 A_2}{l_2 A_1} = \left(\frac{d_2^2}{d_1^2} \right) \pi \frac{d_2^2}{4}$$

$$= \frac{d_2^4}{d_1^4} = \left(\frac{3}{2} \right)^4 = \frac{81}{16}$$

52. (a) 53. (b)

54. (c) For a potentiometer

$$\frac{E_1}{l_1} = \frac{E_2}{l_2} = 1500 \text{ cm}$$

55. (b)

56. (a) In branch, (1) resistors are in series.

$$R_1 = 10 + 10 + 10 = 30 \Omega$$

In branch (3), 20 and 20 are in series

$$\therefore R_3 = 20 + 20 = 40 \Omega$$

The equivalent circuit of (1), (2) and (3) branches are in parallel.

$$\therefore \frac{1}{R_{eq}} = \frac{1}{30} + \frac{1}{20} + \frac{1}{40} = \frac{4 + 6 + 3}{120} = \frac{13}{120}$$

$$E_{eq} = \frac{120}{13} = 9.23 \Omega$$

57. (d)

58. (d) When length of wire is stretched, its area and length changes in such a way that volume always remains same.

Wire A has resistance R , length l , radius r

Wire A when stretched resistance $= R'$, length $= l'$, radius $= r_1$

$\therefore$ Volume of original wire = volume of stretched wire

$$V = V'$$

$$Al = A' l' \Rightarrow \pi r^2 l = \pi r_1^2 l'$$

$$\frac{l}{l'} = \frac{r_1^2}{r^2} = \frac{50}{100} = \frac{1}{2}$$

$$R = \rho \cdot \frac{l}{A}, R' = \rho \cdot \frac{l'}{A'}$$

$$\frac{R}{R'} = \frac{l/A}{l'/A'} = \frac{l}{l'} \times \frac{A'}{A} = \frac{l}{l'} \times \frac{r_1^2}{r^2} = \frac{50}{100} \times \frac{50}{100} = \frac{1}{4}$$

$$R' = 4R = 4 \times 10 = 40 \Omega$$

59. (d)

60. (a) $V_1 = E - Ir = 50 - \frac{50}{220} \times 20 = 50 - 4.5 = 45.5 \text{ V}$

$$\text{Now, } V_2 = 50 - \frac{50}{200} \times 20 = 45$$

% change in potential difference

$$= \frac{(45.5 - 45)}{50} \times 100 = 1\%$$

61. (b) As, $\left(\frac{l-2}{2} \right) 4 = \left(\frac{l-3}{3} \right) 8 \Rightarrow l = 6$

$$\text{Therefore, } r = \left(\frac{l-2}{2} \right) 4 = 8 \Omega$$

62. (c) $R_1 = \tan \theta = R_0 (1 + \alpha T_1)$

$$\text{and } R_2 = \cot \theta = R_0 (1 + \alpha T_2)$$

$$\cot \theta - \tan \theta = R_0 (1 + \alpha T_2) - R_0 (1 + \alpha T_1) = R_0 \alpha (T_2 - T_1)$$

$$\text{or } T_2 - T_1 = \frac{1}{\alpha R_0} (\cot \theta - \tan \theta)$$

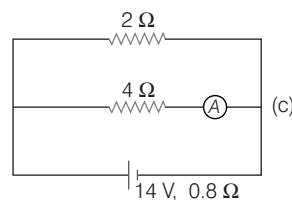
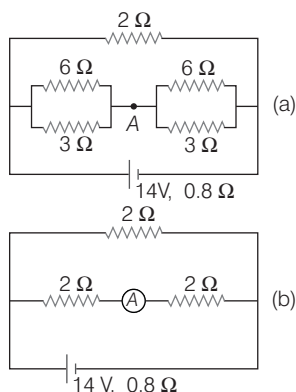
$$= \frac{1}{\alpha R_0} (\cot \theta - \tan \theta)$$

$$= \frac{1}{\alpha R_0} \left(\frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta} \right)$$

$$= \frac{2 \cos 2\theta}{\alpha R_0 \sin \theta} = \frac{2 \cos 2\theta}{\alpha R_0 \sin 2\theta}$$

$$\Rightarrow = \frac{2}{\alpha R_0} \cot 2\theta$$

63. (c) The equivalent circuit of the given circuit will be reduced to as shown in figure.



$$\begin{aligned}\text{Total resistance of the circuit} &= \frac{2 \times 4}{2 + 4} + 0.8 \\ &= \frac{8}{6} + 0.8 = \frac{12.8}{6} \Omega\end{aligned}$$

$$\text{Main current in the circuit} = \frac{14}{(12.8/6)} = \frac{84}{12.8} \text{ A}$$

$$\text{Reading of ammeter} = \frac{84}{12.8} \times \frac{2}{6} = 2.18 \text{ A}$$

BITSAT Archives

1. (c) Ohm's law says $\frac{V}{I} = R = \text{constant}$

2. (c) The current density in the wire is

$$J = \frac{i}{A} = \frac{4}{2 \times 10^{-6}} = 2 \times 10^6 \text{ Am}^{-2}$$

The drift speed is

$$\begin{aligned}v &= \frac{J}{nc} = \frac{2 \times 10^6}{12 \times 10^{28} \times 1.6 \times 10^{-19}} \\ &= \frac{10^6}{6 \times 1.6 \times 10^9} = 1.04 \times 10^{-4} \text{ m/s}\end{aligned}$$

3. (a) $V_A - V_D = -6i - 5i + 10 - 4i$

Here, $V_A = V_D$

Since, points A and D are centred.

$$\begin{aligned}-6i - 5i + 10 - 4i &= 0 \\ \Rightarrow 15i &= 10 \Rightarrow i = \frac{10}{15} = 0.67 \text{ A}\end{aligned}$$

4. (c) Net resistance between ABCD is

$$\begin{aligned}R &= 4 + 1 = 5 \Omega \\ \therefore \text{Current } I &= \frac{V}{R} = \frac{10}{5} = 2 \text{ A}\end{aligned}$$

Potential difference across A and B

$$= I \times 4 = 2 \times 4 = 8 \text{ V}$$

$\therefore$ Two capacitors of $3 \mu\text{F}$ each are in series

$\therefore$ Potential difference across each capacitor

$$= \frac{8}{2} = 4 \text{ V}$$

Charge on each capacitor, $q = CV = 3 \times 4 = 12 \mu\text{C}$

5. (c) If bulb Z is fused, the current stops flowing through Z. The effective resistance of the circuit due to bulbs X and Y in series becomes more as compared to before. Due to which, the current in the circuit decrease.

$$\therefore \text{brightness} \propto (\text{current})^2$$

So, the brightness of bulb X decreases,

Now, bulb Y gets more current than before fusing the bulb, Z.

$\therefore$ Brightness of bulb Y will increase.

6. (c) We can simplify the network as shown in figure.

So, net resistance,

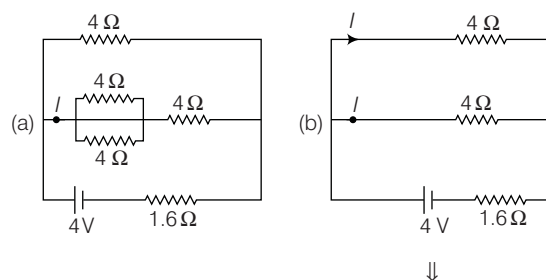
$$R = 2.4 + 1.6 = 4.0 \Omega$$

Therefore, current from the battery

$$i = \frac{V}{R} = \frac{4}{4} = 1 \text{ A}$$

Now, from the circuit (b),

$$4I' = 6I \Rightarrow I' = \frac{3}{2}I$$



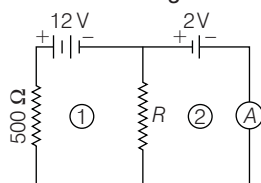
But,

$$i = I + I' = I + \frac{3}{2}I = \frac{5}{2}I$$

$$\therefore 1 = \frac{5}{2}I$$

$$\Rightarrow I = \frac{2}{5} = 0.4 \text{ A}$$

7. (b) Given circuit is shown in figure



In loop (1),

$$12 - 500i_1 - Ri_1 = 0$$

$$\Rightarrow 12 = i_1(500 + R) \quad \dots(i)$$

In loop (2),

$$12 - 500i_1 - 2 = 0 \Rightarrow 10 = 500i_1$$

$$\text{or } i_1 = \frac{1}{50} \text{ A} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$12 \times \frac{1}{i_1} = (500 + R)$$

$$\Rightarrow 12 \times 50 = 500 + R \Rightarrow R = 100 \Omega$$

8. (d) In a metallic conductor of non-uniform cross-section, only the current remains constant along the entire length of the conductor.

9. (a) For a series circuit,

$$\begin{aligned} R_s &= R_1 + R_2 \\ \Rightarrow i^2 R_s &= i^2 R_1 + i^2 R_2 \Rightarrow P_s = P_1 + P_2 \end{aligned}$$

10. (c) Let the potential of the battery be V

$$R_1 = \frac{V}{1}, R_2 = \frac{V}{2}, R_3 = \frac{V}{4}$$

On connecting in series

$$\begin{aligned} R_s &= R_1 + R_2 + R_3 \\ \Rightarrow R_s &= V \left(1 + \frac{1}{2} + \frac{1}{4} \right) \Rightarrow R_s = \frac{7}{4} V \end{aligned}$$

$\therefore$ By definition

$$I = \frac{V}{R_s} \Rightarrow I = \frac{4}{7} \text{ A}$$

11. (a) $N = 24 = mn$

For current to be maximum

$$\begin{aligned} R_{\text{internal}} &= R_{\text{external}} \\ \Rightarrow \frac{mr}{n} &= 3 \Rightarrow \frac{m}{n} (0.5) = 3 \Rightarrow \frac{m}{n} = 6 \end{aligned}$$

$\Rightarrow m = 6n$, substituting the values, we get

$$24 = 6n^2$$

$$\Rightarrow n = 2 \Rightarrow m = 12$$

12. (c)

13. (d) Effective resistance of circuit = $\frac{5 \times 5}{5 + 5} + 15 = 4 \Omega$

Total current in circuit,

$$I = \frac{20}{4} = 5 \text{ A}$$

Current in arm APB or $AQB = \frac{1}{2} = 2.5 \text{ A}$

$$V_A - V_P = 3 \times 2.5 = 7.5 \text{ V}$$

$$V_A - V_Q = 2 \times 2.5 = 5 \text{ V}$$

$$\begin{aligned} V_P - V_Q &= (V_A - V_Q) - (V_A - V_P) \\ &= 5.0 - 7.5 = -2.5 \text{ V} \end{aligned}$$

14. (c) When potential difference applied across a conductor increases, the current flowing through the conductor increases. Therefore, drift velocity of the electrons increases as

$$I \propto v_d$$

15. (d) When a box contains fuse wire, a strong current flows through fuse when circuit is closed. Due to which a very strong heating effect takes place, resisting the breakage of fuse wire.

16. (c) Current through each arm

$$PQR \text{ and } PSQ = 1 \text{ A}$$

$$V_P - V_R = 3 \text{ V}$$

$$V_P - V_S = 7 \text{ V}$$

From Eqs. (i) and (ii), we get

$$V_R - V_S = +4 \text{ V}$$

17. (b) Here, $V < E$

$$\therefore E = V + Ir \quad \dots(i)$$

For first case,

$$E = 12 + \frac{12}{16} r$$

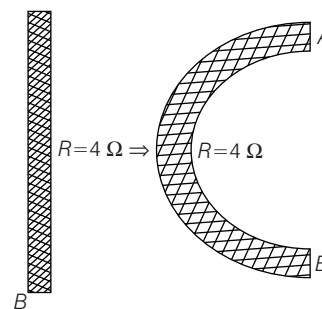
For second case,

$$E = 11 + \frac{11}{10} r \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$12 + \frac{12}{16} r = 11 + \frac{11}{10} r \Rightarrow r = \frac{20}{7} \Omega$$

18. (c)

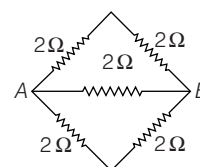


19. (d) Potential difference across PQ , i.e. potential difference across the resistance of 20Ω , which is $V = i \times 20$ and

$$\begin{aligned} i &= \frac{48}{(100 + 100 + 80 + 20)} \\ &= 0.16 \text{ A} \end{aligned}$$

$$\therefore V = 0.16 \times 20 = 3.2 \text{ V}$$

20. (a) Given circuit is a balanced Wheatstone bridge. So, diagonal resistance of 2Ω will be ineffective.



Equivalent resistance of upper arms

$$= 2 + 2 = 4 \Omega$$

Equivalent resistance of lower arms

$$= 2 + 2 = 4 \Omega$$

$$R_{AB} = \frac{4 \times 4}{4 + 4} = 2 \Omega$$

21. (a) Current given by cell

$$I = \frac{E}{R + r}$$

Power delivered in first case

$$P_1 = I^2 R_1 = \left(\frac{E}{R_1 + r} \right)^2 R_1$$

Power delivered in second case

$$P_2 = I^2 R_2 = \left(\frac{E}{R_2 + r} \right)^2 R_2$$

Power delivered is same in the both cases.

$$\left(\frac{E}{R_1 + r} \right)^2 R_1 = \left(\frac{E}{R_2 + r} \right)^2 R_2$$

$$\frac{R_1}{(R_1 + r)^2} = \frac{R_2}{(R_2 + r)^2}$$

$$R_1 (R_2^2 + r^2 + 2R_2 r) = R_2 (R_1^2 + r^2 + 2R_1 r)$$

$$R_1 R_2^2 + R_1 r^2 + 2R_1 R_2 r = R_2 R_1^2 + R_2 r^2 + 2R_1 R_2 r$$

$$R_1 R_2^2 - R_2 R_1^2 = R_2 r^2 - R_1 r^2$$

$$R_1 R_2 (R_2 - R_1) = r^2 (R_2 - R_1)$$

$$r = \sqrt{R_1 R_2}$$

22. (a) $I = \frac{q}{t}$

$$q = I \times t$$

23. (a) Chemical energy reduced

$$= V I t = 6 \times 5 \times 6 \times 60$$

$$= 10800 = 1.08 \times 10^4 \text{ J}$$

24. (b) Let the original resistance is $R \Omega$

$$V = I R$$

$$V = 5 \times R \times = 5 R \quad \dots(i)$$

When 2Ω resistance is inserted, then total resistance

$$= (R + 2) \Omega$$

$$\therefore V = I' (R + 2) = 4 (R + 2) \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$5 R = 4 (R + 2)$$

$$\therefore R = 8 \Omega$$

25. (c) Let S be the large and R be the smaller resistance.

From formula for meter bridge

$$S = \left(\frac{100 - l}{l} \right) R = \frac{100 - 20}{20} R = 4 R$$

$$\text{Again, } S = \left(\frac{100 - l}{l} \right) (R + 15)$$

$$= \frac{100 - 40}{40} (R + 15)$$

$$= \frac{3}{2} (R + 15)$$

$$\therefore 4 R = \frac{3}{2} (R + 15)$$

$$\Rightarrow \frac{8R}{3} - R = 15$$

$$\Rightarrow \frac{5R}{3} = 15$$

$$\Rightarrow R = 9 \Omega$$

26. (b) If we take $R_1 = 4 \Omega$, $R_2 = 12 \Omega$, then in series resistance

$$R_s = R_1 + R_2 = 4 + 12 = 16 \Omega$$

$$\text{In parallel, resistance, } R_p = \frac{4 \times 12}{4 + 12} = 3 \Omega$$

$$\text{So, } R_s = 4 \Omega$$

$$\text{and } R_p = 12 \Omega$$

25

Magnetic Field

Concept of Magnetic Field

In 1820, Oersted observed that a compass needle suffers a deflection when brought near a current carrying wire. This means that electric current (electric charge in motion) gives rise to magnetism.

Biot-Savart's Law

Biot-Savart's law gives the magnitude of magnetic field at any point due to a current carrying conductor. This law is although for an infinitesimally small conductor, yet it can be used for long conductors. According to Biot-Savart's law, magnetic field $d\mathbf{B}$ at any point P due to a current element $i d\mathbf{l}$ is

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{i(d\mathbf{l} \times \hat{\mathbf{r}})}{r^2} = \frac{\mu_0}{4\pi} \cdot \frac{i(d\mathbf{l} \times \mathbf{r})}{r^3}$$
$$d\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{idl \sin \theta}{r^2} \text{ or } d\mathbf{B} = k \frac{idl \sin \theta}{r^2} \quad \left(\because \frac{\mu_0}{4\pi} = k \right)$$

As a moving charge is equivalent to a current, then

$$i d\mathbf{l} = \frac{q}{dt} d\mathbf{l} = q \frac{d\mathbf{l}}{dt} = q\mathbf{v}$$

So, in terms of charge and its velocity, $d\mathbf{B} = \frac{\mu_0}{4\pi} q \frac{(\mathbf{v} \times \mathbf{r})}{r^3}$

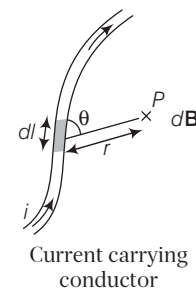
Total field can be found by summing *via* integration.

i.e.
$$B = \int d\mathbf{B} = \frac{\mu_0 i}{4\pi} \cdot \int \frac{dl \sin \theta}{r^2}$$

where, μ_0 = absolute permeability of air or vacuum.

It has exact value of $4\pi \times 10^{-7} \text{ Wb/A} \cdot \text{m}$ or $1.26 \times 10^{-6} \text{ T} \cdot \text{m/A}$.

Its other units are H/m and N/A^2 .



Applications of Biot-Savart's Law

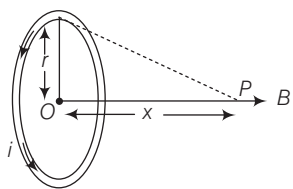
Let us consider few applications of Biot-Savart's law

1. Magnetic Field due to a Circular Current Carrying Loop

If a coil of radius r carries a current i , then magnetic field on its axis at a distance x from its centre is given by

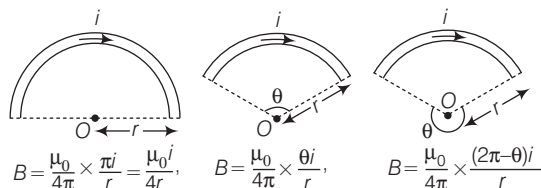
$$B_{\text{axis}} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi N i r^2}{(x^2 + r^2)^{3/2}}$$

where, N = number of turns in the coil.



2. Magnetic Field due to a Current Carrying Circular Arc

Magnetic field at centre O of a circular arc as given by Biot-Savart's law is



If magnetic field at the centre of circular coil is denoted by B_0 , then

$$B_0 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi i}{r}$$

Magnetic field at the centre of arc which is making an angle θ at the centre is $B_{\text{arc}} = \left(\frac{B_0}{2\pi}\right) \cdot \theta$.

3. Magnetic Field in Concentric Circular Loops

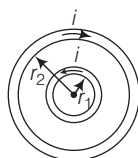
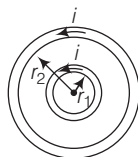
(a) For coplanar and concentric loops, magnetic field is either in same or in different directions.

If currents are in same direction, then magnetic field

$$B_1 = \frac{\mu_0}{4\pi} \times 2\pi i \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

If currents are in opposite directions, then magnetic field

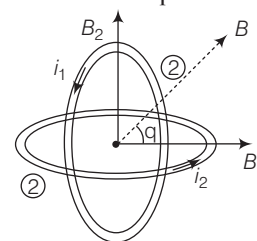
$$B_2 = \frac{\mu_0}{4\pi} \times 2\pi i \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$



(b) For non-coplanar and concentric coils, magnetic field is resultant of fields of both the coils. If planes of coils are perpendicular to each other, then magnetic field at common centre is

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2r} \sqrt{i_1^2 + i_2^2}$$

Also, the direction of B with direction of B_1 is $\tan \theta = \frac{B_2}{B_1}$.



Ampere's Circuital Law

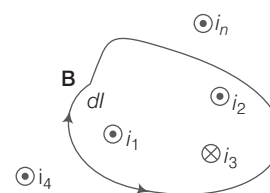
According to Ampere's circuital law, the line integral of the magnetic field B around any closed path is equal to μ_0 times the net current I threading through the area enclosed by the closed path. Mathematically,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \Sigma I$$

Now, consider the diagram shown

Here, $\Sigma I = i_1 + i_2 - i_3$

Hence, $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \cdot (i_1 + i_2 - i_3)$

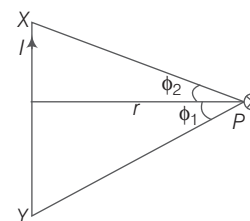


Applications of Ampere's Circuital law

The following are the few important applications of Ampere's circuital law given as below:

1. Magnetic Field due to a Current Carrying Straight Wire

The magnetic field due to a current carrying wire of finite length at a point P situated at a normal distance r is given from Ampere's circuital law, $\int B \cdot dl = \mu_0 \Sigma I$



$$B = \frac{\mu_0 I}{4\pi r} (\sin \phi_1 + \sin \phi_2)$$

If point P lies symmetrically on the perpendicular bisector of wire XY , then $\phi_1 = \phi_2 = \phi$ (say) and hence

$$B = \frac{\mu_0 I}{4\pi r} \cdot 2 \sin \phi = \frac{\mu_0 I \sin \phi}{2\pi r}$$

For a wire of infinite length, $\phi_1 = \phi_2 = 90^\circ$ and hence

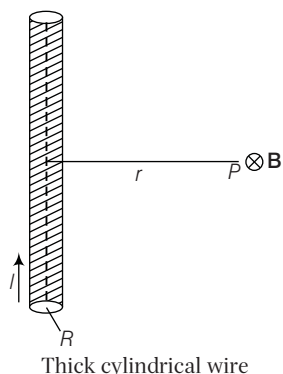
$$B = \frac{\mu_0 I}{2\pi r}$$

2. Magnetic Field due to a Thick (Cylindrical) Wire

Let us consider a thick cylindrical wire of radius R and infinite length, which carries a current I . Then, magnetic field at a point outside the wire $B = \frac{\mu_0 I}{2\pi r}$, where r is the distance of given point from centre of wire and $r > R$.

Magnetic field at a point inside the wire at a distance r from centre of wire ($r < R$) is given by

$$B = \frac{\mu_0 I}{2\pi} \cdot \frac{r}{R^2}$$



Magnetic field inside a hollow current carrying conductor is zero.

3. Magnetic Field due to a Solenoid

For a solenoid coil of infinite length at a point on its axial line, the magnetic field is given as $B = \mu_0 n I$

where, n is the number of turns per unit length.

At the end of solenoid, $B = \frac{1}{2} \mu_0 n I$

At the end field is half of at the centre this is called end effect.

Toroidal Solenoids

For a toroid (i.e. a ring shaped closed solenoid), magnetic field at any point within the core of toroid is $B = \mu_0 n I$,

where, $n = \frac{N}{2\pi R}$ and R = radius of toroid.

Motion of a Charged Particle in a Uniform Magnetic Field

If a particle carrying a positive charge q and moving with velocity v enters a magnetic field B , then it experiences a force F which is given by the expression

$$F = q(\mathbf{v} \times \mathbf{B}) \text{ or } F = qvB \sin \theta$$

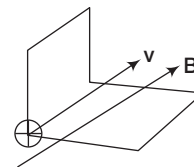
The force F is always perpendicular to both the velocity v and the field B .

Trajectory (path) of the charged particle depends on the angle between v and B .

Case I $\mathbf{v} \parallel \mathbf{B}$, $\theta = 0^\circ$

Then, force on charged particle

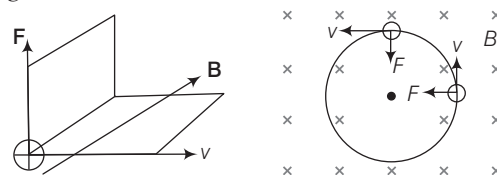
$$F = q(\mathbf{v} \times \mathbf{B}) = 0$$



So, path of particle is a straight line and it passes undeflected through the region of magnetic field.

Case II $\mathbf{v} \perp \mathbf{B}$, $\theta = 90^\circ$

Force is maximum $|F| = Bqv$ and it provides centripetal force and path of the particle is a circle as given below.



$$\text{So, } \frac{mv^2}{r} = Bqv \Rightarrow \frac{v}{r} = \omega = \frac{Bq}{m}$$

$$\Rightarrow T = \text{Time period of rotation} = \frac{2\pi}{\omega} = \frac{2\pi m}{Bq}$$

$$\text{and } f = \text{frequency of rotation} = \frac{1}{T} = \frac{Bq}{2\pi m}$$

which is independent of radius of path and velocity of charged particle.

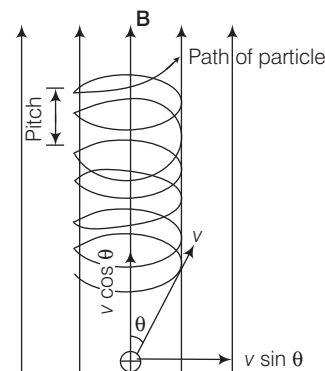
Case III $\mathbf{v}$ is at some angle except 90° with $\mathbf{B}$.

If velocity is at some angle θ with direction of B , then due to parallel component $v \cos \theta$, charged particle moves parallel to the direction of field B .

Also, due to perpendicular component $v \sin \theta$, it moves in a circular path. So, path of particle is spiral in shape.

For the particle, T = time period = $2\pi m / Bq$

$$\text{and } \text{pitch} = \frac{2\pi m}{Bq} \times v \cos \theta$$



Cyclotron

It is a device used to accelerate positively charged particles e.g. proton, deuteron, α -particle and other heavy ions to high energy of 100 MeV or more.

Time taken by charged particle to describe a semi-circular path in the dee is given by

$$t = \frac{\pi r}{v} = \frac{\pi m}{qB}$$

If T is time period of oscillating charged particle, then

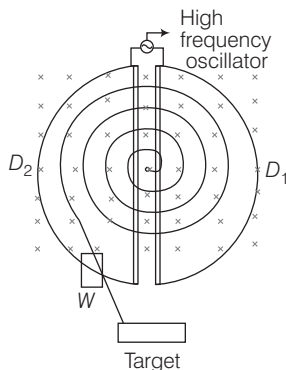
$$T = 2t = \frac{2\pi m}{qB}$$

The cyclotron frequency, D_2

$$v = \frac{1}{T} = \frac{Bq}{2\pi m}$$

Maximum energy gained by the charged particle, $E_{\max} = \left(\frac{B^2 r_0^2 q^2}{2m} \right)$

where, r_0 = maximum radius of the circular path followed by the positive ion.



Magnetic Force on a Current Carrying Conductor

If a current carrying conductor is placed in a magnetic field B , then a small current element $I d\mathbf{l}$ experiences a force given by

$$d\mathbf{F}_m = I d\mathbf{l} \times \mathbf{B}$$

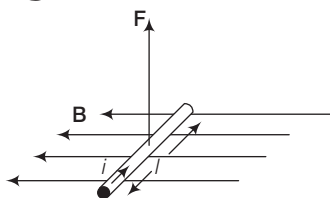
and the total force experienced by whole current carrying conductor will be

$$\mathbf{F}_m = \int d\mathbf{F}_m = \int I(d\mathbf{l} \times \mathbf{B})$$

The direction of force when current element $I d\mathbf{l}$ and B are perpendicular to each other can also be determined by applying Fleming's left hand rule or right hand thumb rule.

Force on a Straight Wire

If a current carrying straight conductor (length l) is placed in a uniform magnetic field B such that it makes an angle θ with the direction of field, then force experienced by it is $F = B i l \sin \theta$.



(i) If $\theta = 0^\circ$, then $F = 0$

(ii) If $\theta = 90^\circ$, then $F_{\max} = B i l$

(iii) In general, $F = i(l \times B)$ or $|F| = B i l \sin \theta$

Force between Two Parallel Current Carrying Conductors

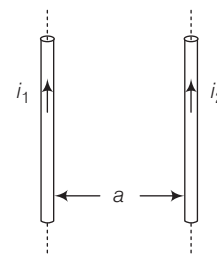
When two long straight conductors carrying currents i_1 and i_2 are placed parallel at distance of a from each other, then a mutual force of attraction acts between them which is given by

$$F_{12} = F_{21} = F = \frac{\mu_0}{4\pi} \cdot \frac{2i_1 i_2}{a} \times l$$

where, l is the length of that portion of the conductor on which force is to be calculated.

Hence, force per unit length

$$\frac{F}{l} = \frac{\mu_0}{4\pi} \cdot \frac{2i_1 i_2}{a} \text{ N/m}$$



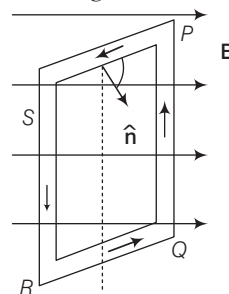
Torque Experienced by a Current Carrying Loop in a Uniform Magnetic Field

In the given figure, when a current carrying coil (loop) having N turns and area A , is placed in a uniform field B , in such a way that the normal ($\hat{n}$) to the coil makes an angle θ with the direction of B , then the coil experiences a torque given by

$$\tau = N B i A \sin \theta$$

$$PQ = RS = a \Rightarrow QR = SP = b \text{ or } \tau = \mathbf{M} \times \mathbf{B}$$

where, $M = N i A$ called magnetic moment of the loop.



Case I τ is zero, when $\theta = 0$, i.e. when the plane of the coil is perpendicular to the field.

Case II τ is maximum, when $\theta = 90^\circ$, i.e. the plane of the coil is parallel to the field. $\tau_{\max} = N B i A$.

The above expression is valid for any shape of coil.

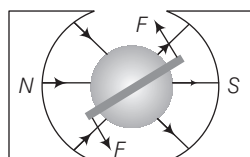
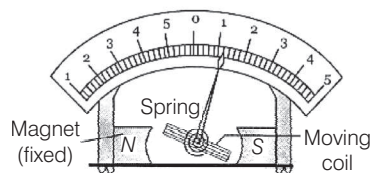
Work Done

If the coil is to be rotated through an angle θ from its equilibrium position ($\theta = 0^\circ$), then required work done, $W = M B (1 - \cos \theta)$. It is maximum when $\theta = 180^\circ$

$$W_{\max} = 2 M B$$

Moving Coil Galvanometer (MCG)

In a moving coil galvanometer, the coil is suspended between the pole pieces of a strong horse-shoe magnet. The pole pieces are made cylindrical and a soft iron cylindrical core is placed within the coil without touching it. This makes the field radial. In such a field, the plane of the coil always remains parallel to the field. When current will pass through this coil, then torque will be experienced by this coil.



Moving coil galvanometer

$$F = NBil \Rightarrow \tau_{\text{def}} = NBiA$$

Suppose, N = number of turns in the coil,

A = area of the coil,

B = magnetic field induction of radial magnetic field in which the coil is suspended.

As the coil deflects, a restoring torque is set up in the suspension fibre. If α is the angle of twist, then the restoring torque is

$$\tau_{\text{rest}} = C\alpha$$

where, C is the torsional constant of the fibre.

When the coil is in equilibrium, then $NBiA = C\alpha$

$$i = \frac{C}{NBA} \alpha \Rightarrow i = k\alpha$$

where, $k = \frac{C}{NBA}$ is the galvanometer constant. This linear relationship between i and α makes the moving coil galvanometer useful for current measurement and detection.

- (i) **Current sensitivity (S_i)** The current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit current flowing through it.

$$S_i = \frac{\alpha}{i} \Rightarrow S_i = \frac{NBA}{C}$$

Thus, in order to increase the sensitivity of a moving coil galvanometer, N , B and A should be increased and C should be decreased.

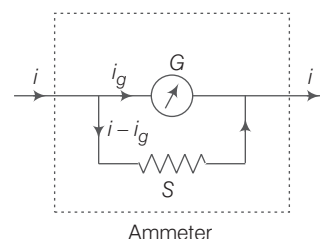
Quartz fibres can also be used for suspension of the coil because they have large tensile strength and very low value of k .

- (ii) **Voltage sensitivity (S_V)** Voltage sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit potential difference applied to it.

$$S_V = \frac{\alpha}{V} = \frac{\alpha}{iR} = \frac{S_i}{R} \Rightarrow S_V = \frac{NBA}{RC}$$

Conversion of Galvanometer into Ammeter

An ammeter is made by connecting a low resistance S in parallel with a moving coil galvanometer G . S is known as shunt.



Ammeter

Then, from circuit

$$i_g \times G = (i - i_g) \times S$$

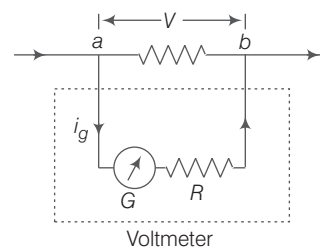
$$\Rightarrow S = \frac{i_g}{i - i_g} G$$

So, $S \ll G$, only a small fraction of current goes through the galvanometer.

Conversion of Galvanometer into Voltmeter

A voltmeter is made by connecting a resistor of high resistance R in series with a moving coil galvanometer G .

From the circuit



Voltmeter

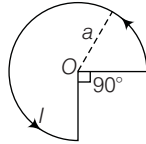
$$i_g = \frac{V}{G + R}$$

$$\Rightarrow G + R = \frac{V}{i_g}$$

$$R = \frac{V}{i_g} - G$$

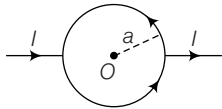
Practice Exercise

1. For the arrangement as shown in the figure, the magnetic induction at the centre is



- a. $\frac{3\mu_0 I}{4a}$ b. $\frac{\mu_0 I}{4\pi a} (1 + \pi)$ c. $\frac{\mu_0 I}{4\pi a}$ d. $\frac{3\mu_0 I}{8a}$

2. If the resistance of the upper half of a rigid loop is twice that of the lower half, the magnitude of magnetic induction at the centre is equal to



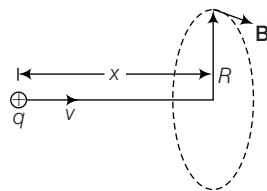
- a. zero b. $\frac{\mu_0 I}{4a}$
c. $\frac{\mu_0 I}{12a}$ d. None of these

3. The earth's magnetic field at a certain point is 0.70 gauss. This field is to be annulled by the magnetic field at the centre of a circular conducting loop 5.0 cm in radius. The required current is about
a. 0.66 A b. 5.6 A c. 0.28 A d. 2.8 A

4. Three circular concentric wires of radii a , $2a$ and $3a$ are carrying currents $3I$, $2I$ and I in same manner. Find the magnetic field at the common centre.

- a. $\frac{13\mu_0 I}{6a}$ b. $\frac{\mu_0 I}{6a}$
c. $\frac{\mu_0 I}{a}$ d. None of these

5. A charge $q (> 0)$ moves towards the centre of a circular loop of radius R along its axis. The magnitude of B along the periphery of the loop is

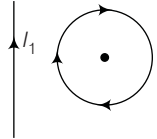


- a. zero b. $\frac{\mu_0}{4\pi} \frac{qvR}{\sqrt{(R^2 + x^2)^3}}$
c. $\frac{qvR}{\sqrt{R^2 + x^2}}$ d. None of these

6. A current carrying loop is free to turn in a uniform magnetic field. The loop will then come into equilibrium when its plane is inclined at

- a. 0° to the direction of the field
b. 45° to the direction of the field
c. 90° to the direction of the field
d. 135° to the direction of the field

7. In the given figure, the loop is fixed but straight wire can move. The straight wire will



- a. remain stationary
b. move towards the loop
c. move away from the loop
d. rotates about the axis

8. The dimensions of $\frac{1}{\sqrt{\mu_0 \epsilon_0}}$ are same as

- a. $\frac{E}{B}$ b. $\frac{B}{E}$
c. $\frac{E^2}{B^2}$ d. $\sqrt{\left(\frac{E}{B}\right)}$

9. A charged particle is moving in a uniform magnetic field and losses 4% of its KE. The radius of curvature of its path changes by

- a. 2%
b. 4%
c. 10%
d. None of the above

10. A charged particle of mass m and charge q , a uniform magnetic field B acting into the plane. The plane is frictional having coefficient of friction μ . The speed of charged particle just before entering into the region is v_0 .

The radius of curvature of the path after the time $\frac{v_0}{2\mu g}$ is

- a. $\frac{mv_0}{qB}$ b. $\frac{mv_0}{2qB}$
c. $\frac{mv_0}{4qB}$ d. None of these

11. A γ -ray photon is passing near a nucleus and breaks into an electron and positron. The region contains a uniform magnetic field B perpendicular to the plane of motion. Find the time after which they again converted into γ -ray. The force of electrostatic interaction and gravitational interaction may be neglected

- a. $\frac{2\pi m}{eB}$ b. $\frac{\pi m}{2eB}$
c. $\frac{4\pi m}{eB}$ d. None of these

12. If a charged particle of charge $5\mu\text{C}$ and mass 5 g is moving with constant speed 5 m/s in a uniform magnetic field B on a curve $x^2 + y^2 = 25$, where x and y are in metre. The value of magnetic field will be

- a. 1 tesla
b. 1 kilo tesla along z -axis
c. 5 kilo tesla along the x -axis
d. 1 kilo tesla along any line in the x - y plane

13. A charged particle of mass m and charge q is accelerated through a potential difference of V volt. It enters a region of uniform magnetic field which is directed perpendicular to the direction of motion of the particle. The particle will move on a circular path of radius given by

a. $\sqrt{\left(\frac{Vm}{qB^2}\right)}$ b. $\frac{2Vm}{qB^2}$
 c. $\sqrt{\left(\frac{2Vm}{q}\right)\left(\frac{1}{B}\right)}$ d. $\sqrt{\left(\frac{Vm}{q}\right)\left(\frac{1}{B}\right)}$

14. A circular flexible loop of wire of radius r carrying a current I is placed in a uniform magnetic field B . If B is doubled, then tension in the loop

- a. remains unchanged b. is doubled
 c. is halved d. becomes 4 times

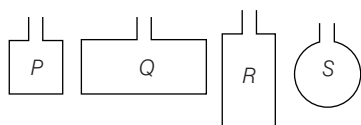
15. A non-relativistic proton beam passes without deviation through the region of space where there are uniform transverse mutually perpendicular electric and magnetic fields with $E = 120$ kV/m and $B = 50$ mT. Then the beam strikes a grounded target. Find the force with which the beam acts on the target if the beam current is equal to $I = 0.80$ mA.

- a. $80 \mu\text{N}$ b. $25 \mu\text{N}$ c. $20 \mu\text{N}$ d. $35 \mu\text{N}$

16. An α -particle and a proton are both simultaneously projected into a region of constant magnetic field, perpendicular to the direction of the field but in opposite directions. After 10^{-8} s, it is found that the velocity of α -particle has changed in the direction by 45° . The angle between the velocity vectors of the α -particle and the proton is

- a. 90° b. 45° c. $45^\circ + 90^\circ$ d. $\frac{(45^\circ + 90^\circ)}{2}$

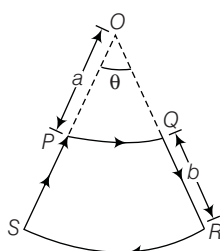
17. Four wires each of length 2 m are bent into four loops P, Q, R and S and then suspended into uniform magnetic field. Some current is passed in each loop. Which statement is correct?



- a. Couple on loop P will be the highest
 b. Couple on loop Q will be the highest
 c. Couple on loop R will be the highest
 d. Couple on loop S will be the highest

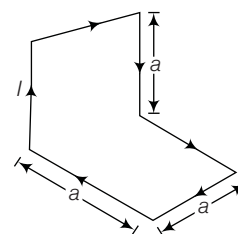
18. The magnetic moment of the current carrying loop shown in the figure is equal to

a. $\frac{I(b^2 + 2ab)\theta}{2}$
 b. $Iab\theta$
 c. $\frac{I(a^2 + ba)\theta}{2}$
 d. None of the above



19. The magnitude of magnetic moment of the current loop in the figure is

- a. Ia^2
 b. $\sqrt{2}Ia^2$
 c. zero
 d. None of the above



20. A length l of a wire is bent to form a circular coil of some turns. A current I is then established in the coil and it is placed in a uniform magnetic field B . The maximum torque that acts on the coil is

- a. IBl^2 b. $4\pi IBl^2$ c. $\frac{Il^2B}{4\pi}$ d. zero

21. The ratio of the energy required to set up in a cube of side 10 cm, a uniform magnetic field of 4 Wb/m^2 and a uniform electric field of 10^6 V/m is

- a. 1.44×10^7 b. 1.44×10^{-5} c. 1.44×10^6 d. 1.44×10^3

22. A magnetic field $4 \times 10^{-3} \text{ kT}$ exerts a force $(4\hat{i} + 3\hat{j}) \times 10^{-10} \text{ N}$ on a particle having a charge 10^{-9} C and going on the x - y plane. The velocity of the particle is

- a. $-75\hat{i} + 100\hat{j}$ b. $-100\hat{i} + 75\hat{j}$
 c. $25\hat{i} + 2\hat{j}$ d. $2\hat{i} + 25\hat{j}$

23. A uniform magnetic field $B = B_0\hat{j}$ exists in space. A particle of mass m and charge q is projected towards X -axis with speed v from a point $(a, 0, 0)$. The maximum value of v for which the particle does not hit the Y - Z plane is

- a. $\frac{Bqa}{m}$ b. $\frac{Bqa}{2m}$ c. $\frac{Bq}{am}$ d. $\frac{Bq}{2am}$

24. Through two parallel wires A and B , 10 A and 2 A of currents are passed respectively in opposite directions. If the wire A is infinitely long the length of the wire B is 2 m, then force on the conductor B , which is situated at 10 cm distance from A , will be

- a. $8 \times 10^{-7} \text{ N}$ b. $8 \times 10^{-5} \text{ N}$
 c. $4 \times 10^{-7} \text{ N}$ d. $4 \times 10^{-5} \text{ N}$

25. Two parallel long wires A and B carry currents I_1 and I_2 ($I_2 < I_1$). When I_1 and I_2 are in the same direction, the magnetic field at a point mid-way between the wires is $10 \mu\text{T}$. If I_2 is reversed, the field becomes $30 \mu\text{T}$. The ratio I_1/I_2 is

- a. 1 b. 3
 c. 2 d. 4

26. A circular coil of 100 turns and effective diameter 20 cm carries a current of 0.5 A. It is to be turned in a magnetic field $B = 2 \text{ T}$ from a position in which θ equals zero to one in which θ equals 180° . The work required in this process is

- a. π joule b. 2π joule
 c. 4π joule d. 8π joule

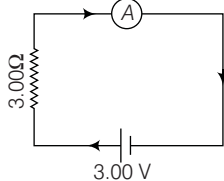
27. Two long parallel wires are placed on a smooth horizontal table. They have equal and opposite charges. Work required to increase separation between wires from a to $2a$ if magnitude of charge per unit length on them is λ , will be
- a. $\frac{\lambda^2 \ln 2}{4\pi\epsilon_0}$ b. $\frac{\lambda^2}{\pi\epsilon_0} \ln 2$ c. $\frac{\lambda^2}{4\pi\epsilon_0 a}$ d. $\frac{\lambda^2}{2\pi\epsilon_0} \ln 2$

28. Two long straight parallel conductors are separated by a distance of 5 cm and carrying current 20 A. What work per unit length of a conductor must be done to increase the separation between conductors to 10 cm, if the current flows in the same direction?
- a. $8 \times 10^{-5} \log_e 2$ b. $\log_e 2$
c. $10^{-7} \log_e 2$ d. None of these

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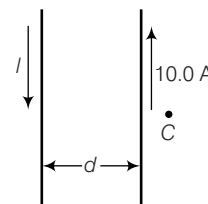
1. A narrow beam of protons and deuterons, each having the same momentum, enters a region of uniform magnetic field directed perpendicular to their direction of momentum. The ratio of the radii of the circular paths described by them is [2014]
a. 1:2 b. 1:1 c. 2:1 d. 1:3
2. A solenoid of length 1.0 m has a radius of 1 cm and has a total of 1000 turns wound on it. It carries a current of 5 A. If an electron was to move with a speed of 10^4 ms^{-1} along the axis of this current carrying solenoid, then force experienced by this electron is [2014]
a. 2 N b. 1.2 N
c. zero d. 2.5 N
3. Magnetic field [2014]
a. can increase the speed of charged particle
b. can accelerate a charged particle
c. Both a and b are correct
d. Both a and b are incorrect
4. A square coil of side 10 cm has 20 turns and carries a current of 12 A. The coil is suspended vertically and the normal to the plane of the coil, makes an angle θ with the direction of a uniform horizontal magnetic field of 0.80 T. If the torque, experienced by the coil, equals 0.96 N-m, the value of θ is [2014]
a. 0° b. $\frac{\pi}{2}$ rad c. $\frac{\pi}{3}$ rad d. $\frac{\pi}{6}$ rad

5. For the circuit (figure), the current is to be measured. The ammeter shown is a galvanometer with a resistance $R_G = 60.00 \Omega$ converted to an ammeter by a shunt resistance $r_S = 0.02 \Omega$. The value of the current is [2014]
a. 0.79 A b. 0.29 A
c. 0.99 A d. 0.8 A

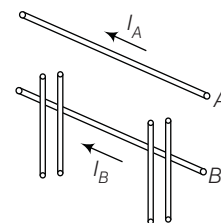


6. A square shaped current loop of side length L and carrying current I lies in a uniform magnetic field B acting perpendicular to the plane of square loop and directed inward. The net magnetic force acting on current loop is [2013]
a. IBL b. $4IBL$ c. zero d. $2IBL$

7. Two parallel conductors carry current in opposite directions as shown in figure. One conductor carries a current of 10.0 A. Point C is a distance $\frac{d}{2}$ to the right of the 10.0 A current. If $d = 18 \text{ cm}$ and I is adjusted so that the magnetic field at C is zero, the value of the current I is [2013]
a. 10.0 A b. 30.0 A
c. 8.0 A d. 18.0 A



8. Two long, parallel conductors carry currents in the same direction, as shown in figure. Conductor A carries a current of 100 A and is held firmly in position. Conductor B carries a current I_B and is allowed to slide freely up and down (parallel to A) between a set of non-conducting guides. The mass per unit length of conductor B is 0.1 g/cm and the distance between the two conductors is 5 cm. If system of conductors is in equilibrium, the value of current I_B is [2013]
a. 250 A b. 240 A c. 220 A d. 230 A



9. A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field B . The magnitude of B (in tesla) is (assume, $g = 9.8 \text{ ms}^{-2}$) [2012]
a. 2 b. 1.5 c. 0.55 d. 0.65
10. The intensity of the magnetic induction field at the centre of a single turn circular coil of radius 5 cm carrying current of 0.9 A is [2012]
a. $36\pi \times 10^{-7} \text{ T}$ b. $9\pi \times 10^{-7} \text{ T}$
c. $36\pi \times 10^{-6} \text{ T}$ d. $9\pi \times 10^{-6} \text{ T}$
11. The maximum current that can be measured by a galvanometer of resistance 40Ω is 10 mA. It is converted into a voltmeter that can read upto 50 V. The resistance to be connected in series with the galvanometer (in ohms) is [2012]
a. 2010 b. 4050
c. 5040 d. 4960

12. A galvanometer of resistance 100Ω gives full scale deflection with 0.01 A current. How much resistance should be connected in parallel to convert it into an ammeter of range 10 A ? [2011]
 a. 0.100Ω b. 1.00Ω c. 10.00Ω d. 100.00Ω
13. A galvanometer of 50Ω resistance has 25 divisions. A current of $4 \times 10^{-4}\text{ A}$ gives a deflection of one division. To convert this galvanometer into a voltmeter having a range of 25 V , it should be connected with a resistance of [2011]
 a. 2500Ω as a shunt b. 2950Ω as a shunt
 c. 2550Ω in series d. 2450Ω in series
14. The cyclotron frequency of an electron gyrating in a magnetic field of 1 T is approximately [2011, 2007]
 a. 28 MHz b. 280 MHz c. 2.8 GHz d. 28 GHz
15. A current carrying circular coil is bent so as to convert it into a double loop, both the loops being concentric and are carrying current in the same direction. If B is the initial magnetic field at the centre, the final magnetic field at the centre will be [2010]
 a. zero b. B c. $2B$ d. $4B$
16. In order to increase the current sensitivity of moving coil galvanometers, the [2010]
 a. number of turns of the coil should be increased
 b. the strength of the magnetic field should be increased
 c. area of coil should be increased
 d. All of the above
17. In an ammeter 10% of main current is passing through the galvanometer. If the resistance of the galvanometer is G , then the shunt resistance, in ohms is [2005]
 a. $9G$ b. $\frac{G}{9}$ c. $90G$ d. $\frac{G}{90}$
18. Due to magnetic field of earth, charged particles coming from outer space. [2009]
 a. required greater kinetic energy to reach the equator than the poles
 b. required lesser kinetic energy to reach the equator than the poles
 c. can never reach the equator
 d. can never reach the poles
19. In a galvanometer, 5% of the total current in the circuit passes through it. If the resistance of the galvanometer is G , the shunt resistance S connected to galvanometer is [2008]
 a. $19G$ b. $\frac{G}{19}$ c. $20G$ d. $\frac{G}{20}$
20. Two concentric coils of 10 turns each are placed in the same plane. Their radii are 20 cm and 40 cm and carry 0.2 A and 0.3 A current respectively in opposite directions. The magnetic induction (in tesla) at the centre is [2008]
 a. $\frac{3}{4}\mu_0$ b. $\frac{5}{4}\mu_0$ c. $\frac{7}{4}\mu_0$ d. $\frac{9}{4}\mu_0$
21. A current carrying small loop behaves like a small magnet. If A be its area and M its magnetic moment, the current in the loop will be [2007]
 a. M/A b. A/M
 c. MA d. AM^2
22. A magnet of magnetic moment 20 CGS units is freely suspended in a uniform magnetic field of intensity 0.3 CGS unit. The amount of work done in deflecting it by an angle of 30° in CGS unit is [2007]
 a. 6 b. $3\sqrt{3}$
 c. $3(2 - \sqrt{3})$ d. 3
23. In a magnetic field of 0.05 T area of coil changes from 101 cm^2 to 100 cm^2 without changing the resistance which is 2Ω . The amount of charge that flow during this period is [2006]
 a. $25 \times 10^{-6}\text{ C}$
 b. $2 \times 10^{-6}\text{ C}$
 c. 10^{-6} C
 d. $8 \times 10^{-6}\text{ C}$
24. An electric field of 1500 V/m and a magnetic field of 0.40 Wb/m^2 act on a moving electron. The minimum uniform speed along a straight line, the electron could have is [2005]
 a. $16 \times 10^{15}\text{ m/s}$
 b. $6 \times 10^{-16}\text{ m/s}$
 c. $3.75 \times 10^3\text{ m/s}$
 d. $3.75 \times 10^2\text{ m/s}$

Answer with Solutions

Practice Exercise

1. (d)

2. (c) $\because I_1 R_1 = I_2 R_2$

$$\frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{R_2}{2R_2} = \frac{1}{2}$$

$$\therefore I_2 = 2I_1$$

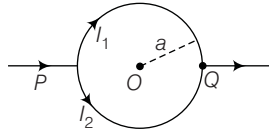
$$\text{and } I = I_1 + I_2 = I_1 + 2I_1 = 3I_1$$

$$\therefore I_1 = \frac{I}{3} \text{ and } I_2 = \frac{2}{3}I$$

$\therefore$ Magnetic field at the centre of coil

$$= \frac{\mu_0 I_1}{4\pi a} \times \pi - \frac{\mu_0 I_2}{4\pi a} \times \pi$$

$$\therefore B = \frac{\mu_0 \pi}{4\pi a} (I_1 - I_2) = \frac{\mu_0}{4a} \left(\frac{I}{3} - \frac{2}{3}I \right) = \frac{\mu_0}{4a} \left(-\frac{I}{3} \right) = -\frac{\mu_0 I}{12a} = \frac{\mu_0 I}{12a}$$



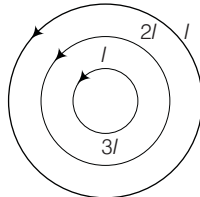
3. (b)

4. (a) $B = B_1 + B_2 + B_3$

$$= \frac{\mu_0 (3I)}{2a} + \frac{\mu_0 (2I)}{2(2a)} + \frac{\mu_0 I}{2(3a)}$$

$$= \frac{3\mu_0 I}{2a} + \frac{\mu_0 I}{2a} + \frac{\mu_0 I}{6a}$$

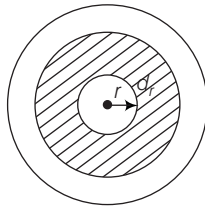
$$= \frac{9\mu_0 I}{6a} + \frac{3\mu_0 I}{6a} + \frac{\mu_0 I}{6a} = \frac{13\mu_0 I}{6a}$$



5. (d) $\mathbf{B} = \frac{\mu_0}{4\pi} q \frac{\mathbf{v} \times \mathbf{r}}{r^3}$

$$B = \frac{\mu_0 q v r \sin \theta}{4\pi (R^2 + x^2)^{\frac{3}{2}}}$$

$$B = \frac{\mu_0 q}{4\pi} \frac{vR}{(R^2 + x^2)^{\frac{3}{2}}} [\because r \sin \theta = R]$$



6. (a)

7. (b)

8. (a)

9. (a) $r = \frac{mv}{qB}$

$$KE = K = \frac{1}{2} mv^2$$

$$\therefore mv = \sqrt{2Km}$$

$$\therefore r = \frac{mv}{qB} = \frac{\sqrt{2Km}}{qB} \text{ or } r \propto \sqrt{K}$$

or $r = eK^{\frac{1}{2}}$, where e is a constant.

$$\text{or } \frac{dr}{dr} = e \frac{dK^{\frac{1}{2}}}{dK} = \frac{e}{2} \frac{ec}{\sqrt{K}} \text{ or } \frac{ec}{2\sqrt{K}} \frac{dK}{dr} = \sqrt{K}$$

$$\text{or } e \frac{\Delta K}{\Delta r} = 2\sqrt{K} \text{ or } \frac{\Delta r}{r} = \frac{c\Delta K}{2\sqrt{K}c\sqrt{K}} = \frac{\Delta K}{2K}$$

$$\text{or } \frac{\Delta r}{r} \times 100 = \frac{\Delta K}{2K} \times 100 = 2\% \quad \left[\because \frac{\Delta K}{K} \times 100 = 4\% \right]$$

10. (b) Here, centripetal acceleration is provided by magnetic force but tangential acceleration is provided by force of friction

$$\frac{mv^2}{r} = qvB$$

$$\therefore r = \frac{mv}{qB}$$

$$\text{Tangential acceleration} = a = \frac{-\mu mg}{m}$$

$$v = -\mu g \left(\frac{v_0}{2\mu g} \right) = \frac{v_0}{2}$$

$$\therefore r = \frac{mv}{qB} = \frac{mv_0}{2qB}$$

11. (a) Both positron and electron are moving on circular paths.

$$\therefore T = \frac{2\pi m}{eB}$$

12. (b)

13. (c)

14. (b)

15. (c) Since, proton moves with constant velocity. So, magnetic force on the proton is balanced by electric force.

$$\therefore qE = qvB \text{ or } E = vB$$

$$\text{The force on the target is } F = \frac{\Delta p}{\Delta t} = \frac{m v n}{\Delta t}$$

where, n = number of protons striking the target in time dt .

m = mass of proton

v = speed of each proton before striking the target

$$I = \frac{ne}{\Delta t} \Rightarrow \frac{n}{\Delta t} = \frac{I}{e}$$

$$\therefore F = \frac{m v I}{e} = \frac{m I \left[\frac{E}{B} \right]}{e}$$

$$F = \frac{1.6 \times 10^{-27} \times 120 \times 1000 \times 0.8 \times 10^{-3}}{50 \times 10^{-3} \times 1.6 \times 10^{-19}}$$

$$= 20 \times 10^{-6} \text{ N} = 20 \mu\text{N}$$

$$16. (c) \omega_\alpha = \frac{(2e)B}{4m} = \frac{1}{2} \frac{eB}{m}$$

$$\omega_p = \frac{eB}{m}$$

$$\theta_\alpha = 45^\circ$$

$\theta_p = 90^\circ$ from their original directions.

17. (d) Concept Couple = $I(\mathbf{A} \times \mathbf{B})$

More area means more couple.

$\therefore$ For a given perimeter, circle has maximum area.

18. (a) Magnetic moment

$$M = IA = I(A_2 - A_1)$$

$$= I \left\{ \frac{(a+b)^2}{2} \theta - \frac{a^2}{2} \theta \right\} = \frac{I(b^2 + 2ab)\theta}{2}$$

19. (b)

20. (c) $\tau = NIAB = N\pi r^2 B$

But, $N \times 2\pi r = l$

$$\therefore r = \frac{l}{2\pi N}$$

$$\therefore \tau = N\pi \left(\frac{l}{2\pi N} \right)^2 B = N\pi \frac{l^2}{4\pi^2 N^2} B = \frac{l^2 B}{4\pi N}$$

$$\therefore \tau_{\max} = \frac{l^2 B}{4\pi} \quad [N_{\min} = 1]$$

21. (c) $U_E = \frac{1}{2} \epsilon_0 E^2 \times \text{volume}$

$$U_B = \frac{B^2}{2\mu_0} \times \text{volume}$$

$$\therefore \frac{U_B}{U_E} = \left(\frac{B}{E} \right)^2 \frac{1}{\mu_0 \epsilon_0} = \left(\frac{4}{10^6} \right)^2 (3 \times 10^8)^2$$

$$= \frac{16}{10^{12}} \times 9 \times 10^{16} = 144 \times 10^4 = 1.44 \times 10^6$$

22. (a) From Lorentz force,

$$F = q(v \times B)$$

Given, $F = (4\hat{i} + 3\hat{j}) \times 10^{-10} \text{ N}, q = 10^{-9} \text{ C}$

$$B = 4 \times 10^{-3} \hat{k} \text{ T}$$

$$\therefore (4\hat{i} + 3\hat{j}) \times 10^{-10} = 10^{-9} (a\hat{i} + b\hat{j}) \times (4 \times 10^{-3}) \hat{k}$$

Solving, we get

$$a = -75, b = 100 \Rightarrow v = -75\hat{i} + 100\hat{j}$$

23. (a) 24. (b)

25. (c) $\frac{\mu_0}{4\pi} \frac{2I_1}{r} - \frac{\mu_0}{4\pi} \frac{2I_2}{r} = 10 \mu\text{T}$

$$\frac{\mu_0 2I_1}{4\pi r} + \frac{\mu_0 2I_2}{4\pi r} = 30 \mu\text{T}$$

On solving, $I_1 = 20 \text{ A}$ and $I_2 = 10 \text{ A}$

So, $I_1 / I_2 = 2$

26. (b)

27. (d) $W = \int_a^{2a} \frac{\lambda \cdot \lambda dr}{2\pi\epsilon_0 r} = \frac{\lambda^2}{2\pi\epsilon_0} \ln 2$

28. (a) The force per unit length is

$$F_0 = \frac{\mu_0 I_1 I_2}{2\pi r}$$

$$\begin{aligned} \therefore W &= \int F dr = \frac{\mu_0 I_1 I_2}{2\pi} \int_{5 \times 10^{-2}}^{10 \times 10^{-2}} \frac{dr}{r} \\ &= \frac{\mu_0 I_1 I_2}{2\pi} \ln 2 = 2 \times 10^{-7} \times 20 \times 20 \ln 2 \\ &= 8 \times 10^{-5} \ln 2 = 8 \times 10^{-5} \log_e 2 \end{aligned}$$

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1. (b) Since, the radius of circular path of a charged particle in magnetic field is $r = \frac{mv}{qB} = \frac{p}{qB}$

Now, the radius of circular path of charged particle of given momentum p and magnetic field B is given by $r \propto \frac{1}{q}$

But charge on both charged particles, protons and deuterons, is same. Therefore,

$$\frac{r_p}{r_D} = \frac{q_D}{q_p} = \frac{1}{1}$$

2. (c) Here, $L = 1 \text{ m}, N = 1000$

The number of turns per unit length

$$n = N/L = 1000 \text{ turn/m}$$

Magnetic field inside the solenoid

$$B = \mu_0 n I = \mu_0 \times 1000 \times 5 = 2\pi \times 10^{-3} \text{ T}$$

The direction of magnetic field is along the solenoid.

For electron, $q = -e, v = 10^4 \text{ ms}^{-1}$

Magnetic Lorentz force, $F = -evB \sin 0^\circ = 0$ as the angle between B and v is 0° .

3. (b) Magnetic field can accelerate a charged particle by changing the direction of its velocity but it cannot change the speed of charged particle as magnetic force always acts perpendicular to the velocity of charged particle.

4. (d) Area of coil, $A = \text{side}^2 = (0.1)^2 = 0.01 \text{ m}^2$,

Number of turns, $N = 20$, current, $I = 12 \text{ A}$,

Normal to the coil make an angle θ with the direction of B , magnetic field, $B = 0.80 \text{ T}$

Torque, experienced by the coil, $\tau = 0.96 \text{ N-m}$

Since, total torque on the coil, $\tau = (NIA)B \sin \theta$

Substituting the values in above formula, we get

$$0.96 \text{ N-m} = 20 \times 12 \text{ A} \times 0.01 \text{ m}^2 \times 0.80 \text{ T} \times \sin \theta$$

$$\sin \theta = \frac{0.96}{192} = \frac{1}{2}$$

$$\theta = \frac{\pi}{6} \text{ rad}$$

5. (c) $R_G = 60.00 \Omega$; shunt resistance, $r_s = 0.02 \Omega$

Total resistance in the circuit is $R_G + 3 = 63 \Omega$.

Hence, $I = 3 / 63 = 0.048 \text{ A}$

Resistance of the galvanometer converted to an ammeter is,

$$\frac{R_G r_s}{R_G + r_s} = \frac{60 \Omega \times 0.02 \Omega}{(60 + 0.02) \Omega} = 0.02 \Omega$$

Total resistance in the circuit $= 0.02 + 3 = 3.02 \Omega$

Hence, $I = 3 / 3.02 = 0.99 \text{ A}$

6. (c) Let the current be flowing in anti-clockwise direction as shown in figure.

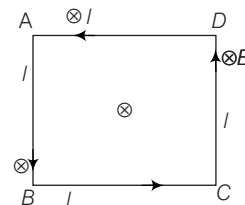
Now, magnetic force on $AD = F_1 = ILB$ inwards

(by Fleming's left hand rule)

Similarly, magnetic force on

$BC = F_2 = ILB$ inwards

Since, two forces are equal in magnitude and opposite in direction, therefore, they cancel out each other.



Also, magnetic force on $CD = F_3 = ILB$ inwards

(by Fleming's left hand rule)

Similarly, magnetic force on $AB = F_4 = ILB$ inwards.

Again, two forces are equal in magnitude and opposite in direction, therefore, they cancel out each other.

7. (b) The magnetic field at C due to first conductor is

$$B_1 = \frac{\mu_0}{2\pi} \frac{I}{3d/2} \quad (\text{since, point } C \text{ is separated by } d + \frac{d}{2} = \frac{3d}{2})$$

from 1st conductor). The direction of field is perpendicular to the plane of paper and directed outwards.

The magnetic field at C due to second conductor is

$$B_2 = \frac{\mu_0}{2\pi} \frac{10}{d/2} \quad (\text{since, point } C \text{ is separated by } \frac{d}{2} \text{ from 2nd}$$

conductor)

The direction of field is perpendicular to the plane of paper and directed inwards.

Since, direction of B_1 and B_2 at point C is in opposite direction and the magnetic field at C is zero, therefore,

$$B_1 = B_2$$

$$\frac{\mu_0}{2\pi} \frac{I}{3d/2} = \frac{\mu_0}{2\pi} \frac{10}{d/2}$$

On solving $I = 30.0 \text{ A}$

8. (a) When system of conductors is in equilibrium. The magnetic force of attraction per unit length between conductors = weight of conductor B per unit length.

$$\frac{\mu_0}{2\pi} \frac{I_A \times I_B}{d} = \frac{mg}{L} = \left(\frac{m}{L}\right)g$$

$$\frac{\mu_0}{2\pi} \frac{I_A \times I_B}{d} = \left(\frac{m}{L}\right)g$$

$$2 \times 10^{-7} \times \frac{100 \times I_B}{0.05} = (0.01 \text{ kg/m}) \times 10$$

On solving, we get $I_B = 250 \text{ A}$

9. (d) Magnetic force on straight wire

$$F = BIl \sin \theta = BIl \sin 90^\circ = BIl$$

For equilibrium of wire in mid-air,

$$F = mg$$

$$Bil = mg$$

$$\therefore B = \frac{mg}{il} = \frac{200 \times 10^{-3} \times 9.8}{2 \times 15} = 0.65 \text{ T}$$

10. (a) The intensity of magnetic induction field

$$B = \frac{\mu_0 i}{2r} = \frac{4\pi \times 10^{-7} \times 0.9}{2 \times 5 \times 10^{-2}}$$

$$B = 36\pi \times 10^{-7} \text{ T}$$

11. (d) To convert a galvanometer into voltmeter, the necessary value of resistance to be connected in series with the galvanometer is

$$R = \frac{V}{I_g} - G = \frac{50}{10 \times 10^{-3}} - 40 = 5000 - 40 = 4960 \Omega$$

12. (a) Shunt is given by

$$S = \frac{I_g \times R_g}{I - I_g} = \frac{0.01 \times 100}{10 - 0.01} = \frac{0.01 \times 100}{9.99} = 0.100 \Omega$$

13. (d) Here, $I_g = 25 \times 4 \times 10^{-4} \text{ A} = 10^{-2} \text{ A}$

To convert the galvanometer into a voltmeter, we must join a series resistance of

$$R = \frac{V}{I_g} - G = \frac{25}{10^{-2}} - 50 = 2500 - 50 = 2450 \Omega$$

14. (d) Cyclotron of frequency, $\nu = \frac{Bq}{2\pi m} = \frac{1 \times 1.6 \times 10^{-19}}{2\pi \times 9.1 \times 10^{-31}}$
 $= 2.8 \times 10^{10} \text{ Hz} = 28 \text{ GHz}$

15. (d) $B_{\text{initial}} = \frac{\mu_0 i}{2r}$

$$B_{\text{final}} = \frac{\mu_0 i}{2\left(\frac{r}{2}\right)} + \frac{\mu_0 i}{2\left(\frac{r}{2}\right)} \Rightarrow B_{\text{final}} = \frac{2\mu_0 i}{r} = 4B_{\text{initial}}$$

16. (d) Current sensitivity of the galvanometer can be increased by all three factors.

17. (b) Shunt resistance $S = \frac{I_g G}{I - I_g} = \frac{0.1 G}{1 - 0.1} = \frac{G}{9}$

18. (c) Due to magnetic force the charged particles get deviated. For positively, charged particles, due to east-west asymmetry of particles, more particles get deviated towards east, then towards west, but since they are deviated, hardly any charged particle will reach equator.

19. (b) Shunt of an ammeter,

$$S = \frac{I_g \times G}{I - I_g} = \frac{5 \times G}{100 - 5} = \frac{G}{19}$$

20. (b) Two coils carry currents in opposite directions, hence net magnetic field at centre will be difference of the two fields.

$$\text{i.e. } B_{\text{net}} = \frac{\mu_0}{4\pi} \cdot 2\pi N \left[\frac{i_1}{r_1} - \frac{i_2}{r_2} \right] = \frac{10\mu_0}{2} \left[\frac{0.2}{0.2} - \frac{0.3}{0.4} \right] = \frac{5}{4} \mu_0$$

21. (a) $M = iA \Rightarrow i = \frac{M}{A}$

22. (c) Work done, $W = MB_{\mu} (1 - \cos \theta)$

$$= 20 \times 0.3 (1 - \cos 30^\circ) = 6 \left(1 - \frac{\sqrt{3}}{2} \right) = 3(2 - \sqrt{3})$$

23. (a) $B = 0.5 \text{ T}$, $A_1 = 101 \text{ cm}^2 = 101 \times 10^{-4} \text{ m}^2$
 $A_2 = 100 \text{ cm}^2 = 100 \times 10^{-4} \text{ m}^2$
 $R = 2 \Omega$

Amount of charge flowing is

$$q = \frac{B\Delta A}{R} = \frac{0.50 \times (101 \times 10^{-4} - 100 \times 10^{-4})}{2}$$

$$= \frac{0.05 \times 1 \times 10^{-4}}{2} = 2.5 \times 10^{-6} \text{ C}$$

24. (c) Here, $E = 1500 \text{ V/m}$, $B = 0.4 \text{ Wb/m}^2$

Minimum speed of electron along the straight line, $v = \frac{E}{B}$

$$= \frac{1500}{0.4} = 3750 = 3.75 \times 10^3 \text{ m/s}$$

26

Magnetostatics

Current Loop as a Magnetic Dipole

A current loop is equivalent to a magnetic dipole. If $A(= \pi a^2)$ be the area of the loop, then the magnitude of its dipole moment is

$$p_m = iA = i\pi a^2$$

where, a is radius of coil and i is current flowing through it.

$$\therefore i = \frac{p_m}{\pi a^2} \quad \dots(i)$$

Magnetic field at the centre of a circular current loop is given by

$$B = \frac{\mu_0 i}{2a} \quad \dots(ii)$$

Putting the value of i from Eq. (i) in Eq. (ii), we get

$$B = \frac{\mu_0}{2\pi} \frac{p_m}{a^3}$$

This is the expression for the magnetic field at the centre of the current loop in terms of its dipole moment. Instead of a circular loop, if there is a circular coil having n turns, its dipole moment would be $p_m = niA = ni\pi a^2$.

Torque on a Magnetic Dipole Placed in a Uniform Magnetic Field

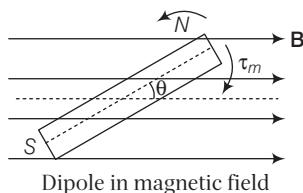
In the given figure, a magnetic dipole is placed in a uniform magnetic field B represented by equidistant parallel lines.

Now, torque exerted on magnetic dipole by magnetic field is

$$\begin{aligned} \Rightarrow \quad \tau_m &= \mathbf{m} \times \mathbf{B} \\ \text{Here,} \quad \tau_m &= \text{magnetic torque} \\ \Rightarrow \quad \tau_m &= mB \sin \theta \end{aligned}$$

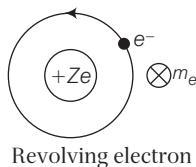
where, $\mathbf{B}$ = magnetic field, $\mathbf{m}$ = magnetic moment of the bar magnet, θ = the angle between $\mathbf{m}$ and $\mathbf{B}$.

So, torque is zero when $\theta = 0^\circ$ or 180° and it is maximum when $\theta = 90^\circ$.



Magnetic Dipole Moment of a Revolving Electron

A revolving electron constitutes a current,



i.e.

$$i = \frac{e}{T}$$

As, T = time period of revolution, $T = \frac{2\pi}{\omega}$,

where, ω = electron's orbital angular speed

$$\therefore i = \text{equivalent current} = \frac{e\omega}{2\pi} = \frac{ev}{2\pi r} \quad (\because v = r\omega)$$

So, magnetic moment associated with this circulating current

$$\mu_l = i\pi r^2 = \frac{evr}{2} = \frac{e}{2m_e}(m_e vr)$$

where, r = radius of an orbit = $\frac{e}{2m_e} \cdot l$,

where, l = angular momentum ($\because l = m_e vr$)

and vector form $\mu_l = \frac{-e}{2m_e} l$

or the angular momentum vector and magnetic dipole moment vectors are opposite in direction.

The ratio $\left| \frac{\mu_l}{l} \right| = \frac{e}{2m_e}$ is called **gyromagnetic ratio**. Its value is 8.8×10^{10} C/kg.

According to Bohr's postulate, $|l| = \frac{nh}{2\pi}$

$$\therefore \mu_l = \frac{nh e}{4\pi m_e}$$

$$\begin{aligned} \text{and } \mu_l (\text{minimum}) &= \frac{e}{4\pi m_e} \cdot h \\ &= 9.27 \times 10^{-24} \text{ A-m}^2 \quad (\because n = 1) \end{aligned}$$

This value is called **Bohr's magneton**.

Potential Energy of a Magnetic Dipole in a Uniform Magnetic Field

When we rotate a dipole with the help of external torque, we have to do some external work.

$$\therefore W_{\text{ext}} = \Delta U \quad \dots(i)$$

$$\Rightarrow W_{\text{ext}} = \int dW = \int \tau d\theta$$

$$= \int_{\theta_1}^{\theta_2} mB \sin\theta d\theta$$

$$W_{\text{ext}} = -mB(\cos\theta_2 - \cos\theta_1) \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$-mB(\cos\theta_2 - \cos\theta_1) = \Delta U \quad \dots(iii)$$

Let at $\theta = 90^\circ$, $U = 0$

So, if $\theta_1 = 90^\circ$, $\theta_2 = \theta$

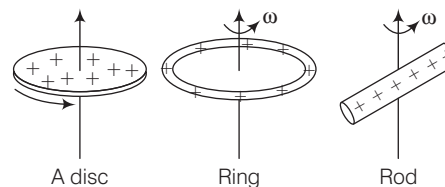
and $U_i = 0$, then

$$W_{\text{ext}} = -mB(\cos\theta_2) = U_f$$

$$\Rightarrow U = -mB\cos\theta = -\mathbf{m} \cdot \mathbf{B}$$

Magnetic Dipole Moment Associated with Charged Rotating Bodies

As a charged rotating body constitutes an equivalent current. So, they have a magnetic dipole moment associated with them. For geometrically symmetric charged bodies, magnetic moment is given by



Charged rotating bodies

$$\mathbf{m} = \frac{qL}{2m} = \frac{ql\omega}{2m} \quad (\because L = I\omega)$$

where,

L = angular momentum,

I = moment of inertia,

ω = angular velocity,

m = mass of rotating body

Bar Magnet

A bar magnet may be viewed as a combination of two magnetic poles, north pole and south pole, separated by some distance. The distance is known as the magnetic length of the given bar magnet.

If a bar magnet is broken, the fragments are independent magnetic dipoles and not isolated magnetic poles.

Magnetic Field due to a Bar Magnet

A bar magnet has a magnetic field around it. Magnetic field is mathematically measured by a vector term $\mathbf{B}$, whose SI unit is 1 tesla (1 T).

The magnetic field in free space, at a point at a distance r from the given bar magnet (or magnetic dipole)

(i) **Along its axial line**

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{2Mr}{(r^2 - l^2)^2}$$

and the direction of $\mathbf{B}$ is the same as the direction of $\mathbf{M}$. For a short dipole (or for a far away point on the axis) when $r \gg l$, the above relation is simplified as

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{2\mathbf{M}}{r^3}$$

(ii) **Along the equatorial line** of a magnetic dipole, the magnetic field $\mathbf{B}$ in free space is given by

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{\mathbf{M}}{(r^2 + l^2)^{3/2}}$$

If $r \gg l$, the relation is modified as $\mathbf{B} = \frac{\mu_0}{4\pi} \frac{\mathbf{M}}{r^3}$

However, along the equatorial line, the direction of $\mathbf{B}$ is opposite to that of $\mathbf{M}$.

In general, in a direction making an angle θ with the magnetic axis, the magnetic field is given by

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{\mathbf{M}}{r^3} \sqrt{(3 \cos^2 \theta + 1)}$$

In these relations, μ_0 is a constant having a value of $4\pi \times 10^{-7} \text{ T mA}^{-1}$ and it is known as the **magnetic permeability of free space**.

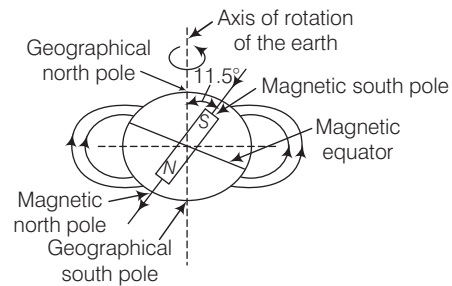
For solenoid $\mathbf{B} = \mu_0 n i$, where n is number of turns of solenoid and i is the current flowing through it.

NOTE Time period of oscillations of a bar magnet in a uniform magnetic field.

$$T = 2\pi \sqrt{\frac{I}{mB}}$$

Earth's Magnetic Field and Magnetic Elements

Earth's magnetic field on the earth's surface is around 0.4 gauss and its value shows variations with time. At present, magnetic south pole is located at northern Canada 70.5° N latitude and 96° W longitude. North magnetic is at 70.5° S and 84° E at Arctic. The magnetic poles are around 2000 km apart.



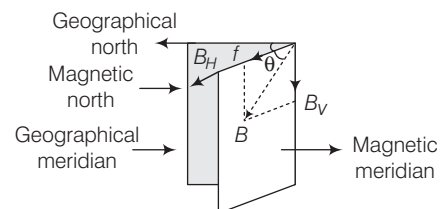
Magnetic equator intersects the geographic equator at 6° W longitude and 174° E latitude. Earth's field reverses itself in a span of around one million years. Declination shows a yearly variation. As per the most established theory, it is due to the rotation of the earth whereby various charged ions present in the molten state in the core of the earth rotate and constitute a current.

Magnetic equator divides the earth into two hemispheres. The hemisphere containing south polarity of the earth's magnetism is called **northern hemisphere**. While the other hemisphere containing north polarity of the earth's magnetism is called the **southern hemisphere**. Direction of the earth's magnetic field is from S (geographical south) to N (geographical north).

The magnitude and direction of the magnetic field of the earth at a place are completely given by certain quantities known as **magnetic elements**.

Three magnetic elements of the earth are

(i) **Magnetic declination (ϕ)** It is the angle between geographic and the magnetic meridian planes.



Declination at a place is expressed at ϕ° E or ϕ° W depending upon whether the north pole of the compass needle lies to the east or to the west of the geographical axis.

(ii) **Angle of inclination or magnetic dip (θ)** It is the angle between the direction of intensity of total magnetic field of the earth and a horizontal line in the magnetic meridian. If λ is magnetic latitude at a place, then angle of dip θ at the place is given by $\tan \theta = 2 \tan \lambda$.

(iii) **Horizontal component of the earth's magnetic field (B_H)** Earth's magnetic field is horizontal only at the magnetic equator.

At any other place, the total intensity can be resolved into horizontal component (B_H) and vertical component (B_V).

Also, $B_H = B \cos \theta$ and $B_V = B \sin \theta$

So, $B = \sqrt{B_H^2 + B_V^2}$ and $\tan \theta = \frac{B_V}{B_H}$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{B_V}{B_H} \right)$$

At equator, $\theta = 0 \Rightarrow B_H = B, B_V = 0$, while at poles $\theta = 90^\circ$

$\Rightarrow B_H = 0, B_V = B$

Important Terms Used in Magnetism

1. Magnetic Induction or Magnetic Flux Density (B)

Whenever a piece of magnetic substance is placed in an external magnetising field, the substance becomes magnetised. If B_0 is the magnetic field in free space, then on placing the given magnetic substance at that place, the magnetic field changes from B_0 to B , where $B = \mu_r B_0$

$\oint \mathbf{B} \cdot d\mathbf{S}$ is magnetic flux which is equal to $\mu_0 \mathbf{m}_{\text{inside}}$, where $\mathbf{m}_{\text{inside}}$ is the net pole strength inside a closed surface.

For a dipole, Gauss's law for magnetism is $\oint \mathbf{B} \cdot d\mathbf{S} = 0$ for a closed surface.

2. Magnetic Permeability (μ)

It is the degree or extent to which the magnetic lines of induction may pass through a given distance.

Magnetic permeability of free space μ_0 has a value of $4\pi \times 10^{-7} \text{ TmA}^{-1}$. However, for a material substance, **absolute permeability (μ)** has a value different than μ_0 .

For any magnetic substance $\frac{\mu}{\mu_0} = \frac{B}{B_0} = \mu_r = \text{relative magnetic permeability of that substance.}$

Relative magnetic permeability μ_r is a unitless and dimensionless term. For magnetic substance like iron, $\mu_r > 1000$

Relative permeability of a diamagnetic substance is less than 1 ($\mu_r < 1$), but it can never be negative. Thus, $0 \leq \mu_r < 1$, for a diamagnetic material.

3. Intensity of Magnetisation (I)

Intensity of magnetisation of a substance is defined as the magnetic moment induced in the substance per unit volume, when placed in the magnetising field.

Thus,

$$I = \frac{M}{V}$$

It is a vector quantity and its SI unit is Am^{-1} .

4. Intensity of Magnetising Field or Magnetic Intensity (H)

It is a measure of the capability of external magnetising field to magnetise the given substance and is mathematically defined as

$$H = \frac{B_0}{\mu_0} \text{ or } H = \frac{B}{\mu} \text{ or } H = \frac{B}{\mu_0} - I$$

Magnetic intensity H is a vector quantity and its SI unit is Am^{-1} .

5. Magnetic Susceptibility (χ_m)

Magnetic susceptibility of a substance is the ratio of the intensity of magnetisation I induced in the substance to the magnetic intensity H . Thus, $\chi_m = \frac{I}{H}$, it is a scalar term and

has no unit or dimension.

Relation (i) $\mathbf{B}I = \mu_0(\mathbf{H} + \mathbf{I})$ and (ii) $\mu_r = 1 + \chi_m$

Magnetic Behaviour of Materials

Diamagnetic Materials

These are materials which show a very small decrease in magnetic flux when placed in a strong magnetising field. Hydrogen, water, copper, zinc, antimony, bismuth, etc. are the examples of diamagnetic materials.

Paramagnetic Materials

These are those materials which show a small increase in the magnetic flux when placed in a magnetising field.

Oxygen, air, platinum, aluminium, etc., are examples of paramagnetic materials.

Ferromagnetic Materials

These are those materials which are strongly attracted by a magnetic field and can themselves be magnetised even in a weak magnetising field. Iron, steel, nickel and cobalt are ferromagnetic materials.

Curie's Law and Curie Temperature

Curie's Law

According to Curie's law, the magnetic susceptibility of paramagnetic substances is inversely proportional to absolute temperature,

$$\text{i.e.} \quad \chi \propto \frac{1}{T}$$

Here, T = absolute temperature.

On increasing temperature, magnetic susceptibility of paramagnetic substances decreases or *vice-versa*.

Curie Temperature or Curie Point

The magnetic susceptibility of these substances decreases on increasing the temperature and above a particular temperature, a ferromagnetic substance behaves like a paramagnetic substance.

This particular temperature is called the Curie temperature of the substance.

e.g. The Curie temperature of iron is 770°C . It follows that at a temperature below 770°C , the iron is ferromagnetic and at a temperature above 770°C , the iron is paramagnetic.

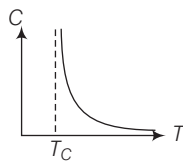
Similarly, the Curie temperature of nickel is 369°C and that of cobalt is 1150°C .

Curie-Weiss Law

At temperature above Curie point, the magnetic susceptibility of ferromagnetic substances is inversely proportional to $(T - T_C)$, i.e.

$$\chi \propto \frac{1}{T - T_C} \text{ or } \chi = \frac{C}{T - T_C}$$

Here, T_C = Curie temperature.

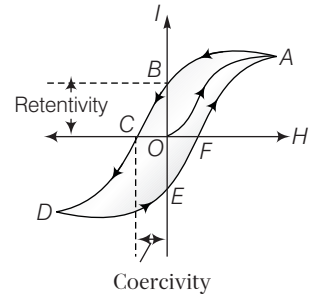


Hysteresis Curve

A great deal of information can be learned about the magnetic properties of a material by studying its hysteresis loop. A hysteresis loop shows the relationship between the induced magnetic flux density (B) and the magnetising force (H). It is often referred as B - H loop.

A ferromagnetic material can be easily magnetised by placing it in an external magnetising field H .

Initially as H is increased, the intensity of magnetisation I developed in the material increases non-linearly along the curve OA and reaches a maximum, known as saturated magnetism. Now on reducing H , I follows the path AB . Point B corresponds to $H = 0$ but I has a finite positive value. This value of I is called **remance** or **retentivity** or **residual magnetisation**.



If direction of H is reversed and is gradually increased, H curve is along BC . Point C represents the situation when I is finally reduced to zero. The reversed value of I , represented by OC , is known as the **coercivity** of the material.

On increasing H in the reverse direction further, graph CD is obtained. Now, if H is taken back from its negative saturation value to its original positive saturation value a similar I - H curve represented by $DEFA$ is traced.

The whole graph $ABCDEFA$ is a closed loop and known as **hysteresis loop**.

Vibration Magnetometer

- (a) If small magnet is placed in magnetic meridian and it vibrates in horizontal plane, then

$$T = 2\pi \sqrt{\frac{I}{MH}}$$

$$\text{where,} \quad I = M \left(\frac{l^2 + b^2}{12} \right)$$

Its breadth is negligible,

$$I = \frac{M_0 l^2}{12}$$

- (b) If magnet is placed parallel to magnetic meridian and oscillates in vertical plane, then

$$T = 2\pi \sqrt{\left(\frac{I}{MBe} \right)}$$

- (c) If magnet is placed perpendicular to magnetic meridian and oscillates in vertical plane

$$T = 2\pi \sqrt{\left(\frac{I}{MV} \right)}$$

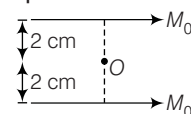
- (d) Comparison of magnetic moment by sum and difference method.

$$\frac{M_1}{M_2} = \frac{T_1^2 + T_2^2}{T_1^2 - T_2^2}$$

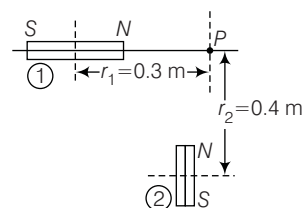
Practice Exercise

- Calculate the force acting between two small magnets, placed in end on position 0.1 m apart from their centres. (Given, magnetic moment of each magnet is 5 Am^2).
a. 0.6 N b. 0.8 N c. 0.15 N d. 0.2 N
- Two similar equal magnetic poles when separated by a distance of 1 m, they repel with a force of 10^{-3} N . The pole strength is
a. 10 Am b. 20 Am c. 50 Am d. 100 Am
- Three similar magnetic south poles each of strength 10 Am are placed at the corners of an equilateral triangle of side 20 cm. Find the magnetic force on one of the pole.
a. $0.25 \times 10^{-3} \text{ N}$ b. 10^{-3} N
c. $10 \times 10^{-3} \text{ N}$ d. None of these
- Six similar magnetic poles are placed on six corners of a regular hexagon of side 10 cm. A south pole of strength 10 Am is placed at the centre of hexagon. Find the magnetic force on the south pole.
a. Zero b. $4\pi \times 10^{-4} \text{ N}$
c. 10 N d. None of these
- A magnetic wire of dipole moment $4\pi \text{ Am}^2$ is bent in the form of semicircle. The new magnetic moment is
a. $4\pi \text{ Am}^2$ b. $8\pi \text{ Am}^2$
c. 4 Am^2 d. None of these
- Current I is flowing in a conducting wire of length l . It is bent in the form of a circular coil of single turn. Its magnetic moment will be
a. $\frac{Il}{4\pi}$ b. $\frac{I^2l}{4\pi}$ c. $\frac{Il^2}{4\pi}$ d. $\frac{I^2l^2}{4\pi}$
- The magnetic induction inside a solenoid is $6.5 \times 10^{-4} \text{ T}$. When it is filled with iron medium, then the induction becomes 1.4 T. The relative permeability of iron will be
a. 1578 b. 2355
c. 1836 d. 2154
- If a magnetising field of 1600 A/m produces a magnetic flux of $14 \times 10^{-5} \text{ Wb}$ in an iron bar of cross-sectional area 0.2 cm^2 . Then,
a. magnetic permeability of iron rod is around 1000
b. magnetic susceptibility is very larger than unity
c. magnetic susceptibility is 340
d. None of the above
- Two dissimilar poles of strength $x \text{ mWb}$ and 2 mWb are separated by a distance 12 cm. If the null point is at a distance of 4 cm from 2 mWb , then calculate x .
a. 5 mWb b. 6 mWb c. 7 mWb d. 8 mWb

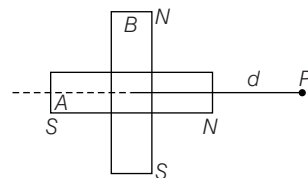
- Two small magnets of each of magnetic moment M_0 is placed parallel to each other (shown in figure). The magnetic field at point O is
a. zero b. $4 \times 10^{-4} \text{ N}$
c. $2 \times 10^{-4} \text{ N}$ d. None of these



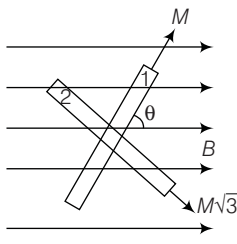
- Two short magnets of magnetic moment 2 Am^2 and 5 Am^2 are placed along two lines drawn at right angle to each other on the sheet of paper as shown in the figure. What is the magnetic field at the point of intersection of their axis?
a. $2.15 \times 10^{-5} \text{ T}$ b. $215 \times 10^{-5} \text{ T}$
c. $2.15 \times 10^{-3} \text{ T}$ d. $21.5 \times 10^{-5} \text{ T}$



- Calculate the magnetic induction at P, for the arrangement shown in the figure, when two similar short magnets of magnetic moment M are joined at the middle. So that, they are mutually perpendicular.
a. $\frac{\mu_0}{4\pi} \frac{M\sqrt{3}}{d^3}$ b. $\frac{\mu_0}{4\pi} \frac{2M}{d^3}$ c. $\frac{\mu_0 M\sqrt{5}}{4\pi d^3}$ d. $\frac{\mu_0 2M}{4\pi d^3}$



- A small magnet of dipole moment M is kept on the arm of a deflection magnetometer set in tan A position at a distance of 0.2 m. If the deflection is 60° , find the value of P ($B_H = 0.4 \times 10^{-4} \text{ T}$).
a. 2.77 Am^2 b. 8 Am^2
c. 0.2 Am^2 d. None of these
- A short bar magnet of magnetic dipole moment $m = 0.32 \text{ JT}^{-1}$ is placed in a uniform external magnetic field of 0.15 T. If bar is free to rotate in the field, then
a. when m is anti-parallel to B , $U = 4.8 \times 10^{-2} \text{ J}$
b. when m is perpendicular to B , $U = 0$
c. when m is perpendicular to B , $U = 4.8 \times 10^{-2} \text{ J}$
d. None of the above

15. A magnet of dipole moment 2 Am^2 is deflected through 30° from magnetic meridian. The required deflecting torque is ($B_H = 0.4 \times 10^{-4} \text{ T}$)
- $0.4 \times 10^{-4} \text{ Nm}$
 - 0.4 Nm
 - $0.2 \times 10^{-4} \text{ Nm}$
 - None of these
16. If the areas under the I - H hysteresis loop and B - H hysteresis loop are denoted by A_1 and A_2 , then
- $A_2 = \mu_0 A_1$
 - $A_2 = A_1$
 - $A_2 = \frac{A_1}{\mu_0}$
 - $A_2 = \mu_0^2 A_1$
17. Consider the plane S formed by the dipole axis and the axis of the earth. Let P be point on the magnetic equator and in S . Let Q be the point of intersection of the geographical and magnetic equators. The declination and dip angles at P and Q are
- 0° and 11.3°
 - 0° and 0°
 - 11.3° and 6.5°
 - 11.3° and 11.3°
18. A magnet of length 14 cm and magnetic moment M , is broken into two parts of length 6 cm and 8 cm respectively. They are put at right angles to each other, with opposite poles together. The magnetic moment of the combination is
- $M/10$
 - M
 - $M/1.4$
 - $2.8 M$
19. A uniform magnetic needle of strength of each pole is 98.1 amp. cm, is suspended from its centre by a thread. When a mass of 50 mg is loaded to its upper end, the needle becomes horizontal, then the vertical component of earth's magnetic induction is ($g = 981 \text{ cm/s}^2$)
- 0.50 gauss
 - 0.25 gauss
 - 0.05 gauss
 - 0.005 gauss
20. M and $M/\sqrt{3}$ are the magnetic dipole moments of the two magnets, which are joined to form a cross figure. Find the inclination of the system with the field, if their combination is suspended freely in a uniform external magnetic field B .
- 
- $\theta = 30^\circ$
 - $\theta = 45^\circ$
 - $\theta = 60^\circ$
 - $\theta = 15^\circ$
21. Calculate the couple acting on a magnet of length 10 cm and pole strength 15 Am, kept in a field of $B = 2 \times 10^{-5} \text{ T}$ at an angle of 30° .
- $1.5 \times 10^{-5} \text{ Nm}$
 - $1.5 \times 10^{-3} \text{ Nm}$
 - $1.5 \times 10^{-2} \text{ Nm}$
 - $1.5 \times 10^{-6} \text{ Nm}$
22. A bar magnet of magnetic moment 2.5 J/T , is placed in magnetic field 0.2 T. What work is done in turning the magnet from parallel to anti-parallel position relative to field direction?
- 1 J
 - 2 J
 - 3 J
 - 4 J

23. Two like poles of strength m_1 and m_2 are far distance apart. Calculate the energy required to bring them r_0 distance apart.
- $\frac{\mu_0 m_1 m_2}{4\pi r_0}$
 - $\frac{\mu_0 m_1 m_2}{8\pi r_0}$
 - $\frac{\mu_0 m_1 m_2}{16\pi r_0}$
 - None of these
24. Calculate the work done in deflecting a small magnet of magnetic moment 10 Am^2 through 180° from a uniform magnetic field of strength $0.4 \times 10^{-4} \text{ T}$.
- $8 \times 10^{-4} \text{ J}$
 - Zero
 - $4 \times 10^{-4} \text{ J}$
 - None of these
25. At a place the values of B_H and B_V are $0.4 \times 10^{-4} \text{ T}$ and $0.3 \times 10^{-4} \text{ T}$ respectively. The resultant earth's magnetic field is
- $0.5 \times 10^{-4} \text{ T}$
 - 10^{-4} T
 - $2 \times 10^{-4} \text{ T}$
 - None of these
26. In previous problem, the angle of dip is
- $\tan^{-1}(0.75)$
 - $\tan^{-1}(0.5)$
 - $\tan^{-1}(0.8)$
 - None of these
27. Calculate the real angle of dip, if a magnet is suspended at an angle of 30° to the magnetic meridian, the dip needle makes an angle of 45° with horizontal.
- $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$
 - $\tan^{-1}(\sqrt{3})$
 - $\tan^{-1}\left(\frac{\sqrt{3}}{\sqrt{2}}\right)$
 - $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$
28. A magnet is cut in three equal parts by cutting it perpendicular to its length. The time period of original magnet is T_0 in a uniform magnetic field B . Then, the time period of each part in the same magnetic field is
- $\frac{T_0}{2}$
 - $\frac{T_0}{3}$
 - $\frac{T_0}{4}$
 - None of these
29. A magnet is cut in four equal parts by cutting it parallel to its length. What will be the time period of each part, if the time period of original magnet in the same field is T_0 ?
- $\frac{T_0}{\sqrt{2}}$
 - $\frac{T_0}{2}$
 - $\frac{T_0}{4}$
 - $4 T_0$
30. The current on the winding of a toroid is 2 A. It has 400 turns and mean circumferential length is 40 cm. With the help of search coil and charge measuring instrument the magnetic field is found to be 1 T. The susceptibility is
- 100
 - 290
 - 398
 - 397
31. The magnetic needle of a tangent galvanometer is deflected at an angle 30° with respect to the magnet. The horizontal component of the earth's magnetic field is $0.34 \times 10^{-4} \text{ T}$ along the plane of the coil. The magnetic intensity is
- $1.96 \times 10^{-4} \text{ T}$
 - $1.96 \times 10^4 \text{ T}$
 - $1.96 \times 10^{-5} \text{ T}$
 - $1.96 \times 10^5 \text{ T}$

32. Calculate the angle of dip, if a dip needle oscillating in a vertical plane makes 40 oscillations per minute in a magnetic meridian and 30 oscillations per minute in a vertical plane at right angle to the magnetic meridian.
 a. $\theta = \sin^{-1}(0.5625)$ b. $\theta = \sin^{-1}(0.325)$
 c. $\theta = \sin^{-1}(0.425)$ d. $\theta = \sin^{-1}(0.235)$
33. Inside a long solenoid wound with 300 turns/m, an iron rod is placed. An iron rod is 0.2 m long, 10 mm in diameter and of permeability 10^3 . Calculate the magnetic moment of the rod, if 0.5 A of current is passed through the rod.
 a. 2.356 SI units b. 1.335 SI units
 c. 3.664 SI units d. 1.664 SI units
34. An iron rod is subjected to cycles of magnetisation at the rate of 50 Hz. Given the density of the rod is $8 \times 10^3 \text{ kg/m}^3$ and specific heat is $0.11 \times 10^{-3} \text{ cal/kg}^\circ\text{C}$. Find the rise in temperature per minute. If the area enclosed by the B - H loop corresponds to energy of 10^{-2} J . Assume there is no radiation losses.
 a. 78°C b. 88°C
 c. 8.1°C d. None of these

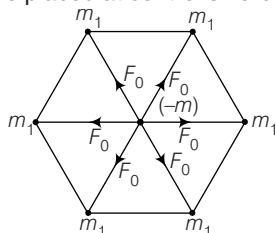
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1. The susceptibility of a magnetism at 300 K is 1.2×10^{-5} . The temperature at which the susceptibility increases to 1.8×10^{-5} is [2014]
 a. 150 K b. 200 K c. 250 K d. 20 K
2. Magnetic moment of bar magnet is M . The work done to turn the magnet by 90° of magnet in direction of magnetic field B will be [2012]
 a. zero b. $\frac{1}{2} MB$ c. $2 MB$ d. MB
3. If $\mathbf{M}$ is magnetic moment and $\mathbf{B}$ is the magnetic field, then the torque is given by [2011]
 a. $\mathbf{M} \cdot \mathbf{B}$ b. $\frac{|\mathbf{M}|}{|\mathbf{B}|}$ c. $\mathbf{M} \times \mathbf{B}$ d. $|\mathbf{M}||\mathbf{B}|$
4. At the magnetic north pole of the earth, the value of horizontal component of earth's magnetic field and angle of dip are, respectively [2009]
 a. zero, maximum b. maximum, minimum
 c. maximum, maximum d. minimum, minimum
5. With a standard rectangular bar magnet the time period of a vibration magnetometer is 4 s. The bar magnet is cut parallel to its length into four equal pieces. The time period of vibration magnetometer when one piece is used (in second) (bar magnet breadth is small) is [2008]
 a. 16 b. 8 c. 4 d. 2
6. The magnetised wire of moment M and length l is bent in the form of semicircle of radius r . Then, its magnetic moment is [2008]
 a. $\frac{2M}{\pi}$ b. $2M$ c. $\frac{M}{\pi}$ d. zero
7. Susceptibility of ferromagnetic substance is [2006]
 a. > 1 b. < 1 c. zero d. 1
8. Among the following properties describing diamagnetism, identify the property that is wrongly stated. [2005]
 a. Diamagnetic material do not have permanent magnetic moment
 b. Diamagnetism is explained in terms of electromagnetic induction
 c. Diamagnetic materials have a small positive susceptibility
 d. The magnetic moment of individual electrons neutralise each other

Answer with Solutions

Practice Exercise

1. (c) 2. (d) 3. (a)
4. (a) From force diagram shown in figure, the net magnetic force is zero. In general, if similar poles are placed on the corners of a regular polygon. Then, the net magnetic force on a pole placed at centre is zero.



5. (c) Here,

$$\begin{aligned} M_i &= 4\pi \\ mL &= 4\pi \\ m &= \frac{4\pi}{L} \end{aligned}$$

The new distance between poles is

$$\begin{aligned} L_1 &= 2r = 2 \frac{L}{2\pi} = \frac{L}{\pi} \\ \therefore M_f &= mL_1 = \frac{mL}{\pi} = \frac{4\pi}{\pi} \\ &= 4 \text{ Am}^2 \end{aligned}$$

6. (b) Length of a conducting wire, $2\pi r = l$, $r = \frac{l}{2\pi}$,

$$\text{Area of coil, } \alpha = \pi r^2 = \frac{\pi \cdot l^2}{4\pi^2} = \frac{l^2}{4\pi}$$

$$\text{So, } m = IA = \frac{l^2}{4\pi}$$

7. (d) $B_{\text{air}} = \mu_0 nI$, $B_{\text{medium}} = \mu_0 \mu_r nI = \mu_r B_0$

$$1.4 = \mu_r \times 6.5 \times 10^{-4}$$

$$\therefore \mu_r = \frac{1.4}{6.5 \times 10^{-4}} = 2154$$

8. (c) Magnetic field, $B = \frac{\phi}{A} = 0.7 \text{ Wb/m}^2$,

$$\mu = \frac{B}{H} = \frac{0.7}{1600} = 4.3 \times 10^{-4} \text{ N/A}^2$$

$$\mu = \mu_0(1 + \chi_m)$$

$$\Rightarrow \chi_m = 340$$

9. (d) $\sqrt{\left(\frac{4}{x^2}\right)} = \frac{4}{4+12} = \frac{4}{16}$

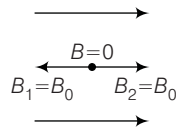
$$\frac{4}{x^2} = \frac{1}{16}$$

$$x = 8 \text{ mWb}$$

10. (a) $B_1 = \frac{\mu_0 M_0}{4\pi r^3}$

$$B_2 = \frac{\mu_0 M_0}{4\pi r^3}$$

$$\therefore B = B_1 - B_2 = 0$$



11. (a) Magnetic field due to magnet (1),

$$B_1 = \frac{\mu_0}{4\pi} \frac{2M_1}{r_1^3} = \frac{10^{-7} \times 2 \times 2}{(0.3)^3} = 1.48 \times 10^{-5} \text{ T}$$

Magnetic field due to magnet (2),

$$B_2 = \frac{\mu_0}{4\pi} \left(\frac{2M_2}{r_2^3} \right) = \frac{10^{-7} \times 2 \times 5}{(0.4)^3} = 1.56 \times 10^{-5} \text{ T}$$

$\therefore$ Net field at P,

$$B = \sqrt{B_1^2 + B_2^2} = 2.15 \times 10^{-5} \text{ T}$$

12. (c)

13. (a) $\therefore B = B_H \tan \theta$

$$\text{or } \frac{\mu_0}{4\pi} \frac{2M}{r^3} = 0.4 \times 10^{-4} \tan 60^\circ$$

$$\therefore \frac{\mu_0 M}{2\pi(0.2)^3} = 0.4 \times 10^{-4} \tan 60^\circ$$

$$M = \frac{0.4 \times 4\pi \times (0.2)^3 \times 10^{-4} \times \sqrt{3}}{2 \times \mu_0}$$

$$= \frac{0.00277 \times 10^{-4}}{10^{-7}}$$

$$\therefore M = 2.77 \text{ Am}^2$$

14. (a)

15. (a)

16. (a) As, $B = \mu_0(H + I)$

$$\Rightarrow dB = \mu_0 dH + \mu_0 dI$$

$$\text{or } \oint H dB = \mu_0 \oint H dH + \mu_0 \oint H dI$$

$$\oint H dH = 0$$

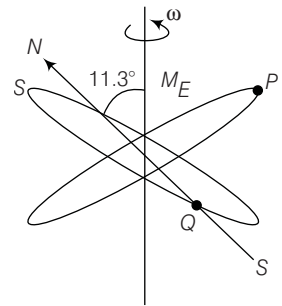
$$\oint H dB = \mu_0 \oint H \cdot dI$$

Area of the B - H loop $= \mu_0 \times$ area of I - H loop

$$\text{i.e. } A_2 = \mu_0 A_1$$

17. (a) P is in the plane S , needle is in north, so the declination is zero.

P is also on the magnetic equator, so the angle of dip $= 0$, because the value of angle of dip at equator is zero Q is also on the magnetic equator, thus the angle of dip is zero. As earth tilted on its axis by 11.3° , thus the declination at Q is 11.3° .



18. (c) Pole strength of the original magnet, $m = \frac{M}{14}$

Effective distance between the poles $= AB$

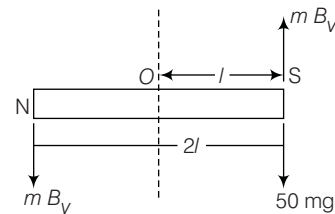
$$= \sqrt{6^2 + 8^2}$$

$$= 10 \text{ cm}$$

$\therefore$ Magnetic moment of the combination

$$M' = m \cdot 2l = \frac{M}{14} \times 10 = \frac{M}{14}$$

19. (b)



Taking moment about O ,

$$Mgl = mB_v L + mB_v L$$

$$Mg = 2mB_v$$

$$\frac{50}{1000} \times 981 = 2 \times 98.1 \times B_v$$

$$B_v = 0.25 \text{ gauss}$$

20. (c) Torque (τ) acting on magnet (1),

$$\tau_1 = MB \sin \theta$$

$$\text{Similarly, } \tau_2 = \sqrt{3} MB \sin \theta$$

$$\therefore \theta = 60^\circ$$

But for equilibrium, $\tau_1 = \tau_2$

$$\therefore MB \sin \theta = \sqrt{3} MB \cos \theta$$

$$\tan \theta = \sqrt{3} = \tan 60^\circ$$

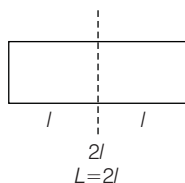
$$\therefore \theta = 60^\circ$$

21. (a) $C = MB \sin \theta$

$$= (m \times 2l) \times 2 \times 10^{-5} \sin 30^\circ$$

$$= 15 \times 10 \times 10^{-2} \times 2 \times 10^{-5} \times \frac{1}{2}$$

$$= 1.5 \times 10^{-5} \text{ Nm}$$



22. (a) 23. (a) 24. (a) 25. (a) 26. (a) 27. (a)

28. (b) In the case of vertical cutting, $T_1 = \frac{\text{Original time period}}{n}$

$$T_1 = \frac{T_0}{3} \text{ (Here, } n = \text{number of equal parts)}$$

29. (a) For each part, $l_1 = \frac{l}{4}$

and $M_1 = \frac{M}{2}$

$$\therefore T = 2\pi \sqrt{\left(\frac{l}{MB}\right)}$$

and $T_1 = 2\pi \sqrt{\left(\frac{l_1}{M_1 B}\right)} = 2\pi \sqrt{\left(\frac{l \times 2}{4M \times B}\right)}$

$$\therefore \frac{T_1}{T} = \frac{1}{\sqrt{2}}$$

$$\therefore T_1 = \frac{T}{\sqrt{2}} = \frac{T_0}{\sqrt{2}}$$

30. (d) As, $n = \frac{400}{2\pi R} = \frac{400}{40 \times 10^{-2}} = 1000$

$$\mu = ni = 1000 \times 2 = 2000$$

$$\Rightarrow \mu_0 \mu_r = \frac{1}{2000} = 5 \times 10^{-4}$$

$$\Rightarrow \mu_r = \frac{5 \times 10^{-4}}{\mu_0} = \frac{5 \times 10^{-4}}{4\pi \times 10^{-7}} = 398$$

$$\Rightarrow \chi = \mu_r - 1 = 397$$

31. (c) 32. (a) 33. (a)

34. (c) Energy loss per unit volume per cycle
= area of hysteresis loop
Energy loss per second per unit volume
= $10^{-2} \times 50 \text{ J}$

Also, heat produced Q in one minute per volume

$$Q = 10^{-2} \times 50 \times 60 \text{ J} = \frac{30}{4.2} \text{ cal}$$

Let θ = rise in temperature

$$\therefore 8 \times 10^3 \times 0.11 \times 10^{-3} \times \theta = \frac{30}{4.2}$$

$$\theta = \frac{30}{4.2 \times 8 \times 0.11} = 8.1^\circ \text{C}$$

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1. (b) $\chi = \frac{C}{T}$

$$\Rightarrow \frac{\chi_1}{\chi_2} = \frac{T_2}{T_1} \Rightarrow \frac{1.2 \times 10^{-5}}{1.8 \times 10^{-5}} = \frac{T_2}{300}$$

$$\Rightarrow T_2 = \frac{12}{18} \times 300 = 200 \text{ K}$$

2. (d) Work done, $W = MB(1 - \cos \theta)$

$$\theta = 90^\circ$$

$$\therefore W = MB$$

3. (c) Torque, $\tau = \mathbf{M} \times \mathbf{B}$

4. (a) At the magnetic north pole, the magnetic needle will point vertically. There is no component of earth's magnetic field in the horizontal direction and the angle of dip (the angle that the resultant magnetic field at the place makes with the horizontal) is 90° .

$$H = 0, \quad \delta = 90^\circ \text{ (maximum)}$$

5. (c) Time period of magnet, $T = 2\pi \sqrt{\frac{I}{MB}}$

When magnet is cut parallel to its length into four equal pieces.

Then,

New magnetic moment, $M' = \frac{M}{4}$

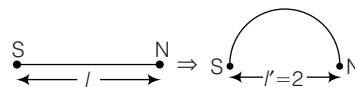
Moment of inertia, $I' = \frac{I}{4}$

$$\therefore \text{New time period, } T' = 2\pi \sqrt{\frac{I'}{M'B}}$$

$$\Rightarrow \text{New time period, } T = T' = 4 \text{ s}$$

6. (a) On bending a wire its pole strength remains unchanged, whereas its magnetic moment changes.
New magnetic moment,

$$M' = m(2r) = m \left(\frac{2l}{\pi} \right) = \frac{2M}{\pi}$$



7. (a) Susceptibility of the ferromagnetic substance is always greater than 1.

8. (c) Diamagnetic materials have negative susceptibility.
Thus, (c) is wrongly stated.

27

Electromagnetic Induction (EMI)

Electromagnetic Induction

Electromagnetic induction is the phenomenon due to which an induced emf is set up in a conductor or in an electric circuit, on changing the magnetic flux linked with it.

Magnetic Flux

The total number of magnetic lines of force passing normally through an area placed in a magnetic field is equal to the **magnetic flux** linked with that area.

Net flux through the surface, $\phi = \int \mathbf{B} \cdot d\mathbf{A} = BA \cos \theta$

where, ϕ = magnetic flux passing through the surface

B = magnetic field at the surface and θ = angle between $\mathbf{B}$ and $d\mathbf{A}$

$d\mathbf{A}$ = a vector perpendicular to the surface and has a magnitude equal to the area dA

For N turns coil, $\phi = NBA \cos \theta$

Magnetic flux is a scalar quantity. Its SI unit is weber (Wb) and CGS unit is maxwell or gauss $\times \text{cm}^2$.

$1 \text{ Wb} = 10^8 \text{ maxwell}$. It is also expressed in tesla $\times \text{m}^2$.

Its dimensional formula is $[\phi] = [\text{ML}^2\text{T}^{-2}\text{A}^{-1}]$.

Faraday's Laws of Electromagnetic Induction

Faraday had introduced two laws of electromagnetic induction which are as follows

Faraday's First Law

This law states that, whenever the number of magnetic lines of forces (magnetic flux) passing through a circuit changes (or a moving conductor cuts the magnetic flux), an emf is produced in the circuit (or emf induces across the ends of the conductor) called **induced emf**. The induced emf persists only as long as the flux is changing.

Faraday's Second Law

This law states that, induced emf in the circuit is equal to the rate of change of magnetic flux linked with the circuit

i.e.
$$e = -\frac{d\phi}{dt}$$

where, e = induced emf, $\frac{-d\phi}{dt}$ = change in magnetic flux.

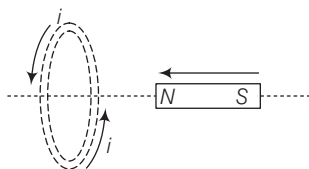
For N turns,
$$e = -\frac{Nd\phi}{dt}$$

Here, negative sign indicates that induced emf opposes the change of flux in the circuit.

Lenz's Law

This law gives the direction of induced emf/induced current. According to this law, the direction of induced emf or induced current in a circuit is such as to oppose the cause that produced it. This law is another form of **law of conservation of energy**.

e.g. If a magnet approaches a coil as shown, then an anti-clockwise current is induced in the coil and the coil opposes approaching magnet. So, some work has to be done to move magnet towards the coil and this work appears in the form of electrical energy.



Motional Emf

Let a conducting rod of length l be moving with a uniform velocity v perpendicular to a uniform magnetic field B , an induced emf is set up. The magnitude of the induced emf will be $|e| = Blv$.

If the rod is moving such that it makes an angle θ with the direction of the magnetic field, then $|e| = Blv \sin \theta$. Hence, for the motion parallel to B , the induced emf is zero.

When a conducting rod moves horizontally, then an induced emf is set up between its ends due to the vertical component of the earth's magnetic field. However, at the magnetic equator, induced emf will be zero because $B_V = 0$.

Motional Emf in a Loop

If a conducting rod moves on two parallel conducting rails then an emf is induced whose magnitude is $|e| = Blv$ and the direction is given by the Fleming's right hand rule.

Induced current, $|I| = \frac{|e|}{R} = \frac{Blv}{R}$

Rotational Emf

Let a conducting rod of length l rotate about an axis passing through one of its ends (that end may be fixed), with an angular velocity ω in a plane perpendicular to the magnetic field B , then an induced emf is set up between the ends of the rod, whose magnitude is given by

$$|e| = \frac{1}{2} Bl^2 \omega$$

Self-Induction

Whenever the electric current passing through a coil or circuit changes, the magnetic flux linked with it will also change. As a result of this, in accordance with Faraday's laws of electromagnetic induction, an emf is induced in the coil or in the circuit which opposes the change that causes it.

This phenomenon is called **self-induction** and the emf induced is called **back emf**, current so produced in the coil is called induced current.

If no magnetic materials are present near the coil, then number of flux linkages with the coil is proportional to the current i , i.e. $N\phi \propto i$ or $N\phi = Li$ (N is the number of turns in coil and $N\phi$ = total flux linkage), where $L = \frac{N\phi}{i}$, coefficient of self-induction. If $i = 1$ A, $N = 1$ then, $L = \phi$ i.e. the coefficient of self-induction of a coil is equal to the flux linked with the coil when the current in it is 1 A and number of turns is one.

By Faraday's second law, induced emf, $e = -N \frac{d\phi}{dt}$, which

gives $e = -L \frac{di}{dt}$

Then, the voltage (V) induced across the inductor,

$$V = |e| = \frac{di}{dt} \times L$$

Its SI unit is $\frac{\text{weber}}{\text{amp}}$.

But its practical unit is henry (H). Its dimensional formula is $[L] = [ML^2T^{-2}A^{-2}]$.

Inductance of a Solenoid

Let us find the inductance of a uniformly wound solenoid having N turns and length l . Assume that l is much longer than the radius of the windings and that the core of the solenoid is air. We can assume that the interior magnetic field due to a current i is uniform and given by equation,

$$B = \mu_0 ni = \mu_0 \left(\frac{N}{l} \right) i$$

where, $n = \frac{N}{l}$ is the number of turns per unit length.

The magnetic flux through each turn is,

$$\phi_B = BS = \mu_0 \frac{NS}{l} i$$

Here, S is the cross-sectional area of the solenoid. Now,

$$L = \frac{N\phi_B}{i} = \frac{N}{i} \left(\frac{\mu_0 NSi}{l} \right) = \frac{\mu_0 N^2 S}{l}$$

$$\therefore L = \frac{\mu_0 N^2 S}{l}$$

This result shows that L depends on dimensions (S, l) and is proportional to the square of the number of turns.

$$L \propto N^2$$

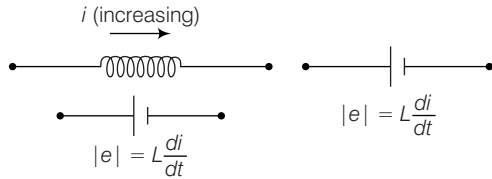
Because $N = nl$, we can also express the result in the form,

$$L = \mu_0 \frac{(nl)^2}{l} S = \mu_0 n^2 Sl = \mu_0 n^2 V \text{ or } L = \mu_0 n^2 V$$

Here, $V = Sl$ is the volume of the solenoid.

Energy Stored in an Inductor

The energy of a capacitor is stored in the form of electric field between its plates. Similarly, an inductor has the capability of storing energy in the form of magnetic field.



An increasing current in an inductor causes an emf between its terminals.

The work done per unit time is power.

$$P = \frac{dW}{dt} = -ei = -Li \frac{di}{dt}$$

$$\text{From, } dW = -dU \text{ or } \frac{dW}{dt} = -\frac{dU}{dt}$$

$$\text{we have, } \frac{dU}{dt} = Li \frac{di}{dt} \text{ or } dU = Li di$$

The total energy U supplied while the current increases from zero to a final value i is,

$$U = L \int_0^i di = \frac{1}{2} Li^2 \therefore U = \frac{1}{2} Li^2$$

Mutual Induction

Whenever the current passing through a coil or circuit changes, the magnetic flux linked with a neighbouring coil or circuit will also change. Hence, an emf will be induced in the neighbouring coil or circuit.

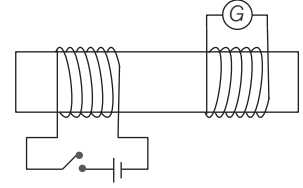
This phenomenon is called **mutual induction**.

In case of mutual inductance for two coils situated close to each other, total flux linked with the secondary coil due to current in the primary coil is

$$N_2 \phi_2$$

$$\text{and } N_2 \phi_2 \propto i_1 \Rightarrow N_2 \phi_2 = M i_1$$

where, N_1 is the number of turns in primary coil; N_2 is the number of turns in secondary coil; ϕ_2 is the flux linked with each turn of secondary coil; i_1 is the current flowing through primary coil; M is the coefficient of mutual induction or mutual inductance.



$$\text{Emf induced in secondary coil, } e_2 = -N_2 \frac{d\phi_2}{dt}; e_2 = -M \frac{di_1}{dt}$$

If $\frac{di_1}{dt} = 1 \text{ A/s}$, then $|e_2| = M$. Hence, coefficient of mutual

induction is equal to the emf induced in the secondary coil when rate of change of current in primary coil is unity.

Mutual Inductance between Two Coils

- (i) If two coils (1 and 2) also called primary and secondary coils are placed close to each other (maximum coupling), then mutual inductance between them

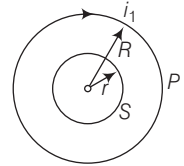
$$M = \frac{B_1 N_2 A_2}{i_1}$$

where, N_1 and N_2 = number of turns in primary and secondary coils respectively, ϕ_2 = flux linked with each turn of secondary coil, $N_2 \phi_2$ = total flux linkage with secondary coils, M = mutual inductance between two coils.

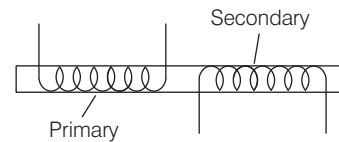
- (ii) If the coils are two concentric coplanar circular coils, then mutual inductance between them

$$M = \frac{\pi \mu_0 N_1 N_2 r^2}{2R}$$

$$\Rightarrow M \propto \frac{r^2}{R}$$



- (iii) For **two solenoids** tightly wound to each other as shown.



Magnetic field inside the primary solenoid,

$$B_1 = \mu_0 n_1 i_1 \quad \left(n_1 = \frac{N_1}{l} \right)$$

$$\text{Thus, } M = \frac{\mu_0 N_1 N_2 A}{l}$$

where, A = area of each solenoid,
 l = length of each solenoid

Combination of Inductors

If two coils of self-inductances L_1 and L_2 are placed quite far apart and are arranged in series, then their equivalent inductance

$$L_s = L_1 + L_2$$

However, if the coils are placed quite close to each other, so as to mutually affect each other, then

$$L_s = L_1 + L_2 \pm 2M$$

Here, M has been written with $\pm$ sign depending on the fact whether currents in the two coils are flowing in same sense or opposite sense.

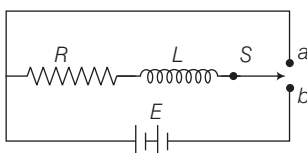
If two coils of self-inductances L_1 and L_2 are connected in parallel, then equivalent inductance L_p is given by

$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} \Rightarrow L_p = \frac{L_1 L_2}{L_1 + L_2}$$

In a parallel arrangement, effect of mutual inductance may be omitted because value of k in this arrangement is quite small.

Growth and Decay of Current in an Inductance Resistance Circuit

Consider a circuit containing a resistance R and inductance L , a two-way key and a battery of emf E connected in series as shown in figure. We consider the following cases



- (a) When the switch is connected to position b , the rise of current in R - L circuit takes place.

The growth of the current in the circuit is given by

$$i = i_0(1 - e^{-t/\tau})$$

where, $i_0 = \frac{E}{R}$ and $\tau = \frac{L}{R}$ = time constant of the circuit.

$$\therefore i = \frac{E}{R} \left[1 - \exp\left(-\frac{R}{L}t\right) \right]$$

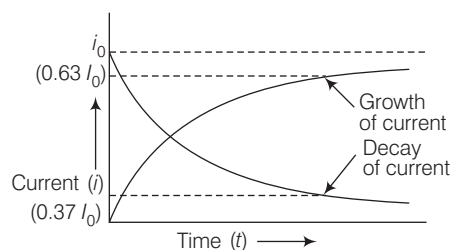
In case of growth of current, when $t = \lambda$, then $i = 0.632i_0$

- (b) When the switch S is connected to position a , decay of the current in R - L circuit takes place. The current in this case is given by

$$i = i_0 \exp\left(-\frac{t}{\tau}\right) = i_0 \exp\left(-\frac{t}{\tau}\right)$$

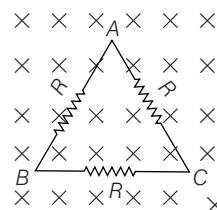
In case of decay of current, when $t = \lambda$, then $i = 0.368i_0$

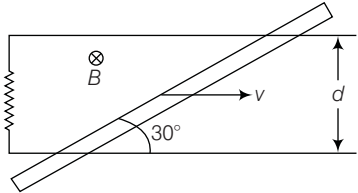
The growth and decay of the current is shown in figure.

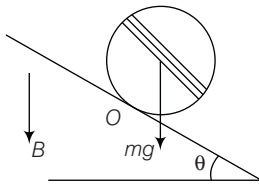


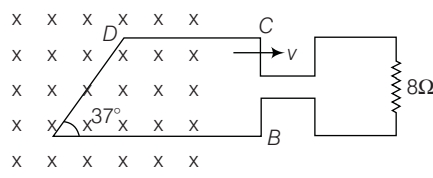
Practice Exercise

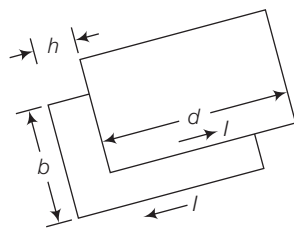
- The magnetic flux ϕ (in weber) in a closed circuit of resistance 10Ω varies with time t (in second) according to equation $\phi = 6t^2 - 5t + 1$. The magnitude of induced current at $t = 0.25$ s
 a. 1.2 A b. 0.8 A c. 0.6 A d. 0.2 A
- A metallic circular loop of radius r is placed in uniform magnetic field B acting perpendicular to the plane of the loop. A naughty boy pulls diametrically opposite corner so that after sometime, the loop changes into an ellipse of major and minor radius a and b . If total resistance of loop is R and it remains constant during the pulling. Find average charge flowing through loop during pulling.
 a. $\frac{B(\pi ab)}{R}$ b. $\frac{B(\pi ab - \pi r^2)}{R}$
 c. $\frac{B\pi r^2}{R}$ d. $\frac{B\pi br}{R}$
- Three resistances of magnitude R each are connected in the form of an equilateral triangle of side a . The combination is placed in a magnetic field $B = B_0 e^{-\lambda t}$ perpendicular to the plane. The induced current in the circuit is given by
 a. $\left(\frac{a^2 \lambda}{2\sqrt{3}R} B_0\right) e^{-\lambda t}$ b. $\left(\frac{a^2 \lambda}{4(\sqrt{3})R} B_0\right) e^{-\lambda t}$
 c. $\left(\frac{a^2 B_0}{\lambda 4\sqrt{3}R}\right) e^{-\lambda t}$ d. $\left(\frac{a^2 B_0 R}{\lambda 4\sqrt{3}}\right) e^{-\lambda t}$



4. A helicopter rises vertically with a speed of 10 ms^{-1} . If helicopter has a length of 10 m and the horizontal component of the earth's magnetic field is $1.5 \times 10^{-3} \text{ Wbm}^{-2}$, the emf induced between the tip of the nose and the tail of the helicopter, is
 a. 0.15 V b. 125 V c. 130 V d. 5 V
5. An air-cored solenoid with length 30 cm, area of cross-section 25 cm^2 and number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10^{-3} s . How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid.
 a. 6.5 V b. 7.4 V c. 8.2 V d. 9.3 V
6. A fan blade of length $2a$ rotates with frequency f cycle per second perpendicular to magnetic field B . Then, potential difference between centre and end of blade is
 a. $\pi B a^2 f$ b. $4\pi B a f$ c. $4\pi a^2 B f$ d. $2\pi a B f$
7. A metal rod AB of length l is rotated with a constant angular velocity ω about an axis passing through O and normal to its length. Potential difference between ends of rod in absence of external magnetic field (where, e = electric charge)
 a. zero b. $\frac{m\omega^2 l^2}{4e}$ c. $\frac{m\omega^2 l^2}{2e}$ d. $\frac{m\omega^2 l^2}{8e}$
8. A wire is sliding as shown in the figure. The angle between the acceleration and velocity of the wire is

 a. 30° b. 40°
 c. 120° d. 90°
9. A fan blade of length $1/\sqrt{\pi}$ metre rotates with frequency 5 cycle per second perpendicular to a magnetic field 10 tesla. What is potential difference between the centre and the end of blade?
 a. -50 V b. +50 V
 c. -2.0 V d. +0.02 V
10. A coil has an area of 0.05 m^2 and has 800 turns. After placing the coil in a magnetic field of strength $4 \times 10^{-5} \text{ Wbm}^{-2}$, perpendicular to the field, the coil is rotated by 90° in 0.1 s. The value of average emf induced is
 a. zero b. 0.016 V
 c. 0.01 V d. 0.032 V

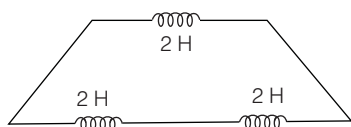
11. In the figure shown, a coil of single turn is wound on a sphere of radius r and mass m . The plane of the coil is parallel to the inclined plane and lies in the equatorial plane of the sphere. If sphere is in rotational equilibrium, the value of B is (current in the coil is I)

 a. $\frac{mg}{\pi I r}$ b. $\frac{mg \sin \theta}{\pi I}$
 c. $\frac{mgr \sin \theta}{\pi I}$ d. None of these

12. The loop $ABCD$ is moving with velocity ' v ' towards right. The magnetic field is 4 T. The loop is connected to a resistance of 8Ω . If steady current of 2 A flows in the loop then value of v if loop has a resistance of 4Ω , (Given, $AB = 30 \text{ cm}$, $AD = 30 \text{ cm}$) is


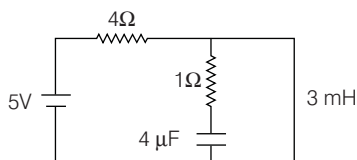
- a. $\frac{50}{3} \text{ m/s}$ b. 20 m/s
 c. 10 m/s d. $\frac{100}{3} \text{ m/s}$
13. Calculate the self-inductance of the air cored solenoid of length 80 cm and has 500 turns and its circular cross-section has diameter of 2 cm.
 a. $150.6 \mu\text{H}$ b. $162.2 \mu\text{H}$
 c. $123.3 \mu\text{H}$ d. $102.5 \mu\text{H}$
14. The inductance per unit length of a double tape line as shown in the figure.


- a. $\frac{\mu_0 h}{b}$ b. $\frac{b}{\mu_0 h}$
 c. $\frac{\mu_0 b}{h}$ d. $\frac{hb}{\mu_0}$
15. What is the mutual inductance of coil and solenoid if a solenoid of length 0.50 m and with 5000 turns of wire has a radius 4 cm and a coil of 700 turns is wound on the middle part of the solenoid?
 a. 44.17 mH b. 48.98 mH
 c. 34.34 mH d. 36.73 mH

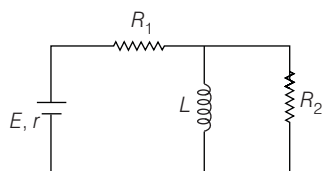
16. When the current changes from $+2\text{ A}$ to -2 A in 0.05 s , an emf of 8 V is induced in a coil. The coefficient of self-induction of the coil is
- 0.1 H
 - 0.2 H
 - 0.4 H
 - 0.8 H
17. A closed circuit consists of a source of emf E and an inductor coil of inductance L , connected in series. The active resistance of whole circuit is R . At the moment $t = 0$, inductance of coil abruptly decreased to L/n . Then, current in the circuit immediately after, is
- zero
 - E/R
 - $\frac{nE}{R}$
 - $\frac{E}{nR}$
18. Three pure inductors each of 2 H are connected as shown in the figure. The equivalent inductance of the circuit is



- $8\text{ H}/6$
 - 6 H
 - 2 H
 - None of these
19. The sum and the difference of self-inductances of two coils are 13 H and 5 H respectively. The maximum mutual inductances of two coils is
- 6 H
 - 5 H
 - $\sqrt{65}\text{ H}$
 - 18 H
20. In the figure, the steady state current through the inductor will be

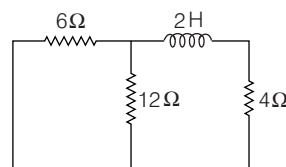


- zero
 - 1 A
 - 1.25 A
 - Cannot be determined
21. Determine the value of time constant for the given circuit.

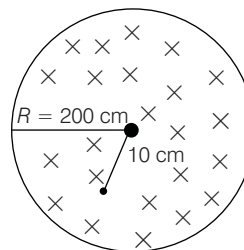


- $\frac{L}{R_1 + r + R_2}$
- $\frac{L}{(R_1 + r)}$
- $\frac{L(R_1 + R_2 + r)}{(R_1 + r)R_2}$
- None of these

22. The time constant for the given circuit is



- 4 s
 - $1/4\text{ s}$
 - 2 s
 - $1/2\text{ s}$
23. With usual notations, the energy dissipation in an ideal inductor is given by
- LI
 - $\frac{1}{2}LI$
 - $\frac{1}{2}LI^2$
 - None of the above
24. A non-conducting ring of radius r has charge per unit length λ . A magnetic field perpendicular to plane of the ring changes at rate dB/dt . Torque experienced by the ring is



- $\lambda\pi r^3 \frac{dB}{dt}$
 - $\lambda 2\pi r^3 \frac{dB}{dt}$
 - $\lambda^2(2\pi r)^2 r \frac{dB}{dt}$
 - zero
25. Figure shows a uniform magnetic field B confined to a cylindrical volume of radius R . If B is increasing at constant rate of 0.01 T/s . Instantaneous acceleration experienced by electron at $r = 10\text{ cm}$ ($< R$) as shown in the figure.
- $8.79 \times 10^{-12}\text{ m/s}^2$
 - $8.79 \times 10^{-11}\text{ m/s}^2$
 - $8.79 \times 10^{-10}\text{ m/s}^2$
 - $8.79 \times 10^{-9}\text{ m/s}^2$
26. Find the energy stored in the magnetic field if current of 5 A produces a magnetic flux of $2 \times 10^{-3}\text{ Wb}$ through a coil of 500 turns.
- 2.5 J
 - 0.25 J
 - 250 J
 - 1.5 J
27. The inductance of a coil in which a current of 0.1 A increasing at the rate of 0.5 A/s represents a power flow of $\frac{1}{2}\text{ watt}$, is
- 2 H
 - 8 H
 - 20 H
 - 10 H

BITSAT Archives

1. Lenz's law of electromagnetic induction corresponds to the [2011]
 - a. law of conservation of charge
 - b. law of conservation of energy
 - c. law of conservation of momentum
 - d. law of conservation of angular momentum
2. A toroidal solenoid with an air core has an average radius of 15 cm, area of cross-section 12 cm^2 and 1200 turns. Ignoring the field variation across the cross-section of the toroid the self-inductance of the toroid is [2011]
 - a. 4.6 mH
 - b. 6.9 mH
 - c. 2.3 mH
 - d. 9.2 mH
3. A coil of inductance L is carrying a steady current I . What is the nature of its stored energy? [2011]
 - a. Magnetic
 - b. Electrical
 - c. Both magnetic and electrical
 - d. Heat
4. If emf induced in a coil is 2 V by changing the current in it from 8 A to 6 A in $2 \times 10^{-3} \text{ s}$. Then, the coefficient of self-induction is [2006]
 - a. $2 \times 10^{-3} \text{ H}$
 - b. 10^{-3} H
 - c. $0.5 \times 10^{-3} \text{ H}$
 - d. $4 \times 10^{-3} \text{ H}$
5. If in a triode valve amplification factor is 20 and plate resistance is $10 \text{ k}\Omega$, then its mutual conductance is [2006]
 - a. 2 milli mho
 - b. 20 milli mho
 - c. (1/2) milli mho
 - d. 200 milli mho
6. The induction coil works on the principle of [2005]
 - a. self-induction
 - b. mutual induction
 - c. Ampere's rule
 - d. Fleming's right hand rule

Answer with Solutions

Practice Exercise

1. (d)
2. (b)

3. (b) $\phi = BA = B_0 e^{-\lambda t} \frac{\sqrt{3}}{4} a^2$

$$e = -\frac{d\phi}{dt} = -\frac{\sqrt{3}}{4} a^2 B_0 \frac{d}{dt} (e^{-\lambda t})$$

$$= -\frac{\sqrt{3}}{4} a^2 B_0 (-\lambda) e^{-\lambda t} = \frac{\sqrt{3}}{4} a^2 B_0 \lambda e^{-\lambda t}$$

$$\therefore I = \frac{e}{3R} = \frac{\sqrt{3}}{12R} a^2 B_0 \lambda e^{-\lambda t} = \left(\frac{a^2 \lambda B_0}{4\sqrt{3}R} \right) e^{-\lambda t}$$

4. (a)

5. (a) Given, length of solenoid $l = 30 \text{ cm} = 30 \times 10^{-2} \text{ m}$

Area of cross-section $A = 25 \text{ cm}^2 = 25 \times 10^{-4} \text{ m}^2$

Number of turns, $N = 500$

Current, $I_1 = 21.5 \text{ A}$, $I_2 = 0$

Brief time $dt = 10^{-3} \text{ s}$

Induced emf in the solenoid

$$e = \frac{d\phi}{dt} = \frac{d}{dt} (BA) \quad [\because \phi = BA]$$

Magnetic field induction B at a point well inside the long solenoid carrying current I is

$$B = \mu_0 n I \quad \left(\text{where, } n = \text{number of turns per unit length} = \frac{N}{l} \right)$$

$$e = NA \frac{dB}{dt} = A \frac{d}{dt} \left(\mu_0 \frac{N}{l} I \right) = A \frac{\mu_0 N}{l} \cdot \frac{dI}{dt}$$

$$e = 500 \times 25 \times 10^{-4} \times 4 \times 3.14 \times 10^{-7} \times \frac{500}{30 \times 10^{-2}} \times \frac{2.5}{10^{-3}}$$

$$= 6.5 \text{ V}$$

6. (a)

7. (b) $eV = m\omega^2 x \Rightarrow V = \frac{m\omega^2 x}{e}$

$$dV = \frac{m\omega^2}{e} dx \Rightarrow V = \frac{m\omega^2}{e} \int_{1/4}^{3/4} x dx = \frac{m\omega^2 l^2}{4e}$$

8. (c) The magnetic force on the wire is perpendicular to wire. (shown in the figure)

$$\therefore \text{Angle} = 30^\circ + 90^\circ = 120^\circ$$

9. (a) $\int_{V_0}^{V_D} de = - \int_0^a B x \omega dx$

$$V_D - V_0 = - \frac{1}{2} B a^2 \omega$$

$$= - \frac{1}{2} \times 10 \times 5 \times 2\pi \times \frac{1}{\pi} = -50 \text{ V}$$

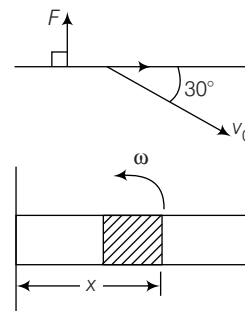
10. (b)

11. (a) Torque due to magnetic force = torque due to gravitational force

or $MB \sin (180 - \theta) = mgr \sin \theta$

or $MB \sin \theta = mgr \sin \theta$

$$\therefore B = \frac{mgr}{M} = \frac{mgr}{\pi r^2} = \frac{mg}{\pi r}$$



12. (d)
- $e = BAD \sin 37^\circ v = 4 \times 0.3 \sin 37^\circ v$

$$I = \frac{4 \times 0.3 \times 0.6 v}{(4 + 8)} = 2 \Rightarrow v = \frac{24 \times 10}{4 \times 0.6 \times 3} = \frac{100}{3} \text{ m/s}$$

13. (c)

14. (a) Neglecting end effects of magnetic field, we have

$$B = \frac{\mu_0 I}{b}$$

Flux ϕ per unit length of the plates is

$$\frac{\mu_0 I}{b} \times h \times l = \frac{\mu_0 h l}{b}$$

$$\text{Also, } \phi = LI \Rightarrow L = \frac{\mu_0 h}{b}$$

15. (a) Flux linked with coil

$$\phi_2 = N_2 B I A = \frac{\mu_0 N_1 N_2 \pi r^2 I}{l} \text{ also } \phi = M I$$

Comparing the equations,

$$M = \frac{\mu_0 N_1 N_2 \pi r^2}{l}$$

$$M = \frac{4 \times 3.14 \times 10^{-7} \times 5000 \times 700 \times 3.14 \times (4 \times 10^{-2})^2}{0.5}$$

$$M = 44.17 \text{ mH}$$

16. (a)
- $e = L \frac{\Delta I}{\Delta t}$

$$\therefore L = \frac{e}{\frac{\Delta I}{\Delta t}} = \frac{8}{\frac{0.05}{0.05}} = \frac{8 \times 0.05}{4} = 0.1 \text{ H}$$

17. (c) Due to change in inductance, magnetic flux remains constant.

But, $\phi = LI = \text{constant}$

$$\text{Here, } L_2 = \frac{L_1}{n} \text{ and } I_1 = \frac{E}{R} \therefore I_2 = \frac{L_1 I_1}{L_2} = \frac{nE}{R}$$

18. (a)

19. (a)
- $L_1 + L_2 = 13$

$$L_1 - L_2 = 5$$

Adding Eqs. (i) and (ii), we get

$$2L_1 = 18 \therefore L_1 = 9 \text{ H}$$

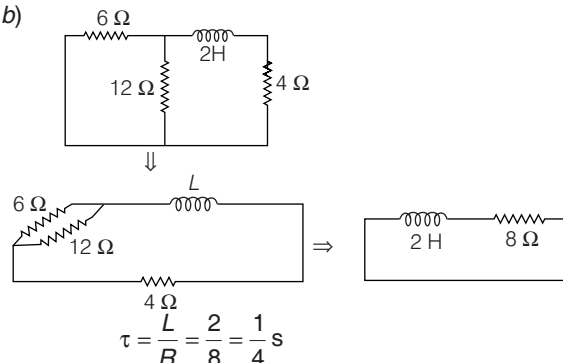
$$\therefore L_2 = L_1 - 5 = 9 - 5 = 4 \text{ H} \therefore M = K \sqrt{L_1 L_2}$$

$$\therefore M_{\max} = K_{\max} \sqrt{9 \times 4} = 1 \times \sqrt{36} = 6 \text{ H} \quad [\because K_{\max} = 1]$$

20. (c)

$$21. (c) \text{ Here, } R_{\text{eq}} = \frac{(R_1 + r)R_2}{R_1 + r + R_2} \therefore \tau = \frac{L}{R} = \frac{L(R_1 + R_2 + r)}{(R_1 + r)R_2}$$

22. (b)



23. (d)

$$24. (a) e = \frac{d\phi}{dt} = \pi r^2 \frac{dB}{dt} \Rightarrow E(2\pi r) = \pi r^2 \frac{dB}{dt}$$

$$E = \frac{1}{2} r \frac{dB}{dt} \Rightarrow dF = \lambda dSE$$

$$\Rightarrow d\tau = r \lambda dSE \Rightarrow \tau = \lambda r E \int dS = \lambda r E 2\pi r$$

$$= 2\pi r^2 \lambda \frac{1}{2} r \frac{dB}{dt} = \pi r^3 \lambda \frac{dB}{dt}$$

$$25. (b) E(2\pi r) = \pi r^2 \frac{dB}{dt} \Rightarrow E = \frac{1}{2} r \frac{dB}{dt}$$

$$a = \frac{eE}{m_e} = \frac{\frac{e}{2} r \left(\frac{dB}{dt} \right)}{m_e} = \frac{1.6 \times 10^{-19} \times \frac{1}{2} \times 0.1 \times 0.01}{9.1 \times 10^{-31}}$$

$$= 8.79 \times 10^{-11} \text{ m/s}^2$$

26. (a)

27. (d)

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1. (b) Lenz's law of electromagnetic induction corresponds to the law of conservation of energy.

2. (c) For a solenoid,

$$B = \mu_0 n I$$

$$\therefore n = \frac{N}{2\pi r}$$

$$\therefore B = \frac{\mu_0 N I}{2\pi r}$$

Flux linked with the solenoid

$$\phi = NBA$$

$$\phi = \frac{\mu_0 N^2 I A}{2\pi r} = \frac{4\pi \times 10^{-7} \times (1200)^2 \times 12 \times 10^{-4}}{2\pi \times 15 \times 10^{-2}}$$

$$L = 2.3 \times 10^{-3} \text{ H}$$

$$= 2.3 \text{ mH}$$

3. (a) Energy is stored in an inductor in the form of magnetic potential energy.

4. (a) Induced emf,
- $e = 2 \text{ V}$

$$i_1 = 8 \text{ A, } i_2 = 6 \text{ A} \Rightarrow \Delta t = 2 \times 10^{-3}$$

Coefficient of self-induction

$$L = \frac{e}{\Delta i / \Delta t} = \frac{-2}{(6 - 8) / 2 \times 10^{-3}} = \frac{-2 \times 2 \times 10^{-3}}{-2} = 2 \times 10^{-3} \text{ H}$$

5. (a) Amplification factor
- $\mu = 20$

$$\text{Plate resistance } R_p = 10 \text{ k}\Omega = 10 \times 10^{-3} \Omega$$

$$\therefore \text{Mutual conductance } g_m = \frac{\mu}{R_p}$$

$$= \frac{20}{10 \times 10^3}$$

$$= 2 \times 10^{-3} \text{ mho or 2 milli mho}$$

6. (b) The induction coil works on the principle of mutual induction.

28

Alternating Current and EM Wave

Alternating Current

An **alternating current** is the current (or voltage) whose magnitude keeps on changing continuously with time, between zero and a maximum value and the direction also reverses periodically.

Alternating current or voltage varying as sine function can be written as

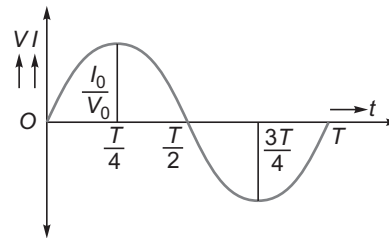
$$i = i_0 \sin \omega t = i_0 \sin 2\pi \nu t = i_0 \sin \frac{2\pi}{T} t$$

and $V = V_0 \sin \omega t = V_0 \sin 2\pi \nu t = V_0 \sin \frac{2\pi}{T} t$

where, i and V = instantaneous values of current and voltage,

i_0 and V_0 = peak values of current and voltage respectively.

ω = angular frequency in rad/s, ν = Frequency (in Hz) and T = time period.



Mean Square Value ($\bar{V}^2$ or $\bar{i}^2$)

The average of square of instantaneous values in one cycle, is called mean square value. It is always positive for one complete cycle.

e.g.
$$\bar{V}^2 = \frac{1}{T} \int_0^T V^2 dt = \frac{V_0^2}{2} \text{ or } \bar{i}^2 = \frac{i_0^2}{2}$$

Root Mean Square (rms) Value

Root mean square of voltage or current in an AC circuit for one complete cycle is called rms value. It is denoted by V_{rms} or i_{rms} .

$$i_{\text{rms}} = \sqrt{\frac{i_1^2 + i_2^2 + \dots}{n}} = \sqrt{i^2} = \sqrt{\frac{\int_0^T i^2 dt}{\int_0^T dt}} = \frac{i_0}{\sqrt{2}}$$

$$= 0.707 i_0 = 70.7\% \text{ of } i_0$$

Similarly, $V_{\text{rms}} = V_0 / \sqrt{2} = 0.707 V_0 = 70.7\% \text{ of } V_0$

The rms value of alternating current is also called **virtual value** or **effective value**.

Mean or Average Value (i_{av} or V_{av})

The average of instantaneous values of current or voltage in one cycle is called its mean value. The average value of alternating quantity for one complete cycle is zero.

The average value of AC over half cycle ($t = 0$ to $T/2$) is

$$i_{\text{av}} = \frac{\int_0^{T/2} i dt}{\int_0^{T/2} dt} = \frac{2i_0}{\pi} = 0.637 i_0 = 63.7\% \text{ of } i_0$$

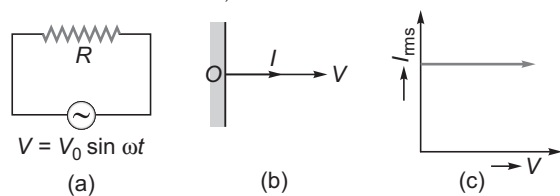
Similarly, $V_{\text{av}} = \frac{2V_0}{\pi} = 0.637 V_0 = 63.7\% \text{ of } V_0$

AC Circuit Elements

The circuits consist of resistor, capacitor and inductor are called pure resistive, pure inductive and pure capacitive circuits. These circuits with their respective phasor diagram are given below.

1. Pure Resistive Circuit

Let an alternating voltage $V = V_0 \sin \omega t$ be applied across a pure resistance R . Then,



$$\text{Current, } I = \frac{V}{R} \text{ or } I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$$

Current and voltage are in the same phase, i.e. current is given by $I = I_0 \sin \omega t$.

2. Pure Inductive Circuit

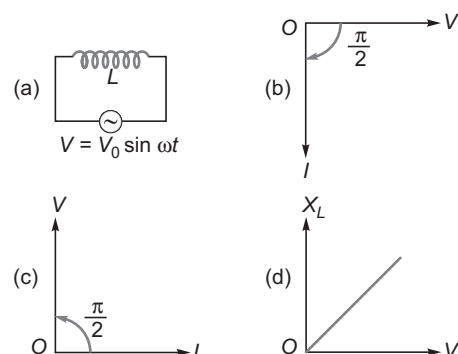
Let an alternating voltage $V = V_0 \sin \omega t$ be applied across a pure inductance L . Then, the average power

$$= V_{\text{rms}} I_{\text{rms}} \cos \frac{\pi}{2} = 0$$

Such a current, for which average power as well as power factor is zero, is called as **wattless current**.

The inductance offers some opposition to the flow of AC. It is known as inductive reactance $X_L = 2\pi v L = L\omega$. Thus, a

pure inductance does not oppose the flow of DC ($\omega = 0$) but opposes the flow of AC.



$$\text{Current flowing, } I = \frac{V}{X_L}$$

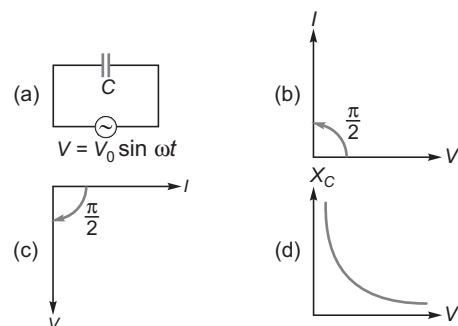
The current decreases with an increase in frequency.

The current lags behind the voltage by $\frac{\pi}{2}$ (or voltage leads the current by $\frac{\pi}{2}$) and is thus given by

$$I = I_0 \sin \left(\omega t - \frac{\pi}{2} \right)$$

3. Pure Capacitive Circuit

Let an alternating voltage $V = V_0 \sin \omega t$ be applied across a pure capacitance C . Then,



The capacitance offers some opposition to the flow of DC current but allows AC to pass through it. The opposition offered is known as the **capacitive reactance**.

$$X_C = \frac{1}{C\omega} \Omega$$

$$= \frac{1}{C \times 2\pi v} \Omega$$

$$\text{Current flowing, } I = \frac{V}{X_C}$$

For DC circuit, frequency $v = 0$ and hence, $X_L = 0$ but $X_C = \infty$

The current increases with an increase in frequency.

The current leads the voltage by $\frac{\pi}{2}$ (or voltage lags behind the current by $\frac{\pi}{2}$) and is thus given by $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$

Out of the three elements in an AC circuit, the resistor is called the **active element** because it always opposes the flow of current under all conditions.

Series AC Circuits

Some of the series AC circuits are given below

1. Series L-R Circuit

The potential difference across a resistance in AC is in phase with current and it leads in phase by 90° with current across the inductor.

For $E = E_0 \sin \omega t$
 $I = \frac{E_0}{Z} \sin(\omega t - \phi)$

where, $Z = \sqrt{R^2 + (\omega L)^2}$

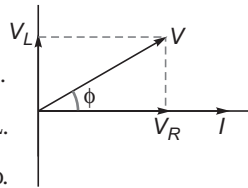
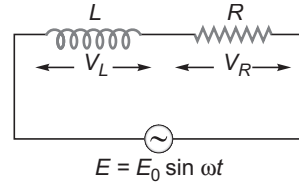
and $\tan \phi = \frac{\omega L}{R}$

$$V = \sqrt{V_R^2 + V_L^2}$$

where, V_R = voltage across resistor R .

V_L = voltage across inductor L .

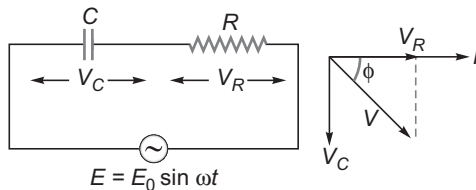
Current lags behind the voltage by ϕ .



2. Series R-C Circuit

Potential difference across a capacitor in AC lags in phase by 90° with the current in the circuit.

For $E = E_0 \sin \omega t$,
 $I = \frac{E_0}{Z} \sin(\omega t + \phi)$



where, $Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

and $\tan \phi = \frac{-1/\omega C}{R}$

Current leads the voltage by ϕ .

$$V^2 = V_R^2 + V_C^2 \Rightarrow V = \sqrt{V_R^2 + V_C^2}$$

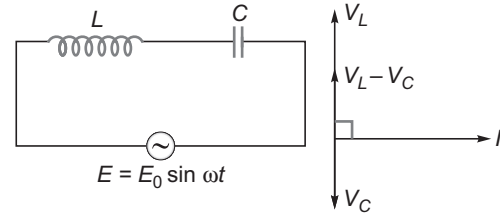
3. Series L-C Circuit

For $E = E_0 \sin \omega t$, $I = \frac{E_0}{Z} \sin(\omega t - \phi)$

where,

$$Z = X_L - X_C$$

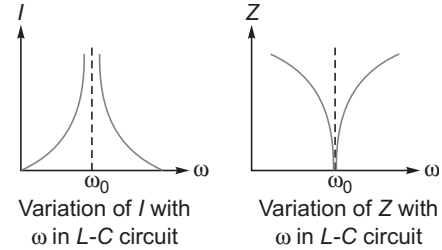
and $\tan \phi = \frac{X_L - X_C}{0} = \infty$



For $X_L > X_C$, $\phi = \frac{\pi}{2}$ and for $X_L < X_C$, $\phi = -\frac{\pi}{2}$

If $X_L = X_C$ i.e. at $\omega = \frac{1}{\sqrt{LC}}$, $Z = 0$ and I_0 becomes infinity.

This condition is termed as the resonant condition and this frequency is termed as natural frequency of the circuit.



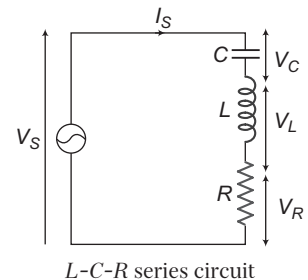
4. L-C-R Series Circuit

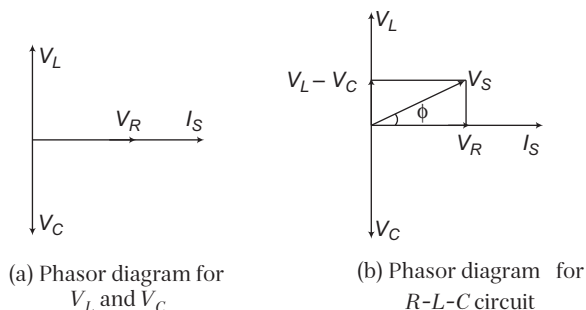
As shown in the figure, inductance L , capacitance C and resistance R are connected together in series and supplied with an alternating voltage. In such an arrangement, the same circuit current flows through all the components of the circuit and V_R , V_L and V_C indicate the voltages across the resistor, inductor and capacitor. Fig. (b) shows the circuit conditions when the inductive reactance (X_L) is greater than the capacitive reactance (X_C).

In this case, since both L and C carry the same current and X_L is greater than X_C , it follows that V_L must be greater than

$$V_C, (V_L = I_S X_L \text{ and } V_C = I_S X_C)$$

Remember that V_C and V_L are in anti-phase to each other due to their 90° leading and lagging relationship with the circuit current I_S . As V_L and V_C directly oppose each other, a resulting voltage is created, which will be the difference between V_C and V_L . This is called the **reactive voltage**.





Phasors for V_L and V_C are in anti-phase. So, the circuit in which V_L is greater than V_C , behaves like an inductive circuit.

The phasors for $(V_L - V_C)$, V_R and V_S in the figure form right angled triangle, a number of properties and values in the circuit can be calculated using Pythagoras theorem or some basic trigonometry. Like

$$V_S^2 = (V_L - V_C)^2 + V_R^2 \quad \text{or} \quad V_S = \sqrt{(V_L - V_C)^2 + V_R^2}$$

Total circuit impedance (Z), $Z = \sqrt{(X_L - X_C)^2 + R^2}$

The phase angle between $(V_L - V_C)$ and V_R can be found using trigonometry

$$\therefore \tan \phi = \frac{(V_L - V_C)}{V_R} \Rightarrow \phi = \tan^{-1} \frac{(V_L - V_C)}{V_R}$$

Also, Ohm's law states that R (or X) = $\frac{V}{I}$

If $(V_L - V_C)$ and V_R are each divided by the current I_S . This allows the phase angle θ to be found using the resistances and reactances.

$$\phi = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

At Resonance (Series Resonant Circuit)

- $X_L = X_C \Rightarrow Z_{\min} = R$, i.e. circuit behaves as resistive circuit.
- $V_L = V_C \Rightarrow V = V_R$, i.e. whole applied voltage appeared across the resistance.
- Phase difference, $\phi = 0^\circ \Rightarrow$ Power factor = $\cos \phi = 1$
- Power consumption, $P = V_{\text{rms}} i_{\text{rms}} = \frac{1}{2} V_0 i_0$
- Current amplitude in the circuit is maximum and it is given by $i_0 = \frac{V_0}{R}$
- This circuit is used for voltage amplification and as selector circuits in wireless telegraphy.

Resonant Frequency (Natural Frequency)

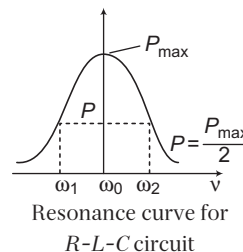
At resonance $X_L = X_C \Rightarrow \omega_0 L = \frac{1}{\omega_0 C}$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

$$\Rightarrow \nu_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz (or cps)}$$

Resonant frequency doesn't depend upon the resistance of the circuit.

Half power frequencies and band width The frequencies at which the power in the circuit is half of the maximum power (the power at resonance), are called half power frequencies.



- (i) The current in the circuit at

Half Power Frequencies (HPF) is $\frac{1}{\sqrt{2}}$ or 0.707 or 70.7% of maximum current (current at resonance).

- (ii) There are two half power frequencies, which are related as

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

Quality Factor

At resonance, $I_0 = \frac{E_0}{R}$

and

$$V_L = I_0 X_L = V_C = I_0 X_C$$

i.e.

$$V_L = \frac{\omega L}{R} \times E_0 = \frac{1}{\omega RC} \times E_0 = Q E_0$$

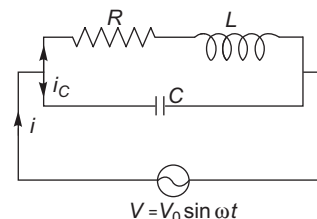
where,

$$Q = \frac{\omega L}{R}$$

or $\frac{1}{\omega RC}$ is termed as the **Quality factor of the circuit**. It determines the sharpness of resonance. Higher the value of Q , sharper is the resonance.

Parallel Resonant Circuit

Figure shows a parallel resonant circuit in which resistor R and inductor L have been connected in series and this combination is connected in parallel with the capacitor C .



To this combination, an alternating source of $V = V_0 \sin \omega t$ is applied.

From the figure,

$$i = i_L + i_C$$

or

$$\begin{aligned} \frac{V}{Z} &= \frac{V}{R + j\omega L} + \frac{V}{-j/\omega C} \\ &= \frac{V}{R + j\omega L} - \frac{\omega CV}{j} \end{aligned}$$

$$= \frac{V}{R + j\omega L} + j(\omega C)V$$

$$\therefore \frac{1}{Z} = \frac{1}{R + j\omega L} + j\omega C$$

$\frac{1}{Z}$ is known as admittance (Y). Thus,

$$Y = \frac{1}{Z} = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

$$\therefore Y = \frac{\sqrt{R^2 + (\omega CR^2 + \omega^3 L^2 C - \omega L)^2}}{R^2 + \omega^2 L^2}$$

The admittance will be minimum when

$$\omega CR^2 + \omega^3 L^2 C - \omega L = 0$$

$$\text{or } \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\therefore f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

is known as resonance frequency. At resonance frequency admittance is minimum and impedance is maximum.

Thus, the parallel circuit does not allow this frequency from the source to pass in the circuit. Due to this reason, the circuit with such a frequency is known as rejector circuit.

We have dynamic resistance

$$Z_{\max} = \frac{1}{Y_{\max}} = \frac{L}{CR}$$

$$\therefore \text{Peak current through the supply} = \frac{V_0}{L/CR} = \frac{V_0 CR}{L}$$

$$\text{The peak current through capacitor} = \frac{V_0}{1/\omega C} = V_0 \omega C$$

$$\text{Hence, } Q\text{-factor} = \frac{V_0 \omega C}{V_0 CR/L} = \frac{\omega L}{R}$$

This is basically the measure of current magnification.

Power in an AC Circuit

Let a voltage $V = V_0 \sin \omega t$ be applied across an AC and consequently a current $I = I_0 \sin(\omega t - \phi)$ flows through the circuit. Then,

(i) **Instantaneous power** = $V I$

= $V_0 I_0 \sin \omega t \sin(\omega t - \phi)$, and its value varies with time.

Here, ϕ is known as **phase difference** between V and I .

(ii) **Average power** over a full cycle of AC is

$$P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos \phi = \frac{1}{2} V_0 I_0 \cos \phi$$

The term $V_{\text{rms}} I_{\text{rms}}$ is known as the apparent or virtual power but $V_{\text{rms}} I_{\text{rms}} \cos \phi$ is called the **true power**.

(iii) The term $\cos \phi$ is known as the **power factor** of the given circuit. Thus, $\cos \phi = \frac{R}{Z}$ = power factor

$$= \frac{\text{true power}}{\text{apparent power}}$$

(iv) For a pure resistive circuit, V and I are in phase ($\phi = 0^\circ$) hence, $\cos \phi = 1$ and average power = $V_{\text{rms}} I_{\text{rms}}$. For a pure inductive or a pure capacitive circuit, current and voltage differ in phase by $\frac{\pi}{2}$ (i.e. $\phi = \frac{\pi}{2}$).

Wattless Current

Average power is given by $P_{\text{av}} = E_{\text{rms}} I_{\text{rms}} \cos \phi$

The phase difference between E_{rms} and I_{rms} is ϕ . We can resolve I_{rms} into two components

$$I_{\text{rms}} \cos \phi \text{ and } I_{\text{rms}} \sin \phi$$

Here, the component $I_{\text{rms}} \cos \phi$ contributes towards power dissipation and the component $I_{\text{rms}} \sin \phi$ does not contribute towards power dissipation. Therefore, it is called **wattless current**.

Choke Coil

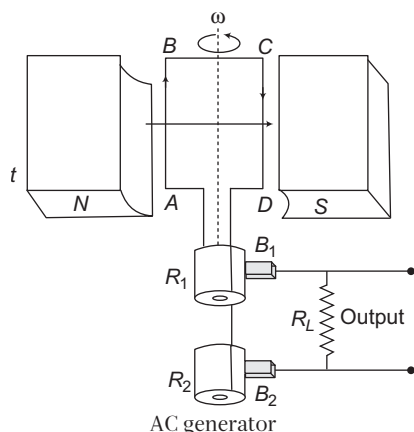
Let us consider a choke coil (used in tube lights) of large inductance, L and low resistance R . The power factor for such a coil is given by

$$\cos \phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} \approx \frac{R}{\omega L} \text{ (as } R \ll \omega L \text{)}$$

As $R \ll \omega L$, $\cos \phi$ is very small. Thus, the power absorbed by the coil $V_{\text{rms}} i_{\text{rms}} \cos \phi$ is very small. On account of its large impedance $Z = \sqrt{R^2 + \omega^2 L^2}$, the current passing through the coil is very small. Such a coil is used in AC circuits for the purpose of adjusting current to any required value without waste of energy. The only loss of energy is due to hysteresis in the iron core, which is much less than the loss of energy in the resistance that can also reduce the current if placed instead of the choke coil.

AC Generator

An electrical machine used to convert mechanical energy into electrical energy is known as AC generator/alternator. It works on the **principle of electromagnetic induction**, i.e. when a coil is rotated in uniform magnetic field, an induced emf is produced in it.



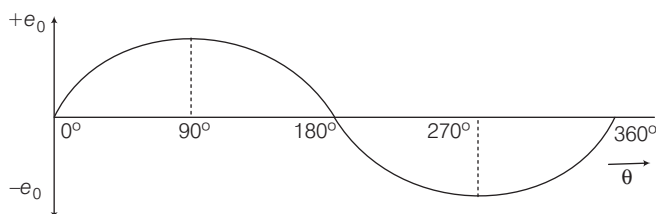
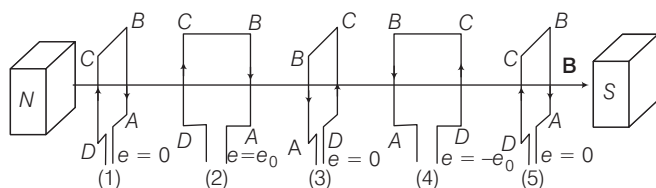
Induced emf,
$$e = -\frac{Nd\phi}{dt} = NBA\omega \sin \omega t = e_0 \sin \omega t$$

where,
$$e_0 = NBA\omega$$

Current in the coil,
$$i = \frac{e}{R} = \frac{e_0}{R} \sin \omega t = i_0 \sin \omega t$$

where, R = Resistance of the circuit.

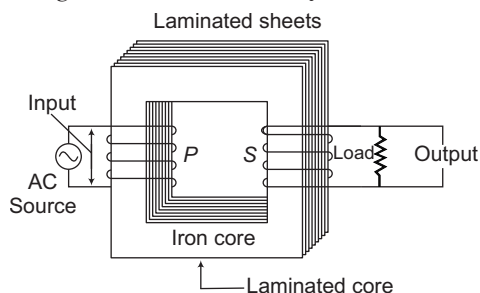
$$\theta = 0^\circ \quad \theta = 90^\circ \quad \theta = 180^\circ \quad \theta = 270^\circ \quad \theta = 360^\circ$$



Output of a single loop of AC generator for one cycle of rotation

Transformer

It is a device which raises or lowers the voltage in AC circuits through mutual induction. It works on the basic **principle of mutual induction** i.e. The alternating current passing through the primary coil creates a continuously changing flux through the core. This changing flux induces an alternating emf in the secondary coil.



Transformer is of two types

Step-up transformer	Step-down transformer
This transformer is used to increase voltage and decrease current.	This transformer is used to decrease voltage and increase current.
$V_S > V_P$	$V_S < V_P$
$N_S > N_P$	$N_S < N_P$
$E_S > E_P$	$E_S < E_P$
$i_S < i_P$	$i_S > i_P$
$R_S > R_P$	$R_S < R_P$
$t_S > t_P$	$t_S < t_P$
$k > 1$	$k < 1$

where, N_P and N_S = Number of turns in primary and secondary coil

V_P and V_S = Applied (input) voltage to primary and secondary (load voltage) coil

e_P and e_S = Induced emf in primary and secondary coil

ϕ = Flux linked with primary as well as secondary coil

i_P and i_S = Current in primary and secondary coil (or load current)

R_P and R_S = Resistance of primary and secondary coil

t_P and t_S = Thickness of turn in primary and secondary coil

As in an ideal transformer, there is no loss of power i.e. $P_{out} = P_{in}$ and $e = V$

So, $V_S i_S = V_P i_P$ and $V_P \approx e_P, V_S \approx e_S$

According to Faraday's law,

$$e_S = -N_S \frac{d\phi}{dt}, e_P = -N_P \frac{d\phi}{dt}$$

Hence,
$$\frac{e_S}{e_P} = \frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{i_P}{i_S} = k;$$

where, k = transformation ratio

Efficiency of transformer (η) Efficiency is defined as the ratio of output power and input power

$$(\eta\%) = \frac{P_{out}}{P_{in}} \times 100$$

i.e.

$$= \frac{V_S i_S}{V_P i_P} \times 100$$

Electromagnetic Waves

Electromagnetic waves are those waves in which electric and magnetic fields vary sinusoidally in space and with time. The electric and magnetic fields are mutually perpendicular to each other and each field is perpendicular to the direction of propagation of the wave.

Characteristics of EM Waves

Maxwell's theory predicted that electromagnetic waves of all frequencies (and hence all wavelengths) propagate in vacuum, with a speed given by $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

where, μ_0 is the magnetic permeability and ϵ_0 is the electric permittivity of vacuum. Now, for the vacuum, $\mu_0 = 4\pi \times 10^{-7} \text{ TmA}^{-1}$ and $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$. Substituting these values in the above relation, we have

$$c = \frac{1}{[(4\pi \times 10^{-7})(8.85 \times 10^{-12})]^{1/2}} \simeq 3.0 \times 10^8 \text{ ms}^{-1}$$

Maxwell's Displacement Current

If the plates of a charged capacitor are joined through a conductor, then the charge arises due to the flow of electrons and thus conduction current flows in the conductor.

Maxwell proposed that as the charge on the plates of the capacitor decreases, the electric field between the plates also decreases with time and the time varying electric field produces magnetic field which signifies that there must be some current flowing between the plates. Maxwell termed this as displacement current (i_d) given as

$$i_d = \epsilon_0 \frac{d\phi_E}{dt}$$

where, ϕ_E = electric flux,

ϵ_0 = permittivity of the free space

Maxwell's Equations

Maxwell in 1862, gave the basic laws of electricity and magnetism in the form of four fundamental equations. In the absence of any dielectric and magnetic material, which are known as Maxwell's equations. It is based on experimental observations followed by all electromagnetic phenomena, may be stated in the integral form as below

1. Gauss's law for electrostatics

- (i) This law gives the total electric flux in terms of charge enclosed by the closed surface.
- (ii) This law states that the electric lines of force start from positive charge and end at negative charge i.e. the electric lines of force do not form a continuous closed path.

$$\text{Mathematically, } \oint_s \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\epsilon_0}$$

2. Gauss's law for magnetism

- (i) This law shows that the number of magnetic lines of force entering a closed surface is equal to the number of magnetic lines of force leaving that closed surface.
- (ii) This law tells that the magnetic lines of force form a continuous closed path.
- (iii) This law also predicts that the isolated magnetic monopole does not exist.

$$\text{Mathematically, } \oint_s \mathbf{B} \cdot d\mathbf{S} = 0$$

3. Faraday's law of electromagnetic induction

- (i) This law gives a relation between electric field and a changing magnetic flux.
- (ii) This law tells that the changing magnetic field is the source of electric field.

$$\text{Mathematically, } \oint \mathbf{E} \cdot d\mathbf{l} = - \frac{d\phi_B}{dt}$$

4. Ampere-Maxwell's law

- (i) This law states that the magnetic field can be produced by a conduction current as well as by displacement current.
- (ii) This law also states that the conduction current and displacement current together have a property of continuity.
- (iii) At an instant, in a circuit, the conduction current is equal to the displacement current.

$$\text{Mathematically, } \oint \mathbf{E} \cdot d\mathbf{l} = \mu_0 \left(i_c + \epsilon_0 \frac{d\phi_E}{dt} \right)$$

These equations are collectively called Maxwell's equations.

Results from Maxwell's Equations

- (i) The average of energy density associated with electromagnetic wave is found to be

$$u_{av} = \frac{1}{2} \epsilon_0 E_0^2 = \text{Electric field energy density}$$

$$u_{av} = \frac{1}{2\mu_0} B_0^2 = \text{Magnetic field energy density}$$

- (ii) Intensity is defined as energy crossing through unit area in unit time.

$$\begin{aligned} \therefore I &= u_{av} c \\ &= \frac{1}{2} \epsilon_0 c E_0^2 \\ I &= \frac{1}{2\mu_0} c B_0^2 \end{aligned}$$

Practice Exercise

1. An AC source of voltage $V = 100 \sin 100\pi t$ is connected to a resistor of resistance 20Ω . The rms value of current through resistor is

a. 10 A
b. $\frac{10}{\sqrt{2}}$ A
c. $\frac{5}{\sqrt{2}}$ A
d. None of these

2. In (Q. No. 1), average value of current for long time is

a. zero
b. $\frac{5}{\sqrt{2}}$ A
c. 10 A
d. None of these

3. In (Q. No. 1), the average value for half cycle is

a. $\frac{10}{\pi}$ A
b. $\frac{5}{\pi}$ A
c. zero
d. None of these

4. In (Q. No. 1), total charge transferred through resistor in long time is

a. zero
b. $\frac{2I_0}{\pi}$
c. $\frac{I_0}{25\pi}$
d. None of these

5. In (Q. No. 1), total charge transferred in $1/100$ second is

a. $\frac{1}{10\pi}$ C
b. $\frac{1}{5\pi}$ C
c. zero
d. None of these

6. In (Q. No. 1), total heat generated in one cycle is

a. $\sqrt{2}$ J
b. 5 J
c. $4\sqrt{2}$ J
d. zero

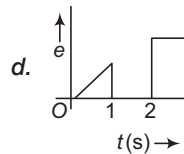
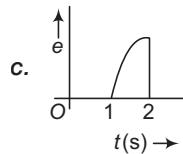
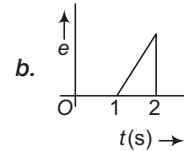
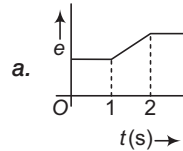
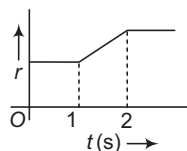
7. In (Q. No. 1), power factor is

a. 1
b. 0
c. $1/2$
d. None of these

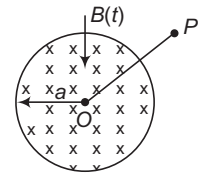
8. A metallic square loop ABCD is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in the figure. An electric field is induced

a. in AD, but not in BC
b. in BC, but not in AD
c. neither in AD nor in BC
d. in both AD and BC

9. A circular coil is placed in a uniform magnetic field such that its plane is perpendicular to field. The radius of coil changes with time as shown in the figure. Then, which of the following graph represent the induced electromotive force in the coil with time?



10. A uniform but time-varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region



a. is zero
b. decreases as $1/r$
c. increases as r
d. decreases as $1/r^2$

11. Calculate the peak and rms value of current in AC circuit. The current is represented by the equation $i = 5 \sin \left(300t - \frac{\pi}{4} \right)$, where t is in seconds and i in ampere.

a. 5 A, 3.535 A
b. 5 A, 5.53 A
c. 3 A, 3.53 A
d. 6.25 A, 5.33 A

12. The average value for half cycle in a 200 V AC source is

a. 180 V
b. 200 V
c. 220 V
d. None

13. Two alternating currents are given by

$$I_1 = I_0 \sin \omega t \text{ and } I_2 = I_0 \cos (\omega t + \phi)$$

The ratio of rms value is

a. 1 : 1
b. 1 : ϕ
c. 1 : 2
d. None of these

14. A current $I = 3 + 8 \sin 100t$ is passing through a resistor of resistance 10Ω . The effective value of current is

a. 5 A
b. 10 A
c. $4\sqrt{2}$ A
d. $3\sqrt{2}$ A

15. An alternative voltage

$$V = 30 \sin 50t + 40 \cos 50t$$

is applied to a resistor of resistance 10Ω . The rms value of current through resistor is

a. $\frac{5}{\sqrt{2}}$ A
b. $\frac{10}{\sqrt{2}}$ A
c. $\frac{7}{\sqrt{2}}$ A
d. 7 A

16. An alternating voltage $V = 140 \sin 50t$ is applied to a resistor of resistance 10Ω . This voltage produces ΔH heat in the resistor in time Δt . To produce the same heat in the same time, required DC current is
 a. 14 A b. about 20 A
 c. about 10 A d. None of these
17. An AC voltage is represented by $e = 220 \sin(100\pi)t$ volt and is applied over a resistance of 110 ohm . Calculate the heat produced in 7 min.
 a. $11 \times 10^3 \text{ cal}$ b. $22 \times 10^3 \text{ cal}$
 c. $33 \times 10^3 \text{ cal}$ d. $25 \times 10^3 \text{ cal}$
18. The reactance of a capacitor connected with DC voltage is
 a. zero b. ∞
 c. 1Ω d. None of these
19. The reactance of an inductor connected with DC voltage is
 a. zero b. ∞
 c. 1Ω d. None of these
20. An AC voltage $e = e_0 \sin 50t - e_0 \cos 100\pi t$ is connected in series with a resistor and capacitor. The steady state current through circuit is found to be $I = I_0 \sin(50\pi t + \phi) + I_0' \cos(100\pi t + \phi_2)$
 Then, the ratio of $\frac{I_0}{I_0'}$ is
 a. greater than 1 b. equal to 1
 c. less than 1 d. None of these
21. An alternating voltage $V = V_0 \sin \omega t$ is connected to a capacitor of capacity C_0 through an AC ammeter of zero resistance. The reading of ammeter is
 a. $\frac{V_0}{\sqrt{2}}$ b. $\frac{V_0}{\omega C \sqrt{2}}$
 c. $\frac{V_0 \omega C}{\sqrt{2}}$ d. None of these
22. Calculate the maximum current in the circuit, if a capacitor of capacitance $1 \mu\text{F}$ is charged to a potential of 2 V and is connected in parallel to an inductor of inductance 10^{-3} H .
 a. $\sqrt{4000} \text{ mA}$ b. $\sqrt{2000} \text{ mA}$ c. $\sqrt{1000} \text{ mA}$ d. $\sqrt{5000} \text{ mA}$
23. In a circuit consisting of inductor (L), capacitor (C) and resistor (R) are in series, if $\omega L < \frac{1}{\omega C}$, then the emf
 a. leads the current b. lags behind the current
 c. is in phase with current d. is zero
24. Find the resonant frequency of a series circuit consist of an inductance $200 \mu\text{H}$, a capacitance of $0.0005 \mu\text{F}$ and a resistance of 10Ω .
 a. 480 kHz b. 503 kHz c. 406 kHz d. 607 kHz
25. Find the frequency of voltage for an AC circuit. The equation of alternating voltage is $V = 200 \sin 314 t$.
 a. 50 Hz b. 60 Hz c. 55 Hz d. 65 Hz
26. An AC circuit with $f = 1000 \text{ Hz}$ consists of a coil of 200 mH and negligible resistance. Calculate the voltage across the coil, if the effective current of 5 mA is flowing.
 a. 7.64 V (rms) b. 7.452 V (rms)
 c. 6.28 V (rms) d. 74.62 V (rms)
27. Find the average power per unit area at distance of 2 m from a small bulb, if the bulb emits 20 W of electromagnetic radiation uniformly in all directions.
 a. 0.69 W/m^2 b. 0.56 W/m^2
 c. 0.78 W/m^2 d. 0.39 W/m^2
28. If a circuit made up of a resistance 1Ω and inductance 0.01 H , an alternating emf of 200 volt at 50 Hz is connected, then find the phase difference between the current and the emf in the circuit.
 a. $\tan^{-1}(\pi)$ b. $\tan^{-1}\left(\frac{\pi}{2}\right)$ c. $\tan^{-1}\left(\frac{\pi}{4}\right)$ d. $\tan^{-1}\left(\frac{\pi}{3}\right)$
29. Choose the correct option. If speed of gamma rays, X-rays and microwaves are v_g, v_x and v_m .
 a. $v_g < v_x < v_m$ b. $v_g > v_x > v_m$
 c. $v_g > v_x < v_m$ d. $v_g = v_x = v_m$
30. A condenser of capacitance of $2.4 \mu\text{F}$, is used in a transmitter to transmit at λ wavelength. If the inductor of 10^{-8} H is used for resonant circuit, then value of λ is
 a. 292 m b. 400 m c. 334 m d. 446 m
31. If a dry cell of emf = 1.5 V is connected across the primary of a step-up transformer of turn ratio 3 : 5, then calculate the voltage developed across the secondary.
 a. 30 V b. 5 V
 c. zero d. None of these
32. If at a certain instant, the magnetic induction of the electromagnetic wave in vacuum is $6.7 \times 10^{-12} \text{ T}$, then the magnitude of electric field intensity will be
 a. $2 \times 10^{-3} \text{ N/C}$ b. $3 \times 10^{-3} \text{ N/C}$
 c. $4 \times 10^{-3} \text{ N/C}$ d. $1 \times 10^{-3} \text{ N/C}$
33. Find the energy of photon of electromagnetic radiation of wavelength $200 \text{ \AA}$.
 a. $1.76 \times 10^{-18} \text{ J}$ b. $0.99 \times 10^{-18} \text{ J}$
 c. $0.54 \times 10^{-18} \text{ J}$ d. $0.63 \times 10^{-18} \text{ J}$
34. Find the speed of light in air, if an electromagnetic wave is travelling in air whose dielectric constant is $k = 1.006$.
 a. $3 \times 10^8 \text{ m/s}$ b. $3.88 \times 10^8 \text{ m/s}$
 c. $2.5 \times 10^8 \text{ m/s}$ d. $4.6 \times 10^8 \text{ m/s}$
35. An object is placed at some distance from a radio station. If the interval between transmission and reception of pulses is $2.66 \times 10^{-2} \text{ s}$, then find the distance.
 a. 4000 km b. 2000 km c. 3000 km d. 2500 km

36. Calculate the wavelength of a radio wave of frequency of 1 MHz.

a. 400 m b. 300 m c. 350 m d. 200 m

37. The electric field in an electromagnetic wave is given by $E = (100 \text{ N/C}) \sin \omega \left(t - \frac{x}{C} \right)$.

If the energy contained in a cylinder of cross-section 10 cm^2 and length 50 cm along the X -axis is $4.4 \times 10^{-8} \text{ J/m}^3$, then find the intensity of the wave.

a. 12.4 W/m^3
b. 13.2 W/m^3
c. 15.7 W/m^3
d. 11.9 W/m^3

38. A radio wave of intensity I is reflected by a surface. Find intensity (I), if pressure exerted on the surface is $2 \times 10^{-8} \text{ N/m}^2$.

a. 3 N/m^2 b. 4 N/m^2 c. 6 N/m^2 d. 7 N/m^2

39. A TV tower has a height of 100 m . Find the area covered by the TV broadcast, if radius of the earth is 6400 km .

a. $380 \times 10^7 \text{ m}^2$ b. $402 \times 10^7 \text{ m}^2$
c. $595 \times 10^7 \text{ m}^2$ d. $440 \times 10^7 \text{ m}^2$

40. An electromagnetic wave with pointing vector 5 W/m^2 is absorbed by a surface of some area. If the force on the surface is 10^{-7} N , then area is

a. 6 m^2 b. 3 m^2 c. 60 m^2 d. 4 m^2

BITSAT Archives

1. Voltage V and current i in AC circuit are given by

$$V = 50 \sin(50t) \text{ volt}, i = 50 \sin\left(50t + \frac{\pi}{3}\right) \text{ mA}$$

The power dissipated in the circuit is

a. 5.0 W b. 2.5 W
c. 1.25 W d. zero

2. The produced rays in sonography are

a. microwaves b. infrared waves
c. sound waves d. ultrasound

3. The ratio of secondary and primary turns of step-up transformer is $4 : 1$. If a current of 4 A is applied to the primary, the induced current in secondary will be

a. 8 A b. 2 A
c. 1 A d. 0.5 A

4. The impedance of a circuit consists of 3Ω resistance and 4Ω reactance. The power factor of the circuit is

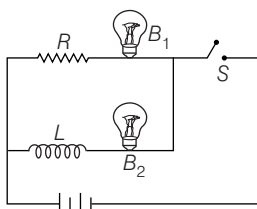
a. 0.4 b. 0.6
c. 0.8 d. 1.0

5. The time taken by the current to rise to 0.63 of its maximum value in a DC circuit containing inductance (L) and resistance (R) depends on

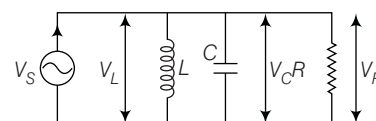
a. L only b. R only
c. $\frac{L}{R}$ d. LR

6. Figure represents two bulbs B_1 and B_2 , resistor R and inductor L . When the switch S is turned off, then

a. both B_1 and B_2 die out promptly
b. both B_1 and B_2 die out with some delay
c. B_1 dies out promptly but B_2 with some delay
d. B_2 dies out promptly but B_1 with some delay



7. An AC source is connected in parallel with an L - C - R circuit as shown. Let I_S , I_L , I_C and I_R denote the currents through and V_S , V_L , V_C and V_R the voltages across the corresponding components. Then,



a. $I_S = I_L + I_C + I_R$
b. $V_S = V_L + V_C + V_R$
c. $(I_L, I_C, I_R) < I_S$
d. I_L, I_C may be greater than I_S

8. The number of turns in primary and secondary coils of a transformer is 50 and 200 , respectively. If the current in the primary coil is 4 A , then current in the secondary coil is

a. 1 A b. 2 A c. 4 A d. 5 A

9. An inductor of 2 H and a resistance of 10Ω are connected in series with a battery of 5 V . The initial rate of change of current is

a. 0.5 A/s b. 2.0 A/s c. 2.5 A/s d. 0.25 A/s

10. The transformation ratio in the step-up transformer is

a. 1 b. greater than one c. less than one d. the ratio greater or less than depends on the other factors

11. In a step-up transformer, if ratio of turns of primary to secondary is $1 : 10$ and primary voltage is 230 V . If the load current is 2 A , then the current in primary is

a. 20 A b. 10 A c. 2 A d. 1 A

12. The square root of the product of inductance and capacitance has the dimension of

a. length b. mass
c. time d. no dimension

Answer with Solutions

Practice Exercise

1. (c) $I_0 = \frac{V_0}{R} = \frac{100}{20} = 5 \text{ A} \quad \therefore I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}} \text{ A}$

2. (a) $I = \frac{V}{R} = \frac{100}{20} \sin 100\pi t = 5 \sin 100\pi t$

$\therefore \bar{I} = \frac{5 \int_0^T \sin 100\pi t \, dt}{\int_0^T dt} = 0$

3. (a) $I_{\text{av}} = \frac{2I_0}{\pi} = \frac{2 \times 5}{\pi} = \frac{10}{\pi} \text{ A}$

4. (a) $q = \int_0^{t=1/50} I \, dt = \int_0^{t=1/50} 100 \sin 100\pi t \, dt = 0$

5. (a) When $t \leq \frac{T}{2}$, i.e. $t \leq \frac{1}{100} \text{ s}$

Then, $q = I_{\text{av}} t = \frac{2I_0}{\pi} t = \frac{10}{\pi} \times \frac{1}{100} = \frac{1}{10\pi}$

6. (b) $\Delta H = I_{\text{rms}}^2 RT = \left(\frac{5}{\sqrt{2}}\right)^2 \times 20 \times \frac{1}{50} = 5 \text{ J}$

7. (a) $\cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + 0}} = 1$

$\Rightarrow \cos \phi = 1$

8. (d) In both AD and BC, because the electrons and positive ions will be separated by magnetic force in these two arms.

9. (b) Flux linked with the circular coil, $\phi = BA$

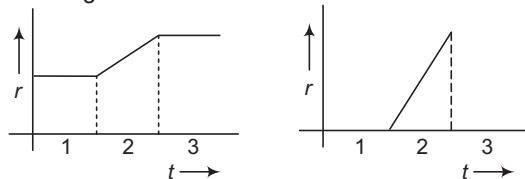
$\frac{d\phi}{dt} = B\pi 2r \frac{dr}{dt} \quad \therefore \frac{d\phi}{dt} \propto \frac{dr}{dt}$

Emf of coil $\propto \frac{dr}{dt}$ and $e \propto r$

In 1 and 3 regions, $r = \text{constant} \Rightarrow \text{emf} = 0$

In 2 region, $\frac{dr}{dt} = \text{constant}$

Emf induced is increasing at constant rate, because r is increasing.



10. (b) Using, $\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\phi}{dt}$

$\Rightarrow E(2\pi r) = \text{constant} \Rightarrow E \propto \frac{1}{r}$

11. (a)

12. (a) $V_{\text{rms}} = \frac{V_0}{\sqrt{2}} \quad \therefore V_0 = \sqrt{2} V_{\text{rms}}$

$\therefore V_{\text{av}} = \frac{2V_0}{\pi} = \frac{2}{\pi} \sqrt{2} V_{\text{rms}} = \frac{2\sqrt{2} \times 200}{\pi} = 180 \text{ V}$

13. (a) $I_{\text{rms}1} = \frac{I_{01}}{\sqrt{2}} = \frac{I_0}{\sqrt{2}}, I_{\text{rms}2} = \frac{I_{02}}{\sqrt{2}} = \frac{I_0}{\sqrt{2}}$

$\therefore \text{Ratio} = 1:1$

14. (a) $I_{\text{effective}}^2 = I_{\text{rms}}^2 \frac{\int_0^T I^2 \, dt}{\int_0^T dt}$

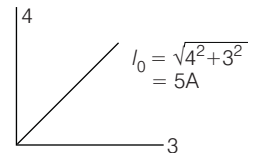
After solving, $I_{\text{effective}} = I_{\text{rms}} = 5 \text{ A}$

15. (a) $I = \frac{V}{R} = \frac{30}{10} \sin 50t + \frac{40}{10} \cos 50t$

or $I = 3 \sin 50t + 4 \cos 50t$

or $I = 3 \sin 50t + 4 \sin \left(50t + \frac{\pi}{2}\right)$

$\therefore I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}} \text{ A}$



16. (c)

17. (b) $H = \frac{I_{\text{rms}}^2 RT}{J} \text{ cal}$

$= \frac{(\sqrt{2})^2 \times 110 \times 7 \times 60}{4.2} \left(\because I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{220/110}{\sqrt{2}} = \sqrt{2} \right)$
 $= 22 \times 10^3 \text{ cal}$

18. (b) For DC source, frequency is zero.

$\therefore X_C = \frac{1}{\omega C} = \frac{1}{0} \times C = \infty$

19. (a) $X_L = \omega L = 0 \times L = 0$

20. (c) $I_0 = \frac{E_0}{\sqrt{R^2 + X_C^2}} = \frac{E_0}{\sqrt{R^2 + \left(\frac{1}{50C}\right)^2}}$

Similarly, $I'_0 = \frac{E_0}{\sqrt{R^2 + \left(\frac{1}{100C}\right)^2}} \quad \text{Hence, } I_0 < I'_0$

21. (c)

22. (a) Charge on capacitor $q_0 = CV, \quad q_0 = 2 \times 10^{-6} \text{ C}$

Here, $q = q_0 \sin \omega t$

For maximum current $= I_0 = \frac{dq}{dt} = \omega q_0$

But $\omega = \frac{1}{\sqrt{LC}} = (10^9)^{1/2}, I_0 = (10^9)^{1/2} (2 \times 10^{-6})$

$= \sqrt{10} \times 10^4 \times 2 \times 10^{-6} = 2\sqrt{10} \times 10^{-2} \text{ A} = \sqrt{4000} \text{ mA}$

23. (a) When $\omega L < \frac{1}{\omega C}$, emf leads the current by $\frac{\pi}{2}$.

24. (b)

25. (a) Comparing $V = 200 \sin 314t$ with the standard equation $V = V_0 \sin \omega t$, we get $\omega = 314 \text{ s}^{-1}$

Linear frequency, $f = \frac{\omega}{2\pi} = \frac{314}{2 \times 3.14} = 50 \text{ Hz}$

26. (c) Inductive reactance,
- $X_L = L\omega = 2\pi fL$

$$\text{also, } \frac{E}{I} = L\omega = X_L \quad \text{or } E = IX_L \Rightarrow E = \frac{5}{1000} \times 1256$$

$$E = 6.28 \text{ volt (rms)}$$

27. (d)
- $I = \frac{P}{4\pi r^2}, \quad I = \frac{20}{4\pi(2)^2} = 0.39 \text{ W/m}^2$

28. (a)
- $\tan \phi = \left(\frac{X_L}{R}\right)$

$$\text{and } X_L = \omega L = (2\pi fL) = (2\pi)(50)(0.01) = \pi \Omega$$

$$\text{also } R = 1 \Omega \Rightarrow \phi = \tan^{-1}(\pi)$$

29. (d)

30. (a) Frequency
- $= n = \frac{c}{\lambda} = \frac{3 \times 10^8}{\lambda}$

$$\text{also, resonant frequency } = n = \frac{1}{2\pi\sqrt{LC}}$$

$$\text{or } LC = \frac{1}{4\pi^2 n^2} \quad \text{or } C = \frac{\lambda^2}{4\pi^2 c^2 L}$$

$$\text{or } \lambda = \sqrt{4\pi^2 c^2 L C}$$

$$\text{or } \lambda = \sqrt{4\pi^2 \times (3 \times 10^8)^2 \times 2.4 \times 10^{-6} \times 10^{-8}} \Rightarrow \lambda = 292 \text{ m}$$

31. (c) Transformer is an AC device, it does not work on DC, so, output voltage is zero.

32. (a)

33. (b)
- $E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{2000 \times 10^{-10}} = 0.0099 \times 10^{-16}$
-
- $E = 0.99 \times 10^{-18} \text{ J}$

34. (a)
- $v = \frac{1}{\sqrt{\mu_0 k \epsilon_0}} \Rightarrow \frac{c}{\sqrt{k}}$
-
- $v = \frac{c}{\sqrt{k}} = \frac{3 \times 10^8}{\sqrt{1.006}} \Rightarrow v \approx 3 \times 10^8 \text{ m/s}$

35. (a)
- $2x = ct$

$$x = \frac{ct}{2} = \frac{3 \times 10^8 \times 2.66 \times 10^{-2}}{2} \Rightarrow x \approx 4000 \text{ km}$$

36. (b)

37. (b)
- $I = \frac{1}{2} \epsilon E_0^2 c = (4.4 \times 10^{-8}) \times (3 \times 10^8) = 13.2 \text{ W/m}^2$

38. (a)

39. (b) Area
- $= \pi r^2 = \pi = ((2hR)^{1/2})^2$

$$\text{Area} = 2\pi Rh = 2 \times \pi \times 6400 \times 10^3 \times 100 = 402 \times 10^7 \text{ m}^2$$

40. (a) We know,
- $P = \frac{I}{c} = \frac{5}{3 \times 10^8}$

$$\text{and } F = P \times A \Rightarrow A = \frac{F}{P}$$

$$A = \frac{10^{-7}}{5} \times 3 \times 10^8 \Rightarrow A = 6 \text{ m}^2$$

BITSAT Archives

1. (c) Given,
- $V = 50 \sin(50 t) \text{ V}$

$$\text{Maximum voltage, } V_0 = 50 \text{ V, } i = 50 \sin\left(50 t + \frac{\pi}{3}\right) \text{ mA}$$

$$\text{Maximum current, } i_0 = 50 \text{ mA} = 50 \times 10^{-3} \text{ A}$$

$$\begin{aligned} \text{Power dissipated, } P &= \frac{i_0}{\sqrt{2}} \times \frac{V_0}{\sqrt{2}} \\ &= \frac{50 \times 50 \times 10^{-3}}{2} \\ &= \frac{2500 \times 10^{-3}}{2} = 1.25 \text{ W} \end{aligned}$$

2. (d) The produced rays in sonography are ultra sound.

3. (c)
- $\frac{I_S}{I_P} = \frac{N_P}{N_S} = \frac{1}{4} \Rightarrow I_S = \frac{1}{4} \times 4 = 1 \text{ A}$

4. (b)
- $\tan \phi = \frac{X}{R} = \frac{4}{3}$

$$\text{Power factor} = \cos \phi = \frac{3}{5} = 0.6$$

5. (c) It depends upon its time constant,
- $\tau = \frac{L}{R}$
- .

6. (c) Bulb
- B_1
- dies out promptly, but bulb
- B_2
- dies out with some delay. This is because of self-induced emf across
- L
- , during decay.

7. (d) In parallel resonant circuit, current through
- L
- and
- C
- may be greater than the source current, i.e.
- I_L, I_C
- may be greater than
- I_S
- .

8. (a) In a transformer,

$$\therefore \frac{N_P}{N_S} = \frac{I_S}{I_P} = \frac{50}{200} = \frac{I_S}{4} \Rightarrow I_S = 1 \text{ A}$$

9. (c)
- $i = i_0 \left(1 - e^{-\frac{Rt}{L}}\right)$

$$\Rightarrow \frac{di}{dt} = \frac{d}{dt} i_0 - \frac{d}{dt} \left(i_0 e^{-\frac{Rt}{L}}\right) = 0 + \frac{i_0 R}{L} e^{-\frac{Rt}{L}}$$

$$\text{Initially, } t = 0$$

$$\Rightarrow \frac{di}{dt} = \frac{i_0 \times R}{L} = \frac{E}{L} = \frac{5}{2} = 2.5 \text{ A/s}$$

10. (b) Transformation ratio,
- $k = \frac{N_S}{N_P} = \frac{V_S}{V_P}$

For step-up transformer,

$$N_S > N_P, \text{ i.e., } V_S > V_P, \text{ hence, } k > 1.$$

11. (a)
- $\frac{N_P}{N_S} = \frac{1}{10}$

$$V_P = 230 \text{ V, } I_S = 2 \text{ A,}$$

$$\frac{I_P}{I_S} = \frac{N_S}{N_P} \Rightarrow \frac{I_P}{2} = \frac{10}{1} \Rightarrow I_P = 20 \text{ A}$$

12. (c) We know
- $f = \frac{1}{2\pi\sqrt{LC}}$
- or
- $\sqrt{LC} = \frac{1}{2\pi f} = \text{time}$

Thus, $\sqrt{LC}$ has the dimension of time.

29

Cathode Ray, Photoelectric Effect of Light and X-Rays

Cathode Rays

It consists of fast moving electrons. If discharge tube is operating at voltage V , then

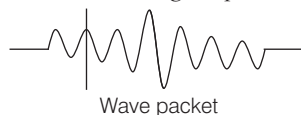
$$E_k = eV = \frac{1}{2}mv^2$$

Dual Nature of Radiation

The concept of wave nature of matter arose from the dual character of the radiation which sometimes behaves as a wave and sometimes as a particle.

In the phenomena where light interacts with itself, such as dispersion, interference, diffraction and polarisation, the wave nature of light dominates. In the phenomena where the light interacts with matter, such as in Photoelectric effect, Compton effect, Raman effect, the nature of particle of light dominates.

Matter waves travel as a wave packet with decreasing amplitude on either side of the present position.



de-Broglie Wavelength Relation

According to de-Broglie, a wave is associated with energy moving particle. These waves are called de-Broglie waves or matter waves.

(i) In the n th orbit, we have

$$L = mvr = \frac{nh}{2\pi} \Rightarrow \lambda = \frac{h}{mv} = \frac{2\pi r}{n}$$

- (ii) Number of waves in n th orbit is $N = \frac{2\pi r}{\lambda} = n$

Thus, the number of de-Broglie waves in n th orbit is n .

Expression for the wavelength associated with charged particles accelerated through a potential difference V

(i) Electron, $\lambda_e = \frac{12.27}{\sqrt{V}} \text{ \AA}$

(ii) Proton, $\lambda_p = \frac{0.286}{\sqrt{V}} \text{ \AA}$

(iii) Deuteron, $\lambda_d = \frac{0.202}{\sqrt{V}} \text{ \AA}$

(iv) α -particle, $\lambda_\alpha = \frac{0.101}{\sqrt{V}} \text{ \AA}$

- (v) The de-Broglie wavelength of a particle of mass m and moving with velocity v is given by

$$\lambda = \frac{h}{mv}$$

- (vi) If m_0 be the rest mass of the particle, then

$$\lambda = \frac{h[1 - v^2/c^2]^{1/2}}{m_0 v}$$

- (vii) The de-Broglie wavelength of a particle of mass m and kinetic energy K is given by

$$\lambda = \frac{h}{\sqrt{2mK}}$$

- (viii) If a particle of mass m carrying charge q_0 is accelerated through a potential V , then its de-Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mq_0V}}$$

- (ix) The de-Broglie wavelength of a gas molecule of mass m at temperature T (in kelvin) is given by

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

Here, k is Boltzmann constant.

Photoelectric Effect

For emission of electrons from a metal surface, a minimum amount of energy (E) is required to overcome the potential barrier (W_0 : work function) provided by the metal. When energy is provided in the form of electromagnetic waves of photons, then emitted electrons are called photoelectrons and this effect is called **photoelectric effect**. As photon energy ($E = h\nu$) is proportional to its frequency, a minimum frequency is required so that photon has energy greater than or equal to work function. This minimum frequency is called **threshold frequency** (ν_0).

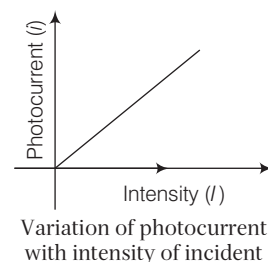
For photoemission to take place, either of the following conditions must be satisfied

$$E \geq W_0 \text{ or } \nu \geq \nu_0 \text{ or } \lambda \leq \lambda_0$$

where, ν_0 = threshold frequency, λ_0 = threshold wavelength.

Dependence of Photocurrent on Intensity

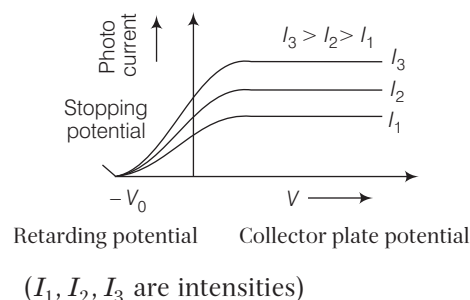
On keeping the frequency of incident radiation and anode potential fixed, if the intensity of incident light is increased, then the photoelectric current increases.



i.e. photocurrent (i) $\propto$ intensity (I)

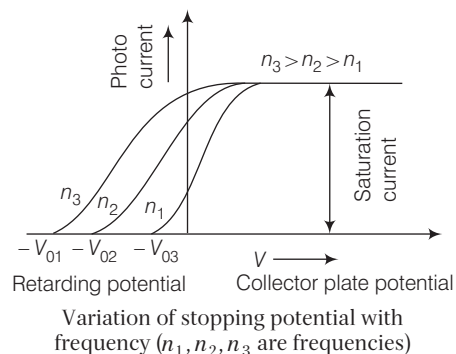
Dependence of Stopping Potential on Intensity

The stopping potential does not change for a given metal and given frequency of radiation, if we change the intensity of incident light.



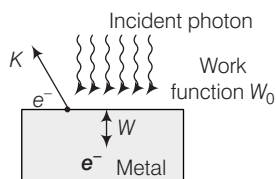
Dependence of Stopping Potential on Frequency

Greater is the frequency of incident radiation, more is the magnitude of stopping potential.



Einstein's Photoelectric Equation

According to Einstein, photoelectric effect is the result of one to one elastic collision between photon and electron in which photon is completely absorbed. So, if an electron in a metal absorbs a photon of energy $E (= h\nu)$, then it uses the energy in the following ways



- (i) Some energy (say W) is used in shifting the electron from interior to surface.
- (ii) Some energy (say W_0) is used in removing electron from metal's surface.
- (iii) Rest energy will appear as kinetic energy (K) of the emitted photoelectrons. Hence, $W + W_0 + K = h\nu$

For the electrons emitting from surface $W = 0$, so kinetic energy of these emitted electrons will be maximum.

Hence, $h\nu = W_0 + K_{\max}$

So, Einstein's photoelectric equation is given by

$$\Rightarrow K_{\max} = h\nu - W_0$$

where, W_0 is a work function.

If V_0 is stopping potential,

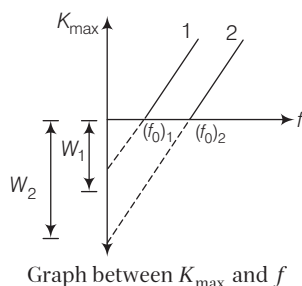
$$\text{then } eV_0 = K_{\max} = h\nu - W_0.$$

Graph between $K_{\max}$ and f

Let us plot a graph between maximum kinetic energy ($K_{\max}$) of photoelectrons and frequency (ν) of incident light.

The equation between $K_{\max}$ and ν is $K_{\max} = h\nu - W$

Comparing it with $y = mx + c$, the graph between $K_{\max}$ and f is a straight line with positive slope and negative intercept.



Graph between $K_{\max}$ and f

From the graph, we observe that

- (i) If $K_{\max} = 0$, then $\nu = \nu_0$
- (ii) Slope of the straight line is h , a universal constant, i.e. if graph is plotted for two different metals 1 and 2, then slope of both the lines is same.
- (iii) The negative intercept of the line is W (the work function) which is characteristic of a metal, i.e. intercepts for two different metals will be different.

If $W_2 > W_1$, then $(\nu_0)_2 > (\nu_0)_1$ as $W = h\nu_0$, where $\nu_0 =$ threshold frequency

Graph between V_0 and ν

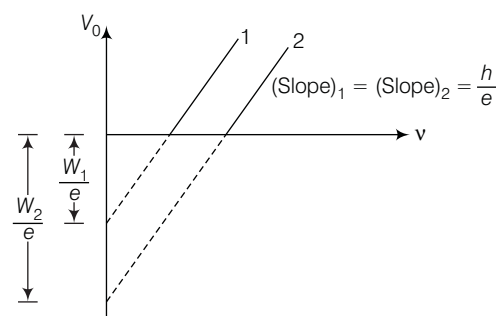
Let us plot a graph between the stopping potential (V_0) and the incident frequency (ν). The equation between them is

$$eV_0 = h\nu - W$$

or

$$V_0 = \left(\frac{h}{e}\right)\nu - \left(\frac{W}{e}\right)$$

Again comparing with $y = mx + c$, the graph between V_0 and ν is a straight line with **positive slope** $\frac{h}{e}$ (a universal constant) and **negative intercept** $\frac{W}{e}$ (which depends on the metal). The corresponding graph is shown as below:



Graph between V_0 and ν

Particle Nature of Light– The Photon

According to Einstein's quantum theory, light propagates in the bundles (packets or quanta) of energy. Each bundle is called a photon which possesses energy.

- (i) **Energy of photon** Energy of each photon is given by

$$E = h\nu = \frac{hc}{\lambda}$$

where, c = speed of light

h = Planck's constant = 6.6×10^{-34} J-s

Energy of photon in electron-volt

$$E(\text{eV}) = \frac{hc}{e\lambda} = \frac{12375}{\lambda(\text{\AA})} \Rightarrow E(\text{eV}) \approx \frac{12400}{\lambda(\text{\AA})}$$

- (ii) **Mass of photon** Actual rest mass of the photon is zero. But its effective mass is given as

$$E = mc^2 = h\nu \Rightarrow m = \frac{E}{c^2} = \frac{h\nu}{c^2} \Rightarrow m = \frac{h}{c\lambda}$$

This mass is also known as **dynamic mass of the photon**.

- (iii) **Momentum of the photon** Momentum,

$$p = m \times c = \frac{E}{c} = \frac{h\nu}{c} \Rightarrow p = \frac{h}{\lambda}$$

- (iv) **Number of emitted photons** The number of photons emitted per second from a source of monochromatic radiation of wavelength λ and power P is given as

$$n = \frac{P}{E}$$

$$= \frac{P}{h\nu}$$

$$\Rightarrow n = \frac{P\lambda}{hc}$$

where, E = energy of each photon.

Bragg's Equations

For normal incidence, $b \sin \phi = n\lambda$

For glancing incidence, $2d \sin \theta = n\lambda$

Also, $\phi + 2\theta = 180^\circ$

Compton Effect

- (a) Compton shift in wavelength

$$\Delta\lambda = \frac{h}{m_0c} (1 - \cos \theta)$$

(Here, $\frac{h}{m_0c} = 0.024 \text{ \AA}$, θ is angle of scattering.)

- (b) KE of recoil electron, $E_k = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$

- (c) Direction of recoil electron, $\tan \theta = \frac{\lambda \sin \theta}{\lambda_0 - \lambda \cos \theta}$

Mosley Law

Frequency ν of characteristic X-rays spectrum

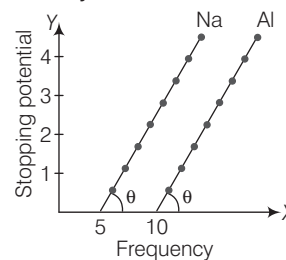
$$\sqrt{\nu} = n(z - \sigma)$$

where, a and σ are constants and screening constant for K_α line, $\sigma = 1$, Screening constant for L_α line, $\sigma = 7.4$.

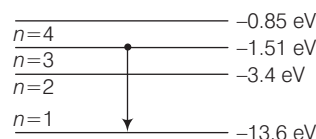
Practice Exercise

- In an oil drop experiment, the following charges (in arbitrary units) were found on a series of oil droplets: 2.30×10^{-15} , 6.90×10^{-15} , 1.38×10^{-14} , 5.75×10^{-15} , 1.955×10^{-14} . The charge on electron (in the same unit) should be
 - 2.30×10^{-15}
 - 1.15×10^{-15}
 - 1.38×10^{-14}
 - 1.955×10^{-14}
- In Wilson cloud chamber experiment, two particles were found to show equal deviation but in opposite directions. The names positron and negatron were given to these particles by Anderson. Negatron should be
 - neutron
 - neutrino
 - electron
 - proton
- An α -particle when accelerated through a potential of V volt has a wavelength λ associated with it, but if a proton in order to have same wavelength λ by what potential difference it must be accelerated?
 - 8 V
 - 6 V
 - 4 V
 - 12 V
- The cathode ray particles originate in a discharge tube from the
 - cathode
 - anode
 - source of high voltage
 - residual gas
- Three particles having charges in the ratio of 2:3:5, produce the same point on the photographic film in Thomson's experiment. Their masses are in the ratio of
 - 2:3:5
 - 5:3:2
 - 15:10:6
 - 3:5:2
- If the velocity of an electron is doubled, its de-Broglie frequency will be
 - half
 - remain same
 - doubled
 - become four times
- An electron is at rest. Its wavelength is
 - 1
 - infinity
 - $\frac{h}{m_e}$
 - it has no wave character
- If the de-Broglie wavelength of a proton is 10^{-13} m , the electric potential through which it must have been accelerated is
 - $4.07 \times 10^4 \text{ V}$
 - $8.2 \times 10^4 \text{ V}$
 - $8.2 \times 10^3 \text{ V}$
 - $4.07 \times 10^5 \text{ V}$
- A moving electron has numerical relation $\lambda = h$. Then,
 - $m_e = \frac{1}{v_e}$
 - $v_e = \frac{1}{m_e}$
 - both a and b
 - None of these
- The de-Broglie wavelength of a bus moving with speed v is λ . Some passengers left the bus at a stoppage. Now, when the bus moves with twice its initial speed, its kinetic energy is found to be twice its initial value. What will be the de-Broglie wavelength, now?
 - λ
 - 2λ
 - $\lambda/2$
 - $\lambda/4$

11. An electron of mass m and charge q is accelerated from rest in a uniform electric field of strength E . The velocity acquired by it as it travels a distance l is
 a. $\sqrt{2Eq/lm}$ b. $\sqrt{2Eq/ml}$ c. $\sqrt{2Em/ql}$ d. $\sqrt{Eq/ml}$
12. Protons of an electromagnetic radiation has an energy 11 keV each. To which region of electromagnetic spectrum does it belong?
 a. X-ray region b. Ultra violet region
 c. Infrared region d. Visible region
13. Two photons of same frequencies moving in same medium have
 a. same linear momenta and wavelengths
 b. same linear momenta and same speeds
 c. same energies and same linear moment
 d. None of the above
14. The voltage applied to an electron microscope to produce electrons of wavelength $0.50 \text{ \AA}$ is
 a. 602 V b. 50 V c. 138 V d. 812 V
15. A certain molecule has an energy level diagram for its vibrational energy in which two levels are 0.014 eV apart. Find the wavelength of the emitted line for the molecule as it falls from one of these levels to the other
 a. $8.9 \times 10^{-5} \text{ m}$ b. $1.2 \times 10^{-6} \text{ m}$
 c. 173.6 m d. $4.6 \times 10^{-7} \text{ m}$
16. How many photons are emitted by a laser source of $5 \times 10^{-3} \text{ W}$ operating at 632.2 nm in 2 second ($h = 6.63 \times 10^{-34} \text{ Js}$)?
 a. 3.2×10^{16} b. 1.6×10^{16}
 c. 4×10^{16} d. None of these
17. Only a fraction of the electrical energy supplied to a tungsten light bulb is converted into visible light. If a 100 W light bulb converts 20% of the electrical energy into visible light ($\lambda = 662.6 \text{ nm}$), then the number of photons emitted by the bulb per second is
 a. 6.67×10^{19} b. 2×10^{28} c. 6×10^{36} d. 6.30×10^{19}
18. Calculate the number of photons emitted by a 60 W bulb per second, if 10% of the electrical energy supplied to an incandescent light bulb is radiated as visible light.
 a. 1.8×10^{19} b. 1.8×10^{16} c. 1.8×10^{11} d. 1.8×10^{21}
19. The momentum of a photon having energy equal to the rest energy of an electron is
 a. zero b. $2.73 \times 10^{-22} \text{ kg ms}^{-1}$
 c. $1.99 \times 10^{-24} \text{ kg ms}^{-1}$ d. infinite
20. A perfectly reflecting solid hemisphere of radius R is placed in the path of a parallel beam of light of large aperture. If the beam carries an intensity I , find the force exerted by the beam on the hemisphere.
 a. $\frac{2\pi R^2 I}{c}$ b. $\frac{\pi R^2 I}{c}$
 c. $\frac{4\pi R^2 I}{c}$ d. None of these
21. An electron (mass m) with an initial velocity $\mathbf{v} = v_0 \hat{i}$ ($v_0 > 0$) is in an electric field $\mathbf{E} = -E_0 \hat{i}$ ($E_0 = \text{constant} > 0$). Its de-Broglie wavelength at time t is given by
 a. $\frac{\lambda_0}{\left(1 + \frac{eE_0 t}{m v_0}\right)}$ b. $\lambda_0 \left(1 + \frac{eE_0 t}{m v_0}\right)$
 c. λ_0 d. $\lambda_0 t$
22. A ruby laser produces radiations of wavelengths 662.6 nm in pulses whose duration are 10^{-9} s . If the laser produces 0.39 J of energy per pulse, how many photons are produced in each pulse?
 a. 1.3×10^9
 b. 1.3×10^{18}
 c. 1.3×10^{27}
 d. 3.9×10^{18}
23. At one time, the metre was defined as 1650763.73 wavelengths of the orange light emitted by a light source containing Kr^{86} atoms. What is the corresponding photon energy of this radiation?
 a. $3.28 \times 10^{-19} \text{ J/quanta}$ b. $1.204 \times 10^{-31} \text{ J/quanta}$
 c. $1.09 \times 10^{-27} \text{ J/quanta}$ d. $4.01 \times 10^{-40} \text{ J/quanta}$
24. From the figure describing photoelectric effect, we may infer correctly that



- a. Na and Al both have the same threshold frequency
 b. Maximum kinetic energy for both the metals depend linearly on the frequency
 c. The stopping potentials are different for Na and Al for the same change in frequency
 d. Al is a better photosensitive material than Na
25. The stopping potential for the photoelectrons emitted from a metal surface of work function 1.7 eV is 10.4 eV . Identify the energy levels corresponding to the transitions in hydrogen atom which will result in emission of wavelength equal to that of incident radiation for the above photoelectric effect.



- a. $n = 3$ to 1 b. $n = 3$ to 2
 c. $n = 2$ to 1 d. $n = 4$ to 1

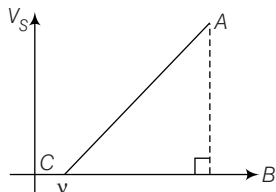
26. The work function of a certain metal is 2.3 eV. If light of wave number $2 \times 10^6 \text{ m}^{-1}$ falls on it, the kinetic energies of fastest and slowest ejected electron will be respectively.
 a. 2.48 eV, 0.18 eV b. 0.18 eV, zero
 c. 2.30 eV, 0.18 eV d. 0.18 eV, 0.18 eV
27. When the electromagnetic radiations of frequencies $4 \times 10^{15} \text{ Hz}$ and $6 \times 10^{15} \text{ Hz}$ fall on the same metal, in different experiments, the ratio of maximum kinetic energy of electrons liberated is 1:3. The threshold frequency for the metal is
 a. $2 \times 10^{15} \text{ Hz}$ b. $1 \times 10^{15} \text{ Hz}$
 c. $3 \times 10^{15} \text{ Hz}$ d. $1.67 \times 10^{15} \text{ Hz}$
28. A surface is irradiated with ultraviolet radiation of wavelength $0.2 \mu\text{m}$. If the maximum velocity of electron liberated from the surface is $8.8 \times 10^5 \text{ m/s}$, then find the work function of the surface.
 a. 3 eV b. 4 eV c. 5 eV d. 6 eV
29. Choose the correct option for the graph between the frequency of incident light and the stopping potential.
 a. It is a parabola b. It is a straight line
 c. It is a hyperbola d. It is a circle
30. Specific heat of water is $4.2 \text{ J/g } ^\circ\text{C}$. If light of frequency $3 \times 10^9 \text{ Hz}$ is used to heat 400 g of water from 20°C to 40°C , the number of moles of photons needed will be
 a. 1.69×10^{29} b. 1.69×10^{28} c. 2.80×10^4 d. 2.80×10^5
31. 10^{-3} W of $5000 \text{ \AA}$ light is directed on a photoelectric cell. If the current in the cell is $0.16 \mu\text{A}$, then the percentage of incident photons which produce photoelectrons, is
 a. 0.4% b. 0.04% c. 20% d. 10%
32. The potential energy of a particle of mass m is given by

$$U(x) = \begin{cases} E_0; & 0 \leq x < 1 \\ 0; & x > 1 \end{cases}$$
 λ_1 and λ_2 are the de-Broglie wavelengths of the particle, when $0 \leq x < 1$ and $x > 1$ respectively. If the total energy of particle is $2E_0$, then the ratio $\frac{\lambda_1}{\lambda_2}$ will be
 a. 2 b. 1 c. $\sqrt{2}$ d. $1/\sqrt{2}$
33. A radiation is incident on a metal surface of work function 2.3 eV. The wavelength of incident radiation is 600 nm, then the number of photoelectrons is
 a. zero b. $> 10^4$
 c. $= 10^4$ d. None of these
34. At its closest approach, the distance between the mars and the earth is found to be 60 million km. When the planets are at this closest distance, how long would it take to send a radio message from a space probe of mars to earth?
 a. 5 s b. 200 s c. 0.2 s d. 500 s
35. Ultraviolet beam of wavelength 280 nm is incident on lithium surface of work function 2.5 eV. The maximum velocity of electron emitted from metal surface is
 a. $8.2 \times 10^5 \text{ m/s}$ b. 10^6 m/s
 c. $7 \times 10^5 \text{ m/s}$ d. None of these
36. In previous problem, the stopping potential is
 a. 1.9 V b. 10 V c. 3 V d. None
37. Threshold frequency for photoelectric effect from a metal surface of work function 4.5 eV is
 a. $1.1 \times 10^9 \text{ Hz}$ b. 540 Hz
 c. $1.1 \times 10^{15} \text{ Hz}$ d. None of these
38. If n_r and n_b are the number of photons of red and blue light respectively with same energy, then
 a. $n_r > n_b$
 b. $n_r < n_b$
 c. $n_r = n_b$
 d. no relation between n_r and n_b
39. The number of photoelectrons emitted per unit time depends on ($\nu > \nu_0$)
 a. threshold frequency
 b. frequency of the incident radiation
 c. intensity of the incident radiation
 d. density of the metal irradiated
40. An X-ray tube is operating at 15 kV. The lower limit of the wavelength of X-rays produced is
 a. $0.82 \times 10^{-7} \text{ m}$
 b. $0.82 \times 10^{-8} \text{ m}$
 c. $0.83 \times 10^{-10} \text{ m}$
 d. $0.82 \times 10^{-13} \text{ m}$
41. Ultraviolet light of wavelength 66.26 nm and intensity 2 W/m^2 falls on potassium surface by which photoelectrons are ejected out. If only 0.1% of the incident photons produce photoelectrons, and surface area of metal surface is 4 m^2 , how many electrons are emitted per second?
 a. 2.67×10^{15}
 b. 3×10^{15}
 c. 3.33×10^7
 d. 4.17×10^{16}
42. The stopping potentials are V_1 and V_2 . Calculate the $(V_1 - V_2)$, if the λ_1 and λ_2 are wavelength of incident lights, respectively.
 a. $\frac{hc}{e} \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right)$
 b. $\frac{hc}{e} \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_1} \right)$
 c. $\frac{e}{hc} \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right)$
 d. $\frac{e}{hc} \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right)$

43. Calculate the work function of the metal, if the kinetic energies of the photoelectrons are E_1 and E_2 , with wavelengths of incident light λ_1 and λ_2
- $\frac{E_1\lambda_1 - E_2\lambda_2}{\lambda_2 - \lambda_1}$
 - $\frac{E_1E_2}{\lambda_1 - \lambda_2}$
 - $\frac{(E_1 - E_2)\lambda_1\lambda_2}{(\lambda_1 - \lambda_2)}$
 - $\frac{\lambda_1\lambda_2E_1}{(\lambda_1 - \lambda_2)E_2}$
44. What is the energy of photon of wavelength 24800 Å?
- 0.5 eV
 - 0.9 eV
 - 1.1 eV
 - 0.75 eV
45. For a certain metal v is five times of v_0 and the maximum velocity of coming out photons is 8×10^6 m/s. If $v = 2v_0$, then maximum velocity of photoelectrons will be
- 4×10^6 m/s
 - 6×10^6 m/s
 - 2×10^6 m/s
 - 1×10^6 m/s
46. A red bulb and violet bulb of equal power emits n_R and n_V number of photons in a given time, then
- $n_R = n_V$
 - $n_R > n_V$
 - $n_R < n_V$
 - $n_R \geq n_V$
47. When a surface 1 cm thick is illuminated with light of wavelength λ , the stopping potential is V_0 , but when the same surface is illuminated by light of wavelength 3λ , the stopping potential is $\frac{V_0}{6}$. Find the threshold wavelength for metallic surface.
- 4λ
 - 5λ
 - 3λ
 - 2λ
48. Photoelectric effect show
- wave-like behaviour of light
 - particle-like behaviour of light
 - both wave-like and particle-like behaviour of light
 - neither wave-like nor particle-like behaviour of light

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1. Graph of stopping potential for most energetic emitted photoelectron (V_s) with frequency of incident radiation on metal is given below. [2014]



The value of $\frac{AB}{BC}$, in graph is

- (h = Planck's constant, e = electronic charge)
- h
 - e
 - $\frac{h}{e}$
 - $\frac{e}{h}$
2. The number of photoelectrons in a photoelectric effect experiment depends on the [2013]
- frequency of light
 - intensity of light
 - Both **a** and **b** are correct
 - Both **a** and **b** are incorrect
3. A and B are two metals with threshold frequencies 1.8×10^{14} Hz and 2.2×10^{14} Hz. Two identical photons of energy 0.825 eV each are incident on them. Then, photoelectrons are emitted by (Taking, $h = 6.6 \times 10^{-34}$ J-s) [2012]
- B alone
 - A alone
 - Neither A nor B
 - Both A and B
4. Energy conversion in a photoelectric cell takes place from [2011]
- chemical to electrical
 - magnetic to electrical
 - optical to electrical
 - mechanical to electrical
5. Of the following, the one which has the largest de-Broglie wavelength for the same speed is [2010]
- electron
 - proton
 - α -particle
 - oxygen atom
6. The energy that should be added to an electron to reduce its de-Broglie wavelength from 1 nm to 0.5 nm is [2005]
- four times the initial energy
 - equal to the initial energy
 - twice the initial energy
 - thrice the initial energy

Answer with Solutions

Practice Exercise

1. (b) 2. (c)

3. (a) $\lambda_p = \lambda_\alpha$

$$\frac{h}{\sqrt{2m_p Q_p V_p}} = \frac{h}{\sqrt{2m_\alpha Q_\alpha V_\alpha}}$$

$$\therefore m_p Q_p V_p = m_\alpha Q_\alpha V_\alpha$$

$$\therefore V_p = \left(\frac{m_\alpha}{m_p}\right) \left(\frac{Q_\alpha}{Q_p}\right) V_\alpha$$

$$V_p = (4)(2) V_\alpha \Rightarrow V_p = 8 \text{ V}$$

4. (a) 5. (a) 6. (c) 7. (d) 8. (b)

9. (c) $\lambda = \frac{h}{m_e v_e}$

$$\therefore m_e v_e = 1 \quad [\because \lambda = h \text{ (given)}]$$

$$\therefore m_e = \frac{1}{v_e}$$

10. (a) Momentum,

$$p = mv = \frac{\frac{1}{2}mv \times v}{\frac{1}{2} \times v} = \frac{2\text{KE}}{v}$$

If KE as well as speed are doubled, momentum p remains unchanged.

$$\therefore \lambda = \frac{h}{p}$$

Hence, de-Broglie wavelength will be unchanged.

11. (a) As, $v^2 = u^2 + 2as$,

$$\text{So, } v^2 = 0 + 2 \frac{qEl}{m} \text{ or } v = \sqrt{\frac{2qEl}{m}}$$

12. (b) 13. (d)

14. (a) de-Broglie wavelength is $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$

But

$$E = eV$$

$$\lambda = \frac{h}{\sqrt{2meV}} \Rightarrow V = \frac{h^2}{2me\lambda^2}$$

$$V = \frac{(6.62 \times 10^{-34})^2}{(0.5 \times 10^{-10})^2 \times 2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}}$$

$$\Rightarrow V = 60198 \text{ V} \approx 602 \text{ V}$$

15. (a)

16. (a) Energy of each photon

$$E_0 = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{632.2 \times 10^{-9}} = 3.15 \times 10^{-19} \text{ J}$$

Total energy = $E = Pt$

$$\therefore nE_0 = Pt$$

$$\therefore n = \frac{Pt}{E_0} = \frac{5 \times 10^{-3} \times 2}{3.15 \times 10^{-19}} = 3.2 \times 10^{16}$$

17. (a) 18. (a)

19. (b) For photon, $p = \frac{h}{\lambda} = \frac{E}{c}$

But from questions,

$$E = \text{rest energy of electron} = m_0 c^2$$

$$\therefore p = \frac{m_0 c^2}{c} = m_0 c = (9.109 \times 10^{-31} \text{ kg}) \times (3 \times 10^8 \text{ ms}^{-1})$$

$$= 2.7327 \times 10^{-22} \text{ kg ms}^{-1} = 2.73 \times 10^{-22} \text{ kg m/s}$$

20. (b) Effected area = πR^2

$$\text{Pressure} = \frac{I}{c}, \quad \text{where, } c \text{ is speed of light}$$

$$\therefore \text{Force} = \frac{\pi R^2 I}{c}$$

21. (a) Acceleration, $a = \frac{eE_0}{m}$

Final velocity after t second,

$$v = v_0 + \frac{eE_0}{m} t \quad (\text{From, } v = u + at)$$

$$\therefore \text{de-Broglie wavelength, } \lambda = \frac{h}{mv} = \frac{h}{m \left(v_0 + \frac{eE_0 t}{m} \right)}$$

$$= \frac{h}{mv_0 \left(1 + \frac{eE_0 t}{mv_0} \right)} = \frac{\lambda_0}{\left(1 + \frac{eE_0 t}{mv_0} \right)}$$

22. (b)

23. (a) Given, $\bar{\nu} = 1650763.73 \text{ m}^{-1}$

$$\text{Now, } E = hc\bar{\nu}$$

$$= (6.626 \times 10^{-34}) \times (3 \times 10^8) \times 1650763.73$$

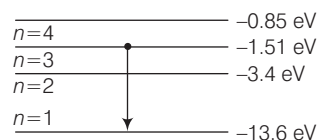
$$= 3.28 \times 10^{-19} \text{ J/quanta}$$

24. (b)

25. (a) If incident radiation has wavelength λ , then

$$eV_0 = \frac{hc}{\lambda} - \phi_0 \text{ or } \frac{hc}{\lambda} = 10.4 + 1.7 = 12.1 \text{ eV}$$

$$\text{or } \lambda = \frac{1240 \text{ eV} \cdot \text{nm}}{12.1 \text{ eV}}$$



and for an excited H_2 atom, energy levels are

$$E_g = E_i - E_f = -1.51 - (-13.6) = 12.1 \text{ eV}$$

So, energy of transition from $n = 3$ to $n = 1$ corresponds to energy of incident photons.

26. (b)

27. (c) $h\nu = h\nu_0 + \text{KE}_{\text{max}}$

For first experiment,

$$h \times 4 \times 10^{15} = h\nu_0 + x$$

For second experiment,

$$h \times 6 \times 10^{15} = h\nu_0 + 3x$$

From Eqs. (i) and (ii), we get $\nu_0 = 3 \times 10^{15}$ Hz

$$\begin{aligned} 28. (d) \text{ We know that } E &= h\nu = \frac{hc}{\lambda} \\ E &= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{0.2 \times 10^{-6}} \\ &= \frac{1.98 \times 10^{-25}}{0.2 \times 10^{-6}} = 9.9 \times 10^{-19} \text{ J} \end{aligned}$$

$$\text{Also, } E - W_0 = \frac{1}{2}mv^2$$

$$9.9 \times 10^{-19} - W_0 = \frac{1}{2} \times 9.1 \times 10^{-31} (8.8 \times 10^5)^2$$

$$\therefore W_0 = 9.9 \times 10^{-19} - 3.52 \times 10^{-19}$$

$$W_0 = 6.38 \approx 6 \text{ eV}$$

$$29. (b) h\nu = h\nu_0 + \frac{1}{2}(mv^2)_{\max}$$

$$\text{But, } \left(\frac{1}{2}mv^2\right)_{\max} = eV_s \therefore V_s = \frac{h\nu}{e} - \frac{h}{e}\nu_0$$

Hence, it is a straight line.

30. (d)

$$31. (b) \text{ Current, } I = \frac{q}{t} = \frac{Ne}{t} = n_e e$$

$$\text{where, } n_e = \text{number of electrons per second} \\ e = 1.6 \times 10^{-19} \text{ C}$$

As, each electron is emitted due to absorption of a photon,

$\therefore n_e = n_2 = \text{number of photons absorbed per second}$

$$\Rightarrow n_r = \frac{I}{e} = \frac{0.16 \times 10^{-6}}{1.6 \times 10^{-19}} \text{ per sec} = 10^{12} \text{ photons/s}$$

And power of source,

$$P = n_1 E, n_1 = \text{number of photons emitted per second}$$

$$\Rightarrow n_1 = \frac{P}{E}$$

$$\text{where, } E = \frac{hc}{\lambda} = \frac{12400}{5000} \text{ eV}$$

$$\Rightarrow E = 2.48 \text{ eV} = 2.48 \times 1.6 \times 10^{-19} \text{ J} = 3.968 \times 10^{-19} \text{ J}$$

$$\Rightarrow n_1 = \frac{P}{E} = 2.52 \times 10^{15} \text{ photons/s}$$

$$\therefore \% \text{ of photons absorbed} = \frac{n_2}{n_1} \times 100$$

$$= \frac{10^{12} \times 100}{2.52 \times 10^{15}} = 0.03968\% \approx 0.04\%$$

$$32. (c) \text{ KE} = \begin{cases} 2E_0 - E_0 = E_0, & \text{for } 0 \leq x \leq 1 \\ 2E_0, & \text{for } x > 1 \end{cases}$$

$$\therefore \lambda_1 = \frac{h}{\sqrt{2mE_0}} \text{ and } \lambda_2 = \frac{h}{\sqrt{4mE_0}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{2}$$

33. (a)

34. (b)

... (i)

... (ii)

$$\begin{aligned} 35. (a) \frac{hc}{\lambda} &= \phi + \frac{1}{2}mv_{\max}^2 \\ \therefore v_{\max} &= \sqrt{\frac{2\left(\frac{hc}{\lambda} - \phi\right)}{m}} = \sqrt{\frac{2\left(\frac{1242 \text{ eV}}{280} - 2.5 \text{ eV}\right)}{9.1 \times 10^{-31}}} \\ &= \sqrt{\left(\frac{2 \times 1.9 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}\right)} = 8.2 \times 10^5 \text{ m/s} \end{aligned}$$

36. (a)

37. (c)

$$38. (a) \xrightarrow{\text{VIBGYOR}} E = n \cdot \frac{hc}{\lambda}$$

From equation,

$$\text{or } n_r \cdot \frac{hc}{\lambda_r} = n_b \cdot \frac{hc}{\lambda_b} \text{ or } \frac{n_r}{n_b} = \frac{\lambda_r}{\lambda_b} \therefore \frac{\lambda_r}{\lambda_b} > 1 \quad (\text{Given})$$

$$\text{Hence, } \frac{n_r}{n_b} > 1 \text{ or } n_r > n_b$$

39. (c)

40. (c)

41. (a) Number of photons falling on metal surface,

$$\begin{aligned} n_p &= \frac{\text{intensity} \times \text{area}}{\text{energy per quanta}} \\ &= \frac{(2 \text{ Js}^{-1} \text{ m}) \times (4 \text{ m}^2)}{\left\{ \frac{(6.626 \times 10^{-34} \text{ Js}) \times (3 \times 10^8 \text{ ms}^{-1})}{(66.26 \times 10^{-9} \text{ m})} \right\}} \\ &= 2.67 \times 10^{18} \text{ per sec} \end{aligned}$$

From equation, $n_e = 0.1\%$ of n_p

$$= \frac{0.1}{100} \times 2.67 \times 10^{18} = 2.67 \times 10^{15} \text{ per sec}$$

$$42. (a) eV_1 = \frac{hc}{\lambda_1} \text{ and } eV_2 = \frac{hc}{\lambda_2} \Rightarrow (V_1 - V_2) = \frac{hc}{e} \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right)$$

$$43. (a) \text{ We know that } E_1 = \frac{hc}{\lambda_1} - W \quad \dots (i)$$

$$E_2 = \frac{hc}{\lambda_2} - W \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

$$\therefore \frac{E_1 + W}{E_2 + W} = \frac{\lambda_2}{\lambda_1} \text{ or } W = \frac{E_1 \lambda_1 - E_2 \lambda_2}{(\lambda_2 - \lambda_1)}$$

44. (a)

$$45. (a) \frac{1}{2}m(8 \times 10^6)^2 = h(5\nu_0 - \nu_0) \quad \dots (i)$$

$$\text{and } \frac{1}{2}mv^2 = h(2\nu_0 - \nu_0) \quad \dots (ii)$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{(8 \times 10^6)^2}{v^2} = \frac{4\nu_0}{\nu_0} \Rightarrow v^2 = \frac{(8 \times 10^6)^2}{4}$$

$$v = \frac{8 \times 10^6}{2} \Rightarrow v = 4 \times 10^6 \text{ m/s}$$

$$46. (b) \text{ We know that, } E = \frac{hc}{\lambda}$$

Let, power = P

then, energy in time t

$$nE = Pt \text{ and } n = \frac{Pt}{E} = \frac{Pt\lambda}{hc}$$

$$\therefore n_R = \frac{Pt\lambda_R}{hc} \quad \dots(i)$$

$$n_V = \frac{Pt\lambda_V}{hc} \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{n_R}{n_V} = \frac{\lambda_R}{\lambda_V}$$

We know, $\lambda_R > \lambda_V$

$$\therefore n_R > n_V$$

$$47. (b) eV_0 = hc \left[\frac{1}{\lambda} - \frac{1}{\lambda_0} \right] \quad \dots(i)$$

$$\frac{eV_0}{6} = hc \left[\frac{1}{3\lambda} - \frac{1}{\lambda_0} \right] \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$6 = \frac{\left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right)}{\left(\frac{1}{3\lambda} - \frac{1}{\lambda_0} \right)}$$

$$\frac{6}{3\lambda} - \frac{6}{\lambda_0} = \frac{1}{\lambda} - \frac{1}{\lambda_0}$$

$$\frac{1}{\lambda} = \frac{5}{\lambda_0}$$

$$\lambda = \frac{\lambda_0}{5}$$

$$\lambda_0 = 5\lambda$$

or
48. (b)

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1. (c) By Einstein's photoelectric equation

$$KE_{\max} = eV_s = h\nu - h\nu_0$$

$$\Rightarrow V_s = \left(\frac{h}{e} \right) \nu - \frac{h\nu_0}{e}$$

Graph of V_s with ν is straight line whose slope = $\frac{h}{e}$

$$\text{Slope of graph} = \frac{AB}{BC} \Rightarrow \frac{AB}{BC} = \frac{h}{e}$$

2. (b) Number of photoelectrons depends on the intensity of light only.

3. (b) Threshold energy of A is

$$\begin{aligned} E_A &= h\nu_A \\ &= 6.6 \times 10^{-34} \times 1.8 \times 10^{14} \\ &= 11.88 \times 10^{-20} \text{ J} \\ &= \frac{11.88 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV} \\ &= 0.74 \text{ eV} \end{aligned}$$

Similarly, $E_B = 0.91 \text{ eV}$

Since, the incident photons have energy greater than E_A but less than E_B .

So, photoelectrons will be emitted from metal A only.

4. (c) In a photoelectric cell, optical energy is being transformed into electrical energy because light photons are being absorbed and photoelectric current is being produced.

5. (a) $\lambda = \frac{h}{mv}$

So, electron is the lightest particle of these four. So, λ_e is maximum.

6. (d) de-Broglie wavelength,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{E_2}{E_1}}$$

$$\Rightarrow \frac{1 \times 10^{-9}}{0.5 \times 10^{-9}} = \sqrt{\frac{E_2}{E_1}}$$

$$\Rightarrow 2 = \sqrt{\frac{E_2}{E_1}}$$

$$\Rightarrow \frac{E_2}{E_1} = 4$$

$$\therefore E_2 = 4E_1$$

$$\therefore \text{Energy to be added} = E_2 - E_1 = 4E_1 - E_1 = 3E_1$$

30

Atomic Structure

Theory of Atom

Atom is the smallest particle of an element which contains all properties of element. Molecule is a single atom or a group of atoms joined by chemical bonds. It is the smallest unit of a chemical compound that can have an independent existence. Nuclei refers to a nucleus of an atom having a given number of nucleons. It is a general term referring to all known isotopes, both stable and unstable of the chemical elements. Thus, O^{16} and O^{17} are different nuclides.

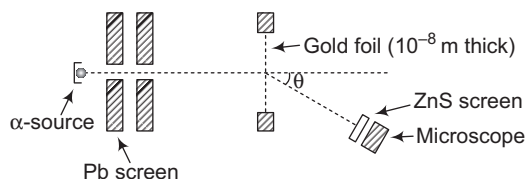
Rutherford's Atomic Model

Rutherford and his two associates, H Geiger and E Marsden performed classical α -particle scattering experiment in order to investigate the structure of the atom.

In this experiment, a beam of 5.5 MeV of α -particles emitted from a ${}^{214}_{33}\text{Bi}$, a radioactive source was made to fall at thin metal foil made of gold.

α -Particle Scattering Experiment

Set up for Rutherford's α -particle scattering experiment is



The scattered α -particles were observed through a rotatable detector consisting of zinc sulphide screen and a microscope. It produces bright light flash which can be seen by microscope.

Main Observations

- (i) Most of the α -particles pass through the foil straight away (undeflected).
- (ii) Some of them are deflected through small angles.
- (iii) Few α -particles (1 in 1000) are deflected through the angle more than 90° .

- (iv) Some α -particles (very few) returned back, i.e. deflected by 180° .
 (v) The number of α -particles scattered at an angle θ by a target is given by

$$N_\theta = \frac{N_0 n t (2Ze^2)^2}{4(4\pi\epsilon_0)^2 r^2 (mv_0^2)^2} \times \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

where, N_0 = total number of α -particles that strike the unit area of the scattered, n = number of target atoms per m^3 , t = thickness of target, Ze = charge on the target nucleus, $2e$ = charge on α -particle, r = distance of screen from the target and v_0 = velocity of α -particle at nearest distance of approach.

The number of α -particles scattered per unit area, N_θ at scattering angle θ varies inversely as $\sin^4 \frac{\theta}{2}$.

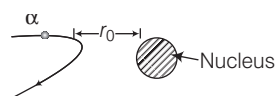
i.e.
$$N_\theta \propto \frac{1}{\sin^4(\theta/2)}$$

- (vi) In an atom, there is maximum empty space and the electrons revolve around the nucleus in the same way as the planets revolve around the sun. Taking electrostatic force as only force of attraction on electrons,

$$\frac{mv^2}{r} = \frac{kZe \cdot e}{r^2}$$

where, Z = number of protons of target nucleus

- (vii) **Distance of closest approach** (Nuclear dimension) The minimum distance from the nucleus upto which the α -particle approaches is called the distance of closest approach (r_0).

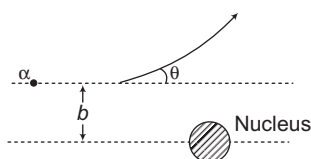


From figure,

$$r_0 = \frac{1}{4\pi\epsilon_0} \cdot \frac{2Ze^2}{E}$$

where, $E = \frac{1}{2}mv^2 = \text{KE of } \alpha\text{-particle}$

- (viii) **Impact parameter** The perpendicular distance of the velocity vector (v) of the α -particle from the centre of the nucleus when it is far away from the nucleus is known as impact parameter. It is given as in the figure.



From figure,
$$b = \frac{Ze^2 \cot\left(\frac{\theta}{2}\right)}{4\pi\epsilon_0 \times \frac{1}{2}mv^2}$$

$$\Rightarrow b \propto \cot\left(\frac{\theta}{2}\right)$$

- (ix) If t is the thickness of the foil and N is the number of α -particles scattered in a particular direction

(θ = constant), then it was observed that $\frac{N}{t} = \text{constant}$

$$\Rightarrow \frac{N_1}{N_2} = \frac{t_1}{t_2}$$

Bohr's Model

Bohr added the following postulates to the Rutherford's model of the atom

- The electrons revolve around the nucleus only in certain permitted orbits, in which the angular momentum of the electron is an integral multiple of $h/2\pi$, where h is the Planck's constant.
- The electrons do not radiate energy while revolving in the permitted orbits.
i.e. the permitted orbits are stationary, non-radiating orbits.
- The energy is radiated only when the electron jumps from an outer permitted orbit to some inner permitted orbit. (Absorption of energy makes the electron jump from inner orbit to outer orbit).
- If energy of the electron in n th and m th orbits be E_n and E_m respectively, then while the electron jumps from n th to m th orbit the radiation frequency ν is emitted, such that $E_n - E_m = h\nu$

This is called the **Bohr's frequency equation**.

- Radius of the orbit of electron in a hydrogen atom in its stable state, corresponding to $n = 1$, is called Bohr's radius. The value of Bohr's radius is $r_0 = 0.529 \text{ \AA} \approx 0.53 \text{ \AA}$.
- The time period of an electron in orbital motion in the Bohr's orbit is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi \times 0.53 \text{ \AA}}{\frac{c}{137}} = 1.52 \times 10^{-16} \text{ s}$$

and the frequency of revolution is

$$f = \frac{1}{T} = 6.5757 \times 10^{15} \text{ cps}$$

Some Characteristics of an Atom

- (i) The **orbital radius** of the electron is

$$r_n = 4\pi\epsilon_0 \frac{n^2 h^2}{4\pi^2 m e^2}$$

(ii) The **orbital velocity** of electron is

$$v_n = \frac{1}{4\pi\epsilon_0} \frac{2\pi e^2}{nh}$$

(iii) The **orbital frequency** is given by

$$f = \frac{1}{T} = \frac{v}{2\pi r} = \frac{me^4}{4\epsilon_0^2 n^3 h^3}$$

(iv) The **total energy** of the orbital electron is

$$E = -\left(\frac{1}{4\pi\epsilon_0}\right)^2 \left[\frac{2\pi me^4}{h^2}\right] \left[\frac{1}{n^2}\right]$$

$$\text{KE} = \frac{me^4}{8n^2 h^2 \epsilon_0^2} \text{ or } \text{PE} = -\frac{me^4}{4n^2 h^2 \epsilon_0^2}$$

(v) The **velocity of the orbital electron** may be written as

$$v_n = \frac{1}{4\pi\epsilon_0} \left[\frac{2\pi e^2}{h}\right] \left(\frac{1}{n}\right) = \frac{c}{n} \left[\frac{1}{4\pi\epsilon_0} \cdot \frac{2\pi e^2}{ch}\right] = \frac{c}{n} \alpha$$

Here, $\frac{1}{4\pi\epsilon_0} \frac{2\pi e^2}{ch} = \alpha$ is called fine structure constant.

It is a dimensionless quantity.

The value of α comes out to be $\frac{1}{137} = 0.0073$.

(vi) The kinetic, potential and total energies of the electron varies with r (the radius of the orbit) as follows

$$\text{KE} = \frac{1}{2} \left[\frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \right], \text{PE} = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$$

$$\text{and } E = -\frac{1}{2} \left[\frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \right]$$

Therefore, they are related to each other as follows

$$\text{KE} = -E \text{ and } \text{PE} = 2E$$

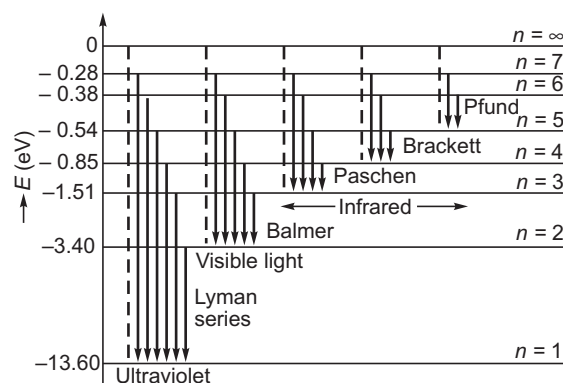
(vii) For a hydrogen atom,

$$r_n \propto n^2, v_n \propto \frac{1}{n} \text{ and } |E| \propto \frac{1}{n^2}$$

The difference in angular momentum associated with the electron in the two successive orbits of hydrogen atom is $\Delta L = (n+1) \frac{h}{2\pi} - \frac{nh}{2\pi} = \frac{h}{2\pi}$.

Hydrogen Spectrum

Hydrogen spectrum consists of spectral lines classified as five spectral series of hydrogen atom. Out of these five, Lyman series lies in the ultraviolet region of spectrum, Balmer series lies in the visible region and the remaining three series, lie in the infrared region of spectrum.



Total number of emission spectral lines from some excited state n , to another energy $n_2 (< n_1)$ is given by

$$\frac{(n_1 - n_2)(n_1 - n_2 + 1)}{2}$$

e.g. Total number of lines from $n_1 = n$ to $n_2 = 1$ are $\frac{n(n-1)}{2}$.

The five spectral series of hydrogen atom are given as below

1. Lyman Series

Spectral lines of Lyman series correspond to the transition of electron from higher energy levels (orbits) $n_i = 2, 3, 4, \dots$ to ground energy level (1st orbit) $n_f = 1$.

For Lyman series, $\frac{1}{\lambda} = \bar{\nu} = R \left[\frac{1}{(1)^2} - \frac{1}{n^2} \right]$, where $n = 2, 3, 4, \dots$

It is found that a term $Rch = 13.6 \text{ eV} = 2.17 \times 10^{-18} \text{ J}$. The term Rch is known as Rydberg's energy.

2. Balmer Series

Electronic transitions from $n_i = 3, 4, 5, \dots$ to $n_f = 2$, give rise to spectral lines of Balmer series.

Thus, for a Balmer series line

$$\frac{1}{\lambda} = \bar{\nu} = R \left[\frac{1}{(2)^2} - \frac{1}{n^2} \right], \text{ where } n = 3, 4, 5, \dots$$

3. Paschen Series

Spectral lines of this series lie in the infrared region and correspond to electronic transition from $n_i = 4, 5, 6, \dots$ to $n_f = 3$.

Thus, $\frac{1}{\lambda} = \bar{\nu} = R \left[\frac{1}{(3)^2} - \frac{1}{n^2} \right]$, where $n = 4, 5, 6, \dots$

4. Brackett Series

It lies in the infrared region and corresponds to transition from

$$n_i = 5, 6, 7, \dots \text{ to } n_f = 4.$$

Thus, for Brackett series

$$\frac{1}{\lambda} = \bar{\nu} = R \left[\frac{1}{(4)^2} - \frac{1}{n^2} \right], \text{ where } n = 5, 6, 7, \dots$$

5. Pfund Series

It lies in the far infrared region of spectrum and corresponds to electronic transitions from higher orbits $n_i = 6, 7, 8, \dots$ to orbit having $n_f = 5$. Thus, we have

$$\frac{1}{\lambda} = \bar{\nu} = R \left[\frac{1}{(5)^2} - \frac{1}{n^2} \right], \text{ where } n = 6, 7, 8, \dots$$

- Radius of the n th orbit in hydrogen atom is the $r_n = 0.53 n^2 \text{ \AA}$.
- If elements with $n > 4$ were not allowed to exist, then total number of elements would be

$$2 \times [1^2 + 2^2 + 3^2 + 4^2] = 60$$

Ionisation Energy and Potential

Ionisation energy of an atom is defined as the energy required to ionise it, i.e. to make the electron jump from its present orbit to infinity.

Thus, ionisation energy of hydrogen atom in the ground state

$$= E_{\infty} - E_1 = 0 - (-13.6 \text{ eV}) = +13.6 \text{ eV}$$

The potential through which an electron is to be accelerated so that it acquires energy equal to the ionisation energy is called the **ionisation potential**.

Therefore, ionisation potential of hydrogen atom in its ground state is 13.6V.

Excitation Energy and Potential

Excitation energy is the energy required to excite an electron from a lower energy level to a higher energy level. The potential through which an electron is accelerated so as to gain requisite ionisation energy is called the **ionisation potential**.

Thus, first excitation energy of hydrogen atom

$$\begin{aligned} &= E_2 - E_1 = -3.4 - (-13.6) \text{ eV} \\ &= +10.2 \text{ eV} \end{aligned}$$

Similarly, second excitation energy of hydrogen atom

$$\begin{aligned} &= E_3 - E_1 = -1.51 - (-13.6) \\ &= 12.09 \text{ eV} \end{aligned}$$

Practice Exercise

1. α -particles are projected towards the nuclei of the following metals with the same kinetic energy. Towards which metal, the distance of closest approach is minimum?
 - a. Cu ($Z = 29$)
 - b. Ag ($Z = 47$)
 - c. Au ($Z = 79$)
 - d. Pd ($Z = 46$)
2. An α -particle accelerated through V volt is fired towards a nucleus. Its distance of closest approach is r . If a proton accelerated through the same potential is fired towards the same nucleus, then distance of closest approach of proton will be
 - a. r
 - b. $2r$
 - c. $r/2$
 - d. $r/4$
3. The distance of closest approach of an α -particle fired towards a nucleus with momentum p is r . What will be the distance of closest approach when the momentum of α -particle is $2p$?
 - a. $2r$
 - b. $4r$
 - c. $r/2$
 - d. $r/4$
4. Which of the following is incorrect regarding Rutherford's atomic model?
 - a. Atom contains nucleus
 - b. Size of nucleus is very small in comparison to that of atom
 - c. Nucleus contains about 90% mass of the atom
 - d. Electrons revolve around the nucleus with a uniform speed
5. In Rutherford's experiment, the number of α -particles scattered through an angle of 90° is 28 per minute. Then, the number of particles scattered through an angle of 60° per minute by the same nucleus is
 - a. 28 per minute
 - b. 112 per minute
 - c. 12.5 per minute
 - d. 7 per minute
6. Find the equivalent current due to motion of electron in first orbit of H-atom.
 - a. $0.7 \times 10^{-3} \text{ A}$
 - b. $9 \times 10^{-3} \text{ A}$
 - c. 10^{-3} A
 - d. None of these

7. If the radius of first Bohr's orbit is x , then de-Broglie wavelength of electron in 3rd orbit is nearly
 a. $2\pi x$ b. $6\pi x$
 c. $9x$ d. $x/3$
8. How many times larger is the spacing between the energy levels with $n = 3$ and $n = 4$, then the spacing between the energy levels with $n = 8$ and $n = 9$ for a hydrogen like atom or ion?
 a. 0.71 b. 0.41
 c. 2.43 d. 14.82
9. The circumference of the second orbit of an atom or ion having single electron, is 4×10^{-9} m. The de-Broglie wavelength of electron revolving in this orbit should be
 a. 2×10^{-9} m b. 4×10^{-9} m
 c. 8×10^{-9} m d. 1×10^{-9} m
10. In each of the following atoms or ions, electronic transition from $n = 4$ to $n = 1$ take place. The frequency of the radiation emitted out will be minimum for
 a. hydrogen atom
 b. deuterium atom
 c. He^+ ion
 d. Li^{2+} ion
11. If an electron is revolving around the hydrogen nucleus at a distance of 0.1 nm, what should be its speed?
 a. 2.188×10^6 m/s b. 1.094×10^6 m/s
 c. 4.376×10^6 m/s d. 1.59×10^6 m/s
12. The angular speed of an electron revolving around the H-nucleus is proportional to
 a. $1/r$ b. $1/r^{3/2}$
 c. $1/r^2$ d. $r^{3/2}$
13. Calculate the angular momentum of the electron in third orbit of hydrogen atom, if the angular momentum in the second orbit of hydrogen atom is L .
 a. L b. $3L$
 c. $\frac{3}{2}L$ d. $\frac{2}{3}L$
14. If an electron is moving around a nucleus of charge $2e$ in a circular orbit of radius 10^{-10} m, then calculate the initial frequency of light emitted by the electron.
 a. 4.2×10^{15} Hz b. 0.36×10^{15} Hz
 c. 3.6×10^{15} Hz d. 4.2×10^{15} Hz
15. An electron of hydrogen atom is revolving in third Bohr's orbit ($n = 3$). How many revolutions will it undergo before making a transition to the second orbit ($n = 2$). Assume the average life time of an excited state of the hydrogen atom is of the order of 10^{-8} s. (Given, Bohr radius = 5.3×10^{-12} m)
 a. 2.5×10^6 revolutions b. 3.5×10^6 revolutions
 c. 4.5×10^6 revolutions d. 1.5×10^6 revolutions
16. If $\left(\frac{0.51 \times 10^{-10}}{4}\right)$ metre is the radius of smallest electron orbit in hydrogen like atom, then this atom is
 a. hydrogen atom b. He^+
 c. Li^{2+} d. Be^{3+}
17. How many different wavelengths may be observed in the spectrum from a hydrogen sample, if the atoms are excited to third excited state?
 a. 3 b. 4 c. 5 d. 6
18. Find the maximum number of photons emitted by an H-atom, if atom is excited to states with principal quantum number four.
 a. 4 b. 3 c. 2 d. 1
19. In (Q. 18) problem, the minimum number of photons emitted by the H-atom is
 a. 1 b. 2 c. 3 d. 4
20. The kinetic energy of an electron in hydrogen atom is 3.40 eV. The minimum energy required to ionise the hydrogen atom is
 a. -3.40 eV b. 6.40 eV c. -6.80 eV d. 3.40 eV
21. Two H-atoms in the ground state collide inelastically. The maximum amount by which their combined kinetic energy is reduced is
 a. 10.20 eV b. 20.40 eV c. 13.6 eV d. 27.2 eV
22. Calculate the ratio of the frequencies of the long wavelength limits of the Balmer and Lyman series of hydrogen.
 a. 27 : 5 b. 5 : 27 c. 4 : 1 d. 1 : 4
23. For a certain atom, there are energy levels A, B, C corresponds to energy values $E_A < E_B < E_C$. Choose the correct option if $\lambda_1, \lambda_2, \lambda_3$ are the wavelength of radiations corresponding to the transition from C to B , B to A and C to A respectively.
 a. $\lambda_3 = \lambda_1 + \lambda_2$ b. $\lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$
 c. $\lambda_1 + \lambda_2 + \lambda_3 = 0$ d. $3\lambda_2 = \lambda_3 + 2\lambda_2$
24. Calculate the energy required to excite an electron in hydrogen atom from the ground state to the next higher state, if the ionisation energy for the hydrogen atom is 13.6 eV.
 a. 3.4 eV b. 10.2 eV c. 12.1 eV d. 1.3 eV
25. Find the wavelength of the emitted radiation, if electron in hydrogen atom jumps from the third orbit to second orbit.
 a. $\lambda = \frac{36}{5R}$ b. $\lambda = \frac{5R}{36}$ c. $\lambda = \frac{5}{R}$ d. $\lambda = \frac{R}{6}$
26. Any radiation in the ultraviolet region of hydrogen spectrum is able to eject photoelectrons from a metal. What should be the maximum value of threshold frequency for the metal?
 a. 3.288×10^{15} Hz b. 2.466×10^{15} Hz
 c. 4.594×10^{14} Hz d. 8.220×10^{14} Hz

27. Balmer gives an equation for wavelength of visible radiation of H-spectrum as $\lambda = \frac{kn^2}{n^2 - 4}$. The value of k in terms of Rydberg constant R is
 a. R b. $4R$ c. $R/4$ d. $4/R$
28. When an electron jumps from higher orbit to the second orbit in He^+ ion, the radiation emitted out will be in ($R = 1.09 \times 10^7 \text{ m}^{-1}$)
 a. ultraviolet region b. visible region
 c. infrared region d. X-ray region
29. Deuterium atoms in the ground state are radiated by photons of energy 12.8 eV. What will be the energy of induced radiation of longest wavelength? Ionisation energy of deuterium is 14.4 eV.
 a. 12.8 eV b. 10.8 eV c. 1.6 eV d. 2.00 eV
30. Calculate the ionisation energy of Li^{2+} atom in ground state.
 a. $13.6 \times 9 \text{ eV}$ b. 13.6 J
 c. 13.6 erg d. 13.6×10^{-19}
31. The first excitation potential of a given atom is 10.2 V, then the ionisation potential is
 a. 10.2 V b. 13.6 V c. 30.6 V d. 20.4 V
32. For a single ionised helium atom, the longest wavelength in ground state will absorb
 a. 912 Å b. 304 Å
 c. 606 Å d. 1216 Å
33. If an electron drops from 4th orbit to 2nd orbit in an H-atom, then
 a. it gains 2.55 eV of potential energy
 b. it gains 2.55 eV of total energy
 c. it emits a 2.55 eV electron
 d. it emits a 2.55 eV photon
34. 29 electrons are removed from Zn-atom ($Z = 30$) by certain means. The minimum energy needed to remove the 30th electron, will be
 a. 12.24 keV b. 408 eV
 c. 0.45 eV d. 765 eV
35. An electron of kinetic energy E_0 is scattered by an atomic hydrogen sample in ground state. Find the minimum value of E_0 , so that a photon of wavelength 656.3 nm may be emitted by H-atom.
 a. 12.09 eV b. 13.6 eV
 c. 14.6 eV d. None of these
36. A H-atom moving with speed v makes a head on collision with a H-atom in rest. Both atoms are in ground state. Find the minimum value of velocity v for which one of atom may excite.
 a. $6.25 \times 10^4 \text{ m/s}$ b. $8 \times 10^4 \text{ m/s}$
 c. $7.25 \times 10^4 \text{ m/s}$ d. $13.6 \times 10^4 \text{ m/s}$
37. A photon of energy 15 eV collides with H-atom. Due to this collision, H-atom gets ionised. The maximum kinetic energy of emitted electron is
 a. 1.4 eV b. 5 eV
 c. 15 eV d. 13.6 eV
38. Find the minimum frequency of light which can ionise a hydrogen atom.
 a. $3.28 \times 10^{15} \text{ Hz}$ b. $5 \times 10^{15} \text{ Hz}$
 c. 91.1 Hz d. None of these
39. In the case of Compton effect, which of the following is applicable?
 a. Energy conservation b. Momentum conservation
 c. Charge conservation d. All of these
40. The number of orbitals in 3rd orbit are
 a. 3 b. 10
 c. 18 d. None of these

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1. In hydrogen atom, an electron jumps from bigger orbit to smaller orbit, so that radius of smaller orbit is one-fourth of radius of bigger orbit. If speed of electron in bigger orbit was v , then speed in smaller orbit is
 a. $\frac{v}{4}$ b. $\frac{v}{2}$ [2014]
 c. v d. $2v$
2. In hydrogen atom, if $\lambda_1, \lambda_2, \lambda_3$ are shortest wavelengths in Lyman, Balmer and Paschen series respectively, then $\lambda_1 : \lambda_2 : \lambda_3$ equals [2013]
 a. 1 : 4 : 9 b. 9 : 4 : 1
 c. 1 : 2 : 3 d. 3 : 2 : 1
3. If λ is the wavelength of hydrogen atom from the transition $n = 3$ to $n = 1$, then what is the wavelength for doubly ionised lithium ion for same transition?
 a. $\frac{\lambda}{3}$ b. 3λ [2006]
 c. $\frac{\lambda}{9}$ d. 9λ
4. In Bohr's atom model, [2005]
 a. the nucleus is of infinite mass and is at rest
 b. electrons in a quantised orbit will not radiate energy
 c. mass of electron remains constant
 d. All of the above

Answer with Solutions

Practice Exercise

1. (a) For distance of closest approach,
(KE) _{α} = PE at distance of closest approach

$$\text{or} \quad \text{KE} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r}$$

where, q_1 = charge of α -particle
 q_2 = charge of nucleus

$$\therefore r = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{\text{KE}} \Rightarrow r \propto q_2$$

Smaller q_2 (= Ze), smaller will be radius of closest approach (r).

2. (a) $\text{KE} = \text{PE}$
or $q_1 V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r}$

or $r = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_2}{V} = \text{constant in this problem}$

3. (d) $\text{KE} = \text{PE}$
or $\frac{p^2}{2m} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r} \Rightarrow r \propto \frac{1}{p^2}$

Hence, momentum of α -particle will be the closest approach of $r/4$.

4. (c)

5. (b) According to Rutherford's scattering formula, if the α -particle scattered at angle θ is directly proportional to $\frac{1}{\sin^4(\theta/2)}$,

Then, $N_\theta = \frac{K}{\sin^4(\theta/2)}$

When $\theta = 90^\circ$, $N_\theta = 28 \text{ min}^{-1}$

$\Rightarrow 28 = \frac{K}{\sin^4(45^\circ)} = 4K \Rightarrow K = 7$

Thus, $N_\theta = \frac{7}{\sin^4(\theta/2)}$

Hence, the number of α -particles scattered at an angle of 60° per minute is $N'_\theta = \frac{7}{\sin^4 30^\circ} = \frac{7}{(1/2)^4} = 7 \times 16 = 112$

6. (a) $I = \frac{e}{T} = \frac{e}{2\pi/\omega} = \frac{e\omega}{2\pi} = \frac{ev}{2\pi r}$

Here, $v = \frac{e^2}{2\epsilon_0 h} = \frac{(1.6 \times 10^{-19})^2}{2 \times 8.85 \times 10^{-12} \times 6.63 \times 10^{-34}} = 2.18 \times 10^6 \text{ m/s}$

$r = n^2 r_0 = (1)^2 \times 0.53 \text{ \AA} = 0.53 \times 10^{-10}$
 $\therefore I = \frac{1.6 \times 10^{-19} \times 2.18 \times 10^6}{3 \times 3.14 \times 0.53 \times 10^{-10}} = 0.7 \times 10^{-3} \text{ A}$

7. (b) $r_3 = r_1 \times n^2 = x \times 3^2 = 9x$

$\therefore 2\pi r = n\lambda$

$\therefore \lambda = \frac{2\pi r}{n} = \frac{2\pi \times 9x}{3} = 6\pi x$

8. (b) $\frac{r_4 - r_3}{r_9 - r_8} = \frac{r_1 \times 4^2 - r_1 \times 3^2}{r_1 \times 9^2 - r_1 \times 8^2} = \frac{16 - 9}{81 - 64} = 0.41$

9. (a)

10. (a)

11. (d)

12. (b) We know that,

$\therefore \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r^2} = \frac{mv^2}{r}$

$\therefore v = \sqrt{\left(\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{mr}\right)} = \frac{k}{\sqrt{r}}$

(Let $k = \frac{Ze^2}{4\pi\epsilon_0 m}$)

Angular speed, $\omega = \frac{v}{r} = \frac{k}{\sqrt{r} \cdot r} = \frac{k}{r^{3/2}}$

13. (c)

14. (c) We know that,

$\frac{mv^2}{r} = \frac{2e^2}{4\pi\epsilon_0 r^2}$ or $v = \left(\frac{2e^2}{4\pi\epsilon_0 mr}\right)^{1/2}$

Also, frequency $= f = \frac{v}{2\pi r} = \left(\frac{2e^2}{4\pi\epsilon_0 mr}\right)^{1/2} \times \frac{1}{2\pi r}$

$\therefore f = \frac{(1.414)(9 \times 10^9)^{1/2} \times 1.6 \times 10^{-19}}{(9.1 \times 10^{-31})^{1/2} \times 2\pi(10^{-10})^{3/2}} \approx 3.6 \times 10^{15} \text{ Hz}$

15. (a) Number of revolutions made in one second $= \frac{v_3}{2\pi r_3}$

(for $n = 3$)

$= \frac{(0.73) \times 10^6}{2 \times \pi \times 47.7 \times 10^{-11}} = 2437 \times 10^{11}$

Number of revolutions taken in 10^{-8} s
 $= 2437 \times 10^{-8} \times 10^{11} \approx 2.5 \times 10^6 \text{ revolutions.}$

16. (d)

17. (d) $N = \frac{n(n-1)}{2}$

For third excited state, $n = 4$

$\therefore N = \frac{4(4-1)}{2} = 6$

18. (b) For maximum number,

$N = n - 1 = 4 - 1 = 3$

19. (a) When transition takes place from $n = 4$ to $n = 1$. One photon will be emitted.

20. (d) Given, KE = 3.40 eV

So, total energy = -3.40 eV

When H-atom is ionised, its minimum total energy will become zero.

So, ionisation energy = $0 - (-3.40) = 3.40$ eV

21. (a) 22. (a)

$$23. (b) \quad E_B - E_A = \frac{hc}{\lambda^2} \Rightarrow E_C - E_B = \frac{hc}{\lambda_2}$$

$$\text{or} \quad \frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \Rightarrow \lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

24. (b) 25. (a)

26. (b) For minimum frequency in Lyman series of H-atom, transition $n = 2$ to $n = 1$ takes place.

Now, $h\nu = \Delta E = E_2 - E_1$

$$\text{or} \quad h\nu = \left[\left(-\frac{13.6}{2^2} \right) - \left(-\frac{13.6}{1^2} \right) \right] \text{eV}$$

$$\nu = \frac{10.2 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$\therefore \nu = 2.466 \times 10^{15} \text{ Hz}$$

If the threshold frequency of metal is ν or less, photoelectron will come out.

27. (d)

28. (b)

29. (d)

30. (a)

31. (b)

32. (b)

$$\lambda = \frac{912}{Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)}$$

$$\lambda = \frac{912}{3} = 304 \text{ Å}$$

33. (d)

34. (a)

35. (a)

36. (a) For $v_{\min}$, collision should be completely inelastic.

According to energy conservation principle,

$$\frac{1}{2} m v_{\min}^2 = \frac{1}{2} m v^2 + \frac{1}{2} m v^2 + \Delta E \quad \dots(i)$$

According to momentum conservation principle,

$$m v_{\min} = m v + m v$$

$$\therefore v = \frac{v_{\min}}{2} \quad \dots(ii)$$

After solving Eqs. (i) and (ii), we get

$$\frac{1}{2} m v_{\min}^2 = 2 \Delta E$$

$$\therefore v_{\min} = \sqrt{\left(\frac{4 \Delta E}{m} \right)}$$

Here, ΔE = minimum excitation energy = 10.2 eV

$$m = 1.67 \times 10^{-27} \text{ kg}$$

$$\therefore v_{\min} = \sqrt{\left(\frac{4 \times 10.2 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27}} \right)} = 6.25 \times 10^4 \text{ m/s}$$

37. (a)

38. (a)

39. (b)

40. (d)

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1. (d) Radius of n th orbit, $r_n \propto n^2$

$$\frac{r_{n \text{ big}}}{r_{n \text{ small}}} = \frac{n_{\text{big}}^2}{n_{\text{small}}^2} = \frac{4}{1} \quad (\text{given})$$

$$\Rightarrow \frac{n_{\text{big}}}{n_{\text{small}}} = 2$$

$$\Rightarrow \frac{n_{\text{small}}}{n_{\text{big}}} = \frac{1}{2}$$

Velocity of electron in n th orbit

$$v_n \propto \frac{1}{n}$$

$$\frac{v_{n \text{ big}}}{v_{n \text{ small}}} = \frac{n_{\text{small}}}{n_{\text{big}}} = \frac{1}{2}$$

$$\Rightarrow v_{n \text{ small}} = 2(v_{n \text{ big}}) = 2v$$

2. (a) For hydrogen atom, $\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$, $n_2 > n_1$

For Lyman series, $n_1 = 1, n_2 = \infty$

$$\Rightarrow \frac{1}{\lambda_1} = R$$

For Balmer series, $n_1 = 2, n_2 = \infty$

$$\Rightarrow \frac{1}{\lambda_2} = \frac{R}{4}$$

For Paschen series, $n_1 = 3, n_2 = \infty$

$$\Rightarrow \frac{1}{\lambda_3} = \frac{R}{9}$$

$$\text{So, } \lambda_1 = \frac{1}{R}, \lambda_2 = \frac{4}{R}, \lambda_3 = \frac{9}{R}$$

$$\lambda_1 : \lambda_2 : \lambda_3 = 1 : 4 : 9$$

3. (c) For wavelength, $\frac{1}{\lambda} = R Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$

Here, transition is same

$$\text{So, } \lambda \propto \frac{1}{Z^2}$$

$$\frac{\lambda_{\text{H}}}{\lambda_{\text{Li}}} = \frac{(Z_{\text{Li}})^2}{(Z_{\text{H}})^2} = \frac{(3)^2}{(1)^2} = 9$$

$$\Rightarrow \lambda_{\text{Li}} = \frac{\lambda_{\text{H}}}{9} = \frac{\lambda}{9}$$

4. (d)

31

Nucleus

Nucleus

In every atom the positive charge and mass is densely concentrated at the centre of the atom forming its nucleus. Nuclear radius is of the order of 10^{-15} m. In nucleus the number of protons is equal to the atomic number of that element and the remaining particles to fulfill the mass number are the neutrons i.e. number of protons = atomic number Z (say) and number of neutrons.

Composition and Size of Nucleus

As mass of an atom is very small, hence we define a new unit of mass, called as 1 atomic mass unit (1u), which is $\frac{1}{12}$ th the mass of one atom of carbon-12.

$$1 \text{ u} = 1.660539 \times 10^{-27} \text{ kg} \simeq 1.66 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV}$$

A nucleus has a structure of its own. It consists of protons and neutrons. Electrons cannot exist inside the nucleus. A proton is a positively charged particle having mass (m_p) of 1.007276 u and charge (+e) = $+1.602 \times 10^{-19}$ C.

Number of protons Z inside the nucleus of an atom is exactly equal to the number of electrons revolving around the nucleus of that atom. This number is called the atomic number. A neutron is a neutral particle having mass $m_n = 1.008665$ u. The number of neutrons in the nucleus of an atom is called the **neutron number** N . The sum of the number of protons and neutrons is called the mass number A . Thus, $A = N + Z$.

Properties of Nucleus

The nucleus properties are described below

- **Nucleus size**
 - (a) Size of the nucleus is of the order of fermi (1 fermi = 10^{-15} m)
 - (b) The radius of the nucleus is given by $R = R_0 A^{1/3}$
where, $R_0 = 1.3$ fermi and A is the mass number.
 - (c) The size of the atom is of the order of 10^{-10} m.

- **Volume** The volume of nucleus is

$$V = \frac{4}{3} \pi R^3 = \frac{4}{3} \pi (R_0 A^{1/3})^3$$

- **Density**

$$\begin{aligned} \text{(a) Density} &= \frac{\text{Mass of nucleus}}{\text{Volume of the nucleus}} = \frac{Am_p}{\frac{4}{3} \pi (R_0 A^{1/3})^3} \\ &= \frac{m_p}{\frac{4}{3} \pi R_0^3} \end{aligned}$$

where, $m_p = 1.6 \times 10^{-27}$ kg = mass of proton and $R_0 = 1.3$ fermi.

- (b) Density of nuclear matter is of the order of 10^{17} kg/m³.
- (c) Density of nuclear matter is independent of the mass number.

Isotopes, Isobars and Isotones

Isotopes

Isotopes of an element are nuclides having same atomic number Z but different mass number A (or different neutron number N). Isotopes of an element have identical electronic configuration and hence, identical chemical properties. ${}^1_1\text{H}$, ${}^2_1\text{H}$, ${}^3_1\text{H}$ and ${}^{11}_6\text{C}$, ${}^{12}_6\text{C}$, ${}^{14}_6\text{C}$, etc., are isotopes.

Isobars

Nuclides having same mass number A but different atomic number Z are called isobars. Isobars have different chemical properties. In isobars number of protons Z as well as number of neutrons N differ but total nucleon (or mass) number $A = N + Z$ is the same. ${}^3_1\text{H}$, ${}^3_2\text{He}$ and ${}^{14}_6\text{C}$, ${}^{14}_7\text{N}$ are isobars.

Isotones

Nuclides with different atomic number Z and different mass number A but same neutron number are called isotones.

Thus, for isotones $N = (A - Z)$ is constant. ${}^3_1\text{H}$, ${}^4_2\text{He}$ and ${}^{198}_{80}\text{Hg}$, ${}^{197}_{79}\text{Au}$ are examples of isotones.

Radioactivity

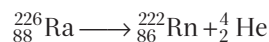
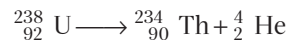
The phenomenon of spontaneous emission of radiations by heavy elements, is called radioactivity.

An unstable nucleus undergoes a decay to form a stable nucleus. Three types of decay occurs in nature, which are defined as follows.

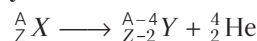
1. α -decay

An α -particle is a helium nucleus. Alpha (α) emission occurs principally with nuclei that are too large to be stable. When a nucleus emits an α -particle, its N and Z values each decrease by two and A decreases by four.

Few examples of α -decay are



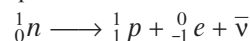
In general, the decay can be written as



where, X = parent nucleus, Y = daughter nuclues

2. β -decay

A beta minus particle (β^-) is like an electron. Emission of β^- involves transformation of a neutron into a proton, an electron and a third particle called an anti-neutrino ($\bar{\nu}$).



β -decay is spontaneous process with a half-life of about few days.

β^- -decay usually occurs with nuclides for which the neutron to proton ratio (N/Z ratio) is too large for stability.

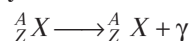
In β^+ -decay, a proton changes into a neutron with the emission of a positron (and a neutrino) $p \rightarrow n + e^+ + \nu$

β^+ -decay usually occurs with nuclides for which the neutron to proton ratio (N/Z ratio) is too small for stability.

3. γ -decay

When a nucleus is placed in an excited state, either by bombardment with high-energy particles or by a radioactive transformation, it can decay to the ground state by emission of one or more photons called gamma rays or gamma ray photon, with typically energies of 10 keV to 5 MeV. This process is called γ -decay.

In general, the decay can be written as



All the known conservative laws are obeyed in γ -decay.

Radioactivity Decay Law

According to Rutherford and Soddy, law for radioactive decay is as follows.

At any instant, the rate of decay of radioactive atoms is proportional to the number of atoms present at that instant.

$$\text{i.e.} \quad -\frac{dN}{dt} \propto N$$

$$\Rightarrow \quad \frac{dN}{dt} = -\lambda N$$

$$\text{So,} \quad N = N_0 e^{-\lambda t}$$

Activity

It is defined as the rate of disintegration (or count rate) of the radioactive substance (or the number of atoms of any material decaying per second)

$$\text{i.e.} \quad R = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t}$$

$$R = R_0 e^{-\lambda t}$$

where, R_0 = activity at $t = 0$, R = activity after time t .

Units of Activity The SI unit of decay rate is becquerel (Bq) but curie (Ci) and rutherford (Rd) are often used in practice.

1 becquerel = 1 disintegration/s, 1 rutherford = 10^6 dis/s,

1 curie = 3.7×10^{11} dis/s.

Half-life ($T_{1/2}$)

Time interval in which the mass of a radioactive substance or the number of its atom reduces to half of its initial value, is called the half-life of the substance.

$$\text{i.e. if } N = \frac{N_0}{2}, \text{ then } t = T_{1/2}$$

$$\text{Hence from,} \quad N = N_0 e^{-\lambda t}$$

$$\text{We have,} \quad \frac{N_0}{2} = N_0 e^{-\lambda(T_{1/2})} \Rightarrow T_{1/2} = \frac{\log_e 2}{\lambda}$$

$$T_{1/2} = \frac{0.693}{\lambda}$$

After n half-lives, number of undecayed atoms

$$\text{i.e.} \quad N = N_0 \left(\frac{1}{2}\right)^n = N_0 \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

Mean (or average) Life (τ)

The time for which a radioactive material remains active, is defined as mean (average) life of that material. It is defined as the sum of lives of all atoms divided by the total number of atoms.

$$\text{i.e.} \quad \tau = \frac{\text{sum of the lives of all the atoms}}{\text{total number of atoms}} = \frac{1}{\lambda}$$

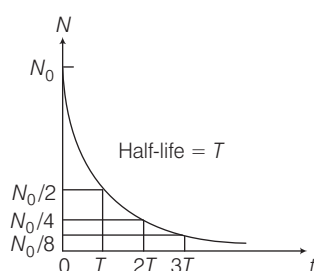
(The average or mean life is the reciprocal of the decay constant)

$$\text{Also, from} \quad T_{1/2} = \frac{0.693}{\lambda}$$

$$\Rightarrow \quad \frac{1}{\lambda} = \tau = \frac{1}{0.693} \cdot (T_{1/2})$$

$$= 1.44 (T_{1/2})$$

Thus, mean life is about 44% more than that of half-life, which gives us $\tau > T_{1/2}$.



Mass Defect and Binding Energy

• The difference in mass of a nucleus and its constituent nucleons is called the mass defect of that nucleus. Thus, mass defect, $\Delta M = Zm_p + (A - Z)m_n - M$ where, M is the mass of a given nucleus.

• **Packing fraction** of an atom is the difference between mass of nucleus and its mass number per nucleon. Thus, packing fraction = $\frac{M - A}{A}$

• The energy equivalent of the mass defect of a nucleus is called its **binding energy**.

Thus, binding energy $\Delta E_b = \Delta M c^2$

$$= [Zm_p + (A - Z)m_n - M]c^2$$

If masses are expressed in atomic mass units,

$$\text{then} \quad \Delta E_b = \Delta M \times 931.5 \text{ MeV}$$

$$= [Zm_p + (A - Z)m_n - M] \times 931.5 \text{ MeV}$$

• Binding energy per nucleon (ΔE_{bn}) is the average energy needed to separate a nucleus into its individual nucleons.

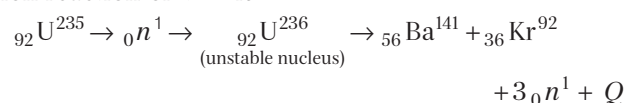
$$\text{Thus, } \Delta E_{bn} = \frac{\Delta E_b}{A}$$

Nuclear Fission and Fusion

Nuclear Fission

The process of a splitting of a heavy nucleus into two lighter nuclei of comparable masses (after bombardment with a energetic particle) with liberation of energy, is called **nuclear fission**.

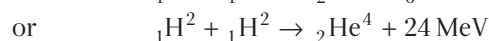
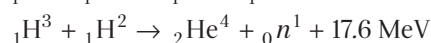
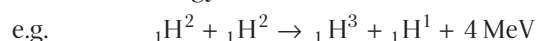
Fission reaction of U^{235} is



The energy released in U^{235} fission is about 200 MeV or 0.8 MeV per nucleon.

Nuclear Fusion

In nuclear fusion, two or more lighter nuclei combine to form a single heavy nucleus. The mass of single nucleus so formed is less than the sum of the masses of parent nuclei. This difference in mass results in the release of tremendous amount of energy.



For fusion, high pressure ($\approx 10^6$ atm) and high temperature (of the order of 10^7 K to 10^8 K) is required and so the reaction is called **thermonuclear fusion**.

Practice Exercise

- The radius of Na^{23} nucleus is
 a. $3.125 \times 10^{-15} \text{ m}$ b. $23 \times 10^{-15} \text{ m}$
 c. $11 \times 10^{-15} \text{ m}$ d. $11 \times 10^{-15} \text{ m}$
- A heavy nucleus (mass number = A) splits into two new nuclei, whose mass numbers are in the ratio 3 : 2. The ratio of radii of these new nuclei is
 a. 3 : 2 b. 2 : 3 c. $3^{1/3} : 2^{1/3}$ d. $2^{1/3} : 3^{1/3}$
- Find the rest mass energy of electron.
 a. 0.8 MeV b. 1.66 amu
 c. 0.5119 MeV d. None of these
- Find the mass of electron in atomic mass unit.
 a. 0.0005498 b. 0.5119 c. 0.5498 d. None
- The atomic mass of Al^{27} is 26.9815 amu. The mass of electron is 0.0005498 amu. The rest mass energy of Al^{27} nucleus is
 a. 1862 MeV b. 25119.78 MeV
 c. 25113.12 MeV d. None of these
- The atomic mass of B^{10} is 10.811 amu. Find the binding energy of B^{10} nucleus. The mass of electron is 0.0005498 amu. The mass of proton is $m_p = 1.007276$ amu. The mass of neutron is $m_n = 1.008665$ amu.
 a. - 678.932 MeV b. 678.932 MeV
 c. 378.932 MeV d. None of these
- Find the binding energy of Na^{23} . Atomic mass of Na^{23} is 22.9898 amu and that of $^1\text{H}_1$ is 1.00783 amu. The mass of neutron = 1.00867 amu.
 a. 931 MeV b. 186.54 MeV
 c. 5.38 MeV d. None of these
- The binding energy per nucleon are 53 MeV, 6.2 MeV and 7.4 MeV for the nucleus with mass numbers, 3, 4 and 5 respectively. If one nucleus of mass number 3 combines with one nucleus of mass number 5 to give two nuclei of mass number 4, then
 a. 0.3 MeV energy is absorbed
 b. 0.3 MeV energy is released
 c. 28.1 MeV energy is absorbed
 d. 3.3 MeV energy is absorbed
- What is the binding energy per nucleon of $_{6}\text{C}^{12}$ nucleus?

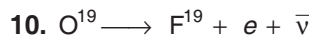
Given, mass of C^{12} (m_c)_m = 12.000 u

Mass of proton (m_p) = 1.0078 u

Mass of neutron (m_n) = 1.0087 u

and 1 amu = 931.4 MeV

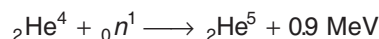
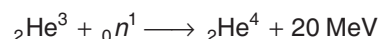
- 5.26 MeV
- 10.11 MeV
- 15.65 MeV
- 7.68 MeV



In this decay, the rest mass energy of O^{19} and F^{19} are 17692.33 MeV and 17687.51 MeV respectively. The Q factor of the decay is

- 4.82 MeV
- 7 MeV
- 17.69 MeV
- None of these

11. The binding energy expressed in MeV is given for the following nuclear reactions



Which of the following conclusion is correct?

- ${}_2\text{He}^4$ is less stable than both ${}_2\text{He}^3$ and ${}_2\text{He}^5$
- ${}_2\text{He}^4$ is less stable than ${}_2\text{He}^3$ but more stable than ${}_2\text{He}^5$
- ${}_2\text{He}^4$ is less stable than ${}_2\text{He}^5$ but more stable than ${}_2\text{He}^3$
- ${}_2\text{He}^4$ is more stable than both ${}_2\text{He}^3$ and ${}_2\text{He}^5$

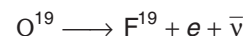
12. Calculate the energy of the reaction,



If the binding energy per nucleon in Li^7 and He^4 nuclei are 5.60 MeV and 7.06 MeV, respectively.

- 19.6 MeV
- 2.4 MeV
- 8.4 MeV
- 17.28 MeV

13. A sample contains 1 kg O^{19} nuclei. The sample decays according to following equation



The mass of sample after one half-life period is

- lesser than 1/2 kg
- equal to 1/2 kg
- slightly less than 1 kg
- equal to 1 kg

14. The number of C^{14} atoms in a sample is 100. The half-life period of C^{14} is 5730 year. The number of C^{14} atoms in the sample after 5730 year.

- must be equal to 50
- must be equal to 100
- may be equal to 90
- must be equal to 90

15. Half-life of an element A is 25 days. After 25 days, three atoms of A become

- 1
- 2
- 3
- all may be

16. The half-life of Tc^{99} is 6 h. The activity of Tc^{99} in a patient, 60 h after receiving an injection containing this radioisotope is at least 0.125 μCi . What was the minimum activity (in μCi) of the sample injected?

- 1.25 μCi
- 12.5 μCi
- 128 μCi
- 125 μCi

17. A radioactive sample has an initial activity of 50 dpm, 20 minute later, the activity is 25 dpm. How many atoms of the radioactive nuclide were there originally?

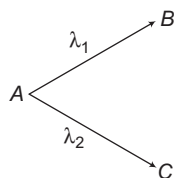
a. 20 b. 1000 c. 1443 d. 2

18. A radioactive sample has a disintegration rate of 36×10^5 disintegrations per minute. The sample itself consisting of 10^{-5} μ mole of the active nuclei. The disintegration constant, λ is given by

a. $6 \times 10^{-7} \text{ s}^{-1}$ b. $6 \times 10^{15} \text{ s}^{-1}$
c. $6 \times 10^9 \text{ s}^{-1}$ d. 10^{-8} s^{-1}

19. A radioactive element undergoes two different types of radioactive disintegration, one with disintegration constant λ_1 and the other with λ_2 . The half-life of the element is

a. $\frac{0.693}{\lambda_1 + \lambda_2}$ b. $\frac{0.693}{(\lambda_1 + \lambda_2)/2}$
c. $0.693 \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$ d. $\frac{0.693 \lambda_1 \lambda_2}{2(\lambda_1 + \lambda_2)}$



20. Half-life period of a given radioactive sample is τ . Its average life would be

a. $\tau \ln 2$ b. $\frac{\tau}{\ln 2}$ c. $\frac{1}{\tau}$ d. $\frac{\ln 2}{\tau}$

21. Choose the correct option, if T_n and T_m denotes the half-value period and the mean-value period, respectively of a radioactive element.

a. $T_n = T_m$ b. $T_n > T_m$ c. $T_n < T_m$ d. $T_n \geq T_m$

22. The half-life of radium is 1600 years. Calculate the number of atoms that will decay from 1 g sample of radium per second (given, atomic weight of radium = 226)

a. 3.6×10^{10} b. 7.2×10^{10} c. 4.2×10^{10} d. 14.6×10^{10}

23. At certain time, the activity of three radioactive materials are in the ratio of 3 : 4 : 5. What will be the mark ratio of their activities at any further date?

a. 1 : 2 : 3 b. 2 : 3 : 4 c. 3 : 4 : 5 d. 5 : 6 : 8

24. 20% of a radioactive substance decay in 10 days. Calculate the amount of the original material left after 30 days.

a. 51.2% b. 62.6% c. 15% d. 21.27%

25. Calculate in how many months, $(3/4)^{\text{th}}$ of the substance will decay, if half-life of the radioactive substance is 2 months.

a. 4 months b. 6 months c. 8 months d. 14 months

26. The half-life of a freshly prepared radioactive sample is 2 hours. If the sample emits radiation of intensity which is 32 times the permissible safe level, then calculate the minimum time taken after which it would be possible to work safely with source.

a. 8 h b. 10 h c. 16 h d. 2 h

27. Calculate the ratio of half-life to the mean life of a radioactive sample. If λ be the decay constant of a radioactive sample.

a. 0.693 b. 0.746
c. $\frac{1}{0.693}$ d. $(0.693)^2$

28. Calculate the number of β -particles, if a radioactive element ${}_{90}\text{X}^{238}$ decays into ${}_{83}\text{Y}^{222}$.

a. 4 b. 6 c. 2 d. 1

29. A radioactive nucleus (initial mass number A and atomic number Z) emits 3α -particles and 2 positrons. the ratio of number of neutrons to that of protons in the final nucleus will be

a. $\frac{A - Z - 8}{Z - 4}$ b. $\frac{A - Z - 4}{Z - 8}$
c. $\frac{A - Z - 12}{Z - 4}$ d. $\frac{A - Z - 4}{Z - 2}$

30. How long will it take for 75% of the atoms of a certain radioactive element, originally present to disintegrate? The half-life of the element is 10 days.

a. 240 days b. 3.6 days
c. 15.6 days d. 4.15 days

31. For measuring the activity of a radioactive sample, a count rate meter is used. At certain observation, count rate meter recorded 5050 counts per minute but after 10 minute later, the count rate showed 2300 counts per minute. Calculate the disintegration constant (λ)

a. 0.065 per min
b. 0.078 per min
c. 0.24 per min
d. 0.868 per min

32. Consider $x \xrightarrow{-\alpha} y \xrightarrow{-\alpha} z$, where half-lives of x and y are z year and one month. The ratio of atoms of x and y when transient equilibrium $[T_{1/2}(x) > T_{1/2}(y)]$ has been established is

a. 1 : 22 b. 1 : 26
c. 26 : 1 d. 23 : 1

33. Analysis of potassium and argon atoms in a moon rock sample by a mass spectrometer shows that the ratio of the number of stable Ar^{40} atoms present to the number of radioactive K^{40} atoms is 7:1. Assume that all the argon were produced by the decay of potassium atoms, with a half-life of 1.25×10^9 year. How old is the rock?

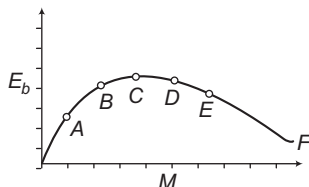
a. 1.25×10^9 yr b. 3.75×10^9 yr
c. 8.75×10^9 yr d. 100×10^{10} yr

34. A radioactive sample decays by two different processes. Half-life for the first process is t_1 and for the second process is t_2 . The effective half-life is

a. $t_1 + t_2$ b. $t_1 - t_2$
c. $(t_1 + t_2)/2$ d. $\frac{t_1 t_2}{t_1 + t_2}$

35.

The alongside is a plot of binding energy per nucleon E_b , against the nuclear mass M , A, B, C, D, E, F correspond to different nuclei. Consider four reactions.



- (i) $A + B \rightarrow C + \varepsilon$ (ii) $C \rightarrow A + B + \varepsilon$
 (iii) $D + E \rightarrow F + \varepsilon$ (iv) $F \rightarrow D + E + \varepsilon$

where, ε is the energy released. In which reactions, is ε positive?

- a. (i) and (iv) b. (i) and (iii) c. (ii) and (iv) d. (ii) and (iii)

36. If ${}_{92}\text{U}^{235}$ reactor takes 30 day to consume 4 kg of fuel and each fission gives 185 MeV of usable energy, then calculate the power output.

- a. 2.75×10^{10} W b. 0.012×10^{10} W
 c. 3.5×10^{10} W d. 7.63×10^{10} W

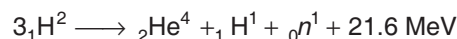
37. Calculate the total energy released during a fission reaction.



The resulting fission fragments are unstable hence, decay into stable end products ${}_{42}^{98}\text{Mo}$ and ${}_{54}^{136}\text{Xe}$ by successive emission of β -particles. Take mass of neutron = 1.0087 amu, mass of ${}_{92}^{235}\text{U}$ = 236.0526 amu, mass of ${}_{42}^{98}\text{Mo}$ = 97.9054 amu and mass of ${}_{54}^{136}\text{Xe}$ = 135.9170 amu.

- a. 198 MeV b. 220 MeV
 c. 185 MeV d. 230 MeV

38. Calculate the energy released per nucleon of the reactant, in the thermonuclear reaction



- a. 21.6 MeV b. 7.2 MeV
 c. 3.6 MeV d. 1.8 MeV

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- Let binding energy per nucleon of nucleus is denoted by E_{bn} and radius of nucleus is denoted as r . If mass number of nuclei A, B are 64 and 125 respectively, then [2014]

a. $r_A < r_B, E_{bnA} < E_{bnB}$ b. $r_A > r_B, E_{bnA} > E_{bnB}$
 c. $r_A = \frac{4}{5} r_B, E_{bnA} < E_{bnB}$ d. $r_A < r_B, E_{bnA} > E_{bnB}$
- Half-lives of elements A and B are 1h and 2 h respectively. Which of the following is correct? [2013]

a. Element A decays slower
 b. Decay constant of A is smaller
 c. If initial number of nuclei are same, then activity of A is more
 d. Mean-life of A is more
- If a radioactive substance reduces to $\frac{1}{16}$ of its original mass in 40 days. Its half-life is [2011]

a. 10 days b. 20 days
 c. 40 days d. None of these
- If the total binding energies of ${}_1\text{H}^2$ and ${}_2\text{He}^4$, ${}_{26}\text{Fe}^{56}$ and ${}_{92}\text{U}^{235}$ nuclei are 2.22, 28.3, 492 and 1786 MeV respectively, identify the most stable nucleus out of the following. [2009]

a. ${}_1\text{H}^2$ b. ${}_2\text{He}^4$
 c. ${}_{26}\text{Fe}^{56}$ d. ${}_{92}\text{U}^{235}$
- Fusion reaction takes place at high temperature because [2006]

a. KE is high enough to overcome repulsion between nuclei
 b. nuclei are most stable at this temperature
 c. nuclei are unstable at this temperature
 d. None of the above
- Two nucleons are at a separation of one fermi. Protons have a charge of $+16 \times 10^{-19}$ C. The net nuclear force between them is F_1 , if both are neutrons, F_2 if both are protons and F_3 if one is proton and the other is neutron. Then, [2005]

a. $F_1 = F_2 > F_3$ b. $F_1 = F_2 = F_3$
 c. $F_1 < F_2 < F_3$ d. $F_1 > F_2 > F_3$
- Mean life of a radioactive sample is 100 s. Then, its half-life (in min) is [2005]

a. 0.693 b. 1
 c. 10^{-4} d. 1.155
- Consider two nuclei of the same radioactive nuclide. One of the nuclei was created in a supernova explosion 5 billion years ago. The probability of decay during the next time is [2005]

a. different for each nuclei
 b. nuclei created in explosion decays first
 c. nuclei created in the reactor decays first
 d. independent of the time of creation

Answer with Solutions

Practice Exercise

1. (a) $R = R_0 A^{1/3} = 1.1 \times 10^{-15} \times (23)^{1/3}$
 $= 3.125 \times 10^{-15} \text{ m}$

2. (c) $\frac{r_1}{r_2} = \frac{r_0 \times (A_1)^{1/3}}{r_0 \times (A_2)^{1/3}} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left\{ \frac{\left(\frac{3}{5} \times A\right)}{\left(\frac{2}{5} \times A\right)} \right\}^{1/3} = \frac{3^{1/3}}{2^{1/3}}$

3. (c) $E = m_e c^2 = 9.1 \times 10^{-31} \times (3 \times 10^8)^2$
 $= 9.1 \times 10^{-31} \times 9 \times 10^{16}$
 $= 81.9 \times 10^{-15} \text{ J}$
 $= \frac{81.9 \times 10^{-15}}{1.6 \times 10^{-19} \times 10^6} \text{ MeV}$
 $= 0.5119 \text{ MeV}$

4. (a) $E = (\text{mass in amu}) \times 931 \text{ MeV}$

$\therefore \text{mass in amu} = \frac{E}{931 \text{ MeV}}$
 $= \frac{0.5119 \text{ MeV}}{931 \text{ MeV}}$
 $= 0.0005498 \text{ amu}$

5. (c) The mass of Al nucleus is

$m_0 = \text{atomic mass} - \text{mass of total electrons}$
 $= 26.9815 - 13 \times 0.0005498$
 $= 26.974353 \text{ amu}$

$\therefore E = m_0 \times 931 \text{ MeV}$
 $E = 26.974353 \times 931 \text{ MeV}$
 $= 25113.12 \text{ MeV}$

6. (a) The mass of B^{10} nucleus

$m_0 = 10.811 - 5 \times 0.0005498$
 $= 10.808251 \text{ amu}$

Binding energy
 $= \{(5m_p + 5m_n) - \text{mass of } B^{10} \text{ nucleus}\} \times 931 \text{ MeV}$
 $= \{(5 \times 1.007276 + 5 \times 1.008665) - 10.808251\} \times 931 \text{ MeV}$
 $= (10.079 - 10.808251) \times 931 \text{ MeV}$
 $= -678.932 \text{ MeV}$

7. (b) The mass of nucleus $= M_{\text{Na}} - 11m_e$

Binding energy
 $= \{11m_p + 12m_n - (M_{\text{Na}} - 11m_e)\} \times 931 \text{ MeV}$
 $= \{11(m_p + m_e) + 12m_n - M_{\text{Na}}\} \times 931 \text{ MeV}$
 $= \{(11M_H + 12m_n - M_{\text{Na}}) \times 931 \text{ MeV}\}$
 $= \{(11 \times 1.00783 + 12 \times 1.00867) - 22.9898\} \times 931 \text{ MeV}$
 $= (11.08613 + 12.10404 - 22.9898) \times 931 \text{ MeV}$
 $= 0.20037 \times 931 \text{ MeV} = 186.54 \text{ MeV}$

8. (d) $X^3 + Y^5 \longrightarrow 2Z^4$

$\Delta E = (3 \times 5.3 + 5 \times 7.4) - 2(4 \times 6.2) = 3.3 \text{ MeV}$

9. (d) Carbon nucleus has 6 protons and 6 neutrons. Total mass of carbon

$(6m_p + 6m_n) = 12.099 \text{ amu}$

and $m_C = 12.000 \text{ amu}$

mass defect $= \Delta m = 0.099 \text{ amu}$

$\frac{\text{Binding energy}}{\text{Nucleons}} = \frac{BE}{A} = \frac{(\Delta m) 931.4}{12}$
 $= \frac{(0.099)(931.4)}{12} = 7.68 \text{ MeV}$

10. (a) 11. (a)

12. (d) Binding energy of Li = 39.20 MeV

Binding energy of $\text{He}^4 = 28.24 \text{ MeV}$

Binding energy of ${}_2\text{He}^4 = 56.48 \text{ MeV}$

$\therefore \text{Energy} = 56.48 - 39.20 = 17.28 \text{ MeV}$

13. (c) 14. (c)

15. (d) After one half-life, out of three atoms of A, 1, 2, or 3 may decay.

16. (c) Number of half-lives, $x = \frac{t}{t_{1/2}} = \frac{60}{6} = 10$

Now, $r = r_0 \left(\frac{1}{2}\right)^x$

$\therefore r_0 + r^{2x} = 0.125 \mu\text{Ci} \times 2^{10} = 128 \mu\text{Ci}$

17. (c) Rate decreases from 50 to 25 dpm in 20 min, hence half-life is 20 min.

Now, $r_0 = \lambda N_0 = \frac{0.693}{t^{1/2}} \times N_0$

or $N_0 = \frac{r_0 \times t_{1/2}}{0.693} = \frac{50 \times 20}{0.693} = 1443$

18. (d)

19. (a) $\lambda_{\text{eff}} = \lambda_1 + \lambda_2$, hence $t_{1/2} = \frac{0.693}{\lambda_1 + \lambda_2}$

20. (b) The relation between half-life ($T_{1/2}$) and mean-life (τ) is

$\tau = 1.44 T_{1/2}$ or $T_{1/2} = \frac{\tau}{\ln 2}$

21. (c) We have, $T_n = \frac{0.693}{\lambda}$

$T_m = \frac{1}{\lambda}$ or $T_n = 0.693 T_m$

$\therefore T_n < T_m$

22. (a) $N_0 = \frac{6.02 \times 10^{23}}{266} = 2.65 \times 10^{21}$

$\lambda = \frac{0.693}{1600 \times 3.16 \times 10^7} = 1.37 \times 10^{-11} \text{ s}^{-1}$

We can assume $N \equiv N_0$ as the half-life is much larger.

$\frac{dN}{dt} = \lambda N_0$

$dN = (1.37 \times 10^{-11})(2.65 \times 10^{21}) \times 1 = 3.6 \times 10^{10}$

23. (c) Activity = $\left| -\frac{dN}{dt} \right| = \lambda N$

$\therefore$ Activity does not depend upon the time.

Hence, the ratio of their activities will be 3 : 4 : 5.

24. (a) Fraction of the substance left

$$= f = 1 - \frac{20}{100} = 0.8$$

Let, $t_1 = 10$ day, $t_2 = 30$ day

fraction, $n = \frac{t_2}{t_1} = \frac{30}{10} = 3$

but $\frac{N}{N_0} = (f)^n = (0.8)^3$

$$\frac{N}{N_0} \times 100 = 0.512 \times 100$$

$$= 51.2\%$$

25. (a)

26. (b) $\frac{1}{32} = \frac{1}{2^x}$

Time to work safely = $5 \times 2 = 10$ h

27. (a) Half-life = $\frac{0.693}{\lambda}$

Average life or mean life = $\frac{1}{\lambda}$

$$\text{Ratio} = \frac{\left(\frac{0.693}{\lambda} \right)}{\left(\frac{1}{\lambda} \right)} = 0.693$$

28. (d) Number of α -particles emitted = $\frac{238 - 222}{4} = 4$

and β^- particle emitted = 1 (as atomic number is 83)

29. (b) 30. (d)

31. (b) $\frac{dM}{dt} = \lambda N_0$

$$\therefore \frac{N_0}{N} = \frac{dN_1/dt}{dN_2/dt} = \frac{5050}{2300} = 2.19$$

Also, $\ln \frac{N_0}{N} = \lambda t$

$\ln(2.19) = \lambda t$

$$\lambda = \frac{\ln(2.19)}{10} = 0.078 \text{ per min}$$

32. (d) $N_2 = \frac{\lambda_1 N_1^0}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t})$

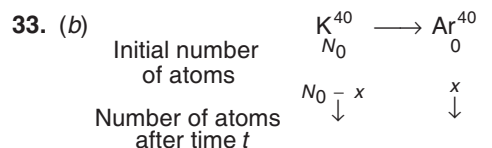
When $(T_{1/2})_1 > (T_{1/2})_2$ at transient equilibrium

$$\lambda_1 < \lambda_2$$

$$e^{-\lambda_2 t} < e^{-\lambda_1 t}$$

$$\therefore N_2 = \frac{\lambda_1 N_1^0 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1} = \frac{\lambda_1 N_1}{\lambda_2 - \lambda_1}$$

$$\therefore \frac{N_1}{N_2} = \frac{\lambda_2 - \lambda_1}{\lambda_1} = \frac{1}{\frac{0.693}{2 \times 12}} = \frac{23}{1}$$



Initial number of atoms N_0

Number of atoms after time t $N_0 - x$

But from question, $\frac{N_0 - x}{x} = \frac{1}{7}$

$$x = \frac{7}{8} N_0$$

$$\therefore (N_0 - x) = \frac{N_0}{8}$$

$$N_0 \rightarrow \frac{N_0}{2} \rightarrow \frac{N_0}{4} \rightarrow \frac{N_0}{8}$$

So, time = $3 \times T_{1/2} = 3.75 \times 10^9$ year

34. (d) As, $\lambda = \lambda_1 + \lambda_2 \Rightarrow \frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2} = \frac{t_2 + t_1}{t_1 t_2}$ or $t = \frac{t_1 t_2}{t_1 + t_2}$

35. (a) First reaction is fusion and 4th reaction is fission.

36. (b) 235 amu of uranium gives energy of 185 MeV.

or $\frac{185}{235} \times 1.6 \times 10^{-13} \text{ J}$

also, $1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg}$

so, energy released by 1 amu or $1.66 \times 10^{-27} \text{ kg}$ of ${}_{92}\text{U}^{235}$

$$= \frac{185 \times 1.6 \times 10^{-13}}{235}$$

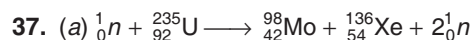
Energy released by 4 kg of ${}_{92}\text{U}^{235}$

$$\Rightarrow W = \frac{185 \times 1.6 \times 10^{-13} \times 4}{1.66 \times 10^{-27} \times 235}$$

$$= 3.035 \times 10^{14} \text{ J}$$

Power of reactor = $\frac{3.035 \times 10^{14}}{30 \times 24 \times 60 \times 60}$

$$= 0.012 \times 10^{10} \text{ W}$$



$$(10087 + 235.0439) = (97.9054 + 135.917 + 2.0174)$$

$$\Delta m = 0.2128$$

$\therefore$ Total energy released during a fission reaction

$$= 0.2128 \times 931 \text{ MeV} = 198 \text{ MeV}$$

38. (c)

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1. (d) $r = r_0 (A)^{1/3}$

r increases with increasing A mass number

So, $r_A < r_B$ as mass number of A is smaller E_{bn} decreases with increasing A for $A > 56$. ^{56}Fe has highest E_{bn} value.

So, E_{bn} for $A = 64$ is larger as compared to E_{bn} for nucleus with $A = 125$

$$E_{bnA} > E_{bnB}$$

2. (c) Let initial number of nuclei of each element = N_0

$$\text{Decay constants, } \lambda_A = \frac{0.693}{1} \text{ h}^{-1}, \lambda_B = \frac{0.693}{2} \text{ h}^{-2}$$

$$\lambda_A > \lambda_B$$

$$\text{Activities, } R_A = \lambda_A N_0$$

$$R_B = \lambda_B N_0$$

$$\Rightarrow R_A > R_B \text{ as } \lambda_A > \lambda_B$$

Less half-life of element A implies faster decay.

$$\text{Mean-life, } \tau = \frac{1}{\lambda}$$

$$\tau_A = \frac{1}{\lambda_A}, \tau_B = \frac{1}{\lambda_B}$$

$$\tau_A < \tau_B \text{ as } \lambda_A > \lambda_B$$

3. (a) From the formula,

$$N = N_0 \left(\frac{1}{2} \right)^n$$

$$\frac{N}{16} = N_0 \left(\frac{1}{2} \right)^n$$

N_0 = original number of atom

$$\left(\frac{1}{2} \right)^4 = \left(\frac{1}{2} \right)^n$$

$\Rightarrow$

$$n = 4$$

4 half-lives

$\therefore$

$$4T_{1/2} = 40$$

$$T_{1/2} = \frac{40}{4} = 10 \text{ days}$$

4. (c) Binding energy per nucleon for $^1\text{H}^2$ and $^2\text{He}^4$, $^{56}_{26}\text{Fe}$ and $^{235}_{92}\text{U}$ are respectively,

$$\frac{2.22}{2}, \frac{28.3}{4}, \frac{492}{56}, \frac{1786}{235}$$

Out of them all $\frac{492}{56}$ is the greatest. Therefore, $^{56}_{26}\text{Fe}$ is the most bound or most stable nucleus.

5. (a) Fusion reaction takes place at high temperature because KE is high enough to overcome repulsion between nuclei.
6. (b) Nuclear forces are charge independent

$$\text{So, } F_1 = F_2 = F_3$$

7. (d) Half-life, $T/2 = \frac{T}{1.44} = \frac{100}{1.44} \text{ s}$

$$= 69.44 \text{ s}$$

$$= \frac{69.44}{100} \approx 1115 \text{ min}$$

8. (d) Radioactive decay does not depend upon the time of creation.

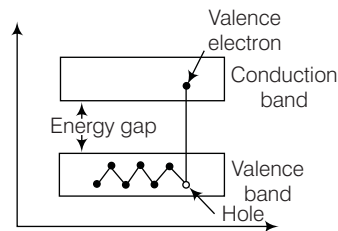
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Semiconductor Devices and Logic Gates

Introduction

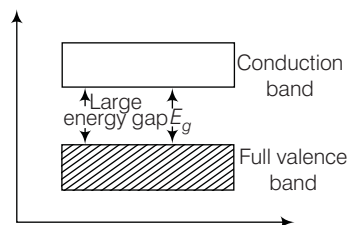
Energy bands When covalent bond is broken down by valence electron, hole is created in valence band and valence electron goes to conduction band.

Conduction band has nothing to do with holes. Conduction electrons are found with free flow in conduction band. Holes exist and flow in valence band. Conduction electrons move twice as fast as the holes.



Insulators Here, valence electrons are bound very tightly to their parent atom and require a large energy to remove them.

- Insulators have a full valence band.
- Conduction band is empty.
- It has a large energy gap (E_g), i.e. 6 eV and above.



Conductors In the conductors, there are plenty of free electrons for conduction, here valence and conduction bands overlap each other.

- Energy gap is absent in conductors.
- No hole is created.
- The flow of current is the flow of electrons.

Semiconductors

Semiconductors are materials whose conductivity and resistivity lie in between metals and insulators.

Its resistivity is $\rho = 10^{-5}$ to $10^6 \Omega \text{ m}^{-1}$ and conductivity, $\sigma = 10^{-5}$ to 10^{-6} Sm^{-1} .

Band gap structure of a semiconductor at $T = 0 \text{ K}$, resembles that of an insulator but with a lower energy gap ($E_g < 3 \text{ eV}$).

For silicon, $E_g \approx 1.1 \text{ eV}$

For germanium, $E_g \approx 0.7 \text{ eV}$

At temperature higher than $T > 0 \text{ K}$, some of the electrons are excited from valence band to conduction band creating an equal number of holes (vacancy for electrons) in valence band.

An increase in temperature results in creation of more electron hole (intrinsic pairs), resulting in an increase in conductivity of the semiconductor.

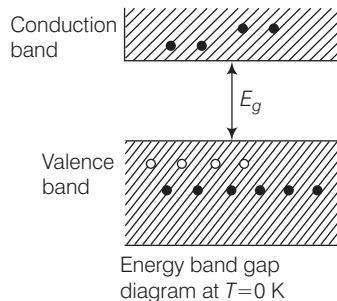
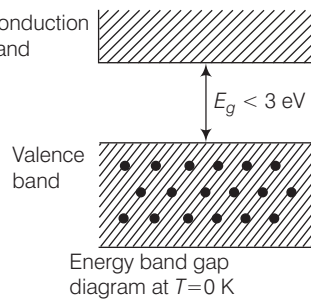
Si and Ge are the good examples of semiconductors. Other than these, we have inorganic semiconductors such as CdS, GaAs, CdSe, organic semiconductors such as anthracene and doped semiconductors such as phalocyanines. For a pure (intrinsic) semiconductor, the number of free electrons in conduction band is equals to number of holes in valence band, i.e. $n_e = n_h = n_i$, where n_i = number of intrinsic pairs.

The highest energy level in the conduction band occupied by electrons at absolute zero temperature is called **fermi level** and the energy corresponding to the fermi level is called **fermi energy** (E_f).

Types of Semiconductors

Semiconductors are of two types

- Intrinsic semiconductor
- Extrinsic semiconductor



- Intrinsic semiconductor** The pure semiconductor is called the intrinsic semiconductor. The conductivity of intrinsic semiconductor is because of their own internal charge carriers. At room temperature, few covalent bonds break because of thermal collision.

For each broken bond, a pair of a free electron in conduction band and a hole in valence band is created. So, intrinsic semiconductor has an equal number of holes and electrons. Therefore, in an intrinsic semiconductor, the conductivity is due to holes and electrons both, which increases with rise of temperature. They are also called **carriers**. Holes move in valence band parallel to the direction of external field. Electrons move in anti-parallel direction to the external field.

- Extrinsic semiconductor** When a small quantity of impurity is mixed in a pure or intrinsic semiconductor, the conductivity of semiconductor increases. Such an **impure semiconductor** is called **extrinsic semiconductor**. The conductivity of resultant crystal depends on the nature and quantity of the impurity added called **dopant**.

In an extrinsic semiconductor, if n_e is the number density of electrons in conduction band and the density of holes in valence band is n_h in a pure semiconductor, then

$$n_e n_h = n_i^2$$

Depending upon the nature of impurity added in intrinsic semiconductor, the extrinsic semiconductors are of two types

- n -type semiconductor or donor and
- p -type semiconductor or acceptor

n -type Semiconductor

To prepare an n -type semiconductor a pentavalent impurity, e.g. P, As, Sb are used as a dopant with Si or Ge. Such an impurity is called donor impurity because each dopant atom provides one free electron.

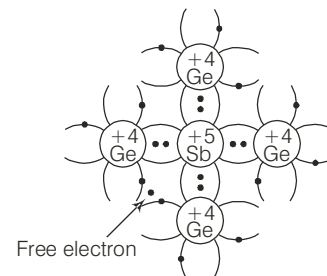
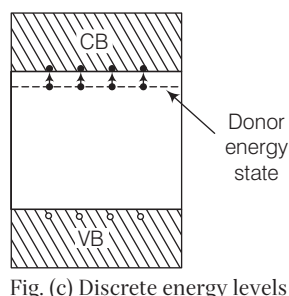
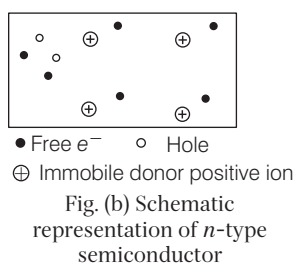


Fig. (a) Formation of n -type semiconductor

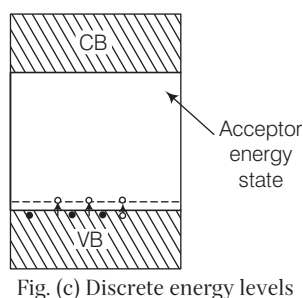
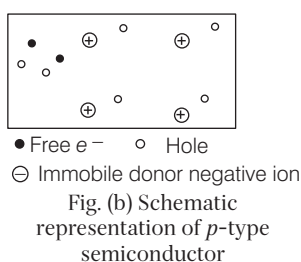
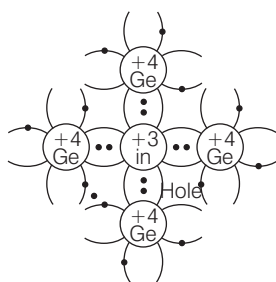
In n -type semiconductor $n_e \gg n_h$, i.e. electrons are majority charge carriers and the holes are minority charge carriers, such that $n_e \cdot n_h = n_i^2$.

An *n*-type semiconductor is electrically neutral and is not negatively charged.



p-type Semiconductor

To prepare a *p*-type semiconductor a trivalent impurity, e.g. B, Al, In, Ga, etc., are used as a dopant with Si or Ge. Such an impurity is called **acceptor impurity** as each impurity atom wants to accept an electron from the crystal lattice. Thus, effectively each dopant atom provides a **hole**. In *p*-type semiconductor $n_h \gg n_e$, i.e. holes are majority charge carriers and electrons are minority charge carriers such that $n_h \cdot n_e = n_i^2$. A *p*-type semiconductor is electrically neutral and is not positively charged. The number of free electrons in a semiconductor varies with temperature as $T^{3/2}$.



Mobility of a Charged Carrier

Electrical mobility The drift velocity acquired by a charged carrier in a unit electric field is called its electrical mobility and is denoted by μ .

In a semiconductor,

Drift velocity of a charged carrier $\propto$ Applied electric field
i.e. $v \propto E$ or $v = \mu E$

$$\therefore \mu = \frac{v}{E}$$

Hence, the electrical mobility (μ) is the drift velocity per unit electric field. Its SI unit is $[\text{m}^2 \text{V}^{-1} \text{s}^{-1}]$.

Conductivity and Resistivity of Semiconductor

(i) **Resistivity** According to Ohm's law, at constant temperature $V \propto I$

$$\text{i.e. } V = IR \quad R = \frac{\rho I}{A}$$

$\Rightarrow$ (where, ρ is resistivity)

$$\therefore \rho = \frac{AR}{I}$$

So, the SI unit of resistivity ρ is $\Omega\text{-m}$.

(ii) The conductivity which is reciprocal of resistivity is given by

$$\sigma = \frac{1}{\rho} \quad \dots(i)$$

$$\sigma = e(n_e \mu_e + n_h \mu_h)$$

So, the SI unit of conductivity σ is Sm^{-1} .

From Eq. (i), the resistivity of the semiconductor is also given by

$$\rho = \frac{1}{e(n_e \mu_e + n_h \mu_h)}$$

Effect of Temperature on the Conductivity of Semiconductors

The conductivity of a semiconductor is given by

$$\sigma = e(n_e \mu_e + n_h \mu_h)$$

As the temperature increases, the mobilities μ_e and μ_h of electrons and holes decreases due to the increase in their collision frequency. But due to the small energy gap of semiconductors, more and more electrons ($n \propto e^{-E_g/2kT}$) from the valence band cross over to the conduction band. The increase in carrier concentration, n_e and n_h is so large that the decrease in the values of μ_e and μ_h has no influence.

In any semiconductor due to thermal effect, generation of free electrons and holes take place. Apart from the process of generation, a process of recombination also occurs simultaneously, in which free electron further recombine with hole. At equilibrium, rate of generation of charge carrier is equal to the rate of recombination of charge carriers.

Larger the value of n_e or n_h , higher is the probability of their recombination.

At equilibrium, $n_e \times n_h = n_i^2$

Semiconductor Diode

A *p-n* junction is obtained by joining a small *p*-type crystal with a small *n*-type crystal without employing any other binding material in between them.

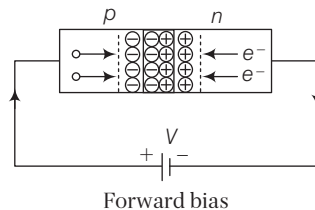
Whenever a p - n junction is formed, electrons from n -region diffuse through the junction into p -region and the holes from p -region diffuse into n -region.

As a result of which neutrality of both n and p -regions is disturbed and a thin layer of immobile negative charged ions appear near the junction in the p -crystal and a layer of positive ions appear near the junction in n -crystal. This layer containing immobile ions is called **depletion layer**. The thickness of depletion layer is approximately of the order of 10^{-6} m.

The potential difference developed across the p - n junction due to diffusion of electrons and holes is called the **potential barrier** V_b (or emf of fictitious battery). For Germanium diode barrier potential is 0.3 V but for Si diode its value is 0.7 V. The barrier electric field developed due to it is of the order of 10^5 Vm $^{-1}$.

Forward Bias

In forward biasing, a positive potential is applied to p -side and negative potential is applied to n -side.



In forward biasing, in a p - n junction, the following changes occur.

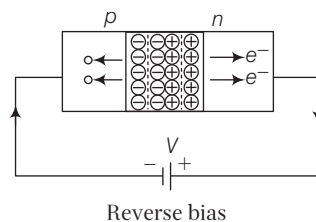
- (i) Potential barrier reduces.
- (ii) Width of depletion layer decreases.
- (iii) p - n junction provides very small resistance.
- (iv) Forward current flows in circuit. (p to n)
- (v) Order of forward current is in mA.
- (vi) Forward current is due to the flow of majority charge carriers.

Reverse Bias

In reverse biasing, a positive potential is applied to n -side and negative potential to p -side.

Due to reverse biasing, the following changes occur.

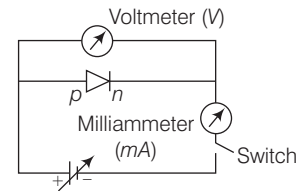
- (i) Potential barrier increases.
- (ii) Width of depletion layer increases.
- (iii) p - n junction provides high resistance.
- (iv) Very small reverse current flows. (n to p)
- (v) Order of reverse current is in μ A.
- (vi) Reverse current is due to the flow of minority charge carriers.



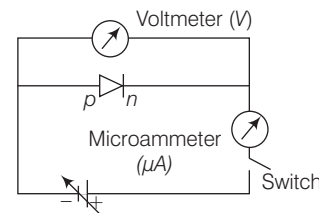
I - V Characteristics of Forward and Reverse Bias Diode

The following circuit diagrams show the arrangement for studying V - I characteristics of a p - n junction diode.

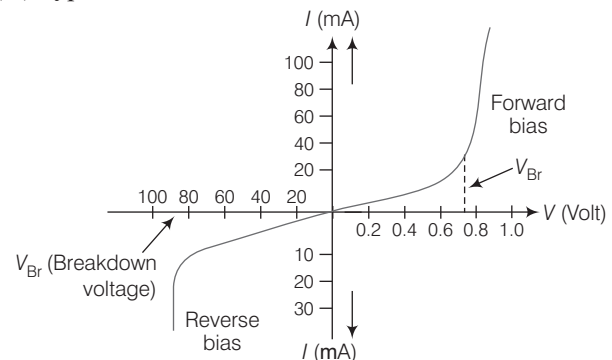
(i) Forward bias



(ii) Reverse bias



(iii) Typical V - I characteristics of a silicon diode



Applications of p - n Junction Diode

Diode as a Rectifier

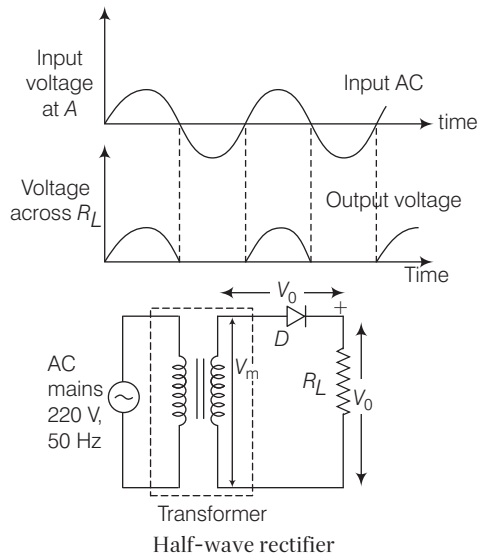
Junction diode allows current to pass only when it is forward biased. This property is used to rectify alternating voltages and the circuit used for this purpose is called as **rectifier** and this process of converting AC to DC is known as **rectification**. Junction diodes are also used to modify waveform.

Half-Wave Rectifier

In half-wave rectifier, a p - n junction is used with a transformer.

In half-wave rectifier of the single phase supply, either the positive or negative half of the AC wave is passed while the other half is blocked. Because, only one-half of the input waveform reaches the output, mean voltage is lower.

Half-wave rectifier requires a single diode in a single phase supply. Half-wave rectifier produces more ripple than full-wave rectifier, and much more filtering is needed to eliminate harmonics of the AC frequency from the output.



During Positive Half Cycle

Diode (D) $\rightarrow$ Forward biased $\rightarrow$ On switch $\rightarrow$ Output due to diode D received.

During Negative Half Cycle

$D \rightarrow$ Reverse biased $\rightarrow$ Off switch $\rightarrow$ No output due to diode D received.

Richardson-Dushman Equation

$$J = At^2 e^{\phi / kT}$$

shows relationship between current density J and absolute temperature T .

where, ϕ = work function of emitting surface in eV.

k = Boltzmann constant.

A = constant = $60 \text{ A/cm}^2 \text{ K}$ for pure metals

T = temperature in K.

Child Langmuir Law For a space charge limited current.

$$I_p \propto (V_p)^{3/2}$$

The no-load output DC voltage of an ideal half-wave rectifier is

$$V_{\text{rms}} = \frac{V_{\text{peak}}}{2}, V_{\text{DC}} = \frac{V_{\text{peak}}}{\pi}$$

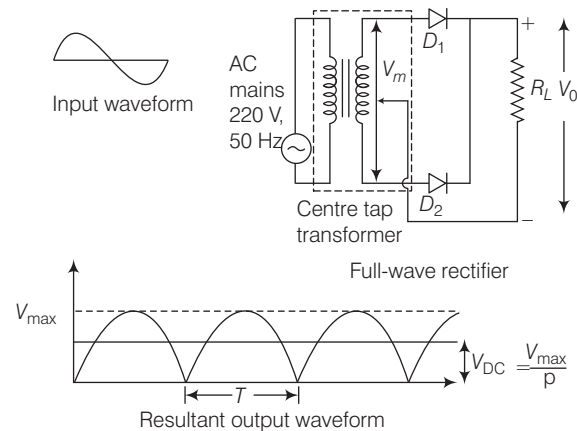
where, $V_{\text{DC}}, V_{\text{av}}$ = DC or average output voltage

V_{peak} = peak value of the input voltage

V_{rms} = root mean square value of output voltage.

Full-Wave Rectifier

In full-wave rectifier two diodes are used, one for each half of the cycle with a centre tapped transformer. Output frequency of a full-wave rectifier is twice that of input.



Full-wave rectifier have some fundamental advantages over their half-wave rectifier counterparts. The average (DC) output voltage is higher than for half-wave, the output of the full-wave rectifier has much less ripple than that of the half-wave rectifier producing a smoother output waveform.

Since, this rectifier produces an output during both half cycles, so it is 100% efficient.

Here,

$$V_{\text{DC}} = \frac{2V_{\text{max}}}{\pi} = 0.637V_{\text{max}}$$

During Positive Half Cycle

$D_1 \rightarrow$ Forward biased $\rightarrow$ On switch $\rightarrow$ Output due to diode D_1

$D_2 \rightarrow$ Reverse biased $\rightarrow$ Off switch $\rightarrow$ No output due to diode D_1

During Negative Half Cycle

$D_1 \rightarrow$ Reverse biased $\rightarrow$ Off switch $\rightarrow$ No output due to diode D_1

$D_2 \rightarrow$ Forward biased $\rightarrow$ On switch $\rightarrow$ Output due to diode D_2

Ripple and Ripple Factor

In the output of rectifier, some AC components are present. They are called **ripple** and their measurement is given by a factor, so it is called **ripple factor**. For good rectifier, the ripple factor must be very low.

(i) For half-wave rectifier ripple factor, $r = 1.21$

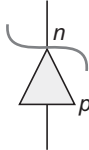
(ii) For full-wave or bridge wave rectifier ripple factor,
 $r = 0.48$

Special Purpose Diodes

The special purpose diode that performs many different functions, e.g. diodes are used to regulate voltage (Zener diode), to produce light, Light Emitting Diode (LED) are given below.

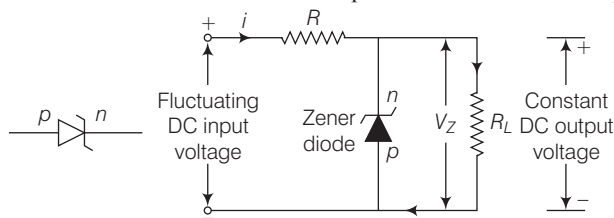
Zener Diode

It is a highly doped p - n junction diode which is not damaged by high reverse current. It is always used in reverse bias in break down voltage region and is chiefly used as a voltage regulator.



Zener Diode as a Voltage Regulator

The following circuit is used for stabilizing voltage across a load R_L . The circuit consists of a series voltage-dropping resistance R and a Zener diode in parallel with the load R_L .



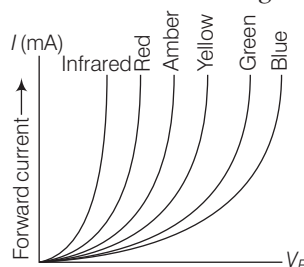
The Zener diode is selected with Zener voltage V_Z equal to the voltage desired across the load. The fluctuating DC input voltage may be the DC output of a rectifier. Whenever the input voltage increases, the excess voltage appears as increased voltage across the resistance R . This causes an increase in the input current i . This increase is taken away by the Zener diode while the current through the load and hence the voltage across it remains constant at V_Z . Likewise a decrease in the input voltage causes a decrease in the input current i . The current through the diode decreases correspondingly, again maintaining the load current.

Light Emitting Diode (LED)

It is a specially designed diode made of GaAsP, GaP, etc. When used in forward biased, it emits characteristic, almost monochromatic light. In reverse biased, it works like a normal diode.

I - V Characteristics of LED

LEDs are current dependent devices with its forward voltage drop (V_F) depending on the forward biased LED current. I - V characteristics of LED is given below



Before LED can emit any form of light it needs a current to flow through it, as it is a current dependent device with their light output intensity being directly proportional to the forward current flowing through the LED.

Photodiode

It is a special diode used in reverse bias which conducts only when light of suitable wavelengths is incident on the junction of diode. The energy of incident light photon must be greater than the band gap of semiconductor. Materials used are Cds, Se, Zns.

Solar Cell

It is a special p - n junction in which one of the semiconductors is made extremely thin so, that solar radiation falling on it reaches junction of diode without any absorption. A solar cell directly converts solar energy into electric energy. Popularly used solar cells, are Ni-cd, PbS cell.

Junction Transistor

Transistor is a three terminal device which is formed, when thin layer of one type of extrinsic semiconductor p or n -type is sandwiched between two thick layers of other two types of n -type or two types of p -type layers extrinsic semiconductor. Each transistor have three terminals which are as follows

- (i) **Emitter** It is the left most part of the transistor. It emits the majority carriers towards the base. It is highly doped and medium in size.
- (ii) **Base** It is the middle part of transistor which is sandwiched by emitter (E) and collector (C). It is lightly doped and very thin in size.
- (iii) **Collector** It is right part of the transistor which collect the majority carrier through base which is emitted by emitter. It has large size and moderately doped.

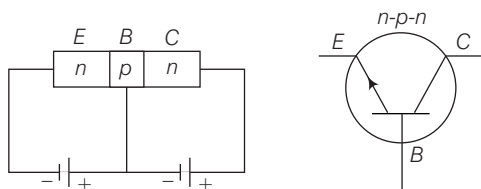
There are two semiconductor junctions, which are defined as below.

- (a) The junction between emitter and base is known as **emitter-base junction** (J_{EB}).
- (b) The junction between base and collector is known as **base-collector junction** (J_{CB}).

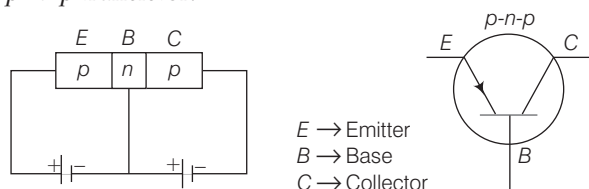
Types of Transistors

Transistors are of two types

- (i) **n - p - n transistor** If a thin layer of p -type semiconductor is sandwiched between two thick layers of n -type semiconductors, then it is known as n - p - n transistor.



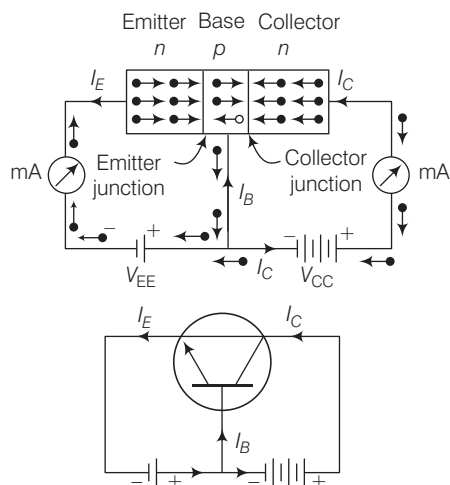
- (ii) **p-n-p transistor** If a thin layer of *n*-type semiconductor is sandwiched between two thick layers of *p*-type semiconductor, then it is known as *p-n-p* transistor.



Transistor Action

The action of both types of transistors, i.e. *n-p-n* and *p-n-p* is similar except that the majority and minority charge carriers in the two cases are of opposite nature.

- (i) **Working of *n-p-n* transistor** The emitter-base junction is forward biased and collector-base junction is kept reversed biased for *n-p-n* transistor in a circuit. Its symbolic representation is shown in given figure.



When the emitter-base junction is forward biased, electrons (majority charge carriers) in emitter are repelled towards base. The barrier of emitter-base junction is reduced and the electrons enter the base. About 5% of these electrons recombine with holes in base region resulting in small current (I_B). The remaining electrons ($\approx 95\%$) enter the collector region, because they are attracted towards the positive terminal of battery (V_{CC}).

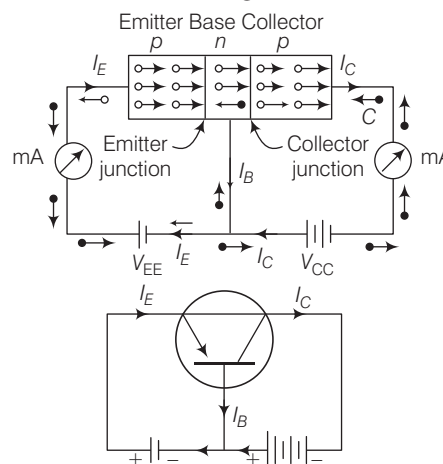
For each electron entering the positive terminal of the battery connected with collector-base junction, an electron from the negative terminal of the battery

connected with emitter-base junction enters the region. The emitter current (I_E) is approximately equal to the collector current (I_C). The base current is the difference between I_E and I_C .

$$\text{i.e.} \quad I_E = I_B + I_C$$

It may be pointed out that the arrows point in the direction of conventional current or hole current in spite of the fact that in the *n-p-n* transistor, the current is carried by electrons.

- (ii) **Working of *p-n-p* transistor** When emitter-base junction is forward biased, holes (majority charge carriers) in the emitter are repelled towards the base and diffuse through the emitter-base junction. The barrier potential of emitter-base junction decreases and holes enter the *n*-region (i.e. base).



A small number of holes ($\approx 5\%$) combines with electrons of *n*-region resulting in small current (I_B). The remaining holes ($\approx 95\%$) enter into the collector region, because they are attracted towards negative terminal of the battery connected with the collector-base junction. These holes constitute the collector current (I_C).

As one hole reaches the collector, it is neutralised by the battery. As soon as one electron and a hole neutralised in collector, a covalent bond is broken in emitter region. As a result, electron-hole pair is produced. The released electron enter the positive terminal of battery (V_{EE}).

Characteristics of a Transistor

In common emitter configuration, the variation of current on the input side with input voltage (I_E versus V_{BE}) is known as the **input characteristics** and the variation in the output current with output voltage (I_C versus V_{CE}) is known as **output characteristics**. From these characteristics, we obtain the values of following parameters.

- Input resistance

$$r_i = \left. \frac{\Delta V_{BE}}{\Delta I_B} \right|_{V_{CE} = \text{constant}}$$

- Output resistance

$$r_o = \left| \frac{\Delta V_{CE}}{\Delta I_C} \right|_{I_B = \text{constant}}$$

- AC current gain

$$\beta = \left| \frac{\Delta I_C}{\Delta I_B} \right|_{V_{CE} = \text{constant}}$$

The current gain for common-emitter configuration β ranges from 20 to 200.

- Transconductance

$$g_m = \frac{\Delta I_C}{\Delta V_{BE}} = \frac{\beta}{r_i}$$

A transistor can be used as an amplifier. The voltage gain of an amplifier will be given by

$$A_V = \frac{V_o}{V_i} = \beta \cdot \frac{R_C}{R_B}$$

where, R_C and R_B are net resistances in collector and base circuits respectively.

In common base configuration, AC current gain is defined as

$$\alpha = \left| \frac{\Delta I_C}{\Delta I_E} \right|_{V_{CE} = \text{constant}}$$

The value of α is slightly less than 1. In fact,
 $0.95 \leq \alpha \leq 1$.

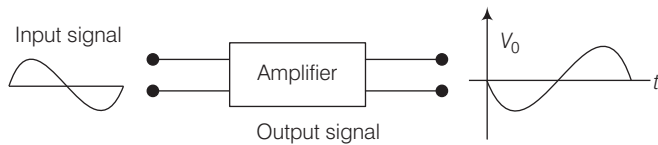
Current gains α and β are correlated as

$$\beta = \frac{\alpha}{1 - \alpha} \quad \text{or} \quad \alpha = \frac{\beta}{1 + \beta}$$

Transistor as an Amplifier

The process of increasing the amplitude of input signal without distorting its wave shape and without changing its frequency is known as **amplification**.

The amplifier is a device which amplifies the power of varying current or alternating current.



As an amplifier, a transistor can be used in the following three configurations.

- Common base amplifier
- Common emitter amplifier
- Common collector amplifier

Common emitter amplifier The given circuit diagram shows an $n-p-n$ transistor which has been used in common-emitter mode. The input and output waveforms are shown along with the circuit.

Current gain in CE configuration, at constant collector to emitter voltage ratio of change in collector current the change in base current. It is denoted by β .

$\therefore$

$$\beta = \left(\frac{\Delta i_C}{\Delta i_B} \right)_{V_{CE} = \text{constant}}$$

The value of β is always greater than 1.

Voltage gain The ratio of change in output voltage to the change in input voltage is called voltage gain. It is denoted by A_V .

$$\text{Voltage gain, } A_V = \frac{\Delta V_C}{\Delta V_B} = \frac{\Delta i_C}{\Delta i_B} \times \frac{R_{out}}{R_{in}}$$

$$\Rightarrow A_V = \beta \times \frac{R_{out}}{R_{in}}$$

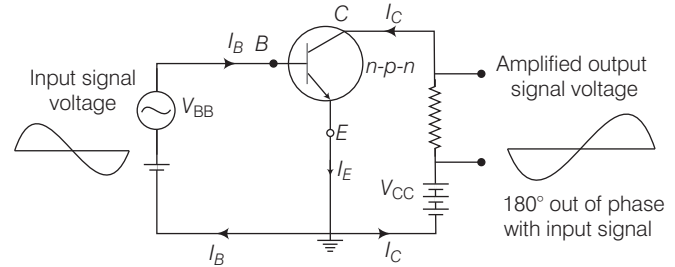
Power gain The ratio of change in output power to the change in input power is called power gain.

$$\therefore \text{Power gain} = \frac{P_{out}}{P_{in}}$$

$$\text{Also, power gain} = \beta^2 \times \frac{R_{out}}{R_{in}}$$

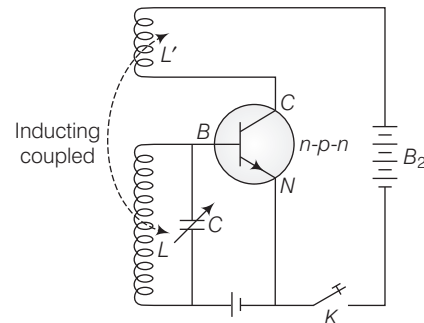
The relation between α and β is given by

$$\beta = \frac{\alpha}{1 - \alpha} \quad \text{or} \quad \alpha = \frac{\beta}{1 + \beta}$$



Transistor as an Oscillator

An electronic oscillator is a device that generates electrical oscillations of constant amplitude and of a desired frequency without any external input. The circuit providing such oscillation is known as a tank and oscillator using positive feedback.



Some of the properties of the oscillator are

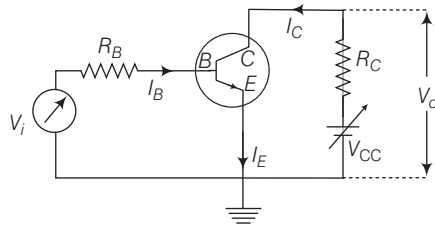
1. Oscillator is using positive feedback.
2. To work as an oscillator

$$|\beta| = 1, \beta \rightarrow \text{feedback factor}$$

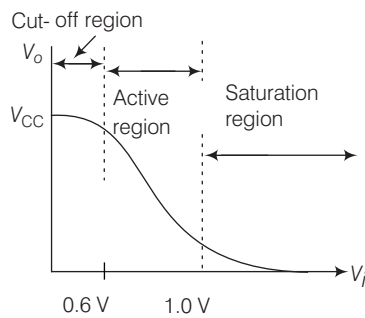
$$3. f = \text{frequency of oscillation} = \frac{1}{2\pi} \times \frac{1}{\sqrt{LC}}$$

Transistor as a Switch

Consider CE configuration of a transistor is shown as below in figure.



Applying Kirchhoff's voltage rule to the input sides of this circuit, we get



$$V_i = I_B \cdot R_B + V_{BE} \quad [V_i = \text{DC input voltage}]$$

and $V_o = V_{CC} - I_C \cdot R_C \quad [V_o = \text{DC output voltage}]$

Now, we can analyse that how V_o changes as V_i increases from zero onwards. In case of silicon transistor, if V_i is less than 0.6 V, I_B will be zero, hence, I_C will be zero and transistor will be said to be in cut-off state and $V_o = V_{CC}$.

When V_i becomes greater than 0.6 V, some I_B flows, so some I_C flows (transistor is in active state now) and output V_o decreases as the term $I_C \cdot R_C$ increases. With increase in V_i , the I_C increases almost linearly and so, V_o decreases linearly till its value becomes less than about 1.0 V.

Logic Gates

A logic gate is a digital electronic circuit which follows a logical relationship between its input and output. A logic gate may have one or more inputs but has only one output. Logic gates follow Boolean algebra, which consists of three basic operations, namely AND ($A \cdot B = Y$), OR ($A + B = Y$) and NOT ($\bar{A} = Y$).

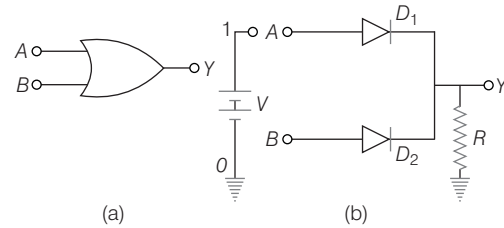
Three Basic Logic Gates

On the basis of types of operations, i.e. OR, AND, NOT etc., there are three basic logic gates. These gates with their detailed descriptions are given below.

OR Gate

It has two inputs, and one output.

Logic symbol of OR gate is shown in Fig.(a). Its Boolean expression is $Y = A + B$ (being read as Y equals A or B).



An OR gate can be realised by using two $p-n$ junction diodes as shown in Fig. (b).

Truth table of OR gate is

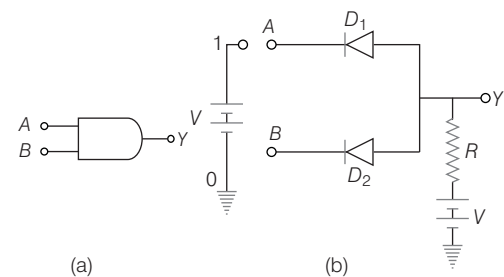
A	B	Y
0	0	0
1	0	1
0	1	1
1	1	1

AND Gate

It also has two inputs and one output.

Boolean expression of AND gate is $Y = A \cdot B$ (being read as Y equals A and B). Truth table of AND gate is

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1



Logic symbol of AND gate is shown in Fig. (a) and the actual circuit arrangement used to realise AND gate in practice has been shown in Fig. (b).

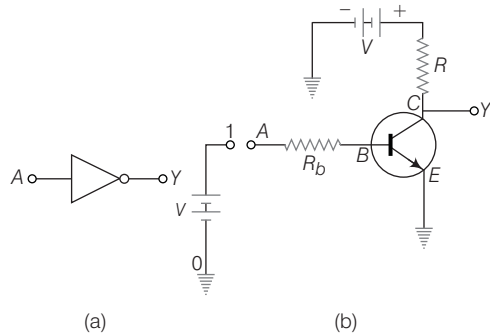
NOT Gate

It has one input and one output.

Boolean expression of NOT gate $\bar{A} = Y$ (being read as A equals Y).

Truth table of the NOT gate is

A	Y
0	1
1	0



The logic symbol and actual circuit for realisation of NOT gate have been shown in above figure.

- | | |
|---|--|
| (i) $A + 0 = A$ | (ii) $A + A = A$ |
| (iii) $A + 1 = 1$ | (iv) $A + \bar{A} = 1$ |
| (v) $A \cdot 0 = 0$ | (vi) $A \cdot A = A$ |
| (vii) $A \cdot 1 = A$ | (viii) $A \cdot \bar{A} = 0$ |
| (ix) $\overline{A + B} = \bar{A} \cdot \bar{B}$ | (x) $\overline{A \cdot B} = \bar{A} + \bar{B}$ |

In the above expressions, A and B are to be assigned the values 0 or 1.

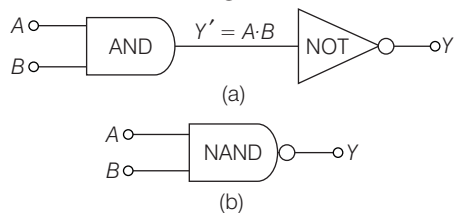
Here, if 0 implies low, then 1 implies high. Similarly, if 0 implies off, then 1 implies on. Also, if 0 stands for down, then 1 stand for up *vice-versa* may also be true. It may be kept in mind that in digital electronics, 0 and 1 represent two states and not two values.

Combination of Logic Gates

Some common combinations obtained from three fundamental gates OR, AND and NOT are as follows

NAND Gate

If the output of AND gate is joined to input of a NOT gate, the combination is a NAND gate.

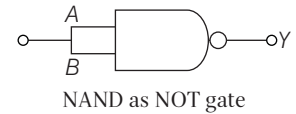


Boolean expression of NAND gate is $\overline{A \cdot B} = Y$. Its logic symbol is shown in above figure.

Truth table of NAND gate is

A	B	Y
0	0	1
1	0	1
0	1	1
1	1	0

If both the inputs of a NAND gate are joined together, it will behave as a NOT gate as shown in figure.

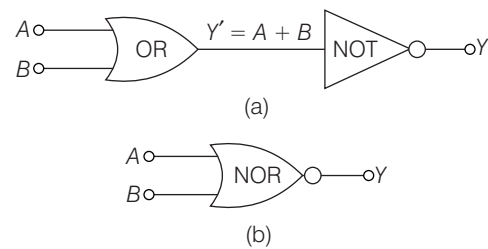


Two/Three NAND gates may be joined so as to have the same effect as a single AND gate or OR gate.

NOR Gate

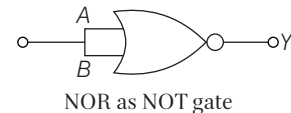
If the output of OR gate is joined to input of a NOT gate, the combination is a NOR gate.

Boolean expression of NOR gate is $\overline{A + B} = Y$. Its logic symbol is shown in figure.



Truth table of NOR gate is

A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

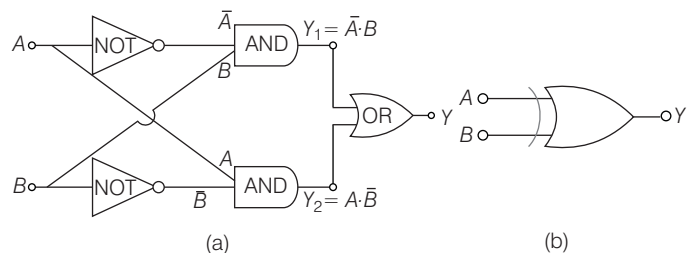


If both inputs of a NOR gate are joined together, it will behave as a NOT gate as shown in figure. Two/ Three NOR gates may be combined so as to have the same effect as a single OR/AND gate.

NAND and NOR gates are known as universal gate.

XOR Gate

The logic gate which gives high output (1) when either input A or input B , but not both of them, are high is called exclusive OR gate or the XOR gate. If both inputs A and B are either high (1) or low (0), then the output is zero.



XOR gate can be obtained by having a combination of two NOT gates, two AND gates and one OR gate as shown in Fig. (a). The logic symbol is shown in Fig. (b).

Boolean expression of XOR gate is

$$Y = A \oplus B = \bar{A} \cdot B + A \cdot \bar{B}$$

Truth table of XOR gate is

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	0

The Boolean expressions obey the commutative law, associative law as well as distributive law.

Commutative law

$$(i) A + B = B + A \quad (ii) A \cdot B = B \cdot A$$

Associative law

$$(iii) A + (B + C) = (A + B) + C$$

$$(iv) (A \cdot B) \cdot C = A \cdot (B \cdot C)$$

Distributive law

$$(v) A \cdot (B + C) = A \cdot B + A \cdot C$$

$$(vi) A + \bar{A} \cdot B = A + B \quad (vii) A + A \cdot B = A$$

$$(viii) A \cdot (A + B) = A \quad (ix) A \cdot (\bar{A} + B) = A \cdot B$$

$$(x) \bar{A} \cdot \bar{B} = \overline{A + B}$$

Practice Exercise

- If the resistivity of copper is $1.7 \times 10^{-6} \Omega\text{-m}$, then the mobility of electrons in copper, if each atom of copper contributes one free electron for conduction is [the atomic weight of copper is 63.54 and density is 8.96 g/cc]
 - $23.36 \text{ cm}^2/\text{Vs}$
 - $503.03 \text{ cm}^2/\text{Vs}$
 - $43.25 \text{ cm}^2/\text{Vs}$
 - $88 \text{ cm}^2/\text{Vs}$
- The conductivity of a semiconductor increases with increase in temperature because
 - number density of free current carriers increases
 - relaxation time increases
 - both number density of carriers and relaxation time increase
 - number density of current carriers increases, relaxation time decreases but effect of decrease in relaxation time is much less than increase in number density
- In an n -type silicon, which of the following statement is true?
 - Electrons are majority charge carriers and trivalent atoms are the dopants
 - Electrons are minority charge carriers and pentavalent atoms are the dopants
 - Holes are minority charge carriers and pentavalent atoms are the dopants
 - Holes are majority carriers and trivalent atoms are the dopants
- Application of a forward bias to a p - n junction
 - increases the number of donors on the n -side
 - increases the electric field in the depletion zone
 - increases the potential difference across the depletion zone
 - widens the depletion zone
- In a three phase full-wave rectifier of 50 Hz, the ripple frequency is
 - 300 Hz
 - 50 Hz
 - 400 Hz
 - 600 Hz
- The dimension of $\sqrt{LC}$ is
 - $(\text{s})^2$
 - volt-s/amp
 - amp-s/volt
 - s
- Given, $\beta = 49$ and $I_e = 2 \text{ mA}$, I_c for a BJT is
 - 2 mA
 - 1.96 mA
 - 98 mA
 - $\frac{2}{49} \text{ mA}$
- A single ended class A transformer coupled amplifier delivers a power of 50 mW. $V_{CC} = 10$ volts. Assuming ideal conditions, the reflected primary resistance is given by
 - 2000 Ω
 - 1000 Ω
 - 500 Ω
 - 200 Ω
- In a vacuum diode, when the plate voltage is 40 V, the plate current is 10 mA. For a plate voltage of 160 V, the plate current is
 - 20 mA
 - 40 mA
 - 80 mA
 - 160 mA
- A triode is operating at $E_c = -4 \text{ V}$, $E_b = 175 \text{ V}$ and $I_b = 10 \text{ mA}$. Around this operating point, the plate characteristics are linear and the parameter values are $\mu = 20$, $g_m = 2.5 \text{ mA/V}$. If E_c is changed to -6 V with $E_b = 175 \text{ V}$, the value of I_b is given by
 - 10 mA
 - 20 mA
 - 15 mA
 - 5 mA
- Given the following set of values for a triode, the value of g_m and μ are given below in the table:

E_c	E_b	I_b
-12 V	-4 V	-4 V
230 V	230 V	150 V
25 mA	35 mA	25 mA

 - 1.25 mA/V and 8
 - 0.8 mA/V and 8
 - 1.25 mA/V and 10
 - 8 mA/V and 10

12. The table gives values of E_b , E_c , I_b for a triode

E_b	E_c	I_b
250 V	-35 V	10 mA
300 V	-45 V	10 mA
300 V	-35 V	25 mA

The value of r_p in $k\Omega$ and g_m in mA/V are given by

- a. 50 and 0.1 b. 10 and 3.0
c. 5 and 1.5 d. 3.33 and 1.5
13. A triode is operating at $E_c = -4$ V, $E_b = 175$ V and $I_b = 10$ mA. Around this operating point, the plate characteristics are linear and the parameters are $\mu = 20$, $r_p = 8$ $k\Omega$. E is changed to -6 V. To obtain the same plate current the value of E_b should be
- a. 40 b. 135 c. 191 d. 215
14. A triode has $r_p = 10$ $k\Omega$ and $\mu = 20$. Table gives the operating point with blanks

E_c	E_b	I_b
-8 V	250 V	8 mA
E_{b1}	250 V	12 mA
E_{c1}	E_{b1}	8 mA

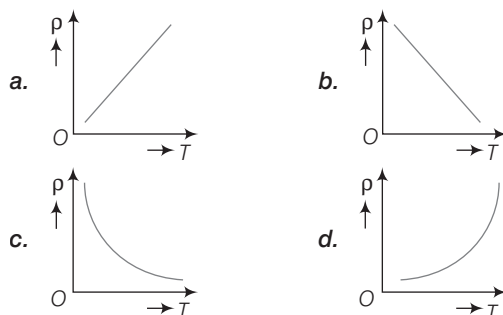
The values of E_{b1} and E_{c1} are

- a. 290 V, -6 V b. 210 V, -10 V
c. 290 V, -10 V d. 210 V, -6 V
15. For a vacuum triode, the values of E_c , E_b and I_b are given as under

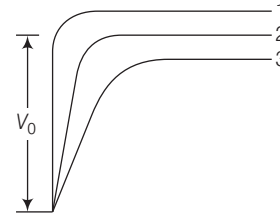
E_c	E_b	I_b
-1 V	100 V	5 mA
-1 V	140 V	9 mA
-6 V	200 V	5 mA

The values of μ and r_p are given by

- a. $\mu = 12$, $r_p = 10$ $k\Omega$ b. $\mu = 12$, $r_p = 20$ $k\Omega$
c. $\mu = 10$, $r_p = 20$ $k\Omega$ d. $\mu = 20$, $r_p = 10$ $k\Omega$
16. The temperature (T) dependence of resistivity (ρ) of a semiconductor is represented by

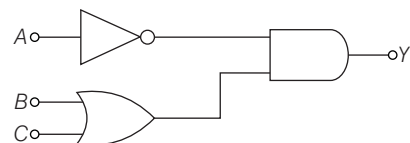


17. In figure, V_0 is the potential barrier across a p - n junction, when no battery is connected across the junction.



- a. 1 and 3 both correspond to forward bias of junction
b. 3 corresponds to forward bias of junction and 1 corresponds to reverse bias of junction
c. 1 corresponds to forward bias and 3 corresponds to reverse bias of junction
d. 3 and 1 both correspond to reverse bias of junction
18. In BJT, maximum current flows in which of the following?
a. Emitter region b. Base region
c. Collector region d. Equal in all the regions
19. A tungsten emitter works at 2500 K. To increase the emission current density by 20%, how much change in the work function is required (Given, $\log 2 = 0.3$, $\log 3 = 0.477$)
a. 0.016 eV b. 0.039 eV c. 2.54 eV d. 0.254 eV
20. In an unbiased p - n junction, holes diffuse from the p -region to n -region because
a. free electrons in the n -region attract them
b. they move across the junction by the potential difference
c. hole concentration in p -region is more as compared to n -region
d. All of the above
21. Zener breakdown in a semiconductor diode occurs when
a. forward current exceeds certain value
b. reverse bias exceeds certain value
c. forward bias exceeds certain value
d. potential barrier is reduced to zero
22. The voltage gain of an amplifier stage is 100. The gain expressed in db is
a. 100 b. 20 c. 40 d. 10
23. If $A = B = 1$, then in terms of Boolean algebra what is not equal to the value of $A \cdot B + A$?
a. $B \cdot A + B$ b. $B + A$ c. B d. None

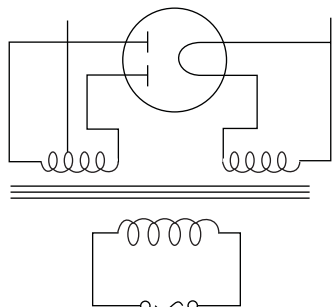
24. The Boolean equation for the circuit given in figure is



- a. $Y = \bar{A} \cdot B + C$ b. $Y = \bar{A} \cdot (\bar{B} + \bar{C})$
c. $Y = \bar{A} \cdot (B + \bar{C})$ d. $Y = \bar{A} \cdot (B + C)$

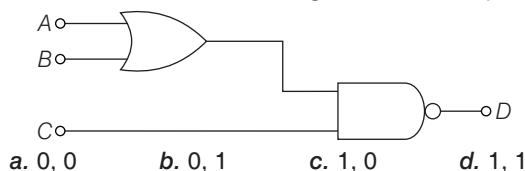
25. In the above circuit, if the polarity is reversed of battery, the current flowing would be
a. 0 mA b. 2 mA c. 5 mA d. 10 mA

26. The given figure shows a rectifier of alternating current ($f = 50$ c/s), the number of pulses of rectified current obtained in 1s is



- a. 50 b. 25 c. 100 d. 200
27. In a sample of pure silicon 10^{13} atom/cm³ is mixed of phosphorus. If all atoms are active, then what will be the resistivity at 20° C if mobility of electron is 1200 cm²/V-s?
a. 0.5209 Ω-cm b. 5.209 Ω-cm
c. 52.09 Ω-cm d. 520.9 Ω-cm

28. For the given combination of gates, if the logic states of inputs A, B and C are as follows. $A = B = C = 0$ and $A = B = 1, C = 0$, then the logic states of output D are



- a. 0, 0 b. 0, 1 c. 1, 0 d. 1, 1
29. The plate voltage of a triode is increased from 225 V to 250 V, the grid voltage is changed from 4 to -4.7 V to maintain plate current constant. The amplification factor of the tube is
a. 35.7 b. 2 c. 70 d. 20

30. In a triode amplifier, the load resistance is equal to the plate resistance r_p . If μ is the amplification factor, the stage gain of the amplifier is

- a. $\frac{\mu}{2}$ b. μ c. 2μ d. $\frac{\mu}{4}$

31. The triodes P and Q have the same amplification factor 40. Their plate resistances are 4 kΩ and 8 kΩ, respectively. If an amplifier circuit is designed using anyone of them and a load resistance is of 8 kΩ, the ratio of the voltage gain obtained from them will be
a. 2 : 3 b. 4 : 3 c. 3 : 4 d. 1 : 2

32. When the plate voltage of a triode is 150 V, its cut-off voltage is -5 V. On increasing the plate voltage to 200 V, the cut-off voltage can be
a. -4.5 V b. -5.0 V
c. +2.3 V d. -6.66 V

33. In a diode vacuum tube, the plate current is 5 mA, When the plate voltage is 160 V, a grid is introduced between the plate and cathode and a voltage of -2 V is applied to it. The plate current will become

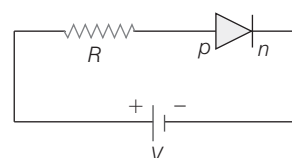
(If $g_m = 5 \times 10^{-3} \Omega^{-1}$)

- a. 20 mA b. 10 mA c. 4 mA d. 7.5 mA

34. In common emitter amplifier, the $\frac{I_c}{I_e}$ is 0.98, then the value of β is

- a. 98 b. 0.98
c. 49 d. None of these

35. For the given circuit of $p-n$ junction diode, which of the following statements is correct?



- a. In forward biasing the voltage across R is V
b. In forward biasing the voltage across R is 2 V
c. In reverse biasing the voltage across R is V
d. In reverse biasing the voltage across R is 2 V

36. When forward bias is applied to a $p-n$ junction, what happens to the potential barrier V_B and the width of charge depleted region x ?

- a. V_B increases, x decreases
b. V_B decreases, x increases
c. V_B increases, x increases
d. V_B decreases, x decreases

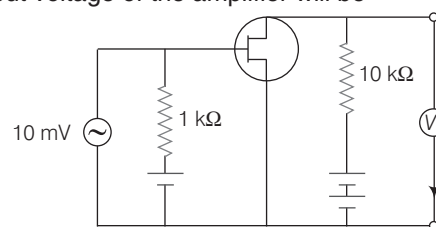
37. Carbon, silicon and germanium atoms have four valence electrons each. Their valence and conduction bands are separated by energy band gaps represented by $(E_g)_C$, $(E_g)_{Si}$ and $(E_g)_{Ge}$, respectively. Which one of the following relationship is true in their case?

- a. $(E_g)_C > (E_g)_{Si}$ b. $(E_g)_C = (E_g)_{Si}$
c. $(E_g)_C < (E_g)_{Ge}$ d. $(E_g)_C < (E_g)_{Si}$

38. What is the plate current in a diode valve under the space charge limited operation, when the plate potential is 60 V ? In a diode valve, the plate current is 320 mA, when the plate potential is 240 V.

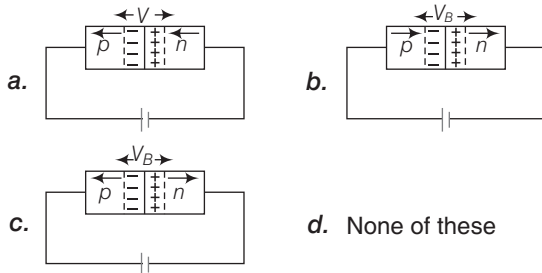
- a. 30 mA b. 20 mA c. 40 mA d. 10 mA

39. In the following common emitter configuration, an $n-p-n$ transistor with current gain $\beta = 100$ is used. The output voltage of the amplifier will be

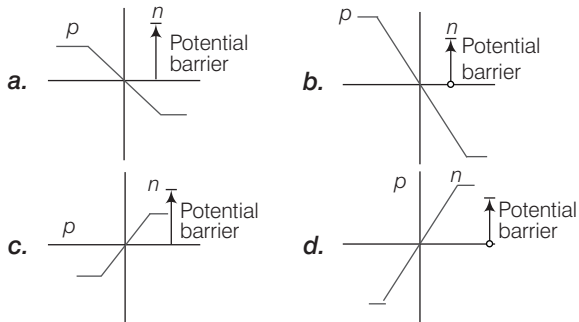


- a. 10 mV b. 0.1 V c. 1.0 V d. 10 V

40. In the case of forward biasing of p - n junction, which one of the following figures correctly depicts the direction of the flow of charge carriers?



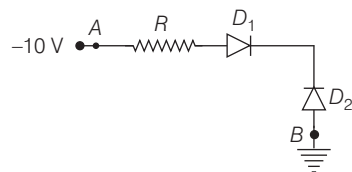
41. In a forward biased p - n junction diode, the potential barrier in the depletion region will be of the form



42. The input resistance of a common emitter transistor amplifier, if the output resistance is $500 \text{ k}\Omega$, the current gain $\alpha = 0.98$ and the power gain is 6.0625×10^6 is

- a. $198 \text{ }\Omega$ b. $300 \text{ }\Omega$
c. $100 \text{ }\Omega$ d. $400 \text{ }\Omega$

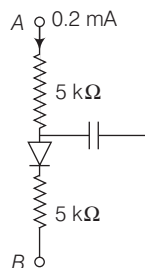
43. In a figure given alongside assuming the diodes to be ideal



- a. D_1 is forward biased and D_2 is reverse biased and hence, current flows from A to B
b. D_2 is forward biased and D_1 is reverse biased and hence, no current flows from B to A and vice-versa
c. D_1 and D_2 are both forward biased and hence current flows from A to B
d. D_1 and D_2 are both reverse biased and hence no current flows from A to B and vice-versa

44. In the circuit shown in figure, if the diode forward voltage drop is 0.3 V , then the voltage difference between A to B is,

- a. 1.3 V
b. 2.3 V
c. zero
d. 0.5 V



45. The exclusive-OR of A, B is represented by $A \oplus B$. An equivalent form is

- a. $AB + \bar{A}\bar{B}$ b. $AB + \overline{A+B}$
c. $\bar{A}\bar{B} + \bar{A}B$ d. None of these

46. Given the truth table relating Y to A, B .

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

Then, Y is given by

- a. $A + B$ b. AB c. $\bar{A}\bar{B}$ d. $\overline{A+B}$

47. Given the following truth table, where A, B are inputs and Y the output

A	B	Y
0	0	1
1	0	1
0	1	1
1	1	0

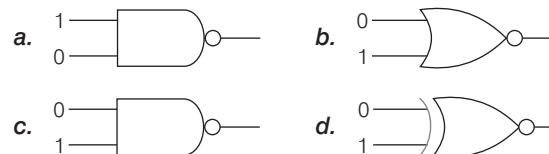
The output Y is

- a. $\bar{A}\bar{B}$ b. $\bar{A}B$ c. AB d. $\bar{A}B$

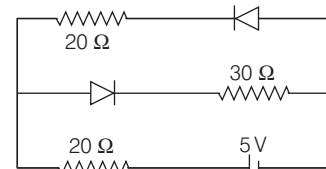
48. What will be the input of A and B for the Boolean expression $(\bar{A} + B) \cdot (A \cdot B) = 1$?

- a. 0, 0 b. 0, 1 c. 1, 0 d. 1, 1

49. Which of the following gates will have an output of 1?

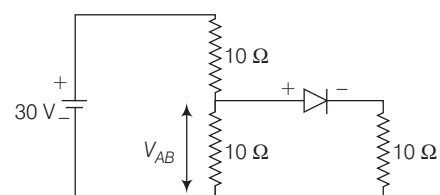


50. Current in the circuit will be



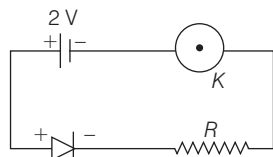
- a. $5/40$ b. $5/50$ c. $5/10$ d. $5/20$

51. Find the value of V_{AB} .



- a. 10 V b. 20 V
c. 30 V d. None of these

52. A two volt battery forward biased and a diode. However, there is a drop of 0.5 V across the diode which is independent of current. Also, a current greater than 10 mA produces large joule loss and damages diode. If diode is to be operated at 5 mA, then the series resistance to be put is

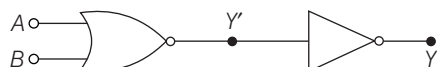


- a. 3 k Ω b. 300 k Ω c. 300 Ω d. 200 k Ω

53. Which of the following statements concerning the depletion zone of an unbiased *p-n* junction is(are) true?

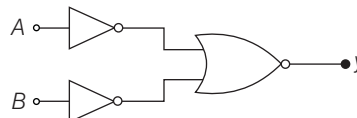
- The width of the zone is independent of the densities of the dopants (impurities)
- The width of the zone is dependent on the densities of the dopants
- The electric field in the zone is produced by ionised dopant atoms
- The electric field in the zone is provided by electrons in the conduction band and the holes in the valence band

54. The circuit shown in figure below will act as



- OR gate
- AND gate
- XOR gate
- None of these

55. The circuit shown below will act as



- AND gate
- OR gate
- NAND gate
- XOR gate

56. In Boolean algebra, $A + B = Y$ implies that

- sum of *A* and *B* is *Y*
- Y* exists when *A* exists or *B* exists or both *A* and *B* exist
- Y* exists only when *A* and *B* both exist
- Y* exists when *A* or *B* exist but not when both *A* and *B* exist

57. In the Boolean algebra, which of the following is wrong?

- $1 + 0 = 1$
- $0 + 1 = 1$
- $1 + 1 = 1$
- None of these

58. The output of a 2-input OR gate is fed to a NOT gate, the new gate obtained is

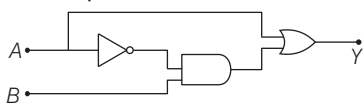
- OR gate
- NOT gate
- NOR gate
- XOR gate

59. Digital circuit can be made by the repetitive use of

- OR gates
- AND gates
- NOT gates
- NAND gates

BITSAT Archives

1. The circuit is equivalent to



[2013]

- AND gate
- OR gate
- Not gate
- None of these

2. Active state of *n-p-n* transistor, in circuit is achieved by

[2013]

- low input voltage
- high input voltage
- both *a* and *b*
- Neither *a* nor *b*

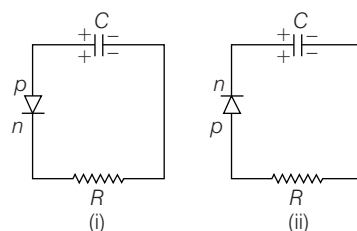
3. If a semiconductor has an intrinsic carrier concentration of $1.41 \times 10^{16}/\text{m}^3$ when doped with $10^{21}/\text{m}^3$ phosphorous atoms, then the concentration of holes / m^3 at room temperature will be

[2009]

- 2×10^{21}
- 2×10^{11}
- 1.41×10^{10}
- 1.41×10^{16}

4. Two identical capacitors each of capacitance *C* are charged to the same potential *V* and are connected in two circuits (i) and (ii) at $t = 0$ as shown. The charges on the capacitor at $t = CR$ are

[2009]



- $\frac{CV}{e}, \frac{CV}{e}$
- CV, CV
- $\frac{VC}{e}, VC$
- $VC, \frac{VC}{e}$

5. A transistor is preferable to a triode valve when used in amplifier because it

- can withstand large changes in temperature
- has a higher input impedance
- can handle larger powers
- does not require powers

which of the above statements is correct? [2009]

- Only (i), (ii) and (iii) are correct
- Only (i) and (iii) are correct
- Only (ii) and (iv) are correct
- Only (vi) is correct

Answer with Solutions

Practice Exercise

$$1. (c) \text{ Mobility of electron, } \mu = \frac{\sigma}{ne} \quad \dots(i)$$

$$\text{Resistivity, } \rho = \frac{1}{\sigma} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\mu = \frac{1}{nep} \quad \dots(iii)$$

n = number of free electrons per unit volume

$$n = \frac{N_0 \times d}{\text{atomic weight}} = \frac{6.023 \times 10^{23} \times 8.96}{63.54} = 8.5 \times 10^{22} \quad \dots(iv)$$

From Eqs. (iii) and (iv), we get

$$\mu = \frac{1}{8.5 \times 10^{22} \times 1.6 \times 10^{-19} \times 1.7 \times 10^{-6}} = 43.25 \text{ cm}^2/\text{Vs}$$

$$2. (d) \quad 3. (c) \quad 4. (a)$$

5. (a) For full-wave rectifier.

For single phase, ripple frequency

$$= 2 \times f \quad (f = \text{frequency of input})$$

$$f' = 2 \times 50 = 100 \text{ Hz} \quad (\text{frequency of supply})$$

For 3-phase, ripple frequency

$$= 3 \times f' = 3 \times 100 = 300 \text{ Hz}$$

$$6. (d) \text{ At resonance, } \omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$\Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

$$2\pi f = \frac{1}{\sqrt{LC}}$$

Thus, dimensions of $\sqrt{LC}$ is second.

$$7. (b) \beta = 49, I_e = 2 \text{ mA}, I_c = ?$$

$$\beta = \frac{I_c}{I_b}, \alpha = \frac{I_c}{I_e}$$

$$\alpha = \frac{\beta}{1 + \beta} = \frac{49}{1 + 49} = \frac{49}{50}$$

$$\frac{49}{50} = \frac{I_c}{I_e}$$

$$I_c = \frac{49 \times 2}{50} \text{ mA}$$

$$I_c = 1.96 \text{ mA}$$

$$8. (b) \text{ For ideal condition, } P = \frac{V^2}{R + R}$$

$$2R = \frac{10 \times 10}{50 \times 10^{-3}} = 2000$$

$$R = 1000 \Omega$$

$$9. (c) V_p = 40 \text{ V}, I_p = 10 \text{ mA}, V'_p = 160 \text{ V}$$

$$\frac{I_p}{I'_p} \propto \frac{(V_p)^3}{(V'_p)^2}$$

$$\frac{10}{I'_p} = \left(\frac{40}{160} \right)^{\frac{3}{2}}$$

$$I'_p = (4)^{\frac{3}{2}} \times 10 = 80 \text{ mA}$$

$$10. (d) E_{c1} = -4 \text{ V}, I_b = 10 \text{ mA}, E_b = 175 \text{ V}, \mu = 20, g_m = 2.5 \text{ mA/V}$$

$$E_{c2} = -6 \text{ V}, E_b = 175 \text{ V}$$

At constant plate potential 175 V

$$g_m = \left(\frac{\Delta I_b}{\Delta I_c} \right)_{V_b}$$

$$2.5 \times 10^{-3} = \frac{\Delta I_b}{-4 + 6}$$

$$5 \times 10^{-3} = \Delta I_b$$

$$I_b - I'_b = 5 \text{ mA}$$

$$I'_b = 10 - 5 = 5 \text{ mA}$$

$$11. (c) \mu = \left(\frac{\Delta V_p}{\Delta V_g} \right)_{I_p} = \frac{80}{8}$$

$$\mu = 10$$

$$g_m = \left(\frac{\Delta I_p}{\Delta V_g} \right)_{I_p} = \frac{10 \text{ mA}}{8} = 1.25 \text{ mA/V}$$

$$12. (d) r_p = \left(\frac{\Delta V_p}{\Delta I_p} \right)_{V_g} = \frac{50}{15} = 3.33 \text{ k}\Omega$$

$$g_m = \left(\frac{\Delta I_p}{\Delta V_g} \right) = \frac{15}{10} = 1.5 \text{ mA/V}$$

$$13. (d) \mu = \left(\frac{\Delta V_b}{\Delta V_g} \right)_{I_p}$$

$$\Delta V_b = 20 \times 2 = 40$$

$$\Delta V_b = V'_b - V_b = 40$$

$$V'_b = 175 + 40 = 215$$

$$14. (d) r_p = \left(\frac{\Delta V_p}{\Delta I_b} \right)$$

$$10 \text{ k}\Omega = \frac{250 - E_{b1}}{4 \text{ mA}}$$

$$10 \times 4 = 250 - E_{b1}$$

$$E_{b1} = 210 \text{ V}$$

$$\mu = \frac{\Delta V_p}{\Delta V_g} = \frac{40}{E_{c1} + 8}$$

$$8 + E_{c1} = \frac{40}{20}$$

$$8 - 2 = -E_{c1}$$

$$E_{c1} = -6 \text{ V}$$

$$15. (d) \mu = \left(\frac{\Delta V_b}{\Delta V_g} \right)_{I_b} = \left(\frac{\Delta V_b}{\Delta V_c} \right)_{I_b} (\text{Max. } \Delta V_p)$$

$$= \frac{200 - 100}{5} = 20$$

$$r_p = \left(\frac{\Delta V_p}{\Delta I_p} \right)_{V_g} (\text{Min. } \Delta V_p) = \frac{140 - 100}{9 - 5} = \frac{40}{4} = 10 \text{ k}\Omega$$

$$16. (c) \quad 17. (b) \quad 18. (a)$$

$$19. (b) \quad 20. (c) \quad 21. (b)$$

$$22. (c)$$

$$23. (d) AB + A = 1 \cdot 1 + 1 = 1 + 1 = 1.$$

Same is true for $B \cdot A + B$ as well as $B + A$.

$$24. (d) Y = \bar{A} \cdot (B + C)$$

$$25. (a) \quad 26. (c)$$

$$27. (d) \frac{1}{\rho} = e (n_e \mu_e + n_h \mu_h) = 520.9 \text{ } \Omega\text{-cm} \quad [\text{By putting values}]$$

$$28. (d) \text{ The output } D \text{ for the given combination}$$

$$D = \overline{(A + B)} \cdot \bar{C} = \overline{(A + B)} + \bar{C}$$

$$\text{If, } A = B = C = D, \text{ then } D = \overline{(0 + 0)} + \bar{0} = \bar{0} + \bar{0} = 1 + 1 = 1$$

$$\text{If } A = B = 1, C = 0, \text{ then } D = \overline{(1 + 1)} + \bar{0} = \bar{1} + \bar{0} = 0 + 1 = 1$$

$$29. (a) V_{p1} = 225 \text{ V}, V_{p2} = 250 \text{ V}, V_{g1} = -4 \text{ V},$$

$$V_{g2} = -4.7 \text{ V}$$

$$\mu = \left(\frac{\Delta V_p}{\Delta V_g} \right)_{I_p} = \frac{25}{0.7} = 35.7$$

$$30. (a) A = \frac{\mu}{1 + \frac{r_p}{R}}$$

$$R = r_p$$

$$A = \frac{\mu}{2}$$

$$31. (b) \mu = 40 = \text{amplification factor}$$

$$\mu_P = \mu_Q = 40$$

$$r_P = 4 \text{ k}\Omega, r_Q = 8 \text{ k}\Omega$$

$$R_L = 8 \text{ k}\Omega$$

$$\text{Voltage gain for triode } A = \frac{\mu R_L}{r_P + R_L}$$

$$\text{For triode } P = A_P = \frac{40 \times 8}{4 + 8}$$

$$A_Q = \frac{40 \times 8}{8 + 8}$$

$$\frac{A_P}{A_Q} = \frac{4 + 8}{40 \times 8} = \frac{16}{12} = \frac{4}{3}$$

$$32. (d) V_{p1} = 150 \text{ V}, V_{p2} = 200 \text{ V}, V_{g1} = -5 \text{ V}, V_{g2} = ?$$

μ = amplification factor of a triode valve, it is a characteristic of valve remains constant.

So long as I_p is constant.

$$\therefore \mu = -\frac{V_{p1}}{V_{g1}} = -\frac{V_{p2}}{V_{g2}}$$

$$\frac{150}{-5} = \frac{200}{V_{g2}}$$

$$V_{g2} = -\frac{20}{3} = -6.66 \text{ V}$$

$$33. (c) V_{g1} = 0$$

$$I_p = \text{plate current} = 5 \text{ mA}$$

$$V_p = \text{plate voltage} = 150 \text{ V}$$

$$V_g = \text{grid voltage} = -2 \text{ V}$$

When $V_g = 0$, it is a diode

When $V_g = -2$, it is a triode

$$\therefore g_m = \left(\frac{\Delta I_p}{\Delta V_g} \right)_{V_p = \text{constant}}$$

$$5 \times 10^{-4} = \frac{I_2 - I_1}{V_2 - V_1} = \frac{I_2 - 5 \times 10^{-3}}{-2 - 0}$$

$$-10 \times 10^{-4} + 5 \times 10^{-3} = I_2$$

$$-1 \times 10^{-3} + 5 \times 10^{-3} = I_2$$

$$I_2 = 4 \text{ mA}$$

$$34. (c) \quad 35. (a) \quad 36. (d) \quad 37. (a)$$

$$38. (c) \text{ Using Child Langmuir law, } I \propto V^{3/2}$$

$$320 \propto (240)^{3/2} \quad \dots(i)$$

$$I \propto (60)^{3/2} \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{320}{I} = (4)^{3/2} \Rightarrow I = \frac{320}{8} \Rightarrow I = 40 \text{ mA}$$

$$39. (c) \quad 40. (c) \quad 41. (d)$$

$$42. (a) \text{ Power gain} = \text{Current gain} \times \text{Voltage gain}$$

$$\text{Voltage gain, } A_V = \beta \frac{R_2}{R_1}$$

$$\text{Current gain } \beta = \frac{\alpha}{1 - \alpha} = \frac{0.98}{1 - 0.98} = 49$$

$$A_V = 49 \left(\frac{500 \times 10^3}{R_1} \right)$$

$$\text{Power gain, } R_1 = 6.0625 \times 10^6$$

$$= 49 \times \left(\frac{500 \times 10^3}{R_1} \right) \times 49 = 198 \text{ } \Omega$$

$$43. (b) \text{ If } V_p - V_n > 0, \text{ then diode is in forward biased. In case of } D_1, V_p - V_n = -10 - 0 = -10 < 0, \text{ so } D_1 \text{ is in reverse biased and for } D_2, V_p - V_n = 0 - (-10) = 10 > 0, \text{ so } D_2 \text{ is in forward biased.}$$

$$44. (b) \text{ Applying KVL across points } A \text{ and } B,$$

$$V_A - 0.2 \times 10^{-3} \times 5 \times 10^3 - 0.3 - 0.2 \times 5 - V_B = 0$$

$$\Rightarrow V_A - V_B = 2.3 \text{ V}$$

45. (c) Exclusive OR,

$$Y = A + B, Y = \overline{AB} + A\overline{B}$$

46. (d)
- $\overline{A+B} = \overline{0+0} = 1$

$$= \overline{0+1} = \overline{0} = 1 + \overline{0} = 0 = \overline{1+1} = 0$$

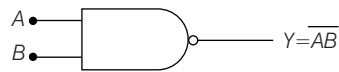
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

47. (d)
- $\overline{AB} = \overline{(0.0)} = \overline{0} = 1$

$$\overline{AB} = \overline{(1.0)} = \overline{0} = 1$$

$$\overline{AB} = \overline{(0.1)} = \overline{0} = 1$$

$$\overline{AB} = \overline{(1.1)} = \overline{1} = 0$$



48. (a) The given Boolean expression can be written as

$$Y = (A+B) \cdot (\overline{AB}) = (\overline{A} \cdot \overline{B}) \cdot (\overline{A} + \overline{B})$$

$$= (\overline{A} \cdot \overline{A}) \cdot \overline{B} + \overline{A}(\overline{B} \cdot \overline{B}) = \overline{A} \cdot \overline{B} + \overline{A} \cdot \overline{B} = \overline{A} \cdot \overline{B}$$

So, the truth table is

A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

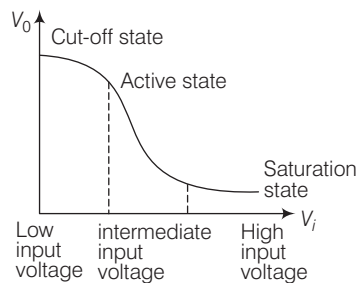
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1. (b)
- $Y = A + (\overline{A} \cdot B) = (A + \overline{A}) \cdot (A + B)$

$$= 1 \cdot (A + B) = A + B \text{ OR gate}$$

Here, t has value 1 for all inputs.

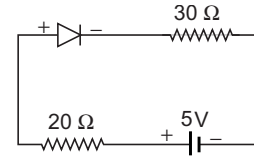
2. (d) Transfer characteristic of transistor is



Active state is achieved at intermediate input voltage.

49. (c) For option (c), it is a NAND gate, its output
- $= \overline{0.1} = \overline{0} = 1$

50. (b) Equivalent circuit is

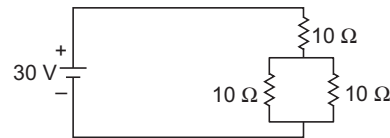


$$R_{eq} = 50 \Omega \Rightarrow I = \frac{V}{R} = \frac{5}{50}$$

51. (a) Equivalent circuit is

$$\therefore R_{eq} = 15 \Omega$$

$$\text{Circuit current, } I = \frac{V}{R} = \frac{30}{15} = 2 \text{ A}$$



$$\text{So, } V_{AB} = I_{AB} \cdot R_{AB} = 1 \times 10 = 10 \text{ V}$$

52. (c) By KVL,
- $2 = 0.5 + IR = 0.5 + 5 \times 10^{-3} R$

$$\therefore R = \frac{1.5}{5 \times 10^{-3}}, R = \frac{1500}{5} = 300 \Omega$$

53. (b) 54. (a) 55. (a)

56. (d) 57. (a) 58. (c)

59. (d) The repetitive use of NAND and NOR gate gives digital circuits.

3. (d) Doping will increase the number of electrons only and not the holes. So, number of holes will be equal to number of intrinsic carrier concentration.

$$= 1.41 \times 10^{16} / \text{m}^3$$

4. (c) In figure (i) the
- $p-n$
- junction diode is forward biased and represents a very low resistance, the capacitor, therefore discharge itself through resistor
- R
- according to relation,

$$q = q_0 e^{-1/CR} \text{ and } q_0 = CV \text{ at } t = CR$$

$$\therefore q = q_0 e^{-1} = \frac{CV}{e}$$

In figure (ii), the $p-n$ junction diode is reverse biased, the capacitor therefore holds the charge intact.

$$\therefore q = q_0 = CV$$

5. (d) A transistor is preferable to a triode valve when used in amplifier because it does not require a power.

33

Universe

Introduction

1. **Solar system** It includes the Sun at centre, eight planets (Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune) revolving around the Sun with 59 natural satellites revolving around the planets.

Mercury and Venus have no satellite, while Saturn have maximum number of satellites twentyone (21).

(a) **Stars** These are very much like the Sun. They may occur as single stars, binary stars or in clusters.

(b) **Galaxies** A large group of stars is called a galaxy. The galaxy containing our Sun and hundred billion (10^{11}) stars is called milky way. There are billions of galaxies. Galaxies may be spiral, elliptical and irregular in shape. Most galaxies are spiral like milky way.

2. Mass (M) of Milky way is given by

$$\frac{v^2}{r} = \frac{GM}{r^2}, \text{ i.e. } M = \frac{v^2 r}{G}$$

where, v = orbital speed of Sun.

r = distance of Sun from centre of galaxy.

The mass M comes out to be nearly 3×10^{41} kg.

3. **Asteroids** The groups of small objects revolving around the Sun between orbits of Mars and Jupiter are called asteroids. It is believed that asteroids are pieces of planets, possibly due to gravitational effect of Jupiter.
4. **Comets** A small rock like material surrounded by large masses like water, ammonia and methane and revolving in highly elliptical orbits around the Sun are called comets.
5. **Meteors and meteorites** These objects which get completely burnt and blazing through the atmosphere are called meteors. The larger objects which are not completely burnt before reaching the earth are called meteorites.
6. **Colour of a star** The colour of a star is determined by Wien's displacement law, according to which

$$\lambda_m = \text{constant}$$

where, λ_m = wavelength corresponding to maximum energy of emission.

Accordingly, a blue star is hottest and the red star is coolest.

The expanding universe and observed red shift are in favour of Big-bang theory.

7. **Pulsating theory** According to this theory, the boundary of universe moves in and out with a period of 8×10^9 years.
8. **The steady state theory** According to this theory, the total number of galaxies in the universe has attained a steady value. If one is lost due to annihilation in observable part of universe, one new is formed. This theory is based on interconversion of mass and energy.
9. **Hubble's law** It states that the speed of recession v is related to distance (r) of galaxy by the relation

$$v = Hr$$

where, H is called Hubble's constant. Its value $10^{17} \text{ kms}^{-1} / \text{million light years}$.

Star

Dust particles, hydrogen and helium gas molecules present in the interstellar space first come together (at about -173°C) to form a cloud. Then, they start contracting. As a result of compression, heating of cloud takes place. When temperature of the core becomes about 10^7 K , the fusion of hydrogen atoms is initiated with the release of energy. This energy keeps the star shining for millions of years.

Death of a Star

A star lasts until the hydrogen in the core of the star is exhausted. The core now starts contracting, it results in the rise in temperature of the star. As a result of rise in temperature, the outer layer of the star expand. Expansion of outer layer brings about cooling effect in them. The process continues till the temperature of outer layer falls enough to make the star appear red. It is then called **red giant**.

At this stage, a violent explosion called **nova** or **supernova** occurs in the star. Due to the explosion its outer layers are thrown into interstellar space leaving behind the core of the star. The core of the star may further end up into one of the following three steller dead materials.

White Dwarf

The core dies as white dwarf, if the original mass of the star was 1.4 solar masses. It was discovered by **S Chandrasekhar** in 1930 and it is known as **Chandrasekhar limit**. The core is composed of protons and electrons. The core keeps on emitting heat and light for millions of years. As it cools steadily, its colour changes from white to yellow, then to red and finally it becomes black. It then becomes invisible forever as black dwarf and neither emits heat nor light.

Neutron Star

The core of the star finishes up as neutron star, if the mass of the star was between 1.5 to 5 solar masses. The

compressed core is made only of neutrons. Neutron stars produce very high magnetic fields. A spinning neutron star emitting electromagnetic waves is called **Pulsar**.

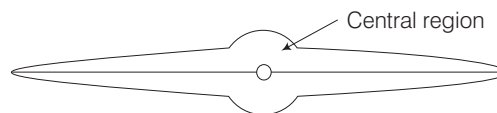
Black Hole

If the original mass of the star is greater than 5 solar masses, the core dies into black holes. The mass of a black hole is greater than the mass of the Sun but its size is very small, therefore the gravitational pull of a black hole is so strong that even the photon or radiation emitted by it, cannot escape from its surface. Since, no radiation is received from a black hole, it cannot be seen through a telescope.

The Milky Way

The Milky way or **Akash-ganga** is the name of the galaxy to which our solar system belongs. It is composed of a few hundred billion stars including our own Sun. The systematic study of the Milky way was undertaken in the eighteenth century by **William Herschel**, the same Herschel who discovered the planet Uranus.

To the naked eye, Milky way looks like a stream of milk across the sky. Some of the important features of Milky way are given below



- (i) Milky way is a convex lens shaped disc with a thick centre and thinning outwards.
- (ii) The Milky way contains about 150 billions of stars.
- (iii) The space in the Milky way is filled with dust and gases called the interstellar matter. About 90% of the interstellar matter is in the form of hydrogen.
- (iv) Some regions of the Milky way appear dark not because they are empty but because the intervening dust and gas obstructs the light from the stars behind them. These dark regions are called **dark nebulae**.
- (v) Sometimes the regions of dust show a great variety of brightness. They shine in the light of the nearby stars and such bright regions of dust are called bright nebulae. Orion is the example of bright nebulae.
- (vi) A remarkable feature of the Milky way is its rotation about its centre. This rotation is not rigid.

Our Sun, alongwith the planetary system is revolving around the centre of the Milky way with a speed $v = 250 \text{ km/s}$. It takes about 250 million years for the Sun to go round once.

- (vii) The mass of Milky way is estimated to be 150 billion solar masses or nearly $3 \times 10^{11} \text{ kg}$. The interstellar matter forms nearly 1.5% of the total mass of the Milky way

Practice Exercise

- Which planet is closest to Sun?
a. Earth b. Mercury c. Mars d. Jupiter
- Which planet is farthest to Sun?
a. Saturn b. Uranus c. Neptune d. Pluto
- The group of small pieces of rock revolving round the Sun between the orbits of Mars and Jupiter are called
a. meteors b. comets c. meteorites d. asteroids
- The tail of a comet points away from the Sun due to
a. centrifugal force
b. electrical repulsion
c. attraction of comet due to other planets
d. radiation pressure
- A star which appears blue will be
a. as hot as the Sun
b. cooler than the Sun
c. very cold indeed
d. much hotter than the Sun
- Sun radiates continuously and maintains its brightness because
a. helium is converted into iron in its core
b. of fusion of hydrogen nuclei in helium
c. fusion of helium in hydrogen
d. burning of carbon in its core
- One astronomical unit (AU) is equal to
a. 10^{-10} m b. 1.496×10^{11} m
c. 9.45×10^{15} m d. 4.964×10^{11} m
- The planet which has no satellite is
a. Neptune b. Mercury
c. Jupiter d. Mars
- The galaxy to which our solar system belongs is called
a. milky way b. radio galaxy
c. solar galaxy d. elliptical galaxy
- Galaxy in which we live is
a. milky way b. radio galaxy
c. circular galaxy d. irregular galaxy
- Milky way is
a. a planet of our solar system
b. a sun
c. one of the stars of solar system
d. one of the enormous galaxies of universe
- The universe is
a. expanding
b. contracting
c. constant in size
d. increasing northwards and decreasing southwards
- Hubble's law is expressed as [v = speed of recession, r = distance r of galaxy, H = Hubble constant]
a. $v = Hr^2$ b. $v = Hr$ c. $v = \frac{H}{r}$ d. $v = \frac{H}{r^2}$
- The group of 100 to 1000 stars held in position by mutual gravitational forces are called
a. galactic clusters b. globular clusters
c. comets d. asteroids
- The most common stars like the Sun are called
a. dwarfs b. white dwarfs
c. milky way d. neutron stars
- Very small stars having diameters $1/5$ th that of sun are called
a. dwarfs b. white dwarfs
c. milky way d. neutron stars
- The brightest planet in the solar system is
a. Mars b. Jupiter c. Venus d. Mercury
- The planet which has no atmosphere is
a. Venus b. Mercury c. Mars d. Earth
- The spectrum of stars is most closely related to
a. colour b. pressure
c. distance from Earth d. mass
- Venus appears brighter than other planets because
a. it is heavier than other planets
b. its density is more than other planets
c. it is nearest to Earth than other planets
d. fusion take place at its surface
- One main characteristic of black hole is that, it
a. emits a photon
b. absorbs a photon
c. changes photon into mass
d. changes all colours into black one
- The spectrum of a star is usually
a. continuous emission spectrum
b. continuous absorption spectrum
c. line absorption spectrum
d. line emission spectrum
- The solar constant at earth's surface is
a. 1.4 watt/m^2 b. 14 watt/m^2
c. 0.14 watt/m^2 d. 1400 watt/m^2
- If r denotes the mean distance of a planet from the sun and T is the time period of planet, then
a. $r \propto T^{2/3}$ b. $r \propto T^{3/2}$
c. $r \propto T^{1/3}$ d. $r \propto T$

25. Which of the following theories is the most satisfactory about the origin of the universe?
 a. Big-Bang theory b. Pulsating theory
 c. Steady state theory d. None of these
26. Hubble's law states that the velocity with which the galaxy is moving away from earth is proportional to
 a. square of distance of milky way
 b. distance of milky way from the Earth
 c. mass of milky way
 d. product of mass of milky way and its distance from earth
27. Albedo is
 a. a star
 b. a galaxy
 c. a constellation
 d. reflecting power of a planet
28. During a total solar eclipse, the disc of the Moon almost completely covers the disc of the sun. If the distance of the Sun from the Earth = 1496×10^{11} m, diameter of the Sun 1393×10^9 m and the distance of the Moon from the Earth = 3.845×10^8 m, then calculate the diameter of the moon.
 a. 1.26×10^6 m b. 4.30×10^6 m
 c. 3.58×10^6 m d. 6.86×10^6 m
29. Suppose the sun shrank from its present size so that its radius is halved. What would energy? (Given that mass of Sun = 1989×10^{30} kg and radius of the Sun = 6.95×10^8 m)
 a. 4.321×10^{50} J b. 2.275×10^{41} J
 c. 1.102×10^{30} J d. 8.123×10^{10} J
30. When the Jupiter is at a distance of 824.7 million km from the earth, its angular diameter is measured to be 35.72 sec of an arc. Calculate the diameter of Jupiter.
 a. 234265.54 km b. 142745.38 km
 c. 312108.82 km d. 121310.12 km

Answer with Solutions

Practice Exercise

- (b) Mercury is the only planet closest to Sun.
- (d) In comparison to the respective distances of all the planets from the Sun, Pluto is farthest to Sun.
- (d) Asteroids are the group of small pieces of rock revolving around the Sun between the orbits of Mars and Jupiter.
- (d) Radiation pressure is only a reason due to which the tail of the comet point away from the Sun.
- (d) A star which appears blue will be much hotter than the Sun.
- (b) Sun radiates continuously and maintain its brightness because of fusion of hydrogen nucleus in helium.
- (b) One astronomical unit is equal to 1.496×10^{11} m.
- (b) Mercury is the planet which has no satellite.
- (a) The galaxy to which our solar system belongs is called milky way.
- (a) Galaxy in which we live is milky way. It is the name of our galaxy to which our solar systems belongs.
- (d) Milky way is one of the enourmous galaxies of universe.
- (a)
- (b)
- (a)
- (a) The most common stars like the Sun are called dwarfs.
- (b) Very small stars having diameter $\frac{1}{5}$ th that of Sun are called white dwarfs. It was discovered by S.chandrasekhar in 1950 and it is known as Chanrdrasekhar limit.
- (c) Venus is the brightest planet in the solar system.
- (b) Mercury is the planet among the given options which has no atmosphere.
- (a) The spectrum of star is most closely related to colour.
- (c) Venus appears brighter than other planets because it is nearest to Earth than the other planets.
- (b) One main characteristic of black hole is that it absorbs a photon.
- (c) The spectrum of a star is usually line absorption spectrum.
- (d) The solar constant at Earth's surface is 1400 walt/m^2 .
- (a) According to Kepler's third law, the square of the time taken by the planet to revolve around the Sun is directly proportional to the cube of semi-major axis of its elliptical path.
 i.e. $T^2 \propto r^3 \Rightarrow r \propto T^{2/3}$
- (a) According to Big-Bang theory of universe, all the constituents of our universe were originally together as a single mass. A big explosion occured at some instant and the single mass burst in to a large number of fragments moving with different velocities.
- (b) Hubble's law states that the velocity with which the galaxy is moving away from the Earth is proportional to distance of milky way from the Earth.
- (d) Albedo is reflecting power of a planet.
- (c) Given, for Sun
 $d = 1393 \times 10^9 \text{ m}$
 $D = 1496 \times 10^{11} \text{ m}$

From the formula,

$$\begin{aligned}\therefore \alpha &= \frac{d}{D} \\ &= \frac{1393 \times 10^9}{1496 \times 10^{11}} \\ &= \frac{1393}{1496} \times 10^{-2} \text{ rad} \\ \text{For moon, } \alpha &= \frac{1393}{1496} \times 10^{-2} \text{ rad}\end{aligned}$$

$$D = 3.845 \times 10^8$$

$$\begin{aligned}\therefore d &= D\alpha \\ 3.845 \times 10^8 \times \frac{1393}{1496} \times 10^{-2} &= 3.58 \times 10^6 \text{ m}\end{aligned}$$

29. (b) Given,

$$M = 1989 \times 30 \text{ kg},$$

$$R = 6.95 \times 10^8 \text{ m}$$

$$G = 6.66 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$$

Present gravitational potential energy of the Sun is given by

$$V = -\frac{3}{5} \frac{GM^2}{R}$$

In case Sun shrank from its present size till its radius is halved $\left(\frac{R}{2}\right)$, then

$$V' = -\frac{3}{5} \frac{GM^2}{R/2} = -\frac{6}{5} \frac{GM^2}{R^2}$$

$\therefore$ Change in gravitation potential energy of the Sun

$$V - V' = -\frac{3}{5} \cdot \frac{GM^2}{R} - \left(-\frac{6}{5} \cdot \frac{GM^2}{R}\right)$$

$$\begin{aligned}V - V' &= \frac{3}{5} \cdot \frac{GM^2}{R} = \frac{3}{5} \times \frac{6.66 \times 10^{-11} \times (1989 \times 10^{30})^2}{6.95 \times 10^8} \\ &= 2.275 \times 10^{41} \text{ J}\end{aligned}$$

30. (b) Given, $\alpha = 35.72 \text{ s of arc}$

$$\text{So, } \alpha = \frac{35.72 \times \pi}{60 \times 60 \times 180} \text{ rad}$$

$$D = 824.7 \times 10^6 \text{ km}$$

Diameter of Jupiter, $d = D\alpha$

$$\begin{aligned}\therefore d &= \frac{824.7 \times 10^6 \times 35.72 \pi}{60 \times 60 \times 180} \\ &= 142745.38 \text{ km}\end{aligned}$$



1

Some Basic Concepts of Chemistry

Introduction

Chemistry is a branch of science which deals with the study of the composition of matter and the chemical changes involved in it. In other words, chemistry is the science of matter and its transformation.

Laws of Chemical Combination

These are the simple statements which have the wide applications during chemical reactions (though some of these laws are not perfect in today's context)

1. Law of Conservation of Mass (Lavoisier, 1774)

According to this law during any chemical or physical change, the total mass of the system (reactants + products) remains constant or we can say that, mass can neither be created nor be destroyed.

2. Law of Definite/Constant Proportion (Proust, 1799)

This law states that a given chemical compound always contains its component elements in a fixed ratio (by weight) and does not depend on its source or method of preparation.

e.g. H_2O obtained from any source always contains H and O in the ratio 1 : 8 (by mass).

3. Law of Multiple Proportion (Dalton, 1808)

It states that if two elements combine to form more than one compound, then the different masses of one element which combine with a fixed mass of the other element, bear a simple ratio to one another.

e.g. Oxides of nitrogen

Fixed weight of N = 28

Compound	N ₂ O	NO	N ₂ O ₃	N ₂ O ₄	N ₂ O ₅
Weight of oxygen	16	32	48	64	80

Ratio = 1 : 2 : 3 : 4 : 5

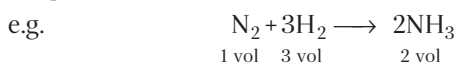
4. Law of Reciprocal Equivalent and Combining Proportion (Richter, 1794)

According to this law, when two or more elements (say X, Y, Z, etc) combine with a fixed mass of another element (say A), then the ratio of their combining masses is same (or a simple multiple) as the proportion in which they combine with each other (XY, YZ, XZ, etc).

e.g. CH₄, CO₂ and H₂O

5. Gay-Lussac's Law of Combining Volumes (Gay-Lussac, 1808)

In a gaseous phase reaction at the same temperature and pressure, the volume of reactant and product gases bear a simple whole number ratio.



Dalton's Atomic Theory

On the basis of laws of chemical combinations, Dalton (1803) proposed atomic theory.

The main postulates of this theory are:

1. Matter is made up of indivisible and indestructible particles, called atoms.
2. Atoms are neither be created nor destroyed in the course of an ordinary chemical reaction.
3. Atoms combine with each other to form compounds in simple whole number ratio.
4. Atom is the smallest portion of matter which takes part in chemical reaction.
5. All atoms of an element have identical mass and similar chemical properties.

Atomic Weight

It may be defined as the relative weight of an atom of an element with respect to 1/12th of the mass of an atom of C-12.

i.e. Atomic weight

$$= \frac{\text{Mass of one atom of element}}{1/12\text{th part of the mass of one atom of C-12}}$$

The weight of one atom of C-12 is 12 amu, where amu stands for atomic mass unit.

$$1 \text{ amu} = \frac{1}{N_A} = 1.667 \times 10^{-24} \text{ g}$$

Sometimes, the amount of a substance is expressed in number of gram atoms. One gram atom contains N_A atoms of an element.

Weight of 1 gram atom of an element whose atomic number is $A = A \text{ g}$

Weight of one atom = $A \text{ amu}$

Molecular Mass

It is the relative mass of one molecule of a compound with respect to 1/12th of the mass of one atom of C-12.

i.e. Molecular mass

$$= \frac{\text{Weight of one molecule of compound}}{1/12\text{th part of the weight of one atom of C-12}}$$

NOTE For compounds, molecular weight is the sum of the atomic weights of all atoms present in the molecule.

Average Atomic/Molecular Mass

If an element exists in the isotopic forms having atomic masses m_1, m_2 and m_3 in the ratio x, y and z respectively, the average atomic mass

$$M_{av} = \frac{m_1 \times x + m_2 \times y + m_3 \times z}{x + y + z}$$

Equivalent Weight

Equivalent weight of an element or of a compound is the weight of an element or of a compound which would combine with or displace (by weight) 1 part of hydrogen or 8 parts of oxygen or 35.5 parts of chlorine.

$$\text{Equivalent weight} = \frac{\text{Atomic weight or molecular weight}}{n\text{-factor}}$$

where, n -factor = acidity (number of ionisable H⁺)

= basicity (number of ionisable OH⁻)

= charge of ions

= number of electrons lost or gained

Mole Concept

Mole is the amount of substance which contains Avogadro's number ($N_A = 6.023 \times 10^{23}$) of particles and has the mass equal to gram atomic mass or gram molecular mass.

$$\text{One mole} = 6.023 \times 10^{23} \text{ particles}$$

$$\text{One mole of atom} = 6.023 \times 10^{23} \text{ atoms}$$

For 1 mole of atoms,

Mass of 1 mole of a substance = Gram atomic mass

For 1 mole of molecules,

Mass of 1 mole of molecules = Gram molecular mass

For 1 mole of ions,

Mass of 1 mole of ions = Gram formula mass

$$\text{Number of moles of particles} = \frac{\text{Number of particles}}{\text{Avogadro's number}}$$

Mole of gas at STP occupies = 22.4 L volume

$$\text{Number of moles of gas} = \frac{\text{Volume of gas at STP (in L)}}{22.4 \text{ L}}$$

Percentage Composition

The percentage of any element or constituent in a compound is the number of parts by mass of that element or constituent present in 100 parts by mass of the compound.

$$\begin{aligned} \text{Mass per cent of an element} \\ = \frac{\text{Mass of element in 1 mole of compound} \times 100}{\text{Mass of 1 mole of compound}} \end{aligned}$$

Dividing percentage by atomic mass, gives molar ratio from which empirical formula is obtained.

$$n = \frac{\text{Molecular mass}}{\text{Empirical formula mass}}$$

Molecular formula = $n \times$ Empirical formula

Molar mass = $2 \times$ Vapour density

Empirical Formula

It shows the simplest relative whole number ratio of atoms of each element present in the molecule of the substance.

e.g. CH is the empirical formula of benzene (C_6H_6), C_6H_6 is its molecular formula.

Molecular Formula

It expresses the actual number of atoms of each element present in the molecule.

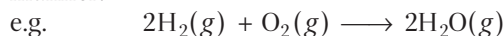
e.g. Molecular formula of hydrazine is NH_2NH_2 or N_2H_4 while its empirical formula is NH_2 .

One more type of chemical formula is seen, i.e. structural formula. It represents the relative position of atoms how they are combined in a molecule.

Chemical Equations and Stoichiometry

A balanced chemical equation with suitable stoichiometric coefficients represents the ratio of number of moles of reactants and products.

The equation provides qualitative and quantitative information about a chemical change in a simple manner.



Mole ratio	2 mol	1 mol	2 mol
Molecule ratio	$2 \times 6.022 \times 10^{23}$ molecules	6.022×10^{23} molecules	$2 \times 6.022 \times 10^{23}$ molecules
Weight ratio	4 g	32 g	36 g
Volume ratio	2 volumes	1 volume	2 volumes

Volume ratio is valid for gaseous state at same temperature and pressure.

Limiting Reagent

The substance which is completely consumed in a reaction, is called limiting reagent. It determines the amount of product.

$$\text{Reaction yield} = \frac{\text{Actual yield} \times 100}{\text{Theoretical yield}}$$

It must be noted that in stoichiometry, if the quantities of two or more reactants are given, the amounts of products formed depend upon the limiting reactant (the reactant which consumed first in the reaction).

Concentration Terms

(a) Mass%

It is defined as the amount of solute in gram present in 100 grams of solution.

$$\text{Mass\% of solute} = \frac{\text{Mass of solute}}{\text{Mass of solution}} \times 100$$

(b) % Mass by Volume

It is defined as the mass of solute present in 100 mL of solution.

(c) ppm

It is defined as the quantity of solute in grams present in 10^6 grams of solution.

(d) Mole Fraction

Suppose n_A moles of solute A and n_B moles of solvent B are present in a solution, then

$$\text{Mole fraction of } A = \chi_A = \frac{n_A}{n_A + n_B}$$

$$\text{Mole fraction of } B = \chi_B = \frac{n_B}{n_A + n_B}$$

$$\chi_A + \chi_B = 1$$

(e) Molarity (M)

It is defined as the number of moles of solute per litre of solution

$$M = \frac{n_A}{V} = \frac{W_A / M_A}{V}$$

where, V = volume of solution in litres

(f) Molality (m)

It is defined as the number of moles of solute present in 1 kg of the solvent

$$m = \left[\frac{W_A}{M_A W_B} \right] \times 1000$$

where, W_A = mass of solute (in gram)
 W_B = mass of solvent (in gram)
 M_A = molecular mass of solute

(g) Normality (N)

It is defined as the number of gram equivalents of solute present in 1 litre of solution.

$$N = \frac{W_A}{E_A V}$$

where, E_A = equivalent mass of A

NOTE Normality (N) = $n \times \text{molarity}$
 where, $n = \frac{\text{Molecular mass}}{\text{Equivalent mass}}$

Practice Exercise

- The answer of the calculation $\frac{2.568 \times 5.8}{4.168}$ in significant figures will be
 a. 3.570 b. 3.6
 c. 3.57 d. 3.579
- Number of significant figures in 78.000 g, 0.0206 g and 3.002 g respectively are
 a. 3, 4, and 5 b. 2, 5, and 4
 c. 3, 3, and 4 d. 5, 3, and 4
- Which of the following statements is true about the science of atoms and molecules?
 a. We can see, weigh and perceive the atoms and molecules through naked eyes
 b. It is possible to count the number of atoms and molecules in a given mass of matter through naked eyes and manually
 c. We can establish a quantitative relationship between the mass and number of these particles (atoms and molecules)
 d. Physical properties of matter can be qualitatively described using numerical values with suitable units
- Which of the following statements about a compound is incorrect?
 a. A molecule of a compound has atoms of different elements
 b. A compound cannot be separated into its constituent elements by physical methods of separation
 c. A compound retains the physical properties of its constituent elements
 d. The ratio of atoms of different elements in a compound is fixed
- Two students performed the same experiment separately and each one of them recorded two readings of mass which are given below. Correct reading of mass is 3.0 g. On the basis of given data, mark the correct option out of the following statements.

Student	Readings	
	(i)	(ii)
A	3.01	2.99
B	3.05	2.95

- Results of both the students are neither accurate nor precise
 - Results of student A are both precise and accurate
 - Results of student B are neither precise nor accurate
 - Result of student B are both precise and accurate
- What is the SI unit of density?
 a. gm^{-3} b. kg/m^3
 c. gcm^{-3} d. kg/cm^3
 - What temperature is 75°F on the Kelvin scale?
 a. 24 K b. 348 K
 c. 297 K d. 215 K
 - A jug contains 2 L of milk. Calculate the volume of the milk in m^3 .
 a. $2 \times 10^{-2} \text{ m}^3$
 b. $2 \times 10^{-1} \text{ m}^3$
 c. $2 \times 10^{-3} \text{ m}^3$
 d. $2 \times 10^{-4} \text{ m}^3$
 - Law of constant composition does not hold good for
 a. endothermic compounds
 b. exothermic compounds
 c. stoichiometric compounds
 d. non-stoichiometric compounds
 - One gram mole of a gas at NTP occupies 22.4 L. This fact is derived from
 a. law of gaseous volumes
 b. Avogadro's hypothesis
 c. Dalton's atomic theory
 d. Berzelius hypothesis

11. One part of element *A* reacts with two parts of another element *B*. 6 parts of element *C* reacts with 4 parts of element *B*. If *A* and *C* combine together, the ratio of their weight be governed by
 - a. law of conservation of mass
 - b. law of reciprocal proportions
 - c. law of definite proportions
 - d. law of multiple proportions
12. In which of the following numbers all zeroes are significant?
 - a. 30.000 b. 0.700 c. 0.00050 d. 0.0030
13. Which of the following is a homogeneous mixture?
 - a. Mixture of soil and water
 - b. Sugar solution
 - c. Mixture of sugar, salt and sand
 - d. Iodised table salt
14. Formation of CO and CO₂ illustrates the law of
 - a. conservation of mass b. multiple proportion
 - c. reciprocal proportion d. constant proportion
15. Which of the following reactions is not correct according to the law of conservation of mass?
 - a. $2\text{Mg}(s) + \text{O}_2(g) \longrightarrow 2\text{MgO}(s)$
 - b. $\text{C}_3\text{H}_8(g) + \text{O}_2(g) \longrightarrow \text{CO}_2(g) + \text{H}_2\text{O}(g)$
 - c. $\text{P}_4(s) + 5\text{O}_2(g) \longrightarrow \text{P}_4\text{O}_{10}(s)$
 - d. $\text{CH}_4(g) + 2\text{O}_2(g) \longrightarrow \text{CO}_2(g) + 2\text{H}_2\text{O}(g)$
16. Carbon dioxide contains 27.27% of carbon, carbon disulphide contains 15.79% of carbon and sulphur dioxide contains 50% of sulphur. This data is an agreement with
 - a. law of conservation of mass
 - b. law of definite proportions
 - c. law of multiple proportions
 - d. law of reciprocal proportions
17. One atom of an element weighs 1.8×10^{-22} g, its atomic mass is
 - a. 18 b. 29.4 c. 108.39 d. 154
18. 27 g of Al (at. mass = 27) will react completely with oxygen equal to
 - a. 24 g b. 8 g c. 40 g d. 10 g
19. Insulin contains 3.4% sulphur. What will be the minimum molecular weight of insulin?
 - a. 94.117 b. 1884 c. 941.176 d. -976
20. Molecular weight of a tribasic acid is *W*. Its equivalent weight will be
 - a. $\frac{W}{2}$ b. *W* c. $\frac{W}{3}$ d. 3*W*
21. Compounds having same empirical formula always have same
 - a. molecular mass
 - b. molecular formula
 - c. number of atoms
 - d. percentage composition by mass
22. What is the equivalent weight of SnCl₂ in the reaction, $\text{SnCl}_2 + \text{Cl}_2 \longrightarrow \text{SnCl}_4$? (mol. wt. of SnCl₂ = 190)
 - a. 95 b. 45
 - c. 60 d. 30
23. 5.6 L of a gas at NTP weighs equal to 8 g. The vapour density of gas is
 - a. 32 b. 16
 - c. 8 d. 40
24. The number of atoms present in 0.1 mole of P₄ (at. mass = 31) are
 - a. 2.4×10^{23} atoms
 - b. same as in 0.05 mole of S₈
 - c. 6.02×10^{22} atoms
 - d. same as in 3.1 g of phosphorus
25. The number of water molecules present in a drop of water (volume = 0.0018 mL) at room temperature is
 - a. 1.084×10^{18} b. 6.023×10^{19}
 - c. 4.84×10^{17} d. 6.023×10^{23}
26. Number of molecules in 4.25 g of NH₃, is
 - a. 1.505×10^{23} b. 6.02×10^{23}
 - c. 3.01×10^{23} d. None of these
27. Choose the wrong statement.
 - a. Molar mass is the mass of one molecule
 - b. Molar mass is the mass of one mole of substance
 - c. 1 mole means 6.023×10^{23} particles
 - d. Molar mass is the molecular mass (g)
28. A sample of AlF₃ contains 3.0×10^{24} F⁻ ions. The number of formula units of the sample are
 - a. 9.0×10^{24} b. 3.0×10^{24}
 - c. 0.75×10^{24} d. 1.0×10^{24}
29. Which of the following pairs contain equal number of atoms?
 - a. 22.4 L (STP) of nitrous oxide and 22.4 L of nitric oxide
 - b. 1 millimole of HCl and 0.5 millimole of H₂S
 - c. 1 mole of H₂O₂ and 1 mole of N₂O₄
 - d. 11.2 cc (STP) of nitrogen and 0.015 g of nitric oxide
30. Vapour density of a gas is 11.2. Volume occupied by 2.4 g of this at STP will be
 - a. 2.4 L b. 2.24 L
 - c. 22.4 L d. 11.2 L
31. Which of the following weigh the most?
 - a. One mole of water
 - b. One gram atom of nitrogen
 - c. One mole of sodium
 - d. One molecule of H₂SO₄
32. The total number of electrons present in 18 mL of water (density = 1g mL⁻¹) is
 - a. 6.02×10^{23} b. 6.02×10^{23}
 - c. 6.02×10^{24} d. 6.02×10^{25}

33. If 1 mL of water contains 20 drops, what is the number of water molecules in the one drop of water?
($A = \text{Avogadro's number}$)
 (a) $\frac{0.5}{18} A$ b. $0.05A$ c. $\frac{0.05}{18} A$ d. $0.5 A$
34. 3 g of an oxide of a metal is converted to chloride completely and it yielded 5 g of chloride. The equivalent weight of the metal is
 a. 33.25 b. 3.325 c. 12 d. 20
35. Assuming that the density of water to be 1g/cm^3 , calculate the volume occupied by one molecule of water.
 a. $2.989 \times 10^{-23} \text{ mL}$
 b. $6.023 \times 10^{23} \text{ cm}^3$
 c. 22400 mL
 d. 18 cm^{-3}
36. If 0.5 mole of BaCl_2 are mixed with 0.2 mole of Na_3PO_4 , the maximum number of moles of $\text{Ba}_3(\text{PO}_4)_2$ that can be formed, is
 a. 0.7 b. 0.5 c. 0.3 d. 0.1
37. If 10^{21} molecules are removed from 200 mg of CO_2 , the number of mole(s) of CO_2 left is/are
 a. 2.88×10^{-3} b. 28.8×10^{-3}
 c. 0.288×10^{-3} d. 1.66×10^{-2}
38. Which of the following pairs of gases contains the same number of molecules?
 a. 16 g of O_2 and 14 g of N_2
 b. 8 g of O_2 and 22 g of CO_2
 c. 28 g of N_2 and 22 g of CO_2
 d. 32 g of O_2 and 32 g of N_2
39. 10 dm^3 of N_2 gas and 10 dm^3 of gas X at the same temperature contain the same number of molecules. The gas X is
 a. CO b. CO_2 c. H_2 d. NO
40. A metal oxide contains 53% metal and carbon dioxide contains 27% carbon. Assuming the law of reciprocal proportions, the percentage of metal in the metal carbide is
 a. 75 b. 25 c. 37 d. 66
41. The number of gram molecules of oxygen in 6.02×10^{24} CO molecule is/are
 a. 10 b. 5 c. 1 d. 0.5
42. The weight of 1×10^{22} molecules of $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$ is
 a. 41.59 g b. 415.9 g
 c. 4.159 g d. None of these
43. The sulphate of a metal M contains 9.87% of M. The sulphate is isomorphous with $\text{ZnSO}_4 \cdot 7\text{H}_2\text{O}$. The atomic weight of M is
 a. 40.3 b. 36.3
 c. 24.3 d. 11.3
44. Rearrange the following (I to IV) in the order of increasing masses and choose the correct answer
(Atomic masses : O = 16, Cu = 63 and N = 14)
 I. 1 molecule of oxygen
 II. 1 atom of nitrogen
 III. $1 \times 10^{-10} \text{ g}$ molecular weight of oxygen
 IV. $1 \times 10^{-10} \text{ g}$ atomic weight of copper
 a. $\text{II} < \text{I} < \text{III} < \text{IV}$ b. $\text{IV} < \text{III} < \text{II} < \text{I}$
 c. $\text{II} < \text{III} < \text{I} < \text{IV}$ d. $\text{III} < \text{IV} < \text{I} < \text{II}$
45. How many moles of magnesium phosphate, $\text{Mg}_3(\text{PO}_4)_2$ will contain 0.25 mole of oxygen atoms?
 a. 0.02 b. 3.125×10^{-2}
 c. 1.25×10^{-2} d. 2.5×10^{-2}
46. If we consider that $1/6$ in place of $1/12$, mass of carbon atom is taken to be the relative atomic mass unit, the mass of one mole of a substance will
 a. to be a function of the molecular mass of the substance
 b. remain unchanged
 c. increase two fold
 d. decrease twice
47. If 6.3 g of NaHCO_3 are added to 15.0 g CH_3COOH solution, the residue is found of weigh 18.0 g. What is the mass of CO_2 released in the reaction?
 a. 4.5 g b. 3.3 g c. 2.6 g d. 2.8 g
48. Consider the following reaction,
 $\text{Na}_2\text{CO}_3 + 2\text{HCl} \longrightarrow 2\text{NaCl} + \text{H}_2\text{O} + \text{CO}_2$
 Equivalent weight of Na_2CO_3 is
 a. $\frac{M}{2}$ b. M c. $2M$ d. $\frac{M}{4}$
49. The weight of lime obtained by heating 200 kg of 95% pure lime stone is
 a. 98.4 kg b. 106.4 kg
 c. 112.8 kg d. 122.6 kg
50. What will be the weight of CO having the same number of oxygen atoms as present in 22 g of CO_2 ?
 a. 28 g b. 22 g c. 44 g d. 72 g
51. An organic compound containing C and H has 92.3% of carbon, its empirical formula is
 a. CH b. CH_3 c. CH_2 d. CH_4
52. Mass of 0.1 mole of methane is
 a. 1 g b. 16 g c. 1.6 g d. 0.1 g
53. A person adds 1.71 g of sugar ($\text{C}_{12}\text{H}_{22}\text{O}_{11}$) in order to sweeten his tea. The number of carbon atoms added are (molecular mass of sugar = 342)
 a. 3.6×10^{22} b. 7.2×10^{21}
 c. 0.05 d. 6.6×10^{22}
54. The one which has least mass, is
 a. 2 g atom of N b. 3×10^{23} atoms of C
 c. 1 mole of S d. 7.0 g of Ag

55. 19.7 kg of gold was recovered from a smuggler. How many atoms of gold were recovered?
 a. 100 b. 6.02×10^{23}
 c. 6.02×10^{24} d. 6.02×10^{25}
56. If 8.4 g of hexane burns completely in oxygen, how many moles of CO_2 is/are produced?
 a. 6 b. 0.6 c. 0.9 d. 1.2
57. The number of atoms in 4.25 g of NH_3 is approximately
 a. 1×10^{23} b. 1.5×10^{23}
 c. 2×10^{23} d. 6×10^{23}
58. Two elements x (at. mass = 75) and y (at. mass = 16) combine to give a compound having 75.8% x. The formula of the compound is
 a. xy b. x_2y c. x_2y_2 d. x_2y_3
59. If we take 44 g of CO_2 and 14 g of N_2 , what will be the mole fraction of CO_2 in the mixture?
 a. 1/5 b. 1/3 c. 2/3 d. 1/4
60. One mole of calcium phosphide on reaction with excess of water gives
 a. one mole of phosphine
 b. two moles of phosphoric acid
 c. two moles of phosphine
 d. one mole of phosphorus pentoxide
61. The volume of water to be added to 100 cm^3 of 0.5 N H_2SO_4 to get decinormal concentration, is
 a. 100 cm^3 b. 450 cm^3
 c. 500 cm^3 d. 400 cm^3
62. 5 mL of N HCl, 20 mL of N/2 H_2SO_4 and 30 mL of N/3 HNO_3 are mixed together and volume made to 1 L. The normality of resulting solution is
 a. 0.45 b. 0.025 c. 0.9 d. 0.05
63. Consider the following reaction,

$$2\text{Al}(s) + 6\text{HCl}(aq) \longrightarrow 2\text{Al}^{3+}(aq) + 6\text{Cl}^{-}(aq) + 3\text{H}_2(g)$$
 Which of the following statements is incorrect?
 a. 6 L HCl (aq) is consumed for every 3 L $\text{H}_2(g)$ produced
 b. 33.6 L $\text{H}_2(g)$ is produced regardless of temperature and pressure for every mole of Al that reacts
 c. 67.2 L $\text{H}_2(g)$ at STP, is produced for every mole of Al that reacts
 d. 11.2 L $\text{H}_2(g)$ at STP, is produced for every mole of HCl (aq) consumed
64. Two solutions of a substance (non-electrolyte) are mixed in the manner, 480 mL of 1.5 M of first solution with 520 mL of 1.2 M of second solution. The molarity of final solution is
 a. 1.20 M b. 1.50 M c. 1.344 M d. 2.70 M
65. Amount of oxalic acid present in a solution can be determined by its titration with KMnO_4 solution in the presence of H_2SO_4 . The titration gives unsatisfactory result when carried out in the presence of HCl because HCl
 a. gets oxidised by oxalic acid to chlorine
 b. furnishes H^+ ions in addition to those from oxalic acid
 c. reduces permanganate to Mn^{2+}
 d. oxidises oxalic acid to carbon dioxide and water
66. Consider the following reaction,

$$\text{CH}_4(g) + 2\text{O}_2(g) \longrightarrow \text{CO}_2(g) + 2\text{H}_2\text{O}(g)$$
 How many moles of methane are required to produce 22 g $\text{CO}_2(g)$ after combustion?
 a. 1 mole b. 0.5 mole
 c. 0.25 mole d. 1.25 moles
67. Which of the following gases will have least volume if 10 g of each gas is taken at same temperature and pressure?
 a. CO_2 b. N_2 c. CH_4 d. HCl
68. A solution is made by dissolving 49 g of H_2SO_4 in 250 mL of water. The molarity of the solution prepared is
 a. 2 M b. 1 M c. 4 M d. 5 M
69. What volume of water is to be added to 100 cm^3 of 0.5 M NaOH solution to make it 0.1 M solution?
 a. 200 cm^3 b. 400 cm^3 c. 500 cm^3 d. 100 cm^3
70. What will be the molality of the solution made by dissolving 10 g of NaOH in 100 g of water?
 a. 2.5 m b. 5 m c. 10 m d. 1.25 m
71. How much of NaOH is required to neutralise 1500 cm^3 of 0.1 N HCl? [Na = 23]
 a. 40 g b. 4 g c. 6 g d. 60 g

BITSAT Archives

1. 10 g of sample of mixture of CaCl_2 and NaCl are treated to precipitate all the calcium as CaCO_3 . This CaCO_3 is heated to convert all the Ca to CaO and the final mass of CaO is 1.62 g. The per cent by mass of CaCl_2 in the original mixture is [2014]
 a. 32.1% b. 16.2% c. 21.8% d. 12.0%
2. The volume respectively of 10 N and 4 N HCl required to make 1 L of 7 N HCl respectively are [2014]
 a. 0.75 L of 10 N HCl and 0.25 L of 4 N HCl
 b. 0.50 L of 10 N HCl and 0.50 L of 4 N HCl
 c. 0.65 L of 10 N HCl and 0.35 L of 4 N HCl
 d. 0.85 L of 10 N HCl and 0.15 L of 4 N HCl

3. 0.1 g of metal combines with 46.6 mL of oxygen at STP. The equivalent weight of metal is [2013]
 a. 12 b. 24
 c. 18 d. 36
4. 5 moles of Ba(OH)_2 are treated with excess of CO_2 . How much Ba(OH)_2 will be formed? [2012]
 a. 39.4 g b. 197 g
 c. 591 g d. 985 g
5. One mole of P_2O_5 undergoes hydrolysis as

$$\text{P}_2\text{O}_5 + \text{H}_2\text{O} \longrightarrow \text{H}_3\text{PO}_4$$
 The normality of the phosphoric acid formed is (Volume of the solution is 1 L) [2011]
 a. 2 b. 12 c. 24 d. 4
6. 1 g of hydrogen is found to combine with 80 g of bromine and 1 g of calcium combines with 4 g of bromine. Equivalent weight of calcium is [2010]
 a. 16 b. 20
 c. 40 d. 80

7. How much water should be added to 200 mL of semi normal solution of NaOH to make it exactly decinormal? [2010]
 a. 200 mL b. 800 mL c. 1000 mL d. 11200 mL

8. Match the following columns. [2008]

Column I		Column II (At STP)	
A	10 g $\text{CaCO}_3 \xrightarrow[\text{Decomposition}]{\Delta}$	1.	0.224 L CO_2
B	1.06 g $\text{Na}_2\text{CO}_3 \xrightarrow{\text{Excess HCl}}$	2.	4.48 L CO_2
C	2.4 g C $\xrightarrow[\text{combustion}]{\text{Excess O}_2}$	3.	0.448 L CO_2
D	0.56 g CO $\xrightarrow[\text{combustion}]{\text{Excess O}_2}$	4.	2.24 L CO_2
		5.	22.4 L CO_2

The correct match is

	A	B	C	D		A	B	C	D
a.	4	1	2	3	b.	5	1	2	3
c.	4	1	3	2	d.	1	4	2	3

Answer with Solutions

Practice Exercise

1. (b) $\frac{2.568 \times 5.8}{4.168} = 3.5735$
 Thus, insignificant figures = 3.6
2. (d)
3. (c)
4. (c) Physical properties of a compound are different from those of elements.
5. (b) Average of reading of student A = $\frac{3.01 + 2.99}{2} = 3.00$

$$B = \frac{3.05 + 2.95}{2} = 3.00$$

Correct reading = 3.0

Results of student A are both precise and accurate.

6. (b) Density of a substance is defined as its amount of mass per unit volume.

$$\text{SI unit of density} = \frac{\text{kg}}{\text{m}^3} \text{ or } \text{kg m}^{-3}$$

7. (c) $75^\circ \text{F} = \frac{9}{5}^\circ \text{C} + 32 = \frac{43 \times 5}{9}^\circ \text{C} = 23.9^\circ \text{C}$
 $= (23.9 + 273) \text{ K}$
 $= 296.9 \approx 297 \text{ K}$

8. (c) $2 \times 10^{-3} \text{ m}^3$

$\therefore$ 1 L = 1000 cm^3
 and 1 m = 100 cm which gives

$$\frac{1 \text{ m}}{100 \text{ cm}} = 1 = \frac{100 \text{ cm}}{100 \text{ cm}}$$

To get m^3 from the above unit factors, the first unit factor is taken and it is cubed.

$$\left(\frac{1 \text{ m}}{100 \text{ cm}} \right)^3$$

$$\Rightarrow \frac{1 \text{ m}^3}{10^6 \text{ cm}^3} = (1)^3 = 1$$

Now, 2 L = $2 \times 1000 \text{ cm}^3$

The above is multiplied by unit factor

$$2 \times 1000 \text{ cm}^3 \times \frac{1 \text{ m}^3}{10^6 \text{ cm}^3} = \frac{2 \text{ m}^3}{10^3} = 2 \times 10^{-3} \text{ m}^3$$

9. (d) Law of constant composition does not hold good for non-stoichiometric compounds.
10. (b) One gram mole of a gas at NTP occupies 22.4 L. This fact is derived from Avogadro's hypothesis.
11. (b) Ratio will be governed by the law of reciprocal proportions.
12. (a) 30.000 has all significant zeroes.
13. (b) Sugar solution is a homogeneous mixture.
14. (b) Formation of CO and CO_2 illustrates the law of multiple proportion.
15. (b) $\text{C}_3\text{H}_8(\text{g}) + \text{O}_2(\text{g}) \longrightarrow \text{CO}_2(\text{g}) + \text{H}_2\text{O}(\text{g})$ is not correct.
 Correct equation is

$$\text{C}_3\text{H}_8(\text{g}) + 5\text{O}_2(\text{g}) \longrightarrow 3\text{CO}_2(\text{g}) + 4\text{H}_2\text{O}(\text{g})$$

16. (d) In
- CO_2

Weight of $\text{CO}_2 = 100$

Weight of carbon = 27.27

Weight of oxygen = 72.73

27.27 g of carbon combines with 72.73 g of oxygen

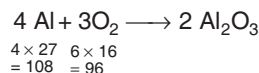
1g of carbon combines with $= \frac{72.73}{27.27} = 2.66$ g of oxygen $\therefore$ Ratio of C and O in $\text{CO}_2 = 1 : 2.66$ Similarly, in CS_2 Ratio of carbon and sulphur in $\text{CS}_2 = 1 : 5.3$ If sulphur and oxygen were to combine to form SO_2 , according to law of reciprocal proportions. They must combine in 5.3 : 2.66 or 2 : 1.In SO_2 , the ratio of S:O is 1:1.Thus, the ratio between the two ratios is $\frac{2}{1} : \frac{1}{1} = 2:1$

Since, the ratio of S : O is a simple whole number ratio, therefore law of reciprocal is proved.

17. (c) Atomic mass =
- $N_A \times \text{mass of one atom}$

$$= 6.022 \times 10^{23} \times 1.8 \times 10^{-22} = 108.39$$

18. (a) The equation of the reaction of aluminium and oxygen, is as

 $\therefore$ 108 g of Al reacts with 96 g of O_2 . $\therefore$ 27 g of Al will react with $= \frac{96 \times 27}{108} = 24$ g of O_2

19. (c) For minimum molecular mass, there must be one S-atom per insulin molecule.

If 3.4 g of S is present, the molecular mass = 100

 $\therefore$ If 32 g of S is present, the molecular mass will be

$$= \frac{100 \times 32}{3.4} = 941.176$$

20. (c) Equivalent weight =
- $\frac{\text{Molecular weight}}{\text{Basicity}} = \frac{W}{3}$

21. (d) Compound having same empirical formula always have same percentage composition by mass.

22. (a)
- $\text{SnCl}_2 + \text{Cl}_2 \longrightarrow \text{SnCl}_4$

Eq. wt. of $\text{SnCl}_2 = \text{Eq. wt. of Cl}_2$

$$\therefore \frac{190}{E_1} = \frac{71}{35.5} \text{ or } E_1 = 95$$

23. (b)
- $\therefore$
- 5.6 L of gas weighs at NTP = 8 g

$$\therefore 22.4 \text{ L of gas will weigh at NTP} = \frac{8 \times 22.4}{5.6} = 32 \text{ g}$$

 $\therefore$ Molecular weight of gas = 32

We know that,

Molecular weight = $2 \times \text{vapour density}$

$$\therefore \text{Vapour density} = \frac{32}{2} = 16$$

24. (a)
- $\therefore$
- 1 mole of
- P_4
- contains phosphorus atoms

$$= 4 \times 6.02 \times 10^{23}$$

 $\therefore$ 0.1 mole of P_4 contains phosphorus atoms

$$= 4 \times 6.02 \times 10^{23} \times 0.1$$

$$= 2.4 \times 10^{23} \text{ atoms}$$

25. (b) Number of moles =
- $\frac{\text{Weight}}{\text{Molecular weight}}$

$$= \frac{0.0018}{18} = 1 \times 10^{-4}$$

$$[\therefore 0.0018 \text{ mL} = 0.0018 \text{ g}]$$

$$\text{Number of water molecules} = 1 \times 10^{-4} \times 6.023 \times 10^{23}$$

$$= 6.023 \times 10^{19}$$

26. (a)
- $\therefore$
- 17 g of
- NH_3
- contains
- $= 6.023 \times 10^{23}$
- molecules of
- NH_3

$$\therefore 4.25 \text{ g of } \text{NH}_3 \text{ will contain} = \frac{6.023 \times 10^{23}}{17} \times 4.25$$

$$= 1.505 \times 10^{23} \text{ molecules of } \text{NH}_3$$

27. (a) Molar mass is the mass of 1 mole or
- 6.022×10^{23}
- molecules of the substance

28. (d)
- $\therefore 3 \text{ F}^- = 1 \text{ formula unit of } \text{AlF}_3$

$$\therefore 3.0 \times 10^{24} \text{ F}^- = 1 \times 10^{24} \text{ formula units of } \text{AlF}_3$$

29. (d) Number of atoms in
- N_2

$$= \frac{11.2 \times 10^{-3} \times 6.023 \times 10^{23} \times 2}{22.4} = 6.023 \times 10^{20}$$

$$\text{Number of atoms in NO} = \frac{0.015 \times 2 \times 6.023 \times 10^{23}}{30}$$

$$= 6.023 \times 10^{20}$$

30. (a) Molecular mass =
- $2 \times \text{vapour density}$

$$= 2 \times 11.2 = 22.4$$

$$\text{Number of moles of gas} = \frac{2.4}{22.4}$$

 $\therefore$ 1 mole occupies = 22.4 L volume

$$\therefore \frac{2.4}{22.4} \text{ mole will occupy} = 22.4 \times \frac{2.4}{22.4}$$

$$= 2.4 \text{ L}$$

31. (c) One mole of water = 18 g

1 g atom of nitrogen = 14 g

1 mole of sodium = 23 g

$$1 \text{ molecule of } \text{H}_2\text{SO}_4 = \frac{98}{6.023 \times 10^{23}} \text{ g}$$

$$= 1.627 \times 10^{-24} \text{ g}$$

Hence, one mole of sodium weigh the most.

32. (c) 18 mL
- $\text{H}_2\text{O} = 18 \text{ g } \text{H}_2\text{O} = 1 \text{ mol}$

$$= 6.02 \times 10^{23} \text{ molecules}$$

$$= 10 \times 6.02 \times 10^{23} \text{ electrons}$$

$$= 6.02 \times 10^{24} \text{ electrons}$$

33. (c) $\therefore 18 \text{ mL} = 18 \text{ g}$ of water contains $= 20 \times 18$ drops

$$= A \text{ molecules}$$

$$\therefore 1 \text{ drop contains} = \frac{A}{20 \times 18} \text{ molecules} = \frac{0.05 A}{18} \text{ molecules}$$

34. (a) $\frac{\text{Weight of metal oxide}}{\text{Weight of metal chloride}}$

$$= \frac{\text{Eq. wt. of metal} + \text{Eq. wt. of oxide}}{\text{Eq. wt. of metal} + \text{Eq. wt. of chloride}}$$

$$\frac{3}{5} = \frac{E + 8}{E + 35.5}$$

$$\text{or } 5E + 40 = 3E + 106.5 \text{ or } 2E = 66.5$$

$$\therefore E = 33.25$$

35. (a) $\therefore$ Density of water $= 1 \text{ g/cm}^3$

$$\therefore 6.023 \times 10^{23} \text{ molecules of } \text{H}_2\text{O} \text{ contain} = 18 \text{ cm}^3 \text{ volume}$$

$$\therefore 1 \text{ molecule of } \text{H}_2\text{O} \text{ will contain} = \frac{18}{6.023 \times 10^{23}} \\ = 2.989 \times 10^{-23} \text{ mL}$$

36. (d) $3\text{BaCl}_2 + 2\text{Na}_3\text{PO}_4 \longrightarrow \text{Ba}_3(\text{PO}_4)_2 + 3\text{NaCl}$

Limiting reactant is Na_3PO_4 .

$$0.2 \text{ mole of } \text{Na}_3\text{PO}_4 \text{ will give} = \frac{1}{2} \times 0.2 \\ = 0.1 \text{ mole of } \text{Ba}_3(\text{PO}_4)_2$$

37. (a) $200 \text{ mg CO}_2 = 0.2 \text{ g} = \frac{0.2}{44} \text{ mol} = 0.00454 \text{ mol}$

$$= 4.54 \times 10^{-3} \text{ mol}$$

$$10^{21} \text{ molecules of } \text{CO}_2 = \frac{10^{21}}{6.02 \times 10^{23}} = 1.66 \times 10^{-3} \text{ mol}$$

$$\therefore \text{Number of moles left} = (4.54 - 1.66) \times 10^{-3} = 2.88 \times 10^{-3}$$

38. (a) $16 \text{ g of } \text{O}_2 = 0.5 \text{ mol} = \frac{N_A}{2} \text{ molecules}$

$$14 \text{ g of } \text{N}_2 = 0.5 \text{ mol} = \frac{N_A}{2} \text{ molecules}$$

39. (a) Number of moles of N_2 and X should be equal. This can be so if X has same molecular weight as N_2 .

CO has same molecular weight as N_2 .

40. (a) In metal oxide, metal $= 53\%$, $\text{O} = 47\%$

In CO_2 , $\text{C} = 27\%$, $\text{O} = 73\%$

$\therefore 73$ parts of oxygen combines with 27 parts of carbon.

$$\therefore 47 \text{ parts of oxygen will combine} = \frac{27}{73} \times 47$$

$$= 17.38 \text{ parts of C}$$

Thus, metal and carbon will be present in the ratio of $53 : 17.38$.

$$\text{Hence, \% of metal} = \frac{53}{53 + 17.38} \times 100 = 75.3\%$$

41. (b) $6.02 \times 10^{24} \text{ CO molecules} = 10 \text{ moles of CO}$

$$= 10 \text{ g atoms of O}$$

$$= 5 \text{ g molecules of } \text{O}_2$$

42. (c) $\therefore 6.02 \times 10^{23} \text{ molecules of } \text{CuSO}_4 \cdot 5\text{H}_2\text{O}$

$$= 63.5 + 32 + 64 + 90 = 249.5 \text{ g}$$

$$\therefore 10^{22} \text{ molecules of } \text{CuSO}_4 \cdot 5\text{H}_2\text{O}$$

$$= \frac{249.5}{6.02 \times 10^{22}} = 4.15 \text{ g}$$

43. (c) As the given sulphate is isomorphous with $\text{ZnSO}_4 \cdot 7\text{H}_2\text{O}$, its formula would be $\text{MSO}_4 \cdot 7\text{H}_2\text{O}$. If m is the atomic weight of M , molecular weight of

$$\text{MSO}_4 \cdot 7\text{H}_2\text{O} = m + 32 + 64 + 126 = m + 222$$

$$\text{Hence, \% of } M = \frac{m}{m + 222} \times 100 = 9.87 \text{ (given)}$$

$$\text{or } 100m = 9.87m + 222 \times 9.87$$

$$\text{or } 90.13m = 222 \times 9.87 \text{ or } m = 24.3$$

44. (a) I. 1 molecule of $\text{O}_2 = \frac{32}{6.022 \times 10^{23}} = 5.3 \times 10^{-23} \text{ g}$

$$\text{II. 1 atom of N} = \frac{14}{6.022 \times 10^{23}} = 2.3 \times 10^{-23} \text{ g}$$

$$\text{III. } 10^{-10} \text{ g molecular weight of oxygen} \\ = 10^{-10} \times 32 = 3.2 \times 10^{-9} \text{ g}$$

$$\text{IV. } 10^{-10} \text{ g atomic weight of copper} \\ = 10^{-10} \times 63 = 6.3 \times 10^{-9} \text{ g}$$

$\therefore$ Order of increasing mass is

$\text{II} < \text{I} < \text{III} < \text{IV}$

45. (b) $\text{Mg}_3(\text{PO}_4)_2 : 1 \text{ mol}$

8 moles of O-atom are contained by 1 mole of $\text{Mg}_3(\text{PO}_4)_2$.

Hence, 0.25 mole of O-atom are contained by

$$= \frac{1}{8} \times 0.25 = 3.125 \times 10^{-2} \text{ mole of } \text{Mg}_3(\text{PO}_4)_2$$

46. (b) Mass of the given amount of a substance is a constant quantity, i.e. remains unchanged.

47. (b) According to law of conservation of mass,

Mass of reactants = Mass of products

$$\therefore 6.3 + 15.0 = 18.0 + x$$

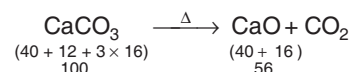
$$\text{or } x = 21.3 - 18.0 = 3.3 \text{ g}$$

48. (a) $\text{Na}_2\text{CO}_3 + 2\text{HCl} \longrightarrow 2\text{NaCl} + \text{H}_2\text{O} + \text{CO}_2$

In the above reaction, equivalent weight of Na_2CO_3 is $\frac{M}{2}$

because 2 moles of Na^+ being transferred per mole of Na_2CO_3 .

49. (b) 200 kg of 95% pure means $\frac{95}{100} \times 200 = 190 \text{ kg}$



$\therefore 100 \text{ g of } \text{CaCO}_3 \text{ on heating gives lime} = 56 \text{ g}$

$$\therefore 190 \text{ g of } \text{CaCO}_3 \text{ on heating will give lime} = \frac{56 \times 190}{100} \\ = 106.4 \text{ g}$$

50. (a) Number of O-atoms in 22 g of
- $\text{CO}_2 = 44 \text{ g}$

Molar mass of $\text{CO}_2 = 44 \text{ g}$ $44 \text{ g} = 1 \text{ mol}$, $22 \text{ g} = 0.5 \text{ mol}$ 1 mole of CO_2 contains $= 2 \times 6.023 \times 10^{23}$ O-atoms0.5 mole of CO_2 will contain $= 6.023 \times 10^{23}$ O-atomsNumber of O-atoms in $\text{CO} = 6.023 \times 10^{23}$ molecules of CO
 $= 1 \text{ mole of CO}$ 1 mole of $\text{CO} = 6.023 \times 10^{23}$ O-atomsMolar mass of $\text{CO} = 12 + 16 = 28 \text{ g}$

Element	% atomic weight	Simplest ratio
C	$\frac{92.3}{12} = 7.69$	$\frac{7.69}{7.69} = 1$
H	$\frac{7.7}{1} = 7.70$	$\frac{7.70}{7.69} = 1$

 $\therefore$ Empirical formula $= \text{CH}$

52. (c) Mass of 1 mole of methane (
- CH_4
-)
- $= 16 \text{ g}$

Mass of 0.1 mole of methane $= 16 \times 0.1 \text{ g} = 1.6 \text{ g}$

53. (a) Number of moles of sugar
- $= \frac{1.71}{342} = 0.005$

 $\therefore$ 1 mole of sugar contains $= 12 \times 6.02 \times 10^{23}$ atoms of carbon $\therefore$ 0.005 mole of sugar will contain $= 12 \times 0.005 \times 6.02 \times 10^{23}$ $= 0.36 \times 10^{23} = 3.6 \times 10^{22}$ C-atoms

54. (b) 2 g atom of
- $\text{N}_2 = 2 \times 14 = 28 \text{ g}$

 3×10^{23} C atoms $= \frac{3 \times 10^{23}}{6 \times 10^{23}} \times 12 = 6 \text{ g}$ 1 mole of S $= 32 \text{ g}$

55. (d) Atoms recovered
- $= \frac{19700}{197} \times N_A$
-
- $= 100 \times 6.02 \times 10^{23} = 6.02 \times 10^{25}$

56. (b)
- $\text{C}_6\text{H}_{12} + 9\text{O}_2 \longrightarrow 6\text{CO}_2 + 6\text{H}_2\text{O}$

 $6 \times 12 + 12 = 84$ $\therefore$ 84 g C_6H_{12} gives $= 6$ moles of CO_2 $\therefore$ 8.4 g C_6H_{12} will give $= \frac{6 \times 8.4}{84} = 0.6$ mole of CO_2

57. (d) Mol. wt. of
- $\text{NH}_3 = 14 + 3 = 17$

 $\therefore$ 17 g of ammonia contains atoms $= 4 \times N_A$ $(\because \text{one molecule of } \text{NH}_3 \text{ has 4 atoms})$ $\therefore$ 4.25 g of ammonia contain atoms $= \frac{4 \times 6.02 \times 10^{23} \times 4.25}{17} = 6.02 \times 10^{23}$ atoms

58. (d) Mole ratio of
- $x = \frac{75.8}{75} = 1.01$

Mole ratio of $y = \frac{24.2}{16} = 1.5$ Simple ratio $= x : y = 1 : 1.5 = 2 : 3 = x_2 y_3$

59. (c) Number of moles of
- $\text{CO}_2 = \frac{44}{44} = 1$

Number of moles of $\text{N}_2 = \frac{14}{28} = 0.5$ $\therefore$ Mole fraction of $\text{CO}_2 = \frac{1}{1 + 0.5} = \frac{1}{1.5}$ or $= \frac{2}{3}$

60. (c)
- $\text{Ca}_3\text{P}_2 + 6\text{H}_2\text{O} \longrightarrow 3\text{Ca}(\text{OH})_2 + 2\text{PH}_3$

61. (d)
- $N_1V_1 = N_2V_2$

i.e. $0.5 \times 100 = 0.1 \times V_2$ or $V_2 = 500 \text{ cm}^3$ $\therefore$ Water to be added to 100 cm^3 solution
 $= 500 - 100 = 400 \text{ cm}^3$

62. (a) Normality equation is

$$N_1V_1 + N_2V_2 + N_3V_3 = N_4(V_1 + V_2 + V_3)$$

$$\text{or } 1 \times 5 + 20 \times \frac{1}{2} + 30 \times \frac{1}{3} = N_4(5 + 20 + 30)$$

(HCl) (H_2SO_4) (HNO_3) $\therefore$ Resulting normality (N_4) $= \frac{25}{55} = 0.45 \text{ N}$

63. (c) 2 moles of Al produce
- $= 3 \times 22.4 \text{ L}$
- of
- H_2
- gas

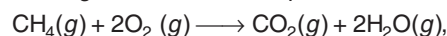
 $\therefore$ 1 mole of Al will produce $= \frac{3 \times 22.4}{2} = 33.62 \text{ L}$ of H_2 gas

64. (c) For solution I, millimoles
- $= MV = 480 \times 1.5 = 720$

For solution II, millimoles $= MV = 520 \times 1.2 = 624$ Total millimoles $= 720 + 624 = 1344$ $\therefore$ Molarity $= \frac{1344}{480 + 520} = 1.344 \text{ M}$

65. (c) Titration of oxalic acid by
- KMnO_4
- in the presence of HCl gives unsatisfactory result because HCl is a better reducing agent than oxalic acid and HCl reduces preferably
- MnO_4^-
- to
- Mn^{2+}
- .

66. (b) According to the chemical equation

44 g of $\text{CO}_2(g)$ is obtained from 16 g of $\text{CH}_4(g)$. $[\because 1 \text{ mole of } \text{CO}_2(g) \text{ is obtained from 1 mole of } \text{CH}_4(g)]$ Mole of $\text{CO}_2(g) = \frac{22 \text{ g } \text{CO}_2(g)}{44 \text{ g } \text{CO}_2(g)} = 0.5$ Hence, 0.5 mole of $\text{CO}_2(g)$ would be obtained from 0.5 mole of $\text{CH}_4(g)$ or 0.5 mole of $\text{CH}_4(g)$ would be required to produce 22 g of $\text{CO}_2(g)$.

67. (a) Number of moles
- $\propto \frac{1}{\text{Molecular mass}}$

Molecular masses : $\text{CO}_2 = 44$, $\text{N}_2 = 28$, $\text{CH}_4 = 16$,
and $\text{HCl} = 36.5$ CO_2 will have least volume.

68. (a) Molarity (
- M
-)
- $= \frac{\text{Weight of solute}}{\text{Molecular weight of solute}} \times \frac{1000}{\text{Volume of solution (mL)}}$
-
- $= \frac{49}{98} \times \frac{1000}{250} = 2 \text{ M}$

69. (b) $M_1V_1 = M_2V_2$

$$0.5 \times 100 = 0.1 \times V_2$$

$$V_2 = 500 \text{ cm}^3$$

Volume of water to be added to 100 cm³ of solution
 $= 500 - 100 = 400 \text{ cm}^3$

70. (a) Molality (m)

$$= \frac{\text{Weight of solute}}{\text{Molar weight of solute}} \times \frac{1000}{\text{Weight of solvent}}$$

$$= \frac{10}{40} \times \frac{1000}{100} = 2.5 \text{ m}$$

71. (c) 1500 cm³ of 0.1 N HCl

$$= \frac{0.1}{1000} \times 1500 = 0.15 \text{ g eq}$$

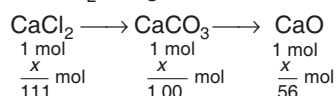
$$= 0.15 \text{ g eq. of NaOH}$$

$$= 0.15 \times 40 = 6 \text{ g NaOH}$$

BITSAT Archives

1. (a) $\text{CaCl}_2 + \text{NaCl} = 10 \text{ g}$

Let, weight of $\text{CaCl}_2 = x \text{ g}$



$$\text{Mole of CaO} = \frac{1.62}{56} \Rightarrow \frac{x}{111} = \frac{1.62}{56}, x = 3.21 \text{ g}$$

$$\% \text{ of CaCl}_2 = \frac{3.21}{10} \times 100 = 32.1\%$$

2. (b) Let, V litre of 10 N HCl be mixed with $(1 - V)$ litre of 4 N HCl to give $(V + 1 - V) = 1 \text{ L}$ of 7 N HCl

As we know that, $N_1V_1 + N_2V_2 = NV$

$$10V + 4(1 - V) = 7 \times 1$$

$$10V + 4 - 4V = 7, 6V = 7 - 4, V = \frac{3}{6} = 0.50 \text{ L}$$

Volume of 10 N HCl = 0.50 L

Volume of 4 N HCl = $1 - 0.50 = 0.50 \text{ L}$

3. (a) 1 mole of $\text{O}_2 = 4$ equivalent of oxygen

22400 mL of $\text{O}_2 = 4$ equivalent of oxygen

$$46.6 \text{ mL of } \text{O}_2 = \frac{4}{22400} \times 46.6 = 0.00832 \text{ eq.}$$

Equivalent of metal = Equivalent of oxygen

$$\frac{\text{Weight}}{\text{Equivalent}} = 0.00832$$

$$\frac{0.1}{E} = 0.00832 \Rightarrow E = \frac{0.1}{0.00832} = 12.0$$

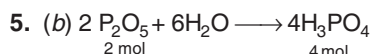


$\therefore$ 5 moles of $\text{Ba(OH)}_2 = 5$ moles of BaCO_3

$\therefore$ Mass of BaCO_3

$$= \text{Moles of BaCO}_3 \times \text{Molecular mass of BaCO}_3$$

$$= 5 \times 197 = 985 \text{ g}$$



$\therefore$ 1 L solution contains 4 moles of H_3PO_4 .

$\therefore$ Molarity of $\text{H}_3\text{PO}_4 = 4 \text{ M}$

Normality = Molarity $\times$ Basicity = $4 \times 3 \text{ N} = 12 \text{ N}$

6. (b) 1g H_2 combines with $\text{Br}_2 = 80 \text{ g}$

Equivalent weight of $\text{Br}_2 = 80$

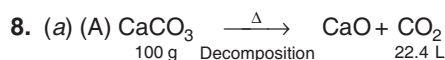
4g of Br_2 combine with = 1 g Ca

$$\therefore 80 \text{ g of Br}_2 \text{ will combine with} = \frac{1 \times 80}{4} = 20 \text{ g Ca}$$

7. (b) $N_1V_1 = N_2V_2, \frac{1}{2} \times 200 = \frac{1}{10} \times V_2$

$$V_2 = \frac{1}{2} \times 200 \times 10 = 1000 \text{ mL}$$

$\therefore$ Volume of water added = $1000 - 200 = 800 \text{ mL}$

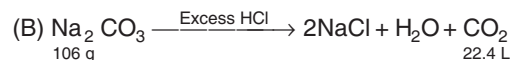


100 g Decomposition 22.4 L

$\therefore$ 100 g CaCO_3 on decomposition gives = 22.4 L CO_2

$\therefore$ 10 g CaCO_3 on decomposition will give

$$= \frac{22.4 \times 10}{100} \text{ L } \text{CO}_2 = 2.24 \text{ L } \text{CO}_2$$

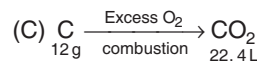


106 g 22.4 L

106 g Na_2CO_3 gives = 22.4 L CO_2

$$1.06 \text{ g Na}_2\text{CO}_3 \text{ will give} = \frac{22.4 \times 1.06}{106} \text{ L } \text{CO}_2 = 0.224$$

L CO_2

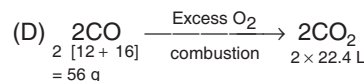


12 g combustion 22.4 L

12 g carbon on combustion gives = 22.4 L CO_2

2.4 g carbon on combustion will give

$$= \frac{22.4 \times 2.4}{12} \text{ L } \text{CO}_2 = 2 \times 2.24 \text{ L } \text{CO}_2 = 4.48 \text{ L } \text{CO}_2$$



2 [12 + 16] combustion 2 $\times$ 22.4 L
 $= 56 \text{ g}$

56 g carbon monoxide on combustion gives

$$= 2 \times 2.24 \text{ L } \text{CO}_2$$

0.56 g carbon monoxide on combustion will give

$$= \frac{2 \times 22.4 \times 0.56}{56} \text{ L } \text{CO}_2$$

$$= 0.448 \text{ L } \text{CO}_2$$

Hence, $A \rightarrow 4, B \rightarrow 1, C \rightarrow 2, D \rightarrow 3$

2

Atomic Structure

Introduction

In the later half of the 19th century and in the beginning of the 20th century the discovery of several scientists like Faraday, Thomson, Rutherford etc. clearly revealed that the atom is divisible and composed of several smallest particles like electrons, protons, neutrons etc. although still it is the smallest particle that takes part in a chemical reaction. After the discovery of these particles, several questions arise, about the nature of these particles, their arrangement in an atom etc. Our study of inorganic chemistry must therefore begin with a review of the structure and properties of atoms.

In this chapter we consider the development of our understanding of atomic structure and the behaviour of electrons in atoms.

Subatomic Particles

A large number of subatomic particles have been discovered so far but only electron, proton and neutron are of great importance among them and hence, are called **fundamental particles**.

Atom

Atomic structure is the study of structure of atom. The word atom is derived from a Greek word 'atomio' which means indivisible and indestructible.

The main facts related to atom are as follows:

1. Atom is the building block particle of matter.
2. Atom is made mainly from three subatomic particles viz. electron, proton and neutron.
3. **Charge to mass ratio (e/m)** is determined by using discharge tube and applying electric and magnetic field. Deviation of particle takes place when electric and magnetic field is applied to it.
4. **Millikan oil drop model** is used to measure charge of the electron and it was found that electrical charge is always an integral multiple of charge on electron.

$$q = ne$$

where,

q = total charge, e = charge of one electron = 1.602×10^{-19} C and

n = positive integer = 0, 1, 2, 3.

Cathode Rays

Cathode rays were discovered by Sir J J Thomson. Cathode rays were a stream of fast moving negatively charged particles, called **electrons**.

The specific charge is the ratio of charge to mass of an electron. It is found by Thomson and denoted as e/m ratio. The e/m ratio of electron was found to be same for all gases.

$$e/m \text{ ratio} = -1.758 \times 10^{11} \text{ C/kg.}$$

Anode Rays (Positive Rays or Canal Rays)

Positive rays were discovered by Goldstein. These rays consist of positively charged particles, called **protons**. Unlike cathode rays, their e/m value depends upon the nature of gas taken in the tube.

The e/m value is maximum, when hydrogen gas is taken in the tube, i.e. $9.58 \times 10^7 \text{ C / kg}$.

Neutrons

Neutrons are neutral particles and discovered by Chadwick. Neutrons are the heaviest particles of the atom.

Types of Atomic Species

There are various types of atomic species depending upon their different distribution of subatomic particles (e , p and n) within an atom.

- (i) **Isotopes** are atoms having same atomic number but different mass number.
e.g. ${}_1\text{H}^1$, ${}_1\text{D}^2$ and ${}_1\text{T}^3$ are three isotopes of hydrogen.
- (ii) **Isobars** are atoms having same mass number but different atomic number.
e.g. ${}_{18}\text{Ar}^{40}$, ${}_{19}\text{K}^{40}$ are isobars.
- (iii) **Isotones** are atoms having same number of neutrons but different atomic number and mass number, e.g. ${}_1\text{H}^3$ and ${}_2\text{He}^4$ are isotones.
- (iv) **Isoelectronic** are atoms or species having same number of electrons.
e.g. F^- , Ne and Na^+ are isoelectronic to each other.
- (v) **Isodiaphers** are atoms/species having same isotopic number and can be calculated as
Isotopic number
= mass number (A) – $2 \times$ atomic number (Z)
e.g. ${}_{19}\text{K}^{39}$ and ${}_{9}\text{F}^{19}$ are isodiaphers.

Atomic Models

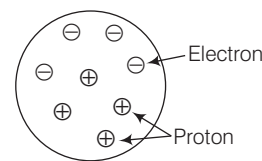
Various atomic models are proposed to explain the structure of atom which are as follows:

Thomson Model (1898)

This is also known by some other names like plum pudding, watermelon and raisin pudding model.

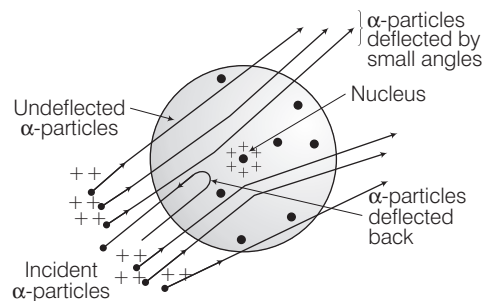
Thomson visualised atom as a pudding or cake of positive charge with electrons embedded into it.

It fails to explain the result of gold foil scattering experiment.



Rutherford Model (α -particle Scattering Experiment)

This model was set up by Rutherford to discover the nucleus. He used gold foil alpha particles set up and found the following results.



Representation of Rutherford Experiment

- (i) Most of the α -particles passed through the atom without any deflection.
- (ii) A small proportion of α -particles were deflected by small angles.
- (iii) A very few α -particles (about 1 in 20,000) returned back to their direction, i.e. reflected by 180° .

Conclusion of Rutherford Experiment

- (i) Atom consists of a very small sized nucleus where the entire mass of atom resides (diameter of nucleus = 10^{-15} m). Nucleus is made up of nucleons (neutrons + protons).
- (ii) Very large space inside the atom is empty which was confirmed when most of α -particles passed straight without any deflection.
- (iii) Volume occupied by the nucleus is very less as compared to total volume of atom.
- (iv) An electron in an atom revolves around the nucleus in stationary orbit and occupies almost all the space around nucleus (same as solar system).

Limitations of Rutherford Model

- Main drawback of this model is that it does not tell anything about electronic structure of atom.
- According to classical theory of electromagnetism, if a charged particle revolves around oppositely charged particle, it loses or radiates energy continuously resulting the decrease in its speed. Hence, the circular orbit gradually becomes spiral and electrons finally falls into the nucleus but in real sense, this does not happen.
- When spectrum of an atom is observed, it has been found that the spectrum consists of well defined lines rather than continuous spectrum (electron radiates energy continuously which gives continuous spectrum).

Development Leading to the Bohr's Model of Atom

Interaction of EMR with matter provides us various important information such as structure, energy, radius etc. EMR have dual nature, i.e. particle as well as wave nature.

Black Body Radiation and Planck's Quantum Theory

A body when heated emits radiation over a wide range of wavelength. The ideal body which emits or absorbs radiations of all possible frequency is known as **black body** and the radiation emitted by black body is called black body radiation.

Quantum is a measure of emission or absorption of energy in discrete quantity. Energy of radiation is directly proportional to frequency of light

$$E \propto \nu \Rightarrow E = h\nu$$

where, h = Planck's constant = 6.625×10^{-34} Js

The emitted energy is integral multiple of $h\nu$ such as $2h\nu, 3h\nu, 5h\nu$, i.e. energy is quantised.

Photoelectric Effect

When the light of certain frequency is allowed to fall on a metal surface, electrons are ejected from metal surface, this phenomenon is known as **photoelectric effect** and such ejected electrons are known as **photoelectrons**.

When a photon (the smallest particle of light) of certain energy strikes electron in an atom, it transfers its energy to electron. Greater the energy of photon greater will be energy transferred to electron.

The minimum amount of energy required to eject an electron from the metal surface is known as **threshold energy** and the frequency is known as **threshold frequency** (ν_0).

Threshold energy (work function) $E = h\nu_0$

Energy of photon = $h\nu$

Energy transferred from photon to electron
= Energy of photoelectrons = $h\nu - h\nu_0 = h(\nu - \nu_0)$

Kinetic energy of photoelectron = $\frac{1}{2}m_e v^2$

Energy transferred from photon to electron
= Kinetic energy of electron

$$\therefore h(\nu - \nu_0) = \frac{1}{2}m_e v^2$$

Bohr's Model

Bohr's model is a quantitative measure of general features of structure of hydrogen atom and its spectrum such as radius, energy, velocity of electron, etc.

Postulates of Bohr's Model

- Electrons revolve around the nucleus in specific circular path known as **orbit** or **stationary energy levels**, i.e. energy of each orbit is constant.
- When an electron absorbs required amount of energy, it undergoes transition from lower energy level to higher energy level and when it emits energy, it comes to the lower energy level.
- Energy absorbed or emitted during transition is given by

$$\Delta E = E_2 - E_1 = hc\bar{\nu} = \frac{hc}{\lambda} \quad \left[\because \bar{\nu} = \frac{1}{\lambda} \right]$$

where, $\bar{\nu}$ = wave number

Frequency of radiation absorbed or emitted during transition is given by ν .

$$\nu = \frac{E_2 - E_1}{h} \quad [\because \nu = c\bar{\nu}]$$

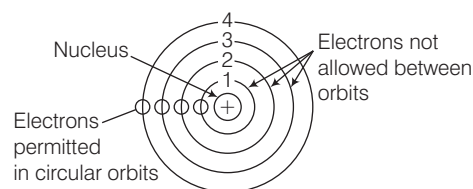
where, E_2 = higher energy level

E_1 = lower energy level

- The angular momentum of an electron present in any orbit must be an integral multiple of $\frac{h}{2\pi}$.

$$mvr = n \times \frac{h}{2\pi}$$

where, $n = 1, 2, 3, \dots$



Bohr's representation of an atom

Bohr's Parameters

(i) Radius of Bohr Orbit

Radius is the distance between nucleus and orbit, it is also known as Bohr's radius. The radius of an orbit is given by

$$\text{Bohr radius, } r_n = \frac{n^2 h^2}{4\pi^2 k m e^2 Z} = 0.529 n^2 \text{ \AA} \quad (\text{for H-atom})$$

where, m = mass of electron, h = Planck's constant,

e = charge of electron, $\pi = 3.14$,

n = orbit number, Z = nuclear charge,

$k = 1$ (in CGS system),

$= 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$ (in MKS system)

(ii) Energy of Electron in Bohr's Orbit

Bohr energy is the sum of kinetic energy and potential energy.

Bohr energy, $E = \text{KE} + \text{PE}$

$$= -\frac{2\pi^2 m Z^2 e^4 k^2}{n^2 h^2} = \frac{-13.6}{n^2} \text{ eV}$$

$$\text{For H-like atom, } E_n = \frac{-13.6}{n^2} \text{ eV or } -13.6 \times \left(\frac{Z^2}{n^2}\right) \text{ eV}$$

where, Z = atomic number

Ionisation energy of electron present in first orbital of H-atom is equal to energy of first orbit of H-atom. As the value of n (orbit number) increases, the value of Bohr's energy increases and becomes zero when $n = \infty$.

(iii) Velocity of an Electron

Velocity of electron in n th orbit is calculated by given formula.

$$v = \frac{2\pi e^2 k Z}{nh} = 2.1847 \times 10^6 \left(\frac{Z}{n}\right) \text{ ms}^{-1}$$

Velocity of electron present in first orbit of H

$$= v_H = 2.18 \times 10^6 \left(\frac{1}{1}\right) = 2.18 \times 10^6 \text{ ms}^{-1}$$

Velocity of electron present in first orbit of He

$$= v_{\text{He}} = 2.18 \times 10^6 \times \frac{2}{1} = 4.36 \times 10^6 \text{ ms}^{-1}$$

i.e. velocity of electron present in first orbit of He is double the velocity of electron present in first orbit of H.

Limitations of Bohr's Model

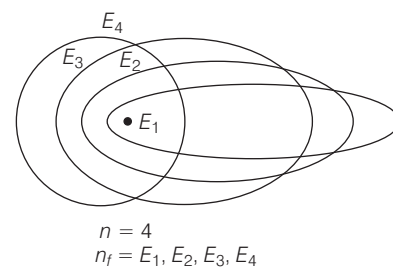
Bohr's model was not able to explain the following phenomena.

- Heisenberg uncertainty principle
- Dual character of particle
- Spectra of multielectron system

- Zeeman effect, i.e. splitting of spectral line in magnetic field.
- Stark effect, i.e. splitting of spectral line in an electric field.
- Fine structure of spectral line.

Bohr-Sommerfeld Model

In order to explain fine structure of H-atom, Sommerfeld suggested a model which states that electrons are present in elliptical orbit along with circular orbit. They revolve around the nucleus in two velocity



components viz. radial velocity and angular velocity.

Radial velocity along the radius vector and angular velocity which is perpendicular to radius vector give rise to radial and angular momentum simultaneously.

$$\text{Radial momentum} = n_r \frac{h}{2\pi}$$

$$\text{Angular momentum} = n_\phi \times \frac{h}{2\pi}$$

where, n_r = radial quantum number

n_ϕ = angular quantum number

Spectrum of Hydrogen Atom

A spectrum is defined as "pictorial representation of arrangement of radiation in the increasing order of wavelength or decreasing order of frequency".

Bohr's Explanation of Hydrogen Spectrum

Line spectrum of hydrogen was obtained when a beam of light was passed through a discharge tube containing hydrogen gas at low pressure.

Line spectrum of hydrogen atom is classified into the following six series:

Name of series		n_1	n_2
Lyman series	← UV	1	2, 3, 4, ...
Balmer series	← visible	2	3, 4, 5, ...
Paschen series	← IR	3	4, 5, 6, ...
Brackett series	← IR	4	5, 6, 7, ...
Pfund series	← IR	5	6, 7, 8, ...
Humphery series	← IR	6	7, 8, 9, ...

Spectrum of H-atom is expressed in terms of wave number $\bar{\nu}$,

$$\bar{\nu} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad \left[\bar{\nu} = \frac{1}{\lambda} \right]$$

where, $R = 1,09,677 \text{ cm}^{-1}$ (Rydberg constant)

n_1 = emission to lower energy level

n_2 = emission from higher energy level

For elements other than hydrogen,

$$\bar{\nu} = R_H Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

where, Z = atomic number of atom

Modern Structure of Atom

Following relationship and principle lead to explain the structure of atom

Dual Nature of Matter

de-Broglie proposed that microscopic particle show dual nature similarly as the radiation. Thus, de-Broglie relation can be stated as "All microscopic particles such as electron, proton, etc, shows wave as well as particle like behaviour, i.e. dual nature of behaviour."

$$\text{Mathematically, } \lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$$

$$\therefore \quad \begin{aligned} E &= q \cdot V \\ \text{and } p &= \sqrt{2mE} \end{aligned}$$

where, h = Planck's constant = $6.626 \times 10^{-34} \text{ Js}$

m = mass of microscopic particle

p = momentum of particle, v = velocity of particle

E = energy of particle, q = charge and

V = potential difference

This equation is known as de-Broglie relation and applicable for microscopic particle only.

Wave like nature of microscopic particle is confirmed by diffraction pattern of electron beam.

Heisenberg Uncertainty Principle

Heisenberg states that exact position and momentum of microscopic particle cannot be determined simultaneously.

$$\text{Mathematically, } \Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\text{or } \Delta x \cdot \Delta v \geq \frac{h}{4\pi m}$$

Quantum Mechanical Model and its Important Features

On the basis of dual nature of matter and Heisenberg uncertainty principle, E Schrödinger, in 1926 gave quantum mechanical model for an atom.

According to this model, yet exact position and momentum of particle cannot be determined although the probability of finding an electron can be determined.

Schrödinger Wave Equation

Schrodinger wave equation of any particle along all three directions may be written as

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0$$

where, ψ = acceptable wave function

E = total energy of electron = KE + PE

V = potential energy of electron

h = Planck's constant

$$\text{or, } \nabla^2 \psi + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0$$

On rearranging we get $H\psi = E\psi$

H = Hamiltonian operator or total energy operator

Now, for hydrogen-atom, it may be solved using wave function in polar coordinates

$$\psi = R(r) \cdot \theta(\theta) \cdot \phi(\phi)$$

Thus, an atomic orbital (ψ) is a product of two factors

- (i) Radial part, i.e. $R(r)$; depends upon distance and tells about the size of atom.
- (ii) Angular part, i.e. $\theta(\theta) \cdot \phi(\phi)$ depends upon angle variable θ and ϕ . It gives an idea about the shape of orbital.

Radial part depends upon principal quantum number (n) and azimuthal quantum number (l), while angular part depends upon azimuthal quantum number (l) and magnetic quantum number (m).

Variation of ψ and ψ^2 with r for 1s and 2s

ψ^2 has physical significance and it measures the probability of finding electron around nucleus.

ψ is a wave function of any orbital along 3 directions of polar coordinates may be written as

$$\psi(r, \theta, \phi) = \underset{\text{Radial part}}{R(r)} \cdot \underset{\text{Angular part}}{\theta(\theta), \phi(\phi)}$$

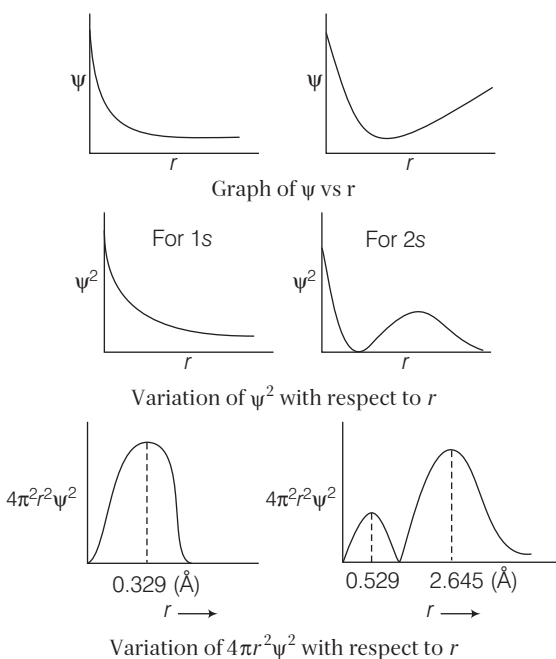
There are two ways to represent probability distribution curve:

- Radial probability distribution curve
- Angular probability distribution curve

Radial probability distribution curve tells about the variation of ψ^2 (probability of finding electron) with respect to radial distance (r), without any reference to its direction from nucleus.

For both 1s and 2s the shape of orbital is spherical, so it is more useful to discuss the probability of finding an electron between the sphere of radius ($r + dr$) and r .

$$\text{Volume of shell} = \frac{4}{3}\pi(r + dr)^3 - \frac{4}{3}\pi r^3 = 4\pi r^2 dr$$



Conclusion of Curves

- Electron finding probability is zero at nucleus.
- As distance from the nucleus increases, probability of finding electron increases as $1s < 2s < 3s < 4s \dots$
- Total number of peaks appearing in the curve for s , p and d -orbitals are equal to n , $(n - 1)$ and $(n - 2)$ respectively.
- The region where the probability density function (ψ^2) becomes zero is called **nodal surface** or simply node.
- For ns -orbital, number of nodes = $n - 1$

Calculation of Number of Nodes

$$\text{Number of radial nodes} = (n - l - 1)$$

$$\text{Number of angular nodes} = l$$

$$\text{Total nodes} = (n - 1)$$

$$\text{Number of nodal planes} = l$$

where,

- $l = 0$ for s -subshell,
- $= 1$ for p -subshell,
- $= 2$ for d -subshell,
- $= 3$ for f -subshell,

Electronic Configuration

Distribution of Electrons in Different Orbits (Shells)

It is the arrangement of electrons in various shells, subshells and orbitals of an atom.

Bohr Burry Scheme

The distribution of electrons into different orbits of an atom was suggested by Bohr and Burry in 1921. According to them, the electrons enter in the orbit in accordance with the following rules

- Maximum number of electrons in a shell is given by formula $2n^2$ ($n = 1, 2, 3, \dots$ for $K, L, M \dots$ shells)
- The outermost shell can have maximum of 8 electrons.

$n l^x$ Types Scheme

It is written as 2, 8, 8, 18, 32 or as $n l^x$ (where, n indicates the principal quantum number, l indicates the azimuthal quantum number or subshell and x is the number of electrons).

Rules for Filling Electrons in Orbitals

Various terms and rules have to be followed while filling up electrons in orbitals:

Aufbau Rule

Aufbau is a German word having mean "building up", i.e. the electrons are filled up in orbitals in the order of increasing energies, i.e. orbital having smaller energy will be filled first followed by higher energy orbitals. The arrangement of orbitals in the increasing order of energy are given as

$$1s, 2s, 2p, 3s, 3p, 4s, 3d, 4p, 5s, 4d, 5p, 4f, 5d, 6p, 7s \dots$$

The subshell having lowest $(n + l)$ value will be filled up first followed by higher value of $(n + l)$. If both the orbitals have same $(n + l)$ value then the orbital having lower value of n will be filled up first, this is known as $(n + l)$ rule.

Pauli Exclusion Principle

According to Pauli exclusion principle, "No two electrons can occupy same orbital with same value of all the four quantum numbers". An orbital can accommodate two electrons with opposite spin as shown in the given figure:



Two electrons can have same value of n , l and m but value of s can never be same.

e.g. in N-atom ($Z=7$): $1s^2, 2s^2, 2p^3$

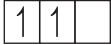
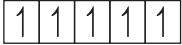
Q.No.	1s	2s	2p _x	2p _y	2p _z
n	1	2	2	2	2
l	0	0	1	1	1
m	0	0	+1	0	-1
s	$+\frac{1}{2}, -\frac{1}{2}$	$+\frac{1}{2}, -\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$

Hund's Rules for Maximum Spin Multiplicity

No electron pairing takes place in p , d and f -orbitals until each orbital in the given subshell contains one electron having parallel spin, e.g. N(7) has electronic configuration $1s^2, 2s^2, 2p_x^1, 2p_y^1, 2p_z^1$, according to Hund's rule and not $1s^2, 2s^2, 2p_x^2, 2p_y^1$.

Extra Stability of Half-filled and Completely Filled Orbitals

In case of half-filled and fully filled electronic configuration, the extra stability is due to the possibility of larger number of exchange of electron in degenerate energy levels which causes evolution of more exchange energy.

	
Incomplete unstable p -orbital	Half-filled stable d -orbital
	
Incomplete unstable f -orbital	Complete fully-filled stable f -orbital

Quantum Numbers

There are four quantum numbers

(i) Principal Quantum Number (n)

It is the measure of the effective volume of the electron cloud. It is represented by ' n '. Greater the value of ' n ' larger is the average distance of an electron in the orbital from the nucleus and therefore, larger is the orbital. The maximum number of electrons possible in a given shell is $2n^2$.

(ii) Angular Momentum or Azimuthal Quantum Number (l)

It gives the shape of the orbitals. Values of l varies from 0 to $(n-1)$.

Value of l	Subshell	Shape of orbital
0	s	Spherical
1	p	Dumb-bell
2	d	Double dumb-bell
3	f	Complicated

Orbital angular momentum,

$$mvr = \frac{h}{2\pi} \sqrt{l(l+1)}$$

(iii) Magnetic Quantum Number (m_l)

It says about orientation or distribution of atomic orbital.

Values of $m_l = -l$ to $+l$

Note

- Total number of orbitals present in any subshell $= 2l + 1$.
- The middle orbital of a subshell has a value of zero. Orbitals to the left of middle orbital have -ve numbers; to the right have +ve numbers.

$M_e = 0$	-1	0	+1	-2	-1	0	+1	+2	-3	-2	-1	0	+1	+2	+3
															
s	p			d				f							
$l = 0$	1			2				3							

(iv) Spin Quantum Number (m_s)

It gives the spin motion of electron and describes the behaviour of electron.

Value of $m_s \Rightarrow$ for clockwise spin ($\uparrow$),

$$s = +\frac{1}{2}$$

For anticlockwise spin ($\downarrow$),

$$s = -\frac{1}{2}$$

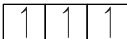
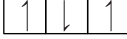
These spin produces spin angular momentum

$$= \sqrt{s(s+1)} \frac{h}{2\pi}$$

Practice Exercise

- The charge on electron was determined by
 - Schrodinger
 - Chadwick
 - Thomson
 - Millikan
- Deflection back of a few particles on hitting thin foil of gold shows that
 - nucleus is heavy
 - nucleus is small
 - Both a and b
 - electrons create hindrance in the movement of α -particles
- A photon of frequency ν has momentum
 - $h\nu/c$
 - hc/ν
 - $\frac{h}{\nu c}$
 - $\frac{\nu c}{h}$
- The energy of one quantum of light with a wavelength of $6500 \text{ \AA}$ ($1 \text{ \AA} = 10^{-10} \text{ cm}$)
 - $9.04 \times 10^{-24} \text{ J}$
 - $3.02 \times 10^{-20} \text{ J}$
 - $3.06 \times 10^{-19} \text{ J}$
 - $6.02 \times 10^{-20} \text{ J}$
- Sodium lamp emits yellow light of wavelength $5800 \text{ \AA}$. The wave number of the light is
 - $1.72 \times 10^6/\text{m}$
 - $17.2 \times 10^8/\text{m}$
 - $5.17 \times 10^{14}/\text{s}$
 - $17.2 \times 10^7/\text{m}$
- The ultraviolet radiation has frequency $6 \times 10^{16}/\text{s}$. The wavelength would be
 - $2 \times 10^6 \text{ cm}$
 - $5 \times 10^{-7} \text{ cm}$
 - $18 \times 10^{-26} \text{ cm}$
 - $2 \times 10^7 \text{ cm}$
- A radiation of $2000 \text{ \AA}$ falls on the metal whose work function is 4.2 eV . Then the kinetic energy of the fastest photo electron is
 - $6.4 \times 10^{-10} \text{ J}$
 - $16 \times 10^{-10} \text{ J}$
 - $1.6 \times 10^{-19} \text{ J}$
 - $3.2 \times 10^{-19} \text{ J}$
- The threshold wavelength for the ejection of electron from metal is 330 nm , then work function for the photoelectric emission is
 - $6 \times 10^{-12} \text{ J}$
 - $6 \times 10^{-19} \text{ J}$
 - $1.2 \times 10^{-20} \text{ J}$
 - $1.2 \times 10^{-18} \text{ J}$
- It is given that $E = \frac{2.859}{\lambda} \text{ cal/mol}$. The energy associated with radiation of wavelength $4 \times 10^{-5} \text{ m}$ will be
 - 71.5 kcal/mol
 - 35.75 kcal/mol
 - 32.0 kcal/mol
 - 7.15 kcal/mol
- The ratio of energy of a photon of $2000 \text{ \AA}$ radiation to that of $4000 \text{ \AA}$ radiation.
 - 2
 - 4
 - $\frac{1}{4}$
 - $\frac{1}{2}$
- Which of the following electromagnetic radiation has maximum wavelength?
 - Infrared
 - Ultraviolet rays
 - X-rays
 - Radiowave
- The kinetic energy of proton, whose mass is 140 times that of electron and potential difference V is
 - 1 KeV
 - 1840 KeV
 - 920 KeV
 - $\frac{1}{1840} \text{ KeV}$
- In first Bohr orbit of hydrogen atom, the velocity of electron would be (given that radius of first Bohr orbit is $0.53 \times 10^{-10} \text{ m}$)
 - $2.2 \times 10^6 \text{ m/s}$
 - $3.3 \times 10^6 \text{ m/s}$
 - $1.1 \times 10^6 \text{ m/s}$
 - $4.4 \times 10^6 \text{ m/s}$
- Bohr atomic model explains
 - finer details of atomic spectra
 - splitting of spectral lines in the presence of magnetic and electric field
 - spectra of hydrogen or hydrogen like atoms
 - intensity of various spectral lines
- The expression for Bohr radius of n th orbit of an atom is
 - $r = \frac{n^2 h^2}{4\pi^2 m Z e^2}$
 - $r = \frac{nh}{4\pi^2 m Z e^2}$
 - $r = \frac{n^2 h^2}{4\pi^2 m Z}$
 - $r = \frac{n^2 h^2}{4\pi^2 m e^2}$
- The potential energy of the electron in the hydrogen atom is -6.8 eV . The excited state in which the electron is present is
 - first
 - second
 - third
 - None of these
- The energy required for the ionisation of excited hydrogen atom would be (in eV)
 - < 13.6
 - > 13.6
 - 13.6
 - None of these
- For hydrogen atom, radius of first Bohr's orbit is a . For Li^{2+} , the radius of first Bohr's orbit is
 - $27a$
 - $3a$
 - $\frac{a}{27}$
 - $\frac{a}{3}$
- If the velocity of an electron in the first orbit of hydrogen atom is approximately $2.2 \times 10^8 \text{ cm/s}$, its velocity in the fourth orbit would be
 - $5.5 \times 10^7 \text{ cm/s}$
 - $4.4 \times 10^7 \text{ cm/s}$
 - $3.3 \times 10^7 \text{ cm/s}$
 - $6.6 \times 10^7 \text{ cm/s}$

20. Energy of an electron in hydrogen atom is given by $E = -\frac{13.6}{n^2}$ eV. If n is changed from 1 to 4, then energy of 1 is
 a. four times that of 4 b. five times that of 4
 c. sixteen times that of 4 d. equal that of 4
21. The radii of nuclei and atoms are known to be of the order of 10^{-13} cm and 10^{-8} cm respectively assuming them to be spherical. The fraction of atomic volume occupied by the nucleus would be
 a. 10^{-15} b. 10^{-17}
 c. 10^{-14} d. 10^{-16}
22. Which of the following statements is false?
 a. Spin quantum number corresponds to two possible directions of spin
 b. No two electrons in an atom may have the same set of values for the four quantum numbers
 c. Number of electrons in an atom is equal to the number of orbitals
 d. The momentum and position of an electron cannot be known simultaneously
23. For hydrogen atom, radius of first Bohr orbit would be
 a. 9.63×10^{-10} m b. 0.43×10^{-10} m
 c. 0.50×10^{-10} m d. 0.53×10^{-10} m
24. The relation between radius of third orbit r_3 and radius of first orbit r_1 in hydrogen atom would be
 a. $r_3 = 3r_1$ b. $r_3 = 9r_1$ c. $r_1 = 3r_3$ d. $r_1 = 9r_3$
25. The electron of hydrogen atom is excited to certain level. When the electron returns to the first Bohr orbit, the wavelength of line emitted, if the energy difference is 11.0 eV would be
 a. 11.25×10^{-7} m b. 2.2×10^{-6} m
 c. 9.1176×10^{-8} m d. 1.22×10^{-7} m
26. Ionisation energy of He^+ is -19.6×10^{-18} J. The energy of the first stationary state ($n = 1$) of Li^{2+} is
 a. 4.41×10^{-16} J atom $^{-1}$ b. -4.41×10^{-17} J atom $^{-1}$
 c. -2.2×10^{-15} J atom $^{-1}$ d. 8.82×10^{-17} J atom $^{-1}$
27. Assuming atom to be a sphere, the radius of atom would be [if the apparent volume of atom of a metal is 1.23×10^{-23} mL]
 a. 1.401 Å b. 1.942 Å c. 1.642 Å d. 1.541 Å
28. In hydrogen an electron transition takes place from $n = 2$ level to $n = 3$ level. The wavelength of the line in the hydrogen spectrum would be
 a. 5.485×10^7 m b. 6.56×10^{-7} m
 c. 4.57×10^{14} m d. 0.529×10^{-10} m
29. The line spectra are characteristics of
 a. molecules in ground state
 b. atoms in ground state
 c. molecules in the excited state
 d. atoms in excited state
30. In Bohr's series of lines of hydrogen spectrum, the third line from the red end corresponds to which one of the following inter orbit jumps of the electron for Bohr's orbit in an atom of hydrogen?
 a. $5 \rightarrow 2$ b. $4 \rightarrow 1$ c. $2 \rightarrow 5$ d. $3 \rightarrow 2$
31. The wavelength of the radiation emitted when the electron jumps from 4th shell to 2nd shell is
 a. 4862 Å b. 2056 Å c. 5241 Å d. 109700 cm
32. First line of Paschen series has wave number ($R_H = 109700 / \text{cm}$)
 a. $2854 \times 10^8 (\text{Å})^{-1}$ b. $6243 \times 10^8 (\text{Å})^{-1}$
 c. $6856 \times 10^8 (\text{Å})^{-1}$ d. $3452 \times 10^8 (\text{Å})^{-1}$
33. Which of the following statement is false?
 a. Violet radiations have a longer wavelength than red radiations
 b. The energy of light with $\lambda = 600$ nm is lower than that of $\lambda = 500$ nm
 c. Spectrum of an atom is known as line spectrum
 d. The wavelength associated with an electron is longer than that of proton if they have the same speed
34. The effect of splitting of spectral lines under the influence of magnetic field is called
 a. photoelectric effect b. Zeeman effect
 c. Raman effect d. Stark effect
35. In the influence of electric field the splitting of spectral lines is called
 a. Zeeman effect b. Stark effect
 c. photoelectric effect d. None of these
36. The visible part of electromagnetic radiations is
 a. 2000 Å to 4000 Å b. 2000 Å to 5000 Å
 c. 4000 Å to 8000 Å d. 1000 Å to 2000 Å
37. In hydrogen spectrum wave number of different lines is given by $\frac{1}{\lambda} = R_H \left[\frac{1}{n_f^2} - \frac{1}{n_i^2} \right]$
 where, $R_H = 1.090678 \times 10^7 \text{ m}^{-1}$
 The wavelength of first line of Lyman series would be
 a. 122×10^{-7} m b. 9.1176×10^{-8} m
 c. 12.2×10^{-7} m d. 1.22×10^{-7} m
38. The mass of a photon of wavelength 1.54 Å is
 a. 2.5×10^{-32} kg b. 1.42×10^{32} kg
 c. Both of these d. None of these
39. The uncertainty in the velocity of moving bullet of mass 10 g, when uncertainty in its position is 10^{-5} m is
 a. 5.2×10^{-28} m/s b. 5.2×10^{-22} m/s
 c. 3×10^{-28} m/s d. 3×10^{-22} m/s
40. The momentum of a particle having de-Broglie wavelength of 6 Å is
 a. 1.1×10^{34} kg-m/s b. 39.6×10^{-34} kg-m/s
 c. 1.1×10^{-24} kg-m/s d. 39.6×10^{-24} kg-m/s

41. When an electron is moving uniformly, it produces
 a. both electric and magnetic fields
 b. an electric field only
 c. a magnetic field only
 d. no such fields
42. Which of the following scientists demonstrated the wave nature of electron?
 a. Davisson
 b. Heisenberg
 c. de-Broglie
 d. Schrödinger
43. Which one of the following statements is most appropriate?
 a. Electron spins around its own axis only
 b. Electron moves around the nucleus in spherical orbits
 c. Electron moves around the nucleus in elliptical orbits
 d. Electron moves around the nucleus in spherical or elliptical orbits spins around its own axis
44. The maximum number of electrons that can be taken by a subshell with $l = 3$ is
 a. 8
 b. 14
 c. 10
 d. 12
45. The spectrum of He^+ is expected to be similar to that of
 a. hydrogen
 b. Li^+
 c. Na
 d. He
46. The orbital with $n = 3$ and $l = 2$ is
 a. $5d$
 b. $3d$
 c. $4d$
 d. $5s$
47. If n and l are respectively the principal and azimuthal quantum numbers, then which of the following expression represent the total number of electrons in any energy level
 a. $\sum_{l=0}^{l=n} 2(2l+1)$
 b. $\sum_{l=1}^{l=n-1} 2(2l+1)$
 c. $\sum_{l=0}^{l=n+1} 2(2l+1)$
 d. $\sum_{l=0}^{l=n-1} 2(2l+1)$
48. Which of the following set of quantum number is not applicable for an electron in an atom?
 a. $n = 2, l = 0, m = 0, s = +1/2$
 b. $n = 1, l = 0, m = 0, s = -1/2$
 c. $n = 1, l = 0, m = 0, s = +1/2$
 d. $n = 1, l = 1, m = 1, s = +1/2$
49. The maximum number of electrons that d -orbital can contain is
 a. 10
 b. 2
 c. 14
 d. 6
50. The shape of the orbital is determined by
 a. spin quantum number
 b. magnetic quantum number
 c. azimuthal quantum number
 d. principal quantum number
51. For the valency electron in copper, the four quantum numbers are
 a. $n = 4, l = 0, m = 0, s = +1/2$
 b. $n = 4, l = 2, m = 2, s = -1/2$
 c. $n = 4, l = -2, m = 2, s = +1/2$
 d. $n = 4, l = 2, m = 0, s = +1/2$
52. What will be the uncertainty in velocity of a cricket ball of 100 g if the uncertainty in its position is $1.65 \text{ \AA}$?
 a. $\frac{10^{-23}}{\pi} \text{ ms}^{-1}$
 b. $\frac{6.6}{\pi} 10^{-45} \text{ ms}^{-1}$
 c. $4.65 \times 10^{-43} \text{ ms}^{-1}$
 d. $\frac{10^{-26}}{\pi} \text{ ms}^{-1}$
53. The electrons, identified by quantum numbers n and l ,
 (i) $n = 4, l = 1$ (ii) $n = 4, l = 0$ (iii) $n = 3, l = 2$
 (iv) $n = 3, l = 1$ can be placed in increasing order of energy, from the lowest to highest, as
 a. (iv) < (ii) < (iii) < (i)
 b. (ii) < (iv) < (i) < (iii)
 c. (i) < (iii) < (ii) < (iv)
 d. (iii) < (i) < (iv) < (ii)
54. An element has atomic number 37. The electronic configuration of the element is
 a. $(2, 8), 3s^2 3p^6 3d^{10}, 4s^2 5s^6 4p^5$
 b. $(2, 8), 3s^2 3p^6 3d^{10}, 4s^2 4p^6, 5s^1$
 c. $(2, 8), 3s^2 3p^6, 4s^2 3d^9 5s^1 4p^5$
 d. None of the above
55. The number of unpaired electrons in ferrous ion is
 a. 5
 b. 2
 c. 4
 d. 3
56. The triad of the nuclei that is isotonic
 a. ${}^{14}_6\text{C}, {}^{14}_7\text{N}, {}^{19}_9\text{F}$
 b. ${}^{14}_6\text{C}, {}^{15}_7\text{N}, {}^{17}_9\text{F}$
 c. ${}^{14}_6\text{C}, {}^{14}_7\text{N}, {}^{17}_9\text{F}$
 d. ${}^{12}_6\text{C}, {}^{14}_7\text{N}, {}^{19}_9\text{F}$
57. In which of the following orbital diagram Aufbau principle is not violated
 a.  
 b.  
 c.  
 d.  
58. Nitrogen has electronic configuration $1s^2 2s^2 2p_x^1 2p_y^1 2p_z^1$. This is because of
 a. Pauli's exclusion principle
 b. $(n+1)$ rule
 c. Hund's rule
 d. Uncertainty principle

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- Which of the following relation is incorrect regarding Bohr's theory? [2014]
 - Velocity of electron $\propto \frac{1}{n}$
 - Frequency of revolution $\propto \frac{1}{n^2}$
 - Radius of orbit $\propto n^2/Z$
 - Force on electron $\propto \frac{1}{n^4}$
- Bohr theory is applicable to [2013]
 - He
 - Li²⁺
 - He²⁺
 - None of these
- If the radius of H is 0.53 Å then what will be the radius of ${}_3\text{Li}^{2+}$? [2012]
 - 0.17 Å
 - 0.36 Å
 - 0.53 Å
 - 0.59 Å
- Which of the following has the largest de-Broglie wavelength, given that all have equal velocity? [2011]
 - CO₂ molecule
 - NH₃ molecule
 - Electron
 - Proton
- The wave number of a spectral line is $5 \times 10^5 \text{ m}^{-1}$. The energy corresponding to this line is [2010]
 - $3.49 \times 10^{-23} \text{ kJ}$
 - $4.45 \times 10^{-24} \text{ J}$
 - $5.50 \times 10^{-22} \text{ J}$
 - $9.93 \times 10^{-23} \text{ kJ}$
- Energy of third orbit of Bohr's atom is [2009]
 - 13.6 eV
 - 3.4 eV
 - 1.5 eV
 - None of these
- An electronic transition in hydrogen atom result in the formation of H α line of hydrogen in Lyman series, the energies associated with the electron in each of the orbits involved in the transition (in kcal mol⁻¹) are [2008]
 - 313.6, -34.84
 - 313.6, -78.4
 - 78.4, -34.84
 - 78.4, -19.6
- The velocities of two particles A and B are 0.05 and 0.02 ms⁻¹ respectively. The mass of B is five times the mass of A. The ratio of their de-Broglie's wavelength is [2008]
 - 2 : 1
 - 1 : 4
 - 1 : 1
 - 4 : 1
- Cr has electronic configuration as [2007]
 - $3s^2 3p^2 3d^4, 4s^1$
 - $3s^2 3p^6 3d^5, 4s^1$
 - $3s^2 3p^6 3d^6$
 - None of these
- The number of waves in an orbit are [2006]
 - n^2
 - n
 - $n-1$
 - $n-2$
- The probability of finding the electron in the orbital is [2006]
 - 100%
 - 90-95%
 - 70-80%
 - 50-60%
- The velocity of electron in first orbit of H-atom as compared to the velocity of light is [2005]
 - $\frac{1}{10}$ th
 - $\frac{1}{100}$ th
 - $\frac{1}{1000}$ th
 - same

Answer with Solutions

Practice Exercise

1. (d)

2. (c)

3. (c) $\lambda = \frac{h}{p}$

where, p = momentum

$$h = \text{Planck's constant} \Rightarrow P = \frac{h}{\lambda} = \frac{h}{c\nu}$$

4. (c) $E = h\nu = \frac{hc}{\lambda} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{6500 \times 10^{-10}}$

$$E = 3.06 \times 10^{-19} \text{ J}$$

5. (a) Wave number = $\frac{1}{\lambda}$

$$= \frac{1}{5800 \times 10^{-10} \text{ m}} = 1.724 \times 10^6 / \text{m}$$

6. (b) $\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{6 \times 10^{16}} = 0.5 \times 10^{-8} \text{ m} = 5 \times 10^{-7} \text{ cm}$

7. (d) Kinetic energy (KE) = $h\nu - W$

$$= \frac{hc}{\lambda} - 4.2 \text{ eV} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{2000 \times 10^{-10}} - 4.2 \times 1.6 \times 10^{-19}$$

$$= 9.9 \times 10^{-19} \text{ J} - 6.7 \times 10^{-19} \text{ J} = 3.2 \times 10^{-19} \text{ J}$$

8. (b) $W = h\nu_0$

$$W = \frac{hc}{\lambda_0} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{330 \times 10^{-9}} = 6 \times 10^{-19} \text{ J}$$

9. (a) Given, $E = \frac{2.859}{\lambda} \text{ cal/mol}$

$$\lambda = 4 \times 10^{-7}$$

$$E = \frac{2.859 \text{ cal/mol}}{4 \times 10^{-5} \text{ m}} = 71.5 \text{ kcal/mol}$$

10. (a) $E = \frac{hc}{\lambda}$

$$\frac{E_1}{E_2} = \frac{\lambda_2}{\lambda_1} = \frac{4000 \text{ Å}}{2000 \text{ Å}} = 2$$

11. (d) Radiowave has maximum wavelength of the order of 10^2 in metre.

12. (a) According to law of conservation of energy, "Sum of potential energy and kinetic energy remain constant".
Now initially proton starts from rest. Hence, kinetic energy of proton at initial point is zero but it has potential energy.

$$PE = qV$$

$$\text{where, } q = 1.6 \times 10^{-19} \text{ C and } V = 1000 \text{ V}$$

Assuming the proton no longer experiences the potential energy and it is all converted to kinetic energy

$$PE^* = 0, KE^* = \frac{1}{2}mv^2$$

$$\text{Since, } KE + PE = \text{Total energy} = PE^* + KE^*$$

$$qV + 0 = 0 + \frac{1}{2}mv^2 \quad \text{or} \quad KE = qV = 1.6 \times 10^{-16} \text{ J} = 1 \text{ KeV}$$

13. (a) From Bohr postulate

$$mvr = \frac{nh}{2\pi} \quad \text{or} \quad v = \frac{nh}{2\pi mr}$$

$$v = \frac{1 \times 6.625 \times 10^{-34}}{2 \times 3.14 \times 9.1 \times 10^{-31} \times 0.53 \times 10^{-10}} = 2.2 \times 10^6 \text{ m/s}$$

14. (c) Spectra of hydrogen or hydrogen like atoms such as He^+ , Li^{2+} were successfully explained by Bohr's model.

15. (a) $r = \frac{n^2 h^2}{4\pi^2 m Z e^2} = R_0 \frac{n^2}{Z} = 0.529 \frac{n^2}{Z} \text{ Å}$

16. (a) Total energy = $E_n = \frac{1}{2}$ (Potential energy)

$$E_n = \frac{-6.8}{2} = -3.4 \text{ eV}$$

$$\text{We know that, } E_n = -\frac{13.6 \text{ eV}}{n^2}$$

$$-3.4 \text{ eV} = \frac{-13.6 \text{ eV}}{n^2}$$

$$n^2 = 4$$

$$n = 2 \text{ First excited state}$$

17. (a) ($n = 2$) $\rightarrow$ Energy of first excited state = -3.4 eV

($n = 3$) $\rightarrow$ Energy of second excited state = -1.51 eV

18. (d) $\frac{r(\text{H})}{r(\text{Li}^{2+})} = \frac{Z_{\text{Li}^{2+}}}{Z_{\text{H}}}$

$$r(\text{Li}^{2+}) = \frac{Z_{\text{H}}}{Z_{\text{Li}^{2+}}} r_{\text{H}} = \frac{1 \times a}{3} = \frac{a}{3}$$

19. (a) $mvr = \frac{nh}{2\pi}$

$$v = \frac{hn}{2\pi mr}$$

$$\frac{v_1}{v_2} = \frac{r_2}{r_1} \Rightarrow v_2 = \left(\frac{r_1}{r_2}\right) v_1 \Rightarrow v_2 = \frac{1}{4} v_1$$

$$v_2 = \frac{1}{4} \times 2.2 \times 10^8$$

$$v_2 = 0.55 \times 10^8 \text{ cm/s}$$

$$= 5.5 \times 10^7 \text{ cm/s}$$

20. (c) $E \propto \frac{1}{n^2}$

$$\frac{E_1}{E_4} = \frac{(4)^2}{(1)^2}$$

$$E_4 = \frac{E_1}{16}$$

21. (a) $\frac{\text{Volume of nucleus}}{\text{Volume of an atom}} = \text{fraction of atom}$
Volume occupied by nucleus

$$\frac{\frac{4}{3}\pi (10^{-13})^3}{\frac{4}{3}\pi (10^{-8})^3} = 10^{-15}$$

22. (c) Number of electrons in an atom is equal to the number of orbitals.

23. (d) $r = \frac{n^2 h^2}{4\pi^2 m e^2 Z}$

$$n = 1, h = 6.62 \times 10^{-34} \text{ Js}^{-1}$$

$$\therefore r = 0.53 \times 10^{-10} \text{ m} = 0.53 \text{ Å}$$

24. (b) $r = \frac{n^2 h^2}{4\pi^2 m e^2 Z}$, i.e. $r \propto n^2$

25. (a) $E_2 - E_1 = \Delta E = \frac{hc}{\lambda}$

$$\lambda = \frac{hc}{\Delta E} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{11.0 \times 1.6 \times 10^{-19}}$$

$$\lambda = 11.25 \times 10^{-7} \text{ m}$$

26. (b) $IE = -E_1$

$$E_1 \text{ for } \text{He}^+ = -19.6 \times 10^{-18} \text{ J atom}^{-1}$$

$$\frac{(E_1)_{\text{He}^+}}{(E_1)_{\text{Li}^{2+}}} = \frac{(Z_{\text{He}^+})^2}{(Z_{\text{Li}^{2+}})^2} \Rightarrow \frac{-19.6 \times 10^{-18}}{(E_1)_{\text{Li}^{2+}}} = \frac{4}{9}$$

$$\text{or } E_1(\text{Li}^{2+}) = \frac{-19.6 \times 9 \times 10^{-18}}{4} = -4.41 \times 10^{-17} \text{ J atom}^{-1}$$

27. (a) $V = \frac{4}{3}\pi r^3$

$$\frac{3V}{4\pi} = r^3 \Rightarrow r = \left(\frac{3V}{4\pi}\right)^{1/3}$$

$$\log r = \frac{1}{3} \log \left[\frac{3V}{4\pi} \right]$$

$$\log r = \frac{1}{3} \log \left[\frac{3 \times 1.23 \times 10^{-23}}{4 \times 3.14} \right]$$

$$28. (b) \frac{1}{\lambda} = 109677 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] = 6.56 \times 10^{-7} \text{ m}$$

29. (d) Radiations obtained by exciting a substance has dark space between two consecutive lines. This is line spectrum.

30. (a) The lines at the red end belong to Balmer series. These are obtained for jumps to $n=2$ from outer orbits (1st line from $n=3$, 2nd line from $n=4$, 3rd line from $n=5$).

31. (a) According to Balmer equation

$$\begin{aligned} \text{Wave number } (\bar{\nu}) &= 109677 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \text{ cm}^{-1} \\ \bar{\nu} &= 109677 \left(\frac{1}{(2)^2} - \frac{1}{(4)^2} \right) \text{ cm}^{-1} \\ &= \frac{109677 \times 3}{16} \text{ cm}^{-1} \\ \lambda &= \frac{1}{\bar{\nu}} = \frac{16}{109677 \times 3} = 4862 \times 10^{-8} \text{ cm} \\ &= 4862 \times 10^{-10} \text{ m} = 4862 \text{ Å} \end{aligned}$$

32. (c) $n_i = 3, n_f = 4$

$$\frac{1}{\lambda} = 109677 \left[\frac{1}{n_i^2} - \frac{1}{n_f^2} \right] = 6856 \times 10^8 (\text{Å})^{-1}$$

33. (a) Violet radiations have a lower wavelength than red radiations.

34. (b) The splitting of spectral lines under the influence of magnetic field is called Zeeman effect.

35. (b) In the influence of electric field the splitting of spectral lines is called Stark effect.

36. (c) The visible part of electromagnetic radiation is 4000 Å to 8000 Å.

37. (d) For first line in Lyman series

$$n_i = 1, n_f = 2$$

$$38. (b) \lambda = \frac{h}{mv} = \frac{h}{\lambda \nu}$$

$$\text{where, } \nu = 3 \times 10^8 \text{ m/s; } h = 6.625 \times 10^{-34} \text{ Js}$$

$$\lambda = 1.54 \times 10^{-10} \text{ m}$$

$$\therefore m = 1.42 \times 10^{-32} \text{ kg}$$

$$39. (a) \Delta x \cdot \Delta p = \frac{h}{4\pi} \text{ or } \Delta x \cdot m \Delta v = \frac{h}{4\pi}$$

$$\begin{aligned} \Delta v &= \frac{h}{4\pi \Delta x m} = \frac{6.626 \times 10^{-34} \text{ Js}}{4 \times 3.14 \times 10^{-5} \text{ m} \times 10^{-2} \text{ kg}} \\ &= 5.2 \times 10^{-28} \text{ m/s} \end{aligned}$$

$$40. (c) p = \frac{h}{\lambda} = \frac{6.62 \times 10^{-34}}{6 \times 10^{-10}}$$

$$p = 1.1 \times 10^{-24} \text{ kg-m/s}$$

41. (a) Moving electron produces both electric and magnetic fields.

42. (c) Electron behaves as a material as well as wave, this was proposed by de-Broglie.

$$\lambda = \frac{h}{mv}$$

43. (d) Electron moves around the nucleus in spherical or elliptical orbits and spins around its own axis.

44. (b) When $l = 3, m = -3, -2, -1, 0, 1, 2, 3$. Each m can have two electrons with opposite spins, thus maximum number of electrons = $2 \times 7 = 14$

45. (a) Both H and He^+ have one electron.

$$46. (b) 3d \text{ orbital } \begin{bmatrix} l=0, 1, 2, 3 \\ =s, p, d, f \end{bmatrix}$$

47. (d) e.g. when $n = 3, l = 0, 1, 2$

$$\begin{aligned} \text{Total electrons} &= \sum_{l=0}^2 2(2l+1) \\ &= 2 + 2[2 \times 1 + 1] + 2[2 \times 2 + 1] \\ &= 2 + 6 + 10 = 18 \text{ electrons} \end{aligned}$$

48. (d) For $n=1, l=1$ not possible

$$\text{Since, } l = 0 \text{ to } (n-1)$$

49. (a) 10 electrons

$$d\text{-orbital } \begin{array}{ccccc} +2 & +1 & 0 & -1 & -2 \\ \uparrow\downarrow & \uparrow\downarrow & \uparrow\downarrow & \uparrow\downarrow & \uparrow\downarrow \end{array}$$

50. (c) The shape of the orbital is determined by azimuthal quantum number.

51. (a) The valence electronic configuration of copper is $3d^{10} 4s^1$

$$\text{Hence, } n=4, l=0, m=0, s=+1/2$$

52. (a) According to Heisenberg uncertainty principle,

$$\Delta x \times m \Delta v = \frac{h}{4\pi}$$

$$1.65 \times 10^{-10} \times \frac{100}{1000} \times \Delta v = \frac{6.625 \times 10^{-34}}{4\pi}$$

$$\Delta v = \frac{6.625 \times 10^{-34+10}}{1.65 \times 4\pi} \times 10 = \frac{10^{-23}}{\pi} \text{ m/s}$$

53. (a) (i) $4p$ (ii) $4s$ (iii) $3d$ (iv) $3p$

$$\text{Order of increasing energy is } 3p < 4s < 3d < 4p$$

54. (b) $\text{Rb} (Z=37); (2, 8), 3s^2 3p^6 3d^{10}, 4s^2 4p^6, 5s^1$

55. (c) Ferrous (Fe^{2+}) ion. Its electronic configuration is d^6 .

$$\begin{array}{cccccc} \uparrow\downarrow & \uparrow\downarrow & \uparrow\downarrow & \uparrow\downarrow & \uparrow\downarrow & \uparrow\downarrow \end{array}$$

$$\text{Number of unpaired electrons} = 4$$

56. (b) Number of neutrons are

$$\text{C} \longrightarrow 14 - 6 = 8$$

$$\text{N} \longrightarrow 15 - 7 = 8$$

$$\text{F} \longrightarrow 17 - 9 = 8$$

57. (b)

$$58. (c) \text{N} - \begin{array}{ccccc} 1s & 2s & 2p_x & 2p_y & 2p_z \\ \uparrow\downarrow & \uparrow\downarrow & \uparrow\downarrow & \uparrow\downarrow & \uparrow\downarrow \end{array}$$

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1. (c) Radius of orbit is directly proportional to ratio of square of principal quantum number and atomic number

i.e. Radius of orbit $\propto \frac{n^2}{Z}$

For H-atom $r_n = \frac{n^2 \times 0.529 \times 10^{-8}}{Z} \text{ cm}$

2. (b) Bohr's theory is applicable to H-like species containing one electron only, e.g. Li^{2+} .
3. (a) The radius of hydrogen atom = 0.53 Å. ${}_3\text{Li}^{2+}$ ion also has only one electron but it has 3 protons in nucleus, hence, its electron feels three times more attraction from nucleus in comparison to hydrogen atom. Thus, the radius of ${}_3\text{Li}^{2+}$ will be

$$= \frac{0.53}{3} = 0.17 \text{ Å}$$

4. (c) $\lambda = h / mv$

For same velocity $\lambda \propto 1/m$

Electron has the least mass, so its wavelength is maximum.

5. (d) Energy $E = \frac{hc}{\lambda} = hc \bar{\nu}$ $\left(\because \bar{\nu} = \frac{1}{\lambda} \right)$
- $$= 6.62 \times 10^{-34} \times 3 \times 10^8 \times 5 \times 10^5$$
- $$= 9.93 \times 10^{-23} \text{ kJ}$$

6. (c) $E_n = \frac{-13.6 Z^2}{n^2} \text{ eV}$

where, Z = atomic number and n = number of orbit

For H, $Z = 1$, and for 3rd orbit $n = 3$

$$\therefore E_3 = -\frac{13.6}{(3)^2} = -\frac{13.6}{9} = -1.5 \text{ eV}$$

7. (b) Energy of an electron in n th orbit,

$$E_n = \frac{2\pi^2 Z^2 e^4}{n^2 h^2}$$

On substituting the values of k , m , e and h , we get

$$E_n = -\frac{2.172 \times 10^{-18} Z^2}{n^2} \text{ J atom}^{-1}$$

$$= \frac{1311.8 Z^2}{n^2} \text{ kJ mol}^{-1} = -\frac{313.52 Z^2}{n^2} \text{ kcal mol}^{-1}$$

[$\therefore 1 \text{ kcal} = 4.184 \text{ kJ}$]

For H-atom,

$$Z = 1$$

For Lyman series,

$$n_1 = 1, n_2 = 2$$

Energy of electron in n_1 orbit

$$= -\frac{313.52 \times (1)^2}{(1)^2} \text{ kcal mol}^{-1}$$

$$= -313.52 \text{ kcal mol}^{-1}$$

$$\approx -313.6 \text{ kcal mol}^{-1}$$

Energy of electron in n_2 orbit

$$= -\frac{313.52 \times (1)^2}{(2)^2} \text{ kcal mol}^{-1}$$

$$= -\frac{313.52}{4} \text{ kcal mol}^{-1}$$

$$= -78.38 \text{ kcal mol}^{-1}$$

8. (a) Given velocity of particle $A = 0.05 \text{ ms}^{-1}$

Velocity of particle $B = 0.02 \text{ ms}^{-1}$

Let the mass of particle $A = x$

$\therefore$ The mass of particle $B = 5x$ de-Broglie's equation is

$$\lambda = \frac{h}{mv}$$

For particle A $\lambda_A = \frac{h}{x \times 0.05} \quad \dots(i)$

For particle B $\lambda_B = \frac{h}{5x \times 0.02} \quad \dots(ii)$

Eq. (i)/(ii) $\frac{\lambda_A}{\lambda_B} = \frac{5x \times 0.02}{x \times 0.05}$

$$\frac{\lambda_A}{\lambda_B} = \frac{2}{1}$$

or $2 : 1$

9. (b) $\text{Cr}(24) = 1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^5, 4s^1$

10. (b) The number of waves in an orbit is equal to n . (principal quantum number)

11. (b) The probability of finding the electron in the orbital is 90-95%.

12. (b)

3

Nuclear Chemistry

Radioactivity

“It is a process in which nuclei of certain elements undergo spontaneous disintegration without excitation by any external means.” And such elements whose atoms disintegrate and emit radiations are called radioactive elements. On applying electric or magnetic field, these radiations were split into three types of rays: α , β and γ -rays.

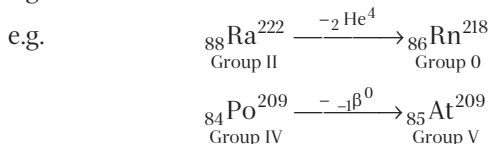
Comparison of α , β and γ -rays

Properties	α -particle or rays	β -particle or rays	γ -rays
Charge and mass	It carries 2 unit positive charge and 4 unit mass.	It carries 1 unit negative charge and has no mass.	These are electromagnetic rays with very short wavelength (approx. 0.05 Å).
Nature	It is represented as helium nucleus or helium ion, ${}_2\text{He}^4$ or He^{2+} .	It is represented as electron ${}_{-1}e^0$.	It is represented as ${}_0\gamma^0$.
Action of a magnetic field	These are deflected towards the cathode.	These are deflected towards the anode.	These are not deflected.
Velocity	2×10^9 cm/s or 2×10^7 m/s ($\frac{1}{10}$ th of velocity of light).	2.36 to 2.83×10^{10} cm/s or 2.36 to 2.83×10^8 m/s ($\frac{1}{3}$ rd to $\frac{9}{10}$ th of velocity of light)	Same as that of light, 3×10^{10} cm / s or $(3 \times 10^8$ m/s)
Ionising power	Very high, nearly 100 times to that of β -rays.	Low, nearly 100 times to that of γ -rays.	Very low
Effect on ZnS plate	They cause luminescence.	Very little effect.	Very little effect
Penetrating power	Low, easily stopped by air.	100 times to that of α -particles.	10,000 times to that of β -particles.
Kinetic energy	High due to their high mass.	Very less due to their small mass.	Zero
Nature of product	Product obtained by the loss of 1 α -particle has atomic number less by 2 units and mass number less by 4 units, e.g. ${}_y A^n \longrightarrow {}_{y-2} A^{n-4} + {}_2\text{He}^4$	Product obtained by the loss of 1 β -particle has atomic number more by 1 unit, without any change in mass number, e.g. ${}_y A^n \longrightarrow {}_{y+1} A^n + {}_{-1}e^0$	There is no change in the atomic number as well as in mass number, e.g. ${}_y A^n \longrightarrow {}_y A^n + {}_0\gamma^0$

NOTE Rate of radioactive disintegration is independent of temperature, pressure and chemical combination, etc.

Group Displacement Law

It was first stated by Fajans and Soddy in 1913. According to them, “in an α -emission, the parent element will be displaced to a group two places to the left and in a β -emission, it will be displaced to a group one place to the right.”



Disintegration Series

There are four radioactive disintegration series:

- The thorium series ($4n$ series)
- The neptunium series ($4n + 1$ series)
- The uranium series ($4n + 2$ series)
- The actinium series ($4n + 3$ series)

Kinetics of Radioactive Disintegration

The rate of radioactive disintegration is found to decrease with time. Radioactive disintegration is found to follow first order kinetics and hence, for first order radioactive disintegration, the rate constant or specific reaction rate (k or λ) is given by

$$\lambda \text{ or } k = \frac{2.303}{t} \log \frac{N_0}{N} \quad \dots(i)$$

where, N_0 = amount of element at start,

N = amount of element remained after t time

and k = disintegration constant (also denoted by λ)

Half-life Period

Half-life is the period, in which a radioactive element disintegrates to half of its original mass. It is represented by $t_{1/2}$.

when, $t = t_{1/2}$ then, $N = \frac{N_0}{2}$,

Hence,

$$k = \frac{2.303}{t_{1/2}} \log \frac{N_0}{N_0/2}$$

$$= \frac{2.303 \log 2}{t_{1/2}}$$

$$t_{1/2} = \frac{0.693}{k} \quad \dots(ii)$$

Average Life Period

It is the reciprocal of disintegration constant and is represented by t_{av} .

Average life $(t_{av}) = \frac{1}{k} = \frac{t_{1/2}}{0.693} = 1.44 \times t_{1/2} \quad \dots(iii)$

Units of Radioactivity

$$1 \text{ curie} = 3.7 \times 10^{10} \text{ dps}$$

$$1 \text{ rutherford} = 10^6 \text{ dps}$$

Becquerel is the SI unit of radioactivity

$$\text{becquerel} = 1 \text{ dps}$$

Nuclear Stability

The stability of the nucleus is decided by the following factors:

(i) Neutron-Proton Ratio

It has been observed that the stability of nucleus depends upon the neutron to proton ratio (n/p).

It has been found that

- n/p ratio for stable nuclei = 1 (for elements with atomic number ≤ 20) and n/p ratio for stable nuclei > 1 (for elements with atomic number > 20)
- When n/p ratio is higher than the ratio required for stability, the nuclei have the tendency to emit β -rays, i.e. a neutron is converted into proton.
- When n/p ratio is lower than the ratio required for stability, the nuclei increase the ratio either by emitting α -particle or by emitting a positron or by K -electron capture.

(ii) Mass Defect and Binding Energy

Mass of an atom is always less than the sum of masses of its constituent nucleons (neutrons and protons). This loss of mass is called mass defect (Δm). This mass defect, in fact, changes into energy according to Einstein equation

$$E = \Delta m \cdot c^2 \quad \dots(iv)$$

We can calculate the binding energy (BE), if mass defect is known as BE equivalent to ($\Delta m = 1 \text{ amu}$) is 931 MeV.

Hence, $\text{BE} = \Delta m \times 931 \text{ MeV} \quad \dots(v)$

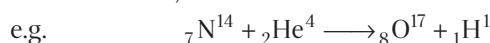
The total binding energy of a nucleus divided by the number of nucleons, is called specific binding energy or binding energy per nucleon. It is denoted by $\bar{B}$.

i.e. $\bar{B} = \frac{\text{Total binding energy of nucleus}}{\text{Number of nucleons}}$

Greater is the value of specific binding energy ($\bar{B}$) of a nucleus, greater is its stability. Nuclei having specific binding energy, ranging from 7.5 – 9.0 MeV per nucleon, are most stable.

Artificial Transmutation

It is defined as the conversion of one element into another or one type of atoms into another. When this conversion is achieved by artificial means, i.e. by means of bombarding with some fundamental particles, it is known as artificial transmutation,



Fundamental particles used in the bombardment of different elements are

α -particle (${}_2\text{He}^4$), proton (${}_1\text{H}^1$), deuteron (${}_1\text{H}^2$) or (${}_1\text{D}^2$) and neutron (${}_0n^1$)

Artificial Radioactivity

Many stable nuclei when bombarded with high speed particles produce unstable nuclei that are radioactive. The radioactivity produced in this manner by artificial means is known as artificial radioactivity or induced radioactivity.

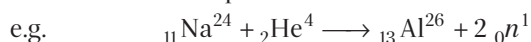
Nuclear Reactions

A nuclear reaction is the process, in which a change in the composition of nucleus occur. It results in the formation of another element. While writing a nuclear reaction, the sum of the mass numbers and atomic numbers should be same on both sides of the reaction.

Nuclear reactions are of various types as given below :

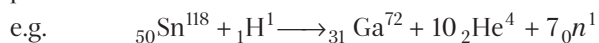
Particle-particle Reactions

In these reactions, bombarding particle (${}_2\text{He}^4$, ${}_0n^1$, ${}_1\text{H}^1$, ${}_1\text{H}^2$, ${}_1\beta^0$, etc.) is absorbed by stable nucleus to become unstable and it emits other particle and form stable nuclei.



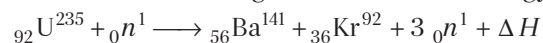
Spallation Reactions

In these reactions, a high speed projectile chips a fragment off from a heavier nucleus and a large number of lighter particles are emitted.



Nuclear Fission

In 1939, German scientists, Otto Hahn and F Strassman discovered these type of reactions. They bombarded U-235 nuclei with slow moving neutrons and found that this nucleus splitted into two lighter nuclei with the liberation of three neutrons and a large amount of heat energy, i.e.



Such reactions are called nuclear fission reactions and the energy produced is called nuclear energy. About 199 MeV of energy is released by the fission of one nucleus of uranium.

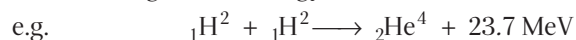
The source of this nuclear energy is actually mass defect (Δm) occurring in the reaction, which changes into the energy according to Einstein equation.

In this reaction, two or three neutrons are also produced, which split other uranium nuclei. Thus, this reaction goes on increasing and then it is called a chain reaction.

In chain reaction, a huge amount of nuclear energy is obtained which can be used either for destructive purpose or for constructive purpose.

Nuclear Fusion

It is a reaction, in which lighter nuclei combine to form a single heavier nucleus. In this process also, some mass defect changes into energy called nuclear fusion energy.



Nuclear fusion differs from nuclear fission in mainly two respects :

- Unlike nuclear fission, nuclear fusion require very high temperature (of the order of 10^6°C) and high pressure.
- Nuclear fusion reaction cannot be controlled in contrast to nuclear fission.

Thus, nuclear fusion is much hazardous. Hydrogen bomb is based on this reaction. Also the source of solar energy is nuclear fusion reactions between hydrogen nuclei, which is continuously occurring on sun.

Other Important Terms

1. Magic Numbers

The nuclei with protons and/or neutrons equal to 2, 8, 20, 28, 50, 82, and 144 have been found much stable. These numbers are called magic numbers.

2. Packing Fraction (PF)

$$\text{PF} = \frac{\text{Actual isotopic mass} - \text{mass number}}{\text{Mass number}} \times 10^4$$

PF depends upon the manner of packing of the nucleons within the nucleus. Its value can be negative, positive or even zero. Higher the negative PF value, more stable is the nucleus.

NOTE

- Lower the packing fraction, greater is the binding energy per nucleon and hence, greater is the stability.
- Packing fraction is highest for H and least for Fe.

3. Uses of Isotopes

${}_6\text{C}^{14}$ is used for radiocarbon dating. ${}_{92}\text{U}^{238}$ is used for determining age of rocks, earth or an ore, etc. Co-60 is used in the treatment of cancer.

${}_{53}\text{I}^{131}$ is used to detect functioning of thyroid gland. ${}_{11}\text{Na}^{24}$ is used to detect blood clot and position of tumour. Besides these uses, various isotopes are used as radioactive tracers in various fields of research.

4. Age of Old Piece of Wood

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{2.303}{t} \times \log \frac{\text{C}^{14} \text{ in living plants}}{\text{C}^{14} \text{ in dead plants}}$$

where, t = age of wood

and $t_{1/2}$ = half-life of C^{14}

It is determined by uranium-lead method using the formula,

$$\lambda = \frac{0.693}{t_{1/2}}$$

$$= \frac{2.303}{t} \log \frac{(\text{U}^{238} + \text{Pb}^{206})}{\text{U}^{238}}$$

NOTE $206 \text{ g Pb}^{206} \equiv 238 \text{ g U}^{238}$
 $\therefore x \text{ g Pb}^{206} \equiv \left(\frac{238}{206} \times x\right) \text{ g U}^{238}$

6. Relation between Disintegration Per Second and Amount of Substance

$$-\frac{dN}{dt} = \lambda \cdot N, \quad -\frac{dN}{dt} = \lambda \cdot \frac{6.023 \times 10^{23} \times w}{M}$$

where, $-\frac{dN}{dt}$ = number of disintegration per sec.

$$N = 6.023 \times 10^{23} \text{ per gram mole of substance}$$

$$= 6.023 \times 10^{23} \times \frac{w}{M} \text{ atoms}$$

w = weight and M = molecular weight

Practice Exercise

- The nuclear radius as compared to the atomic radius, is of the order of
 - 10^{-2}
 - 10^{-15}
 - 10^{-4}
 - 10^{-8}
- Arrange the following particles in the increasing order of their penetration capacity as the projectiles for artificial transmutation of elements :
Proton (${}_1\text{H}^1$) , alpha particle (${}_2\text{He}^4$), deuteron (${}_1\text{H}^2$), neutron (${}_0n^1$)
 - ${}_2\text{He}^4 < {}_1\text{H}^2 < {}_1\text{H}^1 < {}_0n^1$
 - ${}_1\text{H}^1 < {}_1\text{H}^2 > {}_0n^1 < {}_2\text{He}^4$
 - ${}_1\text{H}^1 < {}_1\text{H}^2 < {}_2\text{He}^4 < {}_0n^1$
 - ${}_0n^1 < {}_1\text{H}^2 < {}_2\text{He}^4 < {}_1\text{H}^2$
- Which of the following radiations is most easily stopped by air?
 - β -rays
 - α -rays
 - X-rays
 - γ -rays
- β -particle is emitted in radioactivity by
 - conversion of proton into neutron
 - from outermost orbit
 - conversion of neutron into proton
 - β -particle is not emitted
- ${}_7\text{N}^{13}$ changes to ${}_6\text{C}^{13}$ by the emission of
 - electron
 - neutron
 - positron
 - proton
- Which of the following processes are feasible?
 - ${}_1\text{p}^1 + {}_0n^1 \longrightarrow {}_1\text{D}^2 + \bar{\nu}$
 - ${}_1\text{H}^1 + {}_{-1}\text{e}^0 \longrightarrow {}_0n^1$
 - ${}_0n^1 \longrightarrow {}_1\text{p}^1 + {}_{-1}\text{e}^0 + \bar{\nu}$
 - None of the above
- ${}_{11}\text{Na}^{24}$ is a radioactive and it decays to
 - ${}_{12}\text{Mg}^{24}$ and β -particles
 - ${}_{11}\text{Na}^{21}$ and neutron
 - ${}_{13}\text{F}^{24}$ and positron
 - ${}_9\text{F}^{20}$ and α -particles
- Loss of β -particle is equivalent to
 - decrease of one neutron only
 - increase of one proton only
 - Both (a) and (b)
 - None of the above
- Which of the following processes cause the emission of X-rays?
 - α -emission
 - γ -emission
 - Positron emission
 - Electron capture
- What will be the product, if ${}_{92}\text{U}^{235}$ emits two α -and one β -particle?
 - ${}_{87}\text{Ac}^{211}$
 - ${}_{89}\text{Ac}^{235}$
 - ${}_{89}\text{Ac}^{225}$
 - ${}_{89}\text{Ac}^{227}$
- In the reaction, $\text{Po} \xrightarrow{-\alpha} \text{Pb} \xrightarrow{-\beta} \text{Bi}$, if Bi belongs to group 15, to which group Po belongs?
 - 13
 - 14
 - 15
 - 16
- Which of the following transformation is not correct?
 - ${}_{33}\text{As}^{75} + {}_2\text{He}^4 \longrightarrow {}_{35}\text{Br}^{78} + {}_0n^1$
 - ${}_3\text{Li}^7 + {}_1\text{H}^1 \longrightarrow {}_4\text{Be}^7 + {}_0n^1$
 - ${}_8\text{O}^{16} + {}_0n^1 \longrightarrow {}_6\text{C}^{14} + {}_2\text{He}^4$
 - ${}_{83}\text{Bi}^{209} + {}_1\text{H}^2 \longrightarrow {}_{84}\text{Po}^{210} + {}_0n^1$
- For the nuclear reaction, ${}_0n^1 \longrightarrow {}_1\text{H}^1 + ?$, the missing nuclide is
 - ${}_{-1}\text{e}^0$
 - ${}_1\text{e}^0$
 - ${}_1\text{H}^2$
 - ${}_2\text{He}^4$
- The end product of the series, starting with ${}_{90}\text{Th}^{232}$, is
 - Pb-208
 - Pb-206
 - Bi-209
 - Pb-207
- The number of α -and β -particles emitted in the nuclear reaction, ${}_{90}\text{Th}^{228} \longrightarrow {}_{83}\text{Bi}^{212}$, respectively are
 - 4 α and 1 β
 - 3 α and 7 β
 - 8 α and 1 β
 - 4 α and 7 β

16. The radioactive disintegration follows the kinetics of
 a. zero order b. first order
 c. second order d. third order
17. The amount of substance that gives 3.7×10^7 dps is
 a. one becquerel b. one curie
 c. one millicurie d. one rutherford
18. The weight of 1 curie ${}_{82}\text{Pb}^{214}$ ($t_{1/2} = 26.8$ min) in grams is
 a. 3.1×10^{-8} g b. 1.55×10^{-8} g
 c. 6.2×10^{-8} g d. 3.1×10^{-10} g
19. Radium has atomic weight 226 and a half-life of 1600 yr. The number of disintegrations produced per second from one gram are
 a. 4.8×10^{10} b. 9.2×10^6 c. 3.7×10^{10} d. zero
20. Only 1/8th of the original amount of a radioactive element remains after 96 min. The value of $t_{1/2}$ of this element is
 a. 12.0 min b. 32.0 min c. 24.0 min d. 48.0 min
21. The half-life of radium is 1600 yr, after how much time will 1 g radium be reduced to 125 mg?
 a. 800 yr b. 1600 yr
 c. 3200 yr d. 4800 yr
22. If one starts with 1 curie of radioactive substance ($t_{1/2} = 12$ h), the activity left after a period of 1 week will be about
 a. 1 curie b. 129 microcurie
 c. 60 microcurie d. 8 millicurie
23. 20 mg of C-14 has a half-life of 5760 yr. 100 mg of sample containing C-14 is reduced to 25 mg in
 a. 57600 yr b. 1440 yr
 c. 17280 yr d. 11520 yr
24. In the case of a radioisotope, the value of $t_{1/2}$ and λ are identical in magnitude. The value of $t_{1/2}$ is
 a. 1/0.693 b. $(0.693)^{1/2}$ c. $\frac{0.693}{2}$ d. 0.693
25. The half-life of a radioactive isotope is 3 h. If the initial mass of the isotope was 300 g, the mass which remained undecayed in 18 h would be
 a. 1.17 g b. 9.36 g
 c. 2.34 g d. 4.68 g
26. The half-life period of a radioactive element is 140 day. After 560 days, 1 g of the element will reduce to
 a. 1/8 g b. 1/16 g
 c. 1/4 g d. 1/2 g
27. If 5 g of a radioactive substance has $t_{1/2} = 14$ h, 10 g of the same substance will have $t_{1/2}$ equal to
 a. 70 h b. 14 h c. 28 h d. 50 h
28. Choose the incorrect one.
 a. 1 curie = $3.7 \times 10^{10} \text{ ds}^{-1}$ b. 1 rutherford = 10^6 ds^{-1}
 c. 1 becquerel = 1 ds^{-1} d. 1 fermi = 10^3 ds^{-1}
29. The decay constant of a radioactive sample is λ . The half-life and mean life of the sample respectively are
 a. $\frac{1}{\lambda}, \frac{\ln 2}{\lambda}$ b. $\frac{\ln 2}{\lambda}, \frac{1}{\lambda}$ c. $\lambda \ln 2, \frac{1}{\lambda}$ d. $\frac{\lambda}{\ln 2}, \frac{1}{\lambda}$
30. Which is the most stable nucleus among these?
 a. ${}_2\text{He}^3$ b. ${}_{11}\text{Na}^{24}$ c. ${}_{83}\text{Bi}^{210}$ d. ${}_{82}\text{Pb}^{207}$
31. Sometimes, the ejection of an α -particle does not completely stabilise the nucleus. In such a case, more α -particles may be emitted. The α -decay
 a. raises the n/p ratio and is often followed by β -emission
 b. lowers the n/p ratio and is often followed by positron emission
 c. raises the n/p ratio and is often followed by neutron emission
 d. lowers the n/p ratio and is often followed by γ -rays emission
32. Which of the following nuclear reaction is incorrect?
 a. ${}_{20}\text{Ca}^{40} + {}_0n^1 \longrightarrow {}_{19}\text{K}^{40} + {}_1\text{H}^1$
 b. ${}_{12}\text{Mg}^{24} + {}_2\text{He}^4 \longrightarrow {}_{14}\text{Si}^{27} + {}_0n^1$
 c. ${}_{48}\text{Cd}^{113} + {}_0n^1 \longrightarrow {}_{48}\text{Cd}^{112} + {}_{-1}\text{e}^0$
 d. ${}_{20}\text{Co}^{43} + {}_2\text{He}^4 \longrightarrow {}_{21}\text{Sc}^{46} + {}_1\text{H}^1$
33. The easily fissionable isotope of uranium is with the number
 a. 236 b. 235 c. 237 d. 238
34. Atom bomb is based on the principle of
 a. carbon dating b. nuclear transformation
 c. nuclear fission d. nuclear fusion
35. The compound used in the enrichment of the uranium in nuclear power plant is
 a. UCl_4 b. $\text{UO}_2(\text{NO}_3)_2$
 c. UF_6 d. U_3O_8
36. If uranium (mass number = 238 and atomic number = 92) emits an α -particle, the product has mass number and atomic number respectively are
 a. 236 and 92 b. 236 and 90
 c. 238 and 90 d. 234 and 90
37. In a nuclear reactor, heavy water is used
 a. to provide high speed neutrons
 b. to increase the speed of neutrons
 c. to capture neutrons produced by nuclear fission
 d. to transfer the heat from the nuclear reactor
38. Which one of the following is an exact example of artificial radioactivity?
 a. ${}_0n^1 + {}_{11}\text{Na}^{23} \longrightarrow {}_{11}\text{Na}^{24} + \gamma$; ${}_{11}\text{Na}^{24} + {}_1\text{H}^1 \longrightarrow {}_{12}\text{Mg}^{24} + {}_0n^1$
 b. ${}_2\text{He}^4 + {}_7\text{N}^{14} \longrightarrow {}_8\text{O}^{17} + {}_1\text{H}^1$; ${}_8\text{O}^{17} + {}_0n^1 \longrightarrow {}_7\text{O}^{18} + \gamma$
 c. ${}_2\text{He}^4 + {}_{13}\text{Al}^{27} \longrightarrow {}_{15}\text{P}^{30} + {}_0n^1$; ${}_{15}\text{P}^{30} \longrightarrow {}_{14}\text{Si}^{30} + {}_1\text{e}^0$
 d. ${}_{89}\text{Ac}^{228} \longrightarrow {}_{90}\text{Th}^{228} + \beta$; ${}_{90}\text{Th}^{228} \longrightarrow {}_{88}\text{Ra}^{224} + \alpha$
39. The function of the cadmium rod in a nuclear reactor is
 a. to produce rays b. to produce neutrons
 c. to absorb neutrons d. to speed up neutrons

40. The nuclear reaction ${}^2_1\text{H} + {}^2_1\text{H} \longrightarrow {}^4_2\text{He}$ is called
 a. fission reaction b. fusion reaction
 c. chain reaction d. thermal reaction

41. Which of the following nuclei is not doubly magic?
 a. ${}^4_2\text{He}$ b. ${}^{16}_8\text{O}$ c. ${}^{208}_{82}\text{Pb}$ d. ${}^{238}_{92}\text{U}$

42. Atom *A* possesses higher values of packing fraction than atom *B*. The relative stabilities of *A* and *B* are
 a. *A* is more stable than *B*
 b. *B* is more stable than *A*
 c. *A* and *B* both are equally stable
 d. Stability does not depend on packing fraction

43. Match the Column I and Column II and pick the correct matching from the codes given below.

Column I	Column II
A. Isotope	1. ${}^{228}_{88}\text{Ra}$ and ${}^{228}_{89}\text{Ac}$
B. Isobar	2. ${}^{39}_{18}\text{Ar}$ and ${}^{40}_{19}\text{K}$
C. Isotone	3. ${}^2_1\text{H}$ and ${}^3_1\text{H}$
D. Isosters	4. ${}^{235}_{92}\text{U}$ and ${}^{231}_{90}\text{Th}$
E. Isodiaphers	5. CO_2 and N_2O

The correct matching is

Codes

	A	B	C	D	E
a.	5	4	1	2	3
b.	3	1	2	5	4
c.	2	1	4	5	3
d.	2	5	1	4	3

44. An isobar of ${}^{40}_{20}\text{Ca}$ is

- a. ${}^{40}_{18}\text{Ar}$ b. ${}^{38}_{20}\text{Ca}$
 c. ${}^{42}_{20}\text{Ca}$ d. ${}^{38}_{18}\text{Ar}$

45. An isotope of ${}^{231}_{90}\text{Th}$ can be converted into ${}^{227}_{90}\text{Th}$ by the emission of

- a. one α -particle
 b. one β -particle
 c. two α and one β -particle
 d. one α and 2 β -particles

46. Nuclides having the same difference in mass number and atomic number, are known as

- a. isotopes b. isobars
 c. isotones d. isomers

47. Which of the following statements is wrong?

- a. P^{32} is used in the treatment of leukemia
 b. I^{131} is used the treatment of thyroid gland cancer
 c. Co^{59} cannot be used in the treatment of cancer
 d. Excessive use radioactive elements is responsible of cancerous growth

48. The isotope of carbon used in radiocarbon dating is

- a. ${}^{12}_6\text{C}$ b. ${}^{13}_6\text{C}$
 c. ${}^{14}_6\text{C}$ d. ${}^{15}_6\text{C}$

49. Which one of the following radioisotopes is used in the treatment of blood cancer?

- a. Na^{24} b. I^{131}
 c. Co^{62} d. P^{32}

50. The proper rays for radio carbon dating are

- a. cosmic rays b. X-rays
 c. UV rays d. IR rays

BITSAT Archives

1. 1 g of U-235 is converted into UF_6 . The radioactivity of UF_6 thus obtained is [2011]

- a. zero
 b. less than that of 1 g of U-235
 c. more than that of 1 g of U-235
 d. same as that of 1 g of U-235

2. A sample of radioactive substance shows an intensity of 2.3 millicurie at a time *t* and an intensity of 1.62 millicurie after 600 s. The half-life period of the radioactive metal is [2010]

- a. 1000 s b. 1187 s c. 1200 s d. 1500 s

3. If the mass defect of ${}^{11}_5\text{B}$ is 0.081 u, its average binding energy (in MeV) is [2008]

- a. 8.60
 b. 6.85
 c. 5.60
 d. 5.86

4. When radioactive minerals like cleveite, monozite and pitch blende are heated to 1273 K in vacuo the noble gas obtained is [2007]

- a. Rn b. Kr
 c. He d. Ne

Answer with Solutions

Practice Exercise

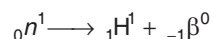
1. (c) The nuclear radius is of the order of 10^{-14} m while that of atom is of the order of 10^{-10} m. Hence,

$$\frac{\text{Nuclear radius}}{\text{Atomic radius}} = \frac{10^{-14}}{10^{-10}} = 10^{-4}$$

2. (a) The penetrating power increases with increase in velocity of particles. Thus, the correct order of penetrating power of given particles is

$${}_0n^1 > {}_1H^1 > {}_1H^2 > {}_2He^4$$

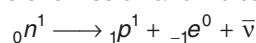
3. (b) α -rays have less penetrating power.
4. (c) It is considered that β -particle is emitted from the nucleus by the conversion of neutron into proton.



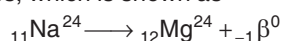
5. (c) ${}_7N^{13} \longrightarrow {}_6C^{13} + {}_{+1}e^0$
Positron

Positron is an anti-particle of electron (${}_{-1}e^0$).

6. (c) The electron does not exist in the nucleus but it is formed at the time of emission and indicates as



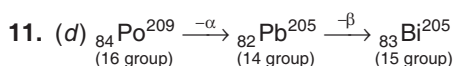
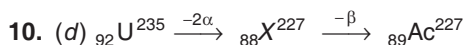
7. (a) ${}_{11}Na^{24}$ is a radioactive isotope and it decays to ${}_{12}Mg^{24}$ and β -particles, which is shown as



8. (c) ${}_0n^1 \longrightarrow {}_1H^1 + {}_{-1}e^0$

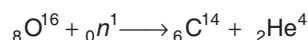
So, it is equivalent to increase of one proton and decrease of one neutron.

9. (d) X-rays are produced when high energy electrons are captured by a metal of high atomic number.



Thus, according to group displacement law, Po belongs to 16 group.

12. (c) Mass number are not balanced in



Therefore, it is not correct transformation

13. (a) ${}_0n^1 \longrightarrow {}_1H^1 + {}_{-1}e^0$

14. (a) ${}_{90}Th^{232}$ is in ($232 = 4 \times 58 = 4n$) series. The end product of this series will be, thus, Pb-208 as ($208 = 4 \times 52 = 4n$).

15. (a) Number of α -particles

$$= \frac{\left(\begin{array}{c} \text{mass number} \\ \text{of emitter} \end{array} - \begin{array}{c} \text{mass number} \\ \text{of end product} \end{array} \right)}{4}$$

$$= \frac{228 - 212}{4} = \frac{16}{4} = 4$$

Number of β -particles = $2 \times \alpha$ -particle

– (difference in atomic number)

$$= 2 \times 4 - (90 - 83) = 8 - 7 = 1$$

Hence, α and β -particles, emitted are 4 and 1 respectively.

16. (b) The radioactive disintegration follows the first order kinetics, for which decay constant is given by

$$\lambda = \frac{2.303}{t} \log \frac{N_0}{N}$$

17. (c) The amount of substance, which give 3.7×10^{10} disintegration per second, is known as 1 curie. Hence,

$$3.7 \times 10^7 \text{ dps} = 1 \text{ millicurie}$$

18. (a) We know that, $\frac{-dN}{dt} = \lambda \frac{6.023 \times 10^{23} \times w}{M}$

$$\text{Here, } \frac{-dN}{dt} = 1 \text{ curie} = 3.7 \times 10^{10} \text{ dps}$$

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{26.8 \times 60} \text{ s}^{-1}$$

$$\therefore 3.7 \times 10^{10} = \frac{0.693}{26.8 \times 60} \times \frac{6.023 \times 10^{23} \times w}{214}$$

$$\text{or } w = 3.1 \times 10^{-8} \text{ g}$$

19. (c) Given, $M = 226$, $t_{1/2} = 1600$ yrs, $w = 1$ g

$$\frac{-dN}{dt} = \lambda \frac{6.023 \times 10^{23} \times w}{M} = \frac{0.693}{t_{1/2}} \times \frac{6.023 \times 10^{23} \times 1}{226}$$

$$= \frac{0.693 \times 6.023 \times 10^{23}}{1600 \times 226 \times (365 \times 24 \times 3600)}$$

$$= 3.7 \times 10^{10} \text{ disintegrations per second}$$

20. (b) Given that, $N_0 = 1$, $N = \frac{1}{8}$, $t = 96$ min

$$\text{We know that, } N = N_0 \left(\frac{1}{2} \right)^{\frac{t}{t_{1/2}}} \text{ or } \frac{1}{8} = \left(\frac{1}{2} \right)^{\frac{96}{t_{1/2}}}$$

$$\text{or } \left(\frac{1}{2} \right)^3 = \left(\frac{1}{2} \right)^{\frac{96}{t_{1/2}}} \text{ or } \frac{96}{t_{1/2}} = 3 \text{ or } t_{1/2} = \frac{96}{3} = 32 \text{ min}$$

21. (d) Given, $t_{1/2} = 1600$ yr, $N_0 = 1$ g, $N = 125$ mg = $\frac{125}{1000}$ g

$$\text{We know that, } N = N_0 \left(\frac{1}{2} \right)^{\frac{t}{t_{1/2}}}$$

$$\frac{125}{1000} = \left(\frac{1}{2} \right)^{\frac{t}{1600}}, \left(\frac{1}{8} \right) \text{ or } \left(\frac{1}{2} \right)^3 = \left(\frac{1}{2} \right)^{\frac{t}{1600}}$$

$$\text{Hence, } \frac{t}{1600} = 3 \text{ or } t = 3 \times 1600 = 4800 \text{ yr}$$

22. (c) Given,
- $t_{1/2} = 12$
- h.

$$t = 1 \text{ week} = 7 \times 24 \text{ h}, N = N_0 \left(\frac{1}{2} \right)^{\frac{t}{t_{1/2}}} = 1 \times \left(\frac{1}{2} \right)^{\frac{7 \times 24}{12}}$$

$$= \left(\frac{1}{2} \right)^{14} = 6.1 \times 10^{-5} \text{ curie} = 60 \text{ microcurie}$$

23. (d) Half-life period for a nucleus is a constant quantity. It does not change with amount or mass. Hence,

$$t_{1/2} = 5760 \text{ yr}, N = N_0 \left(\frac{1}{2} \right)^{\frac{t}{t_{1/2}}}$$

$$\frac{25}{100} = \left(\frac{1}{2} \right)^{\frac{t}{5760}} \text{ or } \left(\frac{1}{2} \right)^2 = \left(\frac{1}{2} \right)^{\frac{t}{5760}}$$

$$\therefore \frac{t}{5760} = 2 \text{ or } t = 2 \times 5760 = 11520 \text{ yr}$$

24. (b) We know that,
- $t_{1/2} = \frac{0.693}{k}$
- or
- $(t_{1/2})^2 = 0.693$
- (as,
- $t_{1/2} = k$
-)

$$\text{or } t_{1/2} = (0.693)^{1/2}$$

25. (d)
- $N = N_0 \times \left(\frac{1}{2} \right)^n = 300 \times \left(\frac{1}{2} \right)^6 = 4.68 \text{ g}$

26. (b) Number of half-lives (
- n
-) =
- $\frac{\text{Total time}}{t_{1/2}} = \frac{560}{140} = 4$

$$\text{Amount left} = \left(\frac{1}{2} \right)^n \times \text{Initial amount} = \left(\frac{1}{2} \right)^4 \times 1 \text{ g} = \frac{1}{16} \text{ g}$$

27. (b)
- $t_{1/2} = \frac{0.693}{\lambda}$
- and it is independent of initial amount or mass.

28. (d) The nuclear radius is expressed in fermi (Fm) units.

$$1 \text{ fermi} = 10^{-13} \text{ cm} = 10^{-15} \text{ m}$$

29. (b)
- $t_{1/2} = \frac{\log_e 2}{\lambda}$
- and average life =
- $\frac{1}{\lambda}$

30. (d)
- n/p
- ratio for
- ${}_2\text{He}^3 = \frac{1}{2} = 0.5 (< 1)$

$$n/p \text{ ratio for } {}_{11}\text{Na}^{24} = \frac{13}{11} = 1.19 \text{ (greater than that of stable isotope } {}_{11}\text{Na}^{23})$$

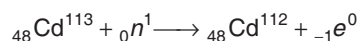
$$n/p \text{ ratio for } {}_{83}\text{Bi}^{210} = \frac{127}{83} = 1.54 (> 1.53)$$

$$n/p \text{ ratio for } {}_{82}\text{Pb}^{207} = \frac{125}{82} = 1.53$$

Hence, ${}_{82}\text{Pb}^{207}$ is the most stable as its n/p ratio lies between the stability range, i.e. (1.00 – 1.53).

31. (a) The
- n/p
- ratio increases in case of
- α
- decay while decreases during
- β
- decay.

32. (c) A nuclear reaction must be balanced in terms of atomic numbers and mass number. Hence, following reaction is unbalanced.



33. (b) The most unstable isotope of uranium is
- ${}_{92}\text{U}^{235}$
- .

34. (c) Atom bomb is based on the principle of nuclear fission.

35. (c) The fuel used in nuclear power plant is enriched in uranium-235 in the form of
- UF_6
- .

36. (d)
- ${}_{92}\text{U}^{238} \longrightarrow {}_{90}\text{Th}^{234} + {}_2\text{He}^4$

37. (d)

38. (c) Nucleus bombarded should be stable and nucleus produced should be radioactive.

39. (c) Cadmium is used to absorb neutron.

40. (b)

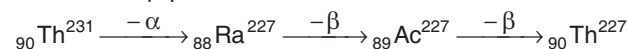
41. (d) The magic numbers for protons are 2, 8, 20, 28, 50, 82, 114 and for neutrons are 2, 8, 20, 28, 50, 82, 126, 184, 196.

42. (b) The atom which has lower value of packing fraction is more stable.

43. (b)

44. (a)

45. (d) Isotopes of an element can be obtained by emitting one
- α
- and
- 2β
- particles from its nucleus.



46. (c)

47. (c)

48. (c)

49. (c)
- Co^{62}
- is used in the treatment of blood cancer.

50. (a)

BITSAT Archives

1. (d) Radioactivity does not depend upon the state of combination so it remains unaffected.

2. (b)
- $\frac{N_0}{N} = \frac{2.30}{1.62}$
- , Now,
- $\lambda = \frac{2.303}{600} \log \frac{2.30}{1.62} = 0.000584$

$$\therefore t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.000584} = 1187 \text{ s}$$

3. (b) Given,
- Δm
- for
- ${}_5\text{B}^{11} = 0.081 \text{ u}$

$$\text{Number of nucleons} = 11$$

$$\text{Binding energy} = 931 \times \Delta m \text{ MeV}$$

$$= 931 \times 0.081 = 75.411 \text{ MeV}$$

$$\text{Average binding energy} = \frac{\text{Binding energy}}{\text{Number of nucleons}}$$

$$= \frac{75.411}{11}$$

$$= 6.85 \text{ MeV}$$

4. (c)

4

Chemical Bonding

Bond or Chemical Bond

A bond is a kind of interatomic, intermolecular or inter-ionic attraction which holds two or more constituents (atoms, molecules or ions) together, e.g. ionic bond, covalent bond etc. It is formed so that the atoms acquire stability.

Valency

The combining capacity of different constituents is called their valency. In case of elements of I A and II A group, it is equal to their group number, i.e. +1 and +2 respectively.

For other elements, it varies from group number to group number – 8,

i.e. Valency = group number to (group number – 8)

Here, group number are in accordance with modern periodic table, e.g. nitrogen an element of group V A, has valency from +5 to –3 (= 5 – 8).

Lewis Octet Rule

The tendency of atoms to acquire 8 electrons in their outermost shell is known as Lewis octet rule.

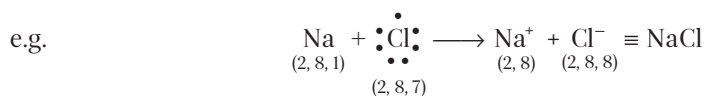
Exceptions

(a) Incomplete octet BF_3 , BeCl_2 etc.

(b) Octet expansion PCl_5 , BF_6 etc.

Ionic Bonding

Electrons are completely transferred from electropositive element to electronegative element forming charged ions which are held together by strong electrostatic forces.



Conditions Favourable for Ionic Bonding

- The electropositive element should have low ionisation energy.
- The electronegative element should have higher electron affinity.
- Lattice energy should be high.

Lattice Energy

A lattice is a regular arrangement of cations and anions in an ionic crystal. The amount of energy released, when free ions combine to form one mole of a crystal lattice, is known as lattice energy.

Factors affecting of Lattice energy

- Size of ions
- Charge on ions

$$\text{Lattice energy (LE)} = -\frac{NAZ^+Z^-e^2}{r} + \frac{ue^2}{r^n}$$

(Born Lande equation)

where, Z^+ , Z^- are the charges on cation and anion respectively.

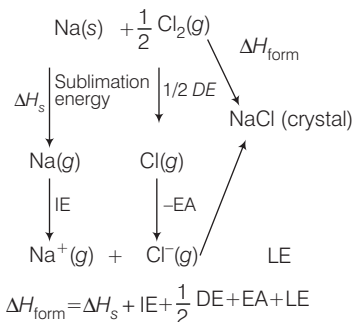
e = electronic charge, N = Avogadro's number

r = interionic distance, A = Madelung constant

u = Born exponent

Born Haber Cycle

e.g. Formation of NaCl



More is the negative value of ΔH_{form} , more stable will be the compound.

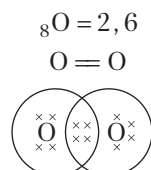
Characteristics of Ionic Compounds

- They are crystalline in nature with 3-D solid aggregates due to non-directional force of attraction.
- They are good conductors of electricity and heat, but only in aqueous phase or molten state. In solid state, there is no free electron.
- They have very high melting and boiling point.
- They are soluble in polar solvents.
- Ionic reactions are very fast due to presence of ions.

Covalent Bonds

Such chemical bonds are formed by sharing of electrons between the elements of almost same electronegativity or between the elements having less difference in electronegativity.

e.g. Formation of O_2 molecule



Dipole Moment (μ)

$$\mu = q \times d$$

where, q = charge

and d = distance between positive and negative charges

Unit $\rightarrow$ Debye, $1 D = 10^{-18}$ esu cm

In symmetric molecules, $\mu = 0$

Higher the value of μ , more is the polarity of bond.

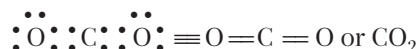
$$\% \text{ Ionic character} = \frac{\text{Observed } \mu}{\text{Calculated } \mu} \times 100$$

Covalency $\rightarrow$ number of electrons contributed by an elements

e.g. Formation of Cl_2 molecule



Formation of CO_2 $\longrightarrow$ molecule



The above structures are known as electron-dot structures or Lewis structure.

Properties of Covalent Compound

- They generally exist in gaseous or liquid form or sometimes as soft solids (e.g. S, P etc). Sometimes, they exist as hard solids, e.g. SiC, BN etc.
- They have low boiling point and melting point except some covalent solids such as SiC, diamond, SiO_2 etc.
- In general, covalent compounds are bad conductors of heat and electricity due to the absence of ions or free electrons. An exception is graphite, since in graphite, electrons can pass from one layer to another.
- Non-polar covalent compounds are soluble in non-polar solvents such as CCl_4 , C_6H_6 , CS_2 etc.
- Covalent substances show slower chemical reactions.
- Covalent substances show isomerism.

Fajan's Rule

The partial covalent character of an ionic bond has been explained by Fajan as follows:

When a cation approaches an anion, the electrons cloud of the anion is deformed which is called polarisation. Greater the polarisation, more is the covalent character.

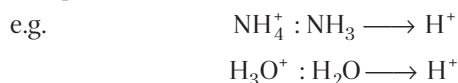
Covalent character $\propto$ polarising power of cation

$$\propto \frac{1}{\text{size of cation}}$$

Polarisability of anion $\propto$ size of anion

Coordinate Covalent (Dative) Bond

A bond in which complete pair of shared electrons is contributed by the same element. Atom contributing its extra electron pair is called donor while other atom is called acceptor and the bond is represented by an arrow ($\rightarrow$) from donor to acceptor.



Theories Developed for Covalent Bonding

1. Valence Bond Theory (VBT)

(Given by Heitler and London)

It explains bond formation in terms of overlapping of orbitals, e.g. the formation of H_2 molecule from two hydrogen atoms involves the overlap of 1s-orbital of two H-atoms which are singly occupied.

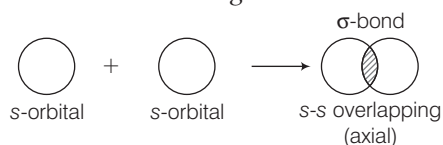
Because of orbital overlap, electron density between the nuclei increases which helps in bringing them closer.

The overlapping of orbitals may results in two types of bonds given below:

Sigma (σ) Bond

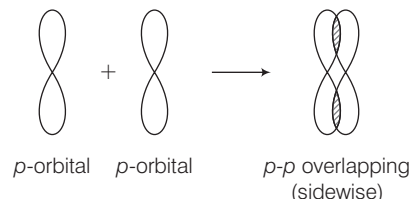
It is the result of end to end overlapping or axial overlapping between s-s, p-p, s-p orbitals, e.g. single bond.

The electron density accumulates between the centres of the atoms being bonded.



Pi (π) Bond

It is formed by incomplete, sidewise or parallel overlapping of orbitals.

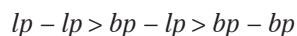


Double bond has one σ -bond and one π -bond. Triple bond has two π -bonds and one σ -bond.

2. Valence Shell Electron Pair Repulsion Theory (VSEPR theory)

Main Postulates

- In polyatomic molecules, one atom is called central atom to which all other atoms (or group of atoms) are linked.
- The shape of any molecule depends upon the total number of valence shell electron pairs on central atom.
- If all the atoms linked to central atom (having no lone pair) are same then molecule is symmetrical.
- The relative order of repulsion is as follows:



Concept of Hybridisation

(Involving s, p, d and f-orbitals)

Intermixing of orbitals of almost similar energy belonging to the same atom to form same number of orbitals of exactly equal energy is known as hybridisation. The new orbital thus formed are known as hybrid orbitals.

Features of Hybridisation

Only valence shell atomic orbitals of an atom or ion takes part in hybridisation. Types of hybridisation along with geometry and bond angles are listed below.

Types of Hybridisation

Number of hybrid orbitals	Hybridisation	Geometry	Bond angle
2	sp	Linear	180°
3	sp^2	Triangular planar	120°
4	sp^3 dsp^2	Tetrahedral Square planar	109.5° 90°
5	sp^3d	Trigonal bipyramidal	$120^\circ, 90^\circ$
6	sp^3d^2 or d^2sp^3	Octahedral	90°
7	sp^3d^3	Pentagonal bipyramidal	$72^\circ, 90^\circ$

The hybridisation of the molecule can be determined by

$$H = \frac{1}{2}[V + Y - C + A]$$

where, V = valence shell electrons of the central metal

Y = number of monovalent atoms

C = total positive charge

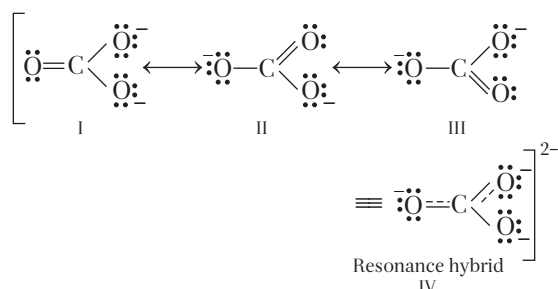
A = total negative charge on the molecule

H	2	3	4	5	6	7
Hybridisation	sp	sp^2	sp^3	sp^3d	sp^3d^2	sp^3d^3

Resonance

Single Lewis structure of some molecules is not adequate to explain the properties. Hence, the molecule could be represented by more than one structural molecule. These structure having similar energy are called resonance structures.

e.g. CO_3^{2-} (carbonate ion) can be represented as

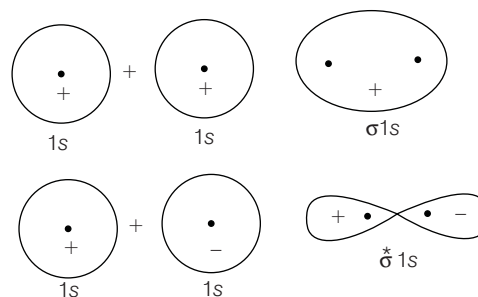


I, II and III are resonating structures of CO_3^{2-} ion. The structure IV is known as resonance hybrid which describes the structure of CO_3^{2-} more accurately.

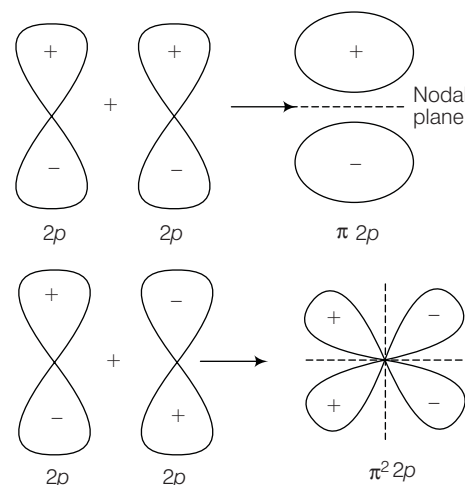
3. Molecular Orbital Theory (1932 Mulliken and Hund)

When the atoms to be bonded come close together, the orbitals of the bonded atoms lose their individual character and overlap to form larger orbitals called molecular orbitals, in which electrons move under the influence of many nuclei. Molecular Orbitals (MO) possess different energy levels and their shapes depend on the atomic orbitals from which they are formed. The combining orbitals must be of comparable energy and should overlap to a larger extent. The filling of electrons in molecular orbital is governed by Aufbau principle, Hund's rule and Pauli's exclusion principle.

Formation of bonding and antibonding molecular orbitals A sigma molecular orbital has cylindrical symmetry around the internuclear axis and has no nodal plane along this axis.



When atomic orbitals overlap laterally, the resulting molecular orbital energetic is called a π -molecular orbital. It has no cylindrical symmetry and contains a nodal plane along the internuclear axis. π -orbital is more energetic than σ -orbital.



Relative Energy of MO

Case I Total $e^- \leq 14$

$$\begin{aligned}\sigma 1s &< \sigma^* 1s < \sigma 2s < \sigma^* 2s < \pi 2p_y \\ &= \pi 2p_z < \sigma 2p_x < \pi^* 2p_y = \pi^* 2p_z < \sigma^* 2p_x\end{aligned}$$

Case II Total $e^- > 14$

$$\begin{aligned}\sigma 1s &< \sigma^* 1s < \sigma 2s < \sigma^* 2s < \sigma 2p_x < \pi 2p_z \\ &= \pi 2p_y < \pi^* 2p_z = \pi^* 2p_y < \sigma^* 2p_x\end{aligned}$$

$$\begin{aligned}\text{Bond order} &= \frac{1}{2} (\text{number of electrons in bonding orbitals} \\ &\quad - \text{number of electrons in anti-bonding orbitals}) \\ &= \frac{1}{2}(N_b - N_a)\end{aligned}$$

If there are unpaired electrons in any molecular orbitals, then the substance is paramagnetic otherwise it is diamagnetic. If $N_b \leq N_a$, the molecule is unstable.

e.g. (a) He_2

Total electrons = 4

Molecular orbital electronic configuration = $\sigma 1s^2 \sigma^* 1s^2$

$$\text{Bond order} = \frac{1}{2}(2 - 2) = 0 \text{ (cannot exist)}$$

(b) O_2

Total electrons = 16

Molecular orbital electronic configuration

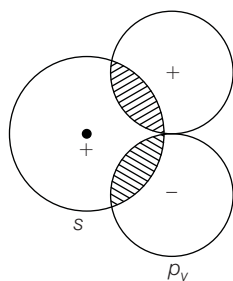
$$\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_x^2 \pi 2p_y^2 \pi 2p_z^2 < \pi^* 2p_y^1 \pi^* 2p_z^1$$

$$\text{Bond order} = \frac{1}{2}(10 - 6) = 2$$

O_2 is paramagnetic molecule as it contains unpaired electrons.

Non-Bonding Combination of AOs

In such an overlap of AOs, any stabilisation due to positive-positive combination is destabilised by an equal amount of positive negative overlap.



Metallic Bond

Attraction between the metallic cations and the mobile electrons which hold the crystal together, is known as metallic bond.

Some properties of metals are :

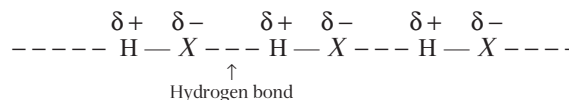
Metallic luster, electrical conductivity, thermal conductivity, malleability, ductility, high tensile strength etc.

Some Weaker Bonds

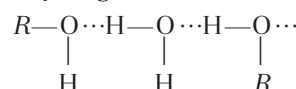
Hydrogen bond

It can be defined as the attractive force which binds hydrogen atom of one molecule with the electronegative atom (F, O or N) of another molecule. Cl has some electronegativity as nitrogen but it does not form strong H-bonds due to its large size.

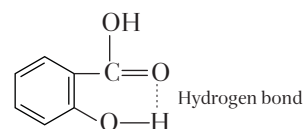
Strong H-bond exist in KHF_2 .



1. Intermolecular hydrogen bond



2. Intramolecular hydrogen bond



Effects of hydrogen bond

- (a) Association
- (b) High melting point and boiling point
- (c) Solubility in water
- (d) Viscosity and surface tension

van der Waals' Forces

These are effective only in non-polar molecules.

$$\begin{aligned} \text{van der Waals' force} &\propto \text{molecular weight} \\ &\propto \text{number of electrons} \\ &\propto \text{atomic weight} \end{aligned}$$

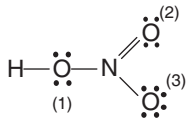
$$\text{Boiling point} \propto \text{van der Waals' forces}$$

NOTE The relative strength of various bond is as follows:

$$\text{Ionic bond} > \text{covalent} > \text{metallic} > \text{hydrogen bond} > \text{van der Waals' forces}$$

Practice Exercise

- Which of the following is not correct?
 - $\text{Na}^+ + \text{Cl}^- \longrightarrow \text{NaCl}$
 - $\text{Ca}^{2+} + 2\text{F}^- \longrightarrow \text{CaF}_2$
 - $\text{Na}^+ + \text{F}^- \longrightarrow \text{NaF}$
 - $\text{Ca}^+ + 2\text{Na}^+ \longrightarrow \text{CaNa}_2$
- Which postulation provide the basic for the modern concepts regarding ion formation by electron transfer and the formation of ionic crystalline compounds?
 - Kossel's postulations
 - Langmuir's postulations
 - Newton's postulations
 - Lewis postulations
- In chlorine atom, how many electrons are short of the argon configuration?
 - One
 - Two
 - Three
 - Four
- Which is not paramagnetic?
 - O_2
 - O_2^+
 - O_2^-
 - O_2^{2-}
- Which of the following is the weakest bond?
 - Hydrogen bond
 - Covalent bond
 - Ionic bond
 - Metallic bond
- Which of the following compounds has the smallest bond angle?
 - H_2O
 - H_2S
 - NH_3
 - CO_2
- The molecule which contains ionic as well as covalent bond, is
 - NH_4Cl
 - H_2O
 - CaCl_2
 - CCl_4
- Consider the following Lewis structure of HNO_3 ,



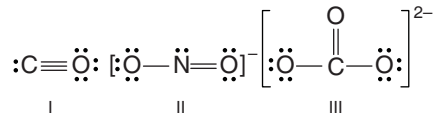
The formal charge on $O_{(1)}$, $O_{(2)}$ and $O_{(3)}$ are given in column II. Match the following and choose the correct option from the codes given below.

Column I		Column II	
A.	O_1	1.	Zero
B.	O_2	2.	-1
C.	O_3	3.	+3

Codes

	A	B	C
a.	1	2	3
b.	2	1	2
c.	1	1	2
d.	3	2	1

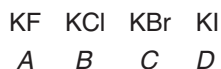
9. Which one has covalent as well as ionic valency?
a. NaCl b. HCl c. H₂O d. NaOH
10. In NO₃⁻ ion, the number of bond pairs and lone pairs of electrons on nitrogen atom respectively are
a. 2 and 2 b. 3 and 1 c. 1 and 3 d. 4 and 0
11. Which one of the following contains both ionic and covalent bonds?
a. C₆H₅Cl b. H₂O
c. NaOH d. CO₂
12. Lewis dot structures of CO, NO₂⁻ and CO₃²⁻ are I, II and III respectively given below.



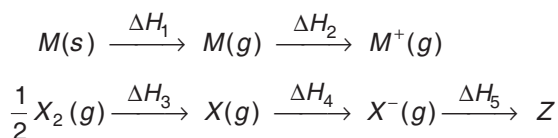
Which of these structure(s) is/are wrong?

- a. Only I
c. Only III
- b. Only II
d. I, II and III
- In the following electron dot structure, calculate the formal charge from left to right nitrogen atom respectively :
- $$\begin{array}{c} \cdot\cdot \\ \cdot\cdot \end{array} \text{N} = \text{N} = \begin{array}{c} \cdot\cdot \\ \cdot\cdot \end{array}$$
- a. -1, -1 and +1
b. -1, +1 and -1
c. +1, -1 and -1
d. +1, -1 and +1
- Lattice energy of a solid increases if
- a. size of ions is small
b. charges of ions are small
c. ions are neutral
d. None of the above
- If the electronic configuration of an element is $1s^2 2s^2 2p^6 3s^2 3p^6 3d^2 4s^2$, four electrons involved in chemical bond formation will be.....
- a. $3p^6$
c. $3p^6, 3d^2$
- b. $3p^6, 4s^2$
d. $3d^2, 4s^2$
- The electronic configuration of the outermost shell of the most electronegative element is
- a. $2s^2 2p^5$
c. $4s^2 4p^5$
- b. $3s^2 3p^5$
d. $5s^2 5p^5$
- An electronic arrangement is said to be stable if its outer shell consists
- a. doublet of electrons
c. octet of electrons
- b. triplet of electrons
d. singlet of electron

18. Which is the most covalent?
 a. C—F b. C—O
 c. C—S d. C—Br
19. Which of the following molecules is correct regarding BeCl_2 ?
 a. It violates octet rule and has sp^2 -hybridisation
 b. It has sp -hybridisation and follow octet rule
 c. It violates octet rule and has linear structure
 d. All of the above are true
20. The correct order of increasing covalent character of the following is
 a. $\text{SiCl}_4 < \text{AlCl}_3 < \text{CaCl}_2 < \text{KCl}$
 b. $\text{KCl} < \text{CaCl}_2 < \text{AlCl}_3 < \text{SiCl}_4$
 c. $\text{AlCl}_3 < \text{CaCl}_2 < \text{KCl} < \text{SiCl}_4$
 d. None of the above
21. The metallic lustre exhibited by sodium is explained by
 a. diffusion of sodium ions
 b. excitation of free protons
 c. oscillation of loose electrons
 d. existence of body centred cubic lattice
22. Arrange the following ionic compounds in order of increasing ionic character:

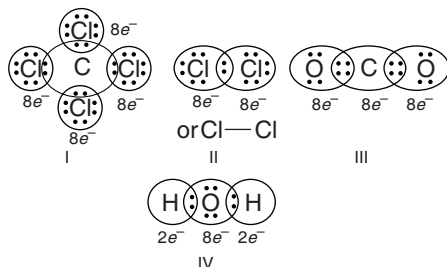


- a. $A < B < C < D$
 b. $D < C < B < A$
 c. $B < A < C < D$
 d. $C < A < B < D$
23. Consider the Born-Haber cycle for the formation of an ionic compound given below.



Here, Z refers to

- a. $M^+X^-(l)$ b. $M^+X^-(s)$
 c. MX_2 d. $M^+X^-(g)$
24. Which of the following structures is/are correct?



- a. I, II and IV b. II, III and IV
 c. II and III d. I, II, III and IV

25. In which of the following molecule/ion, all the bonds are not equal?
 a. XeF_4 b. BF_4^-
 c. C_2H_4 d. SiF_4
26. If the electronegativity difference between two atoms A and B is 2.0, then the percentage of covalent character in the molecule is
 a. 54% b. 46%
 c. 23% d. 72%
27. Which of the following is a favourable factor for cation formation?
 a. Low ionisation potential
 b. High electron affinity
 c. High electron negativity
 d. Small atomic size
28. Match the reaction given in Column I with enthalpy in Column II and choose the correct option from the codes given below.

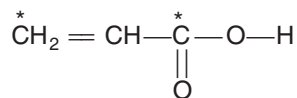
Column I	Column II
A. $\text{Mg} \longrightarrow \text{Mg}^{2+} + 2e^-$	1. Electron gain enthalpy
B. $\text{O} \longrightarrow \text{O}^{2-} - 2e^-$	2. Ionisation enthalpy
C. $M(g) \longrightarrow M^+(g) + e^-$	
D. $X(g) \longrightarrow X^-(g) - e^-$	

Codes

	A	B	C	D
a.	1	2	1	2
b.	2	1	2	1
c.	1	1	2	2
d.	2	2	1	1

29. In the formation of a molecule, only the outer shell electrons take part in chemical combination and are known as
 a. valence electrons
 b. inner electrons
 c. inert electrons
 d. reactive electrons
30. How many double bonds are present in carbon dioxide molecule?
 a. One b. Two c. Three d. Four
31. Which of the following hybridisations is not possible?
 a. sp^3
 b. sp^3d^3
 c. sd^3
 d. None of the above
32. The hybridisation of atomic orbitals of nitrogen in NO_2^+ , NO_3^- and NH_4^+ respectively are
 a. sp^2, sp^3 and sp^2
 b. sp, sp^2 and sp^3
 c. sp^2, sp and sp^3
 d. sp^2, sp^3 and sp

33. What is the type of hybridisation of carbon atoms marked with star?

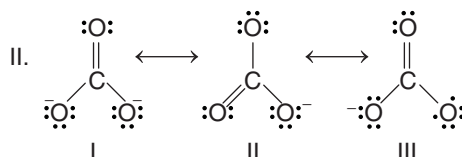


- a. sp^2 and sp b. sp^2 and sp^2
c. sp and sp^2 d. None of these
34. Among the three molecules XeF_4 , SF_4 , SiF_4 , which has/have tetrahedral structure?
a. All three b. SiF_4 and SF_4
c. Only SiF_4 d. Only SF_4
35. What is the structure of XeF_6 ?
a. Tetrahedral b. Distorted octahedral
c. Octahedral d. None of these
36. Which of the molecules has trigonal bipyramidal geometry with bond angles 120° and 90° ?
a. SF_6 b. PCl_5 c. CH_4 d. BF_3
37. Which of the following species has tetrahedral geometry?
a. BH_4^- b. NH_2^-
c. CO_3^{2-} d. H_3O^+
38. The species having pyramidal shape is
a. SO_3 b. BrF_3 c. SiO_3^{2-} d. OSF_2
39. Which of the following molecules have same molecular geometry?
I. CH_4 II. BF_3 III. NH_4^+ IV. SF_4
a. I and II b. III and IV
c. I and III d. I, III and IV
40. The structure of IF_7 is
a. square pyramidal b. trigonal bipyramidal
c. octahedral d. pentagonal bipyramidal
41. Based on VSEPR theory, the number of 90° degree $\text{F}-\text{Br}-\text{F}$ angles in BrF_5 , is
a. 0 b. 1 c. 2 d. 3
42. A σ -bonded molecule MX_3 is T-shaped. The number of non-bonding pairs of electrons is
a. 0
b. 2
c. 1
d. Can be predicted only if atomic number of M is known
43. An atom X has three valence electrons and atom Y has six valence electrons. The compound formed between them will have the formula
a. X_2Y_6 b. XY_2 c. X_2Y_3 d. X_3Y_2

44. The number of oxygen atoms bonded to one phosphorus atom in P_4O_6 is
a. 4 b. 3
c. 6 d. 5

45. In PO_4^{3-} ion, the formal charge on the oxygen atom of $\text{P}-\text{O}$ bond is
a. +1 b. -1 c. -0.75 d. +0.75

46. Which of the following is not the correct representation of resonance?



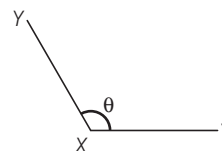
- a. Only I b. Only II
c. Both I and II d. None of these
47. Which of the following molecules represent the resonance?
a. O_3 b. CO_3^{2-}
c. CO_2 d. All of these
48. The structure represents the molecules more accurately, is called
a. resonance hybrid b. canonical structure
c. resonating structure d. None of these
49. Match the following columns and choose the correct option from the codes given below.

Column I		Column II	
A.	BeCl_2	1.	Linear
B.	BF_3	2.	Trigonal planar
C.	CO_2		

Codes

	A	B	C		A	B	C
a.	1	1	2	b.	2	2	1
c.	1	2	1	d.	2	1	2

50. The compound MX_4 is tetrahedral. The number of $\angle \text{XMX}$ angles formed in the compound is
a. three b. four c. five d. six
51. In accordance to molecular theory,
a. O_2^+ is diamagnetic and bond order is more than O_2
b. O_2^+ is diamagnetic and bond order is less than O_2
c. O_2^+ is paramagnetic and bond order is less than O_2
d. O_2^+ is paramagnetic and bond order is more than O_2
52. Which bond angle, θ would result in the maximum dipole moment for a triatomic molecule XY_2 as shown below?



- a. $\theta = 90^\circ$ b. $\theta = 120^\circ$
c. $\theta = 150^\circ$ d. $\theta = 180^\circ$

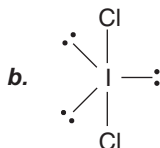
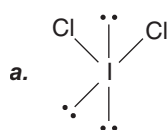
53. The bond angles of NH_3 , NH_4^+ and NH_2^- are in the order

a. $\text{NH}_2^- > \text{NH}_3 > \text{NH}_4^+$ b. $\text{NH}_4^+ > \text{NH}_3 > \text{NH}_2^-$
 c. $\text{NH}_3 > \text{NH}_2^- > \text{NH}_4^+$ d. $\text{NH}_3 > \text{NH}_4^+ > \text{NH}_2^-$

54. The molecule having zero dipole moment is

a. ClF_3 b. CH_4 c. PH_3 d. CH_2Cl_2

55. Which of the following show correct structure of ICl_2 ?



d. None of these

56. The net dipole moment of H_2O molecule is

a. zero b. 1.85 D
 c. 4.90×10^{-30} cm d. 0.80×10^{-30} cm

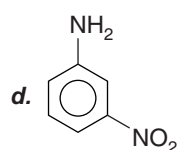
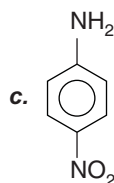
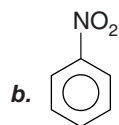
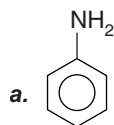
57. Deviation of O—O bond length in ozone molecule from the normal bond length seems as

a. single bond length increases while double bond length decreases
 b. single bond length decreases while double bond length increases
 c. single bond length increases while double bond length remains same
 d. single bond length remains same while double bond length increases

58. The decreasing order of dipole moments of the molecules HF , H_2O , BeF_2 , NF_3 is

a. $\text{HF} > \text{H}_2\text{O} > \text{BeF}_2 > \text{NF}_3$
 b. $\text{H}_2\text{O} > \text{HF} > \text{NF}_3 > \text{BeF}_2$
 c. $\text{BeF}_2 > \text{NF}_3 > \text{HF} > \text{H}_2\text{O}$
 d. $\text{NF}_3 > \text{BeF}_2 > \text{H}_2\text{O} > \text{HF}$

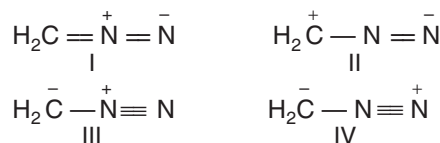
59. Which of the following will have largest dipole moment?



60. Which one of the following pairs of molecules will have permanent dipole moments for both the members?

a. SiF_4 and NO_2 b. NO_2 and CO_2
 c. NO_2 and O_3 d. SiF_4 and CO_2

61. The correct stability order of the following resonance structure is



a. $\text{I} > \text{II} > \text{IV} > \text{III}$ b. $\text{I} > \text{III} > \text{II} > \text{IV}$
 c. $\text{II} > \text{I} > \text{III} > \text{IV}$ d. $\text{III} > \text{I} > \text{IV} > \text{II}$

62. The correct order of bond angles in NH_3 , PCl_3 and BCl_3 is

a. $\text{PCl}_3 > \text{NH}_3 > \text{BCl}_3$ b. $\text{NH}_3 > \text{BCl}_3 > \text{PCl}_3$
 c. $\text{NH}_3 > \text{PCl}_3 > \text{BCl}_3$ d. $\text{BCl}_3 > \text{NH}_3 > \text{PCl}_3$

63. The correct order of decreasing polarity is

a. $\text{HF} > \text{SO}_2 > \text{H}_2\text{O} > \text{NH}_3$
 b. $\text{HF} > \text{H}_2\text{O} > \text{SO}_2 > \text{NH}_3$
 c. $\text{HF} > \text{NH}_3 > \text{SO}_2 > \text{H}_2\text{O}$
 d. $\text{H}_2\text{O} > \text{NH}_3 > \text{SO}_2 > \text{HF}$

64. In case of sodium and chlorine, the electron transfer takes place from

a. sodium to chlorine b. chlorine to sodium
 c. Both (a) and (b) d. None of these

65. The bond order of superoxide ion O_2^{2-} is

a. 2.5 b. 1.5 c. 1.0 d. 2

66. AlCl_3 is covalent while AlF_3 is ionic. This can be justified on the basis of

a. valence bond theory b. crystal structure
 c. lattice energy d. Fajan's rule

67. VSEPR theory was proposed by

a. Pauling b. Sidgwick and Powell
 c. Hund and Mulliken d. GN Lewis

68. The bond order of O_2^- is

a. 2 b. 1.5 c. 1 d. 2.5

69. In the case of Cl_2 , the bond is formed by

a. transfer of electrons b. sharing of electrons
 c. Both (a) and (b) d. None of the above

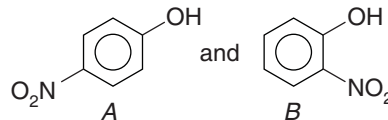
70. Match the following species with chemical formula and choose the correct option from the codes given below.

Column I		Column II	
A.	Compounds	1.	NH_4^+
B.	Atoms	2.	H_2O
C.	Ions	3.	Li_2O
		4.	Ne
		5.	Li^+

Codes

A	B	C	A	B	C
a. 2,3	4	1,5	b. 3	1,2	4,5
c. 2	3,1	4,5	c. 1	2,3	4,5

71. Sodium chloride is soluble in water but not in benzene because
- $\Delta H_{\text{hydration}} < \Delta H_{\text{lattice}}$ (in water)
 $\Delta H_{\text{hydration}} > \Delta H_{\text{lattice}}$ (in benzene)
 - $\Delta H_{\text{hydration}} < \Delta H_{\text{lattice}}$ (in benzene)
 $\Delta H_{\text{hydration}} > \Delta H_{\text{lattice}}$ (in water)
 - $\Delta H_{\text{hydration}} = \Delta H_{\text{lattice}}$ (in water)
 $\Delta H_{\text{hydration}} < \Delta H_{\text{lattice}}$ (in benzene)
 - $\Delta H_{\text{hydration}} < \Delta H_{\text{lattice}}$ (in water)
 $\Delta H_{\text{hydration}} = \Delta H_{\text{lattice}}$ (in benzene)
72. Which of the following bonds is present in BF_4^- ?
- Electrovalent
 - Metallic
 - Dative
 - Hydrogen
73. KF combines with HF to form KHF_2 . The compound contains the species
- K^+ , F^- and H^+
 - K^+ , F^- and HF
 - K^+ and $[\text{HF}_2]^-$
 - $[\text{KHF}]^+$ and F_2
74. Among the following, choose the correct pair which is isostructural and isoelectronic?
- NO_3^- , CO_3^{2-}
 - SO_3 , NO_3^-
 - ClO_3^- , CO_3^{2-}
 - CO_3^{2-} , ClO_3^-
75. When two atoms share two electron pairs, they are said to be joined by a
- single bond
 - double bond
 - triple bond
 - None of these
76. Which of the following molecules have same bond order?
- | | | | | |
|--------------|---------------|-------------|---------------|--------------|
| H_2 | Cl_2 | CO | Br_2 | N_2 |
| I | II | III | IV | V |
- I, II and IV have same bond order
 - III and V have same bond order
 - Both (a) and (b)
 - None of the above
77. An ionic solid is poor conductor of electricity because
- ions do not conduct electricity
 - charge on the ions is uniformly distributed
 - ions have uniform field of influence around it
 - ion occupy fixed position in solids
78. The molecule having one unpaired electron is
- NO
 - CO
 - CN^-
 - CO_2
79. How many corners of a cube would be occupied in the case of noble gas?
- 1
 - 2
 - 4
 - 8
80. The noble gases have a particularly stable outer shell electronic configuration represented as
- ns^2np^6
 - ns^2np^5
 - ns^2np^4
 - ns^2np^8
81. Which of the following has a linear structure?
- CCl_4
 - SO_2
 - C_2H_2
 - C_2H_4
82. In XeF_2 , XeF_4 and XeF_6 , the number of lone pairs of Xe respectively are
- 2, 3 and 1
 - 1, 2 and 3
 - 4, 1 and 2
 - 3, 2 and 1
83. The largest bond angle is in
- AsH_3
 - NH_3
 - H_2O
 - PH_3
84. If the bond enthalpy of O_2 , N_2 and H_2 are 498, 946 and 435.8 kJ mol^{-1} respectively. Choose the correct order of decreasing bond strength.
- $\text{H}_2 > \text{N}_2 > \text{O}_2$
 - $\text{N}_2 > \text{O}_2 > \text{H}_2$
 - $\text{O}_2 > \text{H}_2 > \text{N}_2$
 - $\text{H}_2 > \text{O}_2 > \text{N}_2$
85. The one which has no coordinate bond, is
- HNO_3
 - CO
 - CO_3^{2-}
 - CH_3-NC
86. For the given two compounds, vapour pressure of B at a particular temperature is expected to be



- higher than that of A
- lower than that of A
- same as that of A
- Can be higher or lower depending upon the size of the vessel

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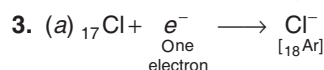
- Which of the following pairs has identical shape?
 - CH_4 and SF_4
 - BCl_3 and ClF_3
 - XeF_2 and ZnCl_2
 - SO_2 and CO_2
 [2014]
- Using MOT, which of the following pairs denote paramagnetic species?
 - B_2 and C_2
 - B_2 and O_2
 - N_2 and C_2
 - O_2 and O_2^{2-}
 [2013]
- Which of the following is isoelectronic with carbon?
 - Na^+
 - Al^{3+}
 - O^{2-}
 - N^+
 [2012]
- Which of the following does not contain any coordinate bond?
 - H_3O^+
 - BF_4^-
 - HF_2^-
 - NH_4^+
 [2011]
- The number of unpaired electrons in nickel carbonyl, is
 - zero
 - one
 - four
 - five
 [2011]
- The highest bond strength is shown by
 - O—O bond
 - S—S bond
 - Se—Se bond
 - Te—Te bond
 [2010]

7. Which of the following has maximum dipole moment?
a. NCl_3 b. NBr_3 c. NH_3 d. NI_3 [2009]
8. Which of the following has giant covalent structure?
a. NaCl b. SiO_2 c. AlCl_3 d. PbO_2 [2009]
9. Which one of the following species has the largest internuclear distance for its ion pair? [2009]
a. NaCl b. NaBr c. LiCl d. KI
10. The pair of species with similar shape is [2009]
a. $\text{PCl}_3, \text{NH}_3$ b. CF_4, SF_4 c. $\text{PbCl}_2, \text{CO}_2$ d. PF_5, IF_5
11. The bond length of HCl molecule is $1.275 \text{ \AA}$ and its dipole moment is 1.03 D . The ionic character of the molecule (in per cent) is
(Charge of the electron = $4.8 \times 10^{-10} \text{ esu}$) [2008]
a. 100 b. 67.3 c. 33.66 d. 16.83
12. Which of the following is a correct set? [2008]
a. $\text{H}_2\text{O}, sp^3$, angular
b. BCl_3, sp^3 , angular
c. NH_4^+, dsp^2 , square planar
d. CH_4, dsp^2 , tetrahedral
13. The isoelectronic pair is [2007]
a. $\text{Cl}_2\text{O}, \text{ICl}_2^-$ b. $\text{Cl}_2^-, \text{ClO}_2$
c. $\text{IF}_2^+, \text{I}_3^-$ d. $\text{ClO}_3^-, \text{ClF}_2^+$
14. Which of the following species has a bond order other than 3? [2006]
a. CO b. CN^- c. NO^+ d. O_2^+
15. The molecular electronic configuration of Be_2 is [2005]
a. $\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2p^2$ b. $KK\sigma 2s^2$
c. $\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2$ d. None of these

Answer with Solutions

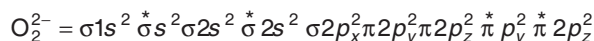
Practice Exercise

1. (d) Both Ca and Na are metals and form positive ions, which repel each other. Hence, the given reaction is not feasible.
2. (a) Kossel's postulations provide the basic for the modern concepts regarding ion formation by electron transfer and the formation of ionic crystalline compounds.



Chlorine is short of one electron only in electronic configuration in comparison with argon.

4. (d) O_2^{2-} has no unpaired electron, hence it is diamagnetic.

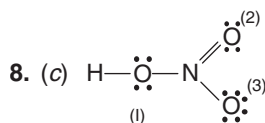
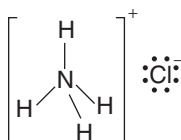


5. (a) H-bond is only an interaction between electronegative element and hydrogen therefore, it is the weakest bond among all.

6. (b) Bond angle of H_2S is smallest because S-atom is large in size and has low electronegativity.

7. (a) 3 N—H bonds are covalent.

1 N—H bond is coordinate and bond between NH_4^+ and Cl^- is ionic.



For $\text{O}_{(1)}$ Formal charge = $6 - \frac{1}{2}(4) - 4 = 0$

For $\text{O}_{(2)}$ Formal charge = $6 - \frac{1}{2}(4) - 4 = 0$

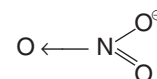
For $\text{O}_{(3)}$ Formal charge = $6 - \frac{1}{2}(2) - 6 = -1$

9. (d) NaOH has following structure



Hence, it has both ionic as well as covalent bonds.

10. (d) For NO_3^- ion,

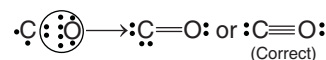


Bond pairs = $4(3\sigma + 1\pi)$

Lone pair = 0

11. (c) In NaOH , Na^+ and OH^- ions are bonded together by ionic bond while in OH^- ion, oxygen and hydrogen atoms are bonded together by covalent bond $\text{Na}^+[\text{O} - \text{H}]^-$.

12. (a) I may be correctly represented as



13. (b)

For $\text{N}_{(1)}$ and $\text{N}_{(3)}$,

Formal charge = $5 - \left[\frac{4}{2} + 4 \right] = 5 - 6 = -1$

For $\text{N}_{(2)}$, Formal charge = $5 - \frac{1}{2} \times 8 - 0 = +1$

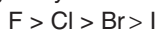
14. (a) According to Born-Landé equation,

$$U = \frac{Z^+ Z^- e^2 N_A M}{r_{\text{node}}} \left(\frac{1}{n} - 1 \right)$$

$\therefore$ Ions should be of small size to have high lattice energy.

15. (d) The outer shell electron takes part in chemical reaction to complete the octet. Therefore, four electrons involved in chemical bond formation will be $3d^2, 4s^2$.

16. (a) The electronegativity order of halogens is



Fluorine is the most electronegative element of the periodic table.

17. (c) Eight electrons (octet) in outer shell show stable electronic arrangement.

18. (c) $\therefore$ Covalent character $\propto \frac{1}{\text{Ionic character}}$
 $\propto \frac{1}{\text{difference in electronegativity}}$

$\therefore$ C—S is the most covalent.

19. (c) For BeCl_2 , $H = \frac{2+2}{2} = 2$

$\Rightarrow$ Hybridisation = sp

$\Rightarrow$ Linear shape

20. (b) EN difference in $\text{SiCl}_4 = 3.0 - 1.8 = 1.2$

in $\text{AlCl}_3 = 3.0 - 1.5 = 1.5$

in $\text{CaCl}_2 = 3.0 - 1.0 = 2.0$

in $\text{KCl} = 3.0 - 0.8 = 2.2$

$\therefore \text{KCl} < \text{CaCl}_2 < \text{AlCl}_3 < \text{SiCl}_4$

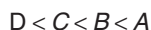
This order is also obtained by applying Fajan's rule.

i.e. covalent character $\propto$ charge on ion.

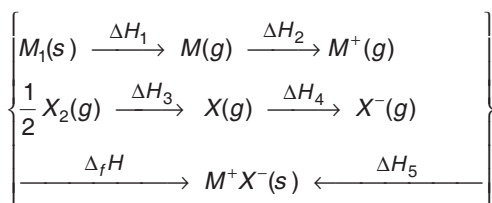
21. (c) Metallic lustre is due to oscillation of loose electrons.

22. (b) Higher the electronegativity difference, higher is the ionic character.

Hence, the correct order is



23. (b) The Born-Haber cycle takes place as follows:



24. (d) I. Represents Lewis-dot structure of CCl_4 .

II. Represents Lewis-dot structure of Cl_2 .

III. Represents Lewis-dot structure of CO_2 .

IV. Represents Lewis-dot structure of H_2O .

25. (c) In C_2H_4 molecule, there are two types of bonds arise from the central metal atom.

C—H (107 pm bond length)

C = C (133 pm bond length)

While in rest of the cases, all bonds from the central atom are similar.

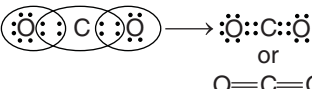
26. (a) If the electronegativity difference between two atoms A and B is 2.0, then the percentage of covalent character in the molecule is 54%.

27. (a) Low ionisation potential indicates that element can easily lose electron to form cation.

28. (b) $\left. \begin{array}{l} \text{Mg} \longrightarrow \text{Mg}^{2+} + 2e^- \\ M(g) \longrightarrow M^+(g) + e^- \end{array} \right\}$ Ionisation enthalpy involved

- $\left. \begin{array}{l} \text{O} \longrightarrow \text{O}^{2-} - 2e^- \\ X(g) \longrightarrow X^- - e^- \end{array} \right\}$ Electron gain enthalpy involved

29. (a) Outer shell is known as valence shell and its electrons are called valence electrons.

30. (b) 

31. (d) sp^3 = tetrahedral geometry

sp^3d^3 = pentagonal bipyramidal geometry

sd^3 = it is possible, e.g. $\rightarrow \text{MnO}_4^-$ (tetrahedral)

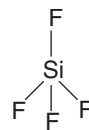
32. (b) Hybridisation in $\text{NO}_2^+ = \frac{1}{2}(5 + 0 - 1) = sp$

Hybridisation in $\text{NO}_3^- = \frac{1}{2}(5 + 0 + 1) = 3 = sp^2$

Hybridisation in $\text{NH}_4^+ = \frac{1}{2}(5 + 4 - 1) = 4 = sp^3$

33. (b)

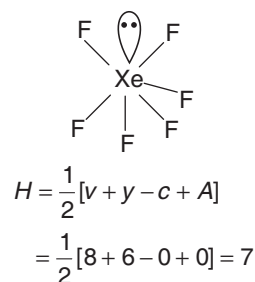
34. (c) For SiF_4



σ -bond = 4, $lp = 0$

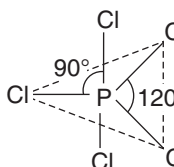
$\therefore$ Structure is tetrahedral.

35. (b) For XeF_6



$\Rightarrow$ Hybridisation = sp^3d^3

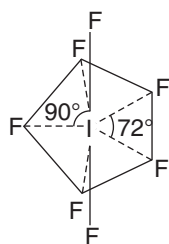
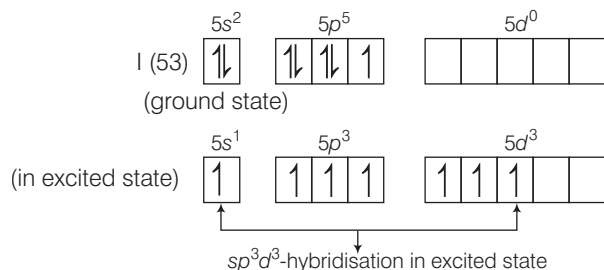
Due to the presence of one lone pair of electron, XeF_6 has distorted octahedral geometry.

36. (b) 

37. (a) BH_4^- has tetrahedral geometry.

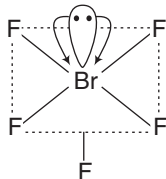
38. (d)

39. (c) CH_4 and NH_4^+ both have tetrahedral shape while BF_3 has trigonal planar and SF_4 has *see-saw* shape.
40. (d) According to VBT, structure of IF_7 can be determined as



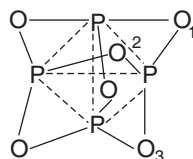
Pentagonal bipyramidal structure

41. (a) In BrF_5 , number of electron pairs
($1lp + 5bp$)

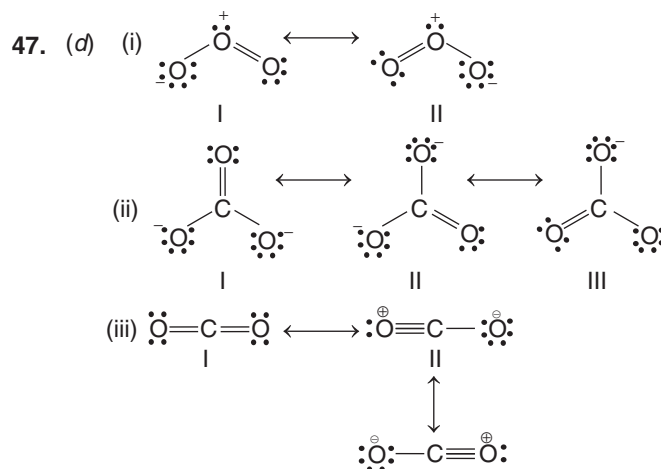


So, the structure is supposed to be square pyramidal but will be distorted because of additional *lp-bp* interaction. Additional *lp-bp* interaction reduced the all bond angle and do not let any angle to be 90° .

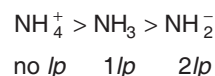
42. (b)
43. (c) If an atom X has three valence electrons and atom Y has six valence electrons, the compound formed between them will have the formula X_2Y_3 like Al_2O_3 .
44. (b) One phosphorus is bonded to three O-atoms as represented by 1,2,3 for P-atom present in the structure given below

Structure of P_4O_6

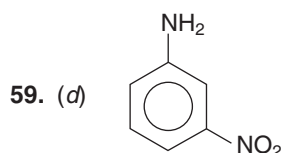
45. (b)
46. (d) Both representation of resonating structures in molecules of CO_2 and CO_3^{2-} are correct.



48. (a)
49. (c)
50. (b) Since, MX_4 is tetrahedral, therefore, total number of $\angle \text{XMX}$ are four.
51. (d) O_2^+ contains 1 unpaired electron and therefore it has bond order of 2.5 while O_2 contains only 2 unpaired electrons. So, it possesses bond order of 2.
52. (a) $\mu = \sqrt{\mu_1^2 + \mu_2^2 + 2\mu_1\mu_2 \cos \theta}$
If $\theta = 90^\circ$, μ is maximum.
Because $\cos 90^\circ = 0$, since, the angle increases from 90 to 180, the value of $\cos \theta$ becomes more and more negative and hence resultant decreases. Thus, dipole moment is maximum, when $\theta = 90^\circ$.
53. (b) As the number of lone pairs of electrons increases, bond angle decreases. Therefore, the order of bond angle is



54. (b)
55. (b) To minimise *lp-lp* repulsion, they exist as shown in option (b).
56. (b) The dipole moment of a molecule is the vector sum of the dipole moments of various bonds.
e.g. In H_2O molecule which has a bent structure, two O—H bonds are oriented at an angle of 104.5° with dipole moment 1.85D.
57. (b)
58. (b) $\text{H}_2\text{O} > \text{HF} > \text{NF}_3 > \text{BeF}_2$
1.85 D 1.78 D 0.23 D 0 D



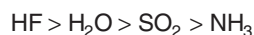
Above compound has the highest dipole moment among the given structures because it is totally unsymmetrical. Usually symmetrical molecules have less dipole moment in comparison to unsymmetrical molecules.

60. (c) NO_2 and O_3 both having irregular geometry due to which they have permanent dipole moments.
61. (b) I has maximum covalent bond and negative charge on electronegative nitrogen, most stable. III has more covalent bond than both II and IV, III is second most stable. II is more stable than IV, since it has negative charge on nitrogen. So, the order is



62. (d)
- | | |
|----------------|---------------|
| NH_3 | (107°) |
| PCl_3 | (93°) |
| BCl_3 | (120°) |

63. (b) $\therefore$ Polarity $\propto$ difference in electronegativity
 $\therefore$ Correct order of polarity is



64. (a) $\text{Na}^+ + \cdot\ddot{\text{Cl}}\cdot \longrightarrow \text{Na}^+ \cdot\ddot{\text{Cl}}\cdot \equiv \text{Na}^+ \text{Cl}^-$

65. (c) $\text{BO} = \frac{N_b - N_a}{2} = \frac{6 - 4}{2} = 1.0$

66. (d) According to Fajan's rule, smaller the anion, lesser will be its polarisability. Hence in AlF_3 , F^- ions are polarised to very little extent, therefore it has more ionic character.

In AlCl_3 , Cl^- are polarised to greater extent (due to larger size), hence AlCl_3 has more covalent character.

67. (b) VSEPR theory was first of all proposed by Sidgwick and Powell in 1940. Later on, it was developed by Gillespie and Nyholm in 1957.

68. (b) $\text{BO} = \frac{1}{2}(6 - 3) = 1.5$

69. (b)

70. (a)

71. (b) For a compound to be soluble, the hydration energy must be greater than the lattice energy. Since, NaCl is soluble in water but insoluble in benzene.

$$\Delta H_{\text{hydration}} < \Delta H_{\text{lattice (in benzene)}}$$

and $\Delta H_{\text{hydration}} > \Delta H_{\text{lattice (in water)}}$

72. (c) BF_4^- can be considered as being made by BF_3 by accepting lone pair of F^- ,

i.e. $\text{F}_3\text{B} \longleftarrow \text{F}^-$ (thus, it has dative bond).

73. (c) F^- forms H-bond with HF, therefore the species $[\text{H} \cdots \text{F} \cdots \text{H}]^-$ or HF^- exists and hence, option (c) is correct.

74. (a) For NO_3^- ion, number of electrons = $7 + 3 \times 8 + 1 = 32$

$$H = \frac{V + M - C + A}{2} = \frac{5 + 1}{2} = 3$$

$\Rightarrow$ Hybridisation = sp^2

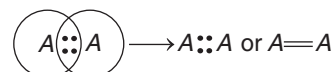
For CO_3^{2-} ion, number of electrons = $6 + 3 \times 8 + 2 = 32$

$$H = \frac{4 + 2}{2} = 3$$

$\Rightarrow$ Hybridisation = sp^2

$\therefore$ Both NO_3^{2-} and CO_3^{2-} are isoelectronic and isostructural species.

75. (b) When two atoms are linked to each other by sharing of electrons pair, they are said to be joined to each other by a double bond.

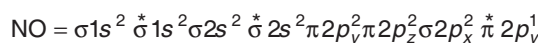


76. (c) In CO (three shared electron pairs between C and O), the bond order is 3. For N_2 , bond order is 3.

Cl_2 , Br_2 and H_2 have same bond order, i.e. 1.

77. (d) An ionic solid is a good conductor of electricity only in molten or solution state, when ions are free to move.

78. (a) NO has 15 electrons in its molecular orbitals which are distributed as follows



As it contains one unpaired electron, hence it is paramagnetic.

79. (d) Eight corners of a cube would be occupied in the case of noble gas.

80. (a) The noble gas have a particular stable outer shell electronic configuration represented as $ns^2 np^6$.

81. (c) C_2H_2 has linear structure.

82. (d) Xe-atom has 8 electrons in its outermost shell. In case of XeF_2 out of these 8 electrons, 2 are used for bond formation while 3 pairs remain as such, that's why it has 3 lone pairs.

In case of XeF_4 , 4 electrons of Xe are used for bonding, therefore number of lone pairs (non-bonding electrons) is 2.

In case of XeF_6 , 6 electrons are involved for bond formation, thus number of lone pair is only 1.

83. (b) The largest bond angle is present in NH_3 (107°). All the given molecules are sp^3 -hybridised but due to the presence of 2 lone pairs in H_2O , its bond angle is less (104.5°). While in PH_3 and AsH_3 , due to less difference in electronegativities, the bonded e^- pair remain between P and H (in PH_3) and As and H (in AsH_3) so that large $lp - bp$ repulsion makes bond angle smaller.

84. (b) Larger the bond dissociation enthalpy, stronger will be the bond in the molecule. So,

$$\text{N}_2 > \text{O}_2 > \text{H}_2$$

$946 \text{ kJ mol}^{-1} \quad 498 \text{ kJ mol}^{-1} \quad 435.8 \text{ kJ mol}^{-1}$

85. (c) Structure of CO_3^{2-} ion is

Hence, it has no coordinate bond. In it, carbon is sp^2 -hybridised and hence the ion has trigonal planar geometry.

86. (a)

BITSAT Archives

1. (c) XeF_2 and ZnCl_2 both have linear shape.
 2. (b) Among given four pairs, B_2 and O_2 are paramagnetic due to presence of unpaired electron.

MO electronic configuration of B_2

$$= \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \pi 2p_z^1 \equiv \pi 2p_z^1$$

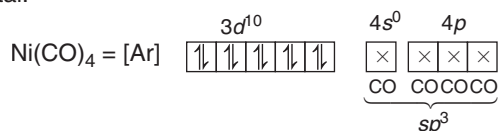
MO electronic configuration of O_2

$$= \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^2 \equiv \pi 2p_x^2 \pi^* 2p_x^1 \equiv \pi 2p_y^1$$

3. (d) $\therefore \text{N}^+ (7 - 1 = 6 e^-)$ is isoelectronic with C as both have 6 electrons.
 4. (c) Among the given, only HF_2^- has H-bonding $[\text{F} \cdots \text{H} \cdots \text{F}]^-$. Rest all the molecules have coordinate bonds.
 5. (a) In nickel carbonyl $\text{Ni}(\text{CO})_4$, Ni is present as Ni^0 .

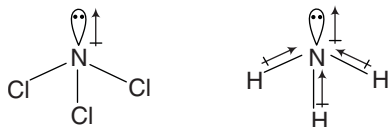
$$\text{Ni} = [\text{Ar}] 3d^8 4s^2$$

CO being strong field ligand, shifts electrons from 4s to 3d orbital.



Therefore, number of unpaired electrons in nickel carbonyl is 0.

6. (b) As the size increases, bond strength decreases. But S—S bond is stronger than O—O bond because of smaller size of O-atoms.
 7. (c) Electronegativity difference between N(3.0) and Cl(3.0) is zero and hence, N—Cl bonds are non-polar. As a result, the overall dipole moment of NCl_3 molecule and its direction is just the dipole moment of the lone pair of electrons.



On the other hand, N—Br, (3.0 – 2.8), N—I (3.0 – 2.5) and N—H (3.0 – 2.1) are polar and hence, contribute towards the overall dipole moment of the respective molecules. Since, the electronegativity difference is highest in case of N—H bonds, therefore NH_3 has the highest dipole moment.

8. (b)

9. (d) Among the given compounds, K^+ is the largest cation and I^- is the largest anion, therefore KI has the largest internuclear distance.

10. (a) Both PCl_3 and NH_3 have pyramidal shape, since both have sp^3 -hybridisation and one lone pair of electrons. In contrast, all other pairs have difference hybridisations and hence, different shapes, i.e. CF_4 has sp^3 while SF_4 has sp^3d -hybridisation, PbCl_2 is ionic while CO_2 has sp -hybridisation, PF_5 has sp^3d while IF_5 has sp^3d^2 -hybridisation.

11. (d) Theoretical value of dipole moment $= e \times d$
 $= 4.8 \times 10^{-10} \times 1.275 \times 10^{-8} \text{ esu cm}$
 $= 6.12 \times 10^{-18} \text{ esu cm}$
 $= 6.12 \text{ D}$

%Ionic character

$$= \frac{\text{Observed dipole moment}}{\text{Theoretical value of dipole moment}} \times 100$$

$$= \frac{1.03}{6.12} \times 100 = 16.83\%$$

12. (a)

13. (d) ClO_3^- and ClF_2^+ contain 34 electrons each hence they are isoelectronic.

14. (d) The configuration of the given species can be written as

CO or CN^- or NO^+ ($14 e^-$)

$$= \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2$$

$$\text{O}_2^+ (8 + 8 - 1) = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2$$

$$\{ \pi 2p_z^2 = \pi 2p_z^2 \pi^* 2p_y^1 \}$$

$$\text{Bond order} = \frac{1}{2} (N_b - N_a)$$

$$\text{CO or } \text{CN}^- \text{ or } \text{NO}^+ = \frac{1}{2} (10 - 4) = 3$$

$$\text{O}_2^+ = \frac{1}{2} (10 - 5) = \frac{5}{2} = 2.5$$

15. (c) Molecular electronic configuration of Be_2 is

$$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2$$

5

Periodic Properties

Classification of Elements

The elements are considered as the basic units of all types of matter. As the more elements were discovered in the recent times, it becomes a very difficult task to study the chemistry of elements and their compounds. In order to remove this difficulty, we need to organise the elements in a systematic pattern.

The efforts to classify the elements are as follows :

Laws for Classifying the Elements

Many attempts were made to classify the known elements from time to time.

These are as follows:

Dobereiner's Triads (1800, Johann Dobereiner)

Dobereiner arranged the elements in several groups of three elements each called **triads**, with similar properties. He found that in triads, the atomic weight of middle element is about half of the sum of the first and third element. This relation is known as **law of triads**.

e.g.

Li	Na	K	Ca	Sr	Ba	Cl	Br	I
7	23	39	40	88	137	35.5	80	127

Law of Octaves (1865, John Alexander Newlands)

Newland arranged the elements on the basis of the increasing order of their atomic weights and noted that the properties of every eighth element resembles the properties of the first element just like the octaves of music. This is known as Newlands law of octaves.

Lothar Meyer's Curve

Lothar Meyer plotted the physical properties such as atomic volume, melting and boiling points etc., with atomic weight and observed a periodic repetition in their properties. In 1868, Lothar Meyer had developed a table of elements which show clear resemblance with the modern periodic table but his work was not published.

Main-group elements

1
IA

2
IIA

3
Li
6.941

4
Be
9.012182

11
Na
22.989768

12
Mg
24.3050

19
K
39.0983

20
Ca
40.078

37
Rb
85.4678

38
Sr
87.62

55
Cs
132.90543

56
Ba
137.327

87
Fr
(223)

88
Ra
(226)

1
H
1.00794

2
He
4.002602

3
Li
6.941

4
Be
9.012182

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22.989768

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Notation for IUPAC Nomenclature of Elements

S.No.	Digit	Name	Abbreviation
1.	0	nil	n
2.	1	un	u
3.	2	bi	b
4.	3	tri	t
5.	4	quad	q
6.	5	pent	p
7.	6	hex	h
8.	7	sept	s
9.	8	oct	o
10.	9	enn	e

To write the name of any element with atomic number more than 100, write the name of each number from the table followed by “ium”.

Atomic no.	Name
101	un-nil-unium 1 0 1
109	un-nil-ennium 1 0 9

Periodic Properties

The elements in the periodic table are arranged in order of increasing atomic number. All of these elements display several other trends and we can use the periodic law and table formation to predict their chemical, physical and atomic properties.

1. Atomic Size or Atomic Radius

Left to right size of atom decreases; down the group size increases.

(i) **Ionic radius** Distance of the outermost shell of an ion from its nucleus.

(ii) **Covalent radius**

(a) In homodiatom molecule, ($A - A$)

$$r = \frac{d_{A-A}}{2}, \quad d_{A-A} = \text{bond length in molecule}$$

(b) In heterodiatom molecule, ($A - B$)

• When ($X_A - X_B$) is very small

$$d_{A-B} = r_A + r_B$$

$$d_{A-B} = \text{bond length}$$

r_A and r_B = covalent radii of A and B respectively

X_A and X_B = electronegativities of A and B

• When ($X_A - X_B$) is considerable

$$d_{A-B} = r_A + r_B - 0.09 (X_A - X_B)$$

(iii) **Metallic radius**, $r' = \frac{d}{2}$

$r' >$ actual size of atom

(iv) **van der Waals' radius**, $r_1 = \frac{d}{2}$, $r_1 > >>$ actual size of atom

Covalent radius < Metallic radius < van der Waals' radius

In the end of the period, atomic radii of inert gases are exceptionally higher because they do not form molecule and their radii are simply van der Waals' radii.

2. Ionisation Enthalpy

It is the energy required when an electron is removed from an isolated gaseous atom. Generally left to right in periods, IE increases; down the group it decreases but half-filled orbitals and fully filled orbitals are stable form and have high IE.

Various factors affecting ionisation energy varies are:

- (i) Atomic size : varies inversely
- (ii) Screening effect : varies inversely
- (iii) Nuclear charge : varies directly
- (iv) Helium has the highest IE_1 while Cs has the lowest.
- (v) IE values of inert gases are exceptionally higher due to stable configuration.

Screening Effect

Shielding (or screening) effect is the repulsion of valence electrons by the electrons in penultimate shell to reduce effective nuclear charge.

$$Z_{\text{eff}} = Z - \sigma$$

where, Z = atomic number, σ = shielding constant

$\sigma = [0.35 \times \text{number of electrons in } n\text{th shell excluding last (valence) electron}] + [0.85 \times \text{number of electrons in } (n-1)\text{th shell}] + [1.0 \times \text{number of electrons in inner shell}]$

3. Electron Gain Enthalpy

It is the energy released when an electron is added in an isolated gaseous atom. Chlorine has the highest electron affinity but oxidising power of fluorine is larger than chlorine.

Various factors affecting electron gain enthalpy are:

- (i) Atomic size : varies inversely
- (ii) Nuclear charge : varies directly
- (iii) Configuration : Half-filled orbitals and fully filled orbitals are stable form, therefore electron gain enthalpy will be low or even sometimes energy is required rather than getting released.

Exception $\text{Cl} > \text{F}$ (electron gain enthalpy)

$\text{S} > \text{O}$

F and O-atoms have small size and high charge density, therefore have lower electron gain enthalpy.

4. Electronegativity

It is the tendency to attract the shared pair of electrons. F is the most electronegative element while Cs is the least.

Difference in the electronegativities of two atoms (A and B)

$$X_A - X_B = 0.208 \sqrt{\Delta}$$

where, Δ = actual bond energy

– energy for 100% covalent bond

$$\therefore \Delta = E_{A-B} - \sqrt{E_{A-A} \times E_{B-B}}$$

Here, E_{A-B} = dissociation enthalpy of A — B (kcal mol^{-1})

E_{A-A} = dissociation enthalpy of A — A (kcal mol^{-1})

E_{B-B} = dissociation enthalpy of B — B (kcal mol^{-1})

Mulliken's approach

$$X_M = \frac{\text{IE} + \text{EA}}{2}$$

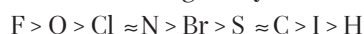
Mulliken values are ≈ 2.8 times greater than Pauling values.

$$X_P = \frac{\text{IE} + \text{EA}}{5.6} \text{ or } X_P = 0.336 [X_M - 0.615]$$

Unit of electronegativity = eV

For noble gases, its value is taken as zero.

Decreasing order of electronegativity



In periods left to right, electronegativity increases.

In groups down the group, electronegativity decreases.

5. Valency

The valency of an element is related to the electronic configuration of its atom and usually determined by electrons present in valence shell (outer shell).

Variation of Valency in a Period

Valency of the elements with respect to hydrogen increases from 1 to 4 upto 14th group and then decreases to 1.

e.g. NaH , CaH_2 , AlH_3 , SiH_4 , PH_3 , H_2S , HCl .

But valency of the elements with respect to oxygen increases from one to seven along a period.

e.g. Na_2O , CaO , Al_2O_3 , SiO_2 , P_4O_{10} , SO_3 , Cl_2O_7

Variation of Valency in a Group

All elements in a group have the same valency as they have same number of electrons in their outer shell. *p*-block elements show variable valency on account of inert pair effect.

Term oxidation state is also being used for covalency, which indicates the actual charge on atom in that particular molecule. It follows the same trend along period or group as valency.

NOTE Tin has maximum number of isotopes (10 in number) and Os has the highest oxidation state or valency (+8).

Variation of Valency in Transition Elements

Transition elements and inner-transition elements show variable valency of 1, 2 or 3 as they use electrons from outer as well as penultimate or inner-penultimate shell.

6. Chemical Reactivity

Reactivity of metals increases with decrease in ionisation energy, electronegativity and increase in atomic radii and electropositive character.

Reactivity of non-metals increases with increase in electronegativity and electron gain enthalpy and decreases with increase in atomic radii.

Variation in a Group

On moving down the group, reactivity of metals increases while for non-metals, it decreases.

Variation in a Period

Reactivity of metals decreases while reactivity of non-metals increases across the period.

Some other Periodic Trends

- Bond energy $\propto$ electronegativity difference
- Bond length $\propto \frac{1}{\text{electronegativity difference}}$
- Acidic character of hydride $\propto$ electronegativity of central atom
- Acidic character of hydra acids increases on moving left to right and top to bottom.
- **Periodicity of oxyacids** Acidic character increases left to right while decreases top to bottom.
- **Nature of oxides** Acidic character increases left to right while decreases top to bottom.

Practice Exercise

- Which of the following has firstly organised the elements according to same trend?
 - Newland
 - Mendeleev
 - Lothar Meyer
 - Dobereiner
- Johann Dobereiner gave the idea of trends among the physical and A..... of several groups of three elements. Here, A refers to
 - atomic number
 - atomic mass
 - chemical properties
 - electronic configuration
- Lothar Meyer plotted the physical properties such as atomic volume, melting point and A..... against atomic weight. Here, A refers to
 - mass
 - boiling point
 - surface tension
 - molecules
- Lothar Meyer drew a graph showing the relation between
 - atomic number and atomic weight
 - atomic number and atomic size
 - atomic weight and atomic volume
 - atomic weight and atomic size
- Which of the following is incorrect explanation about Mendeleev's periodic law?
 - Mendeleev arranged elements in horizontal rows only
 - Mendeleev arranged elements with increasing atomic weight
 - Mendeleev's system of classifying elements was more elaborate than that of Lothar
 - Both (a) and (b)
- Which basic concept was used by Mendeleev for organising the elements?
 - They organise the metals only
 - They organise the non-metals only
 - They organise the both metals and non-metals according to increasing number of masses
 - None of the above
- Which element was named as eka-silicon in Mendeleev classification of elements?
 - Germanium
 - Gallium
 - Thallium
 - Selenium
- The statement that is not correct for the periodic classification of elements, is
 - the properties of the elements are the periodic function of their atomic number
 - non-metallic elements are lesser in number than metallic elements
 - the first ionisation energies of elements along a period do not vary in a regular manner with increase in atomic number
 - for transition elements, the *d*-subshells are filled with electrons monotonically with increase in atomic numbers
- The period number in the long form of the periodic table is equal to
 - magnetic quantum number of any element of the period
 - atomic number of any element of the period
 - maximum principal quantum number of any element of the period
 - maximum azimuthal quantum number of any element of the period
- Which of the following is incorrect?
 - Henry Moseley observed regularities in the characteristics X-ray spectra of elements
 - A plot of frequency of X-ray emitted against atomic number (*Z*) gives a straight line
 - The atomic number of an element is equal to the number of neutron(s) in a neutral atom
 - Electronic configuration of an atom determines the physical and chemical properties
- Match the Column I with Column II and select the correct answer by given codes.

Column I (Number of periods)	Column II (Number of elements)
A. First period	1. 14
B. Third period	2. 02
C. Lanthanoids	3. 14
D. Actinoids	4. 08
	5. 04

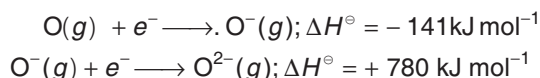
Codes

A	B	C	D	A	B	C	D
a. 2	4	1	5	b. 2	4	1,3	3,1
c. 4	2	1	3	d. None of these			
- Observe the following statements,
 - The physical and chemical properties of elements are the periodic functions of their electronic configuration.
 - Electronegativity of fluorine is less than the electronegativity of chlorine.
 - Electropositive nature decreases from top to bottom in a group.
 The correct answer is
 - I, II and III
 - Only I
 - I and II
 - II and III
- In an element ${}_ZM^{19}$, there are 10 neutrons in the nucleus. It belongs to
 - s-block
 - d-block
 - f-block
 - None of the above

14. The third alkaline earth metal ion contains number of electrons and protons as
 a. $20e^-$, $20p$ b. $18e^-$, $18p$
 c. $18e^-$, $20p$ d. $19e^-$, $20p$
15. A metal having electronic configuration $1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^{10}, 4s^2$ is in
 a. s-block b. d-block
 c. p-block d. None of these
16. Which of the following orbitals are in the process of filling in the 6th period?
 a. $6s, 6p, 6d, 6f$ b. $6s, 5f, 6d, 6p$
 c. $6s, 4f, 5d, 6p$ d. $5s, 5p, 5d$
17. The configuration to second excited state of the element is (O) electronic with O_2 or P^- or Cl^+ is
 a. $[Ne] 3s^2 3p_x^2 3p_y^1 3p_z^1$
 b. $[Ne] 3s^2$
 c. $[Ne] 3s^1 3p_z^1 3p_y^1 3p_x^1 3d_{xy}^1 3d_{yz}^1$
 d. $[Ne] 3s^2 3p_x^1 3p_z^1 3p_{xy}^1$
18. The element having atomic number 33 lies in the group
 a. 16 b. 14 c. 15 d. 13
19. General configuration of ultimate and penultimate shell is $-(n-1)s^2 (n-1)p^6 (n-1)d^x ns^2$. If $n=4$ and $x=5$, the number of protons in the nucleus would be
 a. < 24 b. 25 c. 24 d. > 25
20. Generally, the valency of noble gases is
 a. two b. three c. one d. zero
21. Identify the least stable ion among the following.
 a. Li^- b. Be^- c. B^- d. C^-
22. The electronic configuration of an element is $1s^2, 2s^2 2p^6, 3s^2 3p^3$. The atomic number and the group number of the element X which is just below the above element in the periodic table respectively, are
 a. 23 and 5 b. 23 and 15
 c. 33 and 15 d. 33 and 5
23. What will be the IUPAC name of element having $Z=106$?
 a. Unnilquadium b. Unnilhexium
 c. Unnilheptium d. Ununhexium
24. The one with the largest ionic size is
 a. O^{2-} b. Mg^{2+} c. F^- d. Na^+
25. Fluorine and neon have atomic radii in angstrom given by
 a. 1.60, 1.60 b. 0.72, 0.72
 c. 0.72, 1.60 d. None of these
26. Which of the following alkali metals has smallest size?
 a. Cs b. Rb c. Na d. K
27. Which of the following ionic species has largest size?
 a. $Rb^+(aq)$ b. $Li^+(g)$ c. $Na^+(aq)$ d. $Li^+(aq)$
28. Chloride ion and potassium ion are isoelectronic, then
 a. their sizes are same
 b. Cl^- ion is bigger than K^+ ion
 c. K^+ ion is relatively bigger
 d. their sizes depend on other cation and anion
29. Which one of the following is correct increasing order of size?
 a. $Mg < Na^+ < F^- < Al$ b. $Na^+ < Al < Mg < F^-$
 c. $Na^+ < F^- < Al < Mg$ d. $Na^+ < F^- < Mg < Al$
30. Ionic radii vary in
 a. inverse proportion to the effective nuclear charge
 b. inverse proportion to the square of effective nuclear charge
 c. inverse proportion to the screening effect
 d. direct proportion to the square of the screening effect
31. The first ionisation energy of beryllium is more than that of boron because
 a. boron has higher nuclear charge
 b. boron has only one electron in p-subshell
 c. atomic size of boron is less than that of beryllium
 d. atomic size of boron is more than that of beryllium
32. Among the following, the element with highest ionisation potential is
 a. boron b. carbon
 c. oxygen d. nitrogen
33. The isoelectronic ion having lowest ionisation energy is
 a. S^{2-} b. Ca^{2+} c. K^+ d. Cl^-
34. The values in electron-volt per atom which represent the first ionisation energy of oxygen and nitrogen atom respectively are
 a. 13.6 and 14.6 b. 14.6 and 13.6
 c. 14.6 and 14.6 d. 13.6 and 13.6
35. The pair in which the ionisation energy of first species is less than that of second is
 a. N, P b. Be, Be^+ c. S, P d. N, N^-
36. How many Cs-atoms can be converted to Cs^+ ion by 1 J energy if IE_1 for Cs is 376 kJ mol^{-1} ?
 a. 1.60×10^{23} b. 1.60×10^{15}
 c. 1.60×10^{18} d. 16.0×10^{26}
37. How many joules of energy must be absorbed to convert Li to Li^+ , all the atoms present in 1.00 mg of gaseous Li? IE_1 of Li is $520.3 \text{ kJ mol}^{-1}$ ($Li = 7$).
 a. 0.00743 kJ b. 0.520 kJ
 c. 520 kJ d. 0.0743 kJ
38. The incorrect statements among the following is
 a. the first ionisation potential of Al is less than the first ionisation potential of Mg
 b. the second ionisation potential of Mg is greater than the second ionisation potential of Na
 c. the first ionisation potential of Na is less than the first ionisation potential of Mg
 d. the third ionisation potential of Mg is greater than the third ionisation potential of Al

39. Ionisation enthalpy of Na would be same as
 a. value of electron affinity of Na^+
 b. value of electronegativity of Na
 c. value of ionisation potential of Mg
 d. value of electron affinity of Na
40. Which one of the following statements is incorrect in relation to ionisation enthalpy?
 a. Ionisation enthalpy increases for each successive electrons
 b. The greatest increase in ionisation enthalpy is experienced on removal of electron from core noble gas configuration
 c. End of valence electrons is marked by a big jump in ionisation enthalpy
 d. Removal of electron from orbitals bearing lower n value is easier than from orbitals having higher n value

41. The formation of the oxide ion, $\text{O}^{2-}(\text{g})$ from oxygen atom requires first an exothermic and then an endothermic step as shown below



Thus, process of formation of O^{2-} in gas phase is unfavourable even though O^{2-} is isoelectronic with neon. It is due to the fact that,

- a. oxygen is more electronegative
 b. addition of electron in oxygen results in larger size of the ion
 c. electron repulsion outweighs the stability gained by achieving noble gas configuration
 d. O^- ion has comparatively smaller size than oxygen atom
42. Which one of the following arrangements represent the correct order of electron gain enthalpy (with negative sign) of the given atomic species?
 a. $\text{Cl} < \text{F} < \text{S} < \text{O}$
 b. $\text{O} < \text{S} < \text{F} < \text{Cl}$
 c. $\text{S} < \text{O} < \text{Cl} < \text{F}$
 d. $\text{F} < \text{Cl} < \text{O} < \text{S}$
43. Electronic configurations of four elements A, B, C and D are given below
 A. $1s^2, 2s^2, 2p^6$ B. $1s^2, 2s^2, 2p^4$
 C. $1s^2, 2s^2, 2p^6, 3s^1$ D. $1s^2, 2s^2, 2p^5$
- Which of the following is the correct increasing order of tendency to gain electron?
 a. $A < C < B < D$ b. $A < B < C < D$
 c. $D < B < C < A$ d. $D < A < B < C$
44. In which of following order of arrangement does not agree with variation of property indicated against it?
 a. $\text{Al}^{3+} < \text{Mg}^{2+} < \text{Na}^+ < \text{F}^-$ (Increasing ionic size)
 b. $\text{B} < \text{C} < \text{N} \geq \text{O}$ (Increasing first ionisation enthalpy)
 c. $\text{I} < \text{Br} < \text{Cl} < \text{F}$ (Increasing electron gain enthalpy)
 d. $\text{Li} < \text{Na} < \text{K} < \text{Rb}$ (Increasing metallic radius)

45. The electron affinity values (in kJ mol^{-1}) of three halogens X, Y and Z are -349 , -333 and -325 respectively. Then, X, Y and Z respectively are
 a. F_2 , Cl_2 and Br_2 b. Cl_2 , F_2 and Br_2
 c. Cl_2 , Br_2 and F_2 d. Br_2 , Cl_2 and F_2

46. Which one of the following statements is false?
 a. The electron affinity of chlorine is less than that of fluorine
 b. The electronegativity of fluorine is more than that of chlorine
 c. The electron affinity of bromine is less than that of chlorine
 d. The electronegativity of chlorine is more than that of bromine

47. Which of the following element has highest electronegativity?

a. Oxygen b. Chlorine
 c. Fluorine d. Nitrogen

48. In C, N, O and F, the electronegativity
 a. increases from carbon to fluorine
 b. decreases from carbon to oxygen and then increases
 c. decreases from carbon to fluorine
 d. increases from carbon to oxygen and then decreases

49. Electronegativity of F on Pauling scale is 4.0. What will be its value on Mulliken scale?
 a. 10.0 b. 11.2 c. 8.54 d. 16

50. Two elements whose electronegativities are 1.2 and 3.0 respectively, the bond formed between them would be
 a. ionic b. covalent c. coordinate d. metallic

51. What will be the electronegativity of carbon at Pauling scale? Given that, $E_{\text{H}-\text{H}} = 104.2 \text{ kcal mol}^{-1}$,

$$E_{\text{C}-\text{C}} = 83.1 \text{ kcal mol}^{-1}, E_{\text{C}-\text{H}} = 98.8 \text{ kcal mol}^{-1}$$

[Electronegativity of hydrogen = 2.1]

a. 0.498 b. 0.598 c. 2.134 d. 2.597

52. Which one of the following is incorrect?

a. Non-metals have strong tendency to gain electron
 b. Electronegativity is directly related to non-metallic properties of elements
 c. Electronegativity is inversely proportional to the metallic properties of elements
 d. Increase in electronegativity down the group is accompanied by a decrease in non-metallic properties

53. Pauling's equation for determining the electronegativity of an element, is

(X_A, X_B are electronegativity values of elements A and B respectively, Δ represents the polarity of A—B bond.)

a. $X_A - X_B = 0.208 \sqrt{\Delta}$ b. $X_A + X_B = 0.208 \sqrt{\Delta}$
 c. $X_A - X_B = 0.208 \Delta^2$ d. $X_A - X_B = \sqrt{\Delta}$

54. Match the Column I with Column II and select the correct answer by given codes.

Column I (Elements)	Column II (Properties)
A. $\text{Li}^+ < \text{Al}^{3+} < \text{Mg}^{2+} < \text{K}^+$	1. ΔEA (Electron affinity)
B. $\text{Li}^+ > \text{Al}^{3+} > \text{Mg}^{2+} > \text{K}^+$	2. Ionic radii
C. $\text{Cl} > \text{F} > \text{Br} > \text{I}$	3. EN (Electronegativity)
D. $\text{F} > \text{Cl} > \text{Br} > \text{I}$	4. ENC

Codes

	A	B	C	D		A	B	C	D
a.	2	4	3	1	b.	2	4	1	3
c.	4	2	3	1	d.	4	2	1	3

55. The set that contains pairs of elements that do not belong to same group but show chemical resemblance is
 a. B, Al b. Be, Al c. Hf, Zr d. K, Pb

56. Which pair of elements has same chemical properties?
 a. 13, 22 b. 3, 11 c. 4, 24 d. 2, 4
57. The element of second period which forms most acidic oxide is
 a. carbon b. boron c. fluorine d. nitrogen
58. In periodic table, melting point/ boiling point increases down the group in which of the following group?
 a. Group 13 b. Group 2
 c. Group 17 d. Group 1
59. Considering the elements B, C, N, F and Si, the correct order of their non-metallic character is [NCERT]
 a. $\text{B} > \text{C} > \text{Si} > \text{N} > \text{F}$ b. $\text{Si} > \text{C} > \text{B} > \text{N} > \text{F}$
 c. $\text{F} > \text{N} > \text{C} > \text{B} > \text{Si}$ d. $\text{F} > \text{N} > \text{C} > \text{Si} > \text{B}$
60. Considering the elements B, Al, Mg, and K, the correct order of their metallic character is
 a. $\text{B} > \text{Al} > \text{Mg} > \text{K}$ b. $\text{Al} > \text{Mg} > \text{B} > \text{K}$
 c. $\text{Mg} > \text{Al} > \text{K} > \text{B}$ d. $\text{K} > \text{Mg} > \text{Al} > \text{B}$

BITSAT Archives

1. Elements/ions having same number of electrons are known as isoelectronic species. Arrange the following elements in correct order of atomic/ionic radii and choose the correct option from the four choices given below [2013]
 $\text{O}^{2-}, \text{Na}^+, \text{Mg}^{2+}, \text{F}^-, \text{Al}^{3+}$
 a. $\text{Al}^{3+} < \text{Mg}^{2+} < \text{Na}^+ < \text{F}^- < \text{O}^{2-}$
 b. $\text{Al}^{3+} < \text{Na}^+ < \text{Mg}^{2+} < \text{F}^- < \text{O}^{2-}$
 c. $\text{Al}^{3+} > \text{Mg}^{2+} > \text{Na}^+ > \text{F}^- > \text{O}^{2-}$
 d. None of the above
2. For the properties mentioned, the correct trend for the different species is in [2012]
 a. Strength as Lewis acid— $\text{BCl}_3 > \text{AlCl}_3 > \text{GaCl}_3$
 b. Inert pair effect— $\text{Al} > \text{Ga} > \text{In}$
 c. Oxidising property— $\text{Al}^{3+} > \text{In}^{3+} > \text{Tl}^{3+}$
 d. First ionisation enthalpy— $\text{B} > \text{Al} > \text{Ti}$
3. Which of the following choices represent the correct order of first ionisation enthalpy? [2012]
 a. $\text{B} < \text{C} < \text{N} < \text{O} < \text{F}$ b. $\text{B} > \text{C} > \text{N} > \text{O} > \text{F}$
 c. $\text{B} < \text{C} < \text{N} > \text{O} < \text{F}$ d. $\text{B} < \text{C} < \text{N} > \text{O} > \text{F}$
4. Electron gain enthalpy of fluorine is lower than that of chlorine is due to [2010]
 a. smaller size
 b. smaller nuclear charge
 c. difference in their electronic configurations
 d. its highest reactivity
5. The atomic numbers of elements A, B, C and D are $Z - 1$, Z , $Z + 1$ and $Z + 2$ respectively. If B is a noble gas, choose the correct statement among the the following statements: [2010]
 I. A has higher electron affinity.
 II. C exists in +2 oxidation state.
 III. D is an alkaline earth metal.
 a. I and II b. II and III c. I and III d. I, II and III

Answer with Solutions

Practice Exercise

- (d) Dobereiner has firstly used triad system to organise the elements.
- (c) Johann Dobereiner in early 1800's was the first to consider the idea of trends among properties of elements. By 1829 he noted a similarity among the physical and chemical properties of several groups of three elements (triads).
- (b)
- (c) Lothar Meyer drew a graph showing the relation between atomic weight and atomic volume.
- (a) Mendeleef arranged elements in horizontal rows and vertical columns of a table in increasing order to their atomic weights.
- (c) Mendeleef observed that the periodic properties of an element is a periodic function of atomic masses.
- (a) Germanium was called eka-silicon in Mendeleef's time.

8. (d) 9. (c)
10. (c) The atomic number of elements is equal to the number of electrons/number of protons in a neutral atom.
 $Z = \text{number of } e^- / p^+$
 $A = \text{number of proton} + \text{number of neutron}$
11. (b) $A \rightarrow 2, B \rightarrow 4, C \rightarrow 1, 3, D, \rightarrow 3, 1$
 A. First period contains 2 elements.
 B. Third period contains 8 elements.
 C. D. 14 elements of both sixth and seventh periods are known as lanthanoids and actinoids respectively.
12. (b) The physical and chemical properties of elements are periodic functions of their electronic configuration.
 Electronegativity of fluorine is more than that of chlorine.
 Electropositive nature increases from top to bottom in a group.
13. (d) $Z = 19 - 10 = 9$
 $EC = 1s^2 2s^2 2p^5$, hence, it belongs to p -block.
14. (c) $Ca^{2+}; p = 20, e^- = 18$
15. (b) $Zn(3d^{10} 4s^2)$ is d -block element.
16. (c) The order in which orbitals are filled, is
 $6s \rightarrow 4f \rightarrow 5d \rightarrow 6p$
17. (c) In O_2 or P^- or Cl^+ the electrons present are 16. The element is sulphur ($Z = 16$). The electronic configuration of sulphur in second excited state is given by (c).
18. (c) Arsenic is in group 15.
19. (b) The element having outer electronic configuration $3s^2 3p^6, 3d^5 4s^2$, has atomic number 25. Therefore, the number of protons is 25.
20. (d) They are chemically inert (inactive).
21. (b) Be has fully filled $2s$ -subshell ($2s^2$) and therefore, show least tendency to accept an electron. Thus, Be^- is least stable.
22. (c) Atomic number of the given element = 15
 Group number = $10 + 5$ (valence electrons) = 15
 Period = 3
 Atomic number of the element just below = 33
23. (b) For $Z = 106$, Root of 1 — un, Root of 0 — nil
 Root of 6 — hex + ium, IUPAC name = unnilhexium
24. (a) O^{2-}, F^-, Mg^{2+} and Na^+ are isoelectronic species (each having $10 e^-$). The size of isoelectronic species decreases with increase in nuclear charge (number of protons). Hence, correct order of size is
 $O^{2-} > F^- > Na^+ > Mg^{2+}$
25. (c) 26. (c) 27. (d) 28. (b)
29. (b) Na^+ and F^- ions are isoelectronic, therefore F^- has the largest and Na^+ has the lowest size. Further, Al with higher nuclear charge has lower size than Mg. Thus, the overall order is
 $Na^+ < Al < Mg < F^-$
30. (a)
31. (b) Boron has only one electron in p -subshell, hence it can be lost easily, while beryllium has comparatively stable configuration ($1s^2, 2s^2$) hence, cannot give an electron easily, consequently its IE is greater.
32. (d) Nitrogen has stable half-filled orbital.
33. (b)
34. (a) Ionisation energy of N > ionisation energy of oxygen, since, nitrogen has stable configuration due to half-filled $2p$ -orbitals.
35. (c) P has higher ionisation energy due to stable half-filled configuration.
36. (c) $376 \times 10^3 \text{ J}$ energy produces 6.023×10^{23} ions.
 $\therefore 1 \text{ J}$ energy will produce = $\frac{6.023 \times 10^{23}}{3.76 \times 10^3}$
 $= 1.60 \times 10^{18} \text{ Cs}^+$ ions
37. (d) $1.00 \text{ mg} = 1 \times 10^{-3} \text{ g} = \frac{1 \times 10^{-3}}{7} \text{ mol Li}$
 1 mole of Li is converted by 520.3 kJ.
 $\therefore \frac{1 \times 10^{-3}}{7} \text{ mole is converted to } Li^+ \text{ by}$
 $= \frac{520.3 \times 1 \times 10^{-3}}{7} \text{ kJ} = 0.0743 \text{ kJ}$
38. (b) IE_2 of Mg is lower than that of Na because in case of Mg^{2+} , $3s$ electron has to be removed while in case of Na^+ , an electron from the stable inert gas configuration (neon) has to be removed.
39. (a) $Na \longrightarrow Na^+ + e^-$
 $Na^+ + e^- \longrightarrow -X; \Delta_{eg} = -IE$
40. (d) Higher n value has lower value of IE due to large size.
41. (c) The process of formation of O^{2-} in gas phase is unfavourable even though O^{2-} is isoelectronic with neon because electron repulsion outweighs the stability gained by achieving noble gas configuration.
42. (b)
43. (b) np^5 and ns^1 elements possess a large tendency to accept the electron than fully and half-filled elements
44. (c)
45. (b) The electron affinity for halogens,
 $F = 332.6 \approx 333 \Rightarrow Y$
 $Cl = -348.5 \approx -349 \Rightarrow X$
 $Br = -324.7 \approx -325 \Rightarrow Z$
 X has the highest value of electron affinity. Therefore, the correct order of electron affinity is
 $Cl_2 > F_2 > Br_2$
46. (a) The electron affinity of fluorine is less than that of chlorine, due to very small size of fluorine in which negative charge is highly concentrated and repels the incoming electrons.

47. (c)

48. (a) In a period, it increases from left to right.

49. (b) Electronegativity on Mulliken scale and Pauling scale are given by

$$X_M = X_P \times 2.8 = 4.0 \times 2.8 \\ = 11.2$$

Usually, Mulliken's value of electronegativity are 2.8 times of Pauling value.

50. (a) When the electronegativity difference is more than 1.7, the bond formed will be ionic in nature.

51. (d) $X_C - X_H = 0.208 \sqrt{\Delta}$

$$\text{where, } \Delta = E_{C-H} - \sqrt{E_{C-C} \times E_{H-H}}$$

$$\Delta = 98.8 - \sqrt{83.1 \times 104.2}$$

$\therefore$

$$\Delta = 5.75 \\ X_C - 2.1 = 0.208 \sqrt{5.75}$$

$$X_C - 2.1 = 0.497$$

$$X_C = 2.597$$

52. (d) Decrease in electronegativity down the group is accompanied by a decrease in non-metallic properties.

53. (a) Pauling's equation

$$X_A - X_B = 0.208 \sqrt{\Delta}$$

where, X_A and X_B are electronegativity values of elements A and B and $\Delta = E_{A-B} - \sqrt{E_{A-A} \times E_{B-B}}$

Here, E_{A-B} , E_{A-A} and E_{B-B} are the bond energies of the molecules AB, A_2 and B_2 respectively.

54. (b) $A \rightarrow 2, B \rightarrow 4, C \rightarrow 1, D \rightarrow 3$

A. $Li^+ < Al^{3+} < Mg^{2+} < K^+$

The cation with the greater positive charge will have a smaller radius because of the greater attraction of the electrons to the nucleus. Anion with the greater negative charge will have the larger radius.

$$\text{Positive charge} \propto \frac{1}{\text{ionic radius}}$$

Negative charge $\propto$ ionic radius

B. Greater positive charge, increases ENC in case of isoelectronic species while for same group elements, ENC decreases down the group.

C. $Cl > F > Br > I$

Electron affinity of Cl is highest in halogen family.

D. $F > Cl > Br > I$

Electronegativity of fluorine (F) is higher than that of Cl, Br and I.

55. (b)

56. (b) The pair which belongs to same group, i.e. in which both the elements have same outer electronic configuration has same chemical properties.

$$3 \Rightarrow 1s^2, 2s^1$$

$$11 \Rightarrow 1s^2, 2s^2, 2p^6, 3s^1$$

57. (c)

58. (c)

59. (d)

60. (d)

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1. (a) Elements/ions having equal number of electrons are known as isoelectronic species. Among isoelectronic species, cations having highest charge are smallest while anion having highest charge are largest.

Cation < Neutral atom < Anion

Hence, correct choice is

$$Al^{3+} < Mg^{2+} < Na^+ < F^- < O^{2-}$$

2. (d) As we know on moving down the group, first ionisation enthalpy decreases top to bottom, therefore order of first ionisation enthalpy for group 13 elements is

$$B > Al > Ga > In > Tl$$

3. (d) Ionisation energy is the minimum amount of energy required to remove the outermost electron from an isolated gaseous atom. Quantitatively, it depends on the attraction between electron present on outermost shell and nucleus. Greater the interaction between outermost electron and nucleus, higher will be its ionisation enthalpy. So, correct order of first it must be

$$B < C < N < O < F$$

But due to extra stable half-filled electronic configuration of p-orbital, N has more value of first ionisation enthalpy than oxygen hence, correct order is

$$B < C < N > O > F$$

4. (a) Because of the small size of F, electron-electron repulsions present in its relatively compact 2p-subshell, do not easily allow the addition of an extra electron. On the other hand, Cl because of its comparatively bigger size than F, allows the addition of an extra electron more easily. Thus, the EA of Cl is higher than that of F.

5. (c) $A = Z - 1$

$B = Z \rightarrow$ Noble gas (outer subshells are s or p)

$$C = Z + 1$$

$$D = Z + 2$$

Noble gas with outer s-subshell = He ($1s^2$)

Noble gas with outer p-subshell

$$= ns^2 np^6 \text{ like [Ne]}$$

In periodic table, electron affinity is highest in chlorine of group 17 with electronic configuration of $3s^2 3p^5$.

$$\text{Noble gas} = 3s^2 3p^6 = [\text{Ar}]$$

$$B = [\text{Ar}] \rightarrow Z \rightarrow 3s^2 3p^6 \text{ (Noble gas)}$$

$$A = Z - 1 \rightarrow 3s^2 3p^5 \text{ (Halogen family)}$$

$$C = Z + 1 \rightarrow 3s^2 3p^6, 4s^1 \text{ (Alkali metal)}$$

$$D = Z + 2 \rightarrow 3s^2 3p^6, 4s^2 \text{ (Alkaline earth metal)}$$

6

States of Matter

Matter

Anything that has mass and takes up space is known as, 'matter'. It clearly means, it is everything around us.

In general, matter exists in three states:

- (i) Solids: They have definite volume, a definite shape and hence they are rigid.
- (ii) Liquids: They have definite volume, but do not have definite shape and hence they are non-rigid.
- (iii) Gases: They have neither definite shape, nor definite volume and hence they are non-rigid. It is the simplest state and shows great uniformity in behaviour.

Gaseous State

It is the most disordered state of matter. In this state, matter neither have fixed volume nor fixed shape. It takes the shape and volume of the container in which it is placed.

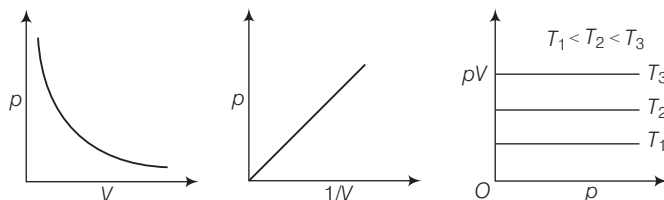
Gas Laws

Gases exhibit dependency on temperature, pressure, volume and mass. The inter relationship of these factors can be analysed through gas laws which are given below:

1. Boyle's Law (1662)

At constant temperature (T), $p \propto \frac{1}{V}$

$$pV = \text{constant (for given moles and } T) \text{ or } p_1V_1 = p_2V_2$$

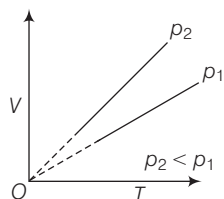


2. Charles' Law (1787)

At constant pressure (p), $V \propto T$

$$\frac{V}{T} = \text{constant (for given } n \text{ and } p)$$

or
$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$



Charles obtained experimentally that for 1°C change in temperature of a gas, the volume get changed by a fraction of $\frac{1}{273}$. Hence, if V_0 be the volume of a gas at 0°C and it is recorded to be V at $t^\circ\text{C}$ then

$$V = V_0 + \frac{1}{273.15} T \quad V_0 = V_0 \left(1 + \frac{T}{273.15} \right) = V_0 \left(\frac{273.15 + T}{273.15} \right)$$

If temperature is lowered, the volume decreases at constant pressure and becomes zero at -273.15°C . Further lowering is not possible thus, it is called the lowest possible temperature or absolute zero of temperature.

$$T(\text{K}) = t(^{\circ}\text{C}) + 273.15$$

3. Gay-Lussac's Law

At constant volume (V)

$$p \propto T$$

$$\frac{p}{T} = \text{constant (for given } n \text{ and } V)$$

or
$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

4. Avogadro's Law

It states that equal volumes of all gases at same pressure and temperature contain equal number of molecules.

$$V \propto n \quad (\text{at given } T \text{ and } p)$$

or
$$\frac{V_1}{n_1} = \frac{V_2}{n_2}$$

5. Ideal Gas Equation

$$pV = nRT \quad \text{or} \quad pV = \frac{w}{M} RT \quad \text{or} \quad pM = dRT$$

where, w = mass of a gas

M = molecular weight of a gas

d = density of a gas

$$R = \text{gas constant} = 0.0821 \text{ L atm K}^{-1} \text{ mol}^{-1} \\ = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$$

6. Dalton's Law of Partial Pressures

Total pressure of a mixture of a number of non-reacting gases is equal to the sum of the pressures exerted by individual gases.

$$p_{\text{total}} = p_1 + p_2 + p_3 + p_4 + \dots$$

- Partial pressure of a gas = Its mole fraction $\times$ total pressure exerted by the mixture
- When a gas is collected over the water, the pressure of the gas

$$p_{\text{observed or moist}} = \text{Pressure of dry gas} \\ + \text{aqueous tension or water VP}$$

- Relative humidity (R_H) is given by $R_H = \frac{\text{Partial pressure of water in air}}{\text{vapour pressure of water}}$.

7. Graham's Law of Diffusion

Rate of diffusion or effusion

$$\propto \frac{1}{\sqrt{\text{Molecular weight}}} \propto \frac{1}{\sqrt{\text{density}}} \\ \text{Density} = \frac{\text{Molecular weight}}{2}$$

$$\text{Rate of effusion (r)} = \frac{\text{Volume effused}}{\text{Time taken}}$$

Hence,
$$\frac{r_1}{r_2} = \frac{t_2}{t_1} = \sqrt{\frac{d_2}{d_1}} = \sqrt{\frac{M_2}{M_1}}$$

If p is not constant, $r \propto p$

$$\therefore \frac{r_1}{r_2} = \frac{p_1}{p_2} \sqrt{\frac{M_2}{M_1}}$$

Kinetic Theory of Gases

The molecular details regarding gases can be visualised with the help of kinetic molecular theory of gases which is based on the following assumptions:

- A gas consists of extremely small discrete molecules so that volume of the molecules is negligible as compared to the total volume of the gas.
- Gas molecules are in constant random motion with high velocities and change directions on collision with other molecules or walls of container.
- The intermolecular forces and the force of gravity on them are negligible.
- The collisions are perfectly elastic, therefore, there is no loss of kinetic energy during collision.
- The pressure of a gas is caused by bombardment of moving molecules against the walls of container.
- In a gas, different molecules have different kinetic energies but the average kinetic energy of molecules is proportional to absolute temperature of the gas. This is known as Maxwell's generalisation.

On the basis of these assumptions, the following mathematical expression is derived :

$$pV = \frac{1}{3} mNu^2$$

or $pV = \frac{1}{3} Mu^2$ ($\because M = m \times N$, for one mole, $m = 1$)

where, u = rms velocity

Average kinetic energy per molecule

$$= \frac{\text{Average KE per mole}}{N}$$

$$= \frac{3}{2} \frac{RT}{N} = \frac{3}{2} KT \quad \left[\because \frac{R}{N} = K \right]$$

where, K = Boltzmann constant

Different Types of Molecular Velocities

Average velocity (v_{av})

It is the average of the velocities of different molecules of the gas at a particular temperature.

Average velocity $\bar{v}$ or $v_{av} = \sqrt{\frac{8RT}{\pi M}}$

Root mean square velocity (v_{rms})

It is the square root of the mean of the square of velocities of various molecules of the gas at a particular temperature.

Root mean square velocity,

$$v_{rms} = \sqrt{\frac{3pV}{M}} = \sqrt{\frac{3RT}{M}}$$

$$= \sqrt{\frac{3p}{d}} \quad [\because d = \text{density of the gas}]$$

Most probable velocity (v_{mp})

It is the velocity possessed by the maximum fraction of molecules of the gas at a particular temperature.

$$v_p \text{ or } v_{mp} = \sqrt{\frac{2RT}{M}} \quad [\because M = \text{Molecule mass of the gas}]$$

$$v_{rms} : v_{av} : v_{mp} :: \sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2} = 1.2248 : 1.1284 : 1$$

$$= 1 : 0.921 : 0.816$$

$$v_{rms} = 1.085 \times v_{av} \text{ or } v_{av} = 0.921 \times v_{rms}$$

$$v_{rms} = 1.224 \times v_{mp} \text{ or } v_{mp} = 0.816 \times v_{rms}$$

$$v_{av} = 1.128 \times v_{mp} \text{ or } v_{mp} = 0.886 \times v_{av}$$

Ideal and Real Gases

The gas which obeys gas law exactly or at standard condition is called an ideal gas while the gases which obey gas laws under moderate conditions of temperature and pressure are called 'Real gases'.

Deviation from ideal behaviour

At very low temperature and high pressure, real gases show deviation from the ideal gas behaviour.

The causes of deviation are that

- at low temperature and high pressure, volume of a real gas is materially larger than that predicted for an ideal gas as the molecules of gas have their own volume.
- intermolecular forces are not negligible.

Compressibility factor (Z)

$$\frac{pV}{RT} = 1 \text{ (for ideal gas)}$$

Different conditions of compressibility factor (Z) for real gases

- $Z < 1$ (as for CH_4, CO_2) can be attributed to predominance of attractive forces among the molecules of these gases at the temperature of experiment.
- $Z > 1$ can be attributed to the dominance of strong repulsive forces among the molecules.

van der Waals' Equation

van der Waals' gave the gas equation for real gases as,

$$\left(p + \frac{an^2}{V^2} \right) (V - nb) = nRT \quad (\text{for } n \text{ mole of gas})$$

$$\left(p + \frac{a}{V^2} \right) (V - b) = RT \quad (\text{for 1 mole of gas})$$

Here, a and b are van der Waals' constants whose values depend on the nature of gas.

Critical Phenomenon and Liquefaction of Gases

The phenomenon of converting a gas into liquid is known as liquefaction at particular temperature, pressure and volume. When a gas changes into liquid, these conditions are known as critical conditions, e.g. T_c, V_c, p_c etc.

$$\text{Critical temperature } (T_c) = \frac{8a}{27Rb}$$

$$\text{Critical pressure } (p_c) = \frac{a}{27b^2}$$

$$\text{Critical volume } (V_c) = 3b$$

Relation between critical p, V and T

$$\frac{p_c V_c}{RT_c} = \frac{3}{8}, Z < 1 \text{ at critical point}$$

At critical point, there is no distinction between gaseous state and liquid state as densities of a substance in gaseous and liquid states are same.

Liquid State

It is the state of matter in which the molecules are held close to each other and execute random motion through intervening spaces. Most of the physical properties of liquids are controlled by the strength of intermolecular attractive forces.

Evaporation

The transformation of a liquid into gaseous state, at surface, is called its evaporation. It occurs at all temperatures but increase with temperature. Evaporation causes a fall in temperature of liquid because the molecules with higher kinetic energy, leaves the surface and hence, average kinetic energy and consequently temperature decreases.

Vapour Pressure

When evaporation takes place, the gaseous state of liquid is called its vapour. These vapours now exert a pressure on the liquid. This pressure is called the vapour pressure. Thus, vapour pressure of a liquid also increases with temperature.

Boiling Point

During evaporation, the temperature at which vapour pressure of liquid becomes equal to atmospheric pressure is called boiling point of that liquid. Obviously, boiling point increases with increases in atmospheric pressure. Liquids having weak attractive forces, have low boiling point.

The temperature of liquid always remains constant during boiling until all the liquid is changed into gaseous vapours.

Surface Tension

The surface tension (γ) is defined as the force acting right angles to the surface along unit length of surface.

Due to the surface tension liquid drops are spherical. The rise of a liquid in a capillary is also due to surface tension. Mathematically, surface tension

$$\gamma = \frac{\text{Work done to expand the surface}}{\text{Change in area}}$$

Units of γ are dynes per cm or Nm^{-1} (in SI system).

Viscosity

The resistance due to which, liquid does not flow with same speed is called viscosity of liquid.

Thus, the liquid with less viscosity flow rapidly (e.g. water, alcohol etc) and *vice-versa*.

The resistance between two layers of liquid (i.e. the force of friction, f) is given by

$$f = \eta \cdot \frac{A \cdot \Delta v}{x}$$

where, η = coefficient of viscosity

A = surface area of each layer of liquid

Δv = velocity difference of two layers

x = distance between two layers

Units of viscosity coefficient (η) are $\text{dynes cm}^{-2} \text{ s}$ or poise.

Reciprocal of viscosity coefficient is known as fluidity of liquid (ϕ), i.e.

$$\phi = \frac{1}{\eta}$$

Solid State

The state of matter which has definite volume and a definite shape due to strong internuclear force of attraction among the constituent particles is known as 'solid'. Solids do not have translatory motion but can have only vibrational motion about their mean positions.

Classification of Solids

On the basis of the arrangement of their atoms or ions or molecules, solids are broadly *classified into two groups* :

1. Crystalline Solids

They have systematic and regular arrangement of particles and sharp melting point. They have flat faces and sharp edges.

On the basis of types of bond present, they are further divided into four groups:

(i) **Molecular crystal** or van der Waals' crystals

e.g. $\text{I}_2(s)$, $\text{H}_2\text{O}(s)$ etc.

(ii) **Atomic crystals** or covalent crystals

e.g. Graphite, diamond etc.

(iii) **Metallic crystals** e.g. All metallic solids etc.

(iv) **Ionic crystals** e.g. NaCl , CsCl , ZnS etc.

2. Amorphous Solids

In these solids, the constituent particles are not arranged in a regular manner, e.g. rubber, glass, pitch, fused silica, plastics, polymer of high molecular mass.

Crystalline solids are anisotropic, i.e. their physical properties have different values in different directions while amorphous solids are isotropic, i.e. their physical properties have same values in different directions.

Bragg's Equation

It is used to calculate interfacial distances (d) of a crystal. In this method, X-rays are used, which are reflected back by crystal faces in a definite manner. Following reaction is the Bragg's equation:

$$n\lambda = 2d \sin\theta$$

where, λ = wave length of X-rays used

n = order of diffraction, θ = angle of diffraction

d = interfacial distance

Space Lattice or Crystal Lattice

An array of lattice points showing arrangement of constituent particles in different position in three dimensional space, is known as space lattice or crystal lattice.

Crystal Systems

All the crystals found in nature, can be grouped into 7 types of crystal systems which have 14 types of unit cell (Bravais lattice) can be tabulated as,

	Crystal system	Axial ratio	Axial angles	Example
1.	Cubic	$a = b = c$	$\alpha = \beta = \gamma = 90^\circ$	NaCl
2.	Tetragonal	$a = b \neq c$	$\alpha = \beta = \gamma = 90^\circ$	TiO ₂
3.	Rhombic	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90^\circ$	Rhombic sulphur
4.	Monoclinic	$a \neq b \neq c$	$\alpha = \gamma = 90^\circ \neq \beta$	Monoclinic sulphur
5.	Triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90^\circ$	K ₂ Cr ₂ O ₇
6.	Trigonal	$a = b = c$	$\alpha = \beta = \gamma = 90^\circ$	Quartz
7.	Hexagonal	$a = b \neq c$	$\alpha = \beta = 90^\circ, \gamma = 120^\circ$	Graphite

Unit Cell

It is the smallest group of lattice points which when repeated in all directions will develop the entire lattice.

It can be

- (i) **Simple unit cell** or **primitive unit cell** in which particles are present only at corners.

$$\therefore \text{Number of atoms per unit cell} = \frac{1}{8} \times 8 = 1$$

- (ii) **Face centred cell** in which the particles are present at the corners as well as at the centre of each face.

$$\therefore \text{Number of atoms per unit cell} = \frac{1}{8} \times 8 + \frac{1}{2} \times 6 = 4$$

- (iii) **Body centred unit cell** in which the particles are present at the corners of the cube as well as one particle is present at the centre within the body.

$$\therefore \text{Number of atoms per unit cell} = \frac{1}{8} \times 8 + 1 = 2$$

Packing in Solids

In solid, each atom is surrounded by certain number of atoms leaving the minimum vacant space.

It is expressed in terms of packing fraction (PF).

$$\text{Packing fraction} = \frac{\text{Total volume of spheres}}{\text{Volume of the unit cell}} \times 100$$

The number of spheres surrounding a particular atom is known as coordination number (CN).

Packing in solids may be divided into the following categories:

One Dimensional Packing

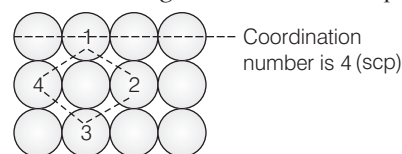
When the spheres are placed in horizontal row, touching each other, an edge of the crystal is formed, this is known as one dimensional packing.



Two Dimensional Packing

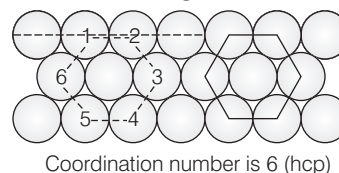
It is of two types:

- (i) **Square Close Packing (scp)** The particles, when placed in the adjacent rows, show a horizontal as well as vertical alignment and form squares.



This is also known as AAA... type arrangement.

- (ii) **Hexagonal Close Packing (hcp)** The particles in every next row are placed in the depression between the particles of the first row. The particles in the third row will be vertically aligned with those in the first row.



This is also known as ABAB... type arrangement.

Three Dimensional Packing

It is of two types:

- (i) **Hexagonal Closed Packing (hcp)** The first layer *A* and second layer *B*, are arranged as *AB AB*... pattern is called hexagonal closed packing (hcp).
- (ii) **Cubic Close Packing (ccp)** When the third layer is placed over the second layer in such a way that spheres cover the octahedral voids, a layer different from layers *A* and *B* is produced. i.e layer *C*.

This pattern of stacking spheres is called *ABC ABC*... pattern or cubic close packing (ccp). It is similar to **face centred cubic (fcc)** packing.

Voids

The empty space between stacked atoms is known as void.

Number of tetrahedral voids

$$= 2 \times \text{Number of octahedral voids}$$

Number of octahedral voids = Number of lattice point

Sizes vary the order,

Trigonal > Tetrahedral > Octahedral

Radius Ratio Rule

$$\text{Radius ratio} = \frac{\text{Radius of cation}}{\text{Radius of anion}}$$

Relation between radius ratio and coordination number

Radius Ratio $\left(\frac{r_+}{r_-}\right)$	Cation occupied	Coordination number	Structure	Example
< 0.155		2	Linear	
0.155 – 0.225	Trigonal void	3	Planar triangular	B ₂ O ₃
0.225 – 0.414	Tetrahedral void	4	Tetrahedral	ZnS
0.414 – 0.732	Octahedral void	6	Octahedral	NaCl
0.732 – 1.0	Cubic void	8	bcc	CsCl

Calculation Involving Unit Cell Parameters

(i) Relation between edge length, radius of sphere and packing fraction

Unit cell	Side/edge length	Radius	Packing fraction
Simple cubic	a	$a/2$	52 %
bcc	a	$(\sqrt{3}/4)a$	68 %
fcc	a	$a/2\sqrt{2}$	74 %

(ii) Determination of density of crystals

$$\text{Density, } d = \frac{Z \times M}{N_A \times a^3(V)}$$

where, Z = number of atoms per unit cell

M = molar mass

a = edge length of unit cell

N_A = Avogadro's number = 6.022×10^{23}

Imperfections as Defects in Solids

These are of following two types:

(i) Stoichiometric Defects

If imperfection in crystals are such that the ratio between the cations and the anions remain the same as in its molecular formula, the defect will be called stoichiometric defects.

These defects are found as Schottky and Frenkel defects.

S. No.	Schottky Defect	Frenkel Defect
1.	It is due to equal number of cations and anions missing from the lattice sites.	It is due to the missing of ions (usually cations) from the lattice sites and they occupy the interstitial sites.
2.	This results in the decrease in density of crystal.	It has no effect on the density of crystal.
3.	This type of defect is found in highly ionic compounds with high coordination number, e.g. NaCl, CsCl, AgBr etc.	This type of defect is found in crystals, where the difference in the size of cations and anions is very large, e.g. AgCl, AgBr, ZnS etc.

(ii) Non-Stoichiometric Defects

When the ratio of cations and anions, due to imperfection, differ from that indicated by their molecular formula, the defects are called non-stoichiometric defects.

***F*-centre** A negative ion may be missing from its lattice site, leaving a hole which is occupied by an electron, thereby maintaining the electrical balance.

The electrons, thus trapped in the anion vacancies are called *F*-centres because they are responsible for imparting colour to the crystals.

Properties of Solids

Some remarkable properties of solids are described below :

Electrical Properties of Solids

Depending upon conduction power, solids are divided into conductors conductivity $10^7 (\Omega\text{m})^{-1}$, semiconductors $(10^{-6} \text{ to } 10^4) (\Omega\text{m})^{-1}$ and insulators $(10^{-20} - 10^{-10}) (\Omega\text{m})^{-1}$.

Superconductivity is the property of the conductor by which they allow electricity to pass through them without any resistance.

The temperature at which a substance starts behaving as superconductor is called transition temperature, e.g. Ba₂Cu₃O₇ at 90K, Bi₂Ca₂Sr₂Cu₃O₁₀ at 105 K, Tl₂Ca₂Ba₂Cu₃O₁₀ at 125 K start behaving as superconductors.

Band Theory to Explain Electrical Property of Solids

Atomic orbitals combine with each other to form molecular orbitals which are close to each other. These are called band. There are two types of band which are valence band and conduction band.

The energy gap between valence band and conduction band is used to explain electrical properties of metal.

- (a) In a conductor, there is no gap between valence band and conduction band.
 (b) In a semiconductor, the gap between valence band and conduction band is small. So, conductivity of semiconductor is low but conductivity can be increased by supplying energy.

There are two types of semiconductors depending upon the doping of foreign elements:

***n*-type semiconductors** Group 14 elements when doped with group 15 elements form *n*-type semiconductors.

***p*-type semiconductors** Group 14 elements when doped with group 13 elements form *p*-type semiconductors.

- (c) In an insulator, the energy gap is so large that it cannot be covered up by supplying energy.

Magnetic properties

Properties	Description	Alignment of magnetic dipole	Examples	Applications
Diamagnetic	Which are feebly repelled by the magnetic fields due to existence of paired electron.	All electrons are paired. $\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow$	TiO ₂ , V ₂ O ₅ , NaCl, C ₆ H ₆	Insulator
Paramagnetic	Attracted by magnetic field due to the presence of unpaired electrons acts as a tiny bar magnet.	At least one unpaired electron. $\uparrow\downarrow\uparrow\uparrow$	O ₂ , Cu ²⁺ , Fe ³⁺ , TiO, Ti ₂ O ₃ , VO, VO ₂ , CuO	Electronic applications
Ferromagnetic	Acts as a permanent magnet even after removing magnetic field. Above curie temperature, no ferromagnetism occurs.	All unpaired electrons are in same direction. $\uparrow\uparrow\uparrow\uparrow\uparrow$	Fe, Ni, Co, CrO ₂	CrO ₂ is used in audio and video tapes.
Anti ferromagnetic	Net dipole moment becomes zero due to equal and opposite alignment.	$\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow$	CoO, Co ₃ O ₄ , MnO, MnO ₂ , M ₂ O ₃ , FeO, Fe ₂ O ₃ , NiO	
Ferrimagnetic	This arises when there is no dipole moment.	$\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow$	Fe ₃ O ₄ , Ferrites	

Practice Exercise

- To which of the following Dalton's law of partial pressure is not applicable?
 - SO₂ and CO₂ at room temperature
 - N₂ and H₂ at room temperature
 - SO₂ and O₂ at room temperature
 - HCl and NH₃ at room temperature
- A sample of gas has a volume of 0.2 litre at 1 atm pressure and 0°C. At the same pressure but at 273°C its volume will become.
 - 0.1 L
 - 0.4 L
 - 0.8 L
 - 0.6 L
- A vessel is filled with a mixture of equal masses of oxygen and nitrogen. What is the ratio of partial pressure of oxygen and nitrogen?
 - $p_{O_2} = 0.5p_{N_2}$
 - $p_{O_2} = 0.875p_{N_2}$
 - $p_{O_2} = p_{N_2}$
 - $p_{O_2} = 1.14p_{N_2}$
- At 25°C and 760 mm of Hg pressure a gas occupies 600 mL volume. What will be its pressure at a height where temperature is 10°C and volume of the gas is 640 mL?
 - 676.6 mm Hg
 - 600 mm Hg
 - 700 mm Hg
 - 752 mm Hg
- The pressure of 1 atmosphere is equal to
 - 760 cm
 - 10^5 Nm^{-2}
 - $10^4 \text{ dyne cm}^{-2}$
 - 1 bar
- A vessel contains a mixture of different types of gases. Which of the following statements is correct?
 - On the average, the heavier molecules have higher speed
 - The average translational energy of different types of molecules is the same
 - The average speed of different molecules is same
 - On an average, the heavier molecules have higher translational energy

7. Diffusion of He gas is
 - a. 4 times faster than CO_2
 - b. 4 times faster than SO_2
 - c. 4 times faster than NO_2
 - d. 4 times faster than ClO_2
8. The vapour density of a gas is 11.2. The volume occupied by 11.2 g of the gas at NTP is
 - a. 22.4 L b. 11.2 L c. 1 L d. 2.24 L
9. The volume of 2.8 g of carbon monoxide at 27°C and 0.821 atm pressure, is
 - a. 30 L b. 3 L c. 0.3 L d. 1.5 L
10. What will be the temperature when the rms velocity is four times of that at 300 K?
 - a. 300 K b. 900 K c. 4800 K d. 1200 K
11. The density of neon is lowest at
 - a. STP b. 0°C , 2 atm
 - c. 273°C , 1 atm d. 273°C , 2 atm
12. When 3.2 g sulphur is vaporised at 450°C and 723 mm Hg pressure, the vapours occupy a volume of 780 mL. What is the molecular formula of S vapours?
 - a. S_2 b. S_4 c. S_6 d. S_8
13. At what temperature, the rms velocity of SO_2 be same as that of O_2 at 303 K?
 - a. 273 K b. 606 K c. 303 K d. 403 K
14. The value of compression factor, Z for critical constants, is
 - a. $\frac{1}{2}$ b. $\frac{3}{4}$
 - c. $\frac{2}{3}$ d. $\frac{3}{8}$
15. The ratio between the root mean square velocity of H_2 at 50 K and that of O_2 at 800 K, is
 - a. 4 b. 2 c. 1 d. $1/4$
16. A vessel contains He and H_2 in the molar ratio 1 : 5. The ratio of mean translational kinetic energies, at the same temperature, is
 - a. 1 : 1 b. 1 : 2 c. 2 : 1 d. 1 : 5
17. The ratio of root mean square velocity to average velocity of a gas molecule at a particular temperature is
 - a. 1 : 1.086 b. 2 : 1.86
 - c. 1.086 : 1 d. 2.086 : 1
18. The RMS speed of helium in ms^{-1} (atomic mass = 4.0 g mol^{-1}) at 400 K, is
 - a. 16.8 b. 60
 - c. 168 d. 1580
19. A mixture of gases having different molecular weights is separated by which method?
 - a. Atmolysis
 - b. Metathesis
 - c. Ostwald and Walker method
 - d. Reverse osmosis

20. The units of van der Waals' constants a and b respectively are
 - a. $\text{L}^{-2} \text{ atm}^{-1} \text{ mol}^{-1}$ and L mol^{-2}
 - b. $\text{L}^2 \text{ atm mol}^{-2}$ and $\text{mol}^{-1} \text{ L}$
 - c. L atm mol^2 and mol L
 - d. $\text{L atm}^2 \text{ mol}^{-1}$ and mol L^{-1}
21. If van der Waals' parameters for gases P, Q, R and S are given as

Gas	a ($\text{time}^2 \text{ atm/mol}^2$)	b (time/mol)
P	4.0	0.027
Q	8.0	0.030
R	12.0	0.027
S	6.0	0.024

The gas which has the highest Boyle temperature is

- a. P b. Q c. R d. S
22. A balloon is filled with hydrogen at room temperature. It will burst, if pressure exceeds 0.2 bar. If a 1 bar pressure the gas occupied 2.27 L volume, up to what volume can the balloon be expanded?
 - a. 6.25 L b. 11.35 L c. 8.35 L d. 10.50 L
23. The average velocity of an ideal gas molecule at 300 K is 2 m/s. The average velocity at 1200 K will be
 - a. 6 m/s b. 4 m/s c. 2 m/s d. 8 m/s
24. In van der Waals' equation of the gas law, the constant ' a ' is a measure of
 - a. intermolecular repulsion
 - b. intermolecular attraction
 - c. volume occupied by the molecules
 - d. relative velocity of gas molecules
25. Which of the following liquid will exhibit highest vapour pressure?
 - a. $\text{C}_2\text{H}_5\text{OH}(l)$ b. $\text{NH}_3(l)$ c. $\text{HF}(l)$ d. $\text{H}_2\text{O}(l)$
26. Which of the following relation is correct for an ideal gas regarding its pressure (p) and translational kinetic energy per unit volume (E)?
 - a. $p = \frac{3}{2} E$ b. $p = \frac{2}{3} E$
 - c. $p = \frac{1}{2} E$ d. $p = 2E$
27. The surface tension of which of the following liquids is maximum?
 - a. H_2O b. CH_3OH c. $\text{C}_2\text{H}_5\text{OH}$ d. C_6H_6
28. Which of the following is not a crystalline solid?
 - a. KCl b. CsCl
 - c. Glass d. Rhombic sulphur
29. The arrangement $ABC\ ABC\ \dots$ is referred to as
 - a. octahedral close packing
 - b. hexagonal close packing
 - c. tetrahedral close packing
 - d. cubic close packing

30. How many Cl^- ions are there around Na^+ ion in a NaCl crystal?
a. 3
b. 4
c. 6
d. 8
31. Which of the following statements is not true about NaCl structure?
a. Cl^- ions are in fcc arrangement
b. Na^+ ions have coordination number four
c. Cl^- ions have coordination number six
d. Each unit cell contains 4 NaCl molecules
32. The number of atoms in 100 g of a fcc crystal with density = 10 g cm^{-3} and cell edge as 200 pm, is equal to
a. 3×10^{25}
b. 5×10^{24}
c. 1×10^{25}
d. 2×10^{25}
33. Which of the following expressions is correct for a CsCl unit cell with lattice parameter, a ?
a. $r_{\text{Cs}^+} + r_{\text{Cl}^-} = 2a$
b. $r_{\text{Cs}^+} + r_{\text{Cl}^-} = \frac{a}{\sqrt{2}}$
c. $r_{\text{Cs}^+} + r_{\text{Cl}^-} = \frac{\sqrt{3}}{2} a$
d. $r_{\text{Cs}^+} + r_{\text{Cl}^-} = \frac{3}{2} a$
34. In diamond, the coordination number of carbon is
a. four and its unit cell has eight carbon atoms
b. four and its unit cell has six carbon atoms
c. six and its unit cell has four carbon atoms
d. four and its unit cell has four carbon atoms
35. The molar volume of KCl and NaCl are 37.46 mL and 27.94 mL respectively. The ratio of the unit cube edges of the crystals is
a. 1.296
b. 1.116
c. 1.341
d. 0.950
36. Calcium crystallises in a face centred cubic unit cell with $a = 0.556 \text{ nm}$. Calculate the density if it contained 0.1% Schottky defects.
a. 1.5463 g/cm^3
b. 1.4962 g/cm^3
c. 1.5448 g/cm^3
d. 1.5943 g/cm^3
37. Calculate the ionic radius of a Cs^+ ion, assuming that the cell edge length for CsCl is 0.4123 nm and that the ionic radius of a Cl^- ion is 0.181 nm.
a. 0.176 nm
b. 0.231 nm
c. 0.357 nm
d. 0.116 nm
38. Iron crystallises in a bcc system with a lattice parameter of 2.861 Å. Calculate the density of iron in the bcc system.
(Atomic weight of Fe = 56, $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$)
a. 7.92 g mL^{-1}
b. 8.96 g mL^{-1}
c. 2.78 g mL^{-1}
d. 6.72 g mL^{-1}
39. A monoclinic crystal has dimensions
a. $a \neq b \neq c, \alpha = \gamma = 90^\circ; \beta \neq 90^\circ$
b. $a = b = c, \alpha = \beta = \gamma = 90^\circ$
c. $a = b = c, \alpha = \beta = 90^\circ; \gamma = 120^\circ$
d. $a \neq b = c, \alpha = \beta = \gamma = 120^\circ$
40. In A^+B^- ionic compound, radii of A^+ and B^- ions are 180 and 187 pm respectively. The crystal structure of the compound will be
a. NaCl type
b. CsCl type
c. ZnS type
d. similar to diamond
41. A compound having bcc geometry has atomic mass 50. Calculate the density of the unit cell, if its edge length is 290 pm.
a. 6.81 g cm^{-3}
b. 3.40 g cm^{-3}
c. 13.62 g cm^{-3}
d. None of these
42. If 'a' stands for the edge length of the cubic systems, simple cubic, body centred cubic and face centred cubic, then the ratio of radii of the spheres in these systems respectively will be
a. $\frac{1}{2} a : \sqrt{3} a : \frac{1}{\sqrt{2}} a$
b. $\frac{1}{2} a : \frac{\sqrt{3}}{2} a : \frac{\sqrt{2}}{2} a$
c. $1 a : \sqrt{3} a : \sqrt{2} a$
d. $\frac{1}{2} a : \frac{\sqrt{3}}{4} a : \frac{1}{2\sqrt{2}} a$
43. Calculate the approximate number of unit cells present in 1g of gold. It is well known that gold crystallises in the face centred cubic lattice (atomic mass of gold is 197u).
a. 7.64×10^{20}
b. 6.02×10^{23}
c. 197
d. 4
44. A solid compound contains X, Y and Z atoms in a cubic lattice with X atoms occupying the corners. Y atoms in the body centred positions and Z atoms at the centres of faces of the unit cell. What is the empirical formula of the compound?
a. XY_2Z_3
b. XYZ_3
c. $\text{X}_2\text{Y}_2\text{Z}_3$
d. X_8YZ_6
45. Total volume of atoms present in a fcc unit cell of a metal is (r = atomic radius).
a. $\frac{16}{3} \pi r^3$
b. $\frac{12}{3} \pi r^3$
c. $\frac{24}{3} \pi r^3$
d. $\frac{20}{3} \pi r^3$
46. What will be the per cent fraction of edge length not covered by atom when a metal crystallises in bcc lattice?
a. 11.4%
b. 10.4%
c. 13.4%
d. 12.4%

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- The temperature 30.98°C is called critical temperature (T_c) of carbon dioxide. The critical temperature is the [2014]
 - lowest temperature at which liquid carbon dioxide is observed
 - highest temperature at which gas carbon dioxide is observed
 - highest temperature at which solid carbon dioxide is observed
 - highest temperature at which liquid carbon dioxide is observed
- Point out the correct statement for the set of characteristics of ZnS crystal. [2014]
 - Coordination number (4 : 4) ; ccp; Zn^{2+} ion in the alternate tetrahedral voids
 - Coordination number (6 : 6) ; hcp; Zn^{2+} ion in all tetrahedral voids
 - Coordination number (6 : 4) ; hcp; Zn^{2+} ion in all octahedral voids
 - Coordination number (4 : 4) ; ccp; Zn^{2+} ion in all tetrahedral voids
- KCl crystallises in the same type of lattice as does NaCl. Given that, $r_{\text{Na}^+} / r_{\text{Cl}^-} = 0.55$ and $r_{\text{K}^+} / r_{\text{Cl}^-} = 0.74$. Determine the ratio of the side of the unit cell for KCl to that of NaCl [2013]
 - 0.124
 - 1.1226
 - 0.891
 - 1.414
- A gas is heated through 1°C in a closed vessel and so the pressure increases by 0.4%. The initial temperature of the gas was [2012]
 - -23°C
 - $+23^{\circ}\text{C}$
 - 250°C
 - 523°C
- Barium titanate has the perovskite structure, i.e. a cubic lattice with Ba^{2+} ions at the corners of the unit cell, oxide ions at the face centres and titanium ions at the body centred. The molecular formula of barium titanate is [2012]
 - BaTiO_3
 - BaTiO_4
 - BaTiO_2
 - BaTiO
- 1 L of a gas is at a pressure of 10^{-6}mm of Hg at 25°C . How many molecules are present in the vessel? [2011]
 - 3.2×10^6
 - 3.2×10^{13}
 - 3.2×10^{10}
 - 3×10^4
- An ideal gas cannot be liquefied because [2009]
 - its critical temperature is always above 0°C
 - its molecules are relatively smaller in size
 - it solidifies before becoming a liquid
 - forces operating between its molecules are negligible
- A compound contains P and Q elements. Atoms Q are in ccp arrangement while P occupy all tetrahedral sites, formula of compound is [2009]
 - PQ
 - PQ_2
 - P_2Q
 - P_3Q
- What is the temperature at which the kinetic energy of 0.3 mole of helium is equal to the kinetic energy of 0.4 mole of argon at 400K ? [2008]
 - 400K
 - 873K
 - 533K
 - 300K
- For a crystal, the angle of diffraction (2θ) is 90° and the second order line has a d value of $2.28\text{\AA}$. The wavelength (in $\text{\AA}$) of X -rays used for Bragg's diffraction is [2008]
 - 1.612
 - 2.00
 - 2.28
 - 4.00
- A solid has a structural in which W atom are located at the corners of a cubic lattice, O atom at the centre of edge and Na atoms at the centre of cube. The formula for the compound is [2007]
 - Na_2WO_3
 - Na_2WO_2
 - NaWO_2
 - NaWO_3
- Which of the following is most volatile compound? [2007]
 - HI
 - HCl
 - HBr
 - HF
- The root mean square velocity of a gas is double when temperature is [2007]
 - increased four times
 - increased two times
 - reduced to half
 - reduced to one - fourth
- The molar volume of CO_2 is maximum at [2006]
 - NTP
 - 0°C and 2.0atm
 - 127°C and 1atm
 - 273°C and 2atm
- The RMS speed of hydrogen is $\sqrt{7}$ times the RMS speed of nitrogen. If T is the temperature of the gas, then [2005]
 - $T_{\text{H}_2} = T_{\text{N}_2}$
 - $T_{\text{H}_2} > T_{\text{N}_2}$
 - $T_{\text{H}_2} < T_{\text{N}_2}$
 - $T_{\text{H}_2} = \sqrt{7} T_{\text{N}_2}$
- Which gas has the highest partial pressure in atmosphere? [2005]
 - CO_2
 - H_2O
 - O_2
 - N_2

Answer with Solutions

Practice Exercise

1. (d) Dalton's law is applicable to those mixtures of gases which don't react. HCl and NH_3 react at room temperature react to give NH_4Cl .

2. (b) According to Charles's law, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$$V_2 = \frac{0.2 \times 546}{273} = 0.4 \text{ L}$$

3. (b) Let 32 g of each gas is taken. Then,

$$\text{Total moles} = \frac{32}{32} + \frac{32}{28} = \frac{15}{7}$$

$$\text{Mole fraction of } \text{O}_2 = \frac{1}{15/7} = \frac{7}{15}$$

$$\text{Mole fraction of } \text{N}_2 = \frac{8/7}{15/7} = \frac{8}{15}$$

$$\therefore \frac{p_{\text{O}_2}}{p_{\text{N}_2}} = \frac{\frac{7}{15} \times 1}{\frac{8}{15} \times 1} = \frac{7}{15} \times \frac{15}{8} = \frac{7}{8} = 0.875$$

$$p_{\text{O}_2} = 0.875 p_{\text{N}_2}$$

4. (a) According to combined gas law, $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$

$$\Rightarrow p_2 = \frac{(760 \text{ mm Hg}) \times (600 \text{ mL}) \times (283 \text{ K})}{(640 \text{ mL}) \times (298 \text{ K})} = 676.6 \text{ mmHg}$$

5. (d) 1 atmosphere = $1.01 \times 10^5 \text{ Nm}^{-2} = 1 \text{ bar}$

6. (b) The average translational energy of different types of molecules is the same because it depends only on temperature.

7. (b) According to the Graham's law, $\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$

$$4 = \sqrt{\frac{M_2}{4}} \Rightarrow 16 = \frac{M_2}{4}$$

$$\therefore M_2 = 64$$

64 is the molecular weight of SO_2 hence, (b) is correct.

8. (b) Molecular weight of gas = $2 \times \text{VD} = 2 \times 11.2 = 22.4$

$$\text{Hence, number of moles of gas taken} = \frac{11.2}{22.4} = \frac{1}{2}$$

$$\text{Hence, volume of gas at NTP} = \frac{1}{2} \times 22.4 \text{ L} = 11.2 \text{ L}$$

9. (b) $pV = nRT$ or $V = \frac{nRT}{p} = \frac{2.8 \times 0.0821 \times 300}{28 \times 0.821} = 3 \text{ L}$

10. (c) $U_{\text{rms}} (\text{velocity}) = \sqrt{\frac{3RT}{M}}$

$$\text{Hence, } u \propto \sqrt{T} \Rightarrow \frac{u_1}{u_2} = \sqrt{\frac{T_1}{T_2}} \quad \dots(i)$$

$$\text{From Eq. (i), } \frac{1}{4} = \sqrt{\frac{300}{T_2}}$$

$$\frac{1}{16} = \frac{300}{T_2}$$

$$T_2 = 16 \times 300 = 4800 \text{ K}$$

11. (c) On applying ideal gas equation we get

$$d \propto \frac{p}{T}$$

Therefore, for low density, p should be less while temperature should be high. Hence, at 273°C and 1 atm pressure, density of neon will be minimum.

12. (d) From ideal gas equation,

$$pV = \frac{w}{M} RT$$

$$\therefore M = \frac{3.2 \times 0.0821 \times 760 \times 1000 \times 723}{723 \times 780} = 256$$

$\therefore$ 32 g sulphur has 1 atom of S.

$\therefore$ Molecular formula = S_8

13. (b) $\frac{U_{\text{rms}} (\text{SO}_2)}{U_{\text{rms}} (\text{O}_2)} = \sqrt{\frac{T (\text{SO}_2)}{M (\text{SO}_2)} \times \frac{M (\text{O}_2)}{T (\text{O}_2)}}$

$$\text{i.e. } 1 = \sqrt{\frac{T (\text{SO}_2)}{64} \times \frac{32}{303}} \quad \text{or } T (\text{SO}_2) = 606 \text{ K}$$

14. (d) Compressibility factor (Z_c) = $\frac{p_c V_c}{RT_c} = \frac{a \times 3b \times 27}{27b^2 \times 8a} = \frac{3}{8}$

$$\frac{p_c V_c}{RT_c} = \frac{3}{8}$$

15. (c) $U_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$$\therefore \frac{U_1}{U_2} = \sqrt{\frac{3RT_1}{M_1} \times \frac{M_2}{3RT_2}} = \sqrt{\frac{T_1 M_2}{T_2 M_1}} = \sqrt{\frac{50 \times 32}{800 \times 2}} = 1$$

16. (d) $\text{KE} = \frac{3}{2} pV = \frac{3}{2} nRT$

$$\frac{(\text{KE})_{\text{He}}}{(\text{KE})_{\text{H}_2}} = \frac{\frac{3}{2} RT}{\frac{3}{2} RT} \left(\frac{1}{5} \right) = \frac{1}{5} = 1 : 5$$

17. (c) For a gas at temperature T , M is constant.

$$\therefore \text{Ratio of } \frac{v_{\text{rms}}}{v_{\text{av}}} = \sqrt{\frac{3RT}{M}} \times \sqrt{\frac{\pi M}{8RT}} = \frac{\sqrt{3} \times \sqrt{\pi}}{\sqrt{8}}$$

$$\therefore = \frac{\sqrt{3} \times \sqrt{3.14}}{\sqrt{8}} = \frac{1.086}{1}$$

18. (d) $v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.314 \times 400}{(4 \times 10^{-3}) \text{ kg}}} \approx 1580 \text{ m/s}$

19. (a) Diffusion method is used to separate a mixture of gases having different molecular weight as rate of diffusion varies inversely with molecular mass, i.e.

$$r \propto \frac{1}{\sqrt{M}}$$

This method is called atmolysis.

20. (b) $\therefore$ Units of $a = \frac{pV^2}{n^2} = \frac{\text{atm} \times \text{L}^2}{\text{mol}^2} = \text{L}^2 \text{ atm mol}^{-2}$

$$\therefore \text{Units of } b = \frac{V}{n} = \frac{\text{L}}{\text{mol}} = \text{mol}^{-1} \text{ L}$$

21. (c) $T_B = \frac{a}{Rb} \Rightarrow$ Higher the value of $\frac{a}{b}$, higher will be the Boyle's temperature.

22. (b) According to Boyle's law, $p_1V_1 = p_2V_2$

$$V_2 = \frac{p_1V_1}{p_2} = \frac{1 \text{ bar} \times 2.27 \text{ L}}{0.2 \text{ bar}} = 11.35 \text{ L}$$

23. (b) $v_{\text{av}} = \sqrt{\frac{8RT}{\pi M}} \Rightarrow v_{\text{av}} \propto \sqrt{T}$

$$\frac{2}{V_2} = \sqrt{\frac{300}{1200}} = \sqrt{\frac{1}{4}} = \frac{1}{2} \Rightarrow v_2 = 4 \text{ m/s}$$

24. (b)

25. (b) NH_3 have weakest hydrogen bonding among these. Hence, its molecules have most escaping tendency and therefore NH_3 has greatest vapour pressure among these liquids.

26. (b) $E = \frac{E'}{V} = \frac{1}{2} \frac{mNU_{\text{rms}}^2}{V} = \frac{3}{2} \times \frac{1}{3} \frac{mNU_{\text{rms}}^2}{V} = \frac{3}{2} p$

$$P = \frac{2}{3} E$$

27. (a) Surface tension of H_2O is maximum due to maximum hydrogen bonding among the given.

28. (c) Glass is an amorphous solid.

29. (d)

30. (c) In NaCl , both Na^+ and Cl^- ions, have coordination number 6.

31. (b) NaCl has a fcc structure have CN 6 : 6.

32. (b) We know that, $d = \frac{Z \cdot M}{n \times a^3}$

$$n = \frac{4 \times 100}{10 \times (200 \times 10^{-10})^3} = 5 \times 10^{24}$$

33. (c) CsCl crystal has bcc structure,

$$\Rightarrow r_{\text{Cs}^+} + r_{\text{Cl}^-} = \frac{\sqrt{3}}{2} a$$

34. (a) The space lattice of diamond is fcc and has tetrahedral geometry. Thus, the conventional unit cube contains eight atoms with CN four.

35. (b) $\frac{d_{\text{KCl}}}{d_{\text{NaCl}}} = \left(\frac{37.46}{27.94} \right)^{\frac{1}{3}} = 1.116$

36. (c) $d = \frac{ZM}{N_0 a^3}$; $Z = 4 \times \left(1 - \frac{0.1}{100} \right) = 3.996$

(because it contains 0.1% Schottky defect)

$$d = \frac{3.996 \times 40}{6.02 \times 10^{23} \times (0.556 \times 10^{-7})^3} = 1.5448 \text{ g/cm}^3$$

37. (a) $r_{\text{Cs}^+} + r_{\text{Cl}^-} = \frac{d_{\text{body}}}{2} = \frac{0.7141}{2} \text{ nm} = 0.3571 \text{ nm}$

$$r_{\text{Cs}^+} = 0.3571 - 0.181 = 0.176 \text{ nm}$$

38. (a) $d = \frac{ZM}{N_A a^3}$ (for bcc, $Z = 2$)

$$d_{\text{Fe}} = \frac{(2) \times 56.0 \text{ g mol}^{-1}}{(6.02 \times 10^{23} \text{ mol}^{-1}) (2.861 \times 10^{-8})^3 \text{ cm}^3} = 7.92 \text{ g cm}^{-3}$$

39. (a) Dimensions of a monoclinic crystal lattice are $a \neq b \neq c$ and $\alpha = \gamma = 90^\circ$, $\beta \neq 90^\circ$.

40. (b) $\frac{r^+}{r^-} = \frac{180}{187} = 0.96$, this value lies in the range of 0.732 – 1.000.

$\therefore$ Crystal structure will be of CsCl type.

41. (a) Density of unit cell, $d = \frac{Z \times M}{N_A \times a^3}$

$$d = \frac{2 \times 50}{(290 \times 10^{-10})^3 \times 6.023 \times 10^{23}} = 6.81 \text{ g/cm}^3$$

[$\therefore$ For bcc, $Z = 2$]

42. (d) Ratio of radius of scc, bcc, fcc will be $\frac{a}{2} : \frac{\sqrt{3}}{4} a : \frac{1}{2\sqrt{2}} a$.

43. (a) 1 mole of gold = 197 g = 6.02×10^{23} atoms.

$$\therefore \text{Number of atoms available in 1 g of gold} = \frac{6.02 \times 10^{23}}{197}$$

As fcc unit cell contains 4 atoms.

$$\therefore \text{Number of unit cells present} = \frac{6.02 \times 10^{23}}{197 \times 4} = 7.64 \times 10^{20}$$

44. (b) $Z_x = \frac{1}{8} \times 8 = 1$ (at corner)

$$Z_y = 1 \text{ [At Body centre]}, \quad Z_z = 3 \text{ [At Face centre]}$$

Hence, the formula of compound = XYZ_3

45. (a) Total number of atoms in the unit cell of fcc = 4

$$\therefore \text{Total volume of an atom} = \frac{4}{3} \pi r^3 \times 4 \quad [\therefore v = \frac{4}{3} \pi r^3]$$

$$= \frac{16}{3} \pi r^3$$

46. (c) Edge length not covered by atom = $a - 2r$

$$[\therefore \text{For bcc, } r = \frac{\sqrt{3}}{4} a]$$

$$= a - 2 \times \frac{\sqrt{3}}{4} a = a \left[\frac{2 - \sqrt{3}}{2} \right]$$

$$a \left[\frac{2 - \sqrt{3}}{2} \right] \times 100$$

$$\therefore \% \text{ of fraction not covered} = \frac{a \left[\frac{2 - \sqrt{3}}{2} \right] \times 100}{a} = 0.134 \times 100 = 13.4\%$$

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1. (d)

2. (a) ZnS has zinc blende type structure (i.e. ccp structure). The S^{2-} ions are present at the corners of the cube and at the centre of each face. Zinc ions occupy half of the tetrahedral sites. Each zinc ion is surrounded by four sulphide ions which are disposed towards the corner of regular tetrahedron. Similarly, S^{2-} ion is surrounded by four Zn^{2+} ions.

3. (b) Given that $r_{Na^+}/r_{Cl^-} = 0.55$, $r_{K^+}/r_{Cl^-} = 0.74$

$$\therefore \frac{r_{Na^+}}{r_{Cl^-}} = 0.55$$

$$\therefore \frac{r_{Na^+}}{r_{Cl^-}} + 1 = 0.55 + 1$$

$$\frac{r_{Na^+} + r_{Cl^-}}{r_{Cl^-}} = 1.55 \quad \dots(i)$$

$$\therefore \frac{r_{K^+}}{r_{Cl^-}} = 0.74$$

$$\text{Similarly } \frac{r_{K^+} + r_{Cl^-}}{r_{Cl^-}} = 1.74 \quad \dots(ii)$$

Dividing Eq. (ii) by Eq. (i)

$$\frac{r_{K^+} + r_{Cl^-}}{r_{Na^+} + r_{Cl^-}} = \frac{1.74}{1.55} = 1.1226$$

4. (a) Let $T_1 = T \Rightarrow T_2 = (T + 1)$

$$\text{and } p_1 = p \Rightarrow p_2 = p + \frac{0.4p}{100} = \frac{100.4p}{100}$$

$$\text{From } \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}, \frac{pV}{T} = \frac{100.4p}{100} \times \frac{V}{(T+1)}$$

$$100T + 100 = 100.4T$$

$$T = \frac{100}{0.4} = 250 \text{ K}$$

$$= (250 - 273)^\circ \text{C} = -23^\circ \text{C}$$

5. (a) Number of Ba^{2+} ions at the corner of unit cell

$$= 8 \times \frac{1}{8} = 1$$

$$\text{Number of } O^{2-} \text{ ions at the face centres} = 6 \times \frac{1}{2} = 3$$

$$\text{Number of } Ti^{4+} \text{ ions at the body centre} = 1$$

$$\therefore \text{Molecular formula of barium titanate} = BaTiO_3$$

$$6. (b) \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \Rightarrow \frac{10^{-6} \times 1000}{298} = \frac{760 \times V_2}{273}$$

$$V_2 = 1.2 \times 10^{-6} \text{ cc} \quad (\text{at STP})$$

Number of molecules

$$= \frac{6.02 \times 10^{23}}{22400} \times 1.2 \times 10^{-6} = 3.2 \times 10^{13}$$

7. (d) In the ideal gas, the intermolecular forces of attraction are negligible and hence, it cannot be liquefied.

8. (c) Suppose number of atoms Q in the ccp arrangement = 100

Then, number of tetrahedral sites = 200

As all the tetrahedral sites are occupied by atoms P , therefore their number = 200

Hence, ratio $P : Q = 2 : 1$

i.e. the formula is $P_2 Q$.

9. (c) We know that, kinetic energy = nRT

$$\text{Kinetic energy of helium} = 0.3 \times R \times T \quad \dots (i)$$

$$\text{Kinetic energy of argon} = 0.4 \times R \times 400 \quad \dots (ii)$$

According to question

$$\text{Kinetic energy of helium} = \text{Kinetic energy of argon}$$

$$0.3 \times R \times T = 0.4 \times R \times 400$$

$$T = 533 \text{ K}$$

10. (a) Given, angle of diffraction (2θ) = 90°

$$\theta = 45^\circ$$

Distance between two planes, $d = 2.28 \text{ \AA}$

$$n = 2 \quad [\because \text{second order diffraction}]$$

Bragg's equation is $n\lambda = 2d \sin \theta$

$$2 \times \lambda = 2 \times 2.28 \times \sin 45^\circ \Rightarrow \lambda = 1.612$$

11. (d) In a unit cell, W-atoms at the corner = $\frac{1}{8} \times 8 = 1$

$$\text{O-atoms at the centre of edge} = \frac{1}{4} \times 12 = 3$$

Na-atoms at the centre of the cube = 1

$$W : O : Na = 1 : 3 : 1$$

Hence, formula = $NaWO_3$

12. (b) Boiling point of HF is highest due to H-bonding. For other halogen acids boiling point increases in the order $HCl < HBr < HI$. Therefore, most volatile (with lower boiling point) is HCl.

$$13. (a) v_{rms} = \sqrt{\frac{3RT}{M}} \text{ or } v_{rms} \propto \sqrt{T}$$

$$\frac{v_{rms}}{v'_{rms}} = \sqrt{\frac{T}{T'}} \Rightarrow \frac{1}{2} = \sqrt{\frac{T}{T'}} \Rightarrow T' = 4T$$

$$14. (c) \text{From } v = \frac{nRT}{P}$$

At high T and low p , V is high.

$$15. (c) \frac{u_{rms}(H_2)}{u_{rms}(N_2)} = \sqrt{7} = \left[\frac{T(H_2)M(N_2)}{M(H_2)T(N_2)} \right]^{1/2} = \sqrt{7}$$

$$\text{Hence, } \frac{T(H_2)}{T(N_2)} = 7 \frac{M(H_2)}{M(N_2)} = 7 \left(\frac{2}{28} \right) = \frac{1}{2}$$

$$T_{H_2} < T_{N_2}$$

16. (d) N_2 has the highest partial pressure in atmosphere.

7

Chemical Thermodynamics

Thermodynamics

The various forms of energy are interrelated and under certain conditions these may be transformed from one form into another. The study of these energy transformation forms the subject matter of thermodynamics. It is based on four generalisations, i.e. zeroth, first, second and third law of thermodynamics.

System

It is the part of the universe, which is under the thermodynamic investigation.

Types of System

- (i) **Open system** The system, which can exchange energy as well as matter with the surroundings, e.g. tea in a cup, human body, etc.
- (ii) **Closed system** The system, which can exchange only energy with the surroundings but not the matter, e.g. hot water in a sealed tube.
- (iii) **Isolated system** The system, which is perfectly insulated and hence, cannot exchange energy and matter with the surroundings, e.g. ice in a thermos flask.

Surroundings and Boundary

The part of the universe other than the system, is known as its surroundings. A system is separated from the surroundings by a real or imaginary boundary.

Thermodynamic Properties

Intensive and Extensive Properties

An intensive property is that which does not depend upon the quantity of matter present in the system, e.g. temperature, density, refractive index, boiling point, viscosity, specific heat etc. While extensive properties are those properties, which depend upon the quantity of matter present in the system, e.g. mass, volume, energy, heat capacity, enthalpy, entropy etc.

State functions

These are the properties which depend on the state and not on the path followed to obtain that state, e.g. internal energy, enthalpy, entropy, free energy etc.

Processes

The state of variable can be changed by means of an operation called process.

These are of following types :

- (i) **Adiabatic process** The process in which system does not exchange heat with its surroundings, i.e. $dQ = 0$.
- (ii) **Isothermal process** The process in which temperature remains constant, i.e. $dT = 0$.
- (iii) **Isobaric process** The process in which change of state is brought about at constant pressure, i.e. $dp = 0$.
- (iv) **Isochoric process** The process in which volume of the system remains constant, i.e. $dV = 0$.
- (v) **Cyclic process** This is the process in which a system undergoes a number of different states and finally returns to its initial state. For such a process, change in internal energy and enthalpy is zero, i.e. $dE = 0$ and $dH = 0$.
- (vi) **Reversible process** It is an ideal process and here, every intermediate state is in equilibrium with others, if any. This is a slow process.
- (vii) **Irreversible process** That process, cannot be reversed in which amount of energy increases. All natural processes are irreversible in nature.

First Law of Thermodynamics

It states that energy can neither be created nor be destroyed although, it can be changed from one form to another. Mathematically,

$$\Delta E = q + W \quad \dots(i)$$

where, ΔE = change in internal energy

q = heat absorbed

W = work done

Concept of Work

Work is a medium to transfer energy from a system to surrounding.

1. Work done in isothermal reversible expansion

$$W = -2.303nRT \log \frac{V_2}{V_1} = -2.303nRT \log \frac{p_1}{p_2}$$

where, V_1 = initial volume, V_2 = final volume

p_1 = initial pressure, p_2 = final pressure

R = real gas constant, T = temperature

2. Work done in isothermal irreversible expansion

$$W = -p_{\text{ext}} \times \Delta V$$

If expansion occurs in vacuum, i.e. at zero pressure, the value of work done is zero.

$$W = -p_{\text{ext}} \times \Delta V = 0$$

3. Work done in adiabatic expansion

$$W = \frac{nR}{\gamma - 1} (T_2 - T_1)$$

Work done by the system, $W = -ve$; $\therefore \Delta U = -ve$

Work done on the system, $W = +ve$; $\therefore \Delta U = +ve$

4. Work done in cyclic process is always zero

$$\oint dW = 0$$

Internal Energy (E or U)

The total amount of energy associated with a molecule is known as internal energy. It is denoted by U .

$$U = U_{\text{trans}} + U_{\text{vib}} + U_{\text{elec}} + U_{\text{rot}} + \dots$$

$$T = \text{constant}, \Delta U = 0$$

Internal energy depends on temperature, pressure, volume and quantity of matter.

Heat

It is defined as the quantity of energy, which flows between system and surroundings on account of temperature difference. It is also a path function, i.e. depends upon the path followed.

It is given as

$$H = ms\Delta t$$

where, m = mass of substance

s = specific heat

Δt = temperature difference

Enthalpy

The total heat content of a system at constant pressure is called the enthalpy of the system. Indeed, it is the sum of internal energy and the product of pressure-volume work. It is an extensive quantity and is represented by the symbol H . The equation is

$$H = E + pV$$

$$\Delta H = \Delta E + p\Delta V$$

$$\Delta H = \Delta E + \Delta n_g RT$$

where, ΔH = enthalpy change

Δn_g = gaseous moles of products

– gaseous moles of reactants

If $\Delta n_g = 0$, then $\Delta H = \Delta E$; If $\Delta n_g > 0$ then $\Delta H > \Delta E$ and if $\Delta n_g < 0$ then, $\Delta H < \Delta E$.

For reaction involving solids and liquids only, $\Delta H = \Delta E$.

Heat Capacity

Heat capacity (C) of a system is defined as the amount of heat required to raise the temperature of the system by 1°C .

$$C = \frac{q}{\Delta T}$$

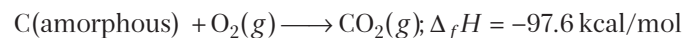
If the system consists of a single substance or a solution and weighs 1 mole, the heat capacity of the system is known as **molar heat capacity**. If the system consists of a single substance or a solution and weighs 1 g, the heat capacity of the system is known as **specific heat** of the system.

$$\sigma_L = C \times m \times \Delta T$$

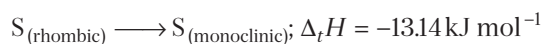
where, m = mass of substance, C = specific heat capacity and ΔT = temperature difference

Enthalpies for Different Types of Reactions

Enthalpy Change of Formation ($\Delta_f H^\circ$) The enthalpy change during the formation of one mole of the compound from constituent element in their standard state is known as enthalpy change of formation.



Enthalpy of Phase Transition ($\Delta_t H$) The enthalpy change during the conversion of one mole of a substance from one phase to another phase is known as enthalpy of phase transition.



Enthalpy of Fusion Change in heat enthalpy when 1 mole of a substance is completely converted from solid state into liquid state at its melting or freezing point is known as enthalpy of fusion.



Enthalpy of Sublimation ($\Delta_{\text{sub}} H$) The enthalpy change during sublimation of 1 mole of solid substance to gaseous substance directly is known as enthalpy of sublimation.

$$\Delta_{\text{sub}} H = \Delta_{\text{fus}} H + \Delta_{\text{vap}} H$$

Enthalpy of Ionisation ($\Delta_{\text{ion}} H^\circ$) The enthalpy change during complete conversion of one mole of weak acid or base into its constituent ions is known as enthalpy of ionisation. It is denoted by $\Delta_{\text{ion}} H^\circ$.



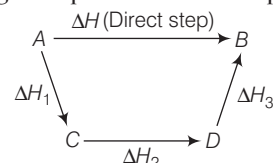
$$\Delta_{\text{ion}} H^\circ = -55.43 \text{ kJ/mol}$$

Thermochemistry

It is the branch of physical chemistry which is concerned with energy change between a chemical system and its surroundings when a change of a phase or a chemical reaction takes place within the system. It is also termed as chemical energetics. It is based on the first law of thermodynamics.

Hess's Law of Constant Heat Summation

Total enthalpy change during the complete course of reaction in a single step and several steps are equal.



$$\Delta H = \Delta H_1 + \Delta H_2 + \Delta H_3$$

Applications

- In determination of enthalpy of reaction, enthalpy of formation, enthalpy of transformation etc.
- In determination of bond energy,

$$\Delta_r H = \Sigma \text{ bond energy of reactants} - \Sigma \text{ bond energy of products}$$

Second Law of Thermodynamics

For a spontaneous process of an isolated system, the entropy change is always positive.

$$\Delta S_{\text{univ}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} > 0$$

Entropy

It is the property of a system which measures the degree of randomness in a system. It is a state function, therefore change in entropy depends only on the initial and final states.

$$\Delta S = S_{\text{final}} - S_{\text{initial}} = \frac{q_{\text{sys rev}}}{T}$$

1. Entropy change for an ideal gas

(i) For isothermal process,

$$\Delta S = 2.303 n R \log \frac{V_2}{V_1} = 2.303 n R \log \frac{p_1}{p_2}$$

(ii) For isobaric process, $\Delta S = 2.303 n C_p \log \frac{T_2}{T_1}$

(iii) For isochoric process, $\Delta S = 2.303 n C_v \log \frac{T_2}{T_1}$

2. Entropy change during phase transition

$$\Delta S = \frac{\Delta H}{T}$$

Criteria of Spontaneity : Gibbs Energy

In most of the chemical reactions, change in enthalpy and entropy occurs simultaneously.

Thus, a new thermodynamic function is introduced to define spontaneity of chemical process known as **Gibbs free energy (G)**.

$$G = H - TS$$

$\therefore$

$$\Delta G = \Delta H - T\Delta S$$

This equation is also known as **Gibbs Helmholtz equation**.

The spontaneity of process is related to Gibbs free energy change as

$\Delta G = +ve$	Non-spontaneous process
$\Delta G = 0$	Equilibrium state
$\Delta G = -ve$	Spontaneous process

NOTE If $\Delta H = +ve$, $\Delta S = +ve$, $\Delta S > \Delta H$
Then, $\Delta G < 0$ hence, reaction becomes spontaneous.

Effect of temperature on spontaneity of process

$\Delta_r H^\circ$	$\Delta_r S^\circ$	$\Delta_r G^\circ$	Reaction will be spontaneous at
-	+	-	All temperature
-	-	-	Low temperature
-	-	+	High temperature
+	+	+	Low temperature
+	+	-	High temperature
+	-	+	All temperature

Percentage efficiency of fuel

$$\% \text{ efficiency} = \frac{\Delta G}{\Delta H} \times 100$$

Third law of Thermodynamics

This law states that the entropy of a substance becomes zero at absolute zero temperature (perfectly crystalline state), i.e.

$$\lim_{T \rightarrow 0} S = 0$$

The importance of the third law is that it is used in calculating absolute entropies of pure substances at different temperatures.

For a solid at TK

$$S_T - S_0 = \Delta S = \int_0^T \frac{C_p \cdot dT}{T}$$

Practice Exercise

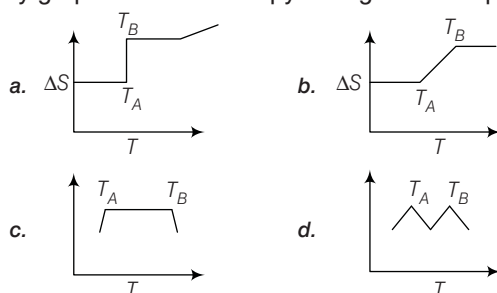
- Thermodynamics is not concerned about
 - energy changes involved in a chemical reaction
 - the extent to which a chemical reaction proceeds
 - the rate at which a reaction proceeds
 - the feasibility of a chemical reaction
- What is ΔE for a system that does 500 cal of work on surrounding and 300 cal of heat is absorbed by the system?
 - 200 cal
 - 300 cal
 - +200 cal
 - +300 cal
- Flying bird is an example of
 - closed system
 - open system
 - isolated system
 - microscopic system
- For an adiabatic process, which of the following is correct?
 - $p\Delta V = 0$
 - $q = +W$
 - $\Delta q = 0$
 - $\Delta E = q$
- A gas expands, isothermally and reversibly. The work done by the gas is
 - zero
 - maximum
 - minimum
 - cannot be determined
- Which of the following statement is correct?
 - The presence of reacting species in a covered beaker is an example of open system
 - There is an exchange of energy as well as matter between the system and the surroundings in a closed system
 - The presence of reactants in a closed vessel made up of copper is an example of a closed system
 - The presence of reactants in a thermos flask or any other closed insulated vessel is an example of a closed system
- 1 mole of CO_2 gas at 300 K is expanded under adiabatic conditions such that its volume becomes 27 times. What is work done? ($\gamma = 1.33$ and $C_V = 6 \text{ cal mol}^{-1}$ for CO_2)
 - 900 cal
 - 1000 cal
 - 1200 cal
 - 1400 cal
- The heat of combustion of benzene determined in a bomb calorimeter is $-870 \text{ kcal mol}^{-1}$ at 298K. The value of ΔE for reaction is
 - $-1740 \text{ kcal mol}^{-1}$
 - $+870 \text{ kcal mol}^{-1}$
 - $-870 \text{ kcal mol}^{-1}$
 - $+1740 \text{ kcal mol}^{-1}$
- Enthalpy of combustion of $\text{C}_6\text{H}_6(l)$ is -3264.64 kJ/mol . The heat produced by burning 3.9 g of benzene is
 - -163.23 kJ
 - -326.4 kJ
 - -32.64 kJ
 - -3.264 kJ
- A molecule with highest bond energy is
 - Br_2
 - F_2
 - Cl_2
 - I_2
- For the reaction $\text{C} + \text{O}_2 \longrightarrow \text{CO}_2$,
 - $\Delta H > \Delta E$
 - $\Delta H < \Delta E$
 - $\Delta H = \Delta E$
 - None of these
- Given that ΔH_{comb} of $\text{C}(s)$, $\text{H}_2(g)$ and $\text{CH}_4(g)$ are -394 , -284 and -892 kJ/mol respectively. The heat of formation of CH_4 is
 - -70 kJ/mol
 - -71.8 kJ/mol
 - -244 kJ/mol
 - -782 kJ/mol

13. For which of the following reactions ΔH is less than ΔE ?
- $\text{HCl}(aq) + \text{NaOH}(aq) \longrightarrow \text{NaCl}(aq) + \text{H}_2\text{O}(l)$
 - $\text{H}_2(g) + \text{I}_2(g) \longrightarrow 2\text{HI}(g)$
 - $\text{C}(s) + \text{O}_2(g) \longrightarrow \text{CO}_2(g)$
 - $\text{N}_2(g) + 3\text{H}_2(g) \longrightarrow 2\text{NH}_3(g)$
14. Enthalpy change, when 1 g water is frozen at 0°C is ($\Delta H_{\text{fus}} = 1.435 \text{ kcal mol}^{-1}$)
- 0.0797 kcal
 - 0.0797 kcal
 - 1.435 kcal
 - 1.435 kcal
15. $\text{S}_{\text{rhombic}} + \text{O}_2(g) \longrightarrow \text{SO}_2(g)$, $\Delta H = -297.5 \text{ kJ}$
 $\text{S}_{\text{monoclinic}} + \text{O}_2(g) \longrightarrow \text{SO}_2(g)$, $\Delta H = -300 \text{ kJ}$
 The data can predict that
- rhombic sulphur is yellow in colour
 - monoclinic sulphur has metallic lustre
 - monoclinic sulphur is more stable
 - $\Delta H_{\text{transition}}$ of S_R of S_M is endothermic
16. When ammonium chloride is dissolved in water, the solution becomes cold. The change is
- endothermic
 - exothermic
 - supercooling
 - None of these
17. If a refrigerator's door is kept opened, then we get
- room cooled
 - room heated
 - more heat is passed out
 - no effect on room
18. Internal energy and pressure of a gas of unit volume are related as
- $p = \frac{2}{3}E$
 - $p = \frac{E}{2}$
 - $p = \frac{3}{2}E$
 - $p = 2E$
19. Latent heat of vaporisation of a liquid at 500 K and 1 atm pressure is 10.0 kcal/mol. What will be the change in internal energy (ΔE) of 3 moles of liquid at same temperature?
- 30 kcal
 - 54 kcal
 - 27.0 kcal
 - 50 kcal
20. Water is brought to boil under a pressure of 1.0 atm. When an electric current of 0.50 A from a 12 V supply is passed for 300 s through a resistance in thermal contact with it, it is found that 0.798 g of water is vaporised. Calculate the molar internal energy change at boiling point (373.15 K).
- 37.5 kJ mol^{-1}
 - 3.75 kJ mol^{-1}
 - 42.6 kJ mol^{-1}
 - 4.26 kJ mol^{-1}
21. Enthalpy of solution of $\text{NaOH}(s)$ in water is $-41.6 \text{ kJ mol}^{-1}$. When NaOH is dissolved in water, the temperature of water
- increases
 - decreases
 - does not change
 - fluctuates indefinitely
22. The heat of combustion of carbon to CO_2 is -393.5 kJ/mol . The heat released upon formation of 35.2 g of CO_2 from carbon and oxygen gas is
- +315 kJ
 - 31.5 kJ
 - 315 kJ
 - +31.5 kJ
23. If the heat of neutralisation for a strong acid-base reaction is -57.1 kJ , what would be the heat released when 350 cm^3 of 0.20 M of a dibasic strong acid is mixed with 650 cm^3 of 0.10 M monoacidic base?
- 57.1 kJ
 - 3.71 kJ
 - 57.1 kJ
 - 0.317 kJ
24. If enthalpies of formation of $\text{C}_2\text{H}_4(g)$, $\text{CO}_2(g)$ and $\text{H}_2\text{O}(l)$ at 250°C and 1 atm pressure be 52, -394 and -286 kJ mol^{-1} respectively, the enthalpy of combustion of $\text{C}_2\text{H}_4(g)$ will be
- 1412 kJ mol^{-1}
 - 1412 kJ mol^{-1}
 - +141.2 kJ mol^{-1}
 - 141.2 kJ mol^{-1}
25. The bond dissociation energies of gaseous H_2 , Cl_2 and HCl are 104.58 and 103 kcal respectively. The enthalpy of formation of HCl gas would be
- 44 kcal
 - 44 kcal
 - 22 kcal
 - 22 kcal
26. The ΔH_f° for $\text{CO}_2(g)$, $\text{CO}(g)$ and $\text{H}_2\text{O}(g)$ are -393.5, -110.5 and $-241.8 \text{ kJ mol}^{-1}$ respectively, the standard enthalpy change (in kJ) for the reaction, $\text{CO}_2(g) + \text{H}_2(g) \longrightarrow \text{CO}(g) + \text{H}_2\text{O}(g)$, is
- 524.1
 - 41.2
 - 262.5
 - 412
27. Heat of formation of H_2O is -188 kJ/mol and H_2O_2 is -286 kJ/mol . The enthalpy change for the reaction $2\text{H}_2\text{O}_2 \longrightarrow 2\text{H}_2\text{O} + \text{O}_2$, is
- 196 kJ
 - 196 kJ
 - 984 kJ
 - 984 kJ
28. The enthalpy of dissolution of $\text{BaCl}_2(s)$ and $\text{BaCl}_2 \cdot 2\text{H}_2\text{O}(s)$ are -20.6 and 8.8 kJ/mol respectively. The enthalpy of hydration for, $\text{BaCl}_2(s) + 2\text{H}_2\text{O} \longrightarrow \text{BaCl}_2 \cdot 2\text{H}_2\text{O}(s)$, is
- 29.4 kJ
 - 29.4 kJ
 - 11.8 kJ
 - 38.2 kJ
29. $\Delta_f U^\circ$ of formation of $\text{CH}_4(g)$ at certain temperature is -393 kJ mol^{-1} . The value of $\Delta_f H^\circ$ is
- zero
 - $< \Delta_f U^\circ$
 - $> \Delta_f U^\circ$
 - equal to $\Delta_f U^\circ$
30. Find the entropy change when 2 moles of ideal gas at 27°C temperature is expanded reversibly from 2 L to 20 L
- 92.1
 - 0
 - 4
 - 9.2
31. When a gas expands from 1.5 L to 6.5 L against a constant pressure of 0.50 atm and during this process, the gas also absorbs 100 J of heat. The change in internal energy is
- 153.3 J
 - 353.3 J
 - 153.3 J
 - 353.3 J
32. The reaction between cyanamide, $\text{NH}_2\text{CN}(s)$ and oxygen was allowed to complete and ΔU at 300 K was observed to be -743 kJ mol^{-1} . The value of ΔH at 300 K for the combustion reaction
- $$\text{NH}_2\text{CN}(s) + \frac{3}{2}\text{O}_2(g) \longrightarrow \text{N}_2(g) + \text{CO}_2(g) + \text{H}_2\text{O}(l)$$
- would be
- 741.75 kJ mol^{-1}
 - 743 kJ mol^{-1}
 - 744.25 kJ mol^{-1}
 - 740.5 kJ mol^{-1}

33. The following data (s) are given as the standard enthalpies of combustion of $C(s)$, $H_2(g)$ and $CH_4(g)$ are $-393.5 \text{ kJ mol}^{-1}$, $-285.8 \text{ kJ mol}^{-1}$ and $-890.4 \text{ kJ mol}^{-1}$ respectively at 298 K. The standard enthalpy of formation of methane $[CH_4(g)]$ is

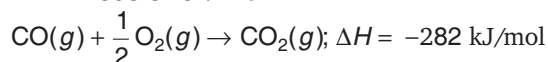
a. $+724.42 \text{ kJ mol}^{-1}$ b. $+74.7 \text{ kJ mol}^{-1}$
c. $-114.82 \text{ kJ mol}^{-1}$ d. $-194.62 \text{ kJ mol}^{-1}$

34. If for a given substance, melting point is T_B and freezing point is T_A then correct variation of entropy by graph between entropy change and temperature is



35. Mark out the enthalpy for the formation of carbon monoxide (CO) Given, $C(s) + \frac{1}{2}O_2(g) \rightarrow CO(g)$;

$$\Delta H = -393.3 \text{ kJ / mol}$$



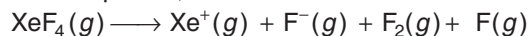
a. 110.5 kJ/mol b. 676.1 kJ/mol
c. 282.8 kJ/mol d. 300.0 kJ/mol

36. In the reaction, $Na(s) \rightarrow Na(g)$

the enthalpy of atomisation is same as the

a. enthalpy of dissociation b. enthalpy of sublimation
c. enthalpy of association d. enthalpy of vaporisation

37. Calculate the bond enthalpy of Xe—F bond as given in the equation,



$$\Delta_r H = 292 \text{ kcal mol}^{-1}$$

Ionisation energy of Xe = $279 \text{ kcal mol}^{-1}$

Bond energy (F—F) = 38 kcal/mol

Electron affinity of F = 85 kcal/mol

a. 8.5 kcal/mol b. 34 kcal/mol
c. 24 kcal/mol d. None of these

38. Calculate the resonance energy of N_2O .

$$\Delta_f H^\circ \text{ of } N_2O = 82 \text{ kJ mol}^{-1}$$

Bond energy of $N=O$ = 607 kJ mol^{-1}

Bond energy of $O=O$ = 498 kJ mol^{-1}

Bond energy of $N=N$ = 418 kJ mol^{-1}

Bond energy of $N \equiv N$ = 946 kJ mol^{-1}

a. 82 kJ mol^{-1}
b. -88 kJ mol^{-1}
c. -82 kJ mol^{-1}
d. $+88 \text{ kJ mol}^{-1}$

39. Use the following data to calculate $\Delta_{\text{lattice}} H^\circ$ for NaBr. $\Delta_{\text{sub}} H^\circ$ for sodium metal = $108.4 \text{ kJ mol}^{-1}$

Ionisation enthalpy of sodium = 496 kJ mol^{-1}

Electron gain enthalpy of bromine = 325 kJ mol^{-1}

Bond dissociation enthalpy of bromine = 192 kJ mol^{-1}

$$\Delta_f H^\circ \text{ for NaBr (s)} = -360.1 \text{ kJ mol}^{-1}$$

a. $-735.5 \text{ kJ mol}^{-1}$ b. $+735.5 \text{ kJ mol}^{-1}$
c. $-735.5 \text{ J mol}^{-1}$ d. $+735.5 \text{ J mol}^{-1}$

40. Find out the standard free energy change at 60°C and at 1 atm if the N_2O_4 is 50% dissociated.

a. $-800.0 \text{ kJ mol}^{-1}$ b. $+800.0 \text{ kJ mol}^{-1}$
c. $-789.89 \text{ JK}^{-1} \text{ mol}^{-1}$ d. $+789.98 \text{ JK}^{-1} \text{ mol}^{-1}$

41. Moles of an ideal gas expand isothermally and reversibly from pressure of 5 atm to 1 atm at 300 K. Calculate the largest mass that can be lifted through a height of 1 m by this expansion.

a. 4092.76 kg b. 8730.9368 kg
c. 4492.76 kg d. 8170.2344 kg

42. At 27°C , one mole of an ideal gas is compressed isothermally and reversibly from a pressure of 2 atm to 10 atm. The value of ΔE and q are ($R = 2 \text{ cal}$)

a. -965.84 cal
b. $-965.84 \text{ cal}, -865.58 \text{ cal}$
c. $865.58 \text{ cal}, -865.58 \text{ cal}$
d. $965.84 \text{ cal}, +865.58 \text{ cal}$

43. The enthalpy of vaporisation of water is 186.5 J/mol . The entropy of its vaporisation will be

a. $0.5 \text{ JK}^{-1} \text{ mol}^{-1}$ b. $1.0 \text{ JK}^{-1} \text{ mol}^{-1}$
c. $1.5 \text{ JK}^{-1} \text{ mol}^{-1}$ d. $2.0 \text{ JK}^{-1} \text{ mol}^{-1}$

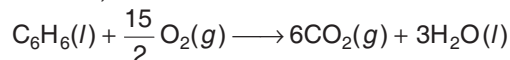
44. The reaction which proceeds in the forward direction is

a. $Fe_2O_3 + 6HCl \rightarrow 2FeCl_3 + 3H_2O$
b. $NH_3 + H_2O + NaCl \rightarrow NH_4Cl + NaOH$
c. $SnCl_4 + Hg_2Cl_2 \rightarrow SnCl_2 + 2HgCl_2$
d. $2CuI + I_2 + 4K^+ \rightarrow 2Cu^+ + 4KI$

45. Ammonium chloride, when dissolved in water, leads to a cooling sensation. The dissolution of ammonium chloride at constant temperature is accompanied by

a. increase in entropy b. decrease in entropy
c. no change in entropy d. no change in enthalpy

46. At 27°C , the reaction



proceeds spontaneously because the magnitude of

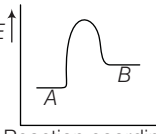
a. $\Delta H = T \cdot \Delta S$ b. $\Delta H > T \cdot \Delta S$
c. $\Delta H < T \cdot \Delta S$ d. $\Delta H > 0$ and $T \cdot \Delta S < 0$

47. The sign of ΔG for the process of melting of ice at 273 K and 1 atm pressure is

a. positive
b. negative
c. neither negative nor positive
d. either negative or positive

48. The initial state A has the temperature T_A , U_A as the internal energy of the system. By applying the mechanical work, new state B is achieved with the temperature T_B and having the internal energy U_B . Given that $T_B > T_A$. What is the correct expression for the change in internal energy (ΔU)?
- $U_B = U_A$
 - $U_B - U_A$
 - $U_A - U_B$
 - None of the above
49. When 1.8 g of steam at the normal boiling point of water is converted into water, at the same temperature, enthalpy and entropy changes respectively will be
[Given, ΔH_{vap} for water = 40.8 kJ mol^{-1}]
- -8.12 kJ , 11.89 JK^{-1}
 - 10.25 kJ , 12.95 JK^{-1}
 - -4.08 kJ , -10.93 JK^{-1}
 - 10.93 kJ , -4.08 JK^{-1}
50. When the heat of a reaction at constant pressure is $-2.5 \times 10^3 \text{ cal}$ and entropy change for the reaction is 7.4 cal deg^{-1} , it is predicted that the reaction at 25°C is
- reversible
 - spontaneous
 - non-spontaneous
 - irreversible
51. What will be the change of entropy $\Delta_r S^\circ$ at 298 K for the reaction in which urea is formed from NH_3 and CO_2 ?
 $2\text{NH}_3(g) + \text{CO}_2(g) \longrightarrow \text{NH}_2\text{CONH}_2(aq) + \text{H}_2\text{O}(l)$
 [Given, the standard entropy of $\text{NH}_2\text{CONH}_2(aq)$, $\text{CO}_2(g)$, $\text{NH}_3(g)$ and $\text{H}_2\text{O}(l)$ are 174.0 , 213.7 , 192.3 and $69.9 \text{ JK}^{-1} \text{ mol}^{-1}$ respectively]
- $200 \text{ JK}^{-1} \text{ mol}^{-1}$
 - $-35.44 \text{ JK}^{-1} \text{ mol}^{-1}$
 - $-354.4 \text{ JK}^{-1} \text{ mol}^{-1}$
 - $425.2 \text{ JK}^{-1} \text{ mol}^{-1}$
52. For an isomerisation reaction $A \rightleftharpoons B$, the temperature dependence of equilibrium constant is given by $\log_e K = 4.0 - \frac{2000}{T}$. The value of ΔS° at 300 K is therefore,
- $4R$
 - $5R$
 - $400R$
 - $2000R$
53. ΔG , in process of melting of ice at -15°C , is
- $\Delta G = -ve$
 - $\Delta G = +ve$
 - $\Delta G = 0$
 - All of these
54. Using following data
 $\text{H}_2\text{O}(l)[373.15 \text{ K}, 1\text{atm}] \longrightarrow \text{H}_2\text{O}(g)[373.15 \text{ K}, 1\text{atm}]; \Delta S_1$
 $\text{H}_2\text{O}(s)[273.15 \text{ K}, 1\text{atm}] \longrightarrow \text{H}_2\text{O}(l)[273.15 \text{ K}, 1\text{atm}]; \Delta S_2$
 Predict which of the following is correct?
- $\Delta S_1 = \Delta S_2$
 - $\Delta S_1 > \Delta S_2$
 - $\Delta S_1 < \Delta S_2$
 - ΔS_1 may be greater or smaller than ΔS_2
55. On the basis of thermochemical equations a , b and c , find out which of the algebraic relationships given in option a . to d . is correct?
- $\text{C}(\text{graphite}) + \text{O}_2(g) \longrightarrow \text{CO}_2(g); \Delta_c H = x \text{ kJ mol}^{-1}$
 - $\text{C}(\text{graphite}) + \frac{1}{2} \text{O}_2(g) \longrightarrow \text{CO}(g); \Delta_c H = y \text{ kJ mol}^{-1}$
 - $\text{CO}(g) + \frac{1}{2} \text{O}_2(g) \longrightarrow \text{CO}_2(g); \Delta_c H = z \text{ kJ mol}^{-1}$
- $z = x + y$
 - $x = y - z$
 - $x = y + z$
 - $y = 2z - x$
56. The heat of atomisation of $\text{P}_4\text{H}_4(g)$ and $\text{PH}_3(g)$ are 355 kcal/mol and 228 kcal/mol respectively. The energy of $\text{P}-\text{P}$ bond is
- $102 \text{ kcal mol}^{-1}$
 - 51 kcal mol^{-1}
 - 26 kcal mol^{-1}
 - $204 \text{ kcal mol}^{-1}$
57. In the given equation,
 $4\text{Fe}(s) + 3\text{O}_2(g) \longrightarrow 2\text{Fe}_2\text{O}_3(s)$
 The entropy change is $-549.4 \text{ JK}^{-1} \text{ mol}^{-1}$ at 298K .
 $(\Delta_r H^\circ = -1648 \times 10^3 \text{ Jmol}^{-1})$. The above reactions is
- spontaneous
 - non-spontaneous
 - Both (a) and (b)
 - None of the above
58. Find out the entropy change in surroundings when 1 mole of $\text{H}_2\text{O}(l)$ is formed under standard conditions
 $\Delta_r H^\circ = -286 \text{ kJ mol}^{-1}$
- $959.7 \text{ JK}^{-1} \text{ mol}^{-1}$
 - $286 \text{ JK}^{-1} \text{ mol}^{-1}$
 - $-959.7 \text{ JK}^{-1} \text{ mol}^{-1}$
 - $-286 \text{ JK}^{-1} \text{ mol}^{-1}$

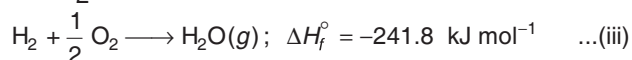
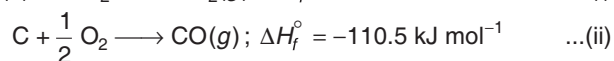
BITSAT Archives

- A swimmer coming out from a pool is covered with a film of water weighing about 18 g. Calculate the internal energy of vaporisation at 100°C. [2014]
[$\Delta_{\text{vap}}H^\circ$ for water at 373 K = 40.66 kJ mol⁻¹]
a. 35.67 kJmol⁻¹ b. 37.56 kJmol⁻¹
c. 36.57 kJmol⁻¹ d. 38.75 kJmol⁻¹
- The heat of combustion of sucrose, C₁₂H₂₂O₁₁(s) at constant volume is 1348.9 kcal mol⁻¹ at 25°C, then the heat of reaction at constant pressure when steam is produced is [2014]
a. - 1348.9 kcal b. - 1342.34 kcal
c. + 1250 kcal d. None of these
- At constant temperature and pressure which one of the following statements is correct for the reaction?
 $\text{CO}(g) + \frac{1}{2} \text{O}_2(g) \longrightarrow \text{CO}_2(g)$ [2013]
a. $\Delta H = \Delta E$
b. $\Delta H < \Delta E$
c. $\Delta H > \Delta E$
d. ΔH is independent of physical state of reactant
- The equilibrium constant K_p for the reaction,
 $\text{N}_2(g) + 3\text{H}_2(g) \rightleftharpoons 2\text{NH}_3(g)$
 $1.6 \times 10^{-4} \text{ (atm)}^{-2}$ at 400°C. What will be the equilibrium constant at 500°C, if heat of the reaction in this temperature range is - 25.14 kcal? [2013]
a. $1.231 \times 10^{-4} \text{ (atm)}^{-2}$ b. $1.876 \times 10^{-7} \text{ (atm)}^{-2}$
c. $1.462 \times 10^{-5} \text{ (atm)}^{-2}$ d. $3.462 \times 10^{-5} \text{ (atm)}^{-2}$
- Given that
 $\text{H}_2\text{O}(l) \longrightarrow \text{H}^+(aq) + \text{OH}^-(aq); \Delta H = 57.32 \text{ kJ}$
 $\text{H}_2(g) + \frac{1}{2} \text{O}_2(g) \longrightarrow \text{H}_2\text{O}(l); \Delta H = - 286.02 \text{ kJ}$
Then, calculate the enthalpy of formation of OH⁻ at 25°C. [2013]
a. - 228.8 kJ b. - 343.52 kJ
c. + 228.8 kJ d. + 343.52 kJ
- Calculate the amount of heat evolved when 500 cm³ of 0.1 M HCl is mixed with 200 cm³ of 0.2 M NaOH. [2013]
a. 57.3 kJ b. 2.865 kJ
c. 2.292 kJ d. 0.573 kJ
- The mutual heat of neutralisation of 40 g NaOH and 60 g CH₃COOH will be [2011]
a. 57.1 kJ b. less than 57.1 kJ
c. more than 57.1 kJ d. 13.7 kJ
- For the dissociation reaction, [2010]
 $\text{H}_2(g) \rightleftharpoons 2\text{H}(g); \Delta H = 162 \text{ kcal}$
heat of atomisation of H is
a. 81 kcal b. 162 kcal c. 208 kcal d. 218 kcal
- Internal energy does not include [2009]
a. vibrational energy
b. rotational energy
c. energy arising by gravitational pull
d. nuclear energy
- A reaction has both ΔH and ΔS negative. The rate of reaction [2009]
a. increases with increase of temperature
b. increases with decrease of temperature
c. remains unaffected by change of temperature
d. cannot be predicted for change in temperature
- One mole of an anhydrous salt AB dissolves in water with the evolution of 21.0 J mol⁻¹ of heat. If the heat of hydration of AB is - 29.4 J mol⁻¹, then the heat of dissociation of hydrated salt AB is [2009]
a. 50.4 Jmol⁻¹ b. 8.4 Jmol⁻¹
c. - 50.4 Jmol⁻¹ d. - 8.4 Jmol⁻¹
- Calculate the ΔH in kJ for the following reaction, [2008]
 $\text{C}(g) + \text{O}_2(g) \longrightarrow \text{CO}_2(g)$
Given that,
 $\text{H}_2\text{O}(g) + \text{C}(g) \longrightarrow \text{CO}(g) + \text{H}_2(g); \Delta H = + 131 \text{ kJ}$
 $\text{CO}(g) + \frac{1}{2} \text{O}_2(g) \longrightarrow \text{CO}_2(g); \Delta H = - 282 \text{ kJ}$
 $\text{H}_2(g) + \frac{1}{2} \text{O}_2(g) \longrightarrow \text{H}_2\text{O}(g); \Delta H = - 242 \text{ kJ}$
a. - 393 b. + 393 c. + 655 d. - 655
- For a reversible reaction $A \rightleftharpoons B$, [2008]
which one of the following statements is wrong from the given energy profile diagram?

a. Activation energy of forward reaction is greater than backward reaction
b. The forward reaction is endothermic
c. The threshold energy is less than that of activation energy
d. The energy of activation of forward reaction is equal to the sum of heat of reaction and the energy of activation of backward reaction
- A 27°C one mole of an ideal gas is compressed isothermally and reversibly from a pressure of 2 atm to 10 atm. The value of ΔE and q are ($R = 2 \text{ cal}$) [2007]
a. 0, - 965.84 cal
b. - 965.58 cal, - 865.58 cal
c. + 865.58 cal, - 865.58 cal
d. + 965.84 cal, + 865.58 cal
- Gibbs free energy G , enthalpy H and entropy S are interrelated as in [2007]
a. $G = H + TS$
b. $G = H - TS$
c. $G - TS = H$
d. $G = S = H$

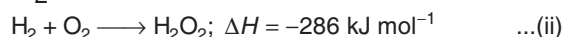
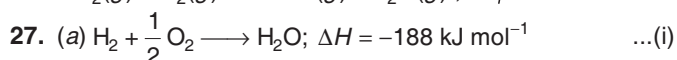
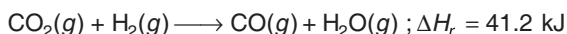
Answer with Solutions

Practice Exercise

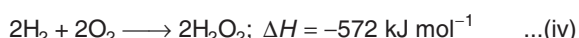
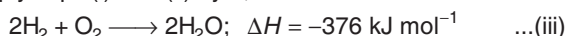
- (c) Thermodynamics is not concerned with the rate at which a reaction proceeds.
- (a) $\Delta E = q + W = 300 + (-500) = -200 \text{ cal}$
- (b) Flying bird gives up energy and takes in air (O_2).
- (c) In adiabatic process, heat is not exchanged with surroundings, hence $\Delta q = 0$
- (b) Work done, during isothermal reversible expansion of a gas, is maximum.
- (c) The presence of reactants in a closed vessel made up of conducting material. e.g. copper or steel is an example of a closed system.
- (c) For adiabatic condition, $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{1}{27}\right)^{1/3} = \frac{1}{3}$
 $\therefore T_2 = 300 \times \frac{1}{3} = 100 \text{ K}$
 Thus, $T_2 < T_1$, hence cooling takes place due to expansion under adiabatic condition.
 $\therefore W = -\Delta E = -C_V(T_2 - T_1)$ [$\Delta E = (-)$ ve, expansion]
 $= -6(100 - 300) = 1200 \text{ cal}$
- (c) The heat change determined in a bomb calorimeter is ΔE , value. Thus, $\Delta E = -870 \text{ kcal mol}^{-1}$
- (a) Since, heat evolved when 78 g benzene is burnt
 $= -3264.6 \text{ kJ}$
 Therefore, heat produced, by burning 3.9 g benzene
 $= \frac{3.9}{78} \times (-3264.6) = -163.23 \text{ kJ}$
- (c) The bond energy of Cl_2 is the highest among the given.
- (c) For the reaction, $\Delta n = 1 - 1 = 0$
 $\therefore \Delta H = \Delta E$
- (a) The formation of CH_4 is represented as:
 $\text{C}(s) + 2\text{H}_2(g) \longrightarrow \text{CH}_4(g)$
 $\Delta H_f = (\Delta H_{\text{comb}})_R - (\Delta H_{\text{comb}})_P$
 $= [-394 + 2(-284) - (-892)] \text{ kJ}$
 $= -962 + 892 = -70 \text{ kJ mol}^{-1}$
- (d) For $\text{N}_2(g) + 3\text{H}_2(g) \longrightarrow 2\text{NH}_3(g)$
 Hence, $\Delta n = -2$
 $\therefore \Delta H < \Delta E$
- (b) $\Delta H = -\frac{1.435}{18} \text{ kcal g}^{-1}$
 $= -0.0797 \text{ kcal g}^{-1}$
- (d) Subtracting second equation from first, we get
 $\text{S}_{(R)} \longrightarrow \text{S}_{(M)} \Delta H = +2.5 \text{ kJ}$; endothermic
- (a) The process is endothermic and takes up heat from solution so that the solution becomes cold.
- (b) The room gets heated because heat is released to the surroundings.
- (a) Internal energy, $E = \frac{3}{2}RT = \frac{3}{2}p \Rightarrow p = \frac{2}{3}E$
- (c) $\Delta H = \Delta E + \Delta n_g RT$, $30 = \Delta E + 3 \times 2 \times 500 \times 10^{-3}$
 $\Delta E = 27 \text{ kcal}$
- (a) $\Delta H = \text{work done} = i \times V \times t = 0.50 \text{ A} \times 12 \text{ V} \times 300 \text{ s}$
 $= 1800 \text{ J} = +1.8 \text{ kJ}$
 Molar enthalpy of vaporisation,
 $\Delta H_m = \frac{\Delta H}{\text{Moles of H}_2\text{O}} = \frac{\Delta H}{n_{\text{H}_2\text{O}}} = \frac{1.8 \text{ kJ}}{\frac{0.798}{18}} = 40.6 \text{ kJ mol}^{-1}$
 $\Delta H_m = \Delta E_m + p \Delta V$
 $\Delta H_m = \Delta E_m + \Delta n_g RT$ [$\Delta n_g = 1$]
 $\Delta H_m = \Delta E_m + RT$
 Molar internal energy change, $\Delta E_m = \Delta H_m - RT$
 $= 40.6 - 8.314 \times 10^{-3} \times 373.15 = 37.5 \text{ kJ mol}^{-1}$
- (a) Since, the process is exothermic, heat is evolved due to this temperature of water increases.
- (c) $\text{C} + \text{O}_2 \longrightarrow \text{CO}_2$; $\Delta H = -393.5 \text{ kJ/mol}$
 $\therefore$ Heat released during the formation of 44 g of CO_2
 $= -393.5 \text{ kJ}$
 Heat released during the formation of 35.2 g (given) of $\text{CO}_2 = -\frac{393.5 \times 35.2}{44} = -315 \text{ kJ}$
- (b) Millimoles of dibasic strong acid
 $= M \times V = 0.20 \times 350 = 70 \text{ mmol}$
 $\therefore$ Amount of H^+ ions in the acid $= 2 \times 70 = 140 \text{ mmol}$
 Similarly, amount of OH^- ions in monobasic strong base
 $= 0.10 \times 650 = 65 \text{ mmol}$
 [Here, OH^- is the limiting reactant]
 $\therefore$ 1 mole of OH^- ions produces $= 57.1 \text{ kJ heat}$
 $65 \times 10^{-3} \text{ mole of OH}^-$ ions will produce
 $= 57.1 \times 65 \times 10^{-3} = 3.71 \text{ kJ}$
- (b) $\text{C}_2\text{H}_4 + 3\text{O}_2 \longrightarrow 2\text{CO}_2 + 2\text{H}_2\text{O}$
 $\Delta H_{\text{reaction}} = [2 \times \Delta H_f^\circ(\text{CO}_2) + 2 \times \Delta H_f^\circ(\text{H}_2\text{O})]$
 $- [\Delta H_f^\circ(\text{C}_2\text{H}_4) + 3 \times \Delta H_f^\circ(\text{O}_2)]$
 $= [2(-394) + 2(-286)] - [52 + 0] = -1412 \text{ kJ mol}^{-1}$
- (c) $\frac{1}{2} \text{H}_2 + \frac{1}{2} \text{Cl}_2 \longrightarrow \text{HCl}$
 $\Delta H = \left[\frac{1}{2} \text{BE}(\text{H}_2) + \frac{1}{2} \text{BE}(\text{Cl}_2) \right] - \text{BE}(\text{HCl})$
 $= \left[\left(\frac{1}{2} \times 104 \right) + \left(\frac{1}{2} \times 58 \right) \right] - 103 = -22 \text{ kcal}$



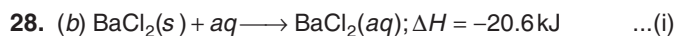
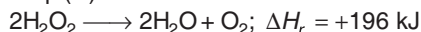
Eq. (ii) + Eq. (iii) - Eq. (i)



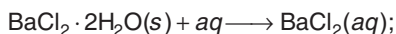
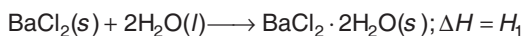
Multiply Eqs. (i) and (ii) by 2,



Eq. (iii) - Eq. (iv)

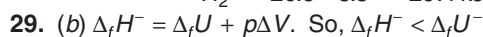


Eq. (i) can be split as



$$\Delta H = H_1 + H_2 = -20.6; H_1 = 8.8 \text{ kJ}$$

$$H_2 = -20.6 - 8.8 = -29.4 \text{ kJ}$$

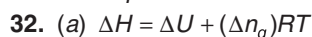


30. (d) $\Delta S = 2.303 nR \log \frac{V_2}{V_1} = 2.303 \times 2 \times 2 \times \log \frac{20}{2} = 9.2$



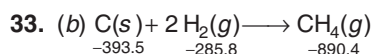
$$= 2.5 \text{ L} \cdot \text{atm} = 2.5 \times 101.3 \text{ J} = 253.25 \text{ J}$$

$$\Delta U = q + W = 100 - 253.25 = -153.25 \text{ J}$$



$$= -743 + \frac{1}{2} \times 8.314 \times 300 \times 10^{-3} \text{ kJ mol}^{-1}$$

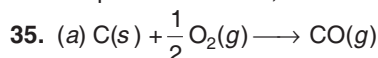
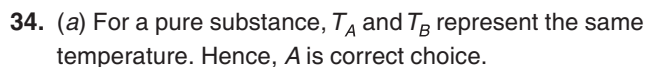
$$= -741.75 \text{ kJ mol}^{-1}$$



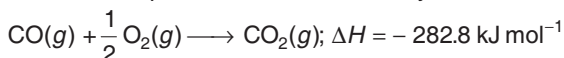
$$\Delta_f H(CH_4(g)) = \Delta_c H^\circ(CH_4) - \Delta_c H^\circ(C) - \Delta_c H^\circ(H_2(g)) \times 2$$

$$= -890.4 - (-393.5) - (-285.8 \times 2)$$

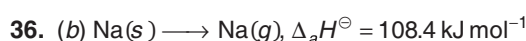
$$= +74.7 \text{ kJ/mol}$$



The above equation can be obtained by subtraction of



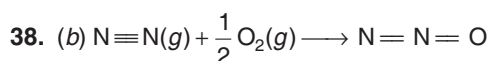
$$\Delta H_f = [+393.3 - (282.8)] = 110.5 \text{ kJ mol}^{-1}$$



The enthalpy of atomisation is same as the enthalpy of sublimation.



$$292 = (x + 279) - (38 + 85), x = 34 \text{ kcal mol}^{-1}$$



Calculated value of $\Delta_f H^\circ =$

$$\Delta_f H^\circ = \left[BE(N \equiv N) + \frac{1}{2} BE(O = O) \right] - [BE(N = N) + BE(N = O)]$$

$$= \left[946 + \frac{1}{2} (498) \right] - [418 + 607] = 170 \text{ kJ mol}^{-1}$$

Resonance energy = Observed $\Delta_f H^\circ$ - Calculated $\Delta_f H^\circ$

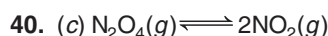
$$= 82 - 170 = -88 \text{ kJ mol}^{-1}$$



$$\Delta_f H^\circ = \Delta_{\text{sub}} H^\circ + IE + \Delta_{\text{diss}} H^\circ + \Delta_{\text{eq}} H^\circ + U$$

$$-360.1 = 108.4 + 496 + 96 + (-325) + U$$

$$U = +735.5 \text{ kJ mol}^{-1}$$



N_2O_4 is 50% dissociated, the mole fraction of the substances is given by

$$x_{N_2O_4} = \frac{1-0.5}{1+0.5}, x_{NO_2} = \frac{2 \times 0.5}{1+0.5}$$

$$p_{N_2O_4} = \frac{0.5}{1.5} \times 1 \text{ atm}, p_{NO_2} = \frac{1}{1.5} \times 1 \text{ atm}$$

The equilibrium constant K_p is given by

$$K_p = \frac{(p_{NO_2})^2}{p_{N_2O_4}} = \frac{1.5}{(1.5)^2(0.5)} = \frac{1}{0.75} = 1.33$$

$$\Delta_a G^\circ = -RT \ln K_p$$

$$= -8.314 \text{ JK}^{-1} \text{ mol}^{-1} \times 333 \text{ K} \times 2.303 \times 0.1239$$

$$= -789.89 \text{ JK}^{-1} \text{ mol}^{-1}$$



$$= -2.303 \times 20 \times 8.314 \text{ JK}^{-1} \text{ mol}^{-1} \times 300 \text{ K} \log \frac{5}{1}$$

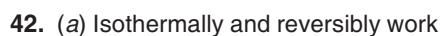
$$= 80.15 \times 10^3 \text{ J}$$

Work done = mgh

$$80.15 \times 10^3 = m \times 9.81 \text{ ms}^{-2} \times 1 \text{ m}$$

$$m = \frac{80.15 \times 10^3 \text{ kgm}^2 \text{ s}^{-2}}{9.81 \text{ ms}^{-2} \times 1 \text{ m}} = 8.17023 \times 10^3$$

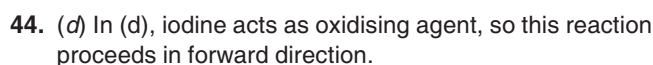
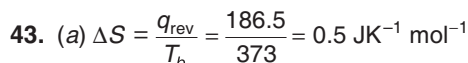
$$= 8170.2344 \text{ kg}$$



$$W = 2.303 nRT \log(p_2 / p_1)$$

$$= 2.303 \times 1 \times 2 \times 300 \log(10/2) = 965.84$$

At constant temperature, $\Delta E = 0$, $q = -W = -965.84 \text{ cal}$



45. (a) In dissolution, solid changes to ions in solution, which have more randomness.
 46. (b) In the reaction, $\Delta S = -ve$
 Hence, for the reaction to be spontaneous, ΔH should be negative and higher than $T \cdot \Delta S$ i.e. $\Delta H > T \cdot \Delta S$
 47. (c) At 273 K, water and ice are in equilibrium, hence, $\Delta G = 0$.

48. (b) The change in the internal energy is $\Delta U = U_B - U_A$.

49. (c) $\Delta H_{\text{condensation}}$ for 1.8 g of steam

$$= (-40.8) \text{ kJ} \times \left(\frac{1.8}{18} \right) \text{ mol} = -4.08 \text{ kJ}$$

$$\Delta S = \frac{\Delta H}{T_b} = \frac{-4.08 \times 10^3 \text{ J}}{373.15 \text{ K}} = -10.93 \text{ JK}^{-1}$$

50. (b) Heat at constant pressure means enthalpy, i.e. we have, $\Delta G = \Delta H - T\Delta S$

$$= -2.5 \times 10^3 - 298 \times 7.4 = -4705 \text{ cal}$$

Hence, the process is spontaneous.

51. (c) For the given change

$$\begin{aligned} \Delta_r S^\circ &= \Sigma n_P S_P^\circ - \Sigma n_R S_R^\circ \\ &= S_{\text{NH}_2\text{CONH}_2}^\circ + S_{\text{H}_2\text{O}}^\circ - [2 \times S_{\text{NH}_3}^\circ + S_{\text{CO}_2}^\circ] \\ &= 174.0 + 69.9 - [2 \times 192.3 + 213.7] \\ &= -354.4 \text{ JK}^{-1} \text{ mol}^{-1} \end{aligned}$$

52. (a) Variation of K with temperature is given by

$$\log K = \frac{\Delta S^\circ}{R} - \frac{\Delta H^\circ}{RT} = 4.0 - \frac{2000}{T}$$

$$\text{On comparing, } \frac{\Delta S^\circ}{R} = 4 \text{ or } \Delta S^\circ = 4R$$

53. (b) Because melting of ice at -15°C is a non-spontaneous process.

54. (b) Vaporisation of water creates more randomness. Hence, has more entropy.

55. (c) $\text{C} + \text{O}_2 \longrightarrow \text{CO}_2(g); \Delta_r H = x \text{ kJ mol}^{-1} \dots \text{(i)}$

$\text{C}(\text{graphite}) + \frac{1}{2} \text{O}_2(g) \longrightarrow \text{CO}(g); \Delta_r H = y \text{ kJ mol}^{-1} \dots \text{(ii)}$

$\text{CO}(g) + \frac{1}{2} \text{O}_2(g) \longrightarrow \text{CO}_2(g); \Delta_r H = z \text{ kJ mol}^{-1} \dots \text{(iii)}$

$$(i) = (ii) + (iii)$$

$$\Rightarrow x = y + z$$

56. (b) P—H bond energy = $\frac{228}{3} = 76 \text{ kcal mol}^{-1}$ For P_4H_4

bond energy of $4(\text{P—H}) + (\text{P—P})$
 $= 355 \text{ kcal mol}^{-1}$

$$\Rightarrow 4 \times 76 + (\text{P—P}) = 355 \text{ kcal mol}^{-1}$$

$$\therefore (\text{P—P}) \text{ bond energy} = 51 \text{ kcal mol}^{-1}$$

57. (a) At temperature, T entropy change of the surroundings is

$$\Delta S_{\text{surr}} = - \frac{\Delta_r H^\circ}{T} \text{ (at constant pressure)}$$

$$= \frac{-(-1648 \times 10^3 \text{ J mol}^{-1})}{298 \text{ K}}$$

$$= 5530 \text{ JK}^{-1} \text{ mol}^{-1}$$

$$\Delta_r S_{\text{total}} = 5530 + (-549.4)$$

$$= 4980.6 \text{ JK}^{-1} \text{ mol}^{-1}$$

Hence, the above reaction is spontaneous.

58. (a) $\text{H}_2(g) + \frac{1}{2} \text{O}_2(g) \longrightarrow \text{H}_2\text{O}(l)$

$$\Delta_r H^\circ = -286 \text{ kJ mol}^{-1}$$

When 1 mole of $\text{H}_2\text{O}(l)$ is formed, 286 kJ of heat is released. This heat is absorbed by the surrounding,

i.e. $q_{\text{surr}} = +286 \text{ kJ mol}^{-1}$

$$\Delta S = \frac{q_{\text{surr}}}{T} = \frac{286 \text{ kJ mol}^{-1}}{298 \text{ K}}$$

$$= 959.7 \text{ JK}^{-1} \text{ mol}^{-1}$$

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1. (b) We can represent the process of evaporation as



Number of moles in 18g $\text{H}_2\text{O}(l)$

$$= \frac{18 \text{ g}}{18 \text{ g mol}^{-1}} = 1 \text{ mol}$$

$$\Delta_{\text{vap}} U = \Delta_{\text{vap}} H^\circ - p\Delta V$$

$$= \Delta_{\text{vap}} H^\circ - \Delta n_g RT$$

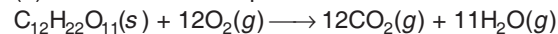
Assume steam behave as an ideal gas

$$\Delta_{\text{vap}} U = (40.66) - (1)(8.314 \times 10^{-3})(373)$$

$$= 40.66 - 3.10$$

$$= 37.56 \text{ kJ mol}^{-1}$$

2. (b) The combustion equation of sucrose is



Here, $\Delta n = 12 + 11 - 12 = 11$

As we know, $\Delta H = \Delta E + \Delta n RT$

$$\Delta H = (-1348.9 \times 10^3) + 11 \times 2 \times 298 = -1342.34 \text{ kcal}$$

3. (b) As we know, $\Delta H = \Delta E + \Delta n_g RT$

For the reaction, $\text{CO}(g) + \frac{1}{2} \text{O}_2(g) \longrightarrow \text{CO}_2(g)$

$$\Delta n = 1 - \left(1 + \frac{1}{2} \right) = -\frac{1}{2}$$

$$\therefore \Delta H = \Delta E - \frac{1}{2} RT$$

Hence, $\Delta H < \Delta E$

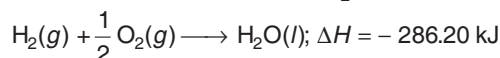
4. (c) Equilibrium constants at different temperature and heat of the reaction are related by the equation

$$2.303 \log \frac{K_{p_2}}{K_{p_1}} = \frac{\Delta H^\circ}{R} \left[\frac{T_2 - T_1}{T_1 T_2} \right]$$

$$\log K_{p_2} = -4.835$$

$$K_{p_2} = 1.462 \times 10^{-5} (\text{atm})^{-2}$$

5. (a) Consider the formation of H_2O



$$\Delta H_f = \Delta H_f [\text{H}_2\text{O}(\text{l})] - \Delta H_f [\text{H}_2(\text{g})] - \frac{1}{2} \Delta H_f [\text{O}_2(\text{g})]$$

$$\therefore \Delta H_f (\text{H}_2\text{O}(\text{l})) = -286.20$$

Now consider the ionisation of H_2O



$$\Delta H_r = \Delta H_f (\text{H}^+(\text{aq}) + \text{OH}^-(\text{aq})) - \Delta H_f (\text{H}_2\text{O}(\text{l}))$$

$$57.32 = 0 + \Delta H_f (\text{OH}^-, \text{aq}) - (-286.20)$$

$$\text{or } \Delta H_f (\text{OH}^-, \text{aq}) = 57.32 - 286.20 \text{ e}$$

$$= -228.8 \text{ kJ}$$

6. (c) $\text{HCl} + \text{NaOH} \longrightarrow \text{NaCl} + \text{H}_2\text{O}$

At $t = 0$,

$$\begin{aligned} \text{Number of moles} &= \frac{500 \times 0.1}{1000} = \frac{200 \times 0.2}{1000} \\ &= 0.05 = 0.04 \end{aligned}$$

$\therefore$ During neutralisation of 1 mole of NaOH by 1 mole of HCl, heat evolved = 57.3 kJ

$\therefore$ To neutralised 0.04 moles of NaOH by 0.04 mole of NaOH, heat evolved

$$= 57.3 \times 0.04$$

$$= 2.292 \text{ kJ}$$

7. (b) $40 \text{ g NaOH} = \frac{40}{40} = 1 \text{ mol NaOH}$

$$60 \text{ g CH}_3\text{COOH} = \frac{60}{60} = 1 \text{ mol CH}_3\text{COOH}$$

Since, acetic acid is a weak acid, some of the heat is utilised to ionise it. So, enthalpy of neutralisation of 1 mole of NaOH by 1 mole CH_3COOH is less than 57.1 kJ.

Enthalpy of neutralisation of a strong acid by a strong base is always 57.1 kJ.

8. (a) $\Delta H = \Delta H_{(\text{product})} - \Delta H_{(\text{reactant})}$

$$162 = 2 \times \Delta H_{\text{H}} - \Delta H_{\text{H}_2}$$

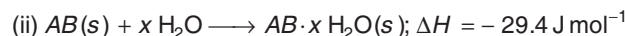
$$\Delta H_{\text{H}} = \frac{162}{2} = 81 \text{ kcal} \quad (\because \Delta H_{\text{H}_2} = 0)$$

9. (c) Thus, internal energy does not include energy due to gravitational pull.

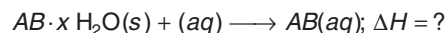
10. (b) $\Delta G = \Delta H - T\Delta S$

Negative ΔH favours the process. Negative ΔS opposes the process. If temperature is decreased, opposing factor decreases. Hence, rate of reaction increases.

11. (b) (i) $\text{AB}(\text{s}) + (\text{aq}) \longrightarrow \text{AB}(\text{aq}); \Delta H = -21 \text{ J mol}^{-1}$



Required equation is



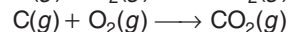
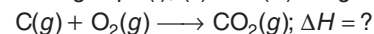
Eq. (i) is equivalent to



$$\Delta H_1 + \Delta H_2 = -21$$

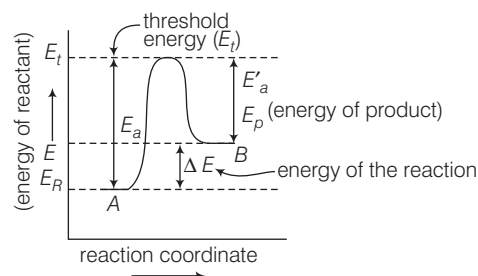
$$-29.4 + \Delta H_2 = -21 \quad \text{or} \quad \Delta H_2 = 8.4 \text{ J mol}^{-1}$$

12. (a) On adding Eqs. (i), (ii) and (iii) we get



$$\Delta H = (131 - 282 - 242) \text{ kJ} = -393 \text{ kJ}$$

13. (c)



where,

E_a = activation energy of forward reaction

E'_a = activation energy of backward reaction

The above energy profile diagram shows that $E_a > E'_a$

The potential energy of the product is greater than that of the reactant, so the reaction is endothermic.

$$E_a = E'_a + \Delta E$$

$$E_t = E_a \text{ or } E_t > E'_a$$

14. (a) Isothermally (at constant temperature) and reversibly work is

$$W = 2.303 nRT \log \frac{p_2}{p_1} = 2.303 \times 1 \times 2 \times 300 \log \frac{10}{2}$$

$$= 965.84$$

at constant temperature, $\Delta E = 0$

$$\Delta E = q + W, q = -W = -965.84 \text{ cal}$$

15. (b) Gibbs free energy G , enthalpy H and entropy S are interrelated as $G = H - TS$

8

Chemical and Ionic Equilibria

Chemical Equilibrium

The state of reversible reactions at which the concentration of the reactants and products do not change with time, is known as chemical equilibrium.

Law of Chemical Equilibrium

This law states that, the rate at which a chemical reaction proceeds, is proportional to the product of active masses of reacting substances with each concentration term raised to the power of stoichiometric coefficient of that species in chemical reaction,

For a reversible reaction, $aA + bB \rightleftharpoons cC + dD$,

Rate of forward reaction, $R_f \propto [A]^a [B]^b = K_f [A]^a [B]^b$ (law of mass action)

Rate of backward reaction, $R_b \propto [C]^c [D]^d = K_b [C]^c [D]^d$

At equilibrium,

$$R_f = R_b$$
$$K_f [A]^a [B]^b = K_b [C]^c [D]^d$$

$$\frac{K_f}{K_b} = \frac{[C]^c [D]^d}{[A]^a [B]^b}$$

$$\frac{K_f}{K_b} = K_C = \text{equilibrium constant}$$

Equilibrium Constants (K_p and K_C)

For a reaction, $aA + bB \rightleftharpoons cC + dD$

Equilibrium constants, $K_C = \frac{[C]^c [D]^d}{[A]^a [B]^b}$, $K_p = \frac{(p_C)^c \times (p_D)^d}{(p_A)^a \times (p_B)^b}$

Relation between K_p and K_C

$$K_p = K_C (RT)^{\Delta n_g}$$

R = gas constant,

T = temperature in kelvin

Δn_g = gaseous moles of products – gaseous moles of reactants

Hence, (i) If $\Delta n_g = 0$, $K_p = K_C$

(ii) If $\Delta n_g = +ve$, $K_p > K_C$

(iii) If $\Delta n_g = -ve$, $K_p < K_C$

Units of Equilibrium Constants

- If $\Delta n_g = 0$, then $K_p = K_C$, no units for both K_C and K_p
- If $\Delta n_g > 0$, unit of K_C is $(\text{mol L}^{-1})^{\Delta n_g}$ and that of K_p is $(\text{atm})^{\Delta n_g}$.
- If $\Delta n_g < 0$, unit of K_C is $(\text{L mol}^{-1})^{\Delta n_g}$ and that of K_p is $(\text{atm})^{\Delta n_g}$.

Characteristics of equilibrium constant

Characteristics	Reaction	Equilibrium constant
If reaction is reversed	$A + B \rightleftharpoons C + D$	$K' = 1/K$
If reaction is divided by factor n	$\frac{1}{n} A + \frac{1}{n} B \rightleftharpoons \frac{1}{n} C + \frac{1}{n} D$	$K' = \sqrt[n]{K}$
If reaction is multiplied by factor n	$nA + nB \rightleftharpoons nC + nD$	$K' = K^n$
If reaction is written in n number of steps	$A + B \xrightleftharpoons{K_1} X + Y$ $X + Y \xrightleftharpoons{K_2} P + Q \dots$ $\xrightleftharpoons{K_n} C + D$	$K' = K_1 \times K_2 \times \dots K_n$

Reaction Quotient (Q)

It is the ratio of the molar concentration or partial pressure of the product species to that of reactant species at any stage in the reaction.

For a general reaction,

$$aA + bB \rightleftharpoons cC + dD$$

$$Q_C = \frac{[C]^c [D]^d}{[A]^a [B]^b} \quad \text{and} \quad Q_p = \frac{p_C^c \cdot p_D^d}{p_A^a \cdot p_B^b}$$

At any stage of the reaction,

- If $Q_C > K_C$, the reaction will proceed in the direction of reactants (reverse reaction).
- If $Q_C < K_C$, the reaction will move in the direction of the products.
- If $Q_C = K_C$, reaction mixture is already at equilibrium.

Relation between K and ΔG°

Gibbs free energy change and reaction quotient are related as

$$\Delta G = \Delta G^\circ + 2.303 RT \log Q_C$$

At equilibrium, $\Delta G = 0$

$$Q_C = K_C$$

$$\Delta G^\circ = -2.303 RT \log K_C$$

or

$$\Delta G^\circ = -2.303 RT \log K_p$$

Significance of ΔG° are given below :

- If $\Delta G^\circ < 0$, $\log K > 0 \Rightarrow K > 1$
Therefore, forward reaction is spontaneous.
- If $\Delta G^\circ > 0$, $\log K < 0 \Rightarrow K < 1$
Therefore, backward reaction is spontaneous.
- If $\Delta G^\circ = 0$, $\log K = 0 \Rightarrow K = 1$
Therefore, reaction is at equilibrium.

Le-Chatelier's Principle

The Le-Chatelier's principle states that "a change in any of the factors that determine the equilibrium conditions of a system will cause the system to change in such a manner so as to reduce or to counteract the effect of the change".

This law is applicable to all physical and chemical equilibria.

Factors Affecting the Equilibrium

Following are the various factors which affect on the state of equilibrium :

- Effect of change in concentration** If concentration of the reactants are increased or products are removed, the equilibrium will shift in the forward direction and *vice-versa*.
- Effect of change in pressure** In a reaction at equilibrium, on increasing pressure, equilibrium shifts in that direction where number of gaseous molecules decreases and *vice-versa*.
- Effect of change in temperature**
 - An increase in temperature for exothermic reactions decreases the concentration of products at equilibrium and *vice-versa*.
 - An increase in temperature for endothermic reactions increases the concentration of products at equilibrium and *vice-versa*.
- Effect of catalyst** Catalyst increases (or decreases) in both forward and backward reactions equally, therefore equilibrium will be attained in less (or more) time. Catalyst does not affect the equilibrium constant.

(v) Effect of the addition of inert gas

- (a) Addition of inert gas to an equilibrium (if $\Delta n = 0$) has no effect.
- (b) Addition of inert gas to an equilibrium (if $\Delta n \neq 0$) has no effect, if addition is made at constant volume, but addition at constant pressure will favour the direction of reaction where total number of moles at equilibrium show an increase.

Weak and Strong Electrolytes and their Ionisation

Weak electrolytes dissociate partially in the solutions and such solutions are poor conductor of electricity, e.g. CH_3COOH , H_3PO_4 , H_3BO_3 , NH_4OH , HCN etc.

Strong electrolytes dissociate completely into their ions in solution and such solutions are very good conductor of electricity, e.g. HCl , H_2SO_4 , NaOH , KOH , NaCl , KCl etc.

Separation of an electrolyte into their ions either on fusion or dissolution is called **ionisation** or **dissociation**.



Degree of Ionisation (α)

$$\alpha = \frac{\text{Number of moles of electrolyte dissociated as ions}}{\text{Total number of moles of electrolyte dissolved}}$$

or
$$\alpha = \frac{\text{Amount of electrolyte dissociated}}{\text{Initial amount}}$$

Ostwald's Dilution Law

It states that degree of dissociation of weak electrolyte is inversely proportional to the square root of concentration.

$$\alpha \propto \frac{1}{\sqrt{c}}, \quad \alpha = \sqrt{\frac{k}{c}} = \sqrt{kV}$$

where, α = degree of dissociation

V = volume containing 1 mole of weak electrolyte

k = dissociation constant

Various Concepts of Acids and Bases and their Ionisation

The various concepts of acids and bases are as follows :

Arrhenius Concept

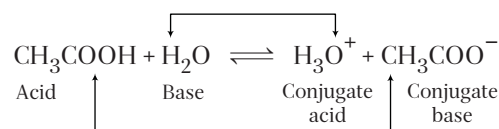
According to Arrhenius concept, all substances which give H^+ ions when dissolved in water are called **acids** while those which ionise in water to furnish OH^- are called **bases**.



Bronsted-Lowry Concept

According to Bronsted-Lowry concept, an acid is a substance (molecule or ion) that can donate a proton and a base, is a substance that can accept a proton from an acid.

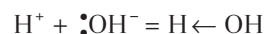
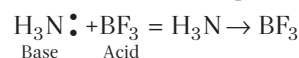
e.g.



Generally, a strong base has a weak conjugate acid and *vice-versa*.

Lewis Acids and Bases Concept

An acid is a species which accepts an electron pair and base which donates an electron pair, e.g.



Relative Strength of Acids and Bases

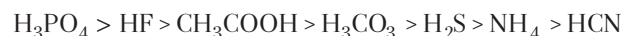
Relative strength of two acids is written as

$$\frac{\text{Strength of acid 1}(\text{HA}_1)}{\text{Strength of acid 2}(\text{HA}_2)} = \frac{\alpha_1}{\alpha_2} = \sqrt{\frac{K_{a1}/C}{K_{a2}/C}} = \sqrt{\frac{K_{a1}}{K_{a2}}}$$

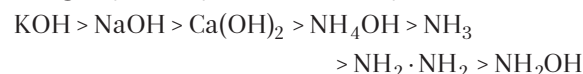
where, K_{a1} and K_{a2} are the dissociation constants of two acids at the same concentration C while α_1 and α_2 are their degrees of dissociation. Similarly, for equimolar weak bases,

$$\frac{\text{Strength of base 1}(\text{B}_1\text{OH})}{\text{Strength of base 2}(\text{B}_2\text{OH})} = \sqrt{\frac{K_{b1}}{K_{b2}}}$$

Relative strength of some of the acids are as follows :

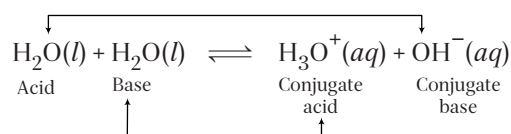


Relative strength of some of the bases are as follows :



Ionisation of Water

For an equilibrium,



The dissociation constant is represented by

$$K = \frac{[\text{H}_3\text{O}^+][\text{OH}^-]}{[\text{H}_2\text{O}]^2}$$

But $[\text{H}_2\text{O}] = 1$, therefore $K_w = [\text{H}_3\text{O}^+] [\text{OH}^-]$

where, K_w = ionic product of water

The value of K_w at 298 K

$$K_w = [\text{H}_3\text{O}^+] [\text{OH}^-] = (1 \times 10^{-7})^2 = 1 \times 10^{-14} \text{ mol}^2 \text{ L}^{-2}$$

Important points related to ionic product of water are :

The value of K_w is temperature dependent. It increases with increase in temperature.

For acidic solutions, $[\text{H}_3\text{O}^+] > [\text{OH}^-]$

For neutral solutions, $[\text{H}_3\text{O}^+] = [\text{OH}^-]$

For basic solutions, $[\text{H}_3\text{O}^+] < [\text{OH}^-]$

pH Scale

pH of a solution is negative logarithm of hydrogen ion concentration of solution.

$$\text{Thus, } \text{pH} = -\log [\text{H}^+] = \log \frac{1}{[\text{H}^+]}$$

$$\text{or } [\text{H}^+] = 10^{-\text{pH}}$$

pOH Scale

pOH represents the OH^- ion concentration.

$$\therefore \text{pOH} = -\log [\text{OH}^-] = \log \frac{1}{[\text{OH}^-]}$$

$$\text{or } [\text{OH}^-] = 10^{-\text{pOH}}$$

$$\text{and } \text{p}K_w = \text{pH} + \text{pOH} = 14 \text{ at } 25^\circ\text{C}$$

Relationship between $[\text{H}^+]$, $[\text{OH}^-]$ and pH

$$\text{pH of a weak monobasic acid base } \text{pH} = \frac{1}{2} [\text{p}K_a - \log C]$$

$$\text{pOH of a weak monoacidic base } \text{pOH} = \frac{1}{2} [\text{p}K_b - \log C]$$

$$\text{pH} = 14 - \text{pOH} = 14 - \frac{1}{2} [\text{p}K_b - \log C]$$

Total $[\text{OH}^-]$ in a mixture of two weak bases

$$[\text{OH}^-] = \sqrt{K_1 C_1 + K_2 C_2}$$

Common Ion Effect

It states that if to the solution of a weak electrolyte, a solution of strong electrolyte is added which furnishes an ion common to that furnished by the weak electrolyte, the ionisation of the weak electrolyte is suppressed.

e.g. In IInd group of qualitative analysis, H_2S is passed in the presence of HCl . This is due to the fact that HCl suppresses the ionisation of weakly dissociated H_2S . Due to this, only sulphides of II group radicals are precipitated. Sulphides of III, IV etc., groups are not precipitated because of their high solubility product.

Salt Hydrolysis and pH of their Solution

The process of salt hydrolysis is actually the reverse process of neutralisation. The reaction of an anion or cation with water accompanied by cleavage of O—H bond is called hydrolysis.

Salt hydrolysis affects the pH of the solution.

Neutral Salts

Salts of strong acids and strong bases (i.e. neutral salts) do not undergo hydrolysis, e.g. NaCl , CaSO_4 etc.

$$\text{pH} = 7$$

Acidic Salts

Salt of a strong acid and weak base, e.g. NH_4Cl , is called acidic salt, which undergoes cationic hydrolysis.

$$\text{pH} < 7$$

$$\text{For such salts, } [\text{H}_3\text{O}^+] = \sqrt{K_h \times C}$$

$$K_h = \frac{K_w}{K_b}$$

$$\text{pH} = 7 - \frac{1}{2} [\text{p}K_b + \log C]$$

where, K_h = hydrolysis constant

K_b = ionisation constant for weak base

C = molar concentration of salt

Basic Salts

Salt of strong base and weak acid, e.g. NaNO_2 , NaCN , CH_3COONa are termed as basic salts.

Such salts undergo anionic hydrolysis.

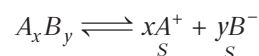
$$\text{pH} > 7$$

$$\text{For basic salts, } [\text{OH}^-] = \sqrt{K_h \times C} \text{ or } K_h = \frac{K_w}{K_a}$$

$$\text{pH} = 7 + \frac{1}{2} [\text{p}K_a + \log C]$$

Solubility Product (S)

It is defined as the product of molar concentration of its ions in a concentrated solution, each concentration terms raised to the power equal to the number of ions produced on dissociation of one molecule of electrolyte.



$$\therefore K_{sp} = [A^+]^x [B^-]^y$$

$$\therefore K_{sp} = S^{x+y}$$

$$\text{or } S = \sqrt{(x+y) K_{sp}}$$

where, S = solubility, K_{sp} = solubility product

Buffer Solution

The solution, which maintains its pH constant or reserve acidic or basic nature even upon addition of small amounts of acid or base, is called buffer solution. The ability of buffer solution to resist changes in pH on addition of acid or base is called buffer action.

Types of buffer solutions —
 — Acidic buffer [HCN + NaCN] (pH < 7)
 — Basic buffer [NH₄OH + NH₄Cl] (pH > 7)

Henderson's Equation for Buffer Solution

(i) For acidic buffer,

$$\text{pH} = \text{p}K_a + \log \frac{[\text{salt}]}{[\text{acid}]}$$

(ii) For basic buffer,

$$\text{pOH} = \text{p}K_b + \log \frac{[\text{salt}]}{[\text{base}]}$$

Buffer Capacity

Buffer capacity

$$= \frac{\text{Number of moles of acid / base added to 1 L of buffer}}{\text{Change in pH}}$$

Buffer capacity is maximum when

(a) [salt] = [acid], pH = pK_a for acidic buffer

(b) [salt] = [base], pH = pK_b for basic buffer

Greater the buffer capacity, larger is its capacity to resist the change in pH value.

Acid-Base Titration

Acid-Base Indicators

An acid-base indicator is a substance which possess one colour in acidic solution and altogether different colours in alkaline medium, i.e. its colour changes with pH.

Ostwald's Theory

According to this theory :

- Indicators are either weak acid or weak base.
- Their unionised molecules possess different colour from those of the ions which they give in the solution.
- An acidic indicator yields a coloured anion while a basic indicator yields a coloured cation in solution.
- Since, they are weak electrolytes, they are not sufficiently ionised in solution. But in the presence of strong acid or alkali, their degree of ionisation is considerably increased and they produce a large number of coloured ions.
- An indicator changes colour when the concentration of hydrogen ion (in mol/L) solution is equal to the ionisation constant of the indicator, i.e. indicator is 50% dissociated.

Practice Exercise

- In which of the following reaction, K_p and K_c are equal?
 - $\text{N}_2(g) + 3\text{H}_2(g) \rightleftharpoons 2\text{NH}_3(g)$
 - $2\text{SO}_2(g) + \text{O}_2(g) \rightleftharpoons 2\text{SO}_3(g)$
 - $\text{N}_2(g) + \text{O}_2(g) \rightleftharpoons 2\text{NO}(g)$
 - $2\text{NO}(g) + \text{O}_2(g) \rightleftharpoons 2\text{NO}_2(g)$
- The number of gram molecules of a substance present in unit volume is termed as
 - activity
 - normal solution
 - molal solution
 - active mass
- The active mass 10 g of CaCO_3 is
 - 1
 - 0.1
 - 10
 - 100
- If the equilibrium constant for the reaction $\text{A} + 2\text{B} \rightleftharpoons 2\text{C}$, is 40 then the equilibrium constant for reaction $\text{C} \rightleftharpoons \text{B} + \frac{1}{2}\text{A}$ will be
 - $1/\sqrt{40}$
 - 1/40
 - $(1/40)^2$
 - 40
- At a certain temperature in $2\text{HI} \rightleftharpoons \text{H}_2 + \text{I}_2$, only 50% HI is dissociated at equilibrium. The equilibrium constant is
 - 1.0
 - 0.25
 - 3.0
 - 0.50
- For the reaction, $2\text{N}_2\text{O}_4 \rightleftharpoons 4\text{NO}_2$, $\frac{-d(\text{N}_2\text{O}_4)}{dt} = K$ and $\frac{d'(\text{NO}_2)}{dt} = K'$, then
 - $K' = 2K$
 - $2K' = K$
 - $K' = K$
 - None of these

7. The equilibrium constant K for the reaction $2\text{HI}(g) \rightleftharpoons \text{H}_2(g) + \text{I}_2(g)$, at room temperature, is 2.85 and that at 698 K is 1.4×10^{-2} . This implies that the forward reaction is
- exothermic
 - endothermic
 - exergonic
 - unpredictable
8. When pressure is applied to the equilibrium system $\text{Ice} \rightleftharpoons \text{water}$, which of the following phenomenon will happen?
- More ice will be formed
 - Water will evaporate
 - Equilibrium will not be established
 - More water will be formed
9. For $\text{N}_2 + 3\text{H}_2 \rightleftharpoons 2\text{NH}_3$, 1 mole of N_2 and 3 moles of H_2 are at 4 atm. Equilibrium pressure is found to be 3 atm. The value of K_p is
- $\frac{1}{(0.5)(0.15)^3}$
 - $\frac{1}{(0.5)(1.5)^3}$
 - $\frac{3 \times 3}{(0.5)(1.5)^3}$
 - None of these
10. The unit of equilibrium constant, (K) for the reaction $\text{A} + \text{B} \rightleftharpoons \text{C}$, would be
- mol L^{-1}
 - mol L
 - $\text{mol}^{-1} \text{L}$
 - $\frac{1}{\text{mol L}}$
11. In any chemical reaction, equilibrium is supposed to be attained when
- mutual opposite reactions undergo
 - velocity of mutual reactions become equal
 - concentration of reactants and resulting products are equal
 - the temperature of mutual opposite reactions become equal
12. When 3 moles of ethyl alcohol are mixed with 3 moles of acetic acid, 2 moles of ester are formed at equilibrium according to the equation
- $$\text{CH}_3\text{COOH}(l) + \text{C}_2\text{H}_5\text{OH}(l) \rightleftharpoons \text{CH}_3\text{COOC}_2\text{H}_5(l) + \text{H}_2\text{O}(l)$$
- The value of the equilibrium constant for the reaction is
- 4
 - 2/9
 - 2
 - 4/9
13. The reaction $2\text{SO}_2(g) + \text{O}_2(g) \rightleftharpoons 2\text{SO}_3(g)$, is carried out in 1 dm^3 and 2 dm^3 vessels respectively. The ratio of the reaction velocities will be
- 1 : 4
 - 4 : 1
 - 1 : 8
 - 8 : 1
14. The equilibrium constant for the reaction $\text{H}_2(g) + \text{I}_2(g) \rightleftharpoons 2\text{HI}(g)$, is 64. If the volume of the container is reduced to half of the original volume, the value of the equilibrium constant will be
- 16
 - 32
 - 64
 - 128
15. The ratio of K_p/K_C for the reaction $\text{CO}(g) + \frac{1}{2}\text{O}_2(g) \rightleftharpoons \text{CO}_2(g)$, is
- 1
 - RT
 - $(RT)^{1/2}$
 - $(RT)^{-1/2}$
16. The equilibrium constant K_p for the reaction $\text{A} \rightleftharpoons 2\text{B}$, is related to degree of dissociation α of A and total pressure p as
- $\frac{4\alpha^2 p}{1-\alpha^2}$
 - $\frac{4\alpha^2 p^2}{1-\alpha^2}$
 - $\frac{4\alpha^2 p^2}{1-\alpha}$
 - $\frac{4\alpha^2 p}{1-\alpha}$
17. At 30°C , K_p for the dissociation reaction $\text{SO}_2\text{Cl}_2(g) \rightleftharpoons \text{SO}_2(g) + \text{Cl}_2$ is $2.9 \times 10^{-2} \text{ atm}$. If the total pressure is 1 atm, the degree of dissociation of SO_2Cl_2 is (assume, $1-\alpha^2 = 1$)
- 87%
 - 13%
 - 0.17%
 - 29%
18. XY_2 dissociates as $\text{XY}_2(g) \rightleftharpoons \text{XY}(g) + \text{Y}(g)$. When the initial pressure of XY_2 is 600 mm Hg, the total equilibrium pressure is 800 mm Hg. Calculate K for the reaction assuming that the volume of the system remains unchanged.
- 50
 - 100
 - 166.6
 - 400.0
19. For these reactions,
- $\text{H}_2(g) + \text{I}_2 \rightleftharpoons 2\text{HI}(g); K_C = \frac{[\text{HI}]^2}{[\text{H}_2][\text{I}_2]}$
 - $2\text{HI}(g) \rightleftharpoons \text{H}_2(g) + \text{I}_2(g); K_C = \frac{[\text{H}_2][\text{I}_2]}{[\text{HI}]^2}$
- K_C and K'_C are related as
- $K = K_C \times K'_C$
 - $K = K_C + K'_C$
 - $K = K'_C - K_C$
 - $K'_C = \frac{1}{K_C}$
20. For the reversible reaction, $\text{N}_2(g) + 3\text{H}_2(g) \rightleftharpoons 2\text{NH}_3(g)$ at 500°C , the value of K_p is 1.44×10^{-5} . When partial pressure is measured in atmospheres, the corresponding value of K_C with concentration in mol L^{-1} is
- $1.44 \times 10^{-5} / (0.082 \times 500)^{-2}$
 - $1.44 \times 10^{-5} / (8.314 \times 773)^{-2}$
 - $1.44 \times 10^{-5} / (0.082 \times 773)^2$
 - $1.44 \times 10^{-5} / (0.082 \times 773)^{-2}$
21. PCl_5 , PCl_3 and Cl_2 are at equilibrium at 500 K and having concentration 1.59 M, 1.59 M and 1.41 M respectively. Calculate the K_C for the following reaction :
- $$\text{PCl}_5 \rightleftharpoons \text{PCl}_3 + \text{Cl}_2$$
- 2.79
 - 3.75
 - 4.57
 - 1.79

22. What is the relation between K_p and K_C for a general reaction, $aA + bB \rightleftharpoons cC + dD$?
- a. $K_p = K_C (RT)^{\Delta n}$ b. $K_C = K_p (RT)^{\Delta n}$
 c. $K_p = \frac{1}{K_C (RT)^{\Delta n}}$ d. $K_C = \frac{1}{K_p (RT)^{\Delta n}}$
23. Consider the following reaction in a closed container at equilibrium. $\text{CaCO}_3(s) \rightleftharpoons \text{CaO}(s) + \text{CO}_2(g)$
 What would be the effect of addition of CaCO_3 on the equilibrium concentration of CO_2 ?
- a. Increases
 b. Decreases
 c. Data is not sufficient to predict it
 d. Remains unaffected
24. According to Le-Chatelier's principle, if heat is given to solid-liquid system, then
- a. quantity of solid will reduce
 b. quantity of solid will increase
 c. temperature will increase
 d. temperature will decrease
25. Which of the following will not change the concentration of ammonia in the equilibrium?
- $$4\text{NH}_3(g) + 5\text{O}_2(g) \rightleftharpoons 4\text{NO}(g) + 6\text{H}_2\text{O}(l); \Delta H = +ve$$
- a. Increase of pressure
 b. Increase of volume
 c. Addition of catalyst
 d. Decrease of temperature
26. The Bronsted acid which gives the weakest conjugate base, is
- a. HF b. H_2S
 c. HCl d. H_2O
27. Which one of the following is the strongest Lewis acid?
- a. BF_3 b. BCl_3 c. BBr_3 d. BI_3
28. Arrange the following acids in decreasing order of acidity :
- (I) H_2SO_3 , (II) H_3PO_3 and (III) HClO_3
- a. I > III > II b. I > II > III
 c. III > I > II d. II > III > I
29. Among the given anions, the strongest Bronsted base is
- a. ClO^- b. ClO_2^- c. ClO_3^- d. ClO_4^-
30. What will be the conjugate bases for the following Bronsted acids?
- HF , H_2SO_4 and HCO_3^-
- a. F^- , SO_4^{2-} and CO_3^{2-} b. F^- , SO_4^{2-} and H_2CO_3
 c. F^- , HSO_4^- and H_2CO_3 d. F^- , HSO_4^- and CO_3^{2-}
31. The vapour density of N_2O_4 at a certain temperature is 30. What is the percentage dissociation of N_2O_4 at this temperature?
- a. 53.3% b. 106.6%
 c. 26.7% d. None of these
32. For the reaction $\text{NO}_2(g) \rightleftharpoons 2\text{NO}(g)$, the degree of dissociation at equilibrium is 0.2 at 1 atmospheric pressure. The equilibrium constant K_p will be
- a. 1/2 b. 1/4
 c. 1/6 d. 1/8
33. If a 0.1 M solution of HCN is 0.01% ionised, the ionisation constant for HCN is
- a. 10^{-9} b. 10^{-7}
 c. 10^{-5} d. 10^{-3}
34. An aqueous solution contains the following ions:
 Hg_2^{2+} , Hg^{2+} , Pb^{2+} and Cd^{2+}
 On adding HCl, which of these will precipitate?
- a. Hg_2Cl_2 and PbCl_2 b. Only Hg_2Cl_2
 c. Only PbCl_2 d. PbCl_2 and HgCl_2
35. What is correct about ionic product of an electrolyte?
- a. It is always equal to its solubility product
 b. It can be less than or equal to K_{sp}
 c. It is always less than K_{sp}
 d. It can be less than or equal to or greater than K_{sp}
36. A solution was prepared by mixing 50 mL of 0.2 M HCl and 50 mL 0.10 M NaOH. The pH of the solution is
- a. 7.0 b. 2.0
 c. 3.0 d. 1.3
37. What is $[\text{H}^+]$ in mol/L of a solution that is 0.20 M in CH_3COONa and 0.10 M CH_3COOH ?
 K_a for $\text{CH}_3\text{COOH} = 1.8 \times 10^{-5}$
- a. 9.0×10^{-6} b. 3.5×10^{-4}
 c. 1.1×10^{-5} d. 1.8×10^{-5}
38. A weak acid, HA, has a K_a of 1.00×10^{-5} . If 0.100 mole of this acid is dissolving in 1 L of water, the percentage of acid dissociated at equilibrium is closed to
- a. 99.0% b. 1.00%
 c. 99.9% d. 0.100%
39. How many gram equivalents of NaOH are required to neutralise 25 cm³ of a decinormal HCl solution?
- a. 0.00125 b. 0.0025
 c. 0.0050 d. 0.025
40. The first and second dissociation constants of an acid H_2A are 1.0×10^{-5} and 5.0×10^{-10} respectively. The overall dissociation constant of the acid will be
- a. 5.0×10^5 b. 5.0×10^{-5}
 c. 5.0×10^{15} d. 5.0×10^{-15}
41. The pH of 0.1 M solution of a weak acid is 3. What is the value of the ionisation constant for the acid?
- a. 0.1 b. 10^{-3}
 c. 10^{-5} d. 10^{-7}
42. What will be the pH at which an acid indicator with $K_a = 1 \times 10^{-5}$ changes colour when the indicator concentration is $1 \times 10^{-3} \text{ M}$?
- a. 5 b. 7 c. 3 d. 6.5

43. The pH of a 10^{-9} M solution of HCl in water is
 a. 8
 b. -8
 c. between 7 and 8
 d. between 6 and 7

44. The following concentrations were obtained for the formation of NH_3 from Na and H_2 at equilibrium at 500 K : $[\text{N}_2] = 1.5 \times 10^{-2}$ M, $[\text{H}_2] = 3.0 \times 10^{-2}$ M and $[\text{NH}_3] = 1.2 \times 10^{-2}$ M

Calculate the equilibrium constant.

- a. 1.08×10^4 b. 3.98×10^2
 c. 1.06×10^3 d. 2.93×10^4

45. Which of the following relations represent correct relation between standard electrode potential and equilibrium constant?

- I. $\log K = \frac{nFE^\circ}{2.303 RT}$ II. $K = e^{\frac{-nFE^\circ}{RT}}$
 III. $\log K = \frac{-nFE^\circ}{2.303 RT}$ IV. $\log K = 0.4342 \frac{nFE^\circ}{RT}$
 a. I, II and III b. II and III
 c. I, II and IV d. I and IV

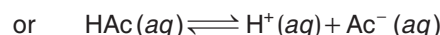
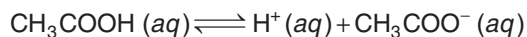
46. If the ionisation constant of acetic acid is 1.8×10^{-5} , at what concentration will it be dissociated to 2%?
 a. 1 M b. 0.018 M c. 0.18 M d. 0.045 M

47. The dissociation constant of acetic acid at a given temperature is 1.69×10^{-5} . The degree of dissociation of 0.01 M acetic acid in the presence of 0.01 M HCl is equal to
 a. 0.41 b. 0.13
 c. 0.169×10^{-2} d. 0.013

48. The concentration of hydrogen ion in a sample of soft drink is 3.8×10^{-3} M. What is its pH?
 a. 4.32 b. 5.12 c. 3.31 d. 2.42

49. The pK_a of acetic acid and pK_b of ammonium hydroxide are 4.76 and 4.75 respectively. Calculate the pH of ammonium acetate solution.
 a. 4.765 b. 5.012 c. 7.005 d. 6.098

50. For a reaction,



Evaluate the pH of the solution resulting on addition of 0.05 M acetate ion to 0.05 M acid solution ($K_a = 1.8 \times 10^{-5}$)

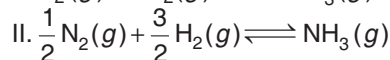
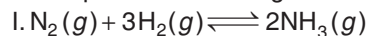
- a. 5.72 b. 3.87 c. 4.24 d. 4.74

51. pH of an acidic buffer is given by

- a. $\text{pH} = \text{p}K_a + \log \frac{[\text{salt}]}{[\text{acid}]}$ b. $\text{pH} = \text{p}K_a - \log \frac{[\text{salt}]}{[\text{acid}]}$
 c. $\text{pH} = \frac{1}{2} \text{p}K_a + \log \frac{[\text{salt}]}{[\text{acid}]}$ d. $\text{pH} = \log K_a + \log \frac{[\text{salt}]}{[\text{acid}]}$

52. The pH of buffer solution containing 4×10^{-3} and 0.4 mole of acetic acid ($\text{p}K_a = 4.76$) and sodium acetate respectively will be
 a. 6.76 b. 4.76 c. 2.76 d. 0.76

53. For the pair of reactions given below,



If at a particular temperature, K_{p1} and K_{p2} are the equilibrium constants for reaction I and II respectively, then

- a. $K_{p1} = 2K_{p2}$ b. $K_{p1} = K_{p2}^2$
 c. $2K_{p1} = K_{p2}$ d. $K_{p1}^2 = K_{p2}$

54. Solubility of BaF_2 in a solution of $\text{Ba}(\text{NO}_3)_2$ will be represented by the concentration term

- a. $[\text{Ba}^{2+}]$ b. $[\text{F}^-]^2$
 c. $\frac{1}{2}[\text{F}^-]$ d. $2[\text{NO}_3^-]$

55. For anionic hydrolysis, pH is given by

- a. $\text{pH} = \frac{1}{2} \text{p}K_a - \frac{1}{2} \text{p}K_b + \frac{1}{2} \log C$
 b. $\text{pH} = \frac{1}{2} \text{p}K_w + \frac{1}{2} \text{p}K_a - \frac{1}{2} \text{p}K_b$
 c. $\text{pH} = \frac{1}{2} \text{p}K_w + \frac{1}{2} \text{p}K_a + \frac{1}{2} \log C$
 d. $\text{pH} = -\frac{1}{2} \text{p}K_w + \frac{1}{2} \text{p}K_a + \frac{1}{2} \text{p}K_b$

56. The solubility of AgBrO_3 in an aqueous solution of NaBrO_3 (as compared to that in water) is
 a. the same
 b. more
 c. less
 d. unpredicted due to a new chemical reaction

57. Which of the following salts undergo anionic hydrolysis?

- a. CuSO_4 b. NH_4Cl
 c. FeCl_3 d. Na_2CO_3

58. How do we differentiate between Fe^{3+} and Cr^{3+} in group III?

- a. By taking excess of NH_4OH
 b. By increasing NH_4^+ ion concentration
 c. By decreasing OH^- ion concentration
 d. Both (a) and (c)

59. What will be the degree of ionisation of 0.05 M acetic acid if its $\text{p}K_a$ value is 4.74 ?

- a. 0.019% b. 1.9% c. 3.0% d. 4.74%

60. A certain buffer solution contains equal concentrations of X^- and HX , K_b for X^- is 10^{-10} . The pH of the buffer solution is

- a. 4 b. 6
 c. 7 d. 14

61. The buffering action of an acidic buffer is maximum when its pH equals to
 a. 5
 b. 7
 c. 1
 d. pK_a
62. The expression for the solubility product of Ag_2CrO_3 will be
 a. $K_{sp} = S^2$
 b. $K_{sp} = 4S^3$
 c. $K_{sp} = 27S^4$
 d. $K_{sp} = S$
63. Amphoteric behaviour is shown by
 a. H_2CO_3 and Al_2O_3
 b. HCO_3^- and H_2O
 c. HCO_3^- and H_3O^+
 d. H_2CO_3 and H_2O
64. The value of pK_w of water
 a. increases with increase in temperature
 b. decreases with rise in temperature
 c. does not change with variation in temperature
 d. increases till $50^\circ C$ and there after decreases
65. At $20^\circ C$, Ag^+ ion concentration in a saturated solution of Ag_2CrO_4 is 1.5×10^{-4} mol/L. At $20^\circ C$, the solubility product of Ag_2CrO_4 will be
 a. 3.3750×10^{-12}
 b. 1.6875×10^{-10}
 c. 1.6875×10^{-12}
 d. 1.6875×10^{-11}
66. If the solubility of Ag_2CrO_4 is S mol/L, its solubility product will be
 a. S^2
 b. S^3
 c. $4S^3$
 d. $2S^3$

BITSAT Archives

1. Calculate the pH at the equivalence point during the titration of 0.1 M, 25 mL CH_3COOH with 0.05 M NaOH solution. [$K_a(CH_3COOH) = 1.8 \times 10^{-5}$] [2014]
 a. 9.63
 b. 8.63
 c. 10.63
 d. 11.63
2. Gastric juice in human stomach has pH value about 1.8 and pH of small intestine is about 7.8. The pK_a value of aspirin is 3.5. Aspirin will be [2013]
 a. ionised in the small intestine and stomach
 b. ionised in the stomach and almost unionised in the small intestine
 c. unionised in small intestine and stomach
 d. completely ionised in small intestine and stomach
3. At $27^\circ C$, K_p value for the reaction $CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$, is 0.1 atm. K_c value for this reaction is [2013]
 a. 4×10^{-3}
 b. 6×10^{-3}
 c. 2×10^{-3}
 d. 9×10^{-3}
4. In the titration of NaOH and HCl, which of the following indicators will be used? [2012]
 a. Methyl orange
 b. Methyl red
 c. Both (a) and (b)
 d. None of (a) and (b)
5. For $2NOBr(g) \rightleftharpoons 2NO(g) + Br_2(g)$ at equilibrium, $p_{Br_2} = \frac{p}{q}$ and p is the total pressure, the ratio $\frac{K_p}{p}$ will be [2012]
 a. $\frac{1}{3}$
 b. $\frac{1}{9}$
 c. $\frac{1}{27}$
 d. $\frac{1}{81}$
6. For the reaction $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$, the unit of K are [2011]
 a. $L \text{ mol}^{-1}$
 b. $L^2 \text{ mol}^{-2}$
 c. mol L^{-1}
 d. No units
7. A sulphuric acid solution has pH = 3. Its normality is [2011]
 a. 1/1000
 b. 1/200
 c. 1/2000
 d. 1/100
8. The pH of 0.01 M HCN solution for which pK_a is 4, is [2011]
 a. 0.47
 b. 1.2
 c. 3.0
 d. 4.0
9. If dissociation constant of ammonia is 10^{-5} , its pK_b and pK_a values respectively are [2010]
 a. 5 and 9
 b. 9 and 5
 c. 7 and 7
 d. 4 and 10
10. $[H^+]$ of 0.2 N CH_3COOH which is 4% dissociated, is [2010]
 a. 0.08 M
 b. 0.12 M
 c. 0.008 M
 d. 0.8 M
11. 50 mL of H_2O is added to 50 mL of 1×10^{-3} M barium hydroxide solution. What is the pH of the resulting solution? [2008]
 a. 3.0
 b. 3.3
 c. 11.0
 d. 11.7
12. **Assertion (A)** The aqueous solution of CH_3COONa is alkaline in nature.
Reason (R) Acetate ion undergoes anionic hydrolysis. [2008]
 a. Both (A) and (R) are true and (R) is the correct explanation of (A)
 b. Both (A) and (R) are true and (R) is not the correct explanation of (A)
 c. (A) is true but (R) is not true
 d. (A) is not true but (R) is true
13. In a 500 mL flask, the degree of dissociation of PCl_5 at equilibrium is 40% and the initial amount is 5 moles. The value of equilibrium constant in mol L^{-1} for the decomposition of PCl_5 is [2008]
 a. 2.33
 b. 2.66
 c. 5.32
 d. 4.66

14. For an equilibrium reaction, $\text{N}_2\text{O}_4(g) \rightleftharpoons 2\text{NO}_2(g)$, the concentrations of N_2O_4 and NO_2 at equilibrium are 4.8×10^{-2} and 1.2×10^{-2} mol/L respectively. The value of K_C for the reaction is [2007]
 a. 3×10^{-3} mol/L b. 3.3×10^{-3} mol/L
 c. 3×10^{-1} mol/L d. 3.3×10^{-1} mol/L
15. In $2\text{HI} \rightleftharpoons \text{H}_2 + \text{I}_2$, the forward reaction is not affected by change in [2006]
 a. catalyst b. pressure
 c. volume d. temperature

16. In which of the following reactions is $K_p < K_C$? [2005]
 a. $\text{I}_2(g) \rightleftharpoons 2\text{I}(g)$
 b. $2\text{BrCl}(g) \rightleftharpoons \text{Cl}_2(g) + \text{Br}_2(g)$
 c. $\text{CO}(g) + 3\text{H}_2(g) \rightleftharpoons \text{CH}_4(g) + \text{H}_2\text{O}(g)$
 d. All of the above
17. The change in pressure will not affect the equilibrium constant for [2005]
 a. $\text{N}_2 + 3\text{H}_2 \rightleftharpoons 2\text{NH}_3$ b. $\text{PCl}_5 \rightleftharpoons \text{PCl}_3 + \text{Cl}_2$
 c. $\text{H}_2 + \text{I}_2 \rightleftharpoons 2\text{HI}$ d. All of these

Answer with Solutions

Practice Exercise

1. (c) Only for this reaction, $\Delta n_g = 2 - 2 = 0$
 Hence, $K_p = K_C (RT)^{\Delta n_g} = K_C (RT)^0 = K_C$
2. (d)
3. (a) Active mass of a pure solid is always taken unity.
4. (a) For $A + 2B \rightleftharpoons 2C$, $K_C = \frac{[C]^2}{[A][B]^2}$... (i)
 For $C \rightleftharpoons B + \frac{1}{2}A$, $K'_C = \frac{[B][A]^{1/2}}{[C]}$... (ii)
 On comparing Eqs. (i) and (ii), we get

$$K'_C = \frac{1}{\sqrt{K_C}} = \frac{1}{\sqrt{40}}$$
5. (b) $2\text{HI} \rightleftharpoons \text{H}_2 + \text{I}_2$
 At equilibrium (1 - 0.5) 0.25 0.25
 $\therefore K_C = \frac{[\text{H}_2][\text{I}_2]}{[\text{HI}]^2} = \frac{0.25 \times 0.25}{(0.5)^2} = \frac{0.25 \times 0.25}{0.25} = 0.25$
6. (a) Rate = $-\frac{1}{2} \frac{d(\text{N}_2\text{O}_4)}{dt} = \frac{1}{4} \frac{d(\text{NO}_2)}{dt}$
 or $\frac{1}{2} K = \frac{1}{4} K'$
 $\Rightarrow K' = 2K$
7. (a)
8. (a) Ice has greater volume than water. Hence, equilibrium will shift towards right. (lesser volume).
9. (b) $\text{N}_2 + 3\text{H}_2 \rightleftharpoons 2\text{NH}_3$
 At equilibrium (1 - x) (3 - 3x) 2x
 Initial pressure = 4 atm
 Total moles at equilibrium = (1 - x) + (3 - 3x) + 2x = 4 - 2x
 Equilibrium pressure = 3 atm

$$\frac{n_2}{n_1} = \frac{p_2}{p_1}$$

$$\Rightarrow \frac{4 - 2x}{4} = \frac{3}{4} \Rightarrow x = 0.5$$

$$\therefore \text{Total moles} = (4 - 2x) = 3$$

Hence,
$$p_{\text{NH}_3} = 3 \times \frac{1}{3} = 1$$

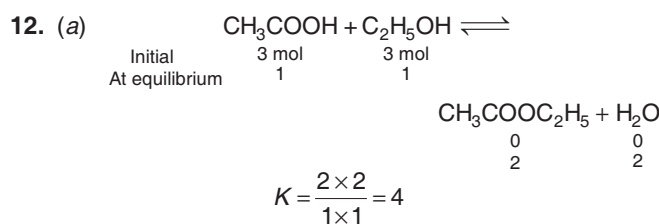
$$p_{\text{N}_2} = 3 \times \frac{1}{6} = 0.5$$

$$p_{\text{H}_2} = 3 \times \frac{1}{2} = 1.5$$

$$K_p = \frac{p_{\text{NH}_3}^2}{p_{\text{N}_2} \cdot p_{\text{H}_2}^3} = \frac{1}{(0.5)(1.5)^3}$$

10. (c) $K = \frac{[C]}{[A][B]} = \frac{\text{mol L}^{-1}}{\text{mol L}^{-1} \times \text{mol L}^{-1}} = (\text{mol L}^{-1})^{-1} = \text{mol}^{-1} \text{L}$

11. (b)



13. (d) $2\text{SO}_2(g) + \text{O}_2(g) \rightleftharpoons 2\text{SO}_3(g)$

For 1 dm³, $R = K[\text{SO}_2]^2 [\text{O}_2]$

$$R = K \left[\frac{1}{1} \right]^2 \left[\frac{1}{1} \right] = 1$$

For 2 dm³, $R = K \left[\frac{1}{2} \right]^2 \left[\frac{1}{2} \right] = \frac{1}{8}$

So, the ratio is 8 : 1.

14. (c) $\text{H}_2(g) + \text{I}_2(g) \rightleftharpoons 2\text{HI}(g)$; $\Delta n_g = 0$

$\therefore$ The reaction and its equilibrium constant are not affected by change in volume.

$$15. (d) \Delta n_g = 1 - \left(1 + \frac{1}{2}\right) = -\frac{1}{2}$$

$$\therefore K_p = K_C (RT)^{\frac{1}{2}} \text{ or } \frac{K_p}{K_C} = (RT)^{-\frac{1}{2}}$$



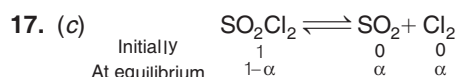
$$\text{Initially} \quad 1 \quad 0$$

$$\text{At equilibrium} \quad 1 - \alpha \quad 2\alpha$$

$$\text{Total moles} = 1 + \alpha$$

$$p_A = \frac{(1 - \alpha) \cdot p}{1 + \alpha}, \text{ and } p_B = \frac{2\alpha}{1 + \alpha} \cdot p$$

$$K_p = \frac{p_B^2}{p_A} = \frac{\left(\frac{2\alpha}{1 + \alpha} \cdot p\right)^2}{\left[\frac{1 - \alpha}{1 + \alpha} p\right]} = \frac{4\alpha^2 p}{1 - \alpha^2}$$



$$\text{At equilibrium} \quad 1 - \alpha \quad \alpha \quad \alpha$$

$$\text{Total moles} = 1 + \alpha$$

$$p_{\text{SO}_2\text{Cl}_2} = \frac{1 - \alpha}{1 + \alpha}, p_{\text{SO}_2} = \frac{\alpha}{1 + \alpha}, p_{\text{Cl}_2} = \frac{\alpha}{1 + \alpha}$$

$$K_p = \left(\frac{\alpha}{1 + \alpha}\right)^2 \bigg/ \left(\frac{1 - \alpha}{1 + \alpha}\right) = \frac{\alpha^2}{1 - \alpha^2}$$

$$= \alpha^2; (1 - \alpha^2 = 1)$$

$$\alpha = \sqrt{K_p} = \sqrt{2.9 \times 10^{-2}} = 0.17$$

$$\therefore \text{Degree of dissociation} = 17\%$$



$$\text{Initially} \quad 600 \text{ mm} \quad 0 \quad 0$$

$$\text{At equilibrium} \quad 600 - p \quad p \quad p$$

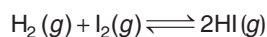
$$\text{Total pressure} = 600 - p + p + p = 600 + p$$

$$\text{or } 600 + p = 800 \text{ mm} \Rightarrow p = 200 \text{ mm}$$

$$\text{and, } p \text{ due to } \text{XY}_2 = 400 \text{ mm}$$

$$K = \frac{200 \times 200}{400} = 100$$

19. (d) Equilibrium constant for the reaction,



$$\text{as, } K_C = \frac{[\text{HI}]^2}{[\text{H}_2][\text{I}_2]} = x$$

The equilibrium constant for the reverse reaction, $2\text{HI}(g) \rightleftharpoons \text{H}_2 + \text{I}_2(g)$, at the same temperature is

$$K'_C = \frac{[\text{H}][\text{I}]_2}{[\text{HI}]^2} = \frac{1}{x} = \frac{1}{K_C}$$

$$\text{Thus, } K'_C = \frac{1}{K_C}$$

$$20. (d) \Delta n_g = 2 - 4 = -2$$

$$K_C = \frac{K_p}{(RT)^{\Delta n}} = \frac{144 \times 10^{-5}}{(0.082 \times 773)^{-2}}$$

21. (d) The equilibrium constant K_C for the reaction can be written as,

$$K_C = \frac{[\text{PCl}_3][\text{Cl}_2]}{[\text{PCl}_5]} = \frac{(1.59)^2}{(1.41)} = 1.79$$

22. (a)

23. (d) This is a heterogeneous equilibrium. Hence, equilibrium constant for it is

$$K_C = [\text{CO}_2]$$

Hence, CaCO_3 has no effect on equilibrium concentration.

24. (b) A reaction which is in equilibrium will shift in reverse or backward direction, when we give heat to the solid, quantity of solid will increase.

25. (c)

26. (c)

27. (d) Among these in BI_3 , $p\pi - p\pi$ back-bonding is least effective as it occurs between $5p - 2p$ (B) orbitals. Hence, BI_3 is the most acidic.

28. (c)

29. (a) $\text{ClO}^- > \text{ClO}_2^- > \text{ClO}_3^- > \text{ClO}_4^-$

30. (d) F^- , HSO_4^- and CO_3^{2-} respectively.

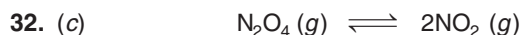
31. (a) The reaction is $\text{N}_2\text{O}_4 \rightleftharpoons 2\text{NO}_2$.

$$\text{Molecular weight of } \text{N}_2\text{O}_4 = 14 \times 2 + 16 \times 4 = 92$$

$$\therefore \text{Vapour density (D) of } \text{N}_2\text{O}_4 = 92 / 2 = 46$$

$$\alpha = \frac{D - d}{(n - 1)d} = \frac{46 - 30}{(2 - 1)30} = \frac{16}{30}$$

$$\therefore \alpha = 0.533 = 53.3\%$$



$$\text{At equilibrium} \quad (1 - 0.2) = 0.8 \quad 2 \times 0.2 = 0.4$$

$$\text{Total moles at equilibrium} = 0.8 + 0.4 = 1.2$$

$$\text{Total pressure} = 1 \text{ atm}$$

$$\text{Hence, } p_{\text{N}_2\text{O}_4} = \frac{0.8}{1.2} = \frac{2}{3}$$

$$\text{and } p_{\text{NO}_2} = \frac{0.4}{1.2} = \frac{1}{3}$$

$$K_p = \frac{p_{\text{NO}_2}}{p_{\text{N}_2\text{O}_4}} = \frac{(1/3)^2}{2/3} = \frac{1}{9} \times \frac{3}{2} = \frac{1}{6}$$



$$\text{Initially} \quad 0.1 \quad 0 \quad 0$$

$$\text{At equilibrium} \quad (0.1 - \alpha) \quad \alpha \quad \alpha$$

$$\therefore K = \frac{[\text{H}^+][\text{CN}^-]}{[\text{HCN}]} = \frac{\alpha \cdot \alpha}{(0.1 - \alpha)}$$

$$\text{As, } \alpha \ll 0.1$$

$$K = \frac{10^{-5} \times 10^{-5}}{0.1} \quad \left(\alpha = 0.1 \times \frac{0.01}{100} \right)$$

$$K = 10^{-9}$$

34. (a) Hg_2^{2+} and Pb^{2+} have been placed in 1st group as their K_{sp} are less than the IIInd group ions (Hg^{2+} , Cd^{2+}).

Hence, only these will precipitate.

35. (d)

36. (d) 50 mL of 0.1 M NaOH $\approx$ 25 mL of 0.2 M HCl

Since, the resultant solution will have volume = 100 mL

$$\text{Molarity} = \frac{25 \times 0.2}{100} = 0.05$$

$$\text{pH} = -\log [\text{H}^+] = -\log 0.05 = 1.3010$$

37. (a) $\text{CH}_3\text{COOH} \rightleftharpoons \text{CH}_3\text{COO}^- + \text{H}^+$

$$K_a = \frac{[\text{CH}_3\text{COO}^-][\text{H}^+]}{[\text{CH}_3\text{COOH}]}$$

$$[\text{CH}_3\text{COOH}] = C - x \approx C = 0.1 \text{ M}$$

$$[\text{CH}_3\text{COO}^-] = 0.2 + x \approx 0.2 \text{ M}$$

$$[\text{H}^+] = \frac{K_a [\text{CH}_3\text{COOH}]}{[\text{CH}_3\text{COO}^-]} = \frac{1.8 \times 10^{-5} \times 0.1}{0.2}$$

$$[\text{H}^+] = 9 \times 10^{-6}$$

38. (b) $\text{HA} \rightleftharpoons \text{H}^+ + \text{A}^-$

$$K_a = \frac{[\text{H}^+][\text{A}^-]}{[\text{HA}]} = \frac{[\text{H}^+]^2}{[\text{HA}]}$$

$$[\text{H}^+] = \sqrt{K_a [\text{HA}]} = \sqrt{1 \times 10^{-5} \times 0.1}$$

$$= 1 \times 10^{-3}$$

$$\alpha = \frac{\text{Actual ionisation}}{\text{Molar concentration}} = \frac{10^{-3}}{0.1} = 10^{-2}$$

$$\% \text{ of acid dissociated} = 10^{-2} \times 100 = 1\%$$

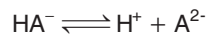
39. (b) For acid-base reactions, g-equivalent of acid (HCl) = g-equivalent of base (NaOH)

$$\therefore \text{g-equivalent of acid} = (25 \times 10^{-3}) \text{ dm}^3 \times \frac{1}{10}$$

$$= 0.0025 = \text{g-equivalent of (NaOH)}$$

40. (d) $\text{H}_2\text{A} \rightleftharpoons \text{H}^+ + \text{HA}^-$

$$K_1 = \frac{[\text{H}^+][\text{HA}^-]}{[\text{H}_2\text{A}]} = 1.0 \times 10^{-5}$$



$$K_2 = \frac{[\text{H}^+][\text{A}^{2-}]}{[\text{HA}^-]} = 5.0 \times 10^{-10}$$

$$K_{\text{overall}} = \frac{[\text{H}^+]^2 [\text{A}^{2-}]}{[\text{H}_2\text{A}]} = K_1 \times K_2$$

$$= 1.0 \times 10^{-5} \times 5.0 \times 10^{-10}$$

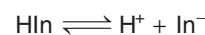
$$= 5.0 \times 10^{-15}$$

41. (c) $\text{HA} \rightleftharpoons \text{H}^+ + \text{A}^-$

$$[\text{H}^+] = 10^{-3} \text{ M}, [\text{A}^-] = 10^{-3} \text{ M}$$

Hence, $K_a = \frac{[\text{H}^+][\text{A}^-]}{[\text{HA}]} = \frac{10^{-3} \times 10^{-3}}{0.1} = 10^{-5}$

42. (a) An acid indicator dissociates at equilibrium as



$$\therefore K_{\text{In}} = \frac{[\text{H}^+][\text{In}^-]}{[\text{HIn}]}$$

At the mid point, $[\text{In}^-] = [\text{HIn}]$

$$\therefore K_{\text{In}} = [\text{H}^+] = 1 \times 10^{-5}$$

$$\therefore [\text{H}^+] = 1 \times 10^{-5} \text{ or } \text{pH} = 5$$

43. (d) As the solution is acidic, $\text{pH} < 7$. This is because $[\text{H}^+]$ from H_2O [10^{-7} M] cannot be neglected in comparison to 10^{-9} M .

44. (b) The equilibrium constant for the reaction $\text{N}_2(\text{g}) + 3\text{H}_2(\text{g}) \rightleftharpoons 2\text{NH}_3(\text{g})$ can be written as

$$K_c = \frac{[\text{NH}_3(\text{g})]^2}{[\text{N}_2(\text{g})][\text{H}_2(\text{g})]^3}$$

$$= \frac{(1.2 \times 10^{-2})^2}{(1.5 \times 10^{-2})(3.0 \times 10^{-2})^3}$$

$$= 0.0355 \times 10^4 = 3.55 \times 10^2$$

$$= 3.98 \times 10^2$$

45. (c) $\Delta G = -2.303 RT \log K$; $-nFE^\circ = -2.303 RT \log K$

$$\log K = \frac{nFE^\circ}{2.303 RT} = 0.4342 \frac{nFE^\circ}{RT} \quad \dots(\text{v})$$

$$\ln K = \frac{nFE^\circ}{RT}$$

$$K = \frac{-nFE^\circ}{e^{RT}} \quad \dots(\text{ii})$$

46. (d) $\text{CH}_3\text{COOH} \rightleftharpoons \text{CH}_3\text{COO}^- + \text{H}^+$

Initially	C	0	0
At equilibrium	$C(1 - \alpha)$	$C\alpha$	$C\alpha$

$$K = \frac{C\alpha \cdot C\alpha}{C(1 - \alpha)} = \frac{C\alpha^2}{1 - \alpha} = C\alpha^2$$

or $C = \frac{K}{\alpha^2} = \frac{1.8 \times 10^{-5}}{(0.02)^2} = 0.045 \text{ M}$

47. (c) Or $\alpha = \frac{K_a}{C} = \frac{1.69 \times 10^{-5}}{0.01} = 0.169 \times 10^{-2}$

48. (d) $\text{pH} = -\log [3.8 \times 10^{-3}] = 2.42$

Therefore, the pH of the soft drink is 2.42 and it is acidic.

49. (c) $\text{pH} = 7 + \frac{1}{2} [\text{p}K_a - \text{p}K_b] = 7 + \frac{1}{2} [4.76 - 4.75]$
- $$= 7.005$$

50. (d) $\text{HAc}(\text{aq}) \rightleftharpoons \text{H}^+(\text{aq}) + \text{Ac}^-(\text{aq})$

Initial conc. (M)	0.05	0	0.05
At equilibrium	$0.05 - x$	x	$0.05 + x$

concentration (M)

$$K_a = \frac{[\text{H}^+][\text{Ac}^-]}{[\text{HAc}]} = \frac{[x](0.05 + x)}{(0.05 - x)}$$

As K_a is small for a very weak acid, $x \ll 0.05$

Hence, $(0.05 + x) \approx (0.05 - x) \approx 0.05$

$$\begin{aligned}\text{Thus, } 1.8 \times 10^{-5} &= \frac{(x)(0.05 + x)}{(0.05 - x)} = \frac{x(0.05)}{(0.05)} = x = [\text{H}^+] \\ &= 1.8 \times 10^{-5} \text{ M} \\ \text{pH} &= -\log(1.8 \times 10^{-5}) = 4.74\end{aligned}$$

51. (a) By Henderson equation

$$\text{pH} = \text{p}K_a + \log \frac{[\text{salt}]}{[\text{acid}]}$$

52. (a) $\text{pH} = \text{p}K_a + \log \frac{[\text{salt}]}{[\text{acid}]} = 4.76 + \log \frac{0.4}{4 \times 10^{-3}} = 6.76$

53. (b) $K_{p_1} = \frac{p_{\text{NH}_3}^2}{p_{\text{N}_2} \cdot p_{\text{H}_2}^3} \quad \dots(i)$

$$K_{p_2} = \frac{p_{\text{NH}_3}}{p_{\text{N}_2}^{1/2} \cdot p_{\text{H}_2}^{3/2}}$$

$$K_{p_2}^2 = \frac{p_{\text{NH}_3}^2}{p_{\text{N}_2} \cdot p_{\text{H}_2}^3} \quad \dots(ii)$$

On dividing Eq.(ii) by Eq(i)

$$\frac{K_{p_1}}{K_{p_2}^2} = 1 \text{ or } K_{p_1} = K_{p_2}^2$$

54. (c) $\text{Ba}(\text{NO}_3)_2$ gives Ba^{2+} and NO_3^- ions, hence, concentration of Ba^{2+} ion increases. To keep K_{sp} constant, $[\text{F}^-]$ decreases.

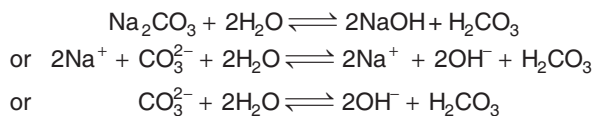
Thus, it is represented as $\frac{1}{2}[\text{F}^-]$.

55. (c) For anionic hydrolysis,

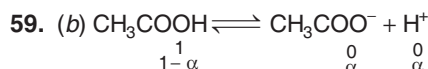
$$\text{pH} = \frac{1}{2}\text{p}K_w + \frac{1}{2}\text{p}K_a + \frac{1}{2}\log C$$

56. (c) NaBrO_3 gives BrO_3^- ions. Hence, $[\text{BrO}_3^-]$ increases. To keep K_{sp} constant, $[\text{Ag}^+]$ decreases.

57. (d) Anionic hydrolysis means anion reacts with water.



58. (d)



$$\text{p}K_a = -\log K_a = 4.74$$

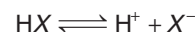
$$\therefore K_a = 1.82 \times 10^{-5}$$

From, $K_a = \frac{C\alpha^2}{(1-\alpha)} = C\alpha^2 \quad (\because 1-\alpha = 1)$

$$\begin{aligned}\alpha &= \sqrt{\frac{K_a}{C}} = \sqrt{\frac{1.82 \times 10^{-5}}{0.05}} \\ &= 0.019 \text{ or } 1.9\%\end{aligned}$$

60. (a) $\text{X}^- + \text{H}_2\text{O} \rightleftharpoons \text{OH}^- + \text{HX}$

$$K_b = \frac{[\text{OH}^-][\text{HX}]}{[\text{X}^-]}$$



$$K_a = \frac{[\text{H}^+][\text{X}^-]}{[\text{HX}]}$$

$$\begin{aligned}\therefore K_a \times K_b &= [\text{H}^+][\text{OH}^-] \\ &= K_w = 10^{-14}\end{aligned}$$

Hence, $K_a = 10^{-4}$

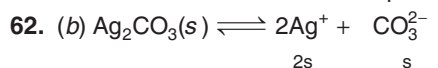
Now, as $[\text{X}^-] = [\text{HX}]$, $\text{pH} = \text{p}K_a = 4$

61. (d) Buffer capacity of an acidic buffer is maximum when the ratio of HA to A^- is unity since, pH of acidic buffer

$$= \text{p}K_a + \log \frac{[\text{A}^-]}{[\text{HA}]}$$

For maximum buffer capacity, $[\text{A}^-] = [\text{HA}]$

$$\therefore \text{pH} = \text{p}K_a$$

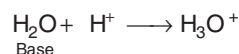
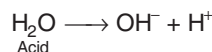
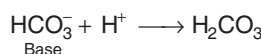
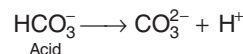


$$K_{\text{sp}} = [\text{Ag}^+]^2[\text{CO}_3^{2-}]$$

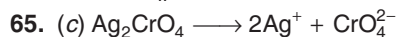
$$= (2\text{S})^2 \text{S}$$

$$K_{\text{sp}} = 4\text{S}^3$$

63. (b) Amphoteric behaviour is that any species which acts as an acid as well as base.



64. (b) On increasing temperature, K_w of water increases and hence $\text{p}K_w$ decreases.



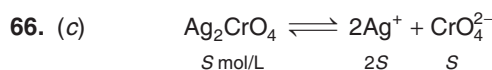
$$[\text{CrO}_4^{2-}] = \frac{1}{2}[\text{Ag}^+]$$

$$= \frac{1.5 \times 10^{-4}}{2} = 0.75 \times 10^{-4}$$

$$K_{\text{sp}} = [\text{Ag}^+]^2[\text{CrO}_4^{2-}]$$

$$= (1.5 \times 10^{-4})^2 (0.75 \times 10^{-4})$$

$$= 1.6875 \times 10^{-12}$$



$$K_{\text{sp}} = [\text{Ag}^+]^2[\text{CrO}_4^{2-}]$$

$$= (2\text{S})^2 (\text{S}) = 4\text{S}^3$$

BITSAT Archives

1. (b) Since, at equivalence point

$$N_1 V_1 = N_2 V_2$$

(for acid) (for base)

$$\therefore \text{Volume of NaOH required to reach equivalence point} = \frac{0.1 \times 25}{0.05} = 50 \text{ mL}$$

$$\therefore \text{Concentration of salt formed} = \frac{\text{Millimolars of acid}}{\text{Total volume in mL}} = \frac{25 \times 0.1}{75} = \frac{0.1}{3}$$

$$\text{Since, } [H^+] = \sqrt{\frac{K_w \times K_a}{C}} = \sqrt{\frac{10^{-14} \times 1.8 \times 10^{-5} \times 3}{0.1}}$$

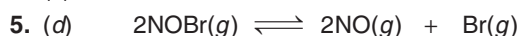
$$\therefore \text{pH} = 8.63$$

2. (a)

3. (a) $K_p = K_c (RT)^{\Delta n}; \Delta n = 1$

$$K_c = \frac{K_p}{RT} = \frac{0.1}{0.082 \times 300} = 4 \times 10^{-3}$$

4. (c)

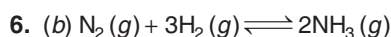


$$p - \left(\frac{2p}{9} + \frac{p}{9} \right) = \frac{2p}{9} - \frac{p}{9}$$

$$= \frac{6p}{9} - \frac{p}{9} = \frac{5p}{9}$$

$$\text{and } K_p = \frac{(p_{\text{NO}})^2 \times (p_{\text{Br}_2})}{(p_{\text{NOBr}})^2} = \frac{\left(\frac{2p}{9} \right)^2 \times \left(\frac{p}{9} \right)}{\left(\frac{6p}{9} \right)^2} = \frac{p}{81}$$

$$\therefore \frac{K_p}{p} = \frac{1}{81}$$



$$K = \frac{[\text{NH}_3]^2}{[\text{N}_2][\text{H}_2]^3} = \frac{\left[\frac{\text{mol}}{\text{L}} \right]^2}{\left[\frac{\text{mol}}{\text{L}} \right] \left[\frac{\text{mol}}{\text{L}} \right]^3} = \left[\frac{\text{mol}}{\text{L}} \right]^{-2} = \text{L}^2 \text{ mol}^{-2}$$

7. (a) pH = 3

$$[H^+] = 1 \times 10^{-3} \text{ mol/L}$$

$$\text{H}_2\text{SO}_4 \longrightarrow 2\text{H}^+ + \text{SO}_4^{2-}$$

$$[\text{H}_2\text{SO}_4] = \frac{1 \times 10^{-3}}{2} = \frac{1}{2000} \text{ M}$$

$$N = 2 \text{ M}$$

$$\text{Normality} = \frac{2}{2000} = \frac{1}{1000} \text{ N}$$

(for H_2SO_4)

8. (c) Given,
- $\text{p}K_a = 4$

$$\therefore K_a = 1 \times 10^{-4}$$

$$[H^+] = \sqrt{K_a \times C} = \sqrt{1 \times 10^{-4} \times 0.01}$$

$$= \sqrt{10^{-6}} = 10^{-3} \text{ M}$$

$$\text{pH} = -\log [H^+] = -\log 10^{-3} = 3$$

9. (a)
- K_b
- of
- NH_3
- is
- 10^{-5}
- . (given)

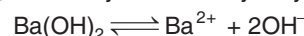
$$\therefore \text{p}K_b = 5 \text{ and } \text{p}K_a = 14 - 5 = 9$$

Hence, $\text{p}K_b$ and $\text{p}K_a$ values are 5 and 9 respectively.

10. (c)
- $\alpha = 4\% = \frac{4}{100} = 0.04$
- , Normality = 0.2

$$\therefore [H^+] = \alpha \times N = 0.04 \times 0.2 = 0.008 \text{ M}$$

11. (c) In water, barium hydroxide is hydrolysed as follows:



$$\text{Concentration of } [\text{OH}^-] = 2 \times 1 \times 10^{-3} \text{ M} = 2 \times 10^{-3} \text{ M}$$

$$\text{pOH} = -\log [\text{OH}^-] = -\log (2 \times 10^{-3}) = 2.69$$

$$\text{pH} + \text{pOH} = 14$$

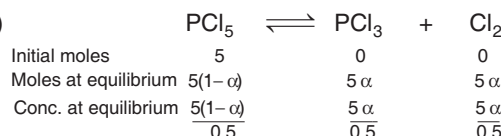
$$\text{pH} = 14 - \text{pOH} = 14 - 2.69 = 11.3 \approx 11.0$$

12. (a)
- $\text{CH}_3\text{COONa} \xrightleftharpoons[\text{in aqueous solution}]{\text{ionisation}} \text{CH}_3\text{COO}^- + \text{Na}^+$
-
- Strong base



Acetate ion undergoes anionic hydrolysis and the resulting solution is slightly basic due to excess of OH^- ions. Hence, both (A) and (R) are true and (R) is the correct explanation of (A).

13. (b)



$$\alpha = 40\% = 0.4$$

$$K_c = \frac{[\text{PCl}_3][\text{Cl}_2]}{[\text{PCl}_5]} = \frac{\left(\frac{5 \times 0.4}{0.5} \right) \left(\frac{5 \times 0.4}{0.5} \right)}{\left(\frac{5 \times 0.6}{0.5} \right)} = 2.66 \text{ mol/L}$$

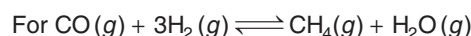
14. (a) According to law of active mass

$$K_c = \frac{[\text{NO}_2]^2}{[\text{N}_2\text{O}_4]} = \frac{[1.2 \times 10^{-2}]^2}{4.8 \times 10^{-2}}$$

$$= 0.3 \times 10^{-2} = 3 \times 10^{-3} \text{ mol/L}$$

15. (a)

16. (c) We have,
- $K_p = K_c (RT)^{\Delta n}$



$$\Delta n = -2$$

$$K_p = \frac{K_c}{(RT)^2}$$

$$K_p < K_c$$

 $\therefore$

17. (d)

9

Chemical Kinetics

Rate of a Reaction

The rate or speed or velocity of a reaction is the rate of change of concentration of reactants or products in unit time. Consider the following reaction,



Rate of reaction can be defined in two ways :

(i) Average rate of reaction, $r_{\text{av}} = -\frac{\Delta[A]}{\Delta t} = \frac{\Delta[B]}{\Delta t}$

(ii) Instantaneous rate of reaction (r_{inst}) can be calculated from r_{av} in the limit $\Delta t \rightarrow 0$ and is represented as

$$r_{\text{inst}} = -\frac{d[A]}{dt} = \frac{d[B]}{dt}$$

In general for a reaction $aA + bB \longrightarrow cC + dD$,

$$\text{Rate} = -\frac{1}{a} \frac{d[A]}{dt} = -\frac{1}{b} \frac{d[B]}{dt} = \frac{1}{c} \frac{d[C]}{dt} = \frac{1}{d} \frac{d[D]}{dt}$$

The rate of chemical reaction depends upon a number of factors such as:

- | | |
|----------------------------|---------------------------|
| (i) Concentration | (ii) Nature of reactants |
| (iii) Temperature | (iv) Presence of catalyst |
| (v) Exposure to radiations | |

Rate Law and Rate Constant

According to the law of mass action, the rate of a chemical reaction is directly proportional to the product of effective concentrations of reacting species, each raised to a suitable power may or may not be equal to the respective stoichiometric coefficients.

For a general reaction, $aA + bB \longrightarrow \text{products}$

$$\text{Rate} = k[A]^a[B]^b$$

where, k is rate constant or velocity constant or specific reaction rate.

The above expression is called **rate law** as it describes the functional dependence of the reaction rate upon concentration of various reactants. Rate law cannot be deduced from balanced equation. It is obtained experimentally.

Molecularity of a Reaction

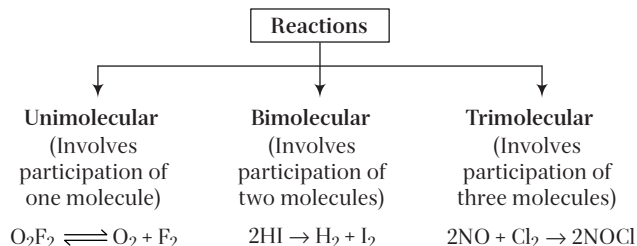
The number of atoms, ions or molecules colliding with each other simultaneously in a single or elementary step of a chemical reaction, is known as molecularity of a reaction.

NOTE

Molecularity of a reaction — is always a whole number.
— can never be zero or fractional.
— does not tell about the rate or mechanism of the reaction.

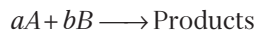
Classification of the Reactions Based on Molecularity

The reaction can be classified on the basis of the number of molecules participating in the reaction.



Order of a Reaction

The sum of the powers of the concentration of the reactants contained in the rate law expression is called order of that chemical reaction. Consider the following reaction,



$$\text{Rate} = k[A]^a [B]^b$$

Order of the reaction = $a + b$

Order of a reaction is an experimental entity.

It may be 0, 1, 2, 3 and even a fraction.

Units of Rate Constant for Different Order of Reactions

The unit of rate constant for different order of reactions can be given by a generalised formula

$$\text{Unit of } k = (\text{mol L}^{-1})^{1-n} \text{ s}^{-1} \quad [\because n = \text{order of a reaction}]$$

e.g. (a) $n = 0$, unit of $k = \text{mol L}^{-1} \text{ s}^{-1}$

(b) $n = 1$, unit of $k = \text{s}^{-1}$

(c) $n = 2$, unit of $k = \text{mol}^{-1} \text{ L s}^{-1}$

(d) $n = 3$, unit of $k = \text{mol}^{-2} \text{ L}^2 \text{ s}^{-1}$

Integrated Rate Expressions

(i) Zero order reaction

(a) The reaction in which the rate of a reaction is independent of the concentration of the reactants is called zero order reaction.

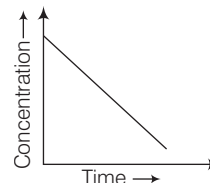
(b) For a zero order reaction,

$$\text{or} \quad k = \frac{1}{t} \{ [A]_0 - [A] \}$$

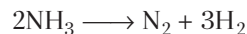
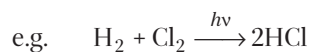
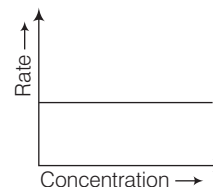
$$\text{Rate} = - \frac{\Delta[R]}{\Delta t} = k$$

$$\text{or,} \quad k = \frac{1}{t} \{ [A]_0 - [A] \}$$

(c) The graph between the concentration of reactants and time is a straight line with slope = $-k$



(d) The graph between the concentration and rate of the reaction is shown as follows



(ii) First order reaction

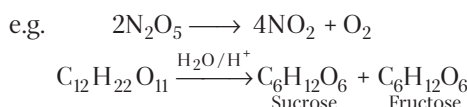
(a) The reaction in which the rate of a reaction depends upon the concentration of one reactant is called first order reaction.

(b) The rate constant for first order reaction is given by

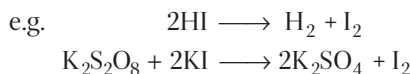
$$k = \frac{2.303}{t} \log_{10} \frac{a}{a-x}$$

where, a = initial concentration of a reactant
($a - x$) = concentration of reactant left after time t .

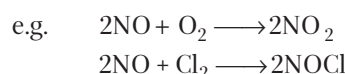
A graph showing the relationship between $\log_{10}\left(\frac{a}{a-x}\right)$ on the vertical axis and Time on the horizontal axis. The graph is a straight line passing through the origin, indicating a linear relationship.


$$k = \frac{2.303}{t(a-b)} \log_{10} \frac{b(a-x)}{a(b-x)}$$

A graph showing the relationship between the reciprocal of the concentration of reactant A, $\frac{1}{[A]}$, and Time. The y-axis is labeled $\frac{1}{[A]}$ and the x-axis is labeled Time. The plot is a straight line with a positive slope, indicating a first-order reaction.


$$k = \frac{1}{t} \cdot \frac{x(2a - x)}{2a^2(a - x)^2}$$

A graph showing the relationship between the reciprocal of the square of the concentration of A, $\frac{1}{[A]^2}$, and Time. The y-axis is labeled $\frac{1}{[A]^2}$ and the x-axis is labeled Time. A straight line starts from the origin (0,0) and extends upwards and to the right, indicating a linear relationship between $\frac{1}{[A]^2}$ and Time.

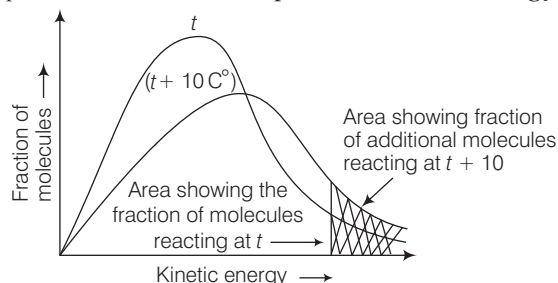

$$t_{1/2} \propto \frac{1}{[A_0]^{n-1}}$$

For first order reaction, $t_{1/2} = \frac{0.693}{k}$

$$t_{87.5\%} = 3t_{1/2}$$
$$t_{1/2n} = \frac{2.303}{k} \log \frac{a}{a-x}$$

When a graph is drawn between kinetic energy and fraction of molecules, a peak of curve is obtained which represents the kinetic energy possessed by the maximum number of molecules in the reaction.

This peak of curve is called **probable kinetic energy**.



When temperature rises to $(t + 10^\circ\text{C})$, the curve broadens and proportion of molecules with higher energies increases. This leads to an increase in the number of effective collisions and hence the rate of the reaction increases.

Arrhenius Equation

The effect of temperature on the rate of a reaction and rate constant (k) was given by Arrhenius in terms of

$$k = Ae^{-E_a/RT}$$

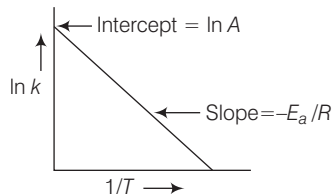
where, A = Arrhenius factor or pre-exponential factor or frequency factor

E_a = activation energy, R = gas constant

The term $e^{-E_a/RT}$ provides the fraction of molecules having energy equal to or greater than the activation energy. Taking natural logarithm of Arrhenius equation,

$$\ln k = \ln A - \frac{E_a}{RT}$$

When a graph is plotted between $\ln k$ and $1/T$, a straight line is obtained.



If at temperature T_1 , the value of rate constant is k_1 and at T_2 , the value of rate constant is k_2 , hence

$$\ln \frac{k_2}{k_1} = \frac{E_a}{R} \left[\frac{T_2 - T_1}{T_1 T_2} \right]$$

or

$$\log \frac{k_2}{k_1} = \frac{E_a}{2.303R} \left[\frac{T_2 - T_1}{T_1 T_2} \right]$$

This equation is used only when rate constants at two different temperatures are provided.

Activation Energy

The minimum amount of energy absorbed by the reactant molecules such that this energy becomes equal to threshold energy, thereby undergoing effective collisions for a reaction to take place is called activation energy.

Mathematically,
Activation energy

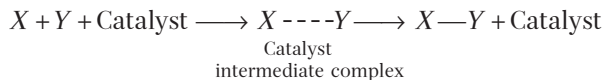
= threshold energy – average energy of the reactants

Hence, lower the activation energy, faster is the reaction and *vice-versa*.

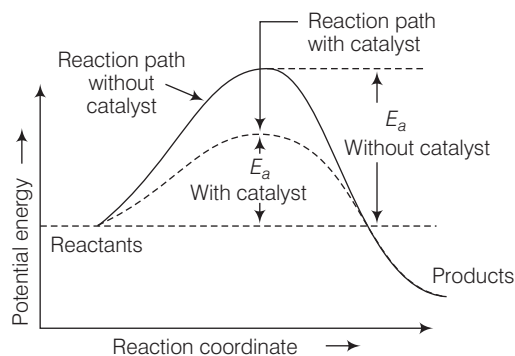
Effect of Catalyst on the Rate of Reaction

The substance which alters the rate of reaction without itself undergoing any chemical change is called **catalyst**.

According to this theory of intermediate complex formation, reactants combine with catalyst through temporary bonds to form an intermediate complex which is short-lived and decomposes to give products and catalyst.



Catalyst provides an alternate pathway by lowering the potential energy barrier through the reduction of activation energy between the reactants and products. Hence, catalyst increases the rate of a reaction.



NOTE Catalyst neither alter the Gibbs free energy change $[\Delta G]$ of the reaction or the enthalpy change (ΔH) of the reaction.

Collision Theory of Bimolecular Gaseous Reactions

This theory is based upon the kinetic theory of gases assuming that the reactant molecules are hard spheres.

The main postulates of this theory are as follows:

- (i) The reaction between the molecules, occurs only when they collide with each other in a proper orientation.
- (ii) Only those collisions result in chemical reaction in which the molecules acquire energy greater than the activation energy.
- (iii) For an effective collision, the colliding molecules must possess threshold energy.

- (iv) An activation energy is supplied to the molecules for attaining this threshold energy. During collisions, this energy is acquired by the molecules as a result of interchange of energies. The rate of a reaction in terms of collision frequency is given by

$$\text{Rate} = Z_{AB} e^{-E_a/RT}$$

where, Z_{AB} refers to the collision frequency of reactants A and B , $e^{-E_a/RT}$ represents the fraction of molecules with energy either equal to or greater than E_a .

For an effective collision, probability or steric factor is introduced in the above equation for proper orientation of colliding molecules.

$$\text{Rate} = pZ_{AB}e^{-E_a/RT}$$

where, p = probability or steric factor or orientation factor

Other Important Terms and Relations

- Temperature coefficient** The ratio of rate constants of a reaction at two temperatures (i.e. at 298 K and 308 K) is known as temperature coefficient i.e.

$$\text{Temperature coefficient} = \frac{\text{Rate constant at 308 K}}{\text{Rate constant at 298 K}}$$

- Quantum efficiency of a photochemical reaction

$$\phi = \frac{\text{Number of molecules reacting in a given time}}{\text{Number of quanta absorbed in the same time}}$$

Practice Exercise

- For a chemical reaction $2X + Y \longrightarrow Z$, the rate of appearance of Z is $0.05 \text{ mol L}^{-1} \text{ min}^{-1}$. The rate of disappearance of X will be
 a. $0.05 \text{ mol L}^{-1} \text{ min}^{-1}$ b. $0.05 \text{ mol L}^{-1} \text{ h}^{-1}$
 c. $0.1 \text{ mol L}^{-1} \text{ min}^{-1}$ d. $0.25 \text{ mol L}^{-1} \text{ min}^{-1}$
- A gaseous reaction $A_2(g) \longrightarrow B(g) + \frac{1}{2}C(g)$, shows increase in pressure from 100 mm to 120 mm in 5 min. The rate of disappearance of A_2 is
 a. 4 mm min^{-1} b. 8 mm min^{-1}
 c. 16 mm min^{-1} d. 2 mm min^{-1}
- For the reaction $2N_2O_4 \rightleftharpoons 4NO_2$, given that $\frac{-d[N_2O_4]}{dt} = K$ and $\frac{d[NO_2]}{dt} = K'$, then
 a. $K' = 2K$ b. $K' = K$
 c. $2K' = K$ d. None of these
- The rate constant of a first order reaction is $2.0 \times 10^{-5} \text{ s}^{-1}$ and the initial concentration is 0.10 mol L^{-1} . The initial rate is
 a. $2.0 \times 10^{-6} \text{ mol L}^{-1} \text{ s}^{-1}$ b. $1.0 \times 10^{-6} \text{ mol L}^{-1} \text{ s}^{-1}$
 c. $1.5 \times 10^{-6} \text{ mol L}^{-1} \text{ s}^{-1}$ d. $0.5 \times 10^{-6} \text{ mol L}^{-1} \text{ s}^{-1}$
- The chemical reaction $2O_3 \longrightarrow 3O_2$, proceeds as follows:

$$O_3 \rightleftharpoons O_2 + O \text{ (fast)}$$

$$O + O_3 \longrightarrow 2O_2 \text{ (slow)}$$
 The rate law expression should be
 a. $r = k'[O_3]^2$ b. $r = k'[O_3]^2 [O_2]^{-1}$
 c. $r = k'[O_3][O_2]$ d. Unpredictable
- The rate for the decomposition of NH_3 on platinum surface is zero order. What are the rates of production of N_2 and H_2 respectively, if $k = 2.5 \times 10^{-4} \text{ mol L}^{-1} \text{ s}^{-1}$?
 a. $1.25 \times 10^{-4} \text{ and } \text{mol L}^{-1} \text{ s}^{-1}$ and $3.75 \times 10^{-4} \text{ and } \text{mol L}^{-1} \text{ s}^{-1}$
 b. $3.75 \times 10^{-4} \text{ and } \text{mol L}^{-1} \text{ s}^{-1}$ and $1.25 \times 10^{-4} \text{ and } \text{mol L}^{-1} \text{ s}^{-1}$
 c. $2.5 \times 10^{-4} \text{ and } \text{mol L}^{-1} \text{ s}^{-1}$ and $3.75 \times 10^{-4} \text{ and } \text{mol L}^{-1} \text{ s}^{-1}$
 d. $1.25 \times 10^{-4} \text{ and } \text{mol L}^{-1} \text{ s}^{-1}$ and $2.5 \times 10^{-4} \text{ and } \text{mol L}^{-1} \text{ s}^{-1}$
- In a reversible reaction $2NO_2 \xrightleftharpoons[k_2]{k_1} N_2O_4$, the rate of disappearance of NO_2 is equal to
 a. $\frac{2k_1}{k_2} [NO_2]^2$ b. $2k_1[NO_2]^2 - 2k_2[N_2O_4]$
 c. $2k_1[NO_2]^2 - k_2[N_2O_4]$ d. $(2k_1 - k_2) [NO_2]$
- For the reaction $R \longrightarrow P$, the concentration of a reactant changes from 0.03 M to 0.02 M in 25 min. Calculate the average rate of reaction using units of time in seconds.
 a. $6.66 \times 10^{-5} \text{ Ms}^{-1}$
 b. $6.6 \times 10^{-6} \text{ Ms}^{-1}$
 c. $5.67 \times 10^{-5} \text{ Ms}^{-1}$
 d. $7.26 \times 10^{-6} \text{ Ms}^{-1}$
- Rate law for the reaction $A + 2B \longrightarrow C$, is found to be $\text{Rate} = k[A][B]$. If the concentration of reactant B is doubled keeping the concentration of A constant, the value of rate constant will be
 a. the same b. doubled
 c. quadrupled d. halved

10. For the reaction $2\text{H}_2 + \text{O}_2 \longrightarrow 2\text{H}_2\text{O}$, the rate law expression is, $r = k[\text{H}_2]^n$. When the concentration of H_2 is doubled, the rate of reaction found to be quadrupled. The value of n is
 a. 0 b. 1 c. 2 d. 3
11. Consider the following reaction,
 $2\text{N}_2\text{O}_5 \longrightarrow 4\text{NO}_2 + \text{O}_2$; $\frac{d[\text{NO}_2]}{dt} = k_2[\text{N}_2\text{O}_5]$,
 $\frac{d[\text{O}_2]}{dt} = k_3[\text{N}_2\text{O}_5]$ and $\frac{d[\text{N}_2\text{O}_5]}{dt} = k_1$
 The relation between k_1 , k_2 and k_3 is
 a. $k_1 = k_2 = k_3$ b. $2k_1 = k_2 = 4k_3$
 c. $2k_1 = 4k_2 = k_3$ d. None of these
12. The rate of formation of SO_3 in the following reaction is 100 g min^{-1} . $2\text{SO}_2 + \text{O}_2 \longrightarrow 2\text{SO}_3$
 The rate of disappearance of O_2 is
 a. 29 g min^{-1} b. 20 g min^{-1}
 c. 50 g min^{-1} d. 200 g min^{-1}
13. Consider the following reaction,
 $2\text{A} + \text{B} + \text{C} \longrightarrow \text{Products}$
 How will the rate of reaction changes when the concentration of A is doubled and that of B is tripled while C is taken in excess?
 a. The rate reduces 8 times of its original value
 b. The rate reduces 12 times of its original value
 c. The rate increases 8 times of its original value
 d. The rate increases 12 times of its original value
14. In the reaction $2\text{A} \longrightarrow \text{Products}$, the concentration of A decreases from 1.0 mol L^{-1} to 0.8 mol L^{-1} in 20 min. Calculate the rate during this interval.
 a. $0.5 \text{ mol L}^{-1} \text{ min}^{-1}$ b. $0.005 \text{ mol L}^{-1} \text{ min}^{-1}$
 c. $0.05 \text{ mol L}^{-1} \text{ min}^{-1}$ d. $0.0005 \text{ mol L}^{-1} \text{ min}^{-1}$
15. The unit of the rate constant of n th order is
 a. $\text{mol}^{1-n} \text{ L}^{n-1} \text{ s}^{-1}$ b. $\text{mol}^{n-1} \text{ L}^{1-n} \text{ s}^{-1}$
 c. $\text{mol}^{n-1} \text{ L}^{n-1} \text{ s}$ d. $\text{mol}^n \text{ L}^{1-n} \text{ s}^{-1}$
16. In the presence of acid, the initial concentration of cane sugar was reduced from 0.2 M to 0.1 M in 5 h and to 0.05 M in 10 h. The reaction must be of
 a. zero order b. first order
 c. second order d. fractional order
17. The rate constant of a reaction is $3.25 \times 10^{-3} \text{ mol}^{-2} \text{ L}^2 \text{ min}^{-1}$. The order of reaction is
 a. zero b. 1 c. 2 d. 3
18. The half-life period of a first order process is 1.6 min. It will be 90% completed in
 a. 0.8 min b. 3.2 min c. 5.3 min d. 1.6 min
19. The hydrolysis of ester in alkaline medium is a
 a. first order reaction with molecularity 1
 b. second order reaction with molecularity 2
 c. first order reaction with molecularity 2
 d. second order reaction with molecularity 1
20. The rate constant of first order reaction whose half-life is 480 s, is
 a. $1.44 \times 10^{-3} \text{ s}^{-1}$ b. 1.44 s^{-1}
 c. $0.72 \times 10^{-3} \text{ s}^{-1}$ d. $2.88 \times 10^{-3} \text{ s}^{-1}$
21. In accordance to Arrhenius equation, the plot of $\log k$ against $\frac{1}{T}$ is a straight line. The slope of the line is equal to
 a. $\frac{-E_a}{R}$ b. $\frac{-2.303}{E_a \cdot R}$
 c. $\frac{-E_a}{2.303R}$ d. $\frac{-E_a}{2.303}$
22. For the first order reaction, $k = 5.48 \times 10^{-4} \text{ s}^{-1}$, the two-third life time for this reaction is
 a. 2005 s b. 1000 s c. 2000 s d. 3005 s
23. The rate constant for a zero order reaction is
 a. $k = \frac{C_0}{2t}$ b. $k = \frac{C_0 - C_t}{t}$
 c. $k = \ln \frac{C_0 - C_t}{t}$ d. $k = \frac{C_0}{C_t}$
24. The time required for 100% completion of a zero order reaction is
 a. ak b. $\frac{a}{2k}$
 c. $\frac{a}{k}$ d. $\frac{2k}{a}$
25. Pieces of wood burn faster than a log of wood of the same mass because
 a. surface area of log of wood is larger and needs more time to burn
 b. pieces of wood have larger surface area
 c. all pieces of wood catch fire at the same time
 d. log of wood has higher density than pieces of the same wood
26. What is the order of a reaction which has a rate expression, i.e. $\text{rate} = k[\text{A}]^{3/2}[\text{B}]^{-1}$?
 a. $\frac{3}{2}$ b. $\frac{1}{2}$
 c. Zero d. None of these
27. If initial concentration is doubled, the time for half-reaction is also doubled, the order of reaction is
 a. zero b. first c. second d. third
28. If the graph of concentration of $[\text{A}]$ vs T for completion of reaction is a straight line, then the order of reaction is
 a. zero b. second c. first d. third
29. For a reaction, temperature increases by 10°C , the equilibrium will be attained faster
 a. 2 times b. same c. $\frac{1}{2}$ times d. 4 times
30. 75% of a first order reaction was completed in 32 min. When was 50% of the reaction completed?
 a. 16 min b. 24 min c. 8 min d. 4 min

31. The rate of a reaction becomes 4 times when temperature is raised from 293 K to 313 K. The activation energy for such reaction would be
 a. 50.855 kJ mol⁻¹
 b. 52.849 kJ mol⁻¹
 c. 54.855 kJ mol⁻¹
 d. 56.855 kJ mol⁻¹

32. Half-life of a hypothetical reaction is found to be inversely proportional to the cube of initial concentration. The order of reaction is
 a. 4
 b. 3
 c. 5
 d. 2

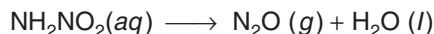
33. H₂O and O-atom react in upper atmosphere bimolecularly to form two OH radicals. ΔH for the reaction is 72 kJ at 500 K and energy of activation is 77 kJ mol⁻¹. E_a for bimolecular recombination of two OH radicals to form H₂O and O-atom, will be
 a. 5 kJ mol⁻¹
 b. 72 kJ mol⁻¹
 c. 77 kJ mol⁻¹
 d. 149 kJ mol⁻¹

34. Calculate the half-life of the first order reaction, $C_2H_4O(g) \longrightarrow CH_4(g) + CO(g)$. If the initial pressure of $C_2H_4O(g)$ is 80 mm and the total pressure at the end of 20 min is 120 mm.
 a. 40 min
 b. 120 min
 c. 20 min
 d. 80 min

35. The half-lives for two samples are 0.1 and 0.8 s whose concentrations are 400 and 50 mol L⁻¹ respectively. The order of the reaction is
 a. 0
 b. 1
 c. 2
 d. 3

36. The rate constant for the first order reaction is 60 s⁻¹. How much time will it take to reduce the concentration of the reactant to 1/16th value?
 a. 4.6×10^4 s
 b. 4.6×10^{-4} s
 c. 4.6×10^{-2} s
 d. 4.6×10^2 s

37. The half-time of the following first order decomposition of nitramide is 2.1 h at 15°C:



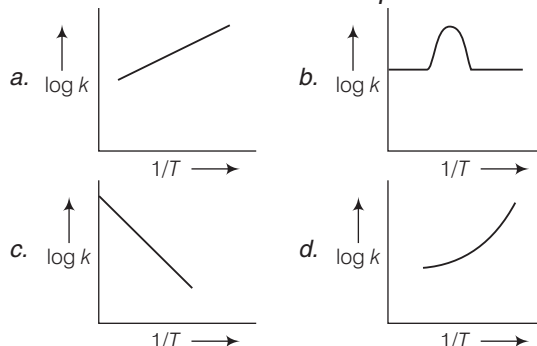
If 6.2 g of nitramide is allowed to decompose then time taken for it to decompose 99%, will be

- a. 2.1 h
 b. 12 h
 c. 13.96 h
 d. 33 h
38. A first order reaction is found to have a rate constant, $k = 4.2 \times 10^{-12} \text{ s}^{-1}$. Find the half-life of the reaction.
 a. $1.26 \times 10^{13} \text{ s}$
 b. $1.65 \times 10^{11} \text{ s}$
 c. $1.65 \times 10^{-11} \text{ s}$
 d. $1.26 \times 10^{-13} \text{ s}$

39. The value of $\log \frac{k_2}{k_1}$ is equal to

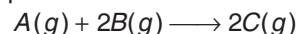
a. $\frac{E_a}{2.303R} \left[\frac{T_2 - T_1}{T_1 T_2} \right]$
 b. $\frac{E_a}{2.303R} \left[\frac{T_1 T_2}{T_2 - T_1} \right]$
 c. $\frac{E_a}{2.303R} \left[\frac{T_2 + T_1}{T_1 T_2} \right]$
 d. $\frac{E_a}{2.303R} \left[\frac{T_1 T_2}{T_1 + T_2} \right]$

40. A graph plotted between $\log k$ vs $\frac{1}{T}$ is represented by



41. Rate of a reaction can be expressed by Arrhenius equation, $k = Ae^{-E_a/RT}$. Here, E is
 a. the total energy of the reacting molecules at a temperature T
 b. the energy above which all the colliding molecules will react
 c. the energy below which colliding molecules will not react
 d. the fraction of molecules with energy greater than the activation energy of the reaction

42. Compounds A and B react according to the following chemical equation:



Concentration of either A or B were changed keeping the concentrations of one of the reactant constants and rates were measured as a function of initial concentration. Following results were obtained.

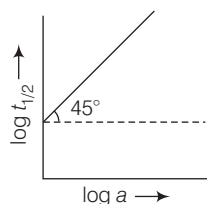
Experiment	Initial concentration of [A] / mol L ⁻¹	Initial concentration of [B] / mol L ⁻¹	Initial rate of formation of [C] / mol L ⁻¹ s ⁻¹
1.	0.30	0.30	0.10
2.	0.30	0.60	0.40
3.	0.60	0.30	0.20

Choose the correct option for the rate equations for this reaction.

- a. Rate = $k[A]^2[B]$
 b. Rate = $k[A][B]^2$
 c. Rate = $k[A][B]$
 d. Rate = $k[A]^2[B]^0$

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1. Following is the graph between $\log T_{50}$ and $\log a$ (a = initial concentration) for a given reaction at 27°C . [2014]



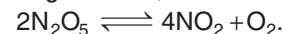
Hence, order is

- a. 1 b. 2 c. 3 d. 0
2. Choose the law that corresponds to data shown for the reaction, $A + B \longrightarrow \text{products}$. [2013]

Experiment	[A]	[B]	Initial rate
1	0.012	0.035	0.1
2	0.024	0.070	0.8
3	0.024	0.035	0.1
4	0.012	0.070	0.8

- a. Rate = $k[B]^3$ b. Rate = $k[B]^4$
 c. Rate = $k[A][B]^3$ d. Rate = $k[A]^3[B]$
3. A reaction takes place in three steps. The rate constants are k_1, k_2 and k_3 . The overall rate constant $k = \frac{k_1 k_3}{k_2}$. If E_1, E_2 and E_3 (energy of activation) are 60, 30 and 10 kJ, respectively, the overall energy of activation is [2013]
- a. 40 kJ b. 30 kJ c. 400 kJ d. 300 kJ

4. Consider the following reaction,



If rate and rate constant for above reaction are $2.40 \times 10^{-5} \text{ mol L}^{-1} \text{ s}^{-1}$ and $3 \times 10^{-5} \text{ s}^{-1}$ respectively, then calculate the concentration of N_2O_5 . [2012]

- a. 1.4 mol L^{-1}
 b. 1.2 mol L^{-1}
 c. 0.04 mol L^{-1}
 d. 0.8 mol L^{-1}
5. For a given reaction, $t_{1/2} = 1/ka$. The order of this reaction is [2011]
- a. 0 b. 1 c. 2 d. 3
6. The time taken for 90% of a first order reaction to be completed is approximately [2011]
- a. 1.1 times that of half-life
 b. 2.2 times that of half-life
 c. 3.3 times that of half-life
 d. 4.4 times that of half-life
7. What is the energy of activation of a reaction if its rate doubles when the temperature is raised from 290 K to 300 K? [2010]
- a. 12 kcal
 b. 15 kcal
 c. 10 kcal
 d. 20 kcal
8. The active mass of solid is generally taken as [2010]
- a. > 1
 b. $= 1$
 c. < 1
 d. 0

Answer with Solutions

Practice Exercise

1. (c) $2X + Y \longrightarrow Z$

Hence, rate expression for the reaction will be

$$-\frac{1}{2} \frac{d[X]}{dt} = \frac{d[Y]}{dt} = + \frac{d[Z]}{dt}$$

$$\therefore -\frac{d[X]}{dt} = 2 \times 0.05 \text{ mol L}^{-1} \text{ min}^{-1}$$

$$= 0.1 \text{ mol L}^{-1} \text{ min}^{-1}$$

2. (b)
- | | | | | | |
|-------------|------------|-------------------|--------|-----|--------------------|
| | $A_2(g)$ | $\longrightarrow$ | $B(g)$ | $+$ | $\frac{1}{2} C(g)$ |
| Initially | 100 mm | | 0 | | 0 |
| After 5 min | (100-p) mm | | p | | $\frac{1}{2}p$ |

$$\text{Total pressure} = 100 - p + p + \frac{1}{2}p = 100 + \frac{1}{2}p$$

$$\therefore \text{Increase in pressure} = \frac{1}{2}p = 120 - 100 = 20 \text{ mm}$$

$$\therefore \text{Decrease in pressure of } A_2 = 2 \times 20 = 40 \text{ mm}$$

$$\therefore \text{Rate of disappearance of } A_2 = \frac{dx}{dt} = \frac{40}{5} = 8 \text{ mm min}^{-1}$$

3. (a) For this reaction,



$$\text{Rate expression} = -\frac{1}{2} \frac{d[N_2O_4]}{dt} = \frac{1}{4} \frac{d[NO_2]}{dt}$$

$$\text{or} \quad -\frac{d[N_2O_4]}{dt} = \frac{1}{2} \frac{d[NO_2]}{dt}$$

$$\text{or} \quad K = \frac{1}{2} K'$$

$$\text{or} \quad K' = 2K$$

4. (a) $\text{Rate} \left(\frac{dx}{dt} \right) = \text{Rate constant } (k) \times \text{Concentration}$

$$= 2.0 \times 10^{-5} \text{ s}^{-1} \times 0.10 \text{ mol L}^{-1}$$

$$= 2.0 \times 10^{-6} \text{ mol L}^{-1} \text{ s}^{-1}$$

5. (b)

6. (a)

7. (b)

8. (b) For the reaction $R \longrightarrow P$,

Average rate of reaction

$$= \frac{\text{Change in concentration of reactant or product}}{\text{Time taken}}$$

$$= \frac{-\Delta[R]}{t} = -\frac{[R_2] - [R_1]}{t} = -\frac{(0.02 - 0.03) \text{ M}}{(25 \times 60) \text{ s}}$$

$$= 6.6 \times 10^{-6} \text{ Ms}^{-1}$$

9. (b) Rate of reactions, $r_1 = k[A][B]$

$$r_2 = k[A][2B]$$

$$r_2 = 2r_1$$

10. (c) $r = k[H_2]^n \quad \dots(i)$

On doubling the concentration of H_2 ,

$$4r = k[2H_2]^n \quad \dots(ii)$$

On dividing Eq. (ii) by Eq. (i), we get,

$$4 = (2)^n$$

$$(2)^2 = (2)^n$$

$$n = 2$$

11. (b) $2N_2O_5 \longrightarrow 4NO_2 + O_2$

$$\text{Rate} = -\frac{1}{2} \frac{d[N_2O_5]}{dt} = \frac{1}{4} \frac{d[NO_2]}{dt} = \frac{d[O_2]}{dt}$$

$$\Rightarrow 2k_1 = k_2 = 4k_3$$

12. (b) $2SO_2 + O_2 \longrightarrow 2SO_3$

$$\text{Rate} = -\frac{d[O_2]}{dt} = \frac{1}{2} \frac{d[SO_3]}{dt}$$

$$-\frac{d[O_2]}{dt} = \frac{1}{2} \left(\frac{100}{80} \right) \text{ mol min}^{-1} = \frac{1}{2} \left(\frac{100}{80} \right) \times 32 \text{ g min}^{-1}$$

$$[\because \text{Molar mass of } O_2 = 32 \text{ g/mol}]$$

$$= 20 \text{ g min}^{-1}$$

13. (d) $2A + B + C \longrightarrow \text{Products}$

$$r_1 = k[A]^2[B]$$

[$\because C$ is taken in excess, rate does not depend on C]

$$r_2 = k[2A]^2[3B]$$

$$\frac{r_2}{r_1} = \frac{12k[A]^2[B]}{k[A]^2[B]} \Rightarrow r_2 = 12r_1$$

14. (b) Rate of reaction = Rate of disappearance of A

$$= -\frac{1}{2} \frac{\Delta(A)}{\Delta t} = -\frac{1}{2} \frac{(0.8 - 1.0) \text{ mol L}^{-1}}{20 \text{ min}}$$

$$= 0.005 \text{ mol L}^{-1} \text{ min}^{-1}$$

15. (a) The units of rate constant of n th order is $\text{mol}^{1-n} \text{ L}^{n-1} \text{ s}^{-1}$

16. (b) For the first order reaction

$$\text{Rate} \propto [A] \Rightarrow \frac{[A]}{\text{Rate}} = \text{Constant } (k)$$

- (i) When $\Delta x = 0.2 - 0.1 = 0.1$, $\Delta t = 5 \text{ h}$, $[A] = 0.2$

$$\text{Hence, } \frac{0.2}{0.1/5} = 10 = k$$

- (ii) When $\Delta x = 0.1 - 0.05 = 0.05$, $\Delta t = 5 \text{ h}$, $[A] = 0.1$

$$\text{Hence, } \frac{0.1}{0.05/5} = 10 = k$$

Hence, reaction follows first order rate law.

17. (d) The unit of rate constant for n th order reaction

$$= (\text{mol})^{1-n} \text{ L}^{n-1} \text{ min}^{-1}$$

On putting $n = 3$, we get the unit of given rate constant.
Hence, the order of reaction is 3.

18. (c) For first order reaction, $t_{1/2} = \frac{0.693}{k}$
 $\Rightarrow k = \frac{0.693}{1.6} = 0.4332 \text{ min}^{-1}$

Now, for 90% completion, $k = \frac{2.303}{t} \log \frac{a}{a-x}$
 $\Rightarrow 0.4332 = \frac{2.303}{t} \log \frac{100}{10} \Rightarrow t = 5.3 \text{ min}$

19. (b) Alkaline hydrolysis of ester (saponification) is a bimolecular and second order reaction.

20. (a) For the first order reaction
 $\Rightarrow k = \frac{0.693}{t_{1/2}} = \frac{0.693}{480 \text{ sec}} = 1.44 \times 10^{-3} \text{ s}^{-1}$

21. (c) Arrhenius equation can be written as

$$\log k = \log A - \frac{E_a}{2.303RT}$$

On comparing this equation with the straight line equation,

$$y = mx + c$$

we get, $y = \log k$, $m = -\frac{E_a}{2.303R}$, $x = \frac{1}{T}$, $C = \log A$

22. (a) For first order reaction, we know that

$$k = \frac{2.303}{t} \log \frac{a}{(a-x)}$$

Hence, $t = \frac{2.303}{5.48 \times 10^{-4}} \times \log \frac{a}{a - \frac{2}{3}a}$
 $= \frac{2.303 \times 10^4}{5.48} \times \log 3$
 $= \frac{2.303 \times 10^4}{5.48} \times 0.4771 = 2005 \text{ s}$

23. (b) For a zero order reaction,

$$C_0 - C_t = kt \Rightarrow k = \frac{C_0 - C_t}{t}$$

24. (c) For a zero order reaction,

$$x = kt$$

For 100% completion of the reaction, $x = a$

Therefore, $a = kt$ or $t = \frac{a}{k}$

25. (b) More the surface area of reactant, more will be the rate of reaction.

26. (b) Order of reaction $= \left(\frac{3}{2}\right) + (-1) = \frac{1}{2}$

27. (a) For n th order reaction, $t_{1/2} \propto a^{1-n}$

Here, $t_{1/2} \propto a$

Hence, $1 - n = 1$ or $n = 0$ (i.e. zero order reaction)

28. (a) In the zero order reaction, concentration decreases linearly with time as the rate of reaction is constant (does not depend on concentration of reactant).

29. (a) Rate of reaction becomes almost twice with an increase of 10°C in temperature (from 298 K to 308 K).

30. (a) For the reaction ,

$$k = \frac{2.303}{t} \log \frac{a}{a-x} = \frac{2.303}{32} \log 4 = 0.04332$$

Hence, time required for 50% completion,

$$t = \frac{2.303}{0.04332} \log \frac{a}{a - \frac{a}{2}}$$

$$= \frac{2.303}{0.04332} \times 0.3010 = 16 \text{ min}$$

31. (b) From the Arrhenius equation,

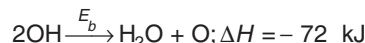
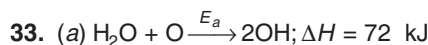
$$\log \frac{k_2}{k_1} = \frac{E_a}{2.303R} \left[\frac{T_2 - T_1}{T_1 \cdot T_2} \right]$$

Hence, $\log 4 = \frac{E_a}{2.303 \times 8.314} \left[\frac{313 - 293}{293 \times 313} \right]$

$$E_a = \frac{11.521}{0.000218} = 52.849 \text{ kJ mol}^{-1}$$

32. (a) $t_{1/2} \propto \frac{1}{a^{n-1}}$

Hence, $t_{1/2} \propto \frac{1}{a^3}$, only when $n = 4$



Also $E_a - E_b = \Delta H$ or $77 - E_b = 72$

$\therefore E_b = 5 \text{ kJ mol}^{-1}$

34. (c)

35. (c) The ratio of two half-lives and their concentrations are

related as $\frac{(t_{1/2})_1}{(t_{1/2})_2} = \left[\frac{a_2}{a_1} \right]^{n-1}$ [Here, n = order]

Therefore, $\frac{0.1}{0.8} = \left[\frac{50}{400} \right]^{n-1}$

On taking log of both sides

$$\log \frac{0.1}{0.8} = (n-1) \log \frac{50}{400}$$

$$1 = n - 1 \Rightarrow n = 2$$

36. (c) For the first order reaction, $k = \frac{2.303}{t} \log \frac{[A_0]}{[A]}$

Then, $k = \frac{2.303}{t} \log \left(\frac{a}{a/16} \right) = 0.0462 = 4.62 \times 10^{-2} \text{ s}$

37. (c) For the first order reaction,

$$k = \frac{0.693}{t_{1/2}} = \frac{0.693}{2.1} = 0.33 \text{ h}^{-1}$$

and

$$k = \frac{2.303}{t} \log_{10} \frac{a}{(a-x)}$$

or

$$t = \frac{2.303}{k} \log_{10} \frac{100}{(100-99)}$$

$$= \frac{2.303}{0.33} \times 2 = 13.96 \text{ h}$$

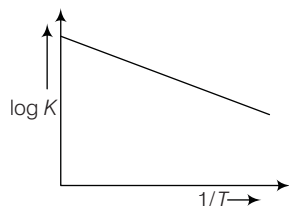
38. (b) For the first order reaction

$$t_{1/2} = \frac{0.693}{k} = \frac{0.693}{4.2 \times 10^{-12} \text{ s}^{-1}}$$

$$= 1.65 \times 10^{11} \text{ s}$$

39. (a) The value of $\log \frac{k_2}{k_1}$ is equal to $\frac{E_a}{2.303 R} \left[\frac{T_2 - T_1}{T_1 T_2} \right]$.

40. (c) On the basis of Arrhenius equation,



41. (c) In the Arrhenius equation for the rate of reaction, E is the energy below which colliding molecules will not react.

42. (b) Let, Rate law, $r = k[A]^x[B]^y$... (i)

On putting values, we get

$$0.10 = K[0.30]^x [0.30]^y \quad \dots (ii)$$

$$0.40 = K[0.30]^x [0.60]^y \quad \dots (iii)$$

$$0.20 = K[0.60]^x [0.30]^y \quad \dots (iv)$$

From Eqs. (ii) and (iii), we get

$$\frac{0.1}{0.4} = \left[\frac{0.30}{0.30} \right]^x \left[\frac{0.30}{0.60} \right]^y$$

$$\frac{1}{4} = \left[\frac{1}{2} \right]^y \Rightarrow y = 2$$

From Eqs. (iii) and (iv), we get

$$\frac{0.40}{0.20} = \left(\frac{0.30}{0.60} \right)^x \left(\frac{0.60}{0.30} \right)^y$$

$$2 = \left(\frac{1}{2} \right)^x (2)^y$$

On putting the values of y , we get

$$2 = \left(\frac{1}{2} \right)^x (2)^2$$

$$\Rightarrow x = 1$$

On putting the values of x and y in Eq. (i), we get

$$r = k[A]^1[B]^2 = k[A][B]^2$$

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1. (d) $t_{1/2} \propto \left(\frac{1}{a} \right)^{n-1}$ or $t_{1/2} = k(a)^{1-n}$
 $\log t_{1/2} = \log k + (1-n) \log a$
 (It represents straight line equation, $y = c + mx$)
 Slope $= (1-n) = \tan 45^\circ = 1$
 $\therefore (1-n) = 1 \Rightarrow n = 0$

2. (a) It is seen that, in experiments (3) and (2), $[A]$ is constant and $[B]$ is doubled and rate becomes 8 times, so order with respect to $[B] = 3$. In experiments (1) and (3), $[B]$ is constant and $[A]$ is doubled, but rate does not change, so order with respect to $[A] = 0$

Thus, rate $= k[B]^3$

3. (a) $k_1 = Ae^{-E_{a1}/RT}$, $k_2 = Ae^{-E_{a2}/RT}$, $k_3 = Ae^{-E_{a3}/RT}$

$$\text{Overall rate} = k = \frac{k_1 k_3}{k_2}$$

$$\text{Therefore, overall } E_a = E_{a1} + E_{a3} - E_{a2}$$

$$= 60 + 10 - 30 = 40 \text{ kJ}$$

4. (d) For a first order reaction, Rate, $= k[N_2O_5]$

$$2.4 \times 10^{-5} = 3 \times 10^{-5} \times [N_2O_5]$$

$$[N_2O_5] = \frac{2.4 \times 10^{-5}}{3 \times 10^{-5}} = 0.8 \text{ mol L}^{-1}$$

5. (c) $t_{1/2} \propto \frac{1}{(a)^{n-1}} \cdot \frac{1}{k^n}$

$$\text{Given, } t_{1/2} \propto \frac{1}{a}$$

$$\text{On comparing, } a^{n-1} = a^1, n-1=1, n=1+1=2$$

6. (c) For a first order reaction,

$$k = \frac{2.303}{t} \log \frac{a}{a-x}$$

$$t = \frac{2.303 \times t_{1/2}}{0.693} \log \frac{100}{(100-90)} \quad \left[\because k = \frac{0.693}{t_{1/2}} \right]$$

$$= 3.3 t_{1/2}$$

= 3.3 times that of half-life

7. (a) $\log \frac{k_2}{k_1} = \frac{E_a}{2.303R} \left[\frac{T_2 - T_1}{T_1 T_2} \right]$

$$\log 2 = \frac{E_a}{2.303 \times 2} \left[\frac{300 - 290}{290 \times 300} \right]$$

$$E_a = 12062 \text{ cal}$$

$$= 12.0 \text{ kcal}$$

8. (b) The active mass of solids is usually taken as unity..

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Solution

Solutions in Chemistry

Solution is a homogeneous mixture of two or more substances on molecular level. A solution of two substances is called a binary solution. The substances forming the solution are called **components** of the solution. As a generalisation, the component present in smaller amount is called **solute** and the other present in larger amount is called **solvent**.

Different Methods for Expressing Concentration of Solutions

1. **Molality (m)** = $\frac{\text{Number of moles of solute}}{\text{Weight of solvent (in kg)}}$
2. **Molarity (M)** = $\frac{\text{Number of moles of solute}}{\text{Volume of solution (in L)}}$

Relation between molality (m) and molarity (M)

$$\text{Molarity (M)} = \frac{m \times d}{1 + \frac{m \times M_2}{1000}}$$

where, M_2 = molar mass of solute

and d = density of the solution

3. **Mole fraction of solute in the solution**

$$\chi_{\text{solute}} = \frac{\text{Moles of solute } (n_{\text{solute}})}{\text{Total moles of solution } (n_{\text{solute}} + n_{\text{solvent}})}$$

Mole fraction of solvent in the solution

$$\chi_{\text{solvent}} = \frac{\text{Moles of solvent } (n_{\text{solvent}})}{\text{Total moles of solution } (n_{\text{solute}} + n_{\text{solvent}})}$$

Sum of mole fractions is always equal to 1,

i.e. $\chi_{\text{solute}} + \chi_{\text{solvent}} = 1$

Relation between molarity (M) and mole fraction (χ) of the solute

$$\chi = \frac{MM_1}{M(M_1 - M_2) + d}$$

where, M_1 = molar mass of solvent

and M_2 = molar mass of solute

4. **Percentage by weight** = $\frac{\text{Weight of solute} \times 100}{\text{Weight of solution}}$

5. **Percentage by volume**

(a) Weight of solute per 100 mL of solution (w/V)

(b) Volume of solute per 100 mL of solution (V/V)

6. **ppm concentration** = $\frac{10^6 \times \text{mass of solute}}{\text{Mass of (solute + solvent)}}$

7. **Normality** = $\frac{\text{Number of gram-equivalents of solute}}{\text{Volume of solution (in L)}}$

Solubility

Maximum amount of a solute that can be dissolved in a given amount of solvent (usually 100 g) at a given temperature is called solubility at that temperature.

Solubility of Gas in Liquid

All gases are soluble in water and in other liquids upto a greater or lesser extent.

The solubility of a gas depends upon the following factors:

- (i) **Nature of the gas** The gases which can be easily liquefied, are more soluble in common solvents, e.g. CO_2 is more soluble than H_2 and O_2 in water.
- (ii) **Nature of the solvent** The gases which are capable of producing ions in aqueous solutions, are much more soluble than in other solvent.
- (iii) **Temperature** The solubility of most of the gases in liquids decreases with rise in temperature.
- (iv) **Pressure** The solubility of a gas in liquid is influenced by pressure. The relation between the solubility and pressure is given by **Henry's law**. It states that 'the partial pressure (p) of the gas in vapour phase is proportional to the mole fraction of the gas (χ) in the solution' and is expressed as $p = k_H \chi$
where, k_H is the Henry's law constant.

Vapour Pressure

The pressure exerted by the vapours of a liquid which are in equilibrium with it at a given temperature, is called vapour pressure. Vapour pressure variation with temperature is given as

$$2.303 \log_{10} \frac{p_2}{p_1} = \frac{\Delta H}{R} \left[\frac{T_2 - T_1}{T_1 T_2} \right]$$

where, p_1 and p_2 are vapour pressures at T_1 and T_2 respectively and ΔH is heat of vaporisation.

Factors Affecting Vapour Pressure

Vapour pressure gets affected by following factors:

Temperature

The vapour pressure of a liquid increases with increase in temperature. This is because on increasing the temperature, kinetic energy of molecules increases that results into the fact that more molecules of the liquid can go into vapour phase.

Effect of Adding Solute

When a liquid contains a solute, some of the solvent molecules are replaced by the solute particles on the liquid surface and therefore, the available surface area for the escape of solvent molecules decreases.

Due to the less available area on the surface of liquid for escape, rate of evaporation and hence, the rate of condensation both lowers.

The vapour pressure of liquid in solution is known as its **partial vapour pressure** and is less than the vapour pressure of the pure liquid at the same temperature.

If p° be the vapour pressure of pure liquid and p_s be that of liquid in solution then lowering of vapour pressure of the liquid = $p^\circ - p_s$

$$\therefore \text{Relative lowering in pressure} = \frac{p^\circ - p_s}{p^\circ}$$

Raoult's Law

The partial vapour pressure of a volatile component is directly proportional to its mole fraction in solution.

For a solution possessing two components A and B , the partial vapour pressure of A is

$$p_A \propto \chi_A$$

$$p_A = k \cdot \chi_A$$

where, χ_A = mole fraction of components A

If $\chi_A = 1$, then k (proportionality constant) = p_A° (vapour pressure of pure component A), thus,

$$p_A = p_A^\circ \chi_A$$

Similarly,

$$p_B = p_B^\circ \chi_B$$

According to Dalton's law,

$$p_{\text{total}} = p_A + p_B$$

$$p_{\text{total}} = \chi_A p_A^\circ + \chi_B p_B^\circ$$

$$= (1 - \chi_B) p_A^\circ + \chi_B p_B^\circ$$

$$= p_A^\circ + (p_B^\circ - p_A^\circ) \chi_B$$

Limitations of Raoult's Law

- It is valid only for very diluted solutions.
- It is valid only for the solution having non-volatile and non-electrolyte solute which exists as a single molecule.
- It is not valid for those solutes which associate or dissociate in the particular solution.

Ideal and Non-ideal Solutions

Ideal Solutions

The solutions which obey Raoult's law over the entire range of concentration are called ideal solutions.

For ideal solutions,

$$\Delta H_{\text{mix}} = 0, \Delta V_{\text{mix}} = 0$$

Solute-solute and solvent-solvent interactions
 $\approx$ solute-solvent interactions

Non-ideal Solutions

The solution which shows deviation from Raoult's law, is called non-ideal solution.

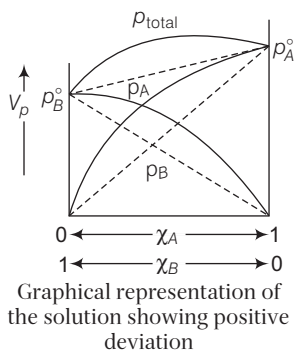
For such solutions, $\Delta H_{\text{mix}} \neq 0, \Delta V_{\text{mix}} \neq 0$

Non-ideal Solutions Showing Positive Deviation

When the observed vapour pressure is more than that expected by Raoult's law, positive deviation is observed.

- For such a deviation, $p_A > p_A^\circ \chi_A$, $p_B > p_B^\circ \chi_B$
 $p_{\text{total}} > p_A^\circ \chi_A + p_B^\circ \chi_B$

In this case, the intermolecular attractive forces between the solute-solvent molecules are weaker than those between the solute-solute and solvent-solvent molecules.



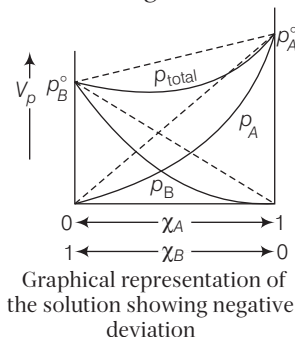
For such solutions, $\Delta H_{\text{mix}} > 0$, i.e. energy is absorbed on mixing and $\Delta V_{\text{mix}} > 0$. These are usually obtained by mixing of polar liquids with non-polar ones.

e.g. Cyclohexane and ethanol, H_2O and $\text{C}_2\text{H}_5\text{OH}$.

- Minimum boiling azeotropes** are formed by those liquid pairs which show positive deviation from ideal behaviour. Such azeotropes have boiling points lower than either of the components.
 e.g. $\text{C}_2\text{H}_5\text{OH}$ (95.57%) + H_2O (4.43%) (by mass)

Non-ideal Solutions Showing Negative Deviation

When the observed vapour pressure is less than that expected by Raoult's law, negative deviation is observed.



For such solutions, $\Delta H_{\text{mix}} < 0$, i.e. energy is released on mixing and $\Delta V_{\text{mix}} < 0$, i.e. attractive forces between unlike molecules are greater than the forces of attraction between like molecules.

e.g. Chloroform and acetone

For a solution showing negative deviation,

$$p_A < p_A^\circ \chi_A \text{ or } p_B < p_B^\circ \chi_B$$

$$p_{\text{total}} < p_A^\circ \chi_A + p_B^\circ \chi_B$$

- Maximum boiling azeotropes** are formed by those liquid pairs which show negative deviation from ideal behaviour. Such azeotropes have boiling points higher than either of the components.
 e.g. H_2O (20.22% by mass) + HCl .

Colligative Properties

The properties which depend only on the number of moles of non-volatile solute, are referred as colligative properties.

There are four types of colligative properties as given below:

1. Relative Lowering of Vapour Pressure

Addition of non-volatile solute leads to the lowering of vapour pressure.

$$\frac{p^\circ - p}{p^\circ} = \chi_{\text{solute}}$$

$$\text{or } \frac{p^\circ - p}{p^\circ} = \frac{n_A}{N} = \frac{w_A \cdot M_B}{M_A \cdot w_B}$$

where, $\frac{p^\circ - p}{p^\circ}$ = relative lowering of vapour pressure

n = moles of solute

N = moles of solvent

p° = vapour pressure of pure solvent

w_B and w_A are masses of solute and solvent respectively

and M_B and M_A are molecular weights of solute and solvent respectively.

2. Elevation in Boiling Point (ΔT_b)

For dilute solutions,

$$\Delta T_b = K_b \times \text{molality}$$

$$\Delta T_b = T_b - T_b^\circ$$

where, T_b = boiling point of solution

and T_b° = boiling point of pure solvent

K_b is ebullioscopic constant or molal elevation constant (K_b depends only on solvent).

$$\therefore M_B = \frac{K_b \times w_B \times 1000}{\Delta T_b \times w_A}$$

$$K_b = \frac{RT_0^2}{1000 \Delta H_V}$$

where, R = gas constant

and ΔH_V = latent heat of vaporisation

3. Depression in Freezing Point (ΔT_f)

For dilute solutions

$$\Delta T_f = K_f \times \text{molality}$$

$$\Delta T_f = T_f^\circ - T_f$$

where, T° = freezing point of pure solvent

T_f = freezing point of solution

K_f = cryoscopic constant or molal depression constant

$$\therefore M_B = \frac{K_f \times w_B \times 1000}{\Delta T_f \times w_A}$$

$$K_f = \frac{RT_0^2}{1000 \Delta H_f}$$

where, H_f = latent heat of fusion

4. Osmosis and Osmotic Pressure

Spontaneous flow of solvent molecules through a semipermeable membrane from a pure solvent to the solution (or from a dilute solution to concentrated solution) is termed as osmosis.

$$\text{Osmotic pressure } (\pi) = \frac{n}{V} RT = CRT$$

where, n = moles of solute, C = molar concentration

V = volume of solution (in litre), R = gas constant

T = temperature (in kelvin)

$$\therefore M_B = \frac{w_B RT}{\pi V}$$

Two solutions having same osmotic pressure at same temperature, are termed as **isotonic solutions**. When two solutions are being compared, the solution with higher osmotic pressure is termed as **hypertonic** and the solution with lower osmotic pressure is termed as **hypotonic**.

Osmotic pressure can be determined quite accurately, hence, it is used in the determination of molecular weights of large proteins and similar substances.

van't Hoff Factor (i)

In 1880, van't Hoff introduced a factor i , known as the van't Hoff factor, to account for the extent of dissociation or association. This factor i is defined as

$$i = \frac{\text{Number of particles after association or dissociation}}{\text{Number of particles before association or dissociation}}$$

$$\text{Degree of dissociation, } \alpha = \frac{i-1}{n-1}$$

where, n = number of particles after dissociation

$$\text{Degree of association, } \alpha = \frac{i-1}{\frac{1}{n}-1}$$

where, n = number of particles after association

Modified Expressions of Colligative Properties

1. Relative lowering of vapour pressure

$$\frac{p^\circ - p}{p^\circ} = i \frac{n_B}{n_B + n_A}; \quad \left(\begin{array}{l} n_B = n \\ n_B + n_A \approx N \end{array} \right)$$

2. Elevation in boiling point

$$\Delta T_b = i \cdot K_b \cdot m \quad \text{where, } m = \text{molality}$$

3. Depression in freezing point

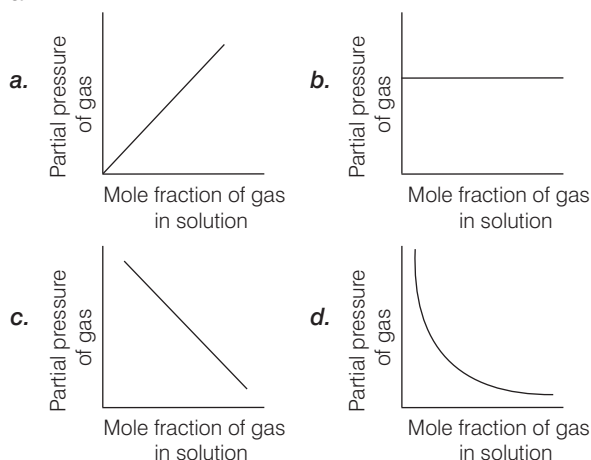
$$\Delta T_f = i \cdot K_f \cdot m$$

4. Osmotic pressure (π) = $i \cdot CRT$

Practice Exercise

- The molarity of 720 g of pure water is
a. 40 M b. 4 M
c. 55.5 M d. unpredictable
- Molarity is expressed as
a. g / L b. L / mol
c. mol / L d. mol / 1000g
- The formula weight of H_2SO_4 is 98. The weight of the acid in 400 mL 0.1 M solution is
a. 2.45 g b. 3.92 g c. 4.90 g d. 9.8 g
- Out of molarity (M), molality (m), normality (N) and mole fraction (χ), those independent of temperature are
a. M and m b. N and χ c. m and χ d. M and χ
- A χ molal solution of a compound in benzene has mole fraction of solute equals to 0.2. The value of χ is
a. 14 b. 3.2 c. 1.4 d. 2
- On mixing 10 mL of acetone with 50 mL of chloroform, the total volume of the solution is
a. < 60 mL b. > 60 mL
c. = 60 mL d. unpredictable
- The normality of 10% (weight / volume) acetic acid is
a. 1 N b. 10 N c. 1.66 N d. 0.83 N
- Normality of 2 M H_2SO_4 is
a. 4 N b. 6 N c. $\frac{N}{4}$ d. $\frac{N}{6}$
- Suppose sea water contains 3.50 weight per cent of NaCl. Calculate the molarity of sea water.
a. 6.2 m b. 0.062 m c. 0.62 m d. 0.0062 m
- Which of the following statements is true about saturated solution?
a. A solution in which no more solute can be dissolved at the same temperature and pressure
b. The solution which is in dynamic equilibrium with undissolved solute and contains maximum amount of solute dissolved in solvent
c. The solution in which more solute can be dissolved at the same temperature
d. Both (a) and (b)
- Calculate the mass of urea (NH_2CONH_2) required in making 2.5 kg of 0.25 molal aqueous solution.
a. 37 g b. 35 g c. 34 g d. 32 g
- One litre of $N/2$ HCl solution is heated in a beaker and it is observed that when volume was reduced to 600 mL, 3.25 g of HCl is lost. Calculate the normality of new solution.
a. 0.50 N b. 0.60 N c. 0.68 N d. 0.70 N
- A substance will be deliquescent if its vapour pressure
a. is equal to the atmospheric pressure
b. is equal to that of water vapour in air
c. is less than that of water vapour in air
d. is greater than that of water vapour in air
- The aqueous solution that has the lowest vapour pressure at a given temperature, is
a. 0.1 M sodium phosphate
b. 0.1 M barium chloride
c. 0.1 M sodium chloride
d. 0.1 M glucose
- For determination of molecular mass, Raoult's law is applicable only to
a. dilute solutions of electrolytes
b. concentrated solutions of electrolytes
c. dilute solutions of non-electrolytes
d. concentrated solutions of non-electrolytes
- 100 mL of liquid A was mixed with 25 mL of liquid B to give non-ideal solution of A-B. The volume of this mixture will be
a. 75 mL
b. 125 mL exact
c. fluctuate between 75 mL and 125 mL
d. close to 125 mL but not to exceed 125 mL
- 6.0 g of urea (molecular weight = 60) was dissolved in 9.9 moles of water. If the vapour pressure of pure water is p_0 , the vapour pressure of solution is
a. $0.10 p_0$ b. $1.10 p_0$ c. $0.90 p_0$ d. $0.99 p_0$
- On mixing 25 mL of CCl_4 with 25 mL of toluene, the total volume of the solution is
a. = 50 mL b. > 50 mL
c. < 50 mL d. unpredictable
- An azeotropic mixture of two liquids has boiling point lower than either of them when it
a. shows a negative deviation from Raoult's law
b. shows no deviation from Raoult's law
c. shows positive deviation from Raoult's law
d. is saturated
- A sugar solution boils at 101°C . The molality of the sugar solution is (Given, $K_b = 0.52^\circ\text{C kg mol}^{-1}$)
a. 1.84 m b. 1.92 m c. 2.02 m d. 4.02 m
- An unknown compound is immiscible with water. It is steam distilled at 98.0°C . At 98.0°C , p and $p_{\text{H}_2\text{O}}^\circ$ are 737 and 707 torr respectively. This distillate was 75% by weight of water. The molecular weight of the unknown will be
a. $318.15 \text{ g mol}^{-1}$ b. 300 g mol^{-1}
c. $306.76 \text{ g mol}^{-1}$ d. None of these

22. The relative lowering of vapour pressure of an aqueous solution containing a non-volatile solute, is 0.0125. The molality of the solution is
a. 0.69 m b. 0.50 m c. 0.80 m d. 0.40 m
23. How many grams of H_2SO_4 is/are to be dissolved to prepare 200 mL aqueous solution having concentration of $[\text{H}_3\text{O}^+]$ ions is 1 M at 25°C temperature?
a. 19.6 g b. 0.98 g c. 4.9 g d. 9.8 g
24. Vapour pressure of pure A is 70 mm of Hg at 25°C . If it forms an ideal solution with B in which mole fraction of A is 0.8 and vapour pressure of the solution is 84 mm of Hg at 25°C , then the vapour pressure of pure B at 25°C is
a. 140 mm b. 70 mm c. 56 mm d. 28 mm
25. Lowering of vapour pressure is highest for
a. 0.1 M BaCl_2 b. 0.1 M glucose
c. 0.1 M MgSO_4 d. urea
26. Which of the following curves represent the Henry's law?

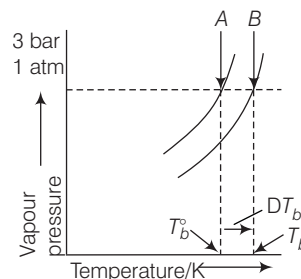


27. If p° and p_s are the vapour pressures of the solvent and solution respectively and n_1 and n_2 are the mole fractions of solvent and solute respectively. Then,
a. $p_s = p^\circ n_1$ b. $p_s = p^\circ n_2$
c. $p^\circ = p_s n_2$ d. $p_s = p^\circ \left(\frac{n_1}{n_2} \right)$
28. Arrange the following in the increasing order of their solubility in *n*-octane based on solute-solvent interaction:
a. $\text{KCl} < \text{CH}_3\text{CN} < \text{CH}_3\text{OH} < \text{Cyclohexane}$
b. $\text{KCl} < \text{Cyclohexane} < \text{CH}_3\text{OH} < \text{CH}_3\text{CN}$
c. $\text{KCl} < \text{CH}_3\text{OH} < \text{CH}_3\text{CN} < \text{Cyclohexane}$
d. $\text{KCl} < \text{Cyclohexane} < \text{CH}_3\text{CN} < \text{CH}_3\text{CN}$
29. Which one of them is more volatile component?
a. CH_2Cl_2 b. CHCl_3
c. Both a and b d. Not able to determine
30. The elevation in boiling point would be highest for
a. 0.08 m BaCl_2 b. 0.10 m glucose
c. 0.10 m KCl d. 0.06 m calcium nitrate

31. A 0.2 molal aqueous solution of weak acid (HX) is 20% ionised. The freezing point of this solution is (Given, $K_f = 1.86^\circ\text{C m}^{-1}$ for water)
a. -0.45°C b. -0.55°C c. -0.90°C d. -0.31°C
32. The van't Hoff factor for a very dilute solution of $\text{Fe}_2(\text{SO}_4)_3$ is
a. 9 b. 5
c. 24 d. None of these
33. The latent heat of vaporisation of water is 9700 cal/mol and if the boiling point is 100°C , the ebullioscopic constant of water is
a. 0.516°C b. 1.026°C c. 10.26°C d. 1.832°C
34. The relative lowering of vapour pressure of an aqueous solution containing a non-volatile solute is 0.0125. The molality of the solution is
a. 0.70 m b. 0.590 m c. 0.80 m d. 0.40 m
35. The osmotic pressure of one molar solution at 27°C is
a. 2.46 atm b. 24.6 atm c. 1.21 atm d. 12.1 atm
36. The freezing point of 0.1M solution of glucose is -1.86°C . If an equal volume of 0.3 M glucose solution is added, the freezing point of the mixture will be
a. -7.44°C b. -5.58°C c. -3.72°C d. -2.79°C
37. The van't Hoff factor for 0.1 M $\text{Ba}(\text{NO}_3)_2$ solution is 2.74. The degree of dissociation is
a. 91.3% b. 87% c. 100% d. 74%
38. In a 0.2 molal aqueous solution of a weak acid HX, the degree of ionisation is 0.3. Taking K_f for water as 1.85, the freezing point of the solution will be nearest to
a. -0.360°C b. -0.260°C
c. $+0.480^\circ\text{C}$ d. -0.481°C
39. Acetic acid exists in benzene solution in the dimeric form. In an actual experiment, the van't Hoff factor was found to be 0.52. Then, the degree of dissociation of acetic acid is
a. 0.48 b. 0.88 c. 0.96 d. 0.52
40. A compound X undergoes tetramerisation in a given organic solvent. The van't Hoff factor is
a. 4.0 b. 0.25 c. 0.125 d. 2.0
41. The freezing point (in $^\circ\text{C}$) of a solution containing 0.1 g of $\text{K}_3[\text{Fe}(\text{CN})_6]$ (molecular weight 329) in 100 g of water ($K_f = 1.86 \text{ K kg mol}^{-1}$) is
a. -2.3×10^{-2} b. -5.7×10^{-2}
c. -5.7×10^{-3} d. -1.2×10^{-2}
42. 0.004 M Na_2SO_4 is isotonic with 0.01 M glucose. Degree of dissociation of Na_2SO_4 is
a. 75% b. 50% c. 25% d. 85%
43. Colligative properties of a solution depends upon
a. nature of solute only
b. nature of both solute and solvent
c. number of solute particles
d. number of solvent particles

44. Among the following 0.10 m aqueous solutions, which one will exhibit the largest freezing point depression?
a. KCl b. $C_6H_{12}O_6$ c. $Al_2(SO_4)_3$ d. K_2SO_4
45. To observe an elevation of boiling point of $0.05^\circ C$, the amount of a solute (molecular weight = 100) to be added to 100 g of water ($K_b = 0.5$) is
a. 2 g b. 0.05 g c. 1 g d. 0.75 g
46. van't Hoff factor can be applied to the
a. relative lowering of vapour pressure of solvent
b. elevation of boiling point
c. osmotic pressure of solution
d. all colligative properties
47. Determine the amount of $CaCl_2$ ($i = 2.47$) dissolved in 2.5 L of water such that its osmotic pressure is 0.75 atm at $27^\circ C$.
a. 1.0 g b. 9.2 g c. 3.42 g d. 2.42 g
48. Find out the osmotic pressure of 0.1 M monobasic acid if $pH = 2.0$ at $25^\circ C$.
a. 2.69 atm b. 26.9 atm
c. 0.269 atm d. None of these

49. Elevation in boiling point was $0.52^\circ C$ when 6 g of a compound X was dissolved in 100 g of water. Molecular weight of X is (K_b of water is $5.2^\circ C$ per 100 g of water)
a. 120 b. 60 c. 600 d. 180
50. What does point A and B represent in the following diagram?

**Point A**

- a. Boiling point of solvent
b. Boiling point of solution
c. Boiling point of solute
d. Boiling point of solvent

Point B

- Boiling point of solution
Boiling point of solvent
Boiling point of solvent
Boiling point of solute

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1. Usually, $CaCl_2$ is preferred over NaCl for cleaning snow on roads particularly in very cold countries. This is because [2014]
a. NaCl makes the road slippery but $CaCl_2$ does not
b. $CaCl_2$ is hygroscopic but NaCl is not
c. $CaCl_2$ is less soluble in H_2O than NaCl
d. eutectic mixture of $CaCl_2/H_2O$ freezes at $-55^\circ C$ while that of $NaCl/H_2O$ freezes at $-18^\circ C$
2. The vapour pressure of a solvent decreased by 10 mm of Hg when a non-volatile solute was added to the solvent. The mole fraction of solute in solution is 0.2, what would be the mole fraction of solvent if the decrease in vapour pressure is 20 mm of Hg? [2013]
a. 0.8 b. 0.6 c. 0.4 d. 0.3
3. If the elevation in boiling point of a solution of 10 g of solute (molecular weight = 100) in 100 g of water is ΔT_b , the ebullioscopic constant of water is [2012]
a. $\Delta T_b/10$ b. ΔT_b c. $10\Delta T_b$ d. $100\Delta T_b$
4. 1.2% NaCl solution is isotonic with 7.2% glucose solution. What will be the van't Hoff factor, i ? [2012]
a. 0.5 b. 1 c. 2 d. 6
5. K_b for water is $0.52 K/m$. Then, 0.1 m solution of NaCl will boil approximately at [2011]
a. $100.52^\circ C$ b. $100.052^\circ C$ c. $101.04^\circ C$ d. $100.104^\circ C$
6. The molal freezing point for water is $1.86^\circ C mol^{-1}$. If 342 g of cane sugar is dissolved in 1000 mL of water, the solution will freeze at [2010]
a. $1.86^\circ C$ b. $-1.86^\circ C$ c. $2.42^\circ C$ d. $-2.42^\circ C$
7. $CaCl_2$ is preferred to NaCl for clearing snow on roads particularly in very cold countries. This is because
a. $CaCl_2$ is less soluble in H_2O than NaCl [2009]
b. $CaCl_2$ is hygroscopic but NaCl is not
c. eutectic mixture of $CaCl_2/H_2O$ freezes at $-55^\circ C$ while that of $NaCl/H_2O$ freezes at $-18^\circ C$
d. NaCl makes the road slippery but $CaCl_2$ does not
8. 100 cc of 1.5% solution of urea is found to have an osmotic pressure of 6.0 atm and 100 cc of 3.42% solution of cane sugar is found to have an osmotic pressure of 2.4 atm. If two solutions are mixed, the osmotic pressure of the resulting solution will be
a. 8.4 atm b. 4.2 atm [2009]
c. 16.8 atm d. 2.1 atm
9. When 25 g of a non-volatile solute is dissolved in 100 g of water, the vapour pressure is lowered by 2.25×10^{-1} mm. If the vapour pressure of water at $20^\circ C$ is 17.5 mm, what is the molecular weight of the solute? [2008]
a. 206 b. 302 c. 350 d. 276
10. A solution of sucrose (molar mass = 342 g/mol) is prepared by dissolving 68.4 g of it per litre of solution, what is its osmotic pressure at 273 K?
($R = 0.082 L atm K^{-1} mol^{-1}$) [2007]
a. 3.92 atm
b. 4.48 atm
c. 5.92 atm
d. 29.4 atm

Answer with Solutions

Practice Exercise

1. (c) Molarity = $\frac{w}{m \cdot V} = \frac{720}{18 \times 720/1000} = 55.5 \text{ M}$

(720 g water = 720 mL water)

2. (c)

3. (b) Molarity = $\frac{w}{m \cdot V \text{ (litre)}}$

Hence, $0.1 = \frac{w}{98 \times \frac{400}{1000}}$

$w = 0.1 \times 98 \times \frac{400}{1000}$
 $= 3.92 \text{ g}$

4. (c) Molality (m) and mole fraction (χ) involves only mass and do not depend upon volume. Hence, these are independent of temperature.

5. (b) Molality and mole fractions of solute (χ_A) are related as

$\chi_A = \frac{m \cdot M_B}{1000 + m \cdot M_B} \Rightarrow 0.2 = \frac{m \times 78}{1000 + m \times 78}$

$200 + 15.6m = 78m$

or $62.4m = 200$
 $m = 3.2$

6. (a) Chloroform and acetone show strong hydrogen bonding. Hence, for such solution ($\Delta V_{\text{mix}} = -\text{ve}$), volume decreases on mixing (Non-ideal solution with negative deviation).

7. (c) Normality = $\frac{w}{E \cdot V} = \frac{10 \text{ g}}{100 \text{ mL}} \times \frac{1}{60}$
 $= \frac{10 \text{ g}}{\frac{100}{1000} \times 60} = 1.66 \text{ N}$

8. (a) Basicity of $\text{H}_2\text{SO}_4 = 2$

Normality = Molarity $\times$ Basicity of acid = $2 \times 2 = 4 \text{ N}$

9. (c) NaCl solution contains 3.5 g of NaCl in 100 g of water.

Water in solution = $100 - 3.5 = 96.5 \text{ g} = 0.0965 \text{ kg}$

Molality = $\frac{3.5}{58.5 \times 0.0965} = 0.62 \text{ m}$

10. (d)

11. (a) Mass of solvent = 1000 g

Molar mass of urea (NH_2CONH_2) = 60 g mol^{-1}

$0.25 \text{ mole of urea} = 0.25 \times 60 = 15 \text{ g}$

Total mass of solution = $100 + 15 = 1.015 \text{ kg}$

$1.015 \text{ kg of solution contain urea} = 15 \text{ g}$

$2.5 \text{ kg of solution} = \frac{15}{1.015} \times 2.5 = 37 \text{ g}$

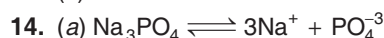
12. (c) 1 L of $N/2$ HCl contains = $\frac{1}{2} \times 36.5 = 18.25 \text{ g}$

HCl now present = $18.25 - 3.25 = 15 \text{ g}$

Now, volume = $600 \text{ mL} = 0.600 \text{ L}$

Normality = $\frac{15}{36.5} \times \frac{1}{0.600 \text{ L}} = 0.685 \text{ N}$

13. (c)



15. (c)

16. (d) For non-ideal solution, $\Delta V_{\text{mix}} < 0$. Hence, total volume will be little less than $(100 + 25 = 125 \text{ mL})$.

17. (d) Mole of urea = $\frac{6}{60} = 0.10$

According to Raoult's law

$\frac{p_{\text{solvent}} - p_{\text{solution}}}{p_{\text{solvent}}} = \chi_{\text{solute}} = \frac{n_A}{n_A + n_B}$

Hence, $\frac{p_0 - p_{\text{solution}}}{p_0} = \frac{0.10}{0.10 + 9.9}$

or $p_0 - p_{\text{solution}} = 0.01 p_0$ or $p_{\text{solution}} = 0.99 p_0$

18. (b) It is a non-ideal solution, hence, $\Delta V_{\text{mix}} = +\text{ve}$

19. (c)

20. (b) We know that $\Delta T_b = i \cdot K_b \cdot m$ (Here, $i = 1$)

$\therefore (101 - 100) = 0.52 \times m$

$\therefore \text{Molality } (m) = \frac{1}{0.52} = 1.92$

21. (a) Since, unknown compound is immiscible with water. Hence, vapour pressure $\propto$ moles

Given, $p_{\text{total}}^\circ = 737 \text{ torr}$

$p_{\text{H}_2\text{O}}^\circ = 707 \text{ torr}$

$\therefore p_{\text{unknown}}^\circ = 737 - 707 = 30 \text{ torr}$

$W_{\text{H}_2\text{O}} = 100 \text{ g}$

$W_{\text{unknown}} = 75 \text{ g}$

$\frac{p_{\text{unknown}}^\circ}{p_{\text{water}}^\circ} = \frac{n_{\text{unknown}}}{n_{\text{H}_2\text{O}}} = \frac{W_{\text{unknown}} \times m_{\text{H}_2\text{O}}}{W_{\text{H}_2\text{O}} \times m_{\text{unknown}}}$

or $\frac{30}{707} = \frac{75.0 \times 18}{100 \times m_{\text{unknown}}}$ or $m_{\text{unknown}} = 318.15 \text{ g mol}^{-1}$

22. (a) $\frac{p^\circ - p_s}{p^\circ} = \frac{m \times M}{1000}$

$0.0125 = \frac{m \times 18}{1000}$ (molecular weight of $\text{H}_2\text{O} = 18$)

$m = \frac{0.0125 \times 1000}{18} = 0.69$

23. (d) Molarity, $M = \frac{w_2 \times 1000}{M_2 \times V \text{ (in mL)}}$

where, w_2 is the mass of H_2SO_4 in g and M_2 is the molar mass of H_2SO_4 .

$$w_2 = \frac{1 \times 98 \times 200}{1000} = 19.6 \text{ g}$$



But according to equation, 1 mole of H_2SO_4 gives 2 moles of $[\text{H}_3\text{O}^+]$ ions. Thus, the amount of H_2SO_4 to prepare 200 mL solution having 1 M concentration of H_3O^+ ions is $19.6/2 = 9.8 \text{ g}$.

24. (a) $p = p_A^\circ \chi_A + p_B^\circ \chi_B$
 $\Rightarrow 84 = 70 \times 0.8 + p_B^\circ \times 0.2$
 $p_B^\circ = \frac{28}{0.2} = 140 \text{ mm}$

25. (a) Lowering of vapour pressure is directly proportional to the van't Hoff factor. For BaCl_2 , $i = 3$ (maximum) thus, BaCl_2 has highest value for relative lowering of vapour pressure.

26. (a) A graph between partial pressure of the gas *versus* mole fraction of the gas in solution is drawn, then we get plot of straight line.

27. (a) Raoult's law $\frac{p_0 - p_s}{p_0} = n_2$

$$1 - \frac{p_s}{p_0} = n_2$$

$$\frac{p_s}{p_0} = 1 - n_2 = n_1$$

$$p_s = n_1 \times p_0$$

28. (c) $\text{KCl} < \text{CH}_3\text{OH} < \text{CH}_3\text{CN} < \text{Cyclohexane}$

29. (a) Since, CH_2Cl_2 is a more volatile component than CHCl_3 , $[p_{\text{CH}_2\text{Cl}_2}^\circ = 415 \text{ mm Hg}$ and $p_{\text{CHCl}_3}^\circ = 200 \text{ mm Hg}]$ and the vapour phase is also richer in $[\chi_{\text{CH}_2\text{Cl}_2} = 0.82$ and $[\chi_{\text{CHCl}_3} = 0.18]$

30. (a) More the number of particles and concentration higher is the elevation in boiling point. Thus, 0.08 m KCl
 $= -0.08 \times 2 = 0.24 \text{ m (highest)}$

31. (a) 32. (b)

33. (a) $K_b = \frac{R \cdot T_b^2}{1000 \cdot L_v} = \frac{2 \times 373 \times 373 \times 18}{1000 \times 9700} = 0.516^\circ \text{C}$

34. (a) Relative lowering of vapour pressure of an aqueous solution containing non-volatile solute a is mole fraction of solute

As $\frac{p_0 - p_s}{p_0} = \frac{n}{n + N} = 0.0125$

$$\Rightarrow \frac{n + N}{N} = \frac{1}{0.0125} - \frac{N}{n} = \frac{1 - 1}{0.0125} = \frac{0.9875}{0.0125}$$

So $\frac{N}{n} = \frac{0.9875}{0.0125}$

Now, molality $= \frac{0.0125 \times 1000}{0.9875 \times 18} = 0.70$

35. (b) $p = \frac{n}{V} RT = MRT \left[\text{Molarity (M)} = \frac{n}{V} \right]$
 $= 1 \times 0.0821 \times 300 = 24.6 \text{ atm}$

36. (c) $K_f = \Delta T_f / M = 1.86 / 0.1 = 18.6$

$$0.1 \times V + 0.3 V = M_3 \times 2V$$

or

$$M_3 = 0.2$$

$$\Delta T_f = 18.6 \times 0.2 = 3.72^\circ \text{C}$$

$$T_f = -3.72^\circ \text{C}$$



Initially	1 mol	0	0
After dissociation	$1 - \alpha$	α	2α

Total number of moles $= 1 + 2\alpha$

$$\therefore i = 1 + 2\alpha$$

or $\alpha = \frac{i - 1}{2} = \frac{2.74 - 1}{2} = 0.87 = 87\%$



Initially	1 mol	0	0
After dissociation	$1 - 0.3$	0.3	0.3

Total moles $= 1 - 0.3 + 0.3 + 0.3 = 1.3$

$$i = \frac{1.3}{1} = 1.3$$

$$\Delta T_f = iK_f m = 1.3 \times 1.85 \times 0.2 = 0.481^\circ \text{C}$$

$$\therefore T_f = 0 - 0.481^\circ \text{C} = -0.481^\circ \text{C}$$



Initially	1	0
After dissociation	$1 - \alpha$	$\alpha / 2$

$$i = 1 - \frac{\alpha}{2}$$

or $\alpha = 2(1 - i) = 2(1 - 0.52) = 0.96$



$$1 - \alpha \quad \frac{\alpha}{4}$$

$$i = \frac{1 - \alpha + \frac{\alpha}{4}}{1} = 1 - \frac{3}{4}\alpha$$

where, α = degree of dissociation $= 1 = 100\%$

$$\therefore i = 1 - \frac{3}{4} = 0.25$$

41. (a) van't Hoff factor (i) $= 4 \{3\text{K}^+ + (\text{Fe}(\text{CN})_6\}^{3-}\}$

$$\text{Molality} = \frac{0.1 \times 1000}{329 \times 100} = \frac{1}{329}$$

$$-\Delta T_f = iK_f m = 4 \times 1.86 \times \frac{1}{329} = 2.3 \times 10^{-2}$$

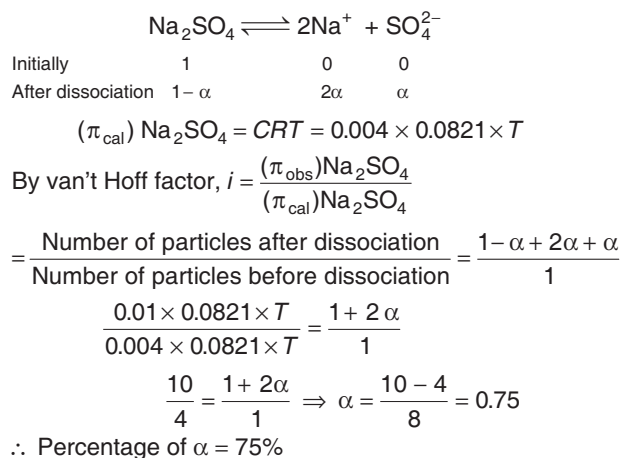
$$\Rightarrow \Delta T_f = -2.3 \times 10^{-2}^\circ \text{C}$$

(As freezing point of water is 0°C)

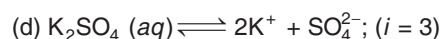
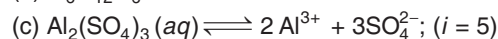
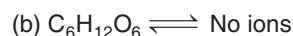
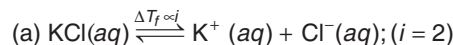
42. (a) $0.004 \text{ M Na}_2\text{SO}_4$ solution is isotonic with 0.01 M solution of glucose, so their osmotic pressures are equal to each other. Osmotic pressure of 0.01 M glucose,

$$\pi_{\text{glucose}} = CRT = 0.01 \times 0.0821 \times T$$

$$= (\pi_{\text{obs}})_{\text{Na}_2\text{SO}_4} = \pi_{\text{glucose}} = 0.01 \times 0.0823 \times T$$



43. (c)

44. (c) As we know, $\Delta T_f = i \times K_f \times m$ where, K_f and m are constants.Hence, $\text{Al}_2(\text{SO}_4)_3$ will exhibit largest freezing point depression due to the highest value of i .

45. (c) Elevation of boiling point,

$$\Delta T_b = \frac{w \times K_b \times 1000}{M \times W(\text{gram})}$$

On substituting values, we get

$$0.05 = \frac{w \times 0.5 \times 1000}{100 \times 100} \text{ or } w = \frac{0.05 \times 100 \times 100}{0.5 \times 1000} = 1\text{g}$$

46. (d)

47. (c) $\pi = iCRT$ $\pi = inRT/V$

$$n = \frac{\pi V}{iRT} = \frac{0.75 \text{ atm} \times 2.5 \text{ L}}{2.47 \times 0.0821 \text{ L atm K}^{-1} \text{ mol}^{-1} \times 300 \text{ K}}$$

$$= 0.0308 \text{ mol}$$

Molar mass of $\text{CaCl}_2 = 111 \text{ g mol}^{-1}$ Amount dissolved = $0.0308 \times 111 = 3.42 \text{ g}$ 48. (a) $\text{HA} \rightleftharpoons \text{H}^+ + \text{A}^-$

$$\begin{array}{ccc} C & 0 & 0 \\ C(1 - \alpha) & C\alpha & C\alpha \end{array}$$

$$[\text{H}^+] = C\alpha = 10^{-2}$$

$$\text{pH} = 2$$

$$C\alpha = 10^{-2} \Rightarrow \alpha = \frac{10^{-2}}{0.1} = 0.1$$

Total number of moles after dissociation

$$= C - C\alpha + C\alpha + C\alpha = C(1 + \alpha)$$

$$\text{van't Hoff factor} = \frac{C(1 + \alpha)}{C} = 1 + \alpha = 1 + 0.1 = 1.1$$

$$\pi = iCRT = 1.1 \times 0.1 \times 0.0821 \times 298 = 2.69 \text{ atm}$$

49. (b) We know that

$$m = \frac{100K_b \cdot w}{\Delta T_b \cdot W}$$

$$= \frac{100 \times 5.2 \times 6}{0.52 \times 100} = 60$$

50. (a) The boiling point of a solution is always higher than that of the boiling point of pure solvent.

 $A \rightarrow$ Boiling point of solvent $B \rightarrow$ Boiling point of solution

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1. (d)

2. (b) From Raoult's law

$$p^\circ - p_s = p^\circ \times \text{mole fraction of solute}$$

$$10 = p^\circ \times 0.2 \Rightarrow 20 = p^\circ \times \chi_2$$

$$\therefore \chi_2 = 0.4 \text{ and } \chi_1 = 1 - 0.4 = 0.6$$

 $\chi_1 =$ mole fraction of solvent

$$3. (b) m = \frac{1000 \times K_b \times w}{W \times \Delta T_b}$$

$$\text{or } K_b = \frac{m \times W \times \Delta T_b}{1000 \times w} = \frac{100 \times 100 \times \Delta T_b}{1000 \times 10} = \Delta T_b$$

4. (c)

5. (d) $\Delta T_b = iK_b \cdot m = 2 \times 0.52 \times 0.1 = 0.104^\circ \text{C}$

$$T_b = 100 + 0.104^\circ \text{C}$$

$$= 100.104^\circ \text{C}$$

$$6. (b) m = \frac{1000 K_f w}{\Delta T_f W}$$

$$\Delta T_f = \frac{1000 K_f w}{mW} = \frac{1000 \times 1.86 \times 342}{342 \times 1000} = 1.86^\circ \text{C}$$

 $\therefore$ The solution will freeze at $0 - 1.86^\circ \text{C} = -1.86^\circ \text{C}$

7. (a)

8. (b) $p_{\text{urea}} = 3.0 \text{ atm}$, $p_{\text{cane sugar}} = 1.2 \text{ atm}$

$$\text{Total} = 4.2 \text{ atm}$$

9. (c)

10. (b) Osmotic pressure (π) = CRT Here, $C =$ concentration of solution

$$C = \frac{n}{V} = \frac{68.4}{342}$$

$$\pi = \frac{68.4}{342} \times 0.082 \times 273 = 4.48 \text{ atm}$$

11

Adsorption and Colloidal System

Concept of Surface Chemistry

Surface chemistry deals with the phenomenon that occurs at the surface or interfaces. There are several properties of substances, particularly of solids, liquids and solids, gases, which depend upon the nature of the surface of interface.

Concept of Adsorption

Retaining of the molecular species on the surface of the solid is known as adsorption. The solid substance on the surface of which adsorption occurs is called **adsorbent**. The molecular species that get adsorbed on the solid surface due to intermolecular attractions are called **adsorbate**.

Rate of adsorption decreases with time whereas rate of absorption remains constant with time.

Characteristics of Adsorption

- (i) Adsorption occurs due to unbalanced residual force on surface which allows to hold the adsorbate particle. These forces are generally chemical forces or physical van der Waals' forces.
- (ii) Adsorption occurs by decreasing ΔG° , ΔS° and ΔH° with condition ΔH has more negative value in comparison to $T\Delta S$.

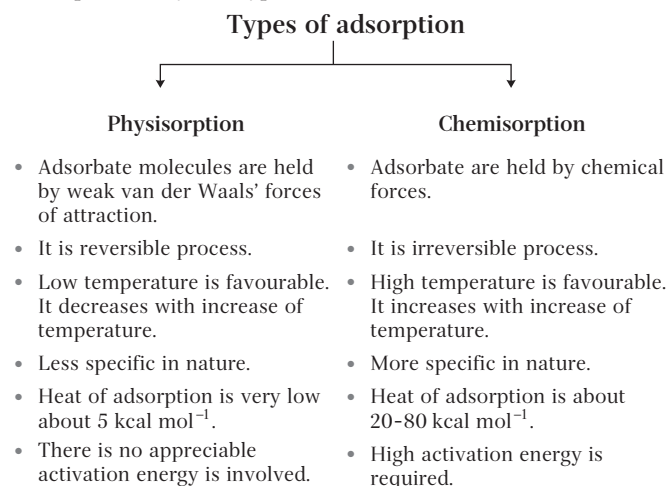
$$\Delta G < 0, \quad \Delta H < 0, \quad \Delta S < 0, \quad -T\Delta S < \Delta H$$

- (iii) Extent of adsorption is directly proportional to surface area, i.e. increases with surface area. That's why finely divided metals and porous materials are better adsorbents.

NOTE At high temperature, physisorption can be converted into chemisorption. $\Delta H = -ve$ hence, adsorption is always an exothermic process.

Classification of Adsorption

Adsorption is of two types:



Adsorption Isotherm

The variation of extent of adsorption with pressure at constant temperature is known as adsorption isotherm.

Freundlich Adsorption Isotherm

According to this isotherm, the extent of adsorption is related to the pressure at constant temperature by the given formula,

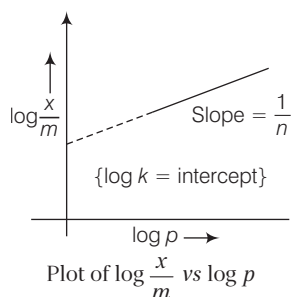
$$\frac{x}{m} = k \cdot p^{1/n} \quad \dots(i)$$

where, $\frac{x}{m}$ = extent of adsorption
 p = pressure

k and n are constants which depend upon the nature of adsorbent and gas.

On taking log on both sides of Eq.(i), we get

$$\log \frac{x}{m} = \log k + \frac{1}{n} \log p$$



The Freundlich isotherm is valid for the value of $\frac{1}{n}$ in between 0 to 1. Probable range is 0.1 to 0.5, where maximum adsorption is observed.

Effect of Temperature on Freundlich Adsorption Isotherm

It has been found that as temperature increases, the extent of adsorption decreases, in

Effects of Pressure on Adsorption

Case I At high pressure, $\frac{1}{n} = 0$ or $\frac{x}{m} = k$ (constant)

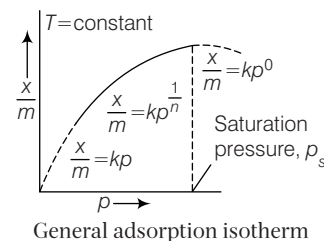
At this condition, graph becomes almost constant and follow zero order kinetics.

Case II At low pressure, $\frac{1}{n} = 1$ or $\frac{x}{m} = kp$

$$\text{i.e. } \frac{x}{m} \propto p$$

Thus, the extent of adsorption is directly proportional to pressure which follow first order kinetics. The graph between extent of adsorption and pressure is a straight line.

Case III At intermediate range of pressure, the value of $\frac{1}{n}$ varies between 0 and 1, the graph can be expressed as



Langmuir Adsorption Isotherm

In order to explain the relation between extent of adsorption and pressure, Langmuir introduced an adsorption isotherm on the basis of the following assumptions :

- Adsorption is monolayered.
- Interaction between adjacent adsorbed molecule is negligible.
- Adsorption is uniform throughout the surface.
- At equilibrium, rate of adsorption is equal to rate of desorption.

On the basis of Langmuir adsorption isotherm, the fraction of surface covered by the adsorbed molecules (θ) represented as

$$(i) \text{ At equilibrium, } \theta = \frac{Kp}{1 + Kp}$$

where, K = equilibrium constant, known as adsorption coefficient

$$= \frac{k_a}{k_d} = \frac{\text{Rate constant for adsorption}}{\text{Rate constant for desorption}}$$

$$(ii) \theta = \frac{K'p}{1 + K'p}, \text{ where, } K' = kK = \text{constant}$$

Pressure Dependence of Langmuir Adsorption

Let us, consider the mathematical relation

$$\frac{x}{m} = \frac{K'p}{1 + Kp} \quad \left(\because \frac{x}{m} \propto \theta \right)$$

Case I At high pressure, $1 + Kp \approx Kp$

$$\frac{x}{m} = \frac{K'p}{Kp} = \frac{K'}{K} = \text{constant}$$

Thus, at high pressure, the extent of adsorption approaches to a limiting value.

Case II At low pressure,

The Kp becomes negligible in comparison to $1 + Kp = kp$

$$\frac{x}{m} = K'p, \quad \frac{x}{m} \propto p$$

Thus, at low pressure, the extent of adsorption is directly proportional to pressure.

Case III When pressure is moderate, then expression is in the form of $\frac{x}{m} = Kp^{1/n}$, where, $\frac{1}{n}$ lies between 0 and 1.

Adsorption from Solutions

Freundlich adsorption isotherm,

$$\frac{x}{m} = kC^{1/n} \quad (n > 1)$$

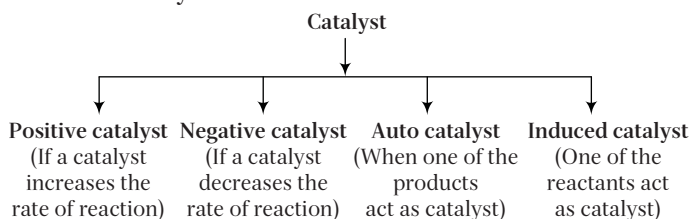
Langmuir adsorption isotherm,

$$\frac{x}{m} = \frac{aC}{(1 + bC)}$$

Langmuir adsorption isotherm is applicable only to chemisorption.

Catalysis

The term catalyst defined as a substance which alter the rate of reaction without taking part in it and phenomenon is called as catalysis.

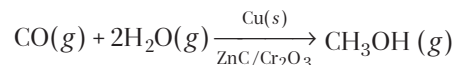
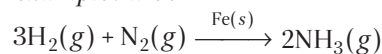
**Types of Catalyst****1. Homogeneous Catalyst**

When the catalyst are in the same phase as the reactants and products then it is called homogeneous catalyst and the process is called **homogeneous catalytic reaction** or **homogeneous catalysis**.

2. Heterogeneous Catalyst

When the catalyst is in different phase than that of the reactants. It is called heterogeneous catalyst.

Important examples are:

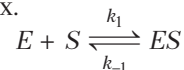
**Enzyme Catalysis**

Enzymes are complex nitrogenous organic compounds which are produced by living plants and animals. They catalyse numerous biochemical reactions. The enzymes are thus, termed as biochemical catalysts and the phenomenon is known as biochemical catalysis.

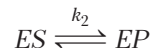
Mechanism

Michaelis and Menton suggested the following steps for enzyme catalysis:

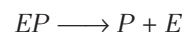
Step I Binding of enzyme to substrate to form an activated complex.



Step II Product formation in the activated complex.



Step III Decomposition of EP into products and enzyme again.

**Colloidal State**

A substance is said to be in the colloidal state, when it is dispersed in another medium in the form of very small particles having diameter between 10^{-4} to 10^{-7} cm, i.e. 1 to 100 nm.

Types of Colloidal System

The dispersed phase or dispersion medium can be a gas, liquid or solid. There are eight types of colloidal system.

These are as follows:

S. No.	Colloidal system	Dispersion phase	Dispersion medium	Examples
1.	Foam	Gas	Liquid	Soda water, froth, shaving cream
2.	Solid foam	Gas	Solid	Foam rubber, cork
3.	Aerosol	Liquid	Gas	Fog, mist, clouds
4.	Emulsion	Liquid	Liquid	Milk, hair cream
5.	Solid emulsion (gel)	Liquid	Solid	Butter, cheese
6.	Aerosol of solids	Solid	Gas	Dust in air, smoke
7.	Sol	Solid	Liquid	Paint, ink, colloidal gold,
8.	Solid sol	Solid	Solid	Ruby glass some gemstones, alloys, rock salt

Classification of Colloids

Colloids can be classified in a number of ways based upon some of their important characteristics as given below :

Based on Affinity of Phase

1. **Lyophilic colloids** represent such colloidal systems in which the particles of dispersed phase have great affinity for the dispersion medium. These are reversible colloids, e.g. gum, gelatin, rubber, proteins etc.
2. **Lyophobic colloids** represent such colloidal systems in which particles of the dispersed phase have no affinity for the dispersion medium. These are irreversible, e.g. sols of metals and their insoluble compounds like sulphides and oxides.

If water is the dispersion medium, the terms used are hydrophilic and hydrophobic colloids.

Lyophobic colloids are less stable due to the presence of electric charge on their particles. On the other hand, lyophilic colloids are stable due to the presence of charge as well as their extensive solvation.

Based on Molecular Size

1. **Multimolecular colloids** are the colloids in which colloidal particles consist of aggregate of atoms or small molecules with diameter less than 10^{-9} m or 1 nm, e.g. a sol of gold, a sol of sulphur.
2. **Macromolecular colloids** are the colloids in which colloidal particles themselves are large molecules.
3. **Associated colloids** or **micelles** are the substances which behave as normal electrolytes at low concentration but as colloids at higher concentration. This is because at higher concentration, they form associated particles called micelles, e.g. soap and synthetic detergents.

The concentration above which micelle formation occurs, is called CMC (critical micelle concentration) and the temperature above which micelle formation occurs is called **Kraft temperature**.

Preparation of Colloid

The various methods of preparation of colloids are as follows:

Dispersion Methods

The breaking down of a large solid particles into small colloidal particles by using mechanical or electrical methods are known as dispersion methods.

- (i) **Mechanical dispersion** It is done by using colloidal mill which causes breaking of large colloidal particles. e.g. Preparation of colloidal graphite (used as lubricant), preparation of printing ink.

- (ii) **Electrical dispersion** It is done by applying electric spark between the electrodes of metal which vaporises metal and converts metal into colloidal size. This process is known as Bredig's arc method.

e.g. Gold sol is prepared by this method.

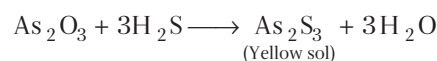
- (iii) **Peptisation** This is a process of conversion of precipitate to the colloidal solution by means of shaking the solution of precipitate in the presence of small amount of electrolyte. Such electrolyte is known as peptising agent.

e.g. $\text{Fe}(\text{OH})_3 + \text{FeCl}_3 \longrightarrow \text{Sol}$

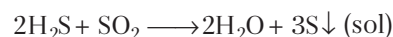
Chemical Methods

Some important examples are as follows:

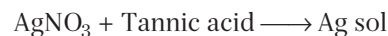
- (i) Double decomposition of As_2O_3



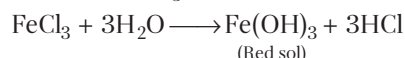
- (ii) Oxidation



- (iii) Reduction of AgNO_3 and AuCl_3 by tannic acid



- (iv) Hydrolysis of FeCl_3



Properties of Colloidal Solutions

The important properties of the colloidal solutions are given below :

Brownian Movement

Colloidal particles are always in a state of rapid random motion, which is termed as Brownian movement.

Tyndall Effect

When a strong and converging beam of light is passed through a colloidal solution, its path becomes visible due to scattering of light by particles. It is called Tyndall effect.

Electrophoresis

The phenomenon involving the migration of colloidal particles under the influence of electric field towards the oppositely charged electrode, is called electrophoresis.

Coagulation or Flocculation

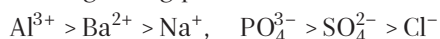
The precipitation of particles of the dispersed phase in a sol is known as coagulation.

The minimum amount of an electrolyte required to cause precipitation of one litre of a colloidal solution is called coagulation value or flocculation value. The reciprocal of coagulation value is regarded as the coagulating power.

Hardy-Schulze Rule

Higher the valency of the active ion, greater will be its power to precipitate the sol.

e.g. Order of coagulating power is

**Gold Number**

Protective power of any lyophilic colloid is determined in terms of gold number which is defined as "the milligrams of lyophobic colloid required to just prevent the precipitation of 10 mL of gold sol on the addition of 10 mL of 10% NaCl solution. Smaller the gold number of hydrophilic colloids, greater is its protective power.

Emulsion

Emulsion is a type of colloidal solution made by the addition or mixing of two immiscible or partially miscible

liquids. Generally, one of the two liquids is water and the liquid other than water is designated as oil.

Thus, the two types of emulsion may be formed as :

1. Oil dispersed in water (O/W type), e.g. milk.
2. Water dispersed in oil (W/O type), e.g. butter, cream.

Characteristics of Emulsion

The main characteristics of emulsion are as follows :

- (i) Commonly emulsions are unstable and stabilised by adding some third substance called emulsifying agents.

Some emulsifying agents are as follows:

- (a) **O/W type emulsion**, e.g. protein, gum, natural and synthetic soaps.
- (b) **W/O type emulsion**, e.g. lamp black, metal salt of fatty acids, long chain alcohols.

- (ii) Droplets of emulsion are often negatively charged.

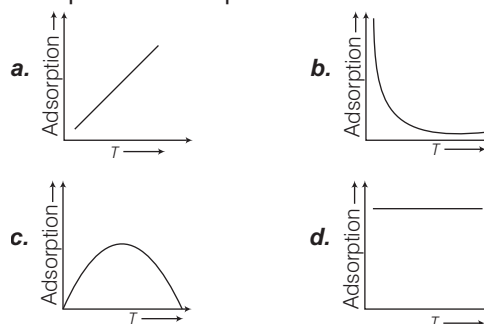
Practice Exercise

1. The substance that gets adsorbed on the surface of the solid is called
 - a. adsorbate
 - b. adsorbent
 - c. micelle
 - d. inner phase
2. Adsorption is multilayered in the case of
 - a. physical adsorption
 - b. chemisorption
 - c. Both (a) and (b)
 - d. None of these
3. Which is favourable for van der Waals' adsorption?
 - a. High temperature, low pressure
 - b. Low temperature, high pressure
 - c. Low temperature, low pressure
 - d. High temperature, high pressure
4. In physical adsorption, the forces associated are
 - a. ionic
 - b. covalent
 - c. van der Waals'
 - d. H-bonding
5. Which adsorption takes place at higher temperatures?
 - a. Physical
 - b. Chemical
 - c. Both (a) and (b)
 - d. None of these
6. Which is adsorbed in minimum amount by the activated charcoal?
 - a. H_2
 - b. CO_2
 - c. SO_3
 - d. CO
7. Which of the following gas is adsorbed in maximum amount by charcoal?
 - a. SO_2
 - b. CO_2
 - c. CO
 - d. Water vapour
8. Which is true about chemisorption?
 - a. It is reversible in nature
 - b. It usually occurs at low temperatures
 - c. It is highly specific in nature
 - d. The attractive forces between adsorbate and adsorbent are van der Waals' forces
9. Adsorption is accompanied by

- a. decrease in entropy
- b. decrease in enthalpy
- c. decrease in free energy
- d. None of these

10. Amount of gas adsorbed per gram of adsorbent increases with pressure, but after certain limit is reached, adsorption becomes constant, it is due to
 - a. multilayers are formed
 - b. desorption takes place
 - c. temperature is increased
 - d. absorption also starts

11. Which of the following is the variation of physical adsorption with temperature?



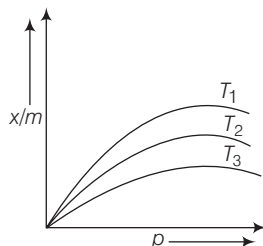
12. According to Freundlich adsorption isotherm, which of the following is correct?

- a. $\frac{x}{m} \propto p^\circ$
- b. $\frac{x}{m} \propto p^1$
- c. $\frac{x}{m} \propto p^{1/n}$
- d. All of the above are correct for different ranges of pressure

13. 1g of charcoal adsorbs 100 mL of 0.5 M CH_3COOH to form a monolayer, and thereby the molarity of CH_3COOH reduces to 0.49 M. What is the surface area of charcoal used by each molecule of acetic acid?

[Surface area of charcoal = $3.01 \times 10^2 \text{ m}^2 / \text{g}$]

- $6.02 \times 10^{-30} \text{ m}^2$
 - $5.00 \times 10^{-19} \text{ m}^2$
 - $3.01 \times 10^{-2} \text{ m}^2$
 - $2.00 \times 10^{-19} \text{ m}^2$
14. Which of the following is not the true about the adsorption?
- During the process of adsorption, residual attractive forces decreases
 - During the adsorption surface energy decreases
 - It is an exothermic process
 - It is an endothermic process
15. Which of the following statements are correct?
- Silica gel adsorbs water molecules.
 - Anhy. CaCl_2 adsorbs the water molecules.
 - Adsorption is a surface phenomenon.
- I and II
 - II and III
 - I and III
 - All of these
16. Which of the following process will be observed, when a chalk stick is dipped in the solution of ink?
- Adsorption
 - Absorption
 - Desorption
 - Both (a) and (b)
17. A mixture of sand and water is an example of
- true solution
 - colloidal solution
 - suspension
 - All of the above
18. Freundlich adsorption isotherm is represented at temperatures T_1 , T_2 and T_3 .



Arrange the temperatures in the increasing order as shown in the graph.

- $T_1 < T_2 < T_3$
 - $T_3 < T_2 < T_1$
 - $T_2 < T_3 < T_1$
 - $T_3 < T_1 < T_2$
19. Identify the gas which is readily adsorbed by activated charcoal?
- H_2
 - N_2
 - SO_2
 - O_2

20. "Rate of adsorption is directly proportional to the fraction of area uncovered and rate of desorption is directly proportional to the fraction of area covered". This statement is true for

- Freundlich adsorption isotherm
 - Langmuir adsorption isotherm
 - BET isotherm
 - None of the above
21. The correct relationship which represents the Freundlich adsorption isotherm is
- $\log \left(\frac{x}{m} \right) = \log k + \frac{1}{n} \log p$
 - $\log \left(\frac{m}{x} \right) = \log k + \frac{1}{n} \log p$
 - $\log \left(\frac{x}{m} \right) = \log p + \frac{1}{k} \log p$
 - $\log \left(\frac{x}{m} \right) = \log c + \frac{1}{n} \log k$

22. Which of the following is correct statement?
- Physisorption occurs at low temperature and chemisorption occurs at high temperature
 - Physisorption occurs at very high temperature and chemisorption occurs at low temperature
 - Physisorption is irreversible and chemisorption is reversible
 - None of the above

23. For Freundlich adsorption isotherm, $\frac{x}{m} = kp^{1/n}$, the value of n is

- always greater than 1
 - always smaller than 1
 - always equal to 1
 - greater than 1 at low temperature and smaller than 1 at high temperature
24. In general, H_2 gas is adsorbed on activated charcoal to a less extent in comparison to the easily liquefiable gases due to
- very strong van der Waals' force and low critical temperature
 - very weak van der Waals' force and low critical temperature
 - very strong van der Waals' force and high critical temperature
 - very weak van der Waals' force and high temperature

25. Which of the following reaction requires catalyst?

- $\text{S} + \text{O}_2 \longrightarrow \text{SO}_2$
 - $2\text{SO}_2 + \text{O}_2 \longrightarrow 2\text{SO}_3$
 - $\text{C} + \text{O}_2 \longrightarrow \text{CO}_2$
 - All of the above
26. Hydrolysis of sugar to glucose and fructose is catalysed by
- invertase
 - zymase
 - lactic bacilli
 - diastase

27. The efficiency of an enzyme in catalysing a reaction is due to its capacity
 a. to form an enzyme substrate complex
 b. to decrease the bond energies of the substrate molecules
 c. to change the shape of the substrate molecule
 d. None of the above
28. In *Zeigler-Natta* polymerisation of ethylene, the active species is
 a. AlCl_3
 b. Et_3Al
 c. CH_2CH_2
 d. Ti^{3+}
29. Following reaction is catalysed by $\text{Br}^-(\text{aq})$

$$2\text{H}_2\text{O}_2(\text{aq}) \longrightarrow 2\text{H}_2\text{O}(\text{l}) + \text{O}_2(\text{g})$$

 This is an example of
 a. homogeneous catalysis
 b. heterogeneous catalysis
 c. Both (a) and (b)
 d. None of these
30. Vanishing cream is an example of
 a. solution
 b. foam
 c. lyophilic solution
 d. emulsion
31. Colloidal solution of gold is prepared by
 a. colloidal mill
 b. double decomposition method
 c. Bredig's arc method
 d. peptisation
32. When a beam of light is passed through colloidal solution,
 a. it gets scattered
 b. it gets adsorbed
 c. it is refracted
 d. it undergoes reflection
33. Fog is a colloidal solution of
 a. gas in gas
 b. solid in gas
 c. liquid in gas
 d. None of these
34. Which property is not shown by colloids?
 a. Adsorption
 b. Tyndall effect
 c. Flocculation
 d. Paramagnetism
35. The path of a beam of light through smoke is visible because
 a. carbon dioxide in the smoke scatters light
 b. carbon dioxide in the smoke absorbs light
 c. carbon particles in the smoke absorb light
 d. carbon particles in the smoke scatter light
36. Four different colloids have the following gold number, Which one has most effective action?
 a. 10
 b. 30
 c. 20
 d. 40
37. In coagulating the colloidal solution of As_2S_3 , which has the minimum coagulation value?
 a. NaCl
 b. KCl
 c. BaCl_2
 d. AlCl_3
38. Which of the following indicates the charge on colloidal particles?
 a. Brownian movement
 b. Electrophoresis
 c. Electrolysis
 d. Tyndall effect
39. Which of the following is not a colloid?
 a. Milk
 b. Blood
 c. Latex
 d. Vinegar
40. Which of the following has minimum gold number?
 a. Gelatin
 b. Egg albumin
 c. Gum arabic
 d. Starch
41. Which of the following represents homogeneous catalysis?
 a. $\text{Oil} + \text{H}_2 \xrightarrow{\text{Ni}} \text{Saturated fat}$
 b. $\text{N}_2 + 3\text{H}_2 \xrightarrow{\text{Fe}} 2\text{NH}_3$
 c. $\text{CH}_3\text{COOH} + \text{C}_2\text{H}_5\text{OH} \xrightarrow{\text{H}^+} \text{CH}_3\text{COOC}_2\text{H}_5 + \text{H}_2\text{O}$
 d. None of the above
42. Which of the following is not a colloid?
 a. Chlorophyll
 b. Smoke
 c. Ruby glass
 d. Milk
43. Lyophilic colloids are stable due to
 a. charge on the particles
 b. large size of particles
 c. small size of particles
 d. layer of dispersion medium on the particle
44. Which has the maximum coagulating power for ferric hydroxide colloid?
 a. AlCl_3
 b. $\text{K}_4[\text{Fe}(\text{CN})_6]$
 c. BaSO_4
 d. MgO
45. Gold number is the index for
 a. protective power of lyophilic colloid
 b. purity of gold
 c. metallic gold
 d. electroplated gold
46. Blue colour of sky and red colour of sunsets are due to
 a. scattering of light from the sun
 b. scattering of light from particles of dust in the atmosphere
 c. refraction of blue light by impurities in sea water
 d. scattering of light due to ozone layer
47. When dilute aqueous solution of AgNO_3 (excess) is added to KI solution, positively charged sol particles of AgI are formed due to adsorption of ion
 a. K^+
 b. Ag^+
 c. I^-
 d. NO_3^-
48. Gold sols and sulphur sols are the examples of
 a. multimolecular colloids
 b. macromolecular colloids
 c. associated colloids
 d. All of the above
49. Colloidion is a
 a. 100% solution of nitrocellulose
 b. 10% solution of nitrocellulose in mixture of alcohol and ether
 c. 4% solution of nitrocellulose in mixture of alcohol and ether
 d. 1% solution of nitrocellulose in mixture of alcohol and ether
50. Lyophilic solution is coagulated by
 a. adding an electrolyte
 b. adding a suitable solvent
 c. Both (a) and (b)
 d. None of the above

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1. The gold number of a few protective colloids are given
 $x = 0.005$, $y = 3.5$, $z = 40$

The protective nature of these colloidal solutions follow the order : [2014]

- a. $z > x > y$ b. $x < y < z$
 c. $z > y > x$ d. $x > y > z$

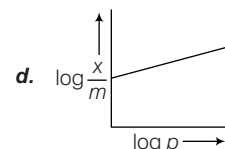
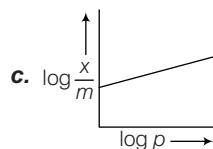
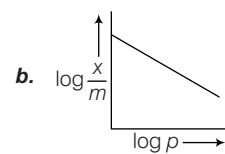
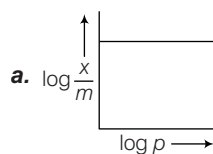
2. When equal volume of each of two sols of AgI, one obtained by adding AgNO_3 to slight excess of KI and another obtained by adding KI to slight excess of AgNO_3 are mixed together. It is observed that [2013]
 a. the sol particles acquired more electric charge
 b. the sols coagulated each other mutually
 c. a true solution is obtained
 d. the two sols stabilised each other

3. Which of the following will be the most effective in the coagulation of $\text{Fe}(\text{OH})_3$ sol? [2012]
 a. $\text{Mg}_3(\text{PO}_4)_2$ b. BaCl_2
 c. NaCl d. KCN

4. Which of the following is an example of homogeneous catalysis? [2010]

- a. Haber's process for the synthesis of NH_3
 b. Catalytic conversion of SO_2 to SO_3 in contact process
 c. Catalytic hydrogenation of oils
 d. Acid hydrolysis of methyl acetate

5. Which of the following graphs represents Freundlich adsorption isotherm? [2008]



Answer with Solutions

Practice Exercise

- (a)
- (a) Adsorption is multilayered in case of physical adsorption while it is almost monolayered in case of chemical adsorption.
- (b) van der Waals' adsorption is the physical adsorption which is favoured by low temperature as the phenomenon is exothermic. Increase in pressure favours the adsorption.
- (c) Adsorbate molecules are held by weak van der Waals' force of attraction.
- (b) Chemical adsorption takes place at higher temperature, in comparison of physical adsorption.
- (a) The gas, which is difficult to liquefy (i.e. having very low critical temperature), is adsorbed in less amount and *vice-versa*. Here, hydrogen has the minimum critical temperature, (-240°C) hence, is adsorbed in minimum amount.
- (a) Stronger the intermolecular forces, easier to liquefy and more strongly. It is adsorbed.
- (c) Chemisorption occurs through formation of chemical bonds, hence it is very specific in nature.
- (d)
- (a) Langmuir showed that at low pressure, the physically adsorbed gas forms only one molecule thick layer. However, above a certain pressure, multimolecular thick layer is formed.
- (b) Adsorption of gases decreases with increase in temperature.
- (d)
- (b) 100 mL of 0.5 CH_3COOH contains
 $\text{CH}_3\text{COOH} = 0.05 \text{ mol}$
 After adsorption, CH_3COOH remained = 0.049 mol
 $\therefore$ Acetic acid adsorbed = $0.001 \text{ mol} = 6.02 \times 10^{20}$ molecules
 $\therefore$ Surface area of charcoal adsorbed by each molecule

$$= \frac{3.01 \times 10^2 \text{ m}^2}{6.02 \times 10^{20}} = 5 \times 10^{-19} \text{ m}^2$$
- (d)
- (b) Anhy. CaCl_2 absorbs moisture and adsorption is a surface phenomenon.
- (d) When a chalk stick is dipped in ink, the surface retains the colour of ink due to adsorption of coloured molecules while the solvent of the ink goes deeper into the stick due to absorption.
- (c)
- (a) $\frac{x}{m}$ shows, extent of adsorption. Extent of adsorption increases by decrease of temperature. Order of temperature
 $T_1 < T_2 < T_3$
- (c) Easily liquefiable gases like SO_2 , NH_3 have greater value of critical temperature than elemental gases, i.e. N_2 , O_2 , H_2 thus, readily get adsorbed.

20. (a) 21. (a)
22. (a) At high temperature, kinetic energy of adsorbate increases and make a strong chemical bond between adsorbate and adsorbent, hence chemisorption occurs.
23. (a) If $n > 1$, then, $\frac{1}{n} < 1$
 $\therefore n$ is always greater than 1.
24. (b) H_2 is a permanent gas, hence, has low value of critical temperature and low van der Waals' force. Due to these factors, it shows low extent of adsorption.
25. (b) 26. (a)
27. (a) Enzyme + Substrate $\longrightarrow$ [Complex intermediate]
 $\longrightarrow$ [Product + enzyme]
28. (d) $(C_2H_5)_3Al + TiCl_4 \longrightarrow$ Active species
 Ti^{3+} has one active site vacant and thus accommodate one alkyl group (as $(C_2H_5)_3Al$ reduces $TiCl_4$ to $TiCl_3$).
29. (a) $2H_2O_2(aq) \xrightarrow{Br^-(aq)} 2H_2O(l) + O_2(g)$
 This is an example of homogeneous catalysis.
30. (d) Vanishing cream is an oil in water (O in W) type emulsion.
31. (c) Colloids of metals are generally prepared by Bredig's arc method.
32. (a)
33. (c) Fog is colloidal solution of water droplets (liquid) dispersed in air (gas) dispersion medium.
34. (d) Paramagnetism is not shown by colloids.
35. (d)
36. (a) Lesser the gold number, greater will be its protecting efficiency.
37. (d) As_2S_3 is a negative sol Hence, $AlCl_3$ with univalent Al^{3+} has least coagulation value.
38. (b) Electrophoresis shows that the colloidal particles have charge on them as they move towards oppositely charged electrode, when electric field is applied on them.
39. (d) Vinegar is approximately 10% solution of acetic acid in water.
40. (a) Gelatin is the best protective colloid as it has the least gold number (about 0.005).
41. (c)
42. (a) Chlorophyll is not a colloid.
43. (d)
44. (b) Ferric hydroxide is a positive sol, $[Fe(OH)_3]Fe^{3+}$.
 Hence, to coagulate it, an anion is needed. $K_4[Fe(CN)_6]$ gives the tetravalent anion $[Fe(CN)_6]^{4-}$, hence it has the greatest coagulating power among these (Hardy-Schulze law).
45. (a) Protective character of various lyophilic substances is expressed in terms of gold number.
46. (b) It is due to scattering of light from particles of dust in the atmosphere.
47. (b) Sol particles possess the tendency to adsorb preferentially the common ion present in solution.
48. (a) Gold sol and sulphur sol are the examples of multimolecular colloids.
49. (c) The usual colloidal solution is a 4% solution of nitrocellulose in the mixture of alcohol and ether.
50. (c) Lyophilic sol is coagulated by adding a suitable solvent or an electrolyte.

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1. (d) Smaller the gold number, greater is its protective power, Hence, the order would be
 $x > y > z$
 Gold number is used for calculating the protective powers of lyophilic colloids.
2. (b)
3. (a) According to Hardy Schulze rule, coagulation power of ions is directly proportional to charge on ion.
 $\therefore Fe(OH)_3$ is positively charged colloid.
 $\therefore$ It will be coagulated by anion.
- (a) $Mg_3(PO_4)_2 \rightleftharpoons 3Mg^{2+} + 2PO_4^{3-}$
 (b) $BaCl_2 \rightleftharpoons Ba^{2+} + 2Cl^-$
 (c) $NaCl \rightleftharpoons Na^+ + Cl^-$
 (d) $KCN \rightleftharpoons K^+ + CN^-$
4. (d) During acid hydrolysis of methyl acetate, reactants and catalyst (H_2SO_4) are in the same phase. Thus, it is an example of homogeneous catalysis.
5. (c) In Freundlich isotherm

12

Redox Reactions

Concepts of Oxidation and Reduction

Loss of electron by an atom is called oxidation or de-electronation while gain of electron by an atom is called reduction or electronation.

Oxidants or oxidising agents are the substances which

- (i) oxidise other,
- (ii) get reduced ,
- (iii) gain electrons (i.e. their oxidation number decreases during a reaction)

Reductants or reducing agents are the substances which

- (i) reduce others,
- (ii) get oxidised,
- (iii) loss electrons (i.e. their oxidation number increases during a reaction)

Oxidation Number

The real or imaginary charge, which an atom appears to have in its combined state is called oxidation number of that atom. Valency of an element is always a whole number. It can neither be zero nor fractional while oxidation number may be positive or negative. It can be zero or fractional.

Fractional oxidation state is only the average oxidation state of an element when two or more of its atoms are present in different oxidation states in a given compound.

Rules for Assigning Oxidation Number

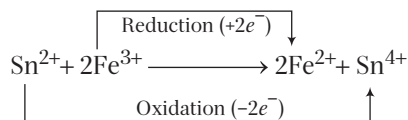
The oxidation number of an element or atom can be calculated with the help of following rules:

- (i) The oxidation number of an element in its elementary state is zero, e.g. H_2 , S_8 and P_4
- (ii) Oxidation number of an ion is equal to the electrical charge present on it.
- (iii) Oxidation number of a compound is zero.
- (iv) Oxidation number of fluorine is always -1 in all of its compounds.
- (v) The oxidation number of alkali metals is always $+1$ and those of alkaline earth metals is $+2$.

- (vi) Oxidation number of hydrogen is +1 except in ionic hydrides, where it is -1.
- (vii) Two oxidation numbers of N are -3 and +3, when it is bonded with less electronegative and more electronegative atoms respectively.
- (viii) Oxidation number of oxygen is -2 except in OF_2 (+2), O_2F_2 (+1), peroxides (-1) and superoxides (-1/2).
- (ix) The oxidation number of halogens is always -1 in metal halides.
- (x) In interhalogen compounds, more electronegative metal of the two halogens gets the oxidation number of -1.

Redox Reactions

The reaction which involves oxidation and reduction as its two half-reactions is called redox reactions. A redox change occurs simultaneously.

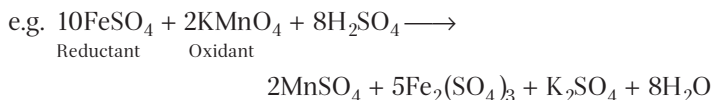


Types of Redox Reactions

These are of three types as follows :

1. Intermolecular Redox Reactions

These involve the reaction between two substances, one of them is oxidant and other is reductant,



2. Intramolecular Redox Reactions

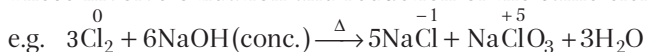
These involve oxidation of one element of a compound as well as reduction of other element of the same compound. Decomposition reactions are also intramolecular redox reactions but to be a redox reaction, it is essential that one of the products of decomposition must be in the elemental state.

e.g.



3. Autoredox or Disproportionation Reactions

These involve oxidation and reduction of the same element,



Balancing of Redox Reaction

1. Oxidation Number Method

In balancing redox reaction, following steps are used:

- (i) Indicate the oxidation number of all the atoms involved in the equation and identify the elements which undergo change in oxidation number.
- (ii) Calculate the increase and decrease in oxidation number per atom with respect to the reactants if more than one atom is involved, then multiply with the number of atoms undergoing the change to calculate the total change in oxidation number.
- (iii) Equate the increase and decrease in oxidation number on the reactant side by multiplying the suitable integers.
- (iv) Balance the equation with respect to all atoms except hydrogen and oxygen.
- (v) Finally, balance hydrogen and oxygen also.
- (vi) In acidic medium, hydrogen and oxygen are balanced by H^+ and H_2O respectively while in basic medium, these are balanced by using OH^- and H_2O .

2. Ion-Electron Method

In this method, the following steps are used :

- (i) Indicate the oxidation number of all the atoms involved in the reaction and determine the oxidising agent and reducing agent.
- (ii) Now, split the reaction into oxidation half-reaction and reduction half-reaction.
- (iii) Now, balance both the half-reactions in the following ways:
 - (a) Balance all the atoms except H and O by multiplying the suitable integer.
 - (b) Now, balance H and O by using H^+ (in acidic medium) or by using OH^- (in alkaline medium) and H_2O .
 - (c) Charge is balanced by using electrons in the side deficient in negative charge.
- (iv) Now, combine both the half-reactions so as to cancel the electrons.

Practice Exercise

- When zinc is added to CuSO_4 solution, copper is precipitated. It is because of
 - reduction of zinc
 - reduction of Cu^{2+}
 - hydrolysis of CuSO_4
 - reduction of SO_4^{2-}
- Which of the following is the most powerful oxidising agent?
 - F_2
 - Cl_2
 - Br_2
 - I_2
- The strongest reducing agent is
 - HNO_3
 - H_2S
 - H_2SO_3
 - SnCl_2
- Which of the following is a reducing agent?
 - NaHCO_3
 - NaHSO_3
 - Na_2O_2
 - NaHSO_4
- In which one of the following reactions, hydrogen is acts as an oxidising agent?
 - With Li to form LiH
 - With I_2 to form HI
 - With N_2 to form NH_3
 - With S to form H_2S
- Which of the following reactions has the underlined substance been reduced?
 - Carbon monoxide + Copper oxide $\longrightarrow$ Carbon dioxide + Copper
 - Copper oxide + Hydrochloric acid $\longrightarrow$ Copper chloride + Water
 - Hydrogen + Iron oxide $\longrightarrow$ Iron + Water
 - Steam + Iron $\longrightarrow$ Iron oxide + Hydrogen
- Nitric oxide acts as a reducing agent in which of the following reaction?
 - $4\text{NH}_3 + 5\text{O}_2 \longrightarrow 2\text{NO} + 6\text{H}_2\text{O}$
 - $2\text{NO} + 3\text{I}_2 + 4\text{H}_2\text{O} \longrightarrow 2\text{NO}_3^- + 6\text{I}^- + 8\text{H}^+$
 - $2\text{NO} + \text{H}_2\text{SO}_3 \longrightarrow \text{N}_2\text{O} + \text{H}_2\text{SO}_4$
 - $2\text{NO} + \text{H}_2\text{S} \longrightarrow \text{N}_2\text{O} + \text{S} + \text{H}_2\text{O}$
- In acidic medium, equivalent weight of $\text{K}_2\text{Cr}_2\text{O}_7$ (molecular weight = M) is
 - $M/3$
 - $M/4$
 - $M/6$
 - $M/2$
- In the reaction, $\text{I}_2 + 2\text{S}_2\text{O}_3^{2-} \longrightarrow 2\text{I}^- + \text{S}_4\text{O}_6^{2-}$, equivalent weight of iodine will be equal to
 - M
 - $M/2$
 - $M/4$
 - $2M$
 (where, M is the molecular weight of iodine)
- The process in which oxidation number increases, is
 - reduction
 - hydrolysis
 - oxidation
 - decomposition
- In which of the following, oxidation number of chlorine is +5?
 - Cl_2O_7
 - ClO_3^-
 - ClO^-
 - ClO_4^-
- Carbon has zero oxidation number in
 - CH_4
 - CH_3Cl
 - CCl_4
 - CH_2Cl_2
- Oxidation states of iodine vary from
 - 1 to +1
 - 1 to +7
 - +3 to +5
 - 1 to +5
- Oxidation number of S in $\text{Na}_2\text{S}_4\text{O}_6$ is
 - 1.5
 - 2.5
 - 3
 - 2
- The brown ring complex compound of iron is formulated as $[\text{Fe}(\text{H}_2\text{O})_5(\text{NO})]\text{SO}_4$. The oxidation state of iron is
 - 1
 - 2
 - 3
 - 0
- Oxidation number of oxygen in F_2O is
 - 1
 - +1
 - +2
 - 2
- The oxidation state of nitrogen in N_3H is
 - $+\frac{1}{2}$
 - +3
 - 1
 - $-\frac{1}{3}$
- The oxidation state of chromium in $\text{Cr}(\text{CO})_6$ is
 - 0
 - +2
 - 2
 - +6
- Phosphorus has the oxidation state +3 in
 - orthophosphoric acid
 - phosphorous acid
 - metaphosphoric acid
 - pyrophosphoric acid
- In which of the following, increasing orders the oxidation number of oxygen has been arranged?
 - $\text{BaO}_2 < \text{O}_3 < \text{OF}_2 < \text{KO}_2$
 - $\text{BaO}_2 < \text{KO}_2 < \text{O}_3 < \text{OF}_2$
 - $\text{OF}_2 < \text{KO}_2 < \text{BaO}_2 < \text{O}_3$
 - $\text{KO}_2 < \text{OF}_2 < \text{O}_3 < \text{BaO}_2$
- The oxidation number of an element in a compound is evaluated on the basis of certain rules. Which of the following rules is not correct in this respect?
 - The oxidation number of hydrogen is always +1
 - The algebraic sum of all the oxidation numbers in a compound is zero
 - An element in the free or the uncombined state bears oxidation number zero
 - In all of its compounds, the oxidation number of fluorine is -1
- In which of the following pairs, there is greatest difference in the oxidation number of the underlined elements?
 - NO_2 and $\underline{\text{N}}_2\text{O}_4$
 - $\underline{\text{P}}_2\text{O}_5$ and $\underline{\text{P}}_4\text{O}_{10}$
 - $\underline{\text{N}}_2\text{O}$ and $\underline{\text{N}}\text{O}$
 - $\underline{\text{S}}\text{O}_2$ and $\underline{\text{S}}\text{O}_3$

23. A compound contains atoms A , B and C . The oxidation number of A is $+2$, of B is $+5$ and of C is -2 . The possible formula of the compound is
 a. ABC_3 b. $B_2(AC_3)_2$
 c. $A_3(BC_4)_2$ d. $A_3(BC_3)_2$
24. Which of the following reactions is a redox reaction?
 a. $\text{NaBr} + \text{HCl} \longrightarrow \text{NaCl} + \text{HBr}$
 b. $\text{HBr} + \text{AgNO}_3 \longrightarrow \text{AgBr} + \text{HNO}_3$
 c. $\text{H}_2 + \text{Br}_2 \longrightarrow 2\text{HBr}$
 d. $\text{Na}_2\text{O} + \text{H}_2\text{SO}_4 \longrightarrow \text{Na}_2\text{SO}_4 + \text{H}_2\text{O}$
25. White phosphorus reacts with caustic soda to form PH_3 and NaH_2PO_2 . This reaction is an example of
 a. oxidation b. reduction
 c. hydrolysis d. disproportionation
26. Following reaction is an example of

$$\text{Ag}^{2+}(\text{aq}) + \text{Ag}(\text{s}) \rightleftharpoons 2\text{Ag}^+(\text{aq})$$

 a. reduction b. oxidation
 c. comproportionation d. disproportionation
27. Which of the following is a redox reaction?
 a. Formation of glucose from CO_2 and water
 b. Reaction of potassium cyanide with silver cyanide
 c. Hydration of rubidium
 d. Reaction of barium chloride with sulphuric acid
28. Identify the disproportionation reaction.
 a. $\text{CH}_4 + 2\text{O}_2 \longrightarrow \text{CO}_2 + 2\text{H}_2\text{O}$
 b. $\text{CH}_4 + 4\text{Cl}_2 \longrightarrow \text{CCl}_4 + 4\text{HCl}$
 c. $2\text{F}_2 + 2\text{OH}^- \longrightarrow 2\text{F}^- + \text{OF}_2 + \text{H}_2\text{O}$
 d. $2\text{NO}_2 + 2\text{OH}^- \longrightarrow \text{NO}_2^- + \text{NO}_3^- + \text{H}_2\text{O}$
29. Which of the following is not an intermolecular redox reaction?
 a. $\text{MgCO}_3 \longrightarrow \text{MgO} + \text{CO}_2$
 b. $\text{O}_2 + 3\text{H}_2 \longrightarrow 2\text{H}_3\text{O}^+$
 c. $\text{K} + \text{H}_2\text{O} \longrightarrow \text{KOH} + \left(\frac{1}{2}\right)\text{H}_2$
 d. $\text{MnBr}_3 \longrightarrow \text{MnBr}_2 + \left(\frac{1}{2}\right)\text{Br}_2$
30. Consider the following reactions,
 I. $2\text{Mn}_2\text{O}_7 \longrightarrow 4\text{MnO}_2 + 3\text{O}_2$
 II. $\text{SnCl}_2 + 2\text{FeCl}_3 \longrightarrow \text{SnCl}_4 + 2\text{FeCl}_2$
 The given reactions are the examples of
 a. intermolecular redox reaction and intramolecular redox reactions respectively
 b. Both reactions I and II are intermolecular redox reactions
 c. Both reactions I and II are intramolecular redox reactions
 d. intramolecular redox reactions and intermolecular redox reaction respectively
31. For the redox reaction,

$$\text{MnO}_4^- + \text{C}_2\text{O}_4^{2-} + \text{H}^+ \longrightarrow \text{Mn}^{2+} + \text{CO}_2 + \text{H}_2\text{O},$$

 the correct coefficients of the reactants for the balanced reaction are

	MnO_4^-	$\text{C}_2\text{O}_4^{2-}$	H^+
a.	2	5	16
b.	16	5	2
c.	5	16	2
d.	2	16	5
32. Consider the following reaction,

$$\text{C}_2\text{H}_6(\text{g}) + n\text{O}_2 \longrightarrow \text{CO}_2(\text{g}) + \text{H}_2\text{O}(\text{l})$$

 In this equation, ratio of the coefficients of CO_2 and H_2O is
 a. 1 : 1 b. 2 : 3 c. 3 : 2 d. 1 : 3
33. Which of the following statements is true regarding the following balanced half-reaction?

$$\text{CN}^- \longrightarrow \text{CNO}^-$$

 a. Carbon is losing two electrons per atom
 b. Oxidation number of carbon increases from $+1$ to $+3$
 c. Oxidation number of nitrogen remains constant
 d. Both (a) and (c)
34. In the ionic equation,

$$\text{BrO}_3^- + 6\text{H}^+ + x\text{e}^- \longrightarrow \text{Br}^{3+} + 3\text{H}_2\text{O},$$

 the value of x is
 a. 6 b. 2 c. 4 d. 3
35. In the redox reaction,

$$x\text{KMnO}_4 + \text{NH}_3 \longrightarrow y\text{KNO}_3 + \text{MnO}_2 + \text{KOH} + \text{H}_2\text{O},$$

 x and y are
 a. $x = 4, y = 6$
 b. $x = 3, y = 8$
 c. $x = 8, y = 6$
 d. $x = 8, y = 3$
36. Consider the following reaction,

$$2\text{Fe}^{3+} + 2\text{I}^- \longrightarrow 2\text{Fe}^{2+} + \text{I}_2$$

 The half-reactions for the given reaction are
 a. $2\text{I}^- \longrightarrow \text{I}_2 + 2\text{e}^-$ and $\text{Fe}^{3+} + \text{e}^- \longrightarrow \text{Fe}^{2+}$
 b. $\text{I}_2 \longrightarrow 2\text{I}^- + 2\text{e}^-$ and $\text{Fe}^{2+} + \text{e}^- \longrightarrow \text{Fe}^{3+}$
 c. $\text{I}_2 \longrightarrow 2\text{e}^- + 2\text{I}^-$ and $\text{Fe}^{2+} \longrightarrow \text{Fe}^{3+} + \text{e}^-$
 d. None of the above

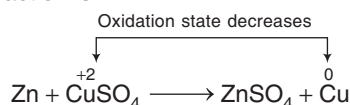
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- The ratio of oxidation states of Cl in potassium chloride to that in potassium chlorate is [2014]
 - $+\frac{1}{5}$
 - $-\frac{1}{5}$
 - $-\frac{2}{5}$
 - $+\frac{3}{5}$
- The oxidation state of sulphur in $\text{Na}_2\text{S}_4\text{O}_6$ is [2013]
 - +6
 - $+\frac{3}{2}$
 - $+\frac{5}{2}$
 - 2
- $2\text{MnO}_4^- + 5\text{H}_2\text{O}_2 + 6\text{H}^+ \longrightarrow 2\text{Z} + 5\text{O}_2 + 8\text{H}_2\text{O}$
Identify Z in the above reaction. [2012]
 - Mn^{2+}
 - Mn^{4+}
 - Mn
 - MnO_2
- The oxidation number of N and Cl in NOClO_4 respectively are [2011]
 - +2 and +7
 - +3 and +7
 - 3 and +5
 - +2 and -7
- Which one of the following reactions represent the oxidising property of H_2O_2 ? [2008]
 - $2\text{KMnO}_4 + 3\text{H}_2\text{SO}_4 + 5\text{H}_2\text{O}_2 \longrightarrow \text{K}_2\text{SO}_4 + 2\text{MnSO}_4 + 8\text{H}_2\text{O} + 5\text{O}_2$
 - $2\text{K}_3[\text{Fe}(\text{CN})_6] + 2\text{KOH} + \text{H}_2\text{O}_2 \longrightarrow 2\text{K}_4[\text{Fe}(\text{CN})_6] + 2\text{H}_2\text{O} + \text{O}_2$
 - $\text{PbO}_2 + \text{H}_2\text{O}_2 \longrightarrow \text{PbO} + \text{H}_2\text{O} + \text{O}_2$
 - $2\text{KI} + \text{H}_2\text{SO}_4 + \text{H}_2\text{O}_2 \longrightarrow \text{K}_2\text{SO}_4 + \text{I}_2 + 2\text{H}_2\text{O}$
- The species that undergoes disproportionation in an alkaline medium is [2006]
 - MnO_4^{2-}
 - ClO_4^-
 - NO_2
 - All of these
- A compound contains X, Y and Z atoms. The oxidation states of X, Y and Z are +2, +2 and -2 respectively. The possible formula of the compound is [2006]
 - XYZ_2
 - $\text{Y}_2(\text{XZ}_3)_2$
 - $\text{X}_3(\text{Y}_4\text{Z})_2$
 - $\text{X}_3(\text{YZ}_4)_3$
- The oxidation number of oxygen in hydrogen peroxide is [2005]
 - +1
 - 1
 - +2
 - 2

Answer with Solutions

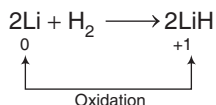
Practice Exercise

- (b) The reaction is



Hence, in this reaction, Cu^{2+} is reduced to Cu.

- (a) Fluorine is the strongest oxidising agent. Its reduction potential (E_{red}°) is +2.87V, hence it is easily reduced to F^- by gaining electron.
- (b) H_2S is the strongest reducing agent among these as the oxidation state of S is minimum in H_2S , i.e. -2.
- (b) NaHSO_3 gives NaHSO_4 during reaction in which the oxidation number of S increase from +4 to +6. Hence, it is a reducing agent.
- (a)

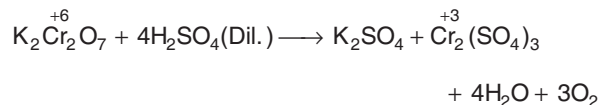


Hence, in this reaction, hydrogen act as oxidising agent.

- (d) Steam (H_2O) is reduced to hydrogen by iron.

- (b) $2\overset{+2}{\text{NO}} + 3\text{I}_2 + 4\text{H}_2\text{O} \longrightarrow 2\overset{+5}{\text{NO}_3^-} + 6\text{I}^- + 8\text{H}^+$
Hence, NO acts as a reducing agent and reduces I_2 to I^- since, the oxidation number of nitrogen changes from +2 in NO to +5 in NO_3^- .

- (c) In acidic medium, $\text{K}_2\text{Cr}_2\text{O}_7$ gives following reaction,



$\therefore$ Decrease in ON of Cr per atom = 3

$\therefore$ Decrease in ON of Cr per molecule = $2 \times 3 = 6$

Hence, equivalent weight of $\text{K}_2\text{Cr}_2\text{O}_7 = \frac{M}{6}$

- (b) $\overset{0}{\text{I}_2} + 2\text{S}_2\text{O}_3^{2-} \longrightarrow \overset{-1}{2\text{I}^-} + \text{S}_4\text{O}_6^{2-}$

$\therefore$ Decrease in ON of iodine per atom = 1

$\therefore$ Decrease in ON of iodine per molecule = $2 \times 1 = 2$

Hence, equivalent weight of iodine

$$\frac{\text{Molecular weight of iodine}}{\text{Total decrease in ON of iodine per molecule}} = \frac{M}{2}$$

10. (c) Oxidation is the process in which oxidation number of effective element in the species increases.

11. (b) ClO_3^-

$$\begin{aligned}x + 3(-2) &= -1 \\x - 6 &= -1 \\x &= 6 - 1 \\x &= +5\end{aligned}$$

12. (d) CH_2Cl_2 ,

$$\begin{aligned}x + 2(+1) + 2(-1) &= 0 \\x + 2 - 2 &= 0 \\x &= 0\end{aligned}$$

13. (b) Iodine shows -1 oxidation state in iodides (KI etc), $+1$, $+3$, $+5$ and $+7$ oxidation states in interhalogen compounds and polyhalides like ICl , ICl_3 , IF_5 , IF_7 etc.

14. (b) $\text{Na}_2\text{S}_4\text{O}_6$, $2(+1) + 4x + 6(-2) = 0$

$$\begin{aligned}4x + 2 - 12 &= 0 \\4x &= 10 \\x &= 2.5\end{aligned}$$

$\Rightarrow$

15. (b) $[\text{Fe}(\text{H}_2\text{O})_5\text{NO}]\text{SO}_4$,

$$\begin{aligned}x + 0 + 0 + (-2) &= 0 \\x &= +2\end{aligned}$$

16. (c) Fluorine is the most electronegative element and oxygen is the second one. Hence, in F_2O , oxygen is the positive element.

$$x + 2(-1) = 0$$

$\Rightarrow$

$$x = +2$$

Oxidation state of oxygen in F_2O (or OF_2) is $+2$.

17. (d) The oxidation state of hydrogen is always $+1$ except in ionic hydrides (where ON of hydrogen is -1).

N_3H (hydrogen acid) is not an ionic compound, hence

$$\begin{aligned}3x + (+1) &= 0 \\3x &= -1, \quad x = -1/3\end{aligned}$$

18. (a) CO (carbonyl) is a neutral ligand, hence oxidation state of Cr in $\text{Cr}(\text{CO})_6$ is zero.

19. (b) Orthophosphoric acid (H_3PO_4), ON of P = $+5$

Phosphorous acid (H_3PO_3), ON of P = $+3$

Metaphosphoric acid (HPO_3), ON of P = $+5$

Pyrophosphoric acid ($\text{H}_4\text{P}_2\text{O}_7$), ON of P = $+5$

20. (b) $\text{BaO}_2 < \text{KO}_2 < \text{O}_3 < \text{OF}_2$

$$\begin{array}{cccc}-1 & -0.5 & 0 & +2\end{array}$$

21. (a) Oxidation number of hydrogen is $+1$ except in ionic hydrides, where it is -1 .

22. (d) (a) NO_2 and N_2O_4 ; Difference = 0

- (b) P_2O_5 and P_4O_{10} ; Difference = 0

- (c) N_2O and NO ; Difference = $+2 - 1 = +1$

- (d) SO_2 and SO_3 ; Difference = $+6 - 4 = +2$

23. (c) The oxidation states of A, B and C are $+2$, $+5$ and -2 respectively.

(a) In ABC_3 , $2 + 5 + 3 \times (-2) = +1 \neq 0$

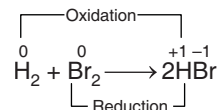
(b) In $\text{B}_2(\text{AC}_3)_2$, $2 \times (+2) + 2\{(+5) + 3(-2)\} = +2 \neq 0$

(c) In $\text{A}_3(\text{BC}_4)_2$, $3 \times (+2) + 2\{(+5) + 4(-2)\} = 0$

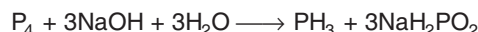
(d) In $\text{A}_3(\text{BC}_3)_2$, $3 \times (+2) + 2\{(+5) + 3(-2)\} = +4 \neq 0$

$\therefore$ Possible formula of compound is $\text{A}_3(\text{BC}_4)_2$.

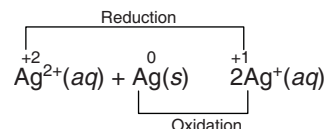
24. (c)



25. (d) Such reactions, in which a compound undergoes both oxidation and reduction simultaneously, are called disproportionation.

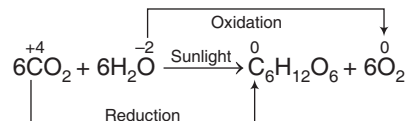


26. (c)



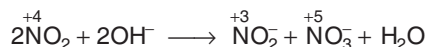
This is an example of comproportionation reaction which is reverse of disproportionation reaction.

27. (a)

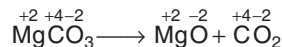


Since, oxidation and reduction both occurs simultaneously in the above equation, so it is a redox reaction.

28. (d) Disproportionation reaction,

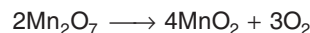


29. (a) Intermolecular redox reactions and redox reactions are same.



As there is no change in oxidation number of atoms, therefore, this reaction is not an intermolecular redox reaction.

30. (d) Same compound, e.g. Mn_2O_7 is dissociated into O_2 and MnO_2 , where oxidation and reduction take place, i.e.



It is an example of intramolecular redox reaction, while reaction, such as



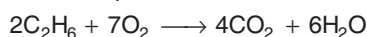
is an example of intermolecular redox reaction.

31. (a) $\text{MnO}_4^- + 8\text{H}^+ + 5\text{e}^- \longrightarrow \text{Mn}^{2+} + 4\text{H}_2\text{O}] \times 2$

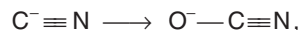


Thus, the coefficients of MnO_4^- , $\text{C}_2\text{O}_4^{2-}$ and H^+ in the above balanced equation respectively are 2, 5, 16.

32. (b) The balanced equation is



33. (d) In the given redox reaction,



Oxidation number of nitrogen is remaining unchanged at -3 .

Oxidation number of carbon is increasing from $+2$ to $+4$.

Hence, (a) and (c) are the only correct response.

Therefore, option (d) is correct.

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1. (b) Oxidation state of Cl in $\text{KCl} = -1$

Oxidation state of Cl in $\text{KClO}_3 = +5$

$$\therefore \text{Ratio of oxidation state of Cl} = \frac{-1}{5}$$

2. (c) Oxidation number of Na = $+1$

Oxidation number of O = -2

Let oxidation number of S = x

$$\therefore 2(\text{ON of Na}) + 4(\text{ON of S}) + 6(\text{ON of O}) = 0$$

$$2(+1) + 4x + 6(-2) = 0$$

$$+2 + 4x - 12 = 0$$

$$4x = +12 - 2$$

$$x = +\frac{10}{4}$$

$$\Rightarrow x = +\frac{5}{2}$$

3. (a) $2\text{MnO}_4^- + 5\text{H}_2\text{O}_2 + 6\text{H}^+ \longrightarrow 2\text{Mn}^{2+} + 5\text{O}_2 + 8\text{H}_2\text{O}$

4. (b) NOClO_4 is actually $\text{NO}^+\text{ClO}_4^-$.

Let the oxidation state of N in NO^+ is x .

$$\text{NO}^+: x + (-2) = +1$$

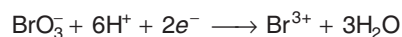
$$x = +1 + 2 = +3$$

Let the oxidation state of Cl in ClO_4^- is y .

$$\text{ClO}_4^-: y + (-2) \times 4 = -1$$

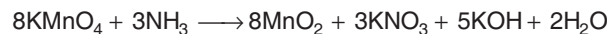
$$y - 8 = -1; y = +7$$

34. (b) Balanced reaction is

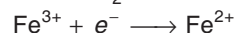
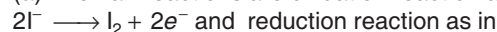


$\therefore$ Value of x is 2.

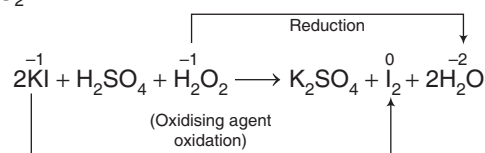
35. (d) The balanced reaction is as follows:



36. (a) The half-reactions are oxidation reaction as in



5. (d) The reaction in which H_2O_2 is reduced while the other reactant is oxidised, represents the oxidising property of H_2O_2 .



6. (c) $2\text{NO}_2(g) + 2\text{OH}^-(aq) \longrightarrow$



This reaction involves disproportionation of NO_2 (+4 state) into NO_2^- (+3 state) and NO_3^- (+5 state).

Therefore, this reaction is an example of disproportionation redox reaction.

7. (a) The oxidation number of X, Y and Z are +2, +2 and -2 respectively.

$$(a) \text{ In } \text{XYZ}_2 = (+2) + (+2) + 2(-2) = 0$$

$$(b) \text{ In } \text{Y}_2(\text{XZ}_3)_2 = 2(+2) + 2(+2) + 6(-2) \neq 0$$

$$(c) \text{ In } \text{X}_3(\text{YZ}_4)_2 = 3(+2) + 8(+2) + 2(-2) \neq 0$$

$$(d) \text{ In } \text{X}_3(\text{YZ}_4)_3 = 3(+2) + 3(+2) + 12(-2) \neq 0$$

8. (b) In H_2O_2 Let the ON of oxygen = x

$$\text{Then, } 2(+1) + 2(x) = 0$$

$$\text{or } 2 + 2x = 0$$

$$\text{or } 2x = -2$$

$$\text{or } x = -1$$

13

Electrochemistry

Electrochemistry is the study of production of electricity from energy released during spontaneous chemical reactions and the use of electrical energy to bring about non-spontaneous chemical transformations.

Section A : Conductor and Conductance

Electrical Conductors

Any substance, through which, we can pass electricity, is called electrical conductor.

Conductors are generally divided into two types with different properties which are tabulated as :

	Metallic conductors (e.g. metals)	Electrolytic conductors (e.g. NaCl)
(a)	In these, conduction takes place by the flow of electrons.	In these, conduction takes place by the flow of ions.
(b)	It does not involve transfer of matter.	It involves transfer of matter.
(c)	In these, conductance decreases with increase in temperature.	In these, conductance increases with increase in temperature.

Conductance and Conductivity of Electrolytic Solutions

The power of an electrolytes conduct electric current is called conductance or conductivity.

The resistance of any conductor varies directly as its length (l) and inversely as its cross-sectional area (a), i.e.

$$R \propto \frac{l}{a}$$

or

$$R = \rho \frac{l}{a}$$

where, ρ is called the resistivity or specific resistance.

If $l = 1 \text{ cm}$ and $a = 1 \text{ cm}^2$ then $R = \rho$

$\therefore$ Specific conductance (κ) = $\frac{1}{\rho} = \frac{l}{a} \times \text{conductance } (C)$

where, $\frac{l}{a}$ = cell constant

The unit of (κ) is $\text{ohm}^{-1} \text{cm}^{-1}$ or S cm^{-1} .

Molar Conductivity (λ_m or μ)

$$\lambda_m \text{ or } \mu = \frac{\kappa \times 1000}{M}$$

where, M = molarity of the solution

μ = molar conductivity

Its SI units are expressed in $\Omega^{-1} \text{cm}^2 \text{mol}^{-1}$ or $\text{mho cm}^2 \text{mol}^{-1}$ or $\text{S cm}^2 \text{mol}^{-1}$.

Equivalent Conductivity (λ_{eq} or Λ)

$$\lambda_{eq} \text{ or } \Lambda = \frac{\kappa \times 1000}{C_{eq} (N)}$$

where, C_{eq} = concentration of solution in equivalent per litre (i.e. normality)

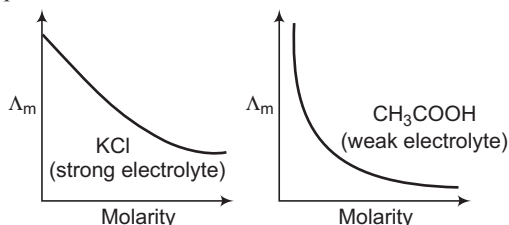
eq = equivalent conductivity

Its SI unit are expressed in $\Omega^{-1} \text{cm}^2 (\text{equiv}^{-1})$ or $\text{mho cm}^2 (\text{equiv}^{-1})$ or $\text{S cm}^2 (\text{equiv}^{-1})$.

Variation of Conductivity and Molar Conductivity with Concentration

Equivalent as well as molar conductivity $\propto$ dilution and specific conductivity $\propto \frac{1}{\text{dilution}}$

Strong electrolytes like KCl, have high value of conductance even at low concentration and there is no rapid increase in their equivalent or molar conductance on dilution.



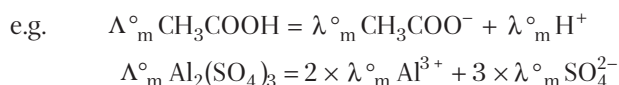
In case of **weak electrolytes**, like acetic acid, have a low value of conductance at high concentration and there is a rapid increase in the value of equivalent conductance (molar conductance) with dilution.

Limiting Molar Conductivity or Infinite Conductivity (Λ_m° or Λ_m^∞)

It is the molar conductivity of electrolyte when concentration of electrolyte approaches zero (i.e. at infinite dilution).

Kohlrausch's Law

It states that molar conductance at infinite dilution for any electrolyte is the sum of contribution of its constituent ions i.e. anions and cations.



Applications of Kohlrausch's Law

- (i) For the determination of equivalent/molar conductivity at infinite dilution.
- (ii) For the determination of degree of dissociation.

$$\text{Degree of dissociation } (\alpha) = \frac{\Lambda_m^c}{\Lambda_m^\circ}$$

- (iii) For the calculation of dissociation constant of a weak electrolyte.

$$K_a = \frac{C\alpha^2}{1-\alpha} = C\alpha^2 \quad [\because \text{For weak electrolyte, } \alpha \ll 1]$$

where, K_a = equilibrium dissociation constant

and C = molar concentration of weak electrolyte

- (iv) For the determination of solubility of sparingly soluble salt.

$$\text{Solubility} = \frac{\kappa \times 1000}{\Lambda_m^\circ}$$

- Transport number of cation (n_c)

$$= \frac{\text{Current carried by cation}}{\text{Total current}} = \frac{u_c}{u_c + u_a}$$

- Transport number of anion (n_a) = $\frac{u_a}{u_c + u_a}$

Section B : Conversion of Electrical Energy into Chemical Energy (Electrolytic Cell)

Electrolysis

It is a process in which electrical energy is used to bring some chemical changes. It is carried out in an electrolytic cell.

Electrolytic cell

⊕ Anode	⊖ Cathode
• Oxidation	• Reduction
• More E_{oxi}° , gets oxidised easily	• More E_{red}° , gets reduced easily

where, E_{oxi}° = standard oxidation potential

E_{red}° = standard reduction potential

- Decreasing order of oxidation potential :
 $I^- > Br^- > Cl^- > OH^- > F^- > NO_2^- > NO_3^- > SO_3^{2-} > SO_4^{2-}$
- Increasing order of reduction potential :
 All metals, more reactive than hydrogen
 $[E_{red}^\circ = \text{less than zero}] < \text{Hydrogen} [E_{red}^\circ = \text{zero}] < \text{metals less reactive than hydrogen} [E_{red}^\circ = +ve] \text{ like Cu, Ag etc.}$
- Besides the ions of electrolyte, if some cations or anions are present in the solution, then ions with lower discharge potential are discharged in preference to those which have high discharge potential.

Faraday's Laws of Electrolysis

There are two laws given by Faraday, are discussed below:

First Law

Deposited mass of the substance is directly proportional to passed charge in a voltameter.

$$w \propto Q$$

$$w = ZQ = Zit$$

where, w = mass, Q = charge (in coulomb)

i = current (in amperes)

t = time (in second)

Z = electrochemical equivalent

$$1 F = 6.022 \times 10^{23} \times 1.6 \times 10^{-19} = 96500 \text{ C (approx.)}$$

Number of gram equivalents = Number of Faraday's of electricity

Second Law

The number of equivalents of any substance produced by a given quantity of electricity during electrolysis, are same.

$$\frac{w_A}{w_B} = \frac{E_A}{E_B}$$

Section C : Conversion of Chemical Energy into Electrical Energy (Electrochemical or Galvanic Cells)

Electrochemical Cell

An electrochemical cell is a system or arrangement in which two electrodes are fitted in the same electrolyte or in two different electrolytes which are joined by a salt bridge.

Galvanic or Voltaic Cells

It is a device in which a redox reaction used to convert chemical energy into electrical energy.

Electrode and Half-Cell

A strip of metal, used in electrochemical studies, is called electrode. The combination of the metal electrode and solution, is called a half-cell.

	Electrolytic cell		Voltaic or Galvanic cell	
	Anode	Cathode	Anode	Cathode
Sign	+	-	-	+
Electron flow	Out	In	Out	In
Half-reaction	Oxidation	Reduction	Oxidation	Reduction

Salt-bridge

It is a U-shaped tube contains a gel permeated with a solution of an inert electrolyte such as Na_2SO_4 . The salt-bridge is necessary to complete the electrical circuit and to maintain electric neutrality in both compartments (by flow of ions).

Representation of a Cell

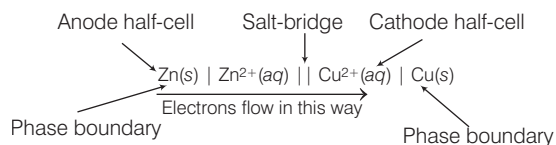
Consider the Daniell cell with following cell reaction :



In writing a cell diagram, following points are considered. We divide the cell into two half-cells.

	Anode	Cathode
Reaction	Oxidation takes place $Zn(s) \longrightarrow Zn^{2+}(aq) + 2e^-$	Reduction takes place $Cu^{2+}(aq) + 2e^- \longrightarrow Cu(s)$
Terminal	Negative	Positive
Side	LHS	RHS
Diagram	$Zn(s) / Zn^{2+}(aq)$	$Cu^{2+}(aq) / Cu(s)$

Complete cell diagram may be represented as follows :



If oxidised or reduced part is a gas, use Pt electrode saturated with that gas.

If oxidised and reduced parts are in ionic state, use Pt electrode.

EMF of Cell

The difference in electrode potential of electrodes due to different flow of electrons, is called electromotive force or cell potential of a cell. This is the driving force for all the cell reactions.

Standard Electrode Potential

Potential difference between the metal and the metal ion in which electrode is dipped, is called electrode potential denoted as E , which is related to density of negative electric charge.

When pressure is 1 bar and concentration is 1 M, electrode potential is called standard electrode potential denoted as E° . Temperature is generally taken as 298 K (i.e. 25°C).

For half-cell reaction $M^{n+} + ne^- \longrightarrow M$,

• If $E_{\text{oxi}}^\circ = x \text{ V}$, then $E_{\text{red}}^\circ = -x \text{ V}$

$E_{\text{Cu}^{2+}/\text{Cu}}^\circ = 0.34 \text{ V}$, then $E_{\text{Cu}^{2+}/\text{Cu}}^\circ = -0.34 \text{ V}$

• The SI unit of cell potential is the volt (V) and the potential of a galvanic cell is defined as the positive quantity.

$$E_{\text{cell}}^\circ = E_{\text{oxi}}^\circ + E_{\text{red}}^\circ, \quad E_{\text{cell}} = E_{\text{oxi}} + E_{\text{red}}$$

Electrochemical Series

The arrangement of metals in decreasing order of tendency to lose electrons, is called electrochemical series or it is the series in which the elements are arranged on the basis of the values of their standard reduction potentials at 25°C.

Element	Electrode reaction (Reduction)	Standard electrode reduction potential E° (volt)
Li	$\text{Li}^+ + e^- \longrightarrow \text{Li}$	-3.05
K	$\text{K}^+ + e^- \longrightarrow \text{K}$	-2.925
Ca	$\text{Ca}^{2+} + 2e^- \longrightarrow \text{Ca}$	-2.87
Na	$\text{Na}^+ + e^- \longrightarrow \text{Na}$	-2.714
Mg	$\text{Mg}^{2+} + 2e^- \longrightarrow \text{Mg}$	-2.37
Al	$\text{Al}^{3+} + 3e^- \longrightarrow \text{Al}$	-1.66
Zn	$\text{Zn}^{2+} + 2e^- \longrightarrow \text{Zn}$	-0.7628
Cr	$\text{Cr}^{3+} + 3e^- \longrightarrow \text{Cr}$	-0.74
Fe	$\text{Fe}^{2+} + 2e^- \longrightarrow \text{Fe}$	-0.44
Cd	$\text{Cd}^{2+} + 2e^- \longrightarrow \text{Cd}$	-0.403
Ni	$\text{Ni}^{2+} + 2e^- \longrightarrow \text{Ni}$	-0.25
Sn	$\text{Sn}^{2+} + 2e^- \longrightarrow \text{Sn}$	-0.14
H_2	$2\text{H}^+ + 2e^- \longrightarrow \text{H}_2$	0.00
Cu	$\text{Cu}^{2+} + 2e^- \longrightarrow \text{Cu}$	+0.337
I_2	$\text{I}_2 + 2e^- \longrightarrow 2\text{I}^-$	+0.535
Ag	$\text{Ag}^+ + e^- \longrightarrow \text{Ag}$	+0.799
Hg	$\text{Hg}^{2+} + 2e^- \longrightarrow \text{Hg}$	+0.885
Br_2	$\text{Br}_2 + e^- \longrightarrow 2\text{Br}^-$	+1.08
Cl_2	$\text{Cl}_2 + 2e^- \longrightarrow 2\text{Cl}^-$	+1.36
Au	$\text{Au}^{3+} + 3e^- \longrightarrow \text{Au}$	+1.50
F_2	$\text{F}_2 + 2e^- \longrightarrow 2\text{F}^-$	+2.87

Nernst Equation

The relationship between electrode potential and concentration of solution is called nernst equation.

$$E_{\text{cell}} \text{ or emf} = E_{\text{cell}}^\circ - \frac{2.303RT}{nF} \log Q$$

where, n = total number of electrons lost or gained

$$Q = \text{reaction coefficient of overall reaction} = \frac{[\text{OS}]}{[\text{RS}]}$$

where, [OS] = concentration of oxidised state

[RS] = concentration of reduced state

$$\therefore E_{\text{cell}} = E_{\text{cell}}^\circ - \frac{0.0591}{n} \log Q$$

- The emf of a standard cell does not change with temperature.
- The standard electrode potential of a half-cell has a fixed value. It does not change, if the half-reaction is multiplied with an integer.

Applications of Nernst Equation

There are two important applications of Nernst equation are given below:

(i) To find equilibrium constant

At equilibrium, $E_{\text{cell}} = \text{zero}$

$$\therefore E_{\text{cell}}^\circ - \frac{2.303RT}{nF} \log K = \frac{0.0591}{n} \log K$$

where, K = equilibrium constant

(ii) To find Gibbs free energy change

$$\Delta G^\circ = -n E_{\text{cell}}^\circ F$$

ΔG° = standard Gibbs free energy change

Relation between Standard Potentials of Half-cells Containing a Metal in Different Oxidation States

If two half-reactions having potentials E_1° and E_2° are combined to give a third half-reaction having a potential E_3° , then

$$E_3^\circ = \frac{n_1 E_1^\circ + n_2 E_2^\circ}{n_3}$$

Batteries

A cell or a battery (arrangement of one or more cells connected in series) is basically a galvanic cell and used where the chemical energy of redox reaction is converted into electrical energy.

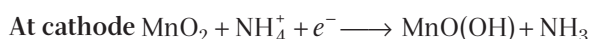
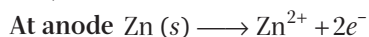
There are two types of batteries :

1. Primary Batteries

The primary batteries are those in which the cell reaction occurs only once and the battery becomes dead after the use over a period of time and cannot be reused again.

- (i) **Dry cell or Leclanche cell** is also called primary voltaic cell.

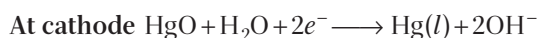
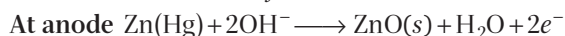
In it, the electrode reactions are



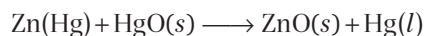
The cell potential is 1.6 V.

- (ii) Another type of dry cell is **mercury cell**.

The electrode reactions for the cell are



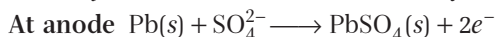
The overall cell reaction is



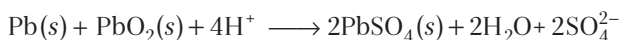
2. Secondary Batteries

These are also called **reversible galvanic** or **voltaic cell**. Secondary batteries are rechargeable because on charging, reaction becomes reverse, e.g. lead storage battery, nickel cadmium cell etc. In **lead storage battery**, a solution of sulphuric acid surrounds the plates and acts as electrolyte. The cell potential is 2 V.

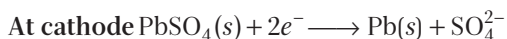
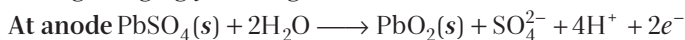
The half-cell reactions, when the battery is being used up are



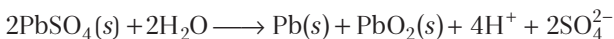
Overall reaction



During charging, following reactions occur



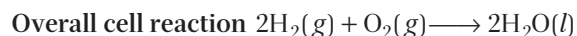
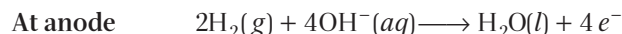
Overall reaction



Fuel Cells

These are another means by which chemical energy may be converted into electrical energy, e.g. H_2 - O_2 fuel cell. This cell was used as a primary source of electrical energy on the moon flights. The overall cell reaction produces water, which was used for drinking by the astronauts.

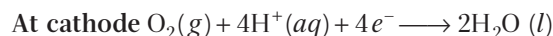
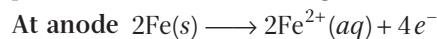
The half-reactions are



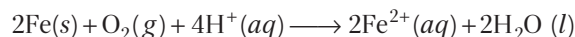
Corrosion and Its Prevention

Corrosion – metal is oxidised by loss of electrons to oxygen and form metal oxide, e.g. conversion of iron to rust $[\text{Fe}_2\text{O}_3 \cdot x\text{H}_2\text{O}]$.

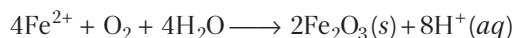
Corrosion of iron, known as **rusting**, occurs in the presence of water and oxygen.



Overall reaction



Fe^{2+} ions further oxidised by atmospheric oxygen to Fe^{3+} ions and form hydrated ferric oxide $[\text{Fe}_2\text{O}_3 \cdot x\text{H}_2\text{O}]$.

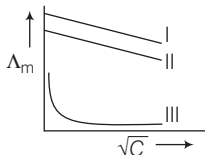


Hydrated ferric oxide (rust)

Rusting of iron can be prevented by the following methods :

- Barrier protection through coating of paints or electroplating.
- Galvanisation or coating of surface with tin metal.
- By the use of antirust solutions.

Practice Exercise

- Two electrodes are fitted in conductance cell 1.5 cm apart while the area of cross-section of each electrode is 0.75 cm^2 . The cell constant is
 a. 1.125 cm^{-1} b. 0.5 cm^{-1}
 c. 2.0 cm^{-1} d. 0.2 cm^{-1}
- Conductivity (unit siemens) is directly proportional to area of the vessel and the concentration of the solution in it and is inversely proportional to the length of the vessel, then the unit of constant of proportionality is
 a. Sm mol^{-1} b. $\text{Sm}^2 \text{ mol}^{-1}$
 c. $\text{S}^{-2} \text{ m}^2 \text{ mol}$ d. $\text{S}^2 \text{ m}^2 \text{ mol}^{-2}$
- Which of the following solutions has the highest equivalent conductance?
 a. 0.01 M NaCl b. 0.05 M NaCl
 c. 0.005 M NaCl d. 0.02 M NaCl
- The resistance of 0.1N solution of a salt is found to be $2.5 \times 10^3 \Omega$. The equivalent conductance of the solution is (Cell constant = 1.15 cm^{-1})
 a. $4.6 \text{ ohm}^{-1} \text{ cm}^2 \text{ equiv}^{-1}$ b. $5.6 \text{ ohm}^{-1} \text{ cm}^2 \text{ equiv}^{-1}$
 c. $6.6 \text{ ohm}^{-1} \text{ cm}^2 \text{ equiv}^{-1}$ d. $7.6 \text{ ohm}^{-1} \text{ cm}^2 \text{ equiv}^{-1}$
- The equivalent conductivity of a solution containing 2.54 g of CuSO_4 per litre, is $91.0 \Omega^{-1} \text{ cm}^2 \text{ eq}^{-1}$. Its conductivity would be
 a. $2.9 \times 10^{-3} \Omega^{-1} \text{ cm}^{-1}$ b. $1.9 \times 10^{-3} \Omega^{-1} \text{ cm}^{-1}$
 c. $2.4 \times 10^{-3} \Omega^{-1} \text{ cm}^{-1}$ d. $3.6 \times 10^{-3} \Omega^{-1} \text{ cm}^{-1}$
- Point out the correct statement.
 a. Equivalent conductance decreases with dilution
 b. Specific conductance increases with dilution
 c. Specific conductance decreases with dilution
 d. Equivalent conductance increases with increase in concentration
- The equivalent conductance of solution is
 [If cell constant is 1.25 cm^{-1} and resistance of $N/10$ solution is $2.5 \times 10^3 \Omega$].
 a. $2.5 \Omega^{-1} \text{ cm}^2 \text{ equiv}^{-1}$
 b. $2.5 \Omega^{-1} \text{ cm}^{-2} \text{ equiv}^{-1}$
 c. $50 \Omega^{-1} \text{ cm}^2 \text{ equiv}^{-1}$
 d. $5.0 \Omega^{-1} \text{ cm}^2 \text{ equiv}^{-1}$
- The increase in the molar conductivity of acetic acid with dilution is due to
 a. decrease in interionic forces
 b. increase in degree of ionisation
 c. increase in self ionisation of water
 d. None of the above
- The increase in the molar conductivity of HCl with dilution is due to
 a. increase in the self ionisation of water
 b. decrease in the self ionisation of water
 c. decrease in the interionic forces
 d. None of the above
- Molar conductance Λ_m is plotted against $\sqrt{C}$ (mol L^{-1}) for three electrolytes (NaCl, HCl, NH_4OH).


Which one of the following is correct?

I	II	III
a. NaCl	HCl	NH_4OH
b. HCl	NaCl	NH_4OH
c. NH_4OH	NaCl	HCl
d. NH_4OH	HCl	NaCl
- The plot of molar conductance vs $\sqrt{C}$ in strong electrolyte, is
 a. circular b. linear
 c. parabolic d. sinusoidal
- The values of $\Lambda_{\text{eq}}^\infty$ for NH_4Cl , NaOH and NaCl are 149.74, 248.1 and $126.4 \Omega^{-1} \text{ cm}^2 \text{ equiv}^{-1}$. The value of $\Lambda_{\text{eq}}^\infty$ of NH_4OH is
 a. $371.44 \Omega \text{ cm}^2 \text{ equiv}^{-1}$
 b. $271.44 \Omega \text{ cm}^2 \text{ equiv}^{-1}$
 c. $71.44 \Omega \text{ cm}^2 \text{ equiv}^{-1}$
 d. data is insufficient to calculate it
- The quantity of electricity required to liberate 0.1 g equivalent of an element at the electrode is
 a. 9650 C b. 96500 C
 c. 965 C d. 96.5 C
- On passing 3 A of electricity for 50 min, 1.8 g of metal deposits. The equivalent mass of metal is
 a. 20.5 b. 25.8 c. 19.3 d. 30.7
- One faraday of electricity will liberate one gram atom of a metal from a solution of
 a. AuCl_3 b. CuSO_4 c. BaCl_2 d. KCl
- The charge required for the reduction of 1 mole of $\text{Cr}_2\text{O}_7^{2-}$ ions to Cr^{3+} is
 a. 96500 C b. $2 \times 96500 \text{ C}$
 c. $3 \times 96500 \text{ C}$ d. $6 \times 96500 \text{ C}$
- On passing 0.1 F of electricity through aluminium chloride, the amount of aluminium metal deposited on cathode is (Atomic weight of Al = 27)

- a. 0.27 g b. 0.3 g c. 0.9 g d. 2.7 g
18. The current of 2 A is passed for 5 h through a molten tin salt to deposit 22.2 g tin. What is the oxidation state of tin in salt? [Atomic weight of Sn = 118.69 g]
a. +2 b. +5 c. +3 d. +4
19. What will be the weight of silver deposited, if 96.5 A of current is passed into aqueous solution of AgNO_3 for 100 s?
a. 1.08 g b. 10.8 g c. 108 g d. 1080 g
20. A certain amount of current liberates 0.5 g of H_2 in 2.0 h. How many gram(s) of oxygen can be liberated by the same current in the same time?
a. 0.5 g b. 8.0 g
c. 4.0 g d. 16.0 g
21. Pick the odd one out.
a. Daniell cell b. Voltaic cell
c. Galvanic cell d. Electrolytic cell
22. The EMF of a cell is
a. sum of two oxidation potentials
b. sum of two potentials
c. difference of two electrode potentials
d. None of the above
23. Consider the following reaction,
 $\text{Cu} | \text{Cu}^{2+} (1\text{M}) || (\text{Zn}^{2+} (1\text{M})) | \text{Zn}$
A cell represented above should have emf
a. positive b. negative
c. zero d. Cannot be predicted
24. The standard reduction potential of Pb and Zn electrodes are -0.126 and -0.763 V respectively. The cell equation will be
a. $\text{Pb}^{2+} + \text{Zn} \longrightarrow \text{Pb} + \text{Zn}^{2+}$
b. $\text{Pb}^{4+} + 2 \text{Zn} \longrightarrow \text{Pb} + 2 \text{Zn}^{2+}$
c. $\text{Zn}^{2+} + \text{Pb} \longrightarrow \text{Zn} + \text{Pb}^{2+}$
d. None of the above
25. Consider the following reaction,
 $\frac{1}{2} \text{H}_2(\text{g}) + \text{AgCl}(\text{s}) \longrightarrow \text{H}^+(\text{aq}) + \text{Cl}^-(\text{aq}) + \text{Ag}(\text{s})$
Above reaction occurs in the galvanic cell
a. $\text{Ag} | \text{AgCl}(\text{s}) | \text{KCl}(\text{aq}) || \text{AgNO}_3(\text{aq}) | \text{Ag}(\text{s})$
b. $\text{Pt} | \text{H}_2(\text{g}) | \text{HCl}(\text{aq}) || \text{AgNO}_3(\text{aq}) | \text{Ag}(\text{s})$
c. $\text{Pt} | \text{H}_2(\text{g}) | \text{HCl}(\text{aq}) || \text{AgCl}(\text{s}) | \text{Ag}(\text{s})$
d. $\text{Pt} | \text{H}_2(\text{g}) | \text{KCl}(\text{aq}) || \text{AgCl}(\text{s}) | \text{Ag}(\text{s})$
26. When the sample of copper with the zinc impurity is to be purified by electrolysis, the appropriate electrodes are

Cathode	Anode
a. Pure zinc	Pure copper
b. Impure zinc	Pure copper
c. Impure zinc	Impure sample
d. Pure copper	Impure sample
27. Cu^+ ion is not stable in aqueous solution because of disproportionation reaction. E° value of disproportionation of Cu^+ is
[$E^\circ_{\text{Cu}^{2+}/\text{Cu}^+} = +0.15$ V, $E^\circ_{\text{Cu}^{2+}/\text{Cu}} = 0.34$ V]
a. -0.38 V b. $+0.19$ V
c. -0.49 V d. $+0.38$ V
28. Calculate the emf of the following cell:
 $\text{Cu}(\text{s}) | \text{Cu}^{2+}(\text{aq}) || \text{Ag}^+(\text{aq}) | \text{Ag}(\text{s})$
Given that, $E^\circ_{\text{Cu}^{2+}/\text{Cu}} = 0.34$ V, $E^\circ_{\text{Ag}^+/\text{Ag}} = -0.80$ V
a. 0.046 V b. 0.46 V c. 0.57 V d. -0.46 V
29. A cell constituted by two electrodes $A(E^\circ_{\text{A}/\text{A}^+} = -0.35$ V) and $B(E^\circ_{\text{B}/\text{B}^+} = 0.42$ V). Calculate the emf of the cell.
a. 0.07 V b. 0.77 V
c. -0.77 V d. -0.07 V
30. Out of Cu, Ag, Fe and Zn, the metal which can displace all others from their salt solution is
a. Ag b. Cu c. Zn d. Fe
31. Which of the following solution will turn blue when placed in copper vessel?
a. AgNO_3 b. NaCl
c. ZnSO_4 d. KNO_3
32. A gas X at 1 atm is bubbled through a solution containing a mixture of 1 M Y^- and 1 M Z^- at 25°C . If the order of reduction potentials is $\text{Z} > \text{Y} > \text{X}$, then
a. Y will oxidise X but not Z
b. Y will oxidise Z but not X
c. Y will oxidise both X and Y
d. Y will reduce both X and Y
33. Using the standard electrode potential, find out the pair between which redox reaction is not feasible.
 E° values: $\text{Fe}^{3+}/\text{Fe}^{2+} = +0.77$, $\text{I}_2/\text{I}^- = +0.54$ V
 $\text{Cu}^{2+}/\text{Cu} = +0.34$ V, $\text{Ag}^+/\text{Ag} = +0.80$ V
a. Fe^{3+} and I^- b. Ag^+ and Cu
c. Fe^{3+} and Cu d. Ag^+ and Fe^{3+}
34. The emf of the cell involving the reaction
 $2\text{Ag}^+(\text{aq}) + \text{H}_2(\text{g}) \longrightarrow 2\text{Ag}(\text{s}) + 2\text{H}^+(\text{aq})$ is 0.80 V. The standard oxidation potential of silver electrode is
a. 0.80 V b. 0.40 V
c. -0.80 V d. 0.20 V
35. For a reaction $\text{A}(\text{s}) + 2\text{B}^+(\text{aq}) \longrightarrow \text{A}^{2+}(\text{aq}) + 2\text{B}$, K_c has been found to be 10^{12} . The E°_{cell} is
a. 0.354 V b. 0.708 V
c. 0.0098 V d. 1.36 V
36. The value of E_{cell} of hydrogen electrode at $\text{pH} = 0$, 298 K and 1 atm, is
a. 0.59 V b. 0 V
c. -0.59 V d. -0.059 V
37. The emf of a galvanic cell is positive when free energy change of reaction is
a. > 0
b. < 0
c. $= 0$

d. no relationship of free energy change and emf

38. What is the value of E°_{cell} in the following reaction?
 $\text{Cr} | \text{Cr}^{3+} (0.1\text{M}) || \text{Fe}^{2+} (0.01\text{M}) | \text{Fe}$

Given, $E^\circ_{\text{Cr}^{3+}/\text{Cr}} = -0.74\text{ V}$, $E^\circ_{\text{Fe}^{2+}/\text{Fe}} = -0.44\text{ V}$

- a. + 0.2606 V b. 0.5212 V
 c. + 0.1303 V d. - 0.2606 V

39. Consider the following reaction,

$\text{Zn(s)} + \text{Cu}^{2+}(0.1\text{M}) \longrightarrow \text{Zn}^{2+}(1\text{M}) + \text{Cu(s)}$ above
 reaction, taking place in a cell, E°_{cell} is 1.10 V. E_{cell} for
 the cell will be $\left(2.303 \frac{RT}{F} = 0.0591\right)$

- a. 1.80 V b. 1.07 V
 c. 0.82 V d. 2.14 V

40. The Edison storage cell is represented as

$\text{Fe(s)} | \text{FeO(s)} | \text{KOH(aq)} | \text{Ni}_2\text{O}_3(\text{s}) | \text{NiO(s)} | \text{Ni(s)}$
 the half-cell reactions are

$\text{Ni}_2\text{O}_3(\text{s}) + \text{H}_2\text{O(l)} + 2\text{e}^- \rightleftharpoons 2\text{NiO(s)} + 2\text{OH}^-;$
 $E^\circ = + 0.40\text{ V}$

$\text{FeO(s)} + \text{H}_2\text{O(l)} + 2\text{e}^- \rightleftharpoons \text{Fe(s)} + 2\text{OH}^-;$

$E^\circ = - 0.87\text{ V}$

What is the maximum amount of electrical energy that
 can be obtained from one mole of Ni_2O_3 ?

- a. 127 kJ b. 245.11 kJ
 c. 90.71 kJ d. 122.55 kJ

41. Consider the following cell reaction .

$\text{Cu(s)} + 2\text{Ag}^+(\text{aq}) \longrightarrow \text{Cu}^{2+}(\text{aq}) + 2\text{Ag(s)}$

$E^\circ_{\text{cell}} = 0.46\text{ V}$ By doubling the concentration of Cu^{2+} ,

E_{cell} is

- a. doubled
 b. halved
 c. increases but less than double
 d. decreases by a small fraction

42. The value of the reaction quotient, Q for the following
 cell is

$\text{Zn(s)} | \text{Zn}^{2+}(0.01\text{M}) || \text{Ag}^+(1.25\text{ M}) | \text{Ag(s)}$

- a. 156 b. 125 c. 1.25×10^{-2} d. 6.4×10^{-3}

43. The standard emf of a cell involving one electron
 change is found to be 0.591 V at 25°C. The
 equilibrium constant of the reaction is
 ($1F = 96500\text{ C mol}^{-1}$)

- a. 1.0×10^1 b. 1.0×10^5
 c. 1.0×10^{10} d. 1.0×10^{30}

44. Hydrogen electrode is placed in the solution whose pH
 is 10. The potential of this electrode will be

- a. + 0.591 V b. - 0.591 V
 c. 0 d. None of these

45. What is the Gibbs energy of the following reaction?

$\text{Zn(s)} + \text{Cu}^{2+}(\text{aq}) \longrightarrow \text{Zn}^{2+}(\text{aq}) + \text{Cu}^{2+}(\text{s});$

$E^\circ_{\text{cell}} = 1.1\text{ V}$

- a. 106135.75 J mol⁻¹ b. 21227 J mol⁻¹
 c. - 212.27 kJ mol⁻¹ d. 21227 J mol⁻¹

46. When a lead storage battery is charged, it acts as

- a. primary cell b. galvanic cell
 c. concentration cell d. electrolytic cell

47. In a hydrogen oxygen fuel cell, combustion of
 hydrogen occurs to

- a. produce high purity water
 b. remove adsorbed oxygen from electrode surface
 c. generate heat
 d. create potential difference between two electrodes

48. In which of the following, the corrosion of iron will be
 most rapid?

- a. In pure water
 b. In pure oxygen
 c. In air and moisture
 d. In air and saline water

49. On the basis of electrochemical theory of aqueous
 corrosion, the reaction occurring at the cathode is

- a. $\text{O}_2(\text{g}) + 4\text{H}^+(\text{aq}) + 4\text{e}^- \longrightarrow 2\text{H}_2\text{O(l)}$
 b. $\text{H}_2(\text{g}) + 2\text{OH}^-(\text{aq}) \longrightarrow 2\text{H}_2\text{O(l)} + 2\text{e}^-$
 c. $\text{Fe}^{2+}(\text{aq}) + 2\text{e}^- \longrightarrow \text{Fe(s)}$
 d. $\text{Fe}^{3+}(\text{aq}) + \text{e}^- \longrightarrow \text{Fe}^{2+}(\text{aq})$

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- A reaction, $\text{Cu}^{2+} + 2e^- \longrightarrow \text{Cu}$ is given. For this reaction, graph between E_{red} versus $\ln[\text{Cu}^{2+}]$ is a straight line of intercept 0.34 V, then the electrode oxidation potential of the half-cell Cu/Cu^{2+} (0.1 M) will be [2014]
 - 0.34
 - $0.34 + \frac{0.0591}{2}$
 - $-0.34 - \frac{0.0591}{2}$
 - $-0.34 + \frac{0.0591}{2}$
- If $E^\circ_{\text{Fe}^{3+}/\text{Fe}}$ and $E^\circ_{\text{Fe}^{2+}/\text{Fe}}$ are -0.36 V and 0.439 V respectively, then the value of $E^\circ_{\text{Fe}^{3+}/\text{Fe}^{2+}}$ is [2013]
 - $(-0.036 - 0.439)$ V
 - $[3(-0.36) + 2(0.439)]$ V
 - $(-0.36 - 0.439)$ V
 - $[(3 - 0.36) - 2(-0.439)]$ V
- The ionic conductance of Ba^{2+} and Cl^- respectively are 127 and $76 \Omega^{-1} \text{cm}^2$ at infinite dilution. The equivalent conductance (in $\Omega^{-1} \text{cm}^2$) of BaCl_2 at infinite dilution will be [2012]
 - $330 \Omega^{-1} \text{cm}^2$
 - $203 \Omega^{-1} \text{cm}^2$
 - $139.5 \Omega^{-1} \text{cm}^2$
 - $51 \Omega^{-1} \text{cm}^2$
- The equilibrium constant (K) for the reaction $\text{Cu}(s) + 2\text{Ag}^+(aq) \longrightarrow \text{Cu}^{2+}(aq) + 2\text{Ag}(s)$, will be [Given, $E^\circ_{\text{cell}} = 0.46$ V] [2011]
 - $K_c = \text{antilog } 15.6$
 - $K_c = \text{antilog } 2.5$
 - $K_c = \text{antilog } 1.5$
 - $K_c = \text{antilog } 12.2$
- E° for Fe/Fe^{2+} is $+0.44$ V and E° for Cu/Cu^{2+} is -0.32 V. Then, in the cell, [2011]
 - Cu oxidises Fe^{2+} ion
 - Cu^{2+} oxidises iron
 - Cu reduces Fe^{2+} ion
 - Cu^{2+} reduces Fe
- A current of 0.5 A when passed through AgNO_3 solution for 193 s deposited 0.108 g Ag. The equivalent weight of silver is [2010]
 - 108
 - 54
 - 10.8
 - 5.4
- When same quantity of electricity is passed through aqueous AgNO_3 and H_2SO_4 solutions connected in series, 5.4×10^{-2} g of H_2 is liberated. What is the mass of silver (in grams) deposited? (Equivalent weight of hydrogen = 1008, silver = 108) [2008]
 - 54
 - 0.54
 - 5.4
 - 10.8
- When electric current is passed through acidified water for 1930 s, 1120 mL of H_2 gas is collected (at STP) at the cathode. What is the current passed in amperes? [2008]
 - 0.05
 - 0.50
 - 5.0
 - 50
- Given, standard electrode potentials $\text{Fe}^{2+} + 2e^- \longrightarrow \text{Fe}$, $E^\circ = -0.440$ V
 $\text{Fe}^{3+} + 3e^- \longrightarrow \text{Fe}$, $E^\circ = -0.036$ V
 The standard electrode potential (E°) for $\text{Fe}^{2+} + e^- \longrightarrow \text{Fe}^{2+}$, is [2007]
 - $+0.772$ V
 - -0.772 V
 - $+0.417$ V
 - -0.417 V
- The specific conductivity of 0.1 N KCl solution is $0.0129 \Omega^{-1} \text{cm}^{-1}$. The resistance of the solution in the cell is 100Ω . The cell constant of the cell will be [2007]
 - 1.10
 - 1.29
 - 0.56
 - 2.80
- The cathodic reaction of a dry cell is represented as $2\text{MnO}_2(s) + \text{Zn}^{2+} + 2e^- \longrightarrow \text{ZnMn}_2\text{O}_4(s)$
 If there are 8 g MnO_2 in the cathodic compartment then the time for which the dry cell will continue to give current of 2 milliampere, is [2006]
 - 25.675 day
 - 51.35 day
 - 12.8 day
 - 6.423 day
- The standard reduction potential of the reaction $\text{H}_2\text{O} + e^- \longrightarrow \frac{1}{2} \text{H}_2 + \text{OH}^-$, at 298 K, is [2005]
 - $E^\circ = \frac{RT}{F} \ln K_w$
 - $E^\circ = \frac{RT}{F} \ln [p_{\text{H}_2}]^{1/2} [\text{OH}^-]$
 - $E^\circ = \frac{RT}{F} \ln \frac{[p_{\text{H}_2}]^{1/2}}{[\text{H}^+]}$
 - $E^\circ = -\frac{RT}{F} \ln K_w$

Answer with Solutions

Practice Exercise

1. (c) Cell constant (x) = $\frac{l}{a} = \frac{1.5}{0.75} = 2 \text{ cm}^{-1}$

2. (b) $\text{Sm}^2 \text{ mol}^{-1}$

3. (c)

4. (a) We know that, $k = C \left(\frac{l}{a} \right)$

$$= \frac{1}{2.5 \times 10^3} \times 1.155 \text{ cm}^{-1} \left(C = \frac{1}{R} \right)$$

$$\Lambda_{\text{eq}} = \frac{k \times 1000}{N} = \frac{1.15 \times 1000}{2.5 \times 10^3 \times 0.1}$$

$$= 4.6 \text{ ohm}^{-1} \text{ cm}^2 \text{ equiv}^{-1}$$

5. (a) $K = \Lambda_{\text{eq}} \times C = (91 \Omega^{-1} \text{ cm}^2 \text{ eq}^{-1}) \left(\frac{2.54}{159/2 \times 1000} \text{ eq} \right)$

$$= 2.9 \times 10^{-3} \Omega^{-1} \text{ cm}^{-1}$$

6. (c)

7. (d) $k = \frac{\text{Cell constant}}{\text{Resistance}} = \frac{1.25}{2.5 \times 10^3} = 5 \times 10^{-4} \Omega^{-1} \text{ cm}^{-1}$

$$\text{Equivalent conductance} = \frac{k \times 1000}{M}$$

$$= \frac{5 \times 10^{-4} \times 1000}{1/10}$$

$$= 5 \Omega^{-1} \text{ cm}^2 \text{ equiv}^{-1}$$

8. (b)

9. (c) Due to increase in volume of water, interionic forces of HCl will decrease. Hence, ionic mobility will increase and consequently, the molar conductivity will also increase.

10. (b)

11. (b) The plot of molar conductance vs $\sqrt{C}$ in strong electrolyte is linear.

12. (b) $\Lambda_{\text{eq}}^{\infty}(\text{NH}_4\text{OH}) = \Lambda_{\text{eq}}^{\infty}(\text{NH}_4\text{Cl}) + \Lambda_{\text{eq}}^{\infty} \text{ NaOH} - \Lambda_{\text{eq}}^{\infty}(\text{NaCl})$

$$= (149.74 + 248.1) - 126.4$$

$$= 271.44 \Omega \text{ cm}^2 \text{ equiv}^{-1}$$

13. (a) Electricity required to liberate 0.1 g equivalent of an element

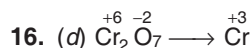
$$= 96500 \times 0.1 = 9650 \text{ C}$$

14. (c) We know that,

$$E = \frac{F \cdot w}{it} = \frac{96500 \times 1.8}{3 \times 50 \times 60} = 19.3$$

15. (d) $\text{KCl} \rightleftharpoons \text{K}^+ + \text{Cl}^-$ and $\text{K}^+ + e^- \longrightarrow \text{K}$

Thus, one mole KCl need one mole electrons to liberate one gram metal atom. One mole electron charge is equal to 96500 C (1 F). Hence, 1 F charge will liberate one gram atom of metal from solution of KCl.



Hence, electrons involved in this reduction will be

$$= 2 \times [(+6) - (+3)] = 2 \times 3 = 6$$

$\therefore$ Charge required for one mole = $6 \times 96500 \text{ C}$

17. (c)

18. (a) Equivalent of tin = $\frac{i \cdot t}{96500}$

or $\frac{22 \cdot 2}{\text{Eq. wt.}} = \frac{2 \times 5 \times 60 \times 60}{96500}$

$\therefore$ Eq. wt. = 59.5

$\therefore$ Valency of tin = $\frac{\text{At. wt.}}{\text{Eq. wt.}} = \frac{118.69}{59.5} = 2$ (an integer)

19. (b) $Q = it = 96.5 \times 100 = 9650 \text{ C}$

96500 C deposited weight of Ag = 108 g

9650 C deposited weight of Ag = $\frac{108 \times 9650}{96500} = 10.8 \text{ g}$

20. (c) According to Faraday's first law, $w = Zit$

Given, it is same for H_2 and O_2 .

$$\frac{W_{\text{H}_2}}{W_{\text{O}_2}} = \frac{Z_{\text{H}_2}}{Z_{\text{O}_2}} = \frac{E_{\text{H}_2}}{E_{\text{O}_2}}$$

$$\frac{0.500}{W_{\text{O}_2}} = \frac{2}{16}, W_{\text{O}_2} = 4.00 \text{ g}$$

21. (d)

22. (c)

23. (b) If E_{red}° of Cu and Zn are known from ECS.

than,

$$E^{\circ} = E_{\text{cathode}}^{\circ} - E_{\text{anode}}^{\circ}$$

$$= E_{\text{right}}^{\circ} - E_{\text{left}}^{\circ}$$

$$= (-0.76) - (+0.34) \text{ (From ECS)}$$

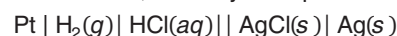
$$= -0.76 - 0.34 = -1.10 \text{ V}$$

24. (c) The E_{red}° of Zn is lower than that of Pb, hence Zn is a better reducing agent than Pb.

$\therefore$ The correct cell reaction will be

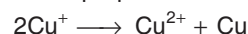


25. (c) H_2 undergoes oxidation and $\text{AgCl}(\text{Ag}^+)$ undergoes reduction. Therefore, cell may be represented as



26. (d) Impure sample is made of the anode and pure copper acts as the cathode.

27. (b) The reaction of disproportionation is



$$\therefore E_{\text{Cu}^+/\text{Cu}^{2+}}^{\circ} = E_{\text{Cu}^+/\text{Cu}^{2+}}^{\circ} + E_{\text{Cu}^{2+}/\text{Cu}}^{\circ}$$

$$= (-E_{\text{Cu}^{2+}/\text{Cu}^+}^{\circ}) + E_{\text{Cu}^{2+}/\text{Cu}}^{\circ}$$

$$= -0.15 + 0.34 = +0.19 \text{ V}$$

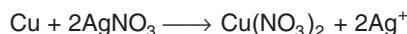
28. (b)

29. (b) Since, reduction potential of electrode, A is lower than that of B.

$$\begin{aligned}\text{We have, } E_{\text{cell}}^{\circ} &= E_{\text{cathode}}^{\circ} - E_{\text{anode}}^{\circ} \\ &= 0.42 - (-0.35) = 0.77 \text{ V}\end{aligned}$$

30. (c) The metal placed above in electrochemical series, can displace the metals placed below, from their salt solution.

Hence, Zn can displace Fe, Ag, Cu.

31. (a) Cu is placed above Ag in electrochemical series. Hence, Cu can displace Ag from AgNO_3 and itself is oxidised to Cu^+ (blue) ions.

32. (a) Greater the reduction potential, stronger is the oxidising agent. Hence, Y is stronger oxidising agent than X but weaker than Z.

33. (d) Ag^+ and Fe^{3+} are not feasible because they have almost same standard electrode potential.

34. (c) From the equation, given cell represented as



$$\begin{aligned}\text{Hence, } E_{\text{cell}} &= E_{\text{Ag}^+/\text{Ag}} - E_{\text{H}^+/\frac{1}{2}\text{H}_2} \\ \text{or } 0.80 &= E_{\text{Ag}^+/\text{Ag}} - 0.0\end{aligned}$$

$$\therefore E_{\text{Ag}^+/\text{Ag}} = 0.80 \text{ V}$$

$$\text{Hence, } E_{\text{oxi}} = -0.80 \text{ V}$$

35. (a) We know that $\Delta G^{\circ} = -2.303 RT \log K_C$

$$= -2.303 RT \log 10^{12} = -12 \times 2.303 RT$$

$$\text{Also, } \Delta G^{\circ} = -nFE_{\text{cell}}^{\circ}$$

$$\text{or } E_{\text{cell}}^{\circ} = \frac{2.303 RT}{nF} \times 12 = \frac{0.059}{2} \times 12; (n=2) = 0.354 \text{ V}$$

36. (b) $E_{\text{cell}} = E^{\circ} - \text{pH} \times 0.059 = 0.00 - (0) \times 0.059$

$$(\text{For hydrogen electrode, } E^{\circ} = 0)$$

$$= 0.0 \text{ V}$$

37. (b) $\Delta G = -nFE$ If $E = +\text{ve}$, then $\Delta G = -\text{ve}$ or < 0

38. (a) The cell reaction is



From Nernst equation,

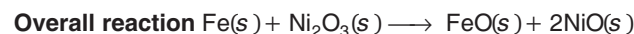
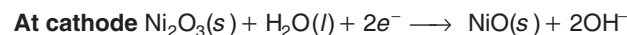
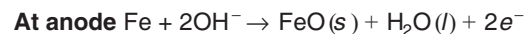
$$\begin{aligned}E_{\text{cell}} &= E_{\text{cell}}^{\circ} - \frac{0.059}{n} \log \frac{[\text{Cr}^{3+}]^2}{[\text{Fe}^{2+}]^3} \\ &= [-0.44 + 0.74] - \frac{0.059}{6} \times \log \frac{(0.1)^2}{(0.01)^3} \\ &= +0.2606 \text{ V}\end{aligned}$$

39. (b)

40. (b) Given, $E_{\text{FeO}/\text{Fe}}^{\circ} = -0.87 \text{ V}$ and $E_{\text{Ni}_2\text{O}_3/\text{NiO}}^{\circ} = 0.40 \text{ V}$

$$\therefore E_{\text{Fe}/\text{FeO}}^{\circ} = +0.87 \text{ V}$$

$$\text{and } E_{\text{NiO}/\text{Ni}_2\text{O}_3}^{\circ} = 0.40 \text{ V}$$

Since, E_{oxi}° for $\text{Fe}/\text{FeO} > E_{\text{oxi}}^{\circ}$ for $\text{NiO}/\text{Ni}_2\text{O}_3$ $\therefore$ Redox changes

$$\text{Hence, } E_{\text{cell}}^{\circ} = E_{\text{anode}}^{\circ} - E_{\text{cathode}}^{\circ} = 0.87 - (-0.40)$$

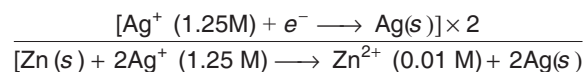
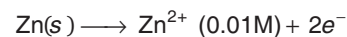
$$\text{or } E_{\text{cell}}^{\circ} = 1.27 \text{ V}$$

$$\begin{aligned}\text{and } \Delta G^{\circ} &= nFE_{\text{cell}}^{\circ} = 2 \times 1.27 \times 96500 \\ &= 245110 \text{ J} \\ &= 245.11 \text{ kJ}\end{aligned}$$

$$41. (d) E_{\text{cell}} = E_{\text{cell}}^{\circ} - \frac{RT}{nF} \ln \frac{[\text{Cu}^{2+}]}{[\text{Ag}^+]^2}$$

Doubling $[\text{Cu}^{2+}]$, decreases the emf by a small fraction.

42. (d) The cell reaction is



$$Q = \frac{[\text{Zn}^{2+}]}{[\text{Ag}^+]^2} = \frac{0.01}{(1.25)^2} = 6.4 \times 10^{-3}$$

43. (c) Relation between K_{eq} and E_{cell}° is

$$E_{\text{cell}}^{\circ} = \frac{0.0591}{n} \log K_{\text{eq}} (\text{at } 298 \text{ K})$$

$$0.591 = \frac{0.0591}{1} \log K_{\text{eq}}$$

$$\therefore K_{\text{eq}} = 1 \times 10^{10}$$

$$44. (b) E_{\text{H}^+/\frac{1}{2}\text{H}_2} = E_{\text{H}^+/\frac{1}{2}\text{H}_2}^{\circ} - \frac{0.0591}{1} \log \frac{1}{10^{-10}}$$

$$= -0.0591 \times 10 = -0.591 \text{ V}$$

45. (c) $\Delta_r G = -2 \times 1.1 \times 96487$

$$= -212.27 \text{ kJ mol}^{-1}$$

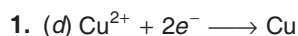
46. (d) When a lead storage battery is charged, it acts as an electrolytic cell.

47. (d) Fuel cell works when potential difference is developed.

48. (d) Corrosion is speed up by air, water, electrolytes and impurities in metal.

49. (a)

BITSAT Archives



$$E_{\text{Cu}^{2+}/\text{Cu}} = E^\circ_{\text{Cu}^{2+}/\text{Cu}} - \frac{0.059}{2} \log \frac{1}{[\text{Cu}^{2+}]}$$

$$= E^\circ_{\text{Cu}^{2+}/\text{Cu}} + \frac{RT}{2F} \ln [\text{Cu}^{2+}]$$

$$E_{\text{Cu}^{2+}/\text{Cu}} = 0.34 + \frac{0.059}{2} \log 0.1 = 0.31 \text{ V}$$

$$E_{\text{Cu}/\text{Cu}^{2+}} = -E_{\text{Cu}^{2+}/\text{Cu}} = -0.34 + \frac{0.059}{2} \text{ V}$$

2. (d) $E^\circ_{\text{Fe}^{3+}/\text{Fe}^{2+}} = 3E^\circ_{\text{Fe}^{3+}/\text{Fe}} - 2E^\circ_{\text{Fe}^{2+}/\text{Fe}}$
 $= [3 - 0.36] - 2(-0.439) \text{ V}$

3. (c) Λ_m^∞ for $\text{BaCl}_2 = \Lambda_m^\infty \text{Ba}^{2+} + 2\Lambda_m^\infty \text{Cl}^-$

$$\therefore \Lambda_{\text{eq}}^\infty \text{ for } \text{BaCl}_2 = \frac{1}{2} \Lambda_m^\infty \text{Ba}^{2+} + \Lambda_m^\infty \text{Cl}^-$$

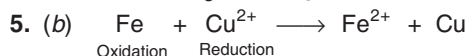
$$= \frac{1}{2} \times 127 + 76 = 139.5 \Omega^{-1} \text{ cm}^2$$

4. (a) $E^\circ = \frac{0.059}{n} \log K_C$

$$0.46 = \frac{0.059}{2} \log K_C$$

$$\log K_C = \frac{0.46 \times 2}{0.059} = 15.6$$

$$K_C = \text{antilog } 15.6$$



$$E_{\text{cell}}^\circ = E^\circ_{\text{Fe}/\text{Fe}^{2+}} + E^\circ_{\text{Cu}^{2+}/\text{Cu}}$$

$$= 0.44 + 0.32$$

$$= 0.76 \text{ V}$$

6. (a) $w = \frac{i \times t \times E}{F} \Rightarrow E = \frac{w \times F}{i \times t}$
 $= \frac{0.108 \times 96500}{0.5 \times 193} = 108$

7. (c) As the reduction potential of hydrogen is lower than that of sodium, it will be discharged at the cathode and chloride will be discharged at the anode.

8. (c) According to Faraday's second law of electrolysis,
 $\frac{\text{Weight of silver deposited}}{\text{Weight of hydrogen liberated}} = \frac{\text{Eq. wt. of silver}}{\text{Eq. wt. of hydrogen}}$

$$\frac{w}{5.04 \times 10^{-2}} = \frac{108}{1.008}$$

$$w = \frac{108 \times 5.04 \times 10^{-2}}{1.008} = 5.4 \text{ g}$$

9. (c) Number of moles of hydrogen collected

$$= \frac{1120 \times 10^{-3}}{22.4} = 0.05$$

$\therefore$ 1 mole of hydrogen is deposited by 2 moles of electrons.

$\therefore$ 0.05 mole of hydrogen will be deposited by 2×0.05
 $= 0.10$ mole of electrons

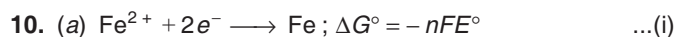
$$\text{Charge, } Q = nF$$

$$= 0.1 \times 96500$$

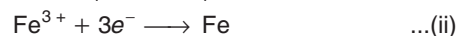
$$\text{Charge, } Q = it$$

$$0.1 \times 96500 = i \times 1930$$

$$i = \frac{0.1 \times 96500}{1930} = 5.0 \text{ A}$$



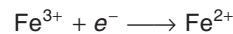
$$\Delta G^\circ = -2 \times F \times (-0.440 \text{ V}) = 0.880 F$$



$$\Delta G^\circ = -3 \times F \times (-0.036)$$

$$= 0.108 F$$

On subtracting Eq. (ii) from Eq. (i) we get



$$\Delta G^\circ = 0.108 F - 0.880 F = -0.772 F$$

$$E^\circ = \frac{-\Delta G^\circ}{nF} = \frac{(-0.772F)}{1 \times F} = +0.772 \text{ V}$$

11. (b) Specific conductivity (κ) = $\frac{1}{R} \times \text{cell constant}$

$$\text{Cell constant} = \kappa \times R = 0.0129 \times 100 = 1.29$$

12. (b) According to Faraday's law of electrolysis

$$m = \left(\frac{It}{F} \right) \frac{M}{Z}$$

or,

$$t = \frac{mZF}{IM}$$

The factor of equivalence can be found dividing the number of electrons by the number of reacted particles.

In case of MnO_2 , the factor is $Z = \frac{2}{2} = 1$

The molar mass of MnO_2 is $M (\text{MnO}_2) = 55 + 16 \times 2$
 $= 87 \text{ g/mol}$

$$\text{Now, } t = \frac{8 \times 1 \times 96500}{0.002 \times 87} = 4.436 \times 10^6 \text{ s} = 1232 = 51.3 \text{ days}$$

13. (a) The standard reduction potential of the cell reaction is

$$E^\circ = \frac{RT}{F} \ln K_w$$

14

Hydrogen

Occurrence of Hydrogen

Dihydrogen (H_2) is the most abundant element in the universe (70% of the total mass of the universe) and is the principal element in the solar atmosphere. The giant planets, Jupiter and Saturn consist of mainly hydrogen.

Position of Hydrogen in Periodic Table

The position of hydrogen in periodic table is uncertain as it shows resemblance with alkali metals as well as with halogens. However, on the basis of electronic configuration ($1s^1$), it is placed above lithium in the periodic table but still it is not considered as the member of that group. It is the lightest element known.

Isotopes and Allotropes of Hydrogen

Hydrogen has three isotopes : protium (1_1H), deuterium or heavy hydrogen (2_1H or D) and tritium (3_1H or T). Out of these isotopes, only tritium is radioactive and emits low energy of β -particles. These three isotopes have different masses, hence, their rates of reaction and equilibrium constants are different. This is known as **isotopic effect**.

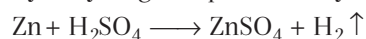
Dihydrogen or Hydrogen Gas (H_2)

Hydrogen being highly reactive, it exists in diatomic form, called the dihydrogen or hydrogen gas.

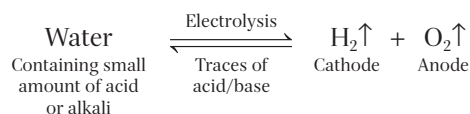
Methods of Preparation of Dihydrogen

Dihydrogen can be prepared by the following methods:

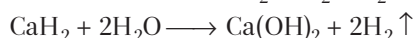
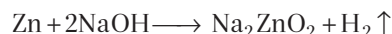
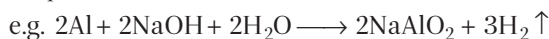
- (i) **Laboratory method** In laboratory, dihydrogen is produced by the reaction of Zn with dil. H_2SO_4 .



- (ii) **By the electrolysis of water** In electrolysis, acidified water is electrolysed by using platinum electrodes as



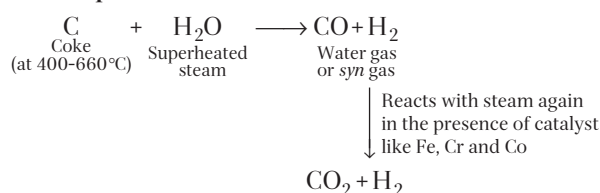
- (iii) High purity (> 99.95%) dihydrogen is obtained by electrolysing warm aqueous barium hydroxide solution between nickel electrodes.
- (iv) By the action of certain active metals on water or any other protic solvent.



- (v) **Industrial methods of preparation of dihydrogen**

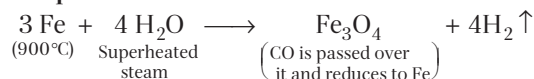
It involve following processes :

- (a) **Bosch process**



The CO_2 is removed either by washing under pressure of 25-30 atm or by reacting with potassium hydroxide.

- (b) **Lane process**



- (c) **In Nelson or Castner Kellner cell**, H_2 is also produced by the electrolysis of brine solution (NaCl) in them.

Properties

Physical Properties of Dihydrogen

Some physical properties are given below:

- (i) Hydrogen is a diatomic $\left(\gamma = \frac{C_p}{C_v} = 1.40\right)$, colourless, tasteless gas with no characteristic odour. It is lighter than air and insoluble in water.
- (ii) Its critical temperature is very low (-236.9°C). It can be liquefied at or below -236.9°C by application of suitable pressure and solidifies at -259.8°C .
- (iii) Certain metals like Ni, Pt, Pd, Fe, Au etc., can adsorb or occlude large volumes of H_2 gas at different temperatures (Pd can occlude 1000 times of its own volume).

Chemical Properties of Dihydrogen

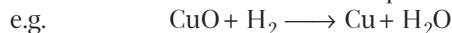
Important chemical reactions of dihydrogen are given below:

- (i) **Nature** Dihydrogen is neutral in nature.
- (ii) **Combustion** Dihydrogen is highly combustible and it burns with pale blue flame in the presence of air.
- $$2\text{H}_2 + \text{O}_2 \longrightarrow 2\text{H}_2\text{O}; \quad (\Delta H = -285.9 \text{ kJ mol}^{-1})$$
- (iii) **Reaction with non-metals** With non-metals (like O_2 , N_2 , F_2 , Cl_2 , Br_2 etc) hydrogen forms covalent

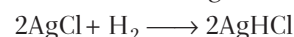
compounds (like H_2O , NH_3 , HX etc) at higher temperatures.

- (iv) **Reaction with metals** Under suitable condition, metals (like Li, Na, K, Cs, Ca etc) react with H_2 to form hydrides which are chiefly ionic, good reducing agents.

- (v) **Reducing action** Dihydrogen has the property to reduce the oxides of less electropositive elements.



Certain metal chlorides also get reduced as



However, hydrogen does not have the ability to reduce the oxides of more electropositive elements like alkali metals and alkaline earth metals.

Uses of Dihydrogen

The important uses of dihydrogen are as follows:

- (i) In the manufacturing of vanaspati fat by the hydrogenation of polyunsaturated vegetable oils like soyabean, cotton seeds etc.
- (ii) In oxyhydrogen and atomic hydrogen torches for cutting and welding of substances.
- (iii) Dihydrogen is considered as very good fuel as it does not cause any pollution and produces greater energy per unit mass of fuel as compared to gasoline or other fuels. H_2 - O_2 fuel cells generate electrical power with conversion efficiency of 70-85%.

Hydrides

Hydrogen combines with a number of metals and non-metals to form as hydrides.

Hydrides are mainly of three types :

Ionic or Saline Hydrides

These are formed by metals which are highly electropositive in character, i.e. alkali and alkaline earth metals (except Be and Mg), e.g. LiH , CaH_2 etc. These are crystalline solids with ionic lattices and behave like salts.

Molecular or Covalent Hydrides

These are formed by elements of comparatively higher electronegativity, generally p -block elements. In most cases, bonds are covalent in character, although in some cases (e.g. HF), bond is partly ionic in character. Molecules are held together by weak van der Waals' forces. These hydrides are soft, have low melting point and boiling point with low electrical conductivity.

These hydrides are of following three types:

- (i) Electron deficient hydrides
- (ii) Electron precise hydrides
- (iii) Electron rich hydrides

Metallic or Interstitial Hydrides

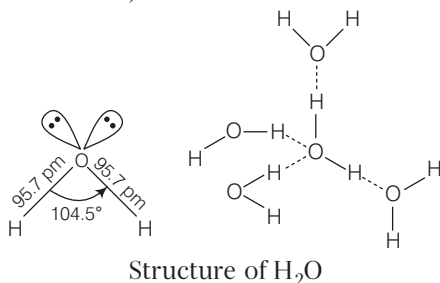
These are formed by Be, Mg (both are *s*-block elements) and transition (*d*-block) elements. These are interstitial compounds as the hydrogen atoms occupy interstitial places in the metal lattices. These are non-stoichiometric compounds (e.g. $\text{VH}_{0.56}$, $\text{NiH}_{0.6-0.7}$ etc.) and give out hydrogen easily and thus, used as reducing agents.

Water (H_2O)

Water is very essential for the existence of life. It constitutes about three-fourth of the earth's surface and two-third of human body.

Structure of Water

Water molecule is a bent molecule with a bond angle of 104.5° . In water molecules, the central atom (oxygen) is sp^3 -hybridised. Because of high electronegativity of oxygen, several H_2O molecules associated through hydrogen bonding ($\text{O}-\text{H}\cdots\text{O}$).



Ice has a highly ordered 3 dimensional H-bonded structure in which each oxygen atom is surrounded tetrahedrally by four other oxygen atoms.

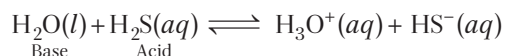
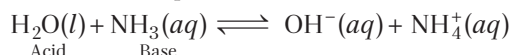
Physical Properties of Water

Water is colourless, odourless and tasteless liquid. Due to its high dielectric constant and polar nature, water is an excellent solvent for polar and ionic substances.

Chemical Properties of Water

The chemical properties of water are below:

- (i) It behaves as an amphoteric substance.



- (ii) It is oxidised to O_2 with fluorine.



- (iii) The association of water is essentially of 5 types namely coordinated water, hydrogen bonded water, lattice water, zeolitic water and clathrate water.

Hard and Soft Water

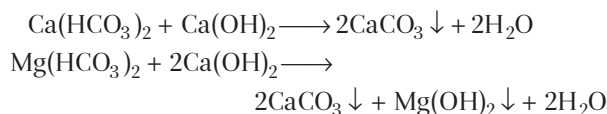
The water which lathers with soap is soft, if not, is hard.

Hardness of water is of two types:

- (i) Temporary hardness
- (ii) Permanent hardness

Temporary Hardness of Water

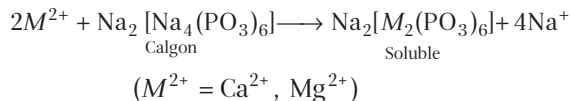
It is due to the presence of magnesium and calcium hydrogen carbonates. It can be removed by **Clark's method**, which involves the addition of calculated amount of lime to hard water.



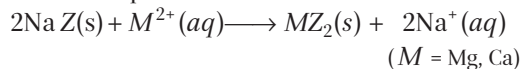
Permanent Hardness of Water

It is due to the presence of soluble salts of magnesium and calcium in the form of chlorides and sulphates.

It can be removed by the following methods:

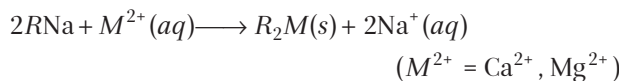


- **Ion-exchange method or Permutit process** is also used to remove permanent hardness.



where, NaZ is sodium aluminium silicate (NaAlSiO_4). This is also known as sodium zeolite. It is regenerated for further use by treating with an aqueous NaCl solution (brine).

- Cation exchange resins contain large organic molecules with $-\text{SO}_3\text{H}$ group and are water insoluble. Ion-exchange resin (RSO_3H) is changed to RNa by treating it with NaCl .



The resin can be regenerated by adding aqueous NaCl solution.

Heavy Water (D_2O)

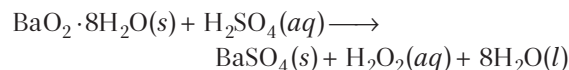
It has quite similar physical and chemical properties to those of H_2O . Dielectric constant of D_2O is lower than that of H_2O and rate of reactions are much slower than H_2O .

It is used as a moderator in nuclear reactions, as trace compound for studying reaction mechanism, for the preparation of deuterium.

Hydrogen Peroxide (H₂O₂)

It is a compound with an oxygen-oxygen single bond. It is also a strong oxidiser.

Preparation



H₂O₂ is commercially represented in terms of volume strength.

$$\begin{aligned} \text{(i) Volume strength} &= 5.6 \times \text{Normality} \\ &= 5.6 \times \frac{\text{Percentage strength}}{\text{Eq. wt. of H}_2\text{O}_2 \text{ (i.e. 17)}} \times 10 \end{aligned}$$

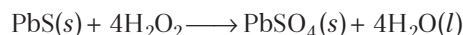
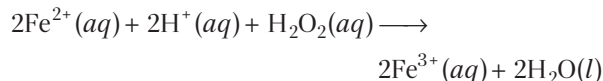
$$\begin{aligned} \text{(ii) Volume strength} &= 11.2 \times \text{Molarity} \\ &= 11.2 \times \frac{\text{Percentage strength}}{\text{Mol. wt. of H}_2\text{O}_2 \text{ (i.e. 34)}} \times 10 \end{aligned}$$

$$\text{(iii) \% strength} = \frac{17}{56} \times \text{Volume strength}$$

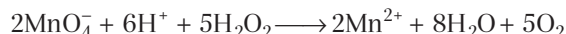
Chemical Properties

The chemical properties are shown below:

(i) Oxidising action in acidic medium



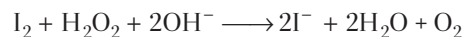
(ii) Reducing action in acidic medium



(iii) Oxidising action in basic medium

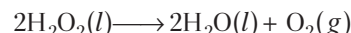


(iv) Reducing action in basic medium



Storage

H₂O₂ decomposes slowly on exposure to light.

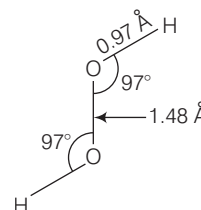


In the presence of metal surfaces or traces of alkali (present in glass containers), the above is catalysed. It is therefore, stored in wax lined glass or plastic vessels in dark. Urea can be added as a stabiliser.

Uses of H₂O₂

1. Aqueous solution of H₂O₂ is used as germicide, antiseptic, preservative for milk and wine, bleaching agent for soft materials.
30% H₂O₂ is called perhydrol. Its volume strength is 100 and molarity is 8.8.
2. It is used as an antichlor and in refreshing old oil paintings which became black.

Structure of H₂O₂



Practice Exercise

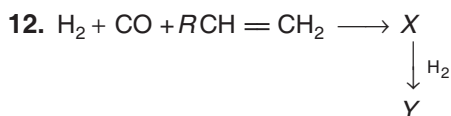
1. Why does H⁺ ion always get associated with other atoms or molecules?
 - a. Ionisation enthalpy of hydrogen resembles to that of alkali metals
 - b. Its reactivity is similar to halogens
 - c. It resembles both alkali metals and halogens
 - d. Loss of an electron from hydrogen atom results in a nucleus of very small size as compared to other atoms or ions. Due to small size, it cannot exist free
2. Which of the following explanation justifies for not placing hydrogen in either the group of alkali metals or halogens?
 - a. Ionisation energy of the hydrogen is too high for group of alkali metals and too low for the halogen group
 - b. Hydrogen atom does not contain any neutron
 - c. Hydrogen is much lighter than the alkali metals or halogens
 - d. Hydrogen can form compounds with almost all other elements
3. *Ortho* and *para* hydrogens differ in the
 - a. number of protons
 - b. molecular weight
 - c. nature of spins of protons
 - d. nature of spins of electrons
4. Consider the following statements about *ortho* and *para*-hydrogen
 - I. In *ortho*-hydrogen, the spin of protons are in the same direction.
 - II. *ortho*-hydrogen is more stable than the *para* form in the ambient condition.
 - III. At ordinary temperature, ordinary hydrogen is a mixture of about 75% *para* and 25% *ortho* forms.
 - IV. Two forms have similar chemical properties but differ in physical properties like specific heat and thermal conductivity.

Which of the statements given above are correct?

- a. I, II and III b. II, III and IV
c. I, III and IV d. I, II and IV
5. Which of the following reactions increase production of dihydrogen from synthesis gas?
 a. $\text{CH}_4(g) + \text{H}_2\text{O}(g) \xrightarrow{1270\text{ K}} \text{CO}(g) + 3\text{H}_2(g)$
 b. $\text{C}(s) + \text{H}_2\text{O}(g) \xrightarrow[1270\text{ K}]{\text{Ni}} \text{CO}(g) + \text{H}_2(g)$
 c. $\text{CO}(g) + \text{H}_2\text{O}(g) \xrightarrow[673\text{ K}]{\text{Catalyst}} \text{CO}_2(g) + \text{H}_2(g)$
 d. $\text{C}_2\text{H}_6 + 2\text{H}_2\text{O} \xrightarrow[1270\text{ K}]{\text{Ni}} 2\text{CO} + 5\text{H}_2$
6. The ionisation of hydrogen atom gives
 a. hydride ion b. hydronium ion
 c. proton d. hydroxyl ion
7. Pure H_2 is obtained by the action of
 a. aluminium with potassium hydroxide
 b. sodium hydride with water
 c. electrolysis of warm solution of $\text{Ba}(\text{OH})_2$ using Ni electrodes
 d. All of the above
8. Hydrogen adsorbed on palladium is known as
 a. atomic H b. ortho H c. occluded H d. heavy H
9. The chemical reactions of dihydrogen is accomplished by the
 a. loss of the only one electron to give H^+
 b. gain of an electron to form H^-
 c. sharing electrons to form a single covalent bond
 d. All of the above
10. Consider the following reactions,
 I. $\text{H}_2(g) + \text{X}_2(g) \longrightarrow 2\text{HX}(g)$ ($\text{X} = \text{F}, \text{Cl}, \text{Br}, \text{I}$)
 II. $2\text{H}_2(g) + \text{O}_2(g) \xrightarrow[\text{heating}]{\text{Catalyst or}} 2\text{H}_2\text{O}(l)$
 III. $3\text{H}_2(g) + \text{N}_2(g) \xrightarrow[200\text{ atm}]{673\text{ K, Fe}} 2\text{NH}_3(g)$
 IV. $\text{H}_2(g) + 2\text{M}(g) \longrightarrow 2\text{MH}(s)$ ($\text{M} = \text{alkali metal}$)
 V. $\text{H}_2(g) + \text{Pd}^{2+}(aq) \longrightarrow \text{Pd}(s) + 2\text{H}^+(aq)$

The correct reactions are

- a. II, III and V b. IV and V
c. I, II and IV d. All of these
11. Hydrogenation of vegetable oils using nickel as a catalyst gives edible fats is known as
 a. coconut oil
 b. soyabean oil
 c. margarine and vanaspati ghee
 d. vanaspati ghee



What are X and Y in the above reaction?

- a. $\text{RCH}_2\text{CH}_2\text{CHO}$ and $\text{RCH}_2\text{CH}_2\text{CH}_2\text{OH}$
 b. $\text{RCH}_2\text{CH}_2\text{CHO}$ and $\text{RCH}_2\text{CH}_2\text{OH}$

- c. $\text{CH}_3\text{CH}_2\text{CHO}$ and $\text{CH}_3\text{CH}_2\text{OH}$
 d. $\text{CH}_3\text{CH}_2\text{CH}_2\text{CHO}$ and $\text{CH}_3\text{CH}_2\text{CH}_2\text{OH}$

13. Dihydrogen is used in the
 I. manufacturing of nitric acid and nitrogenous fertilisers.
 II. manufacturing of vanaspati fat.
 III. manufacturing of methanol.
 IV. preparation of hydrogen chloride.

Choose the correct option.

- a. I, II and IV b. II, III and IV
c. I, II and III d. I, II, III and IV
14. H_2 gas is liberated at cathode and anode both by the electrolysis of the following aqueous solution except in
 a. NaH b. KH
 c. NaCl d. Both a and b
15. Hydrogen is prepared on large scale for an industrial use
 a. by $\text{Zn} + \text{H}_2\text{SO}_4$ b. by $\text{Al} + \text{NaOH}$
 c. by $\text{Na} + \text{C}_2\text{H}_5\text{OH}$ d. from water gas
16. Which of the following is incorrect statement?
 a. s-block elements, except Be and Mg, form ionic hydride
 b. BeH_4 , MgH_2 , CuH_2 , ZnH_2 , CaH_2 and HgH_2 are intermediate hydrides
 c. p-block elements form covalent hydride
 d. d and f-block elements form ionic hydride
17. The correct decreasing order of basic strength of hydrides is
 a. $\text{AsH}_3 > \text{SbH}_3 > \text{PH}_3 > \text{NH}_3$
 b. $\text{SbH}_3 > \text{AsH}_3 > \text{PH}_3 > \text{NH}_3$
 c. $\text{NH}_3 > \text{PH}_3 > \text{AsH}_3 > \text{SbH}_3$
 d. $\text{PH}_3 > \text{AsH}_3 > \text{SbH}_3 > \text{NH}_3$
18. The maximum possible number of hydrogen bonds in a water molecule can form in ice is
 a. 1 b. 2 c. 3 d. 4
19. Consider the following statements about intermolecular and intramolecular hydrogen bonds.
 I. Both types of H-bonds are temperature dependent.
 II. Intramolecular H-bond disappears on increasing the concentration.
 III. Intramolecular H-bond disappears on decreasing the concentration.
 IV. The boiling point of compounds having intramolecular H-bond are lower than that of those compounds which have intermolecular H-bond.

Which of the statements given above are correct?

- a. I, II and IV b. III and IV
c. I, III and IV d. I and II
20. The boiling point of water is high because
 a. water molecule is linear
 b. water molecule is not linear
 c. water molecule possess covalent bond between H and O
 d. water molecules associate due to H-bonding

21. When two ice cubes are pressed over each other, they united to form one cube. Which of the following forces are responsible to hold them together?
- Ionic interaction
 - van der Waals' forces
 - Covalent interaction
 - Hydrogen bond formation
22. If one assume linear structure instead of bent structure of water then which one of the following properties cannot be explained?
- Formation of intermolecular hydrogen bonding in water
 - High boiling point of water
 - Solubility of polar compounds in water
 - Ability of water to form coordinate covalent bonds
23. The strength of an oxo acid ($E—O—H$), where E is the central atom, depends upon the
- electronegativity of E
 - atomic size of E
 - ability of E to share electron pair with O
 - atomic size and electronegativity of E
24. Heavy water (D_2O) freezes at
- $0^\circ C$
 - $3.8^\circ C$
 - $-3.8^\circ C$
 - $38^\circ C$
25. Boiling point of heavy water is
- $100^\circ C$
 - $99^\circ C$
 - $101.4^\circ C$
 - $110^\circ C$
26. Which of the following statements are correct regarding D_2O and H_2O ?
- D_2O reacts with Al_4C_3 at a faster rate than does H_2O .
 - The freezing point of D_2O is higher than that of H_2O .
 - $NaCl$ is more soluble in D_2O than in H_2O .
 - Ionic product of D_2O is smaller than that of H_2O .
- Select the correct answer using the codes given below.
- I and II
 - I and III
 - II and III
 - II and IV
27. Mass percentage of deuterium in heavy water is
- same as that of protium in water
 - 11.1
 - 20.0
 - Cannot be predicted
28. A mixture of hydrazine (N_2H_4) and 58-60% solution of H_2O_2 is used as
- antiseptic
 - fertiliser
 - rocket fuel
 - None
29. Which of the following oxides is a peroxide?
- Na_2O_2
 - MnO_2
 - BaO
 - SO_2
30. Hydrogen peroxide is used as
- an oxidant only
 - a reductant only
 - an acid only
 - All of these
31. Which of the following statements are not correct?
- H_2O_2 oxidises Fe (II) to Fe (III)
 - H_2O_2 can be obtained by the electrolysis of dil. H_2SO_4
 - H_2O_2 reduces Mn (VII) to Mn (II)
 - H_2O_2 is a weak base

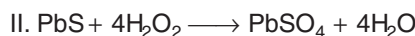
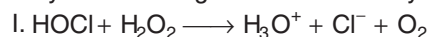
32. Which of the following is not correct regarding the electrolytic preparation of H_2O_2 ?
- Lead is used as cathode
 - 50% H_2SO_4 is used
 - Hydrogen is liberated at anode
 - Sulphuric acid undergoes oxidation
33. Match the Column I with Column II and select the correct option from the codes given below.

	Column I	Column II
A.	10 vol H_2O_2	1. Perhydrol
B.	20 vol H_2O_2	2. 5.358 N
C.	30 vol H_2O_2	3. 1.785 M
D.	100 vol H_2O_2	4. 3.03%

Codes

	A	B	C	D
a.	4	3	2	1
b.	1	2	3	4
c.	1	3	2	4
d.	4	2	3	1

34. Study the following reactions carefully



Point out the correct option.

- In (I), $HOCl$ is reduced and in (II), PbS is oxidised
 - In (I), $HOCl$ is oxidised and in (II), PbS is reduced
 - In both (I) and (II), $HOCl$ and PbS are reduced
 - In both (I) and (II), $HOCl$ and PbS are oxidised
35. 6 volume sample of H_2O_2
- would give 6 volumes of oxygen per unit volume of H_2O_2 sample at STP
 - will contain 6% V/V of H_2O_2
 - will contain 6% W/V of H_2O_2
 - would give 6 volumes of oxygen per unit weight of H_2O_2 sample at STP
36. When zeolite, which is hydrated sodium aluminium silicate, is treated with hard water, the sodium ions are exchanged with
- H^+ ion
 - Ca^{2+} ion
 - SO_4^{2-} ion
 - OH^- ion
37. 1000 g aqueous solution of $CaCO_3$ contains 10 g of calcium carbonate, hardness of the solution is
- 10 ppm
 - 100 ppm
 - 1000 ppm
 - 10000 ppm
38. 100 mL of tap water containing $Ca(HCO_3)_2$ was titrated with $\frac{N}{50}$ HCl with methyl orange as an indicator.
- If 30 mL of HCl was required, the temporary hardness of water, as parts of $CaCO_3$ per 10^6 parts of water was
- 150 ppm
 - 600 ppm
 - 275 ppm
 - 300 ppm
39. Excess of KI and dil. H_2SO_4 were mixed in 50 mL H_2O_2 . Thus, I_2 liberated requires 20 mL of 0.1 N $Na_2S_2O_3$. What will be the strength of H_2O_2 in $g L^{-1}$?
- 0.034
 - 0.68
 - 6.80
 - 5.80

40. H_2O_2 is marked '22.4 volume'. How much of it (in mL) are required to oxidise 3.4 g H_2S gas?
 a. 10 mL b. 70 mL c. 100 mL d. 1000 mL
41. Temporary hardness of water is caused due to the presence of
 a. CaSO_4 b. CaCl_2
 c. CaCO_3 d. $\text{Ca}(\text{HCO}_3)_2$
42. Both temporary and permanent hardness is removed on boiling water with
 a. CaSO_4 b. Na_2CO_3 c. CaCO_3 d. CaO
43. Which one of the following is used for reviving the exhausted permutit?
 a. HCl solution b. 10% CaCl_2 solution
 c. 10% MgCl_2 solution d. 10% NaCl solution

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1. Which one of the following is a covalent hydride? [2014]
 a. CaH_2 b. NaH c. BH_3 d. BeH_2
2. Calcium carbide reacts with heavy water to form [2013]
 a. C_2D_2 b. CaD_2 c. CaD_2O d. CD_2
3. H_2O_2 is always stored in black bottles because [2009]
 a. it is highly unstable
 b. its enthalpy of decomposition is high
 c. it undergoes autoxidation on prolonged standing
 d. None of the above
4. H_2O_2 used in rocket has the concentration [2007]
 a. 50%
 b. 70%
 c. 30%
 d. 90%
5. Calgon used as water softner is [2006]
 a. $\text{Na}_2[\text{Na}_4(\text{PO}_3)_6]$
 b. $\text{Na}_4[\text{Na}_2(\text{PO}_3)_6]$
 c. $\text{Na}_2[\text{Na}_4(\text{PO}_4)_5]$
 d. None of the above

Answer with Solutions

Practice Exercise

1. (d) 2. (a)
3. (c) *ortho* and *para* hydrogens differ in the nature of spins of protons.
4. (d) Statement (III) is incorrect at ordinary temperature (25°C), ordinary hydrogen contains 75% *ortho* and 25% *para* forms.
5. (c) $\text{CO}(\text{g}) + \text{H}_2\text{O}(\text{g}) \xrightarrow[\text{Catalyst}]{673\text{ K}} \text{CO}_2(\text{g}) + \text{H}_2(\text{g})$
6. (c) $\text{H}(\text{g}) \longrightarrow \text{H}^+(\text{g}) + e^-$
7. (d) 8. (c) 9. (d)
10. (d) All reactions are correct.
11. (c)
12. (a) Hydroformylation of olefins yields aldehydes which further undergo reduction to give alcohols.

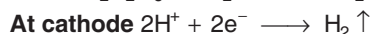
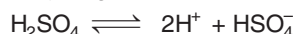
$$\text{H}_2 + \text{CO} + \text{RCH}=\text{CH}_2 \longrightarrow \text{RCH}_2\text{CH}_2\text{CHO}$$

$$\text{H}_2 + \text{RCH}_2\text{CH}_2\text{CHO} \longrightarrow \text{RCH}_2\text{CH}_2\text{CH}_2\text{OH}$$
13. (d)
14. (c) Salt ionisation-3
 (i) $\text{NaH} \rightleftharpoons \text{Na}^+ + \text{H}^-$
 (ii) $\text{HCOONa} \rightleftharpoons \text{HCOO}^- + \text{Na}^+$
 (iii) $\text{NaCl} \rightleftharpoons \text{Na}^+ + \text{Cl}^-$
 (iv) $\text{LiH} \rightleftharpoons \text{Li}^+ + \text{H}^-$
15. (d)
16. (d)
17. (c) The correct order of basic strength is
 $\text{NH}_3 > \text{PH}_3 > \text{AsH}_3 > \text{SbH}_3$
18. (d)
19. (a) Statements I, II and IV are correct.
20. (d) Water molecules are associated with H-bonding, due to which boiling point of water becomes high.
21. (d) Two ice cubes (solid H_2O) united to form one cube due to the formation of H-bond.
22. (b) The high boiling point of water cannot be explained on the basis of bent structure instead of linear structure.
23. (a) Higher electronegativity reduces the strength of acid.
24. (b) Heavy water freezes at 3.8°C .
25. (c) Boiling point of heavy water is 101.4°C .
26. (d) D_2O reacts slower than H_2O due to greater mass, its freezing point is 3.8°C . NaCl is less soluble in D_2O as in H_2O and D_2O does not dissociate therefore, its ionic product is smaller than H_2O .
27. (c) Formula of heavy water is D_2O , i.e. molecular mass = 20

$$\% \text{ of deuterium} = \frac{4 \times 100}{20}$$

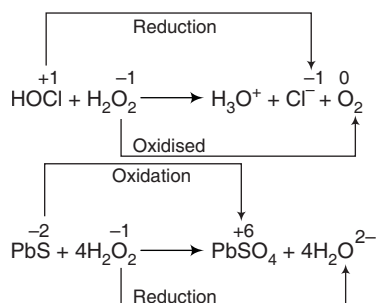
$$= 20\%$$

28. (c) A mixture of H_2O_2 and hydrazine (N_2H_4) is used as a rocket fuel.
29. (a) Na_2O_2 is a peroxide, all others are double oxides (MnO_2 , SO_2) and simple oxide (BaO).
30. (d) H_2O_2 acts as an oxidant, reductant and an acid (weak).
31. (d) H_2O_2 is not a weak base but a weak acid.
32. (c) H_2O_2 can be prepared by electrolysis of 50% H_2SO_4 . In this method, hydrogen is liberated at cathode.



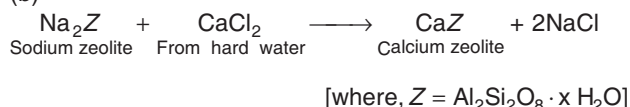
33. (a)

34. (a)



35. (a) 6 volume sample of H_2O_2 means 6 volumes of oxygen are given out per unit volume of H_2O_2 sample at STP.

36. (b)



37. (d) Hardness of solution in ppm = $\frac{10 \times 10^6}{1000} = 10^4$ ppm

38. (d) Meq. of CaCO_3 = Meq. of $\text{Ca}(\text{HCO}_3)_2$ = Meq. of HCl

$$\frac{w}{100/2} \times 1000 = \frac{1}{50} \times 30$$

$$w_{\text{CaCO}_3} = \frac{3 \times 100}{5 \times 2000} = 0.03 \text{ g}$$

$\therefore$ 100 mL of H_2O contains $\text{Ca}^{2+} = 0.03 \text{ g}$

$$10^6 \text{ mL } \text{H}_2\text{O} \text{ contains } \text{Ca}^{2+} = \frac{0.03 \times 10^6}{100} = 300 \text{ ppm}$$

39. (b) For 20 mL of H_2O_2

Meq. of KI = Meq. of H_2O_2 in 50 mL = Meq. of $\text{Na}_2\text{S}_2\text{O}_3$

$$\frac{w \times 1000}{34/2} = 20 \times 0.1$$

$$\therefore w_{\text{H}_2\text{O}_2} \text{ in 50 mL} = \frac{20 \times 0.1 \times 34}{2000} = 0.034$$

$$\therefore w_{\text{H}_2\text{O}_2} \text{ in 1000 mL} = \frac{0.034 \times 1000}{50} = 0.68 \text{ g/L}$$

40. (c) Density of 'x volume' $\text{H}_2\text{O}_2 = \frac{68 \times x}{22400} \text{ g/mL}$

$$\text{Density of '22.4 volume'} = \frac{68 \times 22.4}{22400} = 0.068 \text{ g/mol}$$



34 g H_2S is oxidised by = 68 g H_2O_2

$$3.4 \text{ g } \text{H}_2\text{S} \text{ is oxidised by} = \frac{68 \times 3.4}{34} = 6.8 \text{ g } \text{H}_2\text{O}_2$$

0.068 g H_2O_2 is present in = 1 mL

$\therefore$ 6.8 g H_2O_2 is present in = 100 mL

41. (d) Temporary hardness of water is caused due to the presence of bicarbonates of Ca and Mg.

42. (b) $\text{Ca}(\text{HCO}_3)_2 + \text{Na}_2\text{CO}_3 \longrightarrow \text{CaCO}_3 + 2\text{Na}(\text{HCO}_3)$



Ca^{2+} or Mg^{2+} ions are removed as insoluble carbonates.

43. (d)

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1. (c) Hydrides are binary compounds of hydrogen.

These can be classified into four groups :

(i) Ionic hydrides : NaH , CaH_2 , LiH

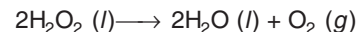
(ii) Covalent hydrides : B_2H_6 , NH_3 , SbH_3

(iii) Polynuclear hydrides : LiAlH_4 , NaBH_4

(iv) Interstitial hydrides are those in which hydrogen is trapped in the interstitial spaces of transition metals.

2. (a)

3. (c) H_2O_2 is always stored in black bottles because H_2O_2 decomposes into water and oxygen in the presence of light.



4. (d) H_2O_2 is used as an oxidant for rocket fuel and has 90% concentration to be used in rockets.

5. (a) Calgon used as water softener. The chemical composition is $\text{Na}_2[\text{Na}_4(\text{PO}_3)_6]$.

15

s-Block Elements

General Introduction of *s*-Block Elements

The components in the long form of the periodic table have been partitioned into four blocks in particular *s*, *p*, *d* and *f*- blocks. The elements of Group 1 (I A) and 2 (II A) have their last electron in *s*-orbital thus, they are called *s*-block elements.

Group 1 Elements : Alkali Metals

The Group 1 elements comprising Li, Na, K, Rb, Cs and Fr are commonly called alkali metals. Francium is radioactive and has a very short life (half-life of 21 min). These are known as alkali metals because their water soluble hydroxides are strongly alkaline. Sodium and potassium are abundant and lithium, rubidium and caesium have much lower abundance.

Electronic Configuration of Alkali Metals

The general electronic configuration of alkali metals is [noble gas] ns^1 ($n = 2$ to 7) .

Group 2 Elements : Alkaline Earth Metals

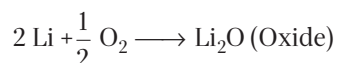
Group 2 elements are basically called alkaline earth metals as their oxides and hydroxides are alkaline in nature and these metal oxides are found in the earth's crust. Calcium and magnesium rank fifth and sixth in abundance respectively in the earth's crust. Strontium and barium have much lower abundances. Beryllium is rare and radium is the rarest of all comprising only 10^{-10} per cent of igneous rocks.

Electronic Configuration of Alkaline Earth Metals

The general electronic configuration of alkaline earth metals is [noble gas] ns^2 ($n = 2$ to 7).

Chemical Properties of s-Block Elements

- (i) **Reaction with oxygen** (air) Except Li, all alkali metals are tarnish in air due to oxide formation at their surface and hence, they are stored in kerosene or paraffin oil.



Superoxide O_2^- ion is stable only in the presence of large cations such as K, Rb, Cs.

Alkaline earth metals get slowly oxidised on exposure to air forming oxides. These oxides are basic in nature. Basic nature of oxides increases on moving down the group.

NOTE Oxide of Be, i.e. BeO is amphoteric in nature due to smaller size of Be^{2+} ion.

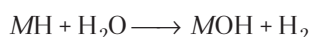
- (ii) **Basic strength of hydroxides** Hydroxides of alkali metals are strongly basic and their basic strength increases down the group. And hydroxides of alkaline earth metals are less basic than alkali metals of the corresponding period.



- (iii) **Reaction with hydrogen** Alkali metals react with hydrogen and form ionic hydrides, M^+H^- .

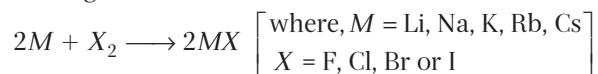


Alkali metal hydrides attacked by water to give hydrogen.



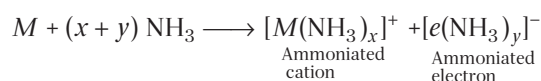
Except Be, all alkaline earth metals react with hydrogen to form ionic hydrides of MH_2 type. The hydride of Mg bears some covalent character in it due to its smaller size.

- (iv) **Reaction with halogens** Alkali metals combine readily with halogens to form halides.



Alkaline earth metals combine with halogens at elevated temperature forming halides, MX_2 . Beryllium halide is covalent while others are ionic. The solubility of halides (except fluoride) decreases on moving top to bottom in the group (Lower solubility of LiF in water is due to its higher lattice energy).

- (v) **Solubility in liquid ammonia** All alkali metals are soluble in liquid ammonia. Dilute alkali metals-ammonia solution is of blue colour. These solutions are strong reducing agents due to the presence of free electrons.



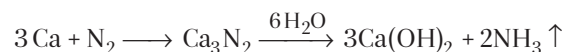
The high conductivity and paramagnetic properties along with blue colour is attributed to the presence of these ammoniated electron as well as ammoniated cation.

Alkaline earth metals are also soluble in liquid ammonia and give bright blue solutions. When these metal-ammonia solutions are evaporated, hexammoniates $[M(\text{NH}_3)_6]$ are formed. The hexammoniates slowly decompose to give amides.



Concentrated solutions of the metals in ammonia impart bronze colour due to the formation of metal clusters and it is diamagnetic.

- (vi) **Reaction with nitrogen and carbon**



- (vii) **Reactivity and electrode potential** Reducing character or reactivity of alkali metals increases from Li to Cs, i.e. $\text{Li} < \text{Na} < \text{K} < \text{Rb} < \text{Cs}$.

Reducing character or reactivity of alkaline earth metals increases from Be to Ba, i.e.



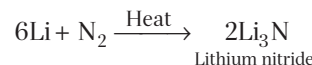
$\text{Mg}(\text{ClO}_4)_2$ is used as drying agent under the name anhydron, it is a strong oxidising agent. So, it is not used with organic material.

Anomalous Properties of Lithium

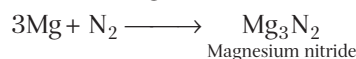
Lithium due to its

- exceptionally small size (like Mg) and
- high polarising power (i.e. charge/radius ratio) (like Mg) exhibits some properties which are different from those of the other members of group 1 but similar to that of magnesium (present diagonally in the following group, i.e. group 2). The property of showing similarity in properties with the element present diagonally opposite in the periodic table is called **diagonal relationship**. These properties are :

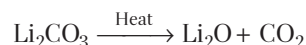
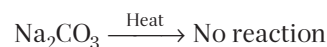
- (a) Alkali metals do not react with nitrogen except Li.

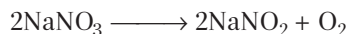
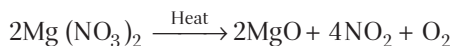
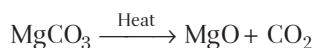


(Li metal is used as scavenger in metallurgy to remove O_2 and N_2 gases.)



- (b) Alkali metal carbonates, nitrates and hydroxides do not decompose on heating into their oxides except lithium.





(c) MgCl_2 and LiCl are deliquescent and crystallise as their hydrates, $\text{LiCl} \cdot 2\text{H}_2\text{O}$ and $\text{MgCl}_2 \cdot 2\text{H}_2\text{O}$.

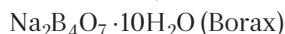
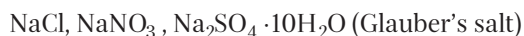
Anomalous behaviour of Beryllium

Beryllium due to its exceptionally small size (like Al) and high polarising power (like Al) shows similarities with aluminium in its properties and this relationship is called diagonal relationship. *These properties are :*

- BeO like Al_2O_3 , is amphoteric and covalent while oxides of other alkaline earth metals are ionic and basic in nature.
- Both BeCl_2 and AlCl_3 are soluble in organic solvents because of covalent nature and both have a bridged polymeric structure.
- Beryllium hydroxide dissolves in excess of alkali to give a beryllate ion, $[\text{Be}(\text{OH})_4]^{2-}$ just as aluminium hydroxide gives aluminate ion, $[\text{Al}(\text{OH})_4]^-$.
- Beryllium and aluminium ions have strong tendency to form complexes, BeF_4^{2-} , AlF_6^{3-} .
- Carbides of Be are covalent and react with water to produce methane gas whereas carbides of other members are ionic and produce acetylene with water.

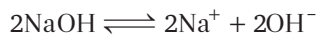
Sodium (Na)

Compounds found in nature



Extraction

1. Castner's Process



Oxygen and H_2O are produced during electrolysis.

2. Down's Process

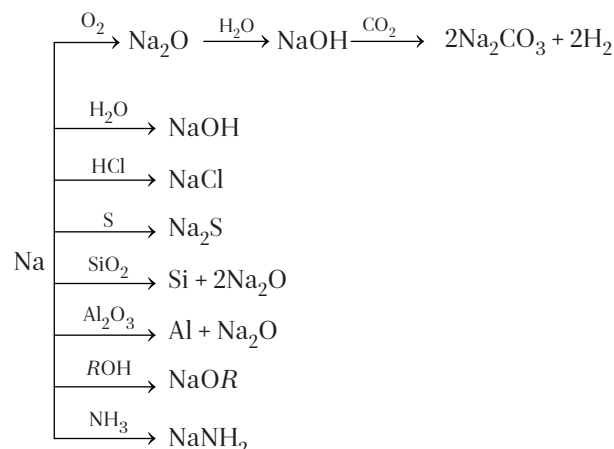
Cathode Fe

Anode Graphite



Fusion temperature is reduced to 600°C by adding CaCl_2 , KE.

Properties



Uses

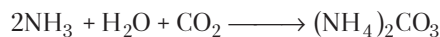
- For extraction C, Be, Mg and Si.
- As lab reagent.
- In high temperature thermometers.

Compounds of Sodium

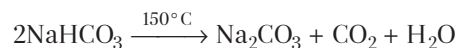
Sodium Carbonate

(Washing Soda, $\text{Na}_2\text{CO}_3 \cdot 10\text{H}_2\text{O}$)

Sodium carbonate is generally prepared by **Solvay process**.



Sodium hydrogen carbonate crystals separate out. These are heated to obtain sodium carbonate.



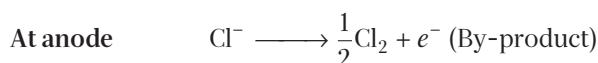
In this process, NH_3 is recovered when the solution containing NH_4Cl is treated with $\text{Ca}(\text{OH})_2$.

Uses of Na_2CO_3

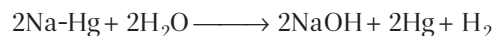
- In laundries and in softening of water as washing soda.
- In the manufacturing of glass, sodium silicate, paper, borax, soap powders, caustic soda, etc.
- In textile and petroleum refining.
- For the preparation of various carbonates of metals.

Sodium Hydroxide (Caustic Soda, NaOH)

Sodium hydroxide is prepared commercially by the electrolysis of sodium chloride in **Castner-Kellner cell** (or mercury cathode cell).



The amalgam is treated with water to give sodium hydroxide and hydrogen gas.



- Sodium hydroxide is white, translucent solid and deliquescent. It is called caustic soda because it breaks down proteins of skin to a pasty mass.
- On exposure to atmosphere, it absorbs moisture and CO_2 . $2\text{NaOH} + \text{CO}_2 \longrightarrow \text{Na}_2\text{CO}_3 + \text{H}_2\text{O}$
- It reacts with metals like Zn, Al, Sn and Pb and evolve H_2 gas.
e.g. $\text{Zn} + \text{NaOH} \longrightarrow \text{Na}_2\text{ZnO}_2 + \text{H}_2$
- It also reacts with metallic salts to form hydroxides. It is used in soap manufacturing, purification of bauxite, manufacturing of rayon, etc.

Sodium Chloride

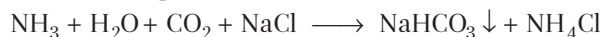
(Common salt or rock salt, NaCl)

The three major sources of NaCl are

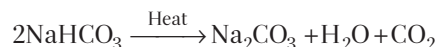
- Sea water (3% NaCl)
- Water of inland lake
- Salt mines
 - It is a white crystalline solid, soluble in water.
 - Pure NaCl is not hygroscopic. It shows hygroscopic nature due to the presence of chlorides of Ca and Mg as an impurity.
 - It is used as the starting material for the preparation of NaOH, Na_2CO_3 , NaHCO_3 , etc.
 - It is used as a preservative for meat, fish, etc. It is also used as a salting out of soap and in making freezing mixtures.

Sodium Hydrogen Carbonate (Baking Soda, NaHCO_3)

It is prepared as an intermediate product in the Solvay ammonia soda process.



- It is a white crystalline solid, sparingly soluble in water.
- Its aqueous solution is alkaline in nature due to hydrolysis.

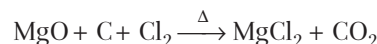
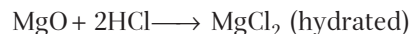
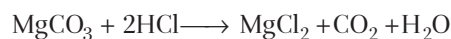


- It is used as a medicine to neutralise acidity in stomach.
- It is used as a constituent of baking powder and in fire extinguisher.

Compounds of Magnesium

Magnesium Chloride ($\text{MgCl}_2 \cdot 6\text{H}_2\text{O}$)

Preparation



Properties

It is a colourless, crystalline, deliquescent and highly soluble in water.

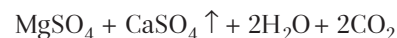
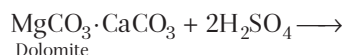
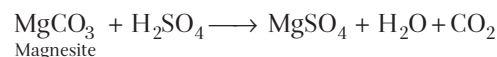


Uses

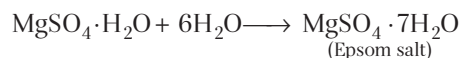
It is used in the preparation of metallic magnesium and in making Sorel cement.

Magnesium Sulphate ($\text{MgSO}_4 \cdot 7\text{H}_2\text{O}$)

Preparation

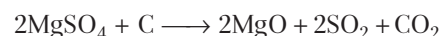
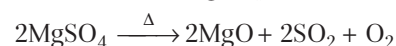
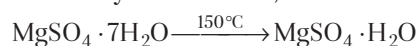


Commercial method



Properties

It is a colourless crystalline solid, soluble in water.



Uses

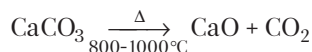
It is used in

- Medicines (as purgative).
- Dyeing and tanning processes.
- Ceramic, cement, match industries.
- Treatment of textile fibres to make them non-inflammable.

Compounds of Calcium

Quicklime (CaO)

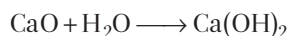
Preparation



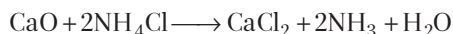
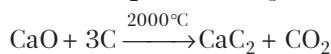
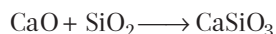
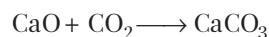
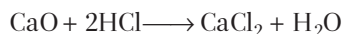
Properties

It is white amorphous powder with melting point 2600°C .

Action of water



Basic nature

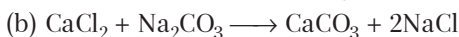
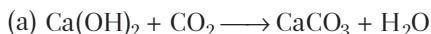


Uses

It is very useful as drying agent, in bleaching powder production (as used for the preparation of slaked lime and as a constituent of mortar).

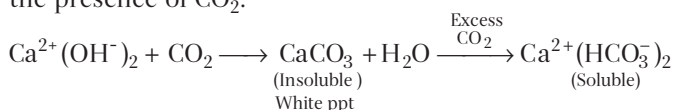
Calcium Carbonate (CaCO_3)

Preparation



Properties

CaCO_3 is a white powder, insoluble in water but dissolves in the presence of CO_2 .



Uses

It is used

1. in manufacturing paints, quicklime, etc.
2. in toothpastes and cosmetics.
3. as a building material.

Calcium Sulphate (Gypsum) ($\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$)

Preparation



Properties

- It is a white crystalline solid, sparingly soluble in water but dissolves in dilute acids.
- $2\text{CaSO}_4 \xrightarrow[\text{heating}]{\text{Strong}} 2\text{CaO} + 2\text{SO}_2 + \text{O}_2$
- $\text{CaSO}_4 + 4\text{C} \longrightarrow \text{CaS} + 4\text{CO}$

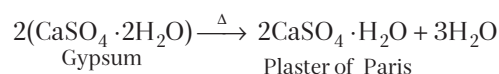
Uses

It is used

1. In the preparation of plaster of Paris.
2. In the manufacturing of cement, H_2SO_4 , $(\text{NH}_4)_2\text{SO}_4$, etc.
3. For glazing paper.

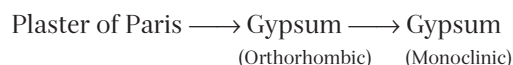
Plaster of Paris ($2\text{CaSO}_4 \cdot \text{H}_2\text{O}$)

Preparation



Properties

It is a white powder, which sets to a hard mass when its paste (with water) is allowed to stand.



Uses

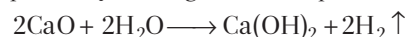
It is used

1. in making chalks, casts in dentistry, etc.
2. in surgery for setting broken/dislocated bones.
3. in making casts for statues, toys etc.

Calcium Hydroxide or Slaked Lime [Ca(OH)_2]

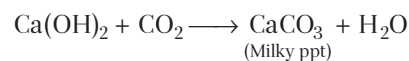
Preparation

It is prepared by adding water to quicklime as

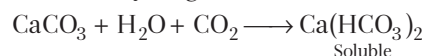


Properties

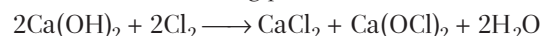
1. It is a white powder, sparingly soluble in water and the aqueous solution is called lime water while the aqueous suspension is called milk of lime. CO_2 has the ability to turn lime water milky as



This milkiness disappears with excess of CO_2 as soluble calcium hydrogen carbonate is formed.



2. Being basic in nature, it reacts with acids and acidic gases forming their salts.
3. On reaction with chlorine, it forms hypochlorite, a constituent of bleaching powder, as



Uses

Slaked lime is used in mortar production, lime water production, disinfectant, bleaching powder production, etc.

Portland Cement

It was discovered by an English Mason, Joseph Aspdin (1824). When the mixture of CaCO_3 , clay and water is heated strongly and allowed to stand, it hardens to a stone like mass.

It resembles with Portland rock (a famous building stone of England) and so, it is named as Portland cement. It is a mixture of aluminates and silicates of Ca with small amount of gypsum.

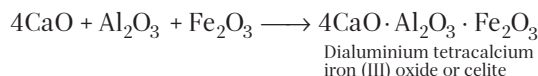
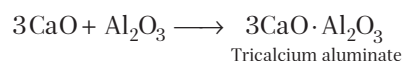
Chemical Composition

CaO = 50-60%, SiO₂ = 20-25%,

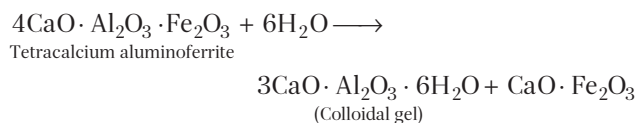
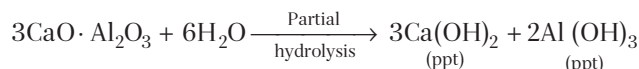
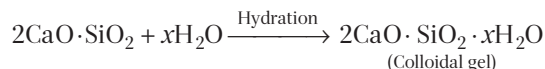
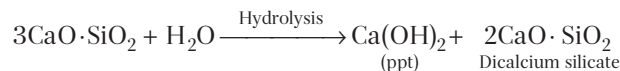
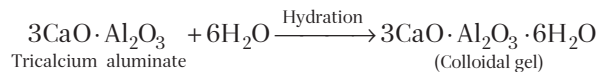
Al₂O₃ = 5-10%, MgO = 2-3%,

Fe₂O₃ = 1-2%, SO₃ = 1-2%

Formation of Constituents of Cement



Setting of Cement Paste of water and cement gets hard on keeping it for some time. This is called setting of cement. Mechanism of setting of cement is as follows:



Practice Exercise

- Which of the following ions forms a hydroxide which is highly soluble in water?
a. Ni²⁺ b. K⁺ c. Zn²⁺ d. Al³⁺
- Solubility of alkaline earth metal sulphates decreases down the group 2 because
a. they become more ionic
b. lattice energy of sulphates does not vary significantly
c. hydration energy decreases rapidly from Be²⁺ to Ba²⁺
d. lattice energy plays more predominant role than hydration energy
- Which of the following compounds has the lowest anion to cation size ratio?
a. LiF b. NaF c. CsI d. CsF
- Which is the most basic among the following?
a. Na₂O b. BaO c. As₂O₃ d. Al₂O₃
- The stability of the following alkali metal chlorides follows the order
a. LiCl > KCl > NaCl > CsCl b. CsCl > KCl > NaCl > LiCl
c. NaCl > KCl > LiCl > CsCl d. KCl > CsCl > NaCl > LiCl
- When CO is passed over solid NaOH heated to 200°C, it forms
a. Na₂CO₃ b. H₂CO₃ c. HCOONa d. All of these
- Which of the following does not illustrate the anomalous properties of lithium?
a. Li is much softer than the other group 1 metals
b. The melting point and boiling point of Li are comparatively high
c. Li forms a nitride Li₃N unlike group 1 metals
d. The ion of Li and its compounds are more heavily hydrated than those of rest of the group 1 elements
- Which of the following pairs cannot exist together in solution?
a. Na₂CO₃ and NaHCO₃ b. NaHCO₃ and NaOH
c. Na₂CO₃ and NaOH d. NaOH and NaCl
- Sodium peroxide, a yellow solid, when exposed to air becomes white due to the formation of
a. H₂O₂ b. Na₂O
c. Na₂O and O₃ d. NaOH and Na₂CO₃
- The products obtained on heating LiNO₃ will be
a. LiNO₂ + O₂ b. Li₂O + NO₂ + O₂
c. Li₃N + O₂ d. Li₂O + OH + O₂
- Identify the correct formula for halides of alkaline earth metals.
a. BaCl₂ · 6H₂O b. CaCl₂ · 6H₂O
c. BaCl₂ · 4H₂O d. SrCl₂ · 4H₂O
- LiF is almost insoluble in water whereas LiCl is soluble not only in water but also in acetone. This is because of
a. high hydration enthalpy of Li⁺
b. low hydration enthalpy of Li⁺
c. more ionic character in LiCl
d. None of the above
- Consider the following statements.
I. BeO is insoluble but BeSO₄ is soluble in water.
II. BaO is insoluble but BaSO₄ is soluble in water.
III. LiI is more soluble than KI in ethanol.
The true statements are
a. I and II b. I and III c. II and III d. I, II and III
- Which of the following statements is/are incorrect regarding the s-block elements?

- I. Francium is highly radioactive element.
 II. Oxides and hydroxides of alkali metals and alkaline earth metals are not alkaline in nature.
 III. Sodium and potassium are the only two *s*-block elements which are found in large proportion in biological fluids.
 IV. Biological function of *s*-block elements is due to maintenance of ion balance and nerve impulse condition.
- a. Only I b. I and III c. II and III d. Only III

15. Match the Column I with Column II and choose the correct option from the codes given below.

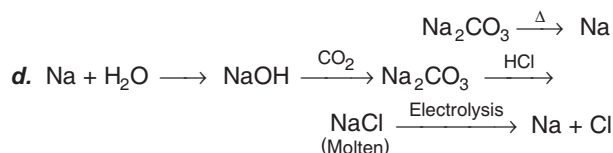
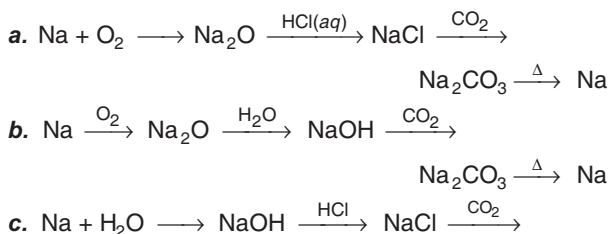
Column I	Column II
A. Sodium	1. Present in biological fluid
B. Beryllium	2. Radioactive element
C. Francium	3. Lower abundance
D. Calcium	4. Alkali metal
	5. Alkaline earth metal

Codes

	A	B	C	D		A	B	C	D
(a)	1	4	2	5	(b)	1	2	3	4
(c)	3	1	2	5	(d)	4	5	2	1

16. Which of the following is/are correct pair(s) regarding diagonally related elements?
 a. Beryllium and aluminium b. Lithium and magnesium
 c. Sodium and aluminium d. Both a. and b.
17. The decomposition temperature is maximum for
 a. MgCO_3 b. CaCO_3
 c. BaCO_3 d. SrCO_3
18. Which of the following metal salts has highest conductivity in aqueous medium?
 a. Li^+ b. Cs^+ c. Na^+ d. K^+
19. In case of alkali metals, the covalent character decreases in the order
 a. $\text{MI} > \text{MBr} > \text{MCl} > \text{MF}$
 b. $\text{MCl} > \text{MI} > \text{MBr} > \text{MF}$
 c. $\text{MF} > \text{MCl} > \text{MBr} > \text{MI}$
 d. $\text{MF} > \text{MCl} > \text{MI} > \text{MBr}$
20. Correct order of thermal stability of K_2CO_3 , CaCO_3 , MgCO_3 and BeCO_3 is
 a. $\text{BeCO}_3 < \text{MgCO}_3 < \text{CaCO}_3 < \text{K}_2\text{CO}_3$
 b. $\text{K}_2\text{CO}_3 < \text{MgCO}_3 < \text{CaCO}_3 < \text{BeCO}_3$
 c. $\text{MgCO}_3 < \text{BeCO}_3 < \text{CaCO}_3 < \text{K}_2\text{CO}_3$
 d. $\text{BeCO}_3 < \text{MgCO}_3 < \text{K}_2\text{CO}_3 < \text{CaCO}_3$

21. Which of the following sequence of chemical reaction is correct?



22. The ore of potassium is
 a. bauxite b. dolomite c. carnallite d. cryolite
23. Which of the following is duralumin alloy?
 a. $\text{Mg} + \text{Cu} + \text{Al} + \text{Mn}$ b. $\text{Mg} + \text{Cu}$
 c. $\text{Cu} + \text{Al}$ d. $\text{Mg} + \text{Al}$
24. When sodium is reacted with limited supply of oxygen, the main compound formed is
 a. Na_2O_4 b. Na_2O c. Na_3O d. Na_2O_2
25. Metallic magnesium is prepared by
 a. electrolysis of molten MgCl_2
 b. displacement of Mg by iron from MgSO_4 solution
 c. electrolysis of aqueous solution of $\text{Mg}(\text{NO}_3)_2$
 d. reduction of MgO by coke
26. The element which is not present in asbestos, is
 a. Si b. Ca c. Ba d. Mg
27. The molarity of Na^+ , when the average concentration of Na^+ in human blood serum is about 3.4 g L^{-1} , is
 a. 8.4 b. 2.3 c. 0.68 d. 0.15
28. Caesium is the most reactive metal in the alkali metal series because
 a. it exerts considerable attractive force on valence electrons
 b. it is a heavier metal
 c. its incomplete shell is nearest to the nucleus
 d. its valence electron has the highest principal quantum number than the valence electron of any of the others
29. When oxygen reacts with potassium, which of the following is preferentially formed?
 a. K_2O_4 b. KO_2
 c. K_2O_3 d. K_2O
30. Which of the following is different from other three oxides?
 a. Cr_2O_3 b. MgO
 c. SnO d. ZnO
31. Nitrogen dioxide cannot be prepared by heating
 a. KNO_3 b. $\text{Pb}(\text{NO}_3)_2$
 c. $\text{Cu}(\text{NO}_3)_2$ d. AgNO_3
32. The correct decreasing order of ionisation enthalpy of alkali metals is
 a. $\text{Na} > \text{Li} > \text{K} > \text{Rb}$ b. $\text{Rb} < \text{Na} < \text{K} < \text{Li}$
 c. $\text{Li} > \text{Na} > \text{K} > \text{Rb}$ d. $\text{K} < \text{Li} < \text{Na} < \text{Rb}$
33. The gas evolved on heating Na_2CO_3 is
 a. CO_2 b. water vapour
 c. CO d. No gas is evolved
34. A mixture contains two moles of Na_2CO_3 and 1 mole of Li_2CO_3 . What will be the volume of CO_2 formed on heating this mixture and the data is converted to STP?
 a. 22.4 L b. 44.8 L c. 50.2 L d. 11.2 L

35. An aqueous solution of sodium carbonate is alkaline because sodium carbonate is a salt of
 a. weak acid and weak base
 b. strong acid and weak base
 c. weak acid and strong base
 d. strong acid and strong base
36. Sodium is heated in air at 350°C to form A. Compound A when reacts with carbon dioxide forms sodium carbonate and Y. Here, Y is
 a. hydrogen peroxide b. hydrogen
 c. ozone d. oxygen
37. On reacting with NaOH, which gives inflammable gas?
 a. Zn b. S c. I_2 d. NH_4Cl
38. When washing soda is heated,
 a. CO_2 is released b. $\text{CO} + \text{CO}_2$ is released
 c. CO is released d. water vapour is released
39. In the synthesis of sodium carbonate, the recovery of ammonia is done by treating NH_4Cl with $\text{Ca}(\text{OH})_2$. The by-product obtained in this process is
 a. CaCl_2 b. NaCl c. NaOH d. NaHCO_3
40. Solvay process is not used to prepare KHCO_3 . Which of the following reactions will produce KHCO_3 ?
 a. $\text{KHSO}_4 + \text{KCl} \longrightarrow$
 b. Magnesia process
 c. $\text{KNO}_3 + \text{H}_2\text{O} \longrightarrow$
 d. Calcium carbonate and silver chloride
41. The chemical name of soda ash is
 a. sodium bicarbonate b. sodium hydroxide
 c. sodium carbonate d. None of these
42. In the electrolytic process for the manufacturing of NaOH from NaCl solution, the ion discharged at the anode is
 a. OH^- b. O^-
 c. Cl^- d. All of these
43. Anhydrous magnesium chloride is prepared by
 a. dissolving Mg in dil. HCl
 b. dissolving MgO in dil. HCl
 c. passing Cl_2 over red hot mixture of $\text{MgO} + \text{C}$
 d. All of the above
44. A substance which gives brick red flame and breaks down on heating to give oxygen and a brown gas, is
 a. magnesium nitrate
 b. calcium nitrate
 c. barium nitrate
 d. strontium nitrate
45. A chemical A is used for the preparation of washing soda to recover ammonia. When CO_2 is bubbled through an aqueous solution of A, the solution turns milky. It is used in white washing due to disinfectant nature. What is the chemical formula of A?
 a. $\text{Ca}(\text{HCO}_3)_2$ b. CaO c. $\text{Ca}(\text{OH})_2$ d. CaCO_3
46. Element A burns in nitrogen to give an ionic compound B. Compound B reacts with water to give C and D. The solution of C becomes milky on bubbling carbon dioxide. The element A is
 a. Li b. Mg c. Ca d. Be
47. The metal ion which plays an important role in muscle contraction is
 a. Be^{2+} b. Mg^{2+} c. Ca^{2+} d. Ba^{2+}
48. Mix calcium sulphate with conc. HCl and forms a paste. Bring a pinch of this paste near to the flame, what colour will you obtain?
 a. Brick red b. Apple green
 c. Golden yellow d. Crimson red
49. In curing cement plasters, water is sprinkled from time to time. This helps in
 a. converting sand into silicic acid
 b. keeping it cool
 c. developing interlocking needle like crystals of hydrated silicates
 d. hydrating sand and gravel mixed with cement
50. What will be the oxidation states of nitrogen and alkali metal respectively when the nitrogen and alkali metal react with each other?
 a. -3 and +1 b. -1 and +1
 c. -1 and -1 d. -3 and -1
51. What will you get when you heat hydrated CaSO_4 to 125°C instead of 200°C ?
 a. $\text{CaO} + \text{SO}_3$ b. CaSO_4
 c. $\text{CaSO}_4 \cdot \frac{3}{4} \text{H}_2\text{O}$ d. $(\text{CaSO}_4)_2 \cdot \text{H}_2\text{O}$
52. The ease of adsorption of the hydrated alkali metal ions on an ion-exchange resins follows the order
 [CBSE-AIPMT 2012]
 a. $\text{Li}^+ < \text{K}^+ < \text{Na}^+ < \text{Rb}^+$ b. $\text{Rb}^+ < \text{K}^+ < \text{Na}^+ < \text{Li}^+$
 c. $\text{K}^+ < \text{Na}^+ < \text{Rb}^+ < \text{Li}^+$ d. $\text{Na}^+ < \text{Li}^+ < \text{K}^+ < \text{Rb}^+$
53. Which of the following represent the correct composition of Sorel cement?
 a. Portland cement + MgO b. $\text{CaSiO}_3 \cdot \text{MgCO}_3$
 c. $\text{MgCl}_2 \cdot \text{CaSiO}_3$ d. $\text{MgCl}_2 \cdot 5\text{MgO} \cdot x\text{H}_2\text{O}$
54. Formula of plaster of Paris is
 a. $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ b. $2(\text{CaSO}_4 \cdot 1/2 \text{H}_2\text{O})$
 c. $2\text{CaO} \cdot \text{H}_2\text{O}$ d. CaCO_3
55. Which of the following is not contained by Portland cement?
 a. Ca_3PO_4 b. CaSiO_3 c. CaSiO_4 d. $\text{Ca}_3\text{Al}_2\text{O}_6$
56. The name given to the mixture of MgCl_2 and MgO is
 a. double salt b. Portland cement
 c. Sorel cement d. None of these
57. Which of the following statements is correct?
 a. Plaster of Paris can be obtained by hydration of gypsum
 b. Gypsum is obtained by heating plaster of Paris
 c. Plaster of Paris is obtained from gypsum by oxidation

- d.** Plaster of Paris contains higher percentage of calcium than gypsum
- 58.** Except lime, a major constituent of Portland cement is
a. silica **b.** alumina **c.** iron oxide **d.** magnesium
- 59.** Bleaching powder is obtained by the reaction of chlorine with
a. dry CaO
b. moist slaked lime
- c.** concentrated solution of Ca(OH)_2
d. dilute solution of Cu(OH)_2
- 60.** Which of the following ions is/are responsible for biological functions such as maintenance of ion balance and nerve impulse conduction?
a. Na^+ **b.** K^+
c. Mg^{2+} **d.** All of these

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- 1.** Salt $A + S \xrightarrow{\text{BaCl}_2} B \xrightarrow{\text{BaCl}_2}$ White precipitate
 A is paramagnetic in nature and contains about 55% K. Thus, A is [2013]
a. K_2O **b.** K_2O_2
c. KO_2 **d.** K_2SO_4
- 2.** The decomposition temperature is maximum for [2012]
a. MgCO_3 **b.** CaCO_3
c. BaCO_3 **d.** SrCO_3
- 3.** When some amount of zinc is treated separately with excess of sulphuric acid and excess of sodium hydroxide solution, the ratio of volumes of hydrogen evolved is [2012]
a. 1 : 1 **b.** 1 : 2 **c.** 2 : 1 **d.** 2 : 3
- 4.** The product(s) of the reaction, [2011]
 $\text{Na}_2\text{CO}_3 + \text{CO}_2 + \text{H}_2\text{O} \longrightarrow$ is/are
a. $2\text{NaOH} + \text{CO}_2$ **b.** $\text{Na}_2\text{CO}_3 + \text{H}_2\text{CO}_3$
c. 2NaHCO_3 **d.** None of these
- 5.** Sodium carbonate reacts with SO_2 in aqueous medium to give [2011]
a. NaHSO_3 **b.** $\text{Na}_2\text{S}_2\text{O}_3$
c. NaHSO_4 **d.** Na_2SO_4
- 6.** Anhydrous, a drying agent is [2010]
a. $\text{Mg(ClO}_4)_2$ **b.** $\text{Sr(ClO}_4)_2$
c. $\text{Ca(ClO}_4)_2$ **d.** $\text{Ba(ClO}_4)_2$
- 7.** $\text{Mg} + \text{NO}_2$ on burning give [2010]
a. N_2 **b.** NO
c. N_2O **d.** N_2O_5
- 8.** A metal X on heating in nitrogen gas gives Y. Y on treatment with water gives a colourless gas which when passed through CuSO_4 solution gives a blue colour. Y is [2009]
a. $\text{Mg(NO}_3)_2$ **b.** Mg_3N_2
c. NH_3 **d.** MgO
- 9.** Which of the following statements are correct for alkali metal compounds?
 I. Superoxides are paramagnetic in nature.
 II. The basic strength of hydroxides increases down the group.
 III. The conductivity of chlorides in their aqueous solutions decreases down the group.
 IV. The basic nature of carbonates in aqueous solution is due to cationic hydrolysis. [2008]
a. I, II and III **b.** I and II
c. II, III and IV **d.** III and IV
- 10.** Setting of plaster of Paris involves [2007]
a. oxidation with atmospheric oxygen
b. combination with atmospheric CO_2
c. dehydration
d. hydration to yield another hydrate
- 11.** The ionic conductance is least for [2007]
a. Cs^+ **b.** Rb^+ **c.** K^+ **d.** Na^+
- 12.** Which one of the following substances is used in the laboratory for a fast drying of neutral gases? [2007]
a. Phosphorus pentoxide
b. Active charcoal
c. Anhydrous calcium chloride
d. Na_2PO_4
- 13.** Which out of the following compounds is called photographers fixer? [2007]
a. Na_2SO_3 **b.** $\text{Na}_2\text{S}_2\text{O}_3 \cdot 5\text{H}_2\text{O}$
c. Na_2SO_2 **d.** Na_2S
- 14.** Which of the following is called Berthelot's salt? [2006]
a. $(\text{NaPO}_3)_6$ **b.** NaOCl **c.** KClO_3 **d.** KHF_2
- 15.** Milk of magnesia is used as [2005]
a. antichlor **b.** antacid
c. antiseptic **d.** food preservative
- 16.** Solvay process is used for the manufacturing of [2005]
a. NaOH **b.** Na_2CO_3
c. NH_3 **d.** NaCl

Answer with Solutions

Practice Exercise

- (b) Alkali metal hydroxides, i.e. KOH is highly soluble in water.
- (c) Due to very large size of sulphate ions, the magnitude of lattice energy of alkaline earth metal sulphates remains almost constant. Thus, their solubility is only governed by hydration energy which decreases from Be^{2+} to Ba^{2+} .
- (d) For anion to cation ratio to be low, the anion must be small and cation must be large, i.e. Cs^+F^- .
- (a) Alkali metal oxides (Na_2O) are most basic followed by alkaline earth metals. Al_2O_3 is however amphoteric.
- (d) Stability of a compound depends upon its enthalpy of formation ΔH_f . The more negative value of ΔH_f shows more stability of a compound.

Thus, KCl is more stable and LiCl is least stable.

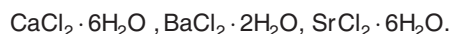
$$\Delta H_f \text{ for LiCl} = -408.8 \text{ kJ mol}^{-1}$$

$$\Delta H_f \text{ for NaCl} = -412.5 \text{ kJ mol}^{-1}$$

$$\Delta H_f \text{ for CsCl} = -433 \text{ kJ mol}^{-1}$$

$$\Delta H_f \text{ for KCl} = -436 \text{ kJ mol}^{-1}$$

- (c) $\text{NaOH} + \text{CO} \xrightarrow{200^\circ\text{C}} \text{HCOONa}$
(Solid) Sodium formate
- (a) Li is not softer than the other group 1 metals. Actually, Li is harder than other alkali metals.
- (b) NaHCO_3 and NaOH both reacts to form Na_2CO_3 .
- (d) In the presence of moisture and CO_2 , Na_2O_2 is converted into NaOH and Na_2CO_3 .
- (b) $2\text{LiNO}_3 \xrightarrow{\Delta} \text{Li}_2\text{O} + 2\text{NO}_2 + \frac{1}{2} \text{O}_2$
- (b) The correct formula of halides of alkaline earth metals is



- (a) LiF is almost insoluble in water due to its high lattice energy but LiCl is soluble in water due to high hydration energy of Li^+ ions. LiCl is also soluble in acetone due to its predominantly covalent nature (because covalent character increases with increase in the size of anion).
- (b) If lattice energy > hydration energy, the compound (BeO) is insoluble in water.
If lattice energy < hydration energy, the compound (BeSO_4) is soluble in water.
LiI is more covalent because Li^+ is smallest and polarised I^- (anion) to maximum extent, so it is more soluble than KI in ethanol.
- (c) Statements II and III are incorrect because oxides and hydroxides of alkaline earth metals and alkali metals are alkaline in nature. Apart from Na and K, Ca and Mg are also found in biological fluid.

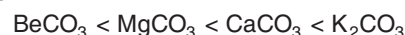
- (d) $\text{A} \rightarrow 4, \text{B} \rightarrow 5, \text{C} \rightarrow 2, \text{D} \rightarrow 1$

- (d)

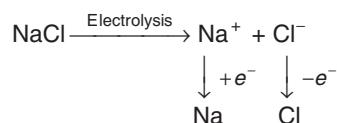
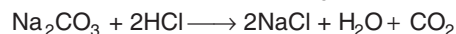
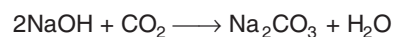
- (c) Greater the electropositive character, more will be the stability and high decomposition temperature.

Among the given options, Ba has maximum electropositive character and hence, highest decomposition temperature.

- (b) Cs^+ has maximum ionic mobility due to low value of hydration enthalpy. It is a large size ion.
- (a) Fajan's rule states that more is the polarising power of anion, greater is the covalent character.
- (a) As the basicity of metal hydroxide increases down the group, thermal stability of their carbonates increases in the same order. As we know, group 1 compounds are more stable than group 2 compounds. Hence, correct order is



- (d) $2\text{Na} + 2\text{H}_2\text{O} \longrightarrow 2\text{NaOH} + \text{H}_2 \uparrow$



In the case of options (a), (b) and (c), the last step on heating

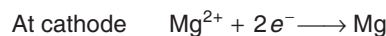


- (c) $\text{KCl} \cdot \text{MgCl}_2 \cdot 6\text{H}_2\text{O}$ (carnallite) is an ore of potassium.

- (a) Cu - 4%, Mg - 0.05%,
Mn - 0.5%, Al - 95%

- (b) $4\text{Na} + \text{O}_2 \longrightarrow 2\text{Na}_2\text{O}$

- (a) Electrolysis of molten MgCl_2



- (c) Asbestos is $\text{CaMg}_3(\text{SiO}_3)_4$.

- (d) Molarity (M)

$$= \frac{\text{Weight of solute / litre of solution}}{\text{Molecular weight of solute}}$$

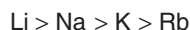
$$= \frac{3.4 \text{ g L}^{-1}}{21.2} = 0.15$$

- (d) When valence electron is in the highest quantum shell, the ionisation energy is lowest and hence, reactivity is highest.

- (b) $\text{K} + \text{O}_2 \longrightarrow \text{KO}_2$ (superoxide)

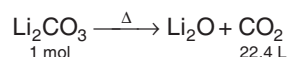
- (b) MgO is basic while other three are amphoteric oxides.

31. (a) Only nitrates of heavy metals and lithium decompose on heating to produce NO_2 .
32. (c) The correct decreasing order of ionisation enthalpy of alkali metals are as follows



33. (d) On heating Na_2CO_3 , no gas is evolved.

34. (a) Decomposition of Li_2CO_3

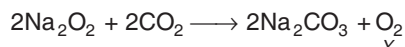


Na_2CO_3 is thermally stable, i.e. it does not give CO_2 gas on heating.

Hence, volume of CO_2 formed on heating the mixture at STP = 22.4 L

35. (c) An aqueous solution of sodium carbonate is alkaline in nature because it is a salt of weak acid and strong base.

36. (d) $\text{Na} + \text{O}_2 \xrightarrow{350^\circ\text{C}} \underset{\text{A}}{\text{Na}_2\text{O}_2}$

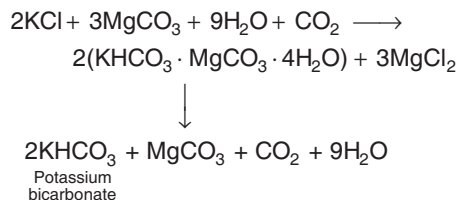


37. (a) On reacting with NaOH solution, Zn gives inflammable gas.

38. (d) $\text{Na}_2\text{CO}_3 \cdot 10\text{H}_2\text{O} \xrightarrow{\Delta} \text{Na}_2\text{CO}_3 \cdot \text{H}_2\text{O} \xrightarrow{\Delta} \underset{\text{Na}_2\text{CO}_3 + \text{H}_2\text{O} \uparrow}{\text{Na}_2\text{CO}_3 + \text{H}_2\text{O} \uparrow}$

39. (a) $2\text{NH}_4\text{Cl} + \text{Ca}(\text{OH})_2 \longrightarrow \text{CaCl}_2 + 2\text{NH}_3 + 2\text{H}_2\text{O}$

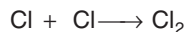
40. (b) **Magnesia Process or Prech's Process** Carbon dioxide is passed into a solution of potassium chloride at 290 K in the presence of hydrated magnesium carbonate, potassium hydrogen carbonate get precipitated.



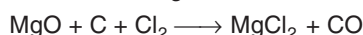
This process is known as magnesia method.

41. (c) Na_2CO_3 (anhydrous) is called soda ash, sodium bicarbonate is called baking soda and sodium hydroxide is called caustic soda.

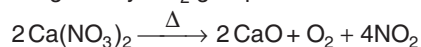
42. (c) $\text{Cl}^- \longrightarrow \text{Cl} + \text{e}^-$



43. (c) Anhydrous MgCl_2 is obtained by passing chlorine gas over red hot mixture of MgO and C .

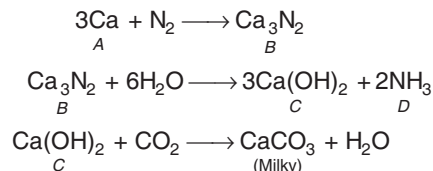


44. (b) Brick red colour is given by Ca which O_2 and brown colour are given by NO_2 group.



45. (c) For the recovery of ammonia, $\text{Ca}(\text{OH})_2$ is used during the preparation of washing soda.

46. (c) Carbon dioxide turns only lime water milky. Thus, the compound C must be $\text{Ca}(\text{OH})_2$ and the element A must be Ca . The reactions are as follows:

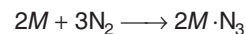


47. (c) Ca^{2+} plays an important role in muscle contraction.

48. (a) The colour of the paste in the flame is brick red.

49. (c) Water is sprinkled on cement plasters to bind hydrated crystals of silicates to one another.

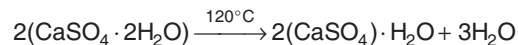
50. (a) Alkali metal reacts with nitrogen to produce nitride.



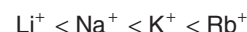
where, oxidation state of $\text{M} = +1$

$$\text{N} = -3$$

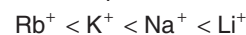
51. (d) On heating hydrated CaSO_4 to 125°C , $(\text{CaSO}_4)_2 \cdot \text{H}_2\text{O}$ is obtained.



52. (b) Ease of adsorption of the hydrated alkali metal ions on an ion exchange resins decreases as the size of alkali metal ions increases. Since, the order of size of alkali metal ions is



Thus, the ease of adsorption follows the order



53. (d) Mixture of MgCl_2 and MgO is called Sorel cement.

Sorel cement = $\text{MgCl}_2 \cdot 5\text{MgO} \cdot x\text{H}_2\text{O}$

54. (b) The formula for plaster of Paris is $\text{CaSO}_4 \cdot \frac{1}{2}\text{H}_2\text{O}$.

55. (a)

56. (c) When a saturated solution of MgCl_2 is mixed with MgO , the resulting paste sets to a hard marble like mass which consists of oxychloride ($\text{MgO} \cdot \text{MgCl}_2 \cdot 6\text{H}_2\text{O}$). The paste is known as Sorel cement.

57. (d) Plaster of Paris is obtained when gypsum is heated to 120°C .

58. (a) Silica 20 -25%, CaO (quicklime) 60 -70%

59. (c) $\text{Ca}(\text{OH})_2 + \text{Cl}_2 \longrightarrow \text{Ca}(\text{OCl})_2 + \text{H}_2\text{O}$

60. (d) Na^+ , K^+ , Ca^{2+} , Mg^{2+} all are responsible for maintenance of ion balance and nerve impulse system in biological system.

16

Metallurgy

Earth Crust

It is the source of many elements. Aluminium is the most abundant metal of earth crust and iron comes second. *The percentages of different elements in earth crust are*

O-49%, Si-26%, Al-7.5%, Fe-4.2%, Ca-3.2%, Na-2.4%, K-2.3%, Mg-2.3%, H-1%

Metals occur in two forms in nature (i) in native state and (ii) in combined state.

Minerals and Ores

The substance (or compound) in the form of metal which is found in nature, is called a mineral and the mineral from which extraction of metal is beneficial and cheap, is called an ore.

Thus, all ores are minerals but all minerals are not ores.

Types of Ores

Depending upon the nature of associated group or atom, ores are of following types:

- (i) **Oxide ores** Haematite (Fe_2O_3), zincite (ZnO) etc.
- (ii) **Sulphide ores** Galena (PbS), cinnabar (HgS), argentite (Ag_2S), ruby silver ($\text{Ag}_2\text{S} \cdot \text{Sb}_2\text{S}_3$) etc.
- (iii) **Carbonate ores** Magnesite (MgCO_3), siderite (FeCO_3) etc.
- (iv) **Sulphate ores** Gypsum ($\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$), Glauber's salt ($\text{Na}_2\text{SO}_4 \cdot 10\text{H}_2\text{O}$) etc.
- (v) **Silicate ores** Willemite (Zn_2SiO_4), feldspar ($\text{NaAlSi}_3\text{O}_8$) etc.

Nitrate ores are rare because all nitrates are water soluble and at higher temperature, they decompose into oxides or metal.

Gangue or Matrix

The impurities associated with the ore, are called gangue or matrix.

Metallurgy

Extraction of a metal from its ores, is known as metallurgy. Metallurgy of a metal includes several metallurgical operations depending upon the nature of metal, its ore and impurities.

Steps Involved in the Extraction of Metals

Various steps/processes involved in the extraction of a metal are as follows :

Concentration of Ores

Removal of unwanted materials from the ores, is known as concentration of ores or **ore dressing**.

Some of the important procedures are described below :

Hydraulic Washing (Levigation)

In this process, an upward stream of running water is used to wash the powdered ore. The lighter gangue particles are washed away and the heavier ores are left behind. The oxide ores of iron (Fe_3O_4 and Fe_2O_3) are concentrated by this method.

Magnetic Separation

This is based on the differences in magnetic properties of the ore components. If either the ore or the gangue is capable of being attracted by a magnetic field, this process is used. In electrostatic separation, electrically charged surfaces are used to separate metallic particles from non-metallic particles of ore.

Froth Floatation Method

This method is used for the concentration of sulphide ores. In this process, a suspension of the powdered ore is made with water. Collectors and froth stabilisers are added to it.

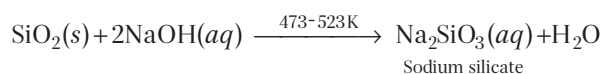
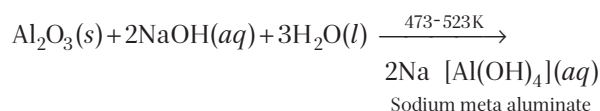
NOTE

- (i) Collectors (e.g. pine oil, fatty acids, xanthates etc.) enhance non-wettability of the mineral particles and froth stabilisers (e.g. cresols, aniline) stabilise the froth.
- (ii) Sometimes, it is possible to separate two sulphide ores by adjusting proportion of oil to water or by using depressants. e.g. In case of an ore containing ZnS and PbS , the depressant used is NaCN .

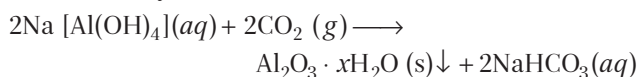
Leaching

It is often used if the ore is soluble in some solvent (i.e. acids, bases or other chemicals) but not the impurities. e.g.

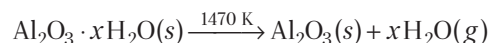
- (i) **Leaching of alumina from bauxite** (Baeyer's process)



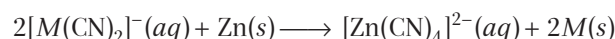
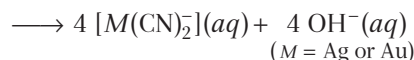
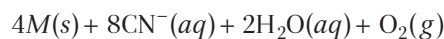
The resulting solution is filtered, cooled and pH is adjusted by neutral with CO_2 causing precipitation of aluminium hydroxide.



Sodium silicate remains in the solution and hydrated alumina is filtered, dried and heated to give back pure Al_2O_3 .



- (ii) **In the metallurgy of silver and gold**



Extraction of Crude Metal from Concentrated Ore

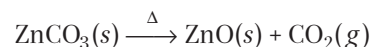
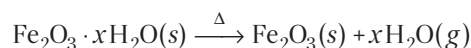
It involves in two major steps:

1. Conversion of Concentrated Ore to Oxide

The following steps are considered under the conversion of concentrated ore to oxide:

Calcination

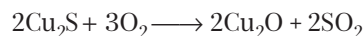
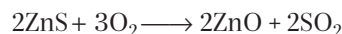
Strong heating of ore in the absence of air, e.g.



Usually carbonates and hydroxides are converted into oxides by this method. Volatile impurities of S, As and P are removed as their volatile oxides.

Roasting

Strong heating of ore in the presence of air, e.g.



The sulphide ores of copper are heated in reverberatory furnace.

2. Reduction of Oxide to Metal

Reduction of the metal oxide to metal, takes place when heated with reducing agents such as C (coke) or CO or even another metal. This process is known as **smelting**.



Some metal oxides get reduced easily while others are very difficult to be reduced.

NOTE Some metals like Fe dissolve the reducing agent used (carbon) in their extraction. This can be removed by heating the impure metal with more of the ore. Misch metal is used as a reducing agent for extraction of pure vanadium. Hydrogen is used as a reducing agent in extraction of pure tungsten and vanadium.

Thermodynamics of Metallurgy

To understand the variation of temperature requirement for thermal reactions and suitable reducing agent for a given metal oxide (M_xO_y), Gibbs energy interpretations are made.

Gibbs equation, $\Delta G = \Delta H - T\Delta S$

where, ΔH = enthalpy change

ΔG = Gibbs free energy

T = temperature

ΔS = entropy change

$$\Delta G^\circ = -2.303 RT \log K$$

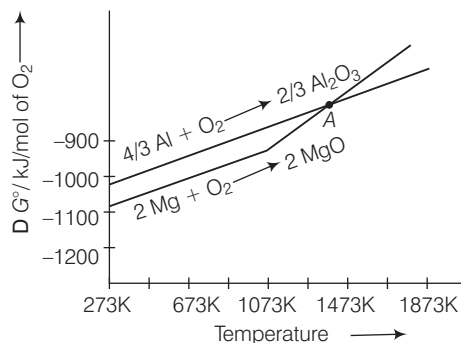
and K = equilibrium constant

If $\Delta G = -ve$, process is spontaneous,

$\Delta G = +ve$, process is non-spontaneous,

$\Delta G = 0$, process is at equilibrium.

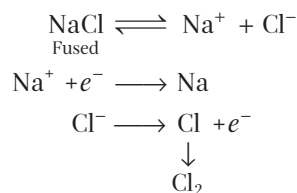
A reaction with positive ΔG can still be made to occur by coupling it with another reaction having large negative ΔG . Such coupling is easily understood through **Ellingham diagram**.



Gibbs Energy (ΔG°) vs T plots (Ellingham Diagram)

From Ellingham diagram, it is evident that metals which have more negative $\Delta_f G^\circ$ of their oxides, can reduce those metal oxides for which $\Delta_f G^\circ$ is less negative.

- Reduction can also be done by using Al, H_2 etc. By Al, oxides of Cr, Fe and Mn are reduced. A mixture of Fe_2O_3 and Al is called **thermite mixture**.
- Highly electropositive metals like Na, K and Al are reduced by the electrolysis of their fused salts. e.g.



Refining

The metal obtained from the above processes is not 100% pure, hence called **crude metal**. From the impure metal, the metal of high purity is obtained by refining. Several techniques are used in refining depending upon the differences in properties of the metal.

Distillation

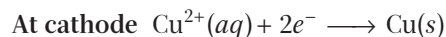
This process is very useful for low boiling metals like zinc, cadmium and mercury. The impure metal is evaporated to obtain the pure metal as distillate.

Liquation

In this method, low melting metals like tin, bismuth and lead can be made to flow on a sloping surface of a reverberatory furnace and heated above their melting point. In this way, it is separated from higher melting impurities.

Electrolytic Refining

Copper is refined by using an electrolytic method.



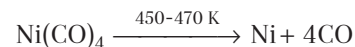
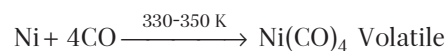
Zone Refining

This method is based on the principle that the impurities are more soluble in the melt than in the solid state of the metal. This method is very useful for producing semiconductors and other metals of very high purity, e.g. germanium, silicon, boron, gallium and indium.

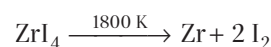
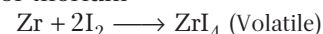
Vapour Phase Refining

In this method, the metal is converted into its volatile compound. It is then decomposed to give pure metal, e.g.

(i) Mond process for refining nickel



(ii) van-Arkel method for zirconium, titanium, vanadium or thorium



Chromatographic method

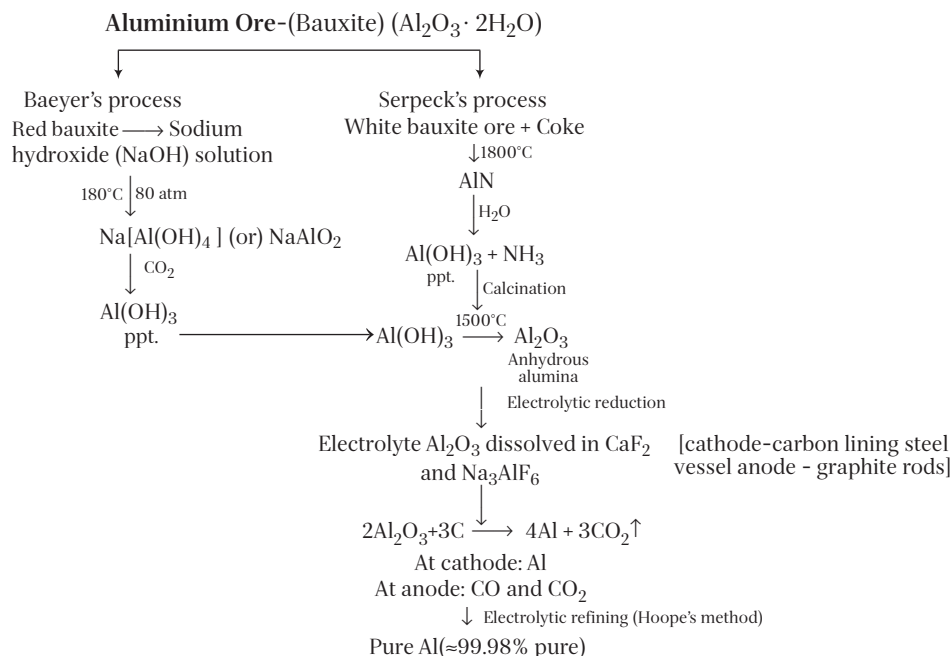
This method is based on the principle that different components of a mixture are adsorbed differently on an adsorbent.

Extraction of Metals

The metals like aluminium, copper, zinc and iron can be extracted from their ores in the following ways:

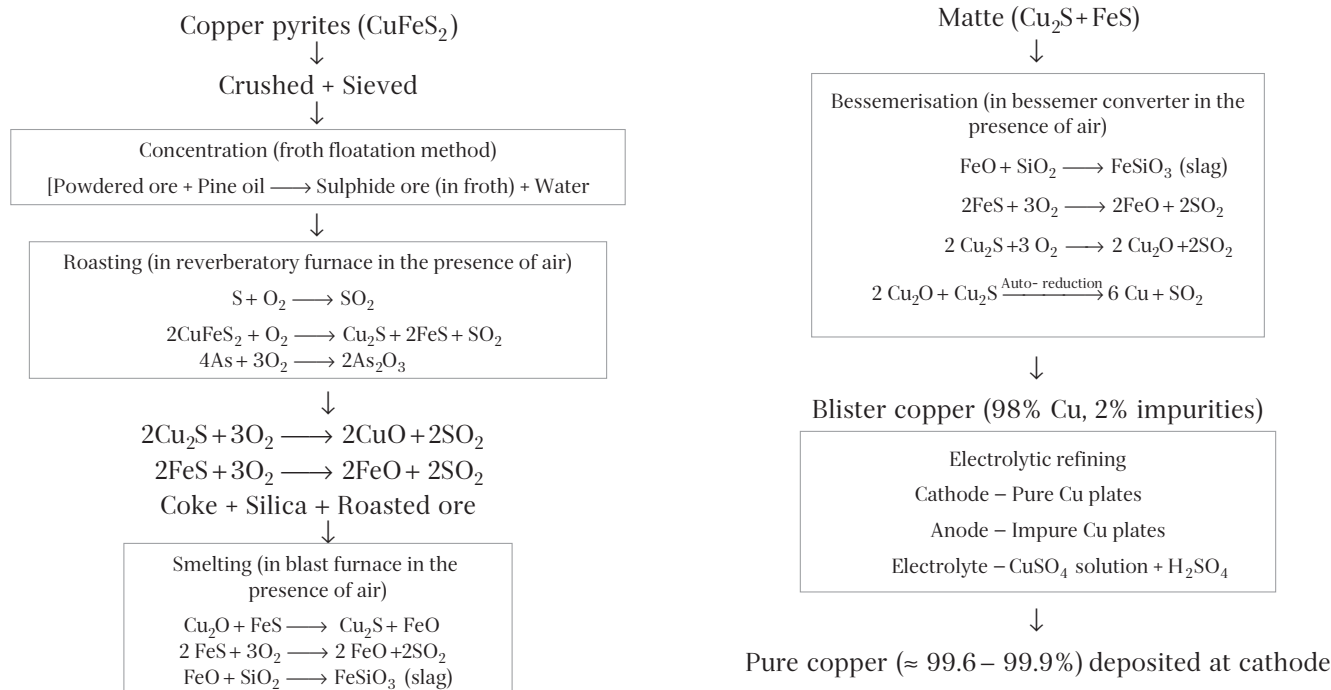
Aluminium Ore-Bauxite ($\text{Al}_2\text{O}_3 \cdot 2\text{H}_2\text{O}$)

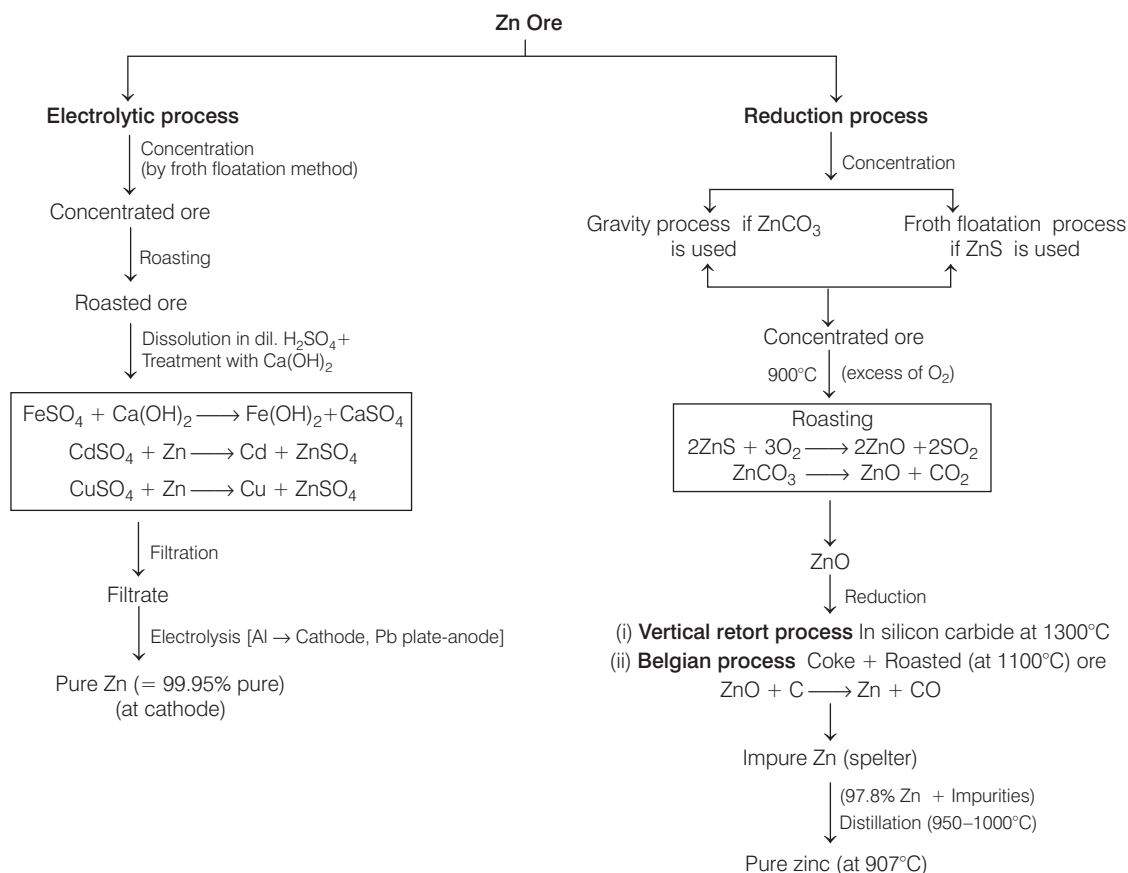
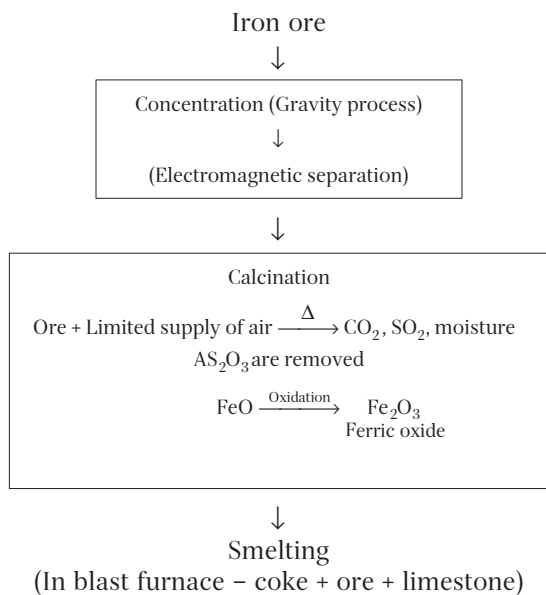
Flow Chart for Al Extraction



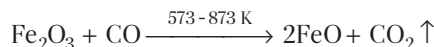
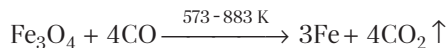
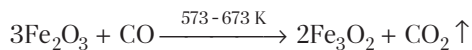
Copper Ore-Copper Pyrites (CuFeS_2)

Flow Chart for Cu Extraction



Zinc Chief Ore–Zinc Blende (ZnS)**Flow Chart for Zinc Extraction****Iron Chief Ore–Haematite (Fe_2O_3)****Flow Chart for Fe Extraction**

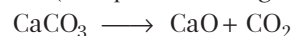
Reduction zone (Upper most part of blast furnace, temperature range 500 to 800 K).



Combustion zone (Temperature 2170 K)

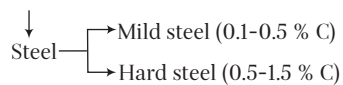


Slag formation zone (Temperature range: $900\text{--}1500\text{ K}$)



Melting zone (Temperature range $1500\text{--}1600\text{ K}$) Molten iron (1600 K as pig iron)

Impurities – 4% C + S, P, Si and Mn in small amount.



Practice Exercise

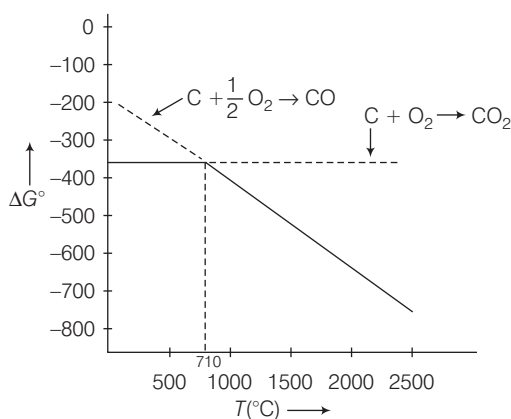
- Among the following statements, the incorrect one is
 - calamine and siderite are carbonates
 - argentite and cuprite are oxides
 - zinc blende and pyrites are sulphides
 - malachite and azurite are ores of copper
- Cassiterite is an ore of
 - Mn
 - Ni
 - Sb
 - Sn
- A metal becomes quite hard due to the presence of small impurity because the impurities
 - reduce the number of mobile electrons
 - reduce the number of slide planes
 - reduce the crystal symmetry
 - change the lattice structure of metals
- Statement I** Nitrate ores are very rare.
Statement II Bond dissociation energy of N_2 is very high.
 Which of these statement(s) is/are true?
 - Only I
 - Only II
 - Neither (a) nor (b)
 - Both (a) and (b)
- The metal found always in the free state, is
 - Au
 - Ag
 - Cu
 - Na
- Which of the following metals is extracted by the electrometallurgical method?
 - Cu
 - Fe
 - Na
 - Ag
- In the froth floatation process for the facilitation of ores, the ore particles float because
 - their surface do not get easily wetted by water
 - they are light
 - they bear electrostatic charge
 - they are not soluble
- The pyrometallurgical operations mainly involve the use of
 - complexation
 - high temperature
 - sulphide ores
 - electrolysis
- Froth floatation process for the concentration of Cu illustrates the practical application of
 - adsorption
 - absorption
 - sedimentation
 - coagulation
- Chemical used as a depressant in separating ZnS from PbS in froth-floatation process, is
 - NaCN
 - NaCl
 - BaCl₂
 - ZnSO₄
- Which of the following ores is not concentrated by froth floatation process?
 - Pyrolusite
 - Pentlandite
 - Zinc blende
 - Copper pyrites
- How do we separate two sulphide ores by froth floatation method?
 - By adding pine oil
 - By adding sodium cyanide
 - By adding foaming agent
 - By passing air
- Commonly used collectors in froth floatation method are
 - pine oil
 - coconut oil
 - fatty acids
 - xanthates
 - I, II, and III
 - II, III, IV
 - I, III, and IV
 - I, II, and IV
- Removal of the unwanted materials from the ore is known as
 - benefaction
 - dressing
 - concentration
 - Either (a) or (b) or (c)
- The process by which lighter earthly particles are made free from heavier particles by washing with water, is called
 - leaching
 - levigation
 - benefication
 - None of these
- Which of the following metals is extracted by auto reduction?
 - Zinc
 - Iron
 - Copper
 - Aluminium
- Among the following pairs of oxides, which pair cannot be reduced by carbon to give the respective metals?
 - Fe₂O₃, ZnO
 - PbO, Fe₃O₄
 - Cu₂O, SnO₂
 - CaO, K₂O
- Which of the following fluxes is used to remove acidic impurities in metallurgical process?
 - Silica
 - Lime stone
 - Sodium chloride
 - Sodium carbonate
- Flux is used to
 - remove all impurities from ores
 - reduce metal oxide
 - remove silica
 - remove silica and undesirable metal oxide
- The process of converting hydrated alumina into anhydrous alumina, is called
 - roasting
 - smelting
 - dressing
 - calcination
- The chemical process in the production of steel from haematite ore involve
 - reduction
 - oxidation
 - reduction followed by oxidation
 - oxidation followed by reduction

22. Which one of the following reactions is an example of auto-reduction?
- $\text{Fe}_3\text{O}_4 + 4\text{C} \longrightarrow 3\text{Fe} + 4\text{CO}_2$
 - $\text{Cu}_2\text{O} + \text{C} \longrightarrow 2\text{Cu} + \text{CO}$
 - $\text{Cu}^{2+}(\text{aq}) + \text{Fe}(\text{s}) \longrightarrow \text{Cu}(\text{s}) + \text{Fe}^{2+}(\text{aq})$
 - $\text{Cu}_2\text{O} + \frac{1}{2}\text{Cu}_2\text{S} \longrightarrow 3\text{Cu} + \frac{1}{2}\text{SO}_2$
23. In order to carry out a reduction process, temperature is selected to make
- ΔG positive
 - ΔH positive
 - ΔG negative
 - ΔH negative
24. Smelting is involved in
- $\text{Fe}_2\text{O}_3 + 3\text{C} \xrightarrow{\text{Heat}} 2\text{Fe} + 3\text{CO}$
 - $2\text{PbS} + 3\text{O}_2 \xrightarrow{\text{Heat}} 2\text{PbO} + 2\text{SO}_2$
 - $\text{Al}_2\text{O}_3 + 2\text{H}_2\text{O} \xrightarrow{\text{Heat}} \text{Al}_2\text{O}_3 + 2\text{H}_2\text{O}$
 - $\text{Zn}_2\text{O}_3 \xrightarrow{\text{Heat}} \text{ZnO} + \text{CO}_2$
25. Specific gravity of slag is
- always less than that of molten metal
 - always higher than that of molten metal
 - same as that of molten metal
 - None of the above
26. The least stable oxide among the following, is
- Sb_2O_3
 - Ag_2O
 - CuO
 - ZnO
27. Which of the following statements about the advantage of roasting of sulphide ore before reduction is not true?
- $\Delta_r G^\circ$ of the sulphide is greater than those of CS_2 and H_2S
 - $\Delta_r G^\circ$ is negative for roasting of sulphide ore to oxide
 - Roasting of sulphide to oxide is thermodynamically feasible
 - Carbon and hydrogen are suitable reducing agents for metal sulphides
28. The value of $\Delta_r G^\circ$ for the formation of Cr_2O_3 is -540 kJ mol^{-1} and that of Al_2O_3 is -827 kJ mol^{-1} . Is the reduction of Cr_2O_3 with Al is feasible reaction?
- The data is incomplete
 - The reaction is feasible
 - The reaction is not feasible
 - The reaction may or may not be feasible
29. Use the relationship $\Delta G^\circ = -nFE^\circ_{\text{cell}}$ to estimate the minimum voltage required to electrolyse Al_2O_3 in the Hall-Heroult process
- $$\Delta_r G^\circ(\text{Al}_2\text{O}_3) = -1520 \text{ kJ mol}^{-1}$$
- $$\Delta_r G^\circ(\text{CO}_2) = -394 \text{ kJ mol}^{-1}$$
- 0.8 V
 - 1.60 V
 - 2.8 V
 - 3.0 V
30. Electrolytic refining's is used to purify which of the following metals?
- Cu and Zn
 - Ge and Si
 - Zr and Ti
 - Zn and Hg
31. Which of the following elements is extracted commercially by the electrolysis of an aqueous solution of its compound?
- Chlorine
 - Bromine
 - Sodium
 - Aluminium
32. For which metal, the property of forming volatile compounds is taken into advantage for its extraction?
- Nickel
 - Iron
 - Cobalt
 - Tungsten
33. Zone refining is based on the principle that
- impurities of low boiling metals can be separated by distillation
 - impurities are more soluble in molten metal than in solid metal
 - different components of a mixture are differently absorbed on an adsorbent
 - vapours of volatile compound can be decomposed into pure metal
34. In electrorefining of copper, some gold is deposited as
- anode mud
 - cathode mud
 - cathode
 - electrolyte
35. Which method of purification is represented by the following equation?
- $$\text{Ti}(\text{s}) + 2\text{I}_2(\text{g}) \xrightarrow{523 \text{ K}} \text{TiI}_4(\text{g}) \xrightarrow{1700 \text{ K}} \text{Ti}(\text{s}) + 2\text{I}_2(\text{g})$$
- Zone refining
 - Cupellation
 - Polling
 - van-Arkel
36. Impure nickel can be purified by
- electrolytic refining
 - Mond-carbonyl process
 - zone-refining process
 - van-Arkel process
37. To obtain pure germanium, which of the following methods of refining is preferred?
- Liquation
 - Zone-refining
 - Electrolytic method
 - Poling
38. Mond's process is used for the purification of
- Ni
 - Ti
 - Zr
 - Hg
39. In the metallurgy of aluminium,
- Al^{3+} is oxidised to Al (s)
 - graphite anode is oxidised to carbon monoxide and carbon dioxide
 - oxidation state of oxygen changes in the reaction at anode
 - oxidation state of oxygen changes in the overall reaction involved in the process
40. Al can be obtained by
- electrolysis of Al_2O_3 dissolved in Na_3AlF_6
 - heating alumina with cryolite
 - reducing Al_2O_3 by coke
 - reducing Al_2O_3 by chromium
41. Aluminothermic process is used for metallurgy of
- Pb
 - Ag
 - Al
 - None of these

42. Which of the following furnaces can be used to get above 3000°C temperature?
 a. Blast furnace b. Arc furnace
 c. Muffle furnace d. Reverberatory furnace
43. The process in which 10 g of green wood is used, is
 a. poling b. pickling
 c. anodising d. galvanising
44. The process used to produce tough pitch copper, is
 a. cupellation b. distillation
 c. poling d. zone-refining
45. In Mc-Arthur Forrest method, silver is extracted from the solution of $\text{Na}[\text{Ag}(\text{CN})_2]$ by the use of
 a. zinc b. magnesium
 c. copper d. iron

BITSAT Archives

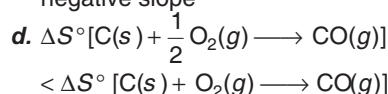
1. Consider the following Ellingham diagram for carbon



Which of the following statements is incorrect for the above Ellingham diagram? [2014]

- a. Upto 710°C , the reaction of formation of CO_2 is energetically more favourable but above 710°C , the formation of CO is preferred
 b. Carbon can be used to reduce any metal oxide at a sufficiently high temperature

c. Carbon reduces many oxides at elevated temperature because ΔG° vs temperature line has a negative slope



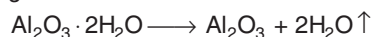
2. In the extraction of Ag, Zn is removed from (Zn-Ag) alloy through [2013]
 a. cupellation b. fractional crystallisation
 c. distillation d. electrolytic refining
3. Pyrolusite is a/an [2011]
 a. oxide ore b. sulphide ore
 c. carbide ore d. None of the above
4. van-Arkel method is based on [2010]
 a. cupellation method b. furnace refining method
 c. poling method d. None of the above
5. Aluminium is present in [2010]
 a. gypsum b. carnallite c. asbestos d. diaspore
6. The extraction of which of the following metals involves basemeiserisation? [2009]
 a. Fe b. Ag c. Al d. Cu

Answer with Solutions

Practice Exercise

- (b) Argentite (Ag_2S) is a sulphide ore and cuprite (Cu_2O) is an oxide ore.
- (d) SnO_2 (tin stone) is also called cassiterite.
- (b) 4. (d)
- (a) Au is always found in free state.
- (c) Na is extracted by the electrometallurgical method.
- (a) In the froth floatation process for the facilitation of ores, the ore particles float because their surface do not get wetted by water.
- (b) High temperature is generally used in the pyrometallurgical process.
- (a) Basically, froth floatation method illustrates the practical application of adsorption.
- (a) PbS can be separated from ZnS in the presence of NaCN. It depresses the floatation property of ZnS by forming a complex $\text{Na}_2[\text{Zn}(\text{CN})_4]$.
- (a) 12. (b) 13. (c)
- (d) Removal of the unwanted materials from the ore is known as benefaction or dressing or concentration.
- (b) Levigation or hydraulic washing.
- (c) $\text{Cu}_2\text{S} + 2\text{Cu}_2\text{O} \longrightarrow 6\text{Cu} + \text{SO}_2$
- (d) Potassium and calcium are strong reductant, hence their oxides cannot be reduced by carbon.
- (b) Lime stone (CaCO_3) is a basic flux. It is used to remove acidic impurities like SiO_2 as CaSiO_3 (slag).

19. (d) Flux removes silica and undesirable metal oxide as silicate.
20. (d) In calcination, moisture or hydrated water is removed on heating.



21. (d) Haematite (Fe_2O_3) having little FeO is removed by acidic flux SiO_2 in the form of slag (FeSiO_3).
22. (d) $\text{Cu}_2\text{O} + \frac{1}{2}\text{Cu}_2\text{S} \longrightarrow 3\text{Cu} + \frac{1}{2}\text{SO}_2$
23. (c) $\Delta G = \Delta H - T\Delta S$
Temperature is chosen such that the sum of ΔG in the following two combined redox processes is negative
- $$\text{MnO} + A_{\text{red}} \longrightarrow \text{XM} + \text{AO}_{\text{oxi}}$$
24. (a) Smelting basically involves the reduction of oxides of metal with carbon or coke at high temperature.
25. (a) Specific gravity of slag is always less than molten metal.
26. (b) The least stable oxide is Ag_2O .
27. (d) Both C and H_2 are not suitable for reducing sulphide ore because $\Delta_f G^\circ$ of metal sulphide is more than that of CS_2 and H_2S which will be formed as a result of reduction.
28. (b) $\frac{4}{3}\text{Cr(s)} + \text{O}_2(\text{g}) \longrightarrow \frac{2}{3}\text{Cr}_2\text{O}_3(\text{s}); \Delta_f G^\circ = -540 \text{ kJ} \dots \text{(i)}$
 $\frac{4}{3}\text{Al(s)} + \text{O}_2(\text{g}) \longrightarrow \frac{2}{3}\text{Al}_2\text{O}_3(\text{s}); \Delta_f G^\circ = -827 \text{ kJ} \dots \text{(ii)}$
 On subtracting Eq. (i) from Eq. (ii)
 $\frac{2}{3}\text{Cr}_2\text{O}_3(\text{s}) + \frac{4}{3}\text{Al(s)} \longrightarrow \frac{2}{3}\text{Al}_2\text{O}_3(\text{s}) + \frac{4}{3}\text{Cr(s)};$
 $\Delta_f G^\circ = -287 \text{ kJ}$
 Since, ΔG° comes out to be negative, the reaction is feasible.
29. (b) $2\text{Al}_2\text{O}_3 + 3\text{C} \longrightarrow 4\text{Al} + 3\text{CO}_2$
 $\Delta_f G^\circ = 3\Delta_f G^\circ(\text{CO}_2) - 2\Delta_f G^\circ(\text{Al}_2\text{O}_3)$
 $= 3 \times (-394) - 2 \times (-1520) = 1858 \text{ kJ}$

$$= 1858 \times 10^3 \text{ J}$$

$$\therefore nFE^\circ = 1858 \times 10^3 \quad (n = 12 \text{ electrons})$$

$$\therefore E^\circ = \frac{1858 \times 10^3}{12 \times 96500} = 1.60 \text{ V}$$

30. (a) Impurities are highly soluble in molten state than their solid state.
31. (a) Electrolysis of aqueous solution of NaCl (brine) gives Cl_2 at anode.
32. (a) Nickel is extracted by Mond's process.
33. (b) CaSiO_3 is called slag which is fusible and lighter than the molten metal and is removed easily.
34. (a)
35. (d) van-Arkel (and de Boer) process is based on the thermal decomposition of a volatile compound like an iodide, which is first formed by direct combination of metal to be purified and iodine. Metal formed is in purest form. Titanium and zirconium are purified by this method.
36. (b) Mond-carbonyl process is used to purify impure Ni.
37. (b) Elements such as Ge, Si, Ga, etc., which are used as semiconductors, are refined by zone-refining.
38. (a) Mond's process is used for the purification of Ni.
39. (b) In the metallurgy of Al, graphite anode is oxidised to CO and CO_2 .
40. (a) Al can be obtained by electrolysis of Al_2O_3 dissolved in Na_3AlF_6 .
41. (d) Thermite process is not used for the metallurgy of Pb, Al and Ag.
42. (c) Muffle furnace is used to get above 3000°C temperature.
43. (a) Poling is the process in which 10 g of green wood is used. It acts as a reducing agent.
44. (c)
45. (b) In Mc-Arthur Forrest method, silver is extracted from the solution of $\text{Na}[\text{Ag}(\text{CN})_2]$ by the use of Mg.

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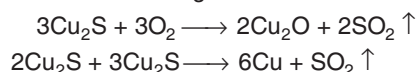
1. (d) Since, $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$
 where, ΔG° = standard Gibbs' free energy of the reaction
 ΔS° = standard entropy of the reaction
 ΔH° = standard enthalpy of the reaction
 T = temperature
 From the above equation, it is clear that ΔG° will be more negative when ΔS° is less negative (or ΔS° is high).
2. (c) The extraction of Ag using (Zn-Ag) alloy is called Parke's process. As zinc is volatile at 920° while Ag is not. Thus, on heating (Zn + Ag) alloy, zinc vaporises while Ag remains at the bottom of the vessel. Hence, Zn is removed from (Zn - Ag) alloy through distillation.

3. (a) Pyrolusite is MnO_2 . Thus, it is an oxide ore.

4. (d)

5. (d) Gypsum $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$
 Carnallite $\text{KCl} \cdot \text{MgCl}_2 \cdot 6\text{H}_2\text{O}$
 Asbestos $\text{CaSiO}_3 \cdot 3\text{MgSiO}_3$
 Diaspore $\text{Al}_2\text{O}_3 \cdot \text{H}_2\text{O}$

6. (d) In a bessemer converter, copper pyrites are oxidised to FeO and Cu_2O . FeO is slagged off. Cu_2O reacts with Cu_2S left unoxidised to give Cu.



17

p-Block Elements-I (Group 13 and 14)

Group-13 Elements : Boron Family

Boron (B), aluminium (Al), gallium (Ga), indium (In) and thallium (Th) are the members of group-13. Boron is non-metal while rest of the members are metals.

Important Properties

General Configuration ns^2, np^1

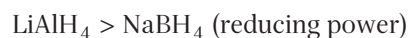
Oxidation State

Stability of +3 oxidation state decreases down the group and that of +1 oxidation state increases down the group due to inert pair effect. Fourth period and onwards effective nuclear charge increases due to poor shielding effect, therefore the radius of gallium (135 pm) is less than that of aluminium (143 pm) and down the group, inert pair effect increases.

Hydrides

Boron hydrides exist in dimeric or polymeric form. The simplest boron hydride is diborane, (B_2H_6). Aluminium forms only one colourless, solid polymeric hydride $(AlH_3)_n$, known as alane. Hydrides of Ga and In are not very much stable. B, Al and Ga form complex anionic hydrides, e.g. $NaBH_4$, $LiAlH_4$ and $LiGaH_4$.

All are strong reducing agents.



Halides

Halides have incomplete octet, therefore, have a high tendency to accept electrons and behave as Lewis acid.



BF_3 is a colourless gas, BCl_3 is a colourless fuming liquid while BI_3 is a white fusible solid. Due to back-bonding and resonance, the B—F bond of BF_3 gets a bond order of 1.33.

- AlCl_3 (anhydrous) is covalent and behaves as Lewis acid because Al^{3+} has small size and high polarising power but AlCl_3 (hydrated) is ionic in nature.
- Al^{3+} has small size due to the presence of vacant d -orbitals, therefore AlCl_3 easily gets hydrolysed in water and form octahedral $[\text{Al}(\text{OH})_6]^{3+}$ ion in which the hybridisation state of Al is sp^3d^2 .

Boron

Boron (B) is the first element of group 13 and occurs in two isotopic forms, i.e. ${}^{10}_5\text{B}$ (18%) and ${}^{11}_5\text{B}$ (82%). Its abundance in the earth crust is less than 0.001% by mass.

The important ores are

Borax	:	$\text{Na}_2 \text{B}_4\text{O}_7 \cdot 10\text{H}_2\text{O}$
Colemanite	:	$\text{Ca}_2 \text{B}_6\text{O}_{11} \cdot 5\text{H}_2\text{O}$
Panderinite	:	$\text{Ca}_2 \text{B}_6\text{O}_{11} \cdot 3\text{H}_2\text{O}$
Boracite	:	$2\text{Mg}_3 \text{B}_8\text{O}_{15} \cdot \text{MgCl}_2$
Kernite	:	$\text{Na}_2 \text{B}_4\text{O}_5(\text{OH})_4$

Properties

The main properties of boron are as follows:

- It is a hard solid (melting point = 2076°C , boiling point = 3927°C) and is a bad conductor of heat and electricity.
- It exists in two allotropic forms namely crystalline and amorphous. Crystalline form is chemically inert and black while amorphous form is chemically active and brown coloured solid.
- Action of acids**

$$\text{B} + 3 \text{HNO}_3 \longrightarrow \text{H}_3 \text{BO}_3 + 3 \text{NO}_2$$

$$2\text{B} + 3 \text{H}_2\text{SO}_4 \longrightarrow 2\text{H}_3\text{BO}_3 + 3 \text{SO}_2$$
- Action of alkalis** On fusion with alkalis, hydrogen is liberated.

$$2\text{B} + 6\text{NaOH} \longrightarrow 2\text{Na}_3\text{BO}_3 + 3 \text{H}_2$$

Uses

It is used as control rods in atomic reactors and as a deoxidiser

Anomalous Behaviour of Boron

Boron shows anomalous behaviour due to its small size and high nuclear charge/size ratio, high electronegativity and non-availability of d -electrons.

The main points of differences are:

- It is a typical non-metal whereas other members are metals.
- It shows allotropy and exists in two forms: crystalline and amorphous. Aluminium is a soft metal and does not exist in different forms.

Similarities with silicon

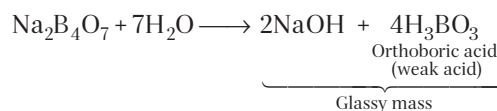
Due to its small size and similar charge/mass ratio, boron shows resemblance with silicon to exhibit diagonal relationship.

Some point of similarities

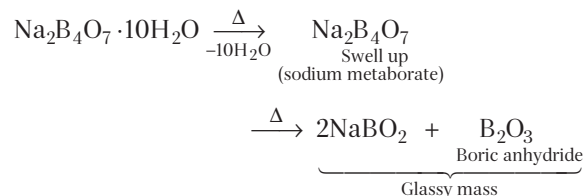
- Both boron and silicon are typical non-metals, having high melting and boiling points, nearly same densities ($\text{B} = 2.35 \text{ g mL}^{-1}$, $\text{Si} = 2.34 \text{ g mL}^{-1}$), low atomic volumes and bad conductor of heat. These are used as **semiconductors**.
- Both exist in amorphous and crystalline state and exhibit allotropy.

Borax ($\text{Na}_2\text{B}_4\text{O}_7 \cdot 10\text{H}_2\text{O}$) [Sodium tetraborate decahydrate]

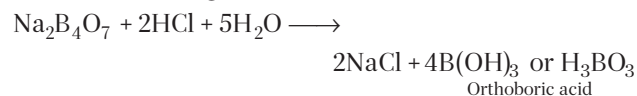
- It is the most important compound of boron. It is white crystalline solid.
- It dissolves in water to give an alkaline solution.



- On heating, it loses the water of crystallisation and swells up to form a mass of a bead.

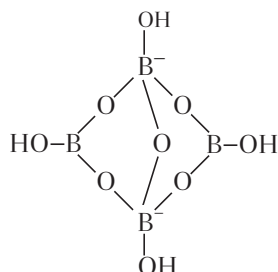


- With acids, it gives orthoboric acid, a weak acid.



(v) **Structure**

Borax contains the tetrahedral units, i.e. $[\text{B}_4\text{O}_5(\text{OH})_4]^{2-}$.

**Boric Acid (H_3BO_3)**

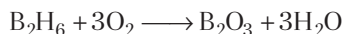
- (i) It is a soft white crystalline solid, soapy to touch and less soluble in cold water.
- (ii) It is a weak monobasic acid. It is not a protonic acid but acts as a Lewis acid.



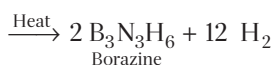
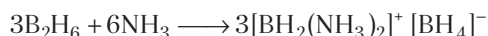
- (iv) **Structure of boric acid** shows that it has a layered structure in which planar BO_3 units are joined by H-bonds.
- (v) It is used as an antiseptic, eye wash, as food preservative, in glass industry and in pottery.

Diborane (B_2H_6)

It is a colourless, highly toxic gas with a boiling point of 180 K. It catches fire spontaneously upon exposure to air.

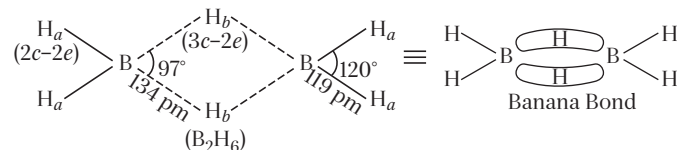


With ammonia, it gives borazine.



Borazine ($\text{B}_3\text{N}_3\text{H}_6$) is known as '**inorganic benzene**'.

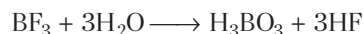
However, at high temperature, inorganic graphite $(\text{BN})_x$ is obtained instead of borazine.

Structure of Diborane**Boron Trifluoride (BF_3)**

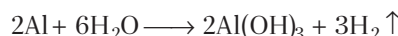
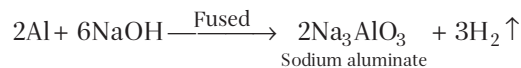
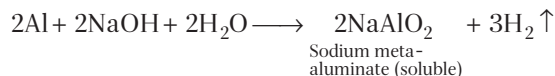
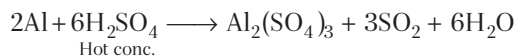
It is a colourless, pungent gas. The melting point and boiling point are -127.1°C and -99.9°C , respectively. It is exceedingly soluble in water, that's why it is collected over mercury.

Properties

- (a) It is an electron deficient compound and can accept a lone pair of electron, thus behaves as Lewis acid.
- (b) BF_3 combines with water to form two types of hydrates, i.e. $\text{BF}_3 \cdot \text{H}_2\text{O}$ (melting point $= 10.18^\circ\text{C}$) and $\text{BF}_3 \cdot 2\text{H}_2\text{O}$ (melting point 6.36°C). It gets hydrolysed in aqueous solutions yielding boric acid and hydrofluoroboric acid.



If BF_3 is present in excess, it is dissolved in HF to give hydrofluoroboric acid $\text{H}[\text{BF}_4]$.

**Reaction of Aluminium with Acids and Alkalies**(a) **Action of water**(b) **Action of alkalies**(c) **Action of acids**

Al becomes passive in HNO_3 due to oxidation and formation of a thin oxide film on its surface.

Thus, conc. HNO_3 may be stored in Al vessels.

Group-14 Elements : Carbon Family

Carbon(C), silicon (Si), germanium (Ge), tin (Sn) and lead (Pb) are the members of group 14. Carbon is the seventeenth most abundant element by mass in the earth's crust. C is non-metal, Si, Ge are metalloids and Sn, Pb are metals.

Important Properties**General Configuration ns^2np^2** **Catenation**

The tendency for catenation is maximum in carbon due to maximum strength of C—C bond (bond energy of C—C = 354 kJ/mol) and it decreases down the group as



Allotropy

All the members except Pb show allotropy.

There are some allotropes of carbon given below :

Diamond

It is the purest, hardest form of carbon with high refractive index and density. In it, each carbon atom (sp^3 -hybridised) is tetrahedrally surrounded by four other carbon atoms. It is three dimensional polymer. It does not conduct electricity as it has no free electrons.

Graphite

It has two dimensional structure. sp^2 -hybridised carbon atom forms three covalent bonds with three other carbon atoms in the same plane and the 4th electron of each carbon remains free and is responsible for electrical conductivity of graphite.

Fullerenes

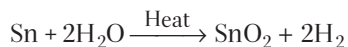
These are the only pure form of carbon. C_{60} molecule contains 12 five membered rings and 20 six membered rings. The five membered rings are connected to six membered rings while six membered rings are connected to both five and six membered rings.

Oxidation States

The common oxidation states are +4 and +2. Carbon also exhibits negative oxidation states, i.e. -4. Down the group, stability of +4 oxidation state decreases and of +2 oxidation state increases due to inert pair effect.

Reactivity Towards Water

Carbon, silicon and germanium are not affected by water. Tin decomposes with steam to form dioxide and dihydrogen gas.



Lead remains unaffected by water, probably because of a protective oxide film formation.

Oxides

CO_2 , SiO_2 and GeO_2 are acidic, whereas SnO_2 and PbO_2 are amphoteric in nature.

- Among monoxides, CO is neutral, GeO is distinctly acidic whereas SnO and PbO are amphoteric.
- Monomeric form of CO_2 is stable due to non-availability of d -orbitals. Carbon has tendency to form a multiple bond ($\text{O}=\text{C}=\text{O}$) but SiO_2 exists in three dimensional polymeric form and has high melting point and it is solid at room temperature.

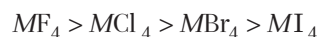
Halides Group

IV A elements form covalent tetrahalides of the type MX_4 (X = halogen) having tetrahedral structure, except PbBr_4 and PbI_4 .

The order of thermal stability of tetrahalides is



Thermal stability and volatility of tetrahalides with a common central atom fall with the increase in the molecular mass of the tetrahalides, e.g.



Except CCl_4 , other tetrachlorides are easily hydrolysed by water due to the availability of empty d -orbitals in them.

Silica (SiO_2)

It is commonly called as silica. It occurs in earth's crust as crystalline quartz, tridymite and cristobalite. Amorphous variety flint is also found.

Methods of Preparation

- It is obtained by heating silicon in oxygen as

$$\text{Si} + \text{O}_2 \longrightarrow \text{SiO}_2$$
- Pure SiO_2 is prepared in its amorphous forms as a white powder by reacting SiCl_4 or SiF_4 with water as

$$\text{SiCl}_4 + 2\text{H}_2\text{O} \longrightarrow \text{SiO}_2 + 4\text{HCl}$$

$$\text{SiF}_4 + 2\text{H}_2\text{O} \longrightarrow \text{SiO}_2 + 4\text{HF}$$

The latter method is normally less used as SiO_2 further reacts with HF to form H_2SiF_6 (fluorosilicic acid).

Silicates

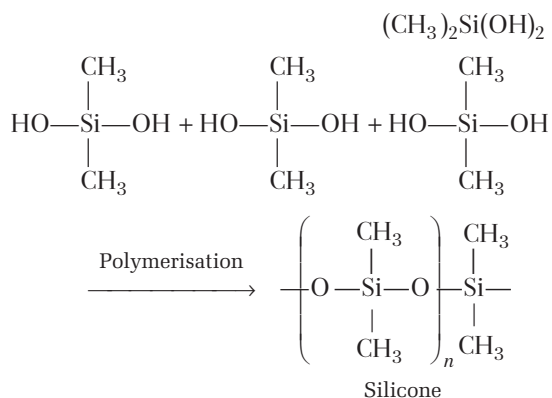
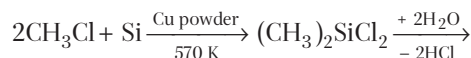
- A large number of silicate minerals exist in nature. Some of the examples are feldspar, zeolites, mica and asbestos. The basic structural unit of silicates is SiO_4^{4-} .
- The SiO_4^{4-} unit is neutralised by positively charged metal ions, if all the four corners are shared with other tetrahedral units.

Silicates are of following types :

- Orthosilicates** Simple silicates containing SiO_4^{4-} tetrahedra.
- Pyrosilicate** Two tetrahedral units share one O-atom to obtain $\text{Si}_2\text{O}_7^{6-}$ anion.
- Cyclic silicates** Two tetrahedral units share two oxygen atoms and form $(\text{SiO}_3^{2-})_n$ or $(\text{SiO}_3)^{2n-}$ anion.
- Chain silicates** Share two oxygen atoms, $(\text{SiO}_3^{2-})_n$ or $(\text{SiO}_3)^{2n-}$ are obtained.
- Sheet silicates** Involve sharing of three O-atoms per tetrahedron to form $(\text{Si}_2\text{O}_5^{2-})_n$.
- Three dimensional silicates** All the four corners (O-atoms) of SiO_4^{4-} tetrahedra are shared with other.

Silicones

- (i) These are synthetic organosilicon compounds which has repeated unit of R_2SiO .
 (ii) These are prepared from alkyl halides.



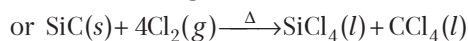
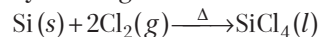
Highly cross-linked silicone polymer is obtained by the hydrolysis of $RSiCl_3$.

- (iii) They are used as sealant, greases electrical insulators and for water proofing of fabrics. Being biocompatible, they are also used in surgical and cosmetic plants.

Silicon tetrachloride (SiCl₄)

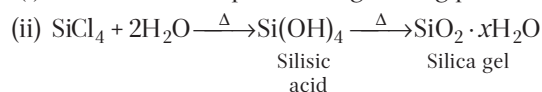
Preparation

It is prepared by heating Si or SiC with Cl₂.



Properties

(i) It is a volatile liquid, having boiling point 330.57 K.



Uses

- (i) Silica gel is used as a catalyst in petroleum industry and as an adsorbent in column chromatography.
 (ii) Si obtained after reduction of silica gel is used for making transistors, computer chips and solar cells.

Zeolites

Zeolites are microporous aluminosilicate having general formula $M_{x/n}[AlO_2]_x[SiO_2]_ymH_2O$.

Uses

- (i) They are used as molecular sieves to separate the molecules of different sizes.
 (ii) They are used as catalyst.

Practice Exercise

- Boron fibres are used in making
 - bullet-proof jacket
 - light composite material for aircraft
 - Both (a) and (b)
 - None of the above
- Match the columns and choose the correct options from the codes given below.

Column I	Column II
A. Orthoboric acid	1. $Na_2B_4O_7 \cdot 4H_2O$
B. Borax	2. $Na_2B_4O_7 \cdot 10H_2O$
C. Kernite	3. H_3BO_3

Codes

A	B	C	A	B	C
a. 1	2	3	b. 3	2	1
c. 2	1	3	d. 1	3	2

- Boron cannot form which one of the following anions?
 - BF_6^{3-}
 - BH_4^-
 - $B(OH)_4^-$
 - BO_2^-

- Heating of an aqueous solution of aluminium chloride to dryness will give
 - $Al(OH)Cl_2$
 - Al_2O_3
 - Al_2Cl_6
 - $AlCl_3$
- When orthoboric acid (H_3BO_3) is heated, the residue is
 - boron
 - metaboric acid
 - boric anhydride
 - borax
- Borax on heating with cobalt oxide forms a blue bead of
 - $Co(BO_2)_2$
 - $CoBO_2$
 - $Co_4(BO_3)_2$
 - $Na_3Co(BO_3)_2$
- Borax bead test is responded by
 - divalent metals
 - trivalent metals
 - light metals
 - metal which forms coloured metaborates
- Which of the following is a component of ruby?
 - $CaCO_3$
 - $MgCO_3$
 - Al_2O_3
 - $Al(OH)_3$
- Tincal is
 - $Na_2CO_3 \cdot 10H_2O$
 - $NaNO_3$
 - $Na_2B_4O_7 \cdot 10H_2O$
 - $NaCl$

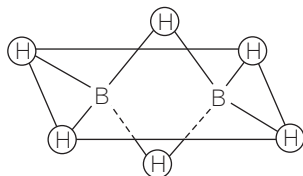
10. Boric acid is used in carom boards for smooth gliding of pawns because
 a. H_3BO_3 molecules are loosely chemically bonded and hence soft
 b. its low density makes it fluffy
 c. it can be powdered to a very small grain size
 d. H-bonding in H_3BO_3 gives it a layered structure

11. Boric acid when burnt with ethyl alcohol gives a green edged flame due to the combustion of
 a. boron trifluoride
 b. metaboric acid
 c. ethyl borate
 d. orthoboric acid

12. In diborane, two H—B—H angles nearly are
 a. 60° , 120°
 b. 95° , 120°
 c. 95° , 150°
 d. 120° , 180°

13. Which of the following statements are correct regarding diborane?

I. Two bridged hydrogen atoms and two boron atoms lie in one plane.



II. Out of six B—H bonds, two bonds can be described in terms of 3 centre-2 electron bonds.

III. Out of six B—H bonds, four B—H bonds can be described in terms of 3 centre-2 electron bonds.

IV. Four terminal B—H bonds are two centre-two electrons regular bonds.

- a. I, II and IV
 b. II, III and IV
 c. I and II
 d. III and IV
14. The species which does not exist, is
 a. $[\text{AlF}_6]^{3-}$
 b. $[\text{GaF}_6]^{3-}$
 c. $[\text{InF}_6]^{3-}$
 d. $[\text{BF}_6]^{3-}$

15. In Al_2Cl_6 ,

I. six Al—Cl bonds are of same length and two of different length.

II. the angle Al—Cl—Al is 87° .

III. four Al—Cl bonds are of same length and two of different length.

IV. the angle Cl—Al—Cl is 93° and 110° .

Which statements are correct?

- a. I, II and III
 b. II, III and IV
 c. I, III and IV
 d. I, II and IV

16. Consider the following statements,

I. BBr_3 is stronger acid than BF_3 .

II. $p\pi-p\pi$ back bonding occurs in the halides of boron but not in the halides of aluminium.

III. Borazine is less reactive than boron.

IV. Al is unstable in air and water.

The set of incorrect statement is

- a. I and II
 b. II and III
 c. III and IV
 d. None of these

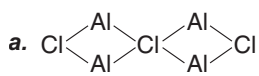
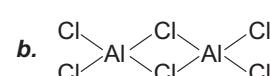
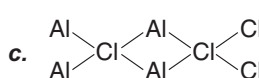
17. B—F bonds in BF_3 are weakly acidic. The main factor responsible is

- a. $p\pi-p\pi$ back bonding
 b. $p\pi-d\pi$ back bonding
 c. three centred- two electron bonds in BF_3
 d. large electronegativity of F

18. Which of the following statements is incorrect for aluminium chloride (Al_2Cl_6) ?

- a. It exists as a dimer in solvents like benzene and carbon disulphide
 b. The aluminium atom is tetrahedrally surrounded by four chlorine atoms
 c. Each aluminium atom forms three covalent bonds and one coordinate bond
 d. In the vapour state, aluminium chloride does not exist as a dimer

19. AlCl_3 achieves stability by forming a dimer. Structure of this dimer is

- a. 
 b. 
 c. 
 d. None of these

20. Which of the following reactions will not give the anhydrous AlCl_3 ?

- a. By heating $\text{AlCl}_3 \cdot 6\text{H}_2\text{O}$
 b. By passing dry HCl gas on heated aluminium powder
 c. By passing dry chlorine gas on heated aluminium powder
 d. By passing dry chlorine gas over a heated mixture of alumina and coke

21. The tendency of Ge, Sn, Pb to show +2 oxidation state, increases in the sequence

- a. $\text{Ge} = \text{Sn} < \text{Pb}$
 b. $\text{Ge} < \text{Sn} < \text{Pb}$
 c. $\text{Ge} > \text{Sn} > \text{Pb}$
 d. $\text{Ge} > \text{Sn} = \text{Pb}$

22. The order of catenation of C, Si, Ge, Sn is

- a. $\text{C} < \text{Si} < \text{Ge} < \text{Sn}$
 b. $\text{C} > \text{Si} > \text{Ge} > \text{Sn}$
 c. $\text{C} \gg \text{Si} > \text{Ge} \approx \text{Sn}$
 d. $\text{C} \approx \text{Si} \gg \text{Ge} \approx \text{Sn}$

23. The hybridisation of the central atom in SiF_6^{2-} , $[\text{GeCl}_6]^{2-}$ and $[\text{Sn}(\text{OH})_6]^{6-}$ is

- a. sp^3d
 b. sp^3d^2
 c. sp^3
 d. sp^3d^3

24. Graphite conducts electricity due to the

- a. highly delocalised nature of π -electrons
 b. highly localised nature of π -electrons
 c. highly polarised nature of π -electrons
 d. None of the above

25. Graphite is a soft, solid lubricant, extremely difficult to melt. The reason for this anomalous behaviour is that graphite

- a. is an allotropic form of diamond
 b. has molecules of variable molecular masses like polymers

- c. has carbon atoms arranged in large plates of rings of strongly bound carbon atoms with weak interplate bonds
d. is a non-crystalline substance
26. Carborundum is obtained when silica is heated at high temperature with
a. carbon
b. carbon monoxide
c. carbon dioxide
d. calcium carbonate
27. The mixture of CO and H₂ is known as
a. producer gas
b. synthesis gas
c. water gas
d. Both (b) and (c)
28. A mixture of CO and N₂ is known as
a. synthesis gas
b. water gas
c. producer gas
d. All of these
29. Water gas is used as an industrial fuel because
a. on combustion, it further produces CO₂
b. on combustion, it loses heat
c. on combustion, it produces CO₂ and liberates heat
d. None of the above
30. Silica gel is used as a/an
a. dehydrating agent (drying agent)
b. dehydrogenating agent
c. reducing agent
d. oxidising agent
31. (Me)₂SiCl₂ on hydrolysis will yield
a. (Me)₂Si=O
b. $[\text{O}-(\text{Me})_2\text{Si}-\text{O}]_n^-$
c. Me₂SiCl(OH)
d. (Me)₂Si(OH)₂
32. Consider the following statements,
I. GeCl₄ in HCl forms H₂[GeCl₆].
II. Trisilyl amine is pyramidal.
III. SnCl₄ is more stable than SnCl₂.
IV. Ge(OH)₂ is amphoteric.
The correct statements are
a. I and IV
b. II and III
c. I and II
d. III and IV
33. Name of the structure of silicates in which three oxygen atoms of [SiO₄]⁴⁻ are shared, is
a. pyrosilicate
b. sheet silicate
c. linear chain silicate
d. three dimensional silicate
34. An inorganic compound X, made of two most occurring elements in the earth's crust and used in building construction, when reacts with carbon, forms a diatomic molecule, which is poisonous in nature. Compound X may be
a. SiO₂
b. Al₂O₃
c. CaO
d. CO₂
35. An inorganic compound containing (3c-2e) and (2c-2e) bonds when reacts with NH₃ at a certain temperature, gives a compound X which is isostructural with benzene and when reacts at high temperature, forms a substance Y. The substance Y is
a. B₂H₆
b. B₃N₃H₆
c. inorganic graphite
d. B₂H₆ · 2NH₃
36. The shape of gaseous SnCl₂ is
a. tetrahedral
b. linear
c. angular
d. T-shaped
37. Mark the oxide which is amphoteric in character.
a. CO₂
b. SiO₂
c. SnO₂
d. CaO
38. SiH₄ and O₂ mixture on bubbling through water and bubbles coming in contact with air,
a. burns with a luminous flame
b. vertex rings of finely divided silica are formed
c. SiH₄ + 2O₂ → SiO₂ + 2H₂O, reaction occurs
d. All of the above
39. Ionisation enthalpy (Δ_iH_i in kJ mol⁻¹) for the elements of group-13 follows the order
a. B > Al > Ga > In > Tl
b. B < Al < Ga < In < Tl
c. B < Al > Ga < In > Tl
d. B > Al < Ga > In < Tl
40. Two elements P and Q react separately with highly electropositive metal to form binary compounds, which upon hydrolysis yield mixture of boranes and silanes. P and Q respectively are
a. B, Al
b. Si, B
c. B, Si
d. Al, B
41. Which of the following is most stable?
a. Sn²⁺
b. Ge²⁺
c. Si²⁺
d. Pb²⁺
42. The correct stability order for boron halides is
a. BF₃ > BCl₃ > BBr₃ > BI₃
b. BCl₃ > BF₃ > BBr₃ > BI₃
c. BI₃ > BBr₃ > BCl₃ > BF₃
d. BBr₃ > BCl₃ > BI₃ > BF₃
43. Aluminium metal is corroded in coastal places near to the sea, it is due to protective oxide film,
a. is removed by sea water
b. reacts with sea water
c. is attacked by salt present in sea water
d. reacts with sand particles
44. Which of the following elements form both neutral as well as acidic oxides?
a. Sn
b. Si
c. C
d. P
45. Al₂O₃ can be converted into anhydrous AlCl₃ by heating from
a. a mixture of Al₂O₃ and carbon in dry Cl₂ gas
b. Al₂O₃ with Cl₂ gas
c. Al₂O₃ and HCl gas
d. Al₂O₃ with NaCl in solid state

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- Which of the following statements are incorrect in context of borax? [2013]
 - It is made up of two triangular BO_3 units and two tetrahedral BO_4 units
 - One mole of borax can be used as a buffer
 - It is a useful primary standard for titrating against acids
 - Aqueous solution of borax can be used as a buffer
- For the properties mentioned, the correct trend for the different species is in [2013]

- Strength as Lewis acid— $\text{BCl}_3 > \text{AlCl}_3 > \text{GaCl}_3$
 - Inert pair effect— $\text{Al} > \text{Ga} > \text{In}$
 - Oxidising property— $\text{Al}^{3+} > \text{In}^{3+} > \text{Tl}^{3+}$
 - First ionisation enthalpy— $\text{B} > \text{Al} > \text{Tl}$
- Which glass has the highest percentage of lead? [2007]
 - Soda glass
 - Flint glass
 - Jena glass
 - Pyrex glass

Answer with Solutions

Practice Exercise

- (c)
- (b) Orthoboric acid is H_3BO_3
 Borax is $\text{Na}_2\text{B}_4\text{O}_7 \cdot 10\text{H}_2\text{O}$
 Kernite is $\text{Na}_2\text{B}_4\text{O}_7 \cdot 4\text{H}_2\text{O}$
- (a) Due to absence of $2d$ -orbital, maximum covalency is four. Thus, BF_6^{3-} is not formed.
- (b) Aqueous solution of AlCl_3 is acidic due to hydrolysis.

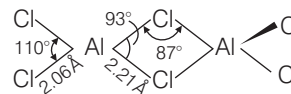
$$\text{AlCl}_3 + 3\text{H}_2\text{O} \rightleftharpoons \text{Al}(\text{OH})_3 + 3\text{HCl}$$
 On strong heating, $\text{Al}(\text{OH})_3$ is converted into Al_2O_3 .

$$2\text{Al}(\text{OH})_3 \xrightarrow{\Delta} \text{Al}_2\text{O}_3 + 3\text{H}_2\text{O}$$
- (c)
$$\text{H}_3\text{BO}_3 \xrightarrow[\text{above } 70\text{ K}]{\Delta} \text{HBO}_2 \xrightarrow{\Delta} \text{B}_2\text{O}_3$$
 Orthoboric acid Metaboric acid Boric anhydride
- (a)
$$\text{Na}_2\text{B}_4\text{O}_7 \longrightarrow 2\text{NaBO}_2 + \text{B}_2\text{O}_3$$

$$\text{B}_2\text{O}_3 + \text{CoO} \longrightarrow \text{CoO} \cdot \text{B}_2\text{O}_3 \text{ or } \text{Co}(\text{BO}_2)_2$$
 Blue bead
- (d) Metals forming coloured bead can be identified by borax bead test.
- (c) Al_2O_3 is a component of ruby ($\text{Al}_2\text{O}_3 + \text{Cr}_2\text{O}_3$), red precious stone.
- (c) Naturally occurring crude borax is called tincal. Thus, it is chemically $\text{Na}_2\text{B}_4\text{O}_7 \cdot 10\text{H}_2\text{O}$.
- (d) Boric acid is used in carom boards for smooth gliding of pawns because H-bonding in H_3BO_3 gives it a layered structure.
- (c)
$$\text{H}_3\text{BO}_3 + 3\text{C}_2\text{H}_5\text{OH} \xrightarrow{-3\text{H}_2\text{O}} (\text{C}_2\text{H}_5)_3\text{BO}_3$$
 Boric acid Ethyl alcohol Ethyl borate
 It burns with green edged flame.
- (b) In diborane, two $\text{H}-\text{B}-\text{H}$ angles are nearly 95° and 120° .
- (a) The correct statements are I, II and IV.

- (d) BF_6^{3-} does not exist because there is no vacant d -subshell in boron.

- (b) The structure of Al_2Cl_6 is shown below



- (c) Borazine is more reactive than boron. Aluminium is stable in air and water.
- (a) It is $p\pi-p\pi$ bonding involving B and F-atom responsible for the acidic nature of boron halides as $\text{BF}_3 < \text{BCl}_3 < \text{BBr}_3 < \text{BI}_3$.
 Smaller, atom shows more back bonding.
- (d)
- (b)
- (a) Hydrated aluminium chloride undergoes hydrolysis to forms Al_2O_3 .

$$2\text{AlCl}_3 \cdot 6\text{H}_2\text{O} \longrightarrow 2\text{Al}(\text{OH})_3 + 6\text{HCl}$$

$$2\text{Al}(\text{OH})_3 \longrightarrow \text{Al}_2\text{O}_3 + 3\text{H}_2\text{O}$$
- (b) The tendency of Ge, Sn, Pb to show +2 oxidation state increases in the sequence $\text{Ge} < \text{Sn} < \text{Pb}$.
- (c) The order of catenation of C, Si, Ge, Sn is $\text{C} \gg \text{Si} > \text{Ge} \approx \text{Sn}$
- (b) The species like, SiF_6^{2-} , $[\text{GeCl}_6]^{2-}$, $[\text{Sn}(\text{OH})_6]^{2-}$ exist where the hybridisation of the central atom is sp^3d^2 .
- (a) Graphite conducts electricity due to the highly delocalised nature of π -electrons.
- (c) Graphite has carbon atoms arranged in large hexagonal layers with weak van der Waals' interactions between the layers.
- (a)
$$\text{SiO}_2 + 3\text{C} \xrightarrow[2300\text{ K}]{\text{High temp.}} \text{SiC} + 2\text{CO}$$
 Carborundum
- (d) Mixture of CO and H_2 is known as water gas as well as *syn* gas.

28. (c) Mixture of CO and N₂ is known as producer gas.
29. (c) Water gas on combustion produces CO₂ and liberate heat.
30. (a) Silica gel is used as dehydrating agent as it removes moisture from atmosphere.
31. (d) (Me)₂SiCl₂ on hydrolysis will yield (Me)₂Si(OH)₂.
32. (a) GeCl₄ in HCl forms H₂[GeCl₆]·Ge(OH)₂ is amphoteric. SnCl₂ is more stable than SnCl₄. Trisilyl amine is planar molecule.
33. (b) Sheet silicates are formed when three oxygen atoms (bridging O-atoms) of each (SiO₄)⁴⁻ unit are shared. .
34. (a) Two most abundant elements in the earth's crust are Si and oxygen and the compound made by them is SiO₂. This compound is used in building construction.
- $$\text{SiO}_2 + 2\text{C} \longrightarrow \text{Si} + \underset{\text{Poisonous}}{2\text{CO}}$$
35. (c)
36. (c) Gaseous SnCl₂ is angular with sp²-hybridisation.
37. (c) SnO₂ is amphoteric. It dissolves in acids as well as in alkalies.
38. (d) All these are characteristics noted during the process.
39. (d) Ionisation enthalpy (Δ_iH_i in kJ mol⁻¹) for the elements of group-13 follows the order
B > Al < Ga > In < Tl
40. (c) P and Q are boron and silicon respectively.
41. (d) Due to inert pair effect, Pb²⁺ is most stable.
42. (a) The stability order for boron halides is explained in terms of back-bonding.
43. (c) It is a reason for given fact.
44. (c) CO is neutral and CO₂ is acidic.
45. (a) Al₂O₃ + 3Cl₂(dry) + 3C → 2AlCl₃ + 3CO

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1. (b) Borax is Na₂B₄O₇·10H₂O in which 2 molecules of water among 10 molecules form a part of structure and exists as Na₂[B₄O₅(OH)₄]·8H₂O.
Methyl orange with pH value of 3.7 is used to detect end point. Aqueous solution of borax acts as buffer because borax is salt of strong base (NaOH) and weak acid (H₃BO₃).
2. (a) The tendency to behave as Lewis acid decreases with the increase in the size down the group. BCl₃ easily accepts a lone pair of electrons from ammonia to form BCl₃·NH₃. Hence, strength as Lewis acid
BCl₃ > AlCl₃ > GaCl₃
3. (b) Flint glass or lead glass has composition of K₂O·PbO·6SiO₂. It is used in making electric bulb and optical instruments

18

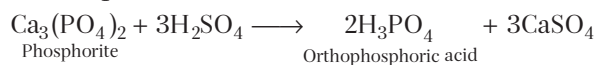
p-Block Elements-II

Group 15 Elements

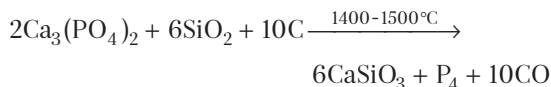
Group 15 (VA) contains 5 elements, namely nitrogen (N), phosphorus (P), arsenic (As), antimony (Sb), bismuth (Bi), collectively called as **pnictogens** taken from Greek word '*pnigmis*' meaning suffocating.

Physical Properties

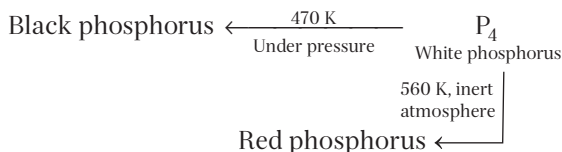
- (i) **Electronic configuration** The general electronic configuration of group 15 is ns^2np^3 .
- (ii) **Atomic and ionic radius** Atomic and ionic radius increase down the group, however arsenic shows exceptionally low value of atomic volume.
- (iii) **Ionisation energy** Ionisation energy of these elements is much higher due to increased nuclear charge and stable exactly half-filled electronic configuration.
- (iv) **Electronegativity** On moving down the group, it decreases due to gradual increase in the atomic radius of the elements.
- (v) **Non-metallic/metallic character** On moving down the group, metallic character increases.
- (vi) **Allotropy** All these elements except bismuth show allotropy.
- (vii) **Oxidation state** Elements of group VA exhibit maximum oxidation state of +5 by losing all the five electrons from their outer shell. +3 oxidation state is shown only when *p*-orbital electrons are used in bonding.
On moving down the group, +3 oxidation state becomes more stable due to inert pair effect. Nitrogen shows +5 to -3 oxidation states.



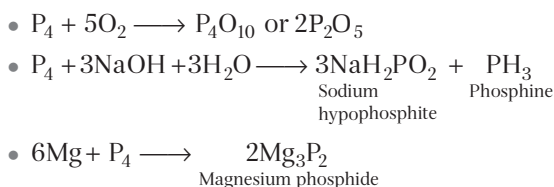
- (ii) **Electrothermal process** Phosphorus is obtained from direct reduction of mineral phosphorite by carbon in the presence of silica.



Allotropy of Phosphorus

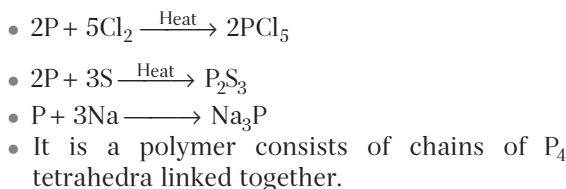


- (i) **White phosphorus** is transparent, soft, poisonous, waxy solid, shows chemiluminescence and chemically more reactive.



- It acts as a strong reducing agent.
 $\text{P}_4 + 10\text{H}_2\text{SO}_4 \longrightarrow 4\text{H}_3\text{PO}_4 + 10\text{SO}_2 + 4\text{H}_2\text{O}$

- (ii) **Red phosphorus** is the stable form of phosphorus. It is odourless, non-poisonous and less reactive.



- (iii) **Black phosphorus** It is thermodynamically most stable at room temperature and has two forms :

- (a) **α -black phosphorus** It is very stable form and does not oxidise unless heated very strongly.
- (b) **β -black phosphorus** It is the only form of P whose structure is definitely known (crystalline).

- (iv) **Scarlet phosphorus** It resembles the red variety in its physical properties and white P in its chemical properties.

- (v) **Violet phosphorus** It is crystalline in nature.

Uses

Red P is widely used in the match industry because of its less reactivity and non-poisonous nature, while yellow P and zinc phosphide are used as a rat poison. ^{32}P is used in the treatment of leukemia and other blood disorders.

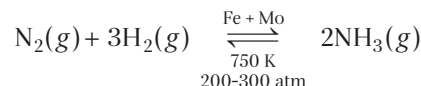
Important Compounds of N and P

Some important compounds of N and P are as follows :

Ammonia

Preparation

It is prepared by Haber's process.



Properties

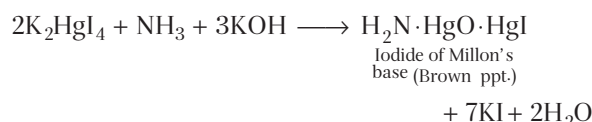
The main properties of ammonia are as follows :

- (i) It is a light colourless gas with characteristic pungent smell and highly soluble in water.



- (iii) Ammonia has a strong tendency to donate its lone pair of electrons to other molecules and thus acts as a strong Lewis base.

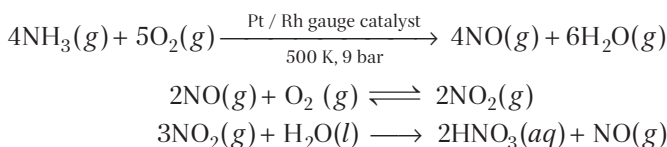
- (iv) It reacts with Nessler's reagent (alkaline K_2HgI_4).



Nitric Acid or Aqua Fortis

Preparation

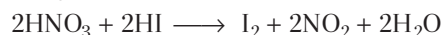
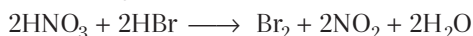
It is prepared by **Ostwald's process**.



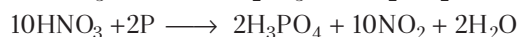
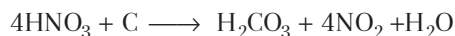
Properties

The main properties of nitric acid are as follows:

- (a) Pure anhydrous HNO_3 is a colourless, water soluble pungent smelling, fuming liquid (boiling point -84.1°C).
- (b) On exposure to light, HNO_3 decomposes to NO_2 which imparts yellow colour.
- (c) It is extremely corrosive in nature.
- (d) In aqueous solution, nitric acid behaves as a strong acid giving H_3O^+ and NO_3^- ions.
- (e) Conc. HNO_3 is a strong oxidising agent.



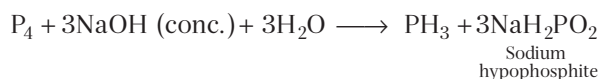
- (f) Hot conc. HNO_3 attacks non-metals also.



Phosphine

Preparation

It is prepared in laboratory by the following reaction:

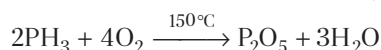


Properties

The main properties of phosphine are as follows:

- It is a colourless, poisonous garlic or rotten fish like smelling gas.
- It is weakly basic like ammonia, gives phosphonium compounds with acids.

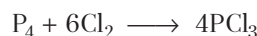
$$\text{PH}_3 + \text{HBr} \longrightarrow \text{PH}_4\text{Br}$$
- Pure PH_3 does not burn spontaneously. However, on heating in air or oxygen forms P_2O_5 .



Phosphorus Trichloride

Preparation

PCl_3 is obtained by passing dry chlorine over heated white P.



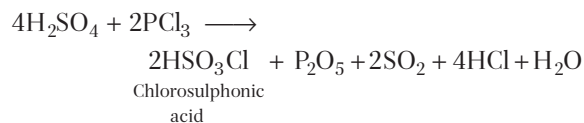
Properties

Following are the properties of PCl_3 :

- PCl_3 is a colourless mobile liquid (boiling point = 349 K, freezing point = 161 K; density = 1.6 g/cm³) with a pungent smell. It fumes in moist air.
- It acts as a reducing agent.

$$3\text{PCl}_3 + \text{S}_2\text{Cl}_2 \longrightarrow \text{PCl}_5 + 2\text{PSCl}_3$$

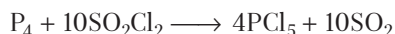
$$3\text{PCl}_3 + \text{SOCl}_2 \longrightarrow \text{PCl}_5 + \text{POCl}_3 + \text{PSCl}_3$$
- It reacts with conc. H_2SO_4 .



Phosphorus Pentachloride (PCl_5)

Preparation

PCl_5 is prepared by the reaction of PCl_3 with excess of dry chlorine of white phosphorus with SO_2Cl_2 .



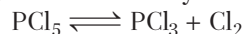
Properties

Following are the main properties of PCl_5 :

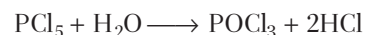
- PCl_5 is a colourless crystalline solid with pungent odour when pure.

- Upon heating, it sublimes below 173 K and can be melted (melting point = 421 K) by heating under pressure.

- It has the ability to dissociate as

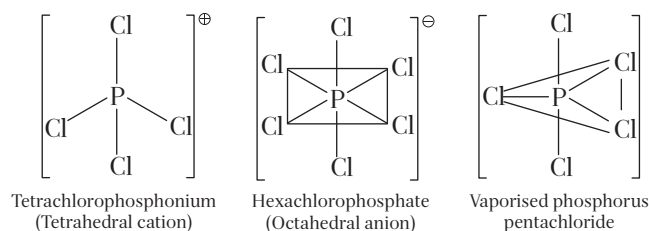


- In moist air, it hydrolyses to POCl_3 and finally gets converted to H_3PO_4 .



Structure

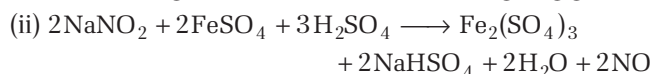
X-ray studies show that solid PCl_5 consists of ionic lattices, i.e. tetrahedral $[\text{PCl}_4]^+$ cations and octahedral $[\text{PCl}_6]^-$ anions. In vapour state, it has trigonal bipyramidal shape (sp^3 -hybridisation).



Oxides of Nitrogen



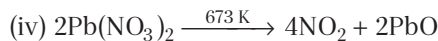
$\text{N}_2\text{O}(g)$ is neutral, colourless gas, with sp hybridisation and linear geometry (it is also called laughing gas).



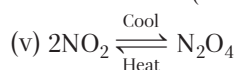
$\text{NO}(g)$ is colourless, neutral gas. It also contains odd number of electrons (paramagnetic) but in solid or liquid state, it exists in dimeric form and have paired electrons (diamagnetic). It is very reactive and harmful to health.



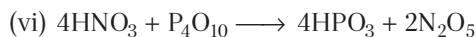
N_2O_3 is blue solid. It is acidic and planar with sp^2 hybridisation.



$\text{NO}_2(g)$ is brown, acidic gas with angular shape and sp^2 hybridisation. It contains odd number of valence electrons (paramagnetic). On dimerisation, it gets converted to stable N_2O_4 molecule with even number of electrons (diamagnetic).

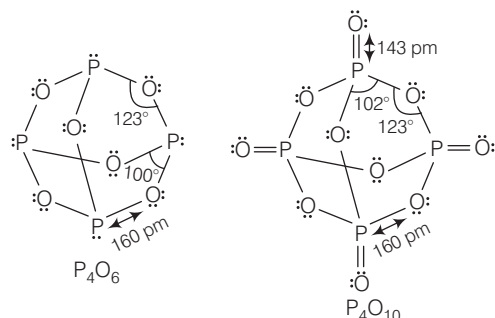


N_2O_4 is colourless solid/liquid. It is acidic with planar geometry and sp^2 hybridisation. Its covalency is four (total number of bonds with central atom).

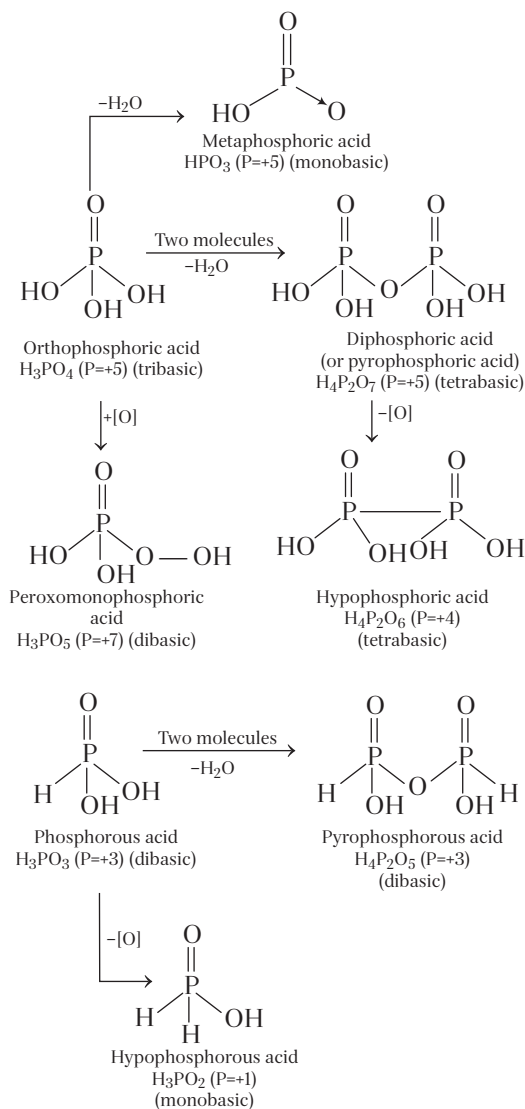


N_2O_5 is colourless, acidic solid with planar shape and sp^2 hybridisation and four covalency.

Structure of Oxides and Oxoacids of Phosphorus



The structures of some important oxyacids of phosphorus are summarised below:



Group 16 Elements

The group VI A contains 5 elements, i.e. oxygen, sulphur, selenium, tellurium and polonium and known as **oxygen family**.

The first four members are non-metals and collectively known as **chalcogens** (metal ores mainly occur in the form of oxide, sulphides etc). The outermost shell electronic configuration is ns^2, np^4 (n = number of shells).

Physical Properties

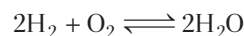
Group 16 elements show following trends in their physical properties :

- Metallic character** On moving down the group, the metallic character increases down the group from oxygen to polonium because of decrease in ionisation energy.
- Oxidation states** Group VIA elements show oxidation state of +2, +4 and +6 due to the promotion of electrons to vacant d -orbitals. But down the group, +4 oxidation state becomes more stable.

Chemical Properties

Following are the main chemical properties of group 16 elements :

- Hydrides** All the elements of the oxygen family form stable hydrides of the type H_2M either by directly combining with hydrogen or by the action of acids on metal sulphides, selenides and tellurides.



H_2O is a liquid due to hydrogen bonding and others are colourless gases with unpleasant smell.

Compounds $H_2O > H_2S > H_2Se > H_2Te$ (all sp^3 -hybridised)

Bond angle $104.5^\circ \ 92.5^\circ \ 91^\circ \ 90^\circ$

The decreasing bond dissociation enthalpy of the $M-H$ bond is due to increase in the size of M which explains the increasing acidic character of hydrides down the group.

- Halides** All the elements of the oxygen family form a number of halides like OF_2 , ClO_2 , S_2F_2 , SF_4 , etc.
- Oxides** These form oxides of the type MO_2 and MO_3 . e.g. SO_2 , TeO_2 , etc.

Anomalous Behaviour of Oxygen

Oxygen, the first member of group 16 differs from rest of the members due to its small size, high electronegativity and non-availability of d -orbitals in the valence shell.

- (i) Oxygen exhibits an oxidation state of -2 only except in O_2F_2 and OF_2 where it has $+1$ and $+2$ state and in peroxides where it has -1 state whereas other members show $+4$ and $+6$ oxidation states in addition to $+2$ oxidation state.
- (ii) Oxygen is capable of forming $p\pi-p\pi$ bonds. The other elements of this group do not show the tendency to exhibit such multiple bonding, sulphur and phosphorus show $p\pi-d\pi$ bonding.

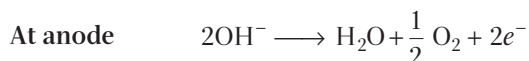
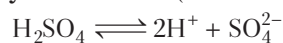
Important Compounds of O and S

Some important compounds of oxygen and sulphur are :

Dioxygen

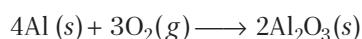
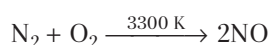
Preparation

- (i) **Laboratory method** $2KClO_3 \xrightarrow[\Delta]{MnO_2} 2KCl + 3O_2$
- (ii) **By electrolysis of water** (Acidified water)



Properties

- (i) Colourless, odourless, tasteless gas which is slightly soluble in water. Liquid oxygen exhibits paramagnetism.
- (ii) It is non-inflammable but a supporter of combustion.
- (iii) $O=O$ bond dissociation energy is high, therefore it reacts with metals or non-metals after external heating to start the reaction.



Ozone

It is formed in the upper layer of atmosphere by the action of UV rays from sun on oxygen. It prevents the UV rays from entering the earth's atmosphere. CFCs, common refrigerants deplete the ozone layer.

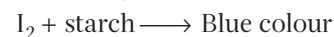
Preparation



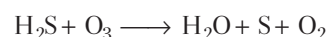
Properties

- It is pale blue gas with pungent odour. It is diamagnetic and poisonous.
- Ozone is unstable; $2O_3 \xrightarrow{\text{Heat}} 3O_2$

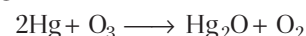
- It acts as a powerful oxidising agent. It liberates iodine from neutral KI solution and the liberated I_2 turns starch paper blue.



- Alkaline KI is oxidised to potassium iodate and periodate.



Mercury loses its meniscus in contact with ozone (tailing of mercury).



Allotropic forms of Sulphur

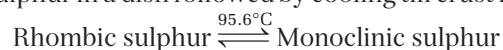
It exists in several allotropic forms.

Rhombic Sulphur (α -sulphur)

It is common crystalline form of sulphur, yellow in colour, melting point 114.5°C and specific gravity 2.06. Its crystals are prepared by evaporating sulphur solution in CS_2 . It is insoluble in water but readily soluble in CS_2 .

Monoclinic Sulphur (β -sulphur)

It is stable above 95.6°C . Its crystals are amber yellow in colour, melting point 119°C , specific gravity 1.98. It is insoluble in CS_2 . It is prepared by melting rhombic sulphur in a dish followed by cooling till crust is formed.



At 95.6°C , both forms are stable. This temperature is called **transition temperature**. S_8 rings in both the forms is puckered and has a crown shape.

Plastic Sulphur

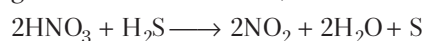
It is obtained by pouring boiling sulphur into cold water. It is amber brown in colour, specific gravity 1.95 and insoluble in CS_2 .

Milk of Sulphur

It is obtained by boiling milk of lime with sulphur and decomposing the products formed with HCl. It is used in medicines.

Colloidal Sulphur

It is prepared by passing H_2S through a solution of an oxidising agent such as nitric acid, etc.



Colloidal sulphur changes into ordinary form on heating.

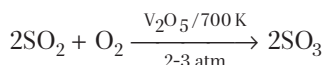
Sulphuric Acid

It is called as 'oil of vitriol' because in early days, it was prepared from ferrous sulphate crystals (green vitriol) and has an oily appearance.

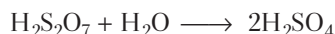
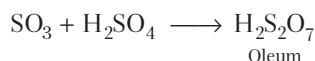
Preparation

Industrial preparations of sulphuric acid are as follows:

(a) Contact process



SO_3 is absorbed in 98% H_2SO_4 to get oleum.



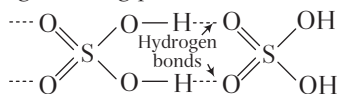
(b) Lead chamber process



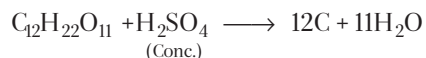
Properties

The main properties of sulphuric acid are as follows:

- Pure sulphuric acid is colourless but commercial acid is yellow in colour due to the presence of impurities.
- H_2SO_4 has associated structure due to intermolecular hydrogen bonding which accounts for its high boiling point and viscosity.



- Conc. H_2SO_4 acts as a dehydrating agent.



The residue of carbon is called **sugar charcoal**.

Uses

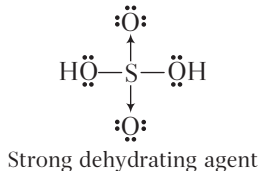
Sulphuric acid is used

- as a dehydrating agent and in petroleum refining.
- for pickling, i.e. cleaning of metal surface before electroplating, in making lead storage batteries.

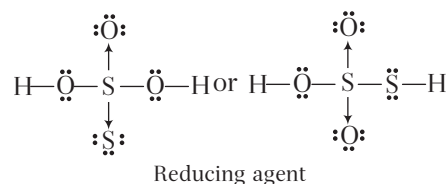
Structure of Oxoacids of Sulphur

Sulphur shows following oxyacids:

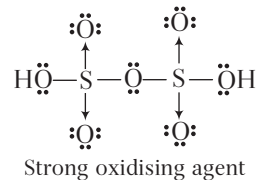
(i) Sulphuric acid (H_2SO_4)



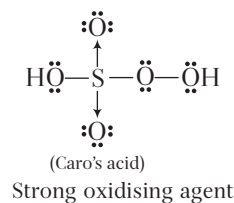
(ii) Thiosulphuric acid ($\text{H}_2\text{S}_2\text{O}_3$)



(iii) Pyrosulphuric acid ($\text{H}_2\text{S}_2\text{O}_7$)



(iv) Peroxomonosulphuric acid (H_2SO_5)



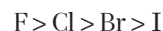
Group 17 Elements

Group VIIA of long form of periodic table consists of the non-metallic elements, fluorine (F), chlorine (Cl), bromine (Br), iodine (I) and astatine (At). These non-metallic elements are collectively known as halogens as their salts are usually found in water (Greek word-Halo means sea salt producer).

Physical Properties

- Electronic configuration** The outermost shell electronic configuration is ns^2, np^5 .
(where, n = number of shells)

- Electronegativity** The halogens are the most electronegative elements. On moving down the group, the electronegativity decreases.

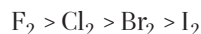


- Oxidation state** All these elements show an oxidation state of -1 . Except fluorine, other elements also show $+1, +3, +5$ and $+7$ oxidation states due to the presence of vacant d -orbitals in their valence shell.

- Non-metallic character** All these elements are non-metallic in nature due to their high ionisation energy and high electronegativities. On moving down the group, the non-metallic character gradually decreases.

Chemical Properties

- (i) **Oxidising power** On moving down the group, the oxidising power decreases from F to I, i.e.



Thus, I^- is the strongest reducing agent, while F^- is the strongest oxidising agent.

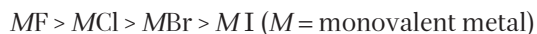
- (ii) **Reaction with oxygen** Halogens form binary compounds with oxygen but most of them are unstable. Decreasing order of stability of oxides formed by halogens is



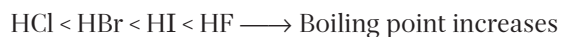
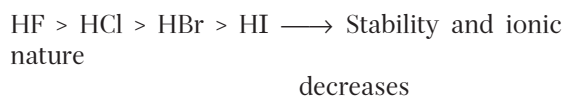
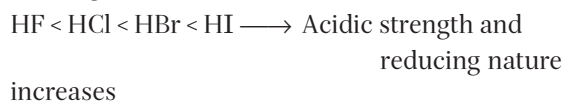
Oxides of Halogens

Oxides of fluorine	OF_2 and O_2F_2
Oxides of chlorine	Cl_2O , ClO_2 , Cl_2O_6 , Cl_2O_7
Oxides of bromine	Br_2O , BrO_2 , BrO_3
Oxides of iodine	I_2O_5

- (iii) **Reaction with metals** Halogens react with metals to form metal halides. The ionic character of the halides decreases in the order



- (iv) **Reaction with hydrogen** Halogens react with hydrogen to form hydrogen halides. Their affinity for hydrogen decreases from F to I.



- (v) **Interhalogen compounds** Halogens react with themselves to produce a number of interhalogen compounds (XX_n , where $n = 1, 3, 5$ or 7).

Examples of Interhalogen Compounds

XX'	ClF , BrF , $BrCl$, ICl , IBr , IF
XX'_3	ClF_3 , BrF_3 , IF_3 , ICl_3 (I_2Cl_6)
XX'_5	ClF_5 , BrF_5 , IF_5
XX'_7	IF_7

- (vi) **Pseudo halogens** There are many uni-negative groups which show characteristics of halide ions. These are called pseudohalides or pseudohalide ions. The covalent dimers of pseudohalide ions are called pseudohalogens.

NOTE Pseudohalide ions, being made up of two heteroatoms can function as ambidentate ligands.

Examples of Pseudohalogens and Their Formulae

Pseudohalide ions	Formulae	Pseudohalogens	Formulae
Cyanide	CN^-	Cyanogen	$(CN)_2$
Cyanate	OCN^-	Oxocyanogen	$(OCN)_2$
Thiocyanate	SCN^-	Thiocyanogen	$(SCN)_2$
Selenocyanate	$SeCN^-$	Selenocyanogen	$(SeCN)_2$
Azidothiocarbonate	$SCSN_3^-$	Azidocarbon disulphide	$(SCSN_3)_2$

Anomalous Behaviour of Fluorine

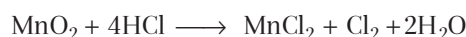
Fluorine differs from the rest of the halogens because of its small size, high charge density, non-availability of d -orbitals and low bond dissociation energy.

Important unique behaviour of fluorine is given below:

- F is very reactive because of its low bond dissociation energy. However, in Cl_2 and Br_2 , $X-X$ bond is stronger.
- HF has a high tendency to form H-bond. Thus, it is a liquid (boiling point = $19^\circ C$) while HCl, HBr and HI are gases under ordinary conditions.
- Hydrofluoric acid being a dibasic acid (H_2F_2) forms two series of salt such as $NaHF_2$, Na_2F_2 while HCl, HBr and HI are monobasic in nature.

Chlorine

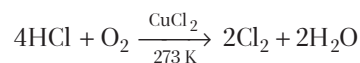
It was discovered in 1774 by Scheele by the action of HCl on MnO_2 .



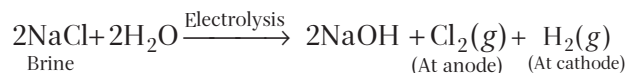
Manufacturing of Chlorine

Chlorine can be manufactured by the following two processes:

Deacon's Process



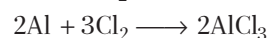
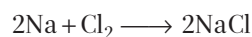
Electrolytic Process



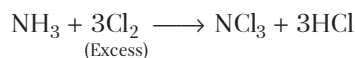
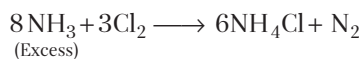
It is also obtained as a by-product in many chemical reactions.

Properties of Chlorine

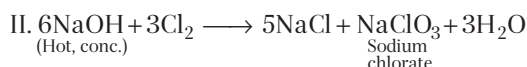
- It is a greenish yellow gas with pungent and suffocating odour which is about 2-5 times heavier than air. It is highly soluble in water.
- Chlorine reacts with a number of metals and non-metals to form chlorides.



- (iii) Chlorine reacts with compounds containing hydrogen, it has great affinity for hydrogen.

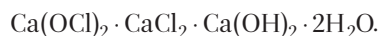


- (iv) I. $2\text{NaOH} + \text{Cl}_2 \longrightarrow \text{NaCl} + \underset{\text{Sodium hypochlorite}}{\text{NaOCl}} + \text{H}_2\text{O}$
(Cold, dilute)

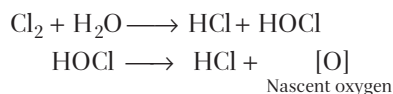


- (v) $2\text{Ca(OH)}_2 + 2\text{Cl}_2 \longrightarrow \text{Ca(OCl)}_2 + \text{CaCl}_2 + 2\text{H}_2\text{O}$

The composition of bleaching powder is



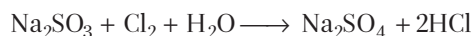
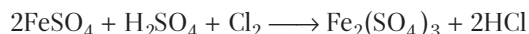
- (vi) Chlorine water on standing loses its yellow colour due to the formation of HCl and HOCl.



Nascent oxygen formed by HOCl is responsible for oxidising and bleaching properties of chlorine.

Coloured substance + [O] $\longrightarrow$ Colourless substance

- (vii) It oxidises ferrous to ferric and sulphite to sulphate, sulphur dioxide to sulphur trioxide and iodine to iodate.



Uses of Chlorine

Some important uses of chlorine are as follows:

- (i) In the manufacturing of dyes, drugs and some organic compounds like DDT, carbon tetrachloride, refrigerants, etc.
- (ii) In the preparation of poisonous gases such as phosgene (COCl_2), tear gas (CCl_3NO_2) and mustard gas ($\text{ClCH}_2\text{CH}_2\text{SCH}_2\text{CH}_2\text{Cl}$).

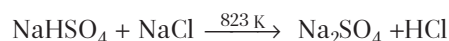
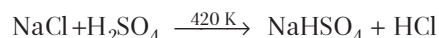
Compounds of Halogens

Hydrogen Chloride (HCl)

It bleaches vegetable or organic matter in the presence of moisture. Bleaching effect of chlorine is permanent.

Preparation of HCl

In laboratory, it is prepared by heating NaCl with conc. H_2SO_4 .

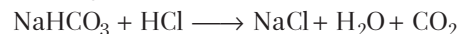
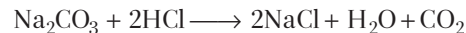


Properties of HCl

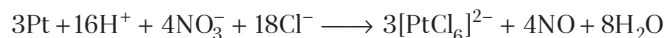
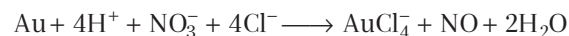
- (i) It is a colourless and pungent smelling gas which is extremely soluble in water.
- (ii) Its boiling point is 189 K and freezing point is 159 K.
- (iii) It gives white fumes of NH_4Cl on reaction with NH_3 .



- (iv) It decomposes salts of weaker acids as:



When three parts of conc. HCl are mixed with one part of conc. HNO_3 , **aqua-regia** is formed. It dissolves noble metals, i.e. Au, Pt.



Uses of HCl

- (i) In the manufacturing of chlorine, NH_4Cl and glucose (from corn starch).
- (ii) In medicines and as a laboratory reagent.
- (iii) For extracting glue from bones and purifying bone black.

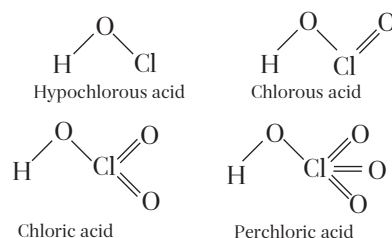
Oxoacids of halogens

Fluorine forms only one oxoacid HOF, known as fluoric (I) acid or hypofluorous acid due to high electronegativity and small size. Chlorine, bromine and iodine form four series of oxoacid. They are stable only in aqueous solutions or in the form of their salts.

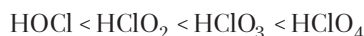
Oxoacids of Halogens

Halic (I) acid (Hypohalous acid)	HOF (Hypofluorous acid)	HOCl (Hypochlorous acid)	HOBr (Hypobromous acid)	HOI (Hypoiodous acid)
Halic (III) acid (Halous acid)	—	HOClO (Chlorous acid)	—	—
Halic (V) acid (Halic acid)	—	HOClO ₂ (Chloric acid)	HOBrO ₂ (Bromic acid)	HOIO ₂ (Iodic acid)
Halic (VII) acid (Perhalic acid)	—	HOClO ₃ (Perchloric acid)	HOBrO ₃ (Perbromic acid)	HOIO ₃ (Periodic acid)

Structures of Oxoacids of Chlorine



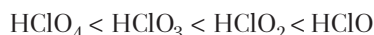
Acidic strength increases as oxidation number increases.



HClO_4 is the strongest acid for the same oxidation number of halide.



The oxidising power of oxoacids of chlorine follows the order



It is because with the increase in the oxidation number of halogen atom in oxoacids, oxidising power decreases.

Group 18 Elements

The zero (or 18) group of the periodic table consists of six gaseous elements namely helium (He), neon (Ne), argon (Ar), krypton (Kr), xenon (Xe) and radon (Rn). Except radon, all of them are present in very small amounts in the atmosphere, hence they are also known as **rare gases** or **noble gases**.

Physical Properties

- (i) **Electronic configuration** These gases have highly stable ns^2np^6 configuration.
- (ii) **Boiling points** On moving down the group, the boiling point increases with increase in size.
- (iii) **Ionisation energy and electron affinity** These gases have stable ns^2np^6 (fully-filled) configuration, thus have no tendency to add or lose electron. Hence, their ionisation energy is very high and electron affinity is zero.
- (iv) **Heat of vaporisation and polarisability** These gases possess very low values of heat of vaporisation due to the presence of very weak van der Waals' forces between their monoatomic molecules.
- (v) **Solubility** Noble gases are slightly soluble in water and their solubility generally increases on moving down the group.
- (vi) **Adsorption** Except helium, all the noble gases are adsorbed by charcoal and the ease of adsorption increases down the group.
- (vii) **Liquefaction** Ease of liquefaction increases down the group from He to Rn due to increase in intermolecular forces.
- (viii) **Conductivity** Noble gases have high electrical conductivity at low pressure.
- (ix) These gases are monoatomic gases ($C_p/C_v = 1.667$).

Chemical Properties

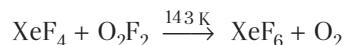
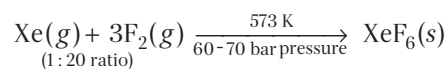
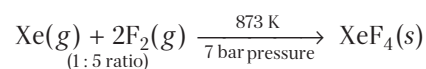
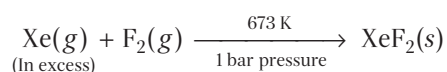
In general, noble gases are not very reactive.

Their inertness to chemical reactivity is attributed to the following reasons:

- (i) The noble gases have completely filled ns^2np^6 electronic configurations in their valence shells.
- (ii) The noble gases have very high ionisation energies.
- (iii) The electron affinities of noble gases are almost zero.

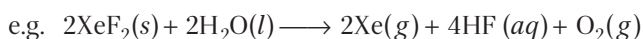
Xenon-Fluorine Compounds

Xenon forms three binary fluorides XeF_2 , XeF_4 and XeF_6 as follows:

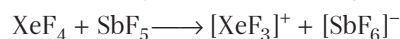
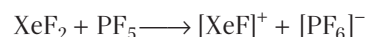


Properties of Xenon-Fluorine Compounds

All fluorides of xenon are colourless crystalline solids and sublime readily at 298 K. They are powerful fluorinating agents. These are readily hydrolysed even by traces of water.



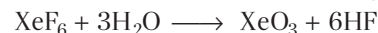
Xenon fluorides may react with fluoride ion acceptors to form cationic species or fluoride ion donors to form fluoro anions.



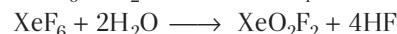
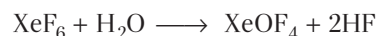
(M = Na, K, Rb or Cs)

Xenon-Oxygen Compounds

Hydrolysis of XeF_4 and XeF_6 with water gives XeO_3 .



Partial hydrolysis of XeF_6 gives oxyfluorides XeOF_4 and XeO_2F_2 .



XeO_3 is a colourless explosive solid and has a pyramidal molecular structure. XeOF_4 is a colourless volatile liquid and has a square pyramidal molecular structure.

Practice Exercise

- When conc. HNO_3 is heated with P_2O_5 , it forms
a. N_2O b. NO c. NO_2 d. N_2O_5
- A hydride of nitrogen which is acidic, is
a. NH_3 b. N_2H_4 c. N_2H_2 d. N_3H
- Extra pure N_2 can be obtained by heating
a. NH_3 with CuO b. NH_4NO_3
c. $(\text{NH}_4)_2\text{Cr}_2\text{O}_7$ d. $\text{Ba}(\text{N}_3)_2$
- The number of σ bonds in P_4O_{10} is
a. 6 b. 16 c. 20 d. 7
- PH_3 , the hydride of phosphorus is
a. metallic b. ionic
c. non-metallic d. covalent
- Among the following compounds, the most acidic is
a. As_2O_3 b. P_2O_5 c. Sb_2O_3 d. Bi_2O_3
- Among the following, the number of compounds that can react with PCl_5 to give POCl_3 , is
 $\text{O}_2, \text{CO}_2, \text{SO}_2, \text{H}_2\text{O}, \text{H}_2\text{SO}_4, \text{P}_4\text{O}_{10}$
a. 1 b. 2 c. 3 d. 4
- Nitrogen shows different oxidation states in the range
a. 0 to +5 b. -3 to +5
c. -5 to +3 d. -3 to +3
- Which of the following acids possess oxidising, reducing and complex forming properties?
a. HCl b. HNO_2 c. H_2SO_4 d. HNO_3
- When conc. H_2SO_4 is heated with P_2O_5 , the acid is converted to
a. sulphur trioxide
b. sulphur dioxide
c. sulphur
d. a mixture of sulphur dioxide and sulphur trioxide
- Strong reducing behaviour of H_3PO_2 is due to
a. low oxidation state of phosphorus
b. presence of two $-\text{OH}$ groups and one $\text{P}-\text{H}$ bond
c. presence of one $-\text{OH}$ group and two $\text{P}-\text{H}$ bonds
d. high electron gain enthalpy of phosphorus
- The molecule having smallest bond angle is
a. NCl_3 b. AsCl_3 c. SbCl_3 d. PCl_3
- Which of the following statements is wrong?
a. The stability of hydrides increases from NH_3 to BiH_3 in group 15 of the periodic table
b. Nitrogen cannot form $d\pi-p\pi$ bond
c. $\text{N}-\text{N}$ single bond is weaker than the $\text{P}-\text{P}$ single bond
d. N_2O_4 has two resonance structures

- Maximum covalency of nitrogen is
a. 3 b. 5 c. 4 d. 6
- Which of the following statements is wrong?
a. $\text{N}-\text{N}$ single bond is stronger than the $\text{P}-\text{P}$ single bond
b. PH_3 can act as a ligand in the formation of coordination compound with transition elements
c. NO_2 is paramagnetic in nature
d. Covalency of nitrogen in N_2O_5 is four
- Which of the following elements can be involved in $p\pi-d\pi$ bonding?
a. Carbon b. Nitrogen
c. Phosphorus d. Boron
- Bond dissociation enthalpy of $E-\text{H}$ (E = element) bonds is given below.

Compound	NH_3	PH_3	AsH_3	SbH_3
$\Delta_{\text{diss}}(E-\text{H})/\text{kJ mol}^{-1}$	389	322	297	255

Which of the following compounds will act as strongest reducing agent?

- NH_3 b. PH_3 c. AsH_3 d. SbH_3
- Density of N_2 gas prepared from air is slightly higher than that of nitrogen prepared by chemical reaction from a compound of nitrogen because aerial nitrogen contains
a. CO_2
b. argon
c. some N_2 molecules analogous to O_2
d. greater amount of N_2 molecules derived from N^{15} isotope
 - Which of the following statements is incorrect?
a. White and red phosphorus react with chlorine at room temperature
b. White phosphorus is metastable while red phosphorus is stable
c. White phosphorus is lighter than red phosphorus
d. White phosphorus is highly poisonous while red phosphorus is not
 - Red phosphorus is chemically less reactive because
a. it does not contain $\text{P}-\text{P}$ bonds
b. it does not contain tetrahedral P_4 molecules
c. it does not catch fire in air even upto 400°C
d. it has a polymeric structure
 - Ammonia (NH_3) on heating with carbon dioxide under pressure gives
a. NH_4HCO_3 b. $(\text{NH}_4)_2\text{CO}_3$
c. $\text{NH}_2\text{COONH}_4$ d. $(\text{NH}_4)_2\text{CO}$

22. Ammonia is mainly manufactured for fertilisers by the reaction

a. $2\text{NH}_4\text{Cl} + \text{Ca}(\text{OH})_2 \longrightarrow \text{CaCl}_2 + 2\text{H}_2\text{O} + 2\text{NH}_3$
 b. by passing an electric discharge in a mixture of N_2 and H_2
 c. by reducing the by-product nitric acid
 d. by passing a mixture of N_2 and H_2 under pressure and moderate temperature over a catalyst

23. In PCl_5 , phosphorus shows

a. sp^2 -hybridisation b. sp^3 -hybridisation
 c. sp^3d -hybridisation d. sp^3d^2 -hybridisation

24. In modern process, manufacturing of phosphorus is occurred by

a. heating a mixture of phosphorite mineral with sand and coke in electric furnace
 b. heating calcium phosphate with coke
 c. heating bone-ash with coke
 d. heating the phosphate mineral with sand

25. Which of the following compounds is not an interpseudo halogen?

a. Cl_2N_3 b. BrCN
 c. ClCN d. ICN

26. The number of hydrogen atoms attached to phosphorus atom in hypophosphorus is

a. three b. one
 c. two d. zero

27. Which of the following statements is true?

a. H_3PO_3 is a stronger acid than H_2SO_3
 b. In aqueous medium, HF is a stronger acid than HCl
 c. HClO_4 is a weaker acid than HClO_3
 d. HNO_3 is a stronger acid than HNO_2

28. In solid state, PCl_5 is a

a. covalent solid
 b. octahedral structure
 c. ionic solid with $[\text{PCl}_6]^+$ octahedral and $[\text{PCl}_4]^-$ tetrahedral
 d. ionic solid with $[\text{PCl}_4]^+$ tetrahedral and $[\text{PCl}_6]^-$ octahedral

29. Ionisation enthalpy of group 15 elements

I. decreases down the group due to the gradual increase in atomic size.
 II. is higher than that of group 14 elements in the corresponding periods.
 III. the order of successive ionisation enthalpies is $\Delta_i H_1 < \Delta_i H_2 < \Delta_i H_3$

Which of the above statements is/are true?

a. I and II b. II and III
 c. I and III d. I, II and III

30. **Assertion** (A) Heavier elements of group 15 do not form $p\pi-p\pi$ bonds.

Reason (R) Their atomic orbitals cannot have effective overlapping due to their large size.

Choose the most suitable option.

a. Both A and R are correct, R is the correct explanation of A
 b. Both A and R are correct; R is not the correct explanation of A
 c. A is correct, R is incorrect
 d. R is correct, A is incorrect

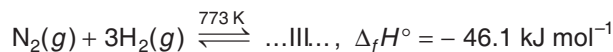
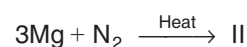
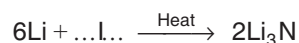
31. Consider the following statements,

I. Covalency of N is restricted to four.
 II. N cannot form $d\pi-p\pi$ bond as the heavier element can.
 III. P and As cannot form $d\pi-d\pi$ bond with transition elements when their compounds like $\text{P}(\text{C}_2\text{H}_5)_3$ and $\text{As}(\text{C}_6\text{H}_5)_3$ act as ligands.

Which of the above statements are true?

a. I and II b. II and III c. I and III d. All of these

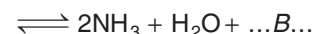
32. Complete the following reactions.



Here, I, II and III refer to

I	II	III
a. N_2	Mg_3N_2	NH_3
b. 2N_2	Mg_3N	NH_3
c. 2N_2	Mg_3N	2NH_3
d. N_2	Mg_3N_2	2NH_3

33. $\text{NH}_2\text{CONH}_2 + 2\text{H}_2\text{O} \longrightarrow \dots\text{A}\dots$



In the above equation, A and B respectively are

a. $(\text{NH}_4)_2\text{CO}_3$ and CO_2
 b. $(\text{NH}_4)_2\text{CO}_3$ and CO
 c. $(\text{NH}_4)\text{CO}_3$ and CO
 d. $(\text{NH}_4)\text{CO}_3$ and CO_2

34. The correct decreasing order for the acidic strength of oxides of nitrogen is

a. $\text{N}_2\text{O}_5 > \text{N}_2\text{O}_3 > \text{N}_2\text{O}_4 > \text{NO} > \text{N}_2\text{O}$
 b. $\text{NO} > \text{N}_2\text{O} > \text{N}_2\text{O}_3 > \text{N}_2\text{O}_4 > \text{N}_2\text{O}_5$
 c. $\text{N}_2\text{O} > \text{NO} > \text{N}_2\text{O}_3 > \text{N}_2\text{O}_4 > \text{N}_2\text{O}_5$
 d. $\text{N}_2\text{O}_5 > \text{N}_2\text{O}_4 > \text{N}_2\text{O}_3 > \text{NO} > \text{N}_2\text{O}$

35. Brown ring test for nitrates depends on

I. the ability of Fe^{2+} to reduce nitrates to nitric oxide.
 II. its reaction with Fe^{2+} to form a brown coloured complex.

Which of the above statements regarding brown test for nitrates is/are true?

a. Only I
 b. Only II
 c. Both I and II
 d. Neither I nor II

36. The correct increasing order of acidic strength of oxyacids of group 15 elements is
 a. $\text{H}_3\text{SbO}_4 < \text{H}_3\text{PO}_4 < \text{H}_3\text{AsO}_4 < \text{HNO}_3$
 b. $\text{H}_3\text{SbO}_4 < \text{H}_3\text{AsO}_4 < \text{H}_3\text{PO}_4 < \text{HNO}_3$
 c. $\text{HNO}_3 < \text{H}_3\text{SbO}_4 < \text{H}_3\text{AsO}_4 < \text{H}_3\text{PO}_4$
 d. $\text{H}_3\text{PO}_4 < \text{H}_3\text{AsO}_4 < \text{H}_3\text{SbO}_4 < \text{HNO}_3$
37. Reaction of HNO_3 with C, P, S and I respectively give
 a. CO, H_3PO_4 , H_2SO_4 and HIO_2
 b. CO_2 , P_2O_5 , SO_2 and I_2O_5
 c. CO_2 , H_3PO_3 , H_2SO_3 and HIO_3
 d. CO_2 , H_3PO_4 , H_2SO_4 and HIO_3
38. Phosphine is used in
 a. holme's signals
 b. smoke screens
 c. Both a. and b.
 d. Neither a. nor b.
39. In trimetaphosphate ion, the number of O-atoms, P—O—P bonds and unit negative charges respectively are
 a. 3, 6 and 3
 b. 9, 6 and 3
 c. 6, 6 and 3
 d. 9, 3 and 3
40. Complete the following reactions:
 I. $8\text{NH}_3 + 3\text{Cl}_2 \longrightarrow \text{I}$
 (Excess)
 II. $\text{NH}_3 + 3\text{Cl}_2 \longrightarrow \text{II}$
 (Excess)
- | | |
|--|---------------------------------------|
| I | II |
| a. $6\text{NH}_4\text{Cl} + \text{N}_2$ | $\text{NCl}_3 + 3\text{HCl}$ |
| b. $\text{NCl}_3 + 3\text{HCl}$ | $6\text{NH}_4\text{Cl} + \text{N}_2$ |
| c. $\text{NCl}_3 + \text{N}_2$ | $6\text{NH}_4\text{Cl} + 3\text{HCl}$ |
| d. $6\text{NH}_4\text{Cl} + 3\text{HCl}$ | $\text{NCl}_3 + \text{N}_2$ |
41. The number of S—S bonds in sulphur trioxide trimer (S_3O_9) is
 a. three
 b. two
 c. one
 d. zero
42. Oxidation states of O in OF_2 and O_2F_2 respectively are
 a. +1 and +2
 b. +1 and +3
 c. +2 and +3
 d. +2 and +1
43. Water is much less volatile than H_2S because
 a. H_2O has a bond angle of nearly 150°
 b. hydrogen is loosely bonded with the sulphur
 c. sulphur atom is less electronegative than oxygen atom
 d. sulphur atom is more electronegative than oxygen atom
44. Correct decreasing order of O—O bond length in O_2 , O_3 and H_2O_2 is
 a. $\text{O}_2 > \text{H}_2\text{O}_2 > \text{O}_3$
 b. $\text{H}_2\text{O}_2 > \text{O}_2 > \text{O}_3$
 c. $\text{H}_2\text{O}_2 > \text{O}_3 > \text{O}_2$
 d. $\text{O}_2 > \text{O}_3 > \text{H}_2\text{O}_2$
45. Uses of ozone (O_3) includes
 I. as an oxidising agent in the manufacturing of KMnO_4 .
 II. as a germicide, disinfectant and for sterilisation.
 III. for bleaching oils, flour, ivory, starch, etc.
 The correct set of uses of ozone is
 a. I and II
 b. II and III
 c. I and III
 d. All of these
46. Angular shape of ozone molecule consists of
 a. 1 σ and 1 π bond
 b. 2 σ and 2 π bonds
 c. 1 σ and 2 π bonds
 d. 2 σ and 1 π bond
47. Consider the following reaction,

$$2\text{SO}_2(g) + \text{O}_2(g) \xrightarrow{\text{Catalyst}} 2\text{SO}_3(g)$$

 Identify the catalyst
 a. V_2O_5
 b. CuCl_2
 c. MnO_2
 d. Either (a) or (b)
48. Peroxoacids of sulphur are
 a. $\text{H}_2\text{S}_2\text{O}_8$ and H_2SO_5
 b. $\text{H}_2\text{S}_2\text{O}_8$ and $\text{H}_2\text{S}_2\text{O}_7$
 c. $\text{H}_2\text{S}_2\text{O}_7$ and $\text{H}_2\text{S}_2\text{O}_6$
 d. H_2SO_5 and $\text{H}_2\text{S}_2\text{O}_7$
49. Consider the following statements,
 I. S—S bond is present in $\text{H}_2\text{S}_2\text{O}_6$.
 II. In peroxodisulphuric acid ($\text{H}_2\text{S}_2\text{O}_8$), sulphur is in +6 oxidation state.
 The correct set of statement is
 a. Only I
 b. Only II
 c. Both I and II
 d. Neither I nor II
50. On treating PCl_5 with H_2SO_4 , sulphuryl chloride (SO_2Cl_2) is formed as the final product. This shows that H_2SO_4
 a. is a derivative of SO_2
 b. is a dibasic acid
 c. has great affinity for water
 d. has two hydroxyl groups in its structure
51. Carborundum is obtained when silica is heated at high temperature with
 a. carbon
 b. carbon monoxide
 c. carbon dioxide
 d. calcium carbonate
52. Red lead is
 a. PbO
 b. Pb_3O_4
 c. PbO_2
 d. Pb_4O_3
53. Which of the following halides is least stable and has doubtful existence?
 a. Cl_4
 b. GeI_4
 c. SnI_4
 d. PbI_4
54. The mixture of conc. HCl and HNO_3 made 3 : 1 ratio contains
 a. ClO_2
 b. NOCl
 c. NCl_3
 d. N_2O_4

55. For making good quality mirrors, plates flint glass are used. These are obtained by floating molten glass over a liquid metal which does not solidify before glass. The metal used can be

a. tin b. sodium
c. magnesium d. mercury

56. When I^- is oxidised by MnO_4^- in alkaline medium, I^- converts into

a. IO_3^- b. I_2 c. IO_4^- d. IO^-

57. The van der Waals' forces in halogens decrease in the order

a. $\text{F}_2 > \text{Cl}_2 > \text{Br}_2 > \text{I}_2$ b. $\text{I}_2 > \text{Br}_2 > \text{Cl}_2 > \text{F}_2$
c. $\text{Br}_2 > \text{Cl}_2 > \text{F}_2 > \text{I}_2$ d. $\text{Cl}_2 > \text{Br}_2 > \text{I}_2 > \text{F}_2$

58. Decreasing order of reducing power of hydrogen halides is

a. $\text{HI} > \text{HBr} > \text{HCl} > \text{HF}$ b. $\text{HF} > \text{HI} > \text{HBr} > \text{HCl}$
c. $\text{HI} > \text{HF} > \text{HBr} > \text{HCl}$ d. None of these

59. Cl_2O is an anhydride of

a. HClO_4 b. HOCl c. HClO_2 d. Cl_2O_3

60. A gas X is passed through water to form a saturated solution. The aqueous solution on treatment with AgNO_3 gives a white precipitate.

The saturated aqueous solution also dissolves Mg ribbon with evolution of colourless gas Y . X and Y respectively are

a. CO_2 and Cl_2 b. Cl_2 and CO_2
c. Cl_2 and H_2 d. H_2 and Cl_2

61. In the oxyacids of chlorine, $\text{Cl}-\text{O}$ bond contains

a. $d\pi-d\pi$ bonding b. $p\pi-d\pi$ bonding
c. $p\pi-p\pi$ bonding d. None of these

62. The stability of the halides of group 16 elements decreases in the order

a. $\text{Br}^- > \text{I}^- > \text{Cl}^- > \text{F}^-$
b. $\text{F}^- > \text{I}^- > \text{Cl}^- > \text{Br}^-$
c. $\text{F}^- > \text{Cl}^- > \text{Br}^- > \text{I}^-$
d. $\text{I}^- > \text{Cl}^- > \text{Br}^- > \text{F}^-$

63. Arrange the following acids in the correct increasing order of their acidic strength:

HCl , HBr , HI , HF

a. $\text{HI} < \text{HBr} < \text{HCl} < \text{HF}$
b. $\text{HF} < \text{HCl} < \text{HBr} < \text{HI}$
c. $\text{HCl} < \text{HBr} < \text{HI} < \text{HF}$
d. $\text{HBr} < \text{HI} < \text{HF} < \text{HCl}$

64. Consider the following statements,

I. Fluorine forms two oxides OF_2 and O_2F_2 .
II. OF_2 is thermodynamically stable at 298 K.
III. O_2F_2 oxidises plutonium to PuF_6 .

The correct set of statements is,

a. I and II b. II and III
c. I and III d. I, II and III

65. Reduction potentials of some ions are given below.

Ion	ClO_4^-	IO_4^-	BrO_4^-
Reduction potential E°/V	$E^\circ = 1.19 \text{ V}$	$E^\circ = 1.65 \text{ V}$	$E^\circ = 1.74 \text{ V}$

Arrange them in decreasing order of oxidising power.

a. $\text{ClO}_4^- > \text{IO}_4^- > \text{BrO}_4^-$ b. $\text{IO}_4^- > \text{BrO}_4^- > \text{ClO}_4^-$
c. $\text{BrO}_4^- > \text{IO}_4^- > \text{ClO}_4^-$ d. $\text{BrO}_4^- > \text{ClO}_4^- > \text{IO}_4^-$

66. Consider the following statements,

I. Among halide ions, iodine is the most powerful reducing agent.

II. HOCl is stronger acid than HOBr .

III. HF is stronger acid than HCl .

IV. Fluorine is the only halogen that does not show variable oxidation states.

The correct set of statements is

a. I, II and III b. II and IV
c. I, II and IV d. I, II, III and IV

67. Among the $\text{C}-\text{X}$ bond (where, $\text{X} = \text{Cl}, \text{Br}, \text{I}$) the correct decreasing order of bond energy is

a. $\text{C}-\text{I} > \text{C}-\text{Cl} > \text{C}-\text{Br}$
b. $\text{C}-\text{I} > \text{C}-\text{Br} > \text{C}-\text{Cl}$
c. $\text{C}-\text{Cl} > \text{C}-\text{Br} > \text{C}-\text{I}$
d. $\text{C}-\text{Br} > \text{C}-\text{Cl} > \text{C}-\text{I}$

68. $4\text{HCl} + \text{O}_2 \xrightarrow{\text{CuCl}_2} 2\text{Cl}_2 + 2\text{H}_2\text{O}$

The above reaction of chlorine preparation is of

a. Deacon's process b. Contact process
c. Either (a) or (b) d. None of these

69. Bleaching powder is an example of

a. a complex salt b. an acidic salt
c. a basic salt d. a mixed salt

70. I. $2\text{NaOH} + \text{Cl}_2 \longrightarrow \dots \text{A} \dots + \dots \text{B} \dots + \text{H}_2\text{O}$
(cold and dil.)

II. $6\text{NaOH} + 3\text{Cl}_2 \longrightarrow \dots \text{C} \dots + \dots \text{D} \dots + 3\text{H}_2\text{O}$
(hot and conc.)

Here, A , B , C and D refer to

A	B	C	D
a. NaCl	NaClO_3	NaOCl	5NaCl
b. 5NaCl	NaClO_3	NaOCl	NaCl
c. NaCl	NaOCl	5NaCl	NaClO_3
d. 5NaCl	NaOCl	NaCl	NaClO_3

71. The reactions in which conc. H_2SO_4 is used as an oxidising agent are

I. $2\text{HI} + \text{H}_2\text{SO}_4 \longrightarrow \text{I}_2 + \text{SO}_2 + 2\text{H}_2\text{O}$

II. $\text{NaCl} + 2\text{H}_2\text{SO}_4 \longrightarrow \text{NaHSO}_4 + \text{HCl}$

III. $\text{CaF}_2 + \text{H}_2\text{SO}_4 \longrightarrow \text{CaSO}_4 + 2\text{HF}$

IV. $\text{Cu} + 2\text{H}_2\text{SO}_4 \longrightarrow \text{CuSO}_4 + \text{SO}_2 + 2\text{H}_2\text{O}$

Choose the correct option.

a. I and II b. II and III
c. I and III d. I and IV

72. Hydrolysis of one mole of peroxodisulphuric acid will yield
 a. two moles of peroxomonosulphuric acid
 b. two moles of sulphuric acid
 c. one mole each of sulphuric acid, peroxomonosulphuric acid and hydrogen peroxide
 d. one mole of sulphuric acid and one mole of peroxomonosulphuric acid
73. Enthalpy of molecular oxygen is almost identical with that of
 a. neon b. argon c. helium d. xenon
74. The ease of liquefaction of noble gases increases in the order
 a. $\text{He} < \text{Xe} < \text{Ar} < \text{Kr} < \text{Ne}$ b. $\text{Xe} < \text{Kr} < \text{Ar} < \text{He} < \text{Ne}$
 c. $\text{He} < \text{Ne} < \text{Ar} < \text{Kr} < \text{Xe}$ d. $\text{Xe} < \text{He} < \text{Ne} < \text{Ar} < \text{Kr}$
75. Among XeO_3 , XeO_2F_2 and XeF_6 , the molecules having same number of lone pairs on Xe are
 a. XeO_3 and XeO_4
 b. XeO_3 and XeO_2F_2
 c. XeO_4 and XeF_6
 d. XeO_3 , XeO_4 and XeF_6
76. In the ring test for NO_3^- ion, a brown ring is formed due to the formation of
 a. $\text{FeSO}_4 \cdot \text{NO}_2$ b. $\text{FeSO}_4 \cdot \text{HNO}_3$
 c. $[\text{Fe}(\text{H}_2\text{O})_5(\text{NO})]^{2+}$ d. $[\text{Fe}(\text{H}_2\text{O})_4(\text{NO})_2]^{2+}$
77. Which of the following statements regarding sulphur is incorrect?
 a. S_2 molecule is paramagnetic
 b. The vapour at 200°C consists mostly of S_8 rings
 c. At 600°C , the gas mainly consists of S_2 molecules
 d. The oxidation state of sulphur is never less than +4 in its compounds
78. Which one of the following reactions of xenon compounds is not feasible?
 a. $\text{XeO}_3 + 6\text{HF} \longrightarrow \text{XeF}_6 + 3\text{H}_2\text{O}$
 b. $3\text{XeF}_4 + 6\text{H}_2\text{O} \longrightarrow 2\text{Xe} + \text{XeO}_3 + 12\text{HF} + 1.5 \text{O}_2$
 c. $2\text{XeF}_2 + 2\text{H}_2\text{O} \longrightarrow 2\text{Xe} + 4\text{HF} + \text{O}_2$
 d. $\text{XeF}_6 + \text{RbF} \longrightarrow \text{Rb}[\text{XeF}_7]$
79. Which of the following acids forms three series of salts?
 a. H_3PO_2 b. H_3BO_3 c. H_3PO_4 d. H_3PO_3
80. The correct decreasing order of the acidic strength of HClO , HClO_2 , HClO_3 , HClO_4 is
 a. $\text{HClO} > \text{HClO}_2 > \text{HClO}_3 > \text{HClO}_4$
 b. $\text{HClO}_4 > \text{HClO}_3 > \text{HClO}_2 > \text{HClO}$
 c. $\text{HClO}_4 > \text{HClO}_2 > \text{HClO} > \text{HClO}_3$
 d. $\text{HClO}_3 > \text{HClO} > \text{HClO}_4 > \text{HClO}_2$
81. The hybrid state of halogen atom is sp^3 in
 a. ClO_4^-
 b. ClO^-
 c. ClO_3^-
 d. All of the above
82. Interhalogen compounds are
 a. covalent molecules
 b. diamagnetic in nature
 c. volatile solids/liquids at 298 K except ClF
 d. All of the above
83. Interhalogen compounds can be used as
 I. non-aqueous solvents.
 II. fluorinating agents.
 The correct use(s) is/are
 a. Only I
 b. Only II
 c. Both I and II
 d. Neither I nor II
84. **Assertion** (A) $\text{X}-\text{X}'$ bond in interhalogens is weaker than $\text{X}-\text{X}$ bond in halogens.
Reason (R) Interhalogen compounds are more reactive than halogens (except fluorine).
 a. Both A and R are correct; R is the correct explanation of A
 b. Both A and R are correct; R is not the correct explanation of A
 c. A is correct; R is incorrect
 d. R is correct; A is incorrect
85. Which of the following statements are correct?
 I. Among halogens, radius ratio between iodine and fluorine is maximum.
 II. Leaving $\text{F}-\text{F}$ bond, all halogens have weaker $\text{X}-\text{X}$ bond than $\text{X}-\text{X}'$ bond in interhalogens.
 III. Among interhalogen compounds, maximum number of atoms are present in iodine fluoride.
 IV. Interhalogen compounds are more reactive than halogen compounds.
 The correct option is
 a. I, II and III
 b. I, III and IV
 c. II, III and IV
 d. I and II
86. **Assertion** (A) SF_6 is known but SCl_6 is not.
Reason (R) Due to small size of F.
 a. Both A and R are correct; R is the correct explanation of A
 b. Both A and R are correct; R is not the correct explanation of A
 c. A is correct; R is incorrect
 d. R is correct; A is incorrect

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- The reaction of P_4 with X leads selectively to P_4O_6 .
 X is [2013]
 - dry O_2
 - moist O_2
 - mixture of O_2 and N_2
 - O_2 in the presence of aqueous $NaOH$
- The acidic strength for the hydrides of group 15 follows the order [2013]
 - $NH_3 > PH_3 > AsH_3 > SbH_3$
 - $NH_3 < PH_3 < AsH_3 < SbH_3$
 - $NH_3 > PH_3 > SbH_3 > AsH_3$
 - $NH_3 < PH_3 < SbH_3 < AsH_3$
- The molecules BF_3 and NF_3 both are covalent compounds but BF_3 is non-polar and NF_3 is polar. The reason is that [2012]
 - boron is a metal and nitrogen is a gas in uncombined state
 - BF_3 bonds have no dipole moment whereas NF_3 bond have dipole moment
 - atomic size of boron is smaller than that of nitrogen
 - BF_3 is symmetrical molecule whereas NF_3 is unsymmetrical
- Sodium carbonate reacts with SO_2 in aqueous medium to give [2011]

a. $NaHSO_3$	b. $Na_2S_2O_3$
c. $NaHSO_4$	d. Na_2SO_4
- The highest bond strength is shown by [2010]

a. $O-O$ bond	b. $S-S$ bond
c. $Se-Se$ bond	d. $Te-Te$ bond
- Which one of the following is pyrophosphoric acid? [2009]

a. H_3PO_4	b. $H_4P_2O_7$
c. $H_4P_2O_5$	d. H_3PO_4
- Which of the following has maximum dipole moment? [2009]

a. NCI_3	b. NBr_3
c. NH_3	d. NI_3
- Chlorine acts as a bleaching agent only in the presence of [2009]

a. dry air	b. moisture
c. sunlight	d. pure oxygen
- Helium-oxygen mixture is used by deep sea divers in the presence of nitrogen-oxygen mixture because [2009]
 - helium is much less soluble in blood than nitrogen
 - nitrogen is much less soluble in blood than helium
 - due to high pressure deep under the sea, nitrogen and oxygen react to give poisonous nitric oxide
 - nitrogen is highly soluble in water
- Distinguishing reagent between silver and lead salt is [2009]
 - H_2S gas
 - dil. HCl solution
 - $[NH_2Cl \text{ (solid)} + NH_4OH]$ solution
 - $[NH_2Cl \text{ (solid)} + (NH_4)CO_3]$ solution
- Identify B in the following reaction: [2008]

$$H_4SiO_4 \xrightarrow[-H_2O]{1000^\circ C} A \xrightarrow[\Delta]{\text{Carbon}} B + CO$$
 - Corundum
 - Quartz
 - Silica
 - Carborundum
- The correct order of reducing abilities of hydrides of group 15 elements is [2008]
 - $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$
 - $NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$
 - $NH_3 < PH_3 > AsH_3 > SbH_3 > BiH_3$
 - $SbH_3 > BiH_3 > AsH_3 > NH_3 > PH_3$
- Which one of the following pentafluorides cannot be formed? [2007]

a. PF_5	b. AsF_5
c. SbF_5	d. BiF_5
- Which of the following is most volatile compound? [2007]

a. HI	b. HCl
c. HBr	d. HF
- SO_2 does not act as a/an [2006]

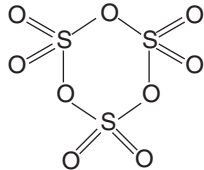
a. bleaching agent	b. oxidising agent
c. reducing agent	d. dehydrating agent
- Which noble gas is not found in atmosphere? [2006]

a. Ne	b. Ar	c. Rn	d. Kr
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- In P_4O_{10} , [2009]
 - second bond in $P=O$ is formed by $p\pi - d\pi$ back bonding
 - $P=O$ bond is formed by $p\pi - p\pi$ bonding
 - $P=O$ bond is formed by $d\pi - d\pi$ bonding
 - $P=O$ bond is formed by $d\pi - d\pi - 3 \sigma$ back bonding
- Dinitrogen pentoxide (N_2O_5), a colourless solid, is prepared by [2005]
 - heating NH_2NO_2 with an excess of oxygen
 - dehydrating HNO_2 with CaO
 - dehydrating HNO_3 with P_4O_{10}
 - heating a mixture of HNO_2 and $Ca(NO_3)_2$
- Incorrect statement for pyrophosphorus acid ($H_4P_2O_5$) is [2005]
 - It contains P in + 5 oxidation state
 - It is dibasic acid
 - It is strongly reducing in nature
 - It contains one $P-O-P$ bond

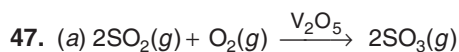
Answer with Solutions

Practice Exercise

- (d) $4\text{HNO}_3 + 2\text{P}_2\text{O}_5 \xrightarrow{\Delta} 2\text{N}_2\text{O}_5 + 4\text{HPO}_3$
- (d) $[\text{N} \equiv \text{N}^+ - \text{N}^-]^- \longrightarrow [\text{N}^- - \text{N}^+ \equiv \text{N}]^-$
- (d) $\text{Ba}(\text{N}_3)_2 \xrightarrow{\text{Heat}} \text{Ba(s)} + 3\text{N}_2(\text{g}) \uparrow$
- (b)
- (d) PH_3 is a covalent hydride.
- (b) Pentoxides are more acidic than trioxides and their acidic character decreases down the group from N to Bi.
- (d) $\text{SO}_2, \text{H}_2\text{O}, \text{H}_2\text{SO}_4, \text{P}_4\text{O}_{10}$
- (b) -3 to +5
- (b) HNO_2 , in its oxidation number of nitrogen is +3 (i.e. in between -3 to +5).
- (a) $2\text{H}_2\text{SO}_4 + 2\text{P}_2\text{O}_5 \longrightarrow 2\text{SO}_3 + 4\text{HPO}_3$
- (c) Due to the presence of one —OH group
- (c) As we move down the group, the repulsion of bond pair and lone pair of central atom decreases.
- (a) $\text{BiH}_3 < \text{SbH}_3 < \text{AsH}_3 < \text{PH}_3 < \text{NH}_3$
Least stable Most stable
- (c) Nitrogen does not show pentacovalency due to non-availability of *d*-orbitals.
- (a)
- (c) Due to availability of *d*-orbitals
- (d) The reducing nature increases from NH_3 to SbH_3 .
- (b) Air contains 1% argon which is heavier than N_2 .
- (a) Red phosphorus is less reactive than white phosphorus.
- (d) It is a reason for the given fact.
- (c) $2\text{NH}_3 + \text{CO}_2 \xrightarrow{\Delta} \text{NH}_2\text{COONH}_4 \xrightarrow{\text{On heating}} \text{NH}_2\text{CONH}_2 + \text{H}_2\text{O}$
- (d) By Haber's process
- (c) P in PCl_5 has sp^3d -hybridisation.
- (a) $\text{Ca}_3(\text{PO}_4)_2 + 3\text{SiO}_2 \longrightarrow 3\text{CaSiO}_3 + \text{P}_2\text{O}_5$
 $2\text{P}_2\text{O}_5 + 10\text{C} \longrightarrow \text{P}_4 + 10\text{CO}$
- (a) Cl_2N_3 is not an interpseudo halogen.
- (c)
- (d) $\text{H} \longrightarrow \text{O} \longrightarrow \text{N}^+ \begin{matrix} \text{O} \\ \text{O}^- \end{matrix}$,
 $\text{H} \longrightarrow \text{O} \longrightarrow \text{N}=\text{O}$
Polarity along O—H in HNO_3 is more in comparison to O—H in HNO_2 .

- (d)
- (d)
- (a) Heavier elements of group 15 do not form $p\pi - p\pi$ bonds as their atomic orbitals are so large and diffuse that they cannot have effective overlapping.
- (a) P and As form $d\pi - d\pi$ bond with transition elements when their compounds like $\text{P}(\text{C}_2\text{H}_5)_3$ and $\text{As}(\text{C}_6\text{H}_5)_3$ act as ligands.
- (d)
- (a) $\text{NH}_2\text{CONH}_2 + 2\text{H}_2\text{O} \longrightarrow (\text{NH}_4)_2\text{CO}_3$
 $\rightleftharpoons 2\text{NH}_3 + \text{H}_2\text{O} + \text{CO}_2$
- (d) $\text{N}_2\text{O}_5 > \text{N}_2\text{O}_4 > \text{N}_2\text{O}_3 > \text{NO} > \text{N}_2\text{O}$
- (c) Both the statements are correct.
 $\text{NO}_3^- + 3\text{Fe}^{2+} + 4\text{H}^+ \longrightarrow \text{NO} + 3\text{Fe}^{3+} + 2\text{H}_2\text{O}$
 $[\text{Fe}(\text{H}_2\text{O})_6]^{2+} + \text{NO} \longrightarrow [\text{Fe}(\text{H}_2\text{O})_5(\text{NO})]^{2+} + \text{H}_2\text{O}$
Brown coloured complex
- (b) $\text{H}_3\text{SbO}_4 < \text{H}_3\text{AsO}_4 < \text{H}_3\text{PO}_4 < \text{HNO}_3$
- (d) Reaction of HNO_3 with C, P, S and I gives $\text{CO}_2, \text{H}_3\text{PO}_4, \text{H}_2\text{SO}_4$ and HIO_3 respectively.
- (c) The spontaneous combustion of phosphine is technically used in Holme's signals. Containers containing calcium carbide and calcium phosphide are pierced and thrown in the sea when the gases evolved burn and serve as a signal. It is also used in smoke screens.
- (d) In trimetaphosphate ion, the number of O-atoms, P—O—P bonds and unit negative charges are 9, 3, 3 respectively.
- (a) (I) $8\text{NH}_3 + 3\text{Cl}_2 \longrightarrow 6\text{NH}_4\text{Cl} + \text{N}_2$
(Excess)
(II) $\text{NH}_3 + 3\text{Cl}_2 \longrightarrow \text{NCl}_3 + 3\text{HCl}$
(Excess)
- (d) S_3O_9 is as

- (d) Oxidation states of O in OF_2 and O_2F_2 are +2 and +1 respectively.
- (c) Water is much less volatile than H_2S because sulphur atom is less electronegative than oxygen atom.
- (c) $\text{H}_2\text{O}_2 > \text{O}_3 > \text{O}_2$
- (d) **Uses of ozone**
As a germicide, disinfectant and for sterilising water. Also used for bleaching oils, ivory, flour, starch, etc. It acts as an oxidising agent in the manufacturing of KMnO_4 .

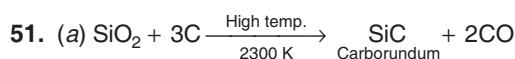
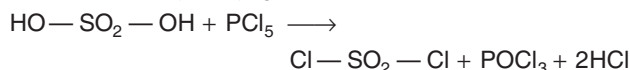
46. (d) Angular shape of ozone molecule consists of two σ and one π -bond.



48. (a) Peroxoacids of sulphur are $\text{H}_2\text{S}_2\text{O}_8$ and H_2SO_5 .

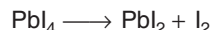
49. (a) In $\text{H}_2\text{S}_2\text{O}_6$, S—S bond is present.

50. (d) When PCl_5 reacts with sulphuric acid, sulphuryl chloride (SO_2Cl_2) is formed as the final product. It shows that it has two hydroxyl groups in its structure.



52. (b) Pb_3O_4 or $\text{PbO}_2 \cdot 2\text{PbO}$ is called red lead.

53. (d) Pb^{4+} will readily oxidise iodide and itself will get reduced to Pb^{2+} . Thus, PbI_2 will be formed,



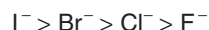
54. (b) $3\text{HCl} + \text{HNO}_3 \longrightarrow \text{NOCl} + 2\text{H}_2\text{O} + 2[\text{Cl}]$

55. (a) Melting point of tin is lower than that of glass.

56. (a) $2\text{MnO}_4^- + \text{H}_2\text{O} + \text{I}^- \longrightarrow 2\text{MnO}_2 + 2\text{OH}^- + \text{IO}_3^-$

57. (b) van der Waals' forces increase with increase in the size and molecular weight of the atom.

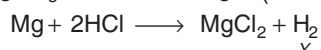
58. (a) The order of reducing power of halide ions is



59. (b) $\text{Cl}_2\text{O} + \text{H}_2\text{O} \longrightarrow 2\text{HClO}$

Chlorine has +1 oxidation state in Cl_2O and HOCl .

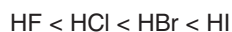
60. (c) $\text{Cl}_2 + \text{H}_2\text{O} \xrightarrow{\text{X}} \text{HOCl} + \text{HCl}$



61. (b) Cl is sp^3 -hybridised having electrons in d -orbitals and p -electrons of oxygen gives rise to $p\pi - d\pi$ bonding to Cl—O bond.

62. (c) Elements of group 16 form a large number of halides of the type EX_6 , EX_4 and EX_2 , where E is an element of the group and X is a halogen. $\text{F}^- > \text{Cl}^- > \text{Br}^- > \text{I}^-$

63. (b) Correct increasing order of their acidic strength is



64. (d) Fluorine forms two oxides, i.e. OF_2 and O_2F_2 . OF_2 is thermodynamically stable at 298 K. These oxides are essentially oxygen fluorides because of the higher electronegativity of fluorine than oxygen. Both are strong fluorinating agents. O_2F_2 oxidises plutonium to PuF_6 and the reaction is used in removing plutonium as PuF_6 from spent nuclear fuel.

65. (d) Greater the SRP value of species, higher will be its oxidising power. Hence, on the basis of the reduction potential of ions, the decreasing order of oxidising power is $\text{BrO}_4^- > \text{ClO}_4^- > \text{IO}_4^-$.

66. (c) HF is a weaker acid than HCl.

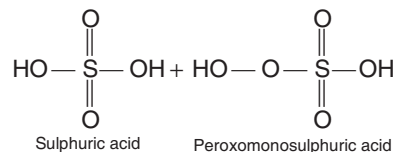
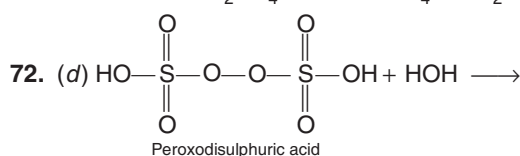
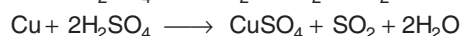
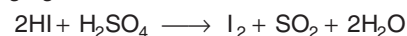
67. (c) Among the C—X bond (where, $X = \text{Cl}, \text{Br}, \text{I}$), the correct decreasing order of bond energy is $\text{C}—\text{Cl} > \text{C}—\text{Br} > \text{C}—\text{I}$

68. (a)

69. (d) A mixed salt is one which gives more than one type of cations or anions, e.g. $\text{Ca}^{2+} + \text{OCl}^- + \text{Cl}^-$.

70. (c)

71. (d) In the following reactions, conc. H_2SO_4 is used as an oxidising agent.



73. (d) Enthalpy of molecular oxygen (1175 kJ mol^{-1}) is almost identical with that of xenon (1170 kJ mol^{-1}).

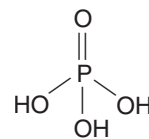
74. (c) 75. (b) 76. (c)

77. (d) Oxidation states of sulphur in H_2S is -2 . Thus, incorrect (valency can be less than 4).

78. (a) XeF_6 has much tendency to hydrolyse. The reverse reaction is more spontaneous.

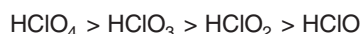


79. (c) Structure of H_3PO_4 is



H_3PO_4 has 3 —OH groups, i.e. has three ionisable H-atoms and hence forms three series of salts. These three possible series of salts for H_3PO_4 are as follows NaH_2PO_4 , Na_2HPO_4 and Na_3PO_4 .

80. (b) The correct decreasing order of the acidic strength of oxoacids of chlorine is



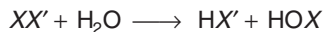
81. (d)

82. (d) Interhalogen compounds are covalent molecules and diamagnetic in nature. These are volatile solids or liquids at 298 K except ClF .

83. (c)

84. (b) Interhalogen compounds are more reactive than halogens (except fluorine). This is because $\text{X}-\text{X}'$ bond in interhalogens is weaker than $\text{X}-\text{X}$ bond. All these

undergo hydrolysis giving halide ion derived from the smaller halogen and a hypohalite (when XX'), halite (when XX'_3), halate (when XX'_5) and perhalate (when XX'_7) anion derived from the larger halogen.



85. (b) Among halogens, radius ratio between iodine and fluorine is maximum. Among interhalogen compounds,

maximum number of atoms are present in iodine fluoride. Interhalogen compounds are more reactive than halogen compounds.

86. (a) SF_6 is known but SCl_6 is not. It is due to the small size of fluorine. Six F^- ions can be accommodated around sulphur while chloride ion is comparatively larger in size, thus there will be interatomic repulsion.

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- (c) The reaction of P_4 and X leads selectively to P_4O_6 because N_2 prevents the further reaction of P_4O_6 into P_4O_{10} .
- (b) $NH_3 < PH_3 < AsH_3 < SbH_3$
- (d) BF_3 is symmetrical planar, although it has polar bonds but resultant dipole moment is zero. In NF_3 , lone pair cause distortion, hence polarity arises.
- (a) $Na_2CO_3 + 2SO_2 + H_2O \longrightarrow 2NaHSO_3 + CO_2$
- (b) As the size increases, bond strength decreases. But S—S bond is stronger than O—O bond because of smaller size of O-atoms.
- (b)
- (c) The electronegativity difference is highest in case of N—H bonds, therefore NH_3 has the highest dipole moment.
- (b) Bleaching action of chlorine is only in the presence of moisture.
- (a) He- O_2 (80%-20%) mixture is used by deep sea divers for artificial respiration. Because of low intermolecular force in He, it is much less soluble in aqueous solutions (as compared to N_2) such as blood and does not cause "caisson sickness" or "bends" by bubbling out of blood

when the worker moves from high pressure (while in deep sea) to atmospheric pressure.

- (b) On adding dil. HCl to silver and lead salts, white precipitate of chlorides of Ag and Pb are obtained which on heating with water are separated from each other as $PbCl_2$ is soluble in hot water but AgCl is not.
- (d) $H_4SiO_4 \xrightarrow[-2H_2O]{1000^\circ C} SiO_2 \xrightarrow[\Delta]{C} \underset{\text{Carborundum}}{SiC} + CO$
- (c) $NH_3 < PH_3 > AsH_3 > SbH_3 > BiH_3$
- (d) The +5 oxidation state of Bi is unstable due to inert pair effect. Thus, BiF_5 cannot be formed.
- (b) Boiling point of HF is highest due to H-bonding. For other halogen acids, boiling point increases in the order $HCl < HBr < HI$. Therefore, most volatile (with lower boiling point) is HCl.
- (d) SO_2 does not act as a dehydrating agent.
- (b) The noble gas which is not found in atmosphere is argon (Ar).
- (a) The terminal bond formed is $p\pi - d\pi$ bonding.
- (c) $P_4O_{10} + 4HNO_3 \longrightarrow 4HPO_3 + 2N_2O_5$
- (a) It contains phosphorus in +3 oxidation state.

19

d- and *f*-Block Elements

d-Block Elements

d-block elements are present from fourth period onwards. There are mainly three series of the transition metals, (a) 3*d*-series (Sc to Zn), (b) 4*d*-series (Y to Cd) and (c) 5*d*-series (La to Hg, omitting Ce to Lu).

d-block elements are known as **transition elements** because their position in the periodic table is in between the *s*-block and *p*-block elements.

Electronic Configuration

3rd group	4th group	5th group	6th group	7th group
$ns^2(n-1)d^1$	$ns^2(n-1)d^2$	$ns^2(n-1)d^3$	$ns^1(n-1)d^5$	$ns^2(n-1)d^5$
Sc	Ti	V	Cr	Mn
Y	Zr	Nb	Mo	Tc
La	Hf	Ta	W	Re
8th group	9th group	10th group	11th group	12th group
$ns^2(n-1)d^6$	$ns^2(n-1)d^7$	$ns^2(n-1)d^8$	$ns^1(n-1)d^{10}$	$ns^2(n-1)d^{10}$
Fe	Co	Ni	Cu	Zn
Ru	Rh	Pd	Ag	Cd
Os	Ir	Pt	Au	Hg

- Unlike Cr and Mo, W has electronic configuration as $ns^2(n-1)d^4$.

General Characteristics of *d*-Block Elements

- Transition elements show all the properties of metals like metallic lustre, conductivity, malleability, ductility, high tensile strength except mercury which is liquid at room temperature.
- Their atomic radii is intermediate between *s*- and *p*-block elements.
- These elements possess high melting and boiling points, high enthalpies of vaporisation, atomisation and hydration of ions.
- Their first ionisation enthalpy is higher than that of *s*-block elements but less than that of *p*-block elements.
- These elements are electropositive in nature and show variable oxidation states.
- Generally, these elements are paramagnetic in nature and form coloured compounds.
- These elements form interstitial compounds with elements like C, H, B and N and show catalytic behaviour.

Metallic Character

With the exception of Zn, Cd and Hg, all the transition elements are much harder and less volatile. Although hardness and brittleness are associated with covalent bonding between metal atoms, while the overlapping of unfilled *d*-orbitals of metals causes covalent bonding, the valence *s*-electrons are responsible for metallic bonding. Cu, Ag and Au are particularly outstanding in their thermal and electrical conductivities.

Oxidation States

Some common oxidation states of 3*d*-series elements are shown below:

Name of the elements	Oxidation states shown by 3 <i>d</i> -series
Sc	+3
Ti	+2, +3, +4
V	+2, +3, +4, +5
Cr	+1, +2, +3, +4, +5, +6
Mn	+2, +3, +4, +5, +6, +7
Fe	+2, +3, +4, +5, +6
Co	+2, +3, +4
Ni	+2, +3, +4
Cu	+1, +2
Zn	+2

Ionic Radii

The trend for ionic radii is same as that of atomic radii.

- Ionic radii decreases with increase in oxidation state.
- Ionic radii of cations present in the same oxidation state, decreases with increase in atomic number.

Catalytic Properties

Transition metals have two incomplete outermost shells and ability to adopt multiple oxidation states and to form complexes, therefore used as a catalyst. Transition metals also provide larger surface area for the reactant to be adsorbed.

Magnetic Properties

When a magnetic field is applied to substances, mainly two types of magnetic behaviours are observed: diamagnetism and paramagnetism. The magnetic moment is determined by the number of unpaired electrons (*n*) which is given by

$$\mu = \sqrt{n(n+2)} \text{ BM} \quad (\text{BM} = \text{Bohr Magnetron})$$

where, *n* = number of unpaired electrons

If *n* = 0, the substance is diamagnetic.

If *n* ≠ 0, the substance is paramagnetic.

Interstitial Compounds

Transition elements possess a tendency to trap atoms like H, B, C, N in their interstitial spaces of their lattices, e.g. TiH_{1.7}, VH_{0.56}. These compounds are very hard, chemically inert and have higher melting points than that of pure metals.

I. Iron (Fe)

Ores Haematite – Fe₂O₃

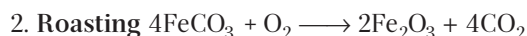
Magnetite – Fe₃O₄

Iron pyrites – FeS₂

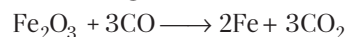
Extraction

1. Concentration

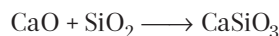
- Gravity separation method
- Magnetic separator



3. Reduction/Smelting



Pig iron



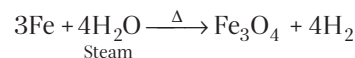
Wrought iron is prepared by 'pudding process.'

Steel is obtained by Bessemer process.

Properties of Iron

It is a grey-white lustrous metal. It is highly tensile, ductile, malleable, magnetic metal etc.

Some important reactions



Compounds of Iron

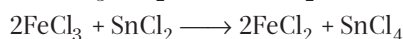
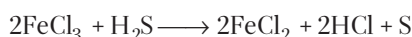
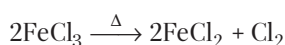
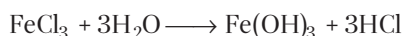
1. Ferric Chloride (FeCl₃)

Preparation

- (a) $2\text{Fe} + 3\text{Cl}_2 \longrightarrow 2\text{FeCl}_3$
 (b) $2\text{Fe} + 4\text{HCl} + \text{Cl}_2 \longrightarrow 2\text{FeCl}_3 + 2\text{H}_2$
 (c) $\text{Fe}_2\text{O}_3 + 6\text{HCl} \longrightarrow 2\text{FeCl}_3 + 3\text{H}_2\text{O}$

Properties

Hydrated FeCl₃·6H₂O is a yellow, deliquescent, crystalline solid, soluble in water, alcohol etc., and anhydrous FeCl₃ is a dark solid.



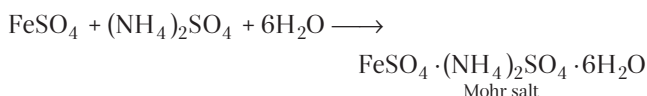
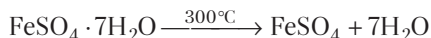
2. Ferrous Sulphate (FeSO₄·7H₂O) (Green Vitriol)

Preparation

- (a) $\text{FeCO}_3 + \text{H}_2\text{SO}_4 \longrightarrow \text{FeSO}_4 + \text{H}_2\text{O} + \text{CO}_2$
 (b) $\text{FeS} + \text{H}_2\text{SO}_4 \longrightarrow \text{FeSO}_4 + \text{H}_2\text{S}$ (Kipp's apparatus)

Properties

It is light green crystalline solid, soluble in water efflorescent and isomorphous with MgSO₄·7H₂O.



II. Copper (Cu)

Ores Copper pyrites – CuFeS₂

Copper glance – Cu₂S

Cuprite – Cu₂O

Malachite – CuCO₃·Cu(OH)₂

Properties of Copper

- Copper is reddish golden coloured metal, soft, malleable, ductile and second best conductor next only to silver, molten metal adsorbs SO₂(g).
- $2\text{Cu} + \text{H}_2\text{O} + \text{CO}_2 + \text{O}_2 \longrightarrow \text{CuCO}_3 \cdot \text{Cu}(\text{OH})_2$
- $8\text{Cu} + 6\text{H}_2\text{O} + 2\text{SO}_2 + 5\text{O}_2 \longrightarrow \text{CuSO}_4 \cdot 3\text{Cu}(\text{OH})_2$
- $\text{Cu} + 4\text{HNO}_3(\text{conc.}) \longrightarrow \text{Cu}(\text{NO}_3)_2 + 2\text{H}_2\text{O} + 2\text{NO}_2$

III. Zinc (Zn)

Ores Zinc blende – (ZnS)

Calamine – ZnCO₃

Zincite – ZnO

Properties of Zinc

- Bluish white crystalline metal, brittle at ordinary temperature, malleable and ductile.
- $2\text{Zn} + \text{O}_2 \xrightarrow{\Delta} 2\text{ZnO}$
- $\text{Zn} + \text{H}_2\text{O} \longrightarrow \text{ZnO} + \text{H}_2$
- $4\text{Zn} + 3\text{H}_2\text{O} + \text{CO}_2 + 2\text{O}_2 \longrightarrow \text{ZnCO}_3 \cdot 3\text{Zn}(\text{OH})_2$
- $\text{Zn} + 4\text{HNO}_3 \xrightarrow{(\text{conc. and hot})} \text{Zn}(\text{NO}_3)_2 + 2\text{NO}_2 + 2\text{H}_2\text{O}$
- $\text{Zn} + 2\text{NaOH} \longrightarrow \text{Na}_2\text{ZnO}_2 + \text{H}_2$

Compounds of Zn

1. Zinc Oxide (ZnO)

Preparation

- (a) $2\text{Zn} + \text{O}_2 \longrightarrow 2\text{ZnO}$
 (b) $\text{Zn}(\text{NO}_3)_2 \xrightarrow{\Delta} 2\text{ZnO} + 4\text{NO}_2 + \text{O}_2$

Properties

- It is white, light powder insoluble in water. On heating, becomes yellow.
- $\text{ZnO} + \text{H}_2 \longrightarrow \text{Zn} + \text{H}_2\text{O}$
- $\text{ZnO} + 2\text{NaOH} \longrightarrow \text{Na}_2\text{ZnO}_2 + \text{H}_2\text{O}$

2. Zinc Sulphide (ZnS)

Preparation

- (a) $\text{ZnSO}_4 + \text{H}_2\text{S} \longrightarrow \text{ZnS} + \text{H}_2\text{SO}_4$
 (b) $2\text{ZnO} + 3\text{S} \xrightarrow{\Delta} 2\text{ZnS} + \text{SO}_2$
 (c) $\text{ZnCl}_2 + \text{H}_2\text{S} + 2\text{NH}_3 \longrightarrow \text{ZnS} + 2\text{NH}_4\text{Cl}$

Properties

- ZnS is a white solid insoluble in water and soluble in mineral acids. It turns yellow in light.
- It also shows phosphorescence.

3. Zinc Sulphate (ZnSO₄·7H₂O) (White Vitriol)



Properties

- It is highly soluble in water.
- $\text{ZnSO}_4 \cdot 7\text{H}_2\text{O} \xrightarrow[100^\circ\text{C}]{\Delta} \text{ZnSO}_4 \cdot 6\text{H}_2\text{O} \xrightarrow[280^\circ\text{C}]{\Delta} \text{ZnSO}_4$
- $\text{ZnSO}_4 \xrightarrow[760^\circ\text{C}]{\Delta} \text{ZnO} + \text{SO}_3$
- It is isomorphous with epsom salt and green vitriol.

IV. Mercury (Hg)

Ore Cinnabar – HgS

Alloy Formation

Alloy is the homogeneous solid solution of two or more metals. Transition metals have approximate same size, therefore in molten form, they can fit to each other's crystalline structures and form homogeneous mixture to form the alloy, e.g. brass (copper-zinc) and bronze (copper-tin), etc.

Other Important Compounds

Copper Sulphate ($\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$) (Blue Vitriol)

Preparation

- (a) $\text{CuCO}_3 \cdot \text{Cu}(\text{OH})_2 + 2\text{H}_2\text{SO}_4 \longrightarrow 2\text{CuSO}_4 + 3\text{H}_2\text{O} + \text{CO}_2$
 (b) $\text{CuO} + \text{H}_2\text{SO}_4 \longrightarrow \text{CuSO}_4 + \text{H}_2\text{O}$

Properties

- It is a deep blue coloured crystalline solid, fairly soluble in water.
- $2\text{CuSO}_4 \xrightarrow{750^\circ\text{C}} 2\text{CuO} + 2\text{SO}_2 + \text{O}_2$
- $\text{CuSO}_4 + 2\text{NH}_4\text{OH} \longrightarrow \text{Cu}(\text{OH})_2 + (\text{NH}_4)_2\text{SO}_4$
- $2\text{CuSO}_4 + 4\text{KI} \longrightarrow \text{Cu}_2\text{I}_2 + \text{I}_2 + 2\text{K}_2\text{SO}_4$
- $2\text{CuSO}_4 + 4\text{KCN} \longrightarrow 2\text{Cu}(\text{CN})_2 + 2\text{K}_2\text{SO}_4$

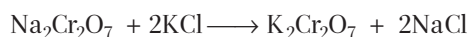
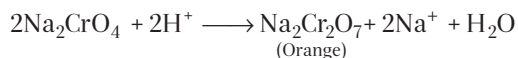
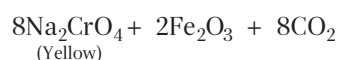
Uses

It is used

- in electroplating, electric batteries, etc.
- in medicine as antiseptic.
- to make Fehling's solution.

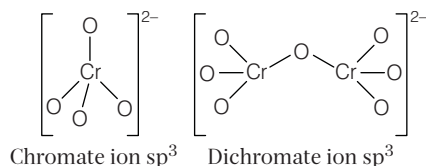
Potassium Dichromate ($\text{K}_2\text{Cr}_2\text{O}_7$)

Preparation



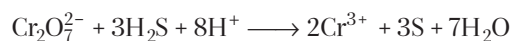
Sodium dichromate is more soluble than potassium dichromate. Chromates and dichromates are interconvertible in aqueous solution depending upon pH of the solutions.

Structure

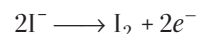
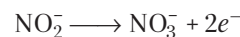
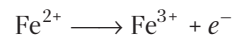
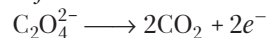


Properties

Sodium and potassium dichromates are strong oxidising agents thus, acidified $\text{K}_2\text{Cr}_2\text{O}_7$ will oxidise iodides to iodine, sulphides to sulphur, tin (II) to tin (IV) and iron (II) salts to iron (III).



The half-reactions of reductants are

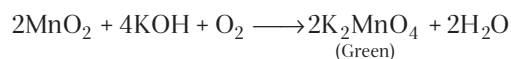


Uses

- $\text{K}_2\text{Cr}_2\text{O}_7$ is used as an oxidising agent in volumetric analysis.
- It is used in mordant dyes, leather industry, photography (for hardening of film), cleaning glass ware.
- It is used in chromyl chloride test.

Potassium Permanganate (KMnO_4)

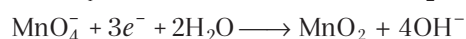
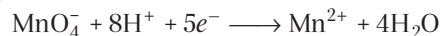
Preparation



Properties

It acts as a strong oxidising agent.

- In the presence of dil. H_2SO_4 , KMnO_4 is reduced to manganous salt.



To acidify KMnO_4 , only H_2SO_4 is used and not HCl or HNO_3 because HCl reacts with KMnO_4 and produce Cl_2 while HNO_3 itself acts as an oxidising agent.

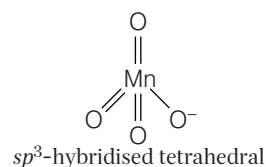
Alkaline or neutral KMnO_4 solution oxidises I^- to IO_3^- , $\text{S}_2\text{O}_3^{2-}$ to SO_4^{2-} , Mn^{2+} to MnO_2 , etc.

Acidic KMnO_4 solution oxidises oxalate to CO_2 , iron (II) to iron (III), nitrites to nitrates and iodides to iodine.

Uses

- In the laboratory preparation of Cl_2 .
- As an oxidising agent, disinfectant.
- In the making of Baeyer's reagent.

Structure of permanganate ion

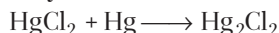


Mercury Halides

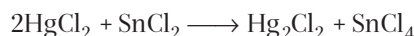
Mercury forms halides in two oxidation states, +1 and +2, e.g. Hg_2Cl_2 , HgCl_2

Preparation

Mercury (I) chloride is prepared by heating a mixture of mercury (II) chloride and mercury in iron vessel.



It can also be obtained by reduction of mercury (II) chloride by reducing agents like tin (II) chloride in limited quantity.

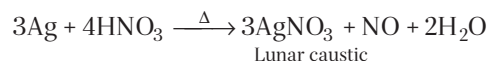


Properties

- (i) $2\text{Hg}_2\text{Cl}_2 \longrightarrow 2\text{HgCl}_2 + 2\text{Hg}$
- (ii) $\text{Hg}_2\text{Cl}_2 + 2\text{NH}_3 \longrightarrow \text{Hg}(\text{NH}_2)\text{Cl} + \text{Hg} + \text{NH}_4\text{Cl}$

Silver Nitrate (Lunar Caustic) (AgNO_3)

Preparation

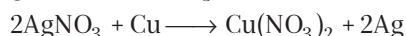


Properties

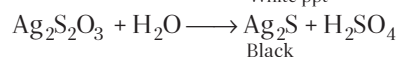
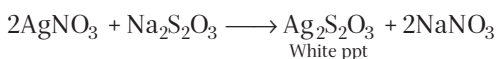
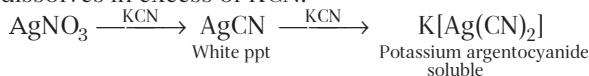
- Colourless crystalline compound soluble in H_2O and alcohol with mp 212°C .
- When exposed to light, it decomposes hence, stored in a brown coloured bottle.



- It is reduced to metallic Ag by more electropositive metals like Cu, Zn, Mg and also by PH_3 .



- It dissolves in excess of KCN.



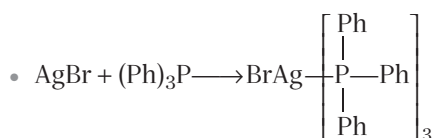
Silver Halide (Silver Bromide)

Preparation

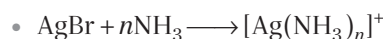


Properties

It is a soft, pale-yellow, water insoluble salt. It has fcc structure.



Tris (triphenylphosphine product)



Tris (triphenylphosphine product)

Photography

Ordinary black and white photographic film consists of a celluloid strip that has been coated with a gelatin emulsion containing silver halides, usually AgBr.

Inner-Transition Elements(f-Block Elements)

Lanthanoids

Properties

General configuration $[\text{Xe}] 4f^{1-14}, 5d^{0-1}, 6s^2$

All the lanthanoids are silvery white soft metals and tarnish rapidly in air.

- Atomic and ionic size, from left to right, decreases due to increase in nuclear charge. This is known as **lanthanoid contraction**.
- The hydroxides are ionic and basic. The basic strength decreases from $\text{Ce}(\text{OH})_3$ to $\text{Lu}(\text{OH})_3$ due to lanthanoid contraction
- Many trivalent lanthanoid ions are coloured both in the solid state and in aqueous solutions. Neither La^{3+} nor Lu^{3+} ion shows any colour but the rest do so. Lanthanoid ions with xf -electrons have similar colour to those with $(14-x)$ f -electrons.
- The lanthanoid ions other than the f^0 type (La^{3+} and Ce^{4+}) and the f^{14} type (Yb^{2+} and Lu^{3+}) are all paramagnetic. The paramagnetism arises to maximum in neodymium.

Oxidation states Ce^{4+} (some elements) is favoured by its noble gas configuration, but it is a strong oxidant reverting to the common +3 state. The E°_{value} for $\text{Ce}^{4+}/\text{Ce}^{3+}$ is +1.74 V, the reaction rate is very slow and hence, $\text{Ce}(\text{IV})$ is a good analytical reagent.

Pr, Nd, Tb and Dy also exhibit +4 state but only in oxides. Eu^{2+} is formed by losing two s -electrons and its f^7 configuration accounts for the formation of this ion. However, Eu^{2+} is a strong reducing agent changing to the common +3 state. Similarly, Yb^{2+} which has f^{14} configuration is a reductant, Tb^{4+} has half-filled f -orbitals and it is an oxidant.

Reactivity Lanthanoids are very reactive metals like alkaline earth metals.

Actinoids

Properties

- General configuration $[\text{Rn}] 5f^{1-14}, 6d^{0-2}, 7s^2$
- Actinoids exhibit a range of oxidation states due to comparable energies of $5f, 6d$ and $7s$ orbitals. The general oxidation state of actinoids is +3.
- All the actinoids are strong reducing agents and very reactive.
- Actinoids also react with oxygen, halogen, hydrogen and sulphur, etc., like lanthanoids.
- Actinoids are radioactive in nature and therefore, it is difficult to study their chemical nature.
- The atomic and ionic radii of actinoids decreases regularly across the series due to poor shielding effect of $5f$ -electrons. This is known as **actinoid contraction**.
- The allocation of electrons to the $5f$ or $6d$ -orbitals in the actinoid atoms is more uncertain than the allocation to

the $4f$ or $5d$ orbitals in the lanthanoid atoms because the external shielding of the $5f$ electrons is less than that of the $4f$ -electrons.

Comparison of Actinoids and Lanthanoids

Lanthanoids	Actinoids
These elements show +2 and +4 oxidation states along with +3.	These elements show variable oxidation states like +4, +5, +6, +7 along with +3.
Most of the ions of lanthanoid are coloured.	The ions of actinoid are also coloured.
These elements show less tendency towards complex formation.	These elements show greater tendency towards complex formation.
Except promethium, these elements are non-radioactive.	These elements are highly radioactive.
These are less basic.	These are more basic.

Practice Exercise

- The incorrect statement for transition elements is
 - the last electron of these elements enters in d -orbital
 - the common oxidation state of these elements is +3
 - the properties of these elements is intermediate between s - and p -block elements
 - the transition elements with smaller atomic radii is scandium
- Zinc does not exhibit variable valency due to
 - presence of complete filled d -orbitals
 - inert pair effect
 - presence of $4s$ -orbital
 - None of the above
- The metal which is considered as transition metal, is
 - zinc
 - cadmium
 - mercury
 - scandium
- The ground state electronic configuration of neutral titanium atom is
 - $[\text{Ar}]4s^2 4p^2$
 - $[\text{Ar}]3d^2 4s^2$
 - $[\text{Ar}]4s^2 p_x^1 p_y^1$
 - $[\text{Ar}]3d^5$
- Among the following electronic configurations of d -orbitals, the electronic configuration showing the highest magnetic moment is
 - $3d^2$
 - $3d^5$
 - $3d^7$
 - $3d^8$
- Magnetic moment
 - increases with increasing number of unpaired electrons
 - gives indication about the number of unpaired electrons present in the atom, molecule or ion
 - its unit is Bohr Magneton (BM). A single unpaired electron has 1.73 BM magnetic moment
 - All of the above
- Which of the following compounds will show magnetic moment of 1.72 BM?
 - $[\text{Ni}(\text{CN})_4]^{2-}$
 - $[\text{CoCl}_6]^{4-}$
 - $[\text{Cu}(\text{NH}_3)_4]^{2+}$
 - TiCl_4
- Interstitial compounds are formed when small atoms are trapped inside the crystal lattice of metals. Which of the following are the characteristic properties of interstitial compounds?
 - They have high melting points in comparison to pure metals.
 - They are very hard.
 - They retain metallic conductivity.
 - They are chemically very reactive.
 - I, II and III
 - II, III and IV
 - I, III and IV
 - I, II and IV
- The element with maximum number of oxidation states in their compounds is
 - Eu
 - La
 - Gd
 - Am
- When Fe metal is rusted, it is
 - isomerised
 - decomposed
 - reduced
 - oxidised
- Ferric sulphate on heating gives
 - SO_2 and SO_3
 - SO_2
 - SO_3
 - S

12. Iron is rendered passive by treatment with conc.
 - a. HCl
 - b. H_2SO_4
 - c. H_3PO_4
 - d. HNO_3
13. Which of the following is an important ore of iron?
 - a. Smaltite
 - b. Garnierite
 - c. Pentlandite
 - d. Haematite
14. The metal which corrodes readily in moist air, is
 - a. gold
 - b. silver
 - c. iron
 - d. nickel
15. Which one of the following forms of iron can give other forms of iron?
 - a. Wrought iron
 - b. Steel
 - c. Pig iron
 - d. Cast iron
16. The oxidation state of iron in $[\text{Fe}(\text{H}_2\text{O})_5\text{NO}]^{2+}$ is
 - a. 1
 - b. 2
 - c. 3
 - d. 4
17. In weak field ligand, which one of the following cations has maximum magnetic moment?
 - a. Fe^{2+}
 - b. Cu^{2+}
 - c. Ni^{2+}
 - d. Co^{2+}
18. In the test for nitrate, the composition of brown ring is
 - a. $\text{FeSO}_4 \cdot \text{N}_2\text{O}$
 - b. $\text{FeSO}_4 \cdot \text{NO}$
 - c. FeSO_4NO_2
 - d. $\text{Fe}(\text{NO}_3)_2$
19. Copper can be extracted from
 - a. kupfer nickel
 - b. dolomite
 - c. malachite
 - d. galena
20. The colour of light absorbed by an aqueous solution of CuSO_4 is
 - a. orange-red
 - b. blue-green
 - c. yellow
 - d. violet
21. For the ions, $\text{Cu}^{2+}(3d^9)$ and $\text{Cu}^+(3d^{10})$, the correct statement is
 - a. Cu^{2+} is more stable than Cu^+
 - b. Cu^{2+} is less stable than Cu^+
 - c. Cu^{2+} and Cu^+ ions are equally stable
 - d. stability of Cu^+ and Cu^{2+} depends on the nature of copper salts
22. When CuSO_4 reacts with aqueous KI, the products are
 - a. $\text{Cu}_2\text{I}_2 + \text{K}_2\text{SO}_4$
 - b. $\text{Cu} + \text{K}_2\text{SO}_4 + \text{I}_2$
 - c. $\text{CuI}_2 + \text{K}_2\text{SO}_4$
 - d. $\text{Cu}_2\text{I}_2 + \text{K}_2\text{SO}_4 + \text{I}_2$
23. Which of the following is formed, when copper (II) sulphate is treated with excess ammonia?
 - a. A black precipitate
 - b. A red precipitate
 - c. A deep blue solution
 - d. A white precipitate turning black
24. When copper nitrate is strongly heated, the compound obtained is
 - a. copper nitrite
 - b. copper
 - c. copper nitride
 - d. copper oxide
25. Hair dye contains
 - a. copper nitrate
 - b. gold chloride
 - c. silver nitrate
 - d. lead nitrate
26. Which of the following ions will finally give a black precipitate with Ag^+ ion?
 - a. SO_3^{2-}
 - b. Br^-
 - c. CrO_4^{2-}
 - d. $\text{S}_2\text{O}_3^{2-}$
27. In the cyanide extraction process of silver from argentite ore, the oxidising and reducing agent used respectively are
 - a. O_2 and CO
 - b. O_2 and Zn dust
 - c. HNO_3 and Zn dust
 - d. HNO_3 and CO
28. Turnbull's blue is a compound, called
 - a. ferricyanide
 - b. ferrous ferrocyanide
 - c. ferrous cyanide
 - d. ferri ferrocyanide
29. Calomel (Hg_2Cl_2) on reaction with ammonium hydroxide gives
 - a. HgO
 - b. Hg_2O
 - c. $\text{NH}_2 - \text{Hg} - \text{Hg} - \text{Cl}$
 - d. HgNH_2Cl
30. Nitriding is the process of surface hardening steel by treating it in an atmosphere of
 - a. NH_3
 - b. O_3
 - c. N_2
 - d. H_2S
31. Potassium dichromate is used
 - a. in leather industry
 - b. as an oxidant for the preparation of many azo compounds
 - c. Both (a) and (b)
 - d. None of the above
32. The structures of chromate and dichromate ions are A and B respectively. Here, A and B refer to
 - a. A - tetrahedral, B - octahedral
 - b. A - tetrahedral, B - two tetrahedral
 - c. A - octahedral, B - two tetrahedral
 - d. A - two octahedral, B - octahedral
33. Consider the following statements.
 - I. Sodium dichromate is less soluble than potassium dichromate.
 - II. Crystals of potassium dichromate are of orange colour.
 - III. The chromates and dichromates are interconvertible in aqueous solution depending upon pH of the solution.
 - IV. The oxidation states of chromate and dichromate are different.
 The correct statements are
 - a. I, II, III and IV
 - b. II and IV
 - c. I and III
 - d. I and II
34. The photographic industry relies on the special light-sensitive properties of ...I... Here, I refers to
 - a. AgCl
 - b. AgBr
 - c. PdCl_2
 - d. either (a) or (b)
35. What happens when FeSO_4 solution reacts with acidified KMnO_4 solution?
 - a. Iron (II) is oxidised
 - b. KMnO_4 is oxidised
 - c. Iron (II) is reduced
 - d. Iron (III) is reduced

36. When pyrolusite is fused with KOH, the colour of the product is
 a. red b. pink
 c. black d. green
37. Which of the following is wrong?
 a. $\text{K}_2\text{Cr}_2\text{O}_7 \longrightarrow$ Orange
 b. $\text{CuSO}_4 \cdot 5\text{H}_2\text{O} \longrightarrow$ Blue
 c. $\text{MnSO}_4 \longrightarrow$ Yellow
 d. $\text{Cr}_2(\text{SO}_4)_3 \longrightarrow$ Purple
38. In which of the following compounds, Mn has highest oxidation state?
 a. K_2MnO_4 b. MnO_2
 c. KMnO_4 d. Mn_3O_4
39. KMnO_4 is used
 a. in analytical chemistry
 b. as a favourite oxidant in preparative organic chemistry
 c. in bleaching of wool, cotton and silk
 d. All of the above
40. On addition of small amount of KMnO_4 to conc. H_2SO_4 , a green oily compound is obtained which is highly explosive in nature. Identify the compound from the following.
 a. Mn_2O_7 b. MnO_2
 c. MnSO_4 d. Mn_2O_3
41. There are 14 elements in actinoid series. Which of the following elements does not belong to this series?
 a. U b. Np
 c. Tm d. Fm
42. In context of the lanthanoids, which of the following statements is not correct?
 a. There is a gradual decrease in the radii of the members with increasing atomic number in the series
 b. All the members exhibit + 3 oxidation state
 c. Because of similar properties, the separation of lanthanoids is not easy
 d. Availability of 4f electrons results in the formation of compounds in + 4 state for all the members of the series
43. Knowing that the chemistry of lanthanoids (Ln) is dominated by its +3 oxidation state, which of the following statements is incorrect?
 a. Because of the large size of the Ln (III) ions, the bonding in its compounds is predominantly ionic in character
 b. The ionic sizes of Ln (III) decrease in general with increasing atomic number
 c. Ln (III) compounds are generally colourless
 d. Ln (III) hydroxides are mainly basic in character
44. The basicity of lanthanoid hydroxides across the lanthanoid series
 a. decreases
 b. increases
 c. first decreases and then increases
 d. first increases and then decreases
45. Among the following lanthanoid ions, the paramagnetic ion is
 a. Yb^{2+} b. Eu^{2+}
 c. Lu^{3+} d. Ce^{4+}
46. The actinoids include the ...A... elements from ...B... to ...C... Here, A, B and C refer to
 a. A - fourteen, B - Th, C - Lr
 b. A - twelve, B - Lr, C - Th
 c. A - thirteen, B - Lr, C - Th
 d. A - fourteen, B - Th, C - Lr
47. Larger number of oxidation states are exhibited by the actinoids than those by the lanthanoids, the main reason being
 a. 4f orbitals are more diffused than the 5f orbitals
 b. lesser energy difference between 5f and 6d than between 4f and 5d orbitals
 c. more energy difference between 5f and 6d than between 4f and 5d orbitals
 d. more reactive nature of the actinoids than the lanthanoids
48. The f-block consists of series.
 a. two b. three
 c. four d. five
49. Actinoids possess
 a. variable valency
 b. 12 elements
 c. all synthetic elements
 d. only short-lived isotopes
50. The lanthanoid contraction is related to
 a. atomic radii
 b. atomic as well as ionic radii, M^{3+}
 c. valence electrons
 d. oxidation states
51. Lanthanoids and actinoids differ from each other because
 a. of the presence of partially filled outermost shells
 b. actinoids are radioactive in nature
 c. they show common oxidation state of + 3
 d. both are known as inner-transition elements
52. The one which is not the characteristic property of transition element, is
 a. diamagnetic behaviour b. formation of complexes
 c. catalytic activity d. variable oxidation states
53. Dichromate ions in alkaline medium exist as
 a. CrO_4^{2-} b. CrO_3
 c. Cr^{3+} d. Cr^{4-}
54. The correct formula for diamine silver chloride is
 a. $[\text{Ag}(\text{NH}_3)_2]\text{Cl}$
 b. $[\text{Ag}(\text{NH}_2)_3]\text{Cl}$
 c. $\text{AgCl} \cdot \text{NH}_3$
 d. $[\text{Ag}(\text{NH}_4)_2]\text{Cl}$
55. Which of the following oxides is used as a whole pigment?
 a. ZnO b. FeO c. NiO d. CuO

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- Mercury is a liquid metal because [2014]
 - it has a completely filled d -orbital that causes d - d overlapping
 - it has completely filled d -orbital that prevents d - d overlapping
 - it has a completely filled s -orbital
 - it has a small atomic size
- When a solution of potassium chromate is treated with an excess of dilute nitric acid, [2013]
 - Cr^{3+} and $\text{Cr}_2\text{O}_7^{2-}$ are formed
 - $\text{Cr}_2\text{O}_7^{2-}$ and H_2O are formed
 - CrO_4^{2-} is reduced to Cr^{3+}
 - CrO_4^{2-} is oxidised to $\text{Cr}_2\text{O}_7^{2-}$
- In the extraction of Ag, Zn is removed from (Zn-Ag) alloy through [2013]
 - cupellation
 - fractional crystallisation
 - distillation
 - electrolytic refining
- Green vitriol is [2012]
 - ferrous sulphate
 - tin oxide
 - zinc oxide
 - ferrous carbonate
- Pyrolusite is a/an [2011]
 - oxide ore
 - sulphide ore
 - carbide ore
 - Not an ore
- When potassium ferrocyanide crystals are heated with conc. H_2SO_4 , the gas evolved is [2011]
 - SO_2
 - NH_3
 - CO_2
 - CO
- van-Arkel method is based on [2010]
 - cupellation method
 - furnace refining method
 - poling method
 - None of the above
- Oil paintings turn blackish after some time. The salt formed is [2010]
 - SnS
 - CuS
 - PbS
 - CdS
- The extraction of which of the following metals involves bessemerisation? [2009]
 - Fe
 - Ag
 - Al
 - Cu
- A metal X on heating in nitrogen gas gives Y . Y on treatment with water gives a colourless gas which when passed through CuSO_4 solution gives a blue colour. Y is [2008]
 - $\text{Mg}(\text{NO}_3)_2$
 - Mg_3N_2
 - NH_3
 - MgO

Answer with Solutions

Practice Exercise

- (d) Scandium is the first element of $3d$ series and possesses largest atomic radii because atomic radii decrease on moving along the period.
- (a) Zinc contains completely filled d -orbitals and exhibit stable electronic configuration, therefore it does not easily lose their electrons and hence, does not exhibit variable valency.
- (d) Zinc, cadmium and mercury of group 12 have full d^{10} configuration in their ground state as well as in their common oxidation states and thus, are not regarded as transition metals.
- (b) The ground state electronic configuration of Ti is $[\text{Ar}] 3d^2 4s^2$
- (b)
- (d) The magnetic moment is determined by the number of unpaired electrons and is calculated by using the 'spin-only' formula, i.e. $\mu = \sqrt{n(n+2)} \text{ BM}$
where, n is the number of unpaired electrons and μ is the magnetic moment in units of Bohr Magnetron (BM).
A single unpaired electron has a magnetic moment of 1.73 Bohr Magnetron (BM).
- (c) Cu has +2 oxidation state in $[\text{Cu}(\text{NH}_3)_4]^{2+}$.
Electronic configuration of Cu^{2+} ($Z = 29$) is $[\text{Ar}]^{18} 3d^9 4s^0$
Number of unpaired electrons (n) = 1
Magnetic moment = $\sqrt{n(n+2)} \text{ BM} = \sqrt{1(1+2)} = 1.73 \text{ BM}$
- (a) Interstitial compounds have high melting points in comparison to pure metals. They are very hard and retain metallic conductivity. They are chemically inert.
- (d)
- (d) Fe is oxidised, $\text{Fe} \longrightarrow \text{Fe}^{2+} + 2e^-$
- (c) $\text{Fe}_2(\text{SO}_4)_3 \xrightarrow{\Delta} \text{Fe}_2\text{O}_3 + 3\text{SO}_3$
- (d) Conc. HNO_3 renders iron passive by forming a thin protective film of Fe_3O_4 on its surface.
- (d) Haematite (Fe_2O_3)
Garnierite is hydrated double silicate of Mg and Si having 5-8% Ni. Pentlandite (NiS) ore associated with copper and iron pyrite.
- (c) Iron is covered by a layer of hydrated iron oxide ($\text{Fe}_2\text{O}_3 \cdot x\text{H}_2\text{O}$), i.e. rust on exposure to moist air.
- (c) The iron produced in the blast furnace is known as pig iron.

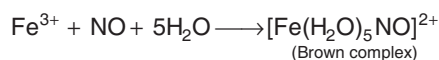
16. (a) H_2O is a neutral ligand but NO possesses +1 oxidation state in the compound of Fe .

Hence, oxidation state of Fe in the given compound is calculated as

$$x + 0 + 1 = 2; x = +1$$

17. (a) Fe^{2+} has maximum number of unpaired electrons.

18. (b) $3\text{Fe}^{2+} + \text{NO}_3^- + 4\text{H}^+ \longrightarrow \text{NO} + 3\text{Fe}^{3+} + 2\text{H}_2\text{O}$

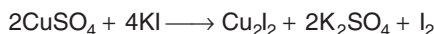
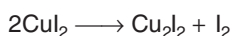


19. (c) Copper can be extracted from malachite $\text{CuCO}_3 \cdot \text{Cu}(\text{OH})_2$.

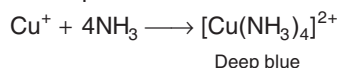
20. (a) The aqueous solution of CuSO_4 consists of the complex $[\text{Cu}(\text{H}_2\text{O})_4]^{2+}$ ion which absorbs in orange-red region and impart deep blue colouration to the solution.

21. (a)

22. (d) $2[\text{CuSO}_4] + 2\text{KI} \longrightarrow \text{CuI}_2 + \text{K}_2\text{SO}_4$



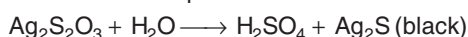
23. (c) Tetraamino cupric ion is formed.



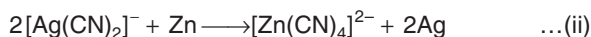
24. (d) $2\text{Cu}(\text{NO}_3)_2 \xrightarrow{\Delta} 2\text{CuO} + 4\text{NO}_2 + \text{O}_2$

25. (c) Hair dye contains silver nitrate.

26. (d) $2\text{Ag} + \text{S}_2\text{O}_3^{2-} \longrightarrow \text{Ag}_2\text{S}_2\text{O}_3$, white ppt which readily changes to yellow, orange, brown and finally black due to the formation of silver sulphide.



27. (b) The reactions involved in the extraction of silver by cyanide process are



In the first reaction, sulphide is oxidised by oxygen to SO_2 and in the second reaction, silver ion is reduced by Zn to Ag . Therefore, O_2 is oxidising agent and Zn is reducing agent.

28. (b) Ferrous ferrocyanide, $\text{Fe}_2[\text{Fe}(\text{CN})_6]$ is Turnbull's blue.

29. (d) $\text{Hg}_2\text{Cl}_2 + 2\text{NH}_3 \longrightarrow \text{HgNH}_2\text{Cl} + \text{Hg} \downarrow + \text{NH}_4\text{Cl}$
- White Black

30. (a) When steel is treated in the presence of NH_3 , iron nitride on the surface of steel is formed which imparts a hard coating. This process is called nitriding.

31. (c) $\text{K}_2\text{Cr}_2\text{O}_7$ is used in leather industry and as an oxidant for the preparation of many azo compounds.

32. (b) The structures of chromate and dichromate ions are tetrahedral and two tetrahedrals, respectively.

33. (c)

34. (b) The photographic industry relies on the special light sensitive properties of AgBr .

35. (a) $5\text{Fe}^{2+} + \text{MnO}_4^- + 8\text{H}^+ \longrightarrow \text{Mn}^{2+} + 4\text{H}_2\text{O} + 5\text{Fe}^{3+}$

36. (d) $\text{MnO}_2 + 2\text{KOH} + [\text{O}] \longrightarrow \text{K}_2\text{MnO}_4 + \text{H}_2\text{O}$

K_2MnO_4 produced is green in colour.

37. (c) MnSO_4 is light pink.

38. (c) It is +7.

In others, (a) + 6 (b) + 4 (d) + 8/3

39. (d) KMnO_4 is used in analytical chemistry, as a favourite oxidant in preparative organic chemistry in bleaching of wool, cotton and silk.

40. (a) On addition of small amount of KMnO_4 to conc. H_2SO_4 , a greeny oily compound is obtained which is highly explosive in nature. This compound is Mn_2O_7 .

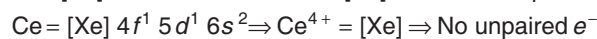
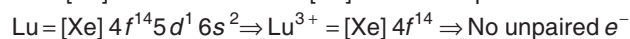
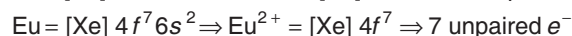
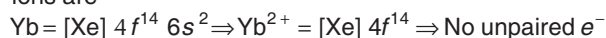
41. (c) The element, Tm does not belong to actinoid series.

42. (d) Formation of + 4 state requires very high energy, thus incorrect.

43. (c) Lanthanoids in +3 oxidation state usually have unpaired electron in f -subshell and impart characteristic colour in solid as well as in solution state due to $f-f$ transition (except lanthanum and lutetium).

44. (a) On moving from left to right across the lanthanoid series, basicity of lanthanoid hydroxides decreases.

45. (b) The valence shell electronic configuration of the given ions are



Hence, Eu^{2+} is paramagnetic as it contains 7 unpaired electrons.

46. (a) The actinoids include the fourteen elements from Th to Lr .

47. (b) Lanthanoids $[\text{Xe}] 4f^{1-14} 5d^{0-1} 6s^2$



Lanthanoids and actinoids use core d and f -orbitals also to show higher oxidation state. As actinoids have comparatively low energy difference between f and d -orbitals, thus show more oxidation states.

48. (a) The f -block consists of two series of lanthanoids and actinoids.

49. (a)

50. (b) Lanthanoid contraction is related to both atomic and ionic radii, M^{3+} .

51. (b) All lanthanoids except promethium are non-radioactive whereas all actinoids are radioactive in nature.

52. (a) Since, the d -orbitals are partially filled in transition metal atoms/ions, they exhibit paramagnetism and not diamagnetism.

53. (a) $2\text{OH}^- + \text{Cr}_2\text{O}_7^{2-} \longrightarrow 2\text{CrO}_4^{2-} + \text{H}_2\text{O}$

54. (a) Diammine means 2NH_3 in the coordination sphere.

55. (a)

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1. (b) The electronic configuration of mercury is $[\text{Xe}] 4f^{10}, 5d^{10}, 6s^2$. Its d -subshell is completely filled, thus, it prevents the overlapping of d -orbitals (d - d overlapping). Hence, it is liquid at room temperature.

2. (b) The reaction of K_2CrO_4 with dilute nitric acid is represented as



3. (c)

4. (a) $\text{FeSO}_4 \cdot 7\text{H}_2\text{O}$ is known as green vitriol.

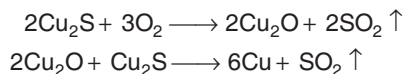
5. (a) Pyrolusite is MnO_2 . Thus, it is an oxide ore.

6. (d) $\text{K}_4[\text{Fe}(\text{CN})_6] + 6\text{H}_2\text{SO}_4 + 6\text{H}_2\text{O} \longrightarrow 2\text{K}_2\text{SO}_4 + \text{FeSO}_4 + 3(\text{NH}_4)_2\text{SO}_4 + 6\text{CO}$

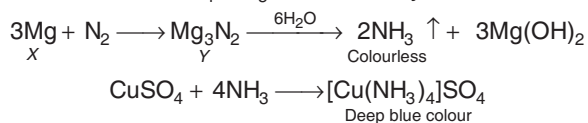
7. (d) van-Arkel method is based on vapour phase refining.

8. (c)

9. (d) In a bessemer converter, copper pyrites are oxidised to FeO and Cu_2O . FeO is slagged off. Cu_2O reacts with Cu_2S left unoxidised to give Cu .



10. (b) Only NH_3 gas gives deep blue solution with CuSO_4 . NH_3 is obtained by the reaction of nitride of metal with water.

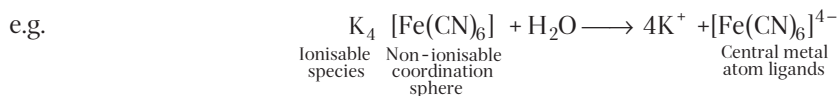


20

Coordination Compounds and Organometallics

Complex or Coordination Compounds

These are those molecular compounds which retain their identity in solid as well as in aqueous solution. In these compounds, metal atoms are bound to a number of anions or neutral molecules by coordinate bonds. A part of these compounds is not dissociated in solution and its behaviour is different than its constituents.



Important Terms

1. Complex Ion

It is an electrically charged species in which central metal atom or ion is surrounded by number of ions or molecules.

2. Ligands

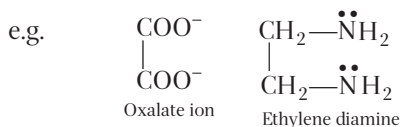
The ions or molecules bound to the central atom/ion in the coordination entity are called ligands.

Types of Ligands

(i) **Unidentate ligand** It is bound to a metal ion through a single donor atom.

e.g. H_2O , NH_3 , CO , Cl^- , NH_2^- etc.

(ii) **Didentate ligand** It is bound to a metal ion through two donor atoms.



(iii) **Polydentate ligand** It is bound to a metal ion through several donor atoms, e.g. ethylene diamine tetra acetate ion $[\text{EDTA}]^{4-}$.

(iv) **Ambidentate ligands** It can ligate through two different atoms.

e.g. $-\text{NO}_2$, $-\text{ONO}$, $-\text{SCN}$, $-\text{NCS}$ etc.

(v) **Chelate ligands** may be di or polydentate ligands which form closed ring with central metal ion. Closed ring is known as chelate ring and this process is called chelation.

More the number of chelate rings in a complex, complex will be more stable. The number of such ligating groups is called the **denticity**.

3. Coordination Number

The number of chemical bonds formed by the ligands with central metal atom or ion is called the coordination number of that complex.

In case of monodentate ligand, it is equal to the number of ligands while in bidentate ligand it is equal to the $2 \times$ number of ligands present in the complex.

4. Coordination Sphere

The central metal ion and the ligands attached to it are enclosed in square bracket which is known as coordination sphere. The ionisable group written outside the bracket is known as counter ions.

5. Oxidation Number of Central Atom

The charge of the complex if all the ligands are removed along with the electron pairs that are shared with the central atom, is called oxidation number of central atom.

Nomenclature of Coordination Compounds

1. Cationic Complex $[\text{Co}(\text{NH}_3)_4\text{Cl}(\text{NO}_2)]\text{NO}_3$

2 1 3
Tetraamminechloronitrocobalt (III) nitrate

- Prefixes mono, di, tri etc.,..... are used to indicate the number of the individual ligands and ligands are named in alphabetical order.
- Central metal atom and its oxidation state is indicated by Roman numeral in parenthesis.
- Name of ionisable anion.

2. Anionic Complex $\text{K}_3[\text{Fe}(\text{CN})_6]$

1 3 2
Potassiumhexacyanoferrate (III)

- Name of ionisable metal.
- Name of ligand in alphabetical order.
- Central metal atom + ate and oxidation state.

3. Neutral Complex $[\text{Pt}(\text{NH}_3)_2\text{Cl}(\text{NO}_2)]$

2 1
Diamminechloronitrito-N-platinum (II)

- Name of ligands in alphabetical order.
- Central metal atom and its oxidation state.

NOTE The oxidation state of Fe is + 2 in sodium nitroprusside, $\text{Na}_2[\text{Fe}(\text{CN})_5\text{NO}]$ and it has the ligand nitrosonium ion, NO^+ . When $\text{Fe}^{2+}(\text{aq})$ is added to $\text{K}_3[\text{Fe}(\text{CN})_6]$, the product obtained is Turnbull's blue but on addition of $\text{Fe}^{3+}(\text{aq})$ to $\text{K}_4[\text{Fe}(\text{CN})_6]$, the product Prussian blue is obtained.

Isomerism in Coordination Compounds

It is a phenomenon, in which compounds have the same molecular formula but different physical and chemical properties on account of different structures. These compounds are called **isomers**.

1. Stereo Isomerism

It occurs due to different arrangements of ligands around central metal atom.

It is of two types, i.e. geometrical isomerism and optical isomerism.

- (i) **Geometrical Isomerism** It arises in heteroleptic complexes due to different possible geometric arrangements of the ligands. Important examples of this behaviour are found in square planar and octahedral complexes.

(a) Square planar complex of formula $[\text{MX}_2\text{L}_2]$

(X and L are unidentate)

Two X ligands may be arranged adjacent to each other in a *cis*-isomer or opposite to each other in a *trans*-isomer.

e.g. $[\text{Pt}(\text{NH}_3)_2\text{Cl}_2]$

(b) Square planar complex of the type $[\text{MABXL}]$ (where A, B, X, L are unidentate) shows three isomers, two *cis* and one *trans*. Such isomerism is not possible for tetrahedral geometry.

e.g. $[\text{Pt}(\text{NH}_3)(\text{Br})(\text{Cl})(\text{Py})]$

(c) Square planar complex of type $\text{M}(\text{XL})_2$, here, XL is unsymmetrical didentate ligand, e.g. $[\text{Pt}(\text{gly})_2]$

(d) Octahedral complexes of formula $[\text{MX}_2\text{L}_4]$ in which the two ligands X may be oriented *cis* or *trans* to each other, e.g. $[\text{Co}(\text{NH}_3)_4\text{Cl}_2]^+$.

(e) Octahedral complexes of formula $[\text{MX}_2\text{A}_2]$ (where X are unidentate ligands and A are didentate ligands) form *cis* and *trans*-isomers, e.g. $[\text{CoCl}_2(\text{en})_2]$.

- (f) **Octahedral coordination entities of the type $[Ma_3b_3]$** like $[\text{Co}(\text{NH}_3)_3(\text{NO}_2)_3]$. If three donor atoms of the same ligands occupy adjacent positions at the corners of an octahedral face, we have the *facial (fac)* isomer. When the positions are around the meridian of the octahedron, we get the *meridional (mer)* isomer.

NOTE Tetrahedral complexes do not show geometrical isomerism. Octahedral complexes of type $[\text{M}(\text{AA})_3]$, MA_6 and MA_5B do not show geometrical isomerism. $\text{MA}_2\text{X}_2\text{Y}_2$ shows both optical and geometrical isomerism.

- (ii) **Optical Isomerism** It arises when mirror images cannot be superimposed on one another. These mirror images are called **enantiomers**. The two forms are called *dextro (d)* and *laevo (l)*.

Optical isomerism is common in octahedral complexes having atleast one didentate ligand.

e.g. $[\text{Co}(\text{en})_3]^{3+}$, $[\text{PtCl}_2(\text{en})_2]^{2+}$ etc.

2. Structural Isomerism

In structural isomerism, isomers have different bonding pattern.

Different types of structural isomerism is as follows :

- (i) **Linkage isomerism** It arises in a coordination compound containing ambidentate ligand.
e.g. $[\text{Co}(\text{NH}_3)_5(\text{NO}_2)]\text{Cl}_2$ and $[\text{Co}(\text{NH}_3)_5(\text{ONO})]\text{Cl}_2$
- (ii) **Coordination isomerism** It arises from the interchange of ligands between cationic and anionic entities of different metal ions present in a complex.
e.g. $[\text{Co}(\text{NH}_3)_6][\text{Cr}(\text{CN})_6]$ and $[\text{Cr}(\text{NH}_3)_6][\text{Co}(\text{CN})_6]$
- (iii) **Ionisation isomerism** When the ionisable anion exchange with anion ligand.
e.g. $[\text{Co}(\text{NH}_3)_5\text{SO}_4]\text{Br}$ and $[\text{Co}(\text{NH}_3)_5\text{Br}]\text{SO}_4$
- (iv) **Solvate isomerism** It is also known as “hydrate isomerism”. In this case water is involved as a solvent.
e.g. $[\text{Cr}(\text{H}_2\text{O})_6]\text{Cl}_3$, $[\text{Cr}(\text{H}_2\text{O})_5\text{Cl}]\text{Cl}_2 \cdot \text{H}_2\text{O}$ and $[\text{Cr}(\text{H}_2\text{O})_4\text{Cl}_2]\text{Cl} \cdot 2\text{H}_2\text{O}$

Bonding in Coordination Compounds

The bonding formation in coordination compounds can be explained by using the two approaches as given below :

Werner's Theory

- (i) In complex compounds, metal atom exhibit two types of valencies—primary valency and secondary valency.
- (ii) Primary valencies are satisfied by anions only while secondary valencies are satisfied by ligands. Primary valency depends upon oxidation number of central metal atom while secondary valency represents the coordination number of central metal atom.

- (iii) Primary valencies are ionisable and are non-directional while secondary valencies are non-ionisable and directional. Therefore, geometry of complex is decided by secondary valencies.

e.g. $[\text{Cr}(\text{H}_2\text{O})_6]\text{Cl}_3$ has primary valency = 3 (ON of Cr) and secondary valency = 6 (CN of Cr)

EAN (Effective Atomic Number) Rule

The EAN is calculated by using following formula :

$$\text{EAN} = Z - x + 2c$$

where, Z = atomic number of metal

x = number of electrons lost by the metal in complex formation (oxidation number)

c = number of pairs of electrons accepted by metal (coordination number)

Complex	EAN ($Z - x + 2c$)	e^- in next inert gas
(Complexes whose central atom obeys EAN rule)		
$[\text{Pd}(\text{NH}_3)_6]^{4+}$	$46 - 4 + 2 \times 6 = 54$	54 (Xe)
$\text{K}_4[\text{Fe}(\text{CN})_6]$	$26 - 2 + 2 \times 6 = 36$	36 (Kr)

Valence Bond Theory

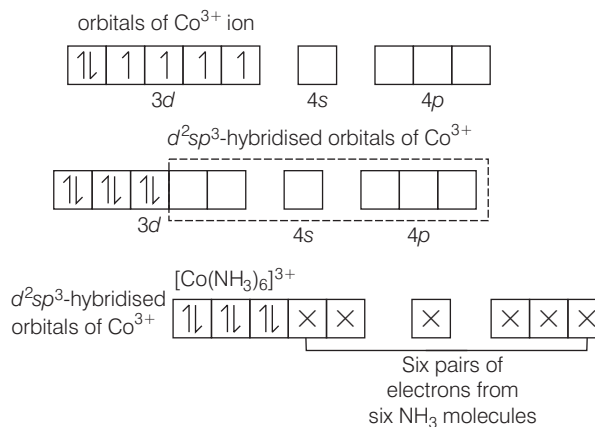
According to this theory, the metal atom or ion under the influence of ligands form inner orbital and outer orbital complex. These hybridised orbitals are allowed to overlap with ligand orbitals that can donate electron pairs for bonding.

1. Six Ligands (unidentate) Octahedral Entity

Such complexes are of the following two types :

- (i) **Inner orbital complexes** (hyperligated complexes) which are formed due to strong field ligands or low spin ligands, has hybridisation d^2sp^3 and shape of complex will be octahedral.

(a) **Inner orbital complex $[\text{Co}(\text{NH}_3)_6]^{3+}$**



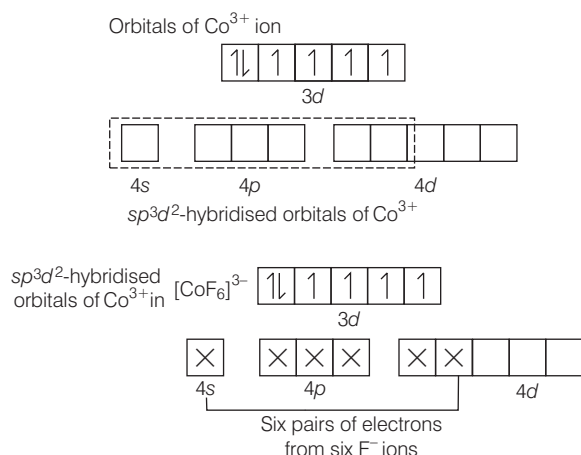
All electrons are paired, therefore complex will be diamagnetic in nature.

- (ii) **Outer orbital complexes** (hypoligated complexes) which are formed due to weak field ligands or high spin ligands, has sp^3d^2 -hybridisation.

Generally, halides (F^- , Cl^- , Br^- , I^-), SCN^- , S^{2-} form outer orbital complexes and other ligands form inner orbital complexes.

e.g.

(b) **Outer Orbital Complex** $[CoF_6]^{3-}$



Complex has unpaired electrons, therefore it will be paramagnetic in nature.

2. Four-ligands (unidentate), Tetrahedral and Square Planar Entity

Tetrahedral complexes can also form inner orbital or outer orbital complexes, characteristics of which are shown below:

Generally, halide (F^- , Cl^- , Br^- , I^-) ligands, $[Ni(CO)_4]$, $[Co(CO)_4]$, $[Zn(NH_3)_4]^{2+}$ complexes form outer orbital complexes and other form inner orbital complexes, e.g.

Type of hybridisation	Geometrical shape	d-orbitals used	CN	Common examples
sp^3	Tetrahedral	—	4	$[Zn(NH_3)_4]^{2+}$ $[Ni(CO)_4]$
dsp^2/sp^2d	Square planar	$d_{x^2-y^2}$	4	$[Ni(CN)_4]^{2-}$
sp^3d	Trigonal bipyramidal	d_{z^2}	5	$[Ni(CN)_5]^{3-}$
d^2sp^3	Octahedral inner orbital complex	$d_{z^2}, d_{x^2-y^2}$	6	$[Fe(CN)_6]^{3-}$
sp^3d^2	Octahedral counter orbital complex	$d_{x^2-y^2}, d_{z^2}$	6	$[Co(H_2O)_6]^{2+}$ $[FeF_6]^{3-}$, $[CoF_6]^{3-}$

Limitations of Valence Bond Theory

It suffers from the following shortcomings:

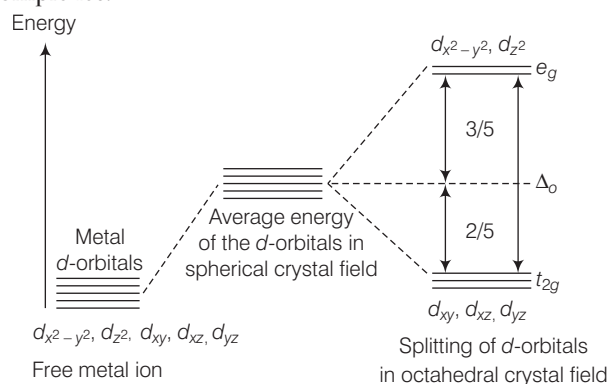
- It involves a number of assumptions.
- It describes bonding in coordination compounds only qualitatively.
- It does not describe the detailed magnetic properties of coordination compounds.
- It does not give a quantitative interpretation of the thermodynamic or kinetic stabilities of coordination compounds.
- It does not make exact predictions regarding the tetrahedral and square planar structures of 4-coordinate complexes.
- It does not distinguish between weak and strong ligands.

Crystal Field Theory (CFT)

The splitting of five d-orbitals into lower and higher energy levels due to approach of ligands, is known as crystal field theory. The five d-orbitals in a gaseous metal atom/ion have same energy.

Crystal Field Splitting in Octahedral Coordination Entities

- Energy separation is denoted by Δ_o (the subscript o, is for octahedral). This is also known as crystal field splitting energy (CFSE).
- The energy of the two e_g orbitals (higher energy orbitals) will increase by $(3/5) \Delta_o$ and that of the three t_{2g} (lower energy orbitals) will decrease by $(2/5) \Delta_o$.
- If $\Delta_o < P$, the fourth electron enters one of the e_g orbitals giving the configuration $t_{2g}^3 e_g^1$. Ligands for which $\Delta_o < p$ are known as weak field ligands and form high spin complexes.



- If $\Delta_o > P$, it becomes more energetically favourable for the fourth electron to occupy a t_{2g} orbital with configuration $t_{2g}^4 e_g^0$. Ligands which produce this effect are known as strong field ligands and form low spin complexes.
- Where, P represents the energy required for electron pairing in a single orbital (pairing energy).

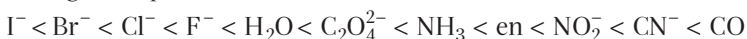
Crystal Field Splitting in Tetrahedral Coordination Entities

In tetrahedral coordination entities, $\Delta_t = (4/9)\Delta_o$.

Consequently, the orbital splitting energies are not sufficiently large for forcing pairing and therefore, low spin configurations are rarely observed. Due to less crystal field stabilisation energy, it is not possible to pair electrons and so all the tetrahedral complexes are high spin.

Spectrochemical Series

An arrangement of ligands in order of increasing crystal field strength is spectrochemical series.

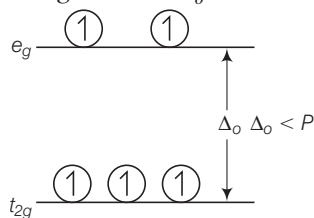


Magnetic Properties

The diamagnetic (repulsion from the applied magnetic field due to the presence of paired electrons) and paramagnetic (attraction towards the applied magnetic field due to the presence of unpaired electrons) behaviour of coordination compound is explained on the basis of crystal field theory.

e.g. Co^{3+} ion shows diamagnetic properties in $[Co(NH_3)_6]^{3+}$ complex ion but in $[CoF_6]^{3-}$, it shows paramagnetic properties. In $[Co(NH_3)_6]^{3+}$ complex ion, NH_3 acts as a strong ligand which causes pairing of electrons.

In this case, energy required to pair up the electrons is smaller than Δ_o . Whereas in $[CoF_6]^{3-}$ complex, F^- acts as a weak ligand that does not allow the pairing of electrons as energy required to pair the electrons is higher than Δ_o .



Colour of the Complexes

The electrons can be excited from lower energy to higher energy state by the absorption of even lower energy radiations. Thus, due to $d-d$ transition of the electrons, coordination compounds show different colours.

Stability Constants

Stability of a complex can be expressed in terms of stability constant, k . If the complex is ML_n and β_n is the overall formation constant, then

$$M + nL \rightleftharpoons ML_n$$

$$\beta_n = \frac{[ML_n]}{[M][L]^n} = k_1 \times k_2 \times k_3 \dots k_n$$

where, $k_1, k_2, \dots, k_n$ are called stepwise formation constants. [Alternatively $1/k$ is known as instability constant.

Application of Coordination Compounds

Estimation of Hardness of Water

EDTA is used in the estimation of hardness of water by volumetric technique as it is hexadentate ligand and forms complex with various metal ions such as Ca^{2+} and Mg^{2+} .

Quantitative Analysis of Metal Ions

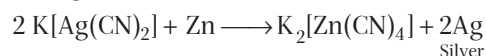
- (i) Separation of group II B sulphides from group II A sulphide is based upon the fact that sulphides of group II B forms complex sulphides with yellow ammonium sulphide that are soluble in water.



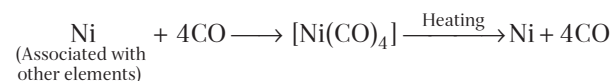
- (ii) Cu^{2+} ion is detected by the formation of deep blue coloured tetraamine copper sulphate complex $[Cu(NH_3)_4]SO_4$ by the addition of NH_3 to Cu^{2+} ions solution.

Extraction of Metals

The property of the formation of coordination compounds by the metals is used in the extraction of metals like Ag and Au.



Complex formed by nickel with carbonyl $[Ni(CO)_4]$ is used for the extraction and purification of nickel.



In Biological System

Cyclic ligands impart extra stability to coordinate compounds. Thus, Mg in chlorophyll and iron in haemoglobin are stable towards dissociation due to the formation of coordination compounds.

Haemoglobin of the blood gets oxygenated through the binding of dioxygen, O_2 to the ferrous ions in haemoglobin.

Organometallics

Compounds, in which a metal atom is directly attached with carbon atom of an organic compound or radical, are called organometallics. These compounds are broadly divided into three types, depending upon the bonding.

- (a) **σ -bonded Organometallics** Grignard reagent $(R-Mg)X$, TEL $(C_2H_5)_4Pb$, TMS (tetramethyl silane), $(CH_3)_4Si$, $(C_2H_5)_2Zn$ etc., are σ -bonded organometallic compounds.

- (b) **π -bonded Organometallics** These are formed mainly by transition metals. Examples are ferrocene, $[\text{Fe}(\eta^2 - \text{C}_5\text{H}_5)_2]$, dibenzene chromium $[\text{Cr}(\eta^6 - \text{C}_6\text{H}_6)_2]$, Zeisse's salt $\text{K}[\text{Pt}(\eta^2 - \text{C}_2\text{H}_4)\text{Cl}_3]$ etc.
- The prefixes η^2 , η^6 and η^2 indicate that 2, 6 and 2-carbon atoms are bound to the metal respectively.
- (c) **σ -and π -bonded Organometallics** Transition metal carbonyls are of this type, e.g. $[\text{Ni}(\text{CO})_4]$, $[\text{Fe}(\text{CO})_5]$ etc.

Some Important Organometallics

- (i) Grignard reagent (RMgX) is very useful in organic synthesis.
- (ii) *Zeigler-Natta* catalyst (TiCl_4 and aluminium) is used in catalysing polymerisation.
- (iii) TEL is used as antiknock compound in petroleum.

Practice Exercise

- According to Lewis, the ligands are
 - acidic in nature
 - basic in nature
 - neither acidic nor basic
 - some are basic and others are acidic
- Which of the following represent hexadentate ligand?
 - 2, 2-bipyridyl
 - DMG
 - Ethylenediamine
 - None of these
- In $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$, copper is coordinated to
 - 5 water molecules
 - 4 water molecules
 - one sulphate ion
 - one water molecule
- Coordination number of Cr is six. A complex ion of Cr with $\text{C}_2\text{O}_4^{2-}$, en and superoxide ion, O_2^- has the formula, $[\text{Cr}(\text{C}_2\text{O}_4)_x(\text{en})_y(\text{O}_2)_z]^-$. The ratio $x:y:z$ will be
 - 1:1:1
 - 1:1:2
 - 1:2:2
 - 2:1:1
- The oxidation number of Fe in $\text{K}_4[\text{Fe}(\text{CN})_6]$ is
 - 2
 - 3
 - 0
 - 1
- Identify the species with an atom in +6 oxidation state present in the following complexes.
 - CrO_2Cl_2
 - $[\text{Cr}(\text{CN})_6]^{3-}$
 - $[\text{NiF}_6]^{2-}$
 - MnO_4^-
- The oxidation state of cobalt in

$$\left[(\text{NH}_3)_4\text{Co} \begin{array}{c} \text{NH} \\ \diagup \quad \diagdown \\ \text{NO}_2 \end{array} \text{Co}(\text{NH}_3)_4 (\text{NO}_3)_4 \right]$$
 is
 - 2
 - 3
 - 4
 - 6
- The coordination number of Co in $[\text{Co}(\text{NH}_3)_5\text{Cl}]\text{Cl}_2$ is
 - 2
 - 5
 - 6
 - 1
- The type of isomerism shown by $[\text{Co}(\text{en})_2(\text{NCS})_2]\text{Cl}$ and $[\text{Co}(\text{en})_2(\text{NCS})\text{Cl}]\text{NCS}$ is
 - coordination
 - ionisation
 - linkage
 - All of the above
- Which of the following compound would exhibit coordination isomerism?
 - $[\text{Cr}(\text{H}_2\text{O})_6]\text{Cl}_3$
 - $[\text{Cr}(\text{NH}_3)_6][\text{Co}(\text{CN})_6]$
 - $[\text{Cr}(\text{en})_2]\text{NO}_2$
 - $[\text{Ni}(\text{NH}_3)_6][\text{BF}_4]_2$
- The planar complex *Mabcd* gives
 - two optical isomers
 - two geometrical isomers
 - three optical isomers
 - three geometrical isomers
- Which will not show geometrical isomerism?
 - $[\text{Co}(\text{NH}_3)_4\text{Cl}_2]^+$
 - $[\text{Co}(\text{en})_3]^{3+}$
 - $[\text{Co}(\text{en})_2\text{Cl}_2]\text{Cl}$
 - $[\text{Cr}(\text{NH}_3)_4\text{Cl}_2]\text{Cl}$
- Geometrical isomerism is shown by
 - $[\text{Ni}(\text{NH}_3)_5\text{Br}]^+$
 - $[\text{Cr}(\text{NH}_3)_4(\text{en})]^{3+}$
 - $[\text{Co}(\text{en})_3]^{3+}$
 - $[\text{Co}(\text{NH}_3)_2(\text{en})_2]^{3+}$
- Optical isomerism is exhibited by
 - $[\text{Cr}(\text{NH}_3)_6]^{3+}$
 - $[\text{Cr}(\text{en})(\text{H}_2\text{O})_4]^{3+}$
 - trans*- $[\text{Cr}(\text{en})_2\text{Cl}_2]^+$
 - $[\text{Cr}(\text{en})_3]^{3+}$
- The number of isomers of $[\text{Cu}''(\text{NH}_3)_4][\text{Pt}''\text{Cl}_4]$ are
 - 6
 - 3
 - 4
 - 5
- Amongst the following pair of enantiomers is given by
 - $[\text{Cr}(\text{NH}_3)_6][\text{Co}(\text{CN})_6]$
 - $[\text{Co}(\text{en})_2\text{Cl}_2]\text{Cl}$
 - $[\text{Co}(\text{NH}_3)_4\text{Cl}_2]\text{NO}_2$
 - $[\text{Pt}(\text{NH}_3)_4][\text{PtCl}_6]$
- $[\text{Co}(\text{NH}_3)_4\text{Cl}_2]\text{NO}_2$ and $[\text{Co}(\text{NH}_3)_4\text{Cl}(\text{NO}_2)]\text{Cl}$ exhibit which type of isomerism?
 - Geometrical
 - Optical
 - Linkage
 - Ionisation
- The difference in colour is due to
 - optical isomerism
 - geometrical isomerism
 - linkage isomerism
 - nuclear isomerism

19. EAN of iron in $K_4[Fe(CN)_6]$ is
 a. 36 b. 34 c. 38 d. 40
20. Which of the following complexes will give white precipitate with $BaCl_2$ solution?
 a. $[Cr(NH_3)_5SO_4]Cl$ b. $[Co(NH_3)_4Cl_2]NO_2$
 c. $[Cr(NH_3)_5Cl]SO_4$ d. Both (a) and (c)
21. Sodium nitroprusside when added to an alkaline solution of sulphide ions produces purple colouration due to the formation of
 a. $Na[Fe(H_2O)_5NOS]$ b. $Na_4[Fe(CN)_5NOS]$
 c. $Na_3[Fe(CN)_5NOS]$ d. $Na_4[Fe(H_2O)_5NOS]$
22. Which of the following has highest conductivity in water?
 a. $Fe_3[Fe(CN)_6]_2$ b. $K_3[Fe(CN)_6]$
 c. $Ag(NH_3)_2Cl$ d. $[Cr(NH_3)_6]Cl_6$
23. How many moles of $AgCl$ would be obtained, when 100 mL of 0.1 M $Co(NH_3)_5Cl_3$ is treated with excess of $AgNO_3$?
 a. 0.01 b. 0.02
 c. 0.03 d. None of these
24. High spin complex $[FeCl_6]^{3-}$ has the d -configuration as
 a. $t_{2g}^3 e_g^2$ b. t_{2g}^5 c. $t_{2g}^2 e_g^3$ d. e_g^5
25. Unpaired electrons in $[FeCl_6]^{3-}$ and $[Fe(CN)_6]^{3-}$ are respectively
 a. 5 and 2 b. 5 and 1 c. 3 and 2 d. 2 and 3
26. Among the complexes, $[Ni(CO)_4]$, $[Ni(CN)_4]^{2-}$ and $[NiCl_4]^{2-}$,
 a. $[Ni(CO)_4]$ is diamagnetic but $[Ni(CO)_4]$ and $[Ni(CN)_4]^{2-}$ are paramagnetic
 b. $[Ni(CN)_4]^{2-}$ and $[Ni(CO)_4]$ are diamagnetic but $[NiCl_4]^{2-}$ is paramagnetic
 c. $[NiCl_4]^{2-}$ and $[Ni(CO)_4]$ are diamagnetic but $[Ni(CN)_4]^{2-}$ is paramagnetic
 d. $[NiCl_4]^{2-}$ and $[Ni(CN)_4]^{2-}$ are diamagnetic but $[Ni(CO)_4]$ is paramagnetic
27. Which of the following shell form an octahedral complex?
 a. d^4 (low spin) b. d^2 (high spin)
 c. d^6 (low spin) d. None of these
28. The geometry of the compound $[Pt(NH_3)_2Cl_2]$ is
 a. square planar
 b. pyramidal
 c. tetrahedral
 d. octahedral
29. Which of the following is paramagnetic?
 a. $[Co(NH_3)_6]^{3+}$
 b. $[Ni(CO)_4]$
 c. $[NiF_4]^{2-}$
 d. $[Ni(CN)_4]^{2-}$
30. The value for crystal field stabilisation energy for an octahedral complex, $[CoCl_6]^{4-}$ is 18000 cm^{-1} . The value for crystal field stabilisation energy for tetrahedral complex is
 a. 16000 cm^{-1} b. 8000 cm^{-1}
 c. 20000 cm^{-1} d. 18000 cm^{-1}
31. The colour of the coordination compounds depends on the crystal field splittings. What will be the correct order of absorption of wavelength of light in the visible region, for the complexes, $[Co(NH_3)_6]^{3+}$, $[Co(CN)_6]^{3-}$ and $[Co(H_2O)_6]^{3+}$?
 a. $[Co(CN)_6]^{3-} > [Co(NH_3)_6]^{3+} > [Co(H_2O)_6]^{3+}$
 b. $[Co(NH_3)_6]^{3+} > [Co(H_2O)_6]^{3+} > [Co(CN)_6]^{3-}$
 c. $[Co(H_2O)_6]^{3+} > [Co(NH_3)_6]^{3+} > [Co(CN)_6]^{3-}$
 d. $[Co(CN)_6]^{3-} > [Co(NH_3)_6]^{3+} > [Co(H_2O)_6]^{3+}$
32. Which of the following species is not expected to be a ligand?
 a. No b. NH_4^+
 c. $NH_2CH_2CH_2NH_2$ d. CO
33. Which one of the following compounds is not colored?
 a. $Na_2[CuCl_4]$
 b. $Na_2[CdCl_4]$
 c. $[Cr(H_2O)_6]Cl_3$
 d. $K_3[Fe(CN)_6]$
34. Which of the following is paramagnetic?
 a. $K_4[Fe(CN)_6]$
 b. $K_3[Fe(CN)_6]$
 c. $Ni(CO)_4$
 d. $[Co(NH_3)_6]Cl_3$
35. The stabilisation of coordination compounds due to chelation is called the chelate effect. Which of the following is the most stable complex species?
 a. $[Fe(CO)_5]$
 b. $[Fe(CN)_6]^{3-}$
 c. $[Fe(C_2O_4)_3]^{3-}$
 d. $[Fe(H_2O)_6]^{3+}$
36. Which of the following is not an example of organometallic compound?
 a. Trimethyl boron
 b. Trimethyl aluminium
 c. Sodium ethoxide
 d. Tetracarbonyl nickel
37. Among the following, which is not the π -bonded organometallic compound?
 a. $K[PtCl_3(\eta^2-C_2H_4)]$ b. $Fe(\eta^5-C_5H_5)_2$
 c. $(CH_3)_4Sn$ d. $Cr(\eta^6-C_6H_6)_2$
38. Which is not a π -bonded complex?
 a. Zeise's salt
 b. Ferrocene
 c. Dibenzene chromium
 d. Tetraethyl lead

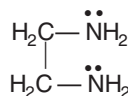
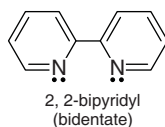
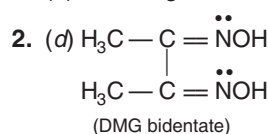
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- Which of the following will not form optical isomers? [2014]
 - $[\text{Co}(\text{en})_3]^{3+}$
 - $[\text{Co}(\text{NH}_3)_3(\text{NO}_2)_3]$
 - $[\text{Pt}(\text{en})_2\text{Cl}_2]^{2+}$
 - $[\text{CrCl}_2(\text{ox})_2]^{3-}$
- The magnitude of Δ_o will be highest in which of the following complex. [2013]
 - $[\text{Cr}(\text{CN})_6]^{3-}$
 - $[\text{Cr}(\text{H}_2\text{O})_6]^{3+}$
 - $[\text{Cr}(\text{NH}_3)_6]^{3+}$
 - $[\text{Cr}(\text{C}_2\text{O}_4)_3]^{3-}$
- Which of the following is an outer d -orbital or high spin complex? [2013]
 - $[\text{Co}(\text{NH}_3)_6]^{3+}$
 - $[\text{Ni}(\text{CN})_4]^{2-}$
 - $[\text{NiCl}_4]^{2-}$
 - $[\text{CoF}_6]^{3-}$
- Which of the following is correct IUPAC name for $\text{K}_2[\text{Cr}(\text{CN})_2\text{O}_2(\text{O})_2\text{NH}_3]$? [2012]
 - Potassium amminecyanoperoxodioxo-chromatic (IV)
 - Potassium amminecyanoperoxodioxo-chromium (V)
 - Potassium amminecyanoperoxodioxo-chromium (VI)
 - Potassium amminedicyanodioxoper-oxochromate (VI)
- Ferrocene is an example of [2009]
 - sandwiched complex
 - π -bonded complex
 - a complex in which all the five carbon atoms of cyclopentadiene anion are bonded to the metal
 - All of the above
- $[\text{Co}(\text{NH}_3)_5\text{SO}_4]\text{Br}$ and $[\text{Co}(\text{NH}_3)_5\text{Br}]\text{SO}_4$ is a pair of isomers. [2008]
 - ionisation
 - ligand
 - coordination
 - hydrate
- The crystal field splitting energy for octahedral (Δ_o) and tetrahedral (Δ_t) complexes is related [2006]
 - $\Delta_t = \frac{4}{9} \Delta_o$
 - $\Delta_t = \frac{1}{2} \Delta_o$
 - $\Delta_o = 2\Delta_t$
 - $\Delta_o = \frac{4}{9} \Delta_t$

Answer with Solutions

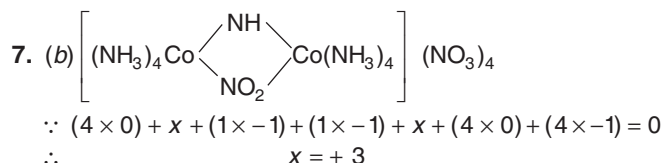
Practice Exercise

- (b) All the ligands are Lewis bases.



- (b) Blue vitriol $[\text{CuSO}_4 \cdot 5\text{H}_2\text{O}]$ is a complex compound in which the coordination number of copper is 4, i.e. Cu^{2+} is attached to 4 water molecules. Hence, it can be correctly written as $[\text{Cu}(\text{H}_2\text{O})_4]\text{SO}_4 \cdot \text{H}_2\text{O}$.
- (b) The ratio of x, y and z be such that, total charge = -1
 $3 + 1(-2) + 1 \times 0 + 2(-1) = -1$
 The ratio of $x : y : z = 1 : 1 : 2$
- (a) $\text{K}_4[\text{Fe}(\text{CN})_6]$
 $4 \times (+1) + x + 6(-1) = 0$
 $4 + x - 6 = 0$
 $x = 2$

- (a) The oxidation state of Cr in CrO_2Cl_2 is +6.



- (c) In the complex $[\text{Co}(\text{NH}_3)_5\text{Cl}]\text{Cl}_2$, total number of the ligands linked to the central atom, cobalt is 6. Therefore, the coordination number of Co is 6.
- (b) These complexes show ionisation isomerism as they give different ions in solution.
- (b) Coordination isomerism is shown by the complexes, which have at least 2 metal atoms so that their ligand may interchange.
- (d) Madc type complex have three geometrical isomers.
- (b) $[\text{Co}(\text{en})_3]^{3+}$ cannot show geometrical isomerism, in it, Co-atom is linked with 3-same ligands.
- (d) Geometrical isomerism is exhibited by octahedral complexes like $[\text{Co}(\text{NH}_3)_2(\text{en})_2]^{3+}$.
- (d) Optical isomerism is exhibited by $[\text{Cr}(\text{en})_3]^{3+}$.
- (a) The possible number of isomers are,
 - $[\text{Cu}(\text{NH}_3)_4][\text{PtCl}_4]$
 - $[\text{Cu}(\text{NH}_3)_2\text{Cl}_2] \cdot [\text{PtCl}_2(\text{NH}_3)_2]$ *cis*
 - $[\text{Cu}(\text{NH}_3)_2\text{Cl}_2] \cdot [\text{PtCl}_2(\text{NH}_3)_2]$ *trans*

- (iv) $[\text{Cu}(\text{NH}_3)_3\text{Cl}][\text{PtCl}_3(\text{NH}_3)]$,
 (v) $[\text{Cu}(\text{NH}_3)\text{Cl}_3][\text{PtCl}(\text{NH}_3)_3]$ and
 (vi) $[\text{CuCl}_4][\text{Pt}(\text{NH}_3)_4]$
16. (b) Pair of enantiomers are non-superimposable mirror images. The complex $[\text{Co}(\text{en})_2\text{Cl}_2]\text{Cl}$ exhibits a pair of enantiomers.
17. (d) Ionisation isomerism is shown.
18. (c) The difference in colour is due to the formation of linkage isomers.
19. (a) $\text{EAN} = z - x + 2c = 26 - 2 + 2 \times 6 = 36$
20. (c) Only $[\text{Cr}(\text{NH}_3)_5\text{Cl}]\text{SO}_4$ gives sulphate ion.

$$[\text{Cr}(\text{NH}_3)_5\text{Cl}]\text{SO}_4 + \text{BaCl}_2 \longrightarrow [\text{Cr}(\text{NH}_3)_5\text{Cl}]\text{Cl}_2 + \text{BaSO}_4$$

White ppt.
21. (b) $\text{Na}_2\text{S} + \text{Na}_2[\text{Fe}(\text{CN})_5\text{NO}] \longrightarrow \text{NH}_4[\text{Fe}(\text{CN})_5\text{NOS}]$
Sodium nitroprusside
Purple
22. (d) $[\text{Cr}(\text{NH}_3)_6]\text{Cl}_6 \xrightarrow{\text{H}_2\text{O}} [\text{Cr}(\text{NH}_3)_6]^{3+} + 6\text{Cl}^-$
 This complex gives 7 ions in solution. Conductivity of an electrolyte is directly proportional to number of ions produced. Hence, the conductivity of this complex will be the maximum.
23. (b) $\text{Co}(\text{NH}_3)_5\text{Cl}_3$ is an octahedral complex ionising in aqueous solution as

$$[\text{Co}(\text{NH}_3)_5\text{Cl}]\text{Cl}_2 \longrightarrow [\text{Co}(\text{NH}_3)_5\text{Cl}]^{2+} + 2\text{Cl}^-$$

$$\text{Moles of } [\text{Co}(\text{NH}_3)_5\text{Cl}]\text{Cl}_2 = \frac{0.1}{1000} \times 100 = 0.01$$

$$\text{Moles of AgCl formed} = 2 \times \text{moles of } \text{Cl}^- \text{ in complex}$$

$$= 0.01 \times 2 = 0.02$$
24. (a) In $[\text{FeCl}_6]^{3-}$ complex, Cl^- is a weak field ligand. It does not cause pairing of electrons in $3d$ subshell of Fe^{3+} . Hence, configuration is $t_{2g}^3 e_g^2$.
25. (b) In the formation of $[\text{FeCl}_6]^{3-}$, Cl^- does not cause pairing because it is a weak field ligand; number of unpaired electrons are five.
 On the other hand, in $[\text{Fe}(\text{CN})_6]^{3-}$ complex, CN^- is a strong field ligand, it causes pairing so the number of unpaired electrons is one.
26. (b) The oxidation state of Ni in $[\text{Ni}(\text{CN})_4]^{2-}$ and $[\text{NiCl}_4]^{2-}$ is +2 while in $[\text{Ni}(\text{CO})_4]$ it is in zero oxidation state.
 Since, CO and CN^- are strong ligands therefore, they cause pairing of the unpaired electrons.
 Therefore, $[\text{Ni}(\text{CO})_4]$ and $[\text{Ni}(\text{CN})_4]^{2-}$ are diamagnetic. Whereas, Cl^- is a weak ligand that does not cause pairing of electrons, resulting in paramagnetic complex. Hence, $[\text{NiCl}_4]^{2-}$ is paramagnetic.
27. (c) d^1 , d^2 , and d^4 configurations cannot give rise to octahedral complexes, as octahedral complexes are formed either by d^2sp^3 -hybridisation in presence of strong ligands, called low spin complex as they have comparatively less unpaired electrons) or by sp^3d^2 -hybridisation (in presence of weak ligands, called high spin complex as they have comparatively more

unpaired electrons). d^6 can give rise to both low spin and high spin octahedral complexes in presence of strong and weak ligands respectively.

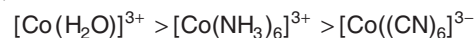
28. (a) The geometry of complex will be square planar.
29. (c) $[\text{NiF}_4]^{2-}$ has 2 unpaired electrons, hence it shows paramagnetic behaviour.
30. (b) The CFSE for tetrahedral complex is $\frac{4}{9} \Delta_o$ where Δ_o is CFSE for octahedral complexes. The value of Δ_o is 18000 cm^{-1} , therefore, the CFSE for tetrahedral complex is $\frac{4}{9}$ of 18000 cm^{-1} that is 8000 cm^{-1} .
31. (c) As we know that, strong field ligand splits the five degenerate energy levels with more energy separation than weak field ligand, i.e. as strength of ligand increases crystal field splitting energy increases.

Hence,
$$\Delta E = \frac{hc}{\lambda}$$

$$\Rightarrow \Delta E \propto \frac{1}{\lambda} \Rightarrow \lambda \propto \frac{1}{\Delta E}$$

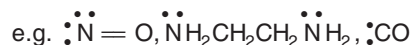
As energy separation increases, the wavelength decreases.

Thus, the correct order is



Here, strength of ligand increases, ΔE increases CFSE increases and λ absorbed decreases.

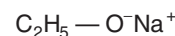
32. (b) Ligand must donate a pair of electron or loosely held electron pair to metal and form a $M-L$ bond.



Among NH_4^+ does not have any pair of electron.

Hence, NH_4^+ is not a ligand.

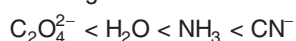
33. (b) Those complexes, in which at least one unpaired electron is found, show colour due to $d-d$ transition of electrons. The complex $\text{Na}_2[\text{CdCl}_4]$ has no unpaired electron in Cd-atom. Hence, it is colourless as no $d-d$ transition can take place.
34. (b) $\text{K}_3[\text{Fe}(\text{CN})_6]$ (potassium ferricyanide) has an unpaired electron in Fe-atom, hence it shows paramagnetic behaviour.
35. (c)
36. (c) Sodium ethoxide is not an organometallic compound as in it sodium metal is attached to oxygen and not to carbon.



37. (c) $(\text{CH}_3)_4\text{Sn}$ is a σ -bonded organometallic compound.
38. (d) Zeise's salt = $\text{K}[\text{Pt}(\eta^2 - \text{C}_2\text{H}_4)\text{Cl}_3]$
 Ferrocene = $[\text{Fe}(\eta^5 - \text{C}_5\text{H}_5)_2]$
 dibenzene chromium = $[\text{Cr}(\eta^6 - \text{C}_6\text{H}_6)_2]$
 These three are π -bonded organometallics only TEL $(\text{C}_2\text{H}_5)_4\text{Pb}$ is a σ -bonded organometallic.

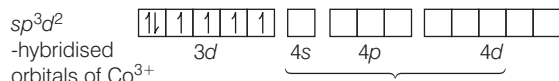
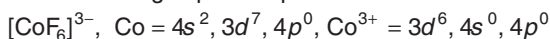
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1. (b) Optical isomerism is shown by only those complexes which lack symmetry. Complex $[\text{Co}(\text{NH}_3)_3(\text{NO}_2)_3]$ shows facial and meridional isomerism. Both isomers of this complex contain plane of symmetry. So, it will not form optical isomers.
2. (a) The crystal field splitting, Δ_o depends upon the field produced by the ligand and charge on the metal ion. In all these complexes of chromium, charge acquired by metal ion is +3. Therefore, Δ_o depends upon the field produced by the ligand. In accordance with the spectrochemical series, the increasing order of field strength is

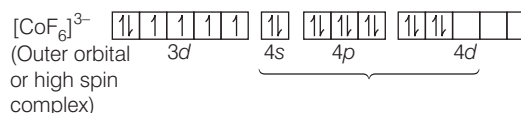


Thus, CN^- is the strong field ligand and will produce highest magnitude of Δ_o .

3. (d) $[\text{Co}(\text{NH}_3)_6]^{3+}$ and $[\text{Ni}(\text{CN})_4]^{2-}$ are inner d -orbital or low spin complex and $[\text{NiCl}_4]^{2-}$ complex has sp^3 -hybridisation and has tetrahedral geometry while $[\text{CoF}_6]^{3-}$ is outer d -orbital or high spin complex.



Here, F^- is a weak ligand so no pairing of electron takes place



4. (d) The IUPAC name of $\text{K}_2[\text{Cr}(\text{CN})_2\text{O}_2(\text{O})_2(\text{NH}_3)]$ is potassium amminedicyanodioxoper-oxochromate (VI).
5. (d) Ferrocene is a sandwich complex compound in which all the five carbon atoms of cyclopentadiene anion are linked to the metal through pi-bonds.
- $[\text{Fe}(\eta^5 - \text{C}_5\text{H}_5)_2]$ Ferrocene $\rightarrow$ it has aromatic character.

6. (a) $[\text{Co}(\text{NH}_3)_5\text{SO}_4]\text{Br} \rightleftharpoons [\text{Co}(\text{NH}_3)_5\text{SO}_4]^+ + \text{Br}^-$
 $[\text{Co}(\text{NH}_3)_5\text{Br}]\text{SO}_4 \rightleftharpoons [\text{Co}(\text{NH}_3)_5\text{Br}]^{2+} + \text{SO}_4^{2-}$

the molecular formula of both of the above compounds is same but on ionisation they give different ions in solution, so they are called ionisation isomers.

7. (a) The crystal field splitting energy for octahedral (Δ_o) and tetrahedral (Δ_t) complexes is related to

$$\Delta_t = \frac{4}{9} \Delta_o$$

21

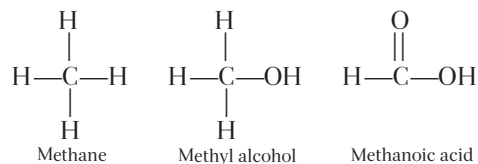
General Organic Chemistry

Concept of Organic Chemistry

Organic chemistry is a branch that deals with the study of C and H compounds and their derivatives. A large number of organic compounds have been synthesised till date because of unique property of C to catenate hence, the classification and nomenclature of organic compounds is necessary.

Tetravalency of Carbon

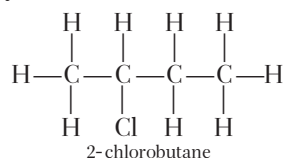
Liebel and van't Hoff studied the tetravalent nature and tetrahedral structure of carbon. The carbon atom is tetravalent which suggests that it forms four covalent bonds with four monovalent atoms or groups. Carbon forms a large number of organic compounds like methane, methyl alcohol, methanoic acid.



Structural Representation of Organic Compounds

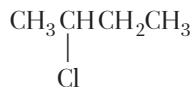
Structural Formula

In this, all the bonds present between any two atoms are shown clearly

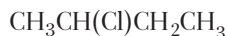


Condensed Formula

In this type of chemical formula, all the bonds are not shown clearly.



or

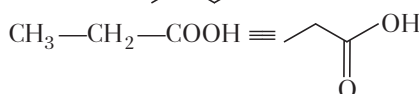


Bond Line Formula

In this, every fold and free terminal represents a carbon and lines represent the bond.



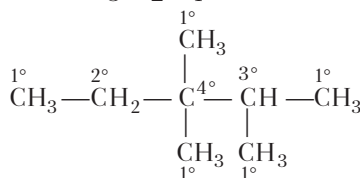
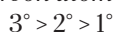
e.g.



Classification of Carbon and Hydrogen Atoms

- **Primary (1°) carbon atom** When carbon atom is attached with other carbon atom only.
- **Secondary (2°) carbon atom** When carbon atom is attached with two other carbon atoms.
- **Tertiary (3°) carbon atom** When carbon atom is attached with three other carbon atoms.
- **Quarternary (4°) carbon atom** When carbon atom is attached with four other carbon atoms.

Reactivity order of carbon atoms are as follows:

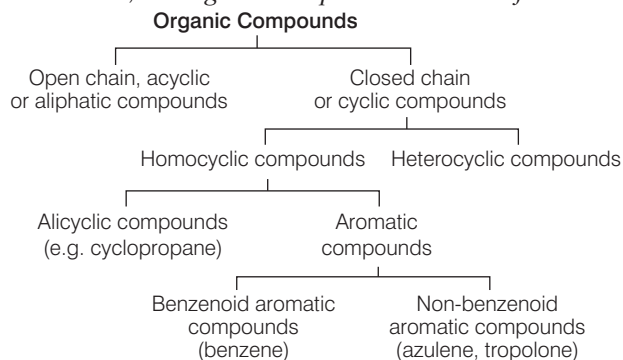


Classification of Organic Compounds

The organic compounds have been classified on the basis of carbon skeleton (structure) or functional groups or the concept of homology, i.e. the series in which two successive members differ by a $-\text{CH}_2$ unit or 14 unit mass.

Classification Based on Structure

On this basis, the organic compounds are classified as:



Classification Based on Functional Groups

The compounds of only carbon and hydrogen are called **hydrocarbons**. Hydrocarbons are considered as the parents of all the organic compounds. All other compounds are obtained from hydrocarbons by the replacement of one or more hydrogen atoms with other atoms or groups.

Various classes of compounds having some of the common functional groups are listed in the following table.

Class	Functional group
Olefins/Alkenes	$\text{>C}=\text{C}<$
Acetylenes/Alkynes	$-\text{C}\equiv\text{C}-$
Halides	$-\text{F}, -\text{Cl}, -\text{Br}, -\text{I}$ (Halo)
Alcohols	$-\text{OH}$ (Hydroxy)
Thiols	$-\text{SH}$
Aldehydes	$-\text{C}-\text{H}$ (Aldehydic) $\parallel$ O
Ketones	O $\parallel$ $-\text{C}-$ (Ketonic)
Ethers	$-\text{C}-\text{O}-\text{C}-$ (Alkoxy)
Carboxylic acid	O $\parallel$ $-\text{C}-\text{OH}$ (Carboxyl)
Sulphonic acid	$-\text{SO}_3\text{H}$
Amides	O $\parallel$ $-\text{C}-\text{NH}_2$ (Amide)
Amines	$-\text{N}<$ (Amino) H
Cyanides/Nitriles	$-\text{C}\equiv\text{N}$ (Cyano)
Nitro compounds	$-\text{N}<$ (Nitro) O
Esters	O $\parallel$ $-\text{C}-\text{O}-\text{C}-$ (Ester)
Acid halides	O $\parallel$ $-\text{C}-\text{X}$ (Acylhalide)

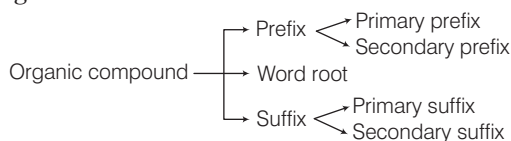
Homologous Series

A homologous series can be defined as a group of compounds in which the members have similar structural features and similar chemical properties and the successive members differ in their molecular formula by CH_2 group.

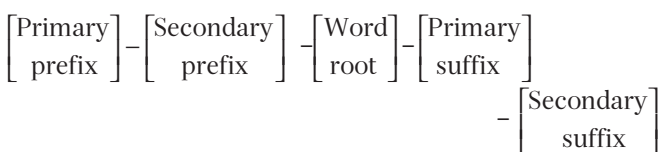
IUPAC Nomenclature for Organic Compounds

The IUPAC name of an organic compound denotes only one molecular structure.

The IUPAC name of any organic compound comprises of the following structure.



General structure of an IUPAC name



Word Root

This indicates the number of carbon atoms present in the parent chain [longest C—C linkages]. This word root includes → meth, eth, prop, but, pent, hex, hept, oct, non, dec.

Suffix

It is used to indicate the nature of C—C linkages whether it is saturated or unsaturated (primary suffix) and type of functional group present in the molecule (secondary suffix).

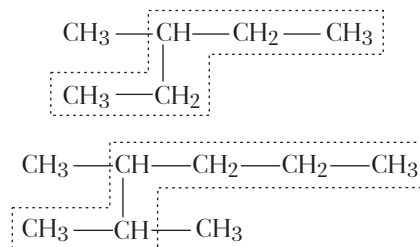
Organic compound	Functional group	Suffix
Alcohols	— OH	—ol
Aldehydes	—CHO (C) HO	Carbaldehyde —al
Ketones	(C) = O	—one
Carboxylic acids	— COOH — (C)OOH	—carboxylic acid —oic acid
Acid amides	—CO NH ₂ (C) O —NH ₂	—carboxamide —amide
Acid halides	$\begin{array}{c} \text{O} \\ \\ \text{—C—X} \end{array}$	—carbonyl halide
	$\begin{array}{c} \text{O} \\ \\ \text{—(C)—X} \\ \text{where, X = halogen} \\ \text{[F, Cl, Br, I]} \end{array}$	—oyl halide
Esters	$\begin{array}{c} \text{O} \\ \\ \text{—C—OR}^* \end{array}$	(R*) carboxylate
	—(C) —OOR	(R*) ...oate
Amines	—NH ₂	—amine
Nitriles	— C ≡ N	—carbonitrile
	— (C) ≡ N	—nitrile
Sulphonic acids	—SO ₂ , —OH	Sulphonic acid

R → any alkyl group

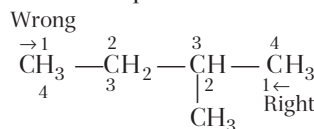
C → when C-atom is included in the main chain.

Rules for IUPAC Nomenclature

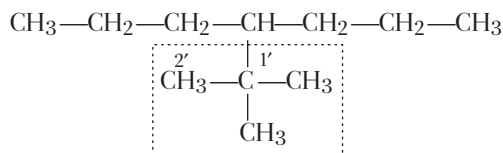
- (i) **Longest chain rule** The longest C—C linkage of a structure is its parent chain. It needs not to be straight chain, but can be of any combination which is longest.



- (ii) **Lowest locant rule** The numbering of the parent chain is done from that side where the value of locant substituent is minimum possible number.



- (iii) **Alphabetical order law** In case of multiple substitutions, the name of the substituents must be arranged in alphabetical order.
- (iv) **First point difference rule** If numbering cannot be resolved by lowest locant rule, i.e. functional groups are at equivalent positions, then the numbering should be done in such a way that functional group coming first in alphabetical order, will be having minimum possible value.
- (v) For complex group (like branched side chain), the nomenclature rules are same, the only difference is that the carbon through which the complex group is associated with main chain, is given number one always. For the purpose of distinction, the locant of side chain are indicated by a prime following them as 1', 2' etc., e.g. consider the structure given below



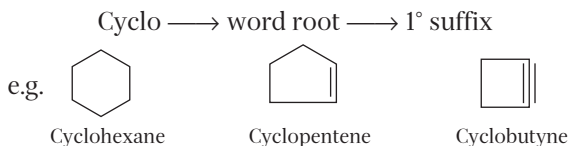
1,1-dimethyl group (complex group)

- (vi) In case of simultaneous presence of double and triple bond, i.e. if bonds are present at equal distance from the ends, then double bond must be preferred. If the position is not symmetrical, then the priority will be decided by lowest locant sum rule and first point difference rule.
- (vii) If the chain is having more than one functional group, the priority of the functional groups is given below
- $$\text{—COOH} > \text{—SO}_3\text{H} > \text{R}(\text{CO})_2\text{O} > \text{—COOR} > \text{—COX} > \\ \text{—CONH}_2 > \text{—CN} > \text{—CHO} > \text{C}=\text{O} > \text{—OH} > \text{NH}_2 > \text{—O—} > \text{C}=\text{C} > \text{C}\equiv\text{C} > \text{R—X} > \text{—NO}_2$$

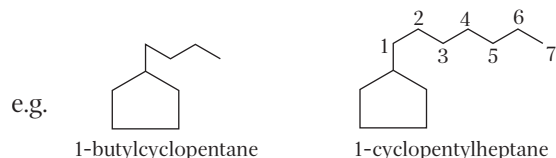
IUPAC Nomenclature for Cyclic Compounds

For the nomenclature of cyclic compounds, general rules are followed:

- (i) The root word is equal to sides of polygon representing the compound. Primary prefix cyclo is used for alicyclic compounds.

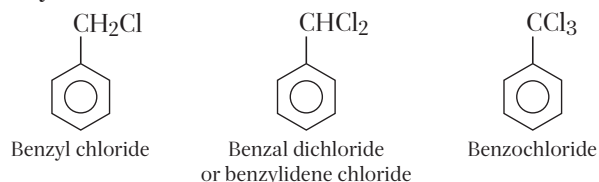


- (ii) If length of side chain is greater than ring size, then ring is treated as alkyl substituent where side chain is substituent.



Nomenclature of Aromatic Compounds

Aromatic compounds are better known by their common name. IUPAC recognises and acknowledges the common nomenclature. Benzenoids are named as derivatives of benzene as per rules mentioned for alicyclic compounds. Benzene with small side chain are also having special name as benzyl, benzal, benzo, etc. Otherwise other common names like toluene, phenol, cresol, resorcinol, etc., are equally used.



Isomerism in Organic Chemistry

Organic compounds having the same molecular formula but different structures or orientations in space are known as isomers and the phenomenon is called isomerism.

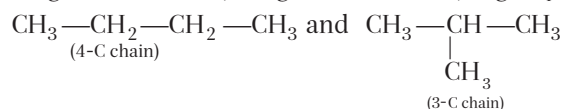
Isomerism can be classified as follows:

1. Structural Isomerism

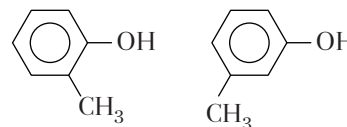
It arises because of the difference in the sequence of covalently bonded atoms in the molecule without reference to space.

It is sub-classified into following types:

- (i) **Chain isomerism** This arises due to the difference in length of C-chain (straight or branched). e.g. C_4H_{10}



- (ii) **Position isomerism** In such isomers, there is difference in the position of functional group or multiple bond or branched chain attached to the parent C-chain.



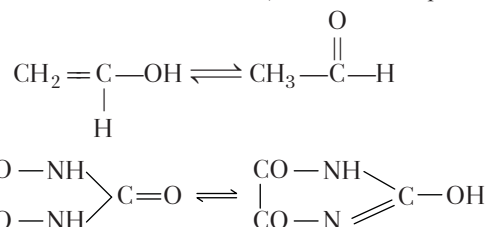
- (iii) **Functional isomerism** Such isomers contain different functional groups.



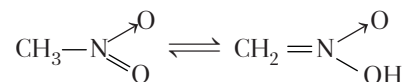
- (iv) **Metamerism** This type of isomerism is due to unequal distribution of substituents on either side of bivalent functional group, e.g. $CH_3CH_2OCH_2CH_3$ and $CH_3CH_2CH_2OCH_3$, $CH_3CH_2NHCH_2CH_3$ and $CH_3CH_2CH_2NHCH_3$.

- (v) **Tautomerism** This arises due to transfer of H-atom from one polyvalent atom to another. Tautomers exist in dynamic equilibrium.

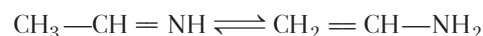
- (a) **Keto-enol tautomerism** (α -H must be present)



- (b) **Nitro-acenitro tautomerism**



- (c) **Imine-enamine tautomerism**



Stereoisomerism

In this type of isomerism, isomers possess same molecular and structural formula but spatial arrangement of atoms is different.

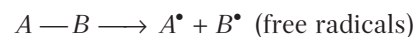
They can be sub-classified in the following types:

Reaction Mechanism

Types of Bond Fissions

1. Homolytic Fission

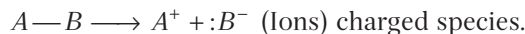
It involves cleavage of covalent bond in such a way that each of the bonded atom takes away one electron out of the shared pair.



It is also called **free-radical cleavage** or **non-polar bond fission**.

2. Heterolytic Fission

The shared pair of electron is taken away by one of the atom (more electronegative atom).



Also called polar bond fission or ionic cleavage.

In the above example,

electronegativity of $B >$ electronegativity of A

Electron Displacement in Covalent Bonds

1. Inductive Effect

The displacement of an electron (shared) pair along the C-chain due to the presence of an electron withdrawing/donating group in the C-chain, is known as inductive effect (I -effect).

It is transmitted along the chain and is permanent.

I -effect can be of the following two types :

- (i) $-I$ -effect Any atom or group of atoms which attracts electrons more strongly than H is said to have negative inductive effect.

Groups introducing $-I$ -effect



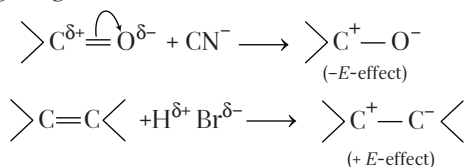
- (ii) $+I$ -effect An atom or group which attracts electrons less strongly as compared to H, is said to have a positive inductive effect.

Groups producing $+I$ -effect



2. Electromeric Effect

It is a temporary effect operates only in π -electrons (i.e. multiple bonded compounds) in the presence of some attacking reagents.

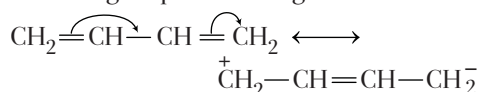


The effect is called $+E$ if transference of π -electrons occurs towards the atom where attacking species attaches.

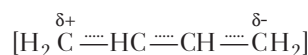
The effect is called $-E$ if transference of electrons occurs away from the atom where the attacking group attaches.

3. Resonance or Mesomeric Effect

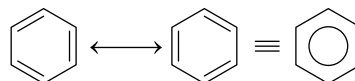
Electron displacement relayed through conjugate system of multiple bonds in the C-chain causing permanent polarisation. It is essential that the atom having a lone pair of electrons or a group containing a π -bond.



The above 2 structures are called resonating structures.

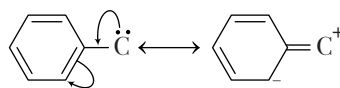
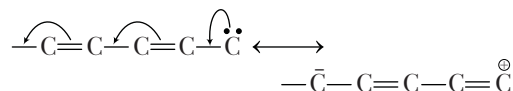


Resonance hybrid (Actual structure)

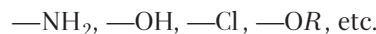


Mesomeric effect is of the following two types :

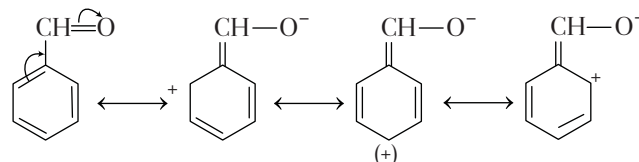
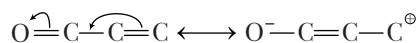
- (a) $+M$ -effect Direction of electron displacement is away from the substituent.



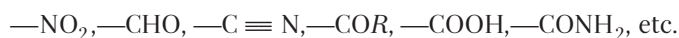
Groups showing $+M$ -effect



- (b) $-M$ -effect Direction of electron displacement is towards the substituent.



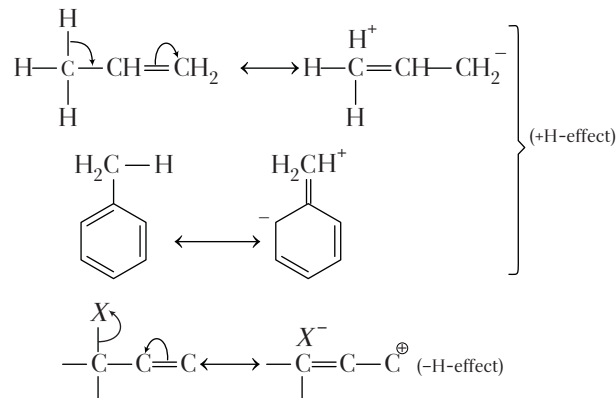
Groups showing $-M$ -effect



4. Hyperconjugation

(Baker-Nathan effect or no bond resonance)

As a result of shifting of σ -electrons, there is a phenomenon of resonance in rest of the molecule. This is a type of σ - π resonance.



Applications of Hyperconjugation

- Relative stability of alkenes.
- Relative stability of alkyl carbonium ions or free radicals.
- The *o*, *p* and *m*-directing influence of some alkyl groups.

Various Reaction Intermediates**1. Carbocation $C^{\oplus}$**

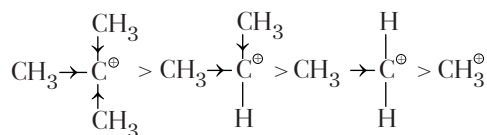
$\begin{array}{c} | \\ -C^{\oplus} \\ | \end{array}$ organic species containing positively charged

C-atom which contains 6 electrons in its valence shell, are known as carbocations. $C^{\oplus}$ has planar geometry.

e.g. $H_3C^{\oplus}$ (Methyl carbocation), $R-\overset{\oplus}{CH_2}$ (1°), $R_2\overset{\oplus}{CH}$ (2°), $R_3\overset{\oplus}{C}$ (3°)

Relative Stability

- Alkyl carbocations** (explained by inductive and hyperconjugation effect)



i.e. $3^{\circ} > 2^{\circ} > 1^{\circ} > \text{CH}_3^{\oplus}$

- Aryl carbocations** (explained by resonance)

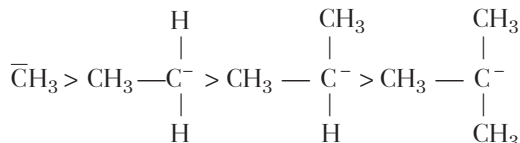
**Carbanion (C^{-})**

Species having such a C-atom that is bearing negative charge and surrounded by octet of electrons. They have pyramidal geometry.

e.g. CH_3^{-} , $(\text{CH}_3)_2\bar{\text{C}}\text{H}$, $(\text{CH}_3)\bar{\text{C}}\text{H}_2$ etc.

Relative Stability

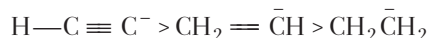
- Alkyl carbanions** (explained by +I-effect)



i.e. $1^{\circ} > 2^{\circ} > 3^{\circ}$

- Benzylic, allylic and triphenyl methyl carbanions** (explained by resonance)

- Stability of C^{-} increases with *s* %.

**3. Free Radical (FR)**

Species having trivalent C-atom bearing no charge but have odd electrons or an unpaired electron, are known as free radicals. They are paramagnetic in nature. Generally, they are planar, formed due to homolytic bond cleavage.

e.g. $-\text{C}-\text{X} \longrightarrow -\text{C}^{\bullet} + \text{X}^{\bullet}$

$\text{CH}_3^{\bullet}$, $\text{CH}_2 = \text{CH}-\text{CH}_2^{\bullet}$, $\text{Cl}^{\bullet}$ etc.

Order of Stability

also $3^{\circ} > 2^{\circ} > 1^{\circ} > \text{CH}_3^{\bullet}$

Free radical generators $\longrightarrow$ Peroxides

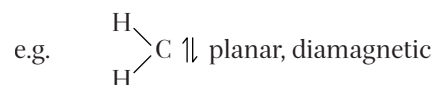
4. Carbene

Species containing divalent C-atom bearing no charge and surrounded by 6 electrons. They are powerful electrophiles.

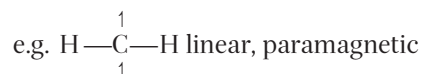
e.g. $:\text{CH}_2$, $:\text{CCl}_2$ etc.

There are two types of carbenes:

- Singlet carbene** (sp^2 -hybridised) Here the unshared electrons are paired and present in the same orbital.



- Triplet carbene** (sp -hybridised) Here the unshared electrons are not paired.



In general, a triplet carbene is more stable than a singlet carbene.

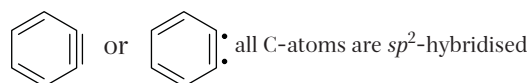
5. Nitrene

It is a neutral intermediate species containing N-atom having one bond pair and two lone pairs. It is electron deficient. They are analogous of carbenes.

e.g. $\text{H}-\ddot{\text{N}}^{\bullet}$, $\text{R}\ddot{\text{N}}^{\bullet}$ etc.

6. Benzyne

It is a neutral highly reactive intermediate in which the aromatic character are not markedly distributed,



Sidewise overlapping of sp^2 -orbitals forms π -bonds outside the ring and out of the plane of π -system of ring. Benzyne is very unstable.

Type of Reagents (or Attacking Species)

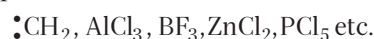
1. Electrophile (E^+)

They are neutral or positively charged species with deficiency of electrons and capable of accepting a pair of electrons (Lewis acid).

Positively charged species



Neutral species



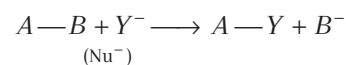
2. Nucleophile (Nu^-)

They are neutral or negatively charged species which are capable of acting as donor of electron pair (Lewis base).

Negatively charged ions



Neutral species



3. Amphibiles

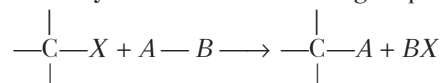
Amphibiles are those which act as both E^+ and Nu^- .



Types of Organic Reactions

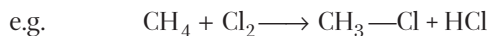
1. Substitution Reaction

Such reactions involve the replacement of an atom or group of atoms by some other atom or group.

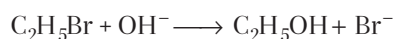
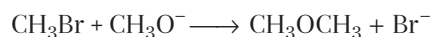
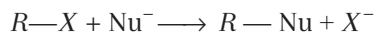


Substitution reaction are of the following types:

(a) Free radical substitution (FR intermediates)

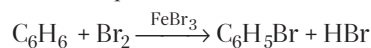


(b) Nucleophilic substitution (Attacking species Nu^-)

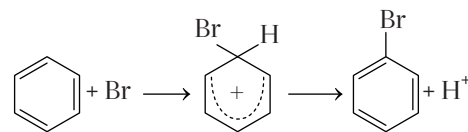


(c) Electrophilic substitution

Substitution in benzene ring follows electrophilic substitution.



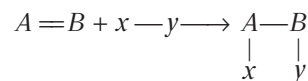
Mechanism



- During E^+ substitution, an electron donating group on benzene ring makes it more reactive. Such groups are $-\text{NH}_2$, $-\text{OH}$, OR , etc.
- A deactivating group is one which makes benzene less reactive. Such groups are $-\text{COOH}$, $>\text{C}=\text{O}$, $-\text{CHO}$, $-\text{NO}_2$, $-\text{CN}$ etc.

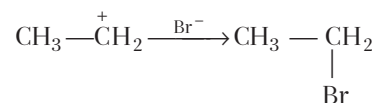
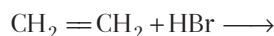
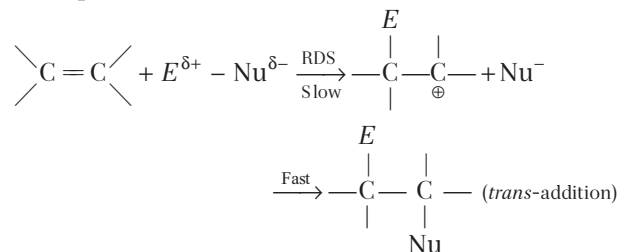
2. Addition Reactions

Reactions in which atoms or group of atoms are added to a molecule, are called addition reactions.

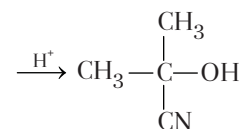
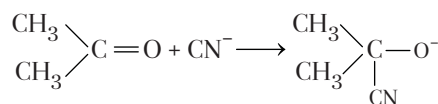
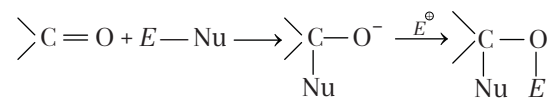


Addition reaction are of the following types :

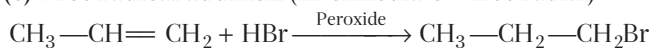
(a) Electrophilic addition reactions



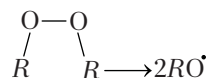
(b) Nucleophilic addition reaction (Intermediate $\rightarrow \text{Nu}^-$)



(c) Free radical addition (Intermediate – free radical)

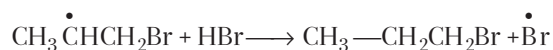
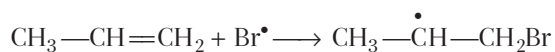


Mechanism

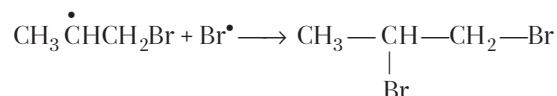


Chain initiation $\text{RO}\cdot + \text{HBr} \longrightarrow \text{ROH} + \text{Br}\cdot$

Chain propagation

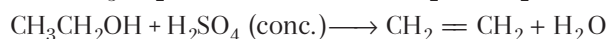
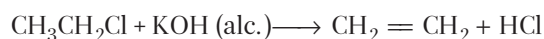
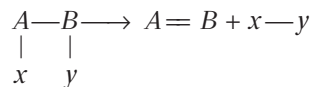


Chain termination $\text{Br}\cdot + \text{Br}\cdot \longrightarrow \text{Br}_2$



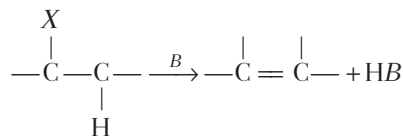
3. Elimination Reactions

Such reaction involves loss of atoms or groups from a molecule to form multiple bonds.



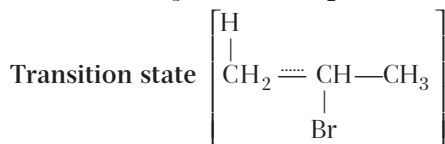
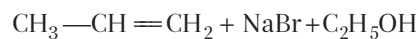
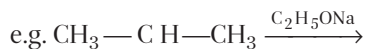
There are two types of mechanism:

(i) E 2 reaction Elimination bimolecular reaction involves one step only.



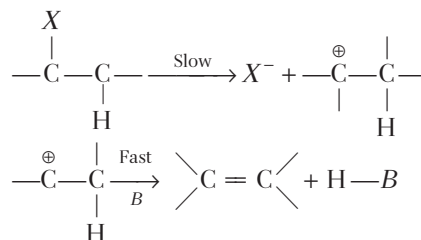
Rate equation $R = k [\text{halide}] [:\text{B}]$

Molecularity = 2, order = 2



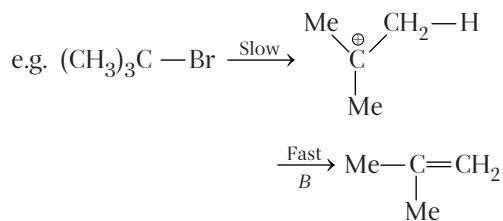
Order of reactivity : $3^\circ > 2^\circ > 1^\circ$

(ii) E 1 reaction (Elimination unimolecular)



Rate equation $R = k[\text{RX}]$

Molecularity = 1, order = 1

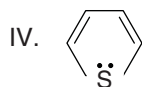
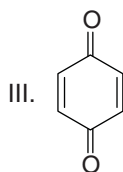
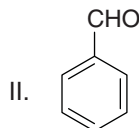
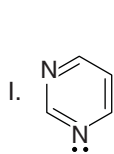


4. Rearrangement Reactions

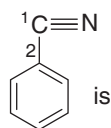
In such reactions, atom's sequence gets reshuffled to form a new structure, e.g. pinacol pinacolone rearrangement, Beckman rearrangement.

Practice Exercise

1. Which of the following are aromatic compounds?



- a. I. and III
b. I, II, and III
c. I, II, and IV
d. III, and IV
2. 3° carbon is present in the compound
a. cyclopropane
b. toluene
c. benzene
d. cyclohexane
3. Choose the compound in which all the carbon atoms have 66.7% *p*-character.
a. $\text{CH}_3-\text{C}\equiv\text{CH}$
b. $\text{CH}_2=\text{C}=\text{CH}-\text{CH}_3$
c. $\text{CH}_2=\text{CH}-\text{CH}=\text{CH}_2$
d. $(\text{CH}_3)_2\text{C}=\text{CH}_2$
4. The bond between carbon atom (1) and carbon atom (2) in compound

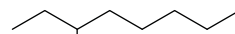
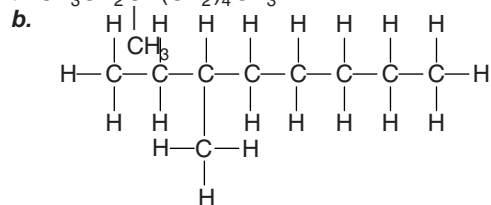
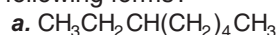


- a. sp^3 and sp^2 -hybridised
b. sp^2 and sp^3 -hybridised
c. sp and sp^2 -hybridised
d. sp and sp -hybridised
5. Electronegativity of carbon atoms depends upon their state of hybridisation. In which of the following compounds, the carbon marked with asterisk, is most electronegative?
a. $\text{CH}_3-\text{CH}_2-\dot{\text{C}}\text{H}_2-\text{CH}_3$
b. $\text{CH}_3-\dot{\text{C}}\text{H}=\dot{\text{C}}\text{H}-\text{CH}_3$
c. $\text{CH}_3-\text{CH}_2-\dot{\text{C}}\equiv\text{CH}$
d. $\text{CH}_3-\text{CH}_2-\text{CH}=\text{CH}_2$
6. The hybridisation of C_2 carbon atom present in $\text{CH}_2=\text{C}=\text{CH}_2$ is
a. sp
b. sp^3
c. sp^2
d. sp^4

7. The kind of valency that exists in CaH_2 and C_2H_2 is

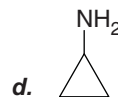
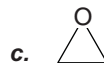
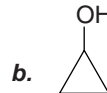
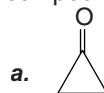
- a. electrovalency in CaH_2 and covalency in C_2H_2
b. electrovalency in both
c. covalency in CaH_2 and electrovalency in C_2H_2
d. covalency in both

8. 3-methyloctane can be represented in which of the following forms?



- d. All of the above

9. Which of the following is a heterocyclic alicyclic compound?



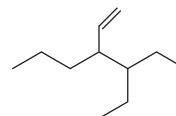
10. The IUPAC name of the isomer of $\text{CH}_3-\text{CH}=\text{NOH}$ would be

- a. methanamide
b. 1-amino-2-propanone
c. ethanamide
d. None of the above

11. Pyruvic aldehyde is nothing but

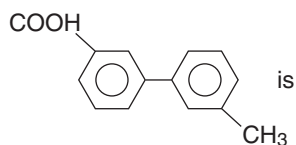
- a. methyl glyoxal
b. ethyl glyoxal
c. glyoxal
d. None of the above

12. The IUPAC name for the following compound is



- a. 4-ethyl-3-propyl hex-1-ene
b. 3-(1-ethylpropyl) hex-1-ene
c. 3-ethyl-4-propyl hex-5-ene
d. 3-ethyl-4-ethylheptane

13. The correct IUPAC name of

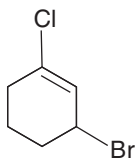


- a. 6-methyl-3-phenyl benzene-1-oic acid
 b. 6-methyl dibenzene-1-oic acid
 c. 3-(3-methyl phenyl) benzene-1-oic acid
 d. 3-(5-methyl phenyl) benzene-1-carboxylic acid

14. The IUPAC name of is

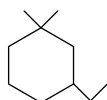
- a. 1-alkoxycyclopent-1-ene
 b. oxocyclohex-2-ene
 c. 2-methoxycyclopent-1-ene
 d. 5-oxocyclopent-1-ene

15. The IUPAC name of the compound shown below is



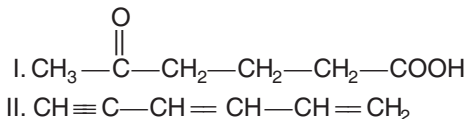
- a. 2-bromo-6-chlorocyclohex-1-ene
 b. 6-bromo-2-chlorocyclohexene
 c. 3-bromo-1-chlorocyclohexene
 d. 1-bromo-3-chlorocyclohexene

16. What is the correct IUPAC name of the following compound?



- a. 3-ethyl-1,1-dimethylcyclohexane
 b. 1-ethyl-3,3-dimethylcyclohexane
 c. 1,1-dimethyl-3-ethylcyclohexane
 d. None of the above

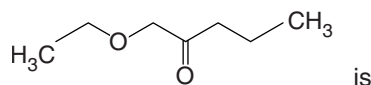
17. The correct IUPAC name of the given structures will be



Choose the correct option.

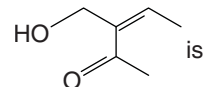
	I	II
a.	Hexane-dione	Hexa-1,3-diene-5-yne
b.	5-oxohexanoic acid	Hexa-1,3-diene-5-yne
c.	5-oxohexanoic acid	Hexadiene-5-yne
d.	3-oxohexanoic acid	Hexadiene-5-yne

18. The IUPAC name of



- a. ethylpentanoate
 b. 1-ethoxy-2-pentanone
 c. 5-ethoxy-4-pentanone
 d. ethyl-2-oxopentyl ether

19. The IUPAC name of



- a. 2-acetyl-1-buten-1-ol
 b. 3-(hydroxymethyl) 3-penten-2-one
 c. 3-ethylidene-4-hydroxy-2-butanone
 d. 3-acetyl-2-buten-4-ol

20. Which skeleton exhibit optical isomerism?

- a. $\text{C}-\text{C}-\text{C}-\text{C}-\text{COOH}$
 b. $\begin{array}{c} \text{C}-\text{C}-\text{C}-\text{C} \\ | \\ \text{COOH} \end{array}$
 c. $\begin{array}{c} \text{C}-\text{C}-\text{C}-\text{COOH} \\ | \\ \text{C} \end{array}$
 d. $\begin{array}{c} \text{C} \\ | \\ \text{C}-\text{C}-\text{COOH} \\ | \\ \text{C} \end{array}$

21. In which isomer of $\text{C}_7\text{H}_7\text{NO}_2$, there are two functional groups?

- a. Phenyl nitromethane
 b. *m*-nitrotoluene
 c. Anthranilic acid
 d. Benzylnitrite

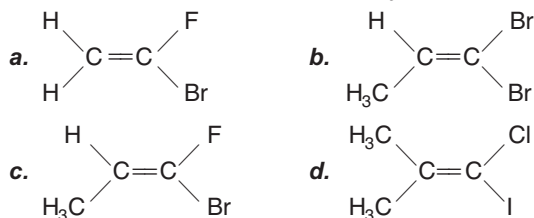
22. The pair of functional group isomers is

- a. CH_3CONH_2 , $\begin{array}{c} \text{CH}_2\text{CHO} \\ | \\ \text{NH}_2 \end{array}$
 b. CH_3COOH , HCOOCH_3
 c. $\text{CH}_3-\text{CH}_2\text{OH}$, CH_3OCH_3
 d. All of the above

23. The compound C_nH_{2n} [$n = 4$] does not exhibit the which of the following isomerism?

- a. Chain
 b. Geometrical
 c. Position
 d. Optical

24. Geometrical isomerism is shown by



25. The compound 2, 3-dichlorobutane exhibits
 a. geometrical isomerism
 b. diastereoisomerism
 c. structural isomerism
 d. optical isomerism
26. The number of constitutional isomers of the formula $C_5H_{11}Br$ is
 a. 4
 b. 8
 c. 6
 d. 10
27. How many optical isomers are possible on monochlorination of 2-methyl butane?
 a. 2
 b. 4
 c. 6
 d. 8
28. The structural formula of 2-oxo-3-methyl-(N-bromo) butanamide is
 a. $CH_3-CH_2-CO-CO-NH-Br$
 b. $CH_3-\underset{\substack{| \\ CH_3}}{CH}-CO-CO-NH-Br$
 c. $CH_3CH-CO-CO-\underset{\substack{| \\ CH_3}}{NOBr}$
 d. $(CH_3)_3C-CO-CO-NHBr$
29. Match the items of Column I with the Column II and choose the correct option from the codes given below.

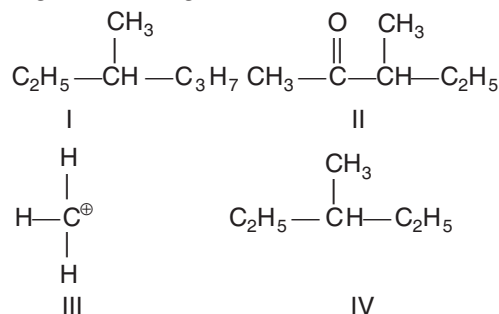
Column I (Structure of compounds)	Column II (Type of isomerism)
A. $CH_3CH_2CH_2CH_2CH_3$ and $CH_3-\underset{\substack{ \\ CH_3}}{CH}-CH_2-CH_3$	1. Chain isomerism
B. $CH_3CH_2CH_2OH$ and $CH_3-\underset{\substack{ \\ OH}}{CH}-CH_3$	2. Position isomerism
C. $CH_3-\overset{\substack{O \\ }}{C}-CH_3$ and $CH_3-CH_2-\underset{\substack{ \\ H}}{C}=O$	3. Metamerism
D. $CH_3OC_3H_7$ and $C_2H_5OC_2H_5$	4. Functional group isomerism

Codes

	A	B	C	D
a.	1	2	3	4
b.	2	3	1	4
c.	4	1	2	3
d.	1	2	4	3

30. Example of geometrical isomerism is
 a. 2-butanol
 b. 2-butene
 c. butanol
 d. 2-butyne

31. Among the following four structures I to IV,



It is true that

- a. all four are chiral compounds
 b. I and II are chiral compounds
 c. III is a chiral compounds
 d. II and IV are chiral compounds
32. The correct statement about the compounds A, B and C is
- $$\begin{array}{c} COOCH_3 \\ | \\ H-C-OH \\ | \\ H-C-OH \\ | \\ COOH \end{array}$$

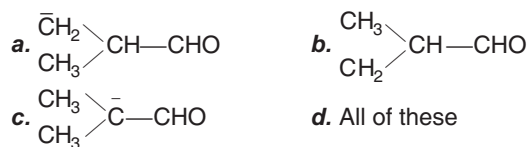
A

$$\begin{array}{c} COOH \\ | \\ H-C-OH \\ | \\ H-C-OH \\ | \\ COOCH_3 \end{array}$$

B

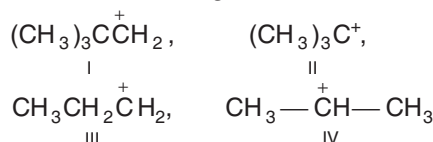
$$\begin{array}{c} COOH \\ | \\ HO-C-H \\ | \\ HO-C-H \\ | \\ COOCH_3 \end{array}$$

C
- a. A and B are identical
 b. A and B are diastereomers
 c. A and C are enantiomers
 d. A and B are enantiomers
33. If $CH_3-\underset{\substack{| \\ CH_3}}{CH}-CHO$ gives up proton, the most stable carbanion would be



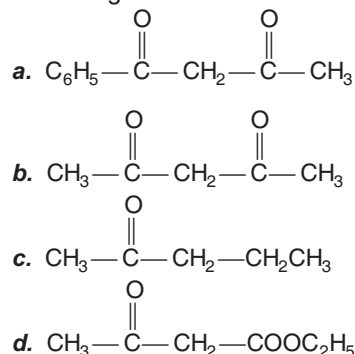
34. In which case, ionic mechanism is not followed?
 a. Reaction of acetylene and bromine water
 b. Reaction of HBr and propene in the presence of peroxide
 c. Reaction of ethyl bromide and alcoholic KOH
 d. Dehydration of ethyl alcohol catalysed by acid
35. Which compound is more sensitive to undergo S_N1 reaction?
 a. 2-bromobutane
 b. 2-bromo-2-methyl propane
 c. 2-methyl-1-bromo propane
 d. Bromoethane
36. The enol form of acetone contains]
 a. 9 σ -bonds, 2 π -bonds and 1 lone pair of electron
 b. 10 σ -bonds, 1 π -bond and 1 lone pair of electrons
 c. 9 σ -bonds, 1 π -bond and 2 lone pairs of electrons
 d. 8 σ -bonds, 2 π -bonds and 2 lone pairs of electrons

37. Consider the following carbocations,

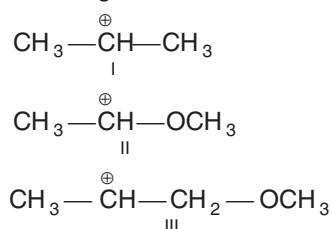


The correct order for the stability of the above carbocations is

- a. I < III < IV < II b. III < IV < I < II
 c. IV < III < II < I d. II < IV < III < I
38. The solvent in which enol form of ethylacetoacetate is maximum, is
 a. CH₃COOH b. aqueous HCl
 c. *n*-hexane d. H₂O
39. Among the following compounds, the compound containing maximum enol contents in their solution, is

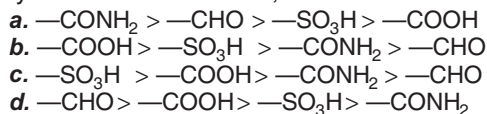


40. What is the correct order of decreasing stability of the following cations?

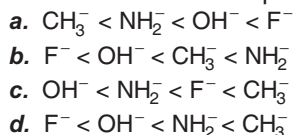


- a. II > I > III b. II > III > I
 c. III > I > II d. I > II > III
41. The correct stability order of the following free radicals is
 a. $(\text{CH}_3)_2\dot{\text{C}}\text{H} < (\text{CH}_3)_3\dot{\text{C}} < (\text{C}_6\text{H}_5)_3\dot{\text{C}} < (\text{C}_6\text{H}_5)_2\dot{\text{C}}\text{H}$
 b. $(\text{C}_6\text{H}_5)_2\dot{\text{C}}\text{H} < (\text{C}_6\text{H}_5)_3\dot{\text{C}} < (\text{CH}_3)_3\dot{\text{C}} < (\text{CH}_3)_2\dot{\text{C}}\text{H}$
 c. $(\text{C}_6\text{H}_5)_3\dot{\text{C}} < (\text{C}_6\text{H}_5)_2\dot{\text{C}}\text{H} < (\text{CH}_3)_3\dot{\text{C}} < (\text{CH}_3)_2\dot{\text{C}}\text{H}$
 d. $(\text{CH}_3)_2\dot{\text{C}}\text{H} < (\text{CH}_3)_3\dot{\text{C}} < (\text{C}_6\text{H}_5)_2\dot{\text{C}}\text{H} < (\text{C}_6\text{H}_5)_3\dot{\text{C}}$
42. Which one of the following species is not an electrophile?
 a. NO_2^+ b. H_3O^+
 c. Cl^+ d. BH_3

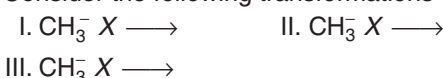
43. The correct decreasing order of priority for the functional groups of organic compounds in the IUPAC system of nomenclature, is



44. Correct order of nucleophilicity is



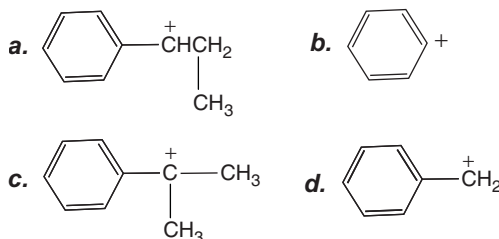
45. Consider the following transformations



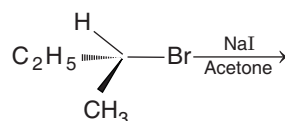
Carbon species formed in I, II and III respectively are

- a. carbocation, carbanion and free radical
 b. free radical, carbocation and carbanion
 c. free radical, carbanion and carbocation
 d. carbanion, carbocation and free radical

46. Which is the most stable carbocation?

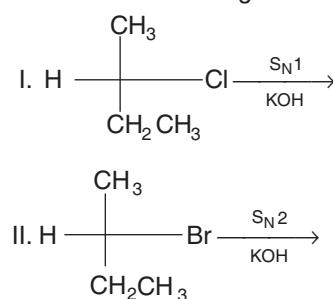


47. Select the correct statement about the following reaction :



- a. it can proceed via S_N2 mechanism
 b. configuration about chiral carbon is retained
 c. a racemic mixture is formed
 d. reaction is stereospecific

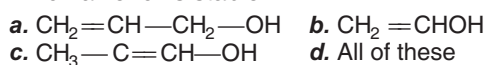
48. Consider the following reactions:



Select the correct statement.

- a. Inversion in both cases takes place
b. Retention in both cases takes place
c. Retention in I and inversion in II takes place
d. Racemisation in I and inversion in II takes place
49. Arrange the following in increasing order of acidic strength :
I. H_2SO_4 II. $(\text{CH}_3)_3\text{CH}(\text{SH})$
III. $\text{CH}_3\text{CH}_2^+\text{OH}_2$ IV. $\text{CH}_3\text{CH}_2\text{CH}_3$
a. $\text{IV} < \text{II} < \text{I} < \text{III}$ b. $\text{IV} < \text{II} < \text{III} < \text{I}$
c. $\text{IV} < \text{I} < \text{II} < \text{III}$ d. $\text{IV} < \text{III} < \text{II} < \text{I}$
50. Choose the correct order of stability of carbocation using concept of hyperconjugation.
-
- a. $\text{I} < \text{II} < \text{III} < \text{IV}$
b. $\text{IV} < \text{III} < \text{II} < \text{I}$
c. $\text{III} < \text{IV} < \text{II} < \text{I}$
d. All of the above
51. Chloroacetic acid is a stronger acid than acetic acid. This can be explained using
a. $-M$ -effect b. $-I$ -effect
c. $+M$ -effect d. $+I$ -effect
52. $\text{CH}_3\text{CH}_2\text{Cl}$ undergoes homolytic fission and produces
a. $\text{CH}_3\dot{\text{C}}\text{H}_2$ and $\dot{\text{Cl}}$ b. $\text{CH}_3\dot{\text{C}}\text{H}_2$ and Cl^-
c. $\text{CH}_3\text{CH}_2^\oplus$ and $\dot{\text{Cl}}$ d. $\text{CH}_3\dot{\text{C}}\text{H}_2$ and Cl^-

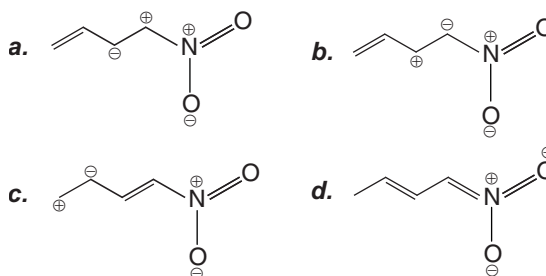
53. Which alkenol is stable?



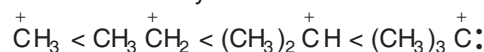
54. Which of the following is not a free radical?



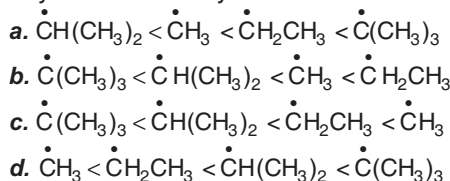
55. Among the following, the least stable resonance structure is



56. Carbocation stability

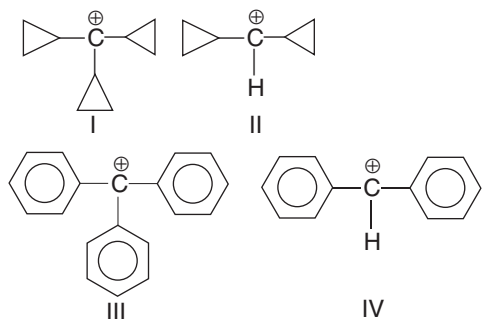


Alkyl radical stability



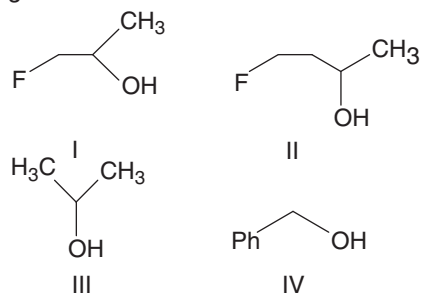
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1. Which of the following is correct order of stability of carbocation? [2014]



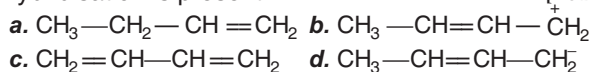
- a. $\text{IV} > \text{III} > \text{II} > \text{I}$
b. $\text{I} > \text{II} > \text{III} > \text{IV}$
c. $\text{III} > \text{II} > \text{I} > \text{IV}$
d. $\text{I} > \text{III} > \text{II} > \text{IV}$

2. Arrange these in correct order of decreasing reactivity.



- a. $\text{I} > \text{II} > \text{III} > \text{IV}$ b. $\text{I} > \text{III} > \text{II} > \text{IV}$ [2013]
c. $\text{IV} > \text{III} > \text{II} > \text{I}$ d. $\text{IV} > \text{III} > \text{I} > \text{II}$

3. In which of the following species only one type of hybridisation is present? [2012]



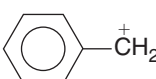
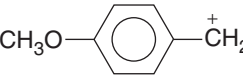
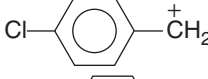
4. Which among the following is likely to show geometrical isomerism? [2011]

a. $\text{CH}_3\text{CH}=\text{NOH}$
 b. $\text{CH}_3\text{CH}=\text{CH}_2$
 c. $\text{CH}_2=\text{CH}-\text{CH}=\text{CCl}_2$
 d. $\text{CH}_3\text{C}(\text{Cl})=\text{C}(\text{CH}_3)_2$

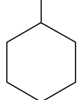
5. The bond dissociation enthalpies of =CH , =CH , —CH follows the order [2010]

a. $sp > sp^2 > sp^3$ b. $sp^3 > sp^2 > sp$
 c. $sp^2 > sp > sp^3$ d. $sp > sp^3 > sp^2$

6. Most stable carbonium ion is [2010]

a. 
 b. 
 c. 
 d. 

7. $\text{CH}_3-\text{CH}-\text{CH}_2-\text{CH}_3$ is

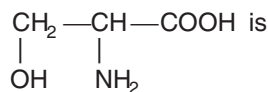


a. 2-phenylbutane
 b. 3-phenylbutane
 c. 3-cyclohexylbutane
 d. 2-cyclohexylbutane

8. The IUPAC name of $\text{C}_2\text{H}_5-\text{O}-\text{CH}(\text{CH}_3)_2$ is

a. ethoxypropane
 b. 1,1-dimethyl ether
 c. 2-ethoxysopropane
 d. 2-ethoxypropane

9. The IUPAC name of the compound, [2007]

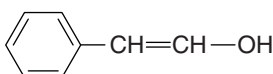
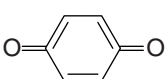
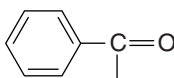
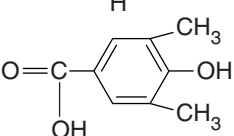


a. 2-amino-3-hydroxy propanoic acid
 b. 1-hydroxy -2-amino propan-3-oic acid
 c. 1-amino-2-hydroxy propanoic acid
 d. 3-hydroxy-2-amino propanoic acid

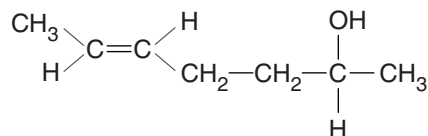
10. The compound which give the most stable carbonium ion on dehydration is [2007]

a. $\text{CH}_3\text{CH}(\text{CH}_3)\text{CH}_2\text{OH}$
 b. $(\text{CH}_3)_3\text{COH}$
 c. $\text{CH}_2=\text{CHCH}_2\text{CH}_2\text{OH}$
 d. $\text{CH}_3\text{CHOHCH}_2-\text{CH}_3$

11. Tautomerism is exhibited by [2007]

a. 
 b. 
 c. 
 d. 

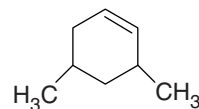
12. The compound whose stereochemical formula is written below, exhibits x geometrical isomers and y optical isomers. [2006]



The values of x and y respectively are

a. 4 and 4 b. 2 and 2 c. 2 and 4 d. 4 and 5

13. The IUPAC name of the following compound is [2006]



a. 3, 5-dimethylcyclohexene
 b. 3, 5-dimethyl-1-cyclohexene
 c. 1, 5-dimethyl-5-cyclohexene
 d. 1, 3-dimethyl -5-cyclohexene

Answer with Solutions

Practice Exercise

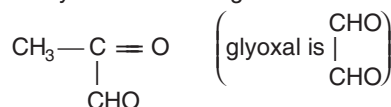
- (c) I, II and IV are aromatic compounds [contain 6π - electrons].
- (b) In toluene, the carbon atom of the ring attached to CH_3 group is 3° .
- (c) $\text{CH}_2=\text{CH}-\text{CH}=\text{CH}_2$

In this compound, each carbon is sp^2 -hybridised.

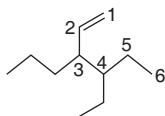
$$\text{Hence, \% of } p\text{-character} = \frac{2}{3} \times 100 = 66.67\%.$$

- (c) C_1 is sp and C_2 is sp^2 -hybridised.
- (c) Triple bonded carbon atoms are more electronegative than double and single bonded carbon atoms.
- (a) When C-atom is double bonded to adjacent carbon atoms, the carbon atoms become linear in shape due to the formation of two π -bonds simultaneously with two other C-atoms. Hence, hybridisation comes out to be sp .
- (a) Electrovalency in CaH_2 and covalency in C_2H_2 .
- (d) 3-methyloctane can be represented by all of the above forms.
- (c) Heterocyclic alicyclic compounds are the compounds in which any atom other than carbon is also present in the ring.
- (c) CH_3CONH_2 is the isomer of the given structure and its IUPAC name is ethanamide.

- (c) Pyruvic aldehyde has following structure

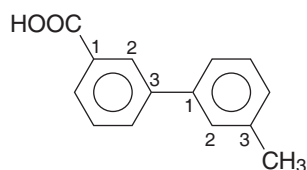


- (a) The given compound is shown as



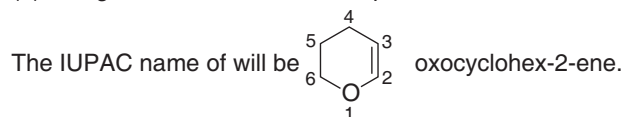
The IUPAC name of above compound is 4-ethyl-3-propylhex-1-ene.

- (c) The given structure of the compound is shown below.

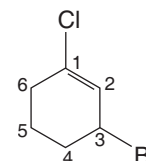


The IUPAC name will be 3-(3-methyl phenyl) benzene-1-oic acid.

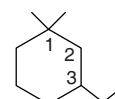
- (b) The given structure of the compound is shown below.



- (c) Unsaturation (double bond) is given priority over halogen. So, the correct IUPAC name is 3-bromo-1-chlorocyclohexene

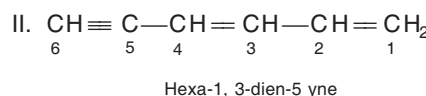


- (a)



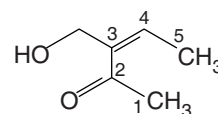
3-ethyl-1,1-dimethylcyclohexane

- (b) I. $\text{CH}_3-\text{C}(=\text{O})-\text{CH}_2-\text{CH}_2-\text{CH}_2-\text{COOH}$
5-oxohexanoic acid



- (b) $\text{CH}_3-\text{CH}_2-\text{O}-\text{CH}_2-\text{C}(=\text{O})-\text{CH}_2-\text{CH}_2-\text{CH}_3$
1-ethoxypentan-2-one

- (b)

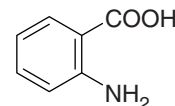


3-(hydroxymethyl) pent-3-ene-2-one

- (b) $\text{CH}_3-\overset{*}{\text{C}}\text{H}(\text{COOH})-\text{CH}_2-\text{CH}_3$

Only this compound has a chiral carbon-atom, hence it will show optical isomerism.

- (c)



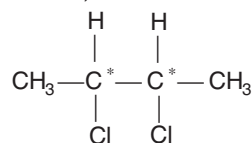
Anthranilic acid

- (c) is an example of position isomer while (b) is of metamerism. $\text{CH}_3\text{CH}_2\text{OH}$ and CH_3OCH_3 are functional group isomers.

- (d) C_4H_8 does not exhibit optical isomerism.

- (c) The compound, $\text{H}_3\text{C}-\text{C}(\text{H})=\text{C}(\text{F})-\text{Br}$ exhibits geometrical isomerism. *E* (*trans*)-*Z* (*cis*) as isomers.

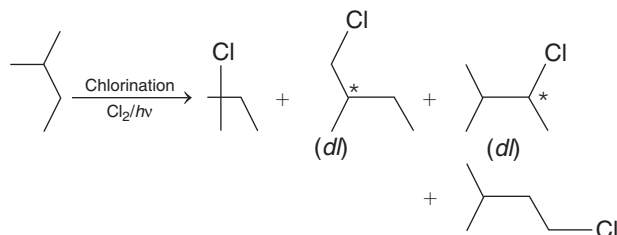
25. (d) The structure for 2, 3-dichlorobutane is



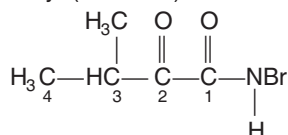
This compound contains two chiral centres (asymmetric carbons). Therefore, it exhibits optical isomerism.

26. (b) The compound $\text{C}_5\text{H}_{11}\text{Br}$ possesses 8 constitutional isomers.

27. (b) On chlorination of 2-methylbutane, 4 optically active isomers are formed.



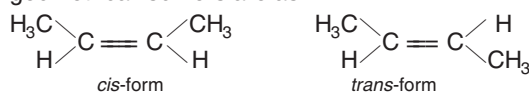
28. (b) 2-oxo-3-methyl-(N-bromo) butanamide is



There is a methyl group at 3rd position and 'Br' at nitrogen atom in the compound.

29. (d)

30. (b) 2-butene contains a double bond and the groups attached with double bonded carbon are different therefore, it exhibits geometrical isomerism. The geometrical isomers are as



31. (b) Chiral compounds have at least one chiral centre, i.e. all four atoms or groups attached to carbon are different.

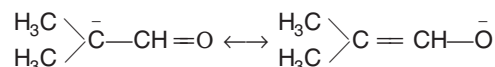
32. (d)

(i) Enantiomers are the pairs of optical isomers which are related as non-superimposable mirror images of each other.

(ii) Diastereomers are the pairs of optical isomers which cannot be related as non-superimposable mirror images of each other.

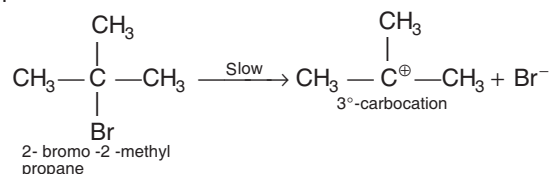
∴ The only correct statement about given structures is that (A) and (B) are enantiomers.

33. (c) This carbanion shows resonance

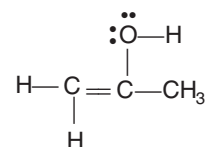


34. (b) In the presence of peroxide, the addition of HBr to propene follows free radical mechanism.

35. (b) Tertiary halides, generally undergo $\text{S}_{\text{N}}1$ reaction, as the tertiary carbocation, obtained in this mechanism, is quite stable.



36. (c) The enol form of acetone is shown below



It shows that this compound possesses 9 σ -bonds, 1 π -bond and 2 lone pairs of electrons on O-atom.

37. (a) Larger the number of α -H-atoms linked to a carbocation, more will be the stable carbocation. Hence, the correct order of stability is $\text{I} < \text{III} < \text{IV} < \text{II}$

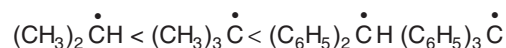
38. (c)

39. (d) The compound must possess only keto group in order to show enol form.

The compound $\text{CH}_3 - \overset{\text{O}}{\parallel}{\text{C}} - \text{CH}_2 - \text{COOC}_2\text{H}_5$ contains keto group along with ester group which favours the enol formation by this keto group.

40. (a) Stability of carbocation increases with the dispersal of positive charge by the donation of electron density through +I-effect. The groups showing +I-effect are $-\text{OCH}_3$, $-\text{CH}_3$, $-\text{CH}_2-\text{OCH}_3$. Thus, the correct stability order of the given carbocations is $\text{II} > \text{I} > \text{III}$

41. (d) Larger the number of phenyl groups linked to a free radical, more the number of resonating structure and greater is the stability of free radical. Thus, the correct order for the stability of free radicals is

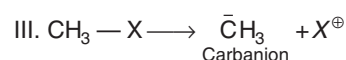
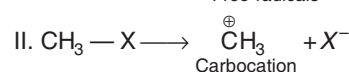


42. (b)

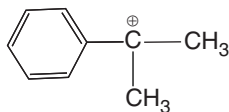
43. (b) $-\text{COOH} > -\text{SO}_3\text{H} > -\text{CONH}_2 > -\text{CHO}$

44. (d) Nucleophilicity $\propto \frac{1}{\text{Electronegativity of atom bearing negative charge}}$

45. (b) I. $\text{CH}_3 - \text{X} \longrightarrow \dot{\text{C}}\text{H}_3 + \text{X}^\bullet$
Free radicals



46. (c)



3° carbocation is more stable

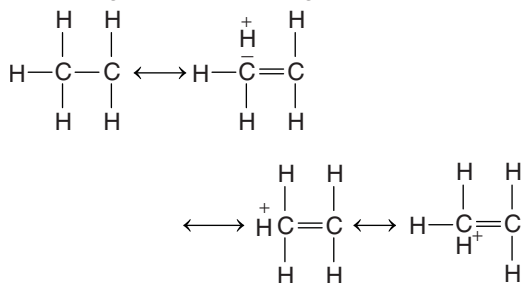
47. (a) The reaction can proceed through S_N2 mechanism and it is stereospecific. A reaction is stereospecific when a particular stereoisomeric form of the starting material reacts in such a way that it gives a stereoisomeric form of the product.

48. (d)

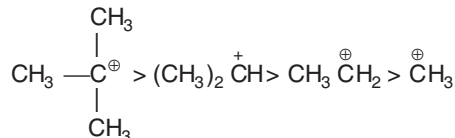
49. (a) Acidic strength depends upon the tendency of a molecule to give a proton. Among the given, $\text{CH}_3\text{CH}_2\text{OH}_2^+$ readily gives a proton, so it is highly acidic. $\text{CH}_3\text{CH}_2\text{CH}_3$ has the least tendency to lose a proton, so its acidic strength is least among the given. Thus, the correct order of acidic strength is



50. (b) The stability of carbocation on the basis of hyperconjugation can be explained as hyperconjugation stabilises the carbocation because electron density from the adjacent π -bond helps in dispersing the positive charge.



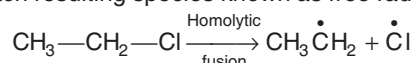
In general, greater the number of alkyl groups attached to a positively charged carbon atom, the greater is the hyperconjugation interaction and stabilisation of the cation. Thus, we have the following relative stability of carbocation



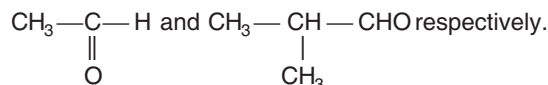
Hence, stability of carbocation is directly proportional to number of alkyl group directly attached to carbocationic carbon.

51. (b)

52. (a) In homolysis, the covalent bond is broken in such a way that each resulting species known as free radical.



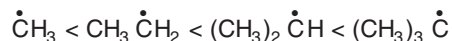
53. (b) and (c) are enols and will tautomerise into



54. (c) Dichlorocarbene ($:\text{CCl}_2$) is an electrophile.

55. (a)

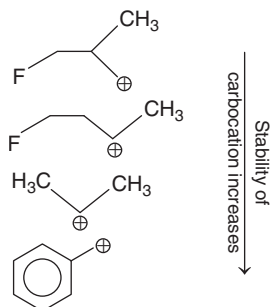
56. (d) The observed order of alkyl radical stability is



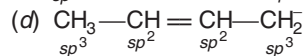
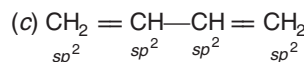
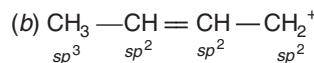
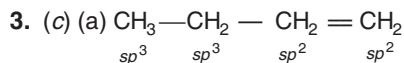
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1. (d)

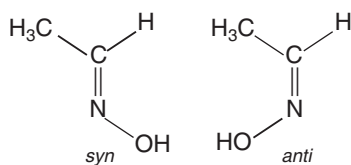
2. (c) This problem includes conceptual mixing of carbocation stability and reactivity of alcohol. Remove the $-\text{OH}$ group by dehydration and then arrange the carbocation in increasing order of correctly. Order of carbocation stability is same as S_N1 reactivity of alkyl halides.



Carbocation stability The carbocation is formed during the reaction of alcohol by removal of $-\text{OH}$. More stable the carbocation, more will be its reactivity of carbocation formed during reaction are as given

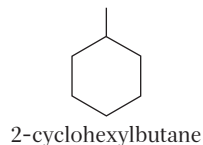


4. (a) Among the given, only $\text{CH}_3\text{CH}=\text{NOH}$ (oxime) satisfy the conditions essential for exhibiting geometrical isomerism. So, it will exhibit *syn-anti* geometrical isomerism.



5. (a)

6. (b)

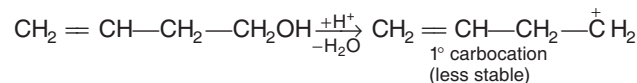
7. (d) $\text{CH}_3-\overset{1}{\text{CH}}-\overset{2}{\text{CH}}-\overset{3}{\text{CH}_2}-\overset{4}{\text{CH}_3}$ 8. (d) $\text{C}_2\text{H}_5-\text{O}-\overset{2}{\text{CH}}(\overset{1}{\text{CH}_3})_2$

2-ethoxypropane

The above compound is an ether and its name is written as alkoxy. Oxy is attached with the lower group. Hence, the IUPAC name of above compound is 2-ethoxypropane.

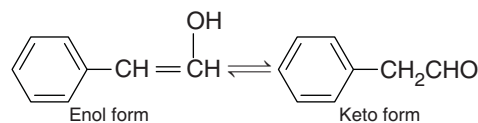
9. (a) $\text{CH}_2-\text{CH}-\text{COOH}$
 $\quad \quad \quad | \quad \quad |$
 $\quad \quad \quad \text{OH} \quad \text{NH}_2$

2-amino-3-hydroxypropanoic acid

10. (b) $(\text{CH}_3)_3\text{C}-\text{OH} \xrightarrow[+\text{H}^+]{-\text{H}_2\text{O}} (\text{CH}_3)_3\text{C}^+$
 Tertiary Alcohol 3° carbocation (more stable)

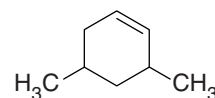
Increasing order of stability of carbocation is

1° carbocation < 2° carbocation < 3° carbocation

11. (a) **Tautomerism** It is functional isomerism in which the isomers are readily interchangeable and maintain a dynamic equilibrium with each other.12. (b) $\text{CH}_3-\text{C}(\text{H})=\text{C}(\text{H})-\text{CH}_2-\text{CH}_2-\text{C}(\text{H})(\text{OH})-\text{CH}_3$

It exhibits 2 geometrical and 2 optical isomers.

13. (a)



IUPAC name : 3, 5-dimethylcyclohexene

22

Purification and Estimation of Organic Compounds

Purification

Methods of Purification of Organic Compounds

The methods employed for the purification of organic compounds are :

Filtration

It is based on the difference in filterability of different components of a solution. This method is used for the separation of insoluble solid components of the mixture from soluble components in a given solution, e.g. separation of sand from salt by dissolving in water.

Crystallisation

This method is used for the purification of organic compounds that are obtained in the pure form of their crystals. This technique is based upon the differences in the solubility of the compound and impurities in different solvents.

The common solvents used for dissolving organic compounds are water, alcohol, ether, benzene, petroleum and chloroform.

Sublimation

This technique is applied to those solids which convert directly into the gaseous state upon heating without converting them into liquid phase and the vapours give directly the pure solid substance.

Organic compounds like naphthalene, anthracene, camphor, benzoic acid, ammonium chloride, salicylic acid, iodine are purified through sublimation.

NOTE *A mixture of two or more sublimates cannot be separated by sublimation technique. Such mixtures are separated only by chemical methods.*

Distillation

It is based upon the difference in the boiling points of two liquids or two components and is generally used, if an organic liquid contains a non-volatile impurity.

Several distillation methods employed to separate mixtures are as follows :

- (i) **Simple distillation** Generally, the non-volatile impurities and organic liquids which differ in their boiling points by 30-50 K are separated by this process, e.g. a mixture of ether and toluene is separated by this method.
- (ii) **Fractional distillation** This technique is employed to those organic liquids which do not possess large difference in their boiling points. It is carried out in a fractionating column which increases the surface area for cooling the vapours of liquid.

e.g. Mixtures of acetone (329 K) and methyl alcohol (338 K), benzene (353 K) and toluene (383 K) are separated through this technique.

- (iii) **Steam distillation** This technique is applied to those compounds which are volatile in steam but insoluble in water and contain non-volatile impurities.

In this type of distillation, the mixture boils at that temperature, when the sum of vapour pressures of organic liquid (p_1) and that of steam (p_2) becomes equal to the atmospheric pressure (p).

Therefore, $p = p_1 + p_2$.

e.g. *ortho* and *para*-nitrophenols aniline, nitrobenzene, sandal wood oil and turpentine oil etc., are purified or obtained by this process.

- (iv) **Vacuum distillation** Certain liquids tend to decompose at a temperature below their boiling point. Such liquids are purified by carrying out their distillation under reduced pressure. Therefore, the liquid boils at lower temperature without undergoing decomposition.

e.g. The boiling point of glycerol is 563 K but it decomposes below this temperature. But when the pressure is reduced to 12 mm, glycerol boils at 453 K without decomposition.

Differential Extraction

In this technique, an aqueous solution of an organic compound, taken in a separating funnel is shaken with a suitable organic solvent in which organic compound is highly soluble. On shaking the funnel, organic layer separates out from an aqueous layer.

The common solvents used are chloroform, (CCl_4), ether, dichloromethane and ethanol. If the extraction is done in small amounts, the extraction will be more efficient and hence, larger amounts of organic compound are extracted.

Chromatography

This technique is used for the separation of an organic compound present in a mixture in very small amount.

Principle of Chromatography

This technique is based on the principle that the components of a given organic mixture is distributed between two phases : stationary and mobile phase.

The stationary phase may be a solid or tightly bound liquid supported over a solid whereas the mobile phase may be a liquid or gas.

The process of distribution depends upon the rate of differential movement of individual components through a stationary phase under the influence of a mobile phase (gas/liquid).

Types of Chromatography

Based upon the nature of stationary and mobile phase, various types of chromatography techniques along with their applications are listed below :

Type of chromatography	Mobile phase	Stationary phase	Application of chromatographic technique
Column (adsorption) chromatography	Liquid	Solid	Used for the purification of organic compounds for both commercial and small scale separations.
High performance liquid chromatography (HPLC)	Liquid	Solid	Used for the identification and characterisation of organic compounds for their quantitative and qualitative analysis.
Thin layer chromatography (TLC)	Liquid	Solid	Only for qualitative analysis.
Gas liquid chromatography (GLC)	Gas	Liquid	For both qualitative and quantitative analysis.
Paper chromatography	Liquid	Solid	For both quantitative and qualitative analysis of polar organic and inorganic compounds.

Estimation of Organic Compounds

Estimation of organic compounds are as follows :

Qualitative Analysis of Organic Compounds

This method involves the conversion of covalently bonded nitrogen, sulphur or halogens present in the organic compounds to corresponding water-soluble ions, in the form of sodium salts.

Detection of Carbon and Hydrogen

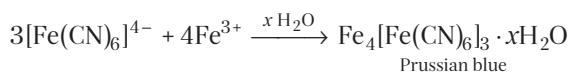
Organic compound when heated with CuO, carbon changes into CO₂ (tested with lime water which develops turbidity) and hydrogen to H₂O (tested with anhydrous copper sulphate which turns blue). This confirms the presence of C and H in organic compound.

Detection of Other Elements

Nitrogen, sulphur, halogens and phosphorus present in an organic compounds are detected by Lassaigne's test. The elements present in the organic compounds are converted from covalent form into the ionic form by fusing the compound with sodium metal.

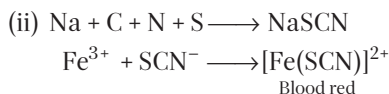
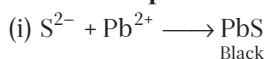


Test for Nitrogen



Lassaigne's test is given by compounds carrying nitrogen and carbon. So, hydrazine (NH₂—NH₂) and hydroxyl amine (NH₂OH) do not give this test.

Test for Sulphur



Test for Halogens

The sodium fusion extract is acidified with nitric acid and then treated with silver nitrate.



AgCl-white ppt, AgBr-dull yellow ppt, AgI-bright yellow ppt

Test for Phosphorus

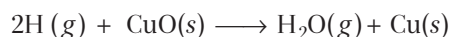
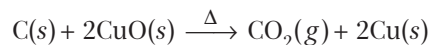
The compound is heated with an oxidising agent (like sodium peroxide), the phosphorus present in the compound is oxidised to phosphate. The solution is boiled with nitric acid and then treated with ammonium molybdate. A yellow colouration or precipitate indicates the presence of phosphorus.

Quantitative Analysis of Elements

After the detection of elements present in the organic compound, the percentage of different constituent elements is determined by the help of suitable methods. The knowledge of molecular mass and percentage composition helps in the determination of the molecular formula of the compound.

Estimation of C and H in an Organic Compound

Liebig method is used to determine the presence of C and H in an organic compound. In this method, known weight of organic compound is heated strongly with dry CuO in an atmosphere of CO₂ free air or oxygen.



Vapours of CO₂ passed through a weighed potash bulb, which results in increased weight of potash by the absorption of CO₂.

While the vapours of H₂O passed through CaCl₂ which leads to increased weight of CaCl₂. The increased weight of potash and CaCl₂ is used to determine the percentage of C and H.

$$\text{(i) \% of C} = \frac{12 \times W_{\text{CO}_2} \times 100}{44 \times W_{\text{organic compound}}}$$

$$\text{(ii) \% of H} = \frac{2 \times W_{\text{H}_2\text{O}} \times 100}{18 \times W_{\text{organic compound}}}$$

Estimation of N in an Organic Compound

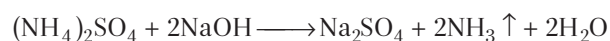
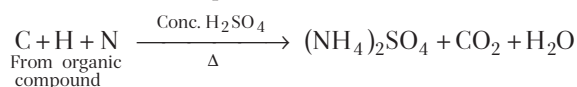
There are two methods used to determine the percentage of N in an organic compound :

(i) Duma's method In this method, a known weight of organic compound is heated with dry CuO in a combustion tube to form CO₂ and water vapour while N₂ is set free. Although, some oxides of nitrogen may be formed but these oxides can be reduced to N₂ when passed through hot reduced copper spiral.

The vapours of N₂, H₂O and CO₂ pass through KOH solution, where N₂ gets collected over KOH solution. While H₂O and CO₂ are absorbed by KOH. The volume of N₂ is recorded from the level of KOH solution in nitrometer tube.

$$\% \text{ of N} = \frac{28 \times V_{\text{N}_2} \text{ at STP} \times 100}{22400 \times W_{\text{organic compound}}}$$

(ii) Kjeldahl's method This method is used for the estimation of N present in food stuffs, soil fertilisers and drugs. In this method, weighed amount of nitrogenous compound is heated strongly with conc. H₂SO₄ in the presence of K₂SO₄ (which raises boiling point of solution), N is converted quantitatively into ammonium sulphate.



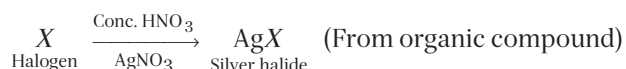
Ammonia evolved is condensed and absorbed by a known volume of standard solution of HCl or H₂SO₄.

The percentage of N present in the compound is calculated by

$$\% \text{ of N} = \frac{14}{1000} \times \frac{V_{\text{acid}} \times N_{\text{acid}}}{W_{\text{organic compound}}} \times 100$$

Estimation of Halogen Present in an Organic Compound

Carius method In this method, halogen atom present in the organic compound is converted into insoluble silver halide which is separated and weighed.



From the weight of the silver halide, the percentage of halogens can be calculated as,

$$\% \text{ of Cl} = \frac{35.5}{143.3} \times \frac{W_{\text{AgCl formed}}}{W_{\text{compound}}} \times 100$$

$$\% \text{ of Br} = \frac{80}{188} \times \frac{W_{\text{AgBr formed}}}{W_{\text{compound}}} \times 100$$

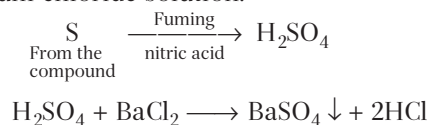
$$\% \text{ of I} = \frac{127}{235} \times \frac{W_{\text{AgI formed}}}{W_{\text{compound}}} \times 100$$

Estimation of Sulphur Present in an Organic Compound

The sulphur present in an organic compound is also estimated through Carius method.

In this method, a weighed amount of organic compound is heated with fuming nitric acid in a Carius tube.

The sulphur of the compound is oxidised to sulphuric acid which is then precipitated as barium sulphate by adding excess of barium chloride solution.



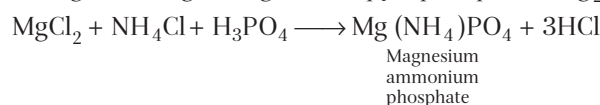
From the weight of barium sulphate obtained, the percentage of sulphur is calculated as

$$\% \text{ of S} = \frac{32}{233} \times \frac{W_{\text{BaSO}_4 \text{ formed}}}{W_{\text{compound}}} \times 100$$

Estimation of Phosphorus Present in an Organic Compound

The phosphorus present in an organic compound is estimated only by Carius method. The weighed amount of organic compound is heated with conc. HNO_3 to convert the phosphorus present in the compound into phosphoric acid. Phosphoric acid reacts with magnesia mixture to form magnesium ammonium phosphate, $\text{Mg}(\text{NH}_4)\text{PO}_4$.

The compound, $\text{Mg}(\text{NH}_4)\text{PO}_4$ is collected, washed, dried and ignited to get magnesium pyrophosphate, $\text{Mg}_2\text{P}_2\text{O}_7$.

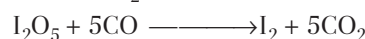
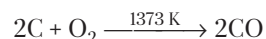
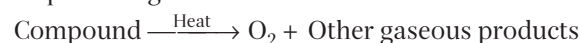


Knowing the weight of $\text{Mg}_2\text{P}_2\text{O}_7$, percentage of P can be calculated as

$$\% \text{ of P} = \frac{62}{222} \times \frac{W_{\text{Mg}_2\text{P}_2\text{O}_7 \text{ formed}}}{W_{\text{compound}}} \times 100$$

Estimation of Oxygen Present in an Organic Compound

Generally, oxygen is estimated as left over percentage, i.e. 100 minus percentage of other elements.



The percentage of oxygen can be derived from the amount of CO_2 or iodine produced.

$$\% \text{ of O} = \frac{32 \times m_1 \times 100}{44 \times m} \%$$

Practice Exercise

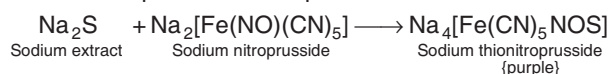
- Absolute alcohol is prepared by
 - fractional distillation
 - Kolbe's method
 - azeotropic distillation
 - vacuum distillation
- Aniline is purified by
 - steam distillation
 - simple distillation
 - vacuum distillation
 - extraction with a solvent
- Name the technique where extraction of compound takes place based on difference in solubility.
 - Differential extraction
 - Chromatography
 - Sublimation
 - Crystallisation
- The technique used for the separation of acetone and methanol, is
 - steam distillation
 - vacuum distillation
 - fractional distillation
 - simple distillation
- The best and latest technique for isolation, purification and separation of organic compound is
 - crystallisation
 - distillation
 - sublimation
 - chromatography
- The compound that does not give blue colour in Lassaigne's test is
 - aniline
 - glycine
 - hydrazine
 - urea
- When FeCl_3 is added to a (FeSO_4 + sodium extract) solution of compound, blood red colour appears. It shows the presence of
 - N and S in the compound
 - S in the compound
 - N and P in the compound
 - Br in the compound
- An organic compound on heating with CuO produces CO_2 but no water. The organic compound may be
 - carbon tetrachloride
 - chloroform
 - methane
 - ethyl iodide
- Which of the following complex formation indicates the presence of sulphur in the organic compound when sodium nitroprusside is added to sodium extract of the compound?
 - $\text{Fe}_4[\text{Fe}(\text{CN})_6]_3$
 - $\text{Na}_2[\text{Fe}(\text{NO})(\text{CN})_5]$
 - $\text{Fe}_4(\text{CNS})_3$
 - $\text{Na}_4[\text{Fe}(\text{CN})_5\text{NOS}]$
- In sulphur estimation, 0.157 g of an organic compound gives 0.4813 g of barium sulphate. The percentage of sulphur in the compound, is
 - 42.10
 - 52.10
 - 21.05
 - 40.01
- In Kjeldahl's method, ammonia from 5 g of food neutralises 30 cm^3 of 0.1 N acid. The percentage of nitrogen in the food is
 - 0.84
 - 0.4
 - 16.8
 - 1.68
- 0.765 g of an acid gives 0.535 g of CO_2 and 0.138 g of H_2O . Then, the ratio of the percentage of carbon and hydrogen is
 - 19 : 2
 - 18 : 11
 - 20 : 17
 - 1 : 7
- 0.59 g of an organic compound produces 112 mL nitrogen at NTP. The percentage of nitrogen in the compound through Duma's method is
 - 23.7
 - 27.5
 - 33.07
 - 16.8
- If 0.1 g of an organic compound containing phosphorus, produces 0.222 g of $\text{Mg}_2\text{P}_2\text{O}_7$, the percentage of phosphorus present in the compound is
 - 31
 - 0.2
 - 66
 - 62
- A compound of carbon, hydrogen and nitrogen contains three elements in the ratio of 9 : 1 : 3.5. The empirical formula of the compound is
 - $\text{C}_2\text{H}_4\text{N}$
 - $\text{C}_3\text{H}_4\text{N}$
 - $\text{C}_3\text{H}_6\text{N}$
 - $\text{C}_2\text{H}_6\text{N}$
- A substance is found to contain 7% nitrogen. The minimum molecular weight of it, is
 - 700
 - 100
 - 200
 - 70
- When FeCl_3 is added to the sodium extract of an organic compound containing both N and S, the red coloured precipitate is formed due to the formation of
 - $[\text{Fe}(\text{CN})_6]^{3-}$
 - $[\text{Fe}(\text{CNS})]^{2+}$
 - $[\text{Fe}(\text{CNS})_2]^+$
 - $[\text{Fe}(\text{CN})_6]^{4-}$

Answer with Solutions

- (c) Absolute alcohol (100% pure ethyl alcohol) is obtained from rectified spirit (95.87% ethanol + 4.13% water) by azeotropic distillation as it is an azeotropic mixture.
- (a) Aniline decomposes on its boiling point and it is also insoluble in water, hence it can be distilled with steam for purification.
- (a) Differential extraction is the technique where extraction of compound takes place based on difference in solubility.
- (c) The compounds acetone and methanol possess equal boiling points therefore, these are separated by fractional distillation technique.
- (d) Chromatography is the best and suitable technique used for the separation of organic compound from their impurities.
- (c) Hydrazine ($\text{NH}_2 \cdot \text{NH}_2$) has no carbon content, hence does not form NaCN in Lassaigne's test. Therefore, it does not give blue colour.
- (a) When N and S, both are present in compound, it forms NaCNS with sodium extract. This gives ferric thiocyanate with FeCl_3 solution, hence blood red colour develops.



- (a) Since, the compound on heating with CuO produces CO_2 which contains carbon. Again, it does not produce water, hence it does not contain hydrogen. So, the organic compound is carbon tetrachloride (CCl_4).
- (d) The formation of sodium thionitroprusside (purple) shows the presence of sulphur.



- (a) Molecular mass of $\text{BaSO}_4 = 137 + 32 + 64 = 233 \text{ g}$
233 g of BaSO_4 contains 32 g of sulphur.

$\therefore$ 0.4813 g of BaSO_4 will contain

$$= \frac{32 \times 0.4813}{233} \text{ g of sulphur}$$

$$\% \text{ of S} = \frac{32 \times 0.4813 \times 100}{233 \times 0.157} = 42.10$$

- (a) From Kjeldahl's method,

$$\% \text{ of N} = \frac{1.4 \times N \times V}{m} = \frac{1.4 \times 0.1 \times 30}{5} = 0.84$$

$$12. (a) \% \text{ of C} = \frac{12}{44} \times \frac{0.535}{0.765} \times 100 = 19.07$$

$$\% \text{ of H} = \frac{2}{18} \times \frac{0.138}{0.765} \times 100 = 2.004$$

Ratio of % of C : H = 19 : 2 (approx.)

- (a) % of N through Duma's method is given by

$$\% \text{ of N} = \frac{28 \times \text{Volume of N}_2 \text{ at STP}}{22400 \times \text{Mass of an organic compound}} \times 100$$

$$= \frac{28 \times 112 \times 100}{22400 \times 0.59} = 23.7$$

- (d) The percentage of P present in the organic compound is given by

$$\% \text{ of P} = \frac{62}{222} \times \frac{\text{Weight of Mg}_2\text{P}_2\text{O}_7}{\text{Weight of organic compound}} \times 100$$

$$= \frac{62}{222} \times \frac{0.222}{0.1} \times 100 = 62$$

- (b) The percentage (or mass) ratio = 9 : 1 : 3.5

$$\therefore \text{Mole ratio} = \frac{9}{12} : \frac{1}{1} : \frac{3.5}{14}$$

$$= 0.75 : 1 : 0.25$$

$$= 3 : 4 : 1$$

Hence, empirical formula = $\text{C}_3\text{H}_4\text{N}$

- (c) Atomic weight of nitrogen is 14. If one atom of nitrogen is present in compound, then it should have molecular weight = $\frac{14}{7} \times 100 = 200$

- (b) When FeCl_3 is added to the sodium extract of an organic compound containing both N and S, a red coloured precipitate of $[\text{Fe(CNS)}]^{2+}$ is formed.



23

Hydrocarbons

Concept of Hydrocarbons

Hydrocarbons means compounds containing carbon and hydrogen only. These may contain only single bonds or double/triple bonds along with single bond. Only single bond containing hydrocarbons are saturated hydrocarbons while that have multiple bond(s) ($=$ or $\equiv$ bond) along with single bonds are unsaturated compounds. Hydrocarbons are mainly obtained from petroleum, which is the major source of commercial energy.

Hydrocarbons are of different types and classified into three main categories :

- (i) Saturated hydrocarbons (alkanes)
- (ii) Unsaturated hydrocarbons (alkenes and alkynes)
- (iii) Aromatic hydrocarbons

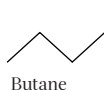
Alkanes

Introduction

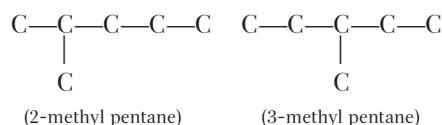
- $\Rightarrow$ Hydrocarbons having C—C single bond.
- $\Rightarrow$ General formula : C_nH_{2n+2}
- $\Rightarrow$ sp^3 -hybrid orbitals having tetrahedral geometry and bond angle $109^\circ 28'$.
- $\Rightarrow$ Also called paraffins (less reactive) as:
 - (a) strong C—C bond
 - (b) non-polar C—H bond

Isomerism

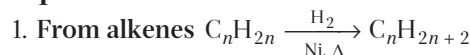
- (a) Chain isomerism



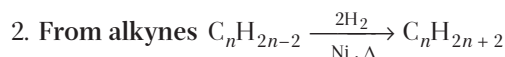
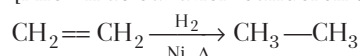
(b) Position isomerism



Preparation



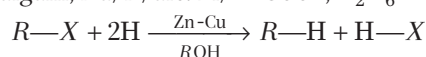
[known as Sabatier-Sanderen's reaction].



3. From alkyl halides

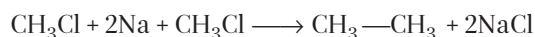
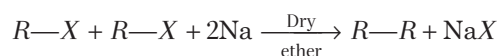
(a) By reducing alkyl halides

Reducing agents Zn - Cu couple LiAlH_4 , Al-Hg amalgam, Na/K/alc. Li/Ni 300° , B_2H_6

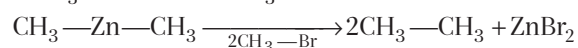
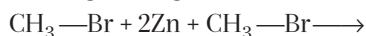


[I_2 is removed as $2\text{P} + 3\text{I}_2 \longrightarrow 2\text{PI}_3$]

(b) Wurtz reaction

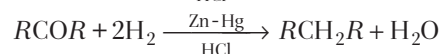
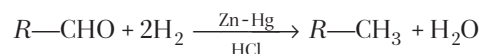
(c) Frankland's method $2\text{RX} + \text{Zn} \longrightarrow \text{R}-\text{R} + \text{ZnX}_2$

[Here, $\text{R}-\text{Zn}-\text{R}$ is formed which on reaction with $\text{R}-\text{X}$ gives higher alkane]

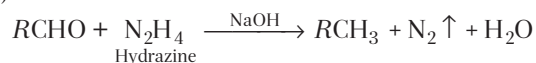


4. From aldehyde and ketone

(a) Clemmensen's reduction



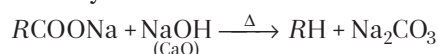
(b) Wolf Kishner reduction



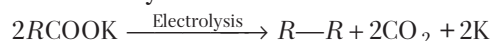
5. From carboxylic acid



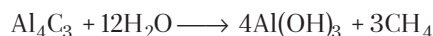
(b) Decarboxylation



(c) Kolbe's electrolysis

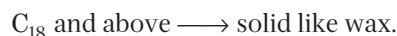
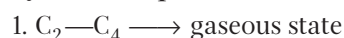


6. From aluminium carbide



[Only CH_4 is formed by this reaction].

Physical Properties



2. Alkanes are colourless, odourless and tasteless.

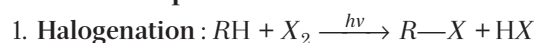
3. Insoluble in water and soluble in organic solvents.

4. The mp of unbranched alkanes having 2, 4, 6, ... (even) carbon atoms is higher than that of alkanes having 3, 5, 7, ... (odd) carbon atoms.

5. Alkanes float on water as they have lower density.

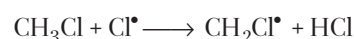
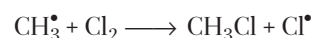
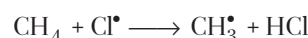
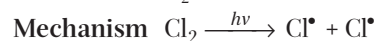
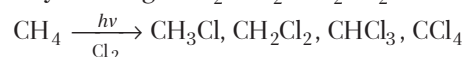
6. Decreasing order of fuel capacity is :
branched > unbranched lower mol. wt. > higher mol. wt.
As the percentage of C in a hydrocarbon increases, its calorific value (fuel capacity) decreases.

Chemical Properties of Alkanes



Reactivity order : $3^\circ > 2^\circ > 1^\circ$ [for C—H bond]

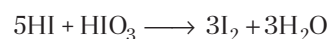
Reactivity of halogen : $\text{F}_2 > \text{Cl}_2 > \text{Br}_2 > \text{I}_2$



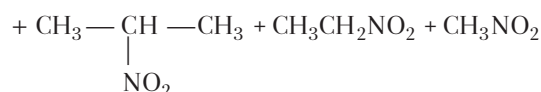
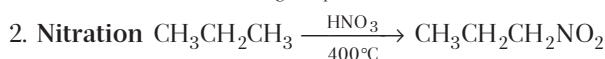
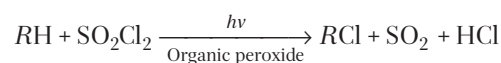
$\Rightarrow$ Iodination is reversible process



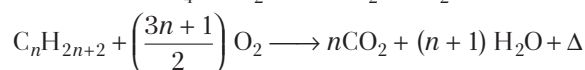
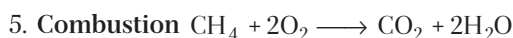
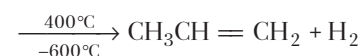
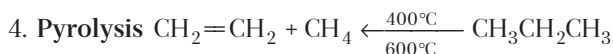
To avoid this HIO_3 is used which destroys HI on its formation.

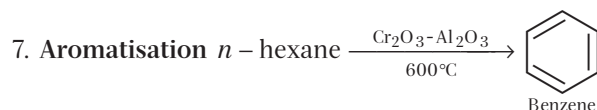
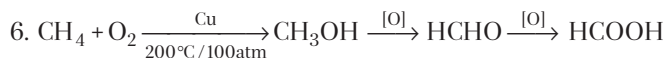


$\Rightarrow$ Chlorination by sulphuryl chloride



3. Sulphonation





Alkenes

Introduction

- ⇒ Hydrocarbons having C=C double bond.
- ⇒ General formula C_nH_{2n} ⇒ sp^2 -hybridisation.
- ⇒ Trigonal planar [(120°) bond angle].

Isomerism

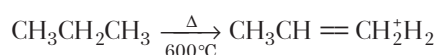
Chain 1-butene, 2-methyl propene

Position 1-butene, 2-butene

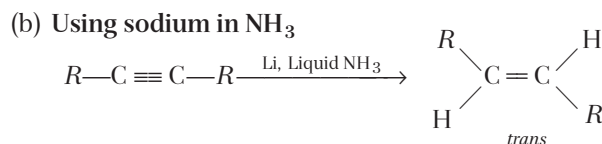
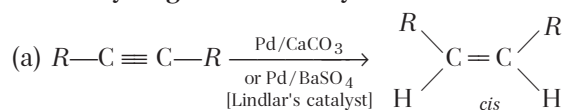
Functional 1-butene, 2-butene,
2-methyl propene, cyclobutane etc.

Geometrical *cis*-2 butene, *trans*-2 butene

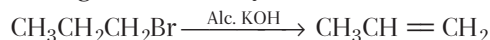
Preparation



2. Partial hydrogenation of alkynes

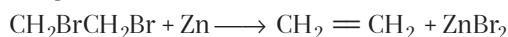


3. Dehydrohalogenation of alkyl halides



Ease of reaction: $3^\circ > 2^\circ > 1^\circ$ [alkyl halide]

By dehalogenation of *vicinal* halide.

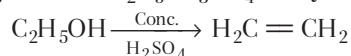


4. Dehydration of alcohols $\text{ROH} \xrightarrow{\text{Conc. H}_2\text{SO}_4} \text{Alkene}$

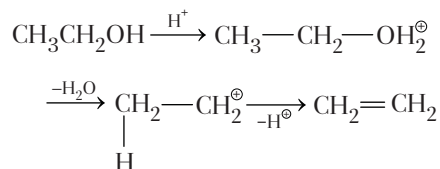
Ease of dehydrogenation: $3^\circ > 2^\circ > 1^\circ$

Dehydrating agents

Al_2O_3 [350°C], P_2O_5 , H_3PO_4 , anhyd. ZnCl_2 , conc. H_2SO_4 .

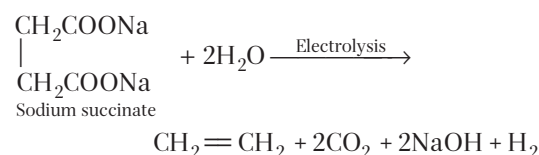


Mechanism



5. Wittig reaction

6. Electrolysis of aqueous solution of sodium salts of carboxylic acids

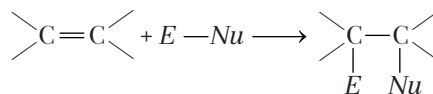


Physical Properties

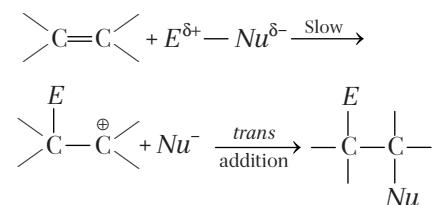
- $\text{C}_2-\text{C}_4 \rightarrow$ gases
 $\text{C}_5-\text{C}_{15} \rightarrow$ liquids $> \text{C}_{15} \rightarrow$ solids
- Insoluble in water, soluble in polar solvents.
- bp $\propto$ molecular weight
- More polar than corresponding alkanes.

Chemical Properties of Alkenes

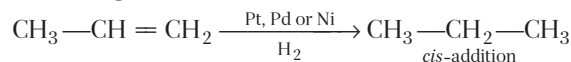
1. Electrophilic addition reactions



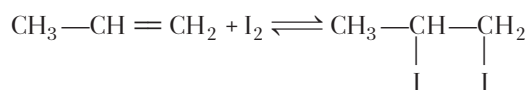
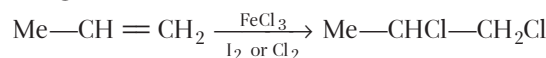
Mechanism



(a) Hydrogenation

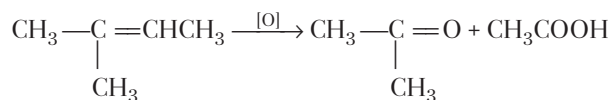
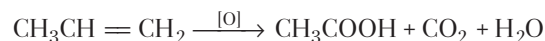


(b) Halogenation



The diagram illustrates the electrophilic addition of bromine to an alkene through three stages:

- Alkene + Br₂ → π-Complex:** An alkene (represented by a C=C double bond with four generic bonds) reacts with a bromine molecule (Br-Br). An arrow points from the C=C double bond to the Br-Br bond, indicating the formation of a π-complex.
- π-Complex → σ-Complex:** The π-complex rearranges to form a σ-complex. In this intermediate, one carbon is bonded to a bromine atom (Br) and the other carbon carries a positive charge (C⁺). A bromide ion (Br⁻) is shown nearby, indicating the heterolytic cleavage of the Br-Br bond.
- σ-Complex + Br⁻ → Dibromide:** The σ-complex reacts with the bromide ion (Br⁻). An arrow points from a lone pair on the bromide ion to the positively charged carbon. The final product is a dibromide, where both carbons are bonded to bromine atoms (Br).

$$\text{CH}_3-\text{CH}=\text{CH}_2 \xrightarrow[\text{Markownikoff's addition}]{\text{HX}} \text{CH}_3-\underset{\text{X}}{\text{CH}}-\text{CH}_3$$
$$\text{CH}_3-\text{CH}=\text{CH}_2 \xrightarrow[\text{Peroxide}]{\text{HBr}} \text{CH}_3-\text{CH}_2-\text{CH}_2-\text{Br}$$
$$\text{CH}_3-\text{CH}=\text{CH}_2 \xrightarrow{\text{H}\ddot{\text{O}}\text{Cl}^+} \text{CH}_3-\underset{\text{OH}}{\text{CH}}-\text{CH}_2\text{Cl}$$
$$\text{CH}_3-\text{CH}=\text{CH}_2 \xrightarrow[\text{H}_2\text{O}]{\text{Dil. H}_2\text{SO}_4} (\text{CH}_3)_2\text{CHOH}$$
$$\text{CH}_3-\text{CH}=\text{CH}_2 \xrightarrow{\text{O}_3} \text{Ozonide} \xrightarrow[\text{Zn}]{\text{H}_2\text{O}} \text{CH}_3\text{CHO} + \text{HCHO}$$
$$\begin{array}{c} \text{CH}_3-\text{CH}=\text{CH}_2 \xrightarrow[\text{H}_2\text{O}^+]{\text{B}_2\text{H}_6} \\ \text{(CH}_3-\text{CH}_2-\text{CH}_2)_3\text{B} \\ \swarrow \text{H}_3\text{O}^+ \quad \searrow \text{OH}^-/\text{H}_2\text{O}_2 \\ \text{CH}_3-\text{CH}_2-\text{CH}_3 \quad \text{H}_3\text{C}-\text{CH}_2-\text{CH}_2-\text{OH} \end{array}$$
$$\text{CH}_3-\text{CH}=\text{CH}_2 \xrightarrow{\text{ON}^{\delta+}-\text{Cl}^{\delta-}} \text{CH}_3-\underset{\text{Cl}}{\underset{|}{\text{CH}}}-\underset{\text{NO}}{\underset{|}{\text{CH}_2}}$$
$$\text{CH}_3-\text{CH}=\text{CH}_2 \xrightarrow[\text{THF}]{\text{Hg}(\text{OCOCH}_3)_2} \begin{array}{c} \text{CH}_3-\text{CH}-\text{CH}_2\text{HgOCOCH}_3 \\ | \\ \text{OCOCH}_3 \end{array}$$
$$\begin{array}{ccc} \text{Demercuration} & & \\ \text{CH}_3-\text{CH}-\text{CH}_2\text{HgOCOCH}_3 & \xrightarrow[\text{NaOH}]{\text{NaBH}_4} & \\ | & & \\ \text{OCOCH}_3 & & \\ & & \text{CH}_3-\text{CH}-\text{CH}_3 + \text{Hg} \\ & & | \\ & & \text{OH} \end{array}$$
$$\begin{array}{ccc} \diagup \text{C}=\text{C} \diagdown & \xrightarrow[\text{OH}^- \text{ cold}]{\text{KMnO}_4} & \begin{array}{c} | \quad | \\ -\text{C}-\text{C}- \\ | \quad | \\ \text{OH} \quad \text{OH} \end{array} \\ & & \downarrow \\ & & \begin{array}{c} | \quad | \\ -\text{C}-\text{C}- \\ | \quad | \\ \text{OH} \quad \text{OH} \end{array} \\ \text{(i) OsO}_4, \text{ pyridine} & \xrightarrow{\quad} & \\ \text{(ii) NaHSO}_3 / \text{H}_2\text{O} & & \end{array}$$
$$\text{CH}_2=\text{CH}_2 \xrightarrow{[\text{O}]} 2\text{CO}_2 + \text{H}_2\text{O}$$


Introduction

- Hydrocarbons having $\text{C} \equiv \text{C}$ triple bond.
- General formula $\text{C}_n\text{H}_{2n-2}$
- sp -hybridisation [$(180)^\circ$ bond angle]
- **Isomerism**

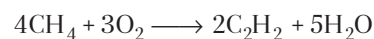
Chain 1-pentyne, 3-methyl-1-butyne.

Position 1-pentyne, 2-pentyne

Functional 1-3 butadiene, 1-2 butadiene, cyclobutene, 1-butyne.

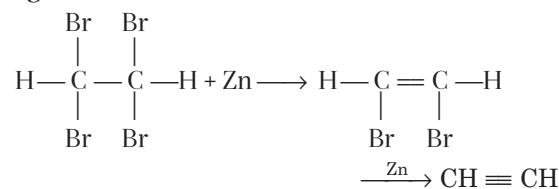
Preparation

- ### 1. From alkanes: Partial oxidation of methane

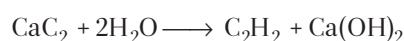
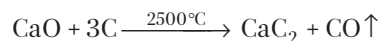


- $$\text{2. Synthesis: } 2\text{C} + \text{H}_2 \xrightarrow[\text{arc. (1100}^\circ\text{)}]{\text{Electric}} \text{HC} \equiv \text{CH}$$
- Ethyne

- ### 3. Dehalogenation of tetrahalides



- #### 4. From calcium carbide



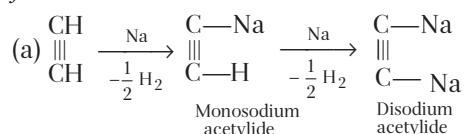
Physical Properties

1. $C_2-C_4 \rightarrow$ gases
 $C_5-C_{14} \rightarrow$ liquids $> C_{14} \rightarrow$ solids.
2. Slightly soluble in water.
3. mp/bp $\propto$ molecular weight.

Chemical Properties of Alkynes

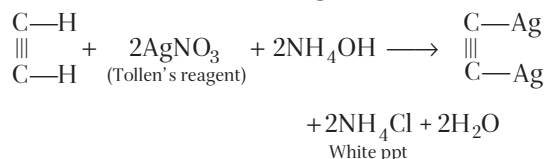
1. **Acidic characters of alkynes** The hydrogen atoms linked to triple bonded carbon atoms is highly acidic in nature.

The reactions involving highly acidic H-atoms are as follows:

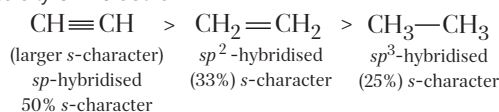


Acetylene also reacts with sodamide to form mono and disodium acetylide and ammonia.

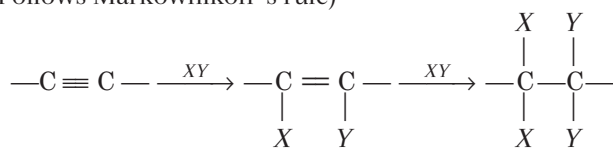
- (b) **Reaction with Tollen's reagent**



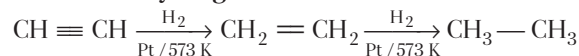
NOTE Greater the *s*-character present in a molecule higher is the acidity of molecule.



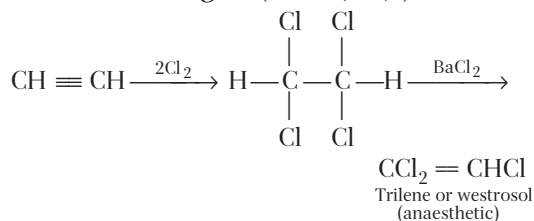
2. **Electrophilic addition reaction**
(Follows Markownikoff's rule)



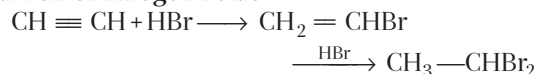
- (a) **Addition of hydrogen**



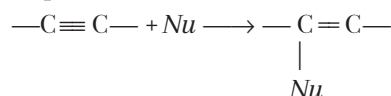
- (b) **Addition of halogens** ($X = \text{Cl}, \text{Br}, \text{I}$)



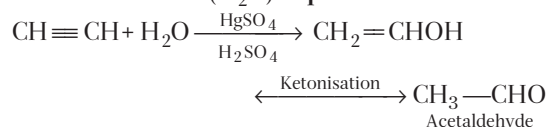
- (c) **Addition of halogen acids**



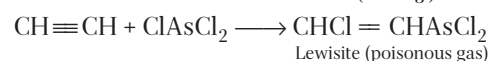
3. **Nucleophilic addition reaction**



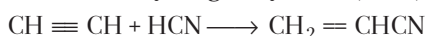
- (a) **Addition of water (H_2O) in presence of acids**



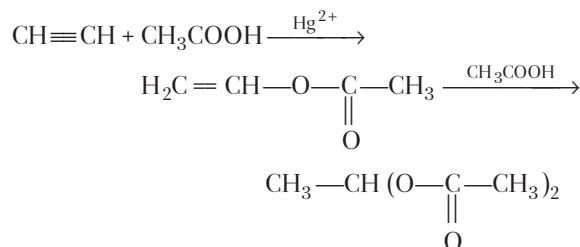
- (b) **Addition of arsenic trichloride (AsCl_3)**



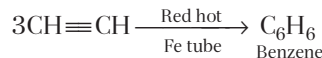
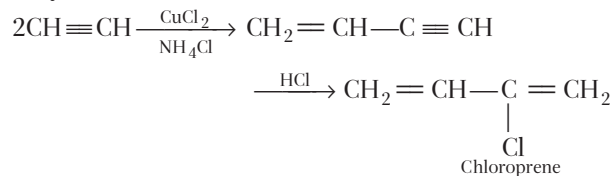
- (c) **Addition of hydrogen cyanide (HCN)**



- (d) **Addition of acetic acid**

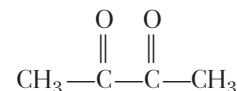
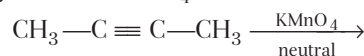


4. **Polymerisation**

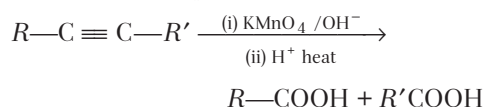


5. **Oxidation of alkynes**

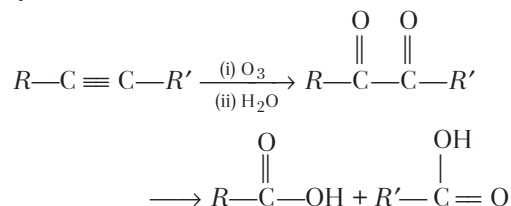
- (a) **By neutral KMnO_4**



- (b) **By acidic KMnO_4 (hot)**



(c) By ozone



Petroleum

The term petroleum originates from the Latin word (Petra-rock, Oleum-oil) meaning rock oil, mineral oil or earth oil. Petroleum is a thick viscous fluorescent liquid with dark green, brown or yellow colours. It was discovered by Colonel Drake in 1859.

Composition

It mostly consists of alkanes (C_1 to C_{40}) cycloparaffins, cycloalkanes and aromatic hydrocarbons. Some organic compounds containing N, S and O are also present.

Cracking

The process of getting lower hydrocarbons by heating hydrocarbons of higher boiling point. Cracking can be carried out in liquid or vapour phase.

Knocking

Knocking is the metallic sound produced due to irregular burning of the fuels and it depends upon the quality of the fuel which can be measured in terms of octane number.

Octane Number

The octane number of any sample is determined by matching its knocking property with a mixture of *iso*-octane and heptane in an experimental engine, e.g. octane number of triptane is 124.

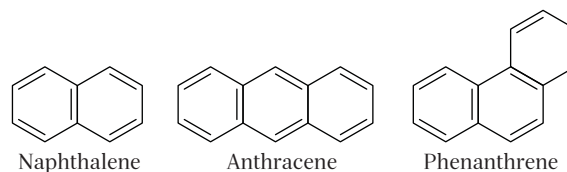
Octane number is the percentage (by volume) of *iso*-octane present in a mixture of *iso*-octane and heptane which has the same knocking performance as the sample fuel.

Cetane Number

The efficiency of diesel oil is expressed in terms of cetane number. Cetane (hexadecane) is assigned cetane number 100 and α -methyl naphthalene is assigned zero. A diesel oil having cetane number 75 would have the same ignition property as a mixture; of 75% cetane and 25% α -methyl naphthalene.

Aromatic Hydrocarbons

The parent compound of aromatic hydrocarbons, is benzene (C_6H_6). Some homologues of benzene are:



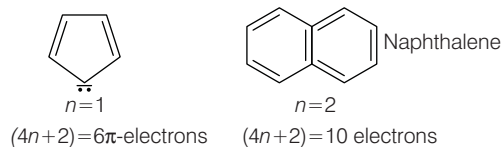
The chief source of aromatic compounds, is still the coaltar, which is a residual component of destructive distillation of coal. The coaltar is further fractionally distilled, when various compounds are obtained at different range of temperature, as shown below in table:

Temperature range	Name of fraction	Major compounds
80°–170°C	Light oil	Benzene, toluene, xylenes etc.
170°–230°C	Middle oil	Phenol, cresol, naphthalene, pyridine etc.
230°–270°C	Heavy oil	Phenols, naphthol etc.
270°–360°C	Green oil or anthracene oil	Anthracene, phenanthrene
—	Pitch (left as residue)	Carbon

Aromaticity

Ring systems whether having benzene ring or not with the following characteristics are said to be aromatic.

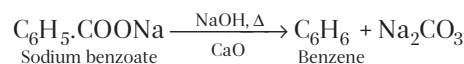
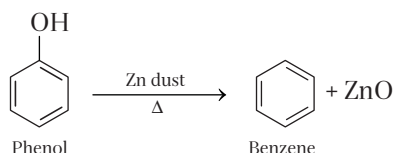
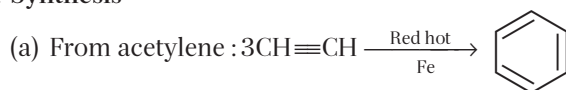
1. Planarity
2. Complete delocalisation of the π - electrons in the ring
3. Presence of $(4n + 2)$ π -electrons in the ring, where n is an integer ($n = 0, 1, 2$). This is known as **Huckel's rule**.



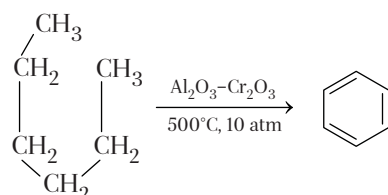
Certain compounds such as pyrrole, furan etc., although do not contain benzene ring, but behaves like benzene. These compounds are called non-benzenoid aromatic compounds.

Benzene

- Discoverer – M. Faraday
- Structure established by – Kekule
- Hybridisation of each carbon – sp^2

Preparation**1. From sodium benzoate****2. From phenol****3. Synthesis**

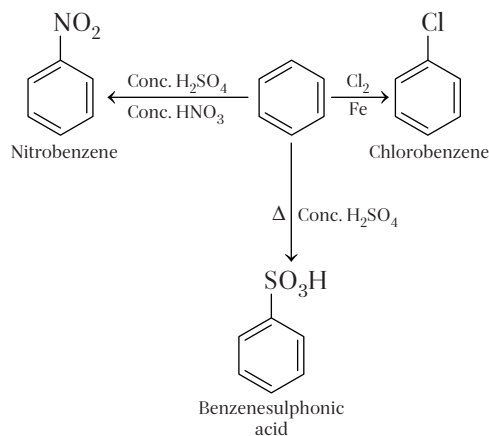
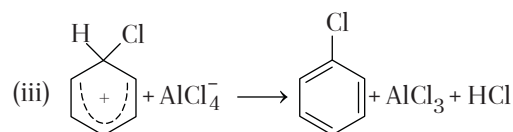
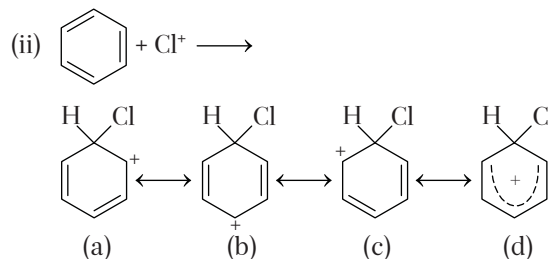
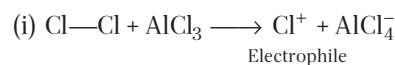
(b) From *n*-hexane

**Physical Properties**

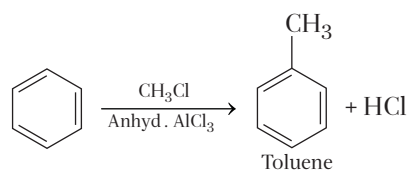
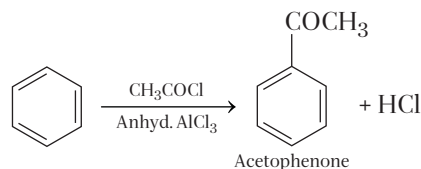
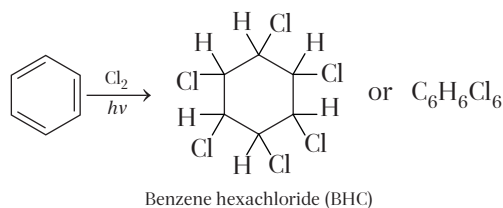
Benzene is a colourless, volatile liquid having characteristic odour, insoluble in water. Boiling point = 80.4°C

Chemical Properties

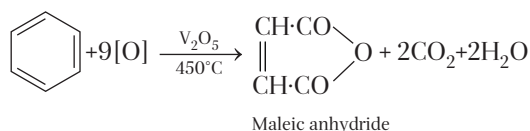
- 1. Electrophilic substitution** Benzene, despite being highly unsaturated compound, gives electrophilic substitution reactions, e.g. halogenation, sulphonation, nitration, Friedel-Crafts reactions etc.

**Mechanism**

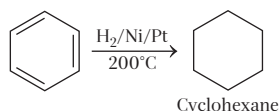
Electrophile in nitration is nitronium ion (NO_2^+) and in sulphonation, it is SO_3H^+ ion.

Friedel-Crafts alkylation**Friedel-Crafts acylation****2. Addition of Chlorine**

Commercially, it is known as gammaxene and is an important insecticide.

3. Oxidation

4. Reduction



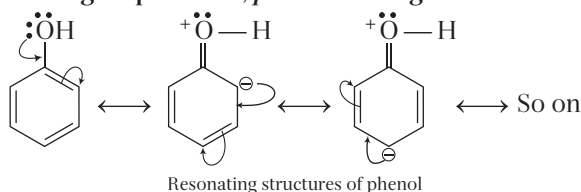
Carcinogenicity and Toxicity

Benzene and polynuclear hydrocarbons containing more than two benzene rings fused together, are toxic and said to possess cancer producing (carcinogenic) property. Such polynuclear hydrocarbons are formed on incomplete combustion of organic material like tobacco, coal and petroleum.

Directive Influence of Functional Group in Monosubstituted Benzene

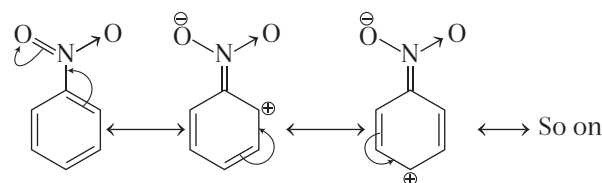
The substituent already present in the benzene ring directs the incoming group either to *ortho* and *para* or to *meta*-position. *o, p*-directing groups increase the electron density on the ring and thus, increase the reaction rate while *m*-directing groups decrease the electron density on the benzene ring and thus, decrease the reaction rate.

e.g.

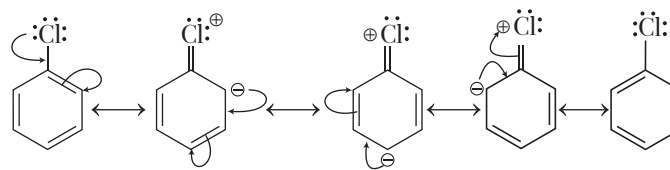
1. —OH group is *ortho, para*-directing.

In case of phenol, electron density is more concentrated on the two *o* and *p*-positions. Therefore, substitution is mainly in these positions.

—NH₂, —NHR, —NHCOCH₃, —OCH₃, —CH₃, —C₂H₅ are *o* and *p*-directing groups.

2. —NO₂ group is *meta*-directing.

- —NO₂, —CN, —CHO, —COR, —COOH, —COOR, —SO₃H are *meta*-directing groups.
- In case of haloarenes, halogens are highly deactivating because of $-I$ -effect but due to resonance, electron density on *o* and *p*-position increases, hence halogens are *o, p*-directing.



Practice Exercise

- Chlorination of *n*-butane gives the product
 - only 2-chlorobutane
 - only 1-chlorobutane
 - mixture of *s*-butyl chloride (excess) + *n*-butyl chloride
 - n*-butyl chloride and isobutyl chloride
- Consider the following reaction,

$$\text{CH}_3-\text{CH}_2-\text{CH}_2-\text{CH}_3 \xrightarrow{\text{AlCl}_3/\text{HCl}} \text{product}$$
 The reaction is named as
 - insertion
 - cracking
 - inversion
 - isomerisation
- The concentrated aqueous solution of potassium salts of acetic acid and propanoic acid are electrolysed. Which of the following hydrocarbons is/are produced?
 - Only CH₃CH₂CH₃
 - CH₃CH₃ and CH₃CH₂CH₂CH₃
 - CH₃CH₂CH₂CH₃, CH₃CH₂CH₃ and CH₃CH₃
 - Only CH₃CH₃
- A mixture of ethyl bromide and methyl bromide is subjected to Wurtz reaction. The mixture of alkanes so formed, consists of
 - propane and butane
 - ethane and propane
 - ethane, propane and butane
 - ethane and butane
- Pure methane can be prepared by
 - Wurtz reaction
 - Kolbe's electrolytic method
 - soda lime decarboxylation
 - reduction with H₂
- The reaction,

$$\text{C}_6\text{H}_5\text{Br} + 2\text{Na} + \text{BrCH}_3 \longrightarrow \text{C}_6\text{H}_5\cdot\text{CH}_3 + 2\text{NaBr}$$
 is known as
 - Wurtz reaction
 - Wurtz-Fittig reaction
 - Friedel-Crafts reaction
 - Berthelot synthesis

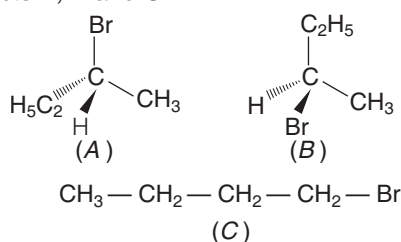
- $$\begin{aligned} \text{I. } \text{CH}_4 + \text{Cl}^\bullet &\xrightarrow{h\nu} \dot{\text{C}}\text{H}_3 + \text{H} - \text{Cl} \\ \text{II. } \text{Cl} - \text{Cl} &\xrightarrow{h\nu} 2\dot{\text{Cl}} \\ \text{III. } \dot{\text{C}}\text{H}_3 + \text{Cl}_2 &\longrightarrow \text{CH}_3 - \text{Cl} + \text{Cl}^\bullet \\ \text{IV. } \dot{\text{Cl}} + \dot{\text{Cl}} &\longrightarrow \text{Cl} - \text{Cl} \\ \dot{\text{C}}\text{H}_3 + \dot{\text{C}}\text{H}_3 &\longrightarrow \text{CH}_3 - \text{CH}_3 \end{aligned}$$

a. II, I, III and IV
b. I, II, III and IV
c. IV, III, II and I
d. II, III, I and IV

- (ii) $(\text{CH}_3)_3\text{CH} \xrightarrow{B} (\text{CH}_3)_3\text{COH}$
Here, A and B respectively are

- a. Zn(Hg)/HCl (conc.) and KMnO₄
 b. KMnO₄ and Zn(Hg)/conc. HCl
 c. H₂ / Ni and KMnO₄
 d. CH₃OH and C₂H₅OH
15. When 2-methyl butane-1-ol is dehydrated to give an alkene, the preferred product is
 a. 2-methyl but-1-ene b. but-1-ene
 c. 2-methyl but-2-ene d. but-2-ene
16. A cylinder of compressed gas that bears no label is supposed to contain either ethane or ethene. Combustion of the sample shows that 16 cm³ of the gas require 48 cm³ of oxygen for complete combustion. This shows that the gas is
 a. only ethane
 b. only ethene
 c. 1 : 1 mixture of two gases
 d. some unknown mixtures of the two gases
17. Cold and dil. KMnO₄ reacts with but-2-ene to form
 a. ethane-1, 2-diol b. butane-1, 4-diol
 c. butane-1, 3-diol d. butane-2, 3-diol
18. The conversion of ClCH=CH—Cl to Cl₂CH—CHCl₂ can be carried out with
 a. Cl₂ b. Cl₂ / hν
 c. Cl₂ / AlCl₃ d. Cl₂ / aq. NaOH
19. *iso*-butyl magnesium bromide with dry ether and ethyl alcohol gives
 a. $\text{CH}_3\underset{\begin{array}{c} | \\ \text{CH}_3 \end{array}}{\text{CH}}\text{CH}_2\text{OH}$ and $\text{CH}_3\text{CH}_2\text{MgBr}$
 b. $\text{CH}_3\underset{\begin{array}{c} | \\ \text{CH}_3 \end{array}}{\text{CH}}\text{CH}_3$ and $\text{MgBr(OC}_2\text{H}_5)$
 c. $\text{CH}_3\underset{\begin{array}{c} | \\ \text{CH}_3 \end{array}}{\text{CH}}\text{CH}=\text{CH}_2$ and Mg(OH)Br
 d. $\text{CH}_3\underset{\begin{array}{c} | \\ \text{CH}_3 \end{array}}{\text{CH}}\text{CH}_3$ and $\text{CH}_3\text{CH}_2\text{OMgBr}$
20. Which of the following will not show geometrical isomerism?
 a. $\begin{array}{ccc} \text{F} & & \text{H} \\ & \diagdown \quad \diagup & \\ & \text{C}=\text{C} & \\ & \diagup \quad \diagdown & \\ \text{Cl} & & \text{D} \end{array}$
 b. $\begin{array}{ccc} \text{F} & & \text{F} \\ & \diagdown \quad \diagup & \\ & \text{C}=\text{C} & \\ & \diagup \quad \diagdown & \\ \text{Cl} & & \text{Cl} \end{array}$
 c. $\begin{array}{ccccc} \text{H}_3\text{C} & & & & \text{C}_2\text{H}_5 \\ & \diagdown & & \diagup & \\ & \text{C}=\text{C} & & & \\ & \diagup & & \diagdown & \\ \text{H}_2\text{C}_2 & & & & \text{CH}_3 \end{array}$
 d. $\begin{array}{ccccc} \text{CH}_3 & & & & \text{CH}_3 \\ & \diagdown & & \diagup & \\ & \text{C}=\text{C} & & & \\ & \diagup & & \diagdown & \\ \text{H}_3\text{C} & & & & \text{C}_2\text{H}_5 \end{array}$
21. Anti-Markownikoff's addition of HBr is not observed in
 a. propene
 b. butene
 c. but-2-ene
 d. pent-2-ene

22. A compound *X* when passed through dil. H_2SO_4 containing HgSO_4 gives a compound *Y* which on reaction with HI and red phosphorus gives C_2H_6 . The compound *X* is
 a. ethene b. ethyne c. 2-butene d. 2-butyne
23. The addition of halogen to an alkene involves the formation of
 a. carbocation as the intermediate
 b. carbanion as the intermediate
 c. free radical as the intermediate
 d. halonium ion as the intermediate
24. The addition of HBr to 1-butene gives a mixture of products *A*, *B* and *C*

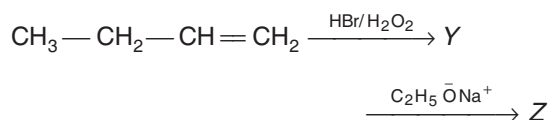


The mixture consists of

- a. *A* and *B* as major and *C* as minor products
 b. *B* as major, *A* and *C* as minor products
 c. *B* as minor, *A* and *C* as major products
 d. *A* and *B* as minor and *C* as major products
25. The principle organic product formed in the reaction.

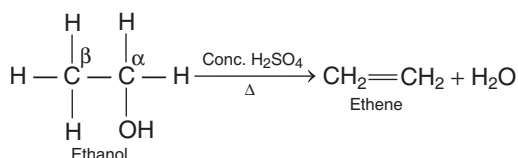


- a. $\text{CH}_3 - \text{CHBr}(\text{CH}_2)_8\text{COOH}$
 b. $\text{CH}_2 = \text{CH}(\text{CH}_2)_8\text{COBr}$
 c. $\text{CH}_2\text{BrCH}_2(\text{CH}_2)_8\text{COOH}$
 d. $\text{CH}_2 = \text{CH}(\text{CH}_2)_7\text{CHBrCOOH}$
26. Identify of *Z* in the sequence



- a. $\text{CH}_3 - \underset{\text{CH}_3}{\text{CH}} - \text{CH}_2 - \text{O} - \text{CH}_2\text{CH}_3$
 b. $\text{CH}_3\text{CH}_2 - \underset{\text{CH}_3}{\text{CH}} - \text{O} - \text{CH}_2\text{CH}_3$
 c. $\text{CH}_3 - (\text{CH}_2)_3 - \text{O} - \text{CH}_2 - \text{CH}_3$
 d. $\text{CH}_3 - (\text{CH}_2)_4 - \text{O} - \text{CH}_3$

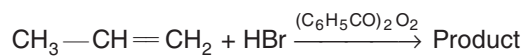
27. The following reaction is an example of



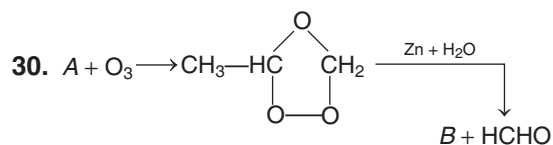
- a. oxidation reaction
 b. reduction reaction
 c. α -elimination reaction
 d. β -elimination reaction

28. According to Markownikoff's rule, the major product formed by addition of HBr with propene is
 a. 1-bromopropane
 b. 1-bromo, 1-methyl ethane
 c. 2-bromopropane
 d. 2,2-dibromopropane

29. What will be the product of the following reaction?

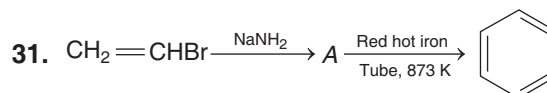


- a. $\text{CH}_3\text{CH}_2\text{CH}_2\text{Br}$ b. $\text{CH}_3\text{CH}(\text{Br})\text{CH}_3$
 c. $\text{CH}_3\text{CBr}_2\text{CH}_3$ d. $\text{CH}_3 - \underset{\text{H}}{\text{C}} = \text{CH}_2$



Here, *A* and *B* respectively are

- a. propene and methanal b. propene and ethanal
 c. propene and ethanol d. propene and ethanal



Here, *A* is

- a. $\text{CH} \equiv \text{CH}$ b. $\text{CH}_3 - \text{C} \equiv \text{CH}$
 c. $\text{CH}_3 - \text{CH}_3$ d. $\text{CH} \equiv \text{C} - \text{NH}_2$

32. Which one of the following compounds will react with two mole of CH_3MgBr ?
 a. CH_3COOH b. $\text{CH}_3 - \text{C} = \text{C} - \text{CH}_3$
 c. $\text{HC} \equiv \text{C} - \text{CH}_2\text{OH}$ d. $\text{CH}_3\text{CH}_2\text{CH}_2\text{OH}$

33. An organic compound on treatment with Br_2/CCl_4 , gives a bromoderivative alkene. The compound will be

- a. $\text{CH}_3\text{CH} = \text{CH}_2$ b. $\text{CH}_3\text{CH} = \text{CHCH}_3$
 c. $\text{HC} \equiv \text{CH}$ d. $\text{CH}_2 = \text{CH}_2$

34. Which of the following shows less reactivity towards Br_2 ?

- a. $\text{CH}_3\text{CH}_2\text{CH} = \text{CH}_2$ b. $\text{CH}_3\text{CH} = \text{CH}_2$
 c. $\text{CH}_3 - \text{C} \equiv \text{C} - \text{CH}_3$ d. $\text{CH} \equiv \text{CH}$

35. Which one of the following does not dissolve in the conc. H_2SO_4 ?

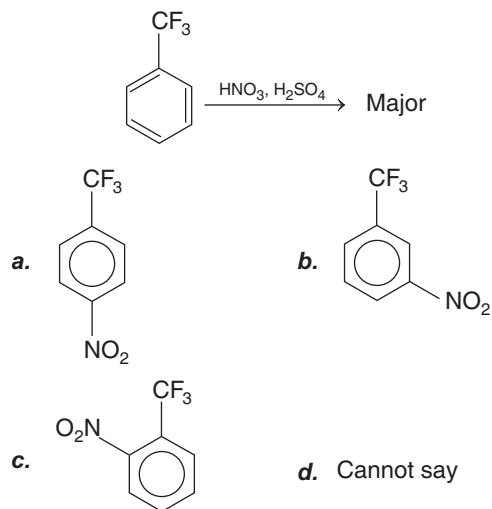
- a. $\text{CH}_2 = \text{CH}_2$ b. $\text{CH} - \text{C} \equiv \text{C} - \text{CH}_3$
 c. $\text{CH}_3\text{CH}_2\text{C} \equiv \text{CH}$ d. $\text{CH} \equiv \text{CH}$

36. *n*-propyl bromide on treatment with ethanolic potassium hydroxide produces

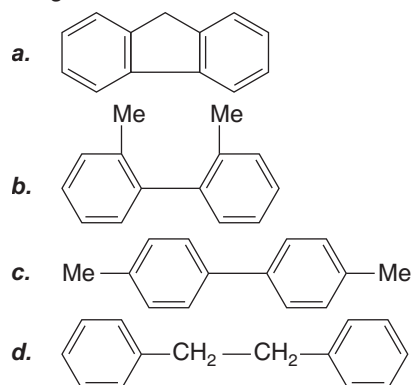
- a. propanol-1 b. propene
 c. propanol-2 d. ethyl propyl ether

37. When an alkyne $RC\equiv CH$ is treated with cuprous ion in an ammoniacal medium one of the products is
 a. $RC\equiv CCu$ b. $CuC\equiv CH$
 c. $CuC\equiv CCu$ d. $RC\equiv CR$
38. Ozonolysis of 2,3-dimethyl-1-butene followed by reduction with zinc and water gives
 a. methanoic acid and 3-methyl-2-butanone
 b. methanal and 2-methyl-2-butanone
 c. methanal and 3-methyl-2-butanone
 d. methanoic acid and 2-methyl-2-butanone
39. Which of the following is the predominant product in the reaction of $HOBr$ with propene?
 a. 2-bromo-1-propanol
 b. 3-bromo-1-propanol
 c. 2-bromo-2-propanol
 d. 1-bromo-2-propanol
40. When propyne is treated with aqueous H_2SO_4 in presence of $HgSO_4$, the major product is
 a. propanal
 b. *n*-propyl hydrogen sulphate
 c. acetone
 d. propanol
41. What is the best way to carry out the following transformation?
 1-pentyne $\longrightarrow$ pentanal
 a. $HgSO_4 / H_2SO_4$
 b. H_2 / Lindlar's catalyst, O_3 , $Zn-H_2O$
 c. HIO_4 / H_2O
 d. $BH_3, H_2O_2 / NaOH$
42. On vigorous oxidation by permanganate solution $(CH_3)_2C=CHCH_2CHO$ gives
 a. $(CH_3)_2CO$ and $OHCCH_2CHO$
 b. $(CH_3)_2C(OH)-CH(OH)-CH_2CHO$
 c. $(CH_3)_2CO$ and $OHCCH_2COOH$
 d. $(CH_3)_2CO$ and $CH_2(COOH)_2$
43. Acidic hydrogen is present in
 a. ethyne b. ethene
 c. benzene d. ethane
44. Which of the following order is correct regarding acidic character of hydrocarbons given below?
 I. $CH\equiv CH > CH_2=CH_2 > CH_3-CH_3$
 II. $HC\equiv CH > CH_3C\equiv CH >> CH_3C\equiv CCH_3$
 a. Only I b. Only II
 c. Both I and II d. None of these
45. The hydrocarbon, which can react with sodium in liquid ammonia is
 a. $CH_3CH_2CH_2C\equiv CCH_2CH_2CH_3$
 b. $CH_3CH_2C\equiv CH$
 c. $CH_3CH\equiv CHCH_3$
 d. $CH_3CH_2C\equiv CCH_2CH_3$

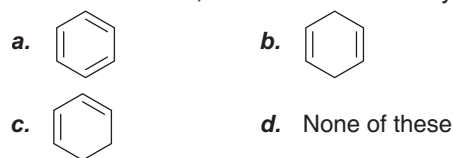
46. Which of the following is non-aromatic?
 a. Benzene b. Tropylium cation
 c. Cyclopentadienyl anion d. Cyclooctatetraene
47. Toluene, on oxidation with $KMnO_4$ gives
 a. benzaldehyde b. phenol
 c. nitrotoluene d. benzoic acid
48. Ozonolysis of benzene gives
 a. 8 moles of glyoxal b. glycol
 c. 6 moles of glyoxal d. 3 moles of glyoxal
49. Benzene was discovered by
 a. Faraday b. Berthelot c. Kekule d. Huckel
50. In nitration of benzene, the active species is
 a. NO_2^- b. ONO^- c. NO_2^+ d. NO_3^-
51. Give the major product of the following reaction.



52. $PhCH_3$ on reaction with $Cl_2 + h\nu$ followed by Na / ether will give



53. The reaction of 1,3-butadiene and acetylene gives



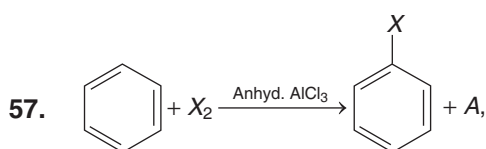
54. When nitrobenzene is treated with Br_2 in the presence of FeBr_3 , the major product formed is *m*-bromonitrobenzene. Statements which are related to obtain the *m*-isomer are
- the electron density on *meta*-carbon is less than that on *ortho* and *para*-positions
 - the intermediate carbonium ion formed after initial attack of Br^+ at the *meta*-position is least destabilised
 - loss of aromaticity when Br^+ attacks at the *ortho* and *para*-positions and not at *meta*-position
 - easier loss of H^+ to regain aromaticity from the *meta*-position than from *ortho* and *para*-positions

55. A hydrocarbon reacts with HI to give X which on reaction with aqueous KOH forms Y . Oxidation of Y gives 3-methyl, 2-butanone. The hydrocarbon is

- $\text{CH}_3\text{CH}=\overset{\text{CH}_3}{\text{C}}-\text{CH}_3$
- $\text{CH}_2=\text{CH}-\underset{\text{CH}_3}{\text{CH}}-\text{CH}_3$
- $\text{CH}_3-\text{CH}_2-\underset{\text{CH}_3}{\text{CH}}-\text{CH}_3$
- $\text{CH}\equiv\text{C}-\underset{\text{CH}_3}{\underset{\text{CH}_3}{\text{CH}}}-\text{CH}_3$

56. A Friedel-Crafts reaction of benzene with chloroform produces

- $\text{C}_6\text{H}_5\text{CHCl}_2$
- $\text{C}_6\text{H}_5-\overset{\text{Cl}}{\underset{\text{H}}{\text{C}}}-\text{C}_6\text{H}_5$
- $\text{C}_6\text{H}_5-\overset{\text{C}_6\text{H}_5}{\underset{\text{H}}{\text{C}}}-\text{C}_6\text{H}_5$
- All of these



Here, A is

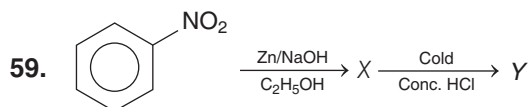
- H_2
- X^-
- HX
- Both b and c

58. Consider the following statements,

- The $-\text{OH}$ group present in the phenol is *ortho* and *para*-directing.
- Directive influence of a functional group in monosubstituted benzene depends on the nature of the substituent already present in the benzene ring.
- The $-\text{OH}$ group activates the benzene ring for the attack by an electrophile.
- Groups such as $-\text{NH}_2$, $-\text{NHR}$, $-\text{NHCOCH}_3$, $-\text{OCH}_3$, $-\text{CH}_3$, $-\text{C}_2\text{H}_5$, etc., are the examples of activating group.

Select the correct option.

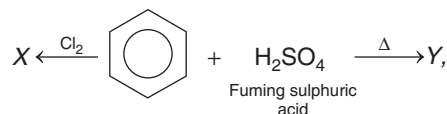
- I and II
- II and III
- II, III and IV
- I, II, III and IV



X and Y are

- $\text{C}_6\text{H}_5\text{NHOH}$, $\text{C}_5\text{H}_4(\text{OH})\text{NH}_2$
- $\text{C}_6\text{H}_5-\text{N}_2\text{H}_4-\text{C}_6\text{H}_5$, $\text{C}_6\text{H}_4-\overset{\text{Cl}}{\underset{\text{Cl}}{\text{N}}}\text{C}_6\text{H}_4-\text{NH}$
- Both (a) and (b)
- None of the above

60. In the reaction,



X and Y are respectively,

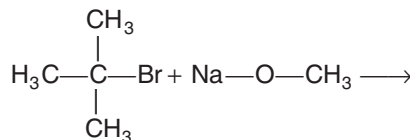
- $\text{C}_6\text{H}_5\text{Cl}$ and $\text{C}_6\text{H}_5\text{SO}_3\text{H}$
- $\text{C}_6\text{H}_5\text{Cl}$ and $\text{C}_6\text{H}_5\text{OH}$
- $\text{C}_6\text{H}_4\text{Cl}_2$ and $\text{C}_6\text{H}_5\text{SO}_2$
- $\text{C}_6\text{H}_5\text{Cl}$ and $\text{C}_6\text{H}_5\text{CHO}$

61. Which of the following organic materials damage DNA of our body?

- Tobacco
- Coal
- Petroleum
- All of the above

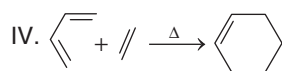
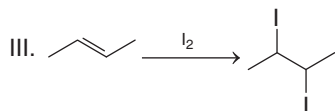
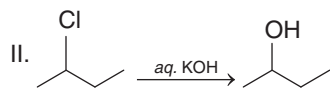
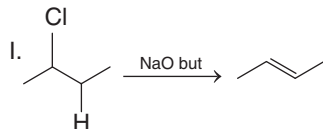
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1. What will be the product of the reaction? [2014]



- a. $\text{CH}_3-\text{C}=\text{CH}_2$
 |
 CH_3
- b. $\text{CH}_3-\text{O}-\text{C}-\text{CH}_3$
 |
 CH_3
- c. $\text{CH}_3-\text{CH}_2-\text{CH}_2-\text{CH}_3$
- d. $\text{CH}_3-\text{CH}-\text{CH}_3$
 |
 CH_2

2. The types of the reactions for these are [2014]

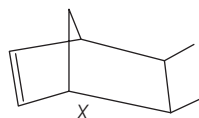


- a.** elimination, substitution, addition, addition
b. addition, elimination, addition, substitution
c. elimination, addition, substitution, addition
d. substitution, elimination, addition, addition

3. Which of the following reaction produces most stable alkene? [2013]

- a.** 2-chloro butane
b. 2, 3-dichloro butane
c. 2, 2-dichloro butane
d. 2, 3-dichloro, 2, 3-dimethyl butane

4. IUPAC name and degree of unsaturation of compound X is [2013]



- a.** 2, 3-dimethyl bicyclo [2,2,1] hept-5 ene, 2
b. 1, 2-dimethyl bicyclo [2,2,1] hept-4 ene, 3
c. 5, 6-dimethyl bicyclo [2,2,1] hept-2 ene, 3
d. 4, 5-dimethyl bicyclo [2,2,1] hept-1 ene, 2

5. Lindane can be obtained by the reaction of benzene with [2012]

- a.** CH_3Cl / anhyd. AlCl_3 **b.** $\text{C}_2\text{H}_5\text{I}$ / anhyd. AlCl_3
c. CH_3COCl / anhyd. AlCl_3 **d.** Cl_2 in sunlight

6. What will be the main product when acetylene reacts with hypochlorous acid? [2012]

- a.** Trichloro acetaldehyde
b. Acetaldehyde
c. Dichloro acetaldehyde
d. Chloro acetaldehyde

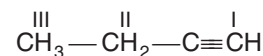
7. In which of the following compounds, the bond length between hybridised carbon atom and other carbon atom is minimum? [2012]

- a.** Butane **b.** Propyne
c. Propene **d.** Butene

8. The treatment of benzene with *iso* -butene in the presence of sulphuric acid gives [2011]

- iso-butylbenzene
- tert-butylbenzene
- n-butylbenzene
- No reaction

9. Which of the following carbon atoms is most electronegative? [2011]



- a. I
b. II
c. III
d. All are equally electronegative

- 10.** The reaction/method that does not give an alkane is
a. catalytic hydrogenation of alkenes [2011]

- b.** hydrolysis of alkyl magnesium bromide
c. Kolbe's electrolytic method
d. dehydrohalogenation of an alkyl halide

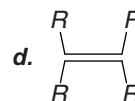
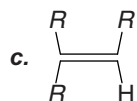
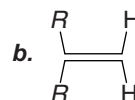
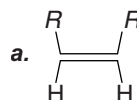
- 11.** The most strained cycloalkane is [2010]

- a.** cyclopropane **b.** cyclobutane
c. cyclopentane **d.** cyclohexane

- 12.** Chain isomers of C_6H_{14} will be [2010]

- a.** 4 **b.** 5 **c.** 6 **d.** 7

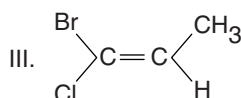
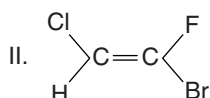
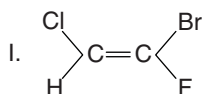
13. Which of the following alkenes will react fastest with H_2 under catalytic hydrogenation conditions? [2009]



14. Which of the following reagents would you prefer to find out whether the hydrocarbon C_3H_4 contains one-triple bond or two-double bonds? [2009]

a. Fehling's solution
b. Ammoniacal $AgNO_3$ or $CuCl$ solution
c. Baeyer's reagent
d. Br_2/CCl_4

15. Which of the following compound(s) has *Z* configuration? [2008]



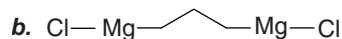
a. Only I b. Only II c. Only III d. I and III

16. $C_7H_8 \xrightarrow{2Cl_2, \text{Heat}} A \xrightarrow{Fe/Br_2} B \xrightarrow{Zn/HCl} C$

Here, the compound *C* is [2008]

a. 3-bromo 2, 4, = 6 -trichlorotoluene
b. *o*-bromotoluene
c. *p*-bromotoluene
d. *m*-bromotoluene

17. What is the product *A* in the following? [2006]



c. Both *a* and *b*

d. None of the above

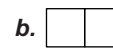
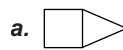
18. How many asymmetric carbon atoms are present in (i) 1, 2-dimethylcyclohexane [2005]

(ii) 3-methylcyclopentene and

(iii) 3-methylcyclohexene

a. two, one, one
b. one, one, one
c. two, none, two
d. two, none, one

19. Bicyclo (1, 1, 0) butane is [2005]

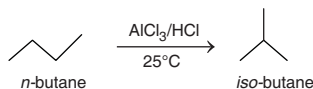


Answer with Solutions

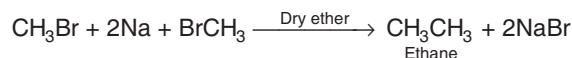
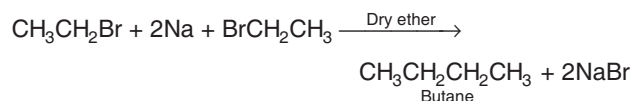
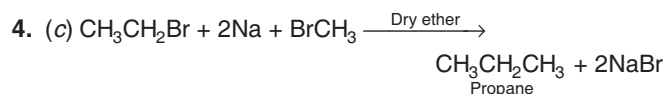
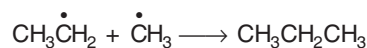
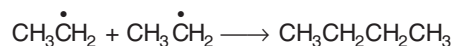
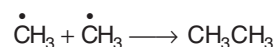
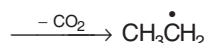
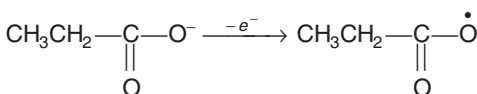
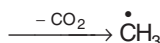
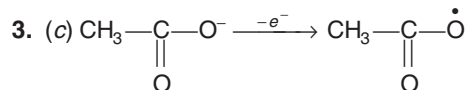
Practice Exercise

1. (c) *Sec*-butyl chloride will be formed in more quantity than *n*-butyl chloride as 2° hydrogen is more reactive.

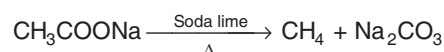
2. (d)



This is known as isomerisation.



5. (c) Methane cannot be prepared by either Wurtz reaction, Kolbe's electrolytic method or by reduction of alkenes with H_2 . While acetic acid salt on heating with soda lime, gives methane.

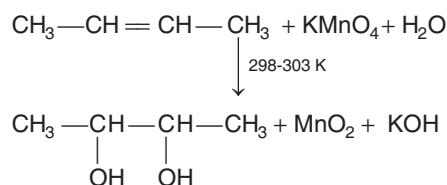


6. (b) This reaction is called Wurtz-Fittig reaction and is a common method of preparing toluene.
7. (c) LPG (Liquified petroleum gas) contains almost 96% *n*-butane.
8. (c) Only red P and conc. HI convert —COOH group to —CH_3 .
9. (a) Boiling point $\propto$ number of carbons.
10. (b) $\text{C—C—C—C} \xrightarrow[\Delta]{\text{Br}_2} \text{C—C—C—C—Br} + \text{C—C—C—Br}$
 C
Major

As 2° free radical is more stable than 1° free radical.

11. (c) With increase in number of branched chains, the molecule attains the shape of a sphere. This results in smaller area of contact and therefore weak intermolecular forces between spherical molecules, which are overcome at relatively lower temperatures.
12. (a)
13. (d) $\text{C}_n\text{H}_{2n+2} + \left(\frac{3n+1}{2}\right) \text{O}_2 \longrightarrow n \text{CO}_2 + (n+1) \text{H}_2\text{O}$
14. (a) $\text{CH}_3\text{COCH}_3 \xrightarrow[2(\text{H})]{\text{Zn(Hg)/HCl}} \text{CH}_3\text{CH}_2\text{CH}_3 + \text{H}_2\text{O}$
 $(\text{CH}_3)_3\text{CH} \xrightarrow{\text{KMnO}_4} (\text{CH}_3)_3\text{COH}$
 3° alcohol
15. (a) Where the structure of the alkyl group permits, rearrangement takes place:
 $\text{CH}_3\text{—CH}_2\text{—CH(CH}_3\text{)—CH}_2\ddot{\text{O}}\text{H} + \text{H}^+ \longrightarrow$
 $\text{CH}_3\text{—CH}_2\text{—C(CH}_3\text{)=CH}_2$
 2-methyl but-2-ene
 (more stable than 2° carbocation)
16. (b) $\text{C}_x\text{H}_y + \left(x + \frac{y}{4}\right) \text{O}_2 \longrightarrow x \text{CO}_2 + \frac{y}{2} \text{H}_2\text{O}$
 1 vol. $\left(x + \frac{y}{4}\right)$ vol
 $16 \text{ cm}^3 16 \left(x + \frac{y}{4}\right) \text{ cm}^3$
 Now, we have $16 \left(x + \frac{y}{4}\right) = 48$
 Putting $x = 2$
 $16 \left(2 + \frac{y}{4}\right) = 48$
 or $y = 4$
 Hence, the hydrocarbon is C_2H_4 .

17. (d) Hydroxylation occurs



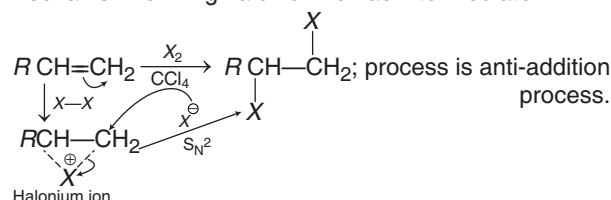
18. (a) $\text{ClCH=CHCl} \xrightarrow{\text{Cl}_2} \text{Cl}_2\text{CH—CHCl}_2$
 $\text{CH}_3\text{—CH—CH}_2\text{MgBr} \xrightarrow{\text{CH}_3} \text{CH}_3\text{—CH(CH}_3\text{)—CH}_3 + \text{Mg} \begin{matrix} \text{Br} \\ \text{OC}_2\text{H}_5 \end{matrix}$
19. (b) $\text{C}_2\text{H}_5\text{OH} \xrightarrow[\text{Dry ether}]{\text{CH}_3\text{—CH—CH}_2\text{MgBr}} \text{CH}_3\text{—CH(CH}_3\text{)—CH}_3 + \text{Mg} \begin{matrix} \text{Br} \\ \text{OC}_2\text{H}_5 \end{matrix}$

20. (d) When more than two similar groups are linked to double bonded carbon atoms, then alkenes does not show any geometrical isomerism.
21. (c) Anti-Markownikoff's addition cannot be seen in symmetrical alkenes, i.e. but-2-ene

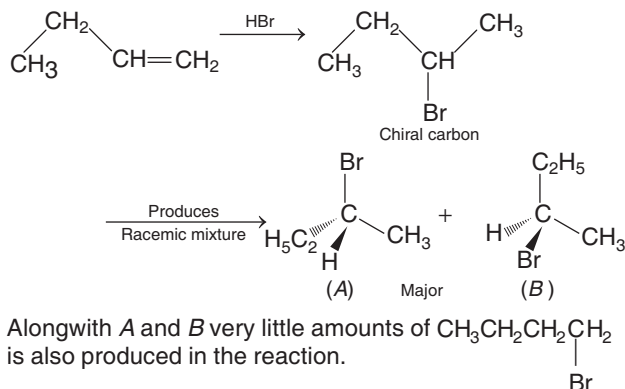
22. (b) $\text{X} \xrightarrow[\text{HgSO}_4]{\text{Dil. H}_2\text{SO}_4} \text{Y} \xrightarrow[\text{HI}]{\text{Red P}} \text{CH}_3\text{—CH}_3$

Clearly, Y is $\text{CH}_3\text{—CHO}$ and X is $\text{CH}\equiv\text{CH}$.

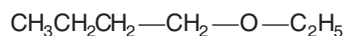
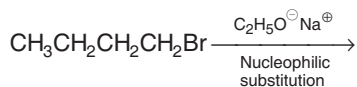
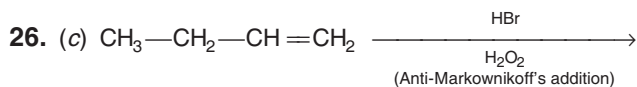
23. (d) Halogenation process proceeds via following mechanism forming halonium ion as intermediate.



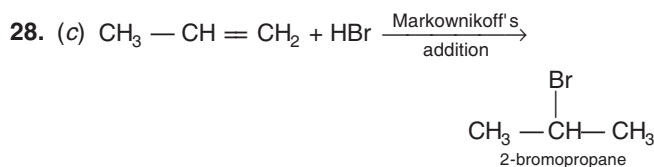
24. (a) HBr selectively adds to the alkenes through Markownikoff's addition.



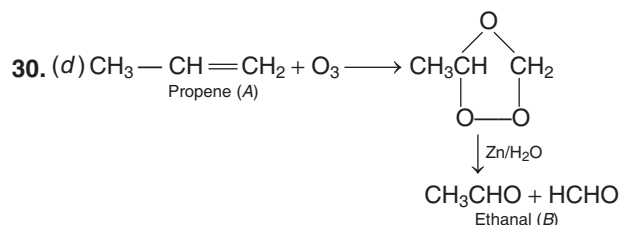
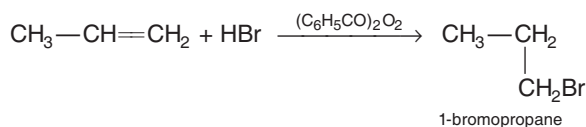
25. (c) $\text{CH}_2=\text{CH—(CH}_2\text{)}_8\text{C(=O)OH} \xrightarrow[\text{Peroxide}]{\text{HBr}} \text{product}$
 via anti-Markownikoff's addition.



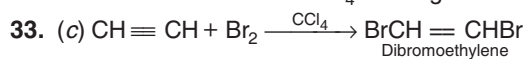
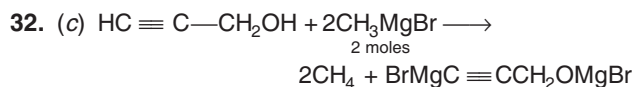
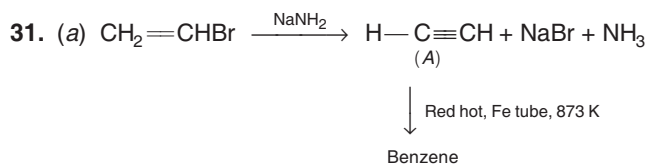
27. (d) This reaction is the example of β -elimination reaction since —OH group takes out one hydrogen atom from the β -carbon atom.



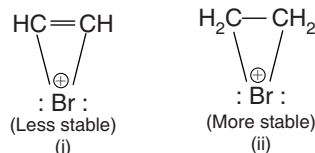
29. (a) Anti-Markownikoff's addition or peroxide effect or Kharash effect



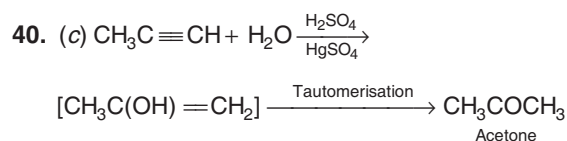
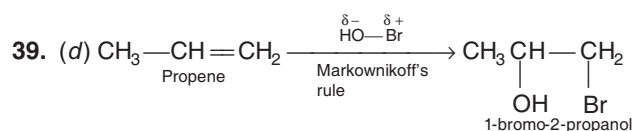
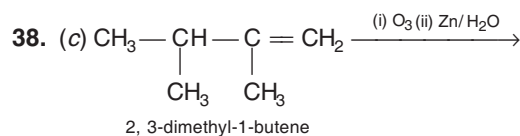
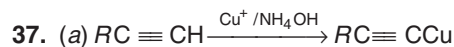
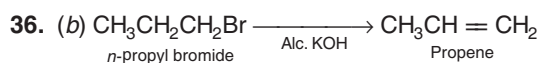
Ozonolysis takes place in alkenes. So, A will be a double bond containing compound.



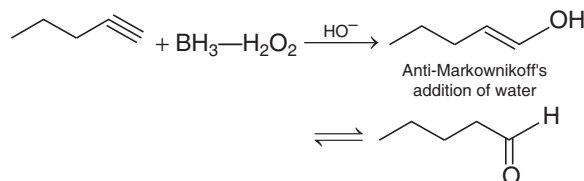
34. (d) The three membered ring of bromonium ion and alkyne has a full double bond, causing it to be more strained and less stable than the one from any alkene.



35. (d)



41. (d)



42. (d) C=C bond is cleaved and oxidised to —COOH, —CHO group is also oxidised to —COOH.

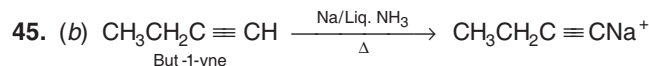
$(\text{CH}_3)_2\text{C=CHCH}_2\text{CHO} \longrightarrow (\text{CH}_3)_2\text{C=O}$
 $+ \text{HOOC—CH}_2\text{—COOH}$

43. (a) Terminal alkynes are acidic in nature.

44. (c) Both are correct. In (i) 1s-character defines correct trend of acidic strength as

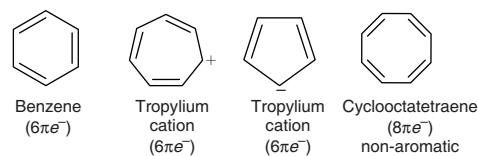
acidic strength $\propto$ % s-character

In (ii) inductive effect defines acidic strength and polarity of C—H bond of alkyne. Group having +I-effect increases electron density around C—H bond hence decreases polarity and acidic strength of alkyne.

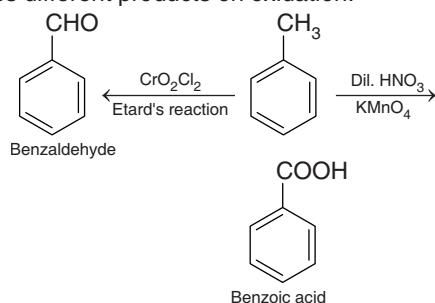


Only terminal alkyne is responsible for reaction with Na in liquid ammonia.

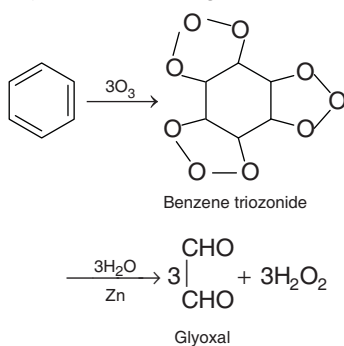
46. (d) According to Huckel's rule planar, closed cyclic structure having $(4n + 2)\pi$ electrons are aromatic.



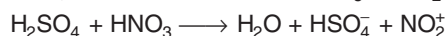
47. (d) Toluene, in presence of different oxidising agent, gives different products on oxidation.



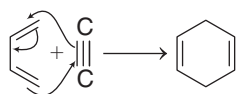
48. (d) Ozonolysis of benzene gives 3 moles of glyoxal.



49. (a) Benzene was discovered by M. Faraday, who obtained this gas by the pyrolysis of whale oil.
50. (c) In nitration of aromatic nucleus, the attacking species is an electrophile, i.e. nitronium ion (NO_2^+). It is obtained by the reaction of conc. HNO_3 and H_2SO_4 .



51. (b)
52. (d) Side chain substitution followed by Wurtz reaction.
53. (b) Diel's Alder reaction in the presence of dienophile, diene will produce the product via 6 MCTS mechanism.



54. (b) $-\text{NO}_2$ is a *m*-directing and deactivating group, hence σ -complex is formed by attack of Br^+ at *meta*-position will be most stable.

55. (b) Hydrocarbon $\xrightarrow{\text{HI}}$ X $\xrightarrow{\text{Alc. KOH}}$ Y $\xrightarrow{[\text{O}]}$ C—C—C—C
|
CH₃
||
O

As per the information Y should be C—C—C—C
|
C
|
OH

$\Rightarrow$ X should be C—C—C—C
|
C

$\Rightarrow$ Hydrocarbon should be C—C—C=C
|
C

56. (c) $\text{C}_6\text{H}_6 + \text{CHCl}_3 \xrightarrow{\text{AlCl}_3} \text{H}_5\text{C}_6-\text{C}(\text{C}_6\text{H}_5)_2-\text{H}$
|
X

57. (d) + $\text{X}_2 \xrightarrow{\text{Anhyd. AlCl}_3}$ + HX

58. (d)

59. (d) $\xrightarrow{\text{Zn/NaOH}}$
|
Cold, conc. HCl
|
Cl
|
H₂N

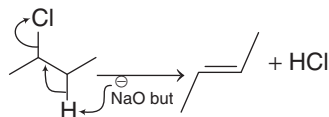
60. (a) $\xrightarrow[\Delta]{\text{H}_2\text{SO}_4 \text{ (Fuming)}}$ $\xrightarrow{\text{Cl}_2}$
YX

61. (d) Tobacco, coal and petroleum damages DNA of our body and causes cancer.

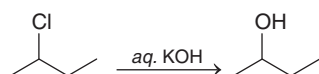
BITSAT Archives

1. (a) When tertiary alkyl halide is treated with sodium alkoxide than elimination reaction competes over substitution reaction because alkoxides are not only nucleophiles but strong base as well. Therefore, alkenes are formed instead of ethers.

2. (a) I. **Elimination reaction**

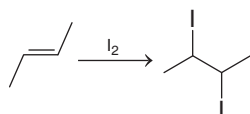


II. **Substitution reaction**



Here, Cl replaces by OH.

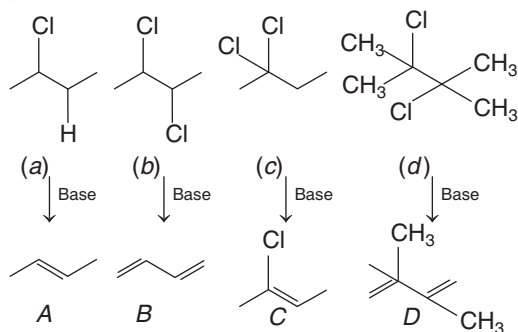
III. **Addition reaction**



IV. **Addition reaction**



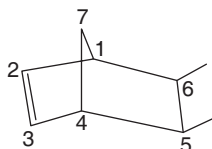
3. (d) According to Saytzeff's rule, more substituted (alkylated) alkene are more stable.



Conjugation Greater the conjugation greater will be the stability of product.

Hence, D has maximum stability, which is stabilised by conjugation as well as Saytzeff's rule.

4. (c)



IUPAC name = 5, 6 -dimethyl bicyclo [2, 2, 1] hept-2-ene

Molecular formula of compound is C₉H₁₄.

Degree of unsaturation can be calculated as

$$u = (C + 1) - \frac{H}{2} + \frac{N}{2}$$

where, u = degree of unsaturation

C = number of carbons

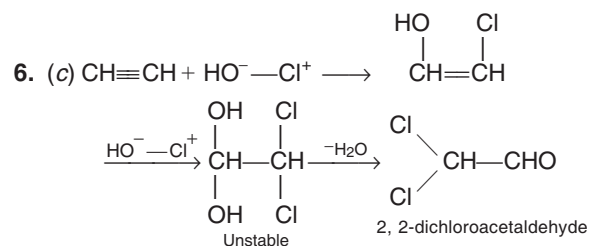
H = number of hydrogen

N = number of nitrogen

Hence, for a compound having molecular formula C₉H₁₄ the degree of unsaturation may be calculated as

$$u = (9 + 1) - \frac{14}{2} = 10 - 7 = 3$$

5. (d)

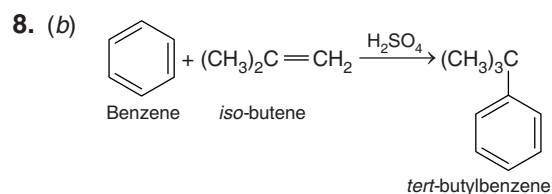


7. (b) We know C—C bond length = 1.54 Å

that C=C bond length = 1.34 Å

C≡C bond length = 1.20 Å

Since, propyne has triple bond, therefore, it has minimum bond length.



9. (a) $\text{CH}_3 - \text{CH}_2 - \text{C} \equiv \text{CH}$
 $\text{sp}^3 \quad \text{sp}^3 \quad \text{sp} \quad \text{sp}$

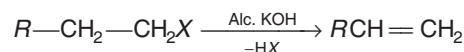
Electronegativity ∝ s -character

In sp -hybrid orbitals, s -character = 50%

and in sp^3 -hybrid orbitals, s -character = 33.3%

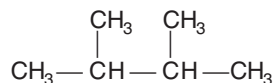
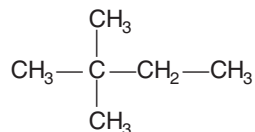
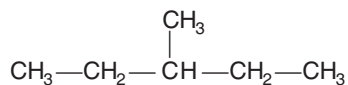
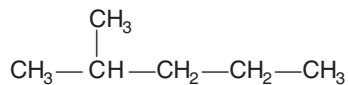
Thus, I is the most electronegative.

10. (d) Dehydrohalogenation of alkyl halide gives alkenes but not alkane.



11. (a) The greatest strain is involved in cyclopropane with bond angle 60°.

12. (b) $\text{CH}_3\text{—CH}_2\text{—CH}_2\text{—CH}_2\text{—CH}_2\text{—CH}_3$

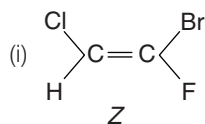


13. (a) During catalytic hydrogenation, the hydrogens are transferred from the catalyst to the less substituted double bond. Thus, option (a) with two *R* groups on the same side of the molecule is correct.

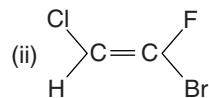
14. (b) If the hydrocarbon C_3H_4 contains a triple bond it must be $\text{CH}_3\text{C}\equiv\text{CH}$. Since, being a terminal alkyne, it will react with ammoniacal AgNO_3 .

AgNO_3 solution, to give a white ppt of silver propynide or with ammoniacal CuCl solution to give a red ppt of copper propynide.

15. (d) When the groups with higher priority (i.e. with high atomic number) are present on same side of double bond, then the configuration is *Z* but when present on opposite side of double bond, the configuration is *E*.

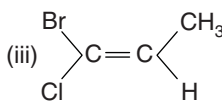


Z
(Priority : $\text{Cl} > \text{H}$ and $\text{Br} > \text{F}$)



E

(Priority : $\text{Cl} > \text{H}$ and $\text{Br} > \text{F}$)

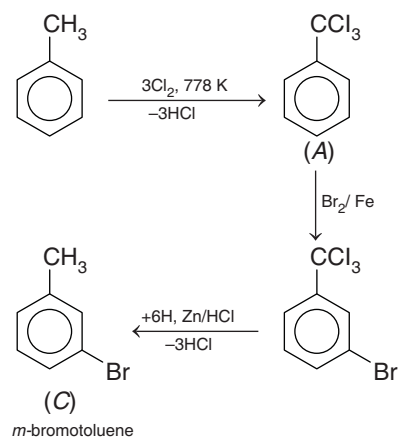


Z

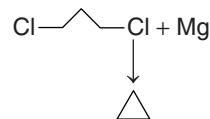
(Priority : $\text{Br} > \text{Cl}$ and $\text{CH}_3 > \text{H}$)

Hence, compounds (i) and (iii) have (*Z*) configuration.

16. (d)



17. (a)



18. (a) two, one, one

19. (c)

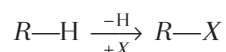


24

Halogen Derivatives of Hydrocarbon

Haloalkane and Haloarene

The replacement of hydrogen atom(s) from a hydrocarbon (aliphatic or aromatic) by halogen atom(s) results in the formation of haloalkane or haloarene. These may be classified as mono, di or tri halo compounds depending upon the number of halogen atoms.



[If R = Alkyl; Haloarene
 R = Aryl; Haloarene
 X = Halogen]

Haloalkane

- (i) General formula : $C_nH_{2n+1}X$ (monohalides)
- (ii) sp^3 -hybrid C, geometry : tetrahedral
- (iii) Bond angle : $109^\circ 28'$

Classification of Haloalkane

1. Based on the number of halogen atoms

(a) Monohalides [CH_3Cl , $(CH_3)_3CCl$ etc]

(b) Dihalides $\left[\begin{array}{cc} \text{CH}_2=\text{C} \begin{array}{l} \nearrow \text{Cl} \\ \searrow \text{Cl} \end{array} & \begin{array}{cc} \text{CH}_2 & - & \text{CH}_2 \\ | & & | \\ \text{Cl} & & \text{Cl} \end{array} \end{array} \right]$
Gem dihalide Vicinal dihalide

(c) Trihalides $CHCl_3$ (chloroform)

(d) Tetrahalides CCl_4 (carbon tetrachloride)

2. Based on the type of C-atom to which halogen atom is attached

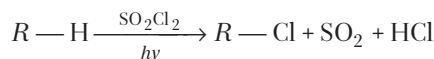
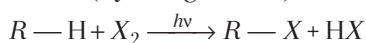
(a) $\text{CH}_3\text{—Cl} \longrightarrow$ primary

(b) $\begin{array}{c} \text{CH}_3 \\ \diagup \\ \text{CH—Cl} \\ \diagdown \\ \text{CH}_3 \end{array} \longrightarrow$ secondary

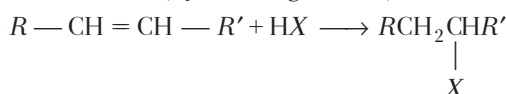
(c) $\begin{array}{c} \text{CH}_3 \\ | \\ \text{CH}_3\text{—C—Cl} \\ | \\ \text{CH}_3 \end{array} \longrightarrow$ tertiary

Preparation of Haloalkane

1. From alkane (By halogenation)



2. From alkene (hydrohalogenation)

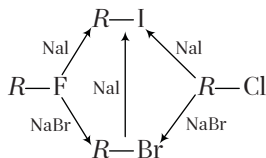


Unsymmetrical alkene gives major product by more stable carbocation according to Markownikoff's rule.

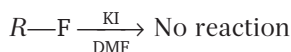


3. From alkyl halide (Halogen exchange reactions)

(a) Finkelstein reaction (In acetone solvent)

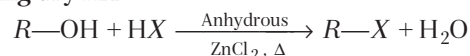


(b) Swart reaction (In polar aprotic solvent)

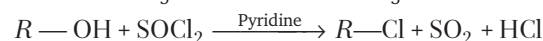
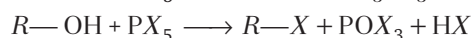


4. From alcohol

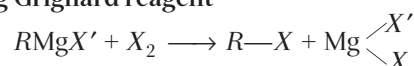
(a) Using dry HX



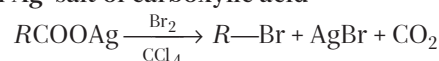
(b) Action of phosphorus halides and SOCl_2 on alcohols



5. Using Grignard reagent

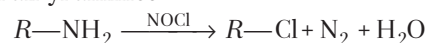


6. From Ag-salt of carboxylic acid

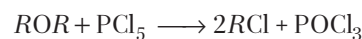


Number of C-atoms decreases by one.

7. From alkyl amines



8. From ether



Physical Properties of Haloalkanes

General physical properties of haloalkanes are as follows:

- CH_3F , CH_3Cl , CH_3Br , $\text{C}_2\text{H}_5\text{F}$ and $\text{C}_2\text{H}_5\text{Cl}$ are found in gaseous state at room temperature.
- Alkyl halides are normally sweet-smelling gases and liquids, whereas odourless in solid state.
- All alkyl halides are insoluble in water.
- Boiling point of alkyl halides increases with increase in the size of halogen atom while decreases with increase in branching.

Chemical Properties of Haloalkanes

Haloalkanes are highly reactive compounds due to the presence of polar R—X bond.

The reactions of haloalkanes have been discussed below:

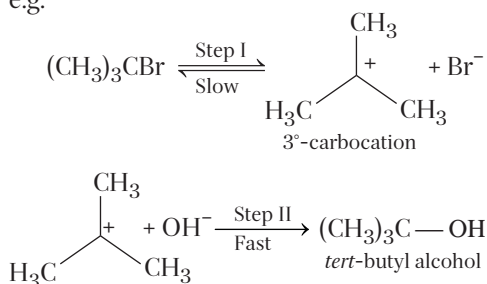
Nucleophilic Substitution Reactions

In haloalkanes, these reactions proceed either by $\text{S}_\text{N}1$ or by $\text{S}_\text{N}2$ type of mechanism.

1. Substitution nucleophilic unimolecular reaction (S_N1)

- Tertiary alkyl group and polar solvent favour S_N1 .
- It follows first order kinetics, rate = k [substrate].
- Carbocation is formed and gets rearranged, if possible, the carbocation must be stable.
- Reaction is completed in two steps.
- Nucleophile can attack from front or back side. Therefore, racemic mixture can be formed.

e.g.

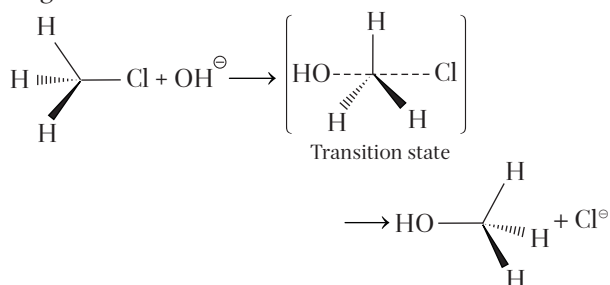


Reactivity order of halides towards S_N1 mechanism is
 $3^\circ > 2^\circ > 1^\circ$

2. Substitution nucleophilic bimolecular reaction (S_N2)

- Primary alkyl group and non-polar solvent favour S_N2 .
- It follows second order kinetics, rate = k [substrate] [nucleophile]
- Carbocation is not formed.
- Reaction is completed in one step.
- It involves complete inversion in configuration as the attack of the nucleophile occurs from the back side of reactant.

e.g.

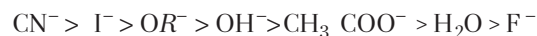


— (Solid bond) – towards the viewers
 (Wedge bond) – away from the viewers

Reactivity order of halides towards S_N2 mechanism is
 $1^\circ > 2^\circ > 3^\circ > \text{neo-pentyl halides}$

In S_N2 mechanism, rate of reactions does not depend upon the strength of attacking nucleophile.

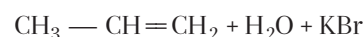
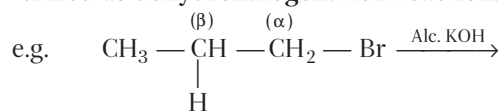
The strength of different nucleophiles is



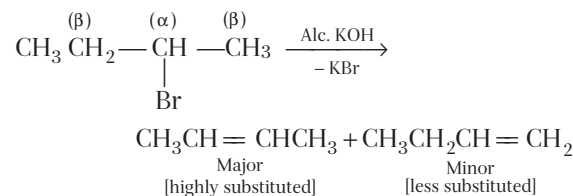
NOTE 3° halides usually proceed through S_N1 type, 1° halides proceed through S_N2 type and 2° halide may proceed either through S_N1 or S_N2 type.

Elimination Reactions [Synthesis of Alkenes]

Reaction of haloalkane with alcoholic KOH solution leads to an elimination of hydrogen atom from β -carbon atom and halogen atom from α -carbon atom. Thus, reaction termed as **dehydrohalogenation reaction**.



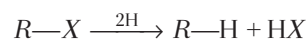
If dehydrohalogenation reactions produce two or more alkenes, the preferred or major product will be that alkene which possess larger number of alkyl groups linked to double bonded C-atoms. This is in accordance to **Saytzeff's rule**.



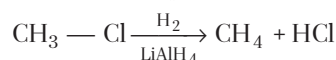
The reactivity order of alkyl halides for dehydrohalogenation reaction is given below



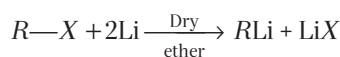
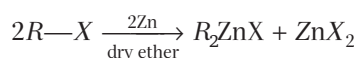
Reduction

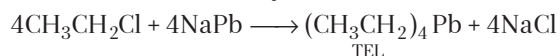


Reducing agents

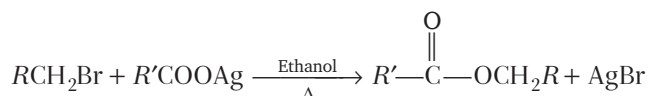


Formation of organo-metallic compounds

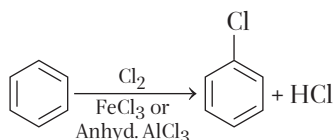
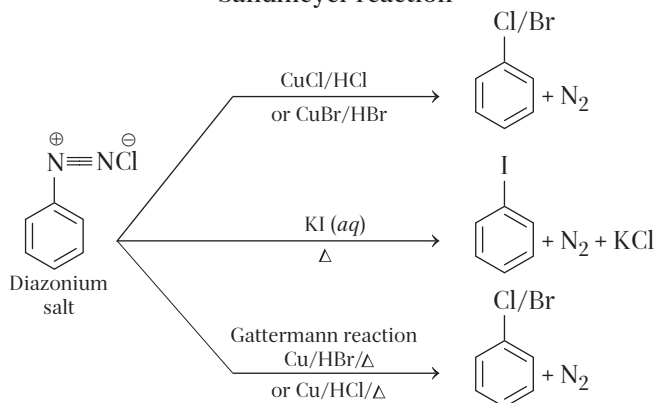
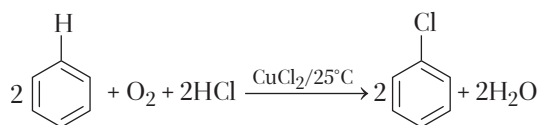


Reaction with NaPb alloy

[TEL $\rightarrow$ tetraethyl lead (antiknocking agent)]

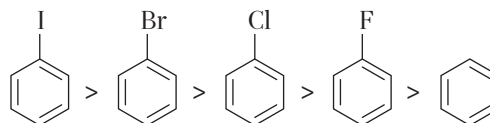
Friedel-Crafts alkylation**Reaction with salts of carboxylic acid****Haloarenes****Preparation of Haloarenes**

General methods for the preparation of haloarenes are as follows:

Halogenation of Benzene/Toluene**Decomposition of Diazonium Salts****Sandmeyer reaction****Raschig Process****Physical Properties of Haloarenes**

- (i) Aryl halides (haloarenes) are heavier than water, though polar in nature but immiscible with water due to the absence of H-bonding with water molecules.

- (ii) For the same aryl group, boiling and melting points increase with an increase in the size of the halogen atom. Boiling and melting points of haloarenes increase with increase in the size of aryl group.



The *p*-isomers possess higher melting points than *ortho* and *meta*-isomers because *p*-isomers fit in the crystal lattice more efficiently than *ortho* and *meta*-isomers.

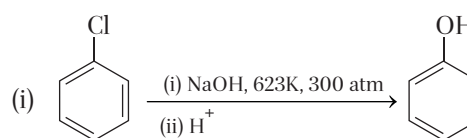
Chemical Properties of Haloarenes

Haloarenes give three types of reactions:

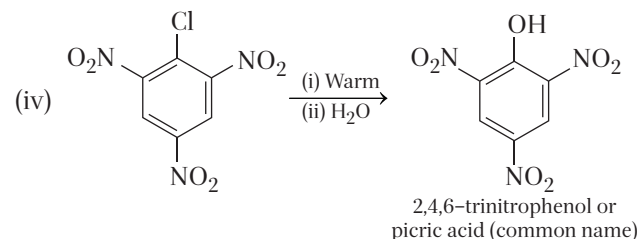
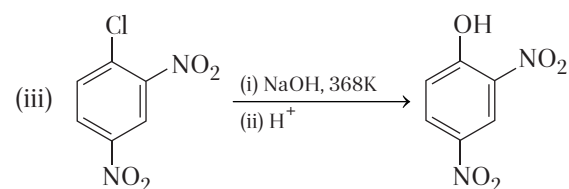
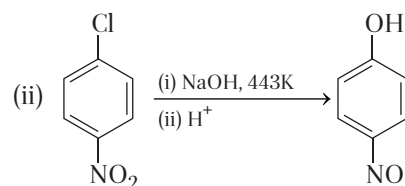
1. Nucleophilic Substitution Reactions

Haloarenes are less reactive than haloalkanes towards nucleophilic substitution reactions.

e.g.

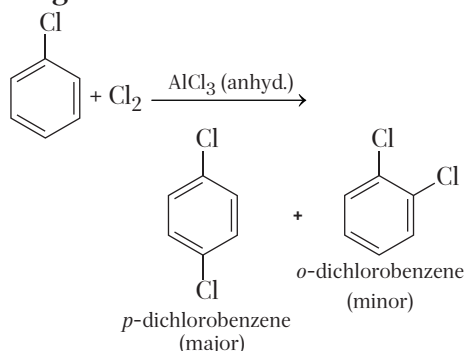


This is known as Dow's process.

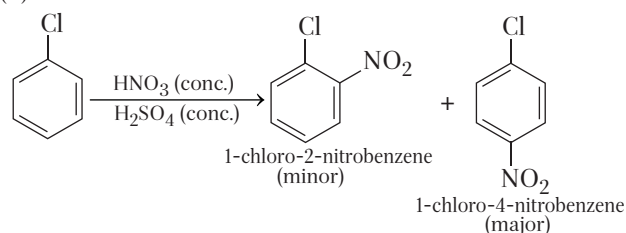
**2. Electrophilic Substitution Reactions**

Haloarenes undergo usual electrophilic substitution reactions of benzene nucleus such as halogenation, nitration, sulphonation and Friedel-Crafts alkylation and acylation.

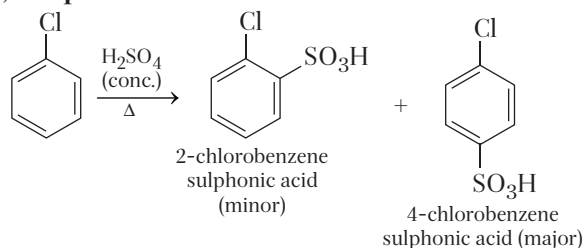
e.g.
(i) **Halogenation**



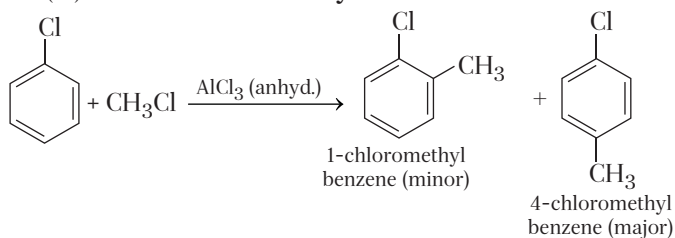
(ii) **Nitration**



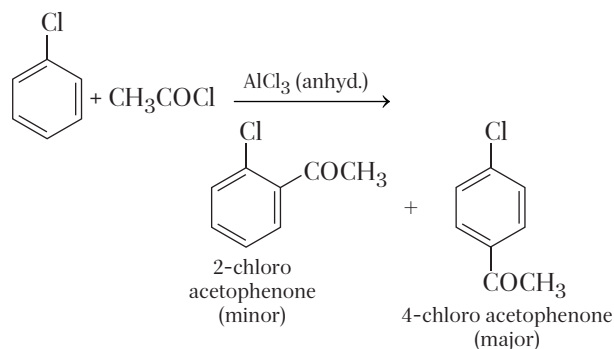
(iii) **Sulphonation**



(iv) **Friedel-Craft's alkylation**



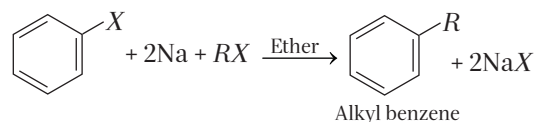
(v) **Friedel-Craft's acylation**



Reactions with Metals

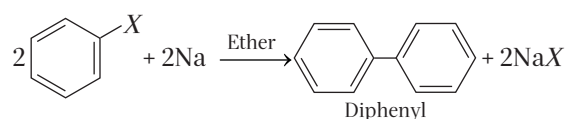
1. Wurtz-Fittig Reaction

A mixture of haloarene and haloalkane, when treated with Na in dry ether gives alkyl arene, e.g.



2. Fittig Reaction

Haloarenes on treatment with Na in dry ether give analogous compounds, e.g.



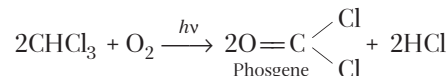
Polyhalogen Compounds

Carbon compounds which contain more than one halogen atom are called polyhalogen compounds.

Some important polyhalogen compounds along with uses and environmental effects are given as :

Trichloromethane (Chloroform), CHCl3

Chloroform oxidises by air slowly in the presence of light to yield phosgene, an extremely poisonous gas.



Therefore, it is stored under closed dark coloured bottles.

Uses

- It is used as a solvent for oils, fats, rubbers and resins.
- It is exclusively used in the production of freons.

Environmental Effects

Major effects of CHCl3 are as follows:

- The presence of CHCl3 in air can cause dizziness, fatigue and headache.
- It may lead to the damage of liver and kidneys when breathed with air.

Triiodomethane (Iodoform)

The chemical formula of iodoform is CHI3.

Uses It is used as an antiseptic for dressing the wounds, due to the liberation of free iodine by iodoform.

Freons

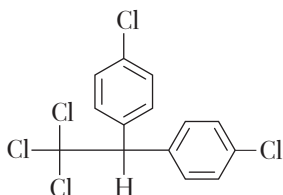
The chlorofluorocarbon compounds of methane and ethane containing no H-atom are called freons. These compounds are highly stable, non-toxic, unreactive and easily liquefiable gases.

Uses It is widely used in the production of aerosol propellants, refrigeration and air conditioning products.

Environmental effects These freons when released into air, diffuse into stratosphere where it initiates radical chain reactions for the depletion of ozone layer.

***p p'*-Dichlorodiphenyl Trichloroethane (DDT)**

The structure of DDT is



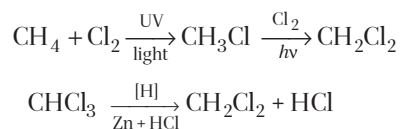
Uses DDT is cheap but powerful insecticide used for sugarcane and fodder crops in order to kill insects and mosquitoes.

Environmental Effects

Most of the species of insects developed resistance towards DDT and it is found to be highly toxic for animals.

It is deposited and stored in the fatty tissues because it is soluble in fat and chemically stable.

Dichloromethane



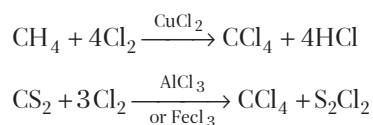
Uses It is widely used as

- (i) Solvent
- (ii) Paint remover
- (iii) Propellant in aerosols
- (iv) Metal cleaning and finishing solvent

Environmental effect

- (i) It harms human nervous system.
- (ii) It causes dizziness, nausea, tingling and numbness in the fingers.
- (iii) Direct human skin contact with CH_2Cl_2 causes intense burning sensation and mild redness of the skin. Also, it can burn the cornea if direct contact with eyes occurs.

Tetrachloro methane (Pyrene)



Uses

- (i) It is used in the manufacturing of refrigerants and propellants for aerosol units.
- (ii) It is used as a cleaning solvent, a degreasing solvent, a spot remover, a fire extinguisher.

Environmental effects

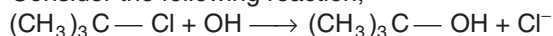
- (i) It causes dizziness, nausea, vomiting.
- (ii) Exposure to CCl_4 may cause irregular heart beat.
- (iii) It depletes the ozone layer which increases the level of UV-rays causing.

Practice Exercise

1. The true order of reactivity towards S_N1 reaction is

a. vinyl chloride > allyl chloride > isopropyl chloride
 b. allyl chloride > isopropyl chloride > vinyl chloride
 c. vinyl chloride > isopropyl chloride > allyl chloride
 d. isopropyl chloride > vinyl chloride > allyl chloride

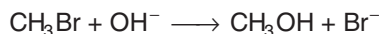
2. Consider the following reaction,



The true statement about the above process is

a. the rate becomes four times of the halide becomes twice
 b. the rate does not change by reducing $[\text{OH}^-]$ to half
 c. the rate does not change by doubling $[\text{halide}]$
 d. the rate becomes twice on doubling the $[\text{OH}^-]$

3. Consider the following reaction,



The true statement about the above process is

a. the rate does not change on doubling $[\text{CH}_3\text{Br}]$ and making $[\text{OH}^-]$ to half
 b. the rate becomes half on doubling $[\text{CH}_3\text{Br}]$
 c. the rate becomes half on doubling $[\text{OH}^-]$
 d. All the above are correct

4. The product of reaction of alcoholic silver nitrite with ethyl bromide is

a. ethyl nitrite b. ethene
 c. nitroethane d. ethyl alcohol

5. Which does not give methyl bromide?

a. $\text{CH}_3\text{OH} + \text{HBr}$ b. $\text{CH}_3\text{OH} + \text{Br}_2$
 c. $\text{CH}_3\text{OH} + \text{PBr}_3$ d. $\text{CH}_3\text{COOAg} + \text{Br}_2$

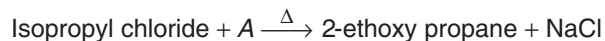
6. Consider the following haloalkanes,

I. CH_3I II. CH_3F
 III. CH_3Cl IV. CH_3Br

The correct sequence of increasing order of dipole moment is

a. $\text{I} < \text{II} < \text{III} < \text{IV}$ b. $\text{IV} < \text{III} < \text{II} < \text{I}$
 c. $\text{I} < \text{IV} < \text{II} < \text{III}$ d. $\text{III} < \text{IV} < \text{I} < \text{II}$

7. Consider the following reaction,



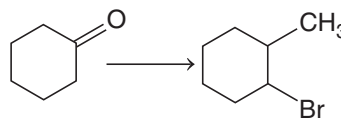
The compound A is

a. $\text{C}_2\text{H}_5\text{Cl}$
 b. $\text{C}_2\text{H}_5\text{ONa}$
 c. CH_2N_2
 d. CH_3ONa

8. In alkaline hydrolysis of a tertiary halide by aqueous alkali, if concentration of alkali is doubled, then the reaction rate

a. will be doubled
 b. will be halved
 c. will remain constant
 d. Cannot say anything

9. Which of the following sequence of reagent is best suited for the reaction shown below?



a. (i) CH_3MgBr , H_3O^+ (ii) H^+ / Δ (iii) $\text{HBr} / \text{H}_2\text{O}_2$
 b. (i) CH_3MgBr , H_3O^+ (ii) H^+ / Δ (iii) HBr
 c. (i) CH_3MgBr , H_3O^+ (ii) HBr
 d. (i) HBr / ROOR (ii) CH_3MgBr , H_3O^+

10. The order of reactivities of the following alkyl halides for a S_N2 reaction is

a. $\text{RF} > \text{RCI} > \text{RBr} > \text{RI}$ b. $\text{RF} > \text{RBr} > \text{RCI} > \text{RI}$
 c. $\text{RCI} > \text{RBr} > \text{RF} > \text{RI}$ d. $\text{RI} > \text{RBr} > \text{RCI} > \text{RF}$

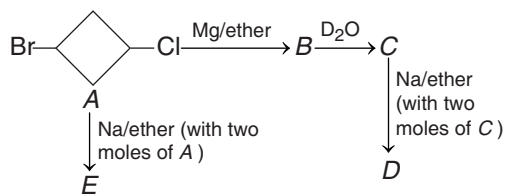
11. Choose the incorrect statement.

a. An S_N1 reaction proceeds with inversion of configuration
 b. An S_N2 reaction proceeds with stereochemical inversion
 c. An S_N2 reaction follows second order kinetics
 d. The reaction of *tert*-butyl bromide with OH^- follows first order kinetics

12. An organic compound $\text{C}_5\text{H}_9\text{Br}$ (A) which readily decolourises bromine water and cold alkaline KMnO_4 solution gives $\text{C}_5\text{H}_{11}\text{Br}$ (B) on catalytic hydrogenation. The reaction of A with alcoholic KOH first and then with NaNH_2 produces C with evolution of NH_3 . C reacts with Lindlar's catalyst to give D and on reaction with Na in liquid NH_3 produces E. D and E are isomers. The compound A is

a. $\text{CH}_3\text{CH}_2\text{CH}=\overset{\text{Br}}{\underset{|}{\text{C}}}-\text{CH}_3$
 b. $\text{CH}_3\text{CH}_2-\overset{\text{Br}}{\underset{|}{\text{C}}}=\text{CHCH}_3$
 c. Both (a) and (b)
 d. Neither (a) nor (b)

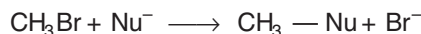
13. Consider the following reaction,



D and E respectively are

- Cl——CH₂OH and Cl——D
 - Cl———Cl and D———D
 - D———D and Cl———Cl
 - None of the above
14. The main product of the reaction of propane with chlorine at 25°C in the presence of sunlight is
- 1-chloropropane
 - 2-chloropropane
 - chloroethane
 - chloromethane
15. Hunsdiecker reaction is an example of
- decarboxylation
 - debromination
 - decarboxylation and bromination
 - bromination
16. When 3-phenyl propene is treated with HBr in the presence of peroxide, the major product formed is
- 1-bromo-3-phenyl propane
 - 1,2-dibromo-3-phenyl propane
 - 2-bromo-1-phenyl propane
 - 3-(*p*-bromo phenyl) propene
17. The alkyl halide that undergoes S_N1 reaction more readily is
- ethyl bromide
 - iso*-propyl bromide
 - vinyl bromide
 - t*-butyl bromide
18. Which alkyl halide from the following would you expect to react more rapidly by S_N2 mechanism?
- CH₃CHCH₂CH₂Br
|
CH₃
 - CH₃CH₂CHCH₂Br
|
CH₃
|
CH₃
 - CH₃CH₂C(CH₃)₂CH₂Br
 - CH₃CH₂CH₂CHBr
|
CH₃

19. Consider the following reaction,



The decreasing order of the rate of the above reaction with Nu[−] is

- PhO[−]
 - AcO[−]
 - HO[−]
 - CH₃O[−]
- IV > III > I > II
 - IV > III > II > I
 - I > II > III > IV
 - II > IV > III > I
20. The compound A in the following reaction sequence is
- $$\text{A} \xrightarrow{\text{PCl}_5} \text{B} \xrightarrow{\text{AlCl}_3, \text{KOH}} \text{C} \xrightarrow{\text{H}_2/\text{Ni}} \text{Propane}$$
- chloroethane
 - ethanol
 - 1,2-dichloroethane
 - None of these
21. Which of the following statements regarding S_N1 reaction shown by alkyl halide is incorrect?
- The added Nu[−] plays no kinetic role in S_N1 reaction
 - S_N1 reaction involves the inversion of configuration of optically active substance
 - S_N1 reaction on chiral starting material ends up with racemisation of the product
 - Polar protic solvent increases the rate of S_N1 reaction
22. The product of which of the following reactions is not a *vicinal* dihalide?
- Reaction of ethylene glycol with P + Br₂
 - Reaction of HCl with ethyne
 - Reaction of HBr with ethyne in the presence of peroxide
 - Reaction of Cl₂ with propene
23. The treatment of CH₃MgX with CH₃—C≡C—H produces
- CH₃—CH=CH₂
 - CH₃C≡C—CH₂
| |
H H
 - CH₃—C=C—CH₂
| |
H H
 - CH₄
24. Consider the following bromides,
- Me—CH₂—CH₂—CH₂—Br
 -
 - Me—CH₂—CH(Br)—CH₂—Me
- The correct order of S_N1 reactivity [AIEEE 2010]
- II > III > I
 - II > I > III
 - III > II > I
 - I > II > III

25. Which of the following reactions come within the framework of elimination?

a. $(\text{CH}_3)_2\text{CHCl} + \text{SH}^\oplus$
 b. $(\text{CH}_3)_3\text{C}-\text{Br} + \text{ethanolic KCN}$
 c. $\text{CH}_3\text{CH}_2\text{CH}_2\text{Cl} + \text{I}^-$
 d. $(\text{CH}_3)_2\text{CHBr} + \text{aq. KOH}$

26. 20% aqueous NaCl solution containing ethyl alcohol when subjected to electrolysis, produce

a. $\text{C}_2\text{H}_5\text{Cl}$
 b. CH_3CHO
 c. CHCl_3
 d. CCl_3CHO

27. Alkaline hydrolysis of which of the following compounds give methyl ketones?

I. Ethylene dichloride
 II. 2, 2-dichlorobutane
 III. Ethylidene chloride
 IV. Isopropylidene chloride

a. I, II and III
 b. II, III and IV
 c. I and III
 d. II and IV

28. When optically active halide is attacked by CN^- , the product obtained is a racemic mixture. Hence, the halide should be

a. primary
 b. secondary
 c. tertiary
 d. None of the above

29. Dipole moment of *cis*-2,3-dichloro-2-butene is than the dipole moment of *cis*-1, 2-dichloroethene.

a. more
 b. less
 c. neither more nor less
 d. None of the above

30. Aryl halides are less reactive towards nucleophilic substitution reaction as compared to alkyl halides due to

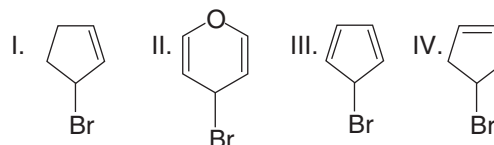
a. the formation of less stable carbonium ion
 b. resonance stabilisation
 c. longer carbon-halogen bond
 d. sp^2 -hybridised C attach to X

31. In the reaction $\text{PhCH}_3 \xrightarrow[\text{heat}]{\text{Cl}_2} \text{X} \xrightarrow{\text{aq. NaOH}} \text{Y}$,

the product Y is

a. *o*-cresol
 b. *p*-cresol
 c. 2, 4-dihydroxytoluene
 d. benzoic acid

32. The increasing order of reactivity of the following bromides in $\text{S}_\text{N}1$ reaction is

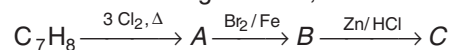


a. $\text{IV} < \text{III} < \text{I} < \text{II}$
 b. $\text{III} < \text{IV} < \text{I} < \text{II}$
 c. $\text{II} < \text{III} < \text{I} < \text{IV}$
 d. $\text{II} < \text{III} < \text{IV} < \text{I}$

33. The Wurtz-Fittig reaction involves condensation of

a. two molecules of aryl halides
 b. one molecule of each of aryl halide and alkyl halide
 c. one molecule of each aryl halide and phenol
 d. two molecules of an alkyl halide

34. Consider the following reaction,



The compound C is

a. *o*-bromotoluene
 b. *m*-bromotoluene
 c. *p*-bromotoluene
 d. 3-bromo-2,4,6-trichlorotoluene

35. *Trans*-2-phenyl-1-bromocyclopentane on reaction with alcoholic KOH produces

a. 4-phenylcyclopentene
 b. 2-phenylcyclopentene
 c. 1-phenylcyclopentene
 d. 3-phenylcyclopentene

36. **Assertion (A)** Exposure of ultraviolet rays to human causes the skin cancer, disorder and disrupt the immune system.

Reason (R) Carbon tetrachloride is released into air, it rises to atmosphere and depletes the ozone layer.

a. Both A and R are correct and R is correct explanation of A
 b. Both A and R are correct but R is not the correct explanation of A
 c. A is correct but R is incorrect
 d. R is correct but A is incorrect

37. IUPAC name of DDT is

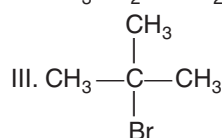
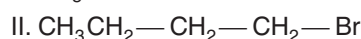
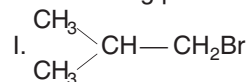
a. 1,1-*bis* (4-chlorophenyl)-2,2,2-trichloroethane
 b. 1,2-*bis* (4-chlorophenyl)-1,1,1-trichloroethane
 c. 2,2-*bis* (4-chlorophenyl)-1,1,1-trichloroethane
 d. 2,1-*bis* (4-chlorophenyl)-1,1,1-trichloroethane

38. Which of the following statements is/are true for chloroform?

a. Chloroform exposure causes damage to heart
 b. People develop sores when skin is immersed in chloroform
 c. Chloroform has no effect on central nervous system
 d. All of the above

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1. Arrange the following compounds in increasing order of their boiling points: [2014]

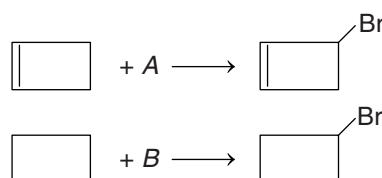


- a. $\text{II} < \text{I} < \text{III}$
 b. $\text{I} < \text{II} < \text{III}$
 c. $\text{III} < \text{I} < \text{II}$
 d. $\text{III} < \text{II} < \text{I}$
2. Which of the following is less acidic among the given halogen compounds? [2013]
- a. CHF_3 b. CHI_3
 c. CHCl_3 d. CHBr_3
3. 2-bromopentane with alcoholic KOH yields a mixture of three alkenes. Which of the following alkene is predominant? [2012]
- a. 1-pentene
 b. *Cis*-2-pentene
 c. *Trans*-2-pentene
 d. *Cis*-1-pentene
4. Toluene on reaction with N-bromo-succinimide gives [2011]
- a. *p*-bromomethylbenzene
 b. *o*-bromomethylbenzene
 c. phenyl bromomethane
 d. *m*-bromomethylbenzene
5. Which of the following will yield a mixture of 2-chlorobutene and 3-chlorobutene on treatment with HCl? [2011]
- a. $\text{CH}_2=\text{C}=\text{CH}-\text{CH}_3$
 b. $\begin{array}{c} \text{H}_2\text{C}=\text{C}-\text{CH}=\text{CH}_2 \\ | \\ \text{CH}_3 \end{array}$
 c. $\text{CH}_2=\text{CH}-\text{CH}=\text{CH}_2$
 d. $\text{HC}\equiv\text{C}-\text{CH}=\text{CH}_2$
6. 3-methyl-2-butanol on treatment with HCl gives (major product) [2010]
- a. 2-chloro-2-methylbutane
 b. 2-chloro-3-methylbutane
 c. 2, 2-dimethylpentane
 d. None of the above

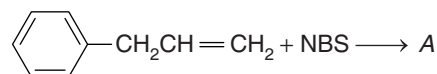
7. The correct order of reactivity of hydrogen halides with ethyl alcohol is [2008]



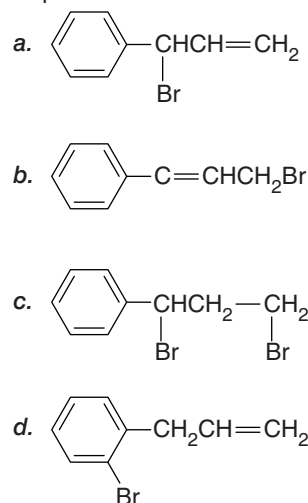
8. Suitable reagents A and B for the following reactions respectively are [2005]



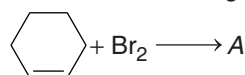
9. Following compound is treated with NBS: [2005]



Compound formed A is



10. Consider the following reaction,



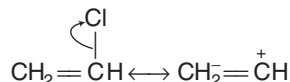
A will have configuration



Answer with Solutions

Practice Exercise

1. (b) Vinyl chloride is relatively less reactive



While allyl chloride is very reactive due to stabilisation of allyl carbonium ion by resonance.

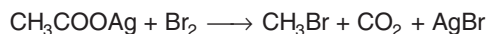
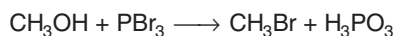


Isopropyl chloride in between them is reactivity towards $\text{S}_{\text{N}}1$.

2. (b) $\text{S}_{\text{N}}1$ reaction depends only upon concentration of substrate.
 3. (a) $\text{S}_{\text{N}}2$ reaction rate is directly proportional to concentration of substrate and nucleophile.
 4. (a) $\text{CH}_3\text{CH}_2\text{Br} + \text{AgNO}_2 \longrightarrow \text{CH}_3\text{CH}_2\text{NO}_2 + \text{AgBr}$

Alcoholic Major

5. (b) $\text{CH}_3\text{OH} + \text{HBr} \longrightarrow \text{CH}_3\text{Br} + \text{H}_2\text{O}$



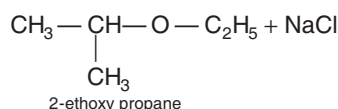
6. (c) $\text{CH}_3\text{I} < \text{CH}_3\text{Br} < \text{CH}_3\text{F} < \text{CH}_3\text{Cl}$

1.6 D 1.79 D 1.84 D 1.94 D

7. (b) $\text{H}_3\text{C}-\text{CH}(\text{Cl})-\text{CH}_3 + \text{NaOC}_2\text{H}_5 \xrightarrow{\Delta}$

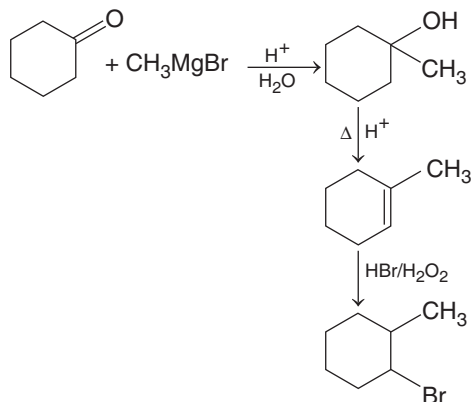
Isopropyl chloride

Sodium ethoxide

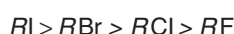


8. (c) Reaction of *t*-alkyl halides with aqueous alkali gives $\text{S}_{\text{N}}1$ reaction and rate of $\text{S}_{\text{N}}1$ reaction is not based on the concentration of nucleophile (alkali). Hence, reaction rate remains constant.

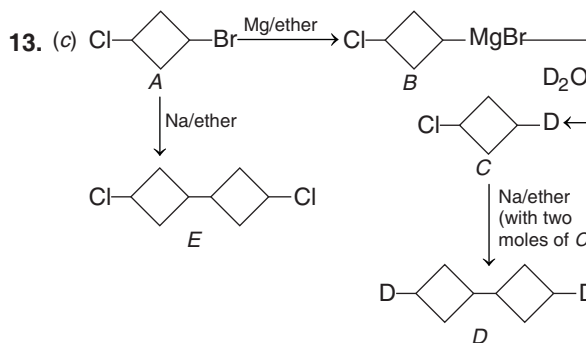
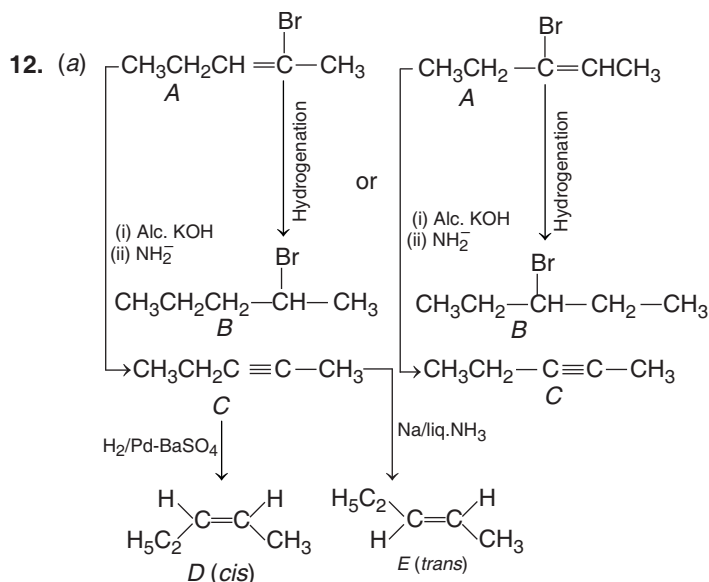
9. (a)



10. (d) The order of reactivities of the alkyl halides for $\text{S}_{\text{N}}2$ reaction is



11. (a) $\text{S}_{\text{N}}1$ reactions are accompanied by racemisation.

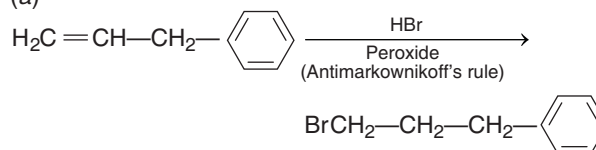


14. (b) $\text{CH}_3-\text{CH}_2-\text{CH}_3 \xrightarrow[25^\circ\text{C}]{\text{Cl}_2} \text{CH}_3-\text{CH}(\text{Cl})-\text{CH}_3 + \text{CH}_3-\text{CH}_2-\text{CH}_2\text{Cl}$
- 2° (Major) 1° (minor)

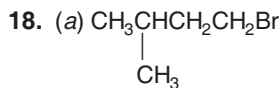
15. (c) $\text{R}-\text{C}(=\text{O})-\text{OH} \xrightarrow[\text{Br} + \text{CCl}_4/\Delta]{\text{aq. KOH}} \text{R}-\text{Br} + \text{CO}_2$

Clearly, it is decarboxylation with bromination.

16. (a)

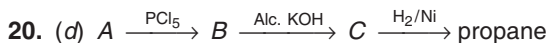
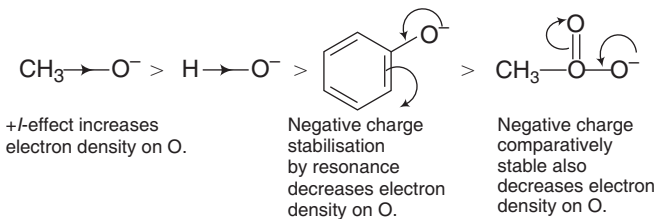


17. (d) $\text{S}_{\text{N}}1$ mechanism involves formation of carbocation intermediate.



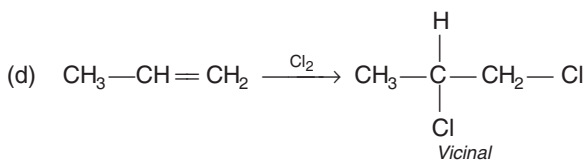
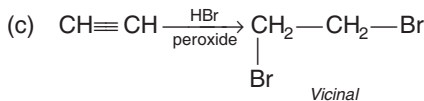
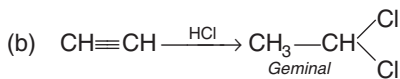
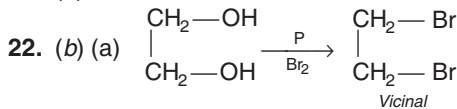
Out of the given compounds, three are primary halides. The presence of methyl group closer to halide group increases the steric hindrance and decreases rate of the reaction. Hence, compound (a) reacts rapidly.

19. (a) Nucleophilicity order is



Since, *C* gives propane ($\text{CH}_3\text{CH}_2\text{CH}_3$), it must be an alkene, $\text{CH}_3\text{CH}=\text{CH}_2$ (*C*). If *C* is formed from *B* through alc.KOH, then *B* must be an alkyl halide, $\text{CH}_3\text{CH}_2-\text{CH}_2\text{Cl}$ which is formed from *A* by PCl_5 . This suggests *A* would be alcohol, $\text{CH}_3\text{CH}_2\text{CH}_2\text{OH}$ (propanol).

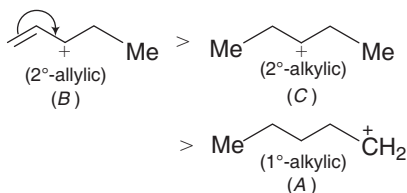
21. (b)



23. (d)

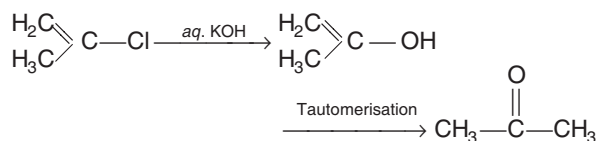
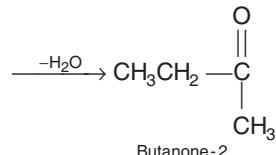
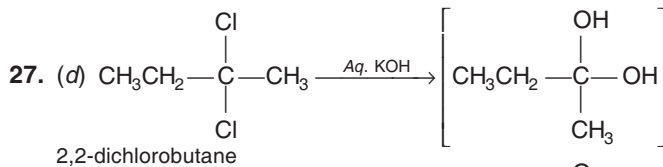


24. (a)



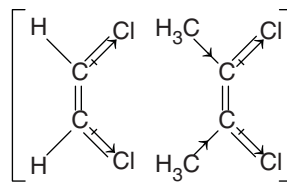
25. (b) $(\text{CH}_3)_3\text{C}-\text{Br}$ undergoes dehydrobromination by ethanolic KCN and produces isobutylene.

26. (c) Electrolysis of aqueous NaCl solution gives NaOH and Cl_2 . These reactants on reaction give chloroform.



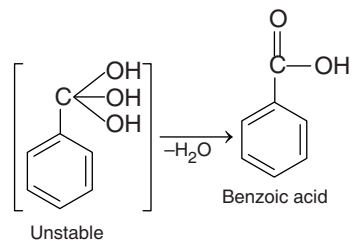
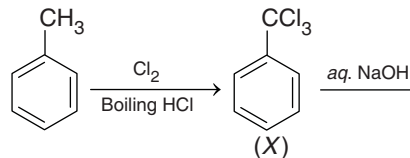
28. (c) Tertiary halide follows S_N1 mechanism which forms racemic mixture.

29. (a) [Hint : It is clear from the figure itself]



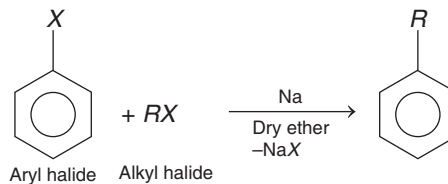
30. (b) All the aryl (aromatic) compounds are less reactive towards nucleophilic substitution because of the presence of π -electrons (resonance) due to which they repel the incoming nucleophile.

31. (d)

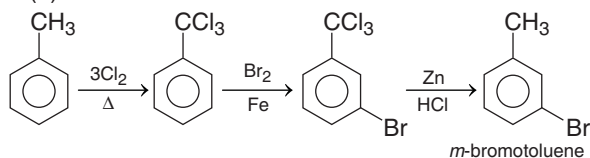


32. (a) II is most reactive as it produces an aromatic carbocation while IV is less reactive as it produces a non-resonance stabilised carbocation. III is least reactive than I as former involve an anti-aromatic carbocation.

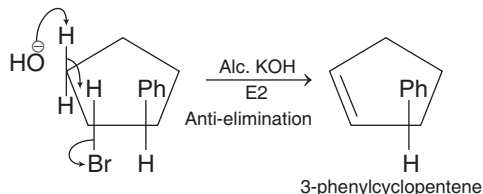
33. (b)



34. (b)



35. (d)



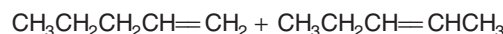
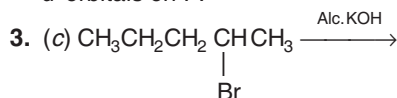
Anti-elimination means —H and —Br both departing groups must be present at dihedral angle of 180° (anti).

36. (a) Carbon tetrachloride rises to atmosphere and deplete the ozone layer. This depletion of ozone layer increases exposure of UV rays to human being which lead to increase of skin cancer, eye disease and disorder with disruption of the immune system.
37. (c) IUPAC name of DDT is 2,2-bis (4-chlorophenyl) 1,1,1-trichloroethane.
38. (b) Chloroform exposure may cause damage to liver and to kidney and some people develop sores when skin is immersed in chloroform.

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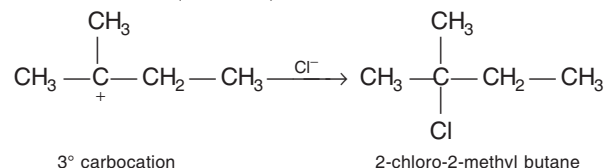
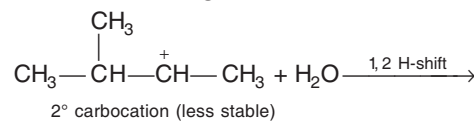
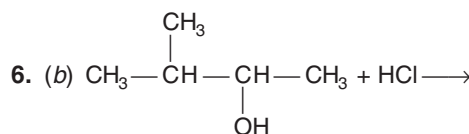
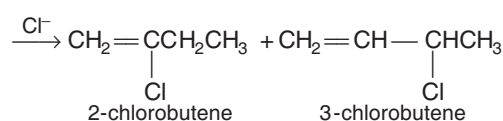
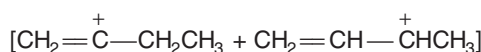
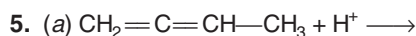
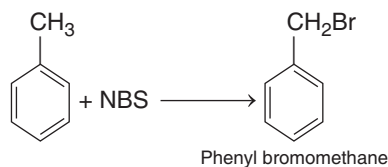
1. (c) Boiling point decreases with increase in branching.
2. (a) Due to stronger $-I$ effect of F than that of Cl, CHF_3 should be more acidic than CHCl_3 . But actually reverse is true.

This is due to $:\text{CCl}_3^-$ left after the removal of a proton from CHCl_3 is stabilised due to presence of d -orbitals in Cl than $:\text{CF}_3^-$ left after the removal of a proton from CHF_3 which is not stabilised due to the absence of d -orbitals on F.

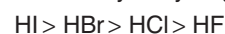


By Saytzeff's rule, substituted alkenes are more stable. Hence, out of *cis* and *trans* forms, *trans* product is more stable.

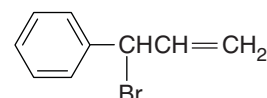
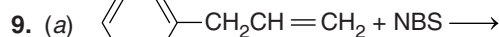
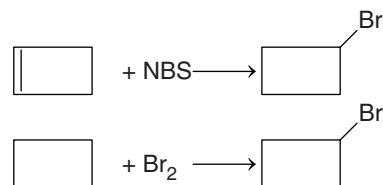
4. (c) NBS (N-bromosuccinimide) causes bromination at allylic position.



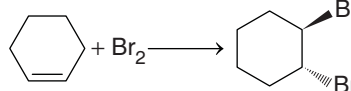
7. (d) Among hydrogen halides, as the size of halide ion increases, its reactivity towards ethyl alcohol also increases. Thus, the order of reactivity of hydrogen halides is



8. (d)



10. (b)



25

Alcohols, Phenols and Ethers

Alcohols

The compounds in which hydroxyl group is attached to C-atoms are called alcohols.

- (i) General formula: $C_nH_{2n+1}OH$ (ii) sp^3 -hybridised carbon, geometry: tetrahedral

Classification

Alcohols are divided into three types, i.e. monohydric, dihydric and trihydric alcohols.

Monohydric Alcohols

Monohydric alcohols may be divided into three types, i.e. 1° , 2° and 3° depending upon the carbon atom to which $—OH$ group is attached.

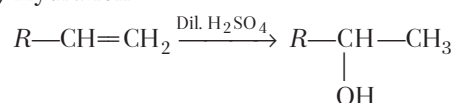
Preparation of Monohydric Alcohols

1. From alkane (Indirect conversion)

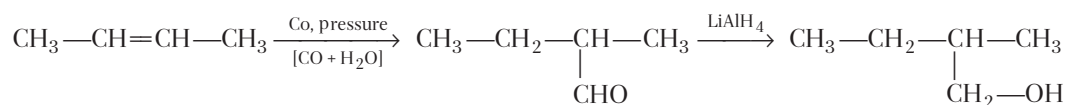


2. From alkene

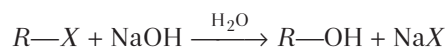
(a) Hydration



(b) Oxo reaction



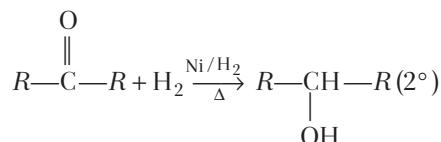
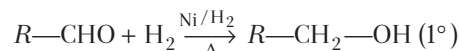
3. From alkyl halide



where, $R = 1^\circ, 2^\circ$ or 3° alkyl, aryl etc.

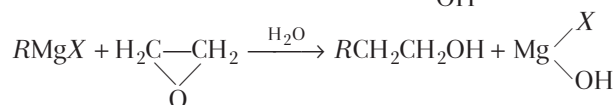
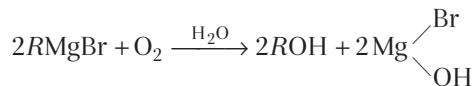
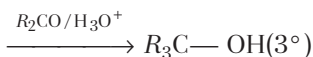
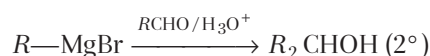
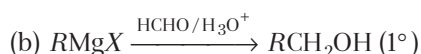
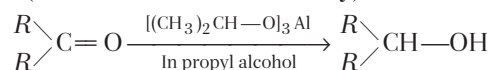
4. From carbonyl compounds

(a) Reduction

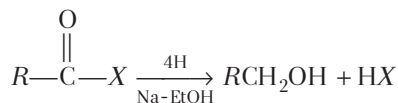
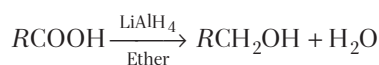


Reducing agents $Ni-H_2$, $LiAlH_4$ (in ether), $NaBH_4$, $Na-EtOH$, $Na-Hg$, H_2O can be used.

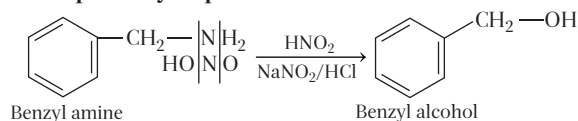
MPV (Meerwein-Ponndorf-Verley) reduction



5. From acid and its derivatives



6. From primary aliphatic amines

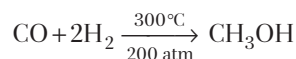
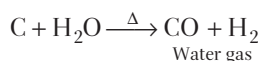


7. Industrial methods

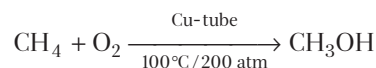
(a) Preparation of methanol

(i) Destructive distillation of wood

(ii) From water gas



(iii) Oxidation of methane



(b) Preparation of ethanol

(i) Fermentation of molasses

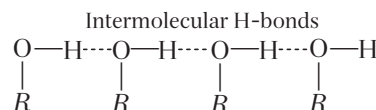
(ii) From starch

Physical Properties of Monohydric Alcohols

Monohydric alcohols have many physical properties which are given below :

(i) **Physical state** Lower alcohols are colourless, toxic liquids with burning taste whereas, higher alcohols are colourless, odourless and wax like solids.

(ii) **Boiling point** The boiling point of alcohols is much higher than that of corresponding hydrocarbons due to the presence of intermolecular H-bonding in alcohols.



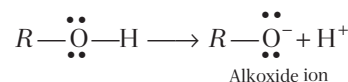
The boiling point for isomeric alcohols follows the order



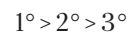
(iii) **Solubility** Lower members of alcohols are soluble in water due to the presence of intermolecular hydrogen bonding.

Acidic Character of Monohydric Alcohols

Alcohols are weak acids due to the presence of alkyl groups (exerting +I-effect) linked to —OH group that increases the electron density on oxygen thereby, decreasing the polarity of O—H bond.



The increasing order of acidic character of alcohols is as follows



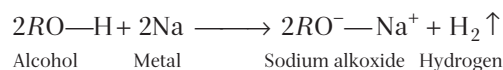
Alcohols are weaker acids than water because alkoxide ion is a strong conjugate base of alcohols as it readily accepts proton from a water molecule.

Chemical Properties of Monohydric Alcohols

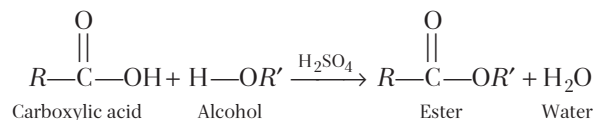
The chemical reactions of alcohols are due to the presence of polar $\rightarrow C-OH$ bond.

Reactions Involving Cleavage of O—H bond

(i) Reaction with metals

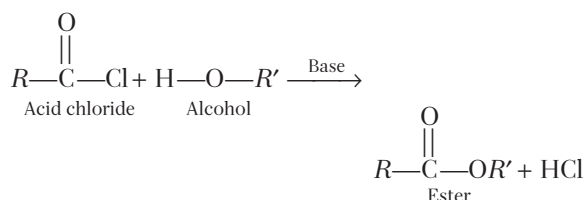


(ii) Reaction with carboxylic acids (Esterification)



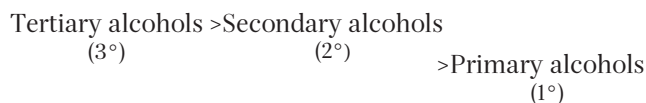
H_2SO_4 acts as a protonating as well as dehydrating agent.

(iii) Reaction with acid derivatives



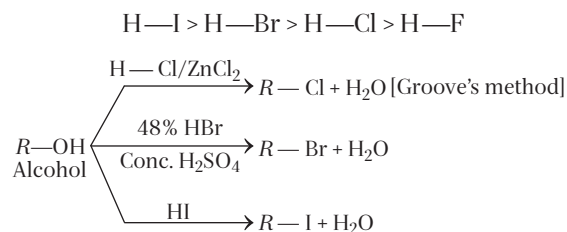
Reactions Involving Cleavage of C—O Bond

The order of reactivity of alcohols towards these type of reactions is as follows:

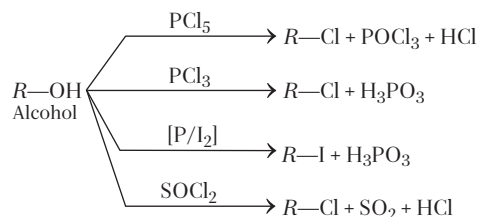


(i) Reaction with halogen acids

The reactivity order for the halogen acids is as follows:



These reactions follow $\text{S}_{\text{N}}1$ mechanism.

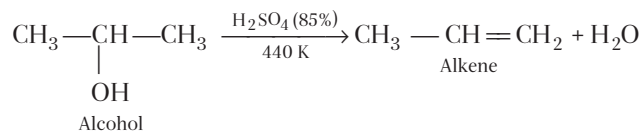
(ii) Reaction with phosphorus halides and thionyl chloride (SOCl_2)

Reactions Due to —OH Group

The reactivity order would be

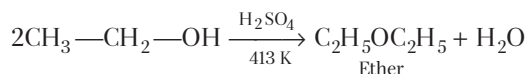
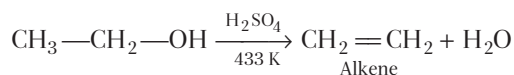
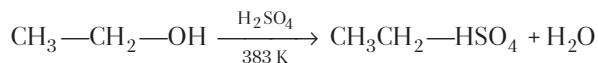


(i) Dehydration

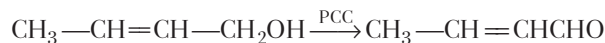
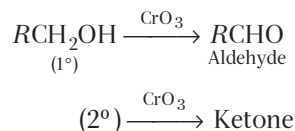


Note The carbocation formed during dehydration undergoes rearrangement resulting in the formation of more than one alkene.

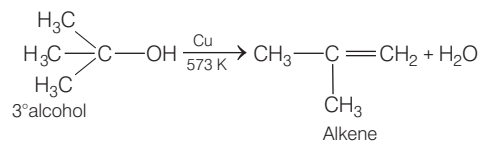
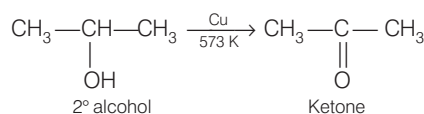
Reaction of ethanol (alcohol) with sulphuric acid at different temperatures



(ii) Oxidation of alcohols Oxidation of alcohols results in the formation of C=O bond.



NOTE $\text{CH}_3\text{CH}_2\text{OH} \xrightarrow[573 \text{ K}]{\text{Cu}} \text{CH}_3\text{CHO}$
1° alcohol Aldehyde



Identification of Primary, Secondary and Tertiary Alcohols

The primary, secondary and tertiary alcohols can be distinguished by

(i) **Victor Meyer's test** Alcohols when treated with P/I_2 and then with AgNO_2 and HNO_2 , a coloured product is produced with NaOH .

Primary alcohol $\longrightarrow$ Blood red product

Secondary alcohol $\longrightarrow$ Blue coloured product

Tertiary alcohol $\longrightarrow$ No colour

(ii) **Lucas test** The equimolar mixture of anhydrous ZnCl_2 and conc. HCl is called **Lucas reagent**. When alcohols are treated with it,

Primary alcohols $\longrightarrow$ No turbidity at room temperature

Secondary alcohols $\longrightarrow$ Turbidity after 5 min

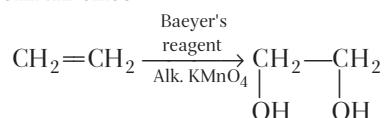
Tertiary alcohols $\longrightarrow$ Immediate turbid solution

Dihydric Alcohols

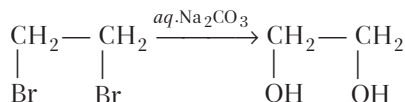
They are the compounds containing two —OH groups either on same carbon atom or different C-atom.
e.g. Ethylene glycol

Methods of Preparation of Ethylene Glycol

1. From alkenes



2. From halides

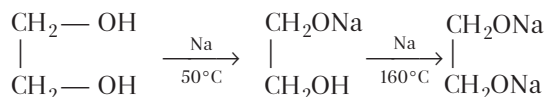


Physical Properties of Ethylene Glycol

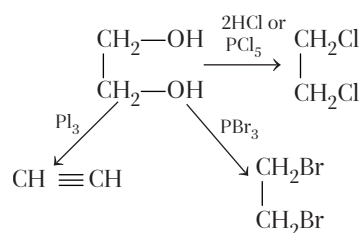
1. Colourless, syrupy liquid.
2. Soluble in water as well as in ethanol.
3. It is toxic as methanol.

Chemical Reactions of Ethylene Glycol

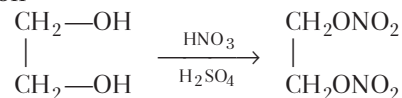
1. Reaction with metals



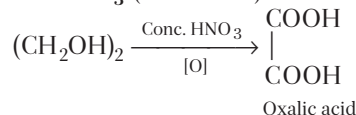
2. Reaction with HCl or phosphorus halides



3. Nitration

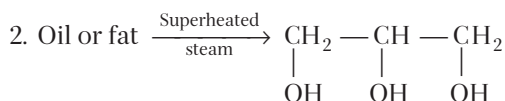
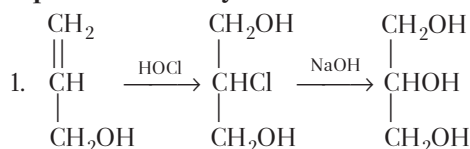


4. Reaction with HNO₃ (Oxidation)



Trihydric Alcohols

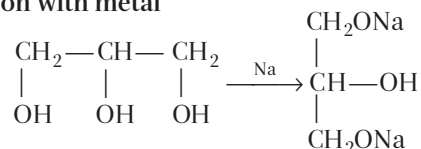
Preparation of Glycerol



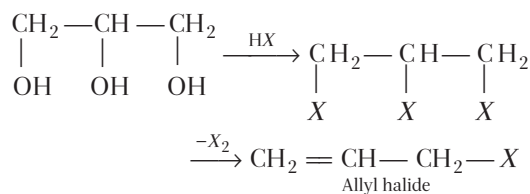
Chemical Reactions of Glycerol

Glycerol has two primary —OH groups and one secondary —OH group, which is little less reactive than the primary —OH groups.

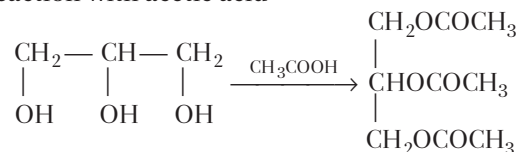
1. Reaction with metal



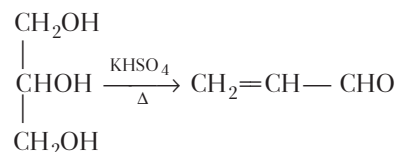
2. Reaction with HX



3. Reaction with acetic acid

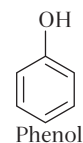


4. Dehydration



Phenols

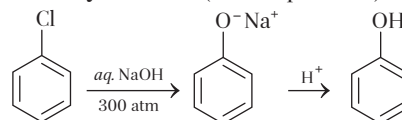
The organic compounds in which hydroxy group (—OH) is directly linked to a benzene ring are called phenols or carbolic acid.



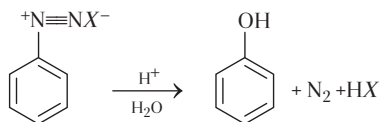
Preparation of Phenols

Phenols are prepared by the following methods:

(i) Hydrolysis of aryl halides (Dow's process)

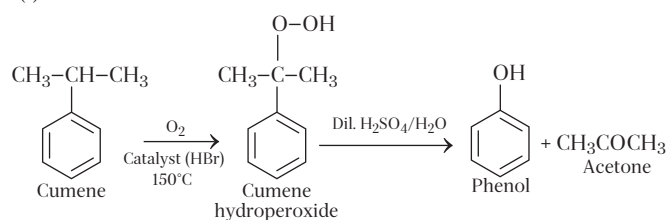


(ii) From diazonium salts

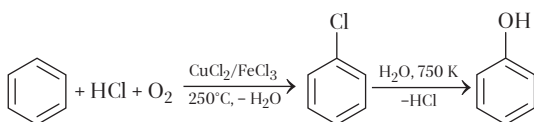
**Industrial Preparation of Phenols**

Phenols for industrial purposes can be manufactured in two ways:

(i) From cumene



(ii) From Raschig process through benzene



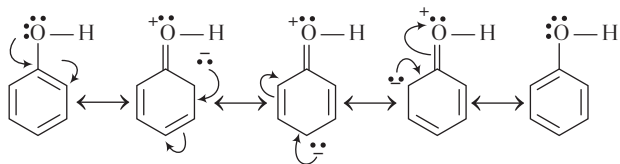
(iii) From coaltar Phenols can be obtained by the middle oil fraction of coaltar distillation.

Physical Properties of Phenols

Phenols are colourless crystalline solids or liquids with carbolic odour. The boiling point of phenols is higher than corresponding alcohols and arenes of compared masses due to the presence of intermolecular H-bonding. Phenols possess high dipole moments.

Acidic Character of Phenols

The acidic character of phenols is due to the conjugation between lone pair of electrons of oxygen and benzene nucleus.

**Effect of Substituents on Phenols**

The presence of electron withdrawing groups like $-\text{NO}_2$, $-\text{CN}$ etc., on the benzene ring stabilises the phenoxide ion by the dispersal of negative charge, thereby, increasing the acidic strength of phenol whereas presence

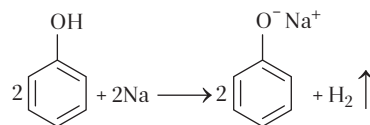
of electron donating groups, like $-\text{CH}_3$ etc., on the benzene ring destabilises the phenoxide ion by increasing the negative charge, thereby, decreasing the acidic strength of phenol.

Chemical Reactions of Phenol

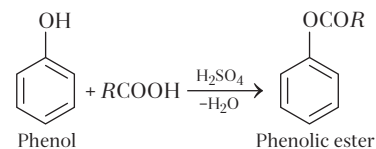
The chemical reactions of phenol are classified into two categories:

Reactions due to the cleavage of O — H bond

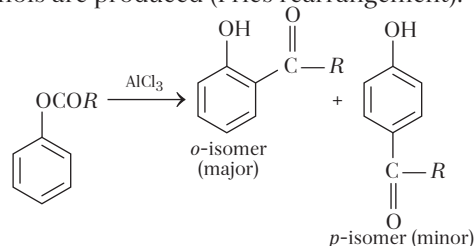
(a) Reaction with metals



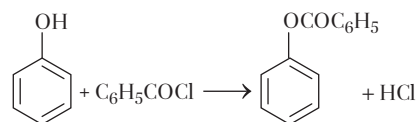
(b) Acylation



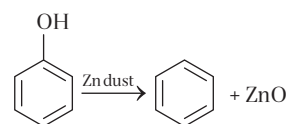
Phenolic esters when heated with AlCl_3 , *o* and *p*-acyl phenols are produced (Fries rearrangement).



(c) **Benzoylation** This reaction is known as Schotten-Baumann reaction.

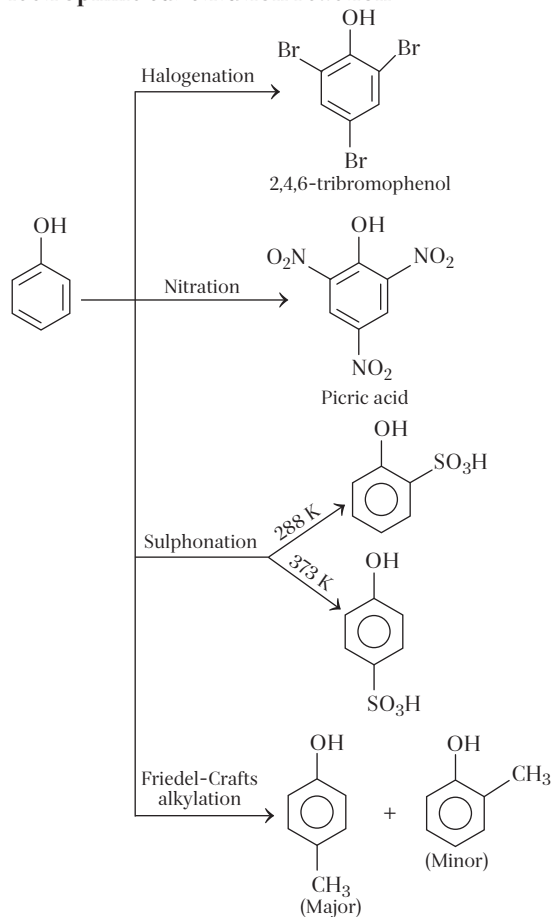


(d) Reaction with Zn dust

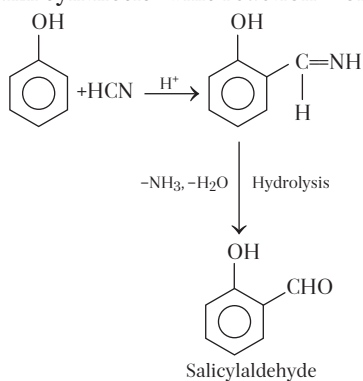
**Reactions due to Benzene Ring**

Phenols undergo the following reactions due to the presence of benzene ring :

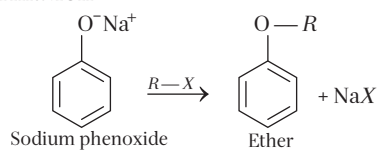
(a) Electrophilic substitution reaction



(b) Gattermann synthesis This reaction works as



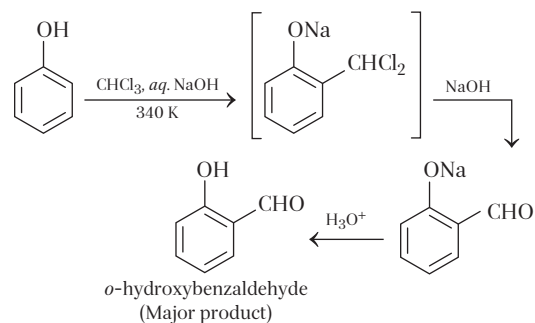
(c) Ether formation



Special Reactions of Phenols

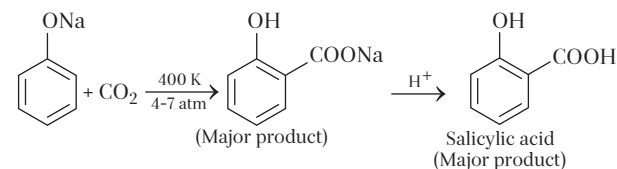
The special reactions of phenols can be summarised as:

(i) Reimer-Tiemann reaction This reaction looks like

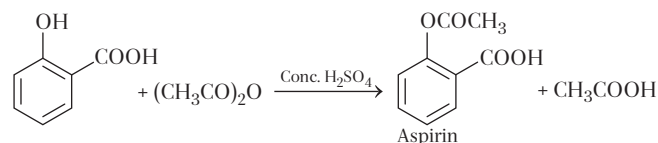


NOTE When CCl_4 in place of CHCl_3 is used in the reaction, salicylic acid is produced in the reaction.

(ii) Kolbe's reaction



NOTE Salicylic acid is used for the production of aspirin (2-acetoxy benzoic acid), a well known analgesic.



Distinguish Tests for Phenols and Alcohols

Phenols can be distinguished from alcohols through the following tests:

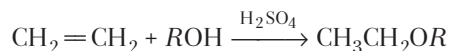
- Neutral FeCl_3 test** Phenols react with neutral FeCl_3 to produce a blue violet or green coloured complex. Alcohols do not react with FeCl_3 .
- Liebermann's test** When sodium nitrite dissolved in conc. H_2SO_4 and it is added to phenol, red/brown colouration is produced which changes to blue-green on the addition of a base. Alcohols do not respond to this test.
- Bromine water test** When bromine water is added to phenol, a white precipitate of 2, 4, 6-tribromophenol is produced. Alcohols do not respond to this test.

Ether

Dialkyl derivative of water is called ether. It is anhydride of alcohol.

Preparation

1. From alkenes

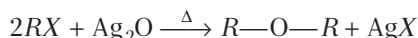


2. From alkyl halides

(a) Williamson ether synthesis

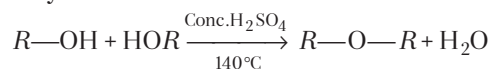


(b) Heating alkyl halides with silver oxide

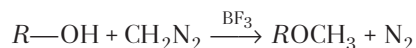


3. From alcohols

(a) Dehydration of alcohols



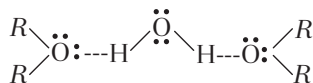
(b) Action of diazomethane on alcohols



Physical Properties of Ethers

Following are the main properties of ethers:

- (i) Dimethyl and ethyl methyl ethers are gases at ordinary temperature. Other lower members are colourless, pleasant smell, volatile liquids.
- (ii) Ethers have lower boiling points in comparison to corresponding alcohols due to the presence of weak dipole-dipole forces among intermolecular ethers.
- (iii) Ethers are soluble in water due to the formation of H-bonding with water molecules.



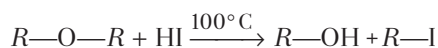
Solubility decreases with increase in the number of C-atoms.

Chemical Properties of Ethers

The chemical properties of ethers are classified into three categories:

Reactions Involving the Cleavage of C—O bond

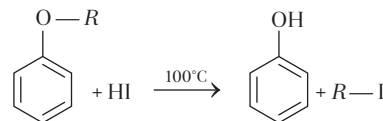
(a) Reaction with halogen acids



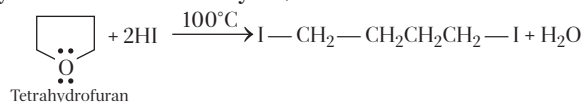
The reactivity order of halogen acids for this reaction are



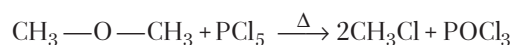
If one R group is phenyl and other is alkyl group, the cleavage of C—O bond produces phenol and alkyl halide.



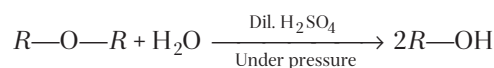
Cyclic ethers cleaved by HI/HBr to form dihaloalkanes.



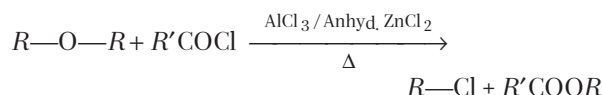
(b) Reaction with PCl_5



(c) Reaction with sulphuric acid



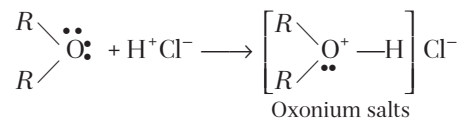
(d) Reaction with acid derivatives



Reactions due to Ethereal Oxygen

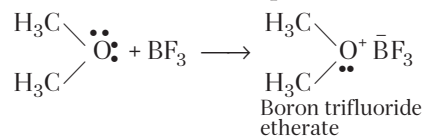
These reactions are due to the presence of two lone pairs on ethereal O-atom.

(a) Reaction with concentrated acids



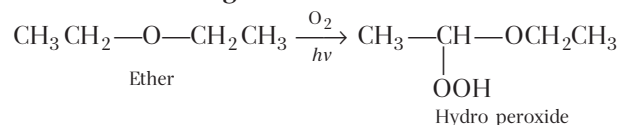
These salts stabilises by the anion of acids.

(b) Formation of coordination complexes

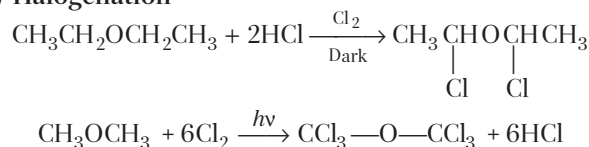


Reactions Due to Alkyl Group

(a) Action of air and light



(b) Halogenation



Practice Exercise

- Appropriate test to make distinction between methanol and methyl carbinol is
 - test with acetic acid
 - test with salicylic acid and conc. H_2SO_4
 - test with sodium
 - None of the above
- Which of the following alcohols is unable to turn orange colour of chromic acid to green?
 - Primary alcohol
 - Secondary alcohol
 - Tertiary alcohol
 - Allyl alcohol
- Match List I with List II and choose the correct code.

List I	List II
A. $\text{CH}_3\text{CH}_2\text{OH} \xrightarrow{[\text{O}]}$ Product-1	1. $\begin{array}{c} \text{H}_2\text{C} - \text{CH}_2 \\ \diagup \quad \diagdown \\ \text{O} \end{array}$ (Functional isomer)
B. $\text{CH}_3\text{COCH}_2\text{CH}_3 \xrightarrow{\text{Reduction}}$ Product-2	2. Optical isomer
C. $\text{CH}_3\text{CONH}_2 \xrightarrow{\text{Na/EtOH}}$ Product-3	3. Ethanol
D. $\text{CH}_2\text{O} \xrightarrow[\text{H}_2\text{O}]{(\text{i}) \text{CH}_3\text{MgI}}$ Product-4	4. Primary amine

Codes

	A	B	C	D
a.	1	2	4	3
b.	3	4	1	2
c.	1	2	3	4
d.	2	3	1	4

- Place the following alcohols in decreasing order of rate of dehydration with conc. H_2SO_4 :
 - $\text{CH}_3\text{CH}_2\text{CH}(\text{OH})\text{CH}_2\text{CH}_2\text{CH}_3$
 - $(\text{CH}_3)_2\text{C}(\text{OH})\text{CH}_2\text{CH}_2\text{CH}_3$
 - $(\text{CH}_3)_2\text{C}(\text{OH})\text{CH}(\text{CH}_3)_2$
 - $\text{CH}_3\text{CH}_2\text{CH}(\text{OH})\text{CH}(\text{CH}_3)_2$
 - $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_2\text{CH}_2\text{CH}_2\text{OH}$
 - $\text{III} > \text{II} > \text{IV} > \text{V} > \text{I}$
 - $\text{III} > \text{II} > \text{IV} > \text{I} > \text{V}$
 - $\text{III} > \text{II} > \text{I} > \text{IV} > \text{V}$
 - $\text{III} > \text{II} > \text{I} > \text{V} > \text{IV}$
- A compound with molecular formula $\text{C}_4\text{H}_{10}\text{O}_3$ is converted by the action of acetyl chloride to a compound of molecular mass 190. The original compound ($\text{C}_4\text{H}_{10}\text{O}_3$) has
 - one $-\text{OH}$ group
 - two $-\text{OH}$ groups
 - three $-\text{OH}$ groups
 - four $-\text{OH}$ groups
- Arrange the following compounds in increasing order of boiling point:

Propan-1-ol, butan-1-ol, butan-2-ol, pentan-1-ol

 - Propan-1-ol < butan-2-ol < butan-1-ol < pentan-1-ol
 - Propan-1-ol < butan-1-ol < butan-2-ol < pentan-1-ol
 - Pentan-1-ol < butan-2-ol < butan-1-ol < propan-1-ol
 - Pentan-1-ol < butan-1-ol < butan-2-ol < propan-1-ol
- An alcohol *A* when heated with copper gives a product *B* not having oxygen atom. *B* on ozonolysis gives two isomeric products *C* and *D*. *C* on oxidation gives a monobasic acid *E*, silver salt of which contains 59.6% Ag. The structure of *A* is
 - $\text{CH}_3 - \underset{\text{CH}_3}{\text{CH}} - \underset{\text{OH}}{\text{CH}} - \text{CH}_2 - \text{CH}_3$
 - $\text{CH}_3 - \underset{\text{CH}_3}{\overset{\text{OH}}{\text{C}}} - \text{CH}_2 - \text{CH}_2 - \text{CH}_3$
 - $\text{CH}_3 - \underset{\text{CH}_3}{\text{CH}} - \text{CH}_2 - \text{CH}_2 - \text{CH}_2\text{OH}$
 - $\text{CH}_3 - \underset{\text{CH}_3}{\text{CH}} - \text{CH}_2 - \text{O} - \text{CH}_2 - \text{CH}_2\text{OH}$
- Rate of dehydration of alcohols follows the order
 - $2^\circ > 1^\circ > \text{CH}_3\text{OH} > 3^\circ$
 - $3^\circ > 2^\circ > 1^\circ > \text{CH}_3\text{OH}$
 - $2^\circ > 3^\circ > 1^\circ > \text{CH}_3\text{OH}$
 - $\text{CH}_3\text{OH} > 1^\circ > 2^\circ > 3^\circ$
- Consider the following reaction,

$$\text{A} \xrightarrow[\text{H}_2\text{SO}_4]{\text{K}_2\text{Cr}_2\text{O}_7} \text{Acetone} \xrightarrow{\text{Oxidation}} \text{Acetic acid}$$

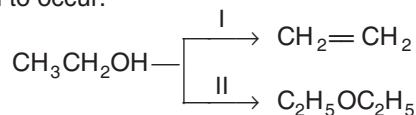
Identify *A* in the above reaction.

 - 1-propanol
 - 2-propanol
 - 2-butanol
 - Ethanol
- The alcohols, 1-propanol and 2-propanol are distinguished by
 - oxidation with conc. H_2SO_4 followed by reaction with Fehling's solution
 - oxidation with alk. KMnO_4 followed by reaction with Fehling's solution
 - reaction with iodine and aq. NaOH
 - reaction with Grignard reagent

11. The order for the acidic strength of 1° , 2° , 3° alcohols, H_2O and $\text{RC}\equiv\text{CH}$ is

a. $\text{RC}\equiv\text{CH} > 3^\circ > 2^\circ > 1^\circ > \text{H}_2\text{O}$
 b. $1^\circ > 2^\circ > 3^\circ > \text{H}_2\text{O} > \text{RC}\equiv\text{CH}$
 c. $\text{H}_2\text{O} > 1^\circ > 2^\circ > 3^\circ > \text{RC}\equiv\text{CH}$
 d. $3^\circ > 2^\circ > 1^\circ > \text{H}_2\text{O} > \text{RC}\equiv\text{CH}$

12. Identify the reaction conditions for the following reaction to occur.



a. $\text{I} \rightarrow \text{H}_2\text{SO}_4, 443 \text{ K}$ $\text{II} \rightarrow \text{H}_2\text{SO}_4, 443 \text{ K}$
 b. $\text{I} \rightarrow \text{H}_2\text{SO}_4, 413 \text{ K}$ $\text{II} \rightarrow \text{H}_2\text{SO}_4, 413 \text{ K}$
 c. $\text{I} \rightarrow \text{H}_2\text{SO}_4, 443 \text{ K}$ $\text{II} \rightarrow \text{H}_2\text{SO}_4, 413 \text{ K}$
 d. $\text{I} \rightarrow \text{H}_2\text{SO}_4, 413 \text{ K}$ $\text{II} \rightarrow \text{H}_2\text{SO}_4, 443 \text{ K}$

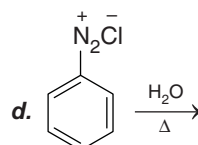
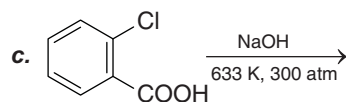
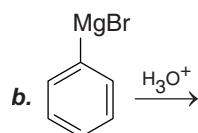
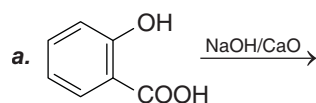
13. Which of the following reagents cannot be used to distinguish between phenol and benzyl alcohol?

a. $\text{Br}_2 / \text{CCl}_4$ b. NaOH
 c. NaHCO_3 d. FeCl_3 (neutral)

14. For the reaction, Phenol $\longrightarrow$ Cyclohexanol, the $s : p$ ratio of the carbon attached to $-\text{OH}$ group changes from ... to ...

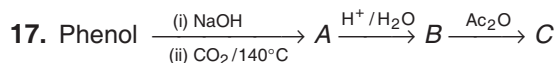
a. $\left(\frac{1}{3} : \frac{2}{3}\right) \left(\frac{1}{4} : \frac{3}{4}\right)$ b. $\left(\frac{1}{3} : \frac{2}{3}\right) \left(\frac{1}{2} : \frac{1}{2}\right)$
 c. $\left(\frac{1}{2} : \frac{1}{2}\right) \left(\frac{1}{3} : \frac{2}{3}\right)$ d. None of these

15. In which of the following reactions, phenol is not obtained?



16. Sodium phenoxide reacts with CO_2 at 400 K and 4.7 atm pressure to give

a. catechol b. salicylaldehyde
 c. sodium salicylate d. benzoic acid



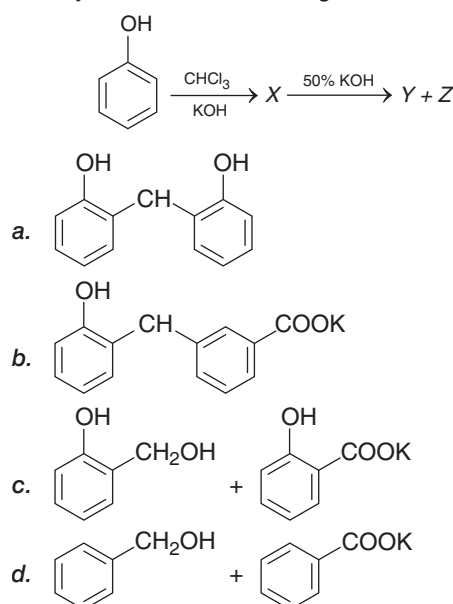
In this reaction, the end product C is

a. salicylaldehyde b. salicylic acid
 c. phenyl acetate d. aspirin

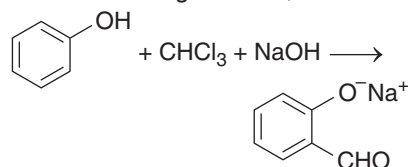
18. Phenol on heating with CHCl_3 and NaOH gives salicylaldehyde. This reaction is called

a. Reimer-Tiemann reaction
 b. Claisen reaction
 c. Cannizzaro reaction
 d. Hell-Volhard-Zelinsky reaction

19. Identify Y + Z in the following reaction,



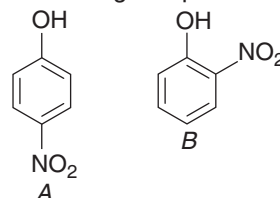
20. Consider the following reaction,



The electrophile involved in the above reaction is

a. dichloromethyl cation, CHCl_2^+
 b. formyl cation, CHO^+
 c. trichloromethyl anion, CCl_3^-
 d. dichlorocarbene, $:\text{CCl}_2$

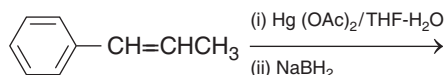
21. Consider the following compounds.



Among the compounds *A* and *B*, the vapour pressure of *B* at a particular temperature is

- higher than that of *A*
- lower than that of *A*
- same as that of *A*
- None of the above

22. Consider the following reaction,

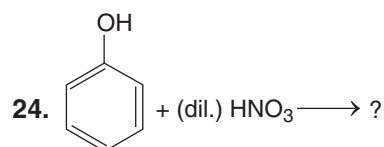


The major product formed in the above reaction is

-
-
-
-

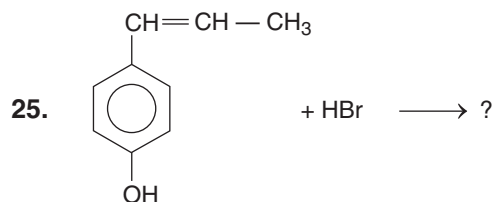
23. Which is/are the correct method(s) to synthesise phenol?

-
-
-
- All of the above



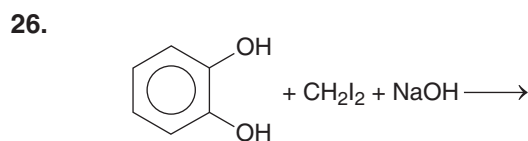
Select the product(s) formed in the above reaction.

-
-
-
- Both (a) and (c)



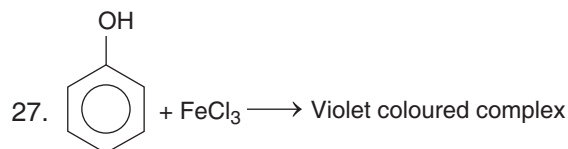
What will be the product formed in the above reaction?

-
-
-
-



In the given reaction, the product is

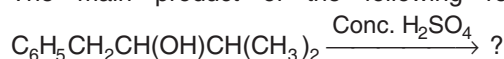
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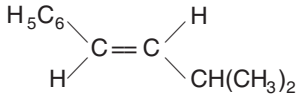
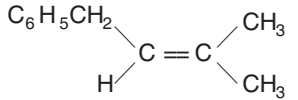
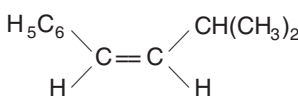
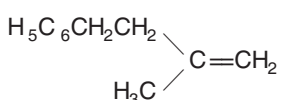


The formula of the complex formed in the above reaction, is

- a. $[\text{Fe}(\text{OC}_6\text{H}_5)_3]^{3-}$ b. $[\text{Fe}(\text{OC}_6\text{H}_5)_3]^{3+}$
 c. $[\text{Fe}(\text{OC}_6\text{H}_5)_6]^{3-}$ d. $[\text{Fe}(\text{OC}_6\text{H}_5)_6]^{3+}$

28. The main product of the following reaction is



- a. 
 b. 
 c. 
 d. 

29. Glycerol on treatment with excess HI gives

- a. 1, 2, 3-triiodopropane b. 1, 3-diiodopropane
 c. 2-iodopropane d. 3-iodopropane

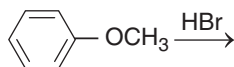
30. Etherates are

- a. ethers
 b. solution in ethers
 c. complexes of ethers with Lewis acid
 d. complexes of ethers with Lewis base

31. Compound $\text{Ph}-\text{O}-\overset{\text{O}}{\parallel}{\text{C}}-\text{Ph}$ can be prepared by the reaction of

- a. phenol and benzoic acid in the presence of NaOH
 b. phenol and benzoyl chloride in the presence of pyridine
 c. phenol and benzoyl chloride in the presence of ZnCl_2
 d. phenol and benzaldehyde in the presence of palladium

32. Consider the following reaction,



The products formed in the above reaction are

- a. -OH and CH_3Br b. -Br and CH_3Br
 c. -OCH₃ and H_2 d. -Br and CH_3OH

33. Anisole on reaction with HI forms

- a. benzene and methyl iodide
 b. phenol and methanol
 c. iodobenzene and methanol
 d. phenol and methyl iodide

34. The reaction of $\text{H}_2\text{C}-\underset{\text{O}}{\text{CH}_2}$ with $\text{H}_2\text{O}/\text{R}-\text{MgX}$

produces

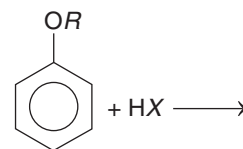
- a. RCHOHR
 b. RCHOHCH_3
 c. $\text{R}_2\text{CHCH}_2\text{OH}$
 d. $\text{RCH}_2\text{CH}_2\text{OH}$

35. **Assertion (A)** The cleavage of C—O bond in ethers takes place under drastic condition with excess of hydrogen halides.

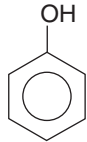
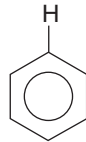
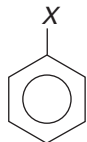
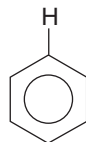
Reason (R) Ethers are the most reactive among all the functional groups.

- a. Both A and R are correct and R is the correct explanation of A
 b. Both A and R are correct but R is not the correct explanation of A
 c. A is correct but R is incorrect
 d. R is correct but A is incorrect

36.

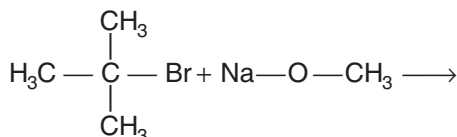


In the above reaction, the products are

- a.  + RX b.  + RX
 c.  + ROH d.  + ROX

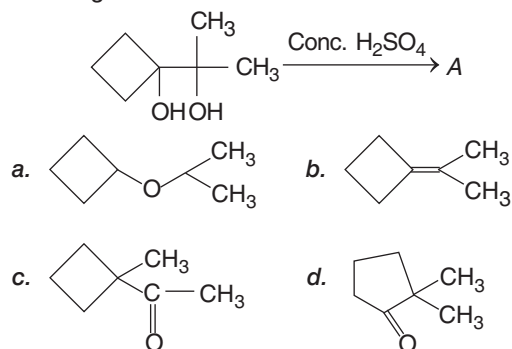
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1. What will be the product of the following reaction? [2014]



- a. $\text{CH}_3-\underset{\text{CH}_3}{\text{C}}=\text{CH}_2$ b. $\text{CH}_3-\text{O}-\underset{\text{CH}_3}{\overset{\text{CH}_3}{\text{C}}}-\text{CH}_3$
 c. $\text{CH}_3-\text{CH}_2-\text{CH}_2-\text{CH}_3$ d. $\text{CH}_3-\underset{\text{CH}_3}{\text{CH}}-\text{CH}_3$

2. Identify the correct product formed during the following reaction: [2014]



3. When 2-methyl propan-2-ol is treated with a mixture of conc. HCl and ZnCl_2 , turbidity appears immediately due to the formation of [2013]

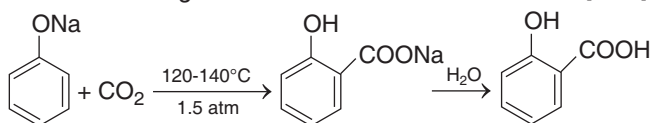
- a. 2-methyl propane
 b. 2-methyl propene
 c. 2-methyl-2-chloropropane
 d. 2-chlorobutane

4. What will be the correct relation between products when 2-methyl cyclohexene is treated with (i) B_2H_6 in the presence of $\text{H}_2\text{O}_2/\text{OH}^-$ and (ii) $\text{H}_2\text{O}/\text{H}_2\text{SO}_4$?

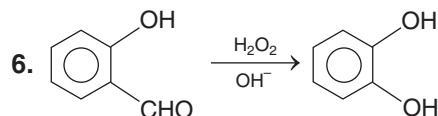
(Also consider stereochemistry of product) [2013]

- a. They are metamers
 b. They are tautomers
 c. They are functional isomers
 d. They are positional isomers

5. The following reaction is known as [2012]



- a. Friedel-Crafts reaction
 b. Kolbe reaction
 c. Reimer-Tiemann reaction
 d. Wittig reaction



[2011]

This reaction is called

- a. Reimer-Tiemann reaction
 b. Liebermann's nitroso reaction
 c. Dakin reaction
 d. Lederer-Manasse reaction

7. 3-methyl-2-butanol on treatment with HCl gives (major product) [2010]

- a. 2-chloro-2-methyl butane
 b. 2-chloro-3-methyl butane
 c. 2, 2-dimethyl pentane
 d. None of the above

8. Phenol can be tested by [2010]

- a. Liebermann's nitroso test
 b. FeCl_3 solution
 c. bromine water
 d. All of the above

9. Phenol reacts with PCl_5 to give mainly [2009]

- a. *p*-chlorophenol
 b. chlorobenzene
 c. *o* and *p*-chlorophenol
 d. triphenyl phosphate

10. The IUPAC name of $\text{C}_2\text{H}_5-\text{O}-\text{CH}(\text{CH}_3)_2$ is

- a. ethoxy propane
 b. 1,1-dimethyl ether
 c. 2-ethoxy isopropane
 d. 2-ethoxy propane [2008]

11. The compound which gives the most stable carbonium ion on dehydration is [2007]

- a. $\text{CH}_3\text{CH}(\text{CH}_3)\text{CH}_2\text{OH}$
 b. $(\text{CH}_3)_3\text{COH}$
 c. $\text{CH}_2=\text{CHCH}_2\text{CH}_2\text{OH}$
 d. $\text{CH}_3\text{CHOHCH}_2-\text{CH}_3$

12. Pinacol is [2006]

- a. 3-methylbutan-2-ol
 b. 2, 3-dimethyl-2, 3-butanediol
 c. 2, 3-dimethyl-2-propanone
 d. None of the above

13. Grignard reagent reacts with HCHO to produce a/an [2005]

- a. secondary alcohol
 b. anhydride
 c. acid
 d. primary alcohol

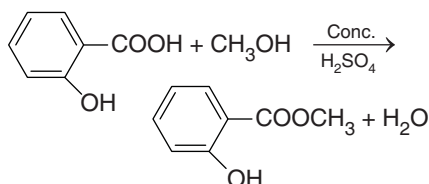
14. The product obtained by heating diethyl ether with HI is [2005]

- a. $\text{C}_2\text{H}_5\text{I}$ b. $\text{C}_2\text{H}_5\text{OH}$
 c. $\text{C}_2\text{H}_5\text{OH} + \text{C}_2\text{H}_5\text{I}$ d. $\text{C}_2\text{H}_5-\text{C}_2\text{H}_5$

Answer with Solutions

Practice Exercise

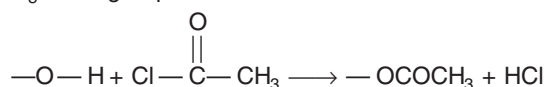
1. (b)



2. (c) Tertiary alcohol offers resistance to oxidation. Hence, Cr^{3+} ions are not formed and the solution does not turn green.
3. (a) Product-1: acetaldehyde (functional isomer of epoxide),
Product-2: *s*-butyl alcohol,
Product-3: ethylamine and
Product-4: ethyl alcohol.
4. (b) The alcohols (II) and (III) both are 3° , but alcohol (III) gives a more substituted alkene. Alcohol (IV) and (I) both are 2° , but alcohol (IV) can give a more substituted alkene and alcohol (V) is 1° . Rate of dehydration of alcohols with conc. H_2SO_4 follows the order

$$3^\circ > 2^\circ > 1^\circ$$

5. (b) In acetylation, replacement of H of O—H occurs by CH_3CO — group.



i.e. H-atom of mass 1 amu is lost and an acetyl group of mass 43 amu is added. Thus, there is a net gain of $43 - 1 = 42$ amu for every acetyl group introduced. Mass of difference of final product and original compound is $190 - 106 = 84$. Hence, number of —OH groups

$$= \frac{84}{42} = 2$$

6. (a) With the increase in molecular mass, the boiling points also increase. Amongst isomeric alcohols, the boiling point decreases with branching due to decrease in surface area.
7. (b) Molecular mass of silver salt of *E* = $\frac{108}{59.6} \times 100 = 181$

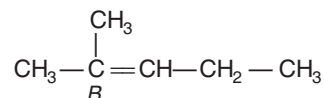
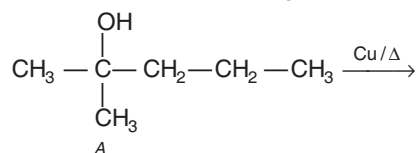
$$\text{RCOOAg} = 181$$

$$R = 181 - (108 + 12 + 32) = 181 - 152 = 29$$

Hence, acid salt is $\text{CH}_3\text{—CH}_2\text{—COOAg}$.

Corresponding to this salt, acid is $\text{CH}_3\text{—CH}_2\text{—COOH}$, which is obtained by oxidation of (C), an ozonolysis product. Thus, (C) is $\text{CH}_3\text{—CH}_2\text{—CH=O}$.

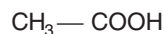
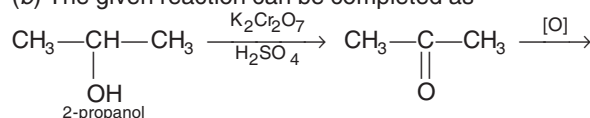
Since, (C) and (D) are isomers, D is $\text{CH}_3\text{—}\overset{\text{CH}_3}{\underset{\text{O}}{\text{C}}}=\text{O}$. On the basis of C and D, we can get product B and finally A.



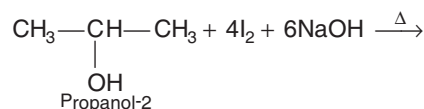
8. (b) The order of dehydration of alcohols is

$$3^\circ > 2^\circ > 1^\circ > \text{CH}_3\text{OH}$$

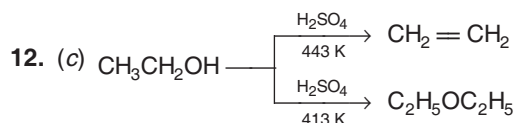
9. (b) The given reaction can be completed as



10. (c) $\text{CH}_3\text{—CH}_2\text{—CH}_2\text{OH} + \text{I}_2 + \text{NaOH} \xrightarrow{\Delta}$ No reaction
Propanol



11. (c) $\text{H}_2\text{O} > 1^\circ > 2^\circ > 3^\circ > \text{R—C}\equiv\text{CH}$

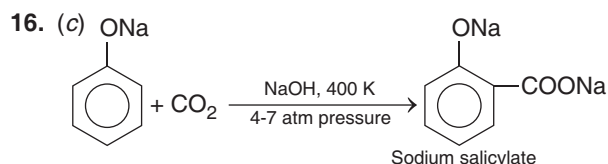


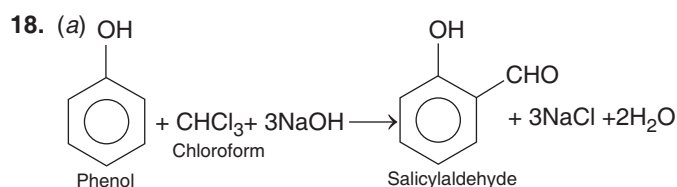
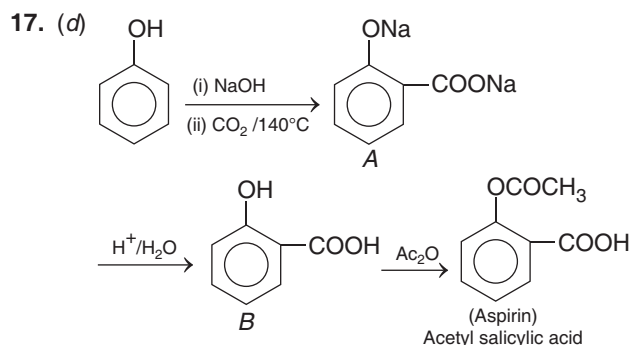
13. (c)

(a) (b) and (d) all react with phenol and none of them reacts with benzyl alcohol. But (c) does not react with either phenol or benzyl alcohol.

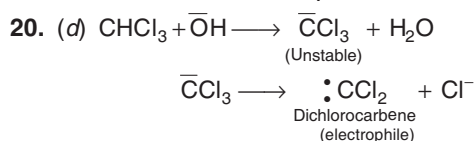
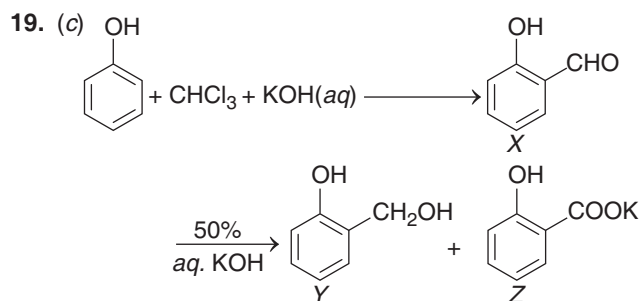
14. (a) Carbon changes its hybridisation state from sp^2 to sp^3 ,
i.e. $\left(\frac{1}{3} : \frac{2}{3}\right)$ to $\left(\frac{1}{4} : \frac{3}{4}\right)$.

15. (b) $\text{C}_6\text{H}_5\text{MgBr} \xrightarrow{\text{H}_3\text{O}^+} \text{C}_6\text{H}_6$

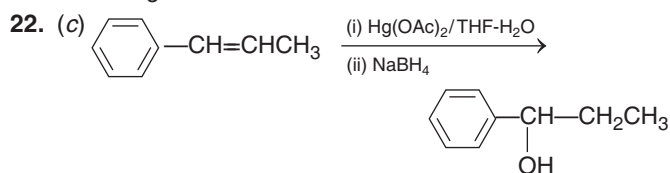




This reaction is called Reimer-Tiemann reaction.

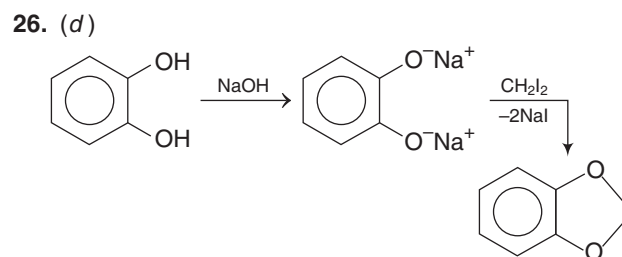
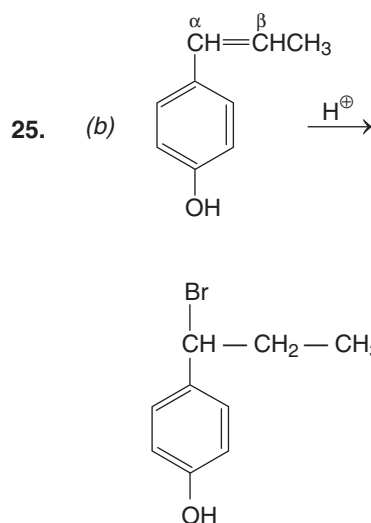
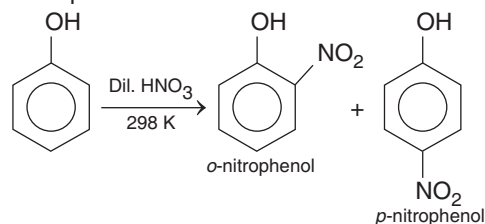


21. (a) In *o*-nitrophenol, intramolecular H-bonding is present between $-\text{OH}$ and $-\text{NO}_2$ groups, whereas *p*-nitrophenol forms intermolecular H-bonding. Thus, vapour pressure of *B* is higher than that of *A*.

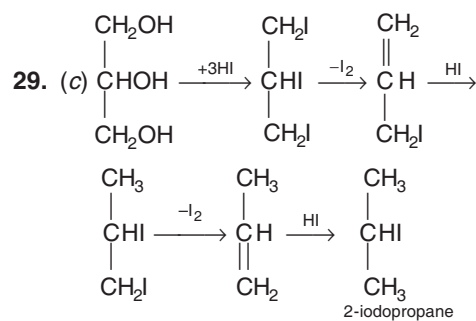
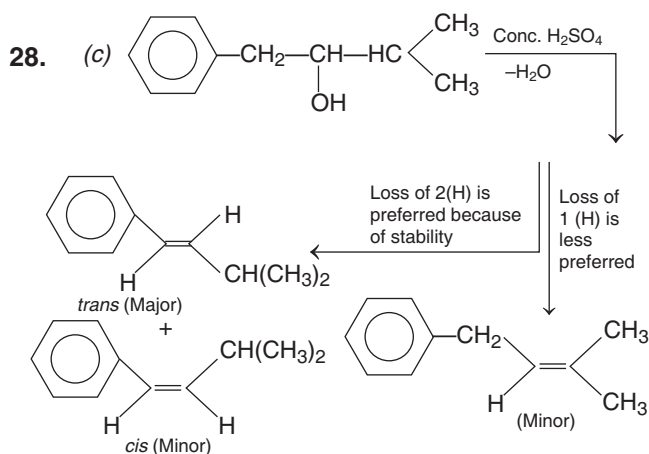


23. (d) All the given methods are correct.

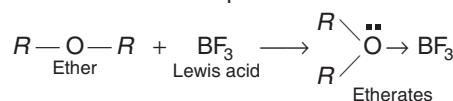
24. (d) Nitration of phenol with dilute nitric acid at low temperature (298 K), phenol yields a mixture of *ortho* and *para*-nitrophenol.



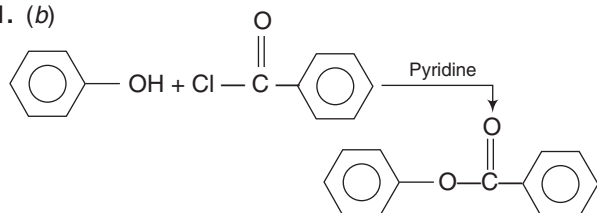
27. (c) $[\text{Fe}(\text{OC}_6\text{H}_5)_6]^{3-}$



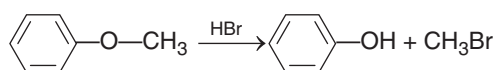
30. (c) Etherates are the complexes of ethers with Lewis acid.



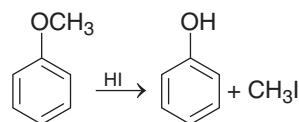
31. (b)



32. (a) The complete reaction is as follows



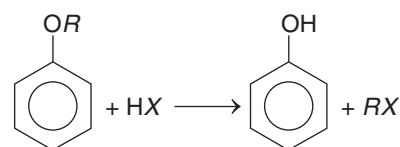
33. (d)



34. (d) $H_2C-CH_2 \xrightarrow[H_2O]{R-MgX} R-CH_2CH_2-OH + MgX(OH)$

35. (c) Ethers are the least reactive among all the functional groups. The cleavage of C—O bond in ethers takes place under drastic conditions with excess of hydrogen halides.

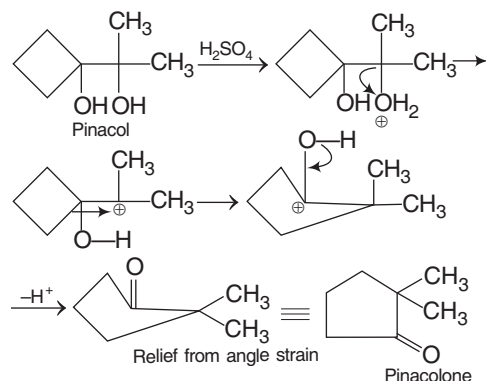
36. (a)



BITSAT Archives

1. (a) When tertiary alkyl halide is treated with sodium alkoxide then elimination reaction completes over substitution reaction because alkoxides are not only nucleophiles but strong base as well. Therefore, alkenes are formed instead of ethers.

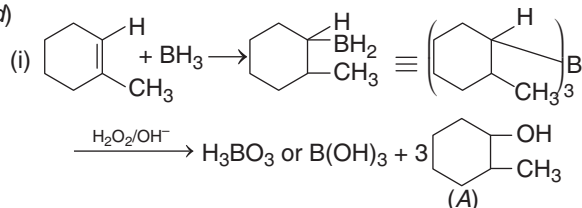
2. (d)



Basis of carbocationic rearrangement is due to relief from angle strain.

3. (c) When 2-methyl propan-1-ol is treated with a mixture of conc. HCl and $ZnCl_2$ (Lucas reagent) then *tert*-alkyl halide is formed and produced turbidity due to its less solubility.

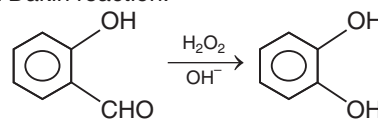
4. (d)



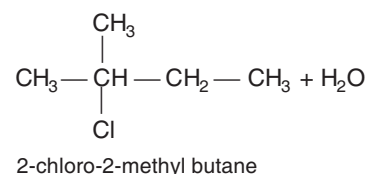
A and B have difference in position of OH only so, A and B are positional isomers.

5. (b) At 120-140°C temperature and 1.5 atm pressure, sodium phenoxide reacts with CO_2 to yield sodium salicylate which on further hydrolysis give to salicylic acid. This reaction is known as Kolbe's reaction.

6. (c) The reduction of —CHO group to —OH by H_2O_2/OH^- is called Dakin reaction.



7. (a) $CH_3-CH(CH_3)-CH(OH)-CH_3 + HCl \longrightarrow$



8. (d) Phenol can be tested by all the given reagents.

9. (d) $3C_6H_5OH + PCl_5 \xrightarrow[\text{Nucleophilic attack}]{-3HCl} (C_6H_5O)_3PCl_2 \xrightarrow[-2HCl]{H_2O} (C_6H_5O)_3PO$ Triphenyl phosphate

10. (d) $C_2H_5-O-\overset{1}{\underset{3}{\overset{2}{CH}}}(CH_3)_2$
2-ethoxy propane

11. (b) $(CH_3)_3C-OH \xrightarrow[-H_2O]{+H^+} (CH_3)_3C^+$
Tertiary alcohol $\rightarrow$ 3° carbocation (more stable)

12. (b)

13. (d) $HCHO + R-MgX \xrightarrow{H_3O^+} R-CH_2OH$
(1° alcohol)

14. (c) $C_2H_5-O-C_2H_5 \xrightarrow[100^\circ C]{HI} C_2H_5-OH + C_2H_5-I$

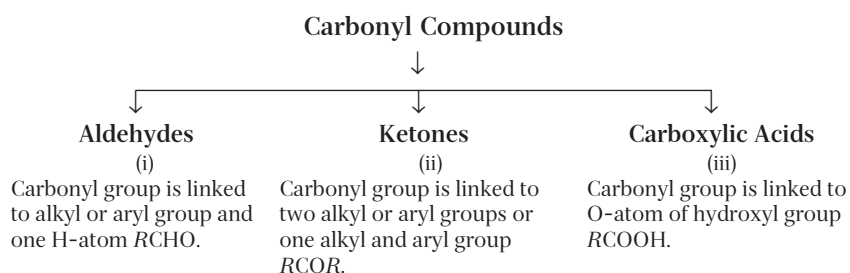
26

Aldehydes and Ketones

Carbonyl Compounds

Carbonyl compounds contain carbon-oxygen double bond, >C=O .

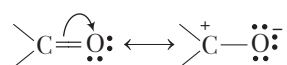
These are divided into three classes:



Nature of >C=O Group

The >C=O group is highly polar in nature due to the high electronegativity of oxygen atom, so that, C-atom acquires partially positive charge and O-atom acquires partially negative charge.

The two canonical forms of carbonyl group are:



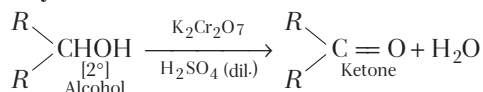
Preparation of Aliphatic and Aromatic Aldehydes and Ketones

The general methods for the preparation for aldehydes and ketones are as follows:

Aldehydes and Ketones

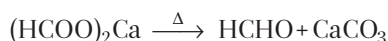
These methods are as follows :

- (i) **From alcohols** The controlled oxidation of primary alcohols, RCH_2OH yields aldehydes while secondary alcohols yield ketones.

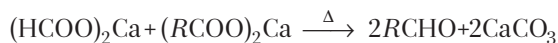


- (ii) **From carboxylic acids**

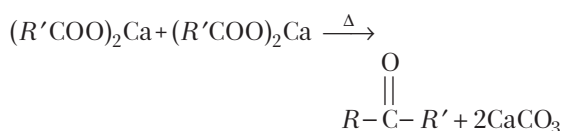
- (a) **Dry distillation of calcium salts of fatty acids** Dry distillation of calcium formate yields formaldehyde only.



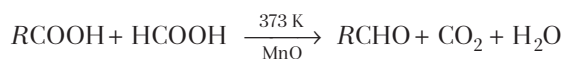
Dry distillation of calcium formate with calcium salts of fatty acids yields other aldehydes.



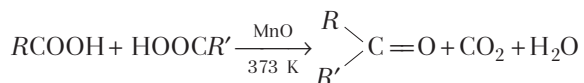
Pyrolysis of a mixture of calcium salt of different fatty acids yields mixed ketones.



- (b) **Catalytic decomposition of acids** Reaction of acids with formic acid yields aldehyde.



Reaction of two molecules of acid yields ketones.

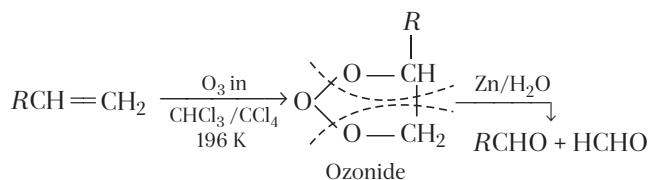


- (iii) **From hydrocarbons** Aldehydes and ketones can be prepared from alkenes and alkynes.

- (a) **Alkenes**

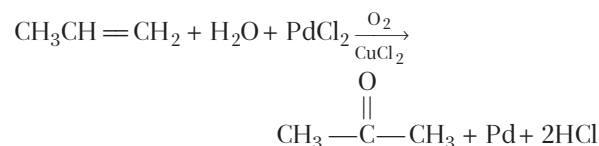
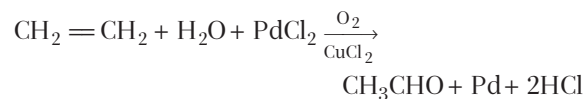
Carbonyl compounds can be prepared from alkenes in the following ways:

Reductive ozonolysis of alkenes



Wacker's process [Oxidation of alkenes with $\text{PdCl}_2/\text{CuCl}_2$]

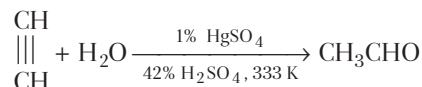
The reaction is as follows:



- (b) **Alkynes**

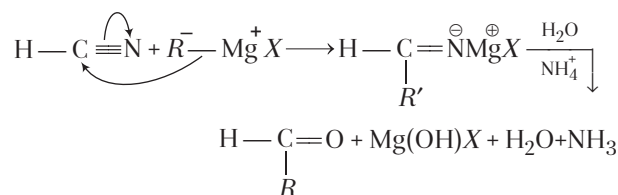
Carbonyl compounds can be prepared from alkynes in the following ways:

Hydration of alkynes The reaction works as



- (iv) **From Grignard's reagents**

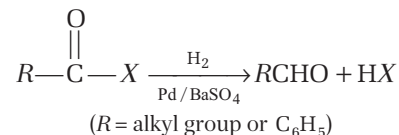
- (a) **Cyanides** Addition of RMgX on HCN yields aldehydes.



Addition of $\text{R}'\text{MgX}$ on RCN gives ketones.

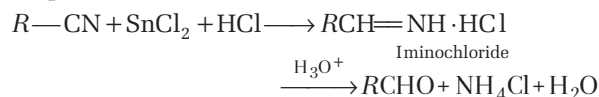
Preparation of Aldehydes Only

- (i) **From acyl chloride**

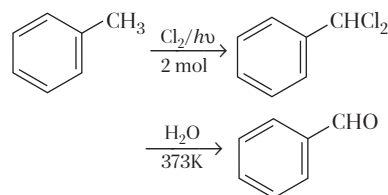


This reaction is called **Rosenmund reduction**.

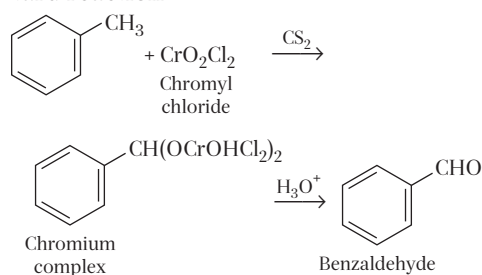
- (ii) **Stephen reaction**



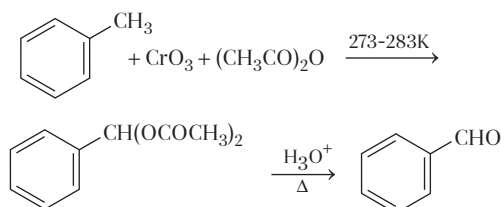
- (iii) **By side chain chlorination followed by hydrolysis**



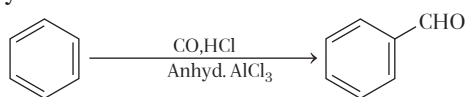
(iv) Etard reaction



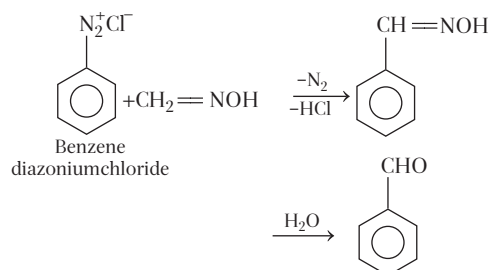
(v) By oxidation of methyl benzene



(vi) By Gattermann-Koch reaction

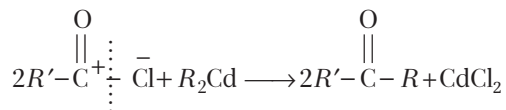


(vii) From diazonium salts

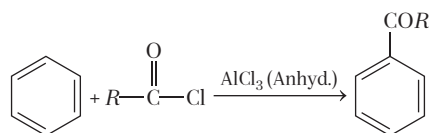


Preparation of Ketones Only

(i) From acid chlorides



(ii) By Friedel-Crafts acylation



Physical Properties of Aldehydes and Ketones

(i) **Boiling points** Boiling points of aldehydes and ketones are higher than that of hydrocarbons and ethers but lower than alcohols of comparable molecular masses. This is because of weak intermolecular interactions due to dipole-dipole interactions and absence of H-bonding.

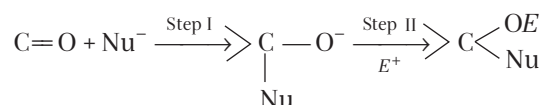
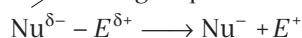
(ii) **Solubility** Solubility decreases with increase in molecular mass or size of alkyl group. Aldehydes and ketones containing upto 4C-atoms are soluble in water.

Chemical Reactions of Aldehydes and Ketones

Due to the presence of polar >C=O group, aldehydes and ketones exhibits the following reactions:

Nucleophilic Addition Reactions

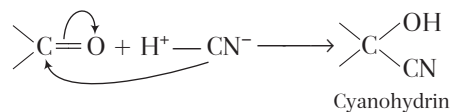
Carbonyl compounds undergo nucleophilic addition reactions in which nucleophile attack electrophilic C-atom of >C=O group.



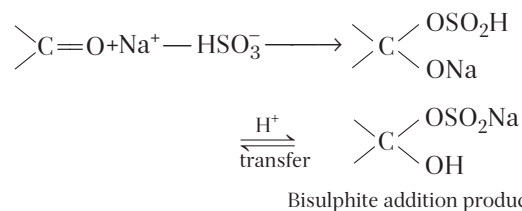
In general, ketones are less reactive in nucleophilic addition reactions as compared to aldehydes. *This is because*

- (i) Aldehydes have only one electron pair releasing group while ketones have two.
- (ii) Steric hindrance is less in aldehydes compared to ketones.

(a) **Addition of HCN** The reaction looks like



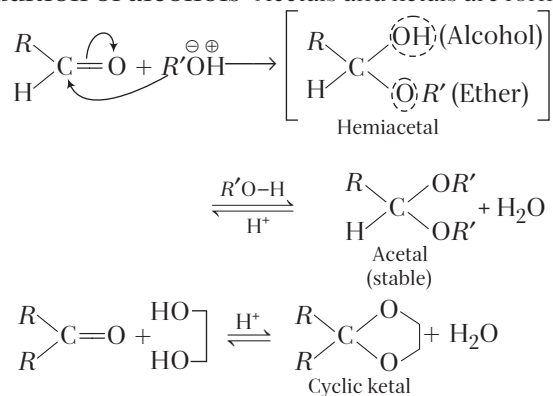
(b) **Addition of NaHSO₃** The reaction looks like



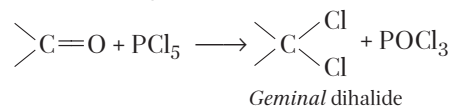
Nucleophilic Addition-Elimination Reaction

In these reactions, additions of nucleophile on electrophilic C-atom of carbonyl group leads to the elimination of O-atom in the form of water, H_2O .

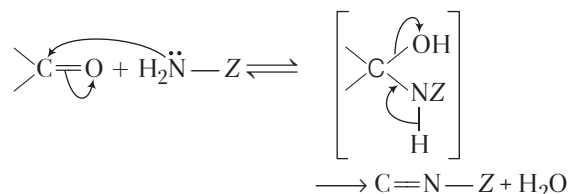
(i) **Addition of alcohols** Acetals and ketals are formed.



(ii) **Addition of PCl_5** Geminal dihalides are produced.



(iii) **Addition of NH_3 and its derivatives $[H_2N-Z]$**



Here, Z may be

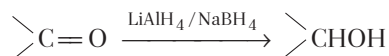

Some N-Substituted Derivatives of Aldehydes and Ketones ($>C=N-Z$)

Z	Reagent name	Carbonyl derivative	Product name
-H	Ammonia	$\begin{array}{c} >C=NH \\ >C=NH \end{array}$	Imine
-R	Amine	$\begin{array}{c} >C=NR \\ >C=NR \end{array}$	Substituted imine (Schiff's base)
-OH	Hydroxyl amine	$\begin{array}{c} >C=N-OH \\ >C=N-OH \end{array}$	Oxime
$-NH_2$	Hydrazine	$\begin{array}{c} >C=N-NH_2 \\ >C=N-NH_2 \end{array}$	Hydrazone
$-HN-\text{C}_6\text{H}_5$	Phenyl hydrazine	$\begin{array}{c} >C=N-NH-\text{C}_6\text{H}_5 \\ >C=N-NH-\text{C}_6\text{H}_5 \end{array}$	Phenyl hydrazone
$-HN-\text{C}_6\text{H}_3(\text{NO}_2)_2$	2,4-dinitrophenyl hydrazine	$\begin{array}{c} >C=N-NH-\text{C}_6\text{H}_3(\text{NO}_2)_2 \\ >C=N-NH-\text{C}_6\text{H}_3(\text{NO}_2)_2 \end{array}$	2,4-dinitrophenyl hydrazone
$-NH-\overset{O}{\parallel}C-NH_2$	Semicarbazide	$\begin{array}{c} >C=N-NH-\overset{O}{\parallel}C-NH_2 \\ >C=N-NH-\overset{O}{\parallel}C-NH_2 \end{array}$	Semicarbazone

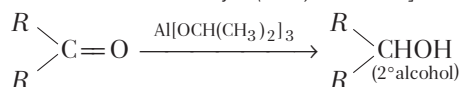
Reduction

Aldehydes and ketones can be reduced to alcohols and alkanes.

(i) Reduction to alcohols



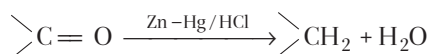
Note Reduction of ketones by aluminium isopropoxide [Meerwein-Ponndorf-Verley's (MPV) reduction]



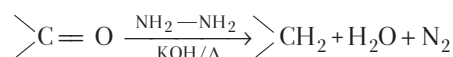
(ii) Reduction to alkanes

The reducing agents may be

(a) Zn-Hg/HCl (Clemmensen's reduction)

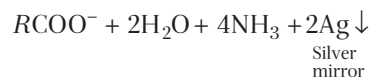
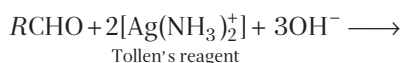


(b) NH_2-NH_2 / KOH (Wolff-Kishner reduction)

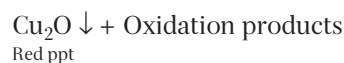
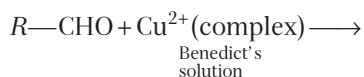
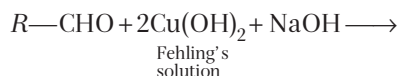


Oxidation

(i) Oxidation of aldehydes

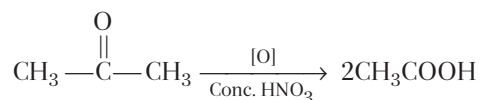


The above reaction is called **Silver mirror test** and used as a distinguish test for aldehydes and ketones.



(ii) Oxidation of ketones

By the help of strong oxidising agents like conc. HNO_3 / $KMnO_4$ / H_2SO_4 / $K_2Cr_2O_7$ / H_2SO_4 , their oxidation involves carbon-carbon bond cleavage.

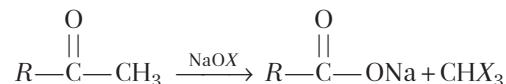


For unsymmetrical ketones, the cleavage takes place in such a way that keto group remains with smaller alkyl group (**popoff's rule**).

(iii) Oxidation with sodium hypohalite (NaOX) or haloform test

This reaction is given by carbonyl

compounds containing $-\overset{\overset{O}{||}}{C}-CH_3$ group that is acetaldehyde or methyl ketone only.

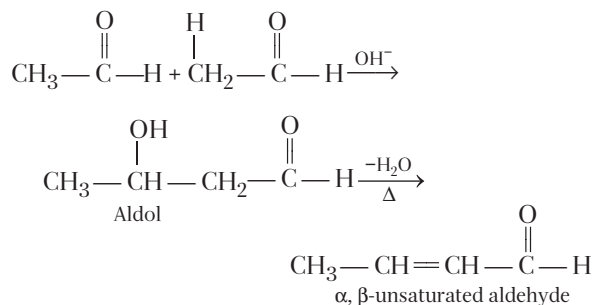


[X = Cl, Br, I]

Reactions Due to α -H-atom

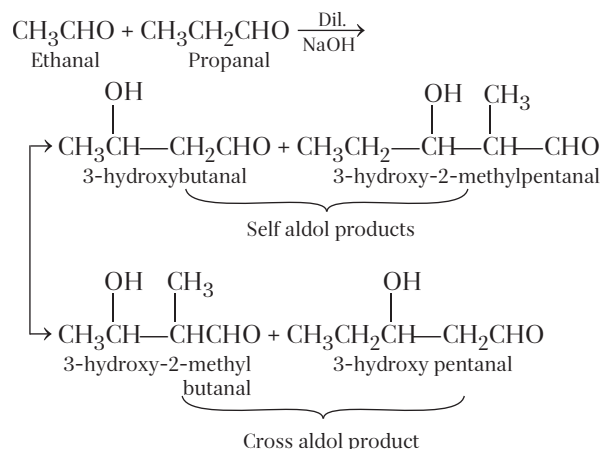
α -H-atom of aldehydes and ketones is highly acidic in nature due to the presence of electron withdrawing effect of carbonyl group.

(i) **Aldol condensation** Aldehydes and ketones possessing atleast one α -H-atom reacts in the presence of dilute alkali [like NaOH as catalyst] to form β -hydroxy aldehydes (aldol)/ β -hydroxyketones (ketol) respectively. Further, aldol and ketol condenses to give α, β -unsaturated carbonyl compounds. This reaction is called aldol condensation.



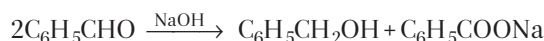
The above reaction is called **Self aldol condensation**.

(ii) **Cross aldol condensation** When aldol condensation is carried out between two different aldehydes/ketones, it is called cross aldol condensation.

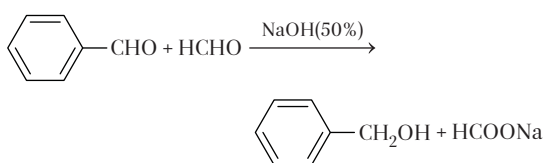


Chemical Reactions of Aldehydes only

- (i) **Cannizzaro reaction** This reaction is shown by those aldehydes which lack α -H-atom and undergo self oxidation-reduction reaction (disproportionation).

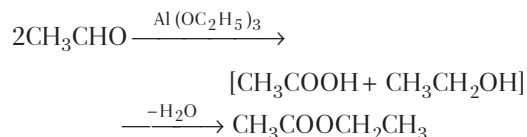


Cross-Cannizzaro reaction



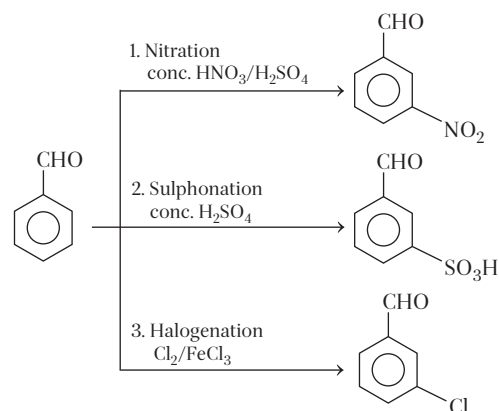
HCHO lacks steric hindrance. Hence, it readily oxidises acid to salt.

- (ii) **Tischenko reaction** The reaction looks like

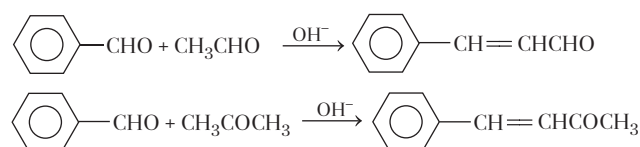


Electrophilic Substitution Reaction

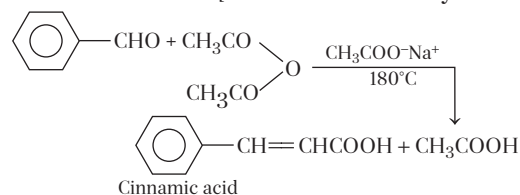
- (i) **Electrophilic substitution reaction due to benzene ring** Aromatic aldehydes undergo electrophilic substitution reactions rarely due to the presence of deactivating carbonyl group. This is *m*-directing group as substituent directed to *m*-position.



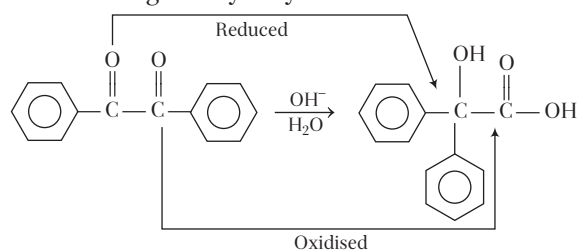
- (ii) **Claisen-Schmidt reaction** The reaction looks like



- (iii) **Perkin condensation** [For aromatic aldehydes only]



- (iv) **Benzylic acid rearrangement** [For aromatic ketones] The reaction is given by only diketones.



Practice Exercise

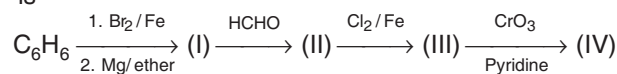
1. Ozonide of $\text{CH}_2=\text{CH}-\text{CH}_2\text{OH}$ on hydrolysis gives

- HCHO , $\text{OHC}-\text{CHO}$
- HCHO , HOCH_2-CHO
- HCHO , $\text{HOC}-\text{CH}_2\text{OH}$
- None of the above

2. Dry distillation of calcium acetate and calcium formate leads to the formation of aldehydes and ... ketones

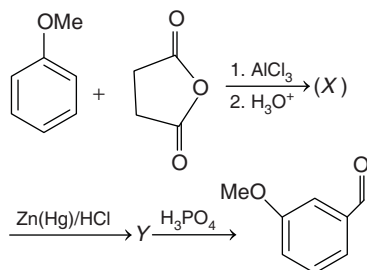
- 2, 1
- 1, 2
- 2, 2
- 1, 1

3. In the following sequence of reactions, the final product is



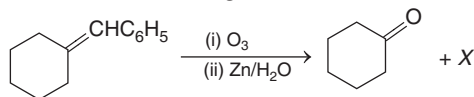
- p*-chlorobenzaldehyde
- p*-chlorobenzylalcohol
- p*-chlorobenzoic acid
- p*-salicylaldehyde

4. Identify the product (Y) in the following reaction sequence:



- $\text{MeO}-\text{C}_6\text{H}_4-(\text{CH}_2)_3\text{COOH}$
- $\text{MeO}-\text{C}_6\text{H}_4-(\text{CH}_2)_2\text{COOH}$
- $\text{MeO}-\text{C}_6\text{H}_4-\text{CH}_2\text{COOH}$
- $\text{MeO}-\text{C}_6\text{H}_4-\text{COOH}$

5. Consider the following reaction,



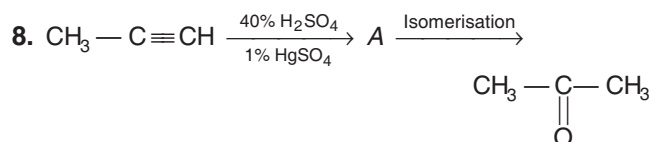
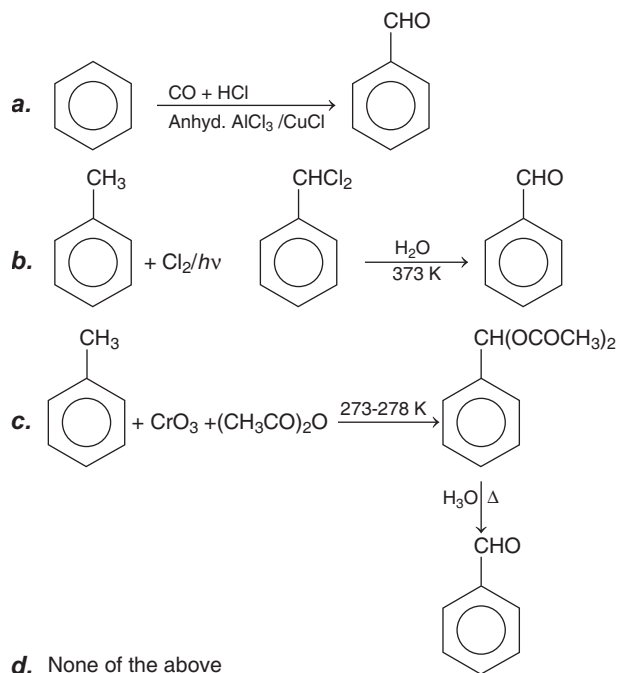
The compounds X is

-
-
-
-

6. The compound formed as a result of oxidation of ethyl benzene by KMnO_4 is

- benzophenone
- acetophenone
- benzoic acid
- benzyl alcohol

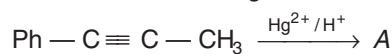
7. Choose the Gattermann-Koch reaction.



Structure of A and type of isomerism in the above reaction respectively are

- prop-1-en-2-ol, metamerism
- prop-1-en-1-ol, tautomerism
- prop-2-en-2-ol, geometrical isomerism
- prop-1-en-2-ol, tautomerism

9. Consider the following reaction,



A is

-
-
-
-

10. Which of the substrate give the same product on the reduction with DIBAL-H?

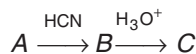
a. $\text{CH}_3 - (\text{CH}_2)_9 - \text{CN}$ and $\text{CH}_3(\text{CH}_2)_9 - \text{COOH}$
 b. $\text{CH}_3 - (\text{CH}_2)_9 - \text{CN}$ and $\text{CH}_3(\text{CH}_2)_9 - \text{COOC}_2\text{H}_5$
 c. $\text{CH}_3 - (\text{CH}_2)_9 - \text{COOH}$ and $\text{CH}_3(\text{CH}_2)_9 - \text{CHO}$
 d. $\text{CH}_3(\text{CH}_2)_9 - \text{COOH}$ and $\text{CH}_3(\text{CH}_2)_9 \text{COOC}_2\text{H}_5$

11. Identify the starting material from which the products
 $\text{H}_3\text{CCH}=\text{C}-\text{CHO}$ and

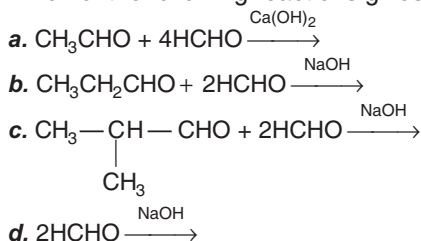


$\text{H}_3\text{C}-\text{CH}_2\text{CH}=\text{CH}-\text{CHO}$ are formed?

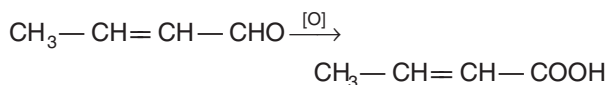
- a. Two molecules of ethanal
 b. Two molecules of propanal
 c. One molecule of ethanal and two molecules of propanal
 d. One molecule of propanal and one molecule of ethanal
12. Both HCHO and CH_3CHO give similar reactions with all the reagents except
- a. Schiff's reagent b. ammoniacal AgNO_3
 c. Fehling solution d. ammonia
13. Which of the following carbonyl compounds gives lactic acid as the end product in the following sequence?



- a. HCHO b. CH_3CHO
 c. $\text{C}_6\text{H}_5\text{CHO}$ d. CH_3COCH_3
14. Oximino acetone is formed in the reaction
- a. acetone + hydroxylamine b. acetone + ammonia
 c. acetone + nitrous acid d. None of the above
15. Which of the following is used to prepare a medicine, which is used in making an important explosive, RDX ?
- a. Acetaldehyde b. Acetone
 c. Formaldehyde d. None of these
16. Which of the following reactions gives pentaerythritol?



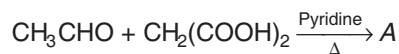
17. Consider the following reaction,



The above reaction is completed by the reagent

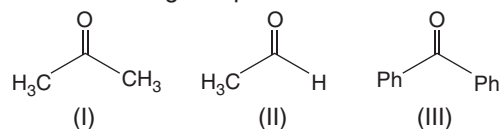
a. alkaline KMnO_4 b. Tollen's reagent
 c. selenium dioxide d. osmium tetroxide

18. Consider the following reaction,

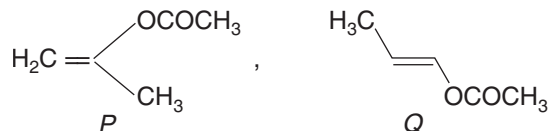


A is

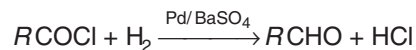
- a. CH_3COOH
 b. $\text{CH}_3\text{CH}_2\text{COOH}$
 c. $\text{CH}_3\text{CH}=\text{CHCOOH}$
 d. $\text{HOOC}-\text{CH}=\text{CH}-\text{COOH}$
19. In the aldol condensation of acetaldehyde and acetone in dilute alkali, the carbanion source will be
- a. acetaldehyde
 b. acetone
 c. Both (a) and (b)
 d. None of the above
20. For distinct between CH_3CHO and $\text{C}_6\text{H}_5\text{CHO}$, the used reagent is
- a. KCN b. HCN c. NH_2OH d. PCl_5
21. Oxidation of ketones with H_2O_2 or with a peroxy acid is called Baeyer-Villiger oxidation. This oxidation reaction forms
- a. carboxylic acid with the fewer number of carbons
 b. an alcohol with the same number of carbons as in the ketone
 c. an ester
 d. carboxylic acid with the same number of carbons as in the parent ketone
22. The order of reactivity of phenyl magnesium bromide with the following compounds is



- a. $\text{II} > \text{III} > \text{I}$
 b. $\text{I} > \text{III} > \text{II}$
 c. $\text{II} > \text{I} > \text{III}$
 d. All react with the same rate
23. The product of acid hydrolysis of P and Q can be distinguished by



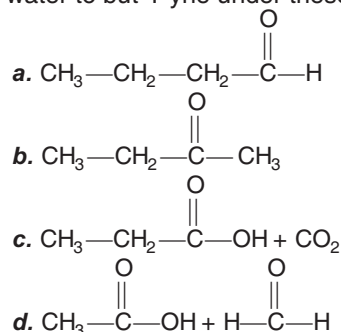
- a. Lucas reagent b. 2, 4-DNP
 c. Fehling's solution d. NaHSO_3
24. Consider the following Rosenmund reaction,



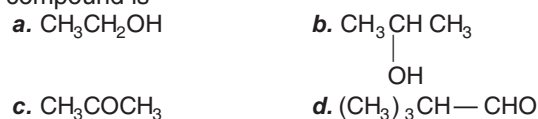
Here, BaSO_4

- a. promotes catalytic activity of Pd
 b. removes the HCl formed in the reaction
 c. deactivates palladium
 d. activates palladium

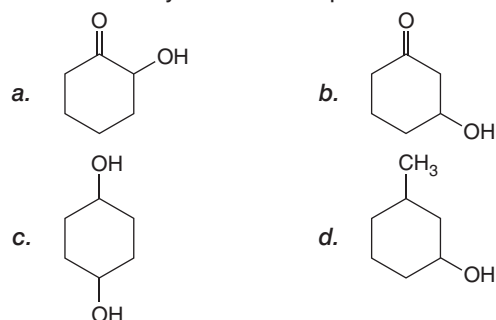
25. Addition of water to alkynes occurs in acidic medium and in the presence of Hg^{2+} ions as a catalyst. Which of the following products will be formed on addition of water to but-1-yne under these conditions?



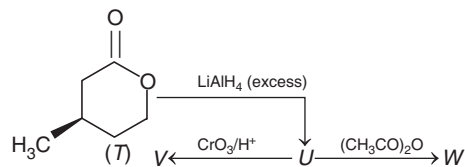
26. An organic compound containing C, H and O gives red colouration with sodium nitroprusside solution but does not reduce Tollen's reagent and yields chloroform on treating with NaOH and Cl_2 . The compound is



27. Maximum dehydration takes place in that of

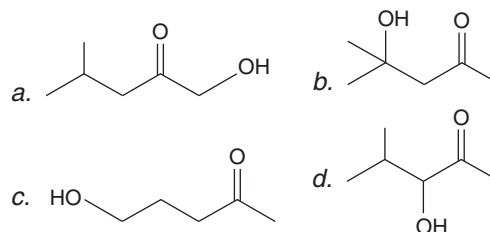


28. With reference to the scheme given, which of the given statements about T, U, V and W is incorrect?



- a. T is soluble in hot aqueous NaOH
 b. V is optically active
 c. Molecular formula of W is $\text{C}_{10}\text{H}_{18}\text{O}_4$
 d. V gives effervescence on treatment with aqueous NaHCO_3

29. Which of the following is the product of aldol condensation?



30. Iodoform can be prepared from all, except

- a. ethyl methyl ketone
 b. isopropyl alcohol
 c. 3-methyl-2-butanone
 d. isobutyl alcohol

31. The Clemmensen reduction of ketones is carried out in the presence of

- a. Zn-Hg with HCl
 b. LiAlH_4
 c. H_2 and Pt as a catalyst
 d. glycol with KOH

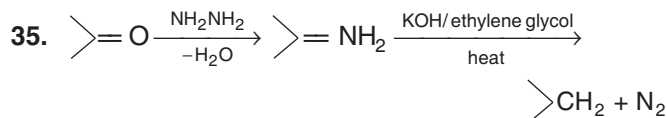
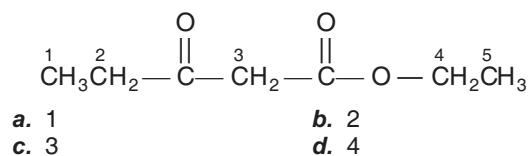
32. Which one of the following aldehydes does not give Cannizzaro's reaction?

- a. Formaldehyde b. Acetaldehyde
 c. Trimethyl acetaldehyde d. Benzaldehyde

33. When α , β -unsaturated carbonyl compounds undergo a ring closure reaction with conjugated dienes, the reaction is called

- a. Claisen rearrangement
 b. Diels Alder reaction
 c. Cannizzaro reaction
 d. Perkin reaction

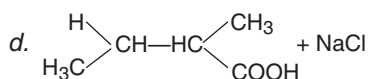
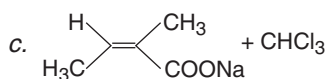
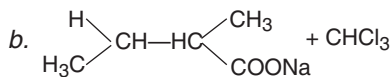
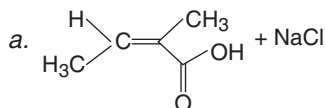
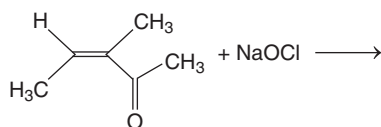
34. The most acidic H-atom in the following ester is



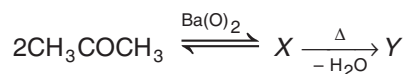
The above reaction is known as

- a. Wolff-Kishner reduction
 b. Clemmensen's reduction
 c. Both (a) and (b)
 d. None of the above

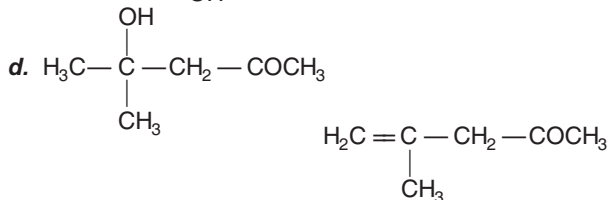
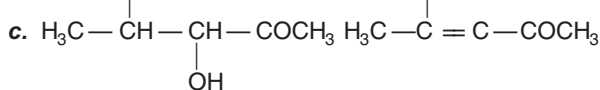
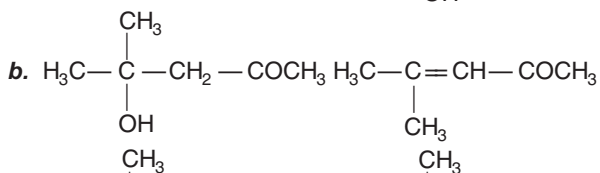
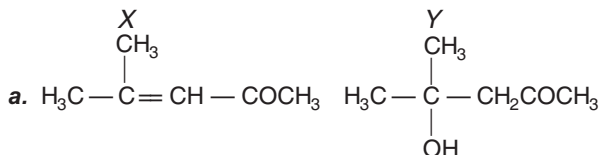
36. Identify the product formed in the following reaction:



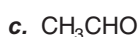
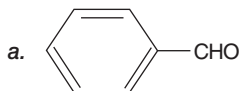
37. Consider the following reaction,



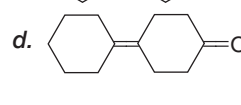
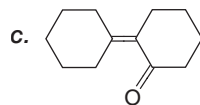
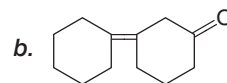
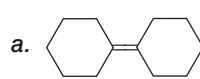
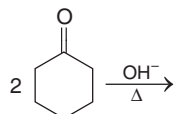
Identify X and Y in the given reaction.



38. Cannizzaro's reaction is not given by



39. The product formed in the following reaction is



40. The compounds, benzaldehyde and acetone are distinguished by

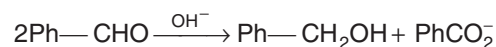
a. Fehling's solution

b. 2, 4-DNP

c. Tollen's reagent

d. sodium hydroxide solution

41. Consider the following Cannizzaro reaction,



In the above reaction, the slowest step of the reaction is

a. the transfer of hydride ion to the carbonyl group

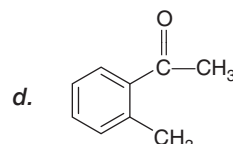
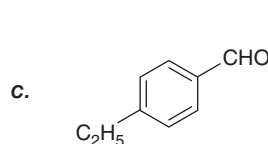
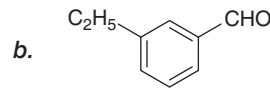
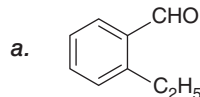
b. the deprotonation of $\text{Ph}-\text{CH}_2\text{OH}$

c. the attack of OH^- at the carbonyl group

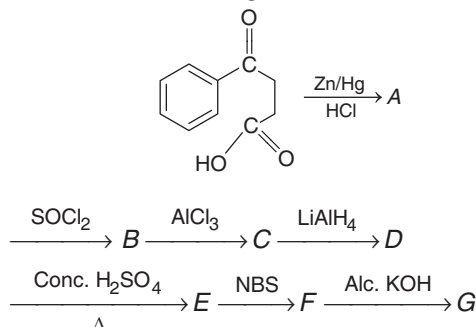
d. the abstraction of proton from the carboxylic acid

42. An aromatic compound, A [$\text{C}_5\text{H}_{10}\text{O}$] undergoes

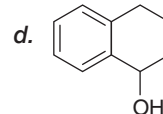
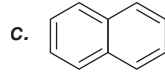
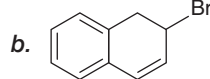
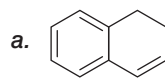
Cannizzaro reaction, forms 2, 4-DNP derivative, reduces Tollen's reagent and produces 1, 2-benzenedicarboxylic acid on vigorous oxidation. The compound A would be



43. Consider the following series of reaction,



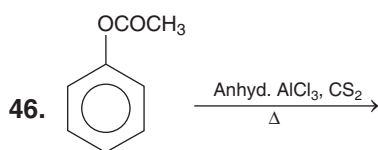
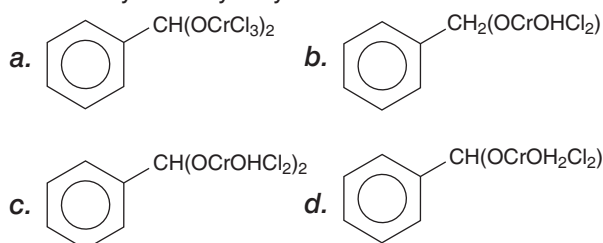
The end product of the above series of reaction is



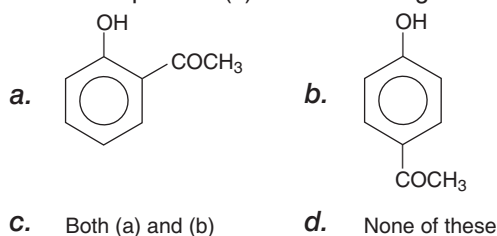
44. The increasing order of the rate of HCN addition to compounds *a-d* is

(i) HCHO
 (ii) CH_3COCH_3
 (iii) PhCOCH_3
 (iv) PhCOPh
a. (i) < (ii) < (iii) < (iv)
b. (iv) < (ii) < (iii) < (i)
c. (iv) < (iii) < (ii) < (i)
d. (iii) < (iv) < (ii) < (i)

45. Select the structure of chromium complex formed when the toluene reacts with chromyl chloride to give benzaldehyde on hydrolysis.



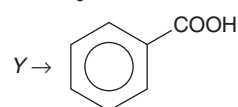
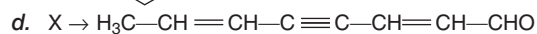
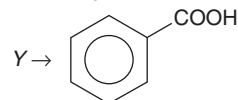
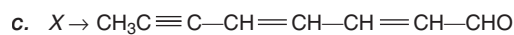
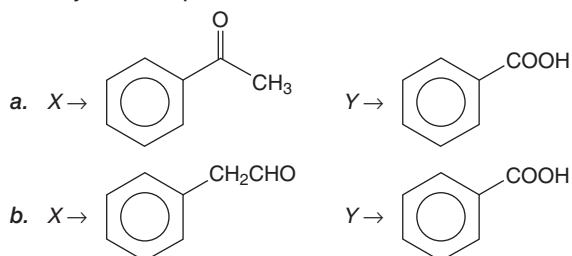
Predict the product (s) formed in the given reaction.



47. An organic compound (*x*) with molecular formula $\text{C}_8\text{H}_8\text{O}$ forms an orange-red precipitate with 2,4-DNP reagent and gives yellow precipitate on heating with iodine in the presence of sodium hydroxide.

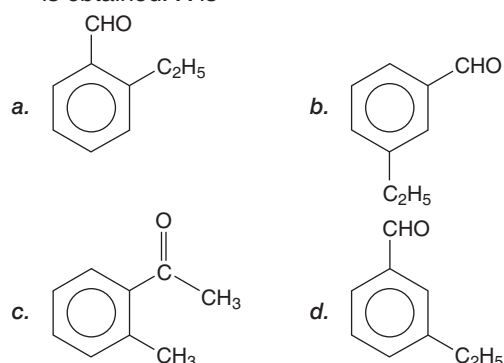
It neither reduces Tollen's or Fehling's reagent, nor does it decolourise bromine water or Baeyer's reagent.

On drastic oxidation with chromic acid, it gives a carboxylic acid (*Y*) having molecular formula $\text{C}_7\text{H}_6\text{O}_2$. Identify the compounds *X* and *Y*.

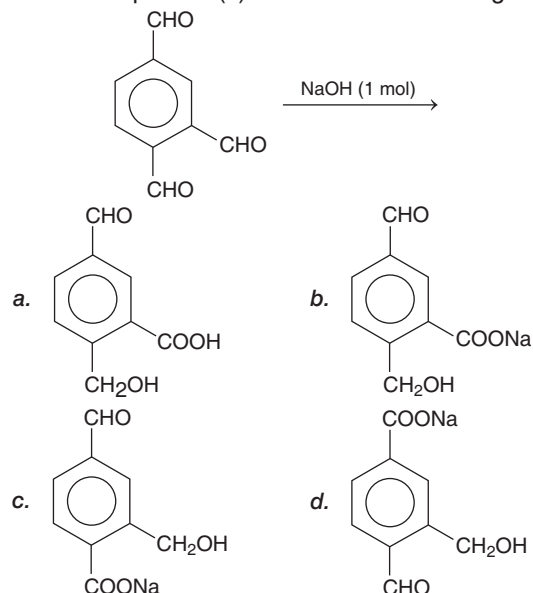


48. An aromatic compound *X* with molecular formula $\text{C}_9\text{H}_{10}\text{O}$ gives the following chemical tests:

I. Forms 2, 4-DNP derivative
 II. Reduces Tollen's reagent
 III. Undergoes Cannizzaro reaction
 IV. On vigorous oxidation, 1,2-benzene dicarboxylic acid is obtained. *X* is



49. Predict the product(s) formed in the following reaction:

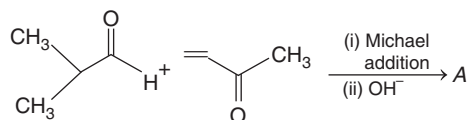


50. Name the reaction which involves the conversion of benzaldehyde to cinnamic acid in the presence of acetic anhydride.

a. Benzoin condensation **b.** Reformatsky reaction
c. Knoevenagel reaction **d.** Perkin's reaction

BITSAT Archives

1. How many chiral centres are possible for the product of following reaction? [2014]



- a. 1
b. 0
c. 3
d. 2

2. Arrange the following compounds in the increasing order of nucleophilic addition reaction: [2014]

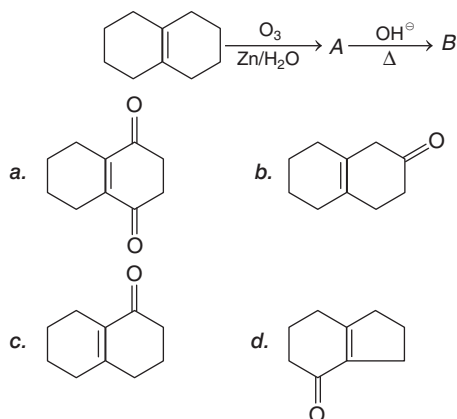
- I. HCHO
II. CH_3COCH_3
III. $\text{C}_6\text{H}_5\text{COCH}_3$
IV. $\text{C}_6\text{H}_5\text{COC}_6\text{H}_5$

- a. $\text{I} < \text{II} < \text{III} < \text{IV}$
b. $\text{IV} < \text{III} < \text{II} < \text{I}$
c. $\text{IV} < \text{II} < \text{III} < \text{I}$
d. $\text{III} < \text{IV} < \text{II} < \text{I}$

3. Which of the following compounds will give positive iodoform test with I_2 and NaOH ? [2013]

- a. $\text{C}_6\text{H}_5\text{COC}_6\text{H}_5$
b. $\text{CH}_3\text{CH}_2\text{CHO}$
c. $\text{C}_6\text{H}_5\text{COCH}_2\text{CH}_3$
d. $\text{C}_6\text{H}_5\text{CH}(\text{OH})\text{CH}_3$

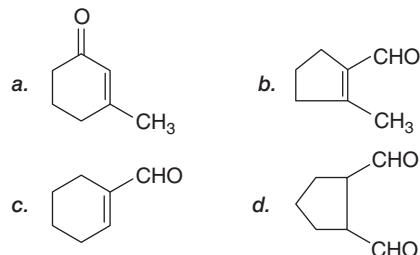
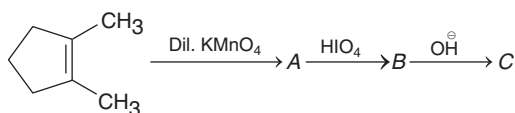
4. What will be the final product of the reaction? [2014]



5. The compound formed as a result of oxidation of propyl benzene by KMnO_4 is [2013]

- a. benzaldehyde
b. benzyl alcohol
c. benzoic acid
d. acetophenone

6. What will be the correct structural formula of product for the following reaction? [2013]



7. Which of the following is process used for the preparation of acetone? [2012]

- a. Waber process
b. Wacker process
c. Wolff-Kishner reduction
d. Gattermann-Koch synthesis

8. What will be the main product when acetylene reacts with hypochlorous acid? [2012]

- a. Trichloro acetaldehyde
b. Acetaldehyde
c. Dichloro acetaldehyde
d. Chloro acetaldehyde

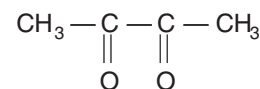
9. Which of the following reagents can be used to prepare benzaldehyde from toluene? [2009]

- a. CrO_3 in $(\text{CH}_3\text{CO})_2\text{O}$
b. $\text{K}_2\text{Cr}_2\text{O}_7$ + conc. H_2SO_4
c. Hot alkaline KMnO_4
d. Conc. HNO_3

10. Acetone on addition to methyl magnesium bromide forms a complex, which on decomposition with acid gives X and $\text{Mg}(\text{OH})\text{Br}$. Which one of the following is X? [2008]

- a. CH_3OH
b. $(\text{CH}_3)_3\text{COH}$
c. $(\text{CH}_3)_2\text{CHOH}$
d. $\text{CH}_3\text{CH}_2\text{OH}$

11. $\text{CH}_3 - \text{C} \equiv \text{CH}_3 - \text{CH}_3 \xrightarrow[\text{(ii) Zn/H}_2\text{O}]{\text{(i) X}}$



In the above reaction, X is [2008]

- a. HNO_3
b. O_2
c. O_3
d. KMnO_4

12. Which of the following ketones will not respond to iodoform test? [2007]

- a. Methyl isopropyl ketone
b. Ethyl isopropyl ketone
c. Dimethyl ketone
d. 2-hexanone

13. Acetone and acetaldehyde can be distinguished by

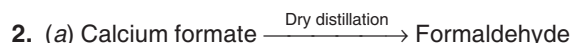
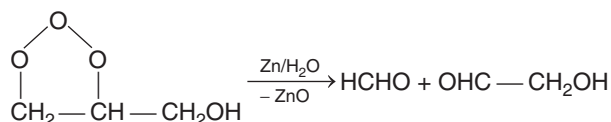
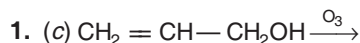
- a. Molisch test
b. Tollen's test
c. Schiff's test
d. iodoform test

14. Cyanohydrin of which of the following forms lactic acid [2005]

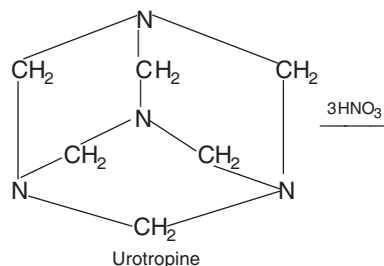
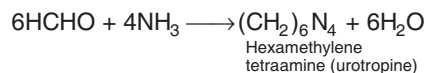
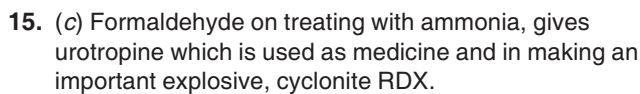
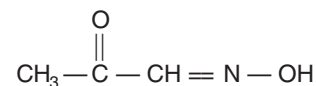
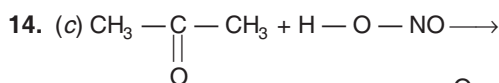
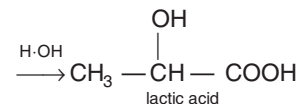
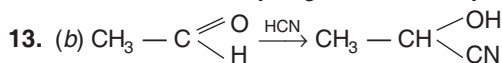
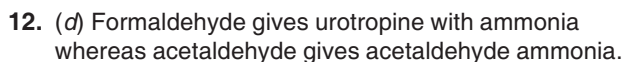
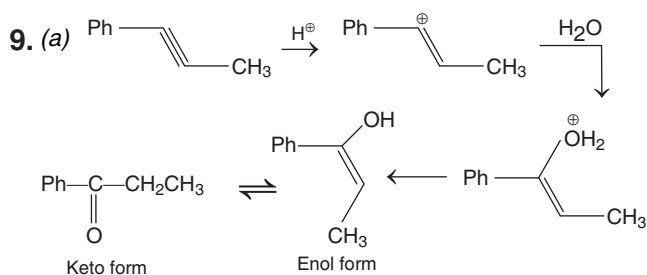
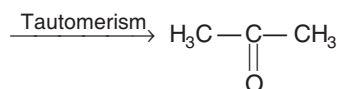
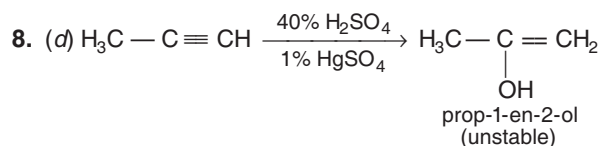
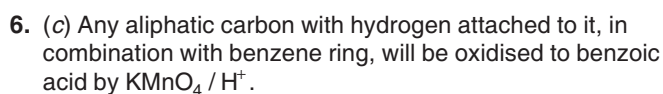
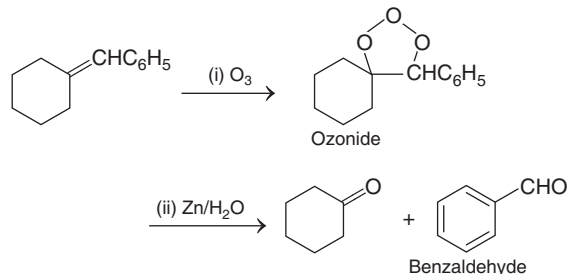
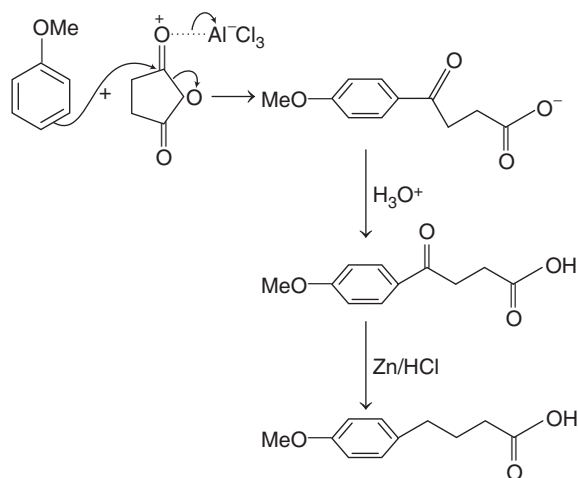
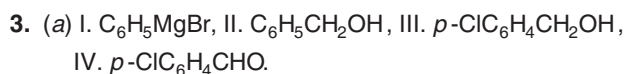
- a. HCHO
b. CH_3CHO
c. $\text{CH}_3\text{CH}_2\text{CHO}$
d. CH_3COCH_3

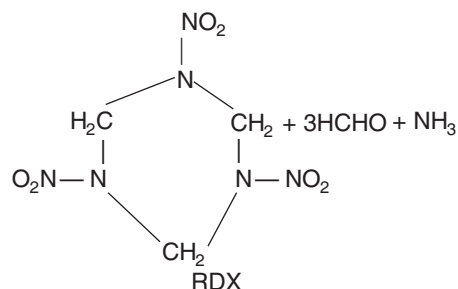
Answer with Solutions

Practice Exercise

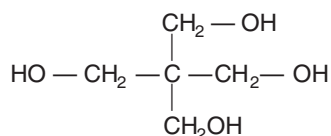


Calcium formate + calcium acetate $\xrightarrow{\Delta}$ Acetaldehyde
i.e. two aldehydes and one ketone are obtained.

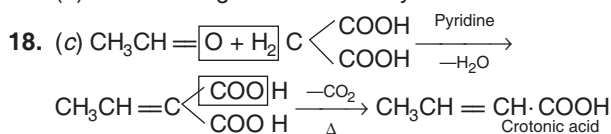




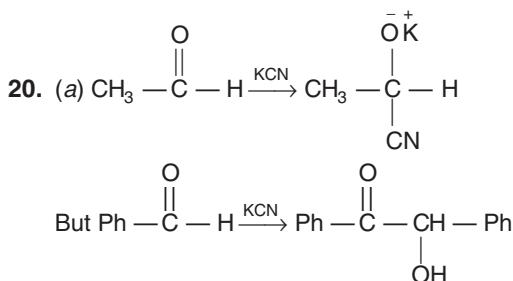
16. (a) It is the product of reaction (a).



17. (b) Tollen's reagent converts only —CHO to —COOH .

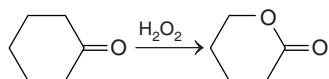


19. (b) Acetone generates carbanion because carbonyl group of acetaldehyde acts as a better nucleophile acceptor than acetone.

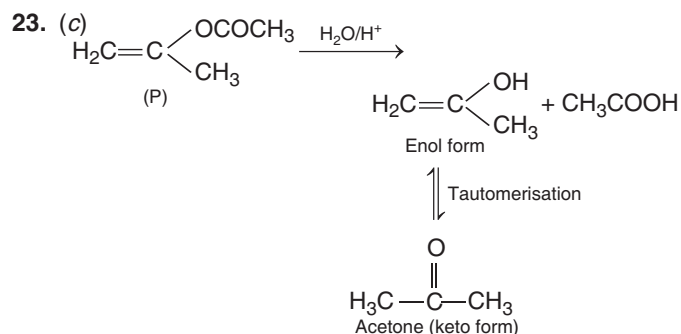


In other reactions, the products are similar.

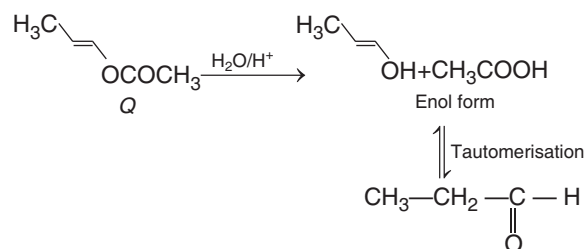
21. (c) $\text{R}_2\text{C}=\text{O} \xrightarrow{\text{H}_2\text{O}_2} \text{R—C(=O)—OR}$
- Cyclic ketones are converted into cyclic esters (lactones).



22. (c) From phenyl magnesium bromide, phenyl is attached with that C-atom of carbonyl group which has low electron density (higher electropositive charge). In carbonyl compounds, aldehydes are more reactive towards nucleophile. In nucleophilic addition reactions because in ketones alkyl groups (due to +I effect) decrease the electropositive charge of carbon of carbonyl group. Hence, attraction of nucleophile decreases. Moreover in the tetrahedral intermediate, aldehydes have less steric repulsion than ketones and also the aldehyde increases the negative charge on oxygen less in comparison of ketones. Therefore, on the basis of above reason, the order of reactivity of acetone (I), acetaldehyde (II) and benzophenone (III) with PhMgBr is $\text{II} > \text{I} > \text{III}$



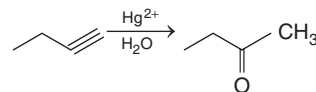
Fehling's solution is not reduced with acetone.



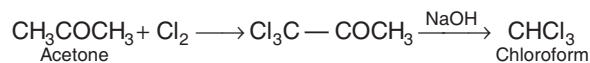
Fehling solution is reduced with aldehyde, i.e. $\text{CH}_3\text{CH}_2\text{CHO}$. Hence, the product of acid hydrolysis of *P* (ketone) and *Q* (aldehyde) can be distinguished by Fehling solution.

24. (c) BaSO_4 decreases the activity of Pd .

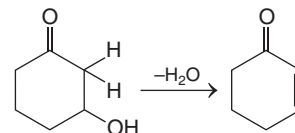
25. (b) But-1-yne on reaction with water in presence of Hg^{2+} ions as a catalyst produces butan-2-one.



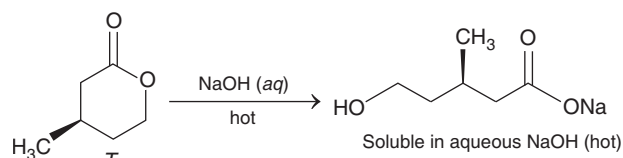
26. (c) CH_3COCH_3 gives red colour with sodium nitroprusside solution but does not reduce Tollen's reagent. Acetone yields chloroform with NaOH/Cl_2 .

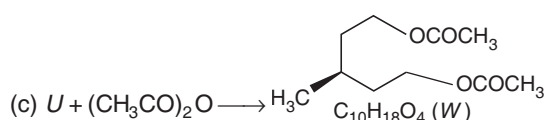
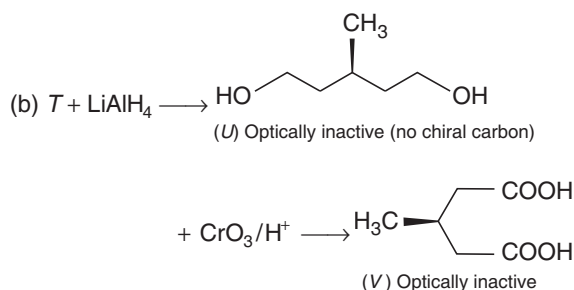


27. (b) Dehydrated product will be conjugated with >C=O

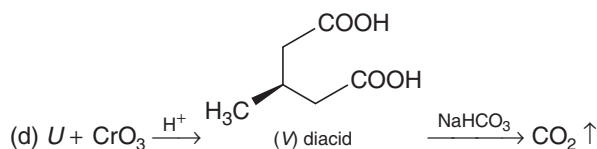


28. (b) (a) *T* is an ester, so get hydrolysed.

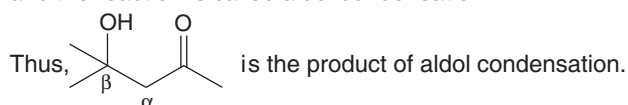




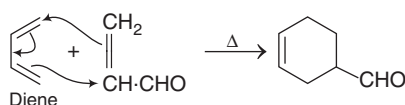
U on treatment with excess $(\text{CH}_3\text{CO})_2\text{O}$, forms a diester (W).



29. (b) When two aldehydes or ketones (or carbonyl compounds) having α -hydrogen atoms react in the presence of a base, they result in the formation of β -hydroxy aldehyde or ketone, called the aldol or ketol and the reaction is called aldol condensation.

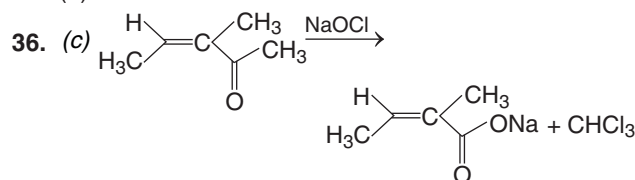


30. (d) All carbonyl compounds of the type $R-\overset{\text{O}}{\parallel}{\text{C}}-\text{CH}_3$ formed iodoform but isobutyl alcohol do not have $R-\overset{\text{O}}{\parallel}{\text{C}}-\text{CH}_3$ group.
31. (a) The reagents used to carry out Clemmensen reduction of ketones are $\text{Zn} - \text{Hg}$ with conc. HCl .
32. (b) Cannizzaro reaction is given by the aldehydes that do not contain α -H-atom. The aldehydes like acetaldehyde, CH_3CHO possess α -H-atom and hence, does not undergo Cannizzaro reaction.
33. (b) The reaction between α, β -unsaturated carbonyl compound and conjugated diene to undergo ring closure is called Diel's Alder reaction.

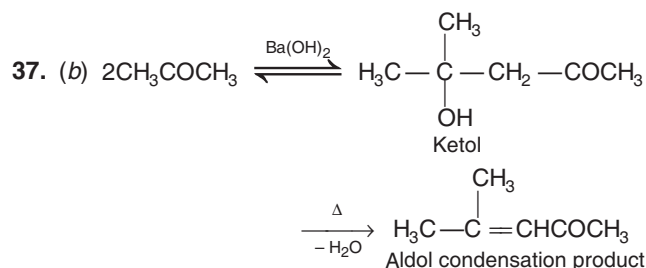


34. (c) The H-atoms linked to C-3 atom are most acidic in nature as these are attached to two electron withdrawing carbonyl groups.

35. (a)

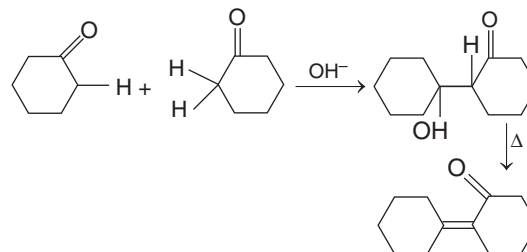


This oxidation does not affect a carbon double bond, if present in the molecule.



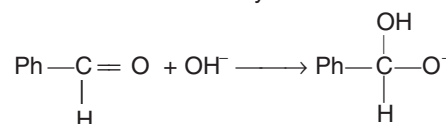
38. (c) Only those aldehydes undergo Cannizzaro reaction in the presence of concentrated base, which have lack of α -H atom. CH_3CHO contains three α -H atoms, that's why it does not undergo Cannizzaro's reaction, rather it undergoes aldol condensation.

39. (c)

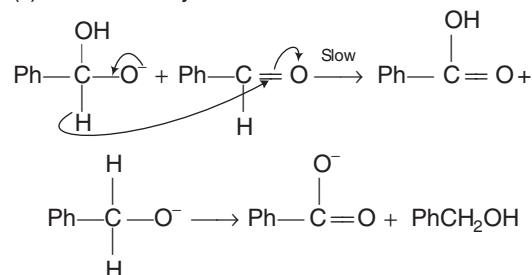


40. (a) Tollen's reagent is used to distinguish between aldehydes and ketones. Aromatic aldehydes do not reduce Fehling solution and Benedict solution because they are weak reagents.
41. (a) The transfer of hydride ion to the carbonyl group is the slowest step of Cannizzaro reaction.

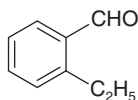
(i) Attack of OH^- on carbonyl carbon



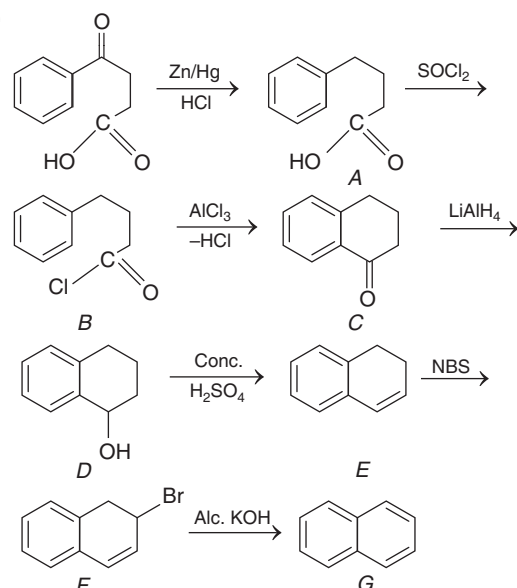
(ii) Transfer of hydride ion



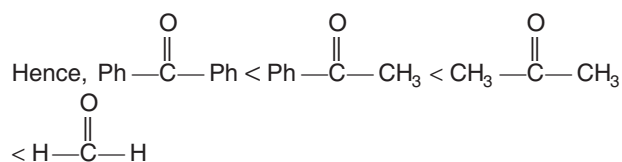
42. (a) The compound A undergoes Cannizzaro reaction shows that it may be an aldehydic compound with no α -H-atom. The formation of 2, 4-DNP suggests the presence of $C=O$ group and reduction of Tollen's reagent confirms aldehydic group in the compound. The production of 1,2-benzene dicarboxylic acid on vigorous oxidation of A shows that aldehydic and ethyl group present at 1, 2-position of benzene ring. Thus, the compound A is



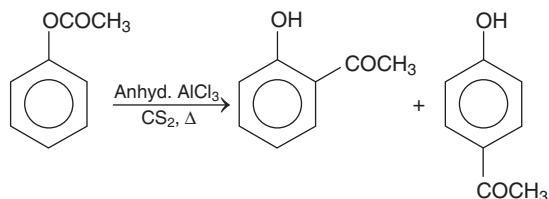
43. (c)



44. (c) Addition of HCN is a nucleophilic addition reaction. Greater the electron deficiency of carbonyl group, higher the rate of reaction.



45. (c)



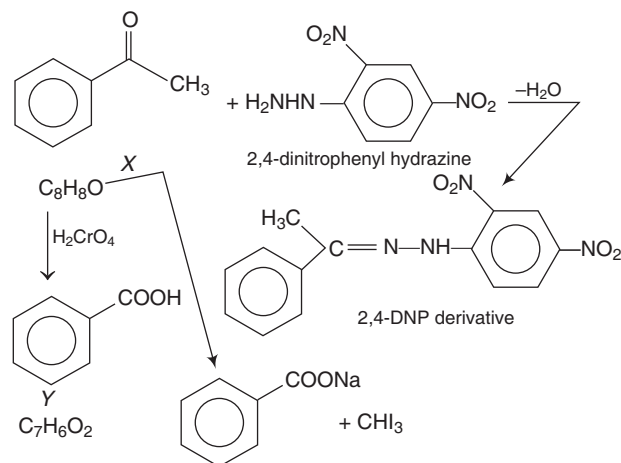
It is the Fries rearrangement.

47. (a) X forms 2, 4-DNP derivative so it is an aldehyde or ketone. X must be ketone as it does not reduce Tollen's or Fehling reagent. X must be methyl ketone as it responds to iodoform test.

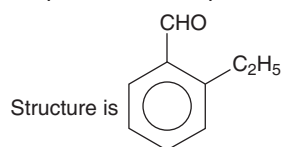
X do not decolourise bromine water which suggests the presence of unsaturation due to an aromatic ring.

Compound Y should be benzoic acid according to the molecular formula of Y. The compound X must be monosubstituted aromatic methyl ketone.

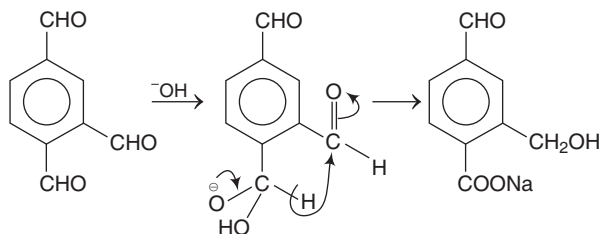
Compound X is phenylmethyl ketone.



48. (a) I. X forms 2, 4-DNP derivatives, it shows that it has a carbonyl compound.
 II. It reduces Tollen's reagent, it shows that it has an aldehyde group.
 III. It undergoes Cannizzaro reaction, that shows the presence of an aldehyde having no α -hydrogen.
 IV. It produces 1, 2-benzenedicarboxylic. It shows that groups are present at 1, 2-position on benzene ring.



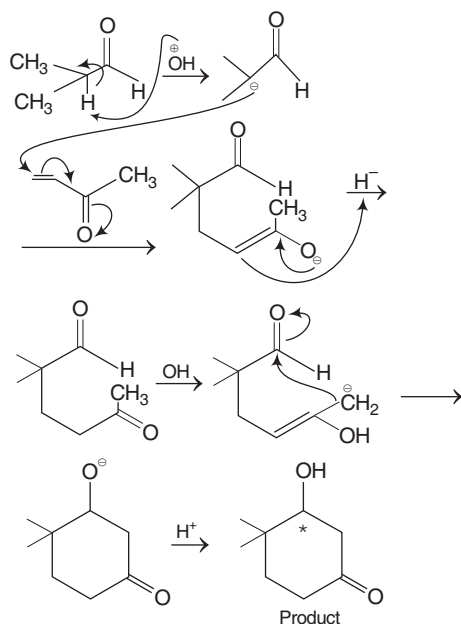
49. (c) OH^- can attack any of the three CHO groups, but nucleophilic addition of OH^- ion occurs on that CHO group which is located at *para*-positions with respect to the other. This is because electron withdrawing groups facilitate the release of hydride ion from the intermediate. The *p*-CHO group is oxidised while the *m*-CHO group is reduced.



50. (d)

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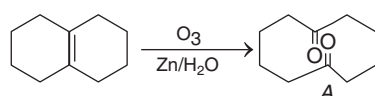
1. (a)



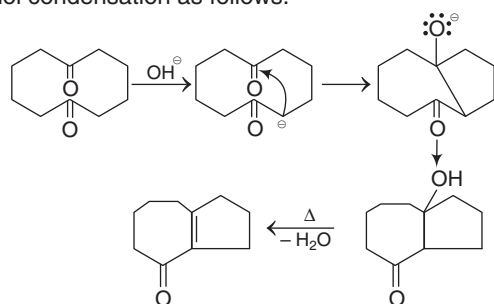
2. (b) Steric hindrance decreases the rate of reaction. This steric hindrance is minimum in methanal and maximum in benzophenone.

3. (d)

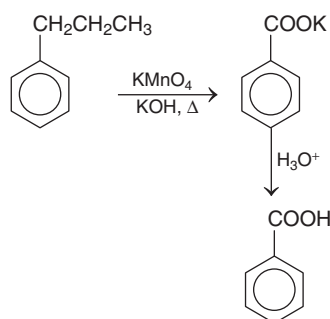
4. (d)



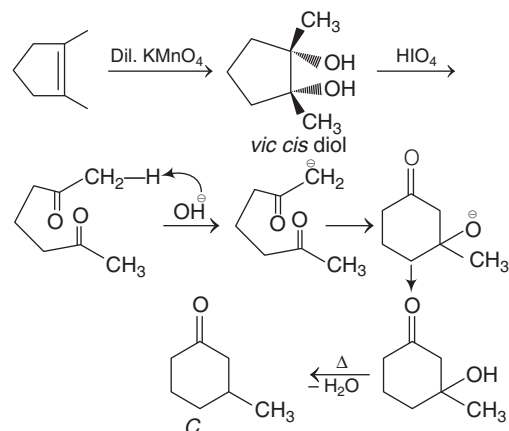
Now, this bicarbonyl compound undergoes intramolecular aldol condensation as follows:



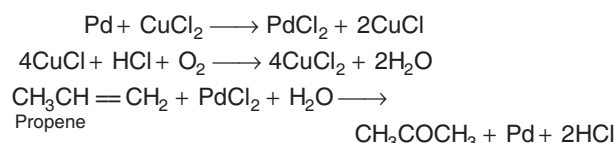
5. (c)



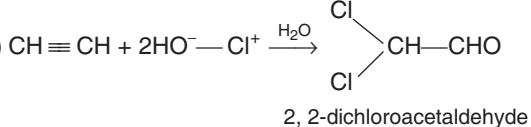
6. (a)



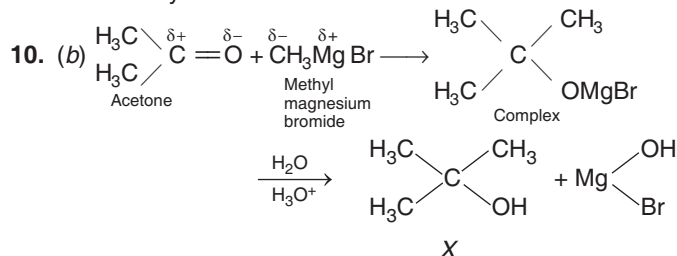
7. (b) In Wacker process, when mixture of propene and air is passed through mixture of Pd and CuCl₂ at high pressure, acetone is formed.



8. (c)

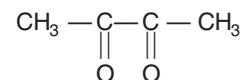


9. (a) K₂Cr₂O₇ + conc. H₂SO₄, hot alkaline KMnO₄ and conc. HNO₃ will all oxidise toluene to benzoic acid while CrO₃ in CH₃(CO₂O) (mild reagent) will oxidise toluene to benzaldehyde.



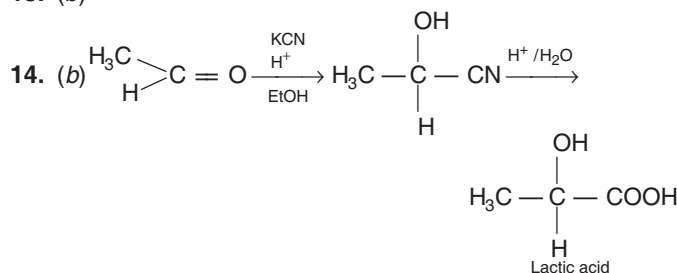
or (CH₃)₃COH 2-methyl propan-2-ol

11. (c) $\text{CH}_3-\text{C}\equiv\text{C}-\text{CH}_3 \xrightarrow[\text{Zn/H}_2\text{O}-\text{H}_2\text{O}_2]{\text{O}_3}$



12. (b)

13. (b)

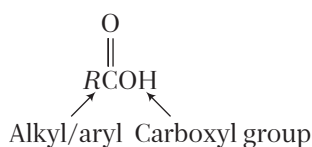


27

Carboxylic Acid and Its Derivatives

Introduction

Carboxylic acids are a large family of organic compounds that contain the functional group called the carboxyl group. Any aliphatic or aromatic compound can have a carboxyl group attached to it and represented by

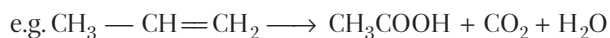
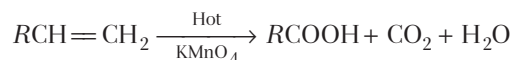


carboxylic acids form numerous important derivatives including esters, amides, anhydrides and acid halides. Carboxylic acids have general formula: $\text{C}_n\text{H}_{2n+1}\text{COOH}$

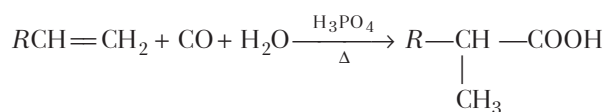
Methods of Preparation

1. From Alkenes

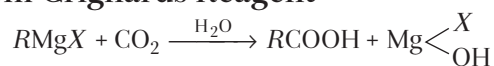
(a) By oxidation



(b) Carboxylation



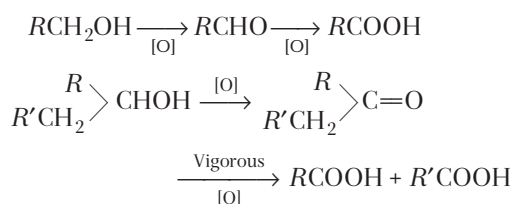
2. From Grignards Reagent



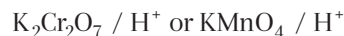
3. From Trihalogen Derivatives



4. By Oxidation of Aldehyde, Ketones and Alcohols

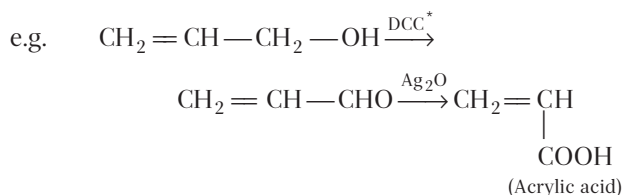


Strong oxidising agents



Mild oxidising agents

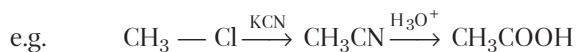
MnO, CuO (Fehling's solution), Ag₂O (Tollen's reagent), FeSO₄ + H₂O₂ (Fenton's reagent)



5. Carboxylation of Sodium Alkoxide



6. By Hydrolysis of Nitriles



Physical Properties of Carboxylic Acids

1. Physical State and Odour

Lower members of aliphatic acids with C₁—C₃ atoms are colourless and pungent smelling liquids. Higher carboxylic acids are colourless oily liquids. Unbranched chain of C₁₂—C₂₄ atoms are called fatty acids.

2. Solubility

Lower members of aliphatic acids with C₁—C₃ atoms are highly soluble in water due to H-bonding. Solubility of carboxylic acids decreases with increases in the number of carbon atoms due to an increase in hydrophobic interactions of the hydrocarbon part.

3. Boiling Points

Boiling points of carboxylic acids is much higher than those of hydrocarbons, aldehydes, ketones and alcohols of comparable masses due to the formation of strong intramolecular H-bonds.

4. Melting Points

The first ten members of carboxylic acids show oscillation or alternation effect in their melting points. The melting point of an acid with even number of C-atoms is higher than that of acids with odd number of C-atoms.

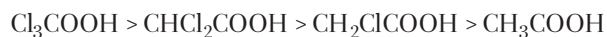
Acidic Strength of Carboxylic Acids

Carboxylic acids are weaker than mineral acids but stronger than phenols because of conjugate base, carboxylate ion is more stabilised than phenoxide ion.

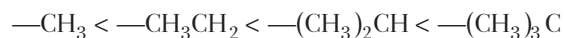
Effect of Substituents on Acidic Strength

The strength of carboxylic acids depends on the substituents bonded to carboxyl group. Electron withdrawing groups like NO₂, CN, F, Cl, Br, OH exerting *-I*-effect decreases the concentration of negative charge from carboxylate ion, thereby making the O—H bond weaker and hence, release of proton easier.

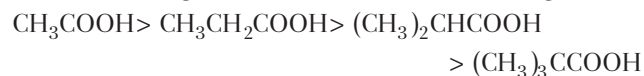
Larger the number of electron withdrawing groups bonded the carboxyl group, greater is the acidic strength of acid.



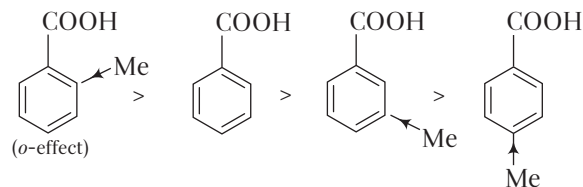
Electron donating groups like —R, —OR exerting *+I*-effect decreases the acidic strength of acids. The *+I*-effect of alkyl groups decreases in the following order

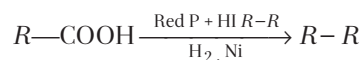
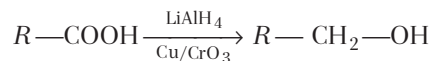


Thus, the relative strength of acids follows the following order

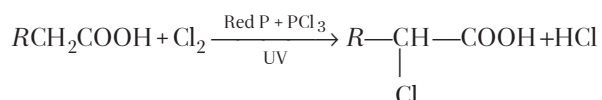
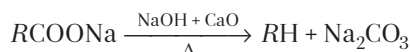
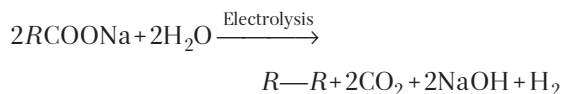
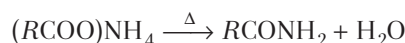
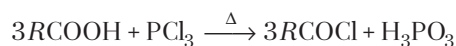
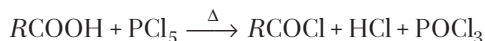
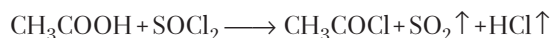
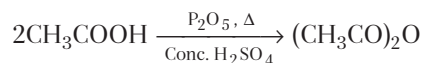
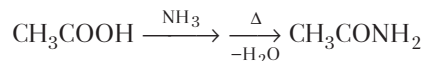
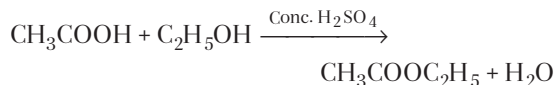


o-substituted benzoic acids are stronger than *para* and *meta*-benzoic acids.

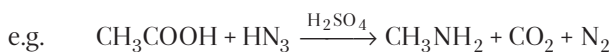
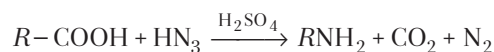
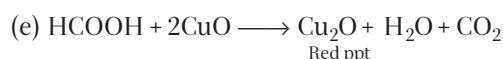
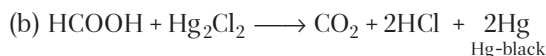
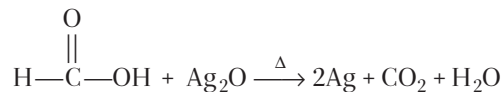


Chemical Reactions**1. Reduction****2. α -Halogenation**

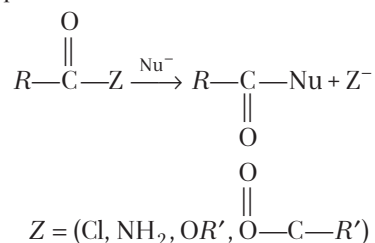
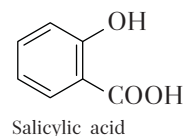
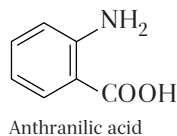
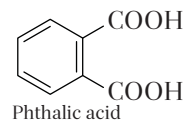
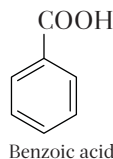
(Hell-Vohland Zelinsky Reaction) HVZ

**3. Reaction of Salts of Acids****(a) Decarboxylation****(b) Kolbe's electrolysis****(c) Action of Heat****4. Formation of Acid Derivatives****(a) Acid chloride****(b) Anhydride****(c) Amide****(d) Esterification****5. Schmidt Reaction**

(Formation of 1°-amines)

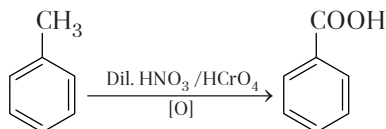
**Some Special Reactions of HCOOH**(a) It contains $-\text{CHO}$ group, hence reduces Tollen's reagent.

The above reactions can be used to differentiate between formic acid and other acids.

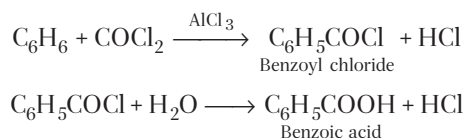
Effect of Carbonyl Group in Acid Derivatives(a) It provides a site for a Nu^- attack.**Reactivity** $\text{RCOX} > \text{RCOOCOR} > \text{RCOOR} > \text{RCONH}_2$ (b) Acidic character of α -hydrogen.Carbonyl group makes α -H active hence, acid derivatives give reactions such as Claisen condensation.**Uses**(a) **Formic acid** Used as reducing agent, for removing hair from skin, formation of resin, insecticide, antiseptic etc.(b) **Acetic acid** Used in synthesis of other organic compounds, as a solvent, in coagulation of rubber latex, for making vinegar, cellulose acetate etc.**Aromatic Acids**

Preparation

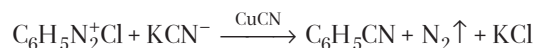
1. By Oxidation of Toluene



2. By Friedel-Crafts Reaction



3. By Sandmeyer Reaction



Chemical Properties

Acidity in Benzoic Acid

Acidity of benzoic acid increases, when any group is introduced at the *ortho* position, due to *ortho*-effect. Hence, *o*-nitrobenzoic acid is the strongest among nitrobenzoic acids.

Aromatic acids are stronger acid than aliphatic acids.



Practice Exercise

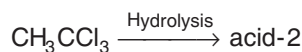
1. Highest bp would be of

- formic acid
- water
- acetic acid
- ethanol

2. Acidic hydrolysis of, which of the following ester will be slowest?

- $\text{CH}_3\text{COOCH}_3$
- $\text{CH}_3\text{COOCH}_2\text{CH}_3$
- $\text{CH}_3\text{COO}-\underset{\text{CH}_3}{\text{CH}}-\text{CH}_3$
- $\text{CH}_3\text{COOC}(\text{CH}_3)_3$

3. $\text{HCN} \xrightarrow{\text{Hydrolysis}}$ acid-1



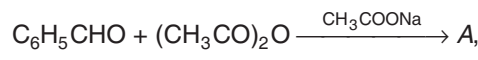
The heat of ionisation of HCN and acid-2 respectively are x kcal and y kcal [$x < y$]. The order of their $\text{p}K_a$ values will be

- $(\text{p}K_a)_{\text{HCN}} > (\text{p}K_a)_{\text{acid-2}}$
- $(\text{p}K_a)_{\text{HCN}} = (\text{p}K_a)_{\text{acid-2}}$
- $(\text{p}K_a)_{\text{HCN}} < (\text{p}K_a)_{\text{acid-2}}$
- All of the above

4. A compound with molecular formula $\text{C}_4\text{H}_{10}\text{O}_4$ on acylation with acetic anhydride gives a compound with molecular formula $\text{C}_{12}\text{H}_{18}\text{O}_8$. How many hydroxyl groups are present in the compound?

- One
- Two
- Three
- Four

5. In the reaction,



product A is

- acetaldehyde
- cinnamic acid
- β -naphthol
- phenol

6. Phenol $\xrightarrow[\text{(ii) CO}_2/140^\circ\text{C}]{\text{(i) NaOH}}$ A $\xrightarrow{\text{H}^+/\text{H}_2\text{O}}$ B $\xrightarrow{\text{Ac}_2\text{O}}$ C

In this reaction, the end product C is

- salicylaldehyde
- salicylic acid
- phenyl acetate
- aspirin

7. $\text{C}_8\text{H}_6\text{O}_4 \xrightarrow{\Delta} \text{X} \xrightarrow{\text{NH}_3} \text{Y}$

The compound X is

- o*-xylene
- phthalic acid
- phthalic anhydride
- salicylic acid

8. The correct order of increasing acidic strength is

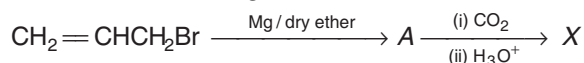
- phenol < ethanol < chloroacetic acid < acetic acid
- ethanol < phenol < chloroacetic acid < acetic acid
- ethanol < phenol < acetic acid < chloroacetic acid
- chloroacetic acid < acetic acid < phenol < ethanol

9. The correct order for the acidic strength of the following compounds is

I. CH_3COOH II. $\text{MeOCH}_2\text{COOH}$
 III. CF_3COOH IV. Me_2CHCOOH

- a. $\text{II} < \text{IV} < \text{I} < \text{III}$
 b. $\text{IV} < \text{I} < \text{III} < \text{II}$
 c. $\text{IV} < \text{I} < \text{II} < \text{III}$
 d. $\text{I} < \text{IV} < \text{III} < \text{II}$

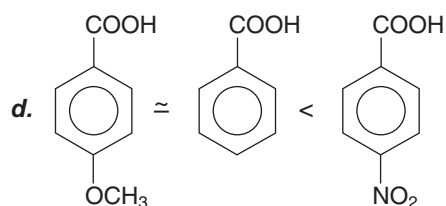
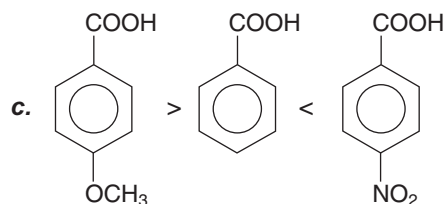
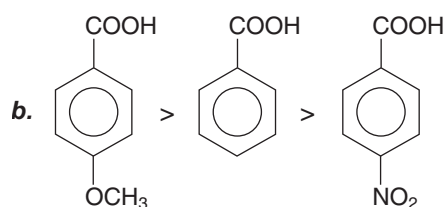
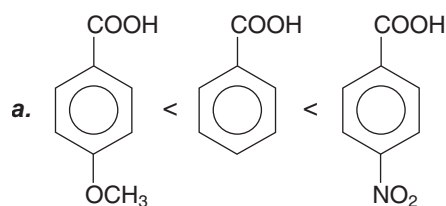
10. Consider the following reaction,



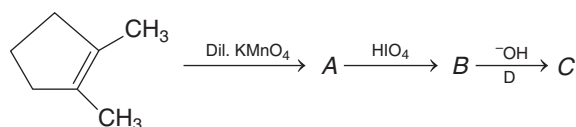
The compound X is

- a. $\text{CH}_2=\text{CHCH}_2\text{COOH}$
 b. $\text{CH}_3\text{CH}_2\text{CH}_2\text{COOH}$
 c. $\text{CH}_3\text{CH}=\text{CH}-\text{COOH}$
 d. $\text{CH}_2=\text{CHCOOH}$

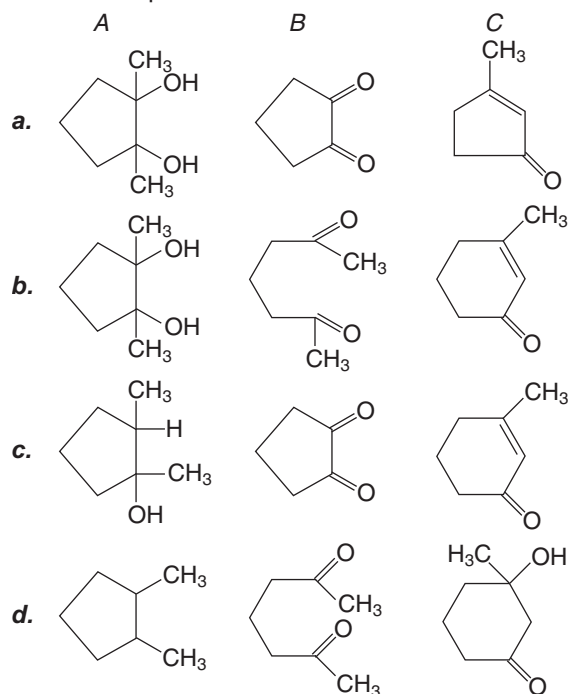
11. Arrange the following derivative of carboxylic acid in their increasing order of their acidity.



12. Select the missing compounds for the given reaction.

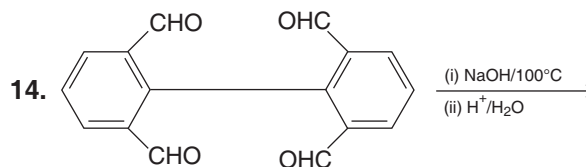


Identify A, B and C for the above reaction and choose the correct option.

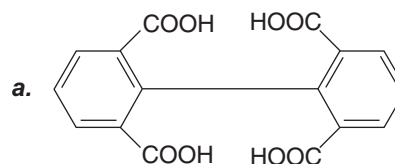


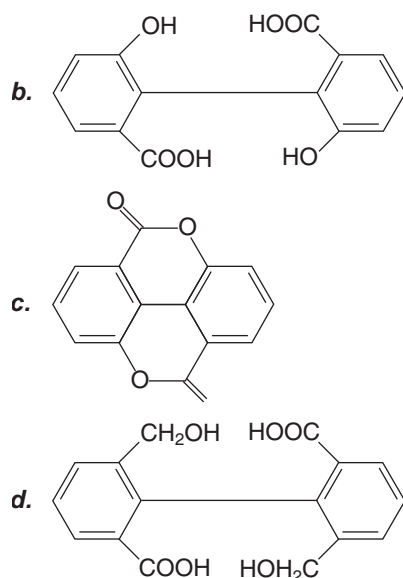
13. Which of the following represents the correct order of the acidic strength in the given compounds?

- a. $\text{CH}_3\text{COOH} > \text{BrCH}_2\text{COOH} > \text{ClCH}_2\text{COOH} > \text{FCH}_2\text{COOH}$
 b. $\text{FCH}_2\text{COOH} > \text{CH}_3\text{COOH} > \text{BrCH}_2\text{COOH} > \text{ClCH}_2\text{COOH}$
 c. $\text{BrCH}_2\text{COOH} > \text{ClCH}_2\text{COOH} > \text{FCH}_2\text{COOH} > \text{CH}_3\text{COOH}$
 d. $\text{FCH}_2\text{COOH} > \text{ClCH}_2\text{COOH} > \text{BrCH}_2\text{COOH} > \text{CH}_3\text{COOH}$



Major product of the above reaction is





15. To distinguish between formic acid and acetic acid, the suitable reagent is

1. acidified KMnO_4
2. Tollen's reagent
3. ethyl alcohol
4. sodium bicarbonate

The correct answer is

a. 1, 2, 3, 4 b. 1, 2, 3
c. 2, 3, 4 d. 1, 2

16. When salicylic acid is heated with acetic anhydride, we get

a. aspirin b. salol
c. paracetamol d. None of these

17. The general formula of the compound obtained as a result of reaction between formic acid and hydroiodic acid is

a. $\text{C}_n\text{H}_{2n+1}\text{OH}$ b. $\text{C}_n\text{H}_{2n+2}$
c. $\text{C}_n\text{H}_{2n}\text{O}$ d. C_nH_{2n}

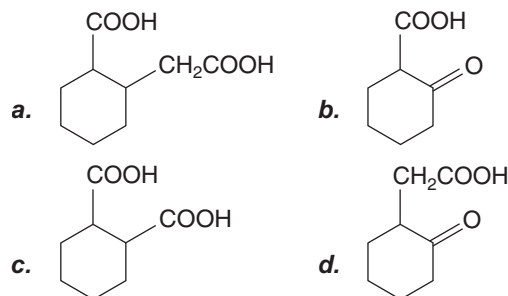
18. Pyruvic acid as

a. α -keto acid
b. β -keto acid
c. γ -keto acid
d. δ -keto acid

19. The product of which of the following reaction is capable of changing orange colour of $\text{Cr}_2\text{O}_7^{2-}$ to green colour of Cr^{3+} .

a. $\text{CH}_2(\text{COOH})_2 \xrightarrow{\Delta}$
b. $\text{CH}_3\text{CN} \xrightarrow{\text{H}_3\text{O}^+}$
c. $\text{HCN} \xrightarrow{\text{H}_3\text{O}^+}$
d. $\text{CH}_3\text{CONH}_2 \xrightarrow{\text{H}_2\text{O}}$

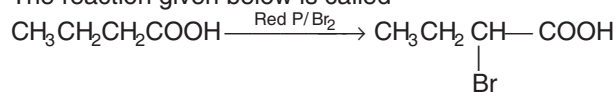
20. The compound that undergoes decarboxylation most readily under mild condition is



21. The acids, HCOOH and CH_3COOH are distinguished by

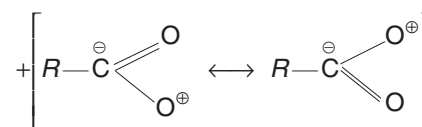
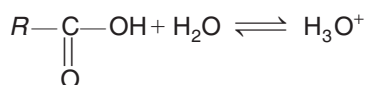
a. reaction with Na metal
b. reaction with NaOH
c. reaction with Tollen's reagent
d. reaction with Schiff's reagent

22. The reaction given below is called



a. Cannizzaro reaction
b. Hell-Volhard-Zelinsky reaction
c. Schrodinger reaction
d. Perkin's reaction

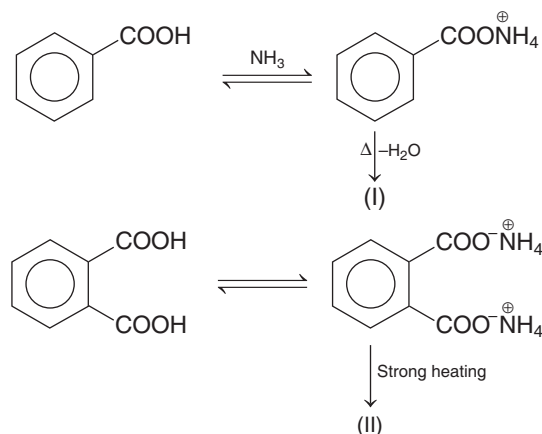
23. Carboxylic acids dissociate to give resonance stabilised carboxylate anion



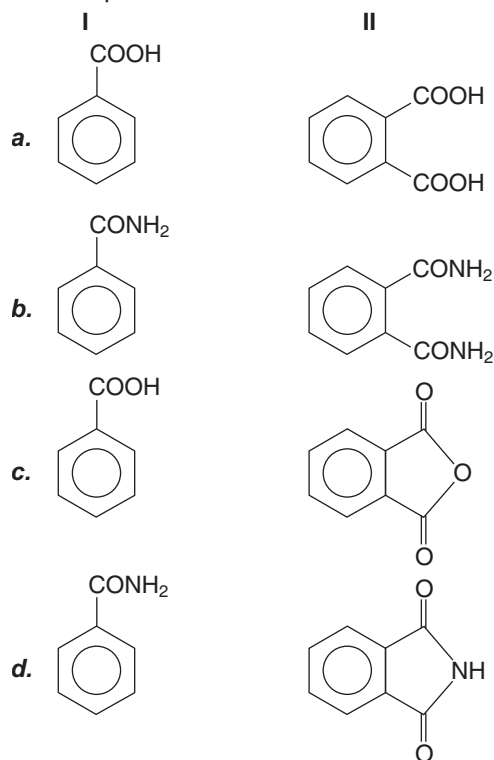
The above statement is

a. true b. false
c. sometimes true d. sometimes false

24. In the given reaction,



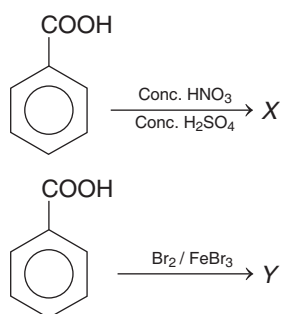
Identify I and II for the above reaction and choose the correct option.



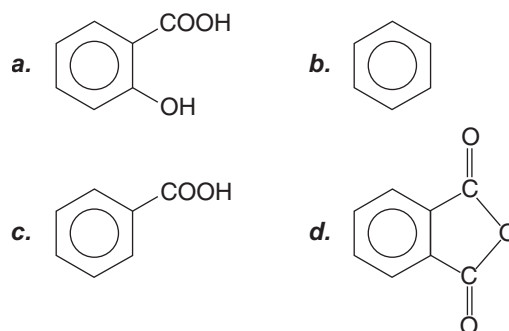
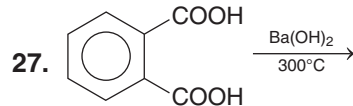
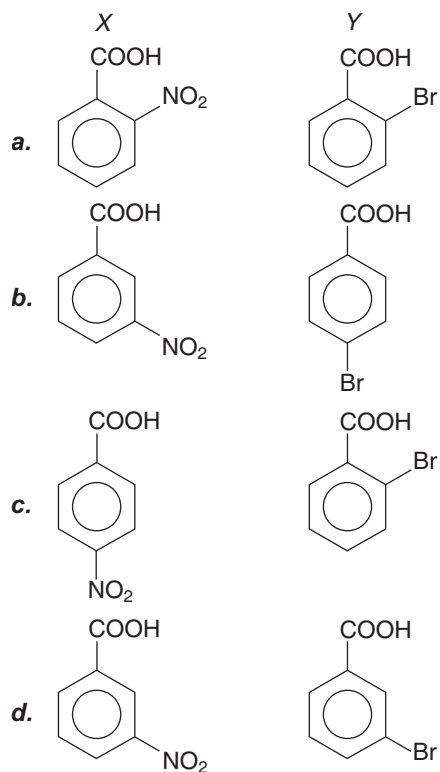
25. The product formed during Hell-Volhard-Zelinsky reaction is

- a. $R-\underset{\substack{| \\ X}}{\text{CH}}-\text{COOH}$
 b. $R-\text{CH}_2-\text{COX}$
 c. $R-\underset{\substack{| \\ X}}{\overset{\substack{| \\ X}}{\text{C}}}-\text{COOH}$
 d. $R-\underset{\substack{| \\ X}}{\text{CH}}-\text{CH}_2-\text{COOH}$

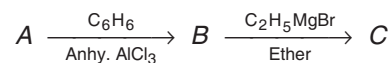
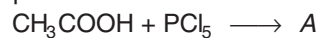
26. In the given reactions:



Identify X and Y for the above reaction.



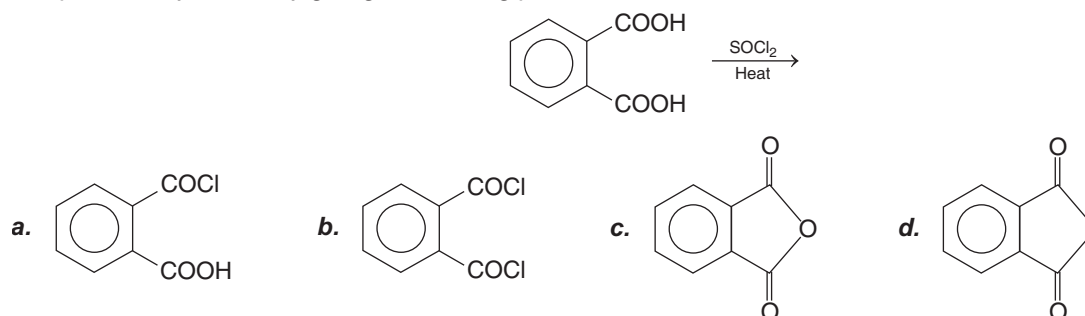
28. In a set of the given reactions, acetic acid yields a product C.



Product C would be

- a. $\text{CH}_3\text{CH}(\text{OH})\text{C}_6\text{H}_5$
 b. $\text{CH}_3-\underset{\substack{| \\ \text{C}_2\text{H}_5}}{\text{C}}(\text{OH})\text{C}_6\text{H}_5$
 c. $\text{CH}_3\text{CH}(\text{OH})\text{C}_2\text{H}_5$
 d. $\text{CH}_3\text{COC}_6\text{H}_5$

29. Complete the synthesis by giving the missing product.



30. Role of 2,4-dichlorophenoxy acetic acid is used as

- a. moth repellent b. insecticide c. fungicide d. herbicide

BITSAT Archives

- Acetic acid on warming with hydrazoic acid in presence of conc. H_2SO_4 gives [2010]
 - CH_3NH_2
 - $\text{CH}_3\text{CH}_2\text{NH}_2$
 - CH_3CONH_2
 - $\text{CH}_3\text{COONH}_4$
- Which of the following on heating does not form an anhydride? [2009]
 - Oxalic acid
 - Succinic acid
 - Glutaric acid
 - Maleic acid
- Acetic acid reacts separately with the following alcohols. The rate of esterification is highest for [2009]
 - CH_3OH
 - $\text{C}_2\text{H}_5\text{OH}$
 - $(\text{CH}_3)_2\text{CHOH}$
 - $(\text{CH}_3)_3\text{COH}$
- Identify A and B in the following reaction. [2008]

$$\text{CH}_3 - \text{CH}_3 \xleftarrow{B} \text{CH}_3\text{COOH} \xrightarrow{A} \text{CH}_3\text{CH}_2\text{OH}$$

A	B
a. $\text{HI} + \text{red P}$	LiAlH_4
b. Ni/Δ	LiAlH_4
c. LiAlH_4	$\text{HI} + \text{red P}$
d. Pd-BaSO_4	$\text{Zn} + \text{HCl}$
- In aqueous solution amino acids mostly exist as [2007]

a. $\text{NH}_2 - \text{CHR} - \text{COOH}$	b. $\text{NH}_2 - \text{CHR} - \text{COO}^-$
c. $\text{NH}_3 - \text{CHR} - \text{COOH}$	d. $\text{H}_3\text{N}^+ \text{CHR} - \text{COO}^-$
- On heating with oxalic acid at 110°C , glycerine gives [2006]

a. glyceryl trioxalate	b. formic acid
c. glyceryl dioxalate	d. None of the above
- Glycerol is oxidised by bismuth nitrate to produce [2005]

a. oxalic acid	b. mesoxalic acid
c. glyceric acid	d. glyoxalic acid

Answer with Solutions

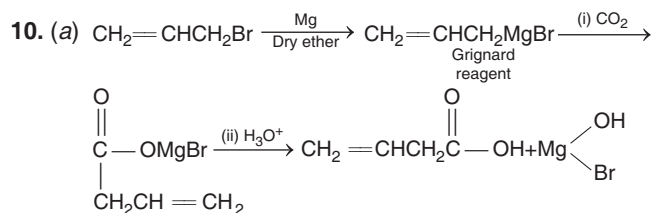
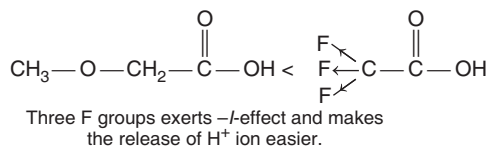
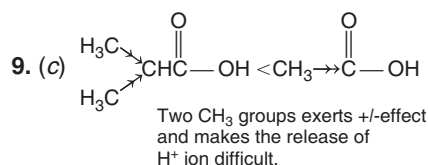
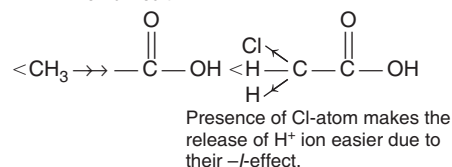
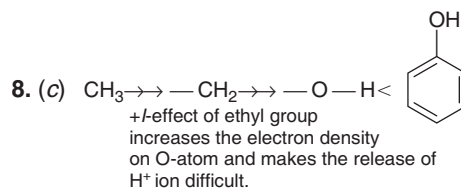
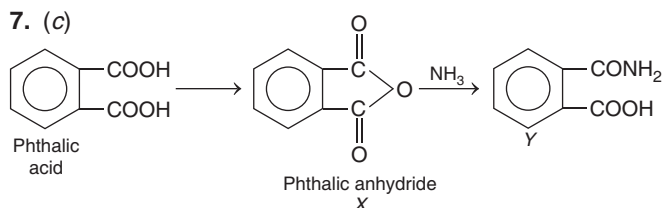
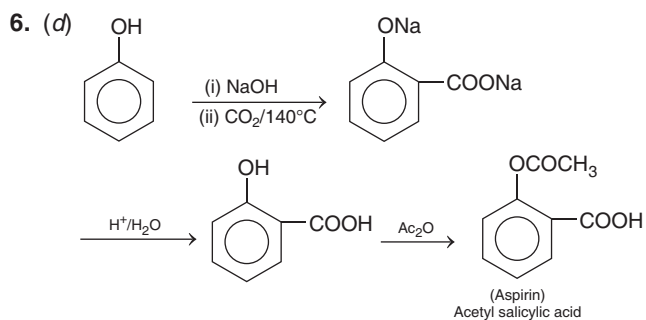
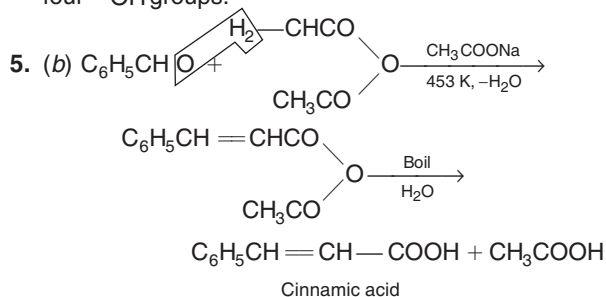
Practice Exercise

- (c) High molecular weight of acetic acid coupled with polar nature due to resonance as well as presence of H-bonding accounts for its high boiling point.
- (d) Alcoholic part of this ester offers maximum steric hinderance. Hence, is hydrolysis will be slowest.
- (a) Acid-2 is acetic acid. HCN is a weaker acid as its heat of ionisation is less than acetic acid. Hence,

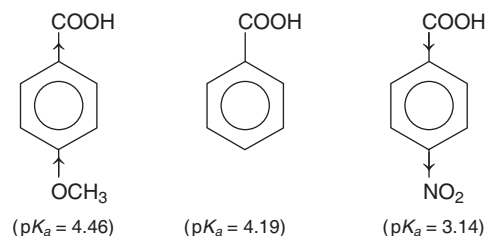
$$(pK_a)_{\text{HCN}} > (pK_a)_{\text{acid-2}}$$

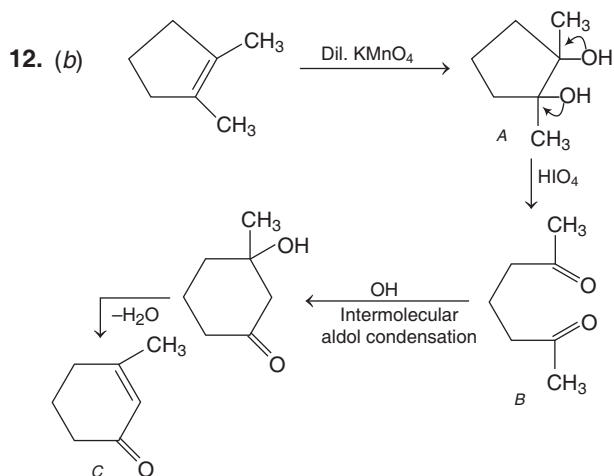
- (d) $R-OH \longrightarrow R-OCOCH_3$

Acylation of one hydroxyl group results in increament of $-COCH_2$ (or C_2H_2O). In the question there is an increament of $(C_8H_8O_4)$, hence there should be present four $-OH$ groups.

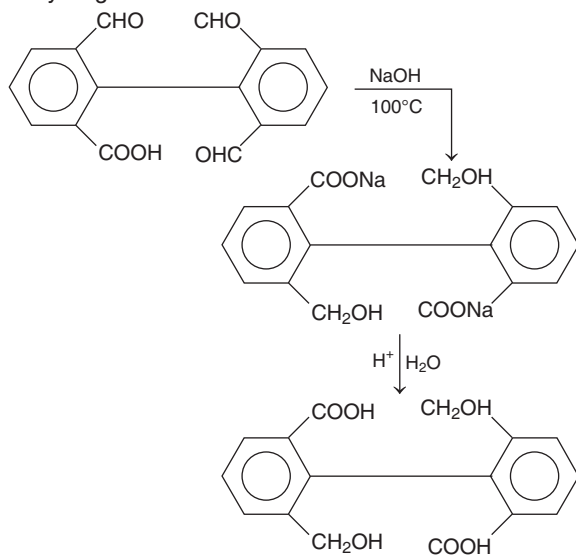


- (a) The presence of electron withdrawing group on the phenyl of aromatic carboxylic acid increases their acidity while electron donating groups decrease their acidity.

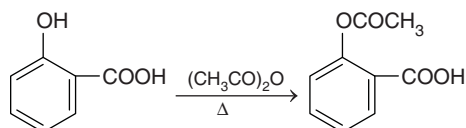




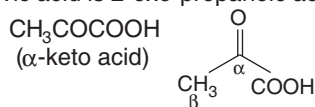
13. (d) The acidity of halogenated acid increases with increase in electronegativity of the halogen present. The correct order is
 $\text{FCH}_2\text{COOH} > \text{ClCH}_2\text{COOH} > \text{BrCH}_2\text{COOH} > \text{CH}_3\text{COOH}$
14. (d) Cannizzaro reaction occurs in the absence of α -hydrogen atom.



15. (d) Formic acid decolourises acidified KMnO_4 solution as well as gives silver mirror with Tollen's reagent.
16. (a) Acetyl salicylic acid is known as aspirin.

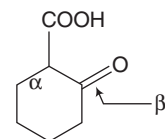


17. (b) The product of the reaction is CH_4 and hence, the general formula is $\text{C}_n\text{H}_{2n+2}$.
18. (a) Pyruvic acid is 2-oxo-propanoic acid.



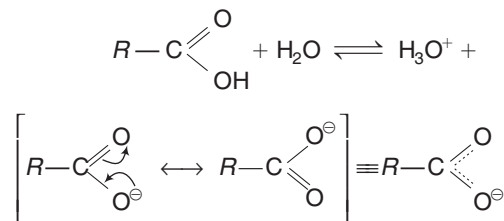
19. (c) Product of 1, 2 and 4 is CH_3COOH , whereas product of 3 is HCOOH which is a reducing agent and will be oxidised by $\text{Cr}_2\text{O}_7^{2-}$.

20. (b) It is a β -keto acid which undergoes decarboxylation in very mild conditions, i.e. on simple heating. Ordinary carboxylic acid requires soda lime catalyst for decarboxylation (β -keto acid must contain an α -H atom).

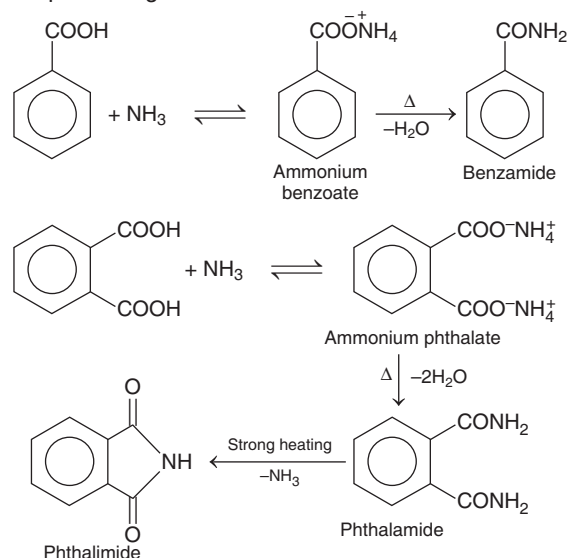


Thus, readily undergoes decarboxylation.

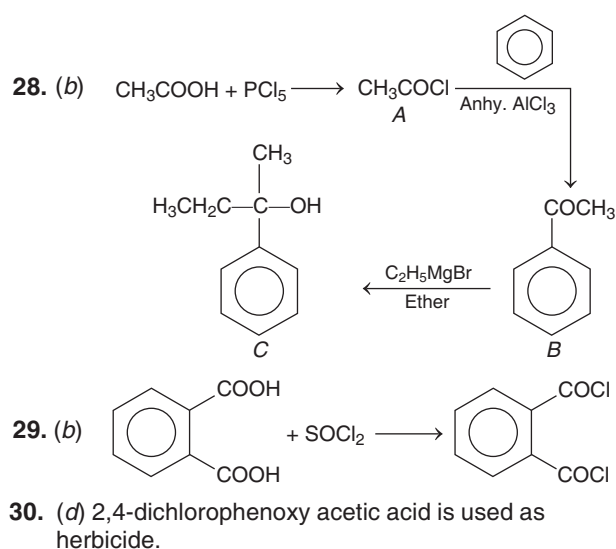
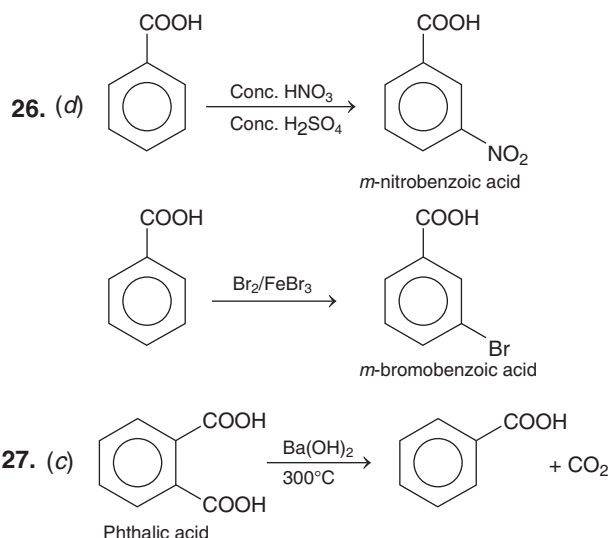
21. (c) HCOOH gives positive Tollen's test while CH_3COOH does not respond to this test.
22. (b) The α -bromination of carboxylic acids is called Hell-Volhard-Zelinsky reaction.
23. (b) Carboxylic acids dissociate in water to give resonance stabilised carboxylate anions and hydronium ions.



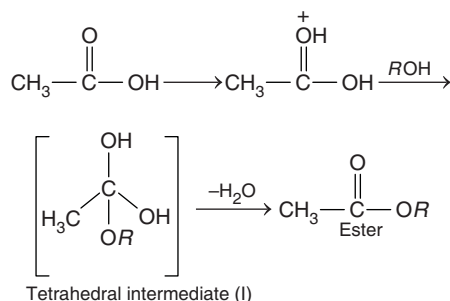
24. (d) Carboxylic acids react with ammonia to give ammonium salt, which on further heating at high temperature gives amides.



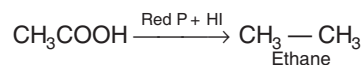
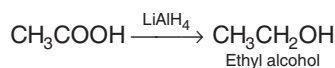
25. (a) Carboxylic acids having an α -hydrogen are halogenated at the α -position on treatment with chlorine or bromine in the presence of a small amount of red phosphorus to give α -halocarboxylic acids. The reaction is known as Hell-Volhard-Zelinsky reaction.

**BITSAT Archives**

1. (a) $\text{CH}_3\text{COOH} \xrightarrow[\text{H}_2\text{SO}_4]{\text{N}_3\text{H}} \text{CH}_3\text{NH}_2 + \text{CO}_2 + \text{N}_2$
2. (a) Therefore, oxalic acid on heating does not form an anhydride. Instead it decomposes to give $\text{CO}_2 + \text{CO} + \text{H}_2\text{O}$.
- $$\begin{array}{c} \text{COOH} \\ | \\ \text{COOH} \end{array} \xrightarrow{\Delta} \text{CO}_2 + [\text{HCOOH}] \xrightarrow{\Delta} \text{CO} + \text{H}_2\text{O}$$
3. (a) As the hydrocarbon part (R) of the alcohol increases, the tetrahedral intermediate (I) formed during esterification becomes more and more crowded and hence, the rate of esterification decreases accordingly. Thus, the smallest alcohol, i.e. CH_3OH reacts at the fastest rate.

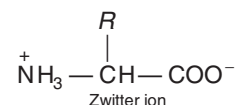


4. (c) Acetic acid on reduction with lithium aluminiumhydride (LiAlH_4) gives ethyl alcohol while on reduction with HI and red P gives ethane.

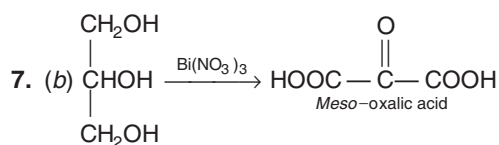
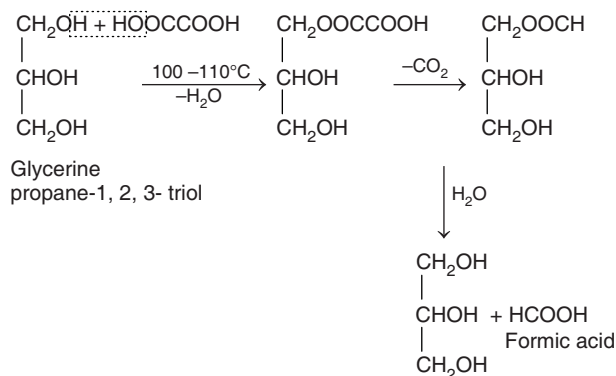


Hence, reagent A and B are respectively LiAlH_4 and HI/red P .

5. (d) In aqueous solutions, amino acids mostly exist as zwitter ions.



6. (b)



28

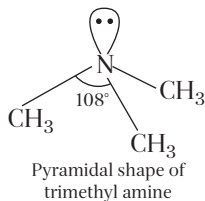
Nitrogen Containing Compounds

Amines

These are the derivatives of ammonia in which one, two or all the three hydrogen atoms are replaced by alkyl or aryl groups. Amines are classified as primary, secondary or tertiary according to the replacement of one, two or three hydrogen atoms from ammonia respectively.

Structure of Amines

Nitrogen in amines, is sp^3 -hybridised. The sp^3 -hybridised orbitals of nitrogen overlap with orbitals of hydrogen or carbon depending upon the composition of amines. The fourth hybridised orbital of nitrogen in all amines has an unshared pair of electrons. Due to the presence of lone pair of electrons, the bond angle C–N–E (where, E is C or H) is less than 109.5° and shape is pyramidal.

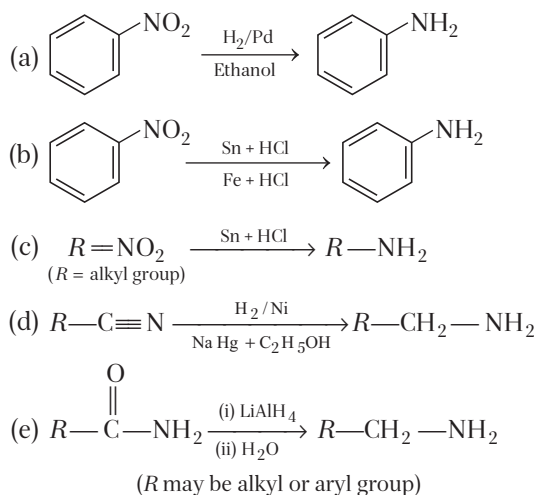


Preparation of Amines

Several methods can be used to prepare aliphatic amines, out of which some are specific for 1°, 2° and 3° amines in which some gives mixtures of all the three.

1. By the Reduction of Nitro Compounds, Nitriles and Amides

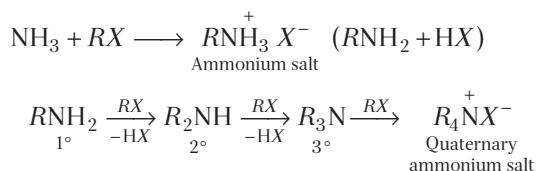
Alkyl nitriles, nitroalkanes, oximes and amides, all on reduction give primary amines. Reduction of nitriles to primary amines by using Na and alcohol is called mendius reaction.



All the above reactions are used in the preparation of primary amines.

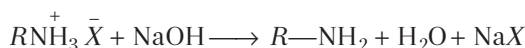
2. By Ammonolysis of Alkyl Halides (Hofmann's Method)

When alkyl halide is heated with alcoholic solution of ammonia in a sealed tube at about 393 K, a mixture of amines is obtained. This reaction is called Hofmann's ammonolysis method.



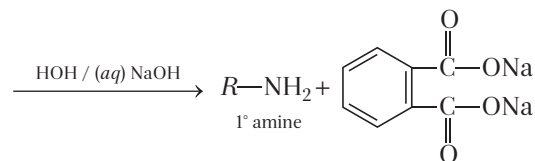
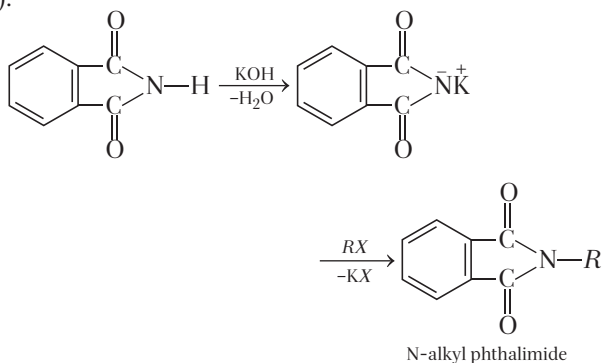
NOTE Ammonolysis of alkyl halides produces a mixture of 1°, 2° and 3° amines alongwith quaternary ammonium salt.

The free amine can be obtained from ammonium salt by the treatment with a strong base.



3. Gabriel Phthalimide Synthesis

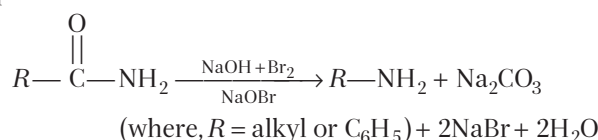
This reaction is used for the preparation of 1° amine (aliphatic as well as aromatic amines having electron withdrawing group at *o*- and *p*-positions, e.g. —NO₂ group).



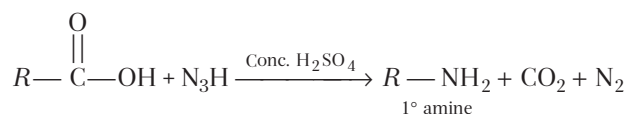
Aromatic primary amines cannot be prepared by this method because aryl halides do not undergo nucleophilic substitution with the anion formed by phthalimide.

4. Hofmann Bromamide Degradation Reaction

Amides on reaction with Br₂ in an aqueous or ethanolic solution of NaOH give 1° amine with one C-atom less than that present in the amide.



5. Schmidt Reaction

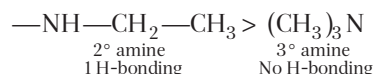
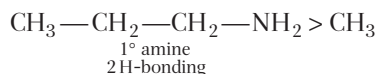


Instead of N₃H and conc. H₂SO₄, a mixture of NaN₃ and conc. H₂SO₄ can also be used.

Physical Properties of Amines

General physical properties of amines are as follows :

- Lower aliphatic amines are gases while lower aromatic amines are liquids.
- Boiling point of amines is higher than hydrocarbons of comparable molecular masses due to the presence of intermolecular H-bonding. However, H-bonding is absent in tertiary amines.
- Aliphatic amines are miscible with water due to intermolecular H-bonding with H₂O molecules.
- The lower aliphatic amines are gases with fishy odour. Aniline and other aryl amines are usually colourless but get coloured on storage due to atmospheric oxidation.
- Boiling points and solubility of isomeric amines decrease with branching, i.e.



- Amines have lower boiling point than corresponding alcohols.

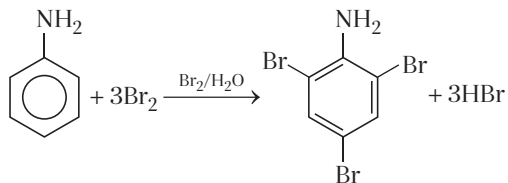
Chemical Properties of Amines

Amines are reactive and behave as nucleophiles due to the presence of one unshared pair of electrons.

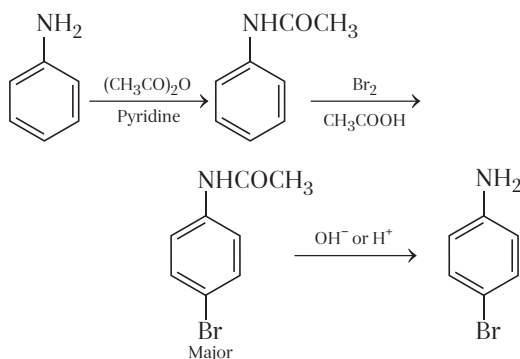
This reaction is used to distinguish 1°, 2° and 3° amines.

Electrophilic Substitution Reaction

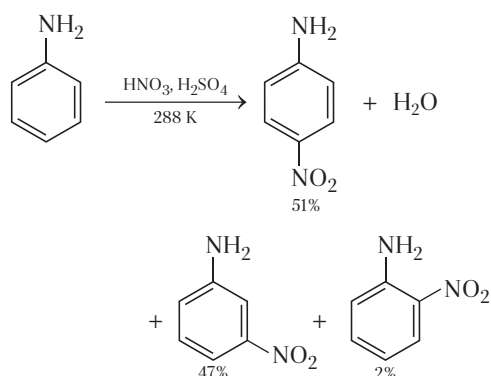
This is an organic reaction in which an atom that is attached to an aromatic system is replaced by an electrophile. *Some of the most important electrophile aromatic substitution, i.e. bromination, nitration, sulphonation etc, are given below :*

1. Bromination

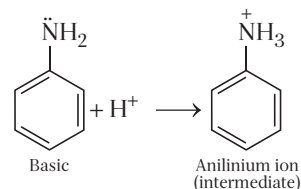
The main problem encountered during electrophilic substitution reactions of aromatic amines is that these have very high reactivity.

2. Selective Bromination**3. Nitration** Aniline can be treated with a nitric acid to give nitrobenzene.

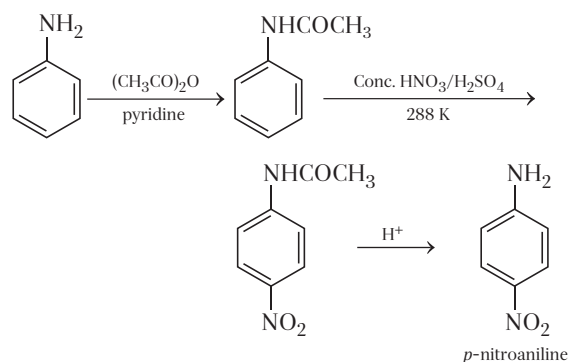
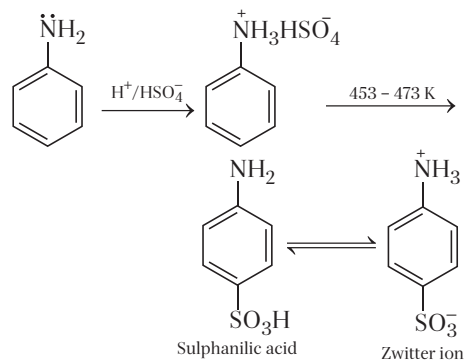
The overall reaction is given below :



This is due to the fact that aniline gets protonated to form anilinium ion which is *meta* directing.



However, by protecting the —NH_2 group through acetylation with acetic anhydride, the nitration can be controlled and *p*-nitro aniline is obtained as a major product.

**4. Sulphonation** Benzene can be treated with a very concentrated sulphuric acid to give sulphanilic acid in a reversible reaction. The overall reaction is given below.**Hinsberg's Test**

Distinction between primary, secondary and tertiary amines is made by $\text{C}_6\text{H}_5\text{SO}_2\text{Cl}$, which is known as Hinsberg's reagent.

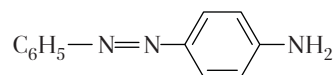
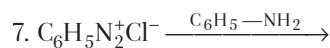
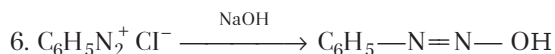
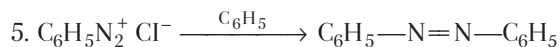
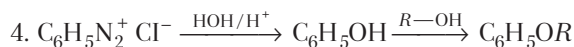
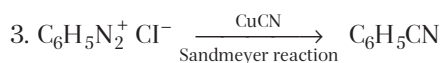
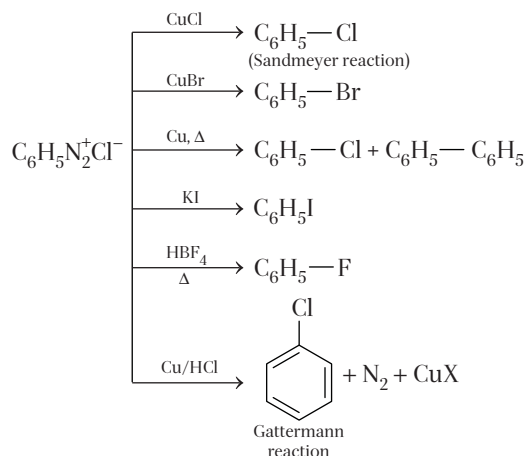
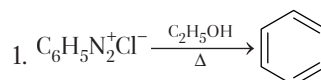
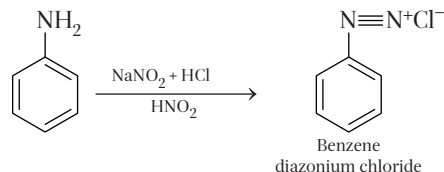
$$\begin{array}{ccc}
 \text{R}-\text{N}-\text{H} + \text{C}_6\text{H}_5-\text{SO}_2-\text{Cl} & \xrightarrow{-\text{HCl}} & \text{C}_6\text{H}_5-\text{SO}_2-\text{N}-\text{R} \\
 | & & | \\
 \text{H} & & \text{H} \\
 \text{(Insoluble mass)} & & \\
 \downarrow \text{NaOH (aq)} & & \\
 \text{C}_6\text{H}_5-\text{SO}_2-\text{N}^--\text{R} & & \text{Na}^+ \\
 \text{Soluble} & &
 \end{array}$$
$$\begin{array}{c}
 \text{O} \\
 \parallel \\
 \text{R}-\text{N}-\text{H} + \text{C}_6\text{H}_5-\text{S}-\text{Cl} \xrightarrow{\quad} \text{HCl} \\
 | \\
 \text{R}
 \end{array}$$

$$\begin{array}{c}
 \text{O} \\
 \parallel \\
 \text{C}_6\text{H}_5-\text{S}-\text{N}-\text{R} \xrightarrow{\text{NaOH (aq)}} \text{Insoluble} \\
 \parallel \quad | \\
 \text{O} \quad \text{R} \\
 \text{Insoluble}
 \end{array}$$

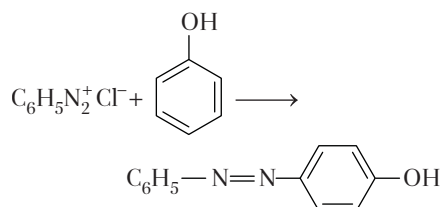
due to the absence of H-atoms

$$\begin{array}{c} \text{H} \qquad \qquad \qquad \text{H} \quad \text{S} \\ | \qquad \qquad \qquad | \quad || \\ \text{C}_2\text{H}_5-\text{N}-\text{H} + \text{S} = \text{C} = \text{S} \longrightarrow \text{C}_2\text{H}_5-\text{N}-\text{C}-\text{SH} \\ \\ \xrightarrow{\text{HgCl}_2} \text{C}_2\text{H}_5-\text{N} = \text{C} = \text{S} + \text{HgS} + 2\text{HCl} \\ \text{Ethyl isothiocyanate} \end{array}$$

Quaternary ammonium salts of long chain aliphatic tertiary amines are used as detergents.

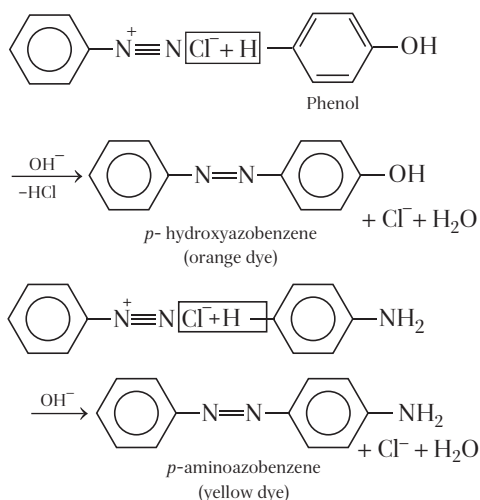


8.



Reactions Involving Retention of Diazo Group

Coupling reactions are the examples of electrophilic substitution reaction.



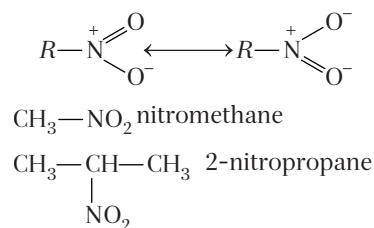
Coupling with phenols is carried out in weakly alkaline medium (pH 9 to 10) because phenate ion produced is coupled with diazonium salt more readily. Above pH 10, diazonium salt reacts with hydroxide ion to form diazotate ion which does not take part in coupling reaction.

Uses of Diazonium Salts

Diazonium salts are used for the preparation of a large number of aromatic compounds viz substitution and coupling compounds.

Nitroalkane

The nitro derivatives of alkanes are called nitroalkanes.



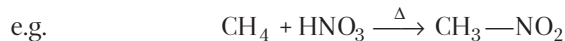
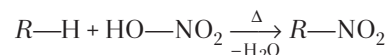
Isomerism



They also show chain, positional and optical isomerism.

Methods of Preparation

1. Nitration of alkane



2. Action of AgNO₂ on alkyl halide

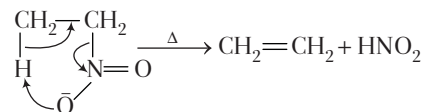


Physical Properties

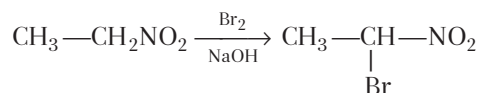
Colourless, pleasant smelling compounds with low solubility in H₂O and have very high boiling point.

Chemical Reactions

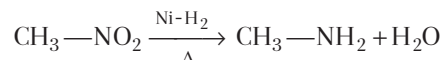
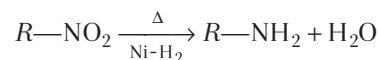
1. Action of heat



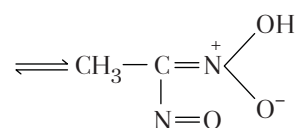
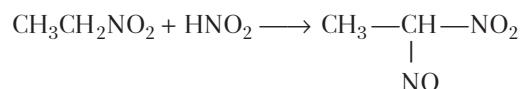
2. Halogenation



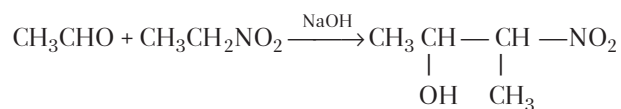
3. Reduction



4. Reaction with HNO₂

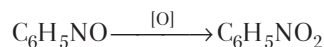
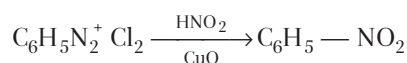
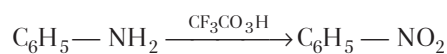
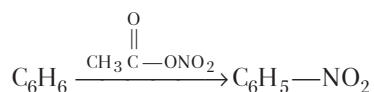


5. Condensation with carbonyl compounds



Nitrobenzene

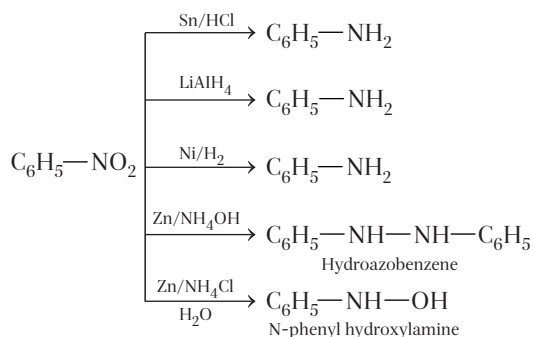
Methods of Preparation



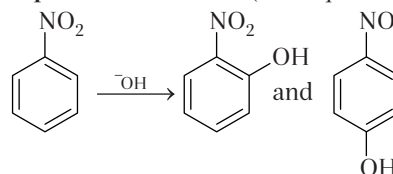
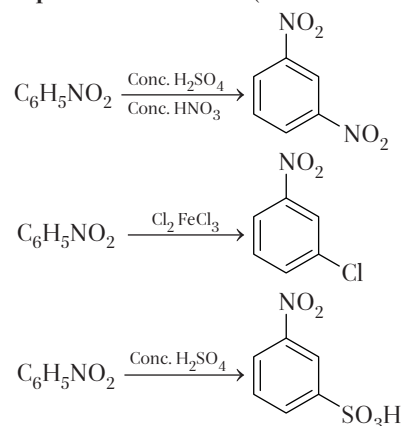
Physical Properties

It has bitter almond odour and is also known as oil of mirbane.

Chemical Properties

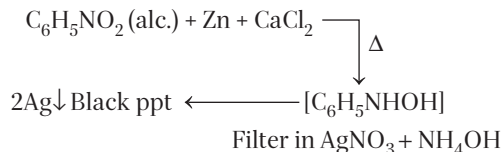


Substitution Reactions

(a) Nucleophilic substitution (*o* and *p* - direction)(b) Electrophilic substitution (*m* - direction)

Test for Nitrobenzene

(Mulliken and Barker's Method)



Uses of Nitrobenzene

It is mainly used in

1. Manufacture of aniline.
2. As solvent in Friedel-Crafts reaction.
3. As an oxidant in the synthesis of quinoline.

Practice Exercise

1. Gabriel phthalimide synthesis can be used for the preparation of amine from

a. $\text{CH}_3\text{CH}_2\text{Br}$ b. $(\text{CH}_3)_3\text{CBr}$
c. $p\text{-CH}_3\text{OC}_6\text{H}_4\text{Br}$ d. $p\text{-CH}_3\text{C}_6\text{H}_4\text{Br}$

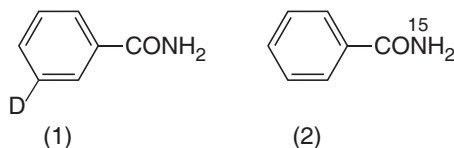
2. Hofmann's bromamide degradation reaction is shown by

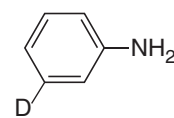
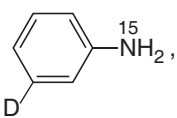
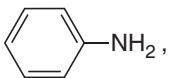
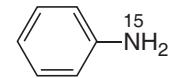
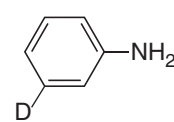
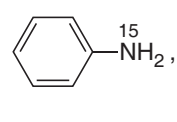
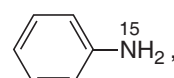
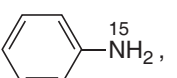
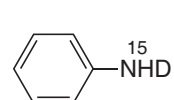
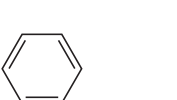
a. ArNH_2
b. ArCONH_2
c. ArNO_2
d. ArCH_2NH_2

3. The positive carbylamine test is shown by

a. $p\text{-methylbenzylamine}$
b. $N,N\text{-dimethylaniline}$
c. $2,4\text{-dimethylaniline}$
d. $N\text{-methyl-}o\text{-methyl aniline}$

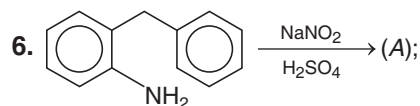
4. What are the constituent amines formed when the mixture of I and II undergoes Hofmann's bromamide degradation?



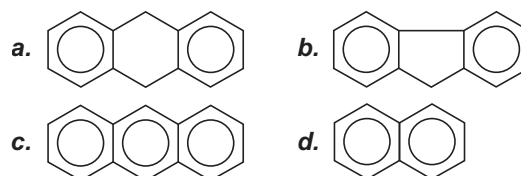
- a.  , 
 , 
b.  , 
c.  , 
d.  , 

5. The primary, secondary and tertiary amines can be best distinguished by

a. mustard oil reaction
b. carbylamine reaction
c. exhaustive alkylation
d. HNO_2 treatment



Product of this reaction is



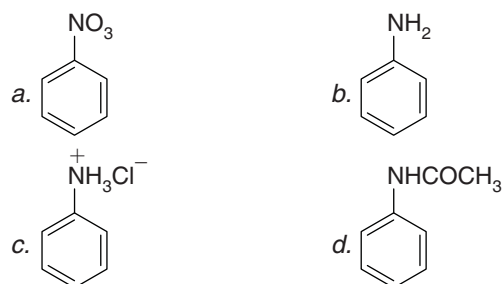
7. Which of the following compounds does not liberate nitrogen with HNO_2 ?

a. Carbamide b. Primary amine
c. Secondary amine d. Alkanamide

8. The compound that is most reactive towards electrophilic nitration is

a. toluene b. benzene
c. benzoic acid d. nitrobenzene

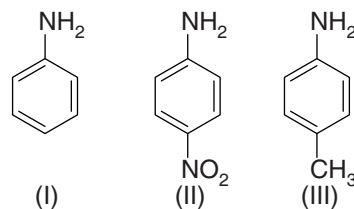
9. Towards electrophilic substitution, the most reactive species will be



10. When aniline is treated with fuming sulphuric acid at 475 K, it gives

a. sulphanilic acid
b. aniline sulphate
c. $o\text{-aminobenzene sulphonc acid}$
d. $m\text{-aminobenzene sulphonc acid}$

11. The correct increasing order of basic strength for the following compounds is [NCERT Exemplar]



a. $\text{II} < \text{III} < \text{I}$ b. $\text{III} < \text{I} < \text{II}$
c. $\text{III} < \text{II} < \text{I}$ d. $\text{II} < \text{I} < \text{III}$

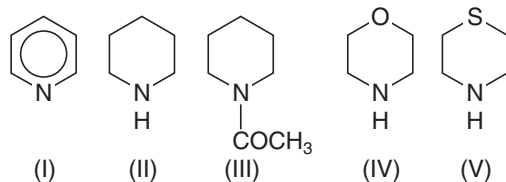
12. Amino group, —NH_2 is *ortho*, *para*-directing group in case of aromatic electrophilic substitution but nitration of aniline produce a good amount of *m*-nitroaniline. This is because

a. —NH_2 gets converted into $\text{—NH}^+\text{NO}_2^-$ which is *m*-directing
 b. NH_2 gets converted into NH_3^+ which is *m*-directing
 c. —NH_2 gets converted into $\text{—NH}^+\text{SO}_4^-$ which is *m*-directing
 d. *ortho*, *para* activity of —NH_2 group is completely destroyed during nitration

13. Reduction of aromatic nitro compounds using Fe and HCl gives

a. aromatic oxime
 b. aromatic hydrocarbon
 c. aromatic primary amine
 d. aromatic amide

14. The relative order of basic character of the following compounds is



a. $\text{II} > \text{I} > \text{III} > \text{IV} > \text{V}$ b. $\text{II} > \text{III} > \text{IV} > \text{V} > \text{I}$
 c. $\text{II} > \text{V} > \text{IV} > \text{I} > \text{III}$ d. $\text{II} > \text{IV} > \text{V} > \text{III} > \text{I}$

15. Which of the following amine does not react with Hinsberg reagent?

a. Neopentyl amine b. Isopropyl amine
 c. Triethyl amine d. Ethyl methyl amine

16. When a primary amine reacts with chloroform in ethanolic KOH, the product is

a. an isocyanide b. an aldehyde
 c. a cyanide d. an alcohol

17. Tertiary nitro compounds do not tautomerise because

a. there is no double bond
 b. there is no α -hydrogen
 c. oxygen is more electronegative than H
 d. All of the above

18. When *p*-toluidine reacts with sodium nitrite and hydrochloric acid at 273 K, a crystalline precipitate is formed, which is boiled with water. The resulting compound obtained is

a. *p*-cresol b. *p*-nitro toluene
 c. phenol d. toluic acid

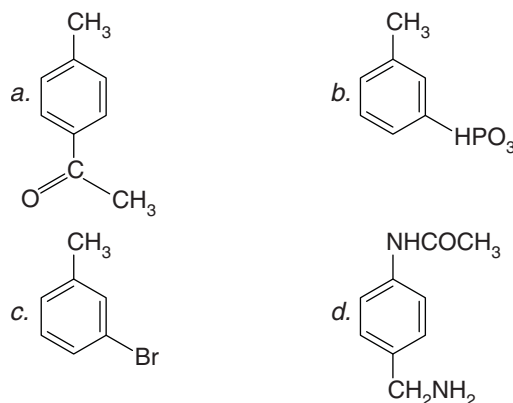
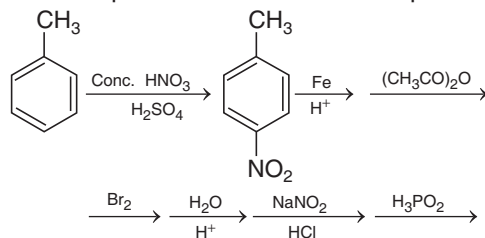
19. The reagent used to distinguish *p*-methylaniline from N-methylaniline is

a. benzenesulphonyl chloride
 b. iodoform in alc. KOH
 c. AgCl
 d. AgNO_3

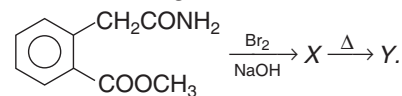
20. The best reagent for converting 2-phenylpropanamide into 2-phenylpropanamine is

a. excess H_2
 b. Br_2 in (aq) NaOH
 c. iodine in the presence of phosphorus
 d. LiAlH_4 in ether

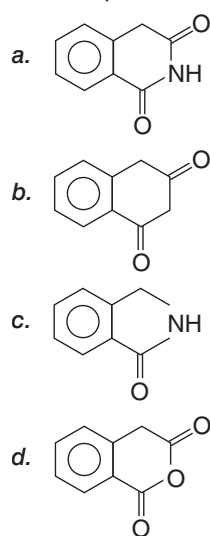
21. Consider the following reaction sequence. The final product of this reaction sequence is



22. Consider the following series of reactions



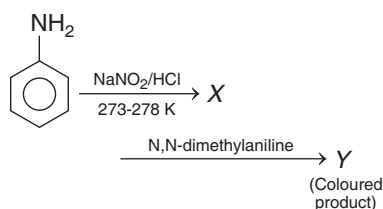
The compound Y is



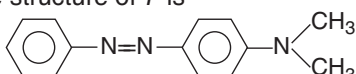
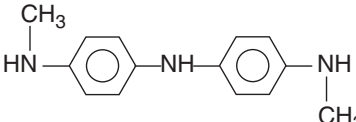
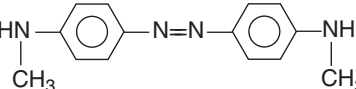
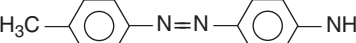
23. Compound A (C_3H_9N) reacts with benzene sulphonyl chloride to form a solid insoluble in alkali. The structure of compound A is

- a. $CH_3-N(CH_3)-CH_3$
 b. $CH_3-CH_2-NH-CH_3$
 c. $CH_3-CH_2-CH_2-NH_2$
 d. All of the above

24. Aniline yields a coloured product Y through the following series of reactions:



The structure of Y is

- a. 
 b. 
 c. 
 d. 

25. Consider the following statements:

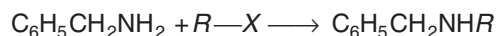
Phenyl diazonium salts form azo dye with

- I. aniline
 II. phenol
 III. N,N-dimethyl aniline
 IV. anisole (methoxybenzene)

The correct statements is

- a. II, III and IV are correct b. I, III and IV are correct
 c. I, II and IV are correct d. I, II and III are correct

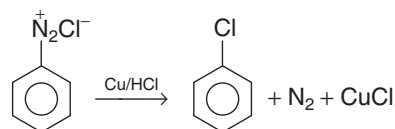
26. Benzylamine may be alkylated as shown in the following equation



Which of the following alkyl halides is best suited for this reaction through S_N1 mechanism?

- a. CH_3Br
 b. C_6H_5Br
 c. $C_6H_5CH_2Br$
 d. C_2H_5Br

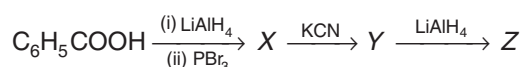
27. Consider the following reaction :



The above reaction is called

- a. carbylamine reaction
 b. Gattermann synthesis
 c. Sandmeyer's reaction
 d. Balz-Schiemann reaction

28. Identify the final product (Z) in the following sequence of reactions



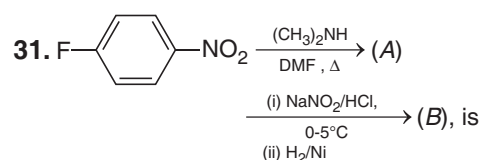
- a. $C_6H_5CH_2NH_2$
 b. $C_6H_5CH_2CH_2NH_2$
 c. $C_6H_5CH_2CH_2NH_2$
 d. $C_6H_5-CH(CH_3)-NH_2$

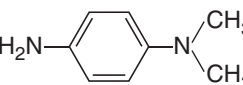
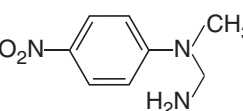
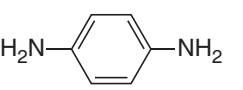
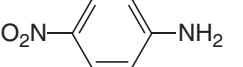
29. Hydrolysis of phenyl isocyanide forms

- a. benzoic acid
 b. formic acid
 c. acetanilide
 d. acetic acid

30. Which of the following on reaction with nitrous acid followed by treatment with NaOH produces a blood red colouration?

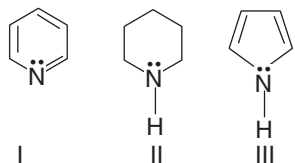
- a. RCH_2NO_2 b. R_3CNO_2
 c. R_2CHNO_2 d. $PhNO_2$



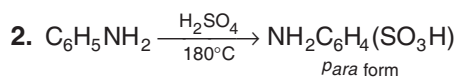
- a. 
 b. 
 c. 
 d. 

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1. Arrange the following in correct order of basicity [2014]



- a. I > II > III
b. III > II > I
c. II > I > III
d. I > III > II

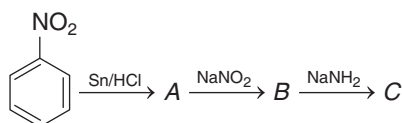


[2014]

The true statement about the product is

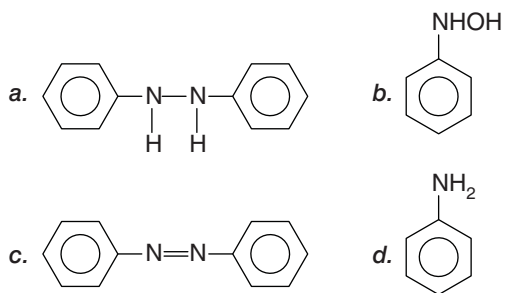
- a. it does not exist as Zwitter ion
b. it does not act as inner salt
c. $-\text{SO}_3$ diminishes the basic character of $-\text{NH}_2$
d. $-\text{NH}_2$ displays a powerful basic character

3. Identify C in the following reaction: [2012]

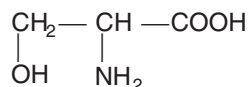


- a. benzamide
b. benzoic acid
c. chlorobenzene
d. aniline

4. The structure of the compound formed, when nitrobenzene is reduced by lithium aluminium hydride (LiAlH_4) is [2008]

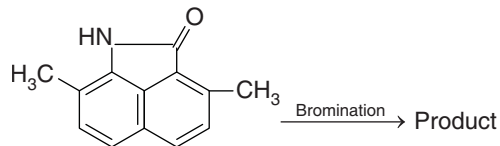


5. The IUPAC name of the compound, [2007]

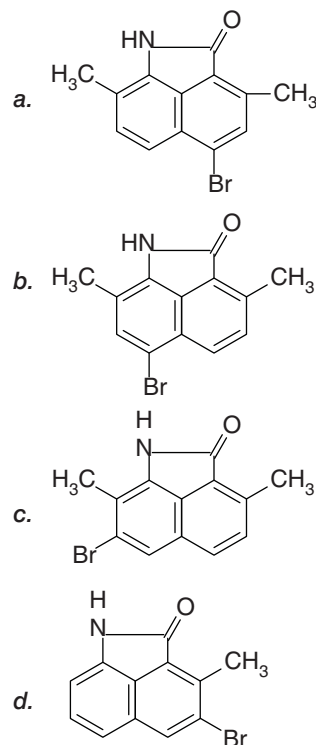


- a. 2-amino-3-hydroxy propanoic acid
b. 1-hydroxy-2-aminopropan-3-oic acid
c. 1-amino-2-hydroxypropanoic acid
d. 3-hydroxy-2-amino propanoic acid

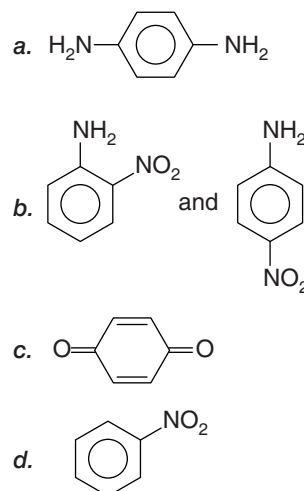
- 6.



[2007]



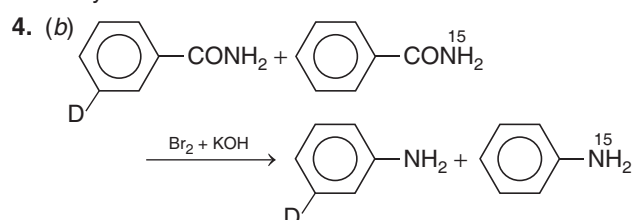
7. Aniline reacts with conc. HNO_3 to give [2007]



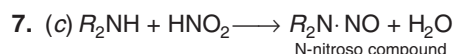
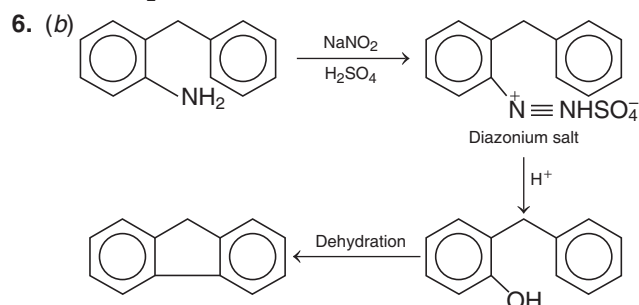
Answer with Solutions

Practice Exercise

- (a) By the use of Gabriel phthalimide synthesis, 1° alkyl halides can be converted into corresponding 1° amines.
- (b) Hofmann's bromamide degradation reaction is shown by amides, i.e. ArCONH_2 .
- (a) Carbylamine test is given by only primary amines. Therefore, *p*-methylbenzylamine gives response to carbylamine test.



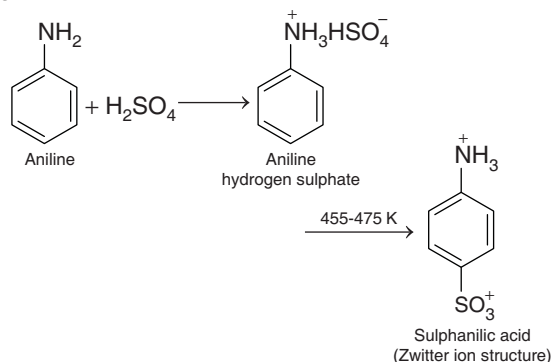
- (d) Primary, secondary and tertiary amines on reaction with HNO_2 produces different products.



- (a) Toluene has electron donating methyl group and hence, reacts fastest.

- (b) $+I$ -effect groups increase electron density on *o* and *p*-positions in benzene ring, hence they favour the electrophilic substitution.

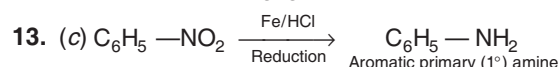
- (a) This is the reason why aniline is most reactive among given compounds.



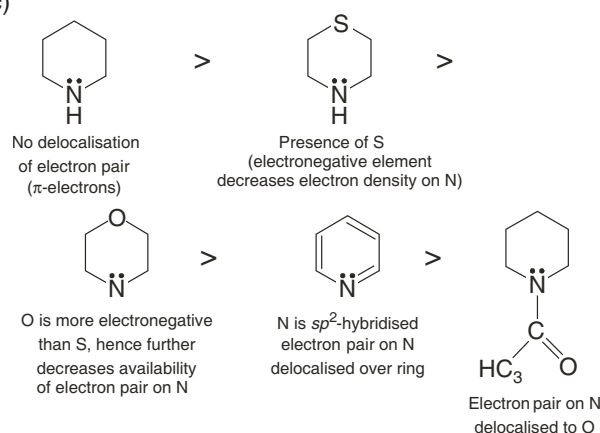
- (d) Electron releasing ($+I$ -effect) groups increase the basicity of amines while electron attracting ($-I$ -effect) groups decrease the basicity of amines.

Hence, the order is $\text{II} < \text{I} < \text{III}$

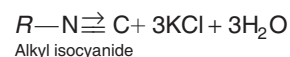
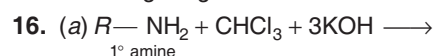
- (b) Protonation of $-\text{NH}_2$ leads to the formation of NH_3^+ which is *m*-directing group.



- (c)

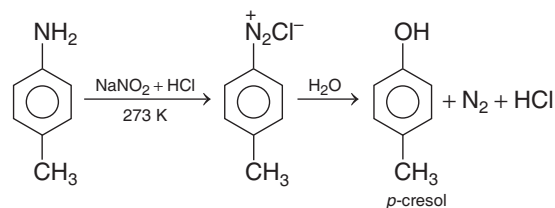


- (c) Triethyl amine is tertiary amine. It does not react with Hinsberg reagent.

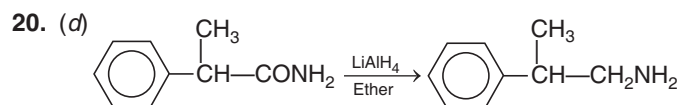


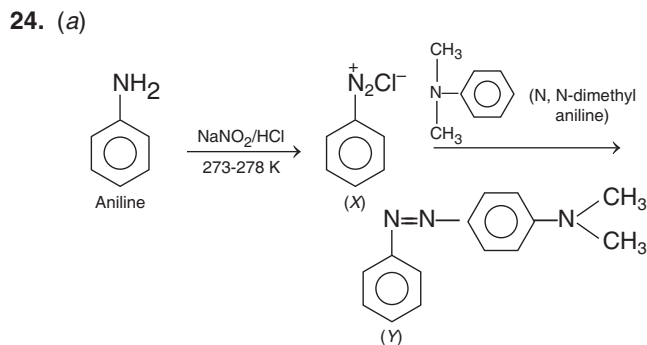
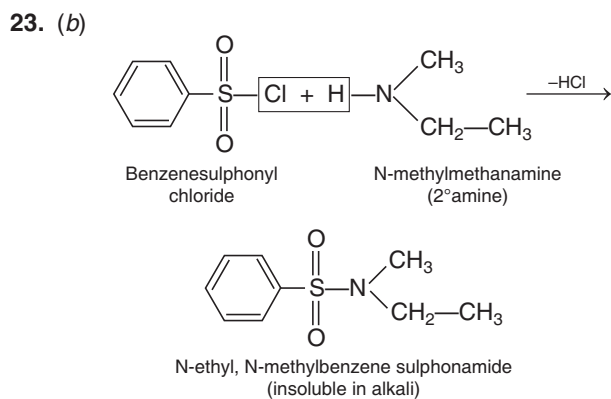
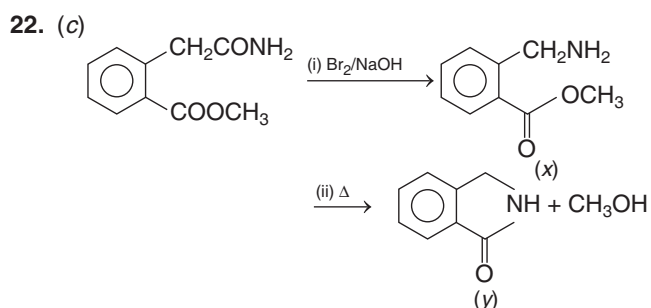
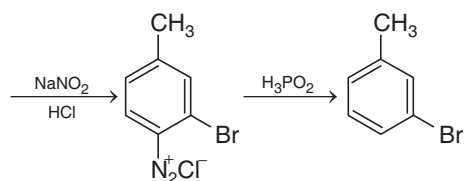
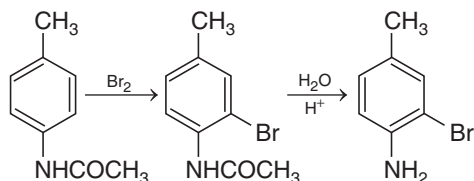
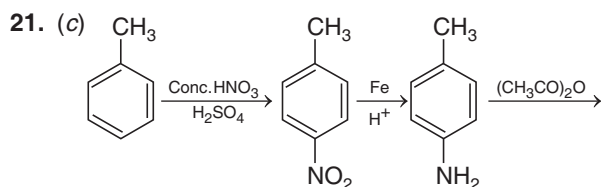
- (b) Tertiary nitro compounds do not tautomerise, since there is no α -hydrogen.

- (a)



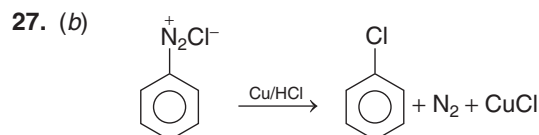
- (a) *p*-methylaniline is a primary amine while *N*-methylaniline is a secondary amine. Primary amines can be distinguished from secondary amines by benzenesulphonyl chloride.



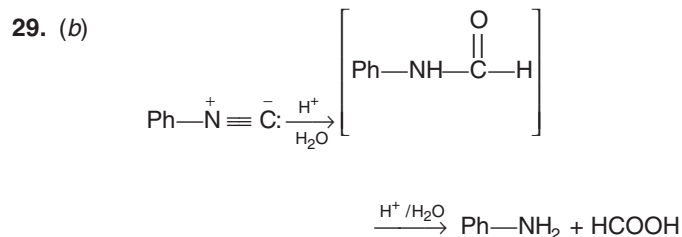
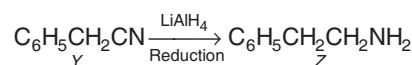
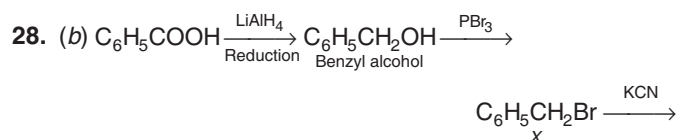


25. (d) $\text{Ph-N}^+\equiv\text{N}$ is a weak electrophile that undergoes diazocoupling only with rings activated by $-\text{OH}$, $-\text{NH}_2$, $-\text{NHR}$ or $-\text{NR}_2$. The ring is not sufficiently activated by $-\text{OCH}_3$ and hence, anisole does not form azodye.

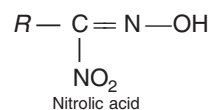
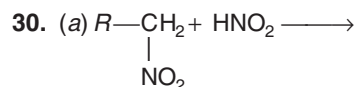
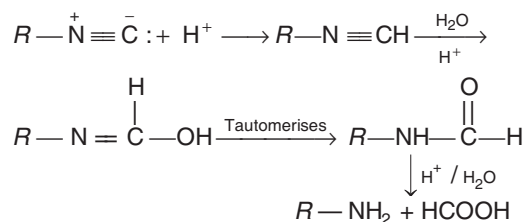
26. (c) The species $\text{C}_6\text{H}_5\text{CH}_2^+$ on ionisation produces, $\text{C}_6\text{H}_5\text{CH}_2$ (benzyl carbocation), therefore, $\text{C}_6\text{H}_5\text{CH}_2\text{Br}$ is best suited for $\text{S}_{\text{N}}1$ reaction.



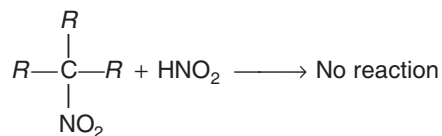
The above reaction is called Gattermann reaction.

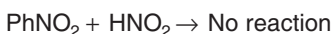
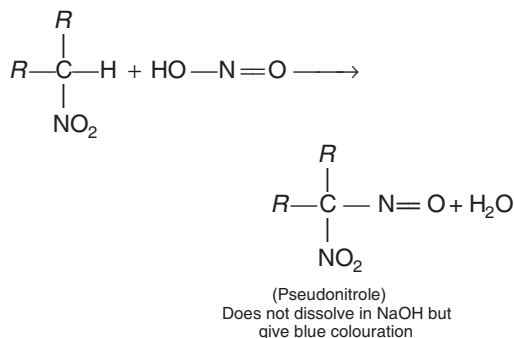


NOTE Isocyanides are hydrolysed only by acids, and not by alkalis. It is because negative charge present on the carbon atom in isocyanides initially attracts electrophile. (i.e. H^+)



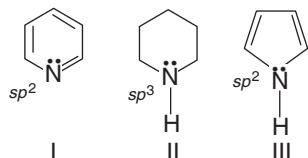
Nitrolic acid dissolve in NaOH to give blood red colouration.





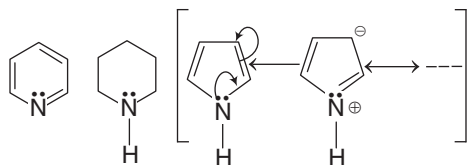
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1. (c) Hybridisation of N-atom in the compounds given below are sp^2 , sp^3 and sp^2 respectively.

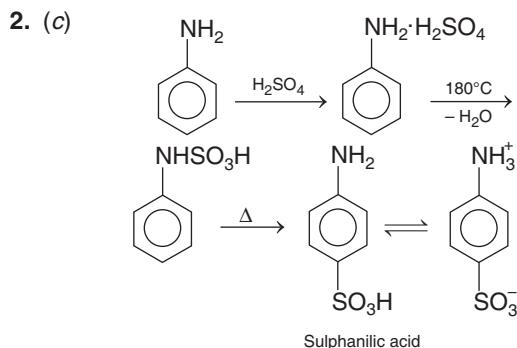


Greater the s -character, more will be electronegativity of N-atom and lesser will be its basicity on this basis, I is less basic than II.

Conjugation If lone pairs of electrons of N are involved in conjugation causes decrease in basicity of compound due to lesser availability of lone pair for donation to show basic nature.

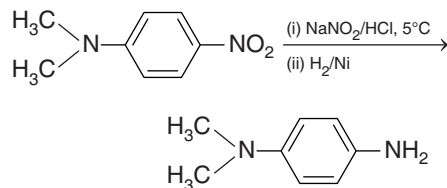
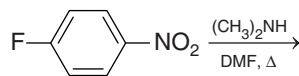


Lone pair involved in formation of aromatic sextet of 6π -electron (least basic).

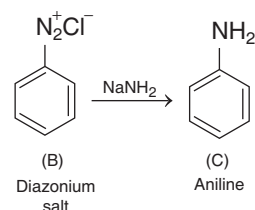
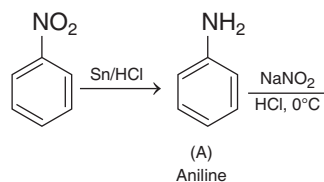


Sulphanilic acid exists as a dipolar ion which has acidic, as well as basic groups in the same molecules. Such ions are called Zwitter ions or inner salts.

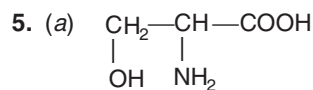
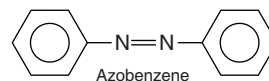
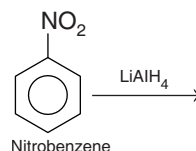
31. (a)



3. (d)

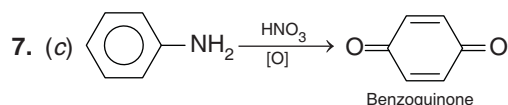


4. (c) Nitrobenzene on reduction with lithium aluminium hydride ($LiAlH_4$) gives azobenzene.



2-amino-3-hydroxy propanoic acid

6. (b) $-NH-$ is stronger electron releasing group than $-CH_3$ group, therefore bromination will take place at p -position with respect to $-NH-$ group.

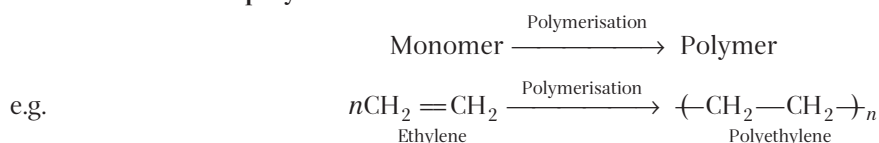


29

Polymers, Biomolecules and Chemistry in Action

Polymers

These are defined as very large molecules having high molecular mass ($10^3 - 10^7$ u). These are formed by joining of repeating structural units, called **monomers**, on a large scale. The monomers units are linked to each other by covalent bonds. The process of formation of polymer from respective monomers is called **polymerisation**.



where, $n = 1000$

Classification

Based on Structure

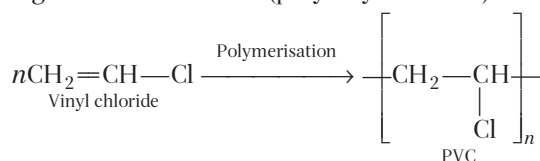
- (i) **Linear polymer** Long straight chain, joining of monomers. They have high melting point, density and tensile strength, e.g. polythene $\text{-(CH}_2\text{—CH}_2\text{)}_n$.
- (ii) **Branched chain polymer** Made up of a main chain and its branches. They have low melting point, density and tensile strength, e.g. low density polythene, PMMA, neoprene etc.
- (iii) **Cross-linked polymer** Joining of linear chains to form a 3-D network. They are hard and brittle in nature, e.g. bakelite.

Based on Synthesis

- (i) **Addition polymers** In this case during linking of monomers, there is no elimination of smaller molecules. Polymerisation takes place usually in the presence of peroxide and follows free radical mechanism.

Molecular weight of polymer = $n \times$ Molecular weight of monomers

e.g. Formation of PVC (polyvinyl chloride)



- (ii) **Condensation polymers** This is a step-growth polymerisation in which polymer is formed by linking of monomer units along with elimination of smaller molecules, such as H_2O , NH_3 , HCl etc.



Based on Source

- (i) **Natural polymers** These are obtained from animals and plants, e.g. starch, protein, natural rubber, nucleic acid, cellulose etc.
- (ii) **Synthetic polymers** Man-made, synthesised in lab, from low weight compounds, e.g. synthetic rubber, nylon, bakelite etc.

Based on Interparticle forces

- (i) **Elastomers** Weakest intermolecular forces. These have elastic character and can be stretched with retainment of structure, e.g. natural rubber.
- (ii) **Fibres** Quite strong interparticle forces such as H-bonds, e.g. nylon, dacron.
- (iii) **Thermoplastics** Polymers which can easily be moulded by heating and subsequent cooling. Intermolecular forces are intermediate to those of elastomers and fibres, e.g. polythene and polystyrene.
- (iv) **Thermosetting polymers** These become hard and infusible on heating due to excessive cross-linking. e.g. bakelite, malamine.

General Methods of Polymerisation

These are the following methods through which monomers combine to give polymers :

Addition Polymerisation

It is the process in which molecules of same or different monomers combine together to give a polymer without the elimination of small molecules. Since, there is an increase in the length of chain (i.e. chain grows) by this mode of polymerisation, it is also known as **chain growth polymerisation**. This mode of polymerisation is observed in molecules having multiple bonds, mainly in $\text{CH}_2=\text{CHy}$ type molecules (where, y may be $-\text{H}$, $-\text{X}$, $-\text{CO}_2\text{R}$, $-\text{CN}$ etc)

The addition or chain growth polymerisation can proceed by the following two mechanisms (i) By the formation of free radicals, i.e. free radical polymerisation and (ii) By the formation of ionic species, i.e. ionic polymerisation.

Condensation Polymerisation

In this mode of polymerisation, there is combination of molecules of same or different monomers in a controlled stepwise manner with the elimination of small molecules such as water, ammonia, alcohol etc., as by-product. These monomers have functional groups such as alcohol, amine or carboxylic acid groups, instead of double bonds. Since, there is a repetitive condensation reaction between bifunctional monomers, the process is also known as step growth polymerisation.

Copolymerisation

When two or more different monomer units are allowed to polymerise, a copolymer is formed which contains multiple units of each monomer, used in the same polymeric chain. The process by which copolymers are formed, is referred to as copolymerisation.

Molecular Weight of Polymers

(a) Number average molecular weight ($\overline{M}_n$)

$$\overline{M}_n = \frac{\text{Total weight of molecules}}{\text{Total number of molecules}} = \frac{n_1M_1 + n_2M_2 + \dots}{n_1 + n_2 + \dots}$$

$$\therefore \overline{M}_n = \frac{\sum M_i n_i}{\sum N_i}$$

(b) Weight average molecular weight ($\overline{M}_w$)

$$\overline{M}_w = \frac{w_1M_1 + w_2M_2 + \dots}{w_1 + w_2 + \dots}$$

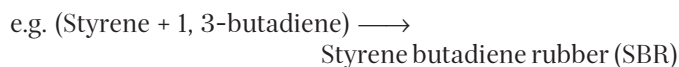
$$\therefore \overline{M}_w > \overline{M}_n$$

Homo and Copolymers

Homopolymers are formed from the same monomers.

e.g. Polythene, PVC etc.

Copolymers are formed from more than one type of monomers.



Atactic, Syndiotactic and Isotactic Polymers

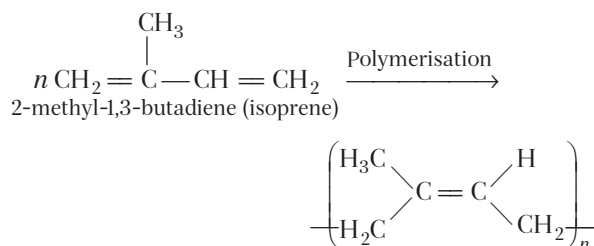
When substituents are arranged in a random fashion, it is called an atactic polymer, e.g. polypropylene. When substituents are arranged in a regular fashion, it is called syndiotactic polymer, e.g. polystyrene. Isotactic polymers are those in which all substituents are on the same side of the chain, e.g. PVC

Some Important Polymers

1. Natural rubber

It is a natural polymer obtained from rubber latex (contains 35% rubber). It is a colloidal dispersion of rubber in water and is obtained from rubber trees (*Hevea brasiliensis*).

It is a polymer of isoprene (2-methyl-1,3-butadiene) and is also known as **cis-1, 4-polyisoprene**.

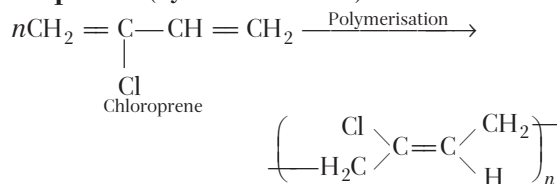


NOTE The average molecular weight ($\bar{M}_w$) of rubber is in the range 1,30,000 - 3,40,000. Its *trans*-form is known as gutta percha.

Natural rubber $\longrightarrow$ *cis*-polyisoprene

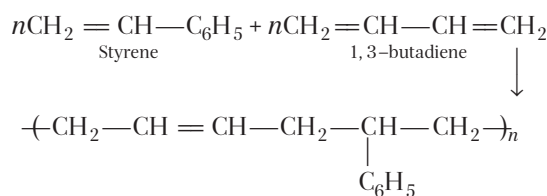
Gutta percha $\longrightarrow$ *trans*-polyisoprene.

2. Neoprene (Synthetic rubber)



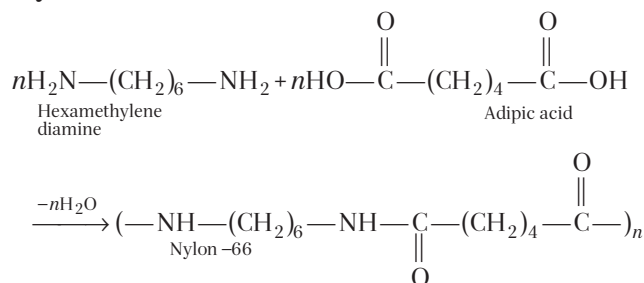
Uses In household and mechanical rubber articles.

3. Buna-S



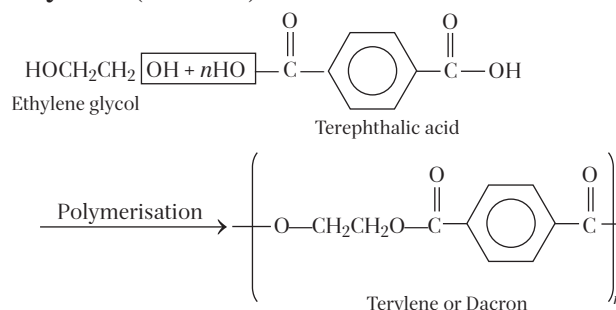
Uses In rubber soles, water proof shoes, tyres, etc.

4. Nylon-66



Uses In carpets, textile fibres, brushes, hosiery, etc.

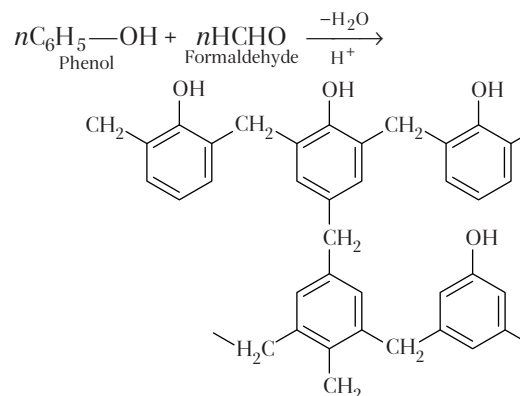
5. Terylene (Dacron)



Properties and Uses

Terylene is highly resistant to the action of chemical and biological agents. Its fibres are quite strong and durable. It can also be blended with wool or cotton to obtain fabrics of desired composition. Terylene is used in the manufacturing of a variety of clothes such as terycot, terywool and terysilk as a result of blending with other yarns. It is also used for preparing magnetic recording tapes, conveyor belts, aprons for industrial workers etc.

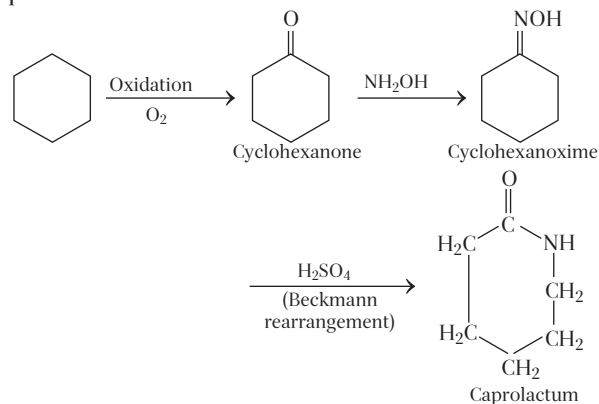
6. Bakelite

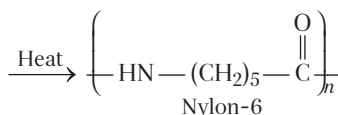
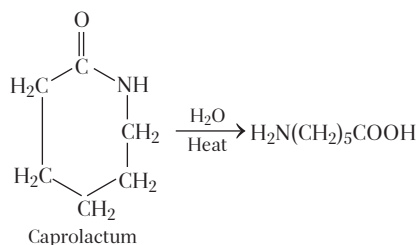


Uses In electrical switches, fuse holders etc.

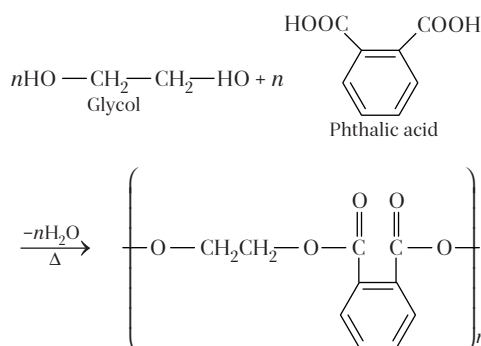
7. Nylon-6 or Perlon

It is obtained by heating caprolactum with water at a high temperature.



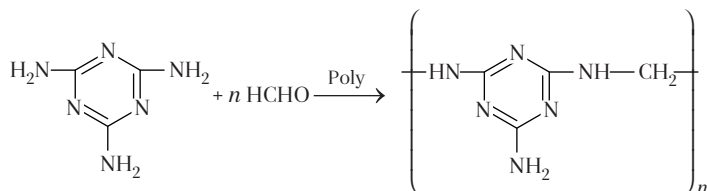


8. Glyptal



Uses In paints and lacquers.

9. Melamine (Formaldehyde resin)

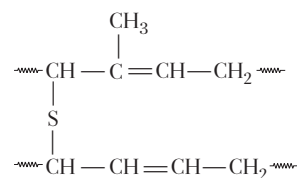
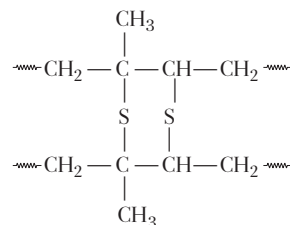


Vulcanisation of Rubber

Natural rubber becomes soft at high temperature (>335 K) and brittle at low temperature (<283 K). It has high water absorbing capacity. It is easily attacked by oxidising agents and soluble in non-polar solvents. To improve these properties, vulcanisation of rubber is carried out.

The process involves the heating of raw rubber with sulphur and appropriate additive at a temperature 373 – 415 K.

Sulphur forms cross links at reactive sites of double bonds and makes the rubber stiffened. Rubber made with 1-3% sulphur is soft and stretchy and rubber made with 3-10% S, is more rigid (used in making tyre).



Biomolecules

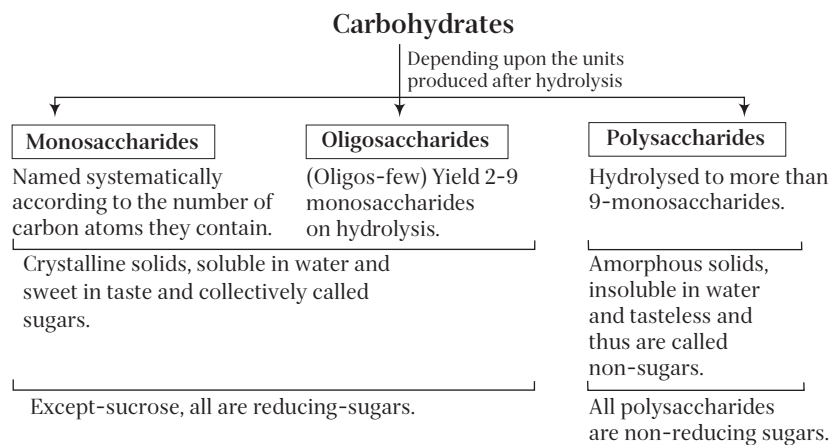
Biomolecules are the complex organic compounds which form the basis of life for growth and maintenance of living organism. e.g. carbohydrates, proteins, vitamin, hormones, etc.

The branch of science which deals with the study of biomolecules and their role in living systems is called biochemistry.

Carbohydrates

These are polyhydric aldehydes and ketones which can be represented by general formula $\text{C}_x(\text{H}_2\text{O})_y$ [where $x, y > 4$].

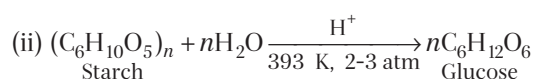
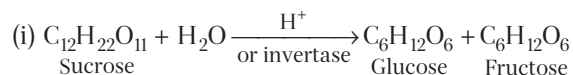
Classification



Monosaccharides

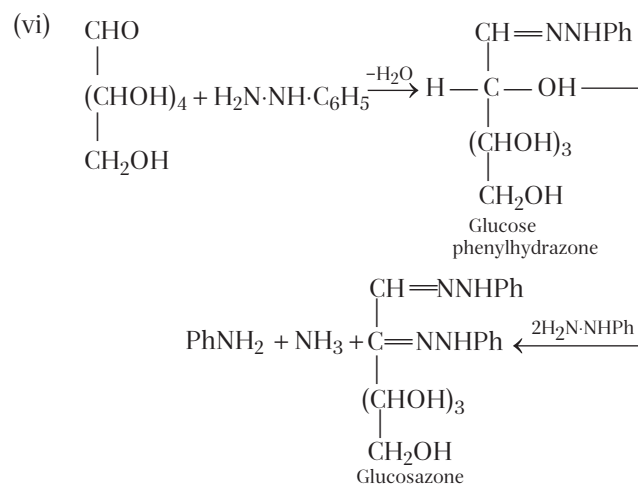
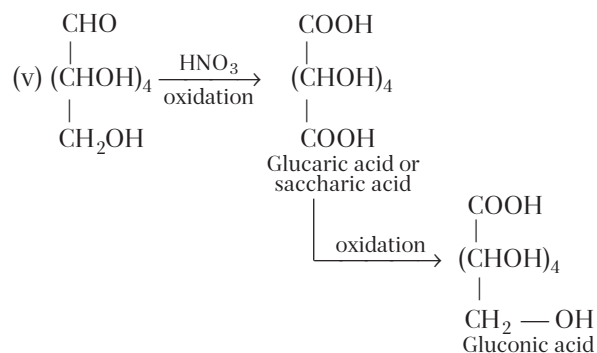
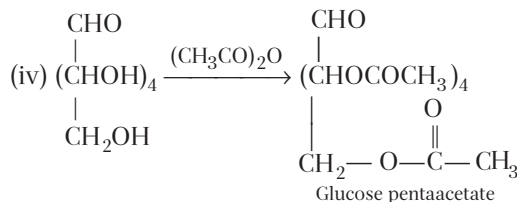
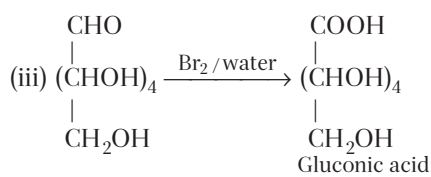
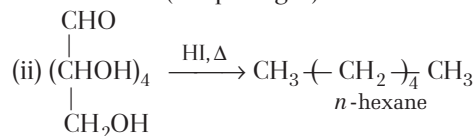
A carbohydrate that cannot be hydrolysed further to give simpler units is called monosaccharide, e.g. glucose, fructose, galactose are hexose while ribose and arabinose are pentose. All monosaccharides are reducing sugars, commonly known as **glucose**.

Preparation of Glucose



Properties of Glucose

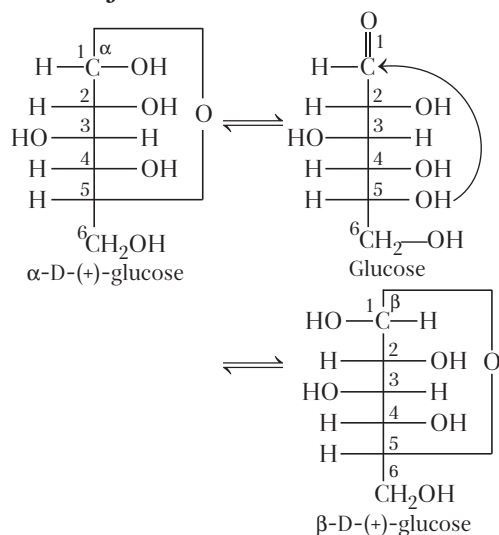
(i) Glucose is an aldohexose and is also known as **dextrose** (Grape sugar).

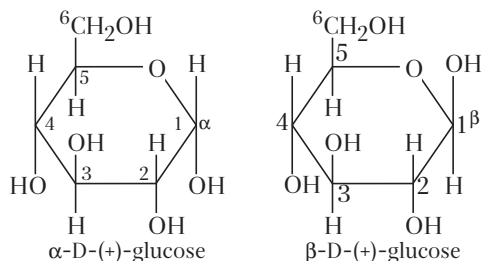


NOTE Glycolysis is the anaerobic degradation of glucose into two molecules of pyruvic acid.

Structure of Glucose

(i) Fisher Projections

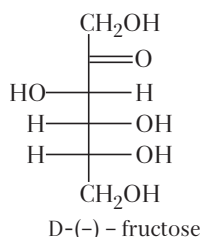


(ii) Haworth Structures

- α and β -D-glucose have different configuration at anomeric (C-1) carbon atom, hence are called **anomers**.
- While the pair of diastereomeric aldoses, e.g. glucose and mannose that differ only in configuration about C-2 are called **epimers**. Glucose and galactose differ in configuration at C4 and are called C4 - epimers.

Structure of Fructose (Laevulose; Fruit Sugar)

It is a functional isomer of glucose and has ketone group.

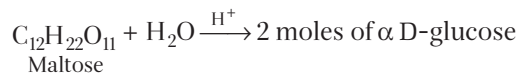
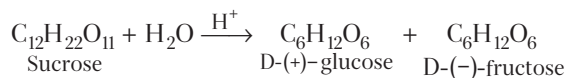
**2. Oligosaccharides**

These are the carbohydrates that yield two to ten monosaccharides units on hydrolysis.

e.g. Disaccharides: sucrose, maltose, lactose etc.

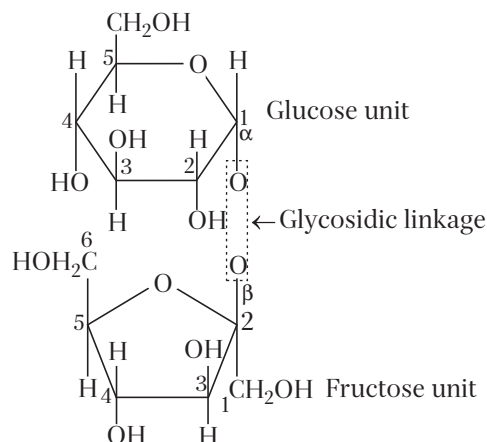
Trisaccharides : raffinose; tetrasaccharides : stachyrose.

- Except sucrose, all other disaccharides are reducing in nature and hence, are called **reducing sugars**.

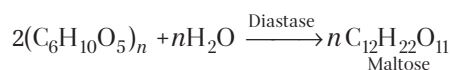
**(a) Sucrose**

It is a non-reducing due to absence of free aldehyde or ketone group. It is cane sugar or table sugar.

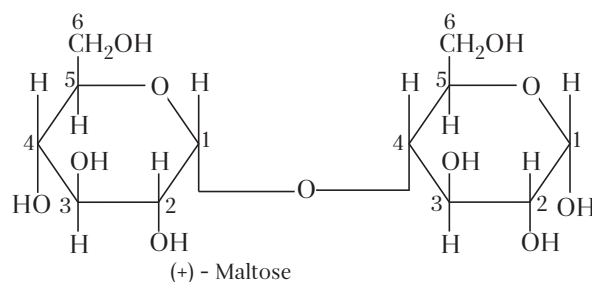
- Sucrose is also known as **invert sugar**. It is due to the fact that on hydrolysis (+) sucrose gets inverted to give a mixture of D-(+) -glucose and D-(-) -fructose.
- In sucrose, free aldehyde or ketone group is absent. It is shown by the facts that it does not form osazone, does not exist in anomeric forms and also does not show mutarotation.

Structure of Sucrose**Maltose (C₁₂H₂₂O₁₁)**

It is obtained by partial hydrolysis of starch by diastase enzyme present in malt, i.e. sprouted barley seeds (hence named as **maltose** or **malt sugar**).



- It is a white crystalline solid (with m.p. 160-165°C), soluble in water and dextrorotatory. When it is hydrolysed with dilute acid or by enzyme maltase, maltose yields two molecules of D-(+)-glucose. Hence, maltose is a condensation product of two α -D-glucose units.



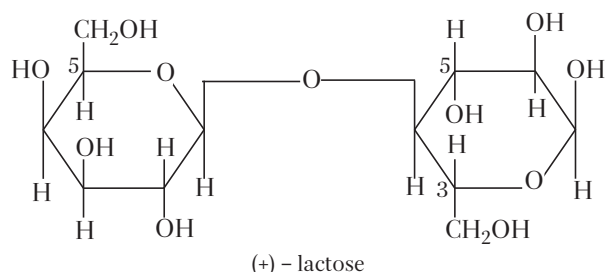
- It is a reducing sugar. It reduces Fehling's solution, Tollen's reagent, it forms an oxime and an osazone and undergoes mutarotation. This indicates that at least one aldehyde group is free in maltose.

Lactose (C₁₂H₂₂O₁₁)

It occurs in the milk of all animals (milk-sugar). It is a white crystalline solid (with m.p. 203°C), soluble in water and is dextrorotatory.

It is hydrolysed by dilute acid or enzyme lactase, to an equimolar mixture of D-(+)-glucose and D-(+)-galactose. It is a reducing sugar, forms an oxime and osazone and also undergoes mutarotation.

It gets hydrolysed by emulsion also, an enzyme which specifically hydrolyses β -glycosidic linkage.



Polysaccharides

Carbohydrates which yield a large number of monosaccharides units on hydrolysis, are called polysaccharides, e.g. starch, cellulose, glycogen, gums arabic etc. Polysaccharides are not sweet in taste, hence they are also called **non-sugars**.

Moreover, all polysaccharides are non-reducing due to the absence of free —CHO or >CO group.

Starch is the main storage polysaccharides of plants. It is a polymer of α -D-glucose units and consists of two components—amylose and amylopectin.

Cellulose is a predominant constituent of cell wall of plant cells. It is a straight chain polysaccharide composed only of β -D-glucose units which are joined together by β -1,4-glycosidic linkage, i.e. the β -glycosidic linkages between C-1 of one glucose and C-4 of the next glucose unit.

Glycogen is the carbohydrate (a condensation polymer of α -D-glucose) which is stored in animal body. When the body needs glucose, enzymes break the glycogen down to glucose.

- Wood contains 50% cellulose, rest being lignin and resins. In cotton, cellulose is 90%, the rest being fats and waxes. Cellulose is not digestible by humans due to the absence of enzyme cellulase in digestive system.
- Starch and cellulose are homopolysaccharides while gums and pectins are heteropolysaccharides. Starch consists of two compounds : amylose (20%) and amylopectin (80%).
- Gums arabic are acidic polysaccharides.

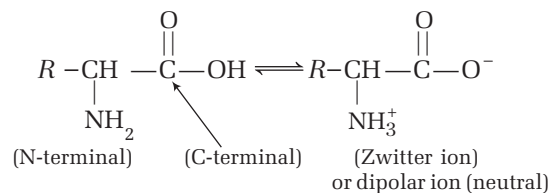
Proteins

These are the polymers of amino acids (the compounds which have both the acid and amino group). The total number of amino acids that have been found in proteins are twenty.

On the basis of their synthesis, amino acids are divided into two classes

1. **Essential amino acids** are the amino acids which cannot be synthesised in the body and must be obtained through diet, e.g. valine, leucine, lysine, isoleucine, arginine etc.

2. **Non-essential amino acids** are the amino acids which can be synthesised in the body, e.g. glycine, alanine, glutamic acid, aspartic acid etc. Amino acids behave like salts rather than simple amines or carboxylic acids. This is due to the presence of both acidic and basic groups in the same molecule.



At a certain pH of the medium, called the **isoelectric point** of an amino acid, the structure behaves as a dipolar ion and does not migrate to any electrode on passing current.

Proteins give biuret test, Millon's test, Ninhydrin test. Those proteins give Mollisch's test, which contain a carbohydrate group. Carbohydrates also give Mollisch's test.

Classification of Proteins

Proteins can be classified into two types on the basis of their molecular shape :

- (i) **Fibrous proteins** Polypeptide chains form fibre like structure, e.g. keratin and myosin etc.
- (ii) **Globular proteins** This structure results when the chains of polypeptides coil around to give a spherical shape. These are usually soluble in water, e.g. insulin and albumins.

Structure of Proteins

N-terminal of one amino acid combines with C-terminal of other amino acid and forms peptide bond (—NHCO—).

In the same way, many amino acids combine to each other and form polypeptide bonds. Polypeptides with fewer amino acids are likely to be called proteins.

Structure and shape of proteins may be discussed at four different levels:

1. Primary Structure

Proteins may have one or more polypeptide chains. Specific sequence of amino acids in a chain gives the primary structure of that protein. Any change in this structure gives a different protein.

2. Secondary Structure

The polypeptide chains are linked by hydrogen bonds. They are found to exist in two different types of structures viz α -helix and β -pleated sheet structure.

3. Tertiary Structure

It has polypeptide bonds, hydrogen bonds, disulphide linkages, van der Waals' forces and electrostatic forces of attraction. It gives rise to two major molecular shapes *viz* fibrous and globular.

4. Quaternary Structure

Some of the proteins are composed of two or more polypeptide chains referred to as subunits. The spatial arrangement of these subunits with respect to each other is known as quaternary structure.

Denaturation of Proteins

Disturbance of hydrogen bonds either by acids or alcohols or heat, results in unfolding of globules. Thus, helix get uncoiled and protein loses its biological activity due to the change in temperature or pH. This is called denaturation of proteins. During denaturation, secondary and tertiary structures of proteins are destroyed while primary structures remain intact.

Enzymes

Enzymes are globular protein bodies, which are biological catalysts. Enzyme inhibitors reduce the activity of a particular enzyme. These are mostly inorganic ions or complex organic molecules.

NOTE Congenital and albinism diseases are caused by the deficiency of the enzymes, phenyl ketone urea and tryosinase respectively.

Vitamins

Organic compounds required in the diet in the small amounts to perform specific biological functions for normal maintenance of optimum growth and health of the organism, are termed as vitamins.

Types of Vitamins

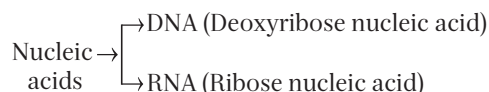
- Fat or oil soluble vitamins, e.g. A, D, E and K.
- Water soluble vitamins, e.g. B group vitamins and vitamin C.

Some Important Vitamins, their Sources and their Deficiency Diseases

Name of Vitamin	Sources	Deficiency Diseases
Vitamin-A (Retinol)	Fish liver oil, carrots, butter and milk	Xerophthalmia (hardening of cornea of eye) night blindness,
Vitamin-B ₁ (Thiamine)	Yeast, milk, green vegetables	Beri-beri (loss of appetite)
Vitamin-B ₂ (Riboflavin)	Milk, egg white, liver, kidney	Cheilosis (fissuring at corners of mouth and lips)
Vitamin-B ₆ (Pyridoxine)	Yeast, milk, egg yolk, cereals	Convulsions, nervousness
Vitamin-B ₁₂ (Cyanocobalamin)	Meat, fish, egg and curd	Pernicious anaemia (RBC deficient in haemoglobin)
Vitamin-C (Ascorbic acid)	Citrus fruit, amla and green leafy vegetables	Scurvy (bleeding gums)
Vitamin-D	Exposure to sunlight, fish and egg yolk	Rickets and osteomalacia
Vitamin-E	Wheat, germ oil, sunflower oil	Increased fragility of RBC and muscular weakness
Vitamin-K	Green leafy vegetables	Increased blood clotting time.

Nucleic Acids

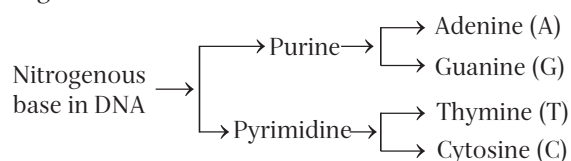
These are the biological polymers. They function as the chemical carriers of cell's genetic information.



DNA (Deoxyribose Nucleic Acid)

- It is the polymer of nucleotide.
- It is a genetic material.
- It has double helical structure.
- **Nucleotide** has deoxyribose sugar, phosphate and nitrogenous base.

- **Nucleoside** has deoxyribose sugar and nitrogenous base. A unit formed by the attachment of a base to 1' position of sugar is known as **nucleoside**.
- In nucleosides, the sugar carbons are numbered as 1', 2', 3' etc., in order to distinguish these from the bases. When nucleoside is linked to phosphoric acid at 5'-position of sugar moiety, we get a nucleotide.
- Nucleotides are joined together by phosphodiester linkage between 5' and 3' carbon.



- DNA has A = T, C ≡ G

RNA (Ribose Nucleic Acid)

- It is also a polymer of nucleotide units but in it, the nucleotide unit contains ribose sugar instead of deoxyribose sugar.
- RNA has uracil (U) instead of thymine (T).
- RNA molecules are of three types. These are messenger RNA (*mRNA*), ribosomal RNA (*rRNA*) and transfer RNA (*tRNA*). They perform different functions.

Biological Functions of Nucleic Acids

- DNA has an ability of self duplication during cell division and identical DNA strand are transferred to daughter cell. In this way, DNA is responsible for maintaining the identity of different species of organisms over million of years.
- RNA molecules synthesised various types of proteins in the cell but the message for the synthesis of a particular type of protein is present in DNA.

Chemicals in Medicines

Drugs are chemicals of low molecular masses, which interact with macromolecular targets (carbohydrates, proteins and nucleic acids) to produce a biological response. When the biological response is therapeutic and useful, these chemicals are called medicines.

Different Classes of Drugs

Some important classes of drugs are given below:

Antacids

Treatment for acidity was administration of antacids. Antacids raise the pH of stomach to some appropriate level. Sodium hydrogen carbonate, a mixture of aluminium and magnesium hydroxide etc., are the commonly used antacids.

Antihistamines

Antihistamines or antiallergic drugs are used as treatment for allergies. Some synthetic drugs, brompheniramine (dimetapp) and terfenadine (seldane), also act as antihistamines. Histamine also stimulate the secretion of gastric juice (in stomach) but antihistamines do not affect this secretion because antiallergic and antacid drugs work on different receptors.

Neurologically Active Drugs

- Tranquiliser** It is a class of chemical compounds used for the treatment of stress, mild and severe mental diseases. These are commonly called **psychotherapeutic** drugs. These are the essential component of sleeping pills.

Some examples according to the action of drugs are

- Antidepressant drugs, e.g. iproniazid, phenelzine etc., are used to reduce depression.

- Chlorodiazepoxide and meprobamate, are relatively mild tranquilisers, suitable for relieving tension. Equanil is used in controlling depression and hypertension.

- Barbiturates such as seconal, luminal, veronal are hypnotic (sleep producing) drugs while valium and serotonin, are non-hypnotic drugs.

- Analgesics** These reduce or abolish pain.

These are classified as follows:

- Non-narcotic (non-addictive) analgesics, e.g. aspirin and paracetamol are analgesics as well as antipyretics (fever reducing).

NOTE Aspirin (acetyl salicylic acid) and paracetamol (N-acetyl-p-amino phenol) are non-narcotic (non-addictive) analgesics. Aspirin inhibits the synthesis of chemicals, known as prostaglandins which stimulate inflammation in the tissue and cause pain. Because of its anti blood clotting action, aspirin is also used in the prevention of heart attacks.

- Narcotic analgesics, e.g. morphine, heroin (morphine diacetate), codeine etc.

Antimicrobials

- Antibiotics** These are the chemicals synthesised from microbes and have either cidal (killing) effect or a static (inhibitory) effect on microbes.

A few examples of the two types of antibiotics are as follows:

- Bactericidal, e.g. penicillin (a narrow spectrum antibiotic), ampicillin and amoxicillin (semisynthetic modification of penicillin), ofloxacin (broad spectrum), aminoglycosides (streptomycin) (broad spectrum) etc.

- Bacteriostatic, e.g. erythromycin, tetracycline, chloramphenicol (a broad spectrum antibiotic) etc.

- Antiseptics** These are applied to the living tissues such as wounds, cuts, ulcers and diseased skin surfaces, e.g. furacine, soframycin, dettol (a mixture of chloroxylenol and α -terpinol), 0.2 per cent solution of phenol. Bithionol (also called bithional) is added to soaps, iodine, iodoform etc.

- Disinfectants** These are applied to inanimate objects such as floors, drainage system, e.g. one per cent solution of phenol, chlorine and SO_2 (in very low concentrations), are disinfectants.

Antifertility Drugs

These have lead to the concept of family planning. Birth control pills essentially contain a mixture of synthetic estrogen and progesterone derivatives.

e.g. Norethindrone (synthetic progesterone) and the estrogen derivative in combination with progesterone derivative, (ethinylestradiol) (novestrol) etc.

Chemicals in Food

Substances which are added to food either to improve its taste and flavour or to preserve it, are called **food additives**.

Main categories of food additives are as follows:

Artificial Sweetening Agents

Some examples of artificial sweetening agents are saccharin (*ortho*-sulphobenzimide), aspartame (nutra sweet), sucralose, alitame etc.

Alitame is high potency sweetener, although it is more stable than aspartame. Sucralose is trichloro derivative of sucrose. Its appearance and taste are sugar like.

It is stable at cooking temperature. Hence, its use is of great value to diabetic persons who need to control in take of calories.

Food Preservatives

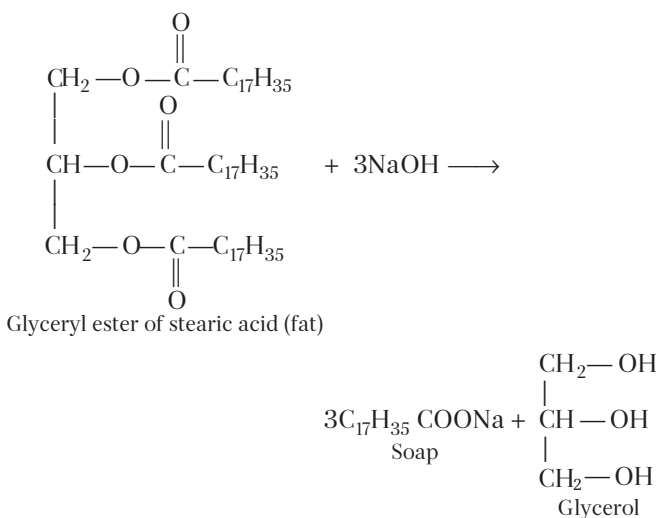
Food preservatives prevent spoilage of food due to microbial growth, e.g. sodium benzoate, salts of sorbic acid and propanoic acid etc.

Cleansing Agents

These are also known as surfactants or surface active agents. In fact, those chemicals which concentrate at the surface of the solution or interfaces or surface films, reduce surface tension of the solution and help in removing dirt and dust by emulsifying grease are known as **surfactants**. Soaps and detergents belong to this class.

Soaps

These are the sodium or potassium salts of higher fatty acids and are prepared by alkaline hydrolysis of fats or oils. Fats or oils are esters of higher fatty acids.



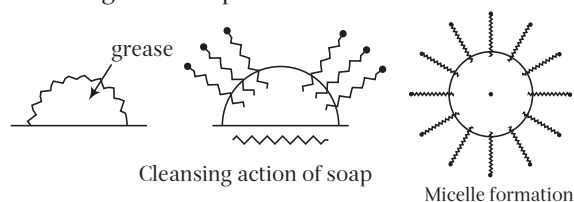
The reaction is known as **saponification**.

Cleansing Action of Soap

On applying soap to a dirty wet cloth, the hydrocarbon part (non-polar part) of soap dissolves in grease or dust while the polar carboxylate part is directed towards water.

Thus, an emulsion is formed between grease particles and water molecules, which appears in the form of foam.

On washing the cloth with excessive water, these dirt or dust or grease particles are washed away from the surface of cloth along with soap and the cloth becomes clean.



- **Advantages** Soaps are biodegradable cleansing agents, therefore they do not cause pollution.
- **Disadvantages** Soaps do not work in hard water because hard water contains calcium and magnesium ions.
- These ions form insoluble calcium and magnesium soaps respectively.

Synthetic detergents

Alkyl benzene sulphonates are the synthetic detergents. These are also called **syndets**.

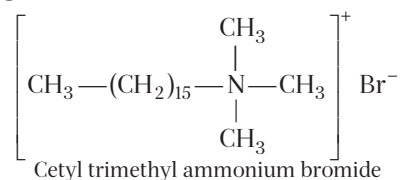
The detergents are classified into following three types on the basis of ionic charge present at the soluble end of their chain:

Anionic detergents

e.g. Sodium dobecyl benzene sulphonate

Anionic detergents are also used in toothpaste.

Cationic detergents



These are used in hair conditioners and are expensive, therefore these are of limited use.

Non-ionic detergents

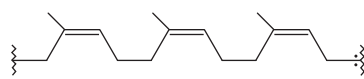
e.g. $\text{CH}_3(\text{CH}_2)_{16}\text{COO}(\text{CH}_2\text{CH}_2\text{O})_n\text{CH}_2\text{CH}_2\text{OH}$

They are used in liquid dishwashing detergents.

Advantages Detergents can be used both in soft and hard water as they give foam even in hard water.

Disadvantages In detergents, hydrocarbon chain is highly branched, so bacteria cannot degrade this easily. In other words, we can say that these are non-biodegradable.

Practice Exercise

- Polymers are also referred to as
 - micromolecules
 - small molecules
 - macromolecules
 - huge molecules
- On the basis of structure of polymers, they can be classified as
 - linear, branched and network polymers
 - natural, synthetic and network polymers
 - natural, synthetic and semi synthetic polymers
 - natural, synthetic and linear polymers
- The correct classification of polymers based on molecular forces is
 - elastomers, fibres and thermosetting polymers
 - elastomers, fibres, thermoplastic and thermosetting polymers
 - homopolymers and copolymers
 - None of the above
- Repeating unit of polystyrene is
 - $\text{—CH}_2\text{—CH—}$
|
Cl
 - $\text{—CH}_2\text{—CH—}$
|
C₆H₅
 - $\text{—CH}_2\text{—CH—}$
|
CN
 - $\text{—CF}_2\text{—CF}_2\text{—}$
- Consider the following polymers,
A = Nylon-66, B = Buna-S, C = Polyethene
Arrange these in the increasing order of their intermolecular forces (lower to higher).
 - $A < B < C$
 - $B < C < A$
 - $B < A < C$
 - $C < A < B$
- Synthetic polymer prepared by using caprolactam is known as
 - terylene
 - teflon
 - nylon-6
 - neoprene
- Relation between number of average molecular mass ($\overline{M}_n$) and weight of average molecular mass ($\overline{M}_w$) of synthetic polymers is
 - $\overline{M}_n < \overline{M}_w$
 - $\overline{M}_n > \overline{M}_w$
 - $\overline{M}_n = \overline{M}_w$
 - $\overline{M}_n > \overline{M}_w$
- Bakelite is obtained from phenol by reacting it with
 - acetaldehyde
 - acetal
 - formaldehyde
 - chlorobenzene
- The structural formula of monomer of poly methyl methacrylate (PMMA) is
 - $\text{CH}_2 = \text{CHCOOCH}_3$
 - $\text{CH}_2 = \text{C}(\text{CH}_3)\text{—COOCH}_3$
 - $\text{CH}_3\text{COOCH}=\text{CH}_2$
 - $\text{CH}_3\text{COOC}(\text{CH}_3)=\text{CH}_2$
- The element present in teflon is
 - fluorine
 - chlorine
 - nitrogen
 - oxygen
- Glyptal is classified as a
 - polyolefin
 - polyester
 - polyamide
 - polyether
- Bakelite is made from phenol and formaldehyde. The initial reaction between them is the example of
 - electrophilic aromatic addition
 - nucleophilic aromatic substitution
 - free radical reaction
 - aldol reaction
- Artificial silk is a
 - polypeptide
 - polysaccharide
 - polythene
 - polyvinyl chloride
- Which one of the following pairs is not correctly matched?
 - Terylene** Condensation polymer of terephthalic acid and ethylene glycol
 - Perspex** A homopolymer of methyl methacrylate
 - Teflon** Thermally stable cross - linked polymer of phenol and formaldehyde
 - Synthetic rubber** A co-polymer of butadiene and styrene
- Identify the following polymer :
 
 - Gutta percha
 - Neoprene
 - Polypropylene
 - Natural rubber
- Orlon is a polymer of
 - styrene
 - tetrafluoro ethylene
 - vinyl chloride
 - acrylonitrile
- Melamine formaldehyde resin is
 - an addition polymer
 - a copolymer
 - a fibre type polymer
 - dimer of melamine and formaldehyde
- Which one of the following compounds is different from the rest?
 - Sucrose
 - Maltose
 - Lactose
 - Glucose
- A carbohydrate which cannot be hydrolysed to simpler compounds, is called
 - monosaccharide
 - disaccharide
 - trisaccharide
 - polysaccharide

- a.** 1–10 **b.** 2–10
c. 4–10 **d.** 5–10

- a.**
- $$\begin{array}{c} \text{CHO} \\ | \\ \text{H} - \text{C} - \text{OH} \\ | \\ \text{HO} - \text{C} - \text{H} \\ | \\ \text{H} - \text{C} - \text{OH} \\ | \\ \text{H} - \text{C} - \text{OH} \\ | \\ \text{CH}_2\text{OH} \end{array}$$
- b.**
- $$\begin{array}{c} \text{CHO} \\ | \\ \text{HO} - \text{C} - \text{H} \\ | \\ \text{H} - \text{C} - \text{OH} \\ | \\ \text{HO} - \text{C} - \text{H} \\ | \\ \text{HO} - \text{C} - \text{H} \\ | \\ \text{CH}_2\text{OH} \end{array}$$
- c.**
- $$\begin{array}{c} \text{CHO} \\ | \\ \text{H} - \text{C} - \text{OH} \\ | \\ \text{H} - \text{C} - \text{OH} \\ | \\ \text{H} - \text{C} - \text{OH} \\ | \\ \text{HO} - \text{C} - \text{H} \\ | \\ \text{CH}_2\text{OH} \end{array}$$
- d.**
- $$\begin{array}{c} \text{CHO} \\ | \\ \text{HO} - \text{C} - \text{H} \\ | \\ \text{HO} - \text{C} - \text{H} \\ | \\ \text{HO} - \text{C} - \text{H} \\ | \\ \text{H} - \text{C} - \text{OH} \\ | \\ \text{CH}_2\text{OH} \end{array}$$

- a.** primary structure of proteins
- b.** secondary structure of proteins
- c.** tertiary structure of proteins
- d.** quaternary structure of proteins

- a.** Glucose and fructose **b.** Glucose and galactose
c. Glucose and arabinose **d.** Lactose and maltose

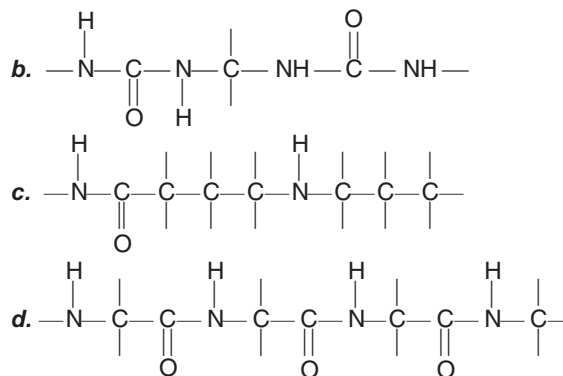
- a.** nucleotide **b.** nucleoside
c. purine base **d.** pyrimidine base

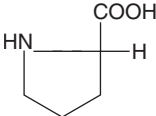
- a.** DNA only **b.** RNA only
c. Both DNA and RNA **d.** protein

- a.** proteins into α -amino acids
- b.** starch into sugar
- c.** glucose into glycogen
- d.** α -amino acids into proteins

- a.** uracil and adenine, cytosine and guanine
b. adenine and guanine, thymine and cytosine
c. adenine and guanine, guanine and uracil
d. adenine and thymine, guanine and cytosine

- $$\text{a. } \begin{array}{ccccccccccccccc} & & \text{H} & & & \text{O} & & & \text{H} & & & & & \\ & & | & & & || & & & | & & & & & \\ \text{a. } & -\text{C} & -\text{N} & -\text{C} & -\text{C} & -\text{C} & -\text{N} & -\text{C} & -\text{C} & -\text{N} & -\text{C} & -\text{C} & -\text{C} & - \\ & | & & | & | & & | & | & | & || & | & | & | & \\ & & & & & & \text{H} & & & \text{O} & & & & \end{array}$$



- | Column I | Column II |
|---------------------------|---|
| A. Asparagine (Asn, N) | 1.  |
| B. Proline (Pro, P) | 2. $\text{HOOC} - \text{CH}_2 - \text{CH}_2 -$ |
| C. Glutamic acid (Glu, E) | 3. $\text{HO} - \text{CH}_2 -$ |
| D. Serine (Ser, S) | 4. $\text{H}_2\text{N} - \text{COCH}_2 -$ |

	A	B	C	D
a.	4	1	2	3
b.	4	2	1	3
c.	4	3	1	2
d.	4	1	3	2

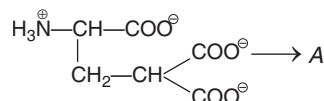
- a.** Vitamin B₂ **b.** Vitamin B₆
c. Vitamin B₁₂ **d.** Vitamin B₁

- a.** aspartame
c. fructose
- b.** cellulose
d. glucose

- a.** uracil **b.** thiamine
c. adenine **d.** guanine

- They are designated by A, B, C and D.
- They are further named as B₁, B₂, B₆ and B₁₂
- Vitamin pills should not be taken without the advice of doctor
- All of the above

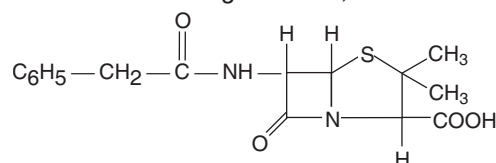
34. Following amino acid has been found in protein prothrombin, but remains undetected due to the formation of another common acid (A)



Identify A.

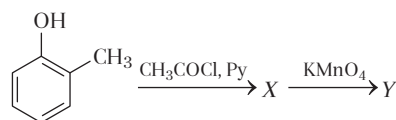
- a. $\text{H}_3\text{N}^+-\text{CH}-\text{COO}^-$
|
 $\text{CH}_2-\text{CH}_2-\text{COO}^-$
- b. $\text{H}_3\text{N}^+-\text{CH}-\text{COO}^-$
|
 $\text{CH}-\text{COO}^-$
|
 COO^-
- c. $\text{H}_2\text{N}-\text{CH}-\text{COO}^-$
|
 CH_2COOH
- d. $\text{H}_2\text{N}-\text{CH}-\text{COOH}$
|
 $\text{CH}-\text{COO}^-$
|
 COO^-
35. Which of the following hormones is responsible for the growth of animals?
a. Auxin
b. Insulin
c. Adrenaline
d. Somatotropin
36. Which of the following is present in DNA?
a. Deoxyribose
b. Starch
c. Riboflavin
d. None of these
37. Which of the following biomolecules contain a non-transition metal ion?
a. Haemoglobin
b. Chlorophyll
c. Insulin
d. Vitamin B₁₂
38. Molecular masses of drugs lie in the range of
a. ~ 5 u – 10 u
b. ~ 50 u – 60 u
c. ~ 100 u – 500 u
d. Above 2000 u
39. Difference in the antiseptics and disinfectants is
a. Antiseptics are used against micro-organisms while disinfectants are used against insects
b. antiseptics are used only over skin while disinfectants can be taken orally also
c. antiseptics merely inhibit the growth and disinfectant kill the micro-organisms
d. antiseptics are used over living tissues while disinfectants cannot be used over living tissues
40. Chloramine -T is a/an
a. antiseptic
b. disinfectant
c. analgesic
d. antipyretic
41. The chemical name of aspirin is
a. methyl salicylate
b. acetylsalicylic acid
c. sodium salicylate
d. salicylic acid
42. Which statement about aspirin is not true?
a. Aspirin belongs to narcotic analgesics
b. It is effective in relieving pain
c. It has antiblood clotting action
d. It is a neurologically active drug

43. Consider the following structure,



It is the structure of

- a. penicillin K
b. penicillin V
c. penicillin G
d. chloramphenicol
44. A drug that is antipyretic as well as analgesic is
a. chlorpromazine hydrochloride
b. *para*-acetamidophenol
c. chloroquine
d. penicillin
45. Consider the following reaction,



The end product Y formed in the above reaction is a well known medicine. Which of the following is incorrect regarding Y?

- a. It has analgesic as well as antipyretic properties
b. It helps to prevent heart attack
c. It has anti-blood clotting property
d. It suppresses the gastric ananoties
46. Tranquillisers are the substances used for the treatment of
a. cancer
b. AIDS
c. mental diseases
d. physical disorders
47. Match the medicines given in Column I with their use given in Column II.

Select the correct option from the codes given below:

Column I	Column II
A. Ranitidine	1. Tranquiliser
B. Furacine	2. Antibiotic
C. Phenelzine	3. Antihistamine
D. Chloramphenicol	4. Antiseptic

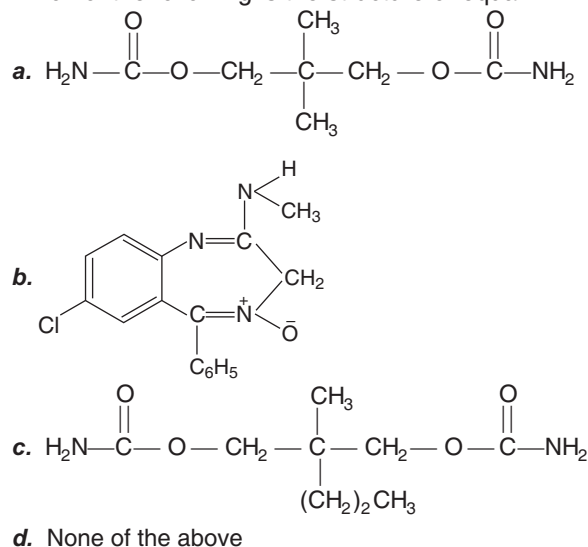
Codes

	A	B	C	D
a.	3	4	1	2
b.	4	1	2	3
c.	1	2	3	4
d.	2	3	4	1

48. Chloramphenicol is used in the treatment of
a. typhoid
b. malaria
c. acidity
d. tuberculosis

49. Arsenic drugs are mainly used in the treatment of
 a. jaundice b. typhoid
 c. syphilis d. cholera
50. Compound which is added to soap to impart antiseptic properties is
 a. sodium lauryl sulphate
 b. sodium dodecylbenzene sulphonate
 c. rosin
 d. bithional
51. The pH value of gastric juice in human stomach is about 1.8 and in the small intestine, it is about 7.8. The pK_a value of aspirin is 3.5. Aspirin will be
 a. ionised in the small intestine and almost unionised in the stomach
 b. unionised in the small intestine and in the stomach
 c. completely ionised in the small intestine and in the stomach
 d. ionised in the stomach and almost unionised in the small intestine

52. Which of the following is the structure of equanil?



BITSAT Archives

1. The catalyst used for olefin polymerisation is [2014]
 a. Ziegler-Natta catalyst
 b. Raney-nickel catalyst
 c. Wilkinson catalyst
 d. Merrified resin
2. A copolymer of ethene and vinyl chloride contains alternate monomers of each type. What is the mass percentage of vinyl chloride in this copolymer? [2014]
 a. 38% b. 69%
 c. 72% d. 82%
3. The number of disulphide linkages present in insulin are [2014]
 a. 1 b. 2 c. 3 d. 4
4. Which of the following statements is not true about the drug barbitol? [2014]
 a. It is used in sleeping pills
 b. It is a non-hypnotic drug
 c. It is tranquilliser
 d. It causes addiction
5. The monosaccharide having anomeric carbon atoms are [2013]
 a. geometrical isomers
 b. α and β -optical isomers
 c. having symmetrical carbon atoms
 d. None of the above
6. In vulcanisation of rubber, [2013]
 a. sulphur reacts to form a new compound
 b. sulphur cross-links are introduced
 c. sulphur forms a very thin protective layer on rubber
 d. All of the above
7. Which of the following antibiotics contain nitro group attached to aromatic nucleus in its structure? [2013]
 a. Tetracyclin b. Penicillin
 c. Streptomycin d. Chloramphenicol
8. Alizarin is an example of [2012]
 a. triaryl dye b. azo dye
 c. vat dye d. anthraquinone dye
9. Which of the following hormones, is responsible for the growth of animals? [2012]
 a. Auxin b. Insulin
 c. Adrenaline d. Somatotropin
10. The polymer polyurethanes are formed by treating diisocyanate with [2012]
 a. butadiene b. isoprene
 c. glycol d. acrylonitrile
11. A synthetic rubber which is resistant to the action of oils, gasoline and other solvents is [2011]
 a. buna-S b. polyisoprene
 c. neoprene d. polystyrene
12. The number of polypeptide chains present in a molecule of haemoglobin is/are [2011]
 a. four b. one c. two d. three
13. The pentose sugar in DNA and RNA has the [2011]
 a. open chain structure b. pyranose structure
 c. furanose structure d. All of these
14. Which of the following is an artificial edible colour? [2011]
 a. Saffron b. Carotene
 c. Tetrazine d. Melamine

15. The well known urinary antiseptic urotropine is formed when formaldehyde reacts with [2011]

a. NH_2OH b. NH_3
c. $\text{NH}_2 \cdot \text{NH}_2$ d. $\text{C}_6\text{H}_5\text{NH} \cdot \text{NH}_2$

16. Correct statement is [2010]

a. keratin is fibrous protein
b. androsterone is male sex hormone
c. vitamin B_1 is antineutritic factor
d. All of the above

17. A certain compound gives negative test with ninhydrin and positive test with Benedict's solution. The compound is [2009]

a. a protein b. a monosaccharide
c. a lipid d. an amino acid

18. Natural rubber and *gutta-percha* respectively are [2009]

a. *cis*-polyisoprene and *trans*-polyisoprene
b. both are *cis*-polyisoprene
c. both are *trans*-polyisoprene
d. *trans*-polychloroprene and *cis*-polychloroprene

19. Match the following. [2008]

List I		List II	
A.	Oxyhaemoglobin	1.	Analgesic
B.	Aspirin	2.	Oxygen carrier
C.	Haemoglobin	3.	Photosynthesis
D.	Chlorophyll	4.	Oil of winter green
		5.	Fe^{2+} paramagnetic

The correct match is

Codes

A	B	C	D	A	B	C	D		
a.	5	1	2	3	b.	4	2	1	3
c.	3	1	2	4	d.	5	2	3	1

20. If $\bar{M}_w$ is the weight average molecular weight and $\bar{M}_n$ is the number average molecular weight of a polymer, the poly dispersity index (PDI) of the polymer is given by [2008]

a. $\frac{\bar{M}_n}{\bar{M}_w}$ b. $\frac{\bar{M}_w}{\bar{M}_n}$ c. $\bar{M}_w \times \bar{M}_n$ d. $\frac{1}{\bar{M}_w \times \bar{M}_n}$

21. Hydrolysis of sucrose with dilute aqueous sulphuric acid yields [2008]

a. 1:1 D- (+)-glucose, D-(-)-fructose
b. 1:2 D- (+)-glucose, D-(-)-fructose
c. 1:1 D- (-)-glucose, D-(+)-fructose
d. 1:2 D- (-)-glucose, D-(+)-fructose

22. Alizarin belongs to the class of [2007]

a. vat dyes b. mordant dyes
c. basic dyes d. reactive dyes

23. 2,4-dichlorophenoxyacetic acid is used as [2007]

a. fungicide b. insecticide
c. herbicide d. moth repellent

24. Bakelite is a product of the reaction between [2007]

a. formaldehyde and NaOH
b. aniline and urea
c. phenol and methanal
d. phenol and chloroform

25. Cellulose is a polymer of [2007]

a. glucose b. fructose
c. ribose d. sucrose

26. Raffinose is [2005]

a. trisaccharide b. monosaccharide
c. disaccharide d. None of these

27. How many hydrogen bonds is/are present between pair of thymine and adenine in DNA? [2005]

a. 1 b. 2
c. 3 d. No bond occurs

28. Dacron is polymer of [2005]

a. glycol and formaldehyde
b. glycol and phenol
c. glycol and phthalic acid
d. glycol and terephthalic acid

29. Natural rubber is a polymer of [2005]

a. styrene b. isoprene
c. ethylene d. butadiene

30. Milk of magnesia is used as [2005]

a. antichlor b. antacid
c. antiseptic d. food preservative

31. Which destroy antigens? [2006]

a. Insulin b. Antibodies
c. Chromoprotein d. Phosphoprotein

32. Nylon-66 is an example of [2006]

a. poly propylene b. polyester
c. polyamide d. polystyrene

33. The purine base present in RNA is [2006]

a. guanine b. thymine c. cytosine d. uracil

34. Glyptal polymer is obtained by the reaction of glycerol with [2006]

a. malonic acid
b. acetic acid
c. phthalic acid
d. maleic acid

Answer with Solutions

Practice Exercise

- (c) Macromolecules is another term used to define polymers.
- (a)
- (b) Polymers can be classified on the basis of molecular forces as elastomers, fibres, thermoplastic and thermosetting polymers.
- (b)
- (b) Nylon-6,6 is a fibre, thus it has strong intermolecular forces like H-bonding. Buna-S is an elastomer, thus it has weakest intermolecular forces. Polythene is a thermoplastic polymer, thus the intermolecular forces present in polythene are in between elastomers and fibres. Thus, the order of intermolecular forces is

$$B < C < A$$

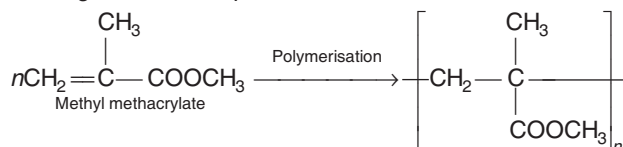
- (c) Caprolactam is the monomer unit of nylon-6.

- (a) Polydispersity index (PDI) of polymer = $\frac{\bar{M}_w}{\bar{M}_n}$

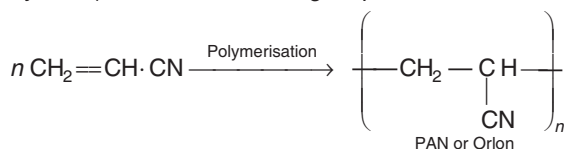
For natural polymers, PDI = 1, i.e. $\bar{M}_w = \bar{M}_n$

For synthetic polymers PDI = 1, i.e. $\bar{M}_w > \bar{M}_n$

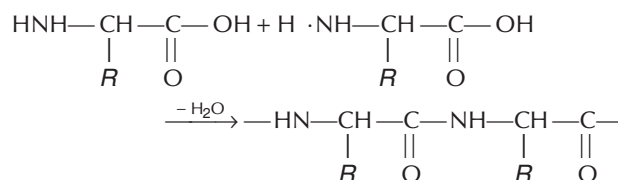
- (c)
- (b) PMMA is the polymer of methyl methacrylate. PMMA is also known as fixiglass or perspex. It is used in making lenses, transparent domes etc.



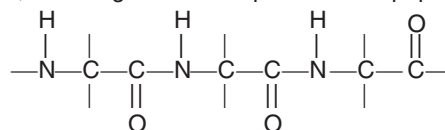
- (a)
- (b)
- (a) The reaction of phenol and formaldehyde, is called Lederer Mannase reaction. It follows electrophilic addition and rearrangement mechanism.
- (b) Chemically treated cellulose (polysaccharide) is called silk.
- (c) Teflon is thermally stable polymer of tetrafluoroethylene ($\text{F}_2\text{C} = \text{CF}_2$).
- (d) Natural rubber is *cis*-polymer.
- (d) Orlon or PAN is the polymer of acrylonitrile (or vinyl cyanide). It is used in making carpets etc.



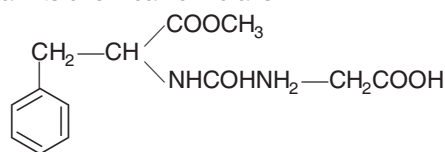
- (b) Melamine-formaldehyde resin is a copolymer.
- (d) Glucose is a monosaccharide while the others are disaccharides. Sucrose is a combination of glucose and fructose. Maltose is a combination of two glucose units. Lactose (or milk sugar) is a combination of glucose and galactose (a hexose sugar).
- (a) Monosaccharide is the simplest carbohydrate which cannot be hydrolyse to simpler compounds.
- (b) Carbohydrates that yield 2-10 monosaccharides units on hydrolysis are called oligosaccharides.
- (b) L-glucose is the mirror image of D-glucose.
- (a) Sequence of amino acids is called primary structure of proteins.
- (a) All monosaccharides which differ in configuration at C_1 and C_2 give the same osazone. Since, glucose and fructose differ from each other only in configuration at C_1 and C_2 therefore, they give the same osazone. All other options given in the questions do not satisfy this condition and hence, do not form the same osazone.
- (b) Adenosine is a nucleoside while adenosine triphosphate is a nucleotide.
- (c) Adenine is a purine base which is common in DNA and RNA.
- (a) Trypsin converts proteins into α -amino acids.
- (d) In DNA, the complementary bases are adenine and thymine, guanina and cytosine.
- (d) The peptide linkage ($-\text{NH}-\text{CO}-$) is formed by the condensation of amino acid molecules.



Hence, following structure represents the peptide chain



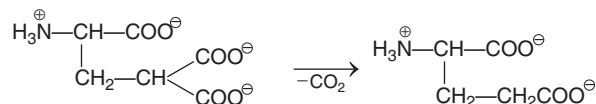
- (a) $A \rightarrow 4$, $B \rightarrow 1$, $C \rightarrow 2$, $D \rightarrow 3$
- (b) Pyridoxin is also known as vitamin B_6 .
- (a) Aspartame is the safest and most popular alternative of sugar. Its chemical formula is



- (a)

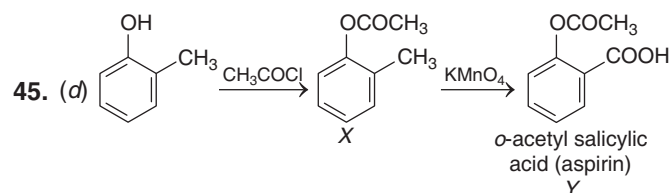
33. (d) Vitamins are designated by alphabets A, B, C, D. Some of them are further named as sub-groups, e.g. B₁, B₂, B₆, and B₁₂ etc. Excess of vitamins is also harmful and vitamin pills should not be taken without the advice of doctor.

34. (a)



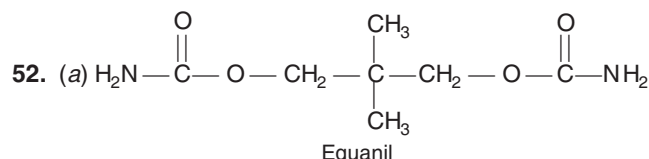
35. (d) Somatotropin is the hormone, secreted by anterior lobe of pituitary gland. It is also called growth hormone as it stimulates protein-synthesis, glycogenesis and some other biological activities. Its deficiency causes midgets or dwarfism.
36. (a) The sugar present in DNA is 2-deoxyribose while that present in RNA is a base.
37. (b) Chlorophyll contains Mg.
38. (c) Molecular masses of drugs lie in the range of $\sim 100 \text{ u} = 500 \text{ u}$.
39. (d) Antiseptics are used over living tissues while disinfectants used for inanimate objects such as floors, toilets, instruments etc.
40. (a) Antiseptic drugs cause destruction of micro-organisms, which produce septic diseases, e.g. dettol, savlon, aeriflavin, iodoform and some dyes such as chloramine-T, methylene blue.
41. (b)
42. (a) Aspirin is a non-narcotic drug.
43. (c) The given structure is of penicillin G.

44. (b) Para-acetamidophenol is used as an antipyretic as well as analgesic.



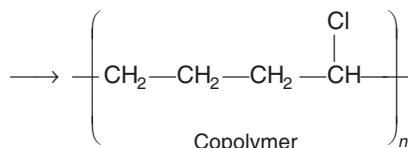
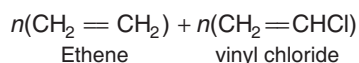
Aspirin increases the gastric anomalies, hence it is banned in various countries.

46. (c) Tranquilisers reduce anxiety and are employed for treatment of mental disease.
47. (a) A $\rightarrow$ 3, B $\rightarrow$ 4, C $\rightarrow$ 1, D $\rightarrow$ 2
48. (a) It is effective in treatment of typhoid.
49. (c) Arsenic drugs, also called arsenical (like atoxyl, salvarsan, neoarsphenamine etc) are mainly used in sleeping sickness, syphilis etc.
50. (d) Bithional impart antiseptic property to soap.
51. (a) Aspirin is a moderate acid ($\text{p}K_a = 3.5$), hence it is almost unionised in stomach due to its strong acidic medium because of HCl. It is due to common ion effect. While in small intestine, the medium is alkaline, hence aspirin will be sufficiently ionised in small intestine.



BITSAT Archives

1. (a) Ziegler-Natta catalyst $[\text{TiCl}_4 + \text{Al}(\text{C}_2\text{H}_5)_3]$ is used as a catalyst in the polymerisation of olefins.
2. (b) The structure of copolymer of ethene and vinyl chloride is shown below



Molecular weight of ethene (CH_2CH_2) = 28

Molecular weight of vinyl chloride (CH_2CHCl) = 62.5

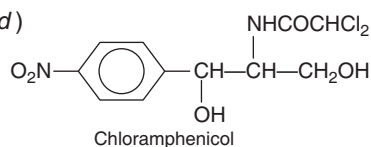
Empirical formula weight of copolymer = $28 + 62.5 = 90.5$

Mass% of vinyl chloride in the copolymer

$$= \frac{62.5 \times 100}{90.5} = 69.06 \approx 69\%$$

3. (b) Insulin is composed of two peptide chains referred to chains A and B. Chain A of 21 residues and chain B of 30 residues are cross-linked by two disulphide bridges.
4. (b) Barbitol is a sleep-producing drug, i.e. hypnotic tranquiliser. It causes addiction.
5. (b) C₁ carbon of monosaccharides is called anomeric carbon. When —OH group is attached with C₁ carbon towards right, it is called α -form and when —OH group is towards left, it is called β -form, such pair of optical isomers which differ in the configuration only around anomeric carbon are called anomers.
6. (b) In vulcanisation of rubber, sulphur cross-links are introduced at the reactive sites of double bonds.

7. (d)

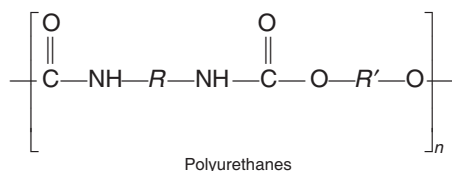
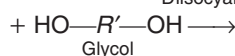


8. (d) Alizarin is an anthraquinone dye. It gives a bright red colour with aluminium and a blue colour with barium.

9. (d) Somatotropin is the hormone, secreted by anterior lobe of pituitary gland. It is also called growth hormone as it stimulates protein synthesis, glycogenesis and some other biological activities. Its deficiency causes midgets or dwarfism.

10. (c) $\text{O}=\text{C}=\text{N}-\text{R}-\text{N}=\text{C}=\text{O}$

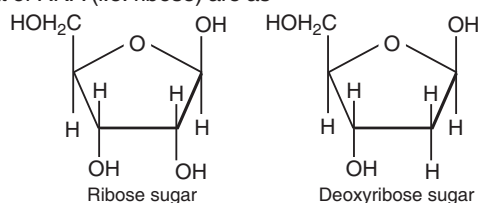
Diisocyanate



11. (c) Neoprene is synthetic rubber. It is a polymer of chloroprene and is resistant to the action of oils, gasoline and other solvents.

12. (a) Haemoglobin molecule contains four polypeptide chains.

13. (c) The structures of sugar of DNA (i.e. deoxyribose) and that of RNA (i.e. ribose) are as



Thus, it is clear that these have furanose structure.

14. (c) Tetrazine is an artificial edible colour.

15. (b) When formaldehyde reacts with ammonia, a well known urinary antiseptic urotropine (also called hexamethylene tetramine) is obtained.

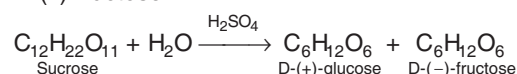
16. (d)

17. (b) Since, it gives a negative test with ninhydrin, it cannot be a protein or an amino acid. Since, it gives a positive test with Benedict's solution, it must be a monosaccharide but not a lipid.

18. (a) Natural rubber is *cis*-polyisoprene while *gutta-percha* is *trans*-polyisoprene.19. (a) Aspirin is used as analgesics as well as antipyretics, i.e. it serve a dual purpose. Chlorophyll is used in photosynthesis. Oxyhaemoglobin contains Fe^{2+} ion, so it is paramagnetic and haemoglobin works as oxygen carrier. Hence $A \rightarrow 5$, $B \rightarrow 1$, $C \rightarrow 2$, $D \rightarrow 3$

20. (b)

21. (a) On hydrolysis with dilute aqueous sulphuric acid, sucrose gives a equimolar mixture of D-(+)-glucose and D-(-)-fructose.



Sucrose is dextrorotatory but after hydrolysis, it gives dextrorotatory glucose and laevorotatory fructose. Laevorotatory fructose is more, so the mixture is laevorotatory.

22. (b) Alizarin is mordant dye. Alizarin gives a bright red colour with aluminium and a blue colour with barium.

23. (c) 2, 4-D or 2, 4-dichlorophenoxyacetic acid is used as a herbicides.

24. (c)

25. (a) Cellulose is a polymer of glucose. β -D-(+)-glucose units are attached to each other by C_1 to C_4 bonds through β -glycosidic linkage in structure of cellulose.

26. (a) Carbohydrates that yield 2 to 0 monosaccharide units on hydrolysis. They are further classified as disaccharides, trisaccharides, etc. Riffinose is trisaccharide.

27. (b) Between pair of thymine and adenine in DNA, two-hydrogen bonds are present.

28. (d) Dacron is polymer of glycol and terephthalic acid.

29. (b) Natural rubber is considered as a linear polymer of isoprene (2-methyl-1, 3-butadiene). It is also called as *cis*-1, 4-polyisoprene.

30. (b) It is a used as an antacid.

31. (c) A chromoprotein is a conjugated protein that contains a pigmented prosthetic group which destroy antigens.

32. (c) It is an example of polyamide. It is prepared by condensation polymerisation of hexamethylene diamine with adipic acid under high pressure and temperature.

33. (d) The purine base present in RNA is uracil

34. (c) Phthalic acid

30

Qualitative Analysis

In inorganic qualitative analysis, an inorganic ionic compound or mixture of compounds, is analysed for its cations and anions (i.e. basic radicals and acid radicals). There are various methods of qualitative analysis like borax bead test, flame test, cobalt nitrate test etc., but the most important method is wet test.

Qualitative Analysis for Acid Radicals

Acids radicals have been grouped in two groups on the basis of decomposing acid (group reagent) :

Group I

On heating the mixture with dil. HCl or dil. H₂SO₄, a gas is evolved which is identified by its properties.

Acid radicals	Evolved gas
(i) Carbonate (CO ₃ ²⁻)	Gas (CO ₂) evolves with brisk effervescence which turns lime water to milky.
(ii) Sulphite (SO ₃ ²⁻)	Gas (SO ₂) with smell of burning sulphur, turns acidic dichromate paper to green [Cr(SO ₄) ₃].
(iii) Sulphide (S ²⁻)	Gas (H ₂ S) with smell of rotten eggs, turns lead acetate paper to black (PbS).
(iv) Nitrite (NO ₂ ⁻)	Brown gas (NO ₂) with pungent odour, turns KI-starch paper to blue.
(v) Acetate (CH ₃ COO ⁻)	Smell of vinegar, aqueous salt + FeCl ₃ → red colour

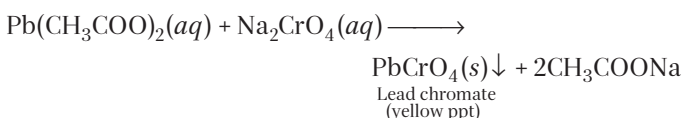
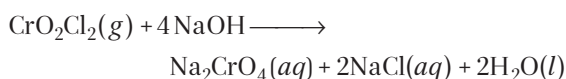
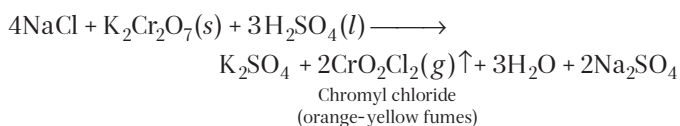
Group II

Mixture is heated with conc. H₂SO₄, when a gas is evolved which is identified by its properties.

Acid radicals	Gas evolved
(i) Chloride (Cl ⁻)	Colourless, pungent gas (HCl), gives white fumes on bringing glass rod moistened with NH ₄ OH.
(ii) Bromide (Br ⁻)	Colourless, pungent gas (HBr) Mixture + MnO ₂ + conc. H ₂ SO ₄ $\xrightarrow{\Delta}$ red vapours (Br ₂)
(iii) Iodide (I ⁻)	Violet vapours (I ₂), turns starch paper to blue.
(iv) Nitrate (NO ₃ ⁻)	Brown, pungent gas (NO ₂).
(v) Oxalate (C ₂ O ₄ ²⁻)	Colourless, odourless gas (CO) evolves, which burns with blue flame.

Some specific tests**Chromyl Chloride Test**

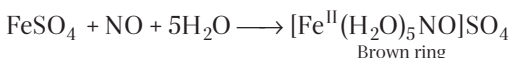
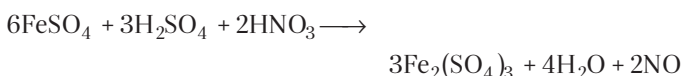
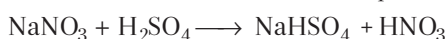
Chloride on heating with solid $K_2Cr_2O_7$ and conc. H_2SO_4 gives orange-yellow vapours of chromyl chloride (CrO_2Cl_2) which on passing over NaOH and then on treating with lead acetate forms yellow precipitate of lead chromate.



Chromyl chloride test is not given by chlorides of Hg, Sn, Ag, Pb and Sb. Bromides and iodides do not give chromyl chloride test.

Ring Test

This test is for the identification of nitrate ion (NO_3^-). To the mixture, freshly prepared $FeSO_4$ solution is added shaken. Now a drop of conc. H_2SO_4 is added by the side of the test tube, a brown ring is formed at the junction of two liquids due to the formation of nitroso ferrous sulphate.

**Test for Some Other Acid Radicals**

- (a) **Sulphate** (SO_4^{2-}) Solution of mixture in dil. HCl + $BaCl_2 \longrightarrow$ white ppt ($BaSO_4$) is insoluble in mineral acids.
- (b) **Phosphate** (PO_4^{3-}) Mixture is boiled with conc. HNO_3 and then ammonium molybdate solution is added when a canary yellow precipitate is obtained.

Interfering Acid Radicals

Borate, oxalate, fluoride and phosphate are called interfering radicals as they interfere in the usual test of basic radicals.

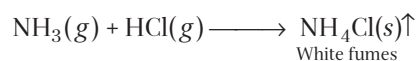
Qualitative Analysis for Basic Radicals

Basic radicals have been grouped in 6 groups on the basis of solubility products of their salts with group reagent.

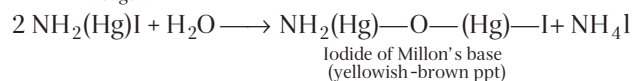
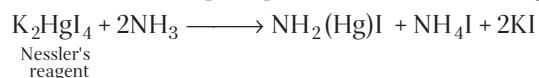
Group	Basic radical	Group reagent	Precipitate
I	Ag^+, Pb^{2+}, Hg_2^{2+}	dil. HCl	$AgCl, PbCl_2, Hg_2Cl_2$ (All white)
II	Hg^{2+}, Pb^{2+}	H_2S (in acidic medium of dil. HCl)	HgS, PbS, CuS (all black), Bi_2S_3 (brown), CdS (yellow)
II A	$Bi^{3+}, Cu^{2+}, Cd^{2+}$ (insoluble in yellow ammonium sulphide)		
II B	$As^{3+}, Sb^{3+}, Sn^{2+}, Sn^{4+}$ (soluble in YAS)		SnS (black), As_2S_3 (yellow), Sb_2S_3 (orange)
III	$Fe^{3+}, Al^{3+}, Cr^{3+}$	NH_4OH (in presence of excess NH_4Cl)	$Fe(OH)_3$ (brown), $Cr(OH)_3$ (green), $Al(OH)_3$ (white gelatinous)
IV	$Co^{2+}, Ni^{2+}, Zn^{2+}$	H_2S (in alkaline medium of NH_4OH)	NiS, CoS (black), ZnS (white)
V	$Ba^{2+}, Ca^{2+}, Sr^{2+}$	$(NH_4)_2CO_3$ (in presence of NH_4OH)	$BaCO_3, CaCO_3, SrCO_3$ (all white)
VI	Mg^{2+}	Na_2HPO_4	$Mg(NH_4)PO_4$ (white)

Test for ammonium (NH_4^+)

Ammonium ion on heating with NaOH gives NH_3 gas which is recognised by taking a glass rod dipped in HCl above the mouth of test tube.



It gives yellowish-brown precipitate with Nessler's reagent.

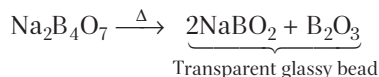
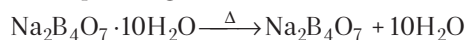
**Some Important Points**

- Before the test of group III, the filtrate of group II is boiled to evolve out H_2S gas so that sulphide of higher group radicals could not precipitate in group III.
- In group III analysis, NH_4Cl is added before NH_4OH to decrease the ionisation of NH_4OH by common ion effect so that only group III radicals could precipitate.

Borax Bead Test

In this test, a platinum wire loop is made and heated. Now it is dipped into borax powder.

It is heated again when borax loses water and decomposes to give a transparent glassy bead of sodium metaborate.



Now mixture is taken over bead and it is again heated in oxidising (non-luminous) flame and then in reducing (luminous flame) and colour is observed in each case.

Different basic radicals give different colours, thus they can be identified.

Basic Radical	Colour (in oxidising flame)	Colour (in reducing flame)
Co^{2+}	Blue	Blue
Cu^{2+}	Light blue or green	Colourless or red
Fe^{3+}	Yellow brown	Bottle green
Cr^{3+}	Green	Green

Practice Exercise

- A substance on treatment with dil. H_2SO_4 liberates a colourless gas which produces (i) turbidity with baryta water and (ii) turns acidified dichromate solution green. The reaction indicates the presence of
 a. CO_3^{2-} b. S^{2-} c. SO_3^{2-} d. NO_2^-
- In qualitative analysis when H_2S is passed through an aqueous solution of salt acidified with dilute HCl a black precipitate is obtained. On boiling the precipitate with dil. HNO_3 , it forms a solution of blue colour. Addition of excess of aqueous solution of NH_3 to this solution gives
 a. deep blue precipitate of $\text{Cu}(\text{OH})_2$
 b. deep blue solution of $[\text{Cu}(\text{NH}_3)_4]^{2+}$
 c. deep blue solution of $\text{Cu}(\text{NO}_3)_2$
 d. deep blue solution of $\text{Cu}(\text{OH})_2 \cdot \text{Cu}(\text{NO}_3)_2$
- The aqueous solution of which of the following salt will be coloured?
 a. $\text{Zn}(\text{NO}_3)_2$ b. LiNO_3
 c. CrCl_3 d. Potash alum
- Which does not give borax bead test?
 a. Fe b. Cu
 c. Cr d. Pb
- A brown ring is formed in the ring test for NO_3^- ion. It is due to the formation of
 a. $[\text{Fe}(\text{H}_2\text{O})_5(\text{NO})]^{2+}$ b. $\text{FeSO}_4 \cdot \text{NO}_2$
 c. $[\text{Fe}(\text{H}_2\text{O})_4(\text{NO}_2)]^{2+}$ d. $\text{FeSO}_4 \cdot \text{HNO}_3$
- Among the following pairs, the pair of salts that gives yellow precipitate separately with aqueous solution of BaBr_2 , is
 a. Na_2CO_3 , AgNO_3 b. K_2SO_4 , K_2CrO_4
 c. K_2SO_4 , AgNO_3 d. AgNO_3 , K_2CrO_4
- Formation of purple colour on the addition of sodium nitroprusside to sodium carbonate extract indicates the presence of
 a. PO_4^{3-} b. SO_3^{2-}
 c. S^{2-} d. $\text{S}_2\text{O}_3^{2-}$
- Which chloride turns black with NH_4OH ?
 a. Lead chloride b. Mercurous chloride
 c. Mercuric chloride d. Silver chloride
- Which among the following is an interfering radical?
 a. Sulphide b. Sulphate
 c. Phosphate d. Nitrite
- Which compound does not dissolve in hot dilute HNO_3 ?
 a. HgS b. CuS c. PbS d. CdS
- A metal nitrate reacts with KI to give a black precipitate which on addition of excess of KI convert into orange colour solution. The cation of metal nitrate is
 a. Hg^{2+} b. Bi^{3+} c. Pb^{2+} d. Cu^+
- A brown precipitate obtained by reaction of alkaline solution of Nessler's reagent and NH_3 are due to the formation of
 a. mercuric hydroxide b. HgI_4^{2-}
 c. $\text{Hg} + \text{Hg}(\text{NH}_2)\text{I}$ d. iodide of Millon's base
- Which of the following on reaction with NaOH and Na_2O_2 gives yellow colour?
 a. $\text{Cr}(\text{OH})_3$ b. $\text{Al}(\text{OH})_3$
 c. $\text{Zn}(\text{OH})_3$ d. None of these
- Consider the following reactions,

$$[\text{A}] + \text{H}_2\text{SO}_4 \longrightarrow [\text{B}]$$
 a colourless gas having irritating smell

$$[\text{B}] + \text{K}_2\text{Cr}_2\text{O}_7 + \text{H}_2\text{SO}_4 \longrightarrow \text{Green solution}$$
 A and B respectively are
 a. Cl^- and HCl b. CO_3^{2-} and CO_2
 c. SO_3^{2-} and SO_2 d. S^{2-} and H_2S
- A solution of metal ion when treated with KI gives a red precipitate which dissolves in excess KI to give a colourless solution. Moreover the solution of metal ion on treatment with a solution of cobalt (II) thiocyanate gives rise to a deep blue crystalline precipitate. The metal ion is
 a. Pb^{2+} b. Hg^{2+} c. Cu^{2+} d. Co^{2+}

16. An aqueous solution contains the ions Hg_2^{2+} , Hg^{2+} , Pb^{2+} and Cd^{2+} . Which of these will precipitate by the addition of dil. HCl ?
 a. Hg_2Cl_2 and PbCl_2 b. Only Hg_2Cl_2
 c. Only PbCl_2 d. PbCl_2 and Hg_2Cl_2
17. Which one of the following does not impart colour to the flame?
 a. LiCl b. KI
 c. MgCl_2 d. CaCl_2
18. Identify the incorrect statement about $\text{Al}_2(\text{SO}_4)_3$.
 a. It is a white salt
 b. On adding NaOH solution to its aqueous solution, a precipitate is formed which get dissolve in excess of NaOH solution
 c. With lead acetate solution also, it gives a precipitate
 d. With silver nitrate, its aqueous solution gives a precipitate
19. On addition of conc. H_2SO_4 to a chloride salt, colourless fumes are evolved but in case of iodide salt, violet fumes come out. This is because
 a. H_2SO_4 reduces HI to I_2 b. HI is of violet colour
 c. HI gets oxidised to I_2 d. HI changes to HIO_3
20. When bismuth chloride is poured into a large excess of water, then white precipitate produced is due to
 a. $\text{Bi}(\text{OH})_3$ b. Bi_2O_3
 c. BiOCl d. Bi_2OCl_2
21. A purple coloured compound is obtained, on treating sodium sulphide with sodium nitroprusside. In this reaction, the oxidation state of Fe
 a. changes from + 3 to + 2
 b. changes from + 2 to + 4
 c. changes from + 2 to + 3
 d. No change in oxidation state
22. Which one of the following statements is correct?
 a. Manganese salts give a violet borax bead test in the reducing flame
 b. From a mixed precipitate of AgCl and AgI , ammonia solution dissolves only AgCl
 c. Ferric ions give a deep green precipitate on adding potassium ferrocyanide solution
 d. On boiling a solution having K^+ , Ca^{2+} and HCO_3^- ions, we get a precipitate of $\text{K}_2\text{Ca}(\text{CO}_3)_2$
23. Which is soluble in NH_4OH ?
 a. PbCl_2 b. AgCl c. PbSO_4 d. CuCO_3
24. Which of the following will not give positive chromyl chloride test?
 a. CuCl_2 b. HgCl_2
 c. ZnCl_2 d. $\text{C}_6\text{H}_5\text{NH}_3\text{Cl}^+$
25. Which of the following does not give flame test?
 a. Ba^{2+} b. Mg^{2+} c. Sr^{2+} d. Ca^{2+}
26. Which of the following will give blood red colour with FeCl_3 in sodium extract?
 a. NH_2CONH_2 b. NH_2CSNH_2
 c. $\text{C}_2\text{H}_5\text{NHNH}_2$ d. $\text{CH}_3\text{C}\equiv\text{N}$
27. Which of the following gives blood red colour with KCNS ?
 a. Cu^{2+} b. Fe^{3+}
 c. Al^{3+} d. Zn^{2+}
28. Which pair of ions gives precipitate when their dilute aqueous solutions are mixed?
 a. CO_3^{2-} , NH_4^+ b. SO_3^{2-} , Na^+
 c. PO_4^{3-} , Fe^{3+} d. Na^+ , S^{2-}
29. The ion that cannot be precipitated by both HCl and H_2S , is
 a. Pb^{2+} b. Cu^+ c. Ag^+ d. Sn^{2+}

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1. In Nessler's reagent for the detection of ammonia, the active species is [2007]
 a. Hg_2Cl_2 b. Mg^{2+}
 c. HgI_2 d. HgI_4^{2-}
2. Which of the following silver salts is insoluble in water? [2005]
 a. AgClO_4 b. Ag_2SO_4
 c. AgF d. AgNO_3

Answer with Solutions

Practice Exercise

- (c) SO_3^{2-} gives SO_2 with dil. H_2SO_4 which gives turbidity with $\text{Ba}(\text{OH})_2$ and turns acidified dichromate solution green due to its reduction to Cr^{3+} ions.
- (b) $3\text{CuS} + 8\text{HNO}_2 \longrightarrow 3\text{Cu}(\text{NO}_3)_2 + 2\text{NO} + 3\text{S} + 4\text{H}_2\text{O}$
 $\text{Cu}^{2+} + 4\text{NH}_4\text{OH} \longrightarrow [\text{Cu}(\text{NH}_3)_4]^{2+} + 4\text{H}_2\text{O}$
Deep blue complex
- (c) Cr^{3+} is a green coloured ion.
- (d) Borax bead test is given only by coloured salts or ions. Hence, Pb does not give borax bead test.
- (a)
- (d) $\text{Ba}^{2+} + \text{K}_2\text{CrO}_4 \longrightarrow \text{BaCrO}_4 \downarrow + 2\text{K}^+$
Yellow
 $\text{Br}^- + \text{AgNO}_3 \longrightarrow \text{AgBr} \downarrow + \text{NO}_3^-$
Yellow
- (c) $\text{Na}_2\text{S} + \text{Na}_2[\text{Fe}(\text{CN})_5(\text{NO})] \longrightarrow \text{Na}_4[\text{Fe}(\text{NOS})\text{CN}]_5$
Sodium nitroprusside Violet (purple)
- (b) It form black precipitate of mercury with NH_4OH
 $\text{Hg}_2\text{Cl}_2 + 2\text{NH}_4\text{OH} \longrightarrow [\text{NH}_2\text{—Hg—Hg—Cl}] + \text{NH}_4\text{Cl} + \text{H}_2\text{O}$
Unstable
 $\downarrow$
 $\text{NH}_2\text{HgCl} \downarrow + \text{Hg} \downarrow$
White Black
- (c) Phosphate causes the precipitation of group IV, V and Mg^{2+} cation in group III as PO_4^{3-} and hence, creates interference.
- (a) HgS is soluble only in aqua-regia (i.e. $3\text{HCl} + \text{HNO}_3$).
- (b) $\text{Bi}(\text{NO}_3)_3(\text{aq}) + 3\text{KI}(\text{aq}) \longrightarrow \text{BiI}_3(\text{s}) \downarrow + 3\text{KNO}_3(\text{aq})$
Black ppt
 $\text{BiI}_3(\text{s}) + \text{KI}(\text{aq}) \longrightarrow \text{K}[\text{BiI}_4]$
Orange
- (d) NH_3 form Millon's iodide base $[\text{I—HgOHg—NH}_2]$ with Nessler's reagent.
- (a) $2\text{Cr}(\text{OH})_3 + 4\text{NaOH} + 3[\text{O}] \longrightarrow 2\text{Na}_2\text{CrO}_4 + 5\text{H}_2\text{O}$
Yellow
- (c) Since, SO_3 has a smell of burning which is irritating, thus, [A] is SO_3^{2-} and [B] is SO_2 , which turns acidified $\text{K}_2\text{Cr}_2\text{O}_7$ solution green, as it get reduced to Cr^{3+} ions.

- (b) $\text{Hg}^{2+} + 2\text{I}^- \longrightarrow \text{HgI}_2$
 $\text{HgI}_2 + 2\text{I}^- \xrightarrow{\text{Excess}} [\text{HgI}_4]^{2-}$
 $\text{Hg}^{2+} + \text{Co}(\text{SCN})_2 \longrightarrow \text{Hg}(\text{SCN})_2 \downarrow$
Blue crystalline ppt
- (a) Hg_2^{2+} and Pb^{2+} are present in I group, hence only these will be precipitated. The solubility product of MgCl_2 and CdCl_2 are higher, hence they will not precipitate.
- (b)
- (d) Aqueous solution of $\text{Al}_2(\text{SO}_4)_3$ does not give precipitate with silver nitrate (AgNO_3) solution.
- (c)
- (c) Bismuth and antimony chloride give white precipitate of oxychloride in water.
 $\text{BiCl}_3 + \text{H}_2\text{O} \longrightarrow \text{BiOCl} + 2\text{HCl}$
Milky
- (d) $\text{Na}_2[\text{Fe}(\text{CN})_5\text{NO}] + \text{Na}_2\text{S} \longrightarrow \text{Na}_4[\text{Fe}(\text{CN})_5\text{NOS}]$
+3
- (b) NH_3 is an inorganic solvent. AgI has much higher covalent character in comparison to AgCl. Thus, AgCl dissolves readily in NH_3 .
- (b) AgCl is completely soluble in NH_4OH while AgBr is sparingly soluble in it. AgI is insoluble in NH_4OH .
- (b) HgCl_2 does not give positive chromyl chloride test due to its covalent nature.
- (b) Mg^{2+} and Be^{2+} do not impart colour to the flame in the flame test.
- (b) Compound (b) contains both N and S, therefore it will form NaSCN which reacts with FeCl_3 to give ferric thiocyanate which is blood red in colour.
- (b) $\text{Fe}^{3+} + 3\text{KCNS} \longrightarrow \text{Fe}(\text{CNS})_3 + 3\text{K}^+$
Ferric thiocyanate (blood red)
- (c) $\text{Fe}^{3+}(\text{aq}) + \text{PO}_4^{3-}(\text{aq}) \longrightarrow \text{FePO}_4 \downarrow$
ppt
- (d) Sn^{2+} can be precipitated only by H_2S and not by HCl. Rest can be precipitated by both reagents.

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- (d) Nessler's reagent gives brown ppt of iodide of Millon's base with ammonium salt.
 $[\text{HgI}_4]^{2-} + \text{NH}_4\text{Cl} + 4\text{OH}^- \longrightarrow \text{NH}_2\text{HgOHgI} + \text{I}^- + \text{Cl}^- + 3\text{H}_2\text{O}$
Iodide of Millon's base (brown ppt)
- (b) Ag_2SO_4 is insoluble in water.

31

Stereochemistry

Isomerism

Compounds having the same molecular formula but differ in structural formulae, properties and spatial arrangement, are called **isomers** and the phenomenon is called isomerism.

Types of Isomerism

Isomerism can be of the following two types:

- (i) Structural isomerism
- (ii) Stereo isomerism

Structural Isomerism

When the isomerism is due to difference in the sequence (connectivity) of atoms within the molecules (without any reference to space), the phenomenon is called structural isomerism. In other words, the structural isomers are the compounds which have the same molecular formula but different structural formulae, hence different properties. The structural isomerism is also called **constitutional isomerism** and categorised as

- (i) Chain isomerism
- (ii) Position isomerism
- (iii) Functional isomerism
- (iv) Metamerism
- (v) Tautomerism
- (vi) Ring chain isomerism
- (vii) Isotopic isomerism

NOTE *Tautomerism and ring chain isomerism are considered as the specialised sub-types of functional isomerism.*

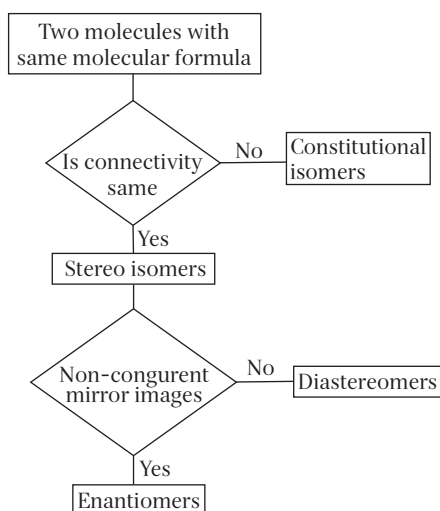
Stereo Isomerism

When the isomerism is caused by different arrangement of atoms or groups in space, i.e. 3D-arrangement, the phenomenon is called stereo isomerism. In other words, the stereo isomerism is shown by those compounds which have the same structural formula (alongwith same molecular formula) but differ in configuration (the term configuration refers to three dimensional arrangement of atoms that characterise a particular compound).

This isomerism is called **stereo isomerism** or **configurational isomerism** and can be categorised as

- (i) Geometrical or *cis/trans*-isomerism
- (ii) Optical isomerism

Caution Point Neither resonance forms nor conformations constitute isomers.



Simple flow chart representing the classification of isomers

Geometrical Isomerism

When a π -bond is present between two atoms, it restricts the free rotation of atoms around its axis. It gives rise to two different structural formulae (properties of these forms differ due to different arrangement of atoms about double bonds in alkenes or about single bonds in cyclic compounds).

This isomerism which arises due to difference in spatial arrangement of groups about the

1. doubly bonded C-atoms in **alkenes**,
2. single bonded C-atoms in **cyclic compounds**,
3. doubly bonded C and N-atoms in **oximes**,
4. doubly bonded N-atoms in **diazo compounds**, is called geometrical isomerism.

Conditions for Exhibiting Geometrical Isomerism

The conditions for exhibiting geometrical isomerism include

1. There should be frozen rotation about two atoms in the molecules, e.g.
 - (i) $>C=C<$, frozen rotation about C—C double bond in alkenes.
 - (ii) $\triangle$, frozen rotation about carbon-carbon single bond in cycloalkanes.

(iii) $>C=N$, frozen rotation about carbon-nitrogen double bond in oximes and imines.

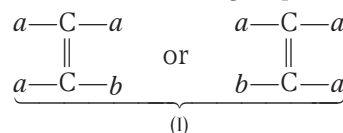
(iv) $-N=N-$, frozen rotation about nitrogen-nitrogen double bond in azo compounds.

2. Both the atoms of above written bonds should contain different substituents.

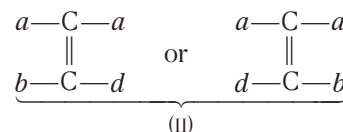
Geometrical Isomerism in Alkenes

The forms in which similar groups are present on the same side of double bond are called ***cis* forms** [Latin *cis* = same side] and the forms, in which similar groups are present on the different sides are called ***trans* forms** [Latin *trans* = across].

These forms show usually similar chemical but different physical properties. However, some forms may show difference in chemical properties as well. Geometrical isomerism is not seen in those forms in which the double bonded carbon atom have same groups attached to it. e.g.



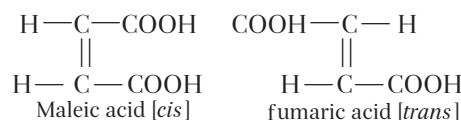
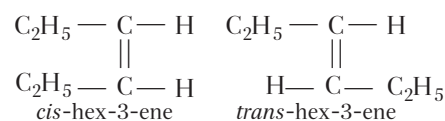
Both the structures are identical



Both the structures are identical

Geometrical figures in (I) and (II) will be identical, hence, no geometrical isomers are possible.

The conversion of *cis*-isomer into *trans*-isomer or *vice-versa* is possible only if either of the isomer is heated to a high temperature or absorbs light. The heat supplies the energy to break the π -bond. Some examples of geometrical isomers are as



These isomers are

- (i) differ in physical properties such as melting point, dipole moment and acidities. The *trans*-isomer has no dipole moment since it is symmetrical, unlike the *cis*-isomer. The *cis*-isomer is stronger acid than the *trans*-isomer.

- (ii) differ even in chemical properties, e.g. *cis* acid undergo dehydration at 100°C and form an anhydride. However, the *trans*-acid does not form an anhydride until heated to 240°C.

Distinction Between *cis* and *trans*-Isomers

1. The *trans*-forms are more stable than the corresponding *cis*-isomer. This is because, in the *cis*-isomer, the bulky groups are on the same side of double bond. The steric repulsion of the groups makes the *cis*-isomer less stable than the *trans*-isomer in which the bulky groups are far apart (they are on the opposite sides of the double bond).
2. Melting point of *trans*-form is higher than the melting point of *cis*-form because of symmetrical packing of *trans*-compounds in its crystal lattice.
3. Boiling point of *cis*-form is higher than that of the *trans*-form because of steric hindrance in *cis*-compound and high polarity of *cis*-compound.
4. Solubility of *cis*-isomer is higher as compared to *trans*-isomer, as molecules of *trans*-isomer are more tightly held in the crystal lattice than the corresponding *cis*-isomer and hence, it is easier to break the crystal lattice of *cis*-isomer as compared to *trans*-isomer.
5. The *cis*-isomer of a symmetrical alkene (alkene in which both the carbon atoms have similar group) has a definite dipole moment, while the *trans*-isomer has zero dipole moment.

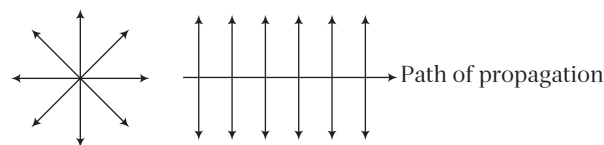
Optical Isomerism

Compounds with similar physical as well as chemical properties and with difference only in their behaviour towards plane polarised light are called optical isomers and this property is called optical isomerism.

Important Terms Related to Optical Isomerism

Before going in detail, it is better to understand some terms related to this isomerism:

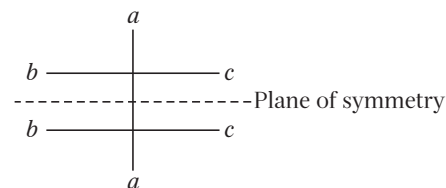
- (a) **Plane polarised light** According to wave theory, an ordinary ray of light is considered to have particulate vibrations in all possible planes perpendicular to its path of propagation. However in polarised light, the particulate vibrations occur in one plane only.



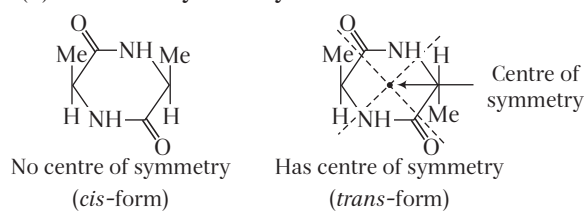
Unpolarised light Polarised light

- (b) **Optical activity** The property of a compound by virtue of which it can rotate the plane of polarised light, is called optical activity.
- (c) **Asymmetric C-atom** C-atom having 4 different groups or atoms attached separately to its four valencies, is called asymmetric carbon atom or **stereocentre**.
- (d) **Chiral molecule** A molecule completely lacking plane of symmetry is called chiral molecule. In other words, it can also be defined as the one which is not superimposable on its mirror reflection or the one in which line of symmetrisation is lacking. An asymmetric C-atom will generally be considered as chiral and we can say that
- (1) A chiral molecule mostly possess at least one chiral carbon atom.
 - (2) A chiral molecule does not possess any element of symmetry like

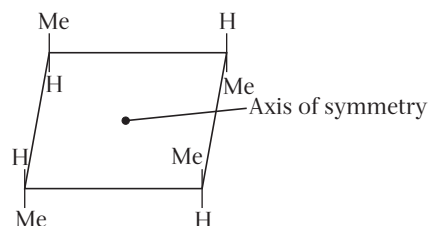
(i) Plane of symmetry



(ii) Centre of symmetry

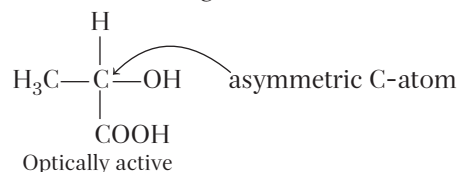


(iii) Axis of symmetry



- (e) **Achiral molecule** A molecule which is superimposable on its mirror image is called achiral.

Usually the compounds having atleast one asymmetric C-atom (the C-atom all the four valencies of which are satisfied by four different groups or atoms) are capable to exhibit optical isomerism, e.g. lactic acid.



The optical isomers can be categorised into the following two types :

(a) Optically Active Compounds

Those compounds which have the ability to rotate the plane of polarised light are called optically active compounds. These compounds are infact chiral, i.e. have absence of elements of symmetry.

Among the optically active compounds, following types are included :

1. **Dextrorotatory** or *d* or (+) forms which rotate the plane of polarised light to the right (clockwise).
2. **Laevorotatory** or *l* or (–) forms which rotate the plane of polarised light to the left (anticlockwise).

(b) Optically Inactive Compounds

Those compounds which do not have such an ability, i.e. which do not rotate the plane of polarised light are called **optically inactive compounds**.

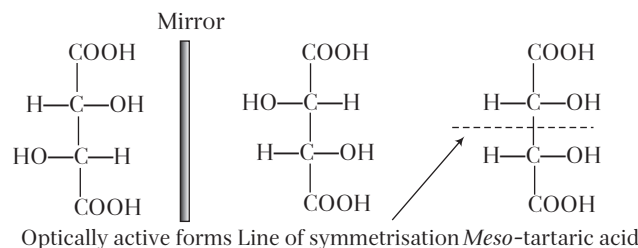
The reasons for the inactivity of these compounds are :

- (i) Presence of element of symmetry
- (ii) Presence of *d* and *l*-forms in equal amounts

On the basis of above reasons, optically inactive compounds can be of two types :

1. **Meso form** It has the plane of symmetry within the molecule, i.e. the one-half of the molecule is dextrorotatory and rest half is laevorotatory. Thus, within the molecule, both halves show equal and opposite optical activity.

As a result, the molecule becomes optically inactive. This phenomenon of optical inactivation in which the rotation produced by one-half of the molecule is cancelled by its another half is called internal compensation, i.e. *meso* forms are said to be optically inactive due to the phenomenon of internal compensation, e.g. the optically active and inactive forms of tartaric acid are as follows :



Optically active forms Line of symmetrisation *Meso*-tartaric acid

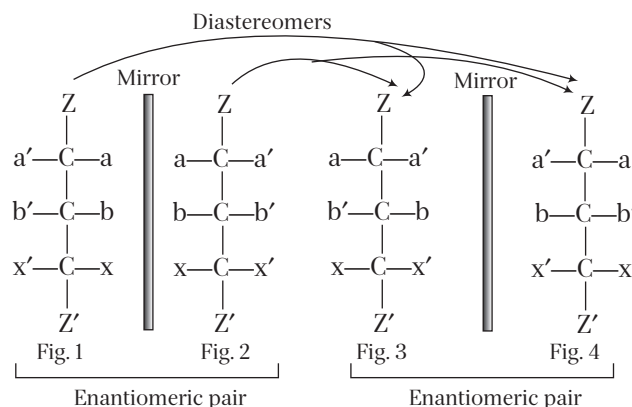
2. **Racemic mixture** The equimolar mixture of two optically active forms (*d* and *l* forms) of a compound is called racemic mixture. The mixture is optically inactive as both the forms neutralises the effect of one another. Such a neutralising effect is called **external compensation**, thus racemic mixtures are optically inactive due to the process of external compensation. The formation of racemic mixture is called **racemisation** while separation of these forms from the mixture is called **resolution**.

Resolution can be achieved by mechanical method, biological method using enzymes, chemical methods (by making their diastereomers) or by chromatographic separation (by using special adsorbents).

Enantiomers and Diastereomers

Optically active compounds which are non-superimposable mirror images of each other are called enantiomers, e.g. in tartaric acid, first two figures written are called enantiomers.

Optical isomers which are not the mirror images of each other are called diastereomers, e.g. in the following representations, the structure of all the four figures is same but spatial arrangement of groups differ as fig. (1) and (2) are enantiomeric, similarly fig. (3) and (4) are enantiomeric.



However, Fig. (1) is diastereomeric with fig. (3) and (4) (same molecular formula, different optical behaviour and

not the mirror images of each other). Similarly, fig. (2) is diastereomeric with fig. (3) and (4). Contrary to enantiomers, two diastereomers have different melting points, boiling points and solubilities. They will have different chemical reactivities towards most reagents, i.e. show difference in physical as well as in chemical properties.

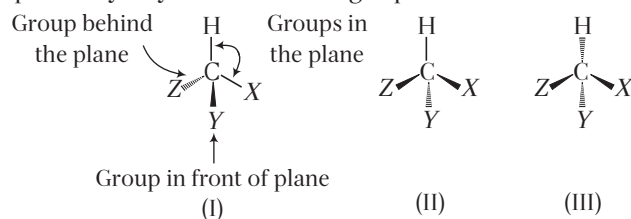
Projection Formulae

Different projection formulae have been given to define the structure a molecule completely. These can be converted into each other.

These are as follows :

(a) Wedge-Dash Formula

The configuration of organic molecules can be visualised by three dimensional (3D) structures, which may be depicted by any of the following representations.



Here the thick solid (solid wedge) line represents the bonds lying above the plane of paper (projecting towards the viewer), thin lines indicate the bonds lying within the plane of paper while the dotted line indicates the bonds lying below the plane of paper. Such a representation is called Wedge-Dash formula.

(b) Fischer Projection Formulae

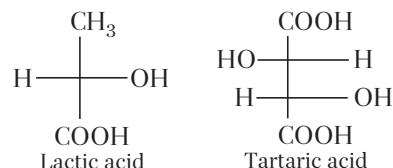
A planar representation of the three dimensional structure is called Fischer projection formula.

Following guidelines are used to draw it :

1. The chain of carbon atoms is arranged vertically in such a way that the most oxidised carbon occupies the top position.
2. The asymmetric carbon atom lies in the plane of paper and is represented at the intersection of crossed lines.
3. The bonds going away from you are represented by vertical lines. In other words, vertical lines represent the groups present behind the plane of paper.

4. The bonds coming towards you, are represented by horizontal lines. In other words, group attached to the horizontal lines are understood to be present above the plane of the paper.

Using the above guidelines, the Fischer projection formulae for lactic acid and tartaric acid are shown as

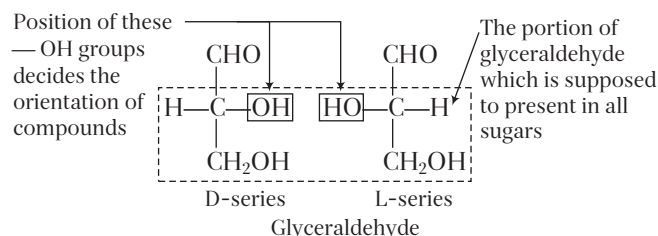


Systems of Nomenclature of Optical Isomers

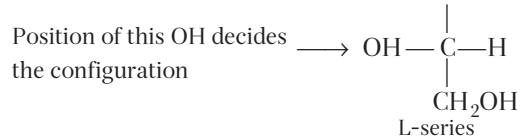
The nomenclature of optical isomers can be done in accordance with

(a) D, L-System of Nomenclature of Partial Configuration

The D, L system of nomenclature is used for optically active polyhydroxy carbonyl compounds (carbohydrates) mainly. This nomenclature was given by **Emil Fischer** who considered glyceraldehyde as the parental sugar.

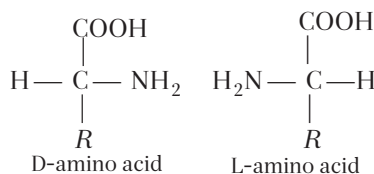


All sugars whose Fischer projection formula shows the —OH group on the right hand side on chiral C-atom adjacent to the terminal —CH₂OH group (i.e. second last C-atom) are called the sugars of D-series. Similarly, if the —OH group is positioned left then it looks like



Here, L stands for laevo, i.e. left hand side and not laevorotatory. All such sugars having this configuration are supposed to belong to the L-series. It must be noted that there is no relation between the sign of rotation (i.e. +, – or *d, l*) and the configuration (D and L) of the enantiomer.

This nomenclature is also used for α -amino acids.
e.g.



(b) *R, S*-System of Nomenclature

The actual three dimensional arrangement of groups in an asymmetrical molecule is called its **absolute configuration**. We can specify the configuration by using the *R, S* system i.e. *rectus* and *sinister* system [Here, *rectus* = right hand side, *sinister* = left hand side].

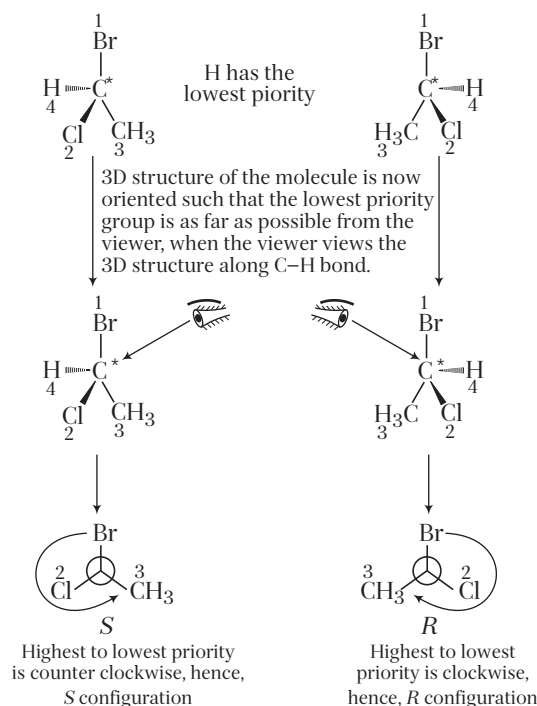
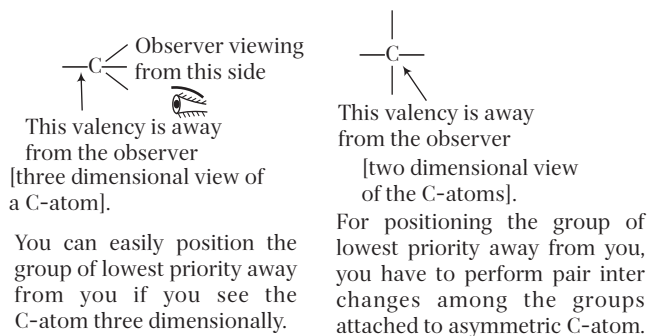
This nomenclature was proposed by **Cahn, Ingold and Prelog** and involves two steps in it:

Step 1 Following a set of sequence rules, a sequence of priority is assigned to four different groups or atoms attached to a chiral C-atom. The most acceptable criterion of priority is based on atomic numbers, i.e. atom with higher atomic number will get higher priority, e.g. in the case of bromochloriodomethane, the sequence of priority is I, Br, Cl, and H.

Step 2 The molecule is next visualised so oriented that the group of lowest priority is directed away from you and the arrangement of remaining groups observed. While observing you have to proceed from the group of highest priority to the group of second priority and then to the third.

- (i) If your eye travels in a clockwise direction, the configuration is called *R* [Latin *rectus* = right].
- (ii) If your eye travels anticlockwise, the configuration is called *S* [Latin *sinister* = left].

Two positioning of the lowest priority group down and away from the observer can be assumed easily in three dimensions but difficult a bit in two dimensions. e.g.



Measurement of Optical Activity

Optical activity of a compound can be detected and measured by means of a **polarimeter**. When a solution of a known concentration of an optically active compound is placed in a polarimeter, the beam of polarised light is rotated through a certain number of degrees either to the right (clockwise) or to the left (anticlockwise). This degree is measured and called **rotation** produced by the solution.

The degree of rotation depends upon the number of molecules of the compound encountered by light along its path. This degree of rotation, if measured, is called **specific rotation** and can be defined as the rotation produced by a solution of 10 cm length and unit concentration (1 g/mL) for the given wavelength (λ) of light at the given temperature.

Thus, for a system in which plane of polarised light is rotated through α degrees at a temperature of $t^\circ\text{C}$ and using the sodium-*D* lines as the source, the specific rotation $[\alpha]_D^t$ is given by

$$[\alpha]_D^t = 100 \alpha / l \times c$$

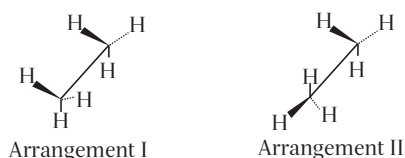
Here, l = length of the fluid filled tube in decimeters
(10 cm = 1 decimetre)

c = number of grams per 100 mL of solution

The specific rotation varies both with light wavelength and temperature.

Conformations or Conformers

The σ electron cloud joining the carbon atoms in alkanes is cylindrically symmetrical about a line joining the two carbon nuclei, hence bond strength should be the same for all possible arrangements. Different arrangements of atoms that can be converted into one another by rotation about single bonds are called conformations. Arrangement I is called the eclipsed conformation; arrangement II is called the staggered conformation. The infinite intermediate conformations in between the above two are called **skew conformations**.



Caution point Conformers are also termed as rotamers and conformational isomerism as rotamerisms.

Conformations of Ethane

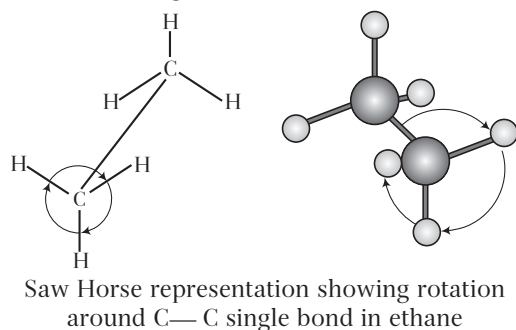
In ethane molecule ($\text{CH}_3\text{—CH}_3$), two carbon atoms are bonded by a single covalent bond. If one of the carbon atoms (methyl group) is kept fixed and the other is rotated about the C—C bond, an infinite number of arrangements of the hydrogen atoms attached to one carbon atom with respect to other are possible. These infinite numbers of different arrangements are called conformations.

To represent these conformations, we can draw three dimensional pictures.

However, chemists represent the conformations in two simple ways as shown below:

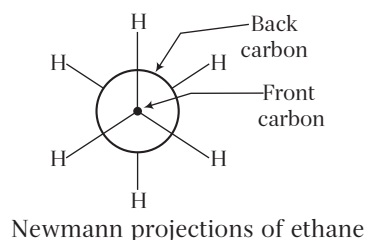
(a) Saw Horse Representation

In it, we view carbon-carbon bond from an oblique angle and indicate spatial arrangements by showing all the C—H bonds as shown in figure.



(b) Newmann Projection

It is a simple method to represent the conformations. In this method, two carbon atoms forming the σ -bond are represented by two circles, one behind the other so that only the front carbon is seen. The C—H bonds of the front carbon are depicted from the centre of the circle while C—H bonds of the back carbon are drawn from the circumference of the circle.



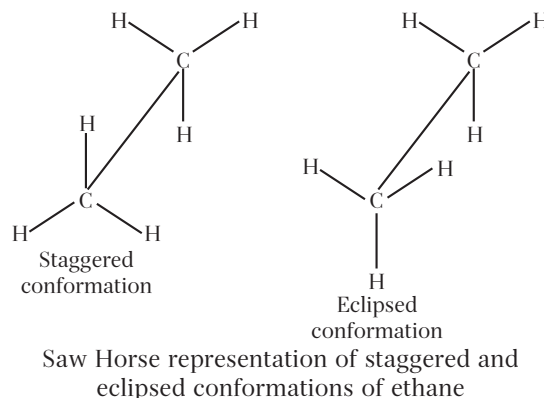
Out of all the conformations possible for ethane, only two extreme conformations are important, which are given below:

(a) Staggered Conformation

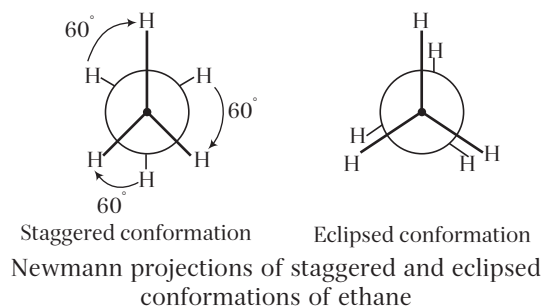
In this arrangement, the hydrogens of the carbon atoms are staggered with respect to one another. As a result, they are at maximum distance apart and have minimum repulsion between them.

(b) Eclipsed Conformation

In this conformation, the hydrogens of one carbon atom are directly behind those of the other. Consequently, the repulsion in these atoms is maximum. The saw horse projections of these conformations are represented in the figure given below.



The Newmann projections for staggered and eclipsed conformations of ethane are shown in figure given below. It is clear that when the staggered conformation is rotated through an angle of 60° , it changes to eclipsed conformation and similarly, when eclipsed conformation is rotated through the same angle, it gives back the staggered conformation.



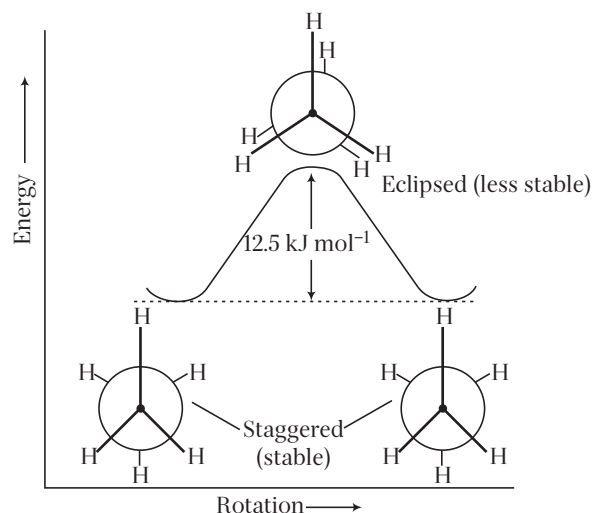
Relative Stabilities of the Conformations of Ethane

Two conformations of ethane differ in their relative stabilities. The staggered conformation has minimum repulsions between the H-atoms attached tetrahedrally to the two carbon atoms. On the other hand, the eclipsed conformation has maximum force of repulsion between H-atoms. Therefore, the staggered conformation is more stable than the eclipsed conformation.

The difference in the energy contents of the staggered and eclipsed conformations is 12.5 kJ mol^{-1} . This small barrier to rotation is also called torsional barrier of the single bond. However, this energy difference is not large enough to prevent rotation. Even at ordinary temperatures, the molecules have thermal or kinetic energy to overcome this energy barrier.

Therefore, the two conformations of ethane go on changing from one form to another thus consequently, it is not possible to isolate the different conformations of ethane.

The variation of energy *versus* rotation about the C—C bond has been shown in the following figure :



The variation of energy *versus* rotation about C—C bond

The difference in the energy of various conformations constitutes an energy barrier in rotation. For free rotation, energy barrier is 0.6 kcal/mol . For restricted rotation, energy barrier is in between > 0.6 and $< 16 \text{ kcal/mol}$.

For frozen rotation, energy barrier is $\geq 16 \text{ kcal/mol}$.

Caution Point The dihedral angle between the C—H bonds on adjacent carbons in staggered and eclipsed conformations of ethane are 180° and 0° respectively whereas between two methyl groups in the gauche and anti conformations of *n*-butane, these are 60° and 180° respectively.

Practice Exercise

1. Which class of compounds can exhibit geometrical isomerism?

- $\text{C}_6\text{H}_5\text{CH}=\text{NOH}$
- $\text{CH}_3\text{CH}=\text{CHCH}_3$
- $\text{HOOCCH}=\text{CH}_2\text{CHCOOH}$
- All of the above

2. Which of the following shows geometrical isomerism?

- 1, 2-dichloroethene
- 1,2-dimethylcyclopropane
- $$\begin{array}{c} \text{CO}-\text{NH} \\ \diagup \quad \diagdown \\ \text{CH}_3\text{CH} \quad \text{CHCH}_3 \\ \diagdown \quad \diagup \\ \text{NH}-\text{CO} \end{array}$$
- All of the above

3. The number of isomers of the compound C_2FCIBrI is

- 3
- 4
- 5
- 6

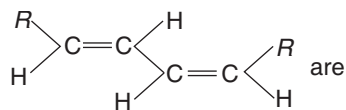
4. Which will form geometrical isomers?

-
- $\text{CH}_3\text{CH}=\text{NOH}$
-
- All of these

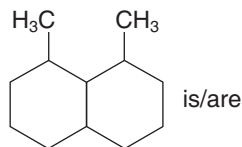
5. In , the double bonds are

- cis, cis
- cis, trans
- trans, cis
- trans, trans

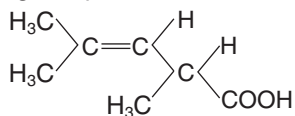
6. Number of geometrical isomers for the molecule



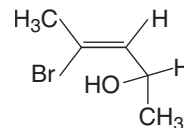
- a. 2
b. 3
c. 6
d. 5
7. Racemic modification can be resolved by
a. the use of enzymes
b. fractional crystallisation
c. fractional distillation
d. None of the above
8. Racemic tartaric acid is optically inactive due to
a. external compensation
b. internal compensation
c. presence of plane of symmetry
d. All of the above
9. (+) and (–) forms of optically active compounds are different in
a. boiling points
b. melting points
c. specific gravity
d. specific rotation
10. Which of the following Fischer projection formula is same as D-glyceraldehyde?
a. $\begin{array}{c} \text{CH}_2\text{OH} \\ | \\ \text{OH} - \text{C} - \text{CHO} \\ | \\ \text{H} \end{array}$
b. $\begin{array}{c} \text{CH}_2\text{OH} \\ | \\ \text{H} - \text{C} - \text{OH} \\ | \\ \text{CHO} \end{array}$
c. $\begin{array}{c} \text{CHO} \\ | \\ \text{H} - \text{C} - \text{OH} \\ | \\ \text{CH}_2\text{OH} \end{array}$
d. $\begin{array}{c} \text{CHO} \\ | \\ \text{H} - \text{C} - \text{CH}_2\text{OH} \\ | \\ \text{OH} \end{array}$
11. How many carbon atoms in the molecule $\text{HOOC}-(\text{CHOH})_2-\text{COOH}$ are asymmetric?
a. 1
b. 2
c. 3
d. None of these
12. Number of chiral centres in



- a. 1
b. 2
c. 3
d. 4
13. The following compound can exhibit
a. tautomerism
b. optical isomerism
c. geometrical isomerism
d. geometrical and optical isomerism



14. Which of the following statements is not correct?
a. A *meso* compound has chiral centres but exhibits no optical activity
b. A *meso* compound has no chiral centre, thus it is optically inactive
c. A *meso* compound has molecules in which one-half of molecule is superimposable on the other even through chiral centre is present in them
d. A *meso* compound is optically inactive because the rotation caused by one-half of molecule is cancelled by the rotation produced by another half
15. An enantiomerically pure acid is treated with racemic mixture of an alcohol having one chiral carbon. The ester formed will be
a. optically active mixture
b. pure enantiomer
c. *meso* compound
d. racemic mixture
16. Which of the following will exhibit geometrical isomerism?
a. Propene
b. Butene-2
c. Butene-1
d. 1,1-dichloro butane
17. Which statement is true?
a. A compound with *R* configuration is the (+) enantiomer
b. If configuration changes from + to –, that essentially means inversion of configuration takes place
c. An achiral molecule reacts always with racemic forms, to give a chiral molecule,
d. By breaking two bonds on the chiral centre, configuration changes
18. The compound whose stereo chemical formula is written below, exhibits *x*-geometrical isomers and *y*-optical isomers. The value of *x* and *y* respectively are

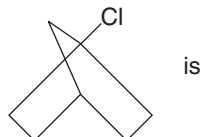


- a. 4 and 4
b. 2 and 2
c. 2 and 4
d. 4 and 2
19. Incorrect statement is
a. ethane can have an infinite number of conformations
b. cyclopropane molecule has considerable angle strain
c. eclipsed form of ethane is less stable than staggered conformation
d. staggered conformation possesses maximum energy
20. At room temperature, the eclipsed and the staggered forms of ethane cannot be isolated because
a. both the conformers are equally stable
b. they interconvert rapidly
c. there is a large energy barrier of rotation about the σ -bond
d. the energy difference between the conformers is large

21. The most stable conformation of ethane chlorohydrin at room temperature is

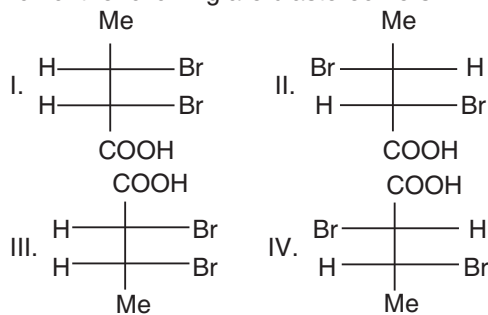
- a. fully eclipsed b. partially eclipsed
c. *gauche* d. staggered

22. The number of chiral carbon atoms present in the molecule



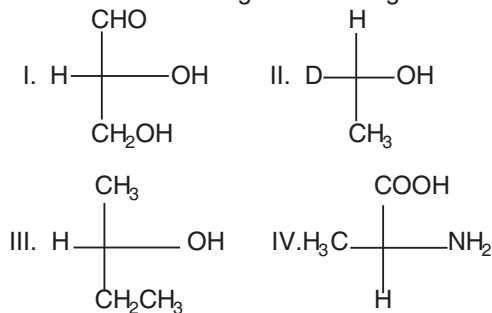
- a. 3 b. 4 c. 2 d. 1

23. Which of the following are diastereomers?

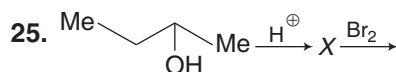


- a. (I) and (III) b. (II) and (IV)
c. (I) and (II) d. None of these

24. The *R*-isomer among the following are

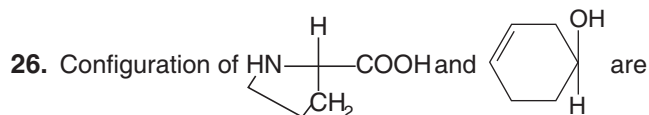


- a. I and II b. II and III
c. III and IV d. I and III



Five compounds with formula $C_4H_8Br_2$. How many structures of *X* are possible?

- a. 2 b. 3
c. 4 d. 5



- a. *R, R* b. *R, S* c. *S, S* d. *S, R*

27. The specific rotation of a pure enantiomer is $+16^\circ$. Its observed rotation if it is isolated from a reaction with 25% racemisation and 75% retention, is

- a. -12°
b. $+12^\circ$
c. $+16^\circ$
d. -16°

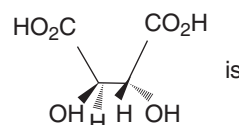
28. Out of the following, the alkene that exhibits optical isomerism is

- a. 3-methyl-2-pentene
b. 4-methyl-1-pentene
c. 3-methyl-1-pentene
d. 2-methyl-2-pentene

29. The alkene that exhibits geometrical isomerism is

- a. propene
b. 2-methyl propene
c. 2-butene
d. 2-methyl-2-butene

30. The absolute configuration of



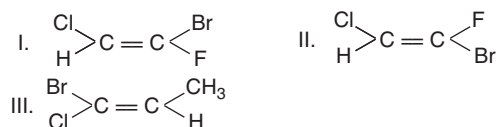
- a. *S, S*
b. *R, R*
c. *R, S*
d. *S, R*

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1. Which among the following is likely to show geometrical isomerism? [2012]

- a. $CH_3CH=NOH$
b. $CH_3CH=CH_2$
c. $CH_2=CH-CH=CCl_2$
d. $CH_3C(Cl)=C(CH_3)_2$

2. Which of the following compound (s) has *Z*-configuration? [2011]

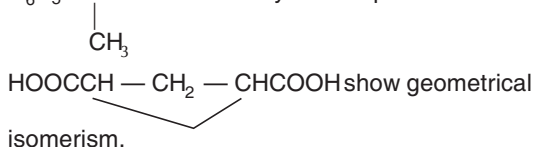


- a. Only I b. Only II c. Only III d. I and III

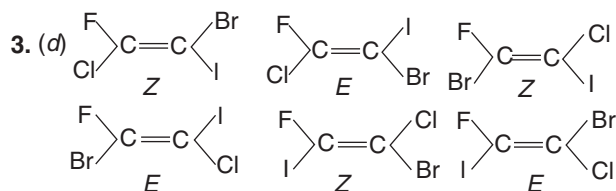
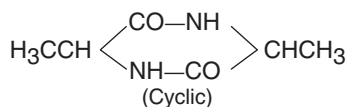
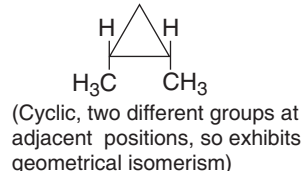
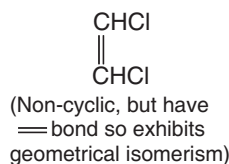
Answer with Solutions

Practice Exercise

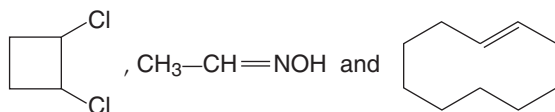
1. (d) All the compounds in which there should be restricted rotation about a bond in the molecule, show geometrical isomerism. Oximes of the type $R-CH=N-OH$, $C_6H_5-C=N-OH$ and cyclic compound like



2. (d) Disubstituted cyclic compounds and disubstituted alkenes show geometrical isomerism.



4. (d) The isomerism which arises due to restricted rotation about a bond in a molecule is known as geometrical isomerism.



All of these form geometrical isomers.

5. (c) In , the double bonds are *trans* and *cis*.

The first and third bonds are identical.

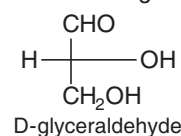
6. (b) When n = even number, then for two identical ends, number of geometrical isomers

$$= 2^{n-1} + 2^{n/2-1} = 2^1 + 2^0 = 3$$

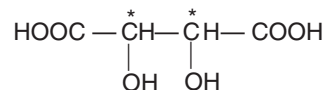
7. (a) The separation of racemic mixture back into *d* and *l* isomers is known as resolution. It can be done by
- (I) mechanical method.
 - (II) biochemical method using enzymes.
 - (III) chemical method (salt formation).

8. (a) Racemic tartaric acid is optically inactive due to external compensation. Racemic tartaric acid is an equimolar mixture of optically active *d* and *l*-forms.
9. (d) If a substance rotates the plane polarised light in clockwise direction, it is dextrorotatory (+). If it rotates the plane polarised light in anticlockwise direction then it is laevorotatory (-). Thus, + / - form differ in specific rotation.

10. (c) The configuration in the Fischer projection of which —OH group is on right hand side, H-atom is on left hand side, —CHO group is on upper side and CH_2OH is on lower side is known as D-configuration.



11. (b) A carbon atom which is attached to four different group is called an asymmetric carbon atom or chiral centre. $HOOC(CHOH)_2COOH$ has two asymmetric carbon (C) atoms.

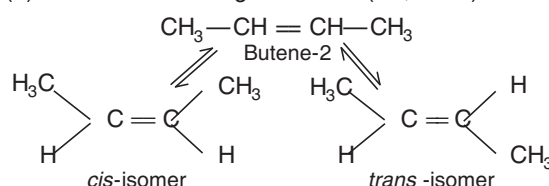


12. (b)
-
- (* is asymmetric C-atom)

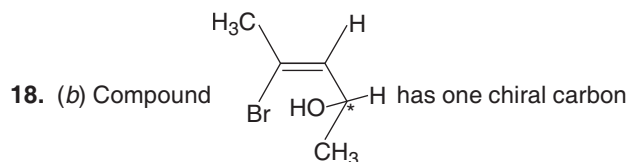
13. (b)
-

The above compound has chiral centre. Hence, it can exhibit optical isomerism while geometrical isomerism is not possible due to the presence of identical groups on double bonded carbon atoms.

14. (d) Compounds which do not show optical activity inspite of the presence of chiral carbon atoms are called *meso*-compounds.
15. (a) If an enantiomerically pure acid is treated with racemic mixture of an alcohol having chiral carbon, the product formed will be optically active mixture.
16. (b) Butene-2 exhibits geometrical (*cis*, *trans*) isomerism.

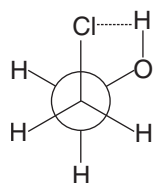


17. (d) We know that by breaking two bonds on the chiral centre, configuration changes.



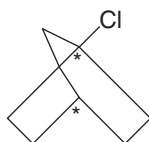
atom and one double bond thus, it has two geometrical (*cis* and *trans*) and two optical isomers.

19. (d) Staggered conformation is most stable due to its minimum energy.
20. (b) Staggered and eclipsed conformers cannot be physically separated because energy difference between them is so small that they most readily interconvert at room temperature.
21. (c)

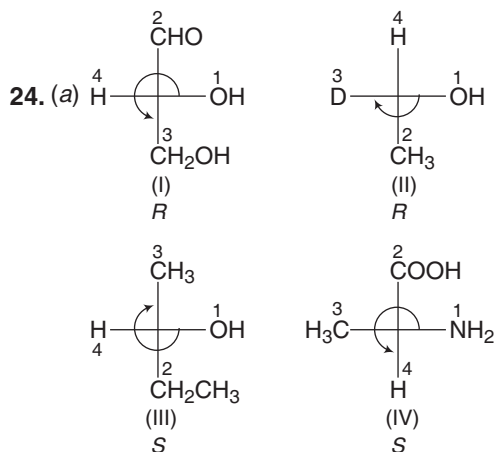


The *gauche* conformation is most stable due to the presence of H-bonding between H-atom of OH and Cl.

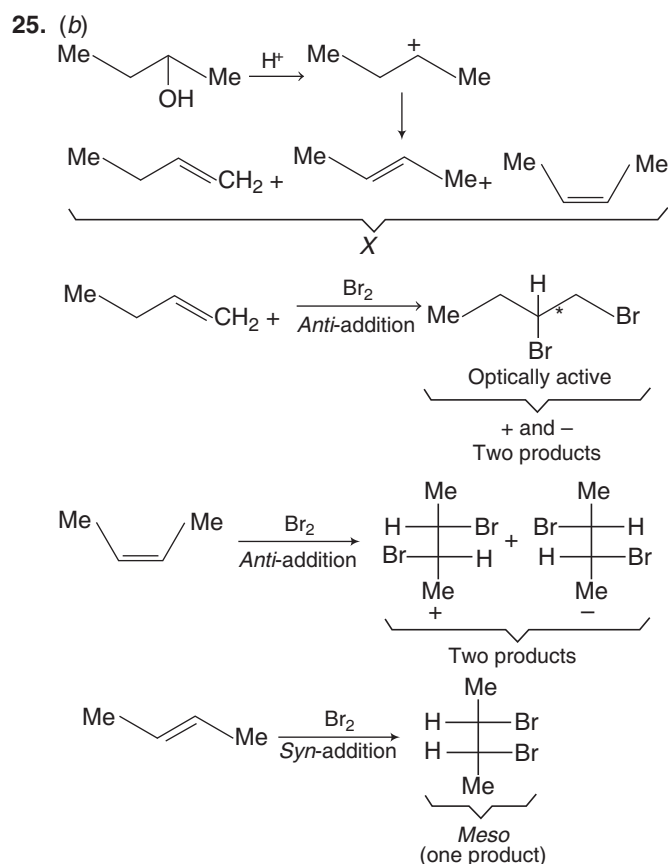
22. (c) Carbon bonded with four different groups is known as chiral carbon atom. In case of given compound, the number of chiral carbon atoms are two.



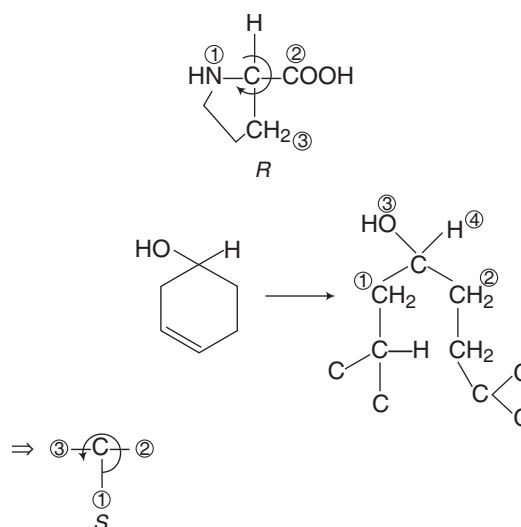
23. (c) Superimposable compounds which are also not the mirror images of each other, are called diastereomers. Thus, I and II are diastereomers.



Note If H is on vertical line, *R* represents clockwise and *S* represents anticlockwise rotation. Opposite is true when H or lower priority group is present on horizontal line.

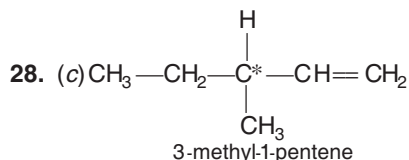


26. (b) Compounds in which the chiral centre is a part of the ring are handed in an analogous fashion.



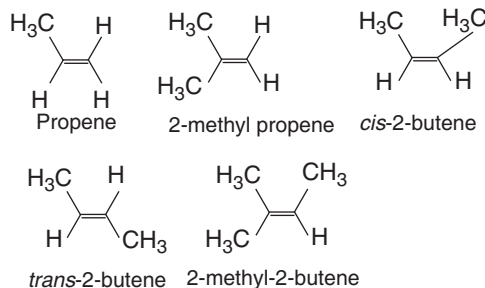
27. (b) % Enantiomeric excess
- $$= \frac{\text{Observed specific rotation}}{\text{Specific rotation of pure enantiomer}} \times 100$$

$$\text{Observed specific rotation} = \frac{3/4}{100} \times (+16^\circ) \times 100 = +12^\circ$$

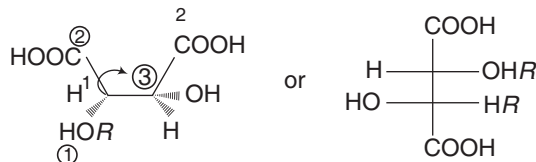


It has one chiral centre.

29. (c)

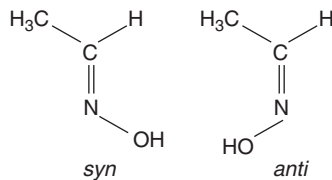


30. (b)

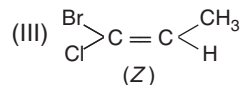
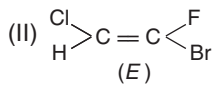
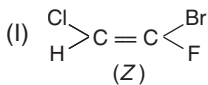


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1. (a) Among the given, only $\text{CH}_3\text{CH}=\text{NOH}$ (oxime) satisfy the conditions essential for exhibiting geometrical isomerism. So, it will exhibit *syn-anti* geometrical isomerism.



2. (d) When the groups with higher priority (i.e. with high atomic number) are present on same side of double bond, then the configuration is *Z* but when present on opposite side of double bond, the configuration is *E*.

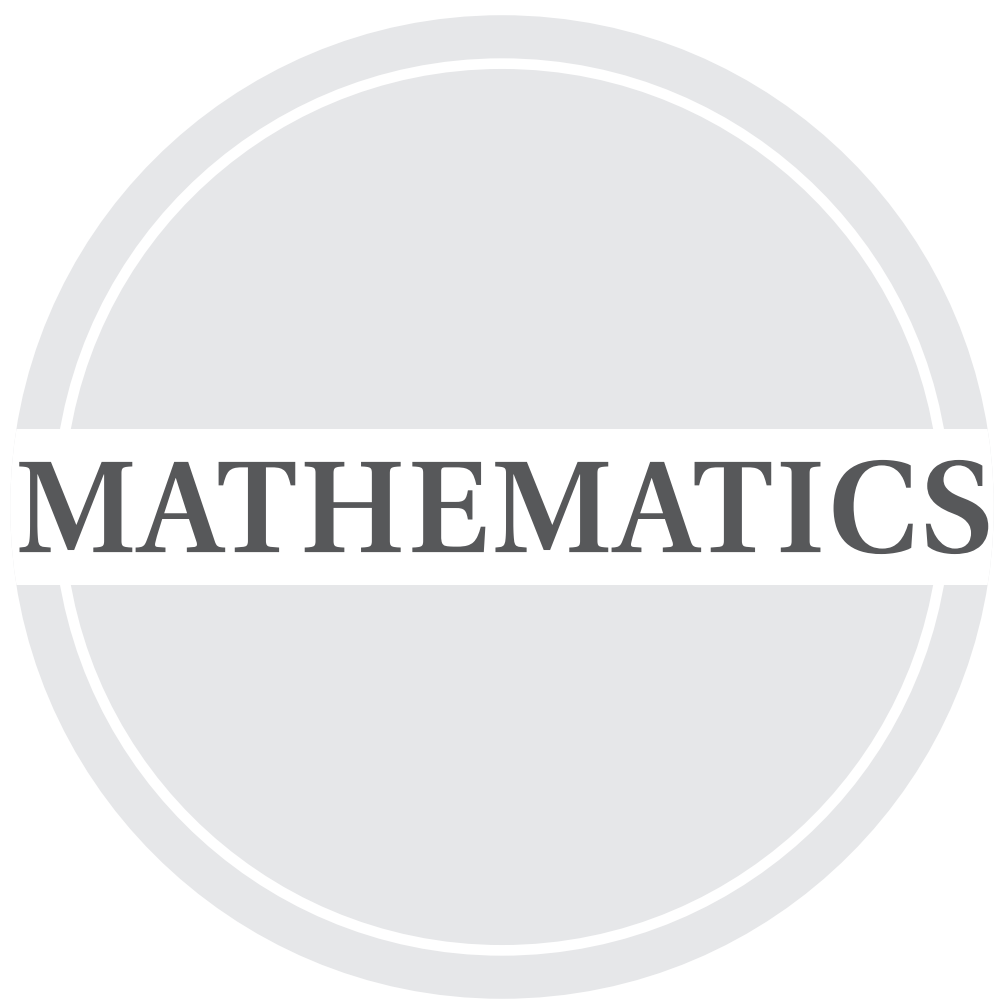


(Priority : $\text{Cl} > \text{H}$ and $\text{Br} > \text{F}$)

(Priority : $\text{Cl} > \text{H}$ and $\text{Br} > \text{F}$)

(Priority : $\text{Br} > \text{Cl}$ and $\text{CH}_3 > \text{H}$)

Hence, compounds (I) and (III) have *Z*-configuration.



1

Complex Numbers

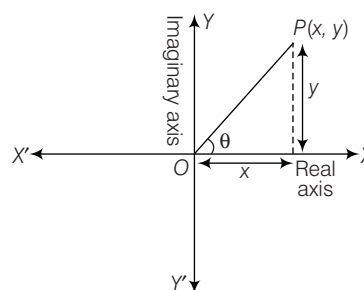
Complex Number and Its Representation

A number in the form of $z = x + iy$, where $x, y \in R, i = \sqrt{-1}$ is called a complex number. The real numbers x and y are respectively called real and imaginary parts of complex number z .

i.e. $x = \text{Re}(z)$ and $y = \text{Im}(z)$

The complex number $z = x + iy$ is represented by a point P , whose coordinates are referred to rectangular axes XOX' and YOY' which are called real and imaginary axes, respectively.

This plane is called argand plane or Gaussian plane. The magnitude of the complex number z is $|z| = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$



A complex number z is said to be purely real or imaginary, if $y = 0$ or $x = 0$, respectively.

NOTE i is neither 0 nor greater than 0 nor less than 0.

Integral Powers of i We know that, $i = \sqrt{-1}, i^2 = -1, i^3 = -i, i^4 = 1$, etc.

In general, $i^{4n} = 1, i^{4n+1} = i, i^{4n+2} = -1, i^{4n+3} = -i, \forall$ any integer n .

The sum of four consecutive powers of i is zero. i.e. $i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0, n \in I$

Algebra of Complex Numbers

If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are two complex numbers, then

(i) Addition of complex numbers is $z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$

Its additive identity is $0 + 0i$.

(ii) Subtraction of complex numbers is $z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$

(iii) Multiplication of complex numbers is $z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$

Its multiplicative identity is $1 + 0i$.

(iv) Division of complex numbers is $\frac{z_1}{z_2} = \frac{(x_1 x_2 + y_1 y_2) + i(x_2 y_1 - x_1 y_2)}{x_2^2 + y_2^2}$

Conjugate of a Complex Number

Complex numbers $z = a + ib$ and $\bar{z} = a - ib$ are called conjugate to each other.

Properties of Conjugates

If z, z_1 and z_2 are complex numbers, then

- (i) $\overline{(\bar{z})} = z$ (ii) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$
- (iii) $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$ (iv) $\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$
- (v) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$
- (vi) $z + \bar{z} = 2\operatorname{Re}(\bar{z}) = 2\operatorname{Re}(z)$
- (vii) $z - \bar{z} = 2i\operatorname{Im}(z)$
- (viii) $z + \bar{z} = 0 \Rightarrow z$ is purely imaginary.
- (ix) $z = \bar{z} \Leftrightarrow z$ is purely real.
- (x) $z_1 \bar{z}_2 + \bar{z}_1 z_2 = 2\operatorname{Re}(\bar{z}_1 z_2) = 2\operatorname{Re}(z_1 \bar{z}_2)$
- (xi) If $z = f(z_1)$, then $\bar{z} = f(\bar{z}_1)$

Modulus of a Complex Number

Let $z = a + ib$, then the modulus of z is the positive real number $\sqrt{a^2 + b^2}$ and is denoted by $|z|$.

i.e. $|z| = \sqrt{a^2 + b^2} = \sqrt{|\operatorname{Re}(z)|^2 + |\operatorname{Im}(z)|^2}$, $|z| \geq 0, \forall z \in \mathbb{C}$

Properties of Moduli

- (i) $|z| \geq 0 \Rightarrow |z| = 0$ iff $z = 0$, i.e. $\operatorname{Re}(z) = \operatorname{Im}(z) = 0$
- (ii) $z\bar{z} = |\bar{z}|^2$
- (iii) $|z_1 z_2| = |z_1| |z_2|$
- (iv) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$
- (v) $|z^n| = |z|^n$
- (vi) $|z_1 + z_2| \leq |z_1| + |z_2|$
- (vii) $|z_1 - z_2| \geq ||z_1| - |z_2||$
- (viii) $||z_1| - |z_2|| \leq |z_1 + z_2| \leq |z_1| + |z_2|$
- (ix) $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\operatorname{Re}(z_1 \bar{z}_2)$
- (x) $|z| = |\bar{z}| = |-z| = |-\bar{z}|$
- (xi) $z_1 \bar{z}_2 + \bar{z}_1 z_2 = 2|z_1||z_2|\cos(\theta_1 - \theta_2)$
where, $\theta_1 = \arg(z_1)$ and $\theta_2 = \arg(z_2)$
- (xii) $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\{|z_1|^2 + |z_2|^2\}$

Argument of a Complex Number

For a complex number $z = x + iy$, the argument or amplitude of z ($z \neq 0$) = $\tan^{-1}\left(\frac{y}{x}\right)$

i.e. solution of the system of equations $\cos \theta = \frac{x}{\sqrt{x^2 + y^2}}$, $\sin \theta = \frac{y}{\sqrt{x^2 + y^2}}$ and is denoted by $\arg(z)$.

The value of θ is found by solving these equations and θ is called the argument or amplitude of z . If $-\pi < \theta \leq \pi$, then θ is called the principal argument of z .

- (i) If $x < 0, y < 0$, then principal $\arg(z)$ lies between $-\pi$ and $-\pi/2$.
- (ii) If $x = 0, y < 0$, then principal $\arg(z)$ is $-\pi/2$.
- (iii) If $x > 0, y < 0$, then principal $\arg(z)$ lies between $-\pi/2$ and 0 .
- (iv) If $x > 0, y = 0$, then principal $\arg(z)$ is 0 .
- (v) If $x > 0, y > 0$, then principal $\arg(z)$ lies between 0 and $\pi/2$.
- (vi) If $x = 0, y > 0$, then principal $\arg(z)$ is $\pi/2$.
- (vii) If $x < 0, y > 0$, then principal $\arg(z)$ is π .

Properties of $\arg(z)$

- (i) $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$
- (ii) $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$
- (iii) $\arg(z)$ is not defined, if $z = 0$
- (iv) $\arg(z^n) = n \arg(z)$ (v) $\arg(z) + \arg(\bar{z}) = 2\pi$
- (vi) If $\arg\left(\frac{z_2}{z_1}\right) = \theta$, then $\arg\left(\frac{z_1}{z_2}\right) = 2k\pi - \theta, k \in \mathbb{I}$

Polar Form of a Complex Number

Expression $r(\cos \theta + i \sin \theta)$ is called the polar form of the complex number $x + iy$.

Here,

$$r = \sqrt{x^2 + y^2}$$

$$\cos \theta = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\Rightarrow \sin \theta = \frac{y}{\sqrt{x^2 + y^2}}$$

Triangle Inequality

In any triangle, sum of any two sides is greater than the third side and difference of any two sides is less than the third side.

- (i) $|z_1 + z_2| \leq |z_1| + |z_2|$ (ii) $|z_1 + z_2| \geq ||z_1| - |z_2||$
- (iii) $|z_1 - z_2| \leq |z_1| + |z_2|$ (iv) $|z_1 - z_2| \geq ||z_1| - |z_2||$

NOTE Complex numbers do not possess the inequality, i.e. $3 + 2i > 1 + 2i$ does not make any sense.

De-Moivre's Theorem

If n is an integer, positive, negative or a rational number, then

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$$

Other Forms of De-Moivre's Theorem

- (i) $(\cos \theta + i \sin \theta)^{-n} = \cos n\theta - i \sin n\theta$
- (ii) $(\cos \theta - i \sin \theta)^n = \cos n\theta - i \sin n\theta$
- (iii) $(\cos \theta - i \sin \theta)^{-n} = \cos n\theta + i \sin n\theta$

Square Roots of a Complex Number

If $z = a + ib$ is a complex number, such that

$\sqrt{a + ib} = x + iy$, where x, y are real numbers, then $a + ib = (x + iy)^2 \Rightarrow a + ib = (x^2 - y^2) + i(2xy)$ and square root of z by equating real and imaginary part can be given

$$\begin{aligned}\sqrt{z} &= \pm \left[\sqrt{\frac{1}{2}(\sqrt{a^2 + b^2} + a)} + i \sqrt{\frac{1}{2}(\sqrt{a^2 + b^2} - a)} \right], b > 0 \\ &= \pm \left[\sqrt{\frac{1}{2}(\sqrt{a^2 + b^2} + a)} - i \sqrt{\frac{1}{2}(\sqrt{a^2 + b^2} - a)} \right], b < 0\end{aligned}$$

n th Roots of Unity

$$(\cos \theta + i \sin \theta)^{1/n} = \cos \left(\frac{2k\pi}{n} \right) + i \sin \left(\frac{2k\pi}{n} \right) = e^{\frac{2\pi k i}{n}}$$

where, $k = 0, 1, 2, \dots, (n-1)$

Thus, n th roots of unity are $1, \alpha, \alpha^2, \alpha^3, \dots, \alpha^{n-1}$

where, $\alpha = e^{i 2\pi/n} = \cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n}$

Cube Roots of Unity

Cube roots of unity are $1, \omega, \omega^2$, where

$$\omega = \frac{-1 + \sqrt{3}i}{2}; \omega^2 = \frac{-1 - \sqrt{3}i}{2} \text{ and } \omega^3 = 1$$

Some Important Results

- The sum of the cube root of unity i.e. $1 + \omega + \omega^2 = 0$ and their product $= 1 \cdot \omega \cdot \omega^2 = \omega^3 = 1$
In general, $(\omega)^{3n} = 1, (\omega)^{3n+1} = \omega$ and $(\omega)^{3n+2} = \omega^2$, where n is an integer.
- Each complex root of unity is the square of the other.
- The three cube roots of unity are the vertices of an equilateral triangle inscribed in the circle $|z| = 1$.
- The four fourth roots of unity are $1, -1, i, -i$

Properties of Roots of Unity

- (i) Sum of the roots of unity in square roots, cube roots, 4th roots or n th roots is zero.
- (ii) They are in GP of common ratio $d = e^{2\pi i/r}$, where $r = 2, 3, 4, \dots, (n-1)$.
- (iii) Sum of their p th powers is zero.

$$\text{i.e. } 1 + \alpha^p + \alpha^{2p} + \dots + \alpha^{(n-1)p} = \frac{1 - (\alpha^n)^p}{1 - \alpha^p} = 0$$

$$\therefore \alpha^n = 1, p \neq kn$$

If $p = kn$, then sum of their p th powers is n .

$$\text{i.e. if } p = kn, \text{ then } \alpha^p = \alpha^{kn} = (\alpha^n)^k = 1.$$

$\therefore$ Each root is 1.

$$\therefore \text{Sum} = 1 + 1 + 1 + \dots + 1 = n$$

Geometrical Applications of Complex Numbers

1. If z_1 and z_2 are two complex numbers, then

- (i) $|z_1 - z_2|$ is the distance between the points affixes z_1 and z_2 .
- (ii) $\frac{mz_2 + nz_1}{m+n}$ is the affix of the point dividing the line joining the points with affixes z_1 and z_2 in the ratio $m : n$ internally, where $m \neq n$.
- (iii) $\frac{mz_2 - nz_1}{m-n}$ is the affix of the point dividing the line joining the points with affixes z_1 and z_2 in the ratio $m : n$ externally, where $m \neq n$.
- (iv) If the affixes of the vertices of the triangle are z_1, z_2 and z_3 , then the affix of its centroid is $\frac{z_1 + z_2 + z_3}{3}$.

2. Three points with affixes z_1, z_2, z_3 are collinear, if

$$\begin{vmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{vmatrix} = 0$$

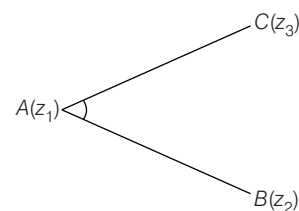
3. (i) $|z - z_1| = r$ represents the circle with centre z_1 and radius r .

(ii) $|z - z_1| < r$ represents the interior of the circle with centre z_1 and radius r .

4. $\left| \frac{z - z_1}{z - z_2} \right| = k$ represents a circle, if $k \neq 1$ and if $k = 1$, then it represent a straight line which is the perpendicular bisector of the line segment joining points with affixes z_1 and z_2 .

5. $(z - z_1)(\bar{z} - \bar{z}_2) + (\bar{z} - \bar{z}_1)(z - z_2) = 0$ represents the circle with line joining points with affixes z_1 and z_2 as a diameter.

6. If z_1, z_2 and z_3 are the affixes of the points A, B, C respectively, then the angle between AB and AC is given by $\arg \left(\frac{z_3 - z_1}{z_2 - z_1} \right)$.



Practice Exercise

- The multiplicative inverse of $(6 + 5i)^2$ is
 a. $\frac{11}{61} - \frac{60}{61}i$ b. $\frac{11}{61} + \frac{60}{61}i$ c. $\frac{9}{61} - \frac{60}{61}i$ d. None
- If $8iz^3 + 12z^2 - 18z + 27i = 0$, then the value of $|z|$ is
 a. $\frac{3}{2}$ b. $\frac{2}{3}$ c. 1 d. $\frac{3}{4}$
- If z_1 and z_2 are two complex numbers such that $|z_1| = |z_2|$ and $\arg(z_1) + \arg(z_2) = \pi$, then z_1 is equal to
 a. $2\bar{z}_2$ b. $\bar{z}_2$
 c. $-\bar{z}_2$ d. None of these
- If $|z - 1| = 1$, then $\arg(z)$ is equal to
 a. $\frac{1}{2} \arg(z)$ b. $\frac{1}{3} \arg(z + 1)$
 c. $\frac{1}{2} \arg(z - 1)$ d. None of these
- If $z_1 = 8 + 4i$, $z_2 = 6 + 4i$ and $\arg\left(\frac{z - z_1}{z - z_2}\right) = \frac{\pi}{4}$, then z satisfies
 a. $|z - 7 - 4i| = 1$ b. $|z - 7 - 5i| = \sqrt{2}$
 c. $|z - 4i| = 8$ d. $|z - 7i| = \sqrt{18}$
- If z_1, z_2 and z_3 are three complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$, then $|z_1 + z_2 + z_3|$ is
 a. equal to 1 b. less than 1
 c. greater than 3 d. equal to 3
- If 1, ω and ω^2 are the three cube roots of unity and α, β and γ are the cube roots of p, q (< 0), then for any x, y and z , the expression $\left(\frac{x\alpha + y\beta + z\gamma}{x\beta + y\gamma + z\alpha}\right)$ is equal to
 a. 1 b. ω
 c. ω^2 d. None of these
- If $\begin{vmatrix} x+1 & \omega & \omega^2 \\ x+\omega & \omega^2 & 1 \\ x+\omega^2 & 1 & \omega \end{vmatrix} = 3$ is an equation of x , where 1, ω and ω^2 are the complex cube roots of unity. Then, the value of x is
 a. 0 b. 1
 c. -1 d. None of these
- If a, b and c are integers not all equal and ω is a cube root of unity (where $\omega \neq 1$), then minimum value of $|a + b\omega + c\omega^2|^2$ is equal to
 a. 0 b. 1 c. $\frac{\sqrt{3}}{2}$ d. $\frac{1}{2}$
- If $z = 1 + \cos \frac{\pi}{5} + i \sin \frac{\pi}{5}$, then $\sin(\arg z)$ is equal to
 a. $\frac{\sqrt{10 - 2\sqrt{5}}}{4}$ b. $\frac{\sqrt{5} - 1}{4}$ c. $\frac{\sqrt{5} + 1}{4}$ d. $\frac{\sqrt{2} - 1}{4}$
- If $\operatorname{Re}\left(\frac{1}{z}\right) = 3$, then z lies on
 a. circle with centre on Y -axis
 b. circle with centre on X -axis not passing through origin
 c. circle with centre on X -axis passing through origin
 d. None of the above
- If a complex number z lies in the interior or on the boundary of a circle of radius 3 and centre at $(-4, 0)$, then the greatest and least values of $|z + 1|$ are
 a. 5, 0 b. 6, 1
 c. 6, 0 d. None of these
- If $\arg(\bar{z}_1) = \arg(z_2)$, then
 a. $z_2 = kz_1^{-1}$ ($k > 0$) b. $z_2 = kz_1$ ($k > 0$)
 c. $|z_2| = |\bar{z}_1|$ d. None of these
- If z_1 and z_2 are two complex numbers such that $\frac{z_1}{z_2} + \frac{z_2}{z_1} = 1$, then
 a. z_1, z_2 are collinear
 b. z_1, z_2 and the origin form a right angled triangle
 c. z_1, z_2 and the origin form an equilateral triangle
 d. None of the above
- If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers, then $\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right)$ is equal to
 a. 0 b. $\pi/2$ c. $3\pi/2$ d. π
- If $a = \cos \alpha + i \sin \alpha, b = \cos \beta + i \sin \beta, c = \cos \gamma + i \sin \gamma$ and $\frac{b}{c} + \frac{c}{a} + \frac{a}{b} = 1$, then $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)$ is equal to
 a. $3/2$ b. $-3/2$ c. 0 d. 1
- The least positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = \frac{2}{\pi}(\sec^{-1} \frac{1}{x} + \sin^{-1} x)$, where $x \neq 0; -1 \leq x \leq 1$, is
 a. 2 b. 4
 c. 6 d. 8
- If $z = \sqrt{(5 + 12i)} + \sqrt{(12i - 5)}$, then the principal value of $\arg(z)$ can be
 a. $\frac{\pi}{4}$ b. $-\frac{\pi}{4}$ c. $-\frac{3\pi}{4}$ d. All of these

19. If $\arg\left(\frac{2-z}{2+z}\right) = \frac{\pi}{6}$, then locus of z is
 a. straight line b. circle
 c. parabola d. None of these
20. If $z_r = \text{cis } \frac{\pi}{2^r}$, then $z_1 z_2 \dots$ is equal to
 a. 1 b. -1 c. -2 d. $-\frac{1}{2}$
21. If z_1, z_2 and z_3 represent the vertices of an equilateral triangle such that $|z_1| = |z_2| = |z_3|$, then
 a. $z_1 + z_2 = z_3$ b. $z_1 + z_2 + z_3 = 0$
 c. $z_1 z_2 = \frac{1}{z_3}$ d. $z_1 - z_2 = z_3 - z_2$
22. For any two complex numbers z_1 and z_2 , we have $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$, then
 a. $\text{Re}\left(\frac{z_1}{z_2}\right) = 0$ b. $\text{Im}\left(\frac{z_1}{z_2}\right) = 0$
 c. $\text{Re}\left(\frac{z_1}{z_2}\right) \neq 0$ d. None of these
23. If α, β are two different complex numbers such that $|\alpha| = 1, |\beta| = 1$, then the expression $\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|$ equals
 a. $\frac{1}{2}$ b. 1 c. 2 d. None
24. The value of $\sum_{n=1}^{13} (i^n + i^{n+1})$, where $i = \sqrt{-1}$, equals
 a. i b. $i - 1$ c. $-i$ d. 0
25. The continued product of the four values of $\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)^{3/4}$ is
 a. -1 b. 1 c. $\frac{3}{2}$ d. $-\frac{1}{2}$
26. If $x_n = \cos\left(\frac{\pi}{2^n}\right) + i \sin\left(\frac{\pi}{2^n}\right), n \in N$, then $x_1 x_2 x_3 \dots$ is equal to
 a. 1 b. -1 c. zero d. None
27. If $\alpha + i\beta = \cot^{-1}(z)$, where $z = x + iy$ and α is a constant, then the locus of z is
 a. $x^2 + y^2 - x \cot 2\alpha - 1 = 0$
 b. $x^2 + y^2 - 2x \cot \alpha - 1 = 0$
 c. $x^2 + y^2 - 2x \cot 2\alpha + 1 = 0$
 d. $x^2 + y^2 - 2x \cot 2\alpha - 1 = 0$
28. If $(a_1 + ib_1)(a_2 + ib_2) \dots (a_n + ib_n) = A + iB$, then $\sum_{i=1}^n \tan^{-1}\left(\frac{b_i}{a_i}\right)$ is equal to
 a. $\frac{B}{A}$ b. $\tan\left(\frac{B}{A}\right)$ c. $\tan^{-1}\left(\frac{B}{A}\right)$ d. $\tan^{-1}\left(\frac{A}{B}\right)$

29. The maximum value of $|z|$, when z satisfy the condition $|z + 3/z| = 3$, is

a. $\frac{2 - \sqrt{21}}{2}$ b. $\frac{3 - \sqrt{20}}{2}$ c. $\frac{3 + \sqrt{21}}{2}$ d. $\frac{3 + \sqrt{20}}{2}$

30. If $i = \sqrt{-1}$, then $4 + 5\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{335}$ is equal to

a. $1 - i\sqrt{3}$ b. $-1 + i\sqrt{3}$ c. $i\sqrt{3}$ d. $-i\sqrt{3}$

Directions (Q. Nos. 31 and 32) Let $z = a + ib = (a, b)$ be any complex number, $\forall a, b \in R$ and $i = \sqrt{-1}$. If $(a, b) \neq (0, 0)$, then $\arg(z) = \tan^{-1}\left(\frac{b}{a}\right)$, where $\arg(z) \leq \pi$

and $\arg(\bar{z}) + \arg(-z) = \begin{cases} \pi, & \text{if } \arg(z) < 0 \\ -\pi, & \text{if } \arg(z) > 0 \end{cases}$

31. If $\arg(z) > 0$, then $\arg(-z) - \arg(z) = \lambda_1$ and if $\arg(z) < 0$, then $\arg(z) - \arg(-z) = \lambda_2$, where
 a. $\lambda_1 + \lambda_2 = 0$ b. $\lambda_1 - \lambda_2 = 0$
 c. $3\lambda_1 - 2\lambda_2 = 0$ d. $2\lambda_1 - 3\lambda_2 = 0$

32. The value of $\sqrt{\{\arg(z) + \arg(\bar{z}) - 2\pi\}\{\arg(-z) + \arg(\bar{z})\}}$, $\forall z = x + iy$, where $i = \sqrt{-1}; x, y > 0$, is
 a. π b. $-\pi$ c. 0 d. not defined

33. The complex numbers z_1, z_2 and z_3 satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$, are the vertices of a triangle which is

a. of area zero b. right angled triangle
 c. equilateral d. obtuse angled triangle

34. Let z_1 and z_2 be n th roots of unity which subtend a right angle at the origin, then n must be of the form
 a. $4k + 1$ b. $4k + 2$ c. $4k + 3$ d. $4k$

35. The value of $\begin{vmatrix} 1+\omega & \omega^2 & 1+\omega^2 \\ -\omega & -(1+\omega^2) & 1+\omega \\ -1 & -(1+\omega^2) & 1+\omega \end{vmatrix}$, where ω is cube

root of unity, is equal to

a. 2ω b. $3\omega^2$ c. $-3\omega^2$ d. 3ω

36. The square roots of $7 + 24i$ are

a. $\pm(4 + 3i)$ b. $\pm(4 - 3i)$ c. $\pm(3 + 4i)$ d. $\pm(3 - 4i)$

37. If $|z_1| = |z_2|$ and $\arg\left(\frac{z_1}{z_2}\right) = \pi$, then $z_1 + z_2$ is equal to

a. 0 b. purely imaginary
 c. purely real d. None of these

38. If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$, then

a. $a = 2, b = -1$ b. $a = 1, b = 0$
 c. $a = 0, b = 1$ d. $a = -1, b = 2$

39. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then

- a. $x = 3, y = 1$
 b. $x = 1, y = 3$
 c. $x = 0, y = 3$
 d. $x = 0, y = 0$

40. Let z_1 and z_2 be two complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{(z_1 + z_2)}{(z_1 - z_2)}$ may be

- a. purely imaginary
 b. real and positive
 c. real and negative
 d. None of the above

BITSAT Archives

1. The complex number $z = x + iy$ which satisfies the equation $\left| \frac{z - 3i}{z + 3i} \right| = 1$, lies on [2014]

- a. the X-axis
 b. the straight line $y = 3$
 c. a circle passing through origin
 d. None of the above

2. The value of $\sum_{k=1}^6 \left(\frac{\sin 2\pi k}{7} - \frac{i \cos 2\pi k}{7} \right)$ is [2013]

- a. -1 b. 0 c. -i d. i

3. If $(x + iy)^{1/3} = 2 + 3i$, then $3x + 2y$ is equal to [2013]

- a. -20 b. -60 c. -120 d. 60

4. Area of the triangle in the argand diagram formed by the complex numbers $z, iz, z + iz$, where $z = x + iy$, is [2012]

- a. $|z|$ b. $|z|^2$
 c. $2|z|^2$ d. $\frac{1}{2}|z|^2$

5. If $z = \cos \theta + i \sin \theta$, then the value of $z^n + \frac{1}{z^n}$ will be [2011]

- a. $\sin 2n\theta$ b. $2 \sin n\theta$ c. $2 \cos n\theta$ d. $\cos 2n\theta$

6. Find the least value of n for which $\left(\frac{1+i}{1-i} \right)^n = 1$. [2010]

- a. 4 b. 3 c. -4 d. 1

7. If ω is a cube root of unity, then $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$ is equal

to [2010, 06]

- a. 1 b. ω c. ω^2 d. 0

8. If $a = \cos \theta + i \sin \theta$, then $\frac{1+a}{1-a}$ is equal to [2009]

- a. $\cot \frac{\theta}{2}$ b. $\cot \theta$
 c. $i \cot \frac{\theta}{2}$ d. $i \tan \frac{\theta}{2}$

9. If ω is a complex cube root of unity, then $\sin \left\{ (\omega^{10} + \omega^{23}) \pi - \frac{\pi}{4} \right\}$ is equal to [2008]

- a. $\frac{1}{\sqrt{2}}$ b. $\frac{1}{2}$
 c. 1 d. $\frac{\sqrt{3}}{2}$

10. If $(\cos \theta + i \sin \theta)(\cos 2\theta + i \sin 2\theta) \dots (\cos n\theta + i \sin n\theta) = 1$, then the value of θ is [2007]

- a. $\frac{2m\pi}{n(n+1)}$ b. $4m\pi$
 c. $\frac{4m\pi}{n(n+1)}$ d. $\frac{m\pi}{n(n+1)}$

11. For all complex numbers z_1 and z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is [2007]

- a. 4 b. 3
 c. 1 d. 2

12. The amplitude of $\frac{1+i\sqrt{3}}{\sqrt{3}+i}$ is [2006]

- a. $\frac{\pi}{6}$ b. $\frac{\pi}{3}$
 c. 0 d. None of these

13. Let $x = \alpha + \beta, y = \alpha\omega + \beta\omega^2, z = \alpha\omega^2 + \beta\omega$, ω being an imaginary cube root of unity. Product of xyz is [2005]

- a. $\alpha^2 + \beta^2$ b. $\alpha^2 - \beta^2$
 c. $\alpha^3 + \beta^3$ d. $\alpha^3 - \beta^3$

Answer with Solutions

Practice Exercise

1. (a) Let $z = (6 + 5i)^2 = 36 + 2 \times 6 \times 5i + 25i^2 = 11 + 60i$

Then, $\bar{z} = 11 - 60i$

and $|z| = \sqrt{(11)^2 + (60)^2} = \sqrt{121 + 3600} = \sqrt{3721} = 61$

$\therefore$ Multiplicative inverse of $z = \frac{\bar{z}}{|z|^2}$

$$= \frac{11 - 60i}{(61)^2} = \frac{11}{(61)^2} - \frac{60}{(61)^2}i$$

2. (a) Given, $8iz^3 + 12z^2 - 18z + 27i = 0$

$\Rightarrow 4z^2(2iz + 3) + 9i(2iz + 3) = 0$

$\Rightarrow (2iz + 3)(4z^2 + 9i) = 0$

$\Rightarrow 2iz + 3 = 0$ or $4z^2 + 9i = 0 \Rightarrow |z| = \frac{3}{2}$

3. (c) Let $z_1 = r(\cos \theta_1 + i \sin \theta_1)$

and $z_2 = r(\cos \theta_2 + i \sin \theta_2)$

Since $|z_2| = |z_1|$

Also, $\arg(z_1) + \arg(z_2) = \pi$

$\therefore \arg(z_2) = \pi - \arg(z_1) \Rightarrow \arg(z_2) = \pi - \theta_1$

$\therefore z_2 = r_1 \{ \cos(\pi - \theta_1) + i \sin(\pi - \theta_1) \}$
 $= r_1 (-\cos \theta_1 + i \sin \theta_1)$
 $= -r_1 (\cos \theta_1 - i \sin \theta_1) = -\bar{z}_1$

$\Rightarrow z_1 = -\bar{z}_2$

4. (c) Given, $|z - 1| = 1$

$\Rightarrow z - 1 = e^{i\theta}$, where $\arg(z - 1) = \theta$... (i)

$\Rightarrow z = e^{i\theta} + 1 \Rightarrow z = 1 + \cos \theta + i \sin \theta$
 $= 2 \cos^2 \frac{\theta}{2} + 2i \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2} \quad [\because e^{i\theta} = \cos \theta + i \sin \theta]$

$\Rightarrow \arg(z) = \frac{\theta}{2} = \frac{1}{2} \arg(z - 1)$ [from Eq. (i)]

5. (b) Hint $\frac{z - z_1}{z - z_2} = \frac{(x - 8) + i(y - 4)}{(x - 6) + i(y - 4)}$

$\Rightarrow \tan^{-1} \left(\frac{y - 4}{x - 8} \right) - \tan^{-1} \left(\frac{y - 4}{x - 6} \right) = \frac{\pi}{4}$

$\Rightarrow \frac{(y - 4) \frac{2}{(x - 8)(x - 6)}}{(x - 8)(x - 6) + (y - 4)^2} = \tan \frac{\pi}{4} = 1$

$\Rightarrow (x - 7)^2 + (y - 5)^2 = (\sqrt{2})^2$

$\Rightarrow |z - 7 - 5i| = \sqrt{2}$

6. (a) $|z_1| = |z_2| = |z_3| = 1$ [given]

Now, $|z_1| = 1 \Rightarrow |z_1|^2 = 1$

$\Rightarrow z_1 \bar{z}_1 = 1$

Similarly, $z_2 \bar{z}_2 = 1, z_3 \bar{z}_3 = 1$

Now, $\left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right| = 1$

$\Rightarrow \left| \frac{\bar{z}_1}{z_1 \bar{z}_1} + \frac{\bar{z}_2}{z_2 \bar{z}_2} + \frac{\bar{z}_3}{z_3 \bar{z}_3} \right| = 1$

and $\left| \frac{\bar{z}_1}{z_1 + z_2 + z_3} \right| = 1$

$\Rightarrow |z_1 + z_2 + z_3| = 1 \Rightarrow |z_1 + z_2 + z_3| = 1$

7. (c) $\because p < 0$, take $p = -q^3$ ($q > 0$)

$\therefore p^{1/3} = q(-1)^{1/3} = -q, -q\omega, -q\omega^2$

Thus, $\alpha = -q, \beta = -q\omega, \gamma = -q\omega^2$

$\therefore$ Given expression $= \frac{x + y\omega + z\omega^2}{x\omega + y\omega^2 + z} = \omega^2$

8. (d) $\begin{vmatrix} x+1 & \omega & \omega^2 \\ x+\omega & \omega^2 & 1 \\ x+\omega^2 & 1 & \omega \end{vmatrix} = 3$

LHS $= \begin{vmatrix} x+1+\omega+\omega^2 & \omega & \omega^2 \\ x+\omega+\omega^2+1 & \omega^2 & 1 \\ x+\omega^2+1+\omega & 1 & \omega \end{vmatrix}$ [applying $C_1 \rightarrow C_1 + C_2 + C_3$]

$= \begin{vmatrix} x+0 & \omega & \omega^2 \\ x+0 & \omega^2 & 1 \\ x+0 & 1 & \omega \end{vmatrix} = x \begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & 1 \\ 1 & 1 & \omega \end{vmatrix}$ [$\because 1 + \omega + \omega^2 = 0$]

$= x \begin{vmatrix} 3 & 1+\omega+\omega^2 & 1+\omega+\omega^2 \\ 1 & \omega^2 & 1 \\ 1 & 1 & \omega \end{vmatrix}$

[applying $R_1 \rightarrow R_1 + R_2 + R_3$]

$= x \begin{vmatrix} 3 & 0 & 0 \\ 1 & \omega^2 & 1 \\ 1 & 1 & \omega \end{vmatrix} = 3x(\omega^3 - 1)$

$\Rightarrow 3x(\omega^3 - 1) = 3 \Rightarrow x(\omega^3 - 1) = 1 \Rightarrow x \times 0 = 1$ [$\because \omega^3 = 1$]

$\therefore x = \frac{1}{0}$ [not defined]

9. (b) $|a + b\omega + c\omega^2|^2 = (a + b\omega + c\omega^2)(a + b\bar{\omega} + c\bar{\omega}^2)$

$= (a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$ [$\because \bar{\omega} = \omega^2$ and $\bar{\omega}^2 = \omega$]

$= a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2} [(a - b)^2 + (b - c)^2 + (c - a)^2]$

So, it has minimum value 1 for $a = b = 1$ and $c = 2$.

10. (b) If $z = 1 + \cos \theta + i \sin \theta$, then $\arg(z) = \frac{\theta}{2}$

$\therefore \arg(z) = \frac{\pi}{2} = \frac{\pi}{10}$

$\Rightarrow \sin(\arg z) = \sin\left(\frac{\pi}{10}\right) = \sin 18^\circ = \frac{\sqrt{5} - 1}{4}$

$$11. (c) \text{ Given, } \operatorname{Re}\left(\frac{1}{z}\right) = 3 \Rightarrow \operatorname{Re}\left(\frac{\bar{z}}{|z|^2}\right) = 3 \quad \left[\because \frac{1}{z} = \frac{\bar{z}}{|z|^2} \right]$$

$$\Rightarrow \frac{x}{x^2 + y^2} = 3 \Rightarrow 3x^2 + 3y^2 - x = 0$$

So, it is a circle whose centre is on X-axis and passes through the origin.

$$12. (c) \text{ Given, } |z + 4| \leq 3$$

$$\text{Now, } |z + 1| = |z + 4 - 3| \leq |z + 4| + |3| \\ = |z + 4| + 3 \leq 3 + 3 = 6$$

Hence, the greatest value of $|z + 1| = 6$

Since, least value of the modulus of a complex number is 0.

$$\therefore |z + 1| = 0 \Rightarrow z = -1$$

$$\text{Now, } |z + 4| = |-1 + 4| = 3$$

So, $|z + 4| \leq 3$ is satisfied by $z = -1$.

$$\therefore \text{Least value of } |z + 1| = 0$$

$$13. (a) \text{ Here, } \bar{z}_1 = \frac{z_1 \bar{z}_1}{z_1} = |z_1|^2 z_1^{-1}$$

$$\Rightarrow \arg(z_1^{-1}) = \arg(\bar{z}_1) = \arg(z_2) \Rightarrow z_2 = kz_1^{-1} (k > 0)$$

$$14. (c) \text{ Given, } \frac{z_1}{z_2} + \frac{z_2}{z_1} = 1 \Rightarrow z_1^2 + z_2^2 = z_1 z_2$$

$$\Rightarrow z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_1 z_3 + z_2 z_3, \text{ where } z_3 = 0$$

So, z_1, z_2 and the origin form an equilateral triangle.

$$15. (a) \text{ Since, } z_2 = \bar{z}_1 \text{ and } z_4 = \bar{z}_3$$

$$\therefore z_1 z_2 = z_1 \bar{z}_1 = |z_1|^2 \text{ and } z_3 z_4 = z_3 \bar{z}_3 = |z_3|^2$$

$$\text{Now, } \arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right) = \arg\left(\frac{z_1 z_2}{z_4 z_3}\right) = \arg\left(\frac{|z_1|^2}{|z_3|^2}\right) \\ = \arg\left(\left(\frac{z_1}{z_3}\right)^2\right) = 0 \quad \left[\because \left|\frac{z_1}{z_3}\right| \text{ is a real number} \right]$$

$$16. (d) \text{ Here, } \frac{a}{b} = \cos(\alpha - \beta) + i \sin(\alpha - \beta),$$

$$\frac{b}{c} = \cos(\beta - \gamma) + i \sin(\beta - \gamma) \text{ and } \frac{c}{a} = \cos(\gamma - \alpha) + i \sin(\gamma - \alpha)$$

$$\therefore \frac{a}{b} + \frac{b}{c} + \frac{c}{a} = 1$$

$$\therefore \cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha) + i[\sin(\alpha - \beta) + \sin(\beta - \gamma) + \sin(\gamma - \alpha)] = 1 + 0i$$

On equating real parts, we get

$$\cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha) = 1$$

$$17. (b) \text{ For } -1 \leq x \leq 1 \text{ and } x \neq 0, \sec^{-1}\left(\frac{1}{x}\right) = \cos^{-1} x$$

$$\therefore \sec^{-1}\left(\frac{1}{x}\right) + \sin^{-1} x = \cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}$$

$$\therefore \left(\frac{1+i}{1-i}\right)^n = \frac{2}{\pi} \times \frac{\pi}{2} = 1$$

$$\Rightarrow \left[\frac{(1+i)^2}{2}\right]^n = 1 \Rightarrow i^n = 1$$

So, the least positive integral value of n is 4.

$$18. (d) \text{ Here, } \sqrt{(5+12i)} = \sqrt{5+2(3)(2i)} = \sqrt{(3+2i)^2} = \pm(3+2i) \\ \text{and } \sqrt{(12i-5)} = \sqrt{-5+2(3)(2i)} \\ = \sqrt{(2+3i)^2} = \pm(2+3i)$$

$$\therefore z = \sqrt{(5+12i)} + \sqrt{(12i-5)}$$

$$= \pm(3+2i) \pm (2+3i)$$

$$\Rightarrow z = 5+5i, -1+i, -5-5i, 1-i$$

Hence, the principal values of $\arg(z)$ are $\frac{\pi}{4}, \frac{3\pi}{4}, -\frac{3\pi}{4}, -\frac{\pi}{4}$.

$$19. (b) \text{ Hint } \arg(2-z) - \arg(2+z) = \frac{\pi}{6}$$

$$\Rightarrow \arg[(2-x) - iy] - \arg[(2+x) + iy] = \frac{\pi}{6}$$

$$\Rightarrow \tan^{-1}\left(\frac{-y}{2-x}\right) - \tan^{-1}\left(\frac{y}{2+x}\right) = \frac{\pi}{6}$$

$$\Rightarrow \frac{4y}{x^2 - 4 + y^2} = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

20. (b) The argument of a product is the sum of the arguments of the factors.

$$\therefore \arg(z_1 \cdot z_2 \dots) = \frac{\pi}{2} + \frac{\pi}{2^2} + \frac{\pi}{2^3} + \dots$$

$$= \frac{\frac{\pi}{2}}{1 - \frac{1}{2}} = \pi \quad \left[\because S_{\infty} = \frac{a}{1-r} \right]$$

$$\therefore z_1 z_2 z_3 \dots = \operatorname{cis} \pi = -1$$

$$21. (b) \text{ Since, } |z_1| = |z_2| = |z_3|$$

$\therefore O$ is the circumcentre of an equilateral $\triangle ABC$.

$$\therefore \frac{x_1 + x_2 + x_3}{3} = 0 = \frac{y_1 + y_2 + y_3}{3}$$

$$\text{where, } z_1 = x_1 + iy_1$$

$$\Rightarrow \frac{x_1 + x_2 + x_3}{3} + i \frac{y_1 + y_2 + y_3}{3} = 0$$

$$\Rightarrow (x_1 + iy_1) + (x_2 + iy_2) + (x_3 + iy_3) = 0$$

$$\Rightarrow z_1 + z_2 + z_3 = 0$$

$$22. (a) \text{ We have, } |z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$$

$$\Rightarrow |z_1|^2 + |z_2|^2 + 2|z_1||z_2|\cos(\theta_1 - \theta_2) = |z_1|^2 + |z_2|^2$$

where, $\theta_1 = \arg(z_1), \theta_2 = \arg(z_2)$

$$\Rightarrow \cos(\theta_1 - \theta_2) = 0 \Rightarrow \theta_1 - \theta_2 = \frac{\pi}{2}$$

$$\Rightarrow \arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2} \Rightarrow \operatorname{Re}\left(\frac{z_1}{z_2}\right) = \frac{|z_1|}{|z_2|} \cos\left(\frac{\pi}{2}\right) = 0$$

$$23. (b) \left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right| = \left| \frac{\beta - \alpha}{\beta \bar{\beta} - \bar{\alpha}\beta} \right| \quad \left[\because |\beta| = 1 \right]$$

$$= \left| \frac{\beta - \alpha}{\beta(\bar{\beta} - \bar{\alpha})} \right| = \frac{1}{|\beta|} \frac{|\beta - \alpha|}{|\bar{\beta} - \bar{\alpha}|} = \frac{|\beta - \alpha|}{|\beta - \alpha|} = 1$$

$$[\because |\beta| = 1 \text{ and } |\bar{\beta} - \bar{\alpha}| = |\beta - \alpha|]$$

$$\text{or } \left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right| = \left| \frac{\beta - \alpha}{1 - \frac{1}{\alpha}\beta} \right| \quad \left[\because \alpha \bar{\alpha} = 1 \Rightarrow \bar{\alpha} = \frac{1}{\alpha} \right]$$

$$= |\alpha| \frac{|\beta - \alpha|}{|\alpha - \beta|} = 1 \cdot 1 = 1 \quad [\because |\beta - \alpha| = |\alpha - \beta|]$$

$$\begin{aligned} 24. (b) \sum_{n=1}^{13} (i^n + i^{n+1}) &= \sum_{n=1}^{13} i^n (1 + i) \\ &= (1 + i)[i + i^2 + i^3 + \dots + i^{13}] \\ &= (1 + i)[(i + i^2 + i^3 + i^4) + (i^5 + i^6 + i^7 + i^8) \\ &\quad + (i^9 + i^{10} + i^{11} + i^{12}) + i^{13}] \quad [\because i^{13} = i] \\ &= (1 + i)[0 + 0 + i] = i + i^2 = i - 1 \end{aligned}$$

$$\begin{aligned} 25. (b) \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)^{3/4} &= \left[\cos \left(2k\pi + \frac{\pi}{3} \right) + i \sin \left(2k\pi + \frac{\pi}{3} \right) \right]^{3/4} \\ &= \left[\cos(6k + 1) \frac{\pi}{3} + i \sin(6k + 1) \frac{\pi}{3} \right]^{3/4}; k = 0, 1, 2, 3, \dots \\ &= \cos \left[(6k + 1) \frac{\pi}{3} \cdot \frac{3}{4} \right] + i \sin \left[(6k + 1) \frac{\pi}{3} \cdot \frac{3}{4} \right] \\ &\quad \text{[by De-Moivre's theorem]} \\ &= \left[\cos(6k + 1) \frac{\pi}{4} \right] + i \left[\sin(6k + 1) \frac{\pi}{4} \right] \end{aligned}$$

$$\therefore \text{Roots are } \cos \frac{\pi}{4} + i \sin \frac{\pi}{4}, (k = 0);$$

$$\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}, (k = 1); \cos \frac{13\pi}{4} + i \sin \frac{13\pi}{4}, (k = 2)$$

$$\text{and } \cos \frac{19\pi}{4} + i \sin \frac{19\pi}{4}, (k = 3)$$

$$\begin{aligned} \text{Products of roots} &= \left[\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right] \left[\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right] \\ &\quad \left[\cos \frac{13\pi}{4} + i \sin \frac{13\pi}{4} \right] \left[\cos \frac{19\pi}{4} + i \sin \frac{19\pi}{4} \right] \\ &= \cos \left(\frac{\pi}{4} + \frac{7\pi}{4} + \frac{13\pi}{4} + \frac{19\pi}{4} \right) + i \sin \left(\frac{\pi}{4} + \frac{7\pi}{4} + \frac{13\pi}{4} + \frac{19\pi}{4} \right) \\ &= \cos 10\pi + i \sin 10\pi = 1 \end{aligned}$$

$$26. (b) \text{ We have, } x_n = e^{i \frac{\pi}{2^n}}$$

$$\therefore x_1 x_2 x_3 \dots = e^{i\pi \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots \right)} = -1$$

$$27. (d) \alpha + i\beta = \cot^{-1}(z) \text{ or } \cot(\alpha + i\beta) = x + iy$$

$$\text{and } \cot(\alpha - i\beta) = x - iy$$

$$\begin{aligned} \therefore \cot 2\alpha &= \cot[(\alpha + i\beta) + (\alpha - i\beta)] \\ &= \frac{\cot(\alpha + i\beta) \cdot \cot(\alpha - i\beta) - 1}{\cot(\alpha + i\beta) + \cot(\alpha - i\beta)} = \frac{(x^2 + y^2 - 1)}{2x} \\ \therefore x^2 + y^2 - 2x \cot 2\alpha - 1 &= 0 \end{aligned}$$

$$28. (c) \text{ Let } a_k + ib_k = r_k (\cos \theta_k + i \sin \theta_k)$$

$$\text{Then, } r_k = \sqrt{a_k^2 + b_k^2} \text{ and } \tan \theta_k = \frac{b_k}{a_k}$$

$$\begin{aligned} \therefore (a_1 + ib_1)(a_2 + ib_2) \dots (a_n + ib_n) &= A + iB \\ \Rightarrow r_1 r_2 \dots r_n [\cos(\theta_1 + \theta_2 + \dots + \theta_n) + i \sin(\theta_1 + \theta_2 + \dots + \theta_n)] \\ &= A + iB \Rightarrow r_1 r_2 r_3 \dots r_n = A^2 + B^2 \end{aligned}$$

$$\text{and } \theta_1 + \theta_2 + \dots + \theta_n = \tan^{-1} \left(\frac{B}{A} \right)$$

$$\begin{aligned} \sum_{i=1}^n \tan^{-1} \left(\frac{b_i}{a_i} \right) &= \tan^{-1} \left(\frac{b_1}{a_1} \right) + \dots + \tan^{-1} \left(\frac{b_n}{a_n} \right) \\ &= \tan^{-1} \left(\frac{B}{A} \right) \end{aligned}$$

$$29. (c) |z| = |z + 3/z - 3/z| \leq |z + 3/z| + |-3/z| = 3 + \frac{3}{|z|}$$

$$\text{Let } |z| = t, t \leq 3 + \frac{3}{t} \Rightarrow \frac{t^2 - 3t - 3}{t} \leq 0 \Rightarrow \left(t - \frac{1}{2} \right)^2 \leq \frac{21}{4}$$

$$\text{So, } \frac{3 - \sqrt{21}}{2} < t \leq \frac{3 + \sqrt{21}}{2}$$

$$\text{Hence, the maximum value of } |z| \text{ is } \frac{3 + \sqrt{21}}{2}.$$

$$30. (c) \text{ Hint The given expression becomes}$$

$$4 + 5\omega^{334} + 3\omega^{335} = 4 + 5\omega + 3\omega^2$$

$$[\because \omega^{334} = (\omega^3)^{111} \cdot \omega \text{ and } \omega^{335} = (\omega^3)^{111} \cdot \omega^2]$$

$$31. (b) \text{ Given, } \arg(z) > 0 \text{ and } \arg(-z) - \arg(z) = \lambda_1, \text{ then}$$

$$(\arg \bar{z}) + \arg(-z) = \lambda_1$$

$$\Rightarrow -\pi = \lambda_1 \Rightarrow \lambda_1 = -\pi$$

$$\text{Again, given } \arg(z) < 0$$

$$\text{Then, } \arg(z) - \arg(-z) = \lambda_2$$

$$\Rightarrow -[-\arg(z) + \arg(-z)] = \lambda_2$$

$$\Rightarrow \lambda_2 = -\pi$$

$$\therefore \lambda_1 = \lambda_2$$

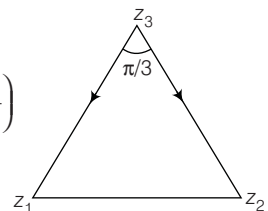
$$\begin{aligned} 32. (a) \sqrt{\{\arg(z) + \arg(-\bar{z}) - 2\pi\} \{\arg(-z) + \arg(\bar{z})\}} \\ = \sqrt{\{\arg(z) + \pi - \arg(z) - 2\pi\} \{\arg(-z) - \arg(z)\}} \\ = \sqrt{(-\pi) - \{\arg(z) - \arg(-z)\}} = \sqrt{(-\pi)(-\pi)} = \pi \end{aligned}$$

$$33. (c) \frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$$

$$= \cos \left(-\frac{\pi}{3} \right) + i \sin \left(-\frac{\pi}{3} \right)$$

$$= e^{-i\frac{\pi}{3}}$$

$$\therefore \left| \frac{z_1 - z_3}{z_2 - z_3} \right| = |e^{-i\pi/3}| = 1$$



$$\text{and angle between } z_1 - z_3 \text{ and } z_2 - z_3 \text{ is } \frac{\pi}{3}.$$

Therefore, triangle is equilateral.

$$34. (d) z = (1)^{1/n} = (\cos 0 + i \sin 0)^{1/n} = \frac{\cos 2r\pi}{n} + i \frac{\sin 2r\pi}{n}$$

or $z = e^{i \frac{2r\pi}{n}}$, where r varies from 0 to $n-1$ and each root is unimodular as $|e^{i\theta}| = 1$.

Let $z_1 = 1$ and $z_2 = e^{\frac{i2k\pi}{n}}$
 where, $z_2 - 0 = (z_1 - 0)e^{\frac{\pi j}{2}}$ [by given condition]
 or $e^{\frac{i2k\pi}{n}} = 1 \cdot e^{\frac{\pi j}{2}}$
 $\therefore n = 4k$

35. (c) Using $1 + \omega + \omega^2 = 0$, then

$$\Delta = \begin{vmatrix} 1+\omega & \omega^2 & -\omega \\ 1+\omega^2 & \omega & -\omega^2 \\ \omega^2+\omega & \omega & -\omega^2 \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2$

$$\Delta = \begin{vmatrix} 0 & \omega^2 & -\omega \\ 0 & \omega & -\omega^2 \\ \omega^2+2\omega & \omega & -\omega^2 \end{vmatrix} = (\omega^2 + 2\omega)(-\omega + \omega^2) = -3\omega^2$$

36. (a) Let $z = 7 + 24i$, then $|z| = 25$, $x = 7$

$$\therefore \text{Square root} = \pm \left[\left(\frac{25+7}{2} \right)^{1/2} + i \left(\frac{25-7}{2} \right)^{1/2} \right] \quad [\because y > 0]$$

$$= \pm (4 + 3i)$$

37. (a) We have, $\arg\left(\frac{z_1}{z_2}\right) = \pi$

$$\Rightarrow \arg(z_1) - \arg(z_2) = \pi$$

$$\Rightarrow \arg(z_1) = \arg(z_2) + \pi$$

$$\text{Let } \arg(z_2) = \theta$$

$$\text{Then, } \arg(z_1) = \pi + \theta$$

$$\therefore z_1 = |z_1|[\cos(\pi + \theta) + i \sin(\pi + \theta)]$$

$$= |z_1|(-\cos \theta - i \sin \theta)$$

$$\text{and } z_2 = |z_2|(\cos \theta + i \sin \theta)$$

$$= |z_1|(\cos \theta + i \sin \theta) \quad [\because |z_1| = |z_2|]$$

$$\text{Hence, } z_1 + z_2 = 0$$

38. (b) Given, $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$

$$\Rightarrow \left[\left(\frac{1-i}{1+i} \right) \times \left(\frac{1-i}{1-i} \right) \right]^{100} = a + ib$$

$$\Rightarrow a + ib = \left[\frac{(1-i)^2}{2} \right]^{100} = \left[\frac{-2i}{2} \right]^{100} = (-i)^{100}$$

$$\Rightarrow a + ib = [(i)^4]^{25} = 1 + 0i \quad [\because (-1)^{100} = 1]$$

$$\text{Hence, } a = 1, b = 0$$

39. (d) We have, $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = \begin{vmatrix} 6i & 0 & 1 \\ 4 & 0 & -1 \\ 20 & 0 & i \end{vmatrix}$

[applying $C_2 \rightarrow C_2 + 3iC_3$]

$$\Rightarrow 0 = 0 + 0i$$

$$\therefore x = 0, y = 0$$

40. (a) Let $z_1 = a + ib = (a, b)$ and $z_2 = c - id = (c, -d)$

where, $b > 0$ and $d > 0$

$$\text{Then, } |z_1| = |z_2|$$

$$\Rightarrow a^2 + b^2 = c^2 + d^2$$

$$\text{Now, } \frac{z_1 + z_2}{z_1 - z_2} = \frac{(a + ib) + (c - id)}{(a + ib) - (c - id)} \quad \dots (i)$$

$$= \frac{[(a + c) + i(b - d)][(a - c) - i(b + d)]}{[(a - c) + i(b + d)][(a + c) - i(b - d)]}$$

$$= \frac{(a^2 + b^2) - (c^2 + d^2) - 2(ad + bc)i}{a^2 + c^2 - 2ac + b^2 + d^2 + 2bd}$$

$$= \frac{-(ad + bc)i}{a^2 + b^2 - ac + bd}$$

[$\because |z_1| = |z_2|$, from Eq. (i)]

So, $\frac{(z_1 + z_2)}{(z_1 - z_2)}$ is purely imaginary.

BITSAT Archives

1. (a) Given, $\left| \frac{z-3i}{z+3i} \right| = 1 \Rightarrow |z-3i| = |z+3i|$

[if $|z-z_1| = |z-z_2|$, then it is a perpendicular bisector of z_1 and z_2]

Hence, perpendicular bisector of $(0, 3)$ and $(0, -3)$ is X -axis.

2. (d) $\sum_{k=1}^6 \left(\frac{\sin 2\pi k}{7} - \frac{i \cos 2\pi k}{7} \right)$

$$= \sum_{k=1}^6 -i \left(\frac{\cos 2\pi k}{7} - \frac{i \sin 2\pi k}{7} \right) = -i \sum_{k=1}^6 e^{i2\pi k/7}$$

$$= -i [e^{i2\pi/7} + e^{i4\pi/7} + \dots + e^{i12\pi/7}]$$

$$= -i \left[e^{i2\pi/7} \frac{(1 - e^{i12\pi/7})}{1 - e^{i2\pi/7}} \right] = -i \left[\frac{e^{i2\pi/7} - e^{i14\pi/7}}{1 - e^{i2\pi/7}} \right]$$

$$= -i \left[\frac{e^{i2\pi/7} - e^{2\pi i}}{1 - e^{i2\pi/7}} \right]$$

$$= -i \left[\frac{e^{i2\pi/7} - 1}{1 - e^{i2\pi/7}} \right] = i$$

3. (c) $(x + iy)^{1/3} = 2 + 3i$

On cubing both sides, we get

$$x + iy = (2 + 3i)^3$$

$$\Rightarrow x + iy = (2)^3 + (3i)^3 + 3 \times 2 \times 3i(2 + 3i)$$

$$\Rightarrow x + iy = 8 - 27i + 18i(2 + 3i)$$

$$\Rightarrow x + iy = 8 - 27i + 36i - 54 \Rightarrow x + iy = -46 + 9i$$

On comparing real and imaginary parts both sides, we get

$$x = -46, y = 9$$

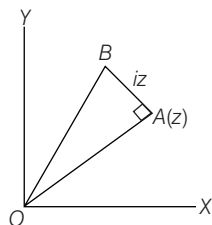
$$\text{Then, } 3x + 2y = 3(-46) + 2(9) = -138 + 18 = -120$$

4. (d) Since, $iz = ze^{i\pi/2}$. This implies that iz is the vector obtained by rotating vector z in anti-clockwise direction through 90° .

$$\therefore OA \perp AB$$

$$\text{So, area of } \triangle OAB = \frac{1}{2} OA \times OB$$

$$= \frac{1}{2} |z| |iz| = \frac{1}{2} |z|^2$$



5. (c) Given, $z = \cos \theta + i \sin \theta$

$$\begin{aligned} \therefore z^n + \frac{1}{z^n} &= (\cos \theta + i \sin \theta)^n + (\cos \theta + i \sin \theta)^{-n} \\ &= \cos n\theta + i \sin n\theta + \cos n\theta - i \sin n\theta \\ &= 2 \cos n\theta \end{aligned}$$

6. (a) Hint $\left(\frac{1+i}{1-i} \times \frac{1+i}{1+i} \right)^n = 1 \Rightarrow i^n = 1$

7. (d) Applying $C_1 \rightarrow C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = \begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ \omega+\omega^2+1 & \omega^2 & 1 \\ \omega^2+\omega+1 & 1 & \omega \end{vmatrix} = \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & \omega \end{vmatrix} = 0$$

8. (c) Since, $a = \cos \theta + i \sin \theta$

$$\begin{aligned} \therefore \frac{1+a}{1-a} &= \frac{1+\cos \theta + i \sin \theta}{1-\cos \theta - i \sin \theta} \\ &= \frac{[(1+\cos \theta) + i \sin \theta][(1-\cos \theta) + i \sin \theta]}{[(1-\cos \theta) - i \sin \theta][(1-\cos \theta) + i \sin \theta]} \\ &= \frac{2i \sin \theta}{(1-\cos \theta)^2 + \sin^2 \theta} = \frac{i \cdot 4 \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2}}{4 \sin^2 \frac{\theta}{2}} = i \cot \frac{\theta}{2} \end{aligned}$$

9. (a) Since, ω is a cube root of unity.

$$\begin{aligned} \therefore \sin \left\{ (\omega^{10} + \omega^{23}) \pi - \frac{\pi}{4} \right\} &= \sin \left\{ (\omega + \omega^2) \pi - \frac{\pi}{4} \right\} \\ &= \sin \left(-\pi - \frac{\pi}{4} \right) \quad [\because 1 + \omega + \omega^2 = 0] \\ &= -\sin \left(\pi + \frac{\pi}{4} \right) = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} \end{aligned}$$

10. (c) We have, $(\cos \theta + i \sin \theta)(\cos 2\theta + i \sin 2\theta) \dots (\cos n\theta + i \sin n\theta) = 1$

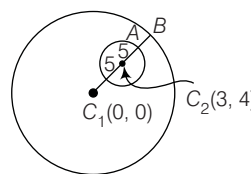
$$\begin{aligned} \therefore \cos(\theta + 2\theta + 3\theta + \dots + n\theta) + i \sin(\theta + 2\theta + 3\theta + \dots + n\theta) &= 1 \\ \Rightarrow \cos \left(\frac{n(n+1)}{2} \theta \right) + i \sin \left(\frac{n(n+1)}{2} \theta \right) &= 1 \end{aligned}$$

On comparing the coefficients of real and imaginary parts both sides, we get

$$\cos \left(\frac{n(n+1)}{2} \theta \right) = 1 \text{ and } \sin \left(\frac{n(n+1)}{2} \theta \right) = 0$$

$$\therefore \frac{n(n+1)}{2} \theta = 2m\pi \Rightarrow \theta = \frac{4m\pi}{n(n+1)}$$

11. (d) The two circles whose centre and radius are $C_1(0, 0)$, $r_1 = 12$, $C_2(3, 4)$, $r_2 = 5$ and it passes through origin, i.e. the centre of C_1 .



Now,

$$C_1 C_2 = \sqrt{3^2 + 4^2} = 5$$

and

$$r_1 - r_2 = 12 - 5 = 7$$

$\therefore$

$$C_1 C_2 < r_1 - r_2$$

Hence, circle C_2 lies inside the circle C_1 .

From figure, the minimum distance between them is

$$AB = C_1 B - C_1 A = r_1 - 2r_2 = 12 - 10 = 2$$

$$\begin{aligned} 12. (a) \text{ Let } z &= \frac{1+i\sqrt{3}}{\sqrt{3}+i} = \frac{1+i\sqrt{3}}{(\sqrt{3}+i)} \times \frac{(\sqrt{3}-i)}{(\sqrt{3}-i)} \\ &= \frac{\sqrt{3}-i+3i+\sqrt{3}}{3+1} = \frac{\sqrt{3}+i}{2} \end{aligned}$$

$$\therefore \text{amp}(z) = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = \frac{\pi}{6}$$

13. (c) Given that, $x = \alpha + \beta$, $y = \alpha\omega + \beta\omega^2$, $z = \alpha\omega^2 + \beta\omega$

Now,

$$\begin{aligned} xyz &= (\alpha + \beta)(\alpha\omega + \beta\omega^2)(\alpha\omega^2 + \beta\omega) \\ &= (\alpha + \beta)(\alpha^2\omega^3 + \alpha\beta\omega^2 + \alpha\beta\omega^4 + \beta^2\omega^3) \\ &= (\alpha + \beta)[\alpha^2 + \alpha\beta(\omega^2 + \omega) + \beta^2] \end{aligned}$$

$$[\because 1 + \omega + \omega^2 = 0 \text{ and } \omega^3 = 1]$$

$$= (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) = \alpha^3 + \beta^3$$

2

Quadratic Equation

Quadratic Equation

An equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$ and $a, b, c, x \in R$, is called a **real quadratic equation**. The numbers a, b and c are called the **coefficients** of the equation. The quantity $D = b^2 - 4ac$ is known as the **discriminant** of the equation $ax^2 + bx + c = 0$ and its roots are given by $x = \frac{-b \pm \sqrt{D}}{2a}$.

An equation of the form $az^2 + bz + c = 0$, where $a \neq 0$ and $a, b, c, z \in C$ (complex), is called a **complex quadratic equation** and its roots are given by $z = \frac{-b \pm \sqrt{D}}{2a}$.

Nature of Roots of Quadratic Equation

Nature of roots of a quadratic equation $ax^2 + bx + c = 0$ implies whether the roots are real or imaginary by analysing the quantity D .

- Let $a, b, c \in R$ and $a \neq 0$, then the equation $ax^2 + bx + c = 0$ has
 - real and distinct roots if and only if $D > 0$.
 - real and equal roots if and only if $D = 0$.
 - complex roots with non-zero imaginary parts if and only if $D < 0$. If $p + iq$ (where, $p, q \in R, q \neq 0$) is one root of $ax^2 + bx + c = 0$, then second root will be $(p - iq)$.
- If $a, b, c \in Q$ and D is a perfect square, then $ax^2 + bx + c = 0$ has **rational roots**.
- If $a, b, c \in Q$ and $p + \sqrt{q}$ (where, $p, q \in Q$) is an **irrational root** of $ax^2 + bx + c = 0$, then other root will be $(p - \sqrt{q})$.
- If $a = 1, b, c \in I$ are roots of $ax^2 + bx + c = 0$, are rational numbers, then these roots must be **integers**.
- If $ax^2 + bx + c = 0$ is satisfied by more than two distinct complex numbers, then it becomes an identity i.e. $a = b = c = 0$.

6. If the roots of $ax^2 + bx + c = 0$ are both positive, then the signs of a and c should be like and opposite to the sign of b .
7. If the roots of $ax^2 + bx + c = 0$ are both negative, then signs of a, b and c should be like.
8. If the roots of $ax^2 + bx + c = 0$ are equal in magnitude but opposite in sign, then $b = 0$ and $c < 0$.
9. If the roots of $ax^2 + bx + c = 0$ are reciprocal to each other, then $c = a$.
10. In the equation $ax^2 + bx + c = 0$ (where, $a, b, c \in R$), if $a + b + c = 0$, then the roots are $1, \frac{c}{a}$ and if $a - b + c = 0$, then the roots are -1 and $-\frac{c}{a}$.

Relation between Roots and Coefficients

1. Quadratic Roots

If α and β are the roots of quadratic equation $ax^2 + bx + c = 0$, then

$$\text{Sum of roots} = \alpha + \beta = -\frac{b}{a}$$

and product of roots $= \alpha\beta = \frac{c}{a}$

2. Cubic Roots

If α, β, γ are the roots of cubic equation $ax^3 + bx^2 + cx + d = 0; a \neq 0$, then

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\beta\gamma + \gamma\alpha + \alpha\beta = \frac{c}{a} \quad \text{and} \quad \alpha\beta\gamma = -\frac{d}{a}$$

3. Biquadratic Roots

If α, β, γ and δ are the roots of biquadratic equation $ax^4 + bx^3 + cx^2 + dx + e = 0; a \neq 0$, then

$$\alpha + \beta + \gamma + \delta = -\frac{b}{a}$$

$$(\alpha + \beta)(\gamma + \delta) + \alpha\beta + \gamma\delta = \frac{c}{a}$$

$$\alpha\beta(\gamma + \delta) + \gamma\delta(\alpha + \beta) = -\frac{d}{a} \quad \text{and} \quad \alpha\beta\gamma\delta = \frac{e}{a}$$

4. Symmetric Roots

If α and β are the roots of quadratic equation $ax^2 + bx + c = 0$, then to find the symmetric function of α and β , we use the following results:

- (i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
- (ii) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
- (iii) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$
- (iv) $\alpha^3 - \beta^3 = (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta)$

5. Common Roots (Conditions)

Suppose that the quadratic equations are $ax^2 + bx + c = 0$ and $a'x^2 + b'x + c' = 0$.

- (i) When one root is common, then the condition is $(a'c - ac')^2 = (bc' - b'c)(ab' - a'b)$.
- (ii) When both roots are common, then the condition is $\frac{a}{a'} = \frac{b}{b'}$.

Formation of an Equation

1. Quadratic Equation

If α and β are the roots of a quadratic equation, then the equation will be of the form

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

2. Cubic Equation

If α, β and γ are the roots of the cubic equation, then the equation will be of the form

$$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0$$

3. Biquadratic Equation

If α, β, γ and δ are the roots of the biquadratic equation, then the equation will be of the form

$$x^4 - (\alpha + \beta + \gamma + \delta)x^3 + (\alpha\beta + \beta\gamma + \gamma\alpha + \alpha\delta + \beta\delta + \gamma\delta)x^2 - (\alpha\beta\gamma + \beta\gamma\delta + \gamma\delta\alpha + \delta\alpha\beta)x + \alpha\beta\gamma\delta = 0$$

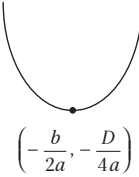
Sign of Quadratic Equation

Let $f(x) = ax^2 + bx + c$ or $y = ax^2 + bx + c$, where $a, b, c \in R$ and $a \neq 0$.

- (i) If $a > 0$ and $D < 0$, then $f(x) > 0, \forall x \in R$.
- (ii) If $a < 0$ and $D < 0$, then $f(x) < 0, \forall x \in R$.
- (iii) If $a > 0$ and $D = 0$, then $f(x) \geq 0, \forall x \in R$.
- (iv) If $a < 0$ and $D = 0$, then $f(x) \leq 0, \forall x \in R$.
- (v) If $a > 0, D > 0$ and $f(x) = 0$ have two real roots α and β (where, $\alpha < \beta$), then $f(x) > 0, \forall x \in (-\infty, \alpha) \cup (\beta, \infty)$ and $f(x) < 0, \forall x \in (\alpha, \beta)$.
- (vi) If $a < 0, D > 0$ and $f(x) = 0$ have two real roots α and β (where, $\alpha < \beta$), then $f(x) < 0, \forall x \in (-\infty, \alpha) \cup (\beta, \infty)$ and $f(x) > 0, \forall x \in (\alpha, \beta)$.

Greatest and Least Values of Quadratic Expression $y = ax^2 + bx + c$

The maximum and minimum values are as follows:

Value of a	Graph	Maximum value	Minimum value
$a > 0$		Not defined	$-\frac{D}{4a}$ at $x = -\frac{b}{2a}$
$a < 0$		$-\frac{D}{4a}$ at $x = -\frac{b}{2a}$	Not defined

Practice Exercise

- If $x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}$, then x equals
 a. $\frac{1 + \sqrt{5}}{2}$ b. $\frac{2 + \sqrt{5}}{2}$ c. $\frac{-1 + \sqrt{5}}{2}$ d. $\frac{-1 - \sqrt{5}}{2}$
- If $a + b + c = 0$, then the roots of the equation $4ax^2 + 3bx + 2c = 0$, where $a, b, c \in R$, are
 a. real and distinct b. imaginary
 c. real and equal d. infinite
- If the roots of the quadratic equation $x^2 - 4x - \log_3 a = 0$ are real, then the least value of a is
 a. 81 b. $\frac{1}{81}$ c. $\frac{1}{64}$ d. None
- If α, β are the roots of $ax^2 + bx + c = 0$ and $\alpha + h, \beta + h$ are the roots of $px^2 + qx + r = 0$, then the value of h is
 a. $\left(\frac{b}{a} - \frac{q}{p}\right)$ b. $\frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$ c. $-\frac{1}{2}\left(\frac{a}{b} - \frac{p}{q}\right)$ d. None
- If the equation $(3x)^2 + (27 \times 3^{1/p} - 15)x + 4 = 0$ has equal roots, then p is equal to
 a. zero b. 2 c. $-\frac{1}{2}$ d. None
- If the equations $x^2 + ax + 12 = 0$, $x^2 + bx + 15 = 0$ and $x^2 + (a + b)x + 36 = 0$ have a common positive root, then the ordered pair (a, b) is
 a. $(-6, -7)$ b. $(-7, -8)$ c. $(-6, -8)$ d. $(-8, -7)$
- The condition that $x^3 - px^2 + qx - r = 0$ may have two of its roots equal to each other but of opposite sign, is
 a. $r = pq$ b. $r = 2p^3 + pq$
 c. $r = p^2q$ d. None of these
- The number of values of k for which the equation $x^2 - 3x + k = 0$ has two distinct roots lying in the interval $(0, 1)$, is
 a. three b. two
 c. infinite d. no value of k will satisfy
- If $ax^2 + 2bx - 3c = 0$ has no real root and $\frac{3c}{4} < a + b$, then the range of c is
 a. $(-1, 1)$ b. $(0, 1)$ c. $(0, \infty)$ d. $(-\infty, 0)$
- If the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then the product of the roots is
 a. $-2(p^2 + q^2)$ b. $-(p^2 + q^2)$
 c. $-\frac{(p^2 + q^2)}{2}$ d. $-pq$
- The harmonic mean of the roots of the equation $(5 + \sqrt{2})x^2 - (4 + \sqrt{5})x + (8 + 2\sqrt{5}) = 0$ is
 a. 2 b. 4 c. 7 d. 8
- The roots of the equation $|x^2 - x - 6| = x + 2$ are
 a. $-2, 1, 4$ b. $0, 2, 4$ c. $0, 1, 4$ d. $-2, 2, 4$

13. In $\triangle PQR, R = \frac{\pi}{2}$. If $\tan \frac{P}{2}$ and $\tan \frac{Q}{2}$ are the roots of the equation $ax^2 + bx + c = 0$, then
 a. $a = b + c$ b. $b = c + a$ c. $c = a + b$ d. $b = c$
14. If the roots of the equation $(a + 1)x^2 - 3ax + 4a = 0$ ($a \neq -1$) are greater than unity, then the values of a are
 a. $\left[-\frac{16}{7}, -1\right)$ b. $[0, -1]$ c. $\left[-\frac{16}{7}, 1\right]$ d. $[0, 1]$
15. The number of real roots of the equation $(x - 1)^2 + (x - 2)^2 + (x - 3)^2 = 0$ is
 a. 2 b. 1 c. 0 d. 3
16. If $y = 3^{x-1} + 3^{-x-1}$ (where, x is real), then the least value of y is
 a. 2 b. 6 c. $\frac{2}{3}$ d. None
17. If the equation $x^2 - (2 + m)x + (m^2 - 4m + 4) = 0$ has coincident roots, then
 a. $m = 0, m = 1$ b. $m = 0, m = 2$
 c. $m = \frac{2}{3}, m = 6$ d. $m = \frac{2}{3}, m = 1$
18. If α, β are the roots of $x^2 + x + 2 = 0$ and γ, δ are the roots of $x^2 + 3x + 4 = 0$, then $(\alpha + \gamma)(\alpha + \delta)(\beta + \gamma)(\beta + \delta)$ is equal to
 a. -18 b. 18 c. 24 d. 44
19. If $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ lies in the interval
 a. $[1, 2]$ b. $\left[0, \frac{1}{2}\right]$ c. $\left[-\frac{1}{2}, 1\right]$ d. $[0, 1]$
20. If $x^2 - 5x + 1 = 0$, then $x^5 + \frac{1}{x^5}$ is equal to
 a. 2424 b. 3232 c. 2525 d. None
21. The value of k for which the number 3 lies between the roots of the equation $x^2 + (1 - 2k)x + (k^2 - k - 2) = 0$, is given by
 a. $k < 2$ b. $2 < k < 5$ c. $2 < k < 3$ d. $k > 5$
22. If α, β and γ are the roots of the equation, $x^3 - x - 1 = 0$, then $\frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma}$ has the value
 a. 0 b. -1 c. -7 d. 1
23. If one root of the equation $x^2 - \lambda x + 12 = 0$ is even prime while $x^2 + \lambda x + \mu = 0$ has equal roots, then μ is equal to
 a. 8 b. 16 c. 24 d. 32
24. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then $(1 + \alpha + \alpha^2)(1 + \beta + \beta^2)$ is equal to
 a. 0 b. positive c. negative d. None
25. The number of real solutions of the equation $(5 + 2\sqrt{6})^{x^2-3} + (5 - 2\sqrt{6})^{x^2-3} = 10$ is
 a. 2 b. 4 c. 6 d. None
26. If $b > a$, then the equation $(x - a)(x - b) - 1 = 0$ has
 a. both roots in $[a, b]$
 b. both roots in $(-\infty, a)$
 c. one root in (b, ∞) and other in $(b, -\infty)$
 d. one root in $(-\infty, a)$
27. If α and β ($\alpha < \beta$) are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then
 a. $0 < \alpha < \beta$ b. $\alpha < 0 < \beta < |\alpha|$
 c. $\alpha < \beta < 0$ d. $\alpha < 0 < |\alpha| < \beta$
28. If the roots of the equation $lx^2 + mx + m = 0$ are in ratio $a : b$, then the value of $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} + \sqrt{\frac{m}{l}}$ is equal to
 a. 0 b. 1 c. 3 d. None
29. For the equation $3x^2 + px + 3 = 0, p > 0$, if one of the roots is square of the other, then p is equal to
 a. $\frac{1}{2}$ b. 1 c. 3 d. $\frac{2}{3}$
30. If $x^2 + 5 = 2x - 4 \cos(a + b)$, where $a, b \in (0, 5)$, is satisfied for atleast one real x , then the maximum value of $a + b$ in $[0, 2\pi]$ is
 a. 3π b. 2π
 c. π d. None of these
31. If $a, b, c \in R$ and $ax^2 + bx + c = 0$ has no real roots, then
 a. $c(a + b + c) > 0$ b. $c - c(a - b - c) > 0$
 c. $c + c(a - b - c) > 0$ d. $c(a - b - c) > 0$
32. The value of P such that the difference of the roots of the equation $x^2 - Px + 8 = 0$ is 2, is
 a. ± 3 b. ± 6 c. ± 2 d. ± 1
33. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then the quadratic equation whose roots are $\frac{\alpha}{1+\alpha}$ and $\frac{\beta}{1+\beta}$, is
 a. $ax^2 - b(1-x) + c(1-x)^2 = 0$
 b. $ax^2 - b(x-1) + c(x-1)^2 = 0$
 c. $ax^2 + b(1-x) + c(1-x)^2 = 0$
 d. $ax^2 + b(x+1) + c(1+x)^2 = 0$
34. If $a + b + c > \frac{9c}{4}$ and the equation $ax^2 + 2bx - 5c = 0$ has non-real complex roots, then
 a. $a > 0, c > 0$ b. $a > 0, c < 0$
 c. $a < 0, c < 0$ d. $a < 0, c > 0$
35. For real values of x , the expression $\frac{(x-b)(x-c)}{(x-a)}$ will assume all real values provided
 a. $a \leq c \leq b$ b. $b \geq a \geq c$
 c. $b \leq c \leq a$ d. $a \geq b \geq c$

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- If α and β are the roots of the equation $x^2 - px + q = 0$, then the value of $(\alpha + \beta)x - \left(\frac{\alpha^2 + \beta^2}{2}\right)x^2 + \left(\frac{\alpha^3 + \beta^3}{3}\right)x^3 + \dots$ is [2014]
 - $\log(1 - px + qx^2)$
 - $\log(1 + px - qx^2)$
 - $\log(1 + px + qx^2)$
 - None of these
- The set of all real x satisfying the inequality $\frac{3 - |x|}{4 - |x|} \geq 0$, is [2013]
 - $[-3, 3] \cup (-\infty, -4) \cup (4, \infty)$
 - $(-\infty, -4) \cup (4, \infty)$
 - $(-\infty, -3) \cup (4, \infty)$
 - $(-\infty, -3) \cup (3, \infty)$
- If N is any four digit number say x_1, x_2, x_3, x_4 , then the maximum value of $\frac{N}{x_1 + x_2 + x_3 + x_4}$ is equal to [2013]
 - 1000
 - $\frac{1111}{4}$
 - 800
 - None
- If $4 - 5i$ is the root of the quadratic equation $x^2 + ax + b = 0$, then (a, b) is equal to [2012]
 - $(8, 41)$
 - $(-8, 41)$
 - $(41, 8)$
 - $(-41, 8)$
- If α and β are the roots of the quadratic equation $4x^2 + 3x + 7 = 0$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is [2012]
 - $-\frac{3}{4}$
 - $-\frac{3}{7}$
 - $\frac{3}{7}$
 - $\frac{4}{7}$
- If α, β are the roots of $ax^2 + bx + c = 0$ and $\alpha + k, \beta + k$ are the roots of $px^2 + qx + r = 0$, then $\frac{b^2 - 4ac}{q^2 - 4pr}$ is equal to [2012]
 - $\frac{a}{p}$
 - 1
 - $\left(\frac{a}{p}\right)^2$
 - 0
- If α and β are the roots of the equation $x^2 - 2x + 4 = 0$, then the value of $\alpha^n + \beta^n$ will be [2011]
 - $i2^{n+1} \sin(n\pi/3)$
 - $2^{n+1} \cos(n\pi/3)$
 - $i2^{n-1} \sin(n\pi/3)$
 - $2^{n-1} \cos(n\pi/3)$
- If the roots of the equation $ax^2 + bx + c = 0$ are real and distinct, then [2010]
 - both roots are greater than $-\frac{b}{2a}$
 - both roots are less than $-\frac{b}{2a}$
 - one of the roots exceeds $-\frac{b}{2a}$
 - None of the above
- The number of solutions of the equation $z^2 + \bar{z} = 0$ is [2010, 09]
 - 1
 - 2
 - 3
 - 4
- If the sum of the roots of the equation $ax^2 + 2x + 3a = 0$ is equal to their product, then the value of a is [2009]
 - $-2/3$
 - -3
 - 4
 - $-1/2$
- If $\alpha + \beta = -2$ and $\alpha^3 + \beta^3 = -56$, then the quadratic equation whose roots are α and β , is [2008]
 - $x^2 + 2x - 16 = 0$
 - $x^2 + 2x + 15 = 0$
 - $x^2 + 2x - 12 = 0$
 - $x^2 + 2x - 8 = 0$
- The cubic equation whose roots are thrice to each of the roots of $x^3 + 2x^2 - 4x + 1 = 0$, is [2008]
 - $x^3 - 6x^2 + 36x + 27 = 0$
 - $x^3 + 6x^2 + 36x + 27 = 0$
 - $x^3 - 6x^2 - 36x + 27 = 0$
 - $x^3 + 6x^2 - 36x + 27 = 0$
- The number of solutions of the equation $1 + \sin x \sin^2 \frac{x}{2} = 0$, in $[-\pi, \pi]$, is [2007]
 - 0
 - 1
 - 2
 - 3
- If one root of the quadratic equation $ax^2 + bx + c = 0$ is equal to n th power of the other root, then the value of $(ac^n)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}}$ is equal to [2007]
 - b
 - $-b$
 - $\frac{1}{b^{n+1}}$
 - $-b^{n+1}$
- If α, β, γ are the roots of the equation $2x^3 - 3x^2 + 6x + 1 = 0$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to [2006]
 - $-\frac{15}{4}$
 - $\frac{15}{4}$
 - $\frac{9}{4}$
 - 4
- If $\sin A, \sin B, \cos A$ are in GP, then the roots of $x^2 + 2x \cot B + 1 = 0$ are always [2005]
 - real
 - imaginary
 - greater than 1
 - equal

Answer with Solutions

Practice Exercise

- (a) Hint $x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}$
 $\Rightarrow x = \sqrt{1 + x} \Rightarrow x^2 = 1 + x \Rightarrow x^2 - x - 1 = 0$
- (a) Here, $D = (3b)^2 - 4 \cdot (4a) \cdot (2c)$
 $= 9b^2 - 32ac = 9(-a-c)^2 - 32ac = 9a^2 - 14ac + 9c^2$
 $= 9c^2 \left[\left(\frac{a}{c}\right)^2 - \frac{14}{9} \cdot \frac{a}{c} + 1 \right] = 9c^2 \left[\left(\frac{a}{c} - \frac{7}{9}\right)^2 - \frac{49}{81} + 1 \right] > 0$
Hence, the roots are real and distinct.
- (b) Since, the roots of the given equation are real.
 $\therefore D > 0 \Rightarrow 16 + 4 \log_3 a \geq 0 \Rightarrow \log_3 a \geq -4$
 $\Rightarrow a \geq 3^{-4} \Rightarrow a \geq \frac{1}{81}$
- (b) Hint $\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$
Also, $\alpha + h + \beta + h = -\frac{q}{p}$ and $(\alpha + h)(\beta + h) = \frac{r}{p}$
- (c) The given equation will have equal roots iff
 $b^2 - 4ac = 0 \Rightarrow (27 \times 3^{1/p} - 15)^2 - 144 = 0$
 $\Rightarrow 27 \times 3^{1/p} - 15 = \pm 12 \Rightarrow 27 \times 3^{1/p} = 27 \text{ or } 3$
Now, $3^{1/p} = 1 \Rightarrow \frac{1}{p} = 0$
Also, $3^{1/p} = 3^{-2} \Rightarrow \frac{1}{p} = -2$
But $\frac{1}{p}$ cannot be zero, so $p = -\frac{1}{2}$.
- (b) We have, $x^2 + ax + 12 = 0$... (i)
 $x^2 + bx + 15 = 0$... (ii)
and $x^2 + (a+b)x + 36 = 0$... (iii)
On adding Eqs. (i) and (ii), we get
 $2x^2 + (a+b)x + 27 = 0$... (iv)
On subtracting Eq. (iii) from Eq. (iv), we get
 $x^2 - 9 = 0 \Rightarrow x = 3, -3$
Thus, common positive root is 3.
 $\therefore (3)^2 + 3a + 12 = 0 \Rightarrow a = -7$
and $9 + 3b + 15 = 0 \Rightarrow b = -8$
Hence, the ordered pair (a, b) is $(-7, -8)$.
- (a) Let α, β, γ be the roots of the given equation such that $\alpha = -\beta$, then
 $\alpha + \beta + \gamma = p \Rightarrow \gamma = p$
Since, γ is a root of the given equation, so it satisfies the equation
 $\gamma^3 - p\gamma^2 + q\gamma - r = 0$
 $\Rightarrow p^3 - p^3 + pq - r = 0 \Rightarrow r = pq$

- (d) Given equation is $x^2 - 3x + k = 0$
 $\therefore$ Discriminant, $D = b^2 - 4ac$
 $= (-3)^2 - 4 \times 1 \times k = 9 - 4k$
For distinct roots, $D > 0$
 $\Rightarrow 9 - 4k > 0 \Rightarrow 9 > 4k \Rightarrow k < \frac{9}{4}$
Every value of k which is less than $\frac{9}{4}$ will give two distinct roots. But these roots will not lie in the interval $(0, 1)$.
- (d) Here, $D = 4b^2 + 12ca < 0$
 $\Rightarrow b^2 + 3ca < 0$... (i)
 $\Rightarrow ca < 0$
If $c > 0$, then $a < 0$
Also, $\frac{3c}{4} < a + b \Rightarrow 3ca > 4a^2 + 4ab$
 $\Rightarrow b^2 + 3ca > 4a^2 + 4ab + b^2 = (2a + b)^2 \geq 0$... (ii)
From Eqs. (i) and (ii), $c > 0$, which is not true.
 $\therefore c < 0$
- (c) Simplified form of given equation is
 $(2x + p + q)r = (x + p)(x + q)$
 $\Rightarrow x^2 + (p + q - 2r)x - (p + q)r + pq = 0$
Since, sum of roots = 0
 $\therefore -(p + q - 2r) = 0 \Rightarrow r = \frac{p + q}{2}$
and product of roots $= -(p + q)r + pq$
 $= -\frac{(p + q)^2}{2} + pq = -\frac{1}{2}(p^2 + q^2)$
- (b) Let α, β be the roots of the given equation.
Then, $\alpha + \beta = \frac{4 + \sqrt{5}}{5 + \sqrt{2}}$ and $\alpha\beta = \frac{8 + 2\sqrt{5}}{5 + \sqrt{2}}$
Let H be the HM of α and β , then
 $H = \frac{2\alpha\beta}{\alpha + \beta} = \frac{16 + 4\sqrt{5}}{4 + \sqrt{5}} = 4$
- (d) We have, $|x^2 - x - 6|$
 $= \begin{cases} x^2 - x - 6, & \text{if } x \leq -2 \text{ or } x \geq 3 \\ -(x^2 - x - 6), & \text{if } -2 < x < 3 \end{cases}$
Case I When $x \leq -2$ or $x \geq 3$, then
 $|x^2 - x - 6| = x + 2 \Rightarrow x^2 - x - 6 = x + 2$
 $\Rightarrow x^2 - 2x - 8 = 0 \Rightarrow (x - 4)(x + 2) = 0 \Rightarrow x = -2, 4$
Case II When $-2 < x < 3$, then
 $|x^2 - x - 6| = x + 2$
 $\Rightarrow -(x^2 - x - 6) = x + 2$
 $\Rightarrow x^2 - 4 = 0 \Rightarrow x = \pm 2$
Hence, roots are $-2, 2$ and 4 .

13. (c) We have, $R = \frac{\pi}{2} \Rightarrow P + Q = \frac{\pi}{2} \Rightarrow \frac{P}{2} + \frac{Q}{2} = \frac{\pi}{4}$

$$\Rightarrow 1 = \tan \frac{\pi}{4} = \frac{\tan \frac{P}{2} + \tan \frac{Q}{2}}{1 - \tan \frac{P}{2} \tan \frac{Q}{2}}$$

$$\Rightarrow 1 = \frac{-\frac{b}{a}}{1 - \frac{c}{a}} = \frac{b}{c-a}$$

$$\Rightarrow a + b = c$$

14. (a) For roots greater than unity,

Discriminant ≥ 0 , sum of roots > 2 and $(a+1)f(1) > 0$

$$\Rightarrow 9a^2 - 16a(a+1) \geq 0, \frac{3a}{a+1} > 2$$

$$\text{and } (a+1)(a+1-3a+4a) > 0$$

$$\Rightarrow a(7a+16) \leq 0, \frac{a-2}{a+1} > 0$$

$$\text{and } (a+1)(2a+1) > 0$$

$$\Rightarrow \frac{-16}{7} \leq a \leq 0, a < -1 \text{ or } a > 2$$

$$\text{and } a < -1 \text{ or } a > -\frac{1}{2}$$

$$\Rightarrow -\frac{16}{7} \leq a < -1$$

15. (c) Since, $(x-1)^2, (x-2)^2, (x-3)^2$ cannot be zero simultaneously for any real value of x .
 $\therefore$ Number of roots = 0

16. (c) Since, $3^{x-1} + 3^{-x-1} = \frac{1}{3}(3^x + 3^{-x})$

$$\geq \frac{1}{3} \cdot 2\sqrt{3^x \cdot 3^{-x}} \quad [\because \text{AM} \geq \text{GM}]$$

$$\Rightarrow 3^{x-1} + 3^{-x-1} \geq \frac{2}{3}$$

17. (c) Hint $b^2 - 4ac = 0$

18. (d) Since, $\alpha + \beta = -1, \alpha\beta = 2, \gamma + \delta = -3, \gamma\delta = 4$

$$\therefore (\alpha + \gamma)(\alpha + \delta)(\beta + \gamma)(\beta + \delta) = (\alpha^2 - 3\alpha + 4)(\beta^2 - 3\beta + 4)$$

$$= 4 - 3\alpha\beta^2 + 4\beta^2 - 3\alpha^2\beta + 9\alpha\beta - 12\beta^2 + 4\alpha^2 - 12\alpha + 16$$

$$= 4 - 3(2)\beta + 4\beta^2 + 4\alpha^2 - 3(2)\alpha + 9(2) - 12(\beta + \alpha) + 16$$

$$= 4 - 6\beta + 4(\alpha^2 + \beta^2) - 6\alpha + 18 + 12 + 16$$

$$= 50 + 6 + 4[(\alpha + \beta)^2 - 2\alpha\beta]$$

$$= 56 - 12 = 44$$

19. (c) Since, $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2} \sum (a-b)^2 \geq 0$

$$\therefore 1 - \sum ab \geq 0 \Rightarrow \sum ab \leq 1 \quad \dots(i)$$

$$\text{Now, } (a+b+c)^2 = \sum a^2 + 2 \sum ab$$

$$\Rightarrow \sum ab \geq -\frac{1}{2} \quad \dots(ii)$$

From Eqs. (i) and (ii), $\sum ab$ lie in $\left[-\frac{1}{2}, 1\right]$.

20. (c) $x + \frac{1}{x} = 5 \Rightarrow x^2 + \frac{1}{x^2} = 5^2 - 2 = 23$

$$\text{and } x^3 + \frac{1}{x^3} = 5^3 - 3 \times 5 = 110$$

$$\text{Now, } \left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right) = 23 \times 110 = 2530$$

$$\therefore x^5 + \frac{1}{x^5} = 2530 - \left(x + \frac{1}{x}\right) = 2525$$

21. (b) Let $f(x) = x^2 + (1-2k)x + k^2 - k - 2$

The number 3 lies between the roots of the given equation, if $f(3) < 0$.

$$\text{Now, } f(3) = 9 + (1-2k)3 + k^2 - k - 2$$

$$\Rightarrow f(3) = k^2 - 7k + 10$$

Hence, $f(3) < 0$

$$\Rightarrow k^2 - 7k + 10 < 0 \text{ or } (k-2)(k-5) < 0$$

$$\therefore k > 2 \text{ and } k < 5 \text{ or } k < 2 \text{ and } k > 5$$

$\therefore k$ lies in the interval $(2, 5)$.

22. (c) Given, α, β and γ are the roots of the given equation.

Then, $\alpha + \beta + \gamma = 0, \alpha\beta + \beta\gamma + \gamma\alpha = -1$ and $\alpha \cdot \beta \cdot \gamma = 1$

$$\text{Let } y = \frac{1+x}{1-x} \Rightarrow x = \frac{y-1}{y+1}$$

$$\Rightarrow y^3 + 7y^2 - y + 1 = 0 \Rightarrow \sum \frac{1+\alpha}{1-\alpha} = -7$$

23. (b) We know that, only even prime is 2, then

$$(2)^2 - \lambda(2) + 12 = 0$$

$$\Rightarrow \lambda = 8 \quad \dots(i)$$

and $x^2 + \lambda x + \mu = 0$ has equal roots.

$$\therefore \lambda^2 - 4\mu = 0 \quad [\because D = 0]$$

$$\Rightarrow (8)^2 - 4\mu = 0 \quad [\text{from Eq. (i)}]$$

$$\Rightarrow \mu = 16$$

24. (b) Given equation is

$$ax^2 + bx + c = 0$$

Since, α, β are the roots of this equation.

$$\therefore \alpha + \beta = \frac{-b}{a} \quad \dots(i)$$

$$\text{and } \alpha\beta = \frac{c}{a} \quad \dots(ii)$$

Now, $(1+\alpha+\alpha^2)(1+\beta+\beta^2)$

$$= 1 + (\alpha + \beta) + \alpha^2 + \beta^2 + \alpha\beta(1 + \alpha + \beta) + \alpha^2\beta^2$$

$$= 1 - \frac{b}{a} + \left(\frac{-b}{a}\right)^2 - \frac{2c}{a} + \frac{c}{a}\left(1 - \frac{b}{a}\right) + \frac{b^2}{a^2}$$

[from Eqs. (i) and (ii)]

$$= \frac{a^2 - ab + b^2 - ac - bc + c^2}{a^2}$$

$$= \frac{(a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (c^2 + a^2 - 2ac)}{2a^2}$$

$$= \frac{(a-b)^2 + (b-c)^2 + (c-a)^2}{2a^2}$$

Here, all terms are in square, therefore it is always positive.

$$25. (b) \text{ We have, } 5 - 2\sqrt{6} = \frac{(5 - 2\sqrt{6})(5 + 2\sqrt{6})}{(5 + 2\sqrt{6})}$$

$$= \frac{25 - 24}{5 + 2\sqrt{6}} = \frac{1}{5 + 2\sqrt{6}}$$

$$\text{Let } (5 + 2\sqrt{6})^{x^2 - 3} = y$$

Then, given equation becomes

$$y + \frac{1}{y} = 10 \Rightarrow y^2 - 10y + 1 = 0$$

$$\Rightarrow y = \frac{10 \pm 4\sqrt{6}}{2} = 5 \pm 2\sqrt{6}$$

When $y = 2\sqrt{6} + 5$, then

$$(5 + 2\sqrt{6})^{x^2 - 3} = (5 + 2\sqrt{6}) \Rightarrow x^2 - 3 = 1$$

$$\Rightarrow x^2 = 4 \Rightarrow x = \pm 2$$

When $y = 5 - 2\sqrt{6}$, then $(5 + 2\sqrt{6})^{x^2 - 3} = (5 - 2\sqrt{6})$

$$\Rightarrow (5 + 2\sqrt{6})^{x^2 - 3} = (5 + 2\sqrt{6})^{-1} \Rightarrow x^2 - 3 = -1$$

$$\Rightarrow x^2 = 2 \Rightarrow x = \pm \sqrt{2}$$

$$\therefore x = 2, -2 - \sqrt{2}, \sqrt{2}$$

26. (d) The given equation can be rewritten as

$$x^2 - (a + b)x + ab - 1 = 0$$

Let α, β be its roots.

$$\therefore \alpha + \beta = a + b, \alpha\beta = ab - 1$$

If one root is less than a , then the other root will be greater than b .

So, we have one root lies in $(-\infty, a)$ and other in (b, ∞) .

27. (b) Clearly, $\alpha + \beta = -b < 0$

$$\alpha\beta = c < 0$$

Now, $\alpha\beta < 0$ and $\alpha < \beta$

$\Rightarrow \alpha$ is a negative root.

$$\alpha + \beta < 0 \text{ and } \alpha < 0 \Rightarrow \beta < |\alpha|$$

$$\therefore \alpha < 0 < \beta < |\alpha|$$

28. (a) Let α, β be the roots of the given equation, then

$$\alpha + \beta = \frac{-m}{l}, \alpha\beta = \frac{m}{l}$$

$$\text{Also } \frac{\alpha}{\beta} = \frac{a}{b}$$

$$\text{Now, } \sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} + \sqrt{\frac{m}{l}} = \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{m}{l}}$$

$$= \frac{\alpha + \beta}{\sqrt{\alpha\beta}} + \sqrt{\frac{m}{l}} = \frac{-m/l}{\sqrt{m/l}} + \sqrt{\frac{m}{l}} = -\sqrt{\frac{m}{l}} + \sqrt{\frac{m}{l}} = 0$$

29. (c) Let α, α^2 be the roots.

$\therefore$ Product of the roots,

$$\alpha \cdot \alpha^2 = 1 \Rightarrow \alpha^3 = 1$$

$$\Rightarrow \alpha = 1, \omega, \omega^2$$

$$\text{Again, } \alpha + \alpha^2 = -\frac{p}{3} \Rightarrow 1 + 1 = -\frac{p}{3}$$

$$\Rightarrow p = -6 \text{ (if } \alpha = 1)$$

But $p > 0$

$\therefore \alpha = 1$ is not possible.

If $\alpha = \omega$, then $\alpha + \alpha^2 = \omega + \omega^2 = -1$

$$\therefore -1 = -\frac{p}{3} \Rightarrow p = 3$$

Again, if $\alpha = \omega^2$, then

$$\alpha + \alpha^2 = \omega^2 + \omega^4 = \omega^2 + \omega = -1$$

$$\therefore -1 = -\frac{p}{3} \Rightarrow p = 3$$

30. (c) We have, $x^2 + 5 = 2x - 4 \cos(a + b)$

$$\Rightarrow x^2 - 2x + 1 + 4 = -4 \cos(a + b)$$

$$\Rightarrow (x - 1)^2 + 4[1 + \cos(a + b)] = 0$$

$$\Rightarrow x = 1 \text{ and } 1 + \cos(a + b) = 0$$

$$\Rightarrow \cos(a + b) = -1 \Rightarrow a + b = \pi, 3\pi, \dots$$

31. (a) Since, the roots of $ax^2 + bx + c = 0$ are non-real. Thus, $2f(x) = ax^2 + bx + c$ will have same sign for every value of x .

$$\Rightarrow f(0) = c, f(1) = a + b + c, f(-1) = a - b + c$$

$$\text{and } f(2) = 4a - 2b + c$$

$$\therefore c \cdot (a + b + c) > 0$$

$$c(a - b + c) > 0 \text{ and } c(4a - 2b + c) > 0$$

32. (b) Hint Using $\alpha - \beta = \pm \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

33. (c) Since, roots of $ax^2 + bx + c = 0$ are α and β .

Hence, roots of $cx^2 + bx + a = 0$ will be $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Now, if we replace x by $x - 1$, then roots of $c(x - 1)^2 + b(x - 1) + a = 0$ will be $1 + \frac{1}{\alpha}$ and $1 + \frac{1}{\beta}$.

Again replace x by $\frac{1}{x}$, we get

$$c(1 - x)^2 + b(1 - x) + ax^2 = 0$$

whose roots are $\frac{\alpha}{1 + \alpha}$ and $\frac{\beta}{1 + \beta}$.

34. (b) Given, $4a + 4b - 5c > 0$

Let $f(x) = ax^2 + 2bx - 5c$, then $f(2) = 4a + 4b - 5c > 0$

Since, $f(x) = 0$ has imaginary roots, therefore $f(x)$ will have same sign as that of a for all $x \in R$.

Since, $f(2) > 0$, therefore $a > 0$ and $c < 0$.

35. (b) Suppose $m = \frac{(x - b)(x - c)}{(x - a)}$

$$\Rightarrow x^2 - (b + c + m)x + (bc + am) = 0$$

Since, x is real, so we must have

$$(b + c + m)^2 - 4(bc + am) \geq 0$$

$$\Rightarrow m^2 + 2(b + c - 20)m + (b - c)^2 \geq 0$$

$$\Rightarrow [m + (b + c - 2a)]^2 + (b - c)^2 - (b + c - 2a)^2 \geq 0$$

Since, m may take any real value, so we must have

$$(b - c)^2 - (b + c - 2a)^2 \geq 0 \Rightarrow (b - a)(a - c) \geq 0$$

$$\Rightarrow (a - b)(a - c) \leq 0 \Rightarrow b \geq a \geq c \text{ or } b \leq a \leq c$$

BITSAT Archives

$$1. (a) \text{ Given, } \left[\alpha x - \frac{1}{2}(\alpha x)^2 + \frac{1}{3}(\alpha x)^3 - \dots \right] + \left[\beta x - \frac{1}{2}(\beta x)^2 + \frac{1}{3}(\beta x)^3 - \dots \right]$$

$$= \log(1 - \alpha x) + \log(1 - \beta x) = \log[1 - (\alpha + \beta)x + \alpha\beta x^2]$$

$$\text{Now, } \alpha + \beta = p \text{ and } \alpha\beta = q$$

$$\therefore \text{ Given series} = \log(1 - px + qx^2)$$

$$2. (a) \text{ Given, } \frac{3 - |x|}{4 - |x|} \geq 0$$

$$\Rightarrow 3 - |x| \leq 0 \text{ and } 4 - |x| < 0$$

$$\text{or } 3 - |x| \geq 0 \text{ and } 4 - |x| > 0$$

$$\Rightarrow |x| \geq 3 \text{ and } |x| > 4$$

$$\text{or } |x| \leq 3 \text{ and } |x| < 4$$

$$\Rightarrow |x| > 4 \text{ or } |x| \leq 3$$

$$\Rightarrow x \in (-\infty, -4) \cup [-3, 3] \cup (4, \infty)$$

$$3. (a) \text{ We have, } \frac{N}{x_1 + x_2 + x_3 + x_4} = \frac{1000x_1 + 100x_2 + 10x_3 + x_4}{x_1 + x_2 + x_3 + x_4} = 1000 - \left(\frac{900x_2 + 990x_3 + 999x_4}{x_1 + x_2 + x_3 + x_4} \right)$$

$$\therefore \text{ Maximum value is } 1000.$$

$$4. (b) \text{ If } 4 - 5i \text{ is the root of } x^2 + ax + b = 0, \text{ then } 4 + 5i \text{ is also the root.}$$

$$\therefore \text{ Sum of roots} = -a = 8 \Rightarrow a = -8$$

$$\text{and product of roots} = b = 16 + 25 = 41$$

$$\therefore (a, b) = (-8, 41)$$

$$5. (b)$$

$$6. (c) \text{ Since, } \alpha, \beta \text{ are the roots of the equation } ax^2 + bx + c = 0.$$

$$\text{Then, } \alpha = -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a} \quad \dots(i)$$

$$\text{and } \beta = -\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\text{Also, } \alpha + k, \beta + k \text{ are the roots of the equation } px^2 + qx + r = 0.$$

$$\text{Then, } \alpha + k = -\frac{q}{2p} + \frac{\sqrt{q^2 - 4pr}}{2p}$$

$$\text{and } \beta + k = -\frac{q}{2p} - \frac{\sqrt{q^2 - 4pr}}{2p}$$

$$\Rightarrow k = -\frac{q}{2p} + \frac{\sqrt{q^2 - 4pr}}{2p} + \frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a} \quad [\text{from Eq. (i)}]$$

$$\text{and } k = -\frac{q}{2p} - \frac{\sqrt{q^2 - 4pr}}{2p} + \frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a} \quad [\text{from Eq. (i)}]$$

$$\Rightarrow \frac{\sqrt{q^2 - 4pr}}{2p} - \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-\sqrt{q^2 - 4pr}}{2p} + \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \frac{\sqrt{q^2 - 4pr}}{p} = \frac{\sqrt{b^2 - 4ac}}{a} \Rightarrow \frac{q^2 - 4pr}{p^2} = \frac{b^2 - 4ac}{a^2}$$

$$\therefore \frac{b^2 - 4ac}{q^2 - 4pr} = \left(\frac{a}{p} \right)^2$$

$$7. (b) \text{ Since, } \alpha \text{ and } \beta \text{ are the roots of } x^2 - 2x + 4 = 0.$$

$$\therefore \alpha + \beta = 2 \text{ and } \alpha\beta = 4$$

$$\text{Now, } (\alpha - \beta) = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} = \sqrt{4 - 16} = 2\sqrt{3}i$$

$$\text{On solving, we get}$$

$$2\alpha = 2 + 2\sqrt{3}i$$

$$\Rightarrow \alpha = 2 \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) = 2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$\text{and } \beta = \frac{2 - 2\sqrt{3}i}{2} = 2 \left(\cos \frac{\pi}{3} - i \sin \frac{\pi}{3} \right)$$

$$\therefore \alpha^n + \beta^n = \left[2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) \right]^n + \left[2 \left(\cos \frac{\pi}{3} - i \sin \frac{\pi}{3} \right) \right]^n$$

$$= 2^n \left[2 \cos \frac{n\pi}{3} \right] = 2^{n+1} \cos \frac{n\pi}{3}$$

$$8. (c) \text{ The roots of the given equation are}$$

$$\alpha = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\text{and } \beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Since, } \alpha, \beta \text{ are real and distinct, therefore } b^2 - 4ac > 0.$$

$$\text{If } a > 0, \text{ then } \beta > -\frac{b}{2a} \text{ and if } a < 0, \text{ then } a > -\frac{b}{2a}. \text{ Thus, one of the roots exceeds } -\frac{b}{2a}.$$

$$9. (d) \text{ Let } z = x + iy, \text{ then}$$

$$z^2 + \bar{z} = 0$$

$$\Rightarrow (x^2 - y^2 + 2ixy) + (x - iy) = 0$$

$$\Rightarrow x^2 - y^2 + x = 0 \text{ and } 2xy - y = 0$$

$$\text{Now, } 2xy - y = 0 \Rightarrow y(2x - 1) = 0$$

$$\Rightarrow y = 0 \text{ or } x = 1/2$$

$$\text{If } y = 0, \text{ then}$$

$$x^2 - y^2 + x = 0$$

$$\Rightarrow x^2 + x = 0$$

$$\Rightarrow x = 0 \text{ or } x = -1$$

$$\text{If } x = 1/2, \text{ then } x^2 - y^2 + x = 0$$

$$\Rightarrow y = \pm \frac{\sqrt{3}}{2}$$

$$\text{Thus, the given equation has four solutions.}$$

10. (a)

11. (d) Given, $\alpha + \beta = -2$ and $\alpha^3 + \beta^3 = -56$

$$\Rightarrow (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta) = -56 \Rightarrow \alpha^2 + \beta^2 - \alpha\beta = 28$$

$$\text{Now, } (\alpha + \beta)^2 = (-2)^2 \Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 4$$

$$\Rightarrow 28 + 3\alpha\beta = 4 \Rightarrow \alpha\beta = -8$$

$\therefore$ Required equation is

$$x^2 - (-2)x + (-8) = 0 \Rightarrow x^2 + 2x - 8 = 0$$

12. (d) Given equation is $x^3 + 2x^2 - 4x + 1 = 0$

Let α, β and γ be the roots of the given equation.

$$\text{Then, } \alpha + \beta + \gamma = -2$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = -4 \text{ and } \alpha\beta\gamma = -1$$

Let the required cubic equation has the roots $3\alpha, 3\beta$ and 3γ .

$$\Rightarrow 3\alpha + 3\beta + 3\gamma = -6$$

$$3\alpha \cdot 3\beta + 3\beta \cdot 3\gamma + 3\gamma \cdot 3\alpha = -36$$

$$\text{and } 3\alpha \cdot 3\beta \cdot 3\gamma = -27$$

$\therefore$ Required equation is

$$x^3 - (-6)x^2 + (-36)x - (-27) = 0$$

$$\Rightarrow x^3 + 6x^2 - 36x + 27 = 0$$

13. (a) Given that, $1 + \sin x \cdot \sin^2 \frac{x}{2} = 0$

$$\therefore 1 + \sin x \left(\frac{1 - \cos x}{2} \right) = 0$$

$$\Rightarrow 2 + \sin x - \sin x \cos x = 0$$

$$\Rightarrow \sin 2x - 2 \sin x = 4$$

Since, the maximum values of $\sin x$ and $\sin 2x$ are 1, which is not possible for any x in $[-\pi, \pi]$.

14. (b) Hint Let α and α^n be the roots of the equation. Then,

$$\alpha + \alpha^n = \frac{-b}{a} \quad \dots(i)$$

$$\text{and } \alpha \cdot \alpha^n = \frac{c}{a} \Rightarrow \alpha^{n+1} = \frac{c}{a}$$

$$\Rightarrow \alpha = \left(\frac{c}{a} \right)^{\frac{1}{n+1}} \quad \dots(ii)$$

15. (a) Hint Here, $\alpha + \beta + \gamma = \frac{3}{2}$... (i)

$$\Rightarrow \alpha^2 + \beta^2 + \gamma^2 + 2[\alpha\beta + \beta\gamma + \gamma\alpha] = \frac{9}{4}$$

16. (a) Since, $\sin A, \sin B$ and $\cos A$ are in GP.

$$\therefore \sin^2 B = \sin A \cos A \quad \dots(i)$$

$$x^2 + 2x \cot B + 1 = 0 \quad [\text{given}]$$

$$\text{Now, } b^2 - 4ac = 4 \cot^2 B - 4$$

$$= \frac{4 \cos^2 B - 4 \sin^2 B}{\sin^2 B}$$

$$= \frac{4(1 - \sin^2 B) - 4 \sin^2 B}{\sin^2 B}$$

$$= \frac{4[1 - 2 \sin^2 B]}{\sin^2 B}$$

$$= \frac{4[1 - 2 \sin A \cos A]}{\sin^2 B} \quad [\text{from Eq. (i)}]$$

$$= 4 \left(\frac{\sin A - \cos A}{\sin B} \right)^2 > 0$$

$\therefore$ Roots are always real.

3

Sequences and Series

Sequences and Series

Sequence is a function whose domain is a subset of N i.e. a set of natural numbers. It displays the images of $1, 2, 3, \dots, n, \dots$ as $f_1, f_2, f_3, \dots, f_n, \dots$, where $f_n = f(n)$. If the terms of a sequence follows certain pattern, then it is called a **progression**. If $a_1, a_2, a_3, \dots, a_n$ is a sequence, then the expression gives the **series** $a_1 + a_2 + a_3 + \dots + a_n$.

Arithmetic Progression (AP)

It is a sequence in which the difference between two consecutive terms is same.

i.e. $a, a + d, a + 2d, a + 3d, \dots$ where, a is the first term and d is the common difference.

n th Term of an AP

Let a be the first term, l be the last term and d be the common difference of a certain sequence in AP. Then, n th term of AP is $T_n = a + (n - 1)d = l$

where,

$$d = T_n - T_{n-1}, n \geq 2, n \in N$$

n th term from last,

$$T'_n = l - (n - 1)d$$

Sum of the First n Terms of an AP

If first term of an AP is a , common difference is d and number of terms is n , then

(i) Sum of n terms, $S_n = \frac{n}{2} [2a + (n - 1)d]$

(ii) When first term and last term are given, then

$$S_n = \frac{n}{2} [a + l]$$

(iii) When last term and the common difference are given, then

$$S_n = \frac{n}{2} [2l - (n - 1)d]$$

Properties of an AP

- If $a_1, a_2, a_3, \dots$ are in AP, then
 - > $a_1 \pm k, a_2 \pm k, a_3 \pm k, \dots$ are also in AP.
 - > $a_1 k, a_2 k, a_3 k, \dots$ and $\frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k}, \dots$ are also in AP.
- If $a_1, a_2, a_3, \dots$ and $b_1, b_2, b_3, \dots$ are two AP's, then
 - > $a_1 \pm b_1, a_2 \pm b_2, a_3 \pm b_3, \dots$ are also in AP.
 - > $a_1 b_1, a_2 b_2, a_3 b_3, \dots$ and $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \dots$ are not in AP.
- If $a_1, a_2, a_3, \dots$ are in AP, then
 - > $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \dots$
 - > $a_r = \frac{a_{r-k} + a_{r+k}}{2}, \forall k; 0 \leq k \leq n-r$
- Three numbers in an AP can be taken as $a-d, a, a+d$.
- Four numbers in an AP can be taken as $a-3d, a-d, a+d, a+3d$
- If n th term of any sequence is linear expression in n such that $t_n = an + b$, then sequence is an AP with common difference a .
- If sum of n terms of any sequence is quadratic expression in n i.e. $S_n = an^2 + bn$, then sequence is an AP.

Geometric Progression (GP)

It is a sequence in which the ratio of any two consecutive terms is same i.e. $a, ar, ar^2, \dots$

where, a is the first term and r is the common ratio.

n th Term of a GP

Let the series be $a, ar, ar^2, ar^3, \dots$

$$\begin{aligned} t_1 &= a = ar^{1-1} \\ t_2 &= ar = ar^{2-1} \\ &\vdots \\ t_n &= ar^{n-1} \end{aligned}$$

(i) So, n th term of a GP,

$$t_n = ar^{n-1} = l \quad \left[\because r = \frac{t_n}{t_{n-1}} \right]$$

(ii) n th term of a GP from the end,

$$t'_n = \frac{l}{r^{n-1}}$$

Sum of n Terms of a GP

Let the first term of a GP be a and common ratio be r , then series is $a, ar, ar^2, \dots, ar^{n-1}$.

$$\text{Let } S = a + ar + ar^2 + \dots + ar^{n-1} \quad \dots(i)$$

$$S \cdot r = ar + ar^2 + \dots + ar^{n-1} + ar^n \quad \dots(ii)$$

On subtracting Eq. (ii) from Eq. (i), we get

$$S(1-r) = a - ar^n, \text{ so}$$

$$(i) S_n = \frac{a(1-r^n)}{1-r}, \text{ when } r < 1$$

$$(ii) S_n = \frac{a(r^n-1)}{r-1}, \text{ when } r > 1$$

(iii) If last term (l) is known, then

$$S_n = \begin{cases} \frac{a-lr}{1-r}, & \text{when } r < 1 \\ \frac{lr-a}{r-1}, & \text{when } r > 1 \end{cases}$$

Sum of Infinite Terms of a GP

When $|r| < 1$ and number of terms is infinite, then

$$\lim_{n \rightarrow \infty} r^n = 0$$

$$\therefore S_\infty = \frac{a}{1-r}, \text{ where } r = \frac{t_n}{t_{n-1}}$$

Properties of a GP

- If $a_1, a_2, a_3, \dots$ are in GP, then
 - > $a_1 k, a_2 k, a_3 k, \dots$ and $\frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k}, \dots$ are also in GP.
 - > $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \frac{1}{a_4}, \dots$ are also in GP.
- If $a_1, a_2, a_3, a_4, \dots$ and $b_1, b_2, b_3, b_4, \dots$ are in GP, then
 - > $a_1 b_1, a_2 b_2, a_3 b_3, \dots$ and $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \dots$ are also in GP.
 - > But $a_1 \pm b_1, a_2 \pm b_2, a_3 \pm b_3, \dots$ are not in GP.
- If $a_1, a_2, a_3, \dots$ are in GP ($a_i > 0, \forall i$), then
 - > $\log a_1, \log a_2, \log a_3, \dots$ are in AP. In this case, the converse is also true.
 - > $a_1^r, a_2^r, a_3^r, \dots$ are also in GP, where $r \in \mathbb{Q}$.
 - > $a_1 a_n = a_2 a_{n-1} = a_3 a_{n-2} = \dots$
 - > $a_r = \sqrt[a_{r-k} \cdot a_{r+k}], \forall k; 0 \leq k \leq n-r$
- If $a_1, a_2, a_3, a_4, \dots, a_{n-1}, a_n$ are in GP, then

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \dots$$

$$= \frac{a_n}{a_{n-1}} = r$$

• Three numbers in GP can be taken as $\frac{a}{r}, a, ar$.

• Four numbers in GP can be taken as $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$.

NOTE If a, b, c are in AP as well as in GP, then $a = b = c$.

Arithmetic Mean (AM)

Let a , x and b be in AP. Then, the arithmetic mean of AP is x and can be written as

$$x = \frac{a+b}{2} = A$$

Similarly, AM of n numbers $a_1, a_2, \dots, a_n$ is

$$\frac{1}{n}(a_1 + a_2 + \dots + a_n)$$

Insertion of n Arithmetic Mean between Two Given Numbers

Let $x_1, x_2, x_3, \dots, x_n$ be n AM's between a and b . Then, $a, x_1, x_2, x_3, \dots, x_n, b$ are in AP and total number of terms are $n+2$. If d is the common difference of an AP, then

$$t_{n+2} = a + (n+2-1)d = b$$

$$d = \frac{b-a}{n+1}$$

Then, $x_r = a + r\left(\frac{b-a}{n+1}\right)$, where $r = 1, 2, 3, \dots$

(i) Sum of n arithmetic means

$$= x_1 + x_2 + x_3 + \dots + x_n = \frac{n}{2}(x_1 + x_n) \quad [\text{sum of AP}]$$

$$= \frac{n}{2}[a + d + b - d] = n\left(\frac{a+b}{2}\right) \quad [\text{AM of } a \text{ and } b]$$

(ii) Sum of n AM's between a and b is nA .

$$\text{i.e. } A_1 + A_2 + A_3 + \dots + A_n = nA$$

(iii) Any three numbers in AP can be taken as $a-d, a, a+d$.

(iv) Any four numbers in AP can be taken as $a-3d, a-d, a+d, a+3d$.

(v) In arithmetic series containing even numbers, we assume, number of terms in such series as $2n$, n th and $(n+1)$ th terms will be two middle terms.

(vi) In arithmetic series containing odd numbers, we assume, number of terms in such series as $(2n+1)$, then $(n+1)$ th term will be middle term.

(vii) Sum of terms from beginning and end of arithmetic series is constant.

NOTE • If p th term of an AP is q and q th term is p , then $T_{p+q} = 0$

• If $S_p = q$ and $S_q = p$ for an AP, then $S_{p+q} = -(p+q)$

> If $S_p = S_q$ for an AP, then $S_{p+q} = 0$.

> If p th term is $\frac{1}{q}$ and q th term is $\frac{1}{p}$, then

$$T_{pq} = 1$$

Geometric Mean (GM)

If three numbers are in GP, then middle one is called the GM between the other two.

Let a , x and b be in GP. Then,

$$\frac{x}{a} = \frac{b}{x}$$

$$\Rightarrow x^2 = ab$$

$$\Rightarrow x = \sqrt{ab} = G$$

Insertion of Geometric Mean between Two Given Numbers

Let a and b be the given numbers and $x_1, x_2, x_3, \dots, x_n$ be n geometric means inserted between them.

Then, $a, x_1, x_2, x_3, \dots, x_n, b$ are in GP.

$$\therefore b = t_{n+2} = ar^{n+1}; r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$x_1 = t_2 = ar$$

$$x_2 = t_3 = ar^2$$

$$\vdots \quad \vdots \quad \vdots$$

$$x_n = t_{n+1} = ar^n$$

$$\therefore x_r = a \left(\frac{b}{a}\right)^{\frac{r}{n+1}}, \text{ where } r = 1, 2, 3, \dots$$

On putting the value of r , we shall find n geometric means. The product of n geometric means between a and b is equal to the n th power of geometric mean of a and b .

$$\text{i.e. } G_1 \cdot G_2 \dots G_n = (ab)^{n/2} = G^n \quad [\because G = \sqrt{ab}]$$

(i) The product of n geometric means between a and $\frac{1}{a}$ is 1.

(ii) Three numbers in GP can be taken as $\frac{a}{r}, a, ar$.

(iii) Four numbers in GP can be taken as $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$.

(iv) Five numbers in GP can be taken as $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$.

NOTE • If A and G are the AM and GM between two numbers a and b , then a, b are given by $[A \pm \sqrt{(A+G)(A-G)}]$

• The product of first n terms of GP is

$$P = a \cdot ar \cdot ar^2 \dots ar^{n-1} = a^n r^{\frac{n(n-1)}{2}}$$

Relation between AM and GM

If a and b are two numbers, then

$$AM = \frac{a+b}{2}$$

and

$$GM = \sqrt{ab}$$

Now,

$$\begin{aligned} AM - GM &= \frac{a+b}{2} - \sqrt{ab} \\ &= \left(\frac{\sqrt{a} - \sqrt{b}}{\sqrt{2}} \right)^2 \geq 0 \end{aligned}$$

$\Rightarrow$

$$AM - GM \geq 0$$

Hence,

$$AM \geq GM$$

NOTE • If A and G are the AM and GM between two positive numbers, then the numbers are $A \pm \sqrt{A^2 - G^2}$.

• If A and G are respectively AM and GM between two positive numbers a and b , then the quadratic equation having a and b as its roots, is $x^2 - 2Ax + G^2 = 0$.

Arithmetico-Geometric Progression (AGP)

A series in which each term is the product of the corresponding terms of an AP and GP, is called arithmetico-geometric progression (AGP).

e.g. $1 + 3x + 5x^2 + 7x^3 + \dots$

Here, $1, 3, 5, \dots$ are in AP and $1, x, x^2, \dots$ are in GP.

Sum of n Terms of AGP

Let $a_1, a_2, \dots, a_n$ be in AP and $b_1, b_2, \dots, b_n$ be in GP.

Then, $a_1b_1, a_2b_2, \dots, a_nb_n$ will be in AGP. If the first term of an AP is a , common difference is d and first term of a GP is b , common ratio is r , then

$ab, (a+d)br, (a+2d)br^2, \dots$ are in AGP.

$$\text{Now, } S_n = \frac{ab}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]br^n}{1-r}, r \neq 1$$

If $-1 < r < 1$, then $\lim_{n \rightarrow \infty} r^n = 0$

and $\lim_{n \rightarrow \infty} nr^n = 0$

$$\therefore \text{Sum to infinity, } S_\infty = \frac{ab}{1-r} + \frac{dbr}{(1-r)^2} \quad [\because -1 < r < 1]$$

Harmonic Progression (HP)

A sequence is said to be a harmonic progression, if reciprocal of terms of the sequence form an AP i.e.

$a_1, a_2, a_3, \dots, a_n$ are in HP, if $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \dots, \frac{1}{a_n}$ are in AP.

n th term of HP from beginning,

$$T_n = \frac{1}{\frac{1}{a_1} + (n-1)\left(\frac{1}{a_2} - \frac{1}{a_1}\right)} = \frac{a_1a_2}{a_2 + (n-1)(a_1 - a_2)}$$

n th term of HP from end,

$$\begin{aligned} T'_n &= \frac{1}{\frac{1}{a_n} - (n-1)\left(\frac{1}{a_2} - \frac{1}{a_1}\right)} \\ &= \frac{a_1a_2a_n}{a_1a_2 - a_n(n-1)(a_1 - a_2)} \end{aligned}$$

Harmonic Mean (HM)

If two numbers a and b are in HP, then harmonic mean (HM) between them is given by $HM = \frac{2ab}{a+b}$.

Insertion of Harmonic Mean

If $a, x_1, x_2, \dots, x_n, b$ are in HP, then $x_1, x_2, \dots, x_n$ are n harmonic means between a and b .

These are the reciprocal of n arithmetic means between $\frac{1}{a}$ and $\frac{1}{b}$.

Hence, $x_1 = \frac{(n+1)ab}{a+nb}$, $x^2 = \frac{(n+1)}{2a+(n-1)b}$, ... and so on.

Relation between AM, GM and HM

Let a and b be two real positive unequal numbers, then AM, GM and HM between them are respectively,

$$A = \frac{a+b}{2}, G = \sqrt{ab}, H = \frac{2ab}{a+b} \Rightarrow G^2 = AH$$

Also,

$$A > G > H$$

Hence, G lies between A and H .

NOTE If A, G and H are arithmetic, geometric and harmonic means between three given numbers a, b and c , then the equation having a, b, c as its roots, is

$$x^3 - 3Ax^2 + \frac{3G^3}{H}x - G^3 = 0$$

Sum upto n Terms of Special Series $\sum n$, $\sum n^2$, $\sum n^3$, etc.

(i) Sum of first n natural numbers

$$\sum_{k=1}^n k = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

(ii) Sum of the squares of first n natural numbers

$$\sum_{k=1}^n k^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6} n(n+1)(2n+1)$$

(iii) Sum of the cubes of first n natural numbers

$$\sum_{k=1}^n k^3 = 1^3 + 2^3 + \dots + n^3 = \left(\sum_{k=1}^n k \right)^2 = \left[\frac{1}{2} n(n+1) \right]^2$$

(iv) Sum of product of first n natural numbers taken two at a time is

$$\frac{1}{2} [\sum n^3 - \sum n^2] = \frac{n(n+1)(n-1)(3n+2)}{24}$$

(v) (a) Sum of first even natural numbers

$$2 + 4 + 6 + \dots + 2n = n(n+1)$$

(b) Sum of first odd natural numbers

$$1 + 3 + 5 + \dots + (2n-1) = n^2$$

(vi) Sum of n terms of series

$$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \dots = \begin{cases} \frac{n(n+1)}{2}, & \text{when } n \text{ is odd} \\ \frac{-n(n+1)}{2}, & \text{when } n \text{ is even} \end{cases}$$

NOTE

- $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$
- $\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \dots + \frac{1}{n(n+1)(n+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$
- $1 \cdot 2 + 2 \cdot 3 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$
- $1 \cdot 2 \cdot 3 \cdot 4 + 2 \cdot 3 \cdot 4 \cdot 5 + \dots + n(n+1)(n+2)(n+3) = \frac{1}{5} n(n+1)(n+2)(n+3)(n+4)$

Practice Exercise

- The number of numbers lying between 100 and 500 that are divisible by 7 but not by 21, is
a. 57 b. 19 c. 38 d. 40
- Let S_n denotes the sum of n terms of an AP, whose first term is a . If the common difference $d = S_n - kS_{n-1} + S_{n-2}$, then k is equal
a. 3 b. 2 c. 5 d. 7
- The sum upto n terms of the sequence $\log a, \log ar, \log ar^2, \dots$ is
a. $\frac{n}{2} \log a^2 r^{n-1}$ b. $\frac{n}{2} \log ar^{n-1}$
c. $\frac{3n}{2} \log ar^{n-1}$ d. $\frac{5n}{2} \log a^2 r^{n-1}$
- If m times the m th term of an AP with non-zero common difference equals n times the n th term of an AP, where $m \neq n$, then $(m+n)$ th term of this AP is
a. mn b. zero
c. $2mn$ d. None
- Let T_r be the r th term of an AP, for $r = 1, 2, 3, \dots$. If for some positive integers m and n , $t_m = \frac{1}{n}$, $t_n = \frac{1}{m}$, then m th term of an AP is equal to
a. $\frac{1}{m}$ b. $\frac{1}{m} + \frac{1}{n}$
c. 1 d. None
- The first, second and middle terms of an AP are a, b, c respectively. Then, their sum is equal to
a. $\frac{2(c-a)}{b-a}$ b. $\frac{2c(c-a)}{b-a} + c$
c. $\frac{2c(b-a)}{c-a}$ d. None of these
- The number of common terms to the two sequences 17, 21, 25, ..., 417 and 16, 21, 26, ..., 466 is
a. 21 b. 19 c. 20 d. 91
- The least value of a for which $5^{1+x} + 5^{1-x}, \frac{a}{2}, 25^x + 25^{-x}$ are three consecutive terms of an AP, is
a. 10 b. 5
c. 12 d. None of these
- Let $a_1, a_2, a_3, \dots$ be in AP with common difference not multiple of 3. Then, the maximum number of consecutive terms so that all are prime number, is
a. 2 b. 3
c. 5 d. infinite
- If $\log_3 2, \log_3 (2^x - 5)$ and $\log_3 \left(2^x - \frac{7}{2} \right)$ are in AP, then x is equal to
a. 2 b. 3
c. 4 d. 2, 3

11. If the sum of n terms of an AP is given by $S_n = a + bn + cn^2$, where a, b and c are independent of n , then
 a. $a \neq 0$
 b. $d \neq 3b$
 c. $d = 2c$
 d. first term of an AP is $b - c$
12. A farmer buys a used tractor of ₹12000. He pays ₹6000 cash and agrees to pay the balance in annual installment of ₹500 plus 12% interest on the unpaid amount. How much will the tractor cost him?
 a. ₹16680 b. ₹16670 c. ₹16681 d. ₹16682
13. In a GP of positive terms, if any term is equal to the sum of the next two terms. Then, the common ratio of the GP is
 a. $\sin 18^\circ$ b. $2 \cos 18^\circ$ c. $\cos 18^\circ$ d. $2 \sin 18^\circ$
14. Let S be the sum, P be the product and R be the sum of the reciprocals of 3 terms of a GP. Then, $P^2 R^3 : S^3$ is equal to
 a. $1 : 1$
 b. $(\text{common ratio})^n : 1$
 c. $(\text{first term})^2 : (\text{common ratio})^2$
 d. None of the above
15. The sum of three numbers in GP is 56. If we subtract 1, 7 and 21 from these numbers in that order, we obtain an AP. Then, the numbers are
 a. 8, 16, 32 b. 8, 16, 30 c. 16, 0, 30 d. 32, 16, 9
16. The product of $(32)(32)^{1/6}(32)^{1/36} \dots$, is equal to
 a. 16 b. 64 c. 32 d. 0
17. If S denotes the sum of infinity and S_n denotes the sum of n terms of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$, such that $S - S_n < \frac{1}{1000}$, then the least value of n is
 a. 8 b. 9 c. 10 d. 11
18. Let a_n be the n th term of a GP of positive integers. If $\sum_{n=1}^{100} a_{2n} = \alpha$ and $\sum_{n=1}^{100} a_{2n+1} = \beta$ such that $\alpha \neq \beta$, then the common ratio is
 a. $\frac{\alpha}{\beta}$
 b. $\frac{\beta}{\alpha}$
 c. $\left(\frac{\alpha}{\beta}\right)^{1/2}$
 d. $\left(\frac{\beta}{\alpha}\right)^{1/2}$
19. Let $S_1, S_2, \dots$ be squares such that for each $n \geq 1$, the length of a side of S_n equals the length of a diagonal of S_{n+1} . If the length of a side of S_1 is 10 cm, then for which of the following values of n is the area of S_n less than 1 sq cm?
 a. 7 b. 8 c. 9 d. 10
20. If one GM, g and two AM's, p and q are inserted between two numbers a and b , then $(2p - q)(p - 2q)$ is equal to
 a. g^2
 b. $-g^2$
 c. $2g$
 d. $3g^2$
21. If a, b, c are in GP and x, y are the AM's between a, b and b, c respectively, then
 a. $\frac{a}{x} + \frac{c}{y} = 2$
 b. $\frac{a}{x} + \frac{2c}{y} = \frac{c}{2a}$
 c. $\frac{1}{x} + \frac{1}{y} = \frac{3}{b}$
 d. $\frac{2}{x} + \frac{1}{2y} = \frac{3}{ac}$
22. If x, y and z are positive integers, then the value of $(x + y)(y + z)(z + x)$
 a. $= 8xyz$
 b. $> 8xyz$
 c. $< 8xyz$
 d. $= 4xyz$
23. The minimum value of the expression $3^x + 3^{1-x}$, $x \in R$, is
 a. 0
 b. $\frac{1}{3}$
 c. 3
 d. $2\sqrt{3}$
24. If A_1, A_2 are two AM's and G_1, G_2 are two GM's between a and b , then $\frac{A_1 + A_2}{G_1 G_2}$ is equal to
 a. $\frac{a+b}{2ab}$
 b. $\frac{2ab}{a+b}$
 c. $\frac{a+b}{ab}$
 d. $\frac{a+b}{\sqrt{ab}}$
25. If a, b, c are in GP, then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, if $d/a, e/b, f/c$ are in
 a. GP
 b. AP
 c. HP
 d. None of these
26. If the sum to infinity of the series $1 + 4x + 7x^2 + 10x^3 + \dots$ is $\frac{35}{16}$. Then, the value of x is
 a. $2/5$
 b. $1/5$
 c. $3/5$
 d. None of these
27. If $\cos(x - y)$, $\cos x$ and $\cos(x + y)$ are in HP, then $\cos x \sec(y/2)$ is equal to
 a. $\pm\sqrt{2}$
 b. $\pm 1/\sqrt{2}$
 c. ± 2
 d. None of the above
28. If AM, GM and HM of first and last terms of the series 25, 26, 27, $\dots$, $N - 1, N$ are the terms of the series, then find the value of N .
 a. 25
 b. 225
 c. 1225
 d. None of the above

BITSAT Archives

1. If p, q, r and s are positive real numbers such that $p + q + r + s = 2$, then $M = (p + q)(r + s)$ satisfies the relation [2014]
 - a. $0 < M \leq 1$
 - b. $1 \leq M \leq 2$
 - c. $2 \leq M \leq 3$
 - d. $3 \leq M \leq 4$
2. The sum of the series [2013]

$$1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \dots + 100 \cdot 2^{99}$$
 is
 - a. $100 \cdot 2^{100} + 1$
 - b. $99 \cdot 2^{100} + 1$
 - c. $99 \cdot 2^{99} - 1$
 - d. $100 \cdot 2^{100} - 1$
3. The sum of the sequence $5 + 55 + 555 + \dots$ upto n terms is [2013]
 - a. $\frac{5}{9} \left[\frac{10(10^n - 1)}{9} + n \right]$
 - b. $\frac{5}{9} \left[\frac{10(10^n - 1)}{9} - n \right]$
 - c. $\frac{5}{9} \left[\frac{10(10^{n+1} - 1)}{9} - n \right]$
 - d. $\frac{5}{9} \left[\frac{10(10^{n-1} - 1)}{9} - n \right]$
4. If a, b, c are in GP and $a^{1/x} = b^{1/y} = c^{1/z}$, then x, y, z are in [2013]
 - a. AP
 - b. GP
 - c. HP
 - d. None of these
5. The sum of n terms of the series $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \dots$ is [2012]
 - a. $n - 1 + 2^{-n}$
 - b. 1
 - c. $n - 1$
 - d. $1 + 2^{-n}$
6. $0.2 + 0.22 + 0.222 + \dots$ upto n terms is equal to [2012]
 - a. $\left(\frac{2}{9}\right) - \left(\frac{2}{81}\right)(1 - 10^{-n})$
 - b. $n - \left(\frac{1}{9}\right)(1 - 10^{-n})$
 - c. $\left(\frac{2}{9}\right) \left[n - \left(\frac{1}{9}\right)(1 - 10^{-n}) \right]$
 - d. $\frac{2}{9}$
7. If AM and HM between two numbers are 27 and 12 respectively, then their GM is [2011]
 - a. 9
 - b. 18
 - c. 24
 - d. 36
8. The value of $\frac{2}{1!} + \frac{2+4}{2!} + \frac{2+4+6}{3!} + \dots$ is [2011]
 - a. e
 - b. $2e$
 - c. $3e$
 - d. None of these
9. Let $a_1, a_2, \dots, a_{10}$ be in AP and $h_1, h_2, \dots, h_{10}$ be in HP. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then $a_4 h_7$ is [2010]
 - a. 2
 - b. 3
 - c. 5
 - d. 6
10. The sum of the infinite series [2009]

$$1 + \frac{2}{3} \cdot \frac{1}{2} + \frac{2}{3} \cdot \frac{5}{6} \cdot \frac{1}{2^2} + \frac{2}{3} \cdot \frac{5}{6} \cdot \frac{8}{9} \cdot \frac{1}{2^3} + \dots,$$
 is
 - a. $2^{1/3}$
 - b. $4^{1/3}$
 - c. $8^{1/3}$
 - d. $2^{1/5}$
11. The HM of two numbers is 4. If their arithmetic mean A and geometric mean G satisfy the relation $2A + G^2 = 27$, then the numbers are [2009]
 - a. 2, 6
 - b. 3, 6
 - c. 1, 3
 - d. 1, 2
12. For any integer $n \geq 1$, $\sum_{k=1}^n k(k+2)$ is equal to [2008]
 - a. $\frac{n(n+1)(n+2)}{6}$
 - b. $\frac{n(n+1)(2n+1)}{6}$
 - c. $\frac{n(n+1)(2n+7)}{6}$
 - d. $\frac{n(n+1)(2n+9)}{6}$
13. In $\triangle ABC$, if $\frac{1}{b+c} + \frac{1}{c+a} = \frac{3}{a+b+c}$, then C is equal to [2008]
 - a. 90°
 - b. 60°
 - c. 45°
 - d. 30°
14. The sum of 24 terms of the series $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \dots$, is [2007]
 - a. 300
 - b. $200\sqrt{2}$
 - c. $300\sqrt{2}$
 - d. $250\sqrt{2}$
15. The sum to n terms of the infinite series $1 \cdot 3^2 + 2 \cdot 5^2 + 3 \cdot 7^2 + \dots$ is [2007]
 - a. $\frac{n}{6}(n+1)(6n^2 + 14n + 7)$
 - b. $\frac{n}{6}(n+1)(2n+1)(3n+1)$
 - c. $4n^3 + 4n^2 + n$
 - d. None of the above
16. If $a = \log_2 3$, $b = \log_2 5$ and $c = \log_7 2$, then $\log_{140} 63$ in terms of a, b, c is [2007]
 - a. $\frac{2ac + 1}{2c + abc + 1}$
 - b. $\frac{2ac + 1}{2a + c + a}$
 - c. $\frac{2ac + 1}{2c + ab + a}$
 - d. None of these
17. When 2^{301} is divided by 5, then the least positive remainder is [2006]
 - a. 4
 - b. 8
 - c. 2
 - d. 6
18. Let α, β, γ and δ be four positive real numbers such that their product is unity, then the least value of $(1+\alpha)(1+\beta)(1+\gamma)(1+\delta)$ is [2005]
 - a. 6
 - b. 16
 - c. 0
 - d. 32

Answer with Solutions

Practice Exercise

1. (c) The numbers between 100 and 500 that are divisible by 7, are 105, 112, 119, 126, ..., 490, 497.

Let such numbers be n .

$$\therefore t_n = a_n + (n-1)d \Rightarrow 497 = 105 + (n-1) \times 7$$

$$\Rightarrow n-1 = 56 \therefore n = 57$$

The numbers between 100 and 500 that are divisible by 21, are 105, 126, 147, ..., 483.

Let such numbers be m .

$$\therefore 483 = 105 + (m-1) \times 21 \Rightarrow 18 = m-1 \Rightarrow m = 19$$

$$\therefore \text{Required number} = n - m = 57 - 19 = 38$$

2. (b) We know that, $t_n = S_n - S_{n-1}$... (i)

$$\text{and } t_{n-1} = S_{n-1} - S_{n-2} \quad \dots (ii)$$

$$\text{Now, } d = t_n - t_{n-1} \Rightarrow d = (S_n - S_{n-1}) - (S_{n-1} - S_{n-2})$$

$$\Rightarrow d = S_n - 2S_{n-1} + S_{n-2}$$

$$\text{Hence, } k = 2$$

3. (a) The sequence can be rewritten as

$$\log a, (\log a + \log r), (\log a + 2 \log r), \dots$$

which is an AP, where $A = \log a$, $D = \log r$

$$\text{Now, } t_n = A + (n-1)D = \log a + (n-1) \log r = l$$

$$\therefore S_n = \frac{n}{2} (A + l) = \frac{n}{2} [\log a + \log a + (n-1) \log r]$$

$$= \frac{n}{2} \log a^2 r^{n-1}$$

4. (b) Hint $ma_m = na_n \Rightarrow m[a + (m-1)d] = n[a + (n-1)d]$

5. (c) Let first term be a and common difference be d of an AP.

$$\text{Then, } T_m - T_n = \frac{1}{n} - \frac{1}{m} \Rightarrow (m-n)d = \frac{m-n}{mn}$$

$$\Rightarrow d = \frac{1}{mn}, \text{ so } a = \frac{1}{mn}$$

$$\therefore T_{mn} = a + (mn-1)d = \frac{1}{mn} + mn \cdot \frac{1}{mn} - \frac{1}{mn} = 1$$

6. (b) We have, first term = a , second term = b , difference (d) = $b - a$ and middle term = c .

This means that there are an odd number of terms in AP.

Let there be $(2n+1)$ terms in AP.

Then, $(n+1)$ th term is the middle term.

$$\text{Middle term, } c = a + nd \Rightarrow c = a + n(b-a) \Rightarrow n = \frac{c-a}{b-a}$$

$$\therefore \text{Sum} = \frac{2n+1}{2} [2a + (2n+1-1)d]$$

$$\therefore S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{1}{2} \left\{ 2 \left(\frac{c-a}{b-a} \right) + 1 \right\} \left\{ 2a + 2 \left(\frac{c-a}{b-a} \right) (b-a) \right\}$$

$$= \frac{1}{2} \left[\frac{2(c-a)}{b-a} + 1 \right] 2c = \frac{2c(c-a)}{b-a} + c$$

7. (c) Let m th term of the first sequence be equal to n th term of the second sequence. Then,

$$17 + 4(m-1) = 16 + 5(n-1) \Rightarrow 4m + 13 = 5n + 11$$

$$\Rightarrow 4m + 2 = 5n \Rightarrow n = m - (1/5)(m-2)$$

Since, n is an integer, so $m-2$ must be a multiple of 5 and must be of the form $5k+2$ with $k \geq 0$.

Then, $n = 4k+2$. The first sequence has 101 terms and the second has 91 terms.

$$\therefore 0 \leq 5k+2 < 101 \text{ and } 0 \leq 4k+2 \leq 91$$

$$\Rightarrow 0 \leq k \leq 19$$

Hence, the given sequence have 20 common terms.

8. (c) $5^{1+x} + 5^{1-x}, \frac{a}{2}, 25^x + 25^{-x}$ are in AP.

$$\Rightarrow 2 \cdot \frac{a}{2} = 5^{1+x} + 5^{1-x} + 25^x + 25^{-x}$$

$$\Rightarrow a = 5 \cdot 5^x + 5 \cdot 5^{-x} + (5^{2x} + 5^{-2x})$$

$$\Rightarrow a = 5 \left(5^x + \frac{1}{5^x} \right) \left(5^{2x} + \frac{1}{5^{2x}} \right)$$

We know that the sum of a positive real number and its reciprocal is always greater than or equal to 2.

$$\therefore \left(5^x + \frac{1}{5^x} \right) \geq 2 \text{ and } \left(5^{2x} + \frac{1}{5^{2x}} \right) \geq 2 \text{ for all } x$$

$$\Rightarrow a = 5 \left(5^x + \frac{1}{5^x} \right) + \left(5^{2x} + \frac{1}{5^{2x}} \right) \geq 5(2) + 2 \Rightarrow a \geq 12, \forall x$$

9. (d) $\because a_2 - a_1 = a_3 - a_2 = \dots$ = common difference

$$\neq \text{multiple of } 3 = d$$

[say]

Then, consecutive terms are

$$a_1, a_1 + d, a_1 + 2d, a_1 + 3d, \dots$$

$\therefore d$ is not a multiple of 3.

$$\text{Let } d = 3\lambda + 1 \text{ or } 3\lambda + 2$$

$\therefore$ Consecutive terms

$$a_1, a_1 + 1 + 3\lambda, a_1 + 2 + 6\lambda, a_1 + 3 + 9\lambda, \dots \text{ are all prime.}$$

Hence, number of terms is infinite.

10. (b) $\because 2 \log_3(2^x - 5) = \log_3 2 + \log_3 \left(2^x - \frac{7}{2} \right)$

$$\therefore (2^x - 5)^2 = 2 \left(2^x - \frac{7}{2} \right)$$

$$\Rightarrow t^2 + 25 - 10t = 2t - 7 \quad [\text{put } 2^x = t]$$

$$\Rightarrow t^2 - 12t + 32 = 0 \Rightarrow (t-8)(t-4) = 0$$

$$\Rightarrow 2^x = 8 \text{ or } 2^x = 4$$

$$\therefore x = 3 \text{ or } x = 2$$

At $x = 2, \log_3(2^x - 5)$ is not defined.

Hence, $x = 3$ is the only solution.

11. (c) $S_n = \frac{n}{2} [2a' + (n-1)d] = a + bn + cn^2$

$$\Rightarrow na' + \frac{n(n-1)}{2} d = a + bn + cn^2$$

$$\Rightarrow \left(a' - \frac{d}{2}\right)n + \frac{n^2 d}{2} = a + bn + cn^2$$

$$\Rightarrow a = 0, b = a' - \frac{d}{2}, c = \frac{d}{2} \Rightarrow d = 2c$$

12. (a) Tractor cost = ₹ 12000, Down payment = ₹ 6000
Balance amount = ₹ 6000

$$\text{Interest on Ist installment} = \frac{6000 \times 12 \times 1}{100} \left[\because I = \frac{P \times R \times T}{100} \right]$$

$$= ₹ 720$$

$$\text{Now, unpaid amount} = 6000 - 500 = ₹ 5500$$

$$\text{Interest on IInd installment} = \frac{5500 \times 12 \times 1}{100} = ₹ 660$$

$$\text{Again, unpaid amount} = 5500 - 500 = ₹ 5000$$

$$\text{Interest on IIIrd installment} = \frac{5000 \times 12 \times 1}{100} = ₹ 600$$

$$\text{Total interest paid by him} \\ = 720 + 660 + 600 + \dots + 12 \text{ terms}$$

which is an AP with

$$a = 720, d = 660 - 720 = -60$$

Therefore, total interest

$$= \frac{12}{2} [2 \times 720 + (12 - 1)(-60)] \\ = 6 [1440 - 11 \times 60] = 6 [1440 - 660] \\ = 6 \times 780 = ₹ 4680$$

$$\text{Hence, total amount or actual cost} \\ = 12000 + 4680 = ₹ 16680$$

13. (d) $\because t_n = t_{n+1} + t_{n+2}$

$$\therefore ar^{n-1} = ar^n + ar^{n+1} \Rightarrow 1 = r + r^2$$

$$\Rightarrow r = \frac{-1 \pm \sqrt{5}}{2} \quad [r > 0]$$

$$\text{Therefore, } r = 2 \frac{\sqrt{5} - 1}{4} = 2 \sin 18^\circ$$

14. (a) Let us take a GP with three terms $\frac{a}{r}, a, ar$. Then,

$$S = \frac{a}{r} + a + ar = \frac{a(r^2 + r + 1)}{r}$$

$$\text{and } P = a^3, R = \frac{r}{a} + \frac{1}{a} + \frac{1}{ar} = \frac{1}{a} \left(\frac{r^2 + r + 1}{r} \right)$$

$$\therefore \frac{P^2 R^3}{S^3} = \frac{a^6 \cdot \frac{1}{a^3} \left(\frac{r^2 + r + 1}{r} \right)^3}{a^3 \left(\frac{r^2 + r + 1}{r} \right)^3} = 1$$

Therefore, the ratio is 1:1.

15. (a) Let three numbers in GP be a, ar, ar^2 .

$$\text{Then, } a + ar + ar^2 = 56 \quad \dots(i)$$

Again, $a - 1, ar - 7, ar^2 - 21$ are in AP.

$$\Rightarrow 2(ar - 7) = (a - 1) + (ar^2 - 21)$$

[if a, b and c are in AP, then $2b = a + c$]

$$\Rightarrow 2ar - 14 = a + ar^2 - 22$$

$$\Rightarrow a + ar^2 - 2ar = -14 + 22$$

$$\Rightarrow a + ar^2 - 2ar = 8 \quad \dots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{a + ar + ar^2}{a + ar^2 - 2ar} = \frac{56}{8}$$

$$\Rightarrow \frac{1 + r + r^2}{1 + r^2 - 2r} = \frac{7}{1}$$

$$\Rightarrow 1 + r + r^2 = 7 + 7r^2 - 14r$$

$$\Rightarrow 6r^2 - 15r + 6 = 0$$

$$\Rightarrow 2r^2 - 5r + 2 = 0 \quad [\text{dividing by 3}]$$

On factorising it by splitting the middle term, we get

$$2r^2 - (4 + 1)r + 2 = 0$$

$$\Rightarrow 2r^2 - 4r - r + 2 = 0$$

$$\Rightarrow 2r(r - 2) - (r - 2) = 0$$

$$\Rightarrow (r - 2)(2r - 1) = 0$$

$$\Rightarrow r = 2, \frac{1}{2}$$

If $r = 2$, then from Eq. (i), we get

$$a + 2a + 4a = 56$$

$$\Rightarrow 7a = 56 \Rightarrow a = 8$$

Then, numbers are $a = 8, ar = 8 \times 2 = 16$

and $ar^2 = 8 \times 4 = 32$

i.e. 8, 16, 32

If $r = \frac{1}{2}$, then from Eq. (i), we get

$$\frac{a}{1} + \frac{a}{2} + \frac{a}{4} = 56 \Rightarrow \frac{4a + 2a + a}{4} = 56$$

$$\Rightarrow \frac{7a}{4} = 56 \Rightarrow a = 32$$

Then, numbers are

$$a = 32, ar = 32 \times \frac{1}{2} = 16$$

$$\text{and } ar^2 = 32 \times \frac{1}{4} = 8$$

i.e. 32, 16, 8

Hence, required numbers are 8, 16, 32 or 32, 16, 8.

16. (b) We have, $(32)(32)^{1/6}(32)^{1/36} \dots$

$$= (32)^{1 + \frac{1}{6} + \frac{1}{36} + \dots} = (32)^t$$

$$\text{where, } t = 1 + \frac{1}{6} + \frac{1}{36} + \dots = \frac{1}{1 - \frac{1}{6}} \quad \left[\because S_\infty = \frac{a}{1 - r} \right]$$

$$= \frac{6}{5}$$

$$\therefore \text{Product} = (2^5)^{6/5} = 2^6 = 64$$

17. (d) $S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$

$$S = \frac{1}{1 - \frac{1}{2}} = 2$$

$$\left[\because S_\infty = \frac{a}{1 - r} \right]$$

$$S_n = \frac{1 \left[1 - \left(\frac{1}{2} \right)^n \right]}{1/2} = 2 - \left(\frac{1}{2} \right)^{n-1} \quad \left[\because S_n = \frac{a(1-r^n)}{1-r} \right]$$

$$\text{Now, } S - S_n = \left(\frac{1}{2} \right)^{n-1} < \frac{1}{1000}$$

$$\Rightarrow n = 9$$

$$\text{but } 2^9 = 512 < 1000$$

$$\text{However, } 2^{10} = 1024 > 1000$$

$$\therefore n - 1 = 10 \Rightarrow n = 11$$

18. (b) Let r be the common ratio of the GP and a be the first term of the GP i.e. $a_1 = a$.

$$\therefore a_2 = ar, a_3 = ar^2, a_4 = ar^3$$

$$\text{Now, } \sum_{n=1}^{100} a_{2n} = a_2 + a_4 + a_6 + \dots + a_{200} = \alpha$$

$$\Rightarrow ar + ar^3 + \dots + ar^{199} = \alpha$$

$$\Rightarrow \frac{ar[1 - (r^2)^{100}]}{1 - r^2} = \alpha$$

$$\Rightarrow \frac{ar(1 - r^{200})}{1 - r^2} = \alpha \quad \dots(i)$$

$$\text{Again, } \sum_{n=1}^{100} a_{2n+1} = a_3 + a_5 + \dots + a_{201} = \beta$$

$$\Rightarrow ar^2 + ar^4 + \dots + ar^{200} = \beta$$

$$\Rightarrow \frac{ar^2[1 - (r^2)^{100}]}{1 - r^2} = \beta$$

$$\Rightarrow \frac{ar^2(1 - r^{200})}{1 - r^2} = \beta \quad \dots(ii)$$

$$\text{Dividing Eq. (ii) by Eq. (i), we get } r = \frac{\beta}{\alpha}$$

19. (b) We have,

$$\text{Length of a side of } S_n = \text{Length of a diagonal of } S_{n+1}$$

$$\Rightarrow \text{Length of a side of } S_n = \sqrt{2} \text{ (Length of a side of } S_{n+1})$$

$$\Rightarrow \frac{\text{Length of a side of } S_{n+1}}{\text{Length of side of } S_n} = \frac{1}{\sqrt{2}} \text{ for all } n \geq 1$$

$$\therefore \text{Sides of } S_1, S_2, \dots, S_n \text{ form a GP with common ratio } \frac{1}{\sqrt{2}}$$

and first term 10.

$$\text{Side of } S_n = 10 \left(\frac{1}{\sqrt{2}} \right)^{n-1} = \frac{10}{(2)^{\frac{n-1}{2}}}$$

$$\therefore \text{Area of } S_n = (\text{Side})^2 = \left(\frac{10}{(2)^{\frac{n-1}{2}}} \right)^2 = \frac{100}{2^{n-1}}$$

$$\text{Now, area of } S_n < 1$$

$$\Rightarrow \frac{100}{2^{n-1}} < 1 \Rightarrow 2^{n-1} > 100$$

$$\Rightarrow n - 1 \geq 7 \Rightarrow n \geq 8$$

Hence, for $n = 8$, the area of S_n will be less than 1 sq cm.

20. (b) Since, $g = \sqrt{ab}$. Also, a, p, q and b are in AP.

$$\text{So, common difference } d \text{ is } \frac{b-a}{3}.$$

$$\therefore p = a + d = a + \frac{b-a}{3} = \frac{2a+b}{3}$$

$$\text{and } q = b - d = b - \frac{b-a}{3} = \frac{a+2b}{3}$$

$$\begin{aligned} \text{Now, } (2p - q)(p - 2q) &= \frac{(4a + 2b - a - 2b)}{3} \cdot \frac{(2a + b - 2a - 4b)}{3} \\ &= -ab = -g^2 \end{aligned}$$

21. (a) Given that, a, b and c are in GP.

$$\text{Then, } b^2 = ac \quad \dots(i)$$

Since, x is AM of a and b .

$$\therefore 2x = a + b \quad \dots(ii)$$

Also, y is AM of b and c .

$$\therefore 2y = b + c \quad \dots(iii)$$

$$\begin{aligned} \text{Now, } \frac{a}{x} + \frac{c}{y} &= a \times \frac{2}{a+b} + c \times \frac{2}{b+c} \\ &= 2 \left[\frac{ab+ac+ac+bc}{ab+ac+b^2+bc} \right] = 2 \quad [\because b^2 = ac] \end{aligned}$$

22. (b) AM > GM, $\frac{x+y}{2} > \sqrt{xy}$, $\frac{y+z}{2} > \sqrt{yz}$ and $\frac{z+x}{2} > \sqrt{zx}$

On multiplying the three inequalities, we get

$$\begin{aligned} \frac{x+y}{2} \cdot \frac{y+z}{2} \cdot \frac{z+x}{2} &> \sqrt{(xy)(yz)(zx)} \\ \therefore (x+y)(y+z)(z+x) &> 8xyz \end{aligned}$$

23. (d) We know that AM $\geq$ GM for positive numbers.

$$\begin{aligned} \therefore \frac{3^x + 3^{1-x}}{2} &\geq \sqrt{3^x \cdot 3^{1-x}} \\ \Rightarrow \frac{3^x + 3^{1-x}}{2} &\geq \sqrt{3^x \cdot \frac{3}{3^x}} \Rightarrow 3^x + 3^{1-x} \geq 2\sqrt{3} \end{aligned}$$

24. (c) Let common difference be d and common ratio be r . Then, a, A_1, A_2, b are in AP, b is the 4th term.

$$\therefore b = a + 3d \quad [\because T_n = a + (n-1)d]$$

$$\Rightarrow d = \frac{b-a}{3}$$

$$\text{So, } A_1 = a + \frac{b-a}{3} = \frac{2a+b}{3}$$

$$\text{and } A_2 = A_1 + d = \frac{2a+b}{3} + \frac{b-a}{3} = \frac{a+2b}{3}$$

Now, a, G_1, G_2, b are in GP and b is the 4th term.

$$\Rightarrow b = ar^3 \Rightarrow r = \left(\frac{b}{a} \right)^{1/3}$$

$$\text{So, } G_1 = ar = a \left(\frac{b}{a} \right)^{1/3} = a^{2/3} b^{1/3}$$

$$\text{and } G_2 = G_1 r = a^{2/3} b^{1/3} \left(\frac{b}{a} \right)^{1/3} = a^{1/3} b^{2/3}$$

$$\text{Then, } \frac{A_1 + A_2}{G_1 \times G_2} = \frac{\frac{2a+b}{3} + \frac{a+2b}{3}}{a^{2/3} b^{1/3} \times a^{1/3} b^{2/3}} = \frac{a+b}{ab}$$

25. (b) Given equation is $ax^2 + 2bx + c = 0$... (i)

Now, a, b, c are in GP.

$$\Rightarrow b^2 = ac$$

Discriminant of the above equation is

$$D = (2b)^2 - 4ac = 4(b^2 - ac) = 0 \quad [\text{from Eq. (i)}]$$

Its roots are equal, let roots be α and α .

$$\text{Then, } \alpha + \alpha = \frac{-2b}{a}$$

$$\Rightarrow \alpha = \frac{-b}{a} \quad \dots (ii)$$

$$\text{Other equation is } dx^2 + 2ex + f = 0 \quad \dots (iii)$$

One common root is α .

$$\Rightarrow d\alpha^2 + 2e\alpha + f = 0 \quad \dots (iv)$$

Put $\alpha = -\frac{b}{a}$ in Eq. (iv).

$$\Rightarrow \frac{db^2}{a^2} - \frac{2eb}{a} + f = 0$$

$$\Rightarrow \frac{b^2}{a} \left(\frac{d}{a} - \frac{2e}{b} + \frac{fa}{b^2} \right) = 0$$

$$\Rightarrow \frac{b^2}{a} \left(\frac{d}{a} - \frac{2e}{b} + \frac{f}{c} \right) = 0$$

$$\left[\because b^2 = ac \Rightarrow c = \frac{b^2}{a} \Rightarrow \frac{1}{c} = \frac{a}{b^2} \right]$$

$$\text{Now, } \frac{b^2}{a} \neq 0$$

$$\text{So, } \frac{d}{a} - \frac{2e}{b} + \frac{f}{c} = 0 \Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in AP.}$$

26. (b) The given series is in AGP, where $1, 4, 7, \dots$ are in AP and $1, x, x^2, \dots$ are in GP.

Here, $a = 1, b = 1, d = 3, r = x$

Using direct formula,

$$S_{\infty} = \frac{ab}{1-r} + \frac{dbr}{(1-r)^2}$$

$$\Rightarrow \frac{35}{16} = \frac{1}{1-x} + \frac{3x}{(1-x)^2} \quad \left[\because S_{\infty} = \frac{35}{16} \right]$$

$$\Rightarrow \frac{35}{16} = \frac{1}{(1-x)^2} (1-x + 3x)$$

$$\Rightarrow 35(1-x)^2 = 16 + 32x$$

$$\Rightarrow 35x^2 - 102x + 19 = 0$$

$$\Rightarrow (7x - 19)(5x - 1) = 0$$

$$\Rightarrow x \neq \frac{19}{5}$$

[since, for infinite series, common ratio (r) < 1]

$$\therefore x = \frac{1}{5}$$

27. (a) Given that $\cos(x-y), \cos x, \cos(x+y)$ are in HP.

$$\text{Then, } \cos x = \frac{2 \cos(x-y) \cos(x+y)}{\cos(x-y) + \cos(x+y)}$$

$$\Rightarrow \cos x = \frac{2(\cos^2 x - \sin^2 y)}{2 \cos x \cos y}$$

$$\Rightarrow \cos^2 x \cos y = \cos^2 x - \sin^2 y$$

$$\Rightarrow \cos^2 x (\cos y - 1) = -\sin^2 y$$

$$\Rightarrow \cos^2 x (1 - \cos y) = \sin^2 y$$

$$\Rightarrow \cos^2 x (1 - \cos y) = 1 - \cos^2 y$$

$$\Rightarrow \cos^2 x (1 - \cos y) = (1 - \cos y)(1 + \cos y)$$

$$\Rightarrow \cos^2 x = 1 + \cos y$$

$$\Rightarrow \cos^2 x = 2 \cos^2 y/2$$

$$\Rightarrow \cos^2 x \sec^2 (y/2) = 2$$

$$\Rightarrow \cos x \sec (y/2) = \pm \sqrt{2}$$

28. (c) Let $A = \text{Arithmetic mean} = \frac{25 + N}{2}$

$$G = \text{Geometric mean} = \sqrt{25} = 5\sqrt{N}$$

$$\text{and } H = \text{Harmonic mean} = \frac{2 \times 25N}{25 + N} = \frac{50N}{25 + N}$$

Let A, G, H be the p th, q th and r th terms of an AP.

$$\text{Then, } 25 + (p-1)1 = \frac{25 + N}{2} \text{ or } N = 2p + 23$$

Since, p is a positive integer, so putting

$p = 1, 2, 3, \dots$, we get

$$N = 25, 27, 29, 31, \dots \quad \dots (i)$$

$$\text{Again, } 25 + (q-1)1 = 5\sqrt{N} \text{ or } \sqrt{N} = \frac{q+24}{5}$$

Since, N is a positive integer, so we give to q those values which make N a positive integer. These values are clearly

$$q = 1, 6, 11, 16, 21, \dots$$

$$\text{Then, } N = 25, 36, 49, 64, 81, \dots \quad \dots (ii)$$

$$\text{Finally, } 25 + (r-1)1 = \frac{50N}{25 + N}$$

$$\Rightarrow 625 + 25N + 25r + Nr - 25 - N = 50N$$

$$\Rightarrow N = \frac{25(24 + r)}{26 - r}$$

We take those positive value of r which make N a positive integer.

These values of r are 1, 21 and 25 only (because higher values of r make N negative).

$$\text{Hence, } N = 25, 225, 1225 \quad \dots (iii)$$

The value of N which satisfies Eqs. (i), (ii) and (iii) is

$$N = 1225$$

BITSAT Archives

1. (a) Since, $AM \geq GM$

$$\therefore \frac{(p+q)+(r+s)}{2} \geq \sqrt{(p+q)(r+s)}$$

$$\Rightarrow \frac{2}{2} \geq \sqrt{M} \Rightarrow \sqrt{M} \leq 1 \Rightarrow M \leq 1$$

Also, $(p+q)(r+s) > 0$ $[\because p, q, r, s > 0]$

$\therefore M > 0$

Hence, $0 < M \leq 1$

2. (b) Let $S = 1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \dots + 100 \cdot 2^{99}$... (i)

It is an arithmetico-geometric series.

On multiplying Eq. (i) by 2 and then subtracting it from Eq. (i), we get

$$\begin{aligned} S &= 1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \dots + 100 \cdot 2^{99} \\ 2S &= 1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \dots + 99 \cdot 2^{99} + 100 \cdot 2^{100} \\ \hline -S &= 1 + 2 + 2^2 + 2^3 + \dots + 2^{99} - 100 \cdot 2^{100} \\ \Rightarrow -S &= \frac{1(2^{100} - 1)}{2 - 1} - 100 \cdot 2^{100} \\ \Rightarrow -S &= 2^{100} - 1 - 100 \cdot 2^{100} \\ \Rightarrow -S &= -1 - 99 \cdot 2^{100} \\ \Rightarrow S &= 99 \cdot 2^{100} + 1 \end{aligned}$$

3. (b) $S_n = 5 + 55 + 555 + \dots$ upto n terms

$$\begin{aligned} &= 5[1 + 11 + 111 + \dots \text{upto } n \text{ terms}] \\ &= \frac{5}{9}[9 + 99 + 999 + \dots \text{upto } n \text{ terms}] \\ &= \frac{5}{9}[(10 - 1) + (10^2 - 1) + (10^3 - 1) + \dots \text{upto } n \text{ terms}] \\ &= \frac{5}{9}[(10 + 10^2 + 10^3 + \dots \text{upto } n \text{ terms}) \\ &\quad - (1 + 1 + 1 + \dots \text{upto } n \text{ terms})] \\ &= \frac{5}{9} \left[\frac{10(10^n - 1)}{10 - 1} - n \right] = \frac{5}{9} \left[\frac{10(10^n - 1)}{9} - n \right] \end{aligned}$$

4. (a) Given, $a^{1/x} = b^{1/y} = c^{1/z}$

Let $a^{1/x} = b^{1/y} = c^{1/z} = k$

Then, $a = k^x, b = k^y, c = k^z$

$\therefore a, b, c$ are in GP.

Therefore, $b^2 = ac \Rightarrow (k^y)^2 = k^x \cdot k^z$

$\Rightarrow k^{2y} = k^{x+z} \Rightarrow 2y = x + z$

$\therefore x, y$ and z are in AP.

5. (a) **Hint** Let $S_n = \left(1 - \frac{1}{2}\right) + \left(1 - \frac{1}{4}\right) + \left(1 - \frac{1}{8}\right) + \dots$
- $$= n - \frac{1}{2} \cdot \frac{(1 - 1/2)}{(1 - 1/2)}$$

6. (c) **Hint** $0.2 + 0.22 + 0.222 + \dots$ n terms

$$= 2(0.1 + 0.11 + 0.111 + \dots n \text{ terms})$$

$$= 2 \left(\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \dots n \text{ terms} \right)$$

$$\begin{aligned} &= \frac{2}{9} \left(\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \dots n \text{ terms} \right) \\ &= \frac{2}{9} \left(1 - \frac{1}{10} + 1 - \frac{1}{100} + 1 - \frac{1}{1000} + \dots n \text{ terms} \right) \end{aligned}$$

7. (b) Let a and b be two numbers, then

$$AM = \frac{a+b}{2} \Rightarrow 27 = \frac{a+b}{2} \Rightarrow a+b = 54 \text{ and } HM = \frac{2ab}{a+b}$$

$$\Rightarrow 12 = \frac{2ab}{54} \Rightarrow ab = 324$$

$$\therefore GM = \sqrt{ab} = \sqrt{324} = 18$$

8. (c) Here, $T_n = \frac{n(n+1)}{n!} = \frac{n-1+2}{(n-1)!} = \frac{1}{(n-2)!} + \frac{2}{(n-1)!}$

$$\begin{aligned} \therefore S &= \sum_{n=1}^{\infty} T_n = \sum_{n=1}^{\infty} \frac{1}{(n-2)!} + 2 \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \\ &= e + 2e = 3e \end{aligned}$$

9. (d) Let d be the common difference of AP.

Then, $a_{10} = 3 \Rightarrow a_1 + 9d = 3$

$\Rightarrow 2 + 9d = 3 \Rightarrow d = 1/9$

$$\therefore a_4 = a_1 + 3d = 2 + \frac{1}{3} = \frac{7}{3}$$

Let D be the common difference of $\frac{1}{h_1}, \frac{1}{h_2}, \dots, \frac{1}{h_{10}}$, then

$$\begin{aligned} h_{10} = 3 &\Rightarrow \frac{1}{h_{10}} = \frac{1}{3} \\ \Rightarrow \frac{1}{h_1} + 9D &= \frac{1}{3} \Rightarrow \frac{1}{2} + 9D = \frac{1}{3} \\ \Rightarrow 9D &= -\frac{1}{6} \Rightarrow D = -\frac{1}{54} \\ \therefore \frac{1}{h_7} &= \frac{1}{h_1} + 6D = \frac{1}{2} - \frac{1}{9} \\ \Rightarrow h_7 &= \frac{18}{7} \\ \therefore a_4 h_7 &= \frac{7}{3} \times \frac{18}{7} = 6 \end{aligned}$$

10. (b) Let $S = 1 + \frac{2}{3} \cdot \frac{1}{2} + \frac{2}{3} \cdot \frac{5}{6} \cdot \frac{1}{2^2} + \frac{2}{3} \cdot \frac{5}{6} \cdot \frac{8}{9} \cdot \frac{1}{2^3} + \dots$

We know that,

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \dots$$

On comparing these two, we get

$$nx = \frac{2}{3} \cdot \frac{1}{2} \quad \dots (i)$$

$$\text{and } \frac{n(n-1)}{2 \cdot 1} = \frac{2}{3} \cdot \frac{5}{6} \cdot \frac{1}{2^2} \quad \dots (ii)$$

From Eqs. (i) and (ii),

$$\frac{n(n-1)}{2 \cdot 1} = \frac{\frac{2}{3} \times \frac{5}{6} \times \frac{1}{4}}{\frac{2}{3} \times \frac{1}{2} \times \frac{2}{3} \times \frac{1}{2}}$$

$$\Rightarrow \frac{n-1}{2n} = \frac{5}{4} \Rightarrow \frac{n-1}{n} = \frac{5}{2}$$

$$\Rightarrow 5n = 2n - 2 \Rightarrow 3n = -2 \Rightarrow n = -\frac{2}{3}$$

On putting the value of n in Eq. (i), we get

$$x = -\frac{1}{2}$$

$$\therefore \text{Sum of series} = \left(1 - \frac{1}{2}\right)^{-2/3} = (4)^{1/3}$$

11. (b) Let the numbers be a and b .

If H is the harmonic mean, then

$$H = 4, 2A + G^2 = 27 \quad [\text{given}] \dots(i)$$

But we know that, $AH = G^2$

$$\therefore 4A = G^2$$

$$\text{From Eq. (i), } 2A + 4A = 27 \Rightarrow A = \frac{27}{6} = \frac{9}{2}$$

$$\Rightarrow \frac{a+b}{2} = \frac{9}{2} \quad \left[\because A = \frac{a+b}{2} \right]$$

$$\Rightarrow a + b = 9 \quad \dots(ii)$$

$$\text{Again, } H = \frac{2ab}{a+b}$$

$$\Rightarrow 4 = \frac{2ab}{9} \quad [\because H = 4]$$

$$\therefore ab = 18 \quad \dots(iii)$$

From Eqs. (ii) and (iii), we get

$$a - b = \sqrt{(a+b)^2 - 4ab}$$

$$= \sqrt{9^2 - 4 \times 18} = \pm 3 \quad \dots(iv)$$

On solving Eqs. (ii) and (iv), we get

$$2a = 9 \pm 3 \text{ and } 2b = 9 \mp 3$$

$$\therefore a = 6, 3 \text{ and } b = 3, 6$$

Hence, numbers are 3 and 6.

12. (c) We have, $\sum_{k=1}^n k(k+2) = \sum_{k=1}^n (k^2 + 2k) = \sum_{k=1}^n k^2 + 2 \sum_{k=1}^n k$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{2 \cdot n(n+1)}{2}$$

$$= n(n+1) \left(\frac{2n+1}{6} + 1 \right) = \frac{n(n+1)(2n+7)}{6}$$

13. (b) Given that, $\frac{1}{b+c} + \frac{1}{c+a} = \frac{3}{a+b+c}$

$$\Rightarrow 1 + \frac{b}{a+c} + 1 + \frac{a}{b+c} = 3$$

$$\Rightarrow b(b+c) + a(a+c) = (a+c)(b+c)$$

$$\Rightarrow b^2 + bc + a^2 + ac = ab + ac + bc + c^2$$

$$\Rightarrow a^2 + b^2 - c^2 = ab$$

$$\text{We know that, } \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{ab}{2ab} = \frac{1}{2}$$

$$\Rightarrow C = 60^\circ$$

14. (c) We have, $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \dots$

$$= 1 \times \sqrt{2} + 2 \times \sqrt{2} + 3 \times \sqrt{2} + 4 \times \sqrt{2} + \dots$$

$$= \sqrt{2}(1 + 2 + 3 + 4 + \dots \text{ upto 24 terms})$$

$$= \sqrt{2} \times \frac{24 \times 25}{2} = 300\sqrt{2} \quad \left[\because \sum n = \frac{n(n+1)}{2} \right]$$

15. (a) Given series is

$$1 \cdot 3^2 + 2 \cdot 5^2 + 3 \cdot 7^2 + \dots$$

This is an arithmetico-geometric series whose n th term is given by

$$T_n = n(2n+1)^2 = 4n^3 + 4n^2 + n$$

$$\therefore S_n = \sum_{n=1}^n T_n = \sum_{n=1}^n (4n^3 + 4n^2 + n)$$

$$= 4 \sum_{n=1}^n n^3 + 4 \sum_{n=1}^n n^2 + \sum_{n=1}^n n$$

$$= 4 \left(\frac{n}{2}(n+1) \right)^2 + \frac{4}{6} n(n+1)(2n+1) + \frac{n}{2}(n+1)$$

$$= n(n+1) \left[n^2 + n + \frac{4}{6}(2n+1) + \frac{1}{2} \right]$$

$$= \frac{n}{6}(n+1)(6n^2 + 14n + 7)$$

16. (d) We have, $\log_{140} 63 = \log_{2^2 \times 5 \times 7} (3 \times 3 \times 7)$

$$= \frac{\log_2(3 \times 3 \times 7)}{\log_2(2^2 \times 5 \times 7)}$$

$$= \frac{\log_2 3 + \log_2 3 + \log_2 7}{2 \log_2 2 + \log_2 5 + \log_2 7}$$

$$= \frac{2a + \frac{1}{c}}{2 + b + \frac{1}{c}} = \frac{2ac + 1}{2c + bc + 1}$$

17. (c) Since, $2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32$. It is clear that unit place is repeated after every four power.

$$\text{Then, } 2^{301} = (2^4)^{75} \cdot 2 = (16)^{75} \cdot 2$$

So, digit at unit's place in $(16)^{75}$ is 6.

$\therefore$ Digit at unit's place in 2^{301}

$$= \text{Digit at unit's place in } (6) \cdot 2 = 2$$

Hence, the remainder, when 2^{301} is divided by 5, is 2.

18. (b) Given that,

$$\alpha\beta\gamma\delta = 1 \quad \dots(i)$$

As, we know $AM \geq GM$

$$\Rightarrow \frac{1+\alpha}{2} \geq \sqrt{\alpha}$$

$$\Rightarrow 1+\alpha \geq 2\sqrt{\alpha} \quad \dots(ii)$$

$$\text{Similarly, } 1+\beta \geq 2\sqrt{\beta} \quad \dots(iii)$$

$$1+\gamma \geq 2\sqrt{\gamma} \quad \dots(iv)$$

$$\text{and } 1+\delta \geq 2\sqrt{\delta} \quad \dots(v)$$

On multiplying Eqs. (ii), (iii), (iv) and (v), we get

$$(1+\alpha)(1+\beta)(1+\gamma)(1+\delta) \geq 16\sqrt{\alpha\beta\gamma\delta}$$

$$\Rightarrow (1+\alpha)(1+\beta)(1+\gamma)(1+\delta) = 16$$

4

Exponential and Logarithmic Series

Exponential Series

The sum of the series $1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots$ is denoted by the number e which is an irrational number lying between 2 and 3.

i.e. $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e = \lim_{x \rightarrow 0} (1 + x)^{1/x}$

1. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$, for all real values of x , $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

2. For $a > 0$, $a^x = 1 + \frac{x}{1!} (\log_e a) + \frac{x^2}{2!} (\log_e a)^2 + \frac{x^3}{3!} (\log_e a)^3 + \dots = e^{x \log_e a}$

3. $e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n!}$

4. $\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots = \sum_{n=0}^{\infty} \frac{x^{2n}}{2n!}$

5. $\frac{e^x - e^{-x}}{2} = \frac{x}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$

6. $\frac{e + e^{-1}}{2} = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots = \sum_{n=0}^{\infty} \frac{1}{2n!}$

7. $\frac{e - e^{-1}}{2} = \frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \dots = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!}$

8. $\sum_{n=0}^{\infty} \frac{1}{n!} = \sum_{n=1}^{\infty} \frac{1}{(n-1)!} = \sum_{n=2}^{\infty} \frac{1}{(n-2)!} = e$

9. $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$

10. $e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots$

11. $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = \lim_{n \rightarrow \infty} (1 + n)^{1/n} = e$

Logarithm

If a is a positive real number other than 1 and $a^x = m$, then x is called the logarithm of m to the base a , written as $\log_a m$. In $\log_a m$, m should always be positive.

- (i) If $m < 0$, then $\log_a m$ will be imaginary and if $m = 0$, then $\log_a m$ will be meaningless.
- (ii) $\log_a m$ exists, if $m, a > 0$ and $a \neq 1$.

Properties of Logarithm

1. $a^{\log_a x} = x$; $a \neq 0, \neq 1, x > 0$
2. $a^{\log_b x} = x^{\log_b a}$; $a, b > 0, \neq 1, x > 0$
3. $\log_a a = 1, a > 0, \neq 1$
4. $\log_a x = \frac{1}{\log_x a}$; $x, a > 0, \neq 1$
5. $\log_a x = \log_a b \log_b x = \frac{\log_b x}{\log_b a}$; $a, b > 0, \neq 1, x > 0$
6. For $m, n > 0$ and $a > 0, \neq 1$, then
 - (i) $\log_a (m \cdot n) = \log_a m + \log_a n$
 - (ii) $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$
 - (iii) $\log_a (m^n) = n \log_a m$
7. For $x > 0, a > 0, \neq 1$
 - (i) $\log_{a^n} (x) = \frac{1}{n} \log_a x$
 - (ii) $\log_{a^n} x^m = \left(\frac{m}{n}\right) \log_a x$

8. For $x > y > 0$
 - (i) $\log_a x > \log_a y$, if $a > 1$
 - (ii) $\log_a x < \log_a y$, if $0 < a < 1$
9. If $a > 1$ and $x > 0$, then
 - (i) $\log_a x > p \Rightarrow x > a^p$
 - (ii) $0 < \log_a x < p \Rightarrow 0 < x < a^p$
10. If $0 < a < 1$, then
 - (i) $\log_a x > p \Rightarrow 0 < x < a^p$
 - (ii) $0 < \log_a x < p \Rightarrow a^p < x < 1$

Logarithmic Series

1. $\log_e (1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$
2. $\log_e (1-x) = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots\right)$, where $-1 \leq x$
3. $\log_e \left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \dots\right)$
4. $\log [(1+x)(1-x)] = 2 - \left(\frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \dots\right)$
5. $\log_e (n+1) - \log_e (n-1) = 2\left(\frac{1}{n} + \frac{1}{3n^3} + \frac{1}{5n^5} + \dots\right)$
6. $\log_e 2 = \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots\right) = 0.693147$
or $0.61 < \log 2 < 0.76$
 $\log_e 2 = \frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 6} + \dots$

Practice Exercise

1. The sum of the series $1 + \frac{1+2}{2!} + \frac{1+2+2^2}{3!} + \frac{1+2+2^2+2^3}{4!} + \dots$ is
a. e^2 b. $e^2 + e$ c. $e^2 - e$ d. $e^2 - e - 1$
2. The sum of the series $\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \dots$ is
a. e^{-1} b. $2e^{-1}$ c. e^{-2} d. $2e^{-2}$
3. The sum of the series $1 + \frac{3}{2!} + \frac{7}{3!} + \frac{15}{4!} + \dots$ is
a. $e(e+1)$ b. $e(1-e)$ c. $e(e-1)$ d. $3e$
4. The sum of the series $\frac{5}{1 \cdot 2 \cdot 3} + \frac{7}{3 \cdot 4 \cdot 5} + \frac{9}{5 \cdot 6 \cdot 7} + \dots$ is
a. $\log \frac{8}{e}$ b. $\log \frac{e}{8}$ c. $\log mn$ d. $\log \frac{m}{n}$
5. $\frac{9}{1!} + \frac{16}{2!} + \frac{27}{3!} + \frac{42}{4!} + \dots$ is equal to
a. $5e$ b. $7e$ c. $9e$ d. $11e - 6$
6. The sum of the series $\frac{1}{1 \cdot 2} + \frac{1 \cdot 3}{1 \cdot 2 \cdot 3 \cdot 4} + \frac{1 \cdot 3 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \dots$ is
a. $e - 1$ b. $e^{1/2} - 1$
c. $e^{1/2} + e$ d. None of these
7. If $\frac{e^{5x} + e^x}{e^{3x}}$ is expanded in a series of ascending power of x and n is an odd natural number, then the coefficient of x^n is
a. $\frac{2^n}{n!}$ b. $\frac{2^{n+1}}{(2n)!}$ c. $\frac{2^{2n}}{(2n)!}$ d. 0

8. The coefficient of x^n in the expansion of $\frac{a+bx+cx^2}{e^x}$ is
 a. $\frac{(-1)^n}{n!} [cn^2 - (b+c)n + a]$
 b. $\frac{(-1)^n}{n!} [cn^2 + (b+c)n + a]$
 c. $\frac{(-1)^n}{n!} [cn^2 + (b+c)n - a]$
 d. None of the above
9. The sum of the infinite series $\frac{2^2}{2!} + \frac{2^4}{4!} + \frac{2^6}{6!} + \dots$ is
 a. $\frac{e^2 + 1}{2e}$ b. $\frac{e^4 + 1}{2e^2}$ c. $\frac{(e^2 - 1)^2}{2e^2}$ d. $\frac{(e^2 + 1)^2}{2e^2}$
10. The sum of the infinite series $1 + \frac{1}{2!} + \frac{1 \cdot 3}{4!} + \frac{1 \cdot 3 \cdot 5}{6!} + \dots$ is
 a. e b. e^2 c. $\sqrt{e}$ d. $\frac{1}{e}$
11. The value of $\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \dots$ is
 a. $e^{1/2}$ b. e^{-1} c. e d. $e^{-1/3}$
12. $1 + \frac{2^3}{2!} + \frac{3^3}{3!} + \frac{4^3}{4!} + \dots$ is equal to
 a. $5e$ b. $4e$ c. $3e$ d. $2e$
13. The value of $1 + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \frac{1+2+3+4}{4!} + \dots$ is
 a. e b. $2e$ c. $\frac{3e}{2}$ d. $\frac{4e}{5}$
14. $\frac{\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots}{1 + \frac{1}{3!} + \frac{1}{5!} + \dots}$ equals
 a. $e + 1$ b. $\frac{e-1}{e+1}$ c. $e-1$ d. 0
15. The coefficient of x^n in the series $1 + \frac{a+bx}{1!} + \frac{(a+bx)^2}{2!} + \frac{(a+bx)^3}{3!} + \dots$ is
 a. $\frac{(ab)^n}{n!}$ b. $e^b \cdot \frac{a^n}{n!}$
 c. $e^a \cdot \frac{b^n}{n!}$ d. $e^{a+b} \cdot \frac{(ab)^n}{n!}$
16. The sum of the series $\sum_{n=1}^{\infty} \frac{2n}{(2n+1)!}$ is
 a. e b. e^{-1} c. $2e$ d. None
17. The coefficient of x^k in the expansion of $\frac{1-2x-x^2}{e^{-x}}$ is
 a. $\frac{1-k-k^2}{k!}$ b. $\frac{k^2+1}{k!}$
 c. $\frac{1-k}{k!}$ d. $\frac{1}{k!}$
18. If $\frac{1}{e^{3x}} (e^x + e^{5x}) = a_0 + a_1x + a_2x^2 + \dots$, then $2a_1 + 2^3a_3 + 2^5a_5 + \dots$ is equal to
 a. e b. e^{-1} c. 1 d. 0
19. If n is not a multiple of 3, then the coefficient of x^n in the expansion of $\log_e (1+x+x^2)$ is
 a. $\frac{1}{n}$ b. $\frac{2}{n}$ c. $-\frac{1}{n}$ d. $-\frac{2}{n}$
20. If α, β are the roots of the equation $x^2 - px + q = 0$, then the value of $(\alpha + \beta)x - \left(\frac{\alpha^2 + \beta^2}{2}\right)x^2 + \left(\frac{\alpha^3 + \beta^3}{2}\right)x^3 - \dots$ is
 a. $\log(1 - px + qx^2)$ b. $\log(1 + px - qx^2)$
 c. $\log(1 + px + qx^2)$ d. None of these
21. The sum of the series $\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \frac{4}{5}x^5 + \dots$ is
 a. $\frac{x}{1+x} + \log(1+x)$ b. $\frac{x}{1-x} + \log(1-x)$
 c. $\frac{-x}{1+x} + \log(1+x)$ d. None of these
22. The sum of the series $\frac{1}{2}\left(\frac{1}{2} + \frac{1}{3}\right) - \frac{1}{4}\left(\frac{1}{2^2} + \frac{1}{3^2}\right) + \frac{1}{6}\left(\frac{1}{2^3} + \frac{1}{3^3}\right) + \dots$ is
 a. $\log_e 2$ b. $\log_e 3$ c. $2\log_e 2$ d. $\frac{1}{2}\log_e 2$
23. The number of solutions of $\log_4(x-1) = \log_2(x-3)$ is
 a. 3 b. 1 c. 2 d. 0
24. The least value of the expression $2\log_{10} x - \log_x 0.01$ for $x > 1$, is
 a. 2 b. 3 c. 5 d. 4
25. If $S = \sum_{n=0}^{\infty} \frac{(\log x)^{2n}}{(2n)!}$, then S equals
 a. $x + x^{-1}$ b. $x - x^{-1}$ c. $\frac{1}{2}(x + x^{-1})$ d. 0
26. If x, y, z are three consecutive positive integers and $x - z + 2 = 0$, then $\frac{1}{2}\log_e x + \frac{1}{2}\log_e z + \frac{1}{2xz+1} + \frac{1}{3}\left(\frac{1}{2xz+1}\right)^3 + \dots$ is equal to
 a. $\log_e x$ b. $\log_e y$ c. $\log_e z$ d. 0
27. The coefficient of n^{-r} in the expansion of $\log_{10} \left(\frac{n}{n-1}\right)$ is
 a. $\frac{1}{r \log_e 10}$ b. $-\frac{1}{r \log_e 10}$
 c. $-\frac{1}{r! \log_e 10}$ d. None of these

28. $\log_e \left(\frac{1+3x}{1-2x} \right)$ is equal to

- a. $-5x - \frac{5x^2}{2} - \frac{35x^3}{3} - \dots$ b. $-5x + \frac{5x^2}{2} - \frac{35x^3}{3} + \dots$
 c. $5x - \frac{5x^2}{2} + \frac{35x^3}{3} - \dots$ d. $5x + \frac{5x^2}{2} + \frac{35x^3}{3} + \dots$

29. The sum of the infinite series $\frac{1}{2} \left(\frac{1}{3} + \frac{1}{4} \right) - \frac{1}{4} \left(\frac{1}{3^2} + \frac{1}{4^2} \right) + \frac{1}{6} \left(\frac{1}{3^3} + \frac{1}{4^3} \right) - \dots$ is

- a. $\frac{1}{2} \log 2$ b. $\log \frac{3}{5}$ c. $\log \frac{5}{3}$ d. $\frac{1}{2} \log \frac{5}{3}$

30. The sum of the series $1 + \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{1}{4^2} + \frac{1}{7} \cdot \frac{1}{4^3} + \dots$ is

- a. $\log_e 1$ b. $\log_e 2$
 c. $\log_e 3$ d. $\log_e 4$

31. The sum of the series $\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{3 \cdot 4 \cdot 5} + \frac{1}{5 \cdot 6 \cdot 7} + \dots$ is

- a. $\log_e 2 - \frac{1}{2}$ b. $\log_e 2$ c. $\log_e 2 + \frac{1}{2}$ d. $\log_e 2 + 1$

32. The sum of the series

$$\log_9 3 + \log_{27} 3 - \log_{81} 3 + \log_{243} 3 - \dots \text{ is}$$

a. $1 - \log_e 2$ b. $1 + \log_e 2$ c. $\log_e 2$ d. $\log_e 3$

33. The sum of the infinite series

$$\left(\frac{1}{3} \right)^2 + \frac{1}{3} \left(\frac{1}{3} \right)^4 + \frac{1}{5} \left(\frac{1}{3} \right)^6 + \dots \text{ is}$$

- a. $\frac{1}{4} \log_e 2$ b. $\frac{1}{2} \log_e 2$ c. $\frac{1}{6} \log_e 2$ d. $\frac{1}{4} \log_e \frac{3}{2}$

34. $\frac{x-y}{x} + \frac{1}{2} \left(\frac{x-y}{x} \right)^2 + \frac{1}{3} \left(\frac{x-y}{x} \right)^3 + \dots$ is equal to

- a. $\log_e(x-y)$ b. $\log_e(x+y)$
 c. $\log_e \left(\frac{x}{y} \right)$ d. $\log_e xy$

35. $\log_e 3 - \frac{\log_e 9}{2^2} + \frac{\log_e 27}{3^2} - \frac{\log_e 81}{4^2} + \dots$ is equal to

- a. $(\log_e 3)(\log_e 2)$ b. $\log_e 3$
 c. $\log_e 2$ d. $\frac{\log_e 5}{\log_e 3}$

BITSAT Archives

1. If $\frac{e^x}{1-x} = B_0 + B_1x + B_2x^2 + \dots + B_nx^n + \dots$, then the value of $B_n - B_{n-1}$ is [2014]

- a. 1 b. $\frac{1}{n}$
 c. $\frac{1}{n!}$ d. None of these

2. The sum of the series $\log_4 2 - \log_8 2 + \log_{16} 2 - \dots$ is [2014, II]

- a. e^2 b. $\log_e 2 + 1$
 c. $\log_e 3 - 2$ d. $1 - \log_e 2$

3. If $a = \sum_{n=0}^{\infty} \frac{x^{3n}}{(3n)!}$, $b = \sum_{n=1}^{\infty} \frac{x^{3n-2}}{(3n-2)!}$ and $c = \sum_{n=1}^{\infty} \frac{x^{3n-1}}{(3n-1)!}$, then the value of $a^3 + b^3 + c^3 - 3abc$ is [2014]

- a. 1 b. 0
 c. -1 d. -2

4. The coefficient of x^n in the expansion of $\log_e \left(\frac{1}{1+x+x^2+x^3} \right)$, when n is odd, is [2013]

- a. $-\frac{2}{n}$ b. $-\frac{1}{n}$
 c. $1/n$ d. None of these

5. The value of $7 \log \left(\frac{16}{15} \right) + 5 \log \left(\frac{25}{24} \right) + 3 \log \left(\frac{81}{80} \right)$ is [2011]

- a. $\log 2$ b. 3
 c. 5 d. 7

6. The value of $\frac{2}{1!} + \frac{2+4}{2!} + \frac{2+4+6}{3!} + \dots$ is [2011]

- a. e b. $2e$
 c. $3e$ d. None of these

7. The value of $\frac{1}{3} + \frac{1}{3 \cdot 3^3} + \frac{1}{5 \cdot 3^5} + \frac{1}{7 \cdot 3^7} + \dots$ is [2010]

- a. $\frac{1}{2} \log_e 2$ b. $\log_e 2$
 c. $\log_e 3$ d. None of these

8. If $a = \log_2 3$, $b = \log_2 5$ and $c = \log_7 2$, then $\log_{140} 63$ in terms of a, b, c is [2007]

- a. $\frac{2ac+1}{2c+abc+1}$
 b. $\frac{2ac+1}{2a+c+a}$
 c. $\frac{2ac+1}{2c+ab+a}$
 d. None of the above

Answer with Solutions

Practice Exercise

1. (c) $T_n = \frac{1 + 2 + 2^2 + \dots + 2^{n-1}}{n!}$

The series in numerator is GP with common ratio 2.

$$\therefore T_n = \frac{1 \cdot (2^n - 1)}{n!} = \frac{2^n - 1}{n!} = \frac{2^n}{n!} - \frac{1}{n!}$$

where, $n = 1, 2, \dots$

Now, $T_1 = \frac{2}{1!} - \frac{1}{1!}$, $T_2 = \frac{2^2}{2!} - \frac{1}{2!}$, $T_3 = \frac{2^3}{3!} - \frac{1}{3!}$

and so on.

On adding these terms, we get

$$T_1 + T_2 + \dots = \left(\frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \dots \right) - \left(\frac{1}{1!} + \frac{1}{2!} + \dots \right)$$

We know that,

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots \Rightarrow e^2 = 1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \dots$$

and $e^2 - 1 = \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \dots$

Also, $\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots = e - 1$

$$\therefore T_1 + T_2 + T_3 + \dots = (e^2 - 1) - (e - 1) = e^2 - e$$

2. (a) We have, $\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \dots = \sum_{n=1}^{\infty} \frac{2n}{(2n+1)!} = \sum_{n=1}^{\infty} \frac{2n+1-1}{(2n+1)!}$

$$\begin{aligned} &= \sum_{n=1}^{\infty} \left[\frac{1}{(2n)!} - \frac{1}{(2n+1)!} \right] \\ &= \frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \frac{1}{7!} + \dots \\ &= \frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots = e^{-1} \end{aligned}$$

3. (c) We have, $1 + \frac{3}{2!} + \frac{7}{3!} + \frac{15}{4!} + \dots$

$\therefore$ Sum of the series is $T_n = \frac{2^n - 1}{n!}$

$$\begin{aligned} \therefore S_n &= \sum_{n=1}^{\infty} T_n = \sum_{n=1}^{\infty} \left[\frac{2^n}{n!} - \frac{1}{n!} \right] \\ &= (e^2 - 1) - (e - 1) = e^2 - e = e(e - 1) \end{aligned}$$

4. (a) We have, $\frac{5}{1 \cdot 2 \cdot 3} + \frac{7}{3 \cdot 4 \cdot 5} + \frac{9}{5 \cdot 6 \cdot 7} + \dots$

$$\begin{aligned} &= \frac{3+2}{1 \cdot 2 \cdot 3} + \frac{5+2}{3 \cdot 4 \cdot 5} + \frac{7+2}{5 \cdot 6 \cdot 7} + \dots \\ &= \left(\frac{1}{1 \cdot 2} + \frac{2}{1 \cdot 2 \cdot 3} \right) + \left(\frac{1}{3 \cdot 4} + \frac{2}{3 \cdot 4 \cdot 5} \right) + \left(\frac{1}{5 \cdot 6} + \frac{2}{5 \cdot 6 \cdot 7} \right) \\ &\quad \left(\frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 6} + \dots \right) + 2 \left(\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{3 \cdot 4 \cdot 5} + \frac{1}{5 \cdot 6 \cdot 7} + \dots \right) \end{aligned}$$

$$\begin{aligned} &= \left(\frac{2-1}{1 \cdot 2} + \frac{4-3}{3 \cdot 4} + \frac{6-5}{5 \cdot 6} + \dots \right) \\ &\quad + \left[\left(\frac{1}{1 \cdot 2} - \frac{1}{2 \cdot 3} \right) + \left(\frac{1}{3 \cdot 4} - \frac{1}{4 \cdot 5} \right) + \dots \right] \\ &= \left(1 - \frac{1}{2} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \left(\frac{1}{5} - \frac{1}{6} \right) \\ &\quad + \dots + \left[\left(1 - \frac{1}{2} \right) - \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) - \left(\frac{1}{4} - \frac{1}{5} \right) + \dots \right] \\ &= \log 2 + 1 - 2 \left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \dots \right) \\ &= \log 2 + 2 \left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \right] - 1 \\ &= \log 2 + 2 \log 2 - 1 = 3 \log 2 - 1 = \log 8 - \log e = \log \frac{8}{e} \end{aligned}$$

5. (d) Let $S_n = 9 + 16 + 27 + 42 + \dots + T_{n-1} + T_n$

Also, $S_n = 9 + 16 + 27 + \dots + T_{n-1} + T_n$

On subtracting, we get

$$0 = 9 + (7 + 11 + 15 + \dots \text{upto } (n-1) \text{ terms}) - T_n$$

i.e. the series becomes AP.

$$\begin{aligned} \therefore T_n &= 9 + \frac{n-1}{2} [2 \cdot 7 + (n-1-1) 4] \\ &= 9 + \frac{n-1}{2} [14 + 4n - 8] \\ &= 9 + (n-1) [3 + 2n] = 2n^2 + n + 6 \end{aligned}$$

$\therefore$ n th term of the given series is given by

$$\begin{aligned} \frac{2n^2 + n + 6}{n!} &= \frac{2n}{(n-1)!} + \frac{1}{(n-1)!} + \frac{6}{n!} \\ &= \frac{2(n-1)+2}{(n-1)!} + \frac{1}{(n-1)!} + \frac{6}{n!} \\ &= \frac{2}{(n-2)!} + \frac{3}{(n-1)!} + \frac{6}{n!} \end{aligned}$$

Putting $n = 2, 3, 4, \dots$ and then adding, we get

$$\begin{aligned} &T_2 + T_3 + T_4 + \dots \\ &= 2 \left[\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \dots \right] + 3 \left[\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots \right] \\ &\quad + 6 \left[\frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots \right] \end{aligned}$$

$$= 2e + 3(e - 1) + 6(e - 2) = 11e - 15$$

$$\therefore T_1 + T_2 + T_3 + \dots = 9 + 11e - 15 = 11e - 6$$

6. (b) We have, $\frac{1}{1 \cdot 2} + \frac{1 \cdot 3}{1 \cdot 2 \cdot 3 \cdot 4} + \frac{1 \cdot 3 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \dots$

$$\begin{aligned} T_n &= \frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \dots (2n)} \\ &= \frac{1 \cdot 2 \cdot 3 \cdot 4 \dots (2n-2) (2n-1) 2n}{[1 \cdot 2 \cdot 3 \dots (2n-1) (2n)] [2 \cdot 4 \cdot 6 \dots (2n)]} \end{aligned}$$

$$= \frac{(2n)!}{(2n)! 2^n n!} = \frac{1}{2^n n!}$$

$$\therefore \text{Required sum} = \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)^n}{n!} = \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)^n}{n!} - 1 = e^{1/2} - 1$$

7. (d) We have, $\frac{e^{5x} + e^x}{e^{3x}} = e^{2x} + e^{-2x}$

$$= \sum_{n=0}^{\infty} \frac{(2x)^n}{n!} + \sum_{n=0}^{\infty} \frac{(-2x)^n}{n!} = 2 \left[1 + \frac{2^2 x^2}{2!} + \frac{2^4 x^4}{4!} + \dots \right]$$

[higher terms will cancel]

Since, this series does not contain any odd power of x .

$\therefore$ Coefficient of $x^n = 0$

8. (a) Given, $\frac{a + bx + cx^2}{e^x} = (a + bx + cx^2) e^{-x}$

$$= (a + bx + cx^2) \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$$

$$= a \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!} + b \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n!} + c \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+2}}{n!}$$

$\therefore$ Coefficient of x^n in the expansion of $\frac{a + bx + cx^2}{e^x}$

$$= a \frac{(-1)^n}{n!} + b \frac{(-1)^{n-1}}{(n-1)!} + c \frac{(-1)^{n-2}}{(n-2)!}$$

$$= \frac{(-1)^n}{n!} [a - bn + cn(n-1)]$$

$$= \frac{(-1)^n}{n!} [cn^2 - (b+c)n + a]$$

9. (c) Hint $\frac{e^2 + e^{-2}}{2} - 1 = \left[\frac{2^2}{2!} + \frac{2^4}{4!} + \dots \right]$

$$\Rightarrow \frac{e^4 + 1 - 2e^2}{2e^2} = \left[\frac{2^2}{2!} + \frac{2^4}{4!} + \dots \right]$$

10. (c) Let $S = 1 + \frac{1}{2!} + \frac{1 \cdot 3}{4!} + \frac{1 \cdot 3 \cdot 5}{6!} + \dots$

$$\therefore T_n = \frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{(2n)!} \times \frac{2 \cdot 4 \dots 2n}{2 \cdot 4 \dots 2n}$$

$$= \frac{(2n)!}{(2n)! 2^n (n)!} = \frac{1}{2^n (n)!}$$

$$\therefore S = 1 + \sum T_n = 1 + \frac{1}{2(1)!} + \frac{1}{2^2(2)!} + \dots = e^{1/2} = \sqrt{e}$$

11. (b) Hint $T_n = \frac{2n}{(2n+1)!} = \frac{1}{2n!} - \frac{1}{(2n+1)!}$

$$\therefore \sum T_n = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots$$

12. (a) Here, $T_n = \frac{n^3}{n!} = \frac{n^2-1}{(n-1)!} + \frac{1}{(n-1)!} = \frac{n+1}{(n-2)!} + \frac{1}{(n-1)!}$

$$= \frac{1}{(n-3)!} + \frac{3}{(n-2)!} + \frac{1}{(n-1)!}$$

$$\therefore \sum T_n = \sum \left(\frac{1}{(n-3)!} + \frac{3}{(n-2)!} + \frac{1}{(n-1)!} \right)$$

$$= e + 3e + e = 5e$$

13. (c) Here, $T_n = \frac{1+2+3+\dots+n}{n!} = \frac{n(n+1)}{2(n)!} \left[\therefore \sum n = \frac{n(n+1)}{2} \right]$

$$= \frac{(n+1)}{2(n-1)!} = \frac{1}{2(n-2)!} + \frac{1}{(n-1)!}$$

Now, $T_1 = 0 + \frac{1}{1!}, T_2 = \frac{1}{2} \cdot \frac{1}{1!} + \frac{1}{1!}, T_3 = \frac{1}{2} \cdot \frac{1}{1!} + \frac{1}{2!}, \dots$

and so on.

$$\therefore S = \sum T_n = \frac{1}{2} \left(1 + \frac{1}{1!} + \frac{1}{2!} + \dots \right) + \left(1 + \frac{1}{1!} + \frac{1}{2!} + \dots \right)$$

$$= \frac{1}{2} e + e = \frac{3}{2} e$$

14. (b) Given, $\frac{\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots}{1 + \frac{1}{3!} + \frac{1}{5!} + \dots} = \frac{\frac{e + e^{-1}}{2} - 1}{\frac{e - e^{-1}}{2}}$

$$= \frac{e^2 + 1 - 2e}{e^2 - 1} = \frac{e-1}{e+1}$$

15. (c) $1 + \frac{(a+bx)}{1!} + \frac{(a+bx)^2}{2!} + \frac{(a+bx)^3}{3!} + \dots = e^{a+bx}$

$\therefore$ Coefficient of x^n in $e^a e^{bx} = e^a \cdot \frac{(b)^n}{n!}$

16. (b) Hint $\sum_{n=1}^{\infty} \frac{2n}{(2n+1)!} = \sum_{n=1}^{\infty} \frac{2n+1-1}{(2n+1)!}$

$$= \sum_{n=1}^{\infty} \frac{1}{(2n)!} - \sum_{n=1}^{\infty} \frac{1}{(2n+1)!}$$

17. (a) We have, $(1-2x-x^2)(e^x)$

$$\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^k}{k!} + \dots \right)$$

$$= \left(1 + x + \frac{x^2}{2!} + \dots + \frac{x^k}{k!} + \dots \right)$$

$$= (1-2x-x^2) - 2 \left(x + x^2 + \frac{x^3}{2!} + \dots + \frac{x^k}{(k-1)!} + \frac{x^{k+1}}{k!} + \dots \right)$$

$$- \left(x^2 + x^3 + \frac{x^4}{2!} + \dots + \frac{x^k}{(k-2)!} \right)$$

$$+ \left(\frac{x^{k+1}}{(k-1)!} + \frac{x^{k+2}}{k!} + \dots \right)$$

$\therefore$ Coefficient of x^k in $\left(\frac{1-2x-x^2}{e^{-x}} \right) = \frac{1}{k!} - \frac{2}{(k-1)!} - \frac{1}{(k-2)!}$

$$= \frac{1}{k!} - \frac{2k}{k!} - \frac{k(k-1)}{k!}$$

$$= \frac{1-k-k^2}{k!}$$

18. (d) Given, $e^{\frac{1}{3x}} (e^x + e^{5x}) = a_0 + a_1 x + a_2 x^2 + \dots$

$$\Rightarrow (e^{-2x} + e^{2x}) = a_0 + a_1 x + a_2 x^2 + \dots$$

$$\Rightarrow 2 \left[1 + \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + \dots \right] = a_0 + a_1 x + a_2 x^2 + \dots$$

$$\Rightarrow a_1 = a_3 = a_5 = \dots = 0$$

$$\therefore 2a_1 + 2^3 a_3 + 2^5 a_5 + \dots = 0$$

19. (a) $\log_e(1+x+x^2) = \log_e \left(\frac{1-x^3}{1-x} \right)$
 $= \log_e(1-x^3) - \log_e(1-x)$
 $= \left[-x^3 - \frac{x^6}{2} - \frac{x^9}{3} - \dots \right] + \left[x + \frac{x^2}{2} + \dots + \frac{x^n}{n} + \dots \right]$
 Since, n is not a multiple of 3.
 $\therefore$ Coefficient of x^n in the first bracket = 0
 $[\because \text{all powers of } x \text{ are multiple of } 3]$
 and coefficient of x^n in the second bracket = $\frac{1}{n}$

Hence, coefficient of $x^n = \frac{1}{n}$

20. (a) Given series can be rewritten as

$$\left[\alpha x - \frac{1}{2}(\alpha x)^2 + \frac{1}{2}(\alpha x)^3 - \dots \right] + \left[\beta x - \frac{1}{2}(\beta x)^2 + \frac{1}{3}(\beta x)^3 - \dots \right]$$

$$= \log(1-\alpha x) + \log(1-\beta x)$$

$$= \log[1-(\alpha+\beta)x + \alpha\beta x^2]$$

Now, $\alpha + \beta = p$ and $\alpha\beta = q$

$\therefore$ Given series = $\log(1-px + qx^2)$

21. (b) We have, $\frac{x^2}{2} + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \frac{4}{5}x^5 + \dots$

$$= \sum_{n=1}^{\infty} \frac{n}{n+1} x^{n+1} = \sum_{n=1}^{\infty} \frac{n+1-1}{n+1} x^{n+1} = \sum_{n=1}^{\infty} \left(1 - \frac{1}{n+1} \right) x^{n+1}$$

$$= \sum_{n=1}^{\infty} x^{n+1} - \sum_{n=1}^{\infty} \frac{x^{n+1}}{n+1} = \frac{x^2}{1-x} + x - \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}$$

$$= \frac{x^2}{1-x} + x + \log(1-x) = \frac{x}{1-x} + \log(1-x)$$

22. (d) Hint

$$\left(\frac{1}{2 \cdot 2} - \frac{1}{4 \cdot 2^2} + \frac{1}{6 \cdot 2^3} - \dots \right) + \left(\frac{1}{2 \cdot 3} - \frac{1}{4 \cdot 3^2} + \frac{1}{6 \cdot 3^3} - \dots \right)$$

$$= \frac{1}{2} \left[\frac{1}{2} - \frac{1}{2} \left(\frac{1}{2} \right)^2 + \frac{1}{3} \left(\frac{1}{2} \right)^3 - \dots \right]$$

$$+ \frac{1}{2} \left[\frac{1}{3} - \frac{1}{2} \left(\frac{1}{3} \right)^2 + \frac{1}{3} \left(\frac{1}{3} \right)^3 - \dots \right]$$

$$= \frac{1}{2} \log \left(1 + \frac{1}{2} \right) + \frac{1}{2} \log \left(1 + \frac{1}{3} \right)$$

23. (b) The given equation can be written as

$$\frac{\log(x-1)}{\log 4} = \frac{\log(x-3)}{\log 2}$$

We know that, $\log 4 = 2 \log 2$

Then, $\log(x-1) = 2 \log(x-3) = \log(x-3)^2$

$\therefore (x-3)^2 - (x-1) = 0$

or $x^2 - 7x + 10 = 0$

$\therefore (x-2)(x-5) = 0$

$\therefore x = 2, 5$

Neglecting $x = 2$ as $x > 3$

Hence, $x = 5$ is the only one solution.

24. (d) $2 \log_{10} x - \log_x 0.01 = 2 \log_{10} x - \frac{\log_{10} 0.01}{\log_{10} x}$
 $= 2 \log_{10} x - \frac{\log_{10} 10^{-2}}{\log_{10} x} = 2 \log_{10} x + \frac{2}{\log_{10} x} \quad [\because \log_e e = 1]$
 $= 2 \left(\log_{10} x + \frac{1}{\log_{10} x} \right) \quad \dots(i)$

Since, $x > 1$, $\log_{10} x > 0$

But $\frac{1}{2} \left(\log_{10} x + \frac{1}{\log_{10} x} \right) \geq \sqrt{\left(\log_{10} x \times \frac{1}{\log_{10} x} \right)} = 1$

$[\because \text{AM} \geq \text{GM}]$

$\therefore \log_{10} x + \frac{1}{\log_{10} x} \geq 2 \quad \dots(ii)$

Thus, $2 \log_{10} x - \log_x 0.01 \geq 4$ [from Eqs. (i) and (ii)]

Hence, minimum value of $2 \log_{10} x - \log_x 0.01 = 4$

25. (c) We have, $S = \sum_{n=0}^{\infty} \frac{(\log x)^{2n}}{(2n)!} = \left(\frac{e^{\log x} + e^{-\log x}}{2} \right) = \frac{x + x^{-1}}{2}$

26. (b)

27. (a) We have, $\log_{10} \left(\frac{n}{n-1} \right) = \log_e \left(\frac{n}{n-1} \right) (\log_{10} e)$

$$= -\log_e \left(\frac{n-1}{n} \right) \log_{10} e = -\log_e \left(1 - \frac{1}{n} \right) \log_{10} e$$

$$= \log_{10} e \left[\frac{1}{n} + \frac{1}{2} \cdot \frac{1}{n^2} + \frac{1}{3} \cdot \frac{1}{n^3} + \dots + \frac{1}{r} \cdot \frac{1}{n^r} + \dots \right]$$

$\therefore$ Coefficient of $n^{-r} = \frac{1}{r} \log_{10} e = \frac{1}{r \log_e 10}$

28. (c) Hint $\log_e \left(\frac{1+3x}{1-2x} \right) = \log_e(1+3x) - \log_e(1-2x)$

29. (d) $\frac{1}{2} \left(\frac{1}{3} + \frac{1}{4} \right) - \frac{1}{4} \left(\frac{1}{3^2} + \frac{1}{4^2} \right) + \frac{1}{6} \left(\frac{1}{3^3} + \frac{1}{4^3} \right) - \dots$
 $= \frac{1}{2} \left[\frac{1}{3} - \frac{1}{2} \left(\frac{1}{3^2} \right) + \frac{1}{3} \left(\frac{1}{3^3} \right) - \dots \right]$
 $+ \frac{1}{2} \left[\frac{1}{4} - \frac{1}{2} \left(\frac{1}{4^2} \right) + \frac{1}{3} \left(\frac{1}{4^3} \right) - \dots \right]$
 $= \frac{1}{2} \left[\log \left(1 + \frac{1}{3} \right) \right] + \frac{1}{2} \left[\log \left(1 + \frac{1}{4} \right) \right]$
 $= \frac{1}{2} \left[\log \left(\frac{4}{3} \right) \times \left(\frac{5}{4} \right) \right] = \frac{1}{2} \log \left(\frac{5}{3} \right)$

30. (c) Since, $\log(1+x) - \log(1-x) = 2 \left[x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \right]$

Put $x = \frac{1}{2}$ on both sides, we get

$$\log \left(\frac{3}{2} \right) - \log \left(\frac{1}{2} \right) = 2 \left(\frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2^3} + \frac{1}{5} \cdot \frac{1}{2^5} + \dots \right)$$

$\Rightarrow \log_e 3 = 1 + \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{1}{4^2} + \dots$

31. (a) Hint $T_n = \frac{1}{(2n-1)(2n)(2n+1)}$
 $= \frac{1}{2} \left[\frac{1}{2n-1} - \frac{1}{2n} \right] - \frac{1}{2} \left[\frac{1}{2n} - \frac{1}{2n+1} \right]$

$$\begin{aligned}
 T_1 &= \frac{1}{2} \left[\frac{1}{1} - \frac{1}{2} \right] - \frac{1}{2} \left[\frac{1}{2} - \frac{1}{3} \right] \\
 T_2 &= \frac{1}{2} \left[\frac{1}{3} - \frac{1}{4} \right] - \frac{1}{2} \left[\frac{1}{4} - \frac{1}{5} \right] \\
 &\vdots \\
 \therefore S &= \frac{1}{2} \left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \dots \right] \\
 &\quad - \frac{1}{2} \left[\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \frac{1}{7} + \dots \right]
 \end{aligned}$$

32. (c) Hint $\log_{3^2} 3 + \log_{3^3} 3 - \log_{3^4} 3 + \log_{3^5} 3 - \dots$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots + 1 - 1$$

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1. (a) We have, $e^x = (1-x)(B_0 + B_1x + B_2x^2 + \dots + B_{n-1}x^{n-1} + B_nx^n + \dots)$

By the expansion of e^x , we get

$$1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + \dots = (1-x)(B_0 + B_1x + B_2x^2 + \dots + B_{n-1}x^{n-1} + B_nx^n + \dots)$$

Equating the coefficient of x^n on both sides, we get

$$B_n - B_{n-1} = \frac{1}{n!}$$

2. (d) Hint $\log_4 2 - \log_8 2 + \log_{16} 2 - \dots$

$$= \frac{1}{\log_2 4} - \frac{1}{\log_2 8} + \frac{1}{\log_2 16} - \dots = \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \dots$$

3. (a) We have, $a = \sum_{n=0}^{\infty} \frac{x^{3n}}{(3n)!}$, $b = \sum_{n=1}^{\infty} \frac{x^{3n-2}}{(3n-2)!}$

and $c = \sum_{n=1}^{\infty} \frac{x^{3n-1}}{(3n-1)!}$

$$\text{Now, } a + b + c = \sum_{n=0}^{\infty} \frac{x^{3n}}{(3n)!} + \sum_{n=1}^{\infty} \frac{x^{3n-2}}{(3n-2)!} + \sum_{n=1}^{\infty} \frac{x^{3n-1}}{(3n-1)!}$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = e^x$$

$$\text{Also, } a + b\omega + c\omega^2 = 1 + \omega x + \frac{\omega^2 x^2}{2!} + \frac{\omega^3 x^3}{3!} + \dots = e^{\omega x}$$

and $a + b\omega^2 + c\omega = e^{\omega^2 x}$, where ω is imaginary cube root of unity.

$$\text{Now, } a^3 + b^3 + c^3 - 3abc$$

$$= (a + b + c)(a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$$

$$= e^x \cdot e^{\omega x} \cdot e^{\omega^2 x} = e^{x(1 + \omega + \omega^2)} = e^{0 \cdot x} = 1$$

4. (b) $\log_e \left(\frac{1}{1+x+x^2+x^3} \right) = \log_e \left(\frac{1-x}{1-x^4} \right)$

$$= \log_e(1-x) - \log_e(1-x^4) = - \sum_{r=1}^{\infty} \frac{x^r}{r} + \sum_{r=1}^{\infty} \frac{x^{4r}}{r}$$

$$\begin{aligned}
 33. (c) & \left(\frac{1}{3} \right)^2 + \frac{1}{3} \left(\frac{1}{3} \right)^4 + \frac{1}{5} \left(\frac{1}{3} \right)^6 + \dots \\
 &= \frac{1}{3} \left[\left(\frac{1}{3} \right) + \frac{1}{3} \left(\frac{1}{3} \right)^3 + \frac{1}{5} \left(\frac{1}{3} \right)^5 + \dots \right] \\
 &= \frac{1}{3} \cdot \frac{1}{2} \log \left(\frac{1 + \frac{1}{3}}{1 - \frac{1}{3}} \right) = \frac{1}{6} \log_e 2
 \end{aligned}$$

34. (c)

$$\begin{aligned}
 35. (a) & \log_e 3 - \frac{\log_e 9}{2^2} + \frac{\log_e 27}{3^2} - \frac{\log_e 81}{4^2} + \dots \\
 &= (\log_e 3) \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \right) = (\log_e 3) \log_e 2
 \end{aligned}$$

When n is odd, there is no term in the second series containing x^n , therefore the coefficient of x^n is zero in the second series and in the first series, the coefficient of x^n is $-\frac{1}{n}$.

Hence, when n is odd, then the coefficient of x^n in the whole expansion is $-\frac{1}{n} + 0 = -\frac{1}{n}$.

$$\begin{aligned}
 5. (a) & 7 \log \left(\frac{16}{15} \right) + 5 \log \left(\frac{25}{24} \right) + 3 \log \left(\frac{81}{80} \right) \\
 &= \log \left[\left(\frac{16}{15} \right)^7 \cdot \left(\frac{25}{24} \right)^5 \cdot \left(\frac{81}{80} \right)^3 \right] = \log 2
 \end{aligned}$$

$$6. (c) \text{ Here, } T_n = \frac{n(n+1)}{n!} = \frac{n-1+2}{(n-1)!} = \frac{1}{(n-2)!} + \frac{2}{(n-1)!}$$

$$\therefore S = \sum_{n=1}^{\infty} T_n = \sum_{n=1}^{\infty} \frac{1}{(n-2)!} + 2 \sum_{n=1}^{\infty} \frac{1}{(n-1)!} = e + 2e = 3e$$

7. (a) Given series is $\frac{1}{3} + \frac{1}{3 \cdot 3^3} + \frac{1}{5 \cdot 3^5} + \frac{1}{7 \cdot 3^7} + \dots$... (i)

Putting $\frac{1}{3} = x$ in Eq. (i), we get

$$\begin{aligned}
 \frac{1}{3} + \frac{1}{3 \cdot 3^3} + \frac{1}{5 \cdot 3^5} + \frac{1}{7 \cdot 3^7} + \dots &= \left(x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots \right) \\
 &= \frac{1}{2} \log_e \left(\frac{1+x}{1-x} \right) = \frac{1}{2} \log_e \left(\frac{1 + \frac{1}{3}}{1 - \frac{1}{3}} \right) \quad \left[\because x = \frac{1}{3} \right] \\
 &= \frac{1}{2} \log_e 2
 \end{aligned}$$

8. (d) Here, $\log_{140} 63 = \log_{2^2 \times 5 \times 7} (3 \times 3 \times 7) = \frac{\log_2 (3 \times 3 \times 7)}{\log_2 (2^2 \times 5 \times 7)}$

$$= \frac{\log_2 3 + \log_2 3 + \log_2 7}{2 \log_2 2 + \log_2 5 + \log_2 7} = \frac{2a + \frac{1}{c}}{2 + b + \frac{1}{c}} = \frac{2ac + 1}{2c + bc + 1}$$

5

Permutations and Combinations

Fundamental Principle of Counting

The fundamental principle of counting is a way to figure out the total number of ways in which different events can occur. If a certain work A can be done in m ways and another work B in n ways, then

- (i) the number of ways of doing the work A or B is $m + n$. [addition principle]
- (ii) the number of ways of doing both the works is mn . [multiplication principle]

Factorial Notation

Let n be a positive integer. Then, the continued product of first n natural numbers is called factorial n . It is denoted by $n!$ or $\underline{n}$.

Thus, $n! = n (n - 1) (n - 2) \dots 3 \cdot 2 \cdot 1$

e.g. $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ and $4! = 4 \times 3 \times 2 \times 1 = 24$

Properties of Factorial Notation

- 1. $0! = 1! = 1$
- 2. Factorials of negative integers and fractions are not defined.
- 3. $n! = n (n - 1)! = n (n - 1) (n - 2)!$
- 4. $\frac{n!}{r!} = n (n - 1) (n - 2) \dots (r + 1)!$

Permutation

Each of different arrangements which can be made by taking some or all of a number of things, is called a permutation and number of permutations of n distinct objects taking r at a time is denoted by ${}^n P_r$.

$${}^n P_r = \frac{n!}{(n-r)!}, \forall 0 \leq r \leq n = n(n-1)(n-2)\dots(n-r+1), \forall n \in N \text{ and } r \in W$$

Properties of ${}^n P_r$

- The number of permutations of n distinct objects taken all at a time is ${}^n P_n = n!$
- ${}^n P_0 = 1$, ${}^n P_1 = n$ and ${}^n P_{n-1} = n!$
- ${}^n P_r = n {}^{n-1} P_{r-1} = r {}^{n-1} P_{r-1} + {}^{n-1} P_r$
- ${}^{n-1} P_r = (n-r) {}^{n-1} P_{r-1}$

Number of Permutations without Repetition

- (i) Arranging n distinct objects, taken r at a time equivalent to filling r places from n things

Number of place	1	2	3	4	...	r
Number of choices	n	$(n-1)$	$(n-2)$	$(n-3)$	...	$n-(r-1)$

Number of arrangements = Number of ways of filling r places

$$\begin{aligned}
 &= n(n-1)(n-2)\dots(n-r+1) \\
 &= \frac{n(n-1)(n-2)\dots(n-r+1)\{(n-r)!\}}{(n-r)!} \\
 &= \frac{n!}{(n-r)!} = {}^n P_r
 \end{aligned}$$

- (ii) The number of arrangements of n different objects taken all at a time

$$= {}^n P_n = n!$$

$$(a) {}^n P_0 = \frac{n!}{n!} = 1, {}^n P_r = n {}^{n-1} P_{r-1}$$

$$(b) 0! = 1, 1/(-r)! = 0 \text{ or } (-r)! = \infty \quad [\because r \in \mathbb{N}]$$

Number of Permutations with Repetition

r -places	1	2	3	4	...	r
Number of choices	n	n	n	n	...	n

- (i) The number of permutations (arrangements) of n different objects, taken r at a time, when each object may occur once, twice, thrice,... upto r times in any arrangement is equal to the number of ways of filling r places, where each place can be filled by anyone of n objects.

Number of permutations = Number of ways of filling r places $= (n)^r$

- (ii) The number of arrangements that can be formed using n objects out of which p are identical (and of one kind), q are identical (and of another kind), r are identical (and of another kind) and the rest are distinct, is $\frac{n!}{p!q!r!}$.

Circular Permutation

If objects are arranged along a closed curve, then permutation is known as circular permutation.

Important Results on Circular Permutation

- (i) The number of circular permutations of n different things taken all at a time is $(n-1)!$. If clockwise and anti-clockwise orders are taken as different.
- (ii) If clockwise and anti-clockwise circular permutations are considered to be the same, then it is $\frac{(n-1)!}{2}$.
- (iii) Number of circular permutations of n different things, taken r at a time, when clockwise and anti-clockwise orders are taken as different, is $\frac{{}^n P_r}{r}$.
- (iv) Number of circular permutations of n different things, taken r at a time, when clockwise and anti-clockwise orders are not different, is ${}^n P_r / 2r$.
- (v) Number of circular permutations of n things, when p are alike and the rest different, taken all at a time distinguishing clockwise and anti-clockwise arrangements is $\frac{(n-1)!}{p!}$.

Combination

Each of the different groups or selections which can be formed by taking some or all of the number of objects, irrespective of their arrangements, is called a combination.

If ${}^n C_r$ denotes the number of combinations of n different

things taken r at a time, then ${}^n C_r = \frac{n!}{r!(n-r)!} = \frac{{}^n P_r}{r!}$, where

$r < n$, $n \in \mathbb{N}$ and $r \in \mathbb{W}$.

Properties of ${}^n C_r$

- ${}^n C_r = {}^n C_{n-r}$
- ${}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r$
- ${}^n C_r = 0$, if $r \notin \{0, 1, 2, 3, \dots, n\}$
- ${}^n P_r = {}^n C_r \cdot r!$

Number of Combinations with Repetition

- (i) Number of ways in which atleast one object is selected out of n distinct objects
 $\Rightarrow {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n = 2^n - 1$
- (ii) The total number of ways in which it is possible to make groups by taking some or all out of $n = (n_1 + n_2 + \dots)$ things, when n_1 are alike (of one kind), n_2 are alike (of second kind) and so on, is $\{(n_1 + 1)(n_2 + 1) \dots\} - 1$.
- (iii) The number of selections of r objects out of n identical objects is 1.
- (iv) Total number of selections of zero or more objects from n identical objects is $n + 1$.
- (v) The number of selections taking atleast one out of $a_1 + a_2 + a_3 + \dots + a_n + k$ objects, where a_1 are alike (of one kind), a_2 are alike (of second kind) and so on $\dots a_n$ are alike (of n th kind) and k is distinct, is $[(a_1 + 1)(a_2 + 1)(a_3 + 1) \dots (a_n + 1)] 2^k - 1$.
- (vi) If $N = p^{a_1} \cdot q^{a_2} \cdot r^{a_3} \dots$, where $p, q, r, \dots$ are different prime numbers and $a_1, a_2, a_3, \dots$ are natural numbers, then
- Total number of divisors of N (including 1 and N)
 $= (a_1 + 1)(a_2 + 1)(a_3 + 1) \dots$
 - Total number of divisors of N (excluding 1 and N)
 $= \{(a_1 + 1)(a_2 + 1)(a_3 + 1) \dots\} - 2$
 - Total number of divisors of N (excluding 1 or N)
 $= \{(a_1 + 1)(a_2 + 1)(a_3 + 1) \dots\} - 1$
 - The sum of these divisors is

$$= (p^0 + p^1 + p^2 + \dots + p^{a_1}) (q^0 + q^1 + q^2 + \dots + q^{a_2}) (r^0 + r^1 + r^2 + \dots + r^{a_3})$$
 - The number of ways in which a composite number N can be decomposed into two factors is
- Case I** When N is not a perfect square, then
 $\frac{1}{2} \{(a_1 + 1)(a_2 + 1) \dots\}$
- Case II** When N is a perfect square, then
 $\frac{1}{2} [\{(a_1 + 1)(a_2 + 1)(a_3 + 1) \dots\} + 1]$
- (f) The number of ways in which a composite number N can be decomposed in two-two factors which are coprime, is 2^{k-1} , where k is the number of different factors of N .

Applications of Permutations and Combinations

The functional and the geometrical applications of permutations and combinations are as given below:

Functional Applications

- The number of all permutations (arrangements) of n different objects taken r at a time,
 - when a particular object is always included in each arrangement, is ${}^{n-1}C_{r-1} \times r!$.
 - when a particular object is never taken in each arrangement, is ${}^{n-1}C_r \times r!$.
- If the sets A has m elements and B has n elements, then
 - the number of functions from A to B is n^m .
 - the number of one-one functions from A to B is nP_m , where $m \leq n$.

Geometrical Applications

- Number of triangles formed from n points, when no three points are collinear, is nC_3 .
- Out of n non-concurrent and non-parallel straight lines, the points of intersection are nC_2 .
- Number of parallelograms in two systems of parallel lines (when 1st system contains m parallel lines and 2nd system contains n parallel lines) = ${}^nC_2 \times {}^mC_2$.
- The number of diagonals in a polygon of n sides, is ${}^nC_2 - n$.
- The number of total triangles formed by joining the n points on a plane of which m are collinear, is ${}^nC_3 - {}^mC_3$.
- The number of total different straight lines formed by joining the n points on a plane of which m are collinear, is ${}^nC_2 - {}^mC_2 + 1$.
- The number of rectangles of any size in a square of $n \times n$ is $\sum_{r=1}^n r^3$ and number of squares of any size is $\sum_{r=1}^n r^2$.

Prime Factors

Any integer greater than 1 can be expressed as product of primes

e.g. $7 = 7^1$, $12 = 2^2 \cdot 3^1$, $360 = 2^3 \cdot 3^2 \cdot 5^1$

Let $n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot p_3^{\alpha_3} \cdot \dots \cdot p_r^{\alpha_r}$, where $p_i, i = 1, 2, \dots, r$ are distinct primes and $\alpha_i, i = 1, 2, \dots, r$ are positive integers.

- (i) Number of divisors of
- n
- is

$$(\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1) \dots (\alpha_r + 1)$$

- (ii) Sum of divisors of
- n
- is

$$\frac{(p_1^{\alpha_1+1}-1)}{(p_1-1)} \frac{(p_2^{\alpha_2+1}-1)}{(p_2-1)} \dots \frac{(p_r^{\alpha_r+1}-1)}{(p_r-1)}$$

- (iii) If
- p
- is a prime and
- p^r
- divides, then
- $r = \left[\frac{n}{p} \right] + \left[\frac{n}{p^2} \right] + \dots$

Division of Objects into Groups

The division of objects into groups are taken place as when the objects are different and identical as given below:

When Objects are Different

1. The number of ways of dividing n different objects into 3 groups of p, q and r ($p + q + r = n$) is

(i) $\frac{n!}{p!q!r!}$; p, q and r are unequal.

(ii) $\frac{n!}{p!2!(q!)^2}$; $q = r$ (iii) $\frac{n!}{3!(p!)^3}$; $p = q = r$

2. The number of ways of dividing n different objects into r groups is

$$\frac{1}{r!} \left[r^n - \binom{r}{1} (r-1)^n + \binom{r}{2} (r-2)^n - \binom{r}{3} (r-3)^n + \dots \right]$$

3. The number of ways of dividing n different objects into r groups taking into account the order of the groups and also the order of objects in each group, is

$${}^{(n+r-1)}P_r = r(r+1)(r+2) \dots (r+n-1).$$

When Objects are Identical

1. The number of ways of dividing n identical objects among r persons, such that each gets 1, 2, 3, ... objects, is the coefficient of x^{n-r} in the expansion of $(1+x+x^2+\dots+x^{k-1})^r$.

2. The number of ways of dividing n identical objects among r persons such that each one may get at most n objects, is $\binom{n+r-1}{r-1}$.

In other words, the total number of ways of dividing n identical objects into r groups, if blank groups are allowed, is ${}^{n+r-1}C_{r-1}$.

3. The total number of ways of dividing n identical objects among r persons and each one of them, receives atleast one item, is ${}^{n-1}C_{r-1}$.

In other words, the number of ways in which n identical things can be divided into r groups such that blank groups are not allowed, is ${}^{n-1}C_{r-1}$.

4. The total number of selections of some or all out of $p + q + r$ items, where p are alike of one kind, q are alike of second kind and rest are alike of third kind, is $[(p+1) \cdot (q+1) \cdot (r+1) - 1]$.

Restricted Selection/Arrangement

1. The number of ways in which r objects can be selected from n different objects, if k particular objects are

(a) always included $= {}^{n-k}C_{r-k}$

(b) never included $= {}^{n-k}C_r$

2. The number of arrangements of n distinct objects taken r at a time, so that k particular objects are

(a) always included $= {}^{n-k}C_{r-k} \cdot r!$

(b) never included $= {}^{n-k}C_r \cdot r!$

Dearrangement

Any change in the given order of the things, is called a dearrangement.

1. If n things form an arrangement in a row, then the number of ways in which they can be dearranged, so that none of them occupies its original place, is

$$n! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \cdot \frac{1}{n!} \right).$$

2. If n things are arranged at n places, then the number of ways to rearrange exactly r things at right places, is

$$\frac{n!}{r!} \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \dots + (-1)^{n-r} \frac{1}{(n-r)!} \right]$$

Practice Exercise

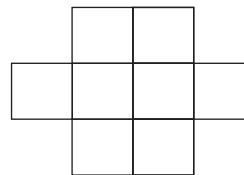
- How many 3-digit numbers can be formed by using 1, 2, 3, 4, 5 without repetition of digits?
 - 12
 - 22
 - 50
 - 60
- A college offers 10 courses in the morning and 7 in the evening. Find the number of ways a student can select exactly one course, either in the morning or in the evening.
 - 10
 - 29
 - 30
 - 17
- The number of words from the letters of the word 'BHARAT' in which B and H will never come together, is
 - 360
 - 240
 - 120
 - None of these
- In how many ways the letters of the word 'ARRANGE' can be arranged without altering the relative positions of vowels and consonants?
 - 36
 - 26
 - 62
 - None
- The sum of the digits in unit place of all the numbers formed with the help of 3, 4, 5 and 6 taken all at a time is
 - 432
 - 108
 - 36
 - 18
- The total number of different combinations of one or more letters which can be made from the letters of the word 'MISSISSIPPI', is
 - 150
 - 148
 - 149
 - None of these
- If $\frac{{}^nP_{r-1}}{a} = \frac{{}^nP_r}{b} = \frac{{}^nP_{r+1}}{c}$, then
 - $b^2 = a(b+c)$
 - $c^2 = a(b+c)$
 - $ab = a^2 + bc$
 - $bc = a^3 + b^2$
- If $m > n$, then the number of ways m men and n women can be seated in a row, so that no two women sit together, is
 - $\frac{m!n!}{(m+n)!}$
 - $\frac{(m+n)!}{m!(n+1)!}$
 - $\frac{m!(n+1)!}{(m-n+1)!}$
 - $\frac{m!(m+1)!}{(m-n+1)!}$
- If all the letters of the word 'QUEST' are arranged in all possible ways and put in dictionary order, then find the rank of the given word.
 - 40
 - 43
 - 45
 - 47
- Eight chairs are numbered 1 to 8. 2 women and 3 men wish to occupy one chair each. First, the women choose the chairs marked 1 to 4 and then the men select from amongst the remaining chairs. The number of seating arrangements is
 - 1250
 - 1360
 - 1440
 - 1560
- In a class of 10 students, there are 3 girls. The number of ways they can be arranged in a row, so that no 2 girls are consecutive, is $k \cdot 8!$, where k is equal to
 - 12
 - 24
 - 36
 - 42
- The number of numbers greater than 1000 but not greater than 4000 that can be formed with the digits 0, 1, 2, 3, 4 when repetition of digits is allowed, is
 - 375
 - 625
 - 125
 - None
- The number of words that can be formed by using the letters of the word 'MATHEMATICS' that start as well as end with T, are
 - 90720
 - 28060
 - 713090
 - None of these
- The total number of permutations of $n (> 1)$ different things taken not more than r at a time, when each thing may be repeated any number of times, is
 - $\frac{n(n^n-1)}{n-1}$
 - $\frac{n^r-1}{n-1}$
 - $\frac{n(n^r-1)}{n-1}$
 - None of these
- Consider 23 different coloured beads in a necklace. In how many ways can the beads be placed in the necklace, so that 3 specific beads always remain together?
 - $\frac{22!3!}{2}$
 - $\frac{21!3!}{2}$
 - $\frac{20!3!}{2}$
 - $20!3! \times 2$
- Six boys and six girls sit along in a line alternatively in x ways and along a circle, (again alternatively in y ways), then
 - $x = y$
 - $y = 12x$
 - $x = 10y$
 - $x = 12y$
- ${}^nC_r + 2 {}^nC_{r-1} + {}^nC_{r-2}$ is equal to
 - ${}^{n+1}C_r$
 - ${}^{n+1}C_{r+1}$
 - ${}^{n+2}C_r$
 - ${}^{n+2}C_{r+1}$
- If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then r is equal to
 - 1
 - 2
 - 3
 - None of these

19. The value of expression ${}^{47}C_4 + \sum_{j=1}^5 {}^{52-j}C_3$ is
 a. ${}^{52}C_4$ b. ${}^{52}C_2$
 c. ${}^{52}C_6$ d. None of these
20. A sports team of 11 students is to be constituted, choosing atleast 5 from Class XI and atleast 5 from Class XII. If there are 20 students in each of these classes, then in how many ways can the team be constituted?
 a. ${}^{20}C_5 \times {}^{20}C_6$ b. $2({}^{20}C_5 \times {}^{20}C_6)$
 c. $2({}^{20}C_5)^2$ d. None of these
21. In how many ways, we can select 4 letters from the letters of the word 'MISSISSIPPI'?
 a. 20 b. 21 c. 22 d. 23
22. There are 5 advanced round candidates in an election and 3 of them are to be elected. A voter can cast any number of votes but not more than three. The number of ways in which he can cast his vote is
 a. 5 b. 15 c. 20 d. 25
23. In how many ways can 6 persons be selected from 4 officers and 8 constables, if atleast one officer has to be included?
 a. 224 b. 672 c. 896 d. None
24. The number of ways in which any four letters can be selected out of the letters of the word 'PCBDCC', is
 a. 8 b. 5 c. 7 d. 6
25. The greatest possible number of points of intersection of 8 straight lines and 4 circles is
 a. 32 b. 64 c. 76 d. 104
26. In a steamer, there are stalls for 12 animals and there are horses, cows and calves (not less than 12 each) ready to be shipped. In how many ways, can the ship load be made?
 a. $3^{12} - 1$ b. 3^{12} c. $(12)^3 - 1$ d. $(12)^3$
27. In an examination of 9 papers, a candidate has to pass in more papers, then the number of papers in which he fails in order to be successful. The number of ways in which he can be unsuccessful, is
 a. 255 b. 256 c. 193 d. 319
28. A box contains 2 white balls, 3 black balls and 4 red balls. The number of ways of drawing 3 balls from the box, if atleast one black ball is included, is
 a. 36 b. 42
 c. 56 d. 64
29. A group of 6 is chosen from 10 men and 7 women so as to contain atleast 3 men and 2 women. The number of ways this can be done, if two particular women refuse to serve on the same group, is
 a. 8000 b. 7800
 c. 7600 d. 7200
30. A candidate is required to answer 7 questions out of 12 questions, which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. Find the number of different ways of doing questions.
 a. 779 b. 781 c. 780 d. 782
31. A committee of 4 persons is to be formed from 2 ladies, 2 old men and 4 young men such that it includes atleast 1 lady, atleast 1 old man and atmost 2 young men. Then, the total number of ways in which this committee can be formed, is
 a. 40 b. 41 c. 16 d. 32
32. Find the number of divisors of 1350. Also, find the sum of all divisors.
 a. 25, 3520 b. 24, 3720 c. 6, 2340 d. None
33. The number of times the digit 5 will be written when listing the integers from 1 to 1000, is
 a. 271 b. 272 c. 300 d. None
34. In a certain test, there are n questions, in this test 2^{n-i} students gave wrong answers to atleast i questions, where $i = 1, 2, 3, \dots, n$. If the total number of wrong answers given is 2047, then n is equal to
 a. 10 b. 11 c. 12 d. 13
35. In an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for atleast one candidate, then the number of ways in which he can vote is
 a. 5040 b. 6210
 c. 385 d. 1110
36. If the letters of the word 'SACHIN' are arranged in all possible ways and these words are written out as in dictionary, then the word 'SACHIN' appears at serial number
 a. 602 b. 603
 c. 600 d. 601
37. How many ways are there, to arrange the letters in the word 'GARDEN' with the vowels in alphabetical order?
 a. 360 b. 240
 c. 120 d. 480
38. The number of ways to sit 3 men and 2 women in a bus such that total number of sitted men and women on each side is 3, is
 a. 5! b. ${}^6C_5 \times 5!$
 c. $6! \times {}^6C_5$ d. $5! + {}^6C_5$
39. Eighteen guests have to be seated half on each side of a long table. Four particular guests desire to sit on one particular side and three others on the other side. The number of sitting arrangements is
 a. ${}^{11}C_7(9!)^2$ b. ${}^{11}C_5(9!)^2$
 c. $7 \cdot (9!)^2$ d. $10!9!$

40. Possible number of words, taking all letters at a time such that in each word both M's are together and both T's are together but both A's are not together, is
- a. $7! \cdot {}^8C_2$ b. $\frac{11!}{2!2!2!} - \frac{10!}{2!2!}$
 c. $\frac{6!4!}{2!2!}$ d. $\frac{9!}{2!2!2!}$
41. There are m points on one straight line AB and n points on another straight line AC , none of them being A . How many triangles can be formed with these points as vertices, if point A is also included?
- a. $\frac{mn}{2}(m+n-2)$ b. ${}^{m+1}C_2 \times {}^nC_1 + {}^{m-1}C_1 \times {}^nC_2$
 c. $\frac{mn}{2}(m+n)$ d. None of these
42. Out of 5 apples, 10 mangoes and 15 oranges, the number of ways of distributing 15 fruits each to two persons is
- a. 56 b. 64 c. 66 d. 72
43. In how many ways 20 identical bananas may be divided among 4 persons and if each person is to be given atleast one banana?
- a. 10626, 4845 b. 1771, 969
 c. 2024, 1140 d. None of these
44. A shopkeeper sells three varieties of the perfumes and he has a large number of bottles of the same size of each variety in his stock. There are 5 places in a row in his showcase. The number of different ways of displaying the three varieties of perfumes in the showcase is
- a. 6
 b. 50
 c. 150
 d. None of the above
45. There are 3 letters and 3 envelopes. The number of ways in which all letters are put in the wrong envelopes, is
- a. 4 b. 2 c. 6 d. 8

BITSAT Archives

1. A student is allowed to select atmost n books from a collection of $(2n+1)$ books. If the number of ways in which he can do this, is 64, then the value of n is [2014]
- a. 6 b. n c. 3 d. None
2. There are 10 points in a plane, out of these 6 are collinear. If n is the number of triangles formed by joining these points, then [2013]
- a. $n \leq 100$ b. $100 < n < 140$
 c. $140 < n \leq 190$ d. $n > 190$
3. $\frac{{}^8C_0}{6} - {}^8C_1 + {}^8C_2 \cdot 6 - {}^8C_3 \cdot 6^2 + {}^8C_4 \cdot 6^3 + \dots + {}^8C_8 \cdot 6^7$ is equal to [2013]
- a. 0 b. 6^7 c. 6^8 d. $\frac{5^8}{6}$
4. The number of ways in which four boys can be seated around a round table in four chairs of different colours, is [2012]
- a. 24 b. 12 c. 23 d. 64
5. The number of ways in which a team of 11 players can be selected from 22 players including 2 of them and excluding 4 of them, is [2012]
- a. ${}^{16}C_{11}$ b. ${}^{16}C_5$ c. ${}^{16}C_9$ d. ${}^{20}C_8$
6. In a cricket championship, there are 36 matches. The number of teams, if each plays 1 match with other are [2009]
- a. 9 b. 10
 c. 8 d. 12
7. ${}^{n-2}C_r + 2{}^{n-2}C_{r-1} + {}^{n-2}C_{r-2}$ equals [2009]
- a. ${}^{n+1}C_r$ b. nC_r c. ${}^nC_{r+1}$ d. ${}^{n-1}C_r$
8. 9 balls are to be placed in 9 boxes and 5 of the balls cannot fit into 3 small boxes. The number of ways of arranging one ball in each of the boxes is [2008]
- a. 18720 b. 18270 c. 17280 d. 12780
9. If ${}^nP_r = 30240$ and ${}^nC_r = 252$, then the ordered pair (n, r) is equal to [2008]
- a. (12, 6) b. (10, 5) c. (9, 4) d. (16, 7)
10. In how many ways can 5 boys and 5 girls sit in a circle so that no two boys sit together? [2007]
- a. $5! \times 5!$ b. $4! \times 5!$ c. $\frac{5! \times 5!}{2}$ d. None
11. Six X's have to be placed in the square of the figure such that each row contains atleast one X. In how many different ways can this be done? [2007]



- a. 27 b. 28 c. 26 d. 35
12. If ${}^nC_{12} = {}^nC_6$, then nC_2 is equal to [2006]
- a. 72 b. 153 c. 306 d. 2556
13. Total number of books is $2n+1$. One is allowed to select a minimum of the one book and a maximum of n books. If total number of selections is 63, then the value of n is [2005]
- a. 3 b. 6
 c. 2 d. None of these

Answer with Solutions

Practice Exercise

1. (d) Making a 3-digit number is equivalent to filling 3 places.

Places 1 2 3

Number of choices 5 4 3

Number of ways of filling all the three places

$$= 5 \times 4 \times 3 = 60$$

Hence, the total possible 3-digit numbers are 60.

2. (d) The student has 10 choices from the morning courses, out of which he can select one course in 10 ways.

For the evening course, he has 7 choices out of which he can select one course in 7 ways.

$\therefore$ Total number of ways = $10 + 7 = 17$

3. (b) There are 6 letters in the word BHARAT, 2 of them are identical.

Hence, total number of words with these letters

$$= \frac{6!}{2!} = 360$$

Also, the number of words in which B and H come together = $\frac{5! \cdot 2!}{2!} = 120$

$$\therefore \text{Required number of words} = 360 - 120 = 240$$

4. (a) $\therefore$ The consonants in their positions can be arranged in

$$\frac{4!}{2!} = 12 \text{ ways}$$

and the vowels in their positions can be arranged in

$$\frac{3!}{2!} = 3 \text{ ways}$$

$\therefore$ Total number of arrangements = $12 \times 3 = 36$

5. (b) The sum of the digits in unit place of all the numbers formed

$$= 3!(3 + 4 + 5 + 6) = 6 \times 18 = 108$$

6. (c) Here, we have 1 M, 4 I, 4 S and 2 P.

Therefore, total number of selections of one or more letters

$$= (1+1)(4+1)(4+1)(2+1) - 1 = 149$$

7. (a) $\therefore \frac{{}^nP_{r-1}}{a} = \frac{{}^nP_r}{b} = \frac{{}^nP_{r+1}}{c}$

From first two terms, $\frac{b}{a} = n - r + 1$

From last two terms, $\frac{c}{b} = n - r$

Hence, $\frac{b}{a} = \frac{c}{b} + 1 \Rightarrow b^2 = a(b + c)$

8. (d) Here, m men can be seated in $m!$ ways and n women can be seated in the $m+1$ gaps between the m men in ${}^{m+1}P_n$ ways.

$$\therefore \text{Number of ways} = m! \cdot {}^{(m+1)}P_n = \frac{m!(m+1)!}{(m-n+1)!}$$

9. (b) Number of words beginning with E = ${}^4P_4 = 24$

Number of words beginning with QE = ${}^3P_3 = 6$

Number of words beginning with QS = 6

Number of words beginning with QT = 6

So, the next word is 'QUEST'.

$$\therefore \text{Rank} = 24 + 6 + 6 + 6 + 1 = 43$$

10. (c) Since, 2 women can choose the seats from 1 to 4 in ${}^4P_2 = 12$ ways and 3 men can choose the remaining 6 seats in ${}^6P_3 = 120$ ways.

$$\therefore \text{Required number} = 12 \times 120 = 1440$$

11. (d) Here, 7 boys can be arranged in row in $7!$ ways. There will be 6 gaps between them and one place before them and one place after them. The 3 girls can be arranged in a row in ${}^8P_3 = 8 \cdot 7 \cdot 6$ ways

$$\therefore \text{Number of ways} = 7! \times 8 \cdot 7 \cdot 6 = 42 \cdot 8!$$

12. (a) The required numbers are of four digits whose left most digit can be 1, 2, 3.

Thus the left most place can be filled in 3 ways.

Since, repetition of digits is allowed, then each of the remaining place can be filled in 5 ways.

Number of ways of filling of the remaining positions

$$= 5 \times 5 \times 5 = 125$$

$\therefore$ Total number of required numbers

$$= 3 \times 125 = 375$$

13. (a) In the word MATHEMATICS, there are 11 letters out of which 2M's, 2A's, 2T's, 1H, 1E, 1C and 1S.

If we feed T at there starting and end the remaining.

T										T
---	--	--	--	--	--	--	--	--	--	---

A letter, out of which (2A's, 2T's) can be arranged by

$$\frac{9!}{2!2!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4} = 90720$$

14. (c) When we arrange one thing at a time, then the number of possible permutations is n . When we arrange them two at a time, the number of possible permutations are $n \times n = n^2$ and so on.

Thus, the total number of permutations are

$$n + n^2 + \dots + n^r = \frac{n(n^r - 1)}{n - 1} \quad [\because n > 1]$$

15. (c) By theory, let us consider 3 beads as one. Hence, we have 21 beads, i.e. $n = 21$.

The number of arrangements = $(n-1)! = (21-1)! = 20!$

Also, the number of ways in which 3 beads can be arranged between themselves is $3! = 3 \times 2 \times 1 = 6$.

$$\text{Thus, the total number of arrangements} = \frac{20! \cdot 3!}{2}$$

16. (d) $x = 6! \times 6! + 6! \times 6! = 2(6!)^2$

$$\begin{aligned} y &= 5! \times 6! \\ \therefore \frac{x}{y} &= 2 \times 6 \Rightarrow x = 12y \end{aligned}$$

17. (c) ${}^nC_r + 2{}^nC_{r-1} + {}^nC_{r-2} = {}^nC_r + {}^nC_{r-1} + {}^nC_{r-1} + {}^nC_{r-2}$
 $= {}^{n+1}C_r + {}^{n+1}C_{r-1} = {}^{n+2}C_r$

18. (c) Given, ${}^nC_{r-1} = 36$, ${}^nC_r = 84$, ${}^nC_{r+1} = 126$

$$\Rightarrow \frac{n!}{(r-1)!(n-r+1)!} = 36 \quad \dots(i)$$

$$\Rightarrow \frac{n!}{r!(n-r)!} = 84 \quad \dots(ii)$$

$$\Rightarrow \frac{n!}{(r+1)!(n-r-1)!} = 126 \quad \dots(iii)$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{(r-1)!(n-r+1)!}{r!(n-r)!} = \frac{84}{36}$$

$$\Rightarrow \frac{n-r+1}{r} = \frac{7}{3}$$

$$\Rightarrow 3n - 10r = -3 \quad \dots(iv)$$

Dividing Eq. (iii) by Eq. (ii), we get

$$\frac{r!(n-r)!}{(r+1)!(n-r-1)!} = \frac{126}{84} \Rightarrow \frac{n-r}{r+1} = \frac{3}{2}$$

$$\Rightarrow 2n - 5r = 3 \quad \dots(v)$$

Now, solving Eqs. (iv) and (v), we get $n = 9$, $r = 3$.

19. (a) **Hint** $({}^{47}C_4 + {}^{47}C_3) + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$
 $= {}^{48}C_4 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

20. (b) There are two cases arises.

Case I Choose six from Class XI and five from class XII
 $\therefore$ Number of ways $= {}^{20}C_6 \times {}^{20}C_5$

Case II Choose five from Class XI and six from Class XII
 $\therefore$ Number of ways $= {}^{20}C_5 \times {}^{20}C_6$

$\therefore$ Required number of ways $= 2({}^{20}C_5 \times {}^{20}C_6)$

21. (b) We have, M, IIII, SSSS, PP

Number of ways of selecting 4 different letters $= {}^4C_4 = 1$

Number of ways of selecting 2 alike letters $= {}^3C_2 = 3$

Number of ways of selecting 3 alike and 1 different letter
 $= {}^2C_1 \times {}^3C_1 = 6$

Number of ways of selecting 4 alike letters $= {}^2C_1 = 2$

Number of ways of selecting 2 alike and 2 different letters
 $= {}^3C_1 \times {}^3C_2 = 9$

$\therefore$ Total number of ways $= 1 + 3 + 6 + 2 + 9 = 21$

22. (d) Since, the voter can cast one or two or three votes.

So, total number of ways in which he can cast his vote
 $= {}^5C_1 + {}^5C_2 + {}^5C_3 = 5 + 10 + 10 = 25$

23. (c) Required number of ways

$$\begin{aligned} &= {}^4C_1 \times {}^8C_5 + {}^4C_2 \times {}^8C_4 + {}^4C_3 \times {}^8C_3 + {}^4C_4 \times {}^8C_2 \\ &= 4 \times 56 + 6 \times 70 + 4 \times 56 + 1 \times 28 = 896 \end{aligned}$$

24. (c) As there are three letters alike, out of six letters and the three others are different.

Selection can be done in the following manner:

(i) All different $= {}^4C_1 = 1$

(ii) 2 alike, 2 different $= {}^1C_1 \cdot {}^3C_2 = 3$

(iii) 3 alike and 1 different $= {}^1C_1 \cdot {}^3C_1 = 3$

Therefore,

total number of ways $= 1 + 3 + 3 = 7$

25. (d) Required number of points

$$= {}^8C_2 \times 1 + {}^4C_2 \times 2 + ({}^8C_1 \times {}^4C_1) \times 2$$

$$= 28 + 12 + 32 \times 2 = 104$$

26. (b) First stall can be filled in 3 ways, second stall can be filled in 3 ways and so on.

$\therefore$ Number of ways of loading steamer

$$= {}^3C_1 \times {}^3C_1 \times \dots \times {}^3C_1 \text{ (12 times)}$$

$$= 3 \times 3 \times \dots \times 3 \text{ (12 times)} = 3^{12}$$

27. (b) Since the candidate is unsuccessful, if he fails in 9 or 8 or 7 or 6 or 5 papers.

$\therefore$ Numbers of ways to be unsuccessful

$$= {}^9C_9 + {}^9C_8 + {}^9C_7 + {}^9C_6 + {}^9C_5$$

$$= {}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + {}^9C_4$$

$$= \frac{1}{2} ({}^9C_0 + {}^9C_1 + \dots + {}^9C_9)$$

$$= \frac{1}{2} (2^9) = 2^8 = 256$$

28. (d) The number of ways of drawing 1 black and 2 non-black balls

$$= {}^3C_1 \cdot {}^6C_2 = 3 \cdot 15 = 45$$

The number of ways of drawing 2 black and 1 non-black ball

$$= {}^3C_2 \cdot {}^6C_1 = 3 \cdot 6 = 18$$

The number of ways of drawing 3 black balls is ${}^3C_3 = 1$

$\therefore$ Number of ways $= 45 + 18 + 1 = 64$

29. (b) Let the men be $M_1, M_2, \dots, M_{10}$ and women be $W_1, W_2, \dots, W_7$.

Let W_1 and W_2 do not want to be in the same group.

The six members group can contain 4 men and 2 women or 3 men and 3 women.

The number of ways of forming 4M, 2W group

$$= {}^{10}C_4 ({}^5C_2 + 2 {}^5C_1) = 4200$$

where, 5C_2 is the number of ways without W_1 and W_2 and

5C_1 is the number of ways with W_1 and without W_2 or with W_2 and without W_1 .

The number of ways of forming 3M, 3W group

$$= {}^{10}C_3 ({}^5C_3 + 2 {}^5C_2) = 3600$$

where, 5C_3 is the number of ways without W_1 and W_2 and

5C_2 is the number of ways with W_1 or W_2 but not both.

$\therefore$ Number of ways $= 4200 + 3600$
 $= 7800$

30. (c) Total number of ways
 = (Attempt 3 from group I and 4 from group II)
 + (Attempt 4 from group I and 3 from group II)
 + (Attempt 5 from group I and 2 from group II)
 + (Attempt 2 from group I and 5 from group II)
 $= {}^6C_3 \times {}^6C_4 + {}^6C_4 \times {}^6C_3 + {}^6C_5 \times {}^6C_2 + {}^6C_2 \times {}^6C_5$
 $= 2({}^6C_3 \times {}^6C_4) + 2({}^6C_5 \times {}^6C_2)$
 $= 2(20 \times 15) + 2(6 \times 15) = 600 + 180 = 780$

31. (b) Ladies = 2, old men = 2, young men = 4

Case I 1 lady, 1 old man, 2 young men
 $= {}^2C_2 \cdot {}^2C_1 \cdot {}^4C_2 = 2 \cdot 2 \cdot 6 = 24$

Case II 2 ladies, 1 old man, 1 young man
 $= {}^2C_2 \cdot {}^2C_1 \cdot {}^4C_1 = 1 \cdot 2 \cdot 4 = 8$

Case III 1 lady, 2 old men, 1 young man
 $= {}^2C_1 \cdot {}^2C_2 \cdot {}^4C_1 = 2 \cdot 1 \cdot 4 = 8$

Case IV 2 ladies, 2 old men, 0 young man
 $= {}^2C_2 \cdot {}^2C_1 \cdot {}^4C_0 = 1 \cdot 1 \cdot 1 = 1$

$\therefore$ Required number of ways = $24 + 8 + 8 + 1 = 41$

32. (b) $\therefore 1350 = 2 \times 3^3 \times 5^2$
 $\therefore$ Number of divisors = $(1+1)(3+1)(2+1) = 24$
 and sum of divisors = $(1+2)(1+3+3^2+3^3)(1+5+5^2)$
 $= 3720$

33. (c) Any number between 1 to 999 is of the form abc when $0 \leq a, b, c \leq 9$. Let us first count the number in which 5 occurs exactly once.

Since 5 can occur at one place in $1 \times {}^3C_1 \times 9 \times 9 = 243$ ways, next 5 can occur in exactly two places in ${}^3C_2 \times 9 = 27$. Lastly, 5 can occur in all three digits in only one way.

Hence, the number of times 5 occurs

$$= 1 \times 243 + 27 \times 2 + 1 \times 3 \\ = 243 + 54 + 3 = 300$$

34. (b) The number of students answering exactly i ($1 \leq i \leq n-1$) questions wrongly is $2^{n-i} - 2^{n-i-1}$.

The number of students answering all n questions wrongly is 2^0 .

Hence, the total number of wrong answers

$$= \sum_{i=1}^{n-1} i(2^{n-i} - 2^{n-i-1}) + n(2^0) = 2047$$

$$\Rightarrow 2^{n-1} + 2^{n-2} + 2^{n-3} + \dots + 2^0 = 2047$$

$$\Rightarrow 2^n - 1 = 2047 \Rightarrow 2^n = 2048 \Rightarrow 2^n = 2^{11} \Rightarrow n = 11$$

35. (c) Required number of ways
 $= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$
 $= 10 + 45 + 120 + 210 = 385$

36. (d) Given, word = SACHIN
 Number of words start with A = $5!$
 Number of words start with C = $5!$

Number of words start with H = $5!$

Number of words start with I = $5!$

Number of words start with N = $5!$

Total number of words

$$= 5! + 5! + 5! + 5! + 5! = 5(5!) = 600$$

Now, add the rank of SACHIN.

$\therefore$ Required rank of SACHIN = $600 + 1 = 601$

37. (a) $\therefore$ Number of letters = 6

Number of vowels = 2 namely A, E

These alphabets can arrange themselves by $2!$ ways.

$$\therefore \text{Number of words} = \frac{6!}{2!} = 360$$

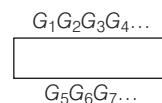
38. (b) Since, 3 men and 2 women equal to 5 and a group of 5 members make $5!$ permutations with each other.

$\therefore$ Number of ways to sit 5 members = $5!$

6 places are filled by 5 members by 6C_5 ways.

$\therefore$ Total number of ways to sit 5 members on 6 seats of a bus = ${}^6C_5 \times 5!$

39. (b) Here, 4 guests G_1, G_2, G_3, G_4 can be seated on a particular side in 4P_4 ways.



Also, 3 guests G_5, G_6, G_7 can be seated on the opposite side in 3P_3 ways.

The other 11 guests can be seated in the remaining 11 seats in $11!$ ways.

$\therefore$ Number of sitting arrangements = ${}^4P_4 \cdot {}^3P_3 \cdot 11!$

$$= \frac{(9!)^2 \cdot 11!}{5! \cdot 6!} = {}^{11}C_5 (9!)^2$$

40. (a) Make a group of both M's and another of T's. Then, except A's, we have S letters remaining. So 'M's, T's and the letters except A's can be arranged in $7!$ ways.

$\therefore$ Total number of arrangements = $7! \times {}^8C_2$

41. (c) A triangle can be constructed in two ways:

(i) By taking two points on AB and one point on AC .

(ii) By taking two points on AC and one point on AB .

If point A is included, then total number of triangles

$$= {}^{m+1}C_2 \times {}^nC_1 + {}^mC_1 \times {}^nC_2 = \frac{mn}{2}(m+n)$$

42. (c) Required number is the coefficient of x^{15} in the product $(1+x+x^2+\dots+x^5)(1+x+x^2+\dots+x^{10})(1+x+x^2+\dots+x^{15})$

$$= (1-x^6)(1-x^{11})(1-x^{16})(1-x)^{-3}$$

$$= (1-x^6-x^{11}+\dots) \left(1 + \binom{3}{1}x + \binom{4}{2}x^2 + \dots \right)$$

$$\text{which is } \binom{17}{15} - \binom{11}{9} - \binom{6}{4} = \binom{17}{2} - \binom{11}{2} - \binom{6}{2} \\ = 136 - 55 - 15 = 66$$

43. (b) Identical bananas may be divided among 4 persons in ${}^{20+4-1}C_{4-1} = {}^{23}C_3 = 1771$ ways

If each person is to be given atleast one banana, then number of ways will be ${}^{20-1}C_{4-1} = {}^{19}C_3 = 969$ ways

Possibilities	Selections	Arrangements
One triplet, two different	${}^3C_1 \times {}^2C_2 = 3$	$3 \times \frac{5!}{3!} = 60$
Two pairs, one different	${}^3C_2 \times {}^1C_1 = 3$	$3 \times \frac{5!}{2! \times 3!} = 90$

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1. (c) **Hint** Number of ways

$$= {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \dots + {}^{2n+1}C_n$$

$$= \frac{1}{2}(2^{2n+1}) = 2^{2n}$$

2. (a) **Case I** Taking 2 points from collinear points and one from non-collinear.

i.e. Number of triangles so formed = ${}^6C_2 \times {}^4C_1$

$$= \frac{6 \cdot 5}{1 \cdot 2} \times \frac{4}{1} = 60$$

Case II Taking 1 point from collinear and two from non-collinear points.

i.e. Number of triangles so formed = ${}^6C_1 \times {}^4C_2$

$$= \frac{6}{1} \times \frac{4 \cdot 3}{1 \cdot 2} = 36$$

Case III All the three points from non-collinear points.

i.e. Number of triangles so formed

$$= {}^4C_3 = {}^4C_1 = \frac{4}{1} = 4$$

$\therefore$ Total number of triangles = $60 + 36 + 4 = 100$

3. (d) $\frac{{}^8C_0}{6} - {}^8C_1 + {}^8C_2 \cdot 6 - {}^8C_3 \cdot 6^2 + {}^8C_4 \cdot 6^3 + \dots + {}^8C_8 \cdot 6^7$
- $$= \frac{1}{6} [{}^8C_0 - {}^8C_1 \cdot 6^1 + {}^8C_2 \cdot 6^2 - {}^8C_3 \cdot 6^3 + \dots + {}^8C_8 \cdot 6^8]$$
- $$= \frac{1}{6} [1 - 6]^8 = \frac{1}{6} \times (-5)^8 = \frac{5^8}{6}$$

4. (a) Required number of ways = $4! = 24$

5. (c) Required number of ways
- $$= {}^{22-4-2}C_{11-2} = {}^{16}C_9$$

6. (a) **Hint** ${}^nC_2 = 36$

7. (b) ${}^{n-2}C_r + 2 {}^{n-2}C_{r-1} + {}^{n-2}C_{r-2}$
- $$= ({}^{n-2}C_r + {}^{n-2}C_{r-1}) + ({}^{n-2}C_{r-1} + {}^{n-2}C_{r-2})$$
- $$= {}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^nC_r \quad [\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r]$$

8. (c) Required number of arrangements
- $$= {}^6P_5 \times 4! = 720 \times 24 = 17280$$

Hence, the required number of ways

$$= 60 + 90 = 150$$

45. (b) The required number of ways

$$= 3! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \right]$$

$$= 3! \left(\frac{2}{6} \right)$$

$$= 3 \times 2 \times \frac{1}{3} = 2$$

9. (b) Given that, ${}^nP_r = 30240$ and ${}^nC_r = 252$

$$\Rightarrow \frac{n!}{(n-r)!} = 30240 \text{ and } \frac{n!}{(n-r)!r!} = 252$$

$$\Rightarrow r! = \frac{30240}{252} = 120 \Rightarrow r = 5 \therefore \frac{n!}{(n-5)!} = 30240$$

$$\Rightarrow n(n-1)(n-2)(n-3)(n-4) = 30240$$

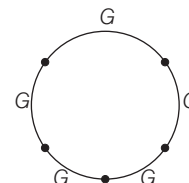
$$\Rightarrow n(n-1)(n-2)(n-3)(n-4)$$

$$= 10(10-1)(10-2)(10-3)(10-4)$$

$$\Rightarrow n = 10$$

Hence, required ordered pair is (10, 5).

10. (b) First, we fix the alternate position of the girls. Five girls can be seated around the circle in $(5-1)! = 4!$. 5 boys can be seated in five vacant places by 5!.



$\therefore$ Required number of ways = $4! \times 5!$

11. (c) In all, we have 8 squares in which six X have to be placed and it can be done in ${}^8C_6 = 28$ ways. But this includes the possibility that either the top or horizontal row does not have any X.

Since we want each row must have atleast one X, these two possibilities are to be excluded.

Hence, required number of ways = $28 - 2 = 26$

12. (b) Given, ${}^nC_{12} = {}^nC_6$ or ${}^nC_{n-12} = {}^nC_6$

$$\Rightarrow n - 12 = 6 \Rightarrow n = 18$$

$$\therefore {}^nC_2 = {}^{18}C_2 = \frac{18 \times 17}{2 \times 1} = 153$$

13. (a) Since, $(1+x)^{2n+1} = C_0 + C_1x + \dots + C_nx^n + C_{n+1}x^{n+1} + \dots + x^{2n+1}C_n$
- $$= 2(C_0 + C_1 + \dots + C_nx^n)$$

Put $x = 1$

$$(1+1)^{2n-1} = 2(C_0 + C_1 + \dots + C_n)$$

$$\Rightarrow 2^{2n} = (C_0 + C_1 + \dots + C_n)$$

$$\Rightarrow 2^{2n} - 1 = C_1 + C_2 + \dots + C_n \Rightarrow 2^{2n} - 1 = 63$$

$$\Rightarrow 2^{2n} = 64 \Rightarrow 2^{2n} = 2^6 \Rightarrow 2n = 6 \Rightarrow n = 3$$

6

Binomial Theorem and Mathematical Induction

Introduction

An expression having two terms connected by either + or – sign, is called a **binomial expression**. e.g. $x + y$, $\left(\frac{1}{x} - \frac{1}{x^3}\right)$, $\left(7x - \frac{3}{4y}\right)$, etc., are all binomial expressions. Similarly, a ‘trinomial expression’ has three terms. In general, multinomial expression has more than two terms.

Binomial Theorem

The expansion of a binomial expression raised to the power of n , where n can be positive, negative or a fraction, is called binomial theorem.

Binomial Theorem for Positive Index

If n is any positive integer, then $(x + a)^n = {}^nC_0x^n + {}^nC_1x^{n-1}a + \dots + {}^nC_na^n = \sum_{r=0}^n {}^nC_r x^{n-r} a^r$, where x and a are real (complex) numbers.

- (i) In the expansion of $(x + a)^n$, it contains $(n + 1)$ terms.
- (ii) In the expansion of $(x + a)^n$, the sum of the powers of x and a in each term is equal to n .
- (iii) The coefficient of terms equidistant from the beginning and the end are equal.
- (iv) The values of the binomial coefficients steadily increase to maximum and then steadily decrease.
- (v) $(x - a)^n = {}^nC_0x^n - {}^nC_1x^{n-1}a + \dots + (-1)^n {}^nC_na^n$
- (vi) $(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \dots + {}^nC_nx^n$

(vii) If n is a positive integer, then the number of terms in $(x + y + z)^n$ is $\frac{(n+1)(n+2)}{2}$.

(viii) The number of terms in the expansion of $(x + a)^n + (x - a)^n = \begin{cases} \frac{n+2}{2}, & \text{if } n \text{ is even} \\ \frac{n+1}{2}, & \text{if } n \text{ is odd} \end{cases}$

(ix) The number of terms in the expansion of $(x + a)^n - (x - a)^n = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even} \\ \frac{n+1}{2}, & \text{if } n \text{ is odd} \end{cases}$

(x) Binomial coefficient ${}^nC_r = \frac{n!}{r!(n-r)!}$ for $0 \leq r \leq n$

So, ${}^nC_0 = 1$,

$${}^nC_1 = n, {}^nC_2 = \frac{n(n-1)}{2!},$$

$${}^nC_3 = \frac{n(n-1)(n-2)}{3!}$$

$${}^nC_n = 1$$

(xi) The coefficients in the expansion follow a certain pattern known as Pascal's triangle.

Pascal's Triangle

From the Pascal's triangle, it is clear that each coefficient of any row is obtained by adding two coefficients of preceding row, one on the immediate left and other on the immediate right and each row has 1 on both extreme sides.

Index of binomial	Coefficients of all terms
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1

General Term in a Binomial Expansion

Let $(r+1)$ th term be the general term in the expansion of $(x + a)^n$.

Then, $T_{r+1} = {}^nC_r x^{n-r} a^r$

If expansion is $(x - a)^n$, then the general term is

$$(-1)^r \cdot {}^nC_r x^{n-r} a^r$$

Middle Term in a Binomial Expansion

The middle term in the expansion of $(a + x)^n$ is as follows :

Case I If n is even, then $\left(\frac{n}{2} + 1\right)$ th term is the middle term.

Case II If n is odd, then $\frac{(n+1)}{2}$ th term and $\frac{(n+3)}{2}$ th term are the middle terms.

r th Term from the End in the Binomial Expansion

In the binomial expansion of $(x + a)^n$, r th term from the end is $(n+1-r+1)$ i.e. $(n-r+2)$ th term.

Method for Finding the Independent Term

Write down general term in the expansion of $(x + a)^n$.

i.e. $T_{r+1} = {}^nC_r x^{n-r} a^r$... (i)

For making a term independent of x , we put $r = n$ in Eq. (i).

So, we get ${}^nC_n a^n$, which is independent of x .

For making a term independent of a , we put $r = 0$ in Eq. (i).

So, we get ${}^nC_0 x^n$, which is independent of a .

NOTE An independent term is also known as constant term.

Greatest Term

If T_r and T_{r+1} are the r th and $(r+1)$ th terms in the expansion of $(1 + x)^n$, then

$$\frac{T_{r+1}}{T_r} = \frac{{}^nC_r x^r}{{}^nC_{r-1} x^{r-1}} = \frac{n-r+1}{r} x$$

Let numerically, T_{r+1} be the greatest term in the above expansion. Then,

$$T_{r+1} \geq T_r \quad \text{or} \quad \frac{T_{r+1}}{T_r} \geq 1$$

$$\therefore \frac{n-r+1}{r} |x| \geq 1 \quad \text{or} \quad r \leq \frac{(n+1)}{(1+|x|)} |x| \quad \dots (i)$$

Now, substituting the values of n and x in Eq. (i), we get $r \leq m + f$ or $r \leq m$, where m is a positive integer and f is a fraction such that $0 < f < 1$.

When n is even, then T_{m+1} is the greatest term and when n is odd, then T_m and T_{m+1} are the greatest terms and both are equal.

Greatest Coefficients in the Expansion of $(x + a)^n$

(i) If n is even, nC_r is greatest when $r = \frac{n}{2}$ i.e. greatest coefficient is ${}^nC_{n/2}$.

(ii) If n is odd, nC_r is greatest when $r = \frac{n-1}{2}$ or $r = \frac{n+1}{2}$ i.e. greatest coefficients are ${}^nC_{\frac{n-1}{2}}$ and ${}^nC_{\frac{n+1}{2}}$.

Properties of Binomial Coefficients

In the binomial expansion of $(1+x)^n$,

$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \dots + {}^nC_rx^r + \dots + {}^nC_nx^n$, where ${}^nC_0, {}^nC_1, \dots, {}^nC_n$ are the coefficients of various powers of x , are called **binomial coefficient** and it is also written as

$$C_0, C_1, \dots, C_n \text{ or } \binom{n}{0} + \binom{n}{1} + \dots + \binom{n}{n}.$$

$$(i) {}^nC_r = {}^nC_s \Rightarrow r = s \text{ or } r + s = n \Rightarrow {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

$$(ii) {}^{n+1}C_{r+1} = \frac{n+1}{r+1} {}^nC_r \quad (iii) \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$$

$$(iv) C_0 + C_1 + C_2 + \dots + C_n = 2^n$$

$$(v) C_0 + C_2 + C_4 + \dots = C_1 + C_3 + C_5 + \dots = 2^{n-1}$$

$$(vi) C_0 - C_1 + C_2 - C_3 + \dots + (-1)^n \cdot C_n = 0$$

$$(vii) C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2 = {}^{2n}C_n = \frac{(2n)!}{(n!)^2}$$

$$(viii) C_0C_r + C_1C_{r+1} + \dots + C_{n-r}C_n = {}^{2n}C_{n-r} = \frac{(2n)!}{(n-r)!(n+r)!}$$

$$(ix) C_1 - 2C_2 + 3C_3 - \dots = 0$$

$$(x) C_0 + 2C_1 + 3C_2 + \dots + (n+1) \cdot C_n = (n+2)2^{n-1}$$

$$(xi) C_0 + \frac{C_1}{2}x + \frac{C_2}{3}x^2 + \frac{C_3}{4}x^3 + \dots + \frac{C_n}{n+1} \cdot x^n = \frac{(1+x)^{n+1} - 1}{(n+1)n}$$

$$(xii) C_0 - C_2 + C_4 - C_6 + \dots = (\sqrt{2})^n \cos \frac{n\pi}{4}$$

$$(xiii) C_1 - C_3 + C_5 - C_7 + \dots = (\sqrt{2})^n \sin \frac{n\pi}{4}$$

Applications of Binomial Theorem

(i) R - f Factor Relation

Here, we are going to discuss problems involving

$$(\sqrt{A} + B)^n = l + f$$

where, l and n are positive integers.

$$0 \leq f \leq 1, |A - B^2| = k \text{ and } |\sqrt{A} - B| < 1$$

(ii) Divisibility Problem

In the expansion of $(1+\alpha)^n = 1 + {}^nC_1\alpha + {}^nC_2\alpha^2 + \dots + {}^nC_n\alpha^n$, we can conclude that $(1+\alpha)^n - 1 = {}^nC_1\alpha + {}^nC_2\alpha^2 + \dots + {}^nC_n\alpha^n$ is divisible by α i.e. it is a multiple of α .

Binomial Theorem for Negative/Rational Index

Let n be a rational number and x be a real number such that $|x| < 1$, then

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$$

e.g. The expansion of $(x+a)^n = a^n \left(1 + \frac{x}{a}\right)^n$

$$= a^n \left\{ 1 + n\left(\frac{x}{a}\right) + \frac{n(n-1)}{2!}\left(\frac{x}{a}\right)^2 + \dots \right\}$$

The above expansion is valid only, when $\left|\frac{x}{a}\right| < 1$.

This expansion is infinite. It terminates after $(n+1)$ terms, if n is +ve integer, otherwise it goes to infinity.

General Term in the Expansion of $(1+x)^n$

General term in the expansion of $(1+x)^n$ is

$$T_{r+1} = \frac{[n(n-1)(n-2)\dots\{n-(r-1)\}]}{r!} x^r$$

Important Expansions and Results

$$(i) (1+x)^n = {}^nC_0x^0 + {}^nC_1x + {}^nC_2x^2 + \dots + {}^nC_rx^r + \dots + {}^nC_{n-1}x^{n-1} + {}^nC_nx^n$$

$$= \sum_{r=0}^n {}^nC_rx^r$$

$$(ii) (1-x)^n = {}^nC_0 x^0 - {}^nC_1 x + {}^nC_2 x^2 - \dots + {}^nC_{n-1} (-1)^{n-1} x^{n-1} + {}^nC_n (-1)^n x^n$$

$$= \sum_{r=0}^n (-1)^r \cdot {}^nC_r x^r$$

$$(iii) (x+a)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} a + {}^nC_2 x^{n-2} a^2 + \dots + {}^nC_r x^{n-r} a^r + \dots + {}^nC_n x^0 a^n$$

$$(iv) (a+x)^n = {}^nC_0 a^n x^0 + {}^nC_1 a^{n-1} x + {}^nC_2 a^{n-2} x^2 + \dots + {}^nC_r a^{n-r} \cdot x^r + \dots + {}^nC_n x^n$$

$$= \sum_{r=0}^n {}^nC_r a^{n-r} \cdot x^r$$

$$(v) (a-x)^n = {}^nC_0 a^n x^0 - {}^nC_1 a^{n-1} x + {}^nC_2 a^{n-2} x^2 - \dots + (-1)^r {}^nC_r a^{n-r} x^r + \dots + (-1)^n {}^nC_n a^{n-n} x^n$$

$$= \sum_{r=0}^n (-1)^r \cdot {}^nC_r a^{n-r} x^r$$

$$(vi) (a+x)^n + (a-x)^n = 2({}^nC_0 a^n x^0 + {}^nC_2 a^{n-2} x^2 + \dots)$$

$$= 2(\text{Sum of the terms at odd places})$$

$$(vii) (a+x)^n - (a-x)^n = 2({}^nC_1 a^{n-1} x + {}^nC_3 a^{n-3} x^3 + \dots)$$

$$= 2(\text{Sum of the terms at even places})$$

$$(viii) (1+x)^{-1} = 1 - x + x^2 - x^3 \dots + (-1)^r x^r + \dots$$

$$(ix) (1-x)^{-1} = 1 + x + x^2 + x^3 \dots + x^r + \dots$$

$$(x) (1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 \dots + (-1)^r (r+1)x^r + \dots$$

$$(xi) (1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 \dots + (r+1)x^r + \dots$$

$$(xii) (1+x)^{-3} = 1 - 3x + 6x^2 - \dots + (-1)^r (r+1)(r+2)x^r + \dots$$

$$(xiii) (1-x)^{-3} = 1 + 3x + 6x^2 + \dots + (r+1)(r+2)x^r + \dots$$

$$(xiv) \text{ If the coefficients of } r\text{th, } (r+1)\text{th, } (r+2)\text{th terms of } (1+x)^n \text{ are in AP, then } n^2 - (4r+1)n + 4r^2 = 2.$$

Principle of Mathematical Induction

In an algebra, there are certain results that are formulated in terms of n , where n is a positive integer. Such results can be proved by specific techniques, which is known as the principle of mathematical induction.

First Principle of Mathematical Induction

It consists of the following three steps.

Step I Actual verification of the proposition for the starting value i .

Step II Assuming the proposition to be true for $k, k \geq i$ and proving that it is true for the value $(k+1)$, which is next higher integer.

Step III To combine the above two steps, let $p(n)$ be a statement involving the natural number n such that

(i) $p(1)$ is true i.e. $p(n)$ is true for $n = 1$.

(ii) $p(m+1)$ is true, whenever $p(m)$ is true i.e. $p(m)$ is true $\Rightarrow p(m+1)$ is true. Then, $p(n)$ is true for all natural numbers n .

Second Principle of Mathematical Induction

The second principle of mathematical induction consists of following steps:

Step I Actual verification of the proposition for the starting values of i and $(i+1)$.

Step II Assuming the proposition to be true for $k-1$ and k and then proving that it is true for the value $k+1; k \geq i+1$.

Step III To combine the above two steps, let $p(n)$ be a statement involving the natural number n such that

(i) $p(1)$ is true i.e. $p(n)$ is true for $n = 1$.

(ii) $p(m+1)$ is true, whenever $p(m)$ is true for all n , where $i \leq n \leq m$. Then, $p(n)$ is true for all natural numbers.

For $a \neq b$, the expression $a^n - b^n$ is divisible by

(a) $a+b$, if n is even.

(b) $a-b$, if n is odd or even.

Practice Exercise

- In the expansion of $\left(2^{1/3} + \frac{1}{3^{1/3}}\right)^n$, if the ratio of 7th term from the beginning to 7th term from the end is 1 : 6, then n is equal to
a. 7 b. 8 c. 9 d. 10
- In the expansion of $\left[\frac{1}{a} + a^{\log_{10} a}\right]^5$, if the value of the third term is 1000, then the value of a is
a. 10 b. 100 c. 1000 d. 99
- If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$, then
a. $\operatorname{Re}(z) = 0$ b. $\operatorname{Im}(z) = 0$
c. $\operatorname{Re}(z) > 0, \operatorname{Im}(z) > 0$ d. $\operatorname{Re}(z) > 0, \operatorname{Im}(z) < 0$
- The expansion of $(x + \sqrt{x^3 - 1})^5 + (x - \sqrt{x^3 - 1})^5$ is a polynomial of degree
a. 5 b. 7 c. 9 d. 6
- The coefficient of x^4 in the expansion of $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$ is
a. $\frac{305}{256}$ b. $\frac{405}{256}$ c. $\frac{504}{256}$ d. 0
- The expression $7^{2n} + 2^{3n-3} \cdot 3^{n-1}$, where $n \in N$, is divisible by
a. 7 b. 25 c. 2 d. 5
- If 7th term in the binomial expansion of $\left(\frac{3}{\sqrt[3]{84}} + \sqrt{3} \ln x\right)^9$, $x > 0$ is equal to 729, then x can be
a. e^2 b. e c. $\frac{e}{2}$ d. $2e$
- If the coefficients of r th, $(r+1)$ th and $(r+2)$ th terms in the binomial expansion of $(1+y)^m$ are in AP, then m and r satisfy the equation
a. $m^2 - m(4r-1) + 4r^2 + 2 = 0$
b. $m^2 - m(4r+1) + 4r^2 - 2 = 0$
c. $m^2 - m(4r+1) + 4r^2 + 2 = 0$
d. $m^2 - m(4r-1) + 4r^2 - 2 = 0$
- Which of the following is middle term in the expansion of $(1+x)^{2n}$?
a. $\frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{n!} 2^n \cdot x^n$ b. $\frac{1 \cdot 2 \cdot 3 \cdot 4 \dots (n+1)}{(n+1)!}$
c. $\frac{1 \cdot 2 \cdot 3 \cdot 4 \dots n}{n!}$ d. None of these
- The middle term in the expansion of $\left(\frac{2}{3}x^2 - \frac{3}{2x}\right)^{20}$ is
a. ${}^{20}C_{10} x^{11}$ b. ${}^{20}C_9 x^{10}$ c. ${}^{20}C_{10} x^{10}$ d. ${}^{20}C_9 x^9$
- If the last term in $\left(\sqrt[3]{2} + \frac{1}{\sqrt{2}}\right)^n$ is $\left[\frac{1}{3(9)^{1/3}}\right]^{\log_3 8}$, then the fifth term is
a. 210 b. 310 c. 200 d. 300
- In the expansion of $(5^{1/6} + 2^{1/8})^{100}$, the number of irrational terms are
a. 93 b. 95 c. 97 d. 99
- The greatest term in the expansion of $\sqrt{3}\left(1 + \frac{1}{\sqrt{3}}\right)^{20}$ is
a. $\binom{20}{7} \frac{1}{27}$ b. $\binom{20}{6} \frac{1}{81}$ c. $\frac{1}{9} \binom{20}{9}$ d. $\binom{20}{8} \frac{1}{3}$
- The greatest term in the expansion of $(3+5x)^{15}$, when $x = \frac{1}{5}$, is
a. ${}^{15}C_3$ b. ${}^{15}C_4 \cdot 3^{11}$ c. ${}^{15}C_{10} \cdot 3^{10}$ d. ${}^{15}C_3 \cdot 3^{13}$
- The term independent of x in the expansion of $\left(3x - \frac{2}{x^2}\right)^{15}$ is
a. $-3003(3^{10})(2^5)$ b. $-3003(3^{10})(2^4)$
c. $3003(3^{10})(2^5)$ d. None of these
- If p is a real number and the middle term in the expansion of $\left(\frac{p}{2} + 2\right)^8$ is 1120, then the value of p is
a. ± 3 b. ± 1 c. ± 2 d. ± 4
- The constant term in the expansion of $\left(1+x+\frac{2}{x}\right)^6$ is
a. 479 b. 517 c. 569 d. 581
- If $(r+1)$ th term in the expansion of $\left(\sqrt[3]{\frac{a}{b}} + \sqrt{\frac{b}{a}}\right)^{21}$ has the same power of a and b , then the value of r is
a. 9 b. 10 c. 8 d. 6
- If the coefficients of x^7 and x^8 are equal in $\left(2 + \frac{x}{3}\right)^n$, then n is
a. 56 b. 55 c. 45 d. 15
- If the coefficient of three consecutive terms in the expansion of $(1+a)^n$ are in the ratio 1 : 7 : 42, then the value of n is
a. 51 b. 53 c. 55 d. 57

21. If the sum of all the coefficients in the expansion of $(1 - 3x + 10x^2)^n$ is α and the sum of all the coefficients in the expansion of $(1 + x^2)^n$ is β , then which of the following is correct?
 a. $\alpha = 3\beta$ b. $\alpha = \beta^3$ c. $\beta = \alpha^3$ d. $\alpha = 2\beta$
22. If $C_0, C_1, C_2, \dots, C_n$ denote the binomial coefficients in the expansion of $(1 + x)^n$, then $C_0^2 + 2C_1^2 + 3C_2^2 + \dots + (n + 1)C_n^2$ equals
 a. $(2n + 1)^{2n}C_n$ b. $(2n - 1)^{2n}C_n$
 c. $\left(\frac{n}{2} + 1\right)^{2n}C_n$ d. $\left(\frac{n}{2} + 1\right)^{2n-1}C_n$
23. If a_1, a_2, a_3 and a_4 are the coefficients of any four consecutive terms in the expansion of $(1 + x)^n$, then the value of $\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4}$ is
 a. $\frac{2a_2}{a_2 + a_3}$ b. $\frac{2a_2}{a_3}$ c. $\frac{a_2}{a_2 + a_3}$ d. $\frac{a_3}{a_2}$
24. The ratio of the coefficient of x^{15} to the term independent of x in the expansion of $\left(x^2 + \frac{2}{x}\right)^{15}$, is
 a. 7 : 16 b. 7 : 64 c. 1 : 4 d. 1 : 32
25. The term independent of x in the expansion of $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$ is
 a. 4 b. 120 c. 210 d. 310
26. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is
 a. -132 b. -144 c. 132 d. 144
27. The value of ${}^{50}C_4 + \sum_{r=1}^6 {}^{56-r}C_3$ is
 a. ${}^{56}C_4$ b. ${}^{56}C_3$ c. ${}^{55}C_3$ d. ${}^{55}C_4$
28. If $\binom{n}{r+1} = 56$, $\binom{n}{r} = 28$ and $\binom{n}{r-1} = 8$, then $n + r$ is equal to
 a. 8 b. 10 c. 12 d. 9
29. If $(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^2 + \dots + a_{12}x^{12}$, then the expression $a_2 + a_4 + \dots + a_{12}$ is equal to
 a. 32 b. 63 c. 64 d. 31
30. The least positive integer n such that $\binom{n-1}{3} + \binom{n-1}{4} > \binom{n}{3}$, is
 a. 6 b. 7 c. 8 d. 9
31. If $C_r = \binom{10}{r}$, then $\sum_{r=1}^{10} C_{r-1} C_r$ is equal to
 a. $\binom{20}{9}$ b. $\binom{20}{10}$ c. $\binom{20}{13}$ d. $\binom{20}{8}$
32. If $n \in N$, then $121^n - 25^n + 1900^n - (-4)^n$ is divisible by
 a. 1904 b. 2000 c. 2002 d. 2006
33. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9, is
 a. 0 b. 2 c. 7 d. 8
34. If $x = (7 + 4\sqrt{3})^{2n} = [x] + f$, then the value of $x(1 - f)$ is
 a. 2^{2n} b. 3^{2n} c. 1^{2n} d. 4
35. The expression $n^3 + 3n^2 + 5n + 3$, $n \in N$, is divisible by
 a. 3 b. 4 c. 5 d. 6
36. The sum of the series ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \dots + {}^{20}C_{10}$ is
 a. $-{}^{20}C_{10}$ b. $\frac{1}{2} {}^{20}C_{10}$ c. 0 d. ${}^{20}C_{10}$
37. The number of terms in the expansion of $(a + b + c)^n$, where $n \in N$, is
 a. $\frac{(n+1)(n+2)}{2}$ b. $n + 1$
 c. $n + 2$ d. $(n + 1)n$
38. The value of $\sum_{r=0}^{n-1} \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}}$ is
 a. $\frac{n}{2}$ b. $\frac{n+1}{2}$ c. $\frac{n-1}{2}$ d. $2n$
39. The sum of the rational terms in the binomial expansion of $(2^{1/2} + 3^{1/5})^{10}$ is
 a. 25 b. 32 c. 9 d. 41
40. If a and b are the coefficients of x^r and x^{n-r} , respectively in the expansion of $(1 + x)^n$, then
 a. $a = b$ b. $a + b = n^2$ c. $a = nb$ d. $a + b = 2^{n/2}$
41. If n is a positive integer, then $(\sqrt{3} + 1)^{2n} - (\sqrt{3} - 1)^{2n}$ is
 a. an irrational number
 b. an odd positive integer
 c. an even positive integer
 d. a rational number other than positive integers
42. Using principle of mathematical induction, the value of the series $\frac{1}{3 \cdot 7} + \frac{1}{7 \cdot 11} + \frac{1}{11 \cdot 14} + \dots + \frac{1}{(4n-1)(4n+3)}$ is
 a. $\frac{n}{3(4n+3)}$ b. $\frac{n}{5(5n+3)}$ c. $\frac{n}{3(7n+3)}$ d. 0
43. In the expansion of $\left(x^3 - \frac{1}{x^2}\right)^{15}$, the constant term is
 a. ${}^{15}C_9$ b. 0 c. ${}^{10}C_9$ d. $-{}^{15}C_9$
44. If $s_n = \sum_{r=0}^n \frac{1}{{}^nC_r}$ and $t_n = \sum_{r=0}^n \frac{r}{{}^nC_r}$, then $\frac{t_n}{s_n}$ is equal to
 a. $\frac{n}{2}$ b. $\frac{n}{2} - 1$ c. $n - 1$ d. $\frac{2n-1}{2}$
45. The number of terms in the expansion of $(1 + 5\sqrt{2}x)^{19} + (1 - 5\sqrt{2}x)^{19}$ is
 a. 10 b. 7 c. 13 d. 4

BITSAT Archives

1. $2^{3n} - 7n - 1$ is divisible by [2014]
a. 64 b. 36 c. 49 d. 25
2. If n is a positive integer, then $n^3 + 2n$ is divisible by [2014]
a. 2 b. 6 c. 15 d. 3
3. The sum of the coefficients in the expansion of $(5x - 4y)^n$, where n is a positive integer, is [2014]
a. 0 b. n c. 1 d. -1
4. If $a = \sum_{n=0}^{\infty} \frac{x^{3n}}{3n!}$, $b = \sum_{n=1}^{\infty} \frac{x^{3n-2}}{(3n-2)!}$ and $c = \sum_{n=1}^{\infty} \frac{x^{3n-1}}{(3n-1)!}$, then the value of $a^3 + b^3 + c^3 - 3abc$ is [2014]
a. 1 b. 0 c. -1 d. -2
5. $10^n + 3(4^{n+2}) + 5$ is divisible by [2014]
a. 7 b. 5 c. 9 d. 17
6. The coefficient of x^n in the expansion of $\log_e \left(\frac{1}{1+x+x^2+x^3} \right)$, when n is odd, is [2013]
a. $-\frac{2}{n}$ b. $-\frac{1}{n}$ c. $\frac{1}{n}$ d. $\frac{2}{n}$
7. $\frac{{}^8C_0}{6} - {}^8C_1 + {}^8C_2 \cdot 6 - {}^8C_3 \cdot 6^2 + {}^8C_4 \cdot 6^3 + \dots + {}^8C_8 \cdot 6^7$ equals [2013]
a. 0 b. 6^7 c. 6^8 d. $\frac{5^8}{6}$
8. If the coefficient of second, third and fourth terms in the expansion of $(1+x)^n$ are in AP, then n is equal to [2012]
a. 7 b. 4 c. 5 d. 6
9. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$, then $C_0 - C_1 + C_2 - C_3 + \dots + (-1)^n \cdot C_n$ is equal to [2011]
a. 3^n b. 2^n c. 1 d. 0
10. The value of $(\sqrt{5} + 1)^5 - (\sqrt{5} - 1)^5$ is [2011]
a. 252 b. 352 c. 452 d. 552
11. The coefficient of x^5 in the expansion of $(2 - x + 3x^2)^6$ is [2010]
a. -4692 b. 4692 c. 2346 d. -5052
12. If $C_1, C_2, C_3, \dots, C_n$ denote the coefficients in the binomial expansion of $(1+x)^n$, then the value of ${}^nC_1 + 2 \cdot {}^nC_2 + 3 \cdot {}^nC_3 + \dots + n \cdot {}^nC_n$ is [2010]
a. $(n-1)2^n$ b. $n \cdot 2^{n-1}$
c. $n \cdot 2^{n+1}$ d. None of these
13. Larger of $99^{50} + 100^{50}$ and 101^{50} equals
a. 101^{50} b. $99^{50} + 100^{50}$
c. Both are equal d. None of these
14. $\frac{1}{n!} + \frac{1}{2!(n-2)!} + \frac{1}{4!(n-4)!} + \dots$ is [2009]
a. $\frac{2^{n-1}}{n!}$ b. $\frac{2^n}{(n+1)!}$ c. $\frac{2^n}{n!}$ d. $\frac{2^{n-2}}{(n-1)!}$
15. When 2^{301} is divided by 5, the least positive remainder is [2006]
a. 4 b. 8
c. 2 d. 6
16. The middle term in the expansion of $\left(x - \frac{1}{x}\right)^{18}$ is [2006]
a. ${}^{18}C_9$ b. $-{}^{18}C_9$ c. ${}^{18}C_{10}$ d. $-{}^{18}C_{10}$

Answer with Solutions

Practice Exercise

1. (c) Hint $\binom{n}{6} (2^{1/3})^{n-6} \left(\frac{1}{3^{1/3}}\right)^6 : \binom{n}{n-6} (2^{1/3})^6 \left(\frac{1}{3^{1/3}}\right)^{n-6} = 1:6$

$$\Rightarrow 6 \cdot 2^{\frac{n}{3}-4} \cdot 3^{\frac{n}{3}-4} = 1$$

2. (b) $\therefore T_3 = 1000$

$$\therefore {}^5C_2 \left(\frac{1}{a}\right)^{5-2} (a^{\log_{10} a})^2 = 1000$$

$$\Rightarrow 10 \cdot (a^{\log_{10} a})^2 \cdot a^{-3} = 1000$$

$$\Rightarrow a^{2\log_{10} a} \cdot a^{-3} = 100$$

$$\Rightarrow [a]^{2\log_{10} a - 3} = 10^2$$

$$\Rightarrow (2\log_{10} a - 3)\log a = \log 10^2$$

$$\Rightarrow 2\log_{10} a - 3 = \frac{2\log 10}{\log a}$$

$$\Rightarrow 2x - 3 = \frac{2}{x} \quad [\text{put } \log_{10} a = x]$$

$$\Rightarrow 2x^2 - 3x = 2 \Rightarrow 2x^2 - 3x - 2 = 0$$

$$\Rightarrow (2x+1)(x-2) = 0 \Rightarrow x = 2 \quad \left[\because x \neq -\frac{1}{2} \right]$$

$$\therefore \log_{10} a = 2$$

$$\Rightarrow a = 10^2 = 100$$

3. (b) On simplification, we get

$$z = 2 \left[{}^5C_0 \left(\frac{\sqrt{3}}{2} \right)^5 + {}^5C_2 \left(\frac{\sqrt{3}}{2} \right)^3 \left(\frac{i}{2} \right)^2 + {}^5C_4 \left(\frac{\sqrt{3}}{2} \right) \left(\frac{i}{2} \right)^4 \right]$$

Since, $i^2 = -1$ and $i^4 = 1$, so z will not contain any i and hence $\text{Im}(z) = 0$.

4. (b) We know that,

$$(a+y)^n + (a-y)^n = 2[{}^nC_0 a^n y^0 + {}^nC_2 a^{n-2} y^2 + \dots]$$

$$\therefore (x + \sqrt{x^3 - 1})^5 + (x - \sqrt{x^3 - 1})^5$$

$$= 2[x^5 + 10x^3(x^3 - 1) + 5x(x^3 - 1)^2]$$

Clearly, it is a polynomial of degree 7.

5. (b) Since, $T_{r+1} = {}^{10}C_r \left(\frac{x}{2} \right)^{10-r} \left(-\frac{3}{x^2} \right)^r$
- $$= {}^{10}C_r x^{10-3r} (-3)^r (2)^{r-10}$$

So, for coefficient of x^4 , $10 - 3r = 4 \Rightarrow r = 2$

$$\therefore T_{2+1} = {}^{10}C_2 x^4 \cdot (-3)^2 \cdot 2^{-10}$$

So, the coefficient of x^4

$$= {}^{10}C_2 (-3)^2 \cdot 2^{-8} = {}^{10}C_2 \frac{9}{2^8} = \frac{5 \times 9 \times 9}{2^8} = \frac{405}{256}$$

6. (b) Let $P(n) = 7^{2n} + 2^{3n-3} \cdot 3^{n-1}$

For $n = 1$, $P(1) = 7^2 + 2^0 \cdot 3^0 = 49 + 1 = 50 = 25 \times 2$

Thus, $P(1)$ is divisible by 25.

Now, assume $P(m)$ is divisible by 25, so that we may write

$$7^{2m} + 2^{3m-3} \cdot 3^{m-1} = 25k$$

where, k is a positive integer.

Then, $P(m+1) = 7^{2(m+1)} + 2^{3(m+1)-3} \cdot 3^{(m+1)-1}$

$$= 7^{2m} \cdot 7^2 + 2^{3m-3} \cdot 3^{m-1} \cdot 3$$

$$= 49 \cdot 7^{2m} + 24 \cdot 2^{3m-3} \cdot 3^{m-1}$$

$$= 49 \cdot 7^{2m} + 24 \cdot 2^{3m-3} \cdot 3^{m-1}$$

$$= (50-1) \cdot 7^{2m} + (25-1) 2^{3m-3} \cdot 3^{m-1}$$

$$= 25(2 \cdot 7^{2m} + 2^{3m-3} \cdot 3^{m-1}) - (7^{2m} + 2^{3m-3} \cdot 3^{m-1})$$

$$= 25(2 \cdot 7^{2m} + 2^{3m-3} \cdot 3^{m-1} - 25k)$$

So, $P(m+1)$ is divisible by 25.

Hence, by mathematical induction, $P(n)$ is divisible by 25 for all positive integers n .

7. (b) $T_7 = {}^9C_6 \left(\frac{3}{\sqrt[3]{84}} \right)^3 (\sqrt{3} \ln x)^6 = 729$

$$\Rightarrow \frac{84 \times 3^3}{84} \times 3^3 \times (\ln x)^6 = 729 \Rightarrow (\ln x)^6 = 1 \Rightarrow x = e$$

8. (b) **Hint** Since, the coefficients of given terms are ${}^mC_{r-1}$, mC_r , ${}^mC_{r+1}$ respectively and they are also in AP.

$$\therefore {}^mC_{r-1} + {}^mC_{r+1} = 2 {}^mC_r$$

$$\Rightarrow \frac{m!}{(r-1)!(m-r+1)!} + \frac{m!}{(r+1)!(m-r-1)!} = 2 \frac{m!}{r!(m-r)!}$$

$$\Rightarrow \frac{1}{(m-r+1)(m-r)} + \frac{1}{(r+1)r} = \frac{2}{r(m-r)}$$

9. (a) Since, $2n$ is even, so the middle term in the expansion of $(1+x)^{2n}$ is $\left(\frac{2n}{2} + 1 \right)$ th, i.e. $(n+1)$ th term which is given by

$$T_{n+1} = {}^{2n}C_n (1)^{2n-n} (x)^n = {}^{2n}C_n x^n = \frac{(2n)!}{n!n!} x^n$$

$$= \frac{2n(2n-1)(2n-2)\dots 4 \cdot 3 \cdot 2 \cdot 1}{n!n!} x^n$$

$$= \frac{1 \cdot 2 \cdot 3 \cdot 4 \dots (2n-2)(2n-1)(2n)}{n!n!} x^n$$

$$= \frac{[1 \cdot 3 \cdot 5 \dots (2n-1)] [2 \cdot 4 \cdot 6 \dots (2n)]}{n!n!} x^n$$

$$= \frac{[1 \cdot 3 \cdot 5 \dots (2n-1)] \cdot 2^n [1 \cdot 2 \cdot 3 \dots n]}{n!n!} x^n$$

$$= \frac{[1 \cdot 3 \cdot 5 \dots (2n-1)] n!}{n!n!} 2^n \cdot x^n = \frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{n!} 2^n \cdot x^n$$

10. (c) For $n = 20$, the middle term is $\left(\frac{20}{2} + 1 \right) = 11$ th term

$$\therefore T_{11} = T_{10+1} = {}^{20}C_{10} \left(\frac{2}{3} x^2 \right)^{20-10} \left(-\frac{3}{2x} \right)^{10} = {}^{20}C_{10} x^{10}$$

11. (a) Since, last term in $(2^{1/3} + 2^{-1/2}y)^n$ is $(n+1)$ th term.

$$\therefore T_{n+1} = {}^nC_n (2^{-1/2}y)^n = \left[\frac{1}{3(9)^{1/3}} \right]^{\log_3 2^3}$$

$$\Rightarrow 2^{-n/2} = [3^{-5/3}]^{\log_3 2^3} \Rightarrow 2^{-n/2} = [3]^{\log_3 2^{3 \times -5/3}}$$

$$\Rightarrow 2^{-n/2} = [3]^{\log_3 2^{-5}} \Rightarrow 2^{-n/2} = 2^{-5} \quad [\because a^{\log_a b} = b]$$

$$\Rightarrow \frac{-n}{2} = -5 \Rightarrow n = 10$$

$$\therefore T_{4+1} = {}^{10}C_4 (2^{1/3})^6 (2^{-1/2})^4 = 210 \cdot (2)^{2-2} = 210$$

12. (c) $\therefore T_{r+1} = {}^{100}C_r [(5)^{1/6}]^{100-r} [(2)^{1/8}]^r$

Since, 5 and 2 are prime numbers, so T_{r+1} is rational, if $\frac{100-r}{6}$ and $\frac{r}{8}$ are integers or $100-r$ is a multiple of 6 and r is a multiple of 8.

So, for r as a multiple of 8, we have

$$r = 0, 8, 16, 24, \dots, 96$$

$$\Rightarrow 100 - r = 100, 92, 84, 76, \dots, 12, 4$$

Also, for r to be divisible by 6, we get the common value from the above $(100-r)$ value as 84, 60, 36, 12 only. So, only four rational terms are possible.

Hence, number of irrational terms = $101 - 4 = 97$

13. (a) Greatest term in the expansion of $(1+x)^n$ is T_{r+1}

$$\text{where, } r = \left[\frac{(n+1)x}{1+x} \right]$$

$$\text{Here, } n = 20 \text{ and } x = \frac{1}{\sqrt{3}}$$

$$\therefore r = \left[\frac{21}{\sqrt{3}+1} \right] = [10.5(\sqrt{3}-1)] = (7.69) \approx 7$$

$$\text{Hence, greatest term is } \sqrt{3} \left(\frac{20}{7} \right) \left(\frac{1}{\sqrt{3}} \right)^7 = \left(\frac{20}{7} \right) \frac{1}{27}.$$

14. (b) The greatest term in $(3 + 5x)^{15} = \frac{(n+1)|5x|}{|3| + |5x|}$

$$= \frac{(15+1)5\left(\frac{1}{5}\right)}{3+5\cdot\frac{1}{5}} = \frac{16 \times 1}{4} = 4$$

So, 4th and 5th terms are the greatest.

$$\therefore T_5 = {}^{15}C_4 \cdot 3^{11}$$

15. (a) The general term of $\left(3x - \frac{2}{x^2}\right)^{15}$ is

$$T_{r+1} = {}^{15}C_r (3x)^{15-r} \left(\frac{-2}{x^2}\right)^r = {}^{15}C_r (3)^{15-r} (-2)^r x^{15-3r}$$

For independent term of x ,

$$\text{put } 15 - 3r = 0 \Rightarrow r = 5$$

$\therefore$ The term independent of x

$$= {}^{15}C_5 (3)^{15-5} (-2)^5 = -3003 (3^{10}) (2^5)$$

16. (c) **Hint** Middle term, $T_5 = {}^8C_4 (p/2)^{8-4} (2^4) = 1120$

$$17. (d) \left(1 + x + \frac{2}{x}\right)^6 = 1 + \binom{6}{1} \left(x + \frac{2}{x}\right) + \binom{6}{2} \left(x + \frac{2}{x}\right)^2 + \dots + \binom{6}{6} \left(x + \frac{2}{x}\right)^6$$

$$\therefore \text{Constant term} = 1 + \binom{6}{2} \binom{2}{1} 2^1 + \binom{6}{4} \binom{4}{2} 2^2 + \binom{6}{6} \binom{6}{3} 2^3$$

$$= 1 + 60 + 360 + 160 = 581$$

18. (a) General term is $T_{r+1} = {}^{21}C_r \left(\sqrt[3]{\frac{a}{b}}\right)^{21-r} \left(\sqrt[3]{\frac{b}{a}}\right)^r$
- $$= {}^{21}C_r a^{7-\frac{r}{2}} b^{\frac{2r}{3}-\frac{7}{2}}$$

$\therefore$ Power of a = Power of b

$$\therefore 7 - \frac{r}{2} = \frac{2}{3}r - \frac{7}{2} \Rightarrow r = 9$$

19. (b) $T_8 = {}^nC_7 (2)^{n-7} \left(\frac{x}{3}\right)^7 = {}^nC_7 \frac{(2)^{n-7}}{3^7} x^7$

$$\text{and } T_9 = {}^nC_8 (2)^{n-8} \left(\frac{x}{3}\right)^8 = {}^nC_8 \frac{(2)^{n-8}}{3^8} x^8$$

$$\text{Therefore, } {}^nC_7 \frac{(2)^{n-7}}{3^7} = {}^nC_8 \frac{(2)^{n-8}}{3^8} \Rightarrow \frac{n!}{7!(n-7)!} \times \frac{8!(n-8)!}{n!}$$

$$= \frac{(2)^{n-8}}{3^8} \cdot \frac{3^7}{(2)^{n-7}} \Rightarrow \frac{8}{n-7} = \frac{1}{6} \therefore n = 55$$

20. (c) Let the three consecutive terms in the expansion of $(1+a)^n$ be $(r-1)$ th, r th and $(r+1)$ th terms.

$$\therefore (r-1)\text{th term is } T_{(r-2)+1} = {}^nC_{r-2} a^{r-2}$$

$$\text{Similarly, } T_{(r-1)+1} = {}^nC_{r-1} a^{r-1} \text{ and } T_{r+1} = {}^nC_r a^r$$

$$\text{But } \frac{{}^nC_{r-2}}{{}^nC_{r-1}} = \frac{1}{7} \Rightarrow n - 8r + 9 = 0 \quad \dots(i)$$

$$\text{and } \frac{{}^nC_{r-1}}{{}^nC_r} = \frac{7}{42} \Rightarrow n - 7r + 1 = 0 \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get $n = 55$

21. (b) On putting $x = 1$ in $(1 - 3x + 10x^2)^n$ and $(1 + x^2)^n$, the required sum of coefficients are

$$\alpha = (1 - 3 + 10)^n = 8^n = (2^n)^3 \text{ and } \beta = (1 + 1)^n = 2^n$$

so, $\alpha = \beta^3$ is correct.

22. (c) Let $S = {}^nC_0^2 + {}^nC_1^2 + {}^nC_2^2 + \dots + (n+1){}^nC_n^2 \quad \dots(i)$

Writing S in the reverse order and using ${}^nC_r = {}^nC_{n-r}$, we get

$$S = (n+1){}^nC_0^2 + {}^nC_1^2 + (n-1){}^nC_2^2 + \dots + {}^nC_n^2 \quad \dots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2S = (n+2) \{ {}^nC_0^2 + {}^nC_1^2 + \dots + {}^nC_n^2 \} \Rightarrow S = \left(\frac{n}{2} + 1\right) 2^n {}^nC_n$$

23. (a) Let a_1, a_2, a_3 and a_4 be the coefficients of four consecutive terms $T_{r+1}, T_{r+2}, T_{r+3}$ and T_{r+4} , respectively.

$$\text{Then, } \frac{a_1}{a_1 + a_2} = \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}} = \frac{{}^nC_r}{{}^{n+1}C_{r+1}}$$

$$[\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}]$$

$$= \frac{n!}{r!(n-r)!} \times \frac{(r+1)!(n-r)!}{(n+1)!} = \left(\frac{r+1}{n+1}\right)$$

$$\text{Similarly, } \frac{a_3}{a_3 + a_4} = \frac{{}^nC_{r+2}}{{}^nC_{r+2} + {}^nC_{r+3}} = \frac{{}^nC_{r+2}}{{}^{n+1}C_{r+3}} = \frac{r+3}{n+1}$$

$$\text{Hence, } \frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{r+1}{n+1} + \frac{r+3}{n+1} = \frac{2r+4}{n+1}$$

$$\text{and } \frac{2a_2}{a_2 + a_3} = \frac{2({}^nC_{r+1})}{{}^nC_{r+1} + {}^nC_{r+2}} = \frac{2({}^nC_{r+1})}{{}^{n+1}C_{r+2}}$$

$$= \frac{2n!}{(r+1)!(n-r-1)!} \times \frac{(r+2)!(n-r-1)!}{(n+1)!}$$

$$= \frac{2(r+2)}{n+1}$$

$$\therefore \frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{2a_2}{a_2 + a_3}$$

24. (d) $T_{r+1} = {}^{15}C_r (x^2)^{15-r} \cdot \left(\frac{2}{x}\right)^r = {}^{15}C_r x^{30-2r} \cdot 2^r \cdot x^{-r}$
- $$= {}^{15}C_r \cdot x^{30-3r} \cdot 2^r$$

For coefficient of x^{15} ,

$$\text{put } 30 - 3r = 15 \Rightarrow 3r = 15 \Rightarrow r = 5$$

$$\therefore \text{Coefficient of } x^{15} = {}^{15}C_5 \cdot 2^5$$

For coefficient of independent of x i.e. x^0 put $30 - 3r = 0$

$$\Rightarrow r = 10$$

$$\therefore \text{Coefficient of } x^0 = {}^{15}C_{10} \cdot 2^{10}$$

$$\text{Hence, their ratio} = \frac{{}^{15}C_5 \cdot 2^5}{{}^{15}C_{10} \cdot 2^{10}} = \frac{{}^{15}C_{10} \cdot 2^5}{{}^{15}C_{10} \cdot 2^{10}} = 1:32$$

25. (c) **Hint** $\left[\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}} \right]^{10}$
- $$= \left[\frac{(x^{1/3})^3 + 1^3}{x^{2/3} - x^{1/3} + 1} - \frac{\{(\sqrt{x})^2 - 1\}}{\sqrt{x}(\sqrt{x} - 1)} \right]^{10}$$
- $$= \left[(x^{1/3} + 1) - \frac{(\sqrt{x} + 1)}{\sqrt{x}} \right]^{10} = (x^{1/3} - x^{-1/2})^{10}$$

$$T_{r+1} = {}^{10}C_r (x^{1/3})^{10-r} (-x^{-1/2})^r = {}^{10}C_r (-1)^r x^{\frac{10-r}{3} - \frac{r}{2}}$$

For independent of x , put

$$\frac{10-r}{3} - \frac{r}{2} = 0$$

26. (b) Here, $(1-x-x^2+x^3)^6 = \{(1-x)-x^2(1-x)\}^6$

$$\begin{aligned} &= \{(1-x)(1-x^2)\}^6 = (1-x)^6 \cdot (1-x^2)^6 \\ &= \left\{ \sum_{r=0}^6 (-1)^r {}^6C_r \cdot x^r \right\} \left\{ \sum_{s=0}^6 (-1)^s {}^6C_s \cdot x^{2s} \right\} \\ &= \sum_{r=0}^6 \sum_{s=0}^6 (-1)^{r+s} \cdot {}^6C_r \cdot {}^6C_s \cdot x^{r+2s} \end{aligned}$$

For coefficient of x^7 , $r+2s=7$

i.e. $(s=1, r=5)$ or $(s=2, r=3)$ or $(s=3, r=1)$

$\therefore$ Coefficient of x^7

$$\begin{aligned} &= \{(-1)^{5+1} \cdot {}^6C_5 \cdot {}^6C_1\} + \{(-1)^{3+2} \cdot {}^6C_3 \cdot {}^6C_2\} \\ &\quad + \{(-1)^{1+3} \cdot {}^6C_1 \cdot {}^6C_3\} \end{aligned}$$

$$= 36 - (20)(15) + 6(20) = 36 - 300 + 120 = -144$$

27. (a) **Hint** ${}^{50}C_4 + {}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 + {}^{51}C_3 + {}^{50}C_3$
 $= ({}^{50}C_3 + {}^{50}C_4) + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$
 $= ({}^{51}C_4 + {}^{51}C_3) + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$

28. (b) **Hint** $\binom{n}{r+1} \div \binom{n}{r} = \frac{56}{28} \Rightarrow \frac{n-r}{r+1} = 2$

$$\text{and } \binom{n}{r} \div \binom{n}{r-1} = \frac{28}{8} \Rightarrow \frac{n-r+1}{r} = \frac{7}{2}$$

29. (a) On putting $x=1$ and $x=-1$ respectively, we get

$$0 = 1 + a_1 + a_2 + \dots + a_{12} \quad \text{and} \quad 64 = 1 - a_1 + a_2 - \dots + a_{12}$$

On adding, we get

$$64 = 2(1 + a_2 + a_4 + \dots + a_{12})$$

$$\therefore a_2 + a_4 + \dots + a_{12} = 31$$

30. (c) Given, $\binom{n-1}{3} + \binom{n-1}{4} > \binom{n}{3}$

$$\Rightarrow \binom{n}{4} > \binom{n}{3} \Rightarrow \frac{\binom{n}{4}}{\binom{n}{3}} > 1$$

$$\Rightarrow \frac{n-4+1}{4} > 1 \Rightarrow n > 7$$

31. (a) Since, $(1+x)^{10} = C_0 + C_1x + C_2x^2 + \dots + C_{10}x^{10}$

$$\text{and } \left(1 + \frac{1}{x}\right)^{10} = C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + \dots + \frac{C_{10}}{x^{10}}$$

So, $C_0C_1 + C_1C_2 + C_2C_3 + \dots + C_9C_{10}$ is the coefficient of x

$$\text{in the product } (1+x)^{10} \left(1 + \frac{1}{x}\right)^{10} = \frac{(1+x)^{20}}{x^{10}}$$

or the coefficient of x^{11} in $(1+x)^{20}$ is

$$\binom{20}{11} = \binom{20}{9}$$

32. (b) **Case I** $121^n - 25^n = (96+25)^n - 25^n$ is divisible by 96.
 and $1900^n - (-4)^n = (1904-4)^n - (-4)^n$ is divisible by 1904.
 Hence, both are divisible by 16.

Case II $121^n - (-4)^n = (125-4)^n - (-4)^n$ is divisible by 125 and $1900^n - 25^n = (1875+25)^n - 25^n$ is divisible by 1875.

Hence, both are divisible by 125.

So, given number is divisible by 16×125 i.e. 2000.

33. (b) We have, $8^{2n} - (62)^{2n+1} = (1+63)^n - (63-1)^{2n+1}$
 $= (1+63)^n + (1-63)^{2n+1}$
 $= [1 + {}^nC_1 \cdot 63 + {}^nC_2 (63)^2 + \dots + (63)^n]$
 $+ [1 - {}^{(2n+1)}C_1 \cdot 63 + {}^{(2n+1)}C_2 (63)^2 - \dots + (-1) (63)^{(2n+1)}]$
 $= 2 + 63 [{}^nC_1 + {}^nC_2 (63) + \dots + (63)^{n-1}$
 $- {}^{(2n+1)}C_1 + {}^{(2n+1)}C_2 (63) - \dots - (63)^{(2n)}]$

So, the remainder is 2.

34. (c) Since, $7 - 4\sqrt{3} = \frac{1}{7 + 4\sqrt{3}}$

$$\therefore 0 < 7 - 4\sqrt{3} < 1 \Rightarrow 0 < (7 - 4\sqrt{3})^{2n} < 1$$

$$\text{Let } F = (7 - 4\sqrt{3})^{2n}$$

$$\begin{aligned} \text{Then, } x + F &= (7 + 4\sqrt{3})^{2n} + (7 - 4\sqrt{3})^{2n} \\ &= 2[{}^{2n}C_0 + {}^{2n}C_2 7^{2n-2} (4\sqrt{3})^2 \\ &\quad + {}^{2n}C_4 7^{2n-4} (4\sqrt{3})^4 + \dots + {}^{2n}C_{2n} (4\sqrt{3})^{2n}] \\ &= 2m \end{aligned}$$

where, m is some positive integer.

$$\Rightarrow [x] + f + F = 2m \Rightarrow f + F = 2m - [x]$$

Since, $0 \leq f < 1$ and $0 < F < 1$, so we get $0 < f + F < 2$.

Also, $f + F$ is an integer, so we must have $f + F = 1$.

$$\begin{aligned} \text{Thus, } x(1-f) &= xF = (7 + 4\sqrt{3})^{2n} (7 - 4\sqrt{3})^{2n} \\ &= (49 - 48)^{2n} = 1^{2n} = 1 \end{aligned}$$

35. (a) For $n=1$, $n^3 + 3n^2 + 5n + 3 = 12$

which is divisible by 3 and 4.

$$\text{For } n=2, n^3 + 3n^2 + 5n + 3 = 33$$

which is divisible by 3 only.

36. (b) We know that,

$$(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1x + \dots + {}^{20}C_{10}x^{10} + \dots + {}^{20}C_{20}x^{20}$$

On putting $x=-1$ in the above expansion, we get

$$\begin{aligned} 0 &= {}^{20}C_0 - {}^{20}C_1 + \dots - {}^{20}C_9 + {}^{20}C_{10} - {}^{20}C_{11} + \dots + {}^{20}C_{20} \\ \Rightarrow 0 &= {}^{20}C_0 - {}^{20}C_1 + \dots - {}^{20}C_9 + {}^{20}C_{10} - {}^{20}C_{11} + \dots + {}^{20}C_{20} \\ \Rightarrow 0 &= 2({}^{20}C_0 - {}^{20}C_1 + \dots - {}^{20}C_9) + {}^{20}C_{10} \\ \Rightarrow {}^{20}C_{10} &= 2({}^{20}C_0 - {}^{20}C_1 + \dots + {}^{20}C_{10}) \\ \Rightarrow {}^{20}C_0 - {}^{20}C_1 + \dots + {}^{20}C_{10} &= \frac{1}{2} {}^{20}C_{10} \end{aligned}$$

37. (a) $\therefore (a+b+c)^n = [a+(b+c)]^n = a^n + {}^nC_1 a^{n-1}(b+c)^1$
 $+ {}^nC_2 a^{n-2}(b+c)^2 + \dots + {}^nC_n (b+c)^n$

So, T_1 has 1 term, T_2 has 2 terms and T_3 has 3 terms.

$$\begin{aligned} \text{Similarly, total number of terms} &= 1 + 2 + 3 + \dots + (n+1) \\ &= \frac{(n+1)(n+2)}{2} \end{aligned}$$

$$38. (a) \text{ Hint } \sum_{r=0}^{n-1} \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}} = \sum_{r=0}^{n-1} \frac{{}^nC_r}{{}^{n+1}C_{r+1}} = \sum_{r=0}^{n-1} \frac{r+1}{n+1}$$

$$39. (d) \text{ Hint General term, } T_{r+1} = {}^{10}C_r (2^{1/2})^{10-r} (3^{1/5})^r \\ = {}^{10}C_r 2^{\left(5-\frac{r}{2}\right)} \cdot 3^{r/5}$$

Clearly, T_{r+1} will be independent of radical sign, if $r/2$ and $r/5$ are integers, where $0 \leq r \leq 10$.

$$\therefore r = 0, 10$$

$$40. (a) \text{ Coefficient of } x^r = {}^nC_r = a$$

$$\text{Coefficient of } x^{n-r} = {}^nC_{n-r} = b$$

$$\text{Since, } {}^nC_r = {}^nC_{n-r}$$

$$\therefore a = b$$

$$41. (a) \text{ Hint } (\sqrt{3} + 1)^{2n} - (\sqrt{3} - 1)^{2n} = 2[{}^{2n}C_1(\sqrt{3})^{2n-1} \\ + {}^{2n}C_3(\sqrt{3})^{2n-3} + {}^{2n}C_5(\sqrt{3})^{2n-5} + \dots + {}^{2n}C_{2n-1}(\sqrt{3})^{2n-(2n-1)}]$$

which is most certainly an irrational number because of odd powers of $\sqrt{3}$ in each of the terms.

$$42. (a) \text{ } n\text{th term will be } \frac{n}{3(4n+3)}$$

$$\text{For } n=1, \text{ LHS} = \frac{1}{3 \cdot 7} = \frac{1}{21} \text{ and RHS} = \frac{1}{3(4+3)} = \frac{1}{21}$$

$$\therefore \text{ LHS} = \text{RHS}$$

Thus, the given result is true for $n=1$, let the given result be true for $n=m \in N$.

$$\therefore \frac{1}{3 \cdot 7} + \frac{1}{7 \cdot 11} + \frac{1}{11 \cdot 14} + \dots + \frac{1}{(4m-1)(4m+3)} = \frac{m}{3(4m+3)}$$

Now, for $n=m+1$,

$$\text{LHS} = \left[\frac{1}{3 \cdot 7} + \frac{1}{7 \cdot 11} + \frac{1}{11 \cdot 14} + \dots + \frac{1}{(4m-1)(4m+3)} \right] \\ + \frac{1}{(4m+3)(4m+7)}$$

$$= \frac{m}{3(4m+3)} + \frac{1}{(4m+3)(4m+7)} = \frac{m(4m+7)+3}{3(4m+3)(4m+7)}$$

$$= \frac{4m^2 + 7m + 3}{3(4m+3)(4m+7)} = \frac{(m+1)(4m+3)}{3(4m+3)(4m+7)}$$

$$\text{LHS} = \frac{m+1}{3(4m+7)} = \frac{m+1}{3[4(m+1)+3]}$$

which is same result as given one with n replaced by $m+1$. Hence, result is true.

$$43. (d) \therefore T_{r+1} = {}^{15}C_r (x^3)^{15-r} \left(\frac{-1}{x^2}\right)^r \\ = {}^{15}C_r x^{45-3r} \cdot \frac{(-1)^r \cdot 1}{x^{2r}} = {}^{15}C_r x^{45-3r-2r} \cdot (-1)^r$$

For constant term, $45 - 3r - 2r = 0$

$$\Rightarrow 45 = 5r \Rightarrow r = 9$$

$$\therefore T_{9+1} = {}^{15}C_9 (-1)^9 = -{}^{15}C_9$$

$$44. (a) \text{ Given, } s_n = \sum_{r=0}^n \frac{1}{{}^nC_r} \Rightarrow s_n = \sum_{r=0}^n \frac{1}{{}^nC_{n-r}} \quad [\because {}^nC_r = {}^nC_{n-r}]$$

$$\Rightarrow ns_n = \sum_{r=0}^n \frac{n}{{}^nC_{n-r}} \Rightarrow ns_n = \sum_{r=0}^n \left(\frac{n-r}{{}^nC_{n-r}} + \frac{r}{{}^nC_{n-r}} \right)$$

$$\Rightarrow ns_n = \sum_{r=0}^n \frac{n-r}{{}^nC_{n-r}} + \sum_{r=0}^n \frac{r}{{}^nC_r}$$

$$\Rightarrow ns_n = \left(\frac{n}{{}^nC_n} + \frac{n-1}{{}^nC_{n-1}} + \dots + \frac{1}{{}^nC_1} \right) + \sum_{r=0}^n \frac{r}{{}^nC_r}$$

$$\Rightarrow ns_n = t_n + t_n \quad \left[\because t_n = \sum_{r=0}^n \frac{r}{{}^nC_r}, \text{ given} \right]$$

$$\Rightarrow ns_n = 2t_n \Rightarrow \frac{t_n}{s_n} = \frac{n}{2}$$

$$45. (a) \text{ We have, } (1+5\sqrt{2}x)^{19} + (1-5\sqrt{2}x)^{19} \\ = 2[{}^{19}C_0 + {}^{19}C_2(5\sqrt{2}x)^2 + \dots + {}^{19}C_{18}(5\sqrt{2}x)^{18}]$$

So, it has only 10 terms.

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$$1. (c) \text{ Let } P(n) = 2^{3n} - 7n - 1$$

$$\Rightarrow P(1) = 0, P(2) = 49$$

$P(1)$ and $P(2)$ are divisible by 49.

$$\text{Let } P(k) = 2^{3k} - 7k - 1 = 49I$$

$$\text{Then, } P(k+1) = 2^{3k+3} - 7k - 8$$

$$= 8(49I + 7k + 1) - 7k - 8$$

$$= 49(8I) + 49k = 49\lambda$$

where, $\lambda = 8I + k$, which is an integer.

$$2. (d) \text{ Let } P(n) = n^3 + 2n$$

$$\text{Then, } P(1) = 1 + 2 = 3$$

$$P(2) = 8 + 4 = 12$$

$$\text{and } P(3) = 27 + 6 = 33$$

Clearly, we see that all these numbers are divisible by 3.

$$3. (c) \text{ Using binomial theorem,}$$

$$(5x - 4y)^n = {}^nC_0(5x)^n + {}^nC_1(5x)^{n-1}(-4y) \\ + {}^nC_2(5x)^{n-2}(-4y)^2 + \dots + {}^nC_n(-4y)^n$$

$\therefore$ Sum of coefficients

$$= {}^nC_0 5^n + {}^nC_1 5^{n-1}(-4) + {}^nC_2 5^{n-2}(-4)^2 + \dots + {}^nC_n(-4)^n \\ = (5 - 4)^n = 1^n = 1$$

$$4. (a) \text{ We have, } a = \sum_{n=0}^{\infty} \frac{x^{3n}}{(3n)!}, b = \sum_{n=1}^{\infty} \frac{x^{3n-2}}{(3n-2)!}$$

$$\text{and } c = \sum_{n=1}^{\infty} \frac{x^{3n-1}}{(3n-1)!}$$

$$\text{Now, } a + b + c = \sum_{n=0}^{\infty} \frac{x^{3n}}{3n!} + \sum_{n=1}^{\infty} \frac{x^{3n-2}}{(3n-2)!} + \sum_{n=1}^{\infty} \frac{x^{3n-1}}{(3n-1)!}$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = e^x$$

$$\therefore a + b\omega + c\omega^2 = 1 + \omega x + \frac{\omega^2 x^2}{2!} + \frac{\omega^3 x^3}{3!} + \dots = e^{\omega x}$$

and $a + b\omega^2 + c\omega = e^{\omega^2 x}$, where ω is imaginary cube root of unity.

$$\text{Now, } a^3 + b^3 + c^3 - 3abc$$

$$= (a + b + c)(a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$$

$$= e^x \cdot e^{\omega x} \cdot e^{\omega^2 x} = e^{x(1 + \omega + \omega^2)} = e^{0 \cdot x} = 1$$

5. (c) For $n = 1$, $10^n + 3 \cdot 4^{n+2} + 5 = 10 + 3 \cdot 4^3 + 5 = 207$, which is divisible by 9.

So, by induction, the result is divisible by 9.

$$\begin{aligned} 6. (b) \log_e \left(\frac{1}{1+x+x^2+x^3} \right) &= \log_e \left(\frac{1-x}{1-x^4} \right) \\ &= \log_e(1-x) - \log_e(1-x^4) = -\sum_{r=1}^{\infty} \frac{x^r}{r} + \sum_{r=1}^{\infty} \frac{x^{4r}}{r} \end{aligned}$$

When n is odd, there is no term in the second series containing x^n , therefore the coefficient x^n is zero in the second series and in the first series, the coefficient of x^n is $-1/n$.

Hence, when n is odd, then the coefficient of x^n in the whole expansion is $-\frac{1}{n} + 0 = -\frac{1}{n}$.

$$\begin{aligned} 7. (d) \frac{{}^8C_0}{6} - {}^8C_1 + {}^8C_2 \cdot 6 - {}^8C_3 \cdot 6^2 + {}^8C_4 \cdot 6^3 + \dots + {}^8C_8 \cdot 6^7 \\ = \frac{1}{6} [{}^8C_0 - {}^8C_1 \cdot 6 + {}^8C_2 \cdot 6^2 - {}^8C_3 \cdot 6^3 + \dots + {}^8C_8 \cdot 6^8] \\ = \frac{1}{6} (1-6)^8 = \frac{1}{6} \times (-5)^8 = \frac{5^8}{6} \end{aligned}$$

$$8. (a) \text{ Since, } T_2 = {}^nC_1 \text{ and } T_3 = {}^nC_2 x, T_4 = {}^nC_3 x^2$$

Also, T_2, T_3, T_4 are in AP.

$$\therefore \frac{{}^nC_1 + {}^nC_3}{2} = {}^nC_2 \Rightarrow {}^nC_1 + {}^nC_3 = 2 \cdot {}^nC_2$$

$$\Rightarrow \frac{n!}{(n-1)!(1)!} + \frac{n!}{(n-3)!3!} = \frac{2n!}{2!(n-2)!}$$

$$\Rightarrow \frac{1}{(n-1)(n-2)} + \frac{1}{3!} = \frac{1}{(n-2)}$$

$$\Rightarrow 1 + \frac{(n-1)(n-2)}{6} = (n-1)$$

$$\Rightarrow 6 + n^2 - 3n + 2 = 6n - 6 \Rightarrow n^2 - 3n - 6n + 8 + 6 = 0$$

$$\Rightarrow n^2 - 9n + 14 = 0 \Rightarrow (n-7)(n-2) = 0$$

$$\Rightarrow n = 7 \text{ or } 2 \quad \therefore n = 7 \quad [\because n \neq 2]$$

$$9. (d) \text{ Given, } (1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n$$

Put $x = -1$, we get

$$C_0 - C_1 + C_2 - \dots + (-1)^n \cdot C_n = 0$$

$$\begin{aligned} 10. (b) \text{ We have, } (\sqrt{5} + 1)^5 - (\sqrt{5} - 1)^5 \\ = 2\{ {}^5C_1(\sqrt{5})^4 + {}^5C_3(\sqrt{5})^2 + {}^5C_5 \cdot 1 \} \\ = 2\{ 5 \times 25 + 50 + 1 \} = 2(176) = 352 \end{aligned}$$

11. (d) The general term in the expansion of

$$(2-x+3x^2)^6 = \frac{6!}{r!s!t!} 2^r (-x)^s (3x^2)^t, \text{ where } r+s+t=6$$

$$= \frac{6!}{r!s!t!} 2^r \times (-1)^s \times 3^t \times x^{s+2t}$$

For the coefficient of x^5 , we must have $s+2t=5$.

$$\text{But } r+s+t=6$$

$$\therefore s=5-2t \text{ and } r=1+t$$

where, $0 \leq r, s, t \leq 6$

Now, for $t=0$; $r=1, s=5$

For $t=1$; $r=2, s=3$

and for $t=2$; $r=3, s=1$

Thus, there are three terms containing x^5 and coefficient of x^5

$$\begin{aligned} &= \frac{6!}{1!5!0!} \times 2 \times (-1)^5 \times 3^0 + \frac{6!}{2!3!1!} \times 2^2 \times (-1)^3 \times 3 \\ &\quad + \frac{6!}{3!1!2!} \times 2^3 \times (-1) \times 3^2 \\ &= -12 - 720 - 4320 = -5052 \end{aligned}$$

12. (b) Since,

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + \dots + {}^nC_n x^n \quad \dots(i)$$

On differentiating Eq. (i) w.r.t. x , we get

$$n(1+x)^{n-1} = {}^nC_1 + 2 \cdot {}^nC_2 x + 3 \cdot {}^nC_3 x^2 + \dots + n \cdot {}^nC_n x^{n-1} \quad \dots(ii)$$

On putting $x=1$ in Eq. (ii), we get

$$\begin{aligned} n(1+1)^{n-1} &= {}^nC_1 + 2 \cdot {}^nC_2 + 3 \cdot {}^nC_3 + \dots + n \cdot {}^nC_n \\ \Rightarrow {}^nC_1 + 2 \cdot {}^nC_2 + 3 \cdot {}^nC_3 + \dots + n \cdot {}^nC_n &= n \cdot 2^{n-1} \end{aligned}$$

13. (a) We have, $101^{50} - 99^{50} = (100+1)^{50} - (100-1)^{50}$

$$\begin{aligned} &= 100^{50} \left(1 + \frac{1}{100} \right)^{50} - 100^{50} \left(1 - \frac{1}{100} \right)^{50} \\ &= 100^{50} \left[2 \left\{ {}^{50}C_1 \cdot \frac{1}{100} + {}^{50}C_3 \left(\frac{1}{100} \right)^3 + \dots \right\} \right] \\ &= \frac{101^{50} - 99^{50}}{100^{50}} = 1 + 2 \left\{ {}^{50}C_3 \left(\frac{1}{100} \right)^3 + \dots \right\} \end{aligned}$$

$$\Rightarrow \frac{101^{50} - 99^{50}}{100^{50}} = 1 + (\text{positive quantity}) > 1$$

$$\Rightarrow 101^{50} - 99^{50} > 100^{50} \Rightarrow 101^{50} > 100^{50} + 99^{50}$$

14. (a) We have, $\frac{1}{n!} + \frac{1}{2!(n-2)!} + \frac{1}{4!(n-4)!} + \dots$

$$= \frac{1}{n!} ({}^nC_0 + {}^nC_2 + {}^nC_4 + \dots) = \frac{2^{n-1}}{n!}$$

15. (c)

16. (b) The general term in the expansion of $\left(x - \frac{1}{x}\right)^{18}$ is given

$$\text{by } T_{r+1} = {}^{18}C_r (x)^{18-r} \left(-\frac{1}{x}\right)^r$$

$$\text{Here, } n = 18$$

So, the middle term is T_{9+1} , where $r = 9$.

$$\therefore T_{9+1} = {}^{18}C_9 (-1)^9 x^{18-2 \cdot 9} = -{}^{18}C_9 x^{18-18} = -{}^{18}C_9$$

7

Matrices

Matrix

In Mathematics, a **matrix** (plural matrices) is a rectangular array of numbers, symbols or expressions, arranged in rows and columns. The individuals in a matrix, are called its **elements** or **entries**. Generally, matrix is, written in following way

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = [a_{ij}]_{m \times n}$$

The order of a matrix A is $m \times n$, where m is the number of rows and n is the number of columns.

NOTE Here, $m \times n$ does not indicate multiplication.

Types of Matrices

The matrices are of following types :

- (i) **Row Matrix** A matrix which have only one row and any number of columns, is called a row matrix.
- (ii) **Column Matrix** A matrix is said to be a column matrix, if it has only one column and any number of rows.
- (iii) **Null Matrix or Zero Matrix** A matrix of order $m \times n$ whose all elements are zero, is called a null matrix of order $m \times n$. It is denoted by O .
- (iv) **Square Matrix** It is a matrix in which number of rows is equal to number of columns. Thus, $m \times n$ matrix is said to be a square matrix, if $m = n$ and is known as a square matrix of order n .
- (v) **Diagonal Matrix** A square matrix is called a diagonal matrix, if all its non-diagonal elements are zero and diagonal elements are not all equal.
- (vi) **Scalar Matrix** A square matrix $A = [a_{ij}]$ is said to be scalar matrix, if
 - (a) $a_{ij} = 0, \forall i \neq j$
 - (b) $a_{ij} = k, \forall i = j$, where $k \neq 0$i.e. a diagonal matrix is said to be a scalar matrix, if the elements of principal diagonal are same.

(vii) **Identity Matrix or Unit Matrix** A square matrix $A = [a_{ij}]$ is said to be a unit or identity matrix, if

(a) $a_{ij} = 0, \forall i \neq j$

(b) $a_{ij} = 1, \forall i = j$

i.e. a diagonal matrix whose elements of principal diagonal are equal to 1 and all remaining elements are zero, is known as unit or identity matrix. It is denoted by I .

(viii) **Upper Triangular Matrix** A square matrix $A = (a_{ij})_{n \times n}$ if $a_{ij} = 0$ for $i > j$ i.e. all elements below the leading diagonal are zero, is called upper triangular matrix.

(ix) **Lower Triangular Matrix** A square matrix $A = (a_{ij})_{n \times n}$, if $a_{ij} = 0$ for $i < j$ i.e. all elements above the leading diagonal are zero, is called lower triangular matrix.

(x) **Trace of a Matrix** The sum of diagonal elements of a square matrix A is called the trace of A and is denoted by $\text{tr}(A)$.

(a) $\text{tr}(\lambda A) = \lambda \text{tr}(A)$ (b) $\text{tr}(A) = \text{tr}(A')$

(c) $\text{tr}(AB) = \text{tr}(BA)$

(xi) **Equal Matrices** Two matrices are said to be equal, if

(a) they have same number of rows and columns.

(b) The elements in the first matrix corresponding the elements of second matrix are equal.

e.g. Let $A = (a_{ij})_{m \times n}$ and $B = (b_{ij})_{m \times n}$

then, $A = B$, if $a_{ij} = b_{ij}, \forall i, j$

Algebra of Matrices

1. Addition of Matrices

Let $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ be two matrices of same order, then

$$A + B = [a_{ij} + b_{ij}], \forall i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n$$

Properties of Addition of Matrices

Let A, B and C be three matrices of same order, then the properties of addition of matrices are given below :

(i) **Commutative** $A + B = B + A$

(ii) **Associative** $(A + B) + C = A + (B + C)$

(iii) **Additive Identity** $A + O = A = O + A$

(iv) **Additive Inverse** $A + (-A) = O = (-A) + A$

(v) **Cancellation Laws**

(a) $A + B = A + C \Rightarrow B = C$ [left cancellation law]

(b) $B + A = C + A \Rightarrow B = C$ [right cancellation law]

2. Subtraction of Matrices

Let A and B be two matrices of same order $m \times n$, then

$$A - B = [a_{ij} - b_{ij}]_{m \times n}$$

3. Multiplication of a Matrix by a Scalar

Let $A = [a_{ij}]$ be an $m \times n$ matrix and k be any scalar. Then, the matrix obtained by multiplying each element of A by k is called the scalar multiple of A by k .

Properties of Multiplication of a Matrix by a Scalar

Let A and B be two matrices of same order, then the properties of multiplication of a matrix by scalar are

(i) $k(A + B) = kA + kB$

(ii) $(k_1 + k_2)A = k_1A + k_2A$

4. Multiplication of Matrices

Let $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{n \times p}$ be two matrices such that number of columns of A is equal to the number of rows of B , then the product matrix is $C = [c_{ij}]$ of order $m \times p$

$$\text{where, } c_{ij} = \sum_{k=1}^n a_{ik} b_{kj}$$

Properties of Multiplication of Matrices

Let A, B and C be three matrices of order $m \times n, n \times p$ and $n \times k$, then the properties of multiplication of matrices are given below :

(i) **Non-commutative** $AB \neq BA$

(ii) **Associative** $(AB)C = A(BC)$

(iii) **Multiplicative Identity** $IA = A = AI$

(iv) **Multiplicative Distributive** $A(B + C) = AB + AC$

Elementary Operation (Transformation) of a Matrix

Two matrices A and B are said to be equivalent, if one is obtained from the other by one or more elementary operations and we write $A \sim B$.

Following elementary operations are given below:

(i) Interchanging any two rows or columns is indicated by

$$R_i \leftrightarrow R_j$$

or $C_i \leftrightarrow C_j$

(ii) Multiplication of the elements of any row or column by a non-zero scalar quantity is indicated by

$$R_i \leftrightarrow kR_i$$

or $C_i \leftrightarrow kC_i$

(iii) Addition of constant multiple of the elements of any row or column to the corresponding element of any other row, is indicated by

$$R_i \rightarrow R_i + kR_j$$

or $C_i \rightarrow C_i + kC_j$

Special Types of Matrices

1. Transpose of a Matrix

Let A be $m \times n$ matrix. Then, $n \times m$ matrix obtained by interchanging the rows and columns of A is called the transpose of A and is denoted by A' or A^C or A^T .

Important Results

- If A and B are two matrices of order $m \times n$, then $(A \pm B)' = A' \pm B'$
- If k is a scalar, then $(kA)' = kA'$
- $(A')' = A$ $(AB)' = B'A'$ $(A^n)' = (A')^n$

2. Idempotent Matrix

A square matrix A is called an idempotent matrix, if it satisfies the relation $A^2 = A$. If A and B are idempotent matrices, then $A + B$ is an idempotent iff $AB = BA$.

3. Nilpotent Matrix

A square matrix A is called nilpotent matrix, if it satisfies the relation $A^k = O$ and $A^{k+1} \neq O$.

4. Orthogonal Matrix

A square matrix A is called an orthogonal matrix, if it satisfies the relation important Results $AA' = I$.

Important Results

- If A and B are orthogonal matrices, then AB is also an orthogonal matrix.
- Every orthogonal matrix is invertible.

5. Involutory Matrix

A square matrix A is called an involutory matrix, if it satisfies the relation $A^2 = I$.

6. Symmetric Matrix

A square matrix A is called symmetric matrix, if it satisfies the relation $A' = A$.

Important Results

- If A and B are symmetric matrices of the same order, then
 - (i) AB is symmetric if and only if $AB = BA$.
 - (ii) $A \pm B$, $AB + BA$ are also symmetric matrices.
- If A is symmetric matrix, then A^{-1} will also be symmetric matrix.

7. Skew-symmetric Matrix

A square matrix A is called skew-symmetric matrix, if it satisfies the relation $A' = -A$. Every square matrix can be uniquely expressed as the sum of symmetric and skew-symmetric matrix.

$$\text{i.e.} \quad A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

Important Results

- If A and B are two skew-symmetric matrices, then
 - (i) $A \pm B$, $AB - BA$ are skew-symmetric matrices.
 - (ii) $AB + BA$ is a symmetric matrix.
- Determinant of skew-symmetric matrix of odd order is zero.

Adjoint of a Matrix

Let $A = [a_{ij}]_{m \times n}$ be a square matrix of order n and C_{ij} be the cofactor of a_{ij} in the determinant $|A|$. Then, the adjoint of A is defined as the transpose of the cofactor matrix and is denoted by $\text{adj}(A)$.

Properties of Adjoint of a Matrix

Let A be a matrix of order n , then

- (i) $(\text{adj } A)A = A(\text{adj } A) = |A| \cdot I_n$
- (ii) $|\text{adj } A| = |A|^{n-1}$, if $|A| \neq 0$
- (iii) $\text{adj}(AB) = (\text{adj } B)(\text{adj } A)$
- (iv) If $|A| = 0$, then $(\text{adj } A)A = A(\text{adj } A) = O$
- (v) $\text{adj}(A^T) = (\text{adj } A)^T$
- (vi) $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$
- (vii) Adjoint of a diagonal matrix is a diagonal matrix.
- (viii) $\text{adj}(\text{adj } A) = |A|^{n-2}A$

Singular and Non-singular Matrix

If A is a square matrix such that $|A| = 0$, then matrix A is singular. However, if $|A| \neq 0$, then the matrix A is called non-singular.

Inverse of a Matrix

Let A be a non-singular (where, $|A| \neq 0$) square matrix. Then, a square matrix B such that $AB = BA = I$ is called inverse of A i.e. $A^{-1} = \frac{1}{|A|} \text{adj}(A)$.

Properties of Inverse of a Matrix

Let A and B be square matrices of same order, then

- (i) A square matrix is invertible if and only if it is non-singular.
 (ii) $(A')^{-1} = (A^{-1})'$ (iii) $(AB)^{-1} = B^{-1}A^{-1}$ (iv) $|A^{-1}| = |A|^{-1}$

Solution of a System of Linear Equations

Let system of linear equations in three variables be

$$a_1x + b_1y + c_1z = d_1, a_2x + b_2y + c_2z = d_2$$

$$\text{and } a_3x + b_3y + c_3z = d_3$$

It can be written in the matrix form as

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \Rightarrow AX = B \Rightarrow X = A^{-1}B$$

Test of Consistency

Non-homogeneous Equations ($B \neq O$)

- If $|A| \neq 0$, then the system of equations is **consistent** and has a unique solution given by $X = A^{-1}B$.
- If $|A| = 0$ and $(\text{adj } A) \cdot B \neq O$, then the system of equations is inconsistent and has no solution.
- If $|A| = 0$ and $(\text{adj } A) \cdot B = O$, then the system of equations is consistent and has infinite number of solutions.

Homogeneous Equations ($B = O$)

- If $|A| \neq 0$, then system of equations has only trivial solution and has one solution.
- If $|A| = 0$, then system of equations has non-trivial solution and has infinite number of solutions.
- If number of equations is less than number of unknowns, then it has non-trivial solution.
- A homogeneous system of equations is never inconsistent.

Practice Exercise

1. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then which of the following is correct?

- a. $(A+B) \cdot (A-B) = A^2 + B^2$ b. $(A+B) \cdot (A-B) = A^2 - B^2$
 c. $(A+B) \cdot (A-B) = I$ d. None of these

2. If $A = \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, then $(I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$ is equal to

- a. A b. I
 c. $I + A$ d. None of these

3. If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, then $I + 2A + 3A^2 + \dots$ is equal to

- a. $\begin{bmatrix} 4 & 1 \\ -4 & 0 \end{bmatrix}$ b. $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$
 c. $\begin{bmatrix} 5 & 2 \\ -8 & -3 \end{bmatrix}$ d. $\begin{bmatrix} 5 & 2 \\ -3 & -8 \end{bmatrix}$

4. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then the value of $A^3 - 6A^2 + 7A + 2I$ is

- a. I b. O
 c. $-2I$ d. $2I$

5. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, then A^n is equal to

- a. $2^{n-1}A - (n-1)I$ b. $nA - (n-1)I$
 c. $2^{n-1}A + (n-1)I$ d. $nA + (n-1)I$

6. If $A = \begin{bmatrix} 4 & 6 & -1 \\ 3 & 0 & 2 \\ 1 & -2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 \\ 0 & 1 \\ -1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$, then the expression which is not defined, is

- a. $A^2 + 2B - 2A$ b. CC'
 c. $B'C$ d. AB

7. If p, q, r are 3 real numbers satisfying the matrix equation, $[p \ q \ r] \begin{bmatrix} 3 & 4 & 1 \\ 3 & 2 & 3 \\ 2 & 0 & 2 \end{bmatrix} = [3 \ 0 \ 1]$, then $2p + q - r$ is equal to

- a. -3 b. -1 c. 4 d. 2

8. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $\det(A^3) = 125$, then α is equal to

- a. ± 1 b. ± 2 c. ± 3 d. ± 5

9. If $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $g(y) = \begin{bmatrix} \cos y & 0 & \sin y \\ 0 & 1 & 0 \\ -\sin y & 0 & \cos y \end{bmatrix}$, then $[f(x)g(y)]^{-1}$ is equal to

- a. $f(-x)g(-y)$ b. $f(x^{-1})g(y^{-1})$
 c. $g(-y)f(-x)$ d. $g(y^{-1})f(x^{-1})$

10. The trace of the matrix $A = \begin{bmatrix} 2 & 5 & 9 \\ 7 & -5 & 3 \\ 2 & 6 & 8 \end{bmatrix}$ is equal to

- a. 6
b. 5
c. 3
d. None of these

11. If $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, then $\det \{\text{adj}(A)\}$ equals

- a. $(14)^2$
b. $(13)^2$
c. $(14)^3$
d. $(13)^3$

12. If A and B are square matrices such that $A^2 = A$, $B^2 = B$ and A, B commute, then

- a. $(AB)^2 = I$
b. $(AB)^2 = AB$
c. $(AB)^2 = O$
d. None of these

13. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 1 \end{bmatrix}$, then α

is equal to

- a. -2
b. 5
c. 2
d. -1

14. If $3A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$ and $A^T A = AA^T = I$, then xy is

equal to

- a. -1
b. 1
c. 2
d. -2

15. If A and B are 3×3 matrices such that $A^2 - B^2 = (A - B)(A + B)$, then

- a. either A or B is zero matrix
b. either A or B is unit matrix
c. $A = B$
d. $AB = BA$

16. If $x = cy + bz$, $y = az + cx$ and $z = bx + ay$, where x, y and z are not all zero, then $a^2 + b^2 + c^2$ is equal to

- a. $1 + 2abc$
b. $1 - 2abc$
c. $1 + abc$
d. $abc - 1$

17. If $A = \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is an orthogonal matrix, then

- a. $a = 1, b = 2$
b. $a = -2, b = 1$
c. $a = 3, b = -1$
d. $a = -2, b = -1$

18. The matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}$ is

- a. idempotent
b. nilpotent
c. involutory
d. orthogonal

19. If $A = \begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}$ and $A + A' = I$, then the value of a is

- a. $\frac{\pi}{6}$
b. $\frac{\pi}{3}$
c. π
d. $\frac{3\pi}{2}$

20. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then A^{-1} is equal to

- a. A
b. A^2
c. A^3
d. A^4

21. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$. If B is

the inverse of matrix A , then α equals

- a. -2
b. 1
c. 2
d. 5

22. If A is skew-symmetric and $B = (I - A)^{-1}(I + A)$, then B is

- a. singular
b. symmetric
c. skew-symmetric
d. orthogonal

23. Let a, b, c be positive real numbers. The following of equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$

$-\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ has

- a. no solution
b. unique solution
c. infinitely many solutions
d. finitely many solutions

24. The equations $x + 2y + 3z = 1$, $x - y + 4z = 0$ and $2x + y + 7z = 1$ has

- a. only one solution
b. only two solutions
c. no solution
d. infinitely many solutions

25. If the equations $a(y + z) = x$, $b(z + x) = y$ and $c(x + y) = z$ have non-trivial solution, then $\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c}$ is equal to

- a. 1
b. 2
c. -1
d. -2

26. The value of λ such that the system $x - 2y + z = -4$, $2x - y + 2z = 2$ and $x + y + \lambda z = 4$ has no solution, is

- a. 0
b. 1
c. $\neq 1$
d. 3

27. If the system of equations $x - ky - z = 0$, $kx - y - z = 0$ and $x + y - z = 0$ has a non-zero solution, then k is equal to

- a. 0, 1
b. 1 - 1
c. -1, 2
d. 2, -2

BITSAT Archives

1. If $\omega (\neq 1)$ is a cube root of unity of

$$A = \begin{bmatrix} 1 + 2\omega^{100} + \omega^{200} & \omega^2 & 1 \\ 1 & 1 + 2\omega^{100} + \omega^{200} & \omega \\ \omega & \omega^2 & 2 + \omega^{100} + 2\omega^{200} \end{bmatrix},$$

then [2014]

- a. A is singular b. $|A| \neq 0$
c. A is symmetric d. None of these

2. If $A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$, then A is [2013]

- a. idempotent b. nilpotent c. involutory d. None

3. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the correct statement is [2011]

- a. $A^2 + 5A - 7I = O$ b. $-A^2 + 5A + 7I = O$
c. $A^2 - 5A + 7I = O$ d. $A^2 + 5A + 7I = O$

4. If $A = (a_{ij})_{2 \times 2}$, where $a_{ij} = i + j$, then A is equal to [2009]

- a. $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 4 \\ 3 & 3 \end{bmatrix}$ d. $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

5. If $A = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$ and $f(t) = t^2 - 3t + 7$, then $f(A) + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix}$ is equal to [2008]

- a. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ c. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ d. $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

6. If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the unit matrix of order 2, then A^2 equals [2007]

- a. $4A - 3I$ b. $3A - 4I$ c. $A - I$ d. $A + I$

7. Let A be orthogonal and non-singular matrix of order n , then the determinant of matrix $(A - I_n)$ is equal to

- a. $|I_n - A|$ b. $|A| |I_n - A|$ [2007]
c. $|A|$ d. $(-1)^n |A| |I_n - A|$

8. The inverse of the matrix $\begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$ is [2006]

- a. $\frac{1}{11} \begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$
c. $\frac{1}{13} \begin{bmatrix} -2 & 5 \\ 1 & 3 \end{bmatrix}$ d. $\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

Answer with Solutions

Practice Exercise

1. (a) Here, $A + B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}$

and $A - B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}$

Now, $A^2 = A \cdot A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 + 1 & 0 + 1 \\ 0 + 1 & 1 + 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

and $B^2 = B \cdot B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$\therefore A^2 - B^2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

$A^2 + B^2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

and $(A + B)(A - B) = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 + 0 & 0 + 0 \\ 0 + 0 & 4 + 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$

Hence, $(A + B)(A - B) \neq A^2 - B^2$

and $(A + B)(A - B) \neq A^2 + B^2$

2. (c) Here, $A = \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$, where $t = \tan\left(\frac{\alpha}{2}\right)$

Now, $\cos \alpha = \frac{1 - \tan^2\left(\frac{\alpha}{2}\right)}{1 + \tan^2\left(\frac{\alpha}{2}\right)} = \frac{1 - t^2}{1 + t^2}$

and $\sin \alpha = \frac{2 \tan\left(\frac{\alpha}{2}\right)}{1 + \tan^2\left(\frac{\alpha}{2}\right)} = \frac{2t}{1 + t^2}$

Now, we have $(I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix} \begin{bmatrix} \frac{1-t^2}{1+t^2} & \frac{-2t}{1+t^2} \\ \frac{2t}{1+t^2} & \frac{1-t^2}{1+t^2} \end{bmatrix} = \begin{bmatrix} \frac{1-t^2+2t^2}{1+t^2} & \frac{-2t+t(1-t^2)}{1+t^2} \\ \frac{-t(1-t^2)+2t}{1+t^2} & \frac{2t^2+1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1+t^2}{1+t^2} & \frac{-t(1+t^2)}{1+t^2} \\ \frac{t(1+t^2)}{1+t^2} & \frac{1+t^2}{1+t^2} \end{bmatrix} = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

Now, $I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix} = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$

$\therefore (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} = I + A$

3. (c) $\therefore A^2 = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

$\therefore I + 2A + 3A^2 + \dots = I + 2A$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ -8 & -4 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -8 & -3 \end{bmatrix}$$

4. (c) **Hint** Here, $A^2 = A \times A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

and $A^3 = A^2 \times A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$

5. (b) Here, $A^2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$$A^3 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$\vdots \quad \vdots \quad \vdots$$

$$A^n = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix} = \begin{bmatrix} n & 0 \\ n & n \end{bmatrix} - \begin{bmatrix} n-1 & 0 \\ 0 & n-1 \end{bmatrix}$$

$$= nA - (n-1)I$$

6. (a) By inspection, matrices A^2 and A are of order 3×3 , while matrix B is of order 3×2 . Therefore, $A^2 + 2B - 2A$ is not defined.

7. (a) $[3p + 3q + 2r \quad 4p + 2q + 0 \quad p + 3q + 2r] = [3 \quad 0 \quad 1]$

$$\Rightarrow 3p + 3q + 2r = 3, 4p + 2q = 0, p + 3q + 2r = 1$$

$$\Rightarrow p = 1, q = -2, r = 3$$

$$\therefore 2p + q - r = 2 - 2 - 3 = -3$$

8. (c) $125 = \det(A^3) = (\det A)^3 = (\alpha^2 - 4)^3$

$$\Rightarrow \alpha^2 - 4 = 5 \Rightarrow \alpha^2 = 9 \Rightarrow \alpha = \pm 3$$

9. (c) We have, $[f(x)g(y)]^{-1} = [g(y)]^{-1}[f(x)]^{-1} = g(-y)f(-x)$

10. (b) $\text{tr}(A) = \text{Sum of principal diagonal elements}$
 $= 2 + (-5) + 8 = 5$

11. (a) $\therefore |\text{adj}(A)| = |A|^{n-1}$ and $\det(A) = 14$

$$\therefore \det\{\text{adj}(A)\} = (14)^2$$

12. (b) $(AB)^2 = (AB) \cdot (AB) = A(BA)B = A(AB)B \quad [\because AB = BA]$
 $= (A \cdot A)(B \cdot B) = A^2 \cdot B^2 = AB$

13. (b) Here, $AA^{-1} = I$

If R_1 of A is multiplied by C_3 of A^{-1} , we get

$$2 - \alpha + 3 = 0 \Rightarrow \alpha = 5$$

14. (c) Since, A is orthogonal. So, each row is orthogonal to the other rows.

$$\therefore R_1 \cdot R_3 = 0$$

$$\Rightarrow x + 4 + 2y = 0$$

and $R_2 \cdot R_3 = 0$

$$\Rightarrow 2x + 2 - 2y = 0$$

On solving, we get $x = -2, y = -1$

$$\therefore xy = 2$$

15. (d) $\therefore A^2 - B^2 = (A - B)(A + B) = A^2 + AB - BA - B^2$

$$\therefore AB = BA$$

16. (b) Given, $-x + cy + bz = 0$

$$cx - y + az = 0$$

and $bx + ay - z = 0$

Then, $\begin{vmatrix} -1 & c & b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$

$$\therefore a^2 + b^2 + c^2 = 1 - 2abc$$

17. (d) **Hint** $AA^T = I$

$$\Rightarrow \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \cdot \frac{1}{3} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9 & 0 & a + 4 + 2b \\ 0 & 9 & 2a + 2 - 2b \\ a + 4 + 2b & 2a + 2 - 2b & a^2 + 4 + b^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

18. (b) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}$, then $A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$

Hence, A is nilpotent matrix of index 2.

19. (b) **Hint** $A + A' = I$

$$\Rightarrow \begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix} + \begin{bmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

20. (c) **Hint** $|A| = 3(-3 + 4) + 3(2 - 0) + 4(-2 + 0)$

$$= 3 \times 1 + 3 \times 2 + 4 \times -2 = 1$$

and $\text{adj}(A) = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}^T = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

$$\therefore A^{-1} = \frac{1}{1} \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \text{ and } A^3 = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

$$21. (d) \text{ Hint } 10A^{-1} = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} \quad [\because B = A^{-1}]$$

$$\Rightarrow 10A^{-1}A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} A$$

$$\Rightarrow 10I = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$22. (d) \text{ Here, } BB^T = (I - A)^{-1}(I + A)(I + A)^T[(I - A)^{-1}]^T \\ = (I - A)^{-1}(I + A)(I - A)(I + A)^{-1} \\ = (I - A)^{-1}(I - A)(I + A)(I + A)^{-1} = I \cdot I = I$$

Hence, B is an orthogonal matrix.

$$23. (b) \text{ Let } \frac{x^2}{a^2} = X, \frac{y^2}{b^2} = Y \text{ and } \frac{z^2}{c^2} = Z. \text{ Then, the given system of equations reduces to}$$

$$X + Y - Z = 1, X - Y + Z = 1, -X + Y + Z = 1$$

$$\text{The coefficient matrix is } A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

Clearly $|A| \neq 0$, so the given system of equations has a unique solution.

$$24. (d) \text{ Here, } |A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & -1 & 4 \\ 2 & 1 & 7 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -3 & 1 \\ 0 & -3 & 1 \end{vmatrix} = 0$$

$$[\text{applying } R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - 2R_1]$$

$$\text{Again, } \text{adj}(A) = \begin{bmatrix} -11 & 1 & 3 \\ -11 & 1 & 3 \\ -11 & -1 & -3 \end{bmatrix} = \begin{bmatrix} -11 & -11 & -11 \\ 1 & 1 & -1 \\ 3 & 3 & -3 \end{bmatrix}$$

$$\text{adj}(A)B = \begin{bmatrix} -11 & -11 & -11 \\ 1 & 1 & -1 \\ 3 & 3 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = O$$

So, the given system is consistent and has infinitely many solutions.

$$25. (b) \text{ Here, } \begin{vmatrix} -1 & a & a \\ b & -1 & b \\ c & c & -1 \end{vmatrix} = 0$$

On applying, $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we get

$$\begin{vmatrix} -1 & a+1 & a+1 \\ b & -(b+1) & 0 \\ c & 0 & -(1+c) \end{vmatrix} = 0$$

On applying $R_1 \rightarrow \frac{R_1}{a+1}$, $R_2 \rightarrow \frac{R_2}{b+1}$, $R_3 \rightarrow \frac{R_3}{c+1}$, we get

$$\begin{vmatrix} -\frac{1}{a+1} & 1 & 1 \\ \frac{b}{b+1} & -1 & 0 \\ \frac{c}{c+1} & 0 & -1 \end{vmatrix} = 0$$

$$\Rightarrow -\frac{1}{a+1} + \frac{b}{b+1} + \frac{c}{c+1} = 0$$

$$\Rightarrow -\frac{1}{a+1} + 1 - \frac{1}{b+1} + 1 - \frac{1}{c+1} = 0$$

$$\Rightarrow \frac{1}{a+1} + \frac{1}{b+1} + \frac{1}{c+1} = 2$$

$$26. (c) \text{ For no solution, } \begin{vmatrix} 1 & -2 & 1 \\ 2 & -1 & 2 \\ 1 & 1 & \lambda \end{vmatrix} \neq 0$$

$$\Rightarrow \lambda \neq 1$$

$$27. (b) \text{ For non-zero solution, } \begin{vmatrix} 1 & -k & -1 \\ k & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & -k & -1 \\ k-1 & k-1 & 0 \\ 0 & k+1 & 0 \end{vmatrix} = 0$$

$$[\text{applying } R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1]$$

$$\Rightarrow k^2 - 1 = 0$$

$$\Rightarrow k = 1, -1$$

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$$1. (d) \text{ Hint } A = \begin{bmatrix} 1+2\omega+\omega^2 & \omega^2 & 1 \\ 1 & 1+\omega^2+2\omega & \omega \\ \omega & \omega^2 & 2+\omega+2\omega^2 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} \omega & \omega^2 & 1 \\ 1 & \omega & \omega \\ \omega & \omega^2 & 1+\omega^2 \end{vmatrix}$$

$$2. (c) \text{ Hint Now, } A^2 = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$3. (c) \text{ Here, } A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\therefore A^2 - 5A + 7I$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$4. (d) A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 1+2 \\ 2+1 & 2+2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$$

5. (b) Given that, $A = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$

and $f(t) = t^2 - 3t + 7$

Then, $A^2 = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} -7 & -12 \\ 24 & 17 \end{bmatrix}$

Now, $f(A) = A^2 - 3A + 7$

$$= \begin{bmatrix} -7 & -12 \\ 24 & 17 \end{bmatrix} - 3 \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -6 \\ 12 & 9 \end{bmatrix}$$

$$\therefore f(A) + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix} = \begin{bmatrix} -3 & -6 \\ 12 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

6. (a) Here, $A^2 = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 4+1 & -2-2 \\ -2-2 & 1+4 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$$

Now, $4A - 3I = 4 \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$

$\therefore A^2 = 4A - 3I$

7. (b) We know that, $A \cdot A^T = I_n$

$\therefore A - I_n = A - AA^T = A(I_n - A^T)$

$\Rightarrow |A - I_n| = |A(I_n - A^T)|$

$$= |A| |I_n - A^T|$$

$$= |A| |I_n - A|$$

8. (a) Let $A = \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$

$|A| = 5 + 6 = 11$

Now, $\text{adj}(A) = \begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|} (\text{adj } A) = \frac{1}{11} \begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$

8

Determinants

Introduction

Every square matrix $A = [a_{ij}]$ of order n can be associated to an expression or a number which is called its determinant. It is denoted by $|A|$ or $\det A$ or Δ . If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A is written as

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \det(A)$$

NOTE

- For matrix A , $|A|$ is read as determinant of A not modulus of A .
- Only square matrices have determinant.

Determinant of a Square Matrix of Order 1

Let $A = [a_{11}]$ be a 1×1 matrix, then the determinant of A is the number a_{11} itself i.e. $|A| = a_{11}$.

Determinant of a Square Matrix of Order 2

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a matrix of order 2×2 , then

$$\det(A) \text{ or } |A| \text{ or } \Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} a_{22} - a_{21} a_{12}$$

Determinant of Matrix of Order 3

Let $[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be a square matrix of order 3.

$$\begin{aligned} \text{Then, } |A| &= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \Rightarrow |A| = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \\ &= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31}) \end{aligned}$$

Minors and Cofactors of a Determinant

Minors and cofactors of determinant can be defined in following ways:

Minor of an Element

Minor of an element is the determinant obtained by deleting the row and column in which that element lies. It is denoted by M_{ij} .

e.g. Given, a 3×3 determinant

$$\begin{vmatrix} a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \\ a_{51} & a_{52} & a_{53} \end{vmatrix}$$

Minor of a_{31} is $M_{31} = \begin{vmatrix} a_{42} & a_{43} \\ a_{52} & a_{53} \end{vmatrix}$

and minor of a_{32} is $M_{32} = \begin{vmatrix} a_{41} & a_{43} \\ a_{51} & a_{53} \end{vmatrix}$

Similarly, it can be calculated for other elements of a determinant.

Cofactor of an Element

Let $A = \{a_{ij}\}$ be an n order square matrix. Then, the cofactor C_{ij} of a_{ij} for A will be $(-1)^{i+j}$ times of M_{ij} (minor of an element of a determinant). It means $C_{ij} = (-1)^{i+j} M_{ij}$.

$$C_{ij} = \begin{cases} -M_{ij} & \text{when } i+j \text{ is odd} \\ M_{ij} & \text{when } i+j \text{ is even} \end{cases}$$

e.g. Cofactor of a_{31} in 3×3 determinant $\begin{vmatrix} a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \\ a_{51} & a_{52} & a_{53} \end{vmatrix}$ will

be C_{31} .

i.e. $C_{31} = (-1)^{3+1} \begin{vmatrix} a_{42} & a_{43} \\ a_{52} & a_{53} \end{vmatrix}$

Properties of Determinants

1. If each element of a row (column) is a zero, then $\Delta = 0$.
2. If two rows (columns) are proportional, then $\Delta = 0$.
3. If any two rows (columns) are interchanged odd times, then Δ becomes $-\Delta$.
4. If the rows and columns are interchanged, then Δ is unchanged i.e. $|A^T| = |A|$.
5. If each element of a row (column) of a determinant is multiplied by a constant k , then the value of the new determinant is k times the value of the original determinant.

i.e. $\begin{vmatrix} ka & kb & kc \\ p & q & r \\ u & v & w \end{vmatrix} = k \begin{vmatrix} a & b & c \\ p & q & r \\ u & v & w \end{vmatrix}$

6. $\begin{vmatrix} a_1 + a_2 & b & c \\ p_1 + p_2 & q & r \\ u_1 + u_2 & v & w \end{vmatrix} = \begin{vmatrix} a_1 & b & c \\ p_1 & q & r \\ u_1 & v & w \end{vmatrix} + \begin{vmatrix} a_2 & b & c \\ p_2 & q & r \\ u_2 & v & w \end{vmatrix}$

7. If a scalar multiple of any row (column) is added to another row (column), then Δ is unchanged.

i.e. $\begin{vmatrix} a & b & c \\ p & q & r \\ u & v & w \end{vmatrix} = \begin{vmatrix} a & b & c \\ p + ka & q + kb & r + kc \\ u & v & w \end{vmatrix}$

8. Let $\Delta(x)$ be a 3rd order determinant having polynomials as its elements.

- (i) If $\Delta(a)$ has two rows (columns) proportional, then $(x-a)$ is a factor of $\Delta(x)$.
- (ii) If $\Delta(a)$ has three rows (columns) proportional, then $(x-a)^2$ is a factor of $\Delta(x)$.

9. Product of two determinants

i.e. $|AB| = |A||B| = |BA| = |AB^T| = |A^T B| = |A^T B^T|$

10. If $\Delta(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ a & b & c \end{vmatrix}$, then

(i) $\sum_{x=1}^n \Delta(x) = \begin{vmatrix} \sum_{x=1}^n f_1(x) & \sum_{x=1}^n f_2(x) & \sum_{x=1}^n f_3(x) \\ \sum_{x=1}^n g_1(x) & \sum_{x=1}^n g_2(x) & \sum_{x=1}^n g_3(x) \\ a & b & c \end{vmatrix}$

(ii) $\prod_{x=1}^n \Delta(x) = \begin{vmatrix} \prod_{x=1}^n f_1(x) & \prod_{x=1}^n f_2(x) & \prod_{x=1}^n f_3(x) \\ \prod_{x=1}^n g_1(x) & \prod_{x=1}^n g_2(x) & \prod_{x=1}^n g_3(x) \\ a & b & c \end{vmatrix}$

11. $\det(kA) = k^n \det(A)$, if A is of order $n \times n$.
12. $\det(A^n) = (\det A)^n$, if $n \in I^+$.
13. $|A^T| = |A|$, where A^T is a transpose of a matrix.

Symmetric Determinant

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = abc + 2fgh - af^2 - bg^2 - ch^2$$

Skew-symmetric Determinant

$$\begin{vmatrix} 0 & h & -g \\ -h & 0 & f \\ g & -f & 0 \end{vmatrix} = 0$$

In skew-symmetric matrix of odd order, the value of determinant is zero.

Product of Determinants

Generally, the product of determinants can be calculated by row column multiplication rule and it is same as the rule of multiplication of two matrices.

$$\text{e.g. Let } \Delta_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ and } \Delta_2 = \begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix}$$

be two determinants, then

$$\Delta_1 \Delta_2 = \begin{vmatrix} a_1\alpha_1 + b_1\alpha_2 + c_1\alpha_3 & a_1\beta_1 + b_1\beta_2 + c_1\beta_3 & a_1\gamma_1 + b_1\gamma_2 + c_1\gamma_3 \\ a_2\alpha_1 + b_2\alpha_2 + c_2\alpha_3 & a_2\beta_1 + b_2\beta_2 + c_2\beta_3 & a_2\gamma_1 + b_2\gamma_2 + c_2\gamma_3 \\ a_3\alpha_1 + b_3\alpha_2 + c_3\alpha_3 & a_3\beta_1 + b_3\beta_2 + c_3\beta_3 & a_3\gamma_1 + b_3\gamma_2 + c_3\gamma_3 \end{vmatrix}$$

Cyclic Determinant

A circulant matrix is one in which each row vector is rotated one element to the right relative to the preceding row vector.

$$1. \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$$

$$2. \begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$$

$$3. \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

$$4. \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$= -(a^3+b^3+c^3-3abc)$$

$$5. \begin{vmatrix} a & bc & abc \\ b & ca & abc \\ c & ab & abc \end{vmatrix} = \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}$$

$$= abc(a-b)(b-c)(c-a)$$

$$6. \begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix} = 0$$

$$7. \begin{vmatrix} 1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha) \\ \cos(\alpha-\beta) & 1 & \cos(\gamma-\beta) \\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1 \end{vmatrix} = 0$$

$$8. \begin{vmatrix} (a_1-b_1)^2 & (a_1-b_2)^2 & (a_1-b_3)^2 \\ (a_2-b_1)^2 & (a_2-b_2)^2 & (a_2-b_3)^2 \\ (a_3-b_1)^2 & (a_3-b_2)^2 & (a_3-b_3)^2 \end{vmatrix} = 0$$

$$= 2(a_1-a_2)(a_2-a_3)(a_3-a_1)(b_1-b_2)(b_2-b_3)(b_3-b_1)$$

Area of Triangles using Determinants

If three vertices of a triangle are (x_1, y_1) , (x_2, y_2) and (x_3, y_3) , then the area of the triangle can be calculated as

$$\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2} |\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}|$$

NOTE Condition for three points to be collinear, is given by

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Solution of System of Linear Equations

For the solution of system of non-homogeneous equation in two or three variables.

$$\text{Let } a_1x + b_1y + c_1z = d_1,$$

$$a_2x + b_2y + c_2z = d_2 \text{ and } a_3x + b_3y + c_3z = d_3$$

be a system of linear equations.

Then, the set of values of variables x, y and z which simultaneously satisfy these three equations, is called a **solution**.

Consistent

If the system of equations has a unique solution or infinite many solutions, then the system of equations is known as consistent.

Inconsistent

If the system of equations has no solution, then the system of equations is known as inconsistent.

Cramer's Rule

A system of simultaneous linear equations can be solved by Cramer's rule, named after the Swiss Mathematician Gabriel Cramer.

(i) **In Two Variables** Let we have two equations

$$a_1x + b_1y = c_1 \text{ and } a_2x + b_2y = c_2$$

Then, its solution will be

$$x = \frac{D_1}{D} \text{ and } y = \frac{D_2}{D}$$

$$\text{where, } D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}, D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$$

$$\text{and } D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}, \text{ provided that } D \neq 0.$$

(ii) **In Three Variables** Let we have three equations

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

Then, its solution will be

$$x = \frac{D_1}{D}, y = \frac{D_2}{D}$$

and

$$z = \frac{D_3}{D}$$

$$\text{where, } D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix},$$

$$D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix},$$

provided that $D \neq 0$.

Conditions for Consistency

Non-homogeneous System

The following cases arise while solving linear equations in two or three variables :

- If $D \neq 0$, then system will be consistent for a unique solution.
- If $D = 0$ and atleast one of D_1, D_2 and D_3 is non-zero, then given system is inconsistent i.e. no solution.
- If $D = 0$ and $D_1 = D_2 = D_3 = 0$, then given system is consistent with infinite many solutions or is inconsistent.

Homogeneous System

The following cases arise while solving linear equations in two or three variables :

- When $D \neq 0$, then system has trivial solution.
i.e. $x = y = z = 0$
- When $D = 0$, then system has non-trivial solution
i.e. infinitely many solutions.

Practice Exercise

1. If $A = \begin{bmatrix} 23 & 1+i & -i \\ 1-i & -31 & 4-5i \\ i & 4+5i & 17 \end{bmatrix}$, then $\det(A)$ is

- complex number with positive real part
- complex number with negative imaginary part
- pure imaginary
- real

2. Let $A_a = \begin{bmatrix} \cos a & -\sin a & 0 \\ \sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then

- $A_{a+\beta} = A_a A_\beta$
- $A_a^{-1} = A_a$
- $A_a^{-1} = -A_a$
- $A_a^2 = -I$

3. The value of third order determinant is 11, then the value of the square of the determinant formed by the cofactors will be

- 11
- 121
- 1331
- 14641

4. If $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = k \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$, then k is equal

to

- 0
- 1
- 2
- 3

5. $\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix} = 0$, if

- x, y and z are in AP
- x, y and z are in GP
- x, y and z are in HP
- xy, yz and zx are in AP

6. If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = k(a+b+c)^3$,

then k is equal to

- 0
- 1
- 2
- 3

7. If a, b and c are cube roots of unity, then

$\begin{vmatrix} e^a & e^{2a} & e^{3a}-1 \\ e^b & e^{2b} & e^{3b}-1 \\ e^c & e^{2c} & e^{3c}-1 \end{vmatrix}$ is equal to

- 0
- e
- e^2
- e^3

8. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and the vectors $(1, a, a^2)$, $(1, b, b^2)$

and $(1, c, c^2)$ are non-coplanar, then abc is equal to

- a. 2 b. -1
c. 1 d. 0

9. If $px^4 + qx^3 + rx^2 + sx + t = \begin{vmatrix} x^2 + 3x & x-1 & x+3 \\ x+1 & -2x & x-4 \\ x-3 & x+4 & 3x \end{vmatrix}$,

where p, q, r, s and t are constants, then t is equal to

- a. 0 b. 1
c. 2 d. -1

10. If $a_1, a_2, a_3, \dots$ are in GP, then

$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$ is equal to

- a. 0 b. 1
c. 2 d. 4

11. If $f(y) = \begin{vmatrix} a & -1 & 0 \\ ay & a & -1 \\ ay^2 & ay & a \end{vmatrix}$, then $f(2y) - f(y)$ is divisible by

- a. y^3 b. 0 c. $2a + 3y$ d. y^2

12. Suppose $\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$, then the value of x

is equal to

- a. 1, 9 b. -1, 9 c. -1, -9 d. 1, -9

13. If $a + b + c = 0$, then one root of

$\begin{vmatrix} a-n & c & b \\ c & b-n & a \\ b & a & c-n \end{vmatrix} = 0$ is

- a. $n = 1$ b. $n = 2$
c. $n = a^2 + b^2 + c^2$ d. $n = 0$

14. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. If $|A^2| = 25$, then $|\alpha|$ equals

- a. 5^2 b. 1 c. $\frac{1}{5}$ d. 5

15. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$, then D is

- a. divisible by neither x nor y
b. divisible by both x and y
c. divisible by x but not y
d. divisible by y but not x

16. In $\triangle ABC$, $\begin{vmatrix} 1 & 1 \\ 1+\sin A & 1+\sin B \\ \sin A + \sin^2 A & \sin B + \sin^2 B \end{vmatrix} \begin{vmatrix} 1 \\ 1+\sin C \\ \sin C + \sin^2 C \end{vmatrix} = 0$,

then $\triangle ABC$ is a/an

- a. right angled triangle b. equilateral triangle
c. isosceles triangle d. scalene triangle

17. If $\Delta = \begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix}$, then using properties

of determinant, Δ is equal to

a. $\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$ b. $\begin{vmatrix} a & c & p \\ d & b & q \\ w & v & u \end{vmatrix}$
c. $(1-x^2) \begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$ d. $(x^2-1) \begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$

18. If $1, \omega, \omega^2$ are the cube roots of unity, then

$\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to

- a. 0 b. 1 c. ω d. ω^2

19. If a, b and c are positive and not all equal, then the

value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

- a. > 0 b. ≥ 0 c. < 0 d. ≤ 0

20. $\begin{vmatrix} n & n+1 & n+2 \\ {}^nP_n & {}^{(n+1)}P_{(n+1)} & {}^{(n+2)}P_{(n+2)} \\ {}^nC_n & {}^{(n+1)}C_{(n+1)} & {}^{(n+2)}C_{(n+2)} \end{vmatrix}$ is equal to

- a. $n(n!)$ b. $(n+1)(n+1)!$
c. $(n+2)(n+2)!$ d. $(n^2+n+1)n!$

21. If the determinant $\begin{vmatrix} \cos 2x & \sin^2 x & \cos 4x \\ \sin^2 x & \cos 2x & \cos^2 x \\ \cos 4x & \cos^2 x & \cos 2x \end{vmatrix}$ is

expanded in powers of $\sin x$, then the constant term is

- a. 0 b. 1 c. -1 d. 2

22. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$,

then $f(50)$ is equal to

- a. 0 b. 50 c. 1 d. -50

23. If ω is an imaginary cube root of unity, then the value

$$\begin{vmatrix} a & b\omega^2 & a\omega \\ b\omega & c & b\omega^2 \\ c\omega^2 & a\omega & c \end{vmatrix}$$

- a. $a^3 + b^3 + c^2 - 3abc$ b. $a^2b - b^2c$
c. 0 d. $a^2 + b^2 + c^2$

24. The number of distinct real roots of the equation

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0 \text{ in } \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \text{ is}$$

- a. 0 b. 1 c. 2 d. 3

25. If x, y and z are positive, then $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$

is equal to

- a. 0 b. 1 c. -1 d. None

26. If $\begin{vmatrix} 1 + \sin^2 \theta & \cos^2 \theta & 4 \sin 4\theta \\ \sin^2 \theta & 1 + \cos^2 \theta & 4 \sin 4\theta \\ \sin^2 \theta & \cos^2 \theta & 1 + 4 \sin 4\theta \end{vmatrix} = 0$, then θ is

equal to

- a. $\frac{7\pi}{24}, \frac{11\pi}{24}$ b. $\frac{5\pi}{24}, \frac{7\pi}{24}$ c. $\frac{11\pi}{24}, \frac{\pi}{24}$ d. $\frac{\pi}{24}, \frac{7\pi}{24}$

27. If α, β and γ are the roots of the equation $x^3 + px + q = 0$, then the value of the determinant

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$$

- a. 0 b. -2 c. 2 d. 4

28. If $a^2 + b^2 + c^2 = -2$ and

$$f(x) = \begin{vmatrix} 1 + a^2x & (1 + b^2)x & (1 + c^2)x \\ (1 + a^2)x & 1 + b^2x & (1 + c^2)x \\ (1 + a^2)x & (1 + b^2)x & 1 + c^2x \end{vmatrix}, \text{ then } f(x) \text{ is a}$$

polynomial of degree

- a. 0 b. 1 c. 2 d. 3

29. If α, β, γ are the roots of $x^3 + ax^2 + b = 0$, then the

$$\text{value of } \begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} \text{ is equal to}$$

- a. $-a^3$ b. $a^3 - ab$ c. a^3 d. $a^2 - 3b$

30. The parameter, on which the value of the determinant

$$\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$$

does not

- depend, is

- a. a b. p c. d d. x

31. If $e^{i\theta} = \cos \theta + i \sin \theta$, $A + B + C = \pi$ and

$$z = \begin{vmatrix} e^{2iA} & e^{-iC} & e^{-iB} \\ e^{-iC} & e^{2iB} & e^{-iA} \\ e^{-iB} & e^{-iA} & e^{2iC} \end{vmatrix}, \text{ then}$$

- a. $\operatorname{Re}(z) = 4$
b. $\operatorname{Im}(z) = 0$
c. $\operatorname{Re}(z) = -4$
d. $\operatorname{Im}(z) = -1$

32. The value of $\begin{vmatrix} 1 & \cos(\beta - \alpha) & \cos(\gamma - \alpha) \\ \cos(\alpha - \beta) & 1 & \cos(\gamma - \beta) \\ \cos(\alpha - \gamma) & \cos(\beta - \gamma) & 1 \end{vmatrix}$ is

- a. $\begin{vmatrix} \cos \alpha & \sin \alpha & 1 \\ \cos \beta & \sin \beta & 1 \\ \cos \gamma & \sin \gamma & 1 \end{vmatrix}^2$ b. $\begin{vmatrix} \sin \alpha & \cos \alpha & 0 \\ \sin \beta & \cos \beta & 0 \\ \sin \gamma & \cos \gamma & 0 \end{vmatrix}^2$
c. $\begin{vmatrix} \cos \alpha & \sin \alpha & 0 \\ \sin \beta & 0 & \cos \beta \\ 0 & \cos \gamma & \sin \gamma \end{vmatrix}^2$ d. None of these

33. If A, B and C are angles of a triangle, then the

$$\text{determinant } \begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix} \text{ is equal to}$$

- a. 0 b. -1
c. 1 d. None of these

34. If $\begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ca - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix} = (a^3 + b^3 + c^3 + kabc)^2$, then the value of k is

- a. 3 b. -3
c. 4 d. 0

35. If $\alpha, \beta \neq 0$ and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1 + f(1) & 1 + f(2) \\ 1 + f(1) & 1 + f(2) & 1 + f(3) \\ 1 + f(2) & 1 + f(3) & 1 + f(4) \end{vmatrix} = k(1 - \alpha)^2(1 - \beta)^2(\alpha - \beta)^2,$$

then k is equal to

- a. $\alpha\beta$ b. $\frac{1}{\alpha\beta}$
c. 1 d. -1

36. Using the factor theorem, it is found that $b + c, c + a$ and $a + b$ are three factors of the determinant

$$\begin{vmatrix} -2a & a + b & a + c \\ b + a & -2b & b + c \\ c + a & c + b & -2c \end{vmatrix}.$$

The constant value of the

determinant, is

- a. 4 b. 2
c. $a + b + c$ d. None of these

37. If the coordinates of the vertices of an equilateral triangle with sides of length a are (x_1, y_1) , (x_2, y_2) and

(x_3, y_3) , then $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2$ is equal to

- a. $\frac{3a^4}{4}$ b. $\frac{3a^5}{4}$
 c. $\frac{3a^2}{2}$ d. $\frac{3a^2}{4}$
38. The equations $\lambda x - y = 2$, $2x - 3y = -\lambda$ and $3x - 2y = -1$ are consistent for
 a. $\lambda = -4$ b. $\lambda = -1, 4$
 c. $\lambda = -1$ d. $\lambda = 1, -4$
39. If the equations are $x + y - 3 = 0$
 $(1 + \beta)x + (2 + \beta)y - 8 = 0$ and $x - (1 + \beta)y + (2 + \beta) = 0$, then the value of β for the consistent solution, is
 a. 2 b. $\frac{16}{3}$
 c. $-\frac{5}{3}$ d. $\frac{5}{3}$
40. If $a > b > c$ and the system of equations $ax + by + cz = 0$, $bx + cy + az = 0$, $cx + ay + bz = 0$ has a non-trivial solution, then both the roots of the quadratic equation $at^2 + bt + c = 0$ are
 a. non-real b. of opposite sign
 c. positive d. complex

41. If the system of these equations

$$m^3x + (m+1)^3y + (m+2)^3z = 0$$

$$m + (m+1)y + (m+2)z = 0$$

$$x + y + z = 0$$

has non-zero solution, then the value of m is equal to

- a. -1 b. 2 c. 3 d. -2
42. The number of values of k for which the linear equations $4x + ky + 2z = 0$, $kx + 4y + z = 0$ and $2x + 2y + z = 0$ possess a non-zero solution, is
 a. 2 b. 1 c. 0 d. 3
43. If the trivial solution is the only solution of the system of equations

$$x - ky + z = 0$$

$$kx + 3y - kz = 0$$

$$3x + y - z = 0$$

Then, the set of all values of k is

- a. $(2, -3)$ b. $R - \{2, -3\}$
 c. $R - \{2\}$ d. $R - \{-3\}$
44. Consider the system of linear equations

$$x_1 + 2x_2 + x_3 = 3$$

$$2x_1 + 3x_2 + x_3 = 3$$

$$3x_1 + 5x_2 + 2x_3 = 1$$

Then, the system has

- a. infinite number of solutions
 b. exactly 3 solutions
 c. a unique solution
 d. no solution

BITSAT Archives

1. The system of equations $x - y + 3z = 4$, $x + z = 2$
 $x + y - z = 0$ has [2013]
 a. a unique solution b. finitely many solutions
 c. infinitely many solutions d. None of these
2. If $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = k(a-b)(b-c)(c-a)$, then k is equal to [2012]
 a. -2 b. 1 c. 2 d. abc
3. $\begin{vmatrix} a+b & a & b \\ a & a+c & c \\ b & c & b+c \end{vmatrix}$ is equal to [2012]
 a. $4abc$ b. abc c. $a^2b^2c^2$ d. $4a^2bc$
4. The system $x + 4y - 2z = 3$, $3x + y + 5z = 7$ [2012]
 and $2x + 3y + z = 5$ has
 a. infinite number of solutions
 b. unique solution
 c. trivial solution
 d. no solution
5. If the three points $(k, 2k)$, $(2k, 3k)$, $(3, 1)$ are collinear, then k is equal to [2012]
 a. -2 b. 1 c. $\frac{1}{2}$ d. $-\frac{1}{2}$
6. The value of the determinant $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ will be [2011, 08]
 a. $(a-b-c)(a^2 + b^2 + c^2)$
 b. $(a+b+c)^3$
 c. $(a+b+c)(ab+bc+ca)$
 d. None of the above
7. If ω is a cube root of unity, then $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$ is equal to [2011, 06]
 a. 1 b. ω
 c. ω^2 d. 0

8. For the equations $x + 2y + 3z = 1$, $2x + y + 3z = 2$ and $5x + 5y + 9z = 4$, [2011]
 a. there is only one solution
 b. there exists infinitely many solutions
 c. there is no solution
 d. None of the above
9. If $C = 2 \cos \theta$, then the value of the determinant $\Delta = \begin{vmatrix} C & 1 & 0 \\ 1 & C & 1 \\ 6 & 1 & C \end{vmatrix}$ is [2007]
 a. $\frac{2 \sin^2 2\theta}{\sin \theta}$
 b. $8 \cos^3 \theta - 4 \cos \theta + 6$
 c. $\frac{2 \sin 2\theta}{\sin \theta}$
 d. $8 \cos^3 \theta + 4 \cos \theta + 6$
10. The solutions of the equation $\begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix} = 0$ are [2006]
 a. 3, -1 b. -3, 1 c. 3, 1 d. -3, -1
11. $\begin{vmatrix} 1 + \sin^2 \theta & \sin^2 \theta & \sin^2 \theta \\ \cos^2 \theta & 1 + \cos^2 \theta & \cos^2 \theta \\ 4 \sin 4\theta & 4 \sin 4\theta & 1 + 4 \sin 4\theta \end{vmatrix} = 0$, then $\sin 4\theta$ equals to [2005]
 a. $\frac{1}{2}$ b. 1 c. $-\frac{1}{2}$ d. -1
12. Let the homogeneous system of linear equations $px + y + z = 0$, $x + qy + z = 0$ and $x + y + rz = 0$, where $p, q, r \neq 1$, have a non-zero solution, then the value of $\frac{1}{1-p} + \frac{1}{1-q} + \frac{1}{1-r}$ is [2005]
 a. -1 b. 0
 c. 2 d. 1
13. Let the determinant of a 3×3 matrix A be 6, then B is a matrix defined by $B = 5A^2$. Then, determinant of B is [2005]
 a. 180 b. 100
 c. 80 d. None of these

Answer with Solutions

Practice Exercise

1. (d) $\bar{A} = \begin{bmatrix} 23 & 1-i & i \\ 1+i & -31 & 4+5i \\ -i & 4-5i & 17 \end{bmatrix}$
 $(\bar{A})^T = A \Rightarrow \det(\bar{A})^T = \det(A)$
 $\Rightarrow \det(\bar{A}) = \det(A) \Rightarrow \overline{\det(A)} = \det(A)$
 So, $\det(A)$ is real.
2. (a) $|A_a| = 1$
 $A_a^c = \begin{bmatrix} \cos a & -\sin a & 0 \\ \sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix}$
 $\text{adj}(A_a) = (A_a^c)^T = \begin{bmatrix} \cos a & \sin a & 0 \\ -\sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix}$
 $\Rightarrow A_a^{-1} = \frac{\text{adj } A_a}{|A_a|} = \begin{bmatrix} \cos a & \sin a & 0 \\ -\sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix} \neq A_a$
 Now, $A_a A_\beta = \begin{bmatrix} \cos a & -\sin a & 0 \\ \sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \beta & -\sin \beta & 0 \\ \sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
 $= \begin{bmatrix} \cos(a+\beta) & -\sin(a+\beta) & 0 \\ \sin(a+\beta) & \cos(a+\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix} = A_{a+\beta}$
3. (d) Required determinant $= |\text{adj } A|^2 = (11^2)^2 = 14641$
4. (c) Let $\Delta = \begin{vmatrix} b & c & a \\ c & a & b \\ a & b & c \end{vmatrix} + \begin{vmatrix} c & a & b \\ a & b & c \\ b & c & a \end{vmatrix}$
 $= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} + \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$
 $\therefore k = 2$
5. (b) Applying $C_1 \rightarrow C_1 - pC_2 - C_3$,
 $\begin{vmatrix} 0 & x & y \\ 0 & y & z \\ -(xp^2 + 2yp + z) & xp + y & yp + z \end{vmatrix} = 0$
 $\Rightarrow 0 = (xp^2 + 2yp + z)(y^2 - xz)$
 $\therefore y^2 = xz$
 So, x, y and z are in GP.
6. (b) Applying $R_1 \rightarrow R_1 + R_2 + R_3$ and then taking common from R_1 , we get
 $\Delta = (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$
 Applying $C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$, we get
 $\Delta = (a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$
 $= (a+b+c)^3 \Rightarrow k = 1$

$$\begin{aligned}
 7. (a) \Delta &= \begin{vmatrix} e^a & e^{2a} & e^{3a} \\ e^b & e^{2b} & e^{3b} \\ e^c & e^{2c} & e^{3c} \end{vmatrix} - \begin{vmatrix} e^a & e^{2a} & 1 \\ e^b & e^{2b} & 1 \\ e^c & e^{2c} & 1 \end{vmatrix} \\
 &= e^a \cdot e^b \cdot e^c \begin{vmatrix} 1 & e^a & e^{2a} \\ 1 & e^b & e^{2b} \\ 1 & e^c & e^{2c} \end{vmatrix} + \begin{vmatrix} e^a & 1 & e^{2a} \\ e^b & 1 & e^{2b} \\ e^c & 1 & e^{2c} \end{vmatrix} \\
 &= 0 \quad [\because a+b+c=0 \Rightarrow e^{a+b+c}=1]
 \end{aligned}$$

$$\begin{aligned}
 8. (b) \text{ Hint } \begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} &= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} \\
 &= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}
 \end{aligned}$$

9. (a) Put $x = 0$ in the given equation, we get

$$t = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix} = -12 + 12 = 0$$

10. (a) $a_n = ar^{n-1}$, where r is the common ratio.

$$\therefore \log a_n = \log a_1 + (n-1) \log r$$

Applying $R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - R_1$ in given determinant.

$$\Rightarrow \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ 3 \log r & 3 \log r & 3 \log r \\ 6 \log r & 6 \log r & 6 \log r \end{vmatrix} = 0$$

[since, R_2 and R_3 are proportional]

11. (c) Applying $R_3 \rightarrow R_3 - yR_2$ and $R_2 \rightarrow R_2 - yR_1$, we get

$$f(y) = \begin{vmatrix} a & -1 & 0 \\ 0 & a+y & -1 \\ 0 & 0 & a+y \end{vmatrix} = a(a+y)^2$$

$$\therefore f(2y) - f(y) = a[(a+2y)^2 - (a+y)^2]$$

$$= a(a+2y-a-y)(a+2y+a+y) = ay(2a+3y)$$

Hence, $f(2y) - f(y)$ is divisible by $(2a+3y)$.

$$12. (d) \text{ Clearly, we can take } \begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$$

Now, applying $C_1 \rightarrow C_1 + (C_2 + C_3)$ and then taking common from C_1 , we get

$$(x+9) \begin{vmatrix} 1 & 3 & 5 \\ 1 & x+2 & 5 \\ 1 & 3 & x+4 \end{vmatrix} = 0$$

Also, applying $R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - R_1$, we get

$$(x+9) \begin{vmatrix} 1 & 3 & 5 \\ 0 & x-1 & 0 \\ 0 & 0 & x-1 \end{vmatrix} = 0$$

$$\text{Thus, } (x+9)(x-1)(x-1) = 0 \Rightarrow x = -9, 1, 1$$

$$13. (d) \text{ We have, } \begin{vmatrix} a-n & c & b \\ c & b-n & a \\ b & a & c-n \end{vmatrix} = 0$$

Now, applying $C_1 \rightarrow C_1 + C_2 + C_3$, we get

$$\begin{aligned}
 &\begin{vmatrix} a+b+c-n & c & b \\ a+b+c-n & b-n & a \\ b+a+c-n & a & c-n \end{vmatrix} = 0 \\
 \Rightarrow &\begin{vmatrix} -n & c & b \\ -n & b-n & a \\ -n & a & c-n \end{vmatrix} = 0 \quad [\because a+b+c=0] \\
 \Rightarrow &-n \begin{vmatrix} 1 & c & b \\ 1 & b-n & a \\ 1 & a & c-n \end{vmatrix} = 0
 \end{aligned}$$

Now, applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we get

$$\begin{vmatrix} 1 & c & b \\ -n & 0 & b-n-c & a-b \\ 0 & a-c & c-b-n \end{vmatrix} = 0$$

$$\Rightarrow -n\{(b-n-c)(c-b-n) - (a-b)(a-c)\} = 0$$

So, $n = 0$ is a root of the given determinant.

$$\begin{aligned}
 14. (c) \text{ Hint } A^2 &= \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix} \\
 &= \begin{bmatrix} 25 & 25\alpha + 5\alpha^2 & 10\alpha + 25\alpha^2 \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{bmatrix}
 \end{aligned}$$

$$15. (b) \text{ Given, } D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$,

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & y \end{vmatrix} = xy$$

Hence, D is divisible by both x and y .

$$\begin{aligned}
 16. (c) \text{ Let } \Delta &= \begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix} \\
 &= - \begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \cos^2 A & \cos^2 B & \cos^2 C \end{vmatrix}
 \end{aligned}$$

[applying $R_3 \rightarrow R_3 - R_2$]

$$= - \begin{vmatrix} 1 & 0 & 0 \\ 1+\sin A & \sin B - \sin A & \sin C - \sin B \\ \cos^2 A & \cos^2 B - \cos^2 A & \cos^2 C - \cos^2 B \end{vmatrix}$$

[applying $C_3 \rightarrow C_3 - C_2$ and $C_2 \rightarrow C_2 - C_1$]

Expanding along R_1 , we get

$$\Delta = (\sin B - \sin A)(\sin^2 C - \sin^2 B) - (\sin C - \sin B)(\sin^2 B - \sin^2 A)$$

$$= (\sin B - \sin A)(\sin C - \sin B)(\sin C - \sin A) = 0$$

[$\because \Delta = 0$, given]

So, we get $A = B$

or $B = C$

or $C = A$

Hence, ΔABC is an isosceles.

17. (c) Applying $R_1 \rightarrow R_1 - x R_2$ to Δ , we get

$$\Delta = \begin{vmatrix} a(1-x^2) & c(1-x^2) & p(1-x^2) \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix}$$

$$= (1-x^2) \begin{vmatrix} a & c & p \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - xR_1$, we get

$$\Delta = (1-x^2) \begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$$

18. (a) Given, $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$

$$= 1(\omega^{3n} - 1) - \omega^n(\omega^{2n} - \omega^{2n}) + \omega^{2n}(\omega^n - \omega^{4n})$$

$$= 1(1 - 1) - 0 + \omega^{2n}(\omega^n - \omega^n) \quad [\because \omega^3 = 1]$$

$$= 0$$

19. (c) Expanding by the first row, we get

$$a(bc - a^2) - b(b^2 - ca) + c(ab - c^2) = 3abc - a^3 - b^3 - c^3$$

$$= -(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= -\frac{(a+b+c)}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

which is negative.

20. (d) Let $\Delta = \begin{vmatrix} n & n+1 & n+2 \\ n! & (n+1)! & (n+2)! \\ 1 & 1 & 1 \end{vmatrix}$

$$= \begin{vmatrix} n & 1 & 1 \\ n! & n \cdot n! & (n+1)(n+1)! \\ 1 & 0 & 0 \end{vmatrix}$$

[applying $C_3 \rightarrow C_3 - C_2$ and $C_2 \rightarrow C_2 - C_1$]

$$= (n+1)(n+1)! - n \cdot n!$$

$$= [(n+1)^2 - n]n! = (n^2 + n + 1) \cdot n!$$

21. (c) Put $x = 0$, we get $\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -1$

22. (a) Taking common factors x from C_2 , $(x+1)$ from C_3 and $(x-1)$ from R_3 , we get

$$f(x) = x(x^2 - 1) \begin{vmatrix} 1 & 1 & 1 \\ 2x & x-1 & x \\ 3x & x-2 & x \end{vmatrix}$$

$$= x(x^2 - 1) \begin{vmatrix} 1 & 0 & 0 \\ 2x & -(x+1) & 1 \\ 3x & -2(x+1) & 2 \end{vmatrix} = 0$$

[applying $C_3 \rightarrow C_3 - C_2$ and $C_2 \rightarrow C_2 - C_1$]

$\therefore f(50) = 0$

23. (c) $\Delta = \begin{vmatrix} a(1+\omega) & b\omega^2 & a\omega \\ b(\omega+\omega^2) & c & b\omega^2 \\ c(\omega^2+1) & a\omega & c \end{vmatrix}$ [applying $C_1 \rightarrow C_1 + C_3$]

$$= \begin{vmatrix} -a\omega^2 & b\omega^2 & a\omega \\ -b & c & b\omega^2 \\ -c\omega & a\omega & c \end{vmatrix}$$

$$= \omega^2 \cdot \omega \begin{vmatrix} -a & b & a\omega^2 \\ -b & c & b\omega^2 \\ -c & a & c\omega^2 \end{vmatrix} = -\omega^3 \begin{vmatrix} a & b & a \\ b & c & b \\ c & a & c \end{vmatrix} = 0$$

24. (b) Applying $C_1 \rightarrow C_1 + C_2 + C_3$ and taking common from C_1 ,

$$\Rightarrow (\sin x + 2 \cos x) \begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} = 0$$

Applying $R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - R_1$,

$$\Rightarrow (\sin x + 2 \cos x) \begin{vmatrix} 1 & \cos x & \cos x \\ 0 & (\sin x - \cos x) & 0 \\ 0 & 0 & (\sin x - \cos x) \end{vmatrix}$$

$$= 0$$

$$\Rightarrow (\sin x + 2 \cos x)(\sin x - \cos x)^2 = 0$$

$$\Rightarrow \tan x = 1$$

$$\therefore x = \frac{\pi}{4}$$

25. (a) $\begin{vmatrix} \frac{\log x}{\log x} & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & \frac{\log y}{\log y} & \frac{\log z}{\log y} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & \frac{\log z}{\log z} \end{vmatrix}$

By taking common factors from the columns and the rows, we get

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

26. (a) Applying $R_1 \rightarrow R_1 - R_3$ and $R_2 \rightarrow R_2 - R_3$

$$\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \sin^2 \theta & \cos^2 \theta & 1 + 4 \sin 4\theta \end{vmatrix} = 0$$

$$\Rightarrow \sin^2 \theta + \cos^2 \theta + 1 + 4 \sin 4\theta = 0 \quad [\text{expanding along } R_3]$$

$$\Rightarrow \sin 4\theta = -\frac{1}{2} \Rightarrow \theta = \frac{7\pi}{24}, \frac{11\pi}{24}$$

27. (a) Since, $\alpha + \beta + \gamma = 0$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$ in the given determinant,

$$\Delta = \begin{vmatrix} \alpha + \beta + \gamma & \beta & \gamma \\ \alpha + \beta + \gamma & \gamma & \alpha \\ \alpha + \beta + \gamma & \alpha & \beta \end{vmatrix} = \begin{vmatrix} 0 & \beta & \gamma \\ 0 & \gamma & \alpha \\ 0 & \alpha & \beta \end{vmatrix} = 0$$

28. (c) Applying $C_1 \rightarrow C_1 + C_2 + C_3$

and put $a^2 + b^2 + c^2 = -2$

$$f(x) = \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - R_1$,

$$f(x) = \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix} = (x-1)^2$$

29. (c) Since α, β, γ are the roots of the given equation.

$$\therefore \alpha + \beta + \gamma = -a, \alpha\beta + \beta\gamma + \gamma\alpha = 0$$

and $\alpha\beta\gamma = -b$

$$\begin{aligned} \text{Now, } \begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} &= -(\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha) \\ &= -(\alpha + \beta + \gamma)\{(\alpha + \beta + \gamma)^2 - 3(\alpha\beta + \beta\gamma + \gamma\alpha)\} \\ &= (a)\{a^2 - 0\} = a^3 \end{aligned}$$

30. (b) Let $\Delta = \begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$

Applying $C_1 \rightarrow C_1 + C_3$

$$\begin{aligned} \Delta &= \begin{vmatrix} 1+a^2 & a & a^2 \\ \cos(p-d)x + \cos(p+d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x + \sin(p+d)x & \sin px & \sin(p+d)x \end{vmatrix} \\ \Rightarrow \Delta &= \begin{vmatrix} 1+a^2 & a & a^2 \\ 2\cos px \cos dx & \cos px & \cos(p+d)x \\ 2\sin px \cos dx & \sin px & \sin(p+d)x \end{vmatrix} \end{aligned}$$

Applying $C_1 \rightarrow C_1 - 2(\cos dx)C_2$

$$\Rightarrow \Delta = \begin{vmatrix} 1+a^2-2a\cos dx & a & a^2 \\ 0 & \cos px & \cos(p+d)x \\ 0 & \sin px & \sin(p+d)x \end{vmatrix}$$

Expanding along C_1 , we get

$$\begin{aligned} \Delta &= (1+a^2-2a\cos dx)[\sin(p+d)x \cos px \\ &\quad - \sin px \cos(p+d)x] \\ &= (1+a^2-2a\cos dx)[\sin\{(p+d)x - px\}] \\ &= (1+a^2-2a\cos dx)[\sin dx] \end{aligned}$$

which is independent if p.

31. (c) $z = e^{iA} \cdot e^{iB} \cdot e^{iC} \begin{vmatrix} e^{iA} & e^{-i(C+A)} & e^{-i(B+A)} \\ e^{-i(C+B)} & e^{iB} & e^{-i(A+B)} \\ e^{-i(B+C)} & e^{-i(A+C)} & e^{iC} \end{vmatrix}$

$$= -1 \begin{vmatrix} e^{iA} & -e^{iB} & -e^{iC} \\ -e^{iA} & e^{iB} & -e^{iC} \\ -e^{iA} & -e^{iB} & e^{iC} \end{vmatrix}$$

Because we know that,

$$e^{i(A+B+C)} = e^{i\pi} = \cos \pi + i \sin \pi = -1$$

$$= -1 \begin{vmatrix} 0 & -2e^{iB} & 0 \\ -2e^{iA} & 0 & 0 \\ -e^{iA} & -e^{iB} & e^{iC} \end{vmatrix}$$

[applying $R_1 \rightarrow R_1 + R_3$ and $R_2 \rightarrow R_2 + R_3$]

$$\begin{aligned} &= -[-2e^{iB}\{+2e^{i(A+C)}\}] \\ &= [4e^{i(A+B+C)}] = 4e^{i\pi} = -4 \end{aligned}$$

32. (b) $\Delta = \begin{vmatrix} \cos \alpha & \sin \alpha & 0 \\ \cos \beta & \sin \beta & 0 \\ \cos \gamma & \sin \gamma & 0 \end{vmatrix} \begin{vmatrix} \cos \alpha & \sin \alpha & 0 \\ \cos \beta & \sin \beta & 0 \\ \cos \gamma & \sin \gamma & 0 \end{vmatrix}$

33. (a) Given, A, B and C are the angles of a triangle.

$$\therefore A + B + C = \pi \Rightarrow A + B = \pi - C$$

Now, let $\cos(A+B) = \cos(\pi - C) = -\cos C$

$$\Rightarrow \cos A \cos B - \sin A \sin B = -\cos C$$

$$\Rightarrow \cos A \cos B + \cos C = \sin A \sin B \quad \dots(i)$$

$$\text{Similarly, } \cos A \cos C + \cos B = \sin A \sin C \quad \dots(ii)$$

$$\text{and } \sin(B+C) = \sin(\pi - A) = \sin A \quad \dots(iii)$$

Now, let $\Delta = \begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$

Expanding along R_1 , we get

$$\begin{aligned} \Delta &= -1(1 - \cos^2 A) + \cos C(\cos C + \cos A \cos B) \\ &\quad + \cos B(\cos B + \cos A \cos C) \\ &= -\sin^2 A + \cos C(\sin A \sin B) + \cos B(\sin A \sin C) \\ &\quad [\text{using Eqs. (i) and (ii) and } \cos^2 A + \sin^2 A = 1] \\ &= -\sin^2 A + \sin A(\sin B \cos C + \cos B \sin C) \\ &= -\sin^2 A + \sin A \sin(B+C) \\ &\quad [\because \sin(x+y) = \sin x \cos y + \cos x \sin y] \\ &= -\sin^2 A + \sin^2 A = 0 \quad [\text{using Eq. (iii)}] \end{aligned}$$

34. (b) We have, $\begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ca - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix}$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} -a & c & b \\ -b & a & c \\ -c & b & a \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$$

$$= [a(bc - a^2) + b(ac - b^2) + c(ab - c^2)]^2$$

$$= [a^3 + b^3 + c^3 - 3abc]^2$$

$$\therefore k = -3$$

35. (c) Use the property that, two determinants can be multiplied row-to-row or row-to-column, to write the given determinant as the product of two determinants and then expand.

$$\text{Given, } f(n) = \alpha^n + \beta^n$$

$$f(1) = \alpha + \beta, f(2) = \alpha^2 + \beta^2,$$

$$f(3) = \alpha^3 + \beta^3, f(4) = \alpha^4 + \beta^4$$

$$\text{Let } \Delta = \begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 \cdot 1 + 1 \cdot 1 + 1 \cdot 1 & 1 \cdot 1 + 1 \cdot \alpha + 1 \cdot \beta & 1 \cdot 1 + 1 \cdot \alpha^2 + 1 \cdot \beta^2 \\ 1 \cdot 1 + 1 \cdot \alpha + 1 \cdot \beta & 1 \cdot 1 + \alpha \cdot \alpha + \beta \cdot \beta & 1 \cdot 1 + \alpha \cdot \alpha^2 + \beta \cdot \beta^2 \\ 1 \cdot 1 + 1 \cdot \alpha^2 + 1 \cdot \beta^2 & 1 \cdot 1 + \alpha^2 \cdot \alpha + \beta^2 \cdot \beta & 1 \cdot 1 + \alpha^2 \cdot \alpha^2 + \beta^2 \cdot \beta^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}^2$$

On expanding, we get

$$\Delta = (1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$$

$$\text{But given, } \Delta = k(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$$

$$\text{Hence, } k(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2 = (1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$$

$$\therefore k = 1$$

36. (a) Let $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = k(b+c)(c+a)(a+b)$

Putting $a = b = c = \lambda$, we get

$$\begin{vmatrix} -2\lambda & 2\lambda & 2\lambda \\ 2\lambda & -2\lambda & 2\lambda \\ 2\lambda & 2\lambda & -2\lambda \end{vmatrix} = k(2\lambda)(2\lambda)(2\lambda)$$

$$\therefore k = \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= -1(0) - 1(-2) + 1(2) = 4$$

37. (a) If (x_1, y_1) , (x_2, y_2) and (x_3, y_3) are the vertices of a triangle, then

$$\text{Area} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \quad \dots(i)$$

Also, we know that, if a is the length of an equilateral triangle, then

$$\text{Area} = \frac{\sqrt{3}}{4} a^2 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{\sqrt{3}}{4} a^2 = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\Rightarrow \frac{\sqrt{3}}{2} a^2 = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\Rightarrow \frac{3}{4} a^4 = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2$$

38. (b) Hint $\begin{vmatrix} \lambda & -1 & -2 \\ 2 & -3 & \lambda \\ 3 & -2 & 1 \end{vmatrix} = 0$

39. (c) $\Delta = 0$ [for consistent solution]

$$\therefore \begin{vmatrix} 1 & 1 & -3 \\ 1+\beta & 2+\beta & -8 \\ 1 & -1-\beta & 2+\beta \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1+\beta & 1 & -5+3\beta \\ 1 & -2-\beta & 5+\beta \end{vmatrix} = 0$$

[applying $C_2 \rightarrow C_2 - C_1$, $C_3 \rightarrow C_3 + 3C_1$]

$$\Rightarrow (5+\beta) + (2+\beta)(-5+3\beta) = 0$$

$$\Rightarrow 3\beta^2 + 2\beta - 5 = 0$$

$$\Rightarrow (\beta-1)(3\beta+5) = 0 \Rightarrow \beta = 1, -\frac{5}{3}$$

40. (b) $\therefore a > b > c$ and given equations are

$$ax + by + cz = 0, bx + cy + az = 0 \text{ and } cx + ay + bz = 0$$

$$\text{For non-trivial solution, } \Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

$$\Rightarrow 3abc - (a^3 + b^3 + c^3) = 0$$

If α and β are the roots of $at^2 + bt + c = 0$, then

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

$$b^2 - 4ac = (-a-c)^2 - 4ac = (a-c)^2 > 0$$

For opposite sign, $|\alpha - \beta| > 0$

$$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta > 0 \Rightarrow \frac{b^2}{a^2} - \frac{4c}{a} > 0$$

$$\Rightarrow b^2 - 4ac > 0$$

$$\Rightarrow (-a-c)^2 - 4ac > 0$$

$$\Rightarrow (a-c)^2 > 0, \text{ which is true.}$$

Hence, roots are real and have opposite sign.

41. (a) We know that, they have non-zero solution, if

$$\Delta = \begin{vmatrix} m^3 & (m+1)^3 & (m+2)^3 \\ m & (m+1) & (m+2) \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Now, applying $C_2 \rightarrow C_2 - C_1$, $C_3 \rightarrow C_3 - C_1$, we get

$$\Delta = \begin{vmatrix} m^3 & 3m^2 + 3m + 1 & 8 + 6m^2 + 12m \\ m & 1 & 2 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$$\Rightarrow 6m^2 + 6m + 2 - 8 - 6m^2 - 12m = 0$$

$$\Rightarrow -6(m+1) = 0$$

$$\Rightarrow m = -1$$

42. (a) Hint $\Delta = 0$

43. (b) Hint $\begin{vmatrix} 1 & -k & 1 \\ k & 3 & -k \\ 3 & 1 & -1 \end{vmatrix} \neq 0$

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1. (c) Let $D = \begin{vmatrix} 1 & -1 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$$= 1(0-1) - 1(1+1) + 3(1)$$

$$= -1 - 2 + 3 = 0$$

$$D_1 = \begin{vmatrix} 4 & -1 & 3 \\ 2 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= 4(0-1) - 1(0+2) + 3(2)$$

$$= -4 - 2 + 6 = 0$$

$$D_2 = \begin{vmatrix} 1 & 4 & 3 \\ 1 & 2 & 1 \\ 1 & 0 & -1 \end{vmatrix}$$

$$= 1(-2) + 4(1+1) + 3(0-2)$$

$$= -2 + 8 - 6 = 0$$

$$D_3 = \begin{vmatrix} 1 & -1 & 4 \\ 1 & 0 & 2 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 1(0-2) - 1(2-0) + 4(1-0)$$

$$= -2 - 2 + 4 = 0$$

Hence, the given system of equations has infinitely many solutions.

2. (b) $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$

[applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$]

$$= (b-a)(c-a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}$$

$$= (b-a)(c-a)(c+a-b-a)$$

$$\Rightarrow (b-a)(c-a)(c-b)$$

$$= k(a-b)(b-c)(c-a) \quad [\text{given}]$$

$$\Rightarrow k = 1$$

44. (d) The given system of linear equations can be put in the matrix form as

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ -8 \end{bmatrix} \quad \left[\begin{array}{l} \text{applying } R_2 \rightarrow R_2 - 2R_1, \\ \text{and } R_3 \rightarrow R_3 - 3R_1 \end{array} \right]$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ -5 \end{bmatrix} \quad [\text{applying } R_3 \rightarrow R_3 - R_2]$$

Clearly, the given system of equations has no solution.

3. (a) $\begin{vmatrix} a+b & a & b \\ a & a+c & c \\ b & c & b+c \end{vmatrix} = \begin{vmatrix} b & -c & b-c \\ a & a+c & c \\ b & c & b+c \end{vmatrix}$

[applying $R_1 \rightarrow R_1 - R_2$]

$$= \begin{vmatrix} 2b & 0 & 2b \\ a & a+c & c \\ b & c & b+c \end{vmatrix} \quad [\text{applying } R_1 \rightarrow R_1 + R_3]$$

$$= \begin{vmatrix} 2b & 0 & 0 \\ a & a+c & c-a \\ b & c & c \end{vmatrix} \quad [\text{applying } C_3 \rightarrow C_3 - C_1]$$

$$= 2b(ac + c^2 - c^2 + ac) = 4abc$$

4. (d) Given system of equations are

$$x + 4y - 2z = 3, 3x + y + 5z = 7 \text{ and } 2x + 3y + z = 5$$

$$\therefore \Delta = \begin{vmatrix} 1 & 4 & -2 \\ 3 & 1 & 5 \\ 2 & 3 & 1 \end{vmatrix} = 1(1-15) - 4(3-10) - 2(9-2)$$

$$= -14 + 28 - 14 = 0$$

$$\text{and } \Delta_2 = \begin{vmatrix} 1 & 3 & -2 \\ 3 & 7 & 5 \\ 2 & 5 & 1 \end{vmatrix} = 1 \neq 0$$

$\therefore$ No solution will exist.

5. (a) Area of triangle $= \frac{1}{2} \begin{vmatrix} k & 2k & 1 \\ 2k & 3k & 1 \\ 3 & 1 & 1 \end{vmatrix} = 0$

[applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$]

$$\Rightarrow \begin{vmatrix} k & 2k & 1 \\ k & k & 0 \\ 3-k & 1-2k & 0 \end{vmatrix} = 0$$

$$\Rightarrow 1[k(1-2k) - k(3-k)] = 0$$

$$\Rightarrow k - 2k^2 - 3k + k^2 = 0$$

$$\Rightarrow -k^2 - 2k = 0$$

$$\Rightarrow -k(k+2) = 0 \Rightarrow k = 0, -2$$

6. (b) Applying $R_1 \rightarrow R_1 + R_2 + R_3$ and taking common from R_1 , we get

$$\Delta = (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$

$$\Delta = (a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$$

$$= (a+b+c)^3$$

7. (d) Applying $C_1 \rightarrow C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = \begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ \omega+\omega^2+1 & \omega^2 & 1 \\ \omega^2+\omega+1 & 1 & \omega \end{vmatrix}$$

$$= \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & \omega \end{vmatrix} = 0 \quad [\because 1+\omega+\omega^2=0]$$

8. (a) The determinant of the coefficient matrix of given system of equation is

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{vmatrix} = 1(9-15) - 2(18-5) + 3(10-5)$$

$$= 3 \neq 0$$

Hence, the system of given equation has unique solution.

9. (b) Given that,

$$\Delta = \begin{vmatrix} C & 1 & 0 \\ 1 & C & 1 \\ 6 & 1 & C \end{vmatrix} = C(C^2-1) - 1(C-6)$$

$$\Rightarrow \Delta = 2 \cos \theta (4 \cos^2 \theta - 1) - (2 \cos \theta - 6)$$

$$[\because C = 2 \cos \theta, \text{ given}]$$

$$= 8 \cos^3 \theta - 4 \cos \theta + 6$$

10. (a) Hint $\begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -3 & -3 & 0 \end{vmatrix} = 0$$

11. (c) Let $A \equiv \begin{vmatrix} 1+\sin^2 \theta & \sin^2 \theta & \sin^2 \theta \\ \cos^2 \theta & 1+\cos^2 \theta & \cos^2 \theta \\ 4 \sin 4\theta & 4 \sin 4\theta & 1+4 \sin 4\theta \end{vmatrix} = 0$

Applying $R_1 \rightarrow R_1 + R_2$

$$\begin{vmatrix} 2 & 2 & 1 \\ \cos^2 \theta & 1+\cos^2 \theta & \cos^2 \theta \\ 4 \sin 4\theta & 4 \sin 4\theta & 1+4 \sin 4\theta \end{vmatrix} = 0$$

Applying $C_1 \rightarrow C_1 - 2C_3, C_2 \rightarrow C_2 - 2C_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ -\cos^2 \theta & 1-\cos^2 \theta & \cos^2 \theta \\ -2-4 \sin 4\theta & -2-4 \sin 4\theta & 1+\sin 4\theta \end{vmatrix} = 0$$

$$\Rightarrow \cos^2 \theta (2+4 \sin 4\theta) + (1-\cos^2 \theta) (2+4 \sin 4\theta) = 0$$

$$\Rightarrow [2 \cos^2 \theta + 4 \cos^2 \theta \sin 4\theta + 2 + 4 \sin 4\theta - 2 \cos^2 \theta - 4 \cos^2 \theta \sin 4\theta] = 0$$

$$\Rightarrow 2 + 4 \sin 4\theta = 0 \Rightarrow \sin 4\theta = -\frac{1}{2}$$

12. (d) Given equations are

$$px + y + z = 0, x + qy + z = 0, x + y + rz = 0$$

Since, the system have a non-zero solution, then

$$\begin{vmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{vmatrix} = 0$$

Applying $C_1 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_2$

$$\begin{vmatrix} p & 1-q & 0 \\ 1 & q-1 & 1-q \\ 1 & 0 & r-1 \end{vmatrix} = 0$$

$$\Rightarrow (1-q)(1-q)(1-r) \begin{vmatrix} \frac{p}{1-q} & 1 & 0 \\ \frac{1}{1-q} & -1 & 1 \\ \frac{1}{1-r} & 0 & -1 \end{vmatrix} = 0$$

$$\Rightarrow (1-p)(1-q)(1-r) \left[\frac{p}{1-q} (1) - 1 \left(-\frac{1}{1-q} - \frac{1}{1-r} \right) \right] = 0$$

Since, $p, q, r \neq 1$

$$\therefore \frac{p}{1-p} + \frac{1}{1-q} + \frac{1}{1-r} = 0$$

$$\Rightarrow \frac{1}{1-p} - 1 + \frac{1}{1-q} + \frac{1}{1-r} = 0$$

$$\Rightarrow \frac{1}{1-p} + \frac{1}{1-q} + \frac{1}{1-r} = 1$$

13. (a) Given that, $\det(A) = 6$... (i)

Now,

$$B = 5A^2$$

$$\Rightarrow \det(B) = \det 5A^2 = 5 \det(A^2) = 5 \det(A)^2$$

$$\Rightarrow = 5(6^2) \quad [\text{from Eq. (i)}]$$

$$\Rightarrow \det(B) = 180$$

9

Sets, Relations and Functions

Sets

A set is a well-defined collection of distinct objects. Here, the term ‘well-defined’ means, it must be possible to decide whether or not the object belongs to the set and the term ‘distinct’ means object should not be repeated. The object in the set is called its **member** or **element**. A set is represented by $\{ \}$. Generally, the sets are denoted by capital letters $A, B, C, \dots$ and its elements are denoted by small letters $a, b, c, \dots$.

If X is any non-empty set and x is a member of X (or belongs to X), then it is expressed as $x \in X$. If x is not a member of X or does not belong to X , then it is expressed as $x \notin X$. e.g. Let A be the set of all vowels in English alphabets. Then, a, e, i, o and u are members of A and we can say that $a \in A, e \in A, i \in A, O \in A$ and $u \in A$.

Representation of Sets

There are two methods to represent a set

- (i) **Roster Method** In this method, elements are listed and put within a the braces $\{ \}$ and separated by commas. This method is also known as **tabular method** or **listing method**.
- (ii) **Set Builder Method** In this method, we list the property or properties satisfied by the elements of set and write it as

$$A = \{x : P(x)\} \text{ or } \{x | P(x)\}$$

It is read as ‘ A is the set of all members x such that x has the property $P(x)$ ’. The symbol ‘:’ or ‘|’ stands for ‘such that’.

This method is also known as **rule method** or **property method**.

e. g. $A = \{1, 2, 3, 4, 5\} = \{x : x \in N \text{ and } x \leq 5\}$

NOTE The order of elements in a set has no importance. e.g. $\{1, 2, 3\}$ and $\{3, 2, 1\}$ are same sets. In a set, repetition of elements is not allowed.

Finite and Infinite Sets Sets containing finite number of elements are called finite sets. e.g. $A = \{a, e, i, o, u\}$

A set containing infinite number of elements is called infinite set. e.g. $A =$ Set of points on a line.

Order of a Finite Set The number of elements in a finite set is called the order of the set A and is denoted by $n(A)$. It is also called cardinal number of the set.

e.g. $A = \{2, 4, 6, 8\}$
 $n(A) = 4$

Null Set or Empty Set A set which contains no element, is called an empty set or null set. It is also called a void set. The null set is denoted by the symbols ϕ .

$\therefore \phi = \{x : x \text{ is an integer between 2 and 3}\}$

NOTE $\{0\}$ is not a null set. Since, it contains 0 as its element.

Singleton Set A set consisting of a single element is called a singleton set.

$A = \{a\}$ is a singleton.

Subset If each element of a set A is also an element of a set B , A is called a subset of B . We write it as $A \subseteq B$. Then, B is called superset of A and we write as $B \supseteq A$.

NOTE (i) Every set is a subset of itself.
 (ii) Empty set is a subset of itself.

Proper Subset of a Set The set A is called the proper subset of B , if and only if each element of A is the element of B and there is atleast one element of B which is not an element of the set A , i.e. A is proper subset of B , if $A \subset B$ and $A \neq B$.

Power Set Elements of a set can also be some sets. Such sets are called power sets of set.

e.g. $\{\phi, \{1\}, \{2\}, \{1, 2\}\}$ is a set whose elements are four sets $\phi, \{1\}, \{2\}$ and $\{1, 2\}$. The set of all subsets of a given set X is called the power set of X and is denoted by $P(X)$.

NOTE If a finite set S has n elements. Then, power set of S has 2^n elements.

Universal Set A set which is such that all the sets under consideration are its subsets, is called the universal set or universe and is denoted by U .

e.g. If $A = \{2, 4, 6, 8\}, B = \{1, 3, 5, 7\}$

Then, $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Complement of a Set with Respect to the Universal Set Complement of a set A is the set which contains all those elements of the universal set which are not in A . It is denoted by A' or $\bar{A}$.

e.g. $U = \{1, 2, 3, 4, 5, 6, 7\}$ and $A = \{2, 4, 5\}$

$A' = \{1, 3, 6, 7\}$

Disjoint Sets Two sets A and B are said to be disjoint or mutually exclusive, if and only if there is no element common to A and B , i.e. $A \cap B = \phi$.

If $A \cap B \neq \phi$, then A and B are said to be intersecting or overlapping sets.

Difference of Sets If A and B are two given sets, then the set of all elements which belongs to A but do not belong to B is called the difference of sets A and B and is denoted by $A - B$.

Thus, $A - B = \{x : x \in A \text{ and } x \notin B\}$

e.g. If $A = \{2, 3, 4, 5, 6, 7\}, B = \{3, 5, 7, 9, 11\}$

Then, $A - B = \{2, 4, 6\}$

Equality of Two Sets Two sets A and B are said to be equal, iff $A \subseteq B$ and $B \subseteq A$.

$\Rightarrow A = B$, i.e. every element of A is an element of B and also every element of B is an element of A .

e.g. If $A = \{1, 2, 4\}, B = \{1, 4, 2, 2, 4, 1\}$, then $A = B$

Equivalent Sets Two sets are said to be equivalent, if they contain the same number of elements, i.e. $n(A) = n(B)$, then sets A and B are equivalent.

NOTE Equal sets are equivalent but equivalent sets are not equal sets.

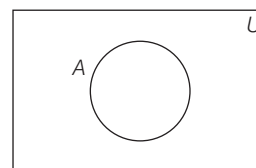
Comparable Sets and Non-comparable Sets Two sets A and B are comparable, if $A \subseteq B$ or $B \subseteq A$.

If neither $A \subseteq B$ nor $B \subseteq A$, then A, B are not comparable.

Venn Diagram

The combination of rectangles and circles is called **Venn Euler diagram** or Venn diagram.

In Venn diagram, the universal set is represented by a rectangular region and a set is represented by circle on some closed geometrical figure, where A is the set and U is the universal set.



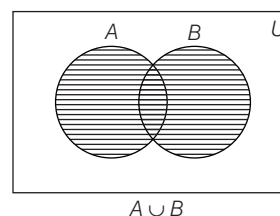
Operations on Sets

1. Union

Let A and B be two sets. The union of A and B is the set of all elements which are in set A or in B or both A and B .

i.e. $A \cup B = \{x : x \in A \text{ or } x \in B\}$

If $A_1, A_2, \dots, A_n$ is a finite family of sets.



Then, union of sets is denoted by

$$\bigcup_{i=1}^n A_i \text{ or } A_1 \cup A_2 \cup \dots \cup A_n$$

2. Intersection

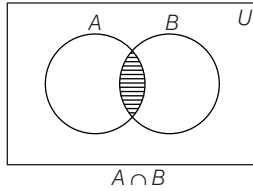
The intersection of A and B is the set of all those elements that belongs to both A and B .

i.e. $A \cap B = \{x : x \in A \text{ and } x \in B\}$

If $A_1, A_2, \dots, A_n$ is a finite family of sets.

Then, their intersection of sets is

denoted by $\bigcap_{i=1}^n A_i$ or $A_1 \cap A_2 \cap A_3 \cap \dots \cap A_n$.

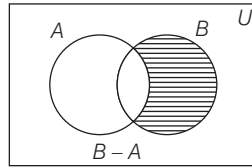
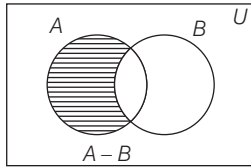


3. Difference

The difference of sets A and B , i.e. $A - B$ is the set of all those elements of A which do not belong to B .

i.e. $A - B = \{x : x \in A \text{ and } x \notin B\}$

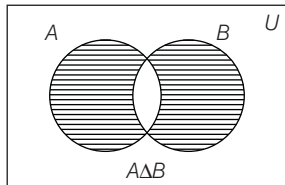
and $B - A = \{x : x \in B \text{ and } x \notin A\}$



4. Symmetric Difference

The symmetric difference of sets A and B is the set $(A - B) \cup (B - A)$ and is denoted by $A \Delta B$.

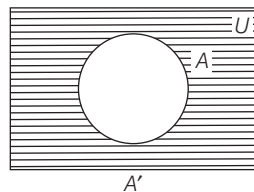
$A \Delta B = (A - B) \cup (B - A)$



5. Complement

Let U be the universal set and A be a set such that $A \subset U$.

Then, complement of A with respect to U is denoted by A' or A^c or $C(A)$ or $U - A$. It is defined as the set of all those elements of U which are not in A .



Laws of Algebra of Sets

If A, B and C are any three sets, then

1. Idempotent Laws

(i) $A \cup A = A$

(ii) $A \cap A = A$

2. Identity Laws

(i) $A \cup \phi = A$

(ii) $A \cap U = A$

3. Distributive Laws

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4. De-Morgan's Law

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

(iii) $A - (B \cap C) = (A - B) \cup (A - C)$

(iv) $A - (B \cup C) = (A - B) \cap (A - C)$

5. Associative Laws

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) $A \cap (B \cap C) = (A \cap B) \cap C$

Cartesian Product of Sets

The set consisting of all distinct ordered pairs (a, b) , where $a \in A$ and $b \in B$ is called the cartesian product of the sets A and B and is denoted by $A \times B$.

Symbolically,

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$$

$$B \times A = \{(b, a) : b \in B \text{ and } a \in A\}$$

NOTE Cartesian product of a finite number of sets $A_1, A_2, A_3, \dots, A_n$ is denoted by

$$A_1 \times A_2 \times A_3 \times \dots \times A_n = \prod_{i=1}^n A_i$$

Some Important Properties

1. $A - (B \cup C) = (A - B) \cap (A - C)$

2. $A - (B \cap C) = (A - B) \cup (A - C)$

3. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

4. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

6. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

7. $(A')' = A$

8. $A \subseteq B \Leftrightarrow B' \subseteq A'$

9. $n(A \times B) = n(A) \times n(B)$

10. $A \times (B \cup C) = (A \times B) \cup (A \times C)$

11. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

12. $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

13. $n(A - B) = n(A) - n(A \cap B)$

14. $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$

15. $n(A^c \cap B^c) = n(A \cup B)^c = n(U) - n(A \cup B)$

16. $n(A^c \cup B^c) = n(A \cap B)^c = n(U) - n(A \cap B)$

Relations

If A and B are two non-empty sets, then a relation from A to B is a subset of the product set $A \times B$.

Let $R \subseteq A \times B$ and $(a, b) \in R$

Then, we say that, a is related to b by the relation R and we write it as ${}_aR_b$.

e.g. If R denotes 'is less than' and $A = \{1, 3, 5\}$ and $B = \{1, 4\}$, then R from A to B is ${}_aR_b$ iff $a < b, \forall a \in A, b \in B$

Then, $R = \{(1, 4), (3, 4)\}$

Domain and Range of a Relation

Let R be the relation defined from A to B , i.e. R is a subset of $A \times B$, then

Domain of $R = \{a : a \in A, (a, b) \in R \text{ for some } b \in B\}$

Range of $R = \{b : b \in B, (a, b) \in R \text{ for some } a \in A\}$

It is obvious that $D \subseteq A$ and $R \subseteq B$

Composition of Relations

Let $R \subseteq A \times B, S \subseteq B \times C$ be two relations, then compositions of the relation R and S is denoted by $SoR \subseteq A \times C$ and is defined by, $(a, c) \in (SoR)$, iff there exists $b \in B$ such that $(a, b) \in R, (b, c) \in S$.

Relation in a Set

A relation in a set A is a subset of the cartesian product $A \times A$.

- 1. Identity Relation** Let A be a set, then the relation $I_A = \{(x, y) : x \in A, y \in R, x = y\}$ is called the identity relation in A , i.e. the identity relation in a set A is the set of all ordered pairs (x, y) of $A \times A : x = y$.

If $A = \{1, 2, 3, 4, 5, 6\}$, then

$$I_A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$$

- 2. Universal Relation** Let A be any set and $R = A \times R$, then R is called the universal relation in A .
- 3. Void Relation in a Set** Consider the set $A \times A$, then every subset of $A \times A$ is a relation in A . Again, the null set ϕ is a subset of $A \times A$. Therefore, the null set ϕ is also a relation in A . This relation is called the void relation in A .
- 4. Inverse Relation** If R is a relation from a set A to a set B , then the relation R^{-1} from set B to the set A is defined as the inverse relation R . Symbolically,

$$R^{-1} = \{(y, x) : (x, y) \in R\}$$

e.g. $R = \{(2, 4), (2, 6), (3, 4), (3, 6)\}$

$$R^{-1} = \{(4, 2), (6, 2), (4, 3), (6, 3)\}$$

Properties of Relation

- 1. Reflexive Relation** A relation R in a set A is a reflexive, if every element of A is related to itself.

i.e. ${}_aR_a, \forall a \in A$

- 2. Symmetric Relation** A relation R is symmetric, if whenever a is related to b , then b is also related to a .

$$(a, b) \in R \Rightarrow (b, a) \in R$$

i.e. ${}_aR_b \Rightarrow {}_bR_a$

The necessary and sufficient condition that a relation R in A be symmetric is that $R = R^{-1}$.

- 3. Anti-symmetric Relations** R is called an anti-symmetric relation, if $(a, b) \in R$ and $(b, a) \in R$.

$$\Rightarrow a = b$$

If $a \neq b$, then a may be related to b and b may be related to a , then both can never be related to each other at a time.

- 4. Transitive Relation** A relation R is transitive, if

$$(a, b) \in R \text{ and } (b, c) \in R \Rightarrow (a, c) \in R$$

i.e. ${}_aR_b \text{ and } {}_bR_c \Rightarrow {}_aR_c$

- 5. Equivalence Relation** A relation on a set A which is reflexive, symmetric and transitive is called an equivalence relation.

$${}_aR_a, {}_aR_b \Rightarrow {}_bR_a \text{ and } {}_aR_b, {}_bR_c \Rightarrow {}_aR_c$$

Some Results on Relations

- The intersection of two equivalence relations on a set is a equivalence relation on the set.
- The union of two equivalence relations on a set is not necessarily an equivalence relation on the set.
- The inverse of an equivalence relation is an equivalence relation.

Binary Operations

A binary operation $*$ on S is a rule that associates with every ordered pair (a, b) of $S \times S$, a unique element $a * b = c \in S$

Thus, if a binary operation on S is denoted by $*$, then the image of $(a, b) \in S \times S$ under the binary operation $*$ is generally written as $a * b$.

e.g. On the set of real numbers, the operation of addition is binary because $\forall a, b \in R, a + b$ is a real number c .

Functions or Mapping

A function f from A to B written as $f : A \rightarrow B$ is a relation in which to every element of A corresponds a unique element of B .

Alternatively Let A and B be two non-empty sets. A function from A to B is a subset of $A \times B$, such that no different ordered pairs in f have the same first element.

Symbolically: $f: A \rightarrow B$, i.e. f is a function A to B .

Domain, Codomain and Range of a Function The set A is called the domain of f , denoted by $\text{dom } f$ and the set B is called the codomain of f . The set of all second elements of the pairs (a, b) of the function f is the range of the function f , so range is a subset of codomain.

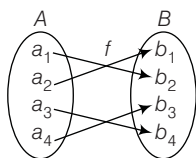
Image and Pre-image of a Function

The variable x of the pair (x, y) which takes on values in the domain is usually called independent variable or x as pre-image of y and y which takes on values in the range is called the dependent variable or y as image of x .

Different Types of Function

1. One-One Function (Injection)

A function $f: A \rightarrow B$ is said to be a one-one function or an injection, if different elements of A have different images in B . Thus, $f: A \rightarrow B$ is one-one, if there exist $a, b \in A$, such that



$$a \neq b \Rightarrow f(a) \neq f(b), \forall a, b \in A$$

$$\text{or } f(a) = f(b) \Rightarrow a = b, \forall a, b \in A$$

Graphically Any line parallel to X -axis cuts the graph of the function at most at one point.

- NOTE**
- The number of functions from a finite set A into finite set $B = \{n(B)\}^{n(A)}$.
 - The number of one-one function that can be defined from a finite set A into finite set B is $\begin{cases} {}^{n(B)}P_{n(A)}, & \text{if } n(B) \geq n(A) \\ 0, & \text{otherwise} \end{cases}$
 - Any function which is entirely increasing or decreasing in the whole of a domain is one-one function.

Methods to Check One-One Function

Method 1 If $f(x) = f(y) \Rightarrow x = y$, then f is one-one.

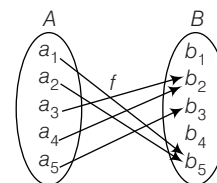
Method 2 A function is one-one, iff no line parallel to X -axis meets the graph of function at more than one point.

Method 3 If $f'(x) \leq 0$ or $f'(x) \geq 0, \forall x \in \text{domain}$, then f is one-one.

2. Many-One Function

A function $f: A \rightarrow B$ is said to be a many-one function, if two or more elements of set A have the same image in B . Thus, $f: A \rightarrow B$ is a many-one function, if there exist $a, b \in A$, such that $a \neq b$ but $f(a) = f(b)$.

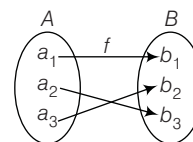
i.e. $f: A \rightarrow B$ is a many-one function, if it is not a one-one function.



Graphically Any line parallel to X -axis, cuts the graph of the function atleast at two points. In three consecutive quadrants, trigonometrical functions are always many-one functions.

3. Onto Function (Surjection)

A function $f: A \rightarrow B$ is said to be an onto, if each element of B has its pre-image in A .



$\therefore$ If $f^{-1}(y) \in A, \forall y \in B$, then function is onto, where f^{-1} is an inverse of f Range of $f = \text{Codomain of } f$.

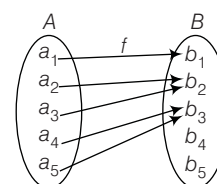
- NOTE**
- The number of onto functions that can be defined from a finite set A containing n elements onto a finite set B containing 2 elements $= 2^n - 2$.
 - The number of onto functions that can be defined from a finite set A out a finite set $B = \text{Number of ways of dividing } n(A) \text{ things into } n(B) \text{ groups. So, that no group is empty, if } n(A) \geq n(B) \text{ and is } 0, \text{ otherwise.}$

Method to Check Onto Function

Find the range $y = f(x)$ and show that range $f(x) = \text{Codomain of } f(x)$.

4. Into Function

A function $f: A \rightarrow B$ is said to be an into function, if there exists atleast one element in B having no pre-image in A , i.e. $f: A \rightarrow B$ is an into function, if it is not an onto function.



5. One-One and Onto Function (Bijection)

A function $f: A \rightarrow B$ is said to be a bijection, if it is one-one as well as onto. Thus, $f: A \rightarrow B$ is a bijection, if

- it is one-one, i.e. $f(x) = f(y) \Rightarrow x = y, \forall x, y \in A$.
- it is onto i.e. $\forall y \in B$, there exists $x \in A$, such that $f(x) = y$.

6. Inverse Function

Let f be defined a function from A to B such that for every element of B , there exists an image. Let y be an arbitrary element of B .

Then, f being onto, there exists an element $x \in A$, such that $f(x) = y$. Also, f being one-one, this x must be unique.

Thus, for each $y \in B$, there exists a unique element $x \in A$, such that $f(x) = y$. So, we may define a function,

$$f^{-1}: B \rightarrow A$$

$$\therefore f^{-1}(y) = x \Leftrightarrow f(x) = y$$

The above function f^{-1} is called the inverse of f .

- Inverse of bijective function is also bijective function.
- If the inverse of f exist, then f is called an invertible function, i.e. a function f is invertible if and only if f is one-one onto.

7. Composite Function

If $f: A \rightarrow B$ and $g: B \rightarrow C$ are two functions, then the composite function of f and g i.e.

$g \circ f: A \rightarrow C$ will be defined as, $g \circ f(x) = g[f(x)]$, $\forall x \in A$.

Generally, $g \circ f \neq f \circ g$.

Practice Exercise

- The number of proper subsets of $\{1, 2, 3\}$ is
a. 8 b. 7 c. 6 d. 5
- Which one of the following is $(A - B) \cup (B - A)$?
a. $(A \cup B) \cup (A - B)$ b. $(A \cup B) \cup (A \cap B)$
c. $(A \cup B) - (A \cap B)$ d. $(A - B) \cap (B - A)$
- Let $A = \{x: x \text{ is a prime factor of } 240\}$, $B = \{x: x \text{ is the sum of any two prime factors of } 240\}$, then
a. $5 \notin A \cap B$ b. $7 \in A \cap B$ c. $8 \in A \cap B$ d. $8 \in A \cup B$
- If $A = \{x: x \text{ is a multiple of } 4 \text{ and } x \in N\}$ and $B = \{x: x \text{ is a multiple of } 6 \text{ and } x \in N\}$, then $A \cap B$ consists of all multiples of
a. 16 b. 12 c. 8 d. 4
- Let A and B be two sets. Then, $(A \cup B)' \cup (A' \cap B)$ is
a. A' b. A
c. B d. None of these
- If $X = \{8^n - 7n - 1; n \in N\}$ and $Y = \{49(n - 1); n \in N\}$, then
a. $X \subset Y$ b. $Y \subset X$
c. $X = Y$ d. None of these
- In a town of 10000 families, it was found that 40% families buy newspaper A , 20% families buy newspaper B , 10% families buy newspaper C , 5% families buy A and B , 3% families buy B and C and 4% by A and C . If 2% families buy all the three newspapers, then the number of families which buy A only is
a. 3100 b. 3300 c. 2900 d. 1400
- If X and Y are two sets, then $X \cap (Y \cup X)'$ equals
a. X b. Y
c. ϕ d. None of these
- Which of the following cannot be the number of elements in the power set of any finite set?
a. 26 b. 32 c. 64 d. 128
- If $A = \{x: x^2 = 1\}$ and $B = \{x: x^4 = 1\}$, then $A \Delta B$ is equal to
a. $\{-i, i\}$ b. $\{-1, 1\}$ c. $\{-1, 1, i, -i\}$ d. $\{1, 1\}$
- A survey shows that 63% of the Americans like cheese where 76% like apples. If $x\%$ of the Americans like both cheese and apples, then
a. $x = 39$ b. $x = 63$
c. $39 \leq x \leq 63$ d. None of these
- Two finite sets have m and n elements. The total number of subsets of first set is 56 more than the total number of subsets of the second set. The values of m and n are
a. 7, 6 b. 6, 3 c. 5, 1 d. 8, 7
- Let X be the universal set for sets A and B . If $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$, then $n(A' \cap B')$ is equal to 300 provided $n(X)$ is equal to
a. 600 b. 700 c. 800 d. 900
- Let A and B be two sets such that $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for same set X . Then,
a. $A = B$ b. $A = X$ c. $B = X$ d. $A \cup B = X$
- If $f(x) = \cos(\log_e x)$, then $f(x)f(y) - \frac{1}{2}[f(x/y) + f(xy)]$ is equal to
a. 0 b. $\frac{1}{2}f(x)f(y)$ c. $f(x + y)$ d. $f(x - y)$
- Let U be the universal set and $A \cup B \cup C = U$. Then, $\{(A - B) \cup (B - C) \cup (C - A)\}'$ is equal to
a. $A \cup B \cup C$ b. $A \cup (B \cap C)$
c. $A \cap B \cap C$ d. $A \cap (B \cup C)$

17. If $A = \{1, 2, 3\}$ and $B = \{3, 6, 8\}$, then $(A \cap B) \times A$ is equal to
 a. $\{(1, 3), (2, 3), (3, 3)\}$ b. $\{(3, 1), (3, 2), (3, 3)\}$
 c. $\{(1, 3), (3, 1), (3, 2)\}$ d. None of these
18. Let R be a relation defined by $R = \{(1, 3), (2, 4), (5, 1)\}$ on the set of natural numbers N . Then, R^{-1} is equal to
 a. $\{(3, 1), (4, 2), (1, 5)\}$ b. $\{(5, 1), (4, 2), (1, 3)\}$
 c. $\{(5, 1), (2, 4), (1, 3)\}$ d. None of these
19. Let A and B be two finite sets having m and n elements, respectively. Then, the total number of mapping from A to B is
 a. mn b. 2^{mn} c. m^n d. n^m
20. The function $f: N \rightarrow N$, where N is the set of natural numbers, defined by $f(x) = 2x + 3$, is
 a. surjective b. not surjective
 c. injective d. None of these
21. Let R be the real line. Consider the following subsets of the plane $R \times R$
 $S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$ and
 $T = \{(x, y) : x - y \text{ is an integer}\}$. Which one of the following is true?
 a. T is an equivalence relation on R but S is not
 b. Neither S nor T is an equivalence relation on R
 c. Both S and T are equivalence relations on R
 d. S is an equivalence relation on R but T is not
22. A natural number a is said to be related to another natural number b , if $|a - b| < 4$. The relation is
 a. reflexive and symmetric
 b. reflexive and transitive
 c. symmetric and transitive
 d. None of the above
23. If $A = \{0, 1, 2, 3, 4, 5\}$ and a relation R is defined by ${}_xR_y$, when $2x + y = 10$. Then, R^{-1} is
 a. $\{(3, 4), (4, 2), (5, 0)\}$ b. $\{(4, 3), (4, 2), (5, 0)\}$
 c. $\{(4, 3), (2, 4), (5, 0)\}$ d. $\{(4, 3), (2, 4), (0, 5)\}$
24. Given that relation $R = \{(1, 2), (2, 3)\}$ on the set $A = \{1, 2, 3\}$, the minimum number of ordered pairs which when added to R make it an equivalence relation, is
 a. 5 b. 6 c. 7 d. 8
25. Let $A = \{1, 2, 3, 4, 5\}$ and R be a relation defined by $R = \{(x, y) : x, y \in A, x + y = 5\}$. Then, R is
 a. reflexive and symmetric but not transitive
 b. an equivalence relation
 c. symmetric but neither reflexive nor transitive
 d. neither reflexive nor symmetric but transitive
26. If a relation R is defined on the set Z of integers as $R = \{(a, b) : a^2 + b^2 = 25\}$. Then, domain of R is
 a. $\{3, 4, 5\}$ b. $\{0, 3, 4, 5\}$
 c. $\{0, \pm 3, \pm 4, \pm 5\}$ d. None of these
27. The domain of the function $f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$ is equal to
 a. $[-3, -2.5) \cup (-2.5, 2]$ b. $[-2, 0) \cup (0, 1]$
 c. $[0, 1]$ d. None of these
28. The range of the function $f(x) = \frac{x-2}{2-x}$, when $x \neq 2$, is
 a. R b. $R - \{1\}$ c. $\{-1\}$ d. $R - \{-1\}$
29. The function $f(x) = \log \frac{1+x}{1-x}$ satisfies the equation
 a. $f(x_1) \cdot f(x_2) = f(x_1 + x_2)$
 b. $f(x+2) - 2f(x+1) + f(x) = 0$
 c. $f(x) + f(x+1) = f(x^2 + x)$
 d. $f(x_1) + f(x_2) = f\left(\frac{x_1 + x_2}{1 + x_1 x_2}\right)$
30. Let $f: R \rightarrow R$ be a function defined by $f(x) = \frac{x-m}{x-n'}$, where $m \neq n$. Then,
 a. f is one-one and onto b. f is one-one and not onto
 c. f is many-one and onto d. f is many-one and into
31. If $f: [1, \infty) \rightarrow [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$, then $f^{-1}(x)$ equals
 a. $\frac{x + \sqrt{x^2 - 4}}{2}$ b. $\frac{x}{1 + x^2}$ c. $\frac{x - \sqrt{x^2 - 4}}{2}$ d. $1 + \sqrt{x^2 - 4}$
32. The mapping $f: N \rightarrow N$ given by $f(n) = 1 + n^2, n \in N$ where N is the set of natural numbers is
 a. one-one and onto b. onto but not one-one
 c. one-one but not onto d. Neither one-one nor onto
33. If $f: R \rightarrow R, f(x) = x^2$, then f is
 a. injective but not surjective
 b. surjective but not injective
 c. bijective
 d. None of the above
34. Let $g(x) = x^2 - 4x - 5$, then
 a. g is one-one on R
 b. g is one-one on $(-\infty, 2)$
 c. g is not one-one on $(-\infty, 4)$
 d. None of the above
35. The number of surjections from $A = \{1, 2, \dots, n\}, n \geq 2$, onto $B = \{a, b\}$, is
 a. nP_2 b. $2^n - 2$
 c. $2^n - 1$ d. None of these
36. If $f(x) = \log \left(\frac{1+x}{1-x} \right)$ and $g(x) = \frac{3x + x^3}{1 + 3x^2}$, then $f \circ g(x)$ equals
 a. $-f(x)$ b. $3f(x)$
 c. $[f(x)]^3$ d. None of these

37. Let $f(x) = (x+1)^2 - 1$, ($x \geq -1$). Then, the set $S = \{x : f(x) = f^{-1}(x)\}$ is
 a. $\left\{0, -1, \frac{-3 + i\sqrt{3}}{2}, \frac{-3 - i\sqrt{3}}{2}\right\}$
 b. $\{0, 1, -1\}$
 c. $\{0, -1\}$
 d. empty
38. If a real valued function $f(x)$ satisfies the functional equation $f(x-y) = f(x)f(y) - f(a-x)f(a+y)$, where a is a given constant and $f(0) = 1$, then $f(2a-x)$ is equal to
 a. $f(-x)$
 b. $f(a) + f(a-x)$
 c. $f(x)$
 d. $-f(x)$
39. If the function $f : [1, \infty) \rightarrow [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is
 a. $\left(\frac{1}{2}\right)^{x(x-1)}$
 b. $\frac{1}{2}(1 + \sqrt{1 + 4 \log_2 x})$
 c. $\frac{1}{2}(1 - \sqrt{1 - 4 \log_2 x})$
 d. Not defined
40. If $f : R \rightarrow (-\infty, 1)$, such that $f(x) = 1 - 2^{-x}$, then $f^{-1}(x)$ is
 a. $1 + \log_2(-x)$
 b. $1 - \log_2(-x)$
 c. $\log_2(1-x)$
 d. $-\log_2(1-x)$
41. Let a binary operation $*$ on Q (set of all rational numbers) be defined by $a * b = a + 2b$ for all $a, b \in Q$. Then,
 a. Q is closed under the given operation
 b. the given operation is commutative
 c. the given operation is associative
 d. Q is not closed under the given operation
42. Let Z be the set of integers and $*$ be a binary operation of Z defined as $a * b = a + b - ab$ for all $a, b \in Z$. The inverse of an element $a(a \neq 1) \in Z$ is
 a. $\frac{a}{a-1}$
 b. $\frac{a}{1-a}$
 c. $\frac{a-1}{a}$
 d. None of the above

BITSAT Archives

1. If $f(x)$ is an odd periodic function with period 2, then $f(4)$ equals [2014]
 a. -4 b. 4 c. 2 d. 0
2. Let $R = \{(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$ be a relation on the set $A = \{3, 6, 9, 12\}$. Then, the relation is [2014]
 a. an equivalence relation
 b. reflexive and symmetric
 c. reflexive and transitive
 d. only reflexive
3. The total number of subsets of a finite set A has 56 more elements than the total number of subsets of another finite set B . What is the number of elements in the set A ? [2014]
 a. 5 b. 6 c. 7 d. 8
4. Let R be the relation on the set R of all real numbers, defined by aRb iff $|a - b| \leq 1$. Then, R is [2013]
 a. reflexive and symmetric only
 b. reflexive and transitive only
 c. equivalence
 d. None of the above
5. If $A = \{x \in C : x^4 - 1 = 0\}$
 $B = \{x \in C : x^2 - 1 = 0\}$
 $C = \{x \in C : x^2 + 1 = 0\}$
 where, C is complex plane. Then, [2013]
 a. $A = B \cup C$ b. $C = A \cap B$
 c. $B = A \cap C$ d. $A = B \cap C$
6. Function $f : (-\infty, -1] \rightarrow (0, e^5]$ defined by $f(x) = e^{x^3 - 3x + 2}$ is [2011]
 a. many-one and onto b. many-one and into
 c. one-one and onto d. one-one and into
7. If the domain of the function $f(x) = x^2 - 6x + 7$ is $(-\infty, \infty)$, then the range of function is [2011, 09]
 a. $(-\infty, \infty)$ b. $[-2, \infty)$ c. $(-2, 3)$ d. $(-\infty, -2)$
8. Let $f : R \rightarrow R$, $g : R \rightarrow R$ be two functions given by $f(x) = 2x - 3$, $g(x) = x^3 + 5$. Then, $(fog)^{-1}$ is equal to [2010]
 a. $\left(\frac{x+7}{2}\right)^{1/3}$ b. $\left(x - \frac{7}{2}\right)^{1/3}$
 c. $\left(\frac{x-2}{7}\right)^{1/3}$ d. $\left(\frac{x-7}{2}\right)^{1/3}$
9. Inverse of function $f(x) = \frac{10^x - 10^{-x}}{10^x + 10^{-x}}$ is [2010]
 a. $\log_{10}(2-x)$ b. $\frac{1}{2} \log_{10} \left(\frac{1+x}{1-x} \right)$
 c. $\frac{1}{2} \log_{10}(2x-1)$ d. $\frac{1}{4} \log_{10} \left(\frac{2x}{2-x} \right)$
10. If $n(U) = 700$, $n(A) = 200$, $n(B) = 300$, $n(A \cap B) = 100$, then $n(A' \cap B')$ is equal to [2009]
 a. 300 b. 350
 c. 400 d. 500

11. Let f be a function with domain $[-3, 5]$ and let $g(x) = |3x + 4|$. Then, the domain of $(fog)(x)$ is [2009]

a. $\left(-3, \frac{1}{3}\right)$ b. $\left[-3, \frac{1}{3}\right)$ c. $\left[-3, \frac{1}{3}\right]$ d. $\left[-3, -\frac{1}{3}\right]$

12. If $f: R \rightarrow R$ and $g: R \rightarrow R$ are defined by $f(x) = |x|$ and $g(x) = (x - 3)$ for $x \in R$, then $\left\{g(f(x)) : -\frac{8}{5} < x < \frac{8}{5}\right\}$ is equal to [2008]

a. $\{0, 1\}$ b. $\{1, 2\}$ c. $\{-3, -2\}$ d. $\{2, 3\}$

13. Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is [2007]

a. a function b. transitive
c. not symmetric d. reflexive

14. Let $A = [-1, 1]$ and $f: A \rightarrow A$ be defined as $f(x) = x|x|$ for all $x \in A$, then $f(x)$ is [2007]

a. many-one and into function
b. one-one and into function
c. many-one and into function
d. one-one and onto function

15. If universal set, $U = \{x | x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$
 $A = \{x | x^2 - 5x + 6 = 0\}$

and $B = \{x | x^2 - 3x + 2 = 0\}$

Then, $(A \cap B)'$ is equal to [2006]

a. $\{1, 3\}$ b. $\{1, 2, 3\}$
c. $\{0, 1, 3\}$ d. $\{0, 1, 2, 3\}$

16. Which of the following statements is not correct for the relation R defined by aRb if and only if b lives within one kilometre from a ? [2006]

a. R is reflexive b. R is symmetric
c. R is not anti-symmetric d. None of these

17. Range of the function $f(x) = \frac{x^2}{x^2 + 1}$ is [2005]

a. $(-1, 0)$ b. $(-1, 1)$ c. $[0, 1)$ d. $(1, 1)$

18. $x^2 = xy$ is relation which is [2005]

a. symmetric b. reflexive
c. transitive d. None of these

19. If $f(x) = ax^2 + bx + c$ and $g(x) = px^2 + qx$ with $g(1) = f(1)$, $g(2) - f(2) = 1$, $g(3) - f(3) = 4$, then $g(4) - f(4)$ is [2005]

a. 0 b. 5
c. 6 d. None of these

Answer with Solutions

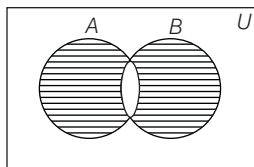
Practice Exercise

1. (b) Total number of subsets $= 2^n = 2^3 = 8$

These include the subset $\{1, 2, 3\}$ which is not the proper subset. Hence, the number of proper subsets $8 - 1 = 7$.

2. (c) We draw a Venn diagram of $(A - B) \cup (B - A)$.

The shaded region is $(A - B) \cup (B - A)$.



This shaded region can also be expressed as $(A \cup B) - (A \cap B)$.

Thus, $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

3. (d) Since, the prime factors of 240 are 2, 3, 5.

Therefore, $A = \{2, 3, 5\}$

and $B = \{5, 8, 7\}$

and $A \cup B = \{2, 3, 5, 7, 8\}$

It can be easily seen that $8 \in A \cup B$.

4. (b) Given that, $A = \{x : x \text{ is a multiple of 4 and } x \in N\}$
 $= \{4, 8, 12, 16, 20, \dots\}$

and $B = \{x : x \text{ is a multiple of 6 and } x \in N\}$
 $= \{6, 12, 18, 24, \dots\}$

$\therefore A \cap B = \{12, 24, \dots\}$
 $= \{x : x \text{ is a multiple of 12 and } x \in N\}$

5. (a) Consider, $(A \cup B)' \cup (A' \cap B) = (A' \cap B') \cup (A' \cap B)$
[using De-Morgan's law]
 $= A' \cap (B' \cup B) - A' \cap U = A'$

6. (a) We have, $8^n - 7n - 1 = (7 + 1)^n - 7n - 1$
 $= ({}^nC_2 7^2 + {}^nC_3 7^3 + \dots + {}^nC_n 7^n)$
 $= 49({}^nC_2 + {}^nC_3 + \dots + {}^nC_n 7^{n-2}), \text{ For } n \geq 2$

For $n = 1$, $8^n - 7n - 1 = 0$

Thus, $8^n - 7n - 1$ is a multiple of 49 for $n \geq 2$ and 0 for $n = 1$. Hence, X consists of all positive integral multiples of 49 of the form $49K_n$,

where $K_n = {}^nC_2 + {}^nC_3 + \dots + {}^nC_n 7^{n-2}$ together with zero.

Therefore, $X \subset Y$.

7. (b) We have, $N = 10000$

$n(A) = 40\% \text{ of } 10000 = 4000, n(B) = 2000$

$n(C) = 1000, n(A \cap B) = 500$

$n(B \cap C) = 300, n(C \cap A) = 400$

$n(A \cap B \cap C) = 200$

We have to find,

$$\begin{aligned} n(A \cap \overline{B} \cap \overline{C}) &= n\{A \cap (B \cap C)^c\} \\ &= n(A) - n[A \cap (B \cup C)] \\ &= n(A) - n[(A \cap B) \cup (A \cap C)] \\ &= n(A) - n\{(A \cap B) \cup (A \cap C) - n(A \cap B \cap C)\} \\ &= 4000 - (500 + 400 - 200) = 3300 \end{aligned}$$

8. (c) $X \cap (Y \cup X)^c = X \cap (Y^c \cap X^c)$
 $= (X \cap X^c) \cap Y^c$
 $= \phi \cap Y^c = \phi$

9. (a) If A is any set having n elements, then the elements in a power set of A are given by 2^n .

Now, here $32 = 2^5$, $128 = 2^7$ and $64 = 2^6$

But 26 cannot be written as some power of 2.

10. (a) Given that, $A = \{x : x^2 = 1\} = \{-1, 1\}$

and $B = \{x : x^4 = 1\} = \{-1, 1, i, -i\}$

Now, $A - B = \phi$ and $B - A = \{-i, i\}$

$\therefore A \Delta B = (A - B) \cup (B - A) = \{-i, i\}$

11. (c) Suppose A refers to the set of Americans who like cheese and B refers those who like apples. Let the population of Americans be 100. So, $n(A) = 63$, $n(B) = 76$

Now, we know that,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\Rightarrow n(A \cap B) = 139 - n(A \cup B)$$

$$\therefore n(A \cap B) \geq 39 \text{ because } n(A \cup B) \leq 100$$

Further, $A \cap B \subset A$ and $A \cap B \subset B$

$$n(A \cap B) \leq n(A)$$

and $n(A \cap B) \leq n(B)$

i.e. $n(A \cap B) \leq 63$

and $n(A \cap B) \geq 39$

So, $39 \leq n(A \cap B) \leq 63$

Hence, $39 \leq x \leq 63$

12. (b) Since, $2^m - 2^n = 56 = 8 \times 7 = 2^3 \times 7$

$$\Rightarrow 2^n(2^{m-n} - 1) = 2^3 \times 7$$

$$\therefore n = 3 \text{ and } 2^{m-n} - 1 = 8 = 2^3$$

$$\Rightarrow m - n = 3$$

$$\Rightarrow m - 3 = 3 \Rightarrow m = 6$$

$$\therefore m = 6 \text{ and } n = 3$$

13. (b) Given, $n(A) = 200$, $n(B) = 300$, $n(A \cap B) = 100$

We know that, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$\therefore n(A \cup B) = 200 + 300 - 100 = 400$$

Also, $n(A' \cap B') = n\{(A \cup B)^c\} = n(X) - n(A \cup B)$

$$\Rightarrow 300 = n(X) - 400$$

$$\Rightarrow n(X) = 700$$

14. (a) Given that, $A \cup X = B \cup X$

$$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$$

$$\Rightarrow (A \cap A) \cup (A \cap X) = (A \cap B) \cup (A \cap X)$$

[using distributive law]

$$\Rightarrow A \cup \phi = (A \cap B) \cup \phi \quad [\because A \cap X = \phi]$$

$$\Rightarrow A = A \cap B \quad \dots (i)$$

Again, consider $A \cup X = B \cup X$

$$\Rightarrow B \cap (A \cup X) = B \cap (B \cup X)$$

$$\Rightarrow (B \cap A) \cup (B \cap X) = (B \cap B) \cup (B \cap X)$$

$$\Rightarrow (B \cap A) \cup \phi = B \cup \phi \quad [\because B \cap X = \phi]$$

$$\Rightarrow A \cap B = B \quad \dots (ii)$$

Thus, from Eqs. (i) and (ii), we get

$$A = B$$

15. (a) We have, $f(x) = \cos(\log_e x)$,

$$\therefore f(x)f(y) = \frac{1}{2}[f(x/y) + f(xy)]$$

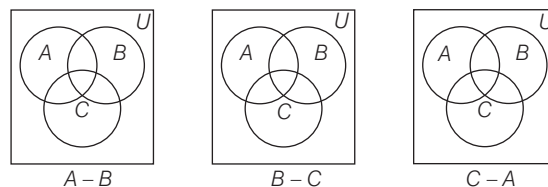
$$= \cos(\log x) \cos(\log y) = \frac{1}{2}[\cos \log(x/y) + \cos(\log xy)]$$

$$= \cos(\log x) \cos(\log y) = \frac{1}{2}[\cos(\log x - \log y)$$

$$+ \cos(\log x + \log y)]$$

$$= \cos(\log x) \cos(\log y) = \frac{1}{2}[2 \cos(\log x) \cos(\log y)] = 0$$

16. (c)



From the figure, it is clear that

$$\{(A - B) \cup (B - C) \cup (C - A)\}' = A \cap B \cap C$$

17. (b) Here, $A = \{1, 2, 3\}$ and $B = \{3, 6, 8\}$

Now, we first find $A \cap B$.

$$\therefore A \cap B = \{1, 2, 3\} \cap \{3, 6, 8\} = \{3\}$$

$$\therefore (A \cap B) \times A = \{3\} \times \{1, 2, 3\} = \{(3, 1), (3, 2), (3, 3)\}$$

18. (a) Here, R is defined by

$$R = \{(1, 3), (2, 4), (5, 1)\}$$

So, the inverse of R , i.e. R^{-1} is given by

$$R^{-1} = \{(3, 1), (4, 2), (1, 5)\}$$

19. (d) Consider any mapping $f: A \rightarrow B$. The image of any given element in A can be anyone of the n elements in B . Therefore, the m elements in A can be assigned images in $n \times n \times n \times \dots \times n = n^m$ ways. This means that these are exactly n^m distinct mappings from A to B .

20. (b) Given, the function is $f(x) = 2x + 3$

For injective function, $f(x) = f(y)$

$$\Rightarrow a = b \quad [\text{Here, } a, b \in N]$$

$$f(x) = f(y)$$

$$2x + 3 = 2y + 3$$

$$\Rightarrow 2x = 2y \Rightarrow x = y$$

For surjective function, each $y \in B$ and $a \in A$,

such that $f(a) = b$

$$f(x) = 2x + 3$$

$$y = 2x + 3$$

$$x = \frac{y - 3}{2}$$

For $y = 0$, $x = -\frac{3}{2}$ and $-\frac{3}{2} \notin M \Rightarrow f(x)$ is not surjective.

21. (a) Since, $(1, 2) \in S$ but $(2, 1) \notin S$

$\therefore S$ is not symmetric.

Hence, S is not an equivalence relation.

Given, $T = \{(x, y) : (x - y) \in I\}$

Now, $x - x = 0 \in I$, T is reflexive.

Again now, $(x - y) \in I$

$\Rightarrow y - x \in I$, T is symmetric.

Let $x - y = l_1$ and $y - z = l_2$

Now, $x - z = (x - y) + (y - z) = l_1 + l_2 \in I$

$\therefore T$ is transitive.

Hence, T is an equivalence relation.

22. (a) aR_b , if $|a - b| < 4$

(i) aRa because $|a - a| = 0 < 4$

Hence, R is reflexive.

(ii) $aR_b \rightarrow bRa$ because

$$|a - b| < 4 \rightarrow |b - a| < 4$$

Hence, R is symmetric.

(iii) Let $|a - b| < 4$ and $|b - c| < 4$

i.e. aR_b, bR_c

Now, $|a - c| = |(a - b) + (b - c)| \leq |a - b| + |b - c|$

$$\Rightarrow |a - c| \leq 4 + 4 \Rightarrow |a - c| \leq 8$$

$\therefore aR_c$

Hence, R is not transitive.

So, the given relation is reflexive and symmetric.

23. (d) The ordered pairs of R are obtained from the relation $2x + y = 10$

x	3	4	5
y	4	2	0

Thus,

$$R = \{(3, 4), (4, 2), (5, 0)\}$$

$$R^{-1} = \{(4, 3), (2, 4), (0, 5)\}$$

24. (c) R is reflexive, if it contains $(1, 1), (2, 2), (3, 3)$

$\therefore (1, 2) \in R, (2, 3) \in R$

$\therefore R$ is symmetric, if $(2, 1), (3, 2) \in R$

Now, $R = \{(1, 1), (2, 2), (3, 3), (2, 1), (3, 2), (2, 3), (1, 2)\}$

R will be transitive, if $(3, 1), (1, 3) \in R$

Thus, R becomes an equivalence relation by adding $(1, 1)$

$(2, 2) (3, 3) (2, 1) (3, 2) (1, 3) (3, 1)$.

Hence, the total number of ordered pairs is 7.

25. (c) We first write the elements of the set R .

i.e. $R = \{(1, 4), (4, 1), (2, 3), (3, 2)\}$

Since, $(1, 1) \notin R \Rightarrow R$ is not reflexive.

Now, as $(1, 4) \in R \Rightarrow (4, 1) \in R$ and $(2, 3) \in R \Rightarrow (3, 2) \in R$

So, R is symmetric.

Now, $(1, 4) \in R$ and $(4, 1) \in R \nRightarrow (1, 1) \in R$

Thus, R is not transitive.

Hence, R is symmetric but neither reflexive nor transitive.

26. (c) Here, R is defined on the set of integers as

$$R = \{(a, b) : a^2 + b^2 = 25\}$$

Now, consider $a^2 + b^2 = 25$

$$\Rightarrow a = \pm \sqrt{25 - b^2} \quad \dots(i)$$

On substituting $b = 0$ in Eq. (i), we get

$$a = \pm \sqrt{25 - 0} = \pm \sqrt{25} = \pm 5$$

On putting $b = \pm 3$ in Eq. (i), we get

$$a = \pm \sqrt{25 - 9} = \pm \sqrt{16} = \pm 4$$

Also, on putting $b = \pm 4$ in Eq. (i), we get

$$a = \pm \sqrt{25 - 16} = \pm \sqrt{9} = \pm 3$$

Again, on putting $b = \pm 5$ in Eq. (i), we get

$$a = \pm \sqrt{25 - 25} = 0$$

Thus, domain of R is $\{0, \pm 3, \pm 4, \pm 5\}$.

27. (b) Given function is $f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

Here, $\log_{10}(1-x) \neq 0$

$$(1-x) \neq 0 \Rightarrow x \neq 0$$

Again, $(1-x) > 0$ [$\because$ log of negative is not possible]

$$1 > 0 + x \Rightarrow x < 1$$

$$\therefore 0 < x < 1 \quad \dots(i)$$

$$\text{Taking } \sqrt{x+2} \geq 0 \Rightarrow x \geq -2$$

$$\therefore -2 \leq x < 0 \quad \dots(ii)$$

From Eqs. (i) and (ii), we can say that

$$x \in [-2, 0) \cup (0, 1]$$

28. (c) We have, $f(x) = \frac{x-2}{2-x} = -\frac{x-2}{x-2} = -1$, if $x \neq 2$

$\therefore$ Range of $f(x) = \{-1\}$

29. (d) Hint $f\left(\frac{x_1+x_2}{1+x_1x_2}\right) = \log \left[\frac{1+\frac{x_1+x_2}{1+x_1x_2}}{1-\frac{x_1+x_2}{1+x_1x_2}} \right]$

$$= \log \left(\frac{1+x_1x_2+x_1+x_2}{1+x_1x_2-x_1-x_2} \right)$$

$$= \log \left[\left(\frac{1+x_1}{1-x_1} \right) \times \left(\frac{1+x_2}{1-x_2} \right) \right]$$

30. (b) For any $x, y \in R$, we have

$$f(x) = f(y) \Rightarrow \frac{x-m}{x-n} = \frac{y-m}{y-n} \Rightarrow x = y$$

So, f is one-one.

Let $\alpha \in R$, such that $f(x) = \alpha$

$$\frac{x-m}{x-n} = \alpha, x = \frac{m-n\alpha}{1-\alpha}$$

Clearly, $x \notin R$ for $\alpha = 1$, so f is not onto.

31. (a) Let $y = x + \frac{1}{x}$

$$\Rightarrow x^2 - xy + 1 = 0$$

$$\Rightarrow x = \frac{y \pm \sqrt{y^2 - 4}}{2}$$

$$\therefore x \in (2, \infty)$$

$$\therefore x = \frac{y + \sqrt{y^2 - 4}}{2}$$

$$\text{Hence, } f^{-1}(x) = \frac{x + \sqrt{x^2 - 4}}{2}$$

32. (c) Since,
- $f(n) = 1 + n^2$

For one-one, $1 + n_1^2 = 1 + n_2^2 \Rightarrow n_1^2 - n_2^2 = 0$

$$\Rightarrow n_1 = n_2 \quad [\because n + n_2 \neq 0]$$

$\therefore f(n)$ is one-one.

$\therefore f(n)$ is not onto.

Hence, $f(n)$ is one-one but not onto.

33. (d) Let
- $x_1, x_2 \in R$
- such that

$$f(x_1) = f(x_2)$$

$$\Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$$

Hence, f is not injective.

Let $y \in R$ be any real number, such that

$$f(x) = y$$

$$\Rightarrow y = x^2 \Rightarrow x = \sqrt{y}$$

$\therefore$ Range of $f = [0, \infty)$,

Codomain of $f = R$

$\Rightarrow$ Codomain $\neq$ Range

Hence, f is not surjective.

34. (b) Let
- $g(x_1) = g(x_2)$

$$\Rightarrow x_1^2 - 4x_1 - 5 = x_2^2 - 4x_2 - 5$$

$$\Rightarrow x_1^2 - x_2^2 = 4(x_1 - x_2)$$

$$\Rightarrow (x_1 - x_2)(x_1 + x_2 - 4) = 0$$

Either $x_1 = x_2$ or $x_1 + x_2 = 4$

Either $x_1 = x_2$ or $x_1 = 4 - x_2$

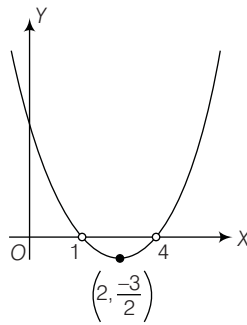
$\therefore$ There are two values of x_1 , for which $g(x_1) = g(x_2)$

$\therefore g(x)$ is not one-one, $\forall x \in R$

Graphically,

The function is one-one either on

$$x \in (-\infty, 2) \text{ or } x \in (2, \infty)$$



35. (b)
- $f: A \rightarrow B$
- , where
- A
- has
- n
- elements and
- B
- has 2 elements. Each element of
- A
- can have image 2 in the elements of
- B
- . Hence, there will be
- $2 \cdot 2 \cdot 2 \dots n$
- times
- $= 2^n$
- mappings.

But, if each element of A either goes to a only or goes to b only where, $a, b \in B$, then in these two cases, the mapping will not be surjective.

Hence, the number of surjective mappings are $2^n - 2$.

36. (b) Hint
- $g(x) = \frac{3x + x^3}{1 + 3x^2} = y$
- [say] ... (i)

$$\therefore f[g(x)] = f(y) = \log \left(\frac{1+y}{1-y} \right)$$

37. (c)
- $S = \{x : f(x) = f^{-1}(x)\}$

where, $f(x) = (x+1)^2 - 1, x \geq -1$

We know that, $f(x) = f^{-1}(x)$ can hold only at those point at which $f(x) = x$

$$\text{or } (x+1)^2 - 1 = x \Rightarrow (x+1)^2 - (x+1) = 0$$

$$\text{or } (x+1)(x+1-1) = 0 \Rightarrow x = -1, 0$$

$$S = \{-1, 0\}$$

38. (d) Given,
- $f(x-y) = f(x)f(y) - f(a-x)f(a+y)$

$$\text{Let } x = 0 = y \Rightarrow f(0) = (f(0))^2 - (f(a))^2$$

$$\Rightarrow 1 = 1 - (f(a))^2 \quad [\because f(0) = 1, \text{ given}]$$

$$\Rightarrow f(a) = 0$$

$$\begin{aligned} \therefore f(2a-x) &= f\{a-(x-a)\} \\ &= f(a)f(x-a) - f(a-a)f(x) \\ &= 0 - f(x) \cdot 1 = -f(x) \end{aligned}$$

39. (b) Let
- $y = 2^{x(x-1)}$
- , where
- $y \geq 1$
- as
- $x \geq 1$

Taking $\log_2$ on both sides, we get

$$\log_2 y = \log_2 2^{x(x-1)} \Rightarrow \log_2 y = x(x-1)$$

$$\therefore \log_a a^x = x \Rightarrow x^2 - x - \log_2 y = 0$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 + 4 \log_2 y}}{2}$$

$$\text{For } y \geq 1, \log_2 y \geq 0 \Rightarrow 4 \log_2 y \geq 0$$

$$\Rightarrow 1 + 4 \log_2 y \geq 1 \Rightarrow \sqrt{1 + 4 \log_2 y} \geq 1$$

$$\Rightarrow -1 \sqrt{1 + 4 \log_2 y} \leq -1 \Rightarrow 1 - \sqrt{1 + 4 \log_2 y} \leq 0$$

$$\text{But } x \geq 1$$

So, $x = 1 - \sqrt{1 + 4 \log_2 y}$, which is not possible.

$$\text{Therefore, } x = \frac{1}{2} (1 + \sqrt{1 + 4 \log_2 y})$$

$$\Rightarrow f^{-1}(y) = \frac{1}{2} (1 + \sqrt{1 + 4 \log_2 y}) \quad [\because x = f^{-1}(y)]$$

$$\Rightarrow f^{-1}(x) = \frac{1}{2} (1 + \sqrt{1 + 4 \log_2 x})$$

40. (d) Let
- $y = 1 - 2^{-x}, 2^{-x} = 1 - y$

$$\Rightarrow -x = \log_2 (1 - y)$$

$$\Rightarrow x = -\log_2 (1 - y)$$

$$\therefore f^{-1}(x) = -\log_2 (1 - x)$$

41. (a)
- $\left. \begin{aligned} a * b &= a + 2b \\ b * a &= b + 2a \end{aligned} \right\} \Rightarrow a * b \neq b * a$

$$(a * b) * c = (a + 2b) * c = a + 2b + 2c$$

$$\text{Again, } a * (b * c) = a * (b + 2c) = a + 2(b + 2c)$$

$$\therefore a * (b * c) \neq (a * b) * c$$

$\therefore$ Associative law does not hold.

$$a, b \in Q, a + 2b \in Q$$

$\therefore Q$ is closed.

42. (a) Let
- e
- be the identity element for the binary operation
- $*$
- defined on
- z
- given by

$$a * b = a + b - ab$$

$$\text{Then, } a * e = a = e * a, \forall a \in z$$

$$\Rightarrow a + e - ae = a, \forall a \in z$$

$$\Rightarrow e(1 - a) = 0, \forall a \in z \Rightarrow e = 0$$

So, 0 is the identity element for the binary operation $*$ on z .

Let x be the inverse of $a \in z$, then

$$a * x = x * a = 0$$

$$\Rightarrow a + x - ax = 0$$

$$\Rightarrow x(1 - a) = -a \Rightarrow x = \frac{a}{a - 1}$$

BITSAT Archives

1. (d) Since, $f(x)$ is an odd periodic function with period 2.

$$\therefore f(-x) = -f(x)$$

$$\text{and } f(x+2) = f(x)$$

$$\therefore f(2) = f(0+2) = f(0)$$

$$\text{and } f(-2) = f(-2+2) = f(0)$$

$$\text{Now, } f(0) = f(-2) = -f(2) = -f(0)$$

$$\Rightarrow 2f(0) = 0$$

$$\text{i.e. } f(0) = 0$$

$$\therefore f(4) = f(2+2) = f(2) = f(0) = 0$$

$$\text{Thus, } f(4) = 0$$

2. (c) R is reflexive as $(3, 3), (6, 6), (9, 9), (12, 12) \in R$.

R is not symmetric as $(6, 12) \in R$ but $(12, 6) \notin R$.

R is transitive as the only pair which needs verification is $(3, 6)$ and $(6, 12) \in R$.

$$\Rightarrow (3, 12) \in R$$

3. (b) Let sets A and B have m and n elements, respectively.

$$\text{Then, } 2^m - 2^n = 56$$

$$\Rightarrow 2^n(2^{m-n} - 1) = 56 = 8 \times 7 = 2^3 \times 7$$

Comparing both sides, we get

$$2^n = 2^3 \text{ and } 2^{m-n} - 1 = 7$$

$$\Rightarrow n = 3 \text{ and } 2^{m-n} = 8$$

$$\Rightarrow 2^{m-n} = 2^3 \Rightarrow m - n = 3$$

$$\Rightarrow m - 3 = 3 \Rightarrow m = 6$$

So, number of elements in A is 6.

4. (a) Since, $|a - a| = 0 < 1$, so $aRa, \forall a \in R$

$\therefore R$ is reflexive.

$$\text{Now, } aRb \Rightarrow |a - b| \leq 1 \Rightarrow |b - a| \leq 1 \Rightarrow bRa$$

$\therefore R$ is symmetric.

But R is not transitive as

$$1R2, 2R3 \text{ but } 1 \not R 3 \quad [\because |1 - 3| = 2 > 1]$$

5. (a) $A = \{1, -1, i - i\}$

$$B = \{1, -1\}$$

$$C = \{i, i\}$$

$$\text{Now, } B \cup C = \{1, -1, i, -i\} = A$$

6. (d) We have, $f(x) = e^{x^3 - 3x + 2}$

$$\text{Let } h(x) = x^3 - 3x + 2$$

$$\therefore h'(x) = 3x^2 - 3 = 3(x^2 - 1)$$

$$\Rightarrow h'(x) \geq 0 \text{ for } x \in (-\infty, -1]$$

$\therefore f(x)$ is an increasing function.

$\therefore f(x)$ is one-one.

$$\text{Now, range of } f(x) = (0, e^4]$$

$$\text{But codomain of } f(x) = (0, e^5]$$

$\therefore f(x)$ is an into function.

7. (b) Now, $x^2 - 6x + 7 = (x - 3)^2 - 2$

It is obvious that, minimum value is -2 and maximum value is ∞ .

8. (d) Since, $f: R \rightarrow R$ and $g: R \rightarrow R$, given by $f(x) = 2x - 3$ and $g(x) = x^3 + 5$, respectively are bijections, therefore f^{-1} and g^{-1} exist.

$$\text{We have, } f(x) = 2x - 3$$

$$\therefore f(x) = y$$

$$\Rightarrow 2x - 3 = y$$

$$\Rightarrow x = \frac{y+3}{2}$$

$$\Rightarrow f^{-1}(y) = \frac{y+3}{2}$$

Thus, f^{-1} is given by

$$f^{-1}(x) = \frac{x+3}{2}, \forall x \in R$$

$$\text{Similarly, } g^{-1}(x) = (x-5)^{1/3}, \forall x \in R$$

$$\text{Now, } (fog)^{-1}(x) = (g^{-1} \circ f^{-1})(x) = g^{-1}(f^{-1}(x))$$

$$= g^{-1}\left(\frac{x+3}{2}\right)$$

$$= \left(\frac{x+3}{2} - 5\right)^{1/3} = \left(\frac{x-7}{2}\right)^{1/3}$$

9. (b) Let $f(x) = y$, then $\frac{10^x - 10^{-x}}{10^x + 10^{-x}} = y$

$$\Rightarrow \frac{10^{2x} - 1}{10^{2x} + 1} = y$$

$$\Rightarrow 10^{2x} = \frac{1+y}{1-y}$$

$$\Rightarrow x = \frac{1}{2} \log_{10} \left(\frac{1+y}{1-y} \right)$$

$$\Rightarrow f^{-1}(y) = \frac{1}{2} \log_{10} \left(\frac{1+y}{1-y} \right)$$

$$\therefore f^{-1}(x) = \frac{1}{2} \log_{10} \left(\frac{1+x}{1-x} \right)$$

10. (a) $n(A' \cap B') = n(A \cup B)'$

$$= n(U) - n(A \cup B)$$

$$= n(U) - n(A) - n(B) + n(A \cap B)$$

$$= 700 - 200 - 300 + 100$$

$$= 300$$

11. (c) $(fog)(x) = f[g(x)] = f(|3x+4|)$

Since, the domain of f is $[-3, 5]$.

$$\therefore -3 \leq |3x+4| \leq 5$$

$$\Rightarrow |3x+4| \leq 5$$

$$\Rightarrow -5 \leq 3x+4 \leq 5$$

$$\Rightarrow -9 \leq 3x \leq 1$$

$$\Rightarrow -3 \leq x \leq \frac{1}{3}$$

$$\therefore \text{Domain of } fog \text{ is } \left[-3, \frac{1}{3}\right]$$

12. (c) Given that, $f(x) = |x|$

and $g(x) = (x-3)$

For $-\frac{8}{5} < x < \frac{8}{5}$, $0 \leq f(x) < \frac{8}{5}$

Now, for $0 \leq f(x) < 1$,

$$g(f(x)) = [f(x) - 3] \\ = -3 \quad [\because -3 \leq f(x) - 3 < -2]$$

Again, for $1 \leq f(x) < 1.6$ $g(f(x)) = -2$
 $[\because -2 \leq f(x) - 3 < -1.4]$

Hence, required set is $\{-3, -2\}$.

13. (c) Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$, then

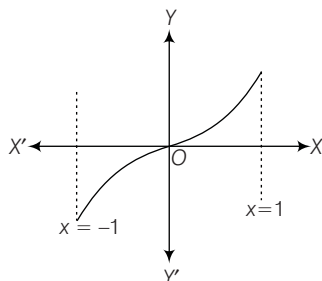
(a) Since, $(2, 4) \in R$ and $(2, 3) \in R$, so R is not a function.

(b) Since, $(1, 3) \in R$ and $(3, 1) \in R$ but $(1, 1) \notin R$, so R is not transitive.

(c) Since, $(2, 3) \in R$ but $(3, 2) \notin R$, so R is not symmetric.

(d) Since, $(1, 1) \notin R$, so R is not reflexive.

14. (d) $f(x) = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$



Since, $-1 \leq x \leq 1$, therefore $-1 \leq f(x) \leq 1$

$\therefore$ Function is one-one and onto.

15. (c) $U = \{x : x^5 - 6x^4 + 11x^3 - 6x^2 = 0\} = \{0, 1, 2, 3\}$

$$A = \{x : x^2 - 5x + 6 = 0\} = \{2, 3\}$$

and $B = \{x : x^2 - 3x + 2 = 0\} = \{2, 1\}$

$$A \cap B = \{2\}$$

$$\therefore (A \cap B)^c = U - (A \cap B) = \{0, 1, 2, 3\} - \{2\} = \{0, 1, 3\}$$

16. (c) R is not anti-symmetric.

17. (c) Given that, $f(x) = \frac{x^2}{x^2 + 1}$

Since, it is an even function, therefore its value is always greater than or equal to 0 and we know that,

$$x^2 < x^2 + 1 \text{ or } \frac{x^2}{x^2 + 1} < 1$$

$\therefore$ Required range is $[0, 1)$.

18. (c) Given that, $x^2 = xy$

Let $x, y \in R$

$$xRy = x^2 = xy \text{ and } yRz = y^2 = yz$$

Now, $x^2y^2 = xy^2z \Rightarrow x^2 = xz \Rightarrow xRz$

$\therefore$ It is transitive.

19. (b) $\because f(x) = ax^2 + bx + c$ and $g(x) = px^2 + qx$

Since, $g(1) = f(1) \Rightarrow p + q = a + b + c \quad \dots(i)$

and $g(2) - f(2) = 1 \Rightarrow 4p + 2q - 4a - 2b - c = 1 \quad \dots(ii)$

Also, $g(3) - f(3) = 4$

$$\Rightarrow 9p + 3q - 9a - 3b - c = 4 \quad \dots(iii)$$

From Eqs. (i) and (ii), we get

$$2p = 2a - c + 1$$

Now, $g(4) - f(4) = 16p + 4q - 16a - 4b - c$

$$= 12p + 4(p + q) - 16a - 4b - c = 6 - 3c$$

10

Linear Inequality

Inequality

Two real numbers or two algebraic expressions related by the symbols $>$, $<$, $\leq$ or $\geq$ form an inequality.

Linear Inequality

An inequality is said to be linear, if each variable occurs in first degree only and there is no term involving the product of the variables. e.g. $ax + b \leq 0$, $ax + by + c$, $ax \leq 4$.

(i) Linear Inequality in One Variable

A linear inequality which has only one variable, is called linear inequality in one variable.

e.g. $ax + b < 0$, where $a \neq 0$, $4x + 7 \geq 0$

(ii) Linear Inequality in Two Variables

A linear inequality which have only two variables, is called linear inequality in two variables.

e.g. $3x + 11y \leq 0$, $4x + 3y > 0$

Important Results on Inequalities

1. If $a > b$ and $b > c$, then $a > c$. Generally, if $a_1 > a_2$, $a_2 > a_3$, ..., $a_{n-1} > a_n$, then $a_1 > a_n$.

2. If $a > b$, then $a \pm c > b \pm c$, $\forall c \in R$.

3. If $a > b$, then (i) for $m > 0$, $am > bm$, $\frac{a}{m} > \frac{b}{m}$ (ii) for $m < 0$, $am < bm$, $\frac{a}{m} < \frac{b}{m}$

4. (i) If $a > b > 0$, then

(a) $a^2 > b^2$

(b) $|a| > |b|$

(c) $\frac{1}{a} < \frac{1}{b}$

(ii) If $a < b < 0$, then

(a) $a^2 > b^2$

(b) $|a| > |b|$

(c) $\frac{1}{a} > \frac{1}{b}$

5. If $a < 0 < b$, then

(i) $a^2 > b^2$, if $|a| > |b|$ (ii) $a^2 < b^2$, if $|a| < |b|$

6. If $a < x < b$ and a, b are positive real numbers, then $a^2 < x^2 < b^2$.
7. If $a < x < b$, where a is negative number and b is a positive number, then
 (i) $0 \leq x^2 < b^2$, if $|b| > |a|$
 (ii) $0 \leq x^2 \leq a^2$, if $|a| > |b|$
8. If $\frac{a}{b} > 0$, then (i) $a > 0$, if $b > 0$ (ii) $a < 0$, if $b < 0$
9. If $a_i > b_i > 0$, where $i = 1, 2, 3, \dots, n$, then
 $a_1 a_2 a_3 \dots a_n > b_1 b_2 b_3 \dots b_n$
10. If $|x| < a$, then
 (i) a is positive, when $-a < x < a$.
 (ii) a is negative, when $x \in \phi$.
11. If $a_i > b_i$, where $i = 1, 2, 3, \dots, n$, then
 $a_1 + a_2 + a_3 + \dots + a_n > b_1 + b_2 + \dots + b_n$
12. If $0 < a < 1$ and n is a positive rational number, then
 (i) $0 < a^n < 1$
 (ii) $a^{-n} > 1$

Arithmetico-Geometric Mean Inequality

If $a, b > 0$ and $a \neq b$, then

$$\frac{a+b}{2} \geq \sqrt{ab}$$

i.e. arithmetic mean $\geq$ geometric mean

Practice Exercise

1. If x satisfies the inequations $2x - 7 < 11$ and $3x + 4 < -5$, then x lies in the interval
 a. $(-\infty, 3)$ b. $(-\infty, 2)$ c. $(-\infty, -3)$ d. $(-\infty, \infty)$
2. If $-3 \leq 4 - \frac{7x}{2} \leq 18$, then
 a. $x \in (-\infty, -4]$ b. $x \in [-4, \infty)$
 c. $x \in [-4, 2]$ d. $x \in [2, \infty)$
3. The solution set of the inequalities $6 \leq -3(2x - 4) < 12$ is
 a. $(-\infty, 1]$ b. $(0, 1]$
 c. $(0, 1] \cup [1, \infty)$ d. $[1, \infty)$
4. The solution set of the inequalities $3x - 7 > 2(x - 6)$ and $6 - x > 11 - 2x$, is
 a. $(-5, \infty)$ b. $[5, \infty)$
 c. $(5, \infty)$ d. $[-5, \infty)$
5. The set of values of x satisfying $2 \leq |x - 3| < 4$, is
 a. $(-1, 1) \cup [5, 7)$
 b. $-4 \leq x \leq 2$
 c. $-1 < x < 7$ or $x \geq 5$
 d. $x < 7$ or $x \geq 5$
6. The number of pairs of consecutive even positive integers, both of which are larger than 8, such that their sum is less than 25, is
 a. 1 b. 2
 c. 3 d. 4
7. IQ of a person is given by the formula

$$IQ = \frac{MA}{CA} \times 100$$
- where, MA is mental age and CA is chronological age. If $80 \leq IQ \leq 140$ for a group of 12 yr children, then the range of their mental age is
 a. $9.8 \leq MA \leq 16.8$ b. $10 \leq MA \leq 16$
 c. $9.6 \leq MA \leq 16.8$ d. $9.6 \leq MA \leq 16.6$
8. A solution of 8% boric is to be diluted by adding 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 L of the 8% solution of the 2% solution will have to be added, is
 a. more than 320 and less than 1000
 b. more than 160 and less than 320
 c. more than 320 and less than 1280
 d. more than 320 and less than 640
9. A solution is to be kept between 68°F and 77°F . The range in temperature in degree Celsius ($^\circ\text{C}$), if the Celsius/Fahrenheit (F) conversion formula is given by $F = \frac{9}{5}C + 32$, is
 a. between 20°C and 22°C
 b. between 20°C and 25°C
 c. between 36°C and 45°C
 d. between 4°C and 5°C
10. If $3 \leq 3t - 18 \leq 18$, then which one of the following is true?
 a. $15 \leq 2t + 1 \leq 20$ b. $8 \leq t < 12$
 c. $8 \leq t + 1 \leq 13$ d. $21 \leq 3t \leq 24$
11. The solution set contained in R of the inequation $3^x + 3^{1-x} - 4 < 0$, is
 a. $(1, 3)$ b. $(0, 1)$
 c. $(1, 2)$ d. $(0, 2)$

12. The minimum value of $f(x) = |3 - x| + 7$ is
 a. 0 b. 6 c. 7 d. 8
13. If $x^2 + 4ax + 2 > 0$ for all values of x , then a lies in the interval
 a. $(-2, 4)$ b. $(1, 2)$
 c. $(-\sqrt{2}, \sqrt{2})$ d. $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
14. The solution set of the inequation $\frac{x+11}{x-3} > 0$ is
 a. $(-\infty, -11) \cup (3, \infty)$ b. $(-\infty, -10) \cup (2, \infty)$
 c. $(-100, -11) \cup (1, \infty)$ d. $(-5, 0) \cup (3, 7)$
15. For $\frac{|x-1|}{x+2} < 1$, x lies in the interval
 a. $(-\infty, -2) \cup \left(-\frac{1}{2}, \infty\right)$ b. $(-\infty, 1) \cup [2, 3]$
 c. $(-\infty, -4)$ d. $\left[-\frac{1}{2}, 1\right]$
16. If a, b and $c > 0$, then the minimum value of $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$ is
 a. 1 b. $3/2$ c. 2 d. $5/2$
17. If $a, b, c > 0$ and $abc = 1$, then the value of $a + b + c + ab + bc + ca$ lies in the interval
 a. $(\infty, -6]$ b. $(-6, 0)$ c. $(0, 6)$ d. $(6, \infty)$
18. The number of positive integers satisfying the inequality ${}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \leq 50$, is
 a. 9 b. 8 c. 7 d. 6
19. The set of admissible values of x such that $\frac{2x+3}{2x-9} < 0$, is
 a. $\left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{9}{2}, \infty\right)$ b. $(-\infty, 0) \cup \left(\frac{9}{2}, \infty\right)$
 c. $\left(-\frac{3}{2}, 0\right)$ d. $\left(-\frac{3}{2}, \frac{9}{2}\right)$
20. The set of values of x for which the inequalities $x^2 - 3x - 10 < 0$ and $10x - x^2 - 16 > 0$ hold simultaneously, is
 a. $(-2, 5)$ b. $(2, 8)$ c. $(-2, 8)$ d. $(2, 5)$
21. The set of values of x satisfying $2 \leq |x-3| < 4$, is
 a. $(-1, 1] \cup [5, 7)$ b. $-4 \leq x \leq 2$
 c. $-1 < x < 7$ or $x \geq 5$ d. $x < 7$ or $x \geq 5$
22. Let $x = \left[\frac{a+2b}{a+b}\right]$ and $y = \frac{a}{b}$, where a and b are positive integers. If $y^2 > 2$, then
 a. $x^2 \leq 2$ b. $x^2 < 2$ c. $x^2 > 2$ d. $x^2 \geq 2$
23. If $\frac{2x}{2x^2+5x+2} > \frac{1}{x+1}$, then
 a. $-2 > x > -1$ b. $-2 \geq x \geq -1$
 c. $-2 < x < -1$ d. $-2 < x \leq -1$
24. If $(x-1)(x^2-5x+7) < (x-1)$, then x belongs to
 a. $(1, 2) \cup (3, \infty)$ b. $(-\infty, 1) \cup (2, 3)$
 c. $(2, 3)$ d. None of these
25. If r is a real number such that $|r| < 1$ and $a = 5(1-r)$, then
 a. $0 < a < 5$ b. $-5 < a < 5$
 c. $0 < a < 10$ d. $0 \leq a < 10$
26. The number of integral solutions of $\frac{x+2}{x^2+1} > \frac{1}{2}$ is
 a. 0 b. 1 c. 2 d. 3
27. The set of all x satisfying the inequality $\frac{4x-1}{3x+1} \geq 1$, is
 a. $\left(-\infty, -\frac{1}{3}\right) \cup \left[\frac{1}{4}, \infty\right)$
 b. $\left(-\infty, -\frac{2}{3}\right) \cup \left[\frac{5}{4}, \infty\right)$
 c. $\left(-\infty, -\frac{1}{3}\right) \cup [2, \infty)$
 d. $\left(-\infty, -\frac{2}{3}\right] \cup [4, \infty)$

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1. If p, q, r and s are positive real numbers such that $p + q + r + s = 2$, then $M = (p + q)(r + s)$ satisfies the relation [2014]
 a. $0 < M \leq 1$
 b. $1 \leq M \leq 2$
 c. $2 \leq M \leq 3$
 d. $3 \leq M \leq 4$
2. The set of all real x satisfying the inequality $\frac{3-|x|}{4-|x|} \geq 0$ is [2013]
 a. $[-3, 3] \cup (-\infty, -4) \cup (4, \infty)$
 b. $(-\infty, -4) \cup (4, \infty)$
 c. $(-\infty, -3) \cup (4, \infty)$
 d. $(-\infty, -3) \cup (3, \infty)$

Answer with Solutions

Practice Exercise

1. (c) We have, $2x - 7 < 11$ and $3x + 4 < -5$

Now, $2x - 7 < 11$

Adding 7 on both sides, we get

$$2x < 11 + 7 \Rightarrow 2x < 18$$

Dividing by 2 on both sides, we get

$$x < 9 \quad \dots(i)$$

and $3x + 4 < -5$

Adding (-4) on both sides, we get

$$3x < -5 - 4 \Rightarrow 3x < -9$$

Dividing by 3 on both sides, we get

$$x < -3 \quad \dots(ii)$$

$\therefore$ Solution set is $(-\infty, -3)$.

2. (c) The given inequality is $-3 \leq 4 - \frac{7x}{2} \leq 18$

Adding (-4) to each term, we get

$$-3 - 4 \leq 4 - \frac{7x}{2} - 4 \leq 18 - 4 \Rightarrow -7 \leq -\frac{7x}{2} \leq 14$$

Multiplying by $\left(\frac{-2}{7}\right)$ to each term, we get

$$-7 \times \left(-\frac{2}{7}\right) \geq -\frac{7}{2}x \times \left(-\frac{2}{7}\right) \geq 14 \times \left(-\frac{2}{7}\right)$$

$$\Rightarrow 2 \geq x \geq -4 \Rightarrow -4 \leq x \leq 2$$

$\therefore$ Solution set is $[-4, 2]$.

3. (b) The given inequality is $6 \leq -3(2x - 4) < 12$

$$\Rightarrow 6 \leq -6x + 12 < 12$$

Adding (-12) to each term, we get

$$6 - 12 \leq -6x + 12 - 12 < 12 - 12 \Rightarrow -6 \leq -6x < 0$$

Dividing by (-6) to each term, we get

$$\frac{-6}{-6} \geq \frac{-6x}{-6} > \frac{0}{-6} \Rightarrow 1 \geq x > 0 \Rightarrow 0 < x \leq 1$$

$\therefore$ Solution set is $(0, 1]$.

4. (c) We have, $3x - 7 > 2(x - 6) \Rightarrow 3x - 7 > 2x - 12$

Transferring the term $2x$ to LHS and the term (-7) to RHS,

$$3x - 2x > -12 + 7 \Rightarrow x > -5 \quad \dots(i)$$

and $6 - x > 11 - 2x$

Transferring the term $(-2x)$ to LHS and the term 6 to RHS,

$$-x + 2x > 11 - 6$$

$$\Rightarrow x > 5 \quad \dots(ii)$$

$\therefore$ Solution set is $(5, \infty)$.

5. (a) We have, $2 \leq |x - 3| < 4$

Case I If $x < 3$, then

$$2 \leq |x - 3| < 4$$

$$\Rightarrow 2 \leq -(x - 3) < 4 \Rightarrow 2 \leq -x + 3 < 4$$

Subtracting 3 from both sides, we get

$$-1 \leq -x < 1$$

Multiplying (-1) on both sides, we get

$$-1 < x \leq 1 \Rightarrow x \in (-1, 1]$$

Case II If $x > 3$, then

$$2 \leq |x - 3| < 4 \Rightarrow 2 \leq x - 3 < 4$$

Adding 3 on both sides, we get

$$5 \leq x < 7$$

Hence, the solution set of given inequality is

$$x \in (-1, 1] \cup [5, 7)$$

6. (a) Let x be the smaller of two consecutive even positive integers. Then, the other even integer is $x + 2$.

Given, $x > 8$ and $x + x + 2 < 25$

$$\Rightarrow x > 8 \text{ and } 2x + 2 < 25$$

$$\Rightarrow x > 8 \text{ and } 2x < 23$$

$$\Rightarrow x > 8 \text{ and } x < \frac{23}{2}$$

$$\Rightarrow x = 10$$

Hence, there exists only one pair of even integers $(10, 12)$.

7. (c) We have,

$$IQ = \frac{MA}{CA} \times 100$$

$$\Rightarrow IQ = \frac{MA}{12} \times 100 \quad [\because CA = 12 \text{ yr}]$$

$$= \frac{25}{3} MA$$

$$\text{Given, } 80 \leq IQ \leq 140 \Rightarrow 80 \leq \frac{25}{3} MA \leq 140$$

$$\Rightarrow 240 \leq 25 MA \leq 420$$

Dividing by 25 to each term, we get

$$\frac{240}{25} \leq MA \leq \frac{420}{25}$$

$$\Rightarrow 9.6 \leq MA \leq 16.8$$

8. (c) Let the 2% boric acid solution be x L.

$\therefore$ Mixture = $(640 + x)$ L

Now, according to the question, following two conditions arise

I. 2% of x + 8% of 640 > 4% of $(640 + x)$

II. 2% of x + 8% of 640 < 6% of $(640 + x)$

From condition I,

$$\frac{2}{100} \times x + \frac{8}{100} \times 640 > \frac{4}{100} \times (640 + x)$$

Multiplying both sides by 100, we get

$$100 \times \left[\frac{2x}{100} + \frac{8}{100} \times 640 \right] > \frac{4}{100} \times (640 + x) \times 100$$

$$\Rightarrow 2x + 8 \times 640 > 4 \times 640 + 4x$$

Transferring the term $4x$ to LHS and the term (8×640) to RHS,

$$2x - 4x > 4 \times 640 - 8 \times 640$$

$$\Rightarrow -2x > 640(4 - 8) \Rightarrow -2x > -4 \times 640$$

Dividing both sides by -2 , we get

$$\frac{-2x}{-2} < \frac{-4 \times 640}{-2} \Rightarrow x < 2 \times 640$$

$$\Rightarrow x < 1280 \quad \dots(i)$$

From condition II,

$$\begin{aligned} \frac{2}{100} \times x + \frac{8}{100} \times 640 &< \frac{6}{100} \times (640 + x) \\ \Rightarrow 100 \times \left[\frac{2x}{100} + \frac{8}{100} \times 640 \right] &< [6 \times 640 + 6x] \times \frac{100}{100} \end{aligned}$$

$$\Rightarrow 2x + 8 \times 640 < 6 \times 640 + 6x$$

Transferring the term $6x$ to LHS and the term (8×640) to RHS,

$$2x - 6x < 6 \times 640 - 8 \times 640$$

$$\Rightarrow -4x < 640(6 - 8)$$

$$\Rightarrow -4x < -2 \times 640$$

Dividing both sides by -4 , we get

$$\frac{-4x}{-4} > \frac{-2 \times 640}{-4}$$

$$\Rightarrow x > 320 \quad \dots(ii)$$

From Eqs. (i) and (ii),

$$320 < x < 1280$$

$$\text{i.e. } x \in (320, 1280)$$

Hence, the number of litres to be added should be greater than 320 L and less than 1280 L.

9. (b) Given that, $68^\circ < F < 77^\circ$ $\dots(i)$

Putting $F = \frac{9}{5}C + 32$ in Eq. (i),

$$\Rightarrow 68^\circ < \frac{9}{5}C + 32 < 77^\circ$$

Adding (-32) in each term, we get

$$68^\circ - 32 < \frac{9}{5}C + 32 - 32 < 77^\circ - 32$$

$$\Rightarrow 36^\circ < \frac{9}{5}C < 45^\circ$$

Multiplying by $\left(\frac{5}{9}\right)$ in each term, we get

$$36^\circ \times \frac{5}{9} < \frac{9}{5}C \times \frac{5}{9} < 45^\circ \times \frac{5}{9}$$

$$\Rightarrow 4^\circ \times 5 < C < 5^\circ \times 5$$

$$\Rightarrow 20^\circ < C < 25^\circ$$

$$\text{i.e. } C \in (20, 25)$$

Hence, temperature in degree Celsius lies between 20°C and 25°C .

10. (c) $3 \leq 3t - 18 \leq 18$

$$\Rightarrow 21 \leq 3t \leq 36$$

$$\Rightarrow 7 \leq t \leq 12 \Rightarrow 8 \leq t + 1 \leq 13$$

11. (b) Given, $3^x + \frac{3}{3^x} - 4 < 0$

$$\Rightarrow 3^{2x} + 3 - 4 \cdot 3^x < 0 \Rightarrow (3^x - 1)(3^x - 3) < 0$$

$$\Rightarrow 1 < 3^x < 3 \Rightarrow 0 < x < 1$$

$\therefore$ The solution set is $(0, 1)$.

12. (c) Minimum value of $f(x)$ is attained at $x = 3$

$\therefore$ Minimum value of $f(x) = 7$

13. (d) $x^2 + 4ax + 2 > 0$

$$\therefore (4a)^2 - 4 \times 2 < 0 \quad [\text{if } f(x) > 0, \text{ then } D < 0]$$

$$\Rightarrow 16a^2 < 8 \Rightarrow a^2 < \frac{1}{2}$$

$$\Rightarrow -\frac{1}{\sqrt{2}} < a < \frac{1}{\sqrt{2}}$$

14. (a) $\frac{x+11}{x-3} > 0$

$$\Rightarrow (x-3)(x+11) > 0 \Rightarrow x < -11, x > 3$$

$$\Rightarrow x \in (-\infty, -11) \cup (3, \infty)$$

15. (a) Given, $\frac{|x-1|}{x+2} - 1 < 0$

Case I When $x < 1$, $|x-1| = 1-x$

$$\therefore \frac{1-x}{x+2} - 1 < 0 \Rightarrow \frac{-2x-1}{x+2} < 0$$

$$\Rightarrow \frac{2x+1}{x+2} > 0 \Rightarrow x < -2 \text{ or } x > -\frac{1}{2}$$

But $x < 1$

$$\therefore x \in (-\infty, -2) \cup \left(-\frac{1}{2}, 1\right) \quad \dots(i)$$

Case II When $x \geq 1$, $|x-1| = x-1$

$$\therefore \frac{x-1}{x+2} - 1 < 0 \Rightarrow \frac{-3}{x+2} < 0$$

$$\Rightarrow \frac{3}{x+2} > 0 \Rightarrow x > -2$$

But $x \geq 1$

$$\therefore x \geq 1, \text{ i.e. } x \in [1, \infty) \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$x \in (-\infty, -2) \cup \left(-\frac{1}{2}, \infty\right)$$

16. (b) Using $AM \geq GM$

$$\frac{\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}}{3} \geq \sqrt[3]{\frac{abc}{(a+b)(b+c)(c+a)}} \quad \dots(i)$$

Again, using $AM \geq GM$

$$\frac{a+b}{2} \geq \sqrt{ab}, \frac{b+c}{2} \geq \sqrt{bc}, \frac{c+a}{2} \geq \sqrt{ca}$$

$$\Rightarrow (a+b)(b+c)(c+a) \geq 8abc$$

$$\Rightarrow \sqrt[3]{\frac{abc}{(a+b)(b+c)(c+a)}} \leq \frac{1}{2}$$

From Eq. (i),

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}$$

17. (d) Using $AM > GM$

$$\therefore \frac{a+b+c}{3} > \sqrt[3]{abc}$$

$$\Rightarrow a+b+c > 3 \quad [\because abc = 1, \text{ given}] \dots(i)$$

Also, $GM > HM$

$$\sqrt[3]{abc} > \frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$$

$$\Rightarrow (1)^{1/3} > \frac{3abc}{bc + ac + ab}$$

$$\Rightarrow ab + bc + ac > 3$$

From Eqs. (i) and (ii), we get

$$a + b + c + ab + bc + ac > 6$$

18. (b) Given, ${}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \leq 50$

$$\Rightarrow \frac{(n-1)!}{3!(n-2)!} - \frac{(n+1)!}{2!(n-1)!} \leq 50$$

$$\Rightarrow \frac{(n+1)!}{3!} \left[\frac{1}{(n-2)!} - \frac{3}{(n-1)!} \right] \leq 50$$

$$\Rightarrow (n+1)! \left(\frac{n-1-3}{(n-1)!} \right) \leq 300$$

$$\Rightarrow (n+1)n(n-4) \leq 300$$

For $n = 8$, it satisfies the above inequality.

But for $n = 1$, it does not satisfy the above inequality.

19. (d) Given, $\frac{2x+3}{2x-9} < 0$

$$\Rightarrow 2x+3 < 0 \text{ and } 2x-9 > 0$$

or $2x+3 > 0 \text{ and } 2x-9 < 0 \text{ and } x \neq \frac{9}{2}$

$$\Rightarrow x < -\frac{3}{2} \text{ and } x > \frac{9}{2} \text{ or } x > -\frac{3}{2} \text{ and } x < \frac{9}{2}$$

and $x \neq \frac{9}{2} \Rightarrow x \in \left(-\frac{3}{2}, \frac{9}{2}\right)$

20. (d) Given inequalities are

$$x^2 - 3x - 10 < 0 \text{ and } 10x - x^2 - 16 > 0$$

$$\Rightarrow (x+2)(x-5) < 0 \text{ and } (x-2)(x-8) < 0$$

$$\Rightarrow x \in (-2, 5) \text{ and } x \in (2, 8)$$

$$\therefore x \in (2, 5)$$

21. (a) Since, $2 \leq |x-3| < 4$

$$\Rightarrow 2 \leq x-3 < 4 \text{ or } 2 \leq -(x-3) < 4$$

$$\Rightarrow 5 \leq x < 7 \text{ or } -1 \leq -x < 1$$

$$\Rightarrow 5 \leq x < 7 \text{ or } -1 < x \leq 1$$

$$\therefore x \in (-1, 1] \cup [5, 7)$$

22. (b) Given that, $x = \left[\frac{a+2b}{a+b} \right]$ and $y = \frac{a}{b}$

$$\therefore x = \frac{a+2b}{a+b} = \frac{\frac{a}{b} + 2}{1 + \frac{a}{b}} = 1 + \frac{1}{\frac{a}{b} + 1}$$

$$\Rightarrow x = 1 + \frac{1}{y+1} \quad [\because y = \frac{a}{b} \text{ and } y^2 > 2, \text{ given}]$$

which shows $x^2 < 2$. $\left[\because \frac{1}{y+1} < 1 \text{ as } y > 1 \right]$

23. (c) Given, $\frac{2x}{(2x+1)(x+2)} - \frac{1}{(x+1)} > 0$

$$\Rightarrow \frac{-3x-2}{(x+1)(x+2)(2x+1)} > 0$$

Equating each factor equal to 0, we have

$$x = -2, -1, -\frac{2}{3}, -\frac{1}{2}$$

It is clear that, $-\frac{2}{3} < x < -\frac{1}{2}$ or $-2 < x < -1$

24. (b) Given, $(x-1)(x^2-5x+7) < (x-1)$

$$\Rightarrow (x-1)(x^2-5x+6) < 0$$

$$\Rightarrow (x-1)(x-2)(x-3) < 0$$

$$\Rightarrow x \in (-\infty, 1) \cup (2, 3)$$

25. (c) Since, $|r| < 1 \Rightarrow -1 < r < 1$

Also, $a = 5(1-r)$

$$\Rightarrow 0 < a < 10 \quad \left[\because \text{at } r = -1, a = 10 \right. \\ \left. \text{and at } r = 1, a = 0 \right]$$

26. (d) Given that, $\frac{x+2}{x^2+1} > \frac{1}{2} \Rightarrow x^2 - 2x - 3 < 0$

$$\Rightarrow (x-3)(x+1) < 0 \Rightarrow -1 < x < 3$$

The integral values of x are 0, 1, 2.

$\therefore$ The number of integral solutions are 3.

27. (c) Given, $\frac{4x-1}{3x+1} - 1 \geq 0 \Rightarrow \frac{x-2}{3x+1} \geq 0$

$$\Rightarrow x-2 \geq 0 \text{ and } 3x+1 > 0 \text{ or } x-2 \leq 0 \text{ and } 3x+1 < 0$$

$$\Rightarrow x \geq 2 \text{ and } x > -\frac{1}{3} \text{ or } x \leq 2 \text{ and } x > -\frac{1}{3}$$

$$\Rightarrow x \in \left(-\infty, -\frac{1}{3}\right) \cup [2, \infty)$$

BITSAT Archives

1. (a) Since, $AM \geq GM$, then

$$\frac{(p+q)+(r+s)}{2} \geq \sqrt{(p+q)(r+s)}$$

$$\Rightarrow \frac{2}{2} \geq \sqrt{M} \Rightarrow \sqrt{M} \leq 1 \Rightarrow M \leq 1$$

Also, $(p+q)(r+s) > 0$ $[\because p, q, r, s > 0]$

$$\therefore M > 0$$

Hence, $0 < M \leq 1$

2. (a) Given, $\frac{3-|x|}{4-|x|} \geq 0$

$$\Rightarrow 3-|x| \leq 0 \text{ and } 4-|x| < 0$$

or $3-|x| \geq 0 \text{ and } 4-|x| > 0$

$$\Rightarrow |x| \geq 3 \text{ and } |x| > 4$$

or $|x| \leq 3 \text{ and } |x| < 4$

$$\Rightarrow |x| > 4 \text{ or } |x| \leq 3$$

$$\Rightarrow x \in (-\infty, -4) \cup [-3, 3] \cup (4, \infty)$$

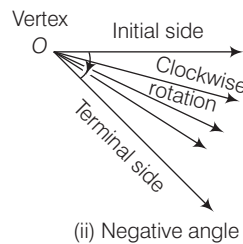
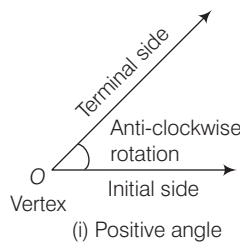
11

Trigonometry

Measure of Angles

The measure of an angle is the amount of rotation performed to get the terminal side from the initial side. The angle is called positive, if its direction of rotation is anti-clockwise and it is called negative, if its direction of rotation is clockwise.

The point of rotation is called the vertex.



System of Measurement of Angles

There are three systems of measurement, which are given below:

Sexagesimal System

In this system, each angle is divided into 90 equal parts and each part is known as a **degree**.

Thus,

$$\begin{aligned} 1 \text{ right angle} &= 90^\circ && [\text{read as 90 degrees}] \\ 1^\circ &= 60' && [\text{read as 60 min}] \\ 1' &= 60'' && [\text{read as 60 s}] \end{aligned}$$

Centesimal System

In this system, each angle is divided into 100 equal parts and one part is known as a **grade**.

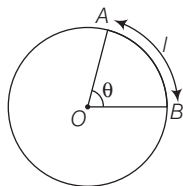
Thus,

$$\begin{aligned} 1 \text{ right angle} &= 100^g && [\text{read as 100 grades}] \\ 1^g &= 100' && [\text{read as 100 min}] \\ 1' &= 100'' && [\text{read as 100 s}] \end{aligned}$$

Circular System

If the angle subtended by an arc of length l to the centre of circle of radius r is θ , then

$$\theta = \frac{l}{r}$$



If the length of arc is equal to the radius of the circle, then the angle subtended at the centre of the circle will be one radian. One radian is denoted by 1^c .

Relation between Different Systems of Measurement of Angles

If measure of an angle is given in degree, then to convert it into radians, multiply the measure of an angle by $\frac{\pi}{180^\circ}$ and if

the measure of an angle is given in radians, then to convert it into degree, write 180° at the place of π .

$$\therefore 1^\circ = \frac{\pi}{180} \text{ rad} = 0.0174 \text{ rad}$$

$$1 \text{ rad} = \frac{180}{\pi} \text{ degree} = 57^\circ 16' 22''$$

Also, $1^g = \frac{\pi}{200} \text{ rad}$

$$1 \text{ rad} = \frac{200}{\pi} \text{ grades}$$

So, if the measure of an angle in degrees, grades and radians, are D , G and C respectively, then

$$D/90 = G/100 = 2C/\pi$$

Trigonometric Ratios

Relation between different sides and angles of a right angled triangle are called trigonometric ratios or T-ratios.

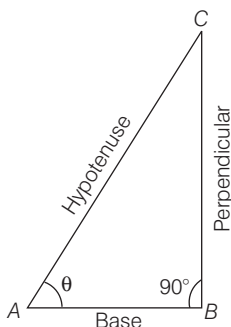
In a right angled $\triangle ABC$, if $\angle CAB = \theta$, then

$$(i) \sin \theta = \frac{BC}{AC} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$(ii) \cos \theta = \frac{AB}{AC} = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$(iii) \tan \theta = \frac{BC}{AB} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$(iv) \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{AC}{BC} = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$$

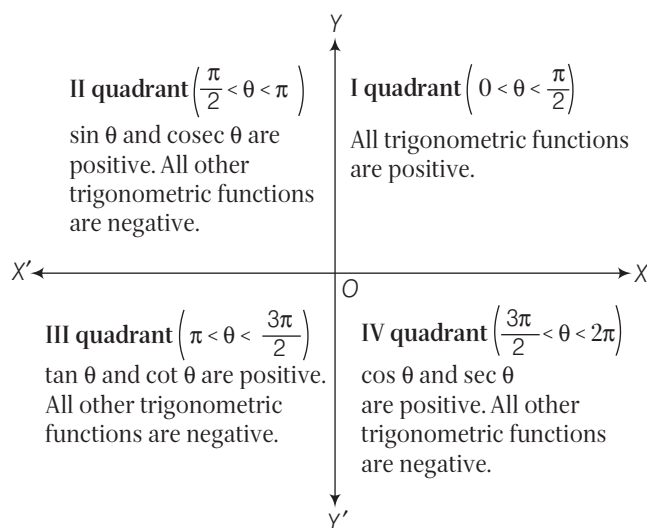


$$(v) \sec \theta = \frac{1}{\cos \theta} = \frac{AC}{AB} = \frac{\text{Hypotenuse}}{\text{Base}}$$

$$(vi) \cot \theta = \frac{1}{\tan \theta} = \frac{AB}{BC} = \frac{\text{Base}}{\text{Perpendicular}}$$

Signs of Trigonometric Functions

The signs of trigonometric functions in different quadrants are given in the following figure:



Trigonometric Ratios of Allied Angles

θ	$\sin \theta$	$\operatorname{cosec} \theta$	$\cos \theta$	$\sec \theta$	$\tan \theta$	$\cot \theta$
$-\theta$	$-\sin \theta$	$-\operatorname{cosec} \theta$	$\cos \theta$	$\sec \theta$	$-\tan \theta$	$-\cot \theta$
$90^\circ - \theta$	$\cos \theta$	$\sec \theta$	$\sin \theta$	$-\operatorname{cosec} \theta$	$\cot \theta$	$\tan \theta$
$90^\circ + \theta$	$\cos \theta$	$\sec \theta$	$-\sin \theta$	$-\operatorname{cosec} \theta$	$-\cot \theta$	$-\tan \theta$
$180^\circ - \theta$	$\sin \theta$	$-\operatorname{cosec} \theta$	$-\cos \theta$	$-\sec \theta$	$-\tan \theta$	$-\cot \theta$
$180^\circ + \theta$	$-\sin \theta$	$-\operatorname{cosec} \theta$	$-\cos \theta$	$-\sec \theta$	$\tan \theta$	$\cot \theta$
$270^\circ - \theta$	$-\cos \theta$	$-\sec \theta$	$-\sin \theta$	$-\operatorname{cosec} \theta$	$\cot \theta$	$\tan \theta$
$270^\circ + \theta$	$-\cos \theta$	$-\sec \theta$	$\sin \theta$	$\operatorname{cosec} \theta$	$-\cot \theta$	$-\tan \theta$
$360^\circ - \theta$	$-\sin \theta$	$-\operatorname{cosec} \theta$	$\cos \theta$	$\sec \theta$	$-\tan \theta$	$-\cot \theta$

Trigonometric Ratios of Some Useful Angles between 0° and 90°

Angle	$0^\circ/0$	$15^\circ/\frac{\pi}{12}$	$18^\circ/\frac{\pi}{10}$	$22.5^\circ/\frac{\pi}{8}$	$30^\circ/\frac{\pi}{6}$	$36^\circ/\frac{\pi}{5}$	$45^\circ/\frac{\pi}{4}$	$54^\circ/\frac{3\pi}{10}$	$60^\circ/\frac{\pi}{3}$	$67.5^\circ/\frac{3\pi}{8}$	$72^\circ/\frac{2\pi}{5}$	$75^\circ/\frac{5\pi}{12}$	$90^\circ/\frac{\pi}{2}$
$\sin \theta$	0	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{2}-\sqrt{2}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}+\sqrt{2}}{2}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	1
$\cos \theta$	1	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{2}+\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{5}+1}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{1}{2}$	$\frac{\sqrt{2}-\sqrt{2}}{2}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	0
$\tan \theta$	0	$\frac{\sqrt{3}-1}{\sqrt{3}+1}$	$\frac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}$	$\sqrt{2}-1$	$\frac{1}{\sqrt{3}}$	$\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}$	1	$\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}$	$\sqrt{3}$	$\sqrt{2}+1$	$\frac{\sqrt{10+2\sqrt{5}}}{\sqrt{5}-1}$	$\frac{\sqrt{3}+1}{\sqrt{3}-1}$	∞
$\cot \theta$	∞	$\frac{\sqrt{3}+1}{\sqrt{3}-1}$	$\frac{\sqrt{10+2\sqrt{5}}}{\sqrt{5}-1}$	$\sqrt{2}+1$	$\sqrt{3}$	$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$	1	$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$	$\frac{1}{\sqrt{3}}$	$\sqrt{2}-1$	$\frac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}$	$\frac{\sqrt{3}-1}{\sqrt{3}+1}$	0
$\sec \theta$	1	$\frac{2\sqrt{2}}{\sqrt{3}+1}$	$\frac{4}{10+2\sqrt{5}}$	$\sqrt{4-2\sqrt{2}}$	$\frac{2}{\sqrt{3}}$	$\sqrt{5}-1$	$\sqrt{2}$	$\frac{4}{\sqrt{10-2\sqrt{5}}}$	2	$\sqrt{4+2\sqrt{2}}$	$\sqrt{5}+1$	$\frac{2\sqrt{2}}{\sqrt{3}-1}$	∞
$\operatorname{cosec} \theta$	∞	$\frac{2\sqrt{2}}{\sqrt{3}-1}$	$\sqrt{5}+1$	$\sqrt{4+2\sqrt{2}}$	2	$\frac{4}{\sqrt{10-2\sqrt{5}}}$	$\sqrt{2}$	$\sqrt{5}-1$	$\frac{2}{\sqrt{3}}$	$\sqrt{4-2\sqrt{2}}$	$\frac{4}{\sqrt{10+2\sqrt{5}}}$	$\frac{2\sqrt{2}}{\sqrt{3}+1}$	1

Some Important Formulae

- (i) $\sin 15^\circ + \cos 15^\circ = \sqrt{\frac{3}{2}} = \sin 75^\circ + \cos 75^\circ$
(ii) $\cos 15^\circ - \sin 15^\circ = \frac{1}{\sqrt{2}} = \sin 75^\circ - \cos 75^\circ$
(iii) $\tan 15^\circ + \cot 15^\circ = 4 = \tan 75^\circ + \cot 75^\circ$
(iv) $\cot 15^\circ - \tan 15^\circ = 2\sqrt{3} = \tan 75^\circ - \cot 75^\circ$

Trigonometric Functions

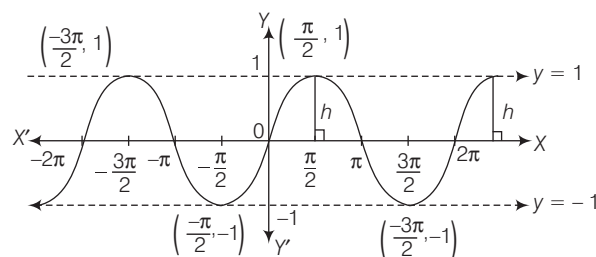
Trigonometric ratios are defined for acute angles as the ratio of the sides of a right angled triangle. The extension of trigonometric ratios to any angle in terms of radian measure (real numbers), are known as trigonometric functions which are given in the following table.

Behaviour of Trigonometric Functions

	I quadrant	II quadrant	III quadrant	IV quadrant
$\sin \theta$	increases from 0 to 1	decreases from 1 to 0	decreases from 0 to -1	increases from -1 to 0
$\cos \theta$	decreases from 1 to 0	decreases from 0 to -1	increases from -1 to 0	increases from 0 to 1
$\tan \theta$	increases from 0 to ∞	increases from $-\infty$ to 0	increases from 0 to ∞	increases from $-\infty$ to 0
$\cot \theta$	decreases from ∞ to 0	decreases from 0 to $-\infty$	decreases from ∞ to 0	decreases from 0 to $-\infty$
$\sec \theta$	increases from 1 to ∞	increases from $-\infty$ to -1	decreases from -1 to $-\infty$	decreases from ∞ to 1
$\operatorname{cosec} \theta$	decreases from ∞ to 1	increases from 1 to ∞	increases from $-\infty$ to -1	decreases from -1 to $-\infty$

Graphs of Trigonometric Functions

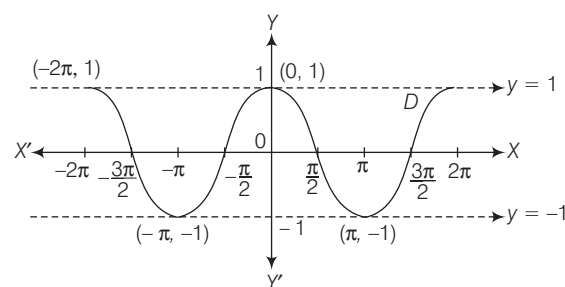
Graph of $\sin x$



Facts related to $\sin x$

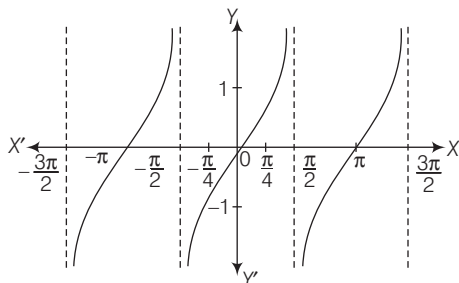
- (a) Domain = R
(b) Range = $[-1, 1]$
(c) Period = 2π
(d) Graph of $\sin x$ is continuous for all real values of x .

Graph of $\cos x$

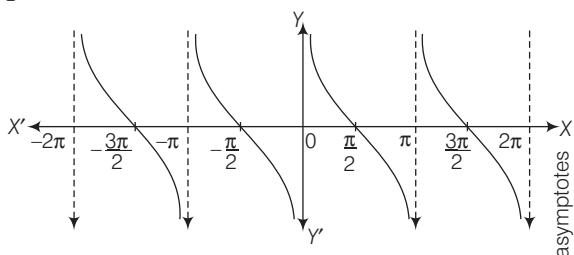


Facts related to $\cos x$

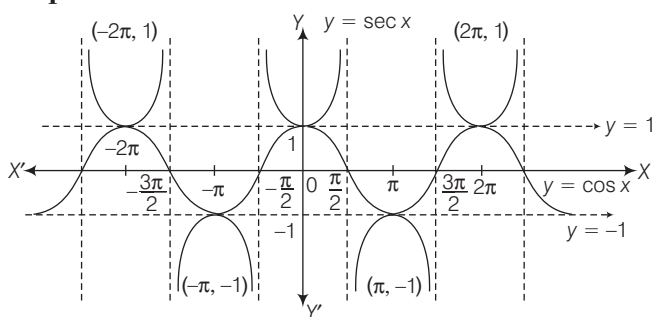
- (a) Domain = R
- (b) Range = $[-1, 1]$
- (c) Period = 2π
- (d) Graph of $\cos x$ is continuous for all real values of x .

Graph of $\tan x$ **Facts related to $\tan x$**

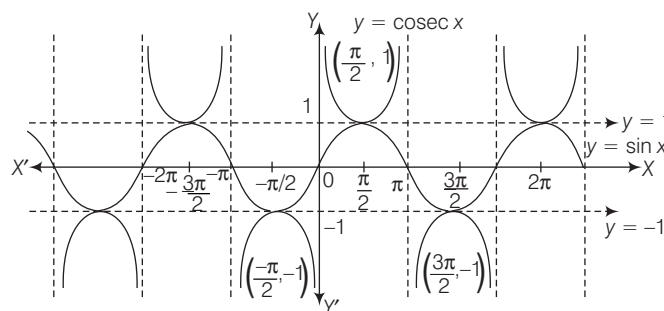
- (a) Domain = $R - (2n + 1)\frac{\pi}{2}, n \in I$
- (b) Range = $(-\infty, \infty)$
- (c) Period = π
- (d) Graph of $\tan x$ is discontinuous at $x = \frac{m\pi}{2}$, where m is an odd integer.

Graph of $\cot x$ **Facts related to $\cot x$**

- (a) Domain = $R - n\pi, n \in I$
- (b) Range = $(-\infty, \infty)$
- (c) Period = π
- (d) Graph of $\cot x$ is discontinuous at $x = m\pi$, where m is an integer.

Graph of $\sec x$ **Facts related to $\sec x$**

- (a) Domain = $R - (2n + 1)\frac{\pi}{2}, n \in I$
- (b) Range = $(-\infty, -1] \cup [1, \infty)$
- (c) Period = 2π
- (d) Graph of $\sec x$ is discontinuous at $x = \frac{m\pi}{2}$, where m is an odd integer.

Graph of $\operatorname{cosec} x$ **Facts related to $\operatorname{cosec} x$**

- (a) Domain = $R - n\pi, n \in I$
- (b) Range = $(-\infty, -1] \cup [1, \infty)$
- (c) Period = 2π
- (d) Graph of $\operatorname{cosec} x$ is discontinuous at $x = m\pi$, where m is an integer.

Trigonometric Identities

An equation involving trigonometric functions which is true for all those angles for which the functions are defined, is called trigonometric identity. Some identities are given below:

- (i) $\sin^2 \theta + \cos^2 \theta = 1$
- (ii) $\sec^2 \theta - \tan^2 \theta = 1$
- (iii) $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$

Trigonometric Ratios between Two Angles**1. Compound Angles**

- (i) $\sin(A + B) = \sin A \cos B + \cos A \sin B$
- (ii) $\sin(A - B) = \sin A \cos B - \cos A \sin B$
- (iii) $\cos(A + B) = \cos A \cos B - \sin A \sin B$
- (iv) $\cos(A - B) = \cos A \cos B + \sin A \sin B$
- (v) $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$(vi) \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$(vii) \cot(A + B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$$

$$(viii) \cot(A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

2. Transformation Formulae

$$(i) 2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

$$(ii) 2 \cos A \sin B = \sin(A + B) - \sin(A - B)$$

$$(iii) 2 \cos A \cos B = \cos(A + B) + \cos(A - B)$$

$$(iv) 2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

3. Multiple Angles

$$(i) \sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$(ii) \cos 2A = \cos^2 A - \sin^2 A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$= 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$(iii) 1 + \sin 2A = (\sin A + \cos A)^2$$

$$(iv) 1 - \sin 2A = (\sin A - \cos A)^2$$

$$(v) \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$(vi) \sin 3A = 3 \sin A - 4 \sin^3 A$$

$$(vii) \cos 3A = 4 \cos^3 A - 3 \cos A$$

$$(viii) \tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

$$(ix) (a) \frac{1 + \tan A}{1 - \tan A} = \tan \left(\frac{\pi}{4} + A \right)$$

$$(b) \frac{1 - \tan A}{1 + \tan A} = \tan \left(\frac{\pi}{4} - A \right)$$

4. Sum or Difference into Product

$$(i) \sin C + \sin D = 2 \sin \frac{C + D}{2} \cos \frac{C - D}{2}$$

$$(ii) \sin C - \sin D = 2 \cos \frac{C + D}{2} \sin \frac{C - D}{2}$$

$$(iii) \cos C + \cos D = 2 \cos \frac{C + D}{2} \cos \frac{C - D}{2}$$

$$(iv) \cos C - \cos D = -2 \sin \frac{C + D}{2} \sin \frac{C - D}{2}$$

$$(v) \sin(A + B) \sin(A - B) = \sin^2 A - \sin^2 B$$

$$(vi) \cos(A + B) \cos(A - B) = \cos^2 A - \sin^2 B$$

$$(vii) \sin(60^\circ - A) \sin(60^\circ + A) = \frac{\sin 3A}{4 \sin A}$$

$$(viii) \cos(60^\circ - A) \cos(60^\circ + A) = \frac{\cos 3A}{4 \cos A}$$

$$(ix) \tan(60^\circ - A) \tan(60^\circ + A) = \frac{\tan 3A}{\tan A}$$

Conditional Identities

- $A + B + C = 180^\circ$
- $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$
- $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$
- $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$
- $\cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$
- $\tan A + \tan B + \tan C = \tan A \tan B \tan C$
- $\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$

Other Important Identities

$$(i) \sin \alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \dots + \sin[\alpha + (n - 1)\beta]$$

$$= \frac{\sin \left[\alpha + (n - 1) \frac{\beta}{2} \right] \cdot \sin \frac{n\beta}{2}}{\sin \frac{\beta}{2}} \quad [\text{here, angles are in AP}]$$

$$(ii) \cos \alpha + \cos(\alpha + \beta) + \cos(\alpha + 2\beta)$$

$$+ \dots + \cos[\alpha + (n - 1)\beta]$$

$$= \frac{\cos \left[\alpha + (n - 1) \frac{\beta}{2} \right] \cdot \sin \frac{n\beta}{2}}{\sin \frac{\beta}{2}} \quad [\text{here, angles are in AP}]$$

Trigonometric Equations

An equation involving one or more trigonometrical ratios of unknown angle is called a **trigonometrical equation**.

e.g. $\sin \theta + \cos^2 \theta = 0$

Principal Solution

The solutions of a trigonometric equations for which $0 \leq \theta < 2\pi$, are known as principal solutions. Numerically, least general values of angle is called principal value.

NOTE The principal angle range for $\sin, \cos, \tan$ and $\cot$ are $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $[0, \pi]$, $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[0, \pi]$ respectively. This can be seen from the graph of these functions.

General Solution

General solution is the complete set of values of the unknown angle satisfying the equation.

The general solutions of trigonometric equations are

- (i) If $\sin x = 0$, then $x = n\pi$, where $n \in \mathbb{Z}$.
 (ii) If $\cos x = 0$, then $x = (2n+1)\frac{\pi}{2}$, where $n \in \mathbb{Z}$.
 (iii) If $\tan x = 0$, then $x = n\pi$, where $n \in \mathbb{Z}$.
 (iv) If $\cot x = 0$, then $x = (2n+1)\frac{\pi}{2}$, where $n \in \mathbb{Z}$.
 (v) If $\sin x = \sin y$, then $x = n\pi + (-1)^n y$, where $n \in \mathbb{Z}$.
 (vi) If $\cos x = \cos y$, then $x = 2n\pi \pm y$, where $n \in \mathbb{Z}$.
 (vii) If $\tan x = \tan y$, then $x = n\pi + y$, where $n \in \mathbb{Z}$.
 (viii) If $\sin^2 x = \sin^2 y$ or $\cos^2 x = \cos^2 y$ or $\tan^2 x = \tan^2 y$, then $x = n\pi \pm y$, where $n \in \mathbb{Z}$.

NOTE

- $\sin \theta = \begin{cases} 1, & \text{if } n \text{ is even} \\ -1, & \text{if } n \text{ is odd} \end{cases}$, where $\theta = (2n+1)\frac{\pi}{2}$
- $\cos \theta = \begin{cases} 1, & \text{if } n \text{ is even} \\ -1, & \text{if } n \text{ is odd} \end{cases}$, where $\theta = n\pi$
- $\sec \theta$ and $\tan \theta$ are not defined for $\theta = (2n+1)\frac{\pi}{2}$, $n \in \mathbb{Z}$.
- $\operatorname{cosec} \theta$ and $\cot \theta$ are not defined for $\theta = n\pi$, $n \in \mathbb{Z}$.

Maximum and Minimum (Greater and Least) Value of $a\sin\theta + b\cos\theta$

For the solution of such equation, we put $a = r \cos \alpha$ and $b = r \sin \alpha$, so that $r^2 = a^2 + b^2$ and $\tan \alpha = \frac{b}{a}$

Now, we get equation in the form
 $r(\sin \theta \cdot \cos \alpha + \cos \theta \cdot \sin \alpha) = c$

$$\Rightarrow \sin(\theta + \alpha) = \frac{c}{r}$$

$$\text{Since, } -1 \leq \sin(\theta + \alpha) \leq 1$$

$$\therefore -1 \leq \frac{c}{r} \leq 1$$

$$\Rightarrow -r \leq c \leq r$$

which is the solution of the given equation.

Hence, maximum value of the equation is

$$a \sin \theta + b \cos \theta = \sqrt{a^2 + b^2}$$

and minimum value of the equation is

$$a \sin \theta + b \cos \theta = -\sqrt{a^2 + b^2}$$

where, a and b are constants.

Inverse Trigonometric Functions

Trigonometric functions are not one-one and onto on their natural domains and ranges, so their inverse do not exist in all values but their inverse may exist in some interval of their domains and ranges.

Thus, we can say that inverse of trigonometric functions are defined within restricted domains of corresponding trigonometric functions. Inverse of f is denoted by f^{-1} .

Let $y = f(x) = \sin x$ be a function.

Then, its inverse is $x = \sin^{-1} y$.

$$\text{i.e. } \sin x \xrightarrow{\text{Inverse}} \sin^{-1} x,$$

NOTE

- $\sin^{-1} y \neq (\sin y)^{-1}$
- $\sin^{-1} y \neq \sin\left(\frac{1}{y}\right)$
- $(\sin^{-1} x) \neq \left(\frac{1}{\sin x}\right)$

Principal Value

The value of an inverse trigonometric function which is numerically least, is called principal value and the set of least values for which the domain is uniquely defined, is called the principal value branch of the function.

Domain and Range of Inverse Trigonometric Functions

Function	Domain	Range	Principal value branch
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, where $y = \sin^{-1} x$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$	$0 \leq y \leq \pi$, where $y = \cos^{-1} x$
$\tan^{-1} x$	$\mathbb{R}$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$, where $y = \tan^{-1} x$
$\operatorname{cosec}^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, where, $y = \operatorname{cosec}^{-1} x$, $y \neq 0$
$\sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi] - \left[\frac{\pi}{2}\right]$	$0 \leq y \leq \pi$, where $y = \sec^{-1} x$, $y \neq \frac{\pi}{2}$
$\cot^{-1} x$	$\mathbb{R}$	$(0, \pi)$	$0 < y < \pi$, where $y = \cot^{-1} x$

Properties of Inverse Trigonometric Functions

Some properties of inverse trigonometric functions are given below:

Property 1 Self Adjusting Properties

- $\sin(\sin^{-1} x) = x, \forall x \in [-1, 1]$
- $\cos(\cos^{-1} x) = x, \forall x \in [-1, 1]$
- $\tan(\tan^{-1} x) = x, \forall x \in \mathbb{R}$
- $\operatorname{cosec}(\operatorname{cosec}^{-1} x) = x, \forall x \in (-\infty, -1] \cup [1, \infty)$
- $\sec(\sec^{-1} x) = x, \forall x \in (-\infty, -1] \cup [1, \infty)$
- $\cot(\cot^{-1} x) = x, \forall x \in \mathbb{R}$
- $\sin^{-1}(\sin x) = x, \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$(viii) \cos^{-1}(\cos x) = x, \forall x \in [0, \pi]$$

$$(ix) \tan^{-1}(\tan x) = x, \forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$(x) \operatorname{cosec}^{-1}(\operatorname{cosec} x) = x, \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$$

$$(xi) \sec^{-1}(\sec x) = x, \forall x \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$$

$$(xii) \cot^{-1}(\cot x) = x, \forall x \in (0, \pi)$$

Property 2 Negative Arguments

$$(i) \sin^{-1}(-x) = -\sin^{-1} x, \forall x \in [-1, 1]$$

$$(ii) \cos^{-1}(-x) = \pi - \cos^{-1} x, \forall x \in [-1, 1]$$

$$(iii) \tan^{-1}(-x) = -\tan^{-1} x, \forall x \in R$$

$$(iv) \operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x, \forall x \in (-\infty, -1] \cup [1, \infty)$$

$$(v) \sec^{-1}(-x) = \pi - \sec^{-1} x, \forall x \in (-\infty, -1] \cup [1, \infty)$$

$$(vi) \cot^{-1}(-x) = \pi - \cot^{-1} x, \forall x \in R$$

Property 3 Reciprocal Arguments

$$(i) \sin^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1} x, \forall x \in (-\infty, -1] \cup [1, \infty)$$

$$(ii) \cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x, \forall x \in (-\infty, -1] \cup [1, \infty)$$

$$(iii) \tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} \cot^{-1} x, & \text{for } x > 0 \\ -\pi + \cot^{-1} x, & \text{for } x < 0 \end{cases}$$

Property 4 Inverse Sum Identities

$$(i) \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, \forall x \in [-1, 1]$$

$$(ii) \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, \forall x \in R$$

$$(iii) \sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}, \forall x \in (-\infty, -1] \cup [1, \infty)$$

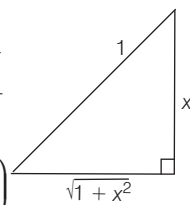
Property 5 Conversion of One Inverse to Another Inverse Trigonometric Function

$$(i) \sin^{-1} x = \cos^{-1} \sqrt{1-x^2}$$

$$= \tan^{-1} \frac{x}{\sqrt{1-x^2}} = \cot^{-1} \frac{\sqrt{1-x^2}}{x}$$

$$= \sec^{-1} \left(\frac{1}{\sqrt{1-x^2}} \right) = \operatorname{cosec}^{-1} \left(\frac{1}{x} \right)$$

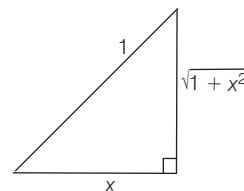
$$(ii) \cos^{-1} x = \sin^{-1} \sqrt{1-x^2} = \tan^{-1} \left(\frac{\sqrt{1-x^2}}{x} \right)$$



$$= \cot^{-1} \left(\frac{x}{\sqrt{1-x^2}} \right)$$

$$= \sec^{-1} \left(\frac{1}{x} \right)$$

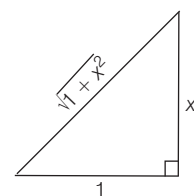
$$= \operatorname{cosec}^{-1} \left(\frac{1}{\sqrt{1-x^2}} \right)$$



$$(iii) \tan^{-1} x = \sin^{-1} \left(\frac{x}{\sqrt{1+x^2}} \right) = \cos^{-1} \left(\frac{1}{\sqrt{1+x^2}} \right)$$

$$= \cot^{-1} \frac{1}{x} = \sec^{-1} \sqrt{1+x^2}$$

$$= \operatorname{cosec}^{-1} \left(\frac{\sqrt{1+x^2}}{x} \right)$$



Property 6 Sum and Difference of Inverse Trigonometric Functions

$$(i) \sin^{-1} x + \sin^{-1} y$$

$$= \begin{cases} \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}), & \text{if } x \geq 0, y \geq 0 \\ & \text{and } x^2 + y^2 \leq 1 \end{cases}$$

$$(ii) \sin^{-1} x - \sin^{-1} y$$

$$= \begin{cases} \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}, \\ & \text{if } -1 \leq x, y \leq 1 \text{ and } x^2 + y^2 \leq 1 \\ & \text{or if } xy > 0 \text{ and } x^2 + y^2 > 1 \end{cases}$$

$$(iii) \cos^{-1} x + \cos^{-1} y = \begin{cases} \cos^{-1}(xy - \sqrt{1-x^2} \cdot \sqrt{1-y^2}), \\ & \text{if } -1 \leq x, y \leq 1 \text{ and } x + y \geq 0 \end{cases}$$

$$(iv) \cos^{-1} x - \cos^{-1} y = \begin{cases} \cos^{-1}(xy + \sqrt{1-x^2} \cdot \sqrt{1-y^2}), \\ & \text{if } -1 \leq x, y \leq 1 \text{ and } x \leq y \end{cases}$$

$$(v) \tan^{-1} x + \tan^{-1} y = \begin{cases} \tan^{-1} \left(\frac{x+y}{1-xy} \right), & \text{if } xy < 1 \end{cases}$$

$$(vi) \tan^{-1} x - \tan^{-1} y = \begin{cases} \tan^{-1} \left(\frac{x-y}{1+xy} \right), & \text{if } xy > -1 \end{cases}$$

Property 7 Inverse Trigonometric Ratios of Multiple Angles

$$(i) 2 \sin^{-1} x = \begin{cases} \sin^{-1}(2x\sqrt{1-x^2}), & \text{if } -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}} \end{cases}$$

$$(ii) 3 \sin^{-1} x = \begin{cases} \sin^{-1}(3x - 4x^3), & \text{if } -\frac{1}{2} \leq x \leq \frac{1}{2} \end{cases}$$

$$(iii) 2\cos^{-1}x = \{\cos^{-1}(2x^2 - 1), \text{ if } 0 \leq x \leq 1$$

$$(iv) 3\cos^{-1}x = \begin{cases} \cos^{-1}(4x^3 - 3x), & \text{if } \frac{1}{2} \leq x \leq 1 \end{cases}$$

$$(v) 2\tan^{-1}x = \begin{cases} \tan^{-1}\left(\frac{2x}{1-x^2}\right), & \text{if } -1 < x < 1 \end{cases}$$

$$(vi) 3\tan^{-1}x = \begin{cases} \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), & \text{if } -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}} \end{cases}$$

$$(vii) 2\tan^{-1}x = \begin{cases} \sin^{-1}\left(\frac{2x}{1+x^2}\right), & \text{if } -1 \leq x \leq 1 \end{cases}$$

$$(viii) 2\tan^{-1}x = \begin{cases} \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), & \text{if } 0 \leq x < \infty \end{cases}$$

Important Results on Inverse Trigonometric Functions

(i) If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, then

$$x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$$

(ii) If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = 3\pi/2$, then

$$xy + yz + zx = 3$$

(iii) If $\sin^{-1}x + \sin^{-1}y = \theta$, then $\cos^{-1}x + \cos^{-1}y = \pi - \theta$

(iv) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, then

$$x^2 + y^2 + z^2 + 2xyz = 1$$

(v) If $\cos^{-1}x + \cos^{-1}y = \theta$, then $\sin^{-1}x + \sin^{-1}y = \pi - \theta$

(vi) If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \theta$, then

$$\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\theta + \frac{y^2}{b^2} = \sin^2\theta$$

Important Result

- If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, then $x + y + z = xyz$
- If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$, then $xy + yz + zx = 1$
- If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$, then $xy + yz + zx = 3$
- If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{\pi}{2}$, then $x^2 + y^2 + z^2 + 2xyz = 1$

Triangle and Its Properties

A triangle has six components, three sides and three angles. The three angles of a $\triangle ABC$ are denoted by letters A, B and C and the sides opposite to these angles by letters a, b and c , respectively.

Following are some well known relations for a $\triangle ABC$

(i) $A + B + C = 180^\circ$ (or π), i.e. sum of all the three angles of a triangle is 180° .

(ii) $a + b > c$, $b + c > a$, $c + a > b$, i.e. in any triangle, the greatest side is always less than the sum of other two sides.

(iii) $|a - b| < c$, $|b - c| < a$ and $|c - a| < b$, i.e. in any triangle, the smallest side is always greater than the difference of other two sides.

Relations between the Sides and Angles of Triangle

Here are some relations between the sides and trigonometrical ratios of the angles of a triangle.

(i) Sine Rule

In $\triangle ABC$, the lengths of the sides are proportional to the sines of the opposite angle.

$$\text{i.e. } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

where, R is the radius of triangle's circumcircle.

(ii) Cosine Rule

The square of one side of a triangle equals the sum of the squares of the other two sides minus twice their product times the cosine of their included angle.

In a $\triangle ABC$,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \text{ or } a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca} \text{ or } b^2 = c^2 + a^2 - 2ca\cos B$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \text{ or } c^2 = a^2 + b^2 - 2ab\cos C$$

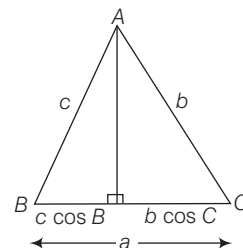
(iii) Projection Formula

In $\triangle ABC$ with sides $a = BC, b = CA$ and $c = BA$, the projection formula is

$$a = c \cos B + b \cos C$$

$$b = a \cos C + c \cos A$$

$$c = a \cos B + b \cos A$$



(iv) Napier's Formula (Napier's Analogy)

$$\text{In } \triangle ABC, \quad \tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c} \cot \frac{A}{2}$$

$$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b} \cot \frac{C}{2} \Rightarrow \tan\left(\frac{C-A}{2}\right) = \frac{c-a}{c+a} \cot \frac{B}{2}$$

(v) Half-angle Formulae

In $\triangle ABC$, if the sides of the triangle are a, b, c and corresponding angles are A, B and C respectively and s is the semi-perimeter, then

$$(a) \sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}} \quad (b) \sin \frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$$

$$(c) \sin \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}} \quad (d) \cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$$

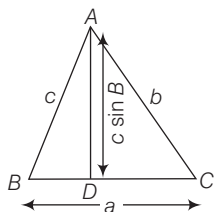
$$(e) \cos \frac{B}{2} = \sqrt{\frac{s(s-b)}{ac}} \quad (f) \cos \frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$$

$$(g) \tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \quad (h) \tan \frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$$

$$(i) \tan \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

Area of Triangle

From the figure given below, area of $\triangle ABC$ can be calculated in the following ways.



- (i) When two sides and angle between them is known, then

$$\Delta = \frac{1}{2} \times AD \times BC = \frac{1}{2} \times c \sin B \times a$$

$$\Rightarrow \Delta = \frac{1}{2} ac \sin B$$

$$\text{Similarly, } \Delta = \frac{1}{2} ab \sin C$$

$$\text{and } \Delta = \frac{1}{2} bc \sin A$$

- (ii) When all the three sides are known, then

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)},$$

which is also known as Heron's formula.

- (iii) By using sine rule,

$$\sin A = \frac{a}{2R}$$

$$\Rightarrow a = 2R \sin A$$

$$\therefore \Delta = \frac{abc}{4R}$$

$$\therefore \Delta = \frac{(2R \sin A)(2R \sin B)(2R \sin C)}{4R}$$

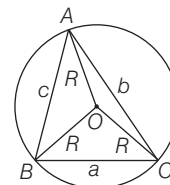
$$\Rightarrow \Delta = 2R^2 \sin A \sin B \sin C$$

Circles Connected with Triangles

Circles can be connected with the triangles in the following ways.

1. Circumcircle of a Triangle

A circle which passes through the vertices of a $\triangle ABC$, is called circumcircle.

**Circumcentre (O)**

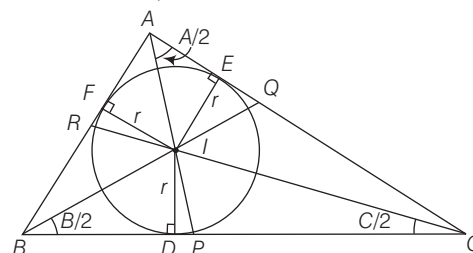
The centre of circumcircle is the point of intersection of the perpendicular bisectors of the sides and is called the circumcentre. Its radius is denoted R .

$$(i) R = \frac{a}{2 \sin A} = \frac{b}{2 \sin B} = \frac{c}{2 \sin C} \quad (ii) R = \frac{abc}{4\Delta}$$

NOTE Circumcentre of a right angled triangle is the mid-point of the hypotenuse.

2. Incircle or Inscribed Circle of a Triangle

The circle that can be inscribed within a triangle so as to touch each of its sides, is called inscribed circle or incircle.

**Incentre (I)**

The centre of incircle is the point of intersection of bisectors of the angles of the triangle. It is denoted by I .

Inradius (r)

The radius of inscribed circle of a triangle is called the inradius. It is denoted by r and is given by

$$(i) r = \frac{\Delta}{s}, \text{ where } \Delta = \frac{1}{2} r (a + b + c)$$

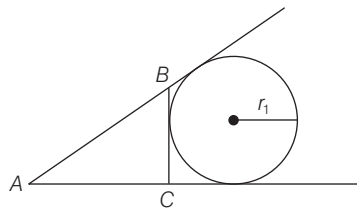
$$(ii) r = (s-a) \tan \frac{A}{2}, r = (s-b) \tan \frac{B}{2}, r = (s-c) \tan \frac{C}{2}$$

$$(iii) r = \frac{a \sin \frac{B}{2} \cdot \sin \frac{C}{2}}{\cos \frac{A}{2}}, r = \frac{b \sin \frac{C}{2} \cdot \sin \frac{A}{2}}{\cos \frac{B}{2}}, r = \frac{c \sin \frac{A}{2} \cdot \sin \frac{B}{2}}{\cos \frac{C}{2}}$$

$$(iv) r = 4R \sin \frac{A}{2} \cdot \sin \frac{B}{2} \cdot \sin \frac{C}{2}$$

3. Escribed Circle

The circle touches BC and the two sides AB and AC produced of $\triangle ABC$ externally, is called the escribed circle opposite A . Its radius is denoted by r_1 .



Similarly, r_2 and r_3 denote the radii of the escribed circles opposite angles B and C , respectively. Hence, r_1, r_2 and r_3 are called the exradius of $\triangle ABC$. Here,

$$(i) \quad r_1 = \frac{\Delta}{s-a} = s \tan \frac{A}{2} = 4R \sin \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$$

$$= \frac{a \cos \frac{B}{2} \cos \frac{C}{2}}{\cos \frac{A}{2}}$$

$$(ii) \quad r_2 = \frac{\Delta}{s-b} = s \tan \frac{B}{2} = 4R \sin \frac{B}{2} \cos \frac{C}{2} \cos \frac{A}{2}$$

$$= \frac{b \cos \frac{C}{2} \cos \frac{A}{2}}{\cos \frac{B}{2}}$$

$$(iii) \quad r_3 = \frac{\Delta}{s-c} = s \tan \frac{C}{2} = 4R \sin \frac{C}{2} \cos \frac{A}{2} \cos \frac{B}{2}$$

$$= \frac{c \cos \frac{A}{2} \cos \frac{B}{2}}{\cos \frac{C}{2}}$$

Regular Polygon

A polygon which has all its sides as well as its angles equal, is known as regular polygon. If the polygon has n sides, then

Sum of the internal angle $= (n-2)\pi$

and each angle $= \frac{(n-2)\pi}{n}$

NOTE In the regular polygon, the circumcentre and the incentre are same.

Radii of the Inscribed and Circumscribing Circles of a Regular Polygon

Let the point O be both the incentre and circumcentre of polygon as shown in figure.

Now, if $OB = OC = R$, $OL = r$

and a = Length of side of polygon

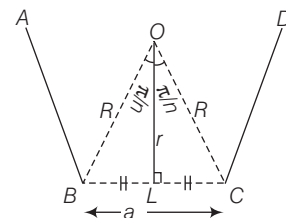
Then, $R = \frac{a}{2} \operatorname{cosec} \left(\frac{\pi}{n} \right)$

and $r = \frac{a}{2} \cot \left(\frac{\pi}{n} \right)$

$\therefore$ Area of regular polygon

$$= \frac{1}{4} n a^2 \cdot \cot \left(\frac{\pi}{n} \right)$$

$$= n r^2 \cdot \tan \frac{\pi}{n} = \frac{n}{2} R^2 \sin \frac{2\pi}{n}$$

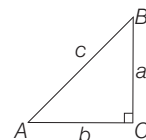


Solution of Triangle

If three elements including atleast one side is known, then remaining three elements can be uniquely determined. This is known as solution of triangle.

Solution of Right Angled Triangle

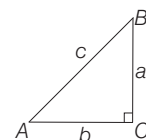
Given elements	Required elements
1. $a, b, \angle C = 90^\circ$	(i) $\tan A = \frac{a}{b}, \sin A = \frac{a}{c}, \cos A = \frac{b}{c}$
	(ii) $B = 90^\circ - A$
	(iii) $c = \frac{a}{\sin A}$ or $c^2 = a^2 + b^2$
2. $a, \angle A = \text{Acute angle}$ and $\angle C = 90^\circ$	(i) $B = 90^\circ - A$
	(ii) $b = a \cot A$
	(iii) $c = \frac{a}{\sin A}$
3. $c, \angle A, \angle C = 90^\circ$	(i) $B = 90^\circ - A$
	(ii) $b = c \cos A$
	(iii) $a = \sqrt{c^2 - b^2}$ or $a = c \sin A$



Solution of Oblique Triangle

$\triangle ABC$ is called oblique triangle, if none of its angles is right angle.

Given elements	Required elements
1. Three sides a, b, c	(i) $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ where, $2s = a + b + c$
	(ii) $\sin A = \frac{2\Delta}{bc}, \sin B = \frac{2\Delta}{ac}, \sin C = \frac{2\Delta}{ab}$
	(iii) $\tan \frac{A}{2} = \frac{\Delta}{s(s-a)}, \tan \frac{B}{2} = \frac{\Delta}{s(s-b)},$ $\tan \frac{C}{2} = \frac{\Delta}{s(s-c)}$
	(iv) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$ $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$



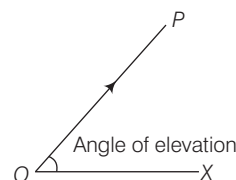
Given elements	Required elements
2. When two sides a, b and included $\angle C$ are given	(i) $\Delta = \frac{1}{2}ab \sin C$ (ii) $\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b} \cot \frac{C}{2}$ (iii) $c = \frac{a \sin C}{\sin A}$
3. When one side a and two angles A and B are given	(i) $\angle C = 180^\circ - (\angle A + \angle B)$ (ii) $b = \frac{a \sin B}{\sin A}$ and $c = \frac{a \sin C}{\sin A}$ (iii) $\Delta = \frac{1}{2}ac \sin B$
4. When two sides a, b and $\angle A$ opposite to one side are given	$\sin B = \frac{b}{a} \sin A$ (i) If $\angle A$ is acute and $a < b \sin A$, then $\sin B > 1$. No triangle is possible. (ii) If $\angle A$ is acute, $a = b \sin A$, $\sin B = 1$, $\angle B = 90^\circ$, then triangle is possible right angled at B . (iii) If A is acute, $a > b \sin A$, $\sin B < 1$, then two values of $\angle B$ say B_1 and B_2 such that $B_1 + B_2 = 180^\circ$, side c can be obtained as $c = \frac{a \sin C}{\sin A}$
5. When exradius r_1, r_2, r_3 of ΔABC are given	Use the formulae $r_1 = \frac{\Delta}{s-a}, r_2 = \frac{\Delta}{s-b}, r_3 = \frac{\Delta}{s-c}$

Heights and Distance

The problems related with heights and distance are solved by using the properties of triangles.

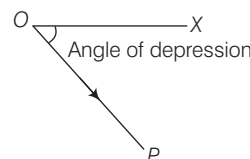
Some terms which are used in heights and distance, are given below:

- (i) **Angle of elevation** Let P be an object and OX be a horizontal line. An observer at O to perceive the object P has to elevate his eye from the direction OX to the direction OP .



So, $\angle XOP$ is the angle of elevation of P at O .

- (ii) **Angle of depression** Let P be an object and OX be a horizontal line. An observer at O to perceive the object P has to depress his eye from the direction OX to the direction OP .



So, $\angle XOP$ is the angle of depression of P at O .

Practice Exercise

- Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm. (Use $\pi = \frac{22}{7}$)
 a. $11^\circ 36'$ b. $12^\circ 36'$ c. $12^\circ 30'$ d. $12^\circ 34'$
- If the three angles of a quadrilateral are $60^\circ, 60^\circ$ and $\frac{5\pi}{6}$ radian. Then, the fourth angle is
 a. 60° b. 96°
 c. 96° d. None of these
- Which of the following is correct?
 a. $\sin 1^\circ > \sin 1$ b. $\sin 1^\circ < \sin 1$
 c. $\sin 1^\circ = \sin 1$ d. $\sin 1^\circ = \frac{\pi}{18^\circ} \sin 1$
- Given, a right angled ΔABC , whose base is 6 and perpendicular height is 8. Find the trigonometric ratios of $\sin A, \cos A$ and $\tan A$, respectively.
 a. $\frac{4}{5}, \frac{3}{5}, \frac{4}{3}$ b. $\frac{2}{5}, \frac{3}{5}, \frac{4}{5}$
 c. $\frac{4}{5}, \frac{1}{5}, \frac{2}{3}$ d. None of these
- The value of $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$ is
 a. 2 b. 3
 c. 4 d. None of these
- The value of $\log_3 \tan 1^\circ + \log_3 \tan 2^\circ + \dots + \log_3 \tan 89^\circ$ is
 a. 3 b. 1
 c. 2 d. 0
- If the angle of elevation of the top of a hill from each of the vertices A, B and C of a horizontal triangle is α . Then, the height of the hill is
 a. $\frac{1}{2}b \tan \alpha \sec B$ b. $\frac{1}{2}b \tan \alpha \operatorname{cosec} A$
 c. $\frac{1}{2}c \tan \alpha \sin C$ d. $\frac{1}{2}a \tan \alpha \operatorname{cosec} A$

8. If $\cos A = \tan B$, $\cos B = \tan C$ and $\cos C = \tan A$, then $\sin A$ is equal to
 a. $\sin 18^\circ$ b. $2 \sin 18^\circ$ c. $2 \cos 18^\circ$ d. $2 \cos 36^\circ$
9. If $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$, then $\sin 81^\circ$ is equal to
 a. $\frac{\sqrt{5}+1}{4}$ b. $\frac{\sqrt{3+\sqrt{5}}+\sqrt{5-\sqrt{5}}}{4}$
 c. $\frac{\sqrt{10+2\sqrt{5}}}{4}$ d. None of these
10. $\sec^2 \theta = \frac{4xy}{(x+y)^2}$ is true, if
 a. $x+y \neq 0$ b. $x=y, x \neq 0$
 c. $x=y$ d. $x \neq 0, y=0$
11. The value of $(1+\tan 1^\circ)(1+\tan 44^\circ)(1+\tan 2^\circ)(1+\tan 43^\circ) \dots (1+\tan 22^\circ)(1+\tan 23^\circ)$ is
 a. 1 b. 2^{22} c. 3^8 d. 2^{11}
12. The value of $\tan 10^\circ + \tan 35^\circ + \tan 10^\circ \tan 35^\circ$ is
 a. 0 b. $\frac{1}{2}$ c. -1 d. 1
13. If $\alpha + \beta = \frac{\pi}{2}$ and $\beta + \gamma = \alpha$, then $\tan \alpha$ is equal to
 a. $2(\tan \beta + \tan \gamma)$ b. $\tan \beta + \tan \gamma$
 c. $\tan \beta + 2 \tan \gamma$ d. $2 \tan \beta + \tan \gamma$
14. If $2 \cos x + 2 \cos 3x = \cos y$ and $2 \sin x + 2 \sin 3x = \sin y$, then the value of $\cos 2x$ is
 a. $-\frac{7}{8}$ b. $\frac{1}{8}$ c. $-\frac{1}{8}$ d. $\frac{7}{8}$
15. If $\cos 5\theta = a \cos \theta + b \cos^3 \theta + c \cos^5 \theta + d$, then
 a. $a=20$ b. $b=-30$
 c. $a+b+c=2$ d. $a+b+c+d=1$
16. If $\sin \alpha + \sin \beta = l$, $\cos \alpha + \cos \beta = m$ and $\tan \frac{\alpha}{2} \tan \frac{\beta}{2} = n (n \neq 1)$, then $\frac{1+n}{1-n}$ is equal to
 a. $\frac{l^2 - m^2}{2m}$ b. $\frac{l^2 + m^2}{2m}$
 c. $\frac{l^2 + m^2}{m}$ d. None of these
17. Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^n b_r \sin^r \theta$ for every value of θ , then
 a. $b_0 = 1, b_1 = 3$ b. $b_0 = 0, b_1 = n$
 c. $b_0 = -1, b_1 = n$ d. $b_0 = 0, b_1 = n^2 - n + 3$
18. The value of $3 \left[\sin^4 \left(\frac{3\pi}{2} - \alpha \right) + \sin^4 (3\pi + \alpha) \right] - 2 \left[\sin^6 \left(\frac{\pi}{2} + \alpha \right) + \sin^6 (5\pi - \alpha) \right]$ is
 a. 0 b. 1
 c. 3 d. $\sin 4\alpha + \sin 6\alpha$
19. $\cot \theta + \cot \left(\theta + \frac{\pi}{3} \right) + \cot \left(\theta + \frac{2\pi}{3} \right)$ is equal to
 a. $-\cot 3\theta$ b. $\cot 3\theta$
 c. $2 \cot 3\theta$ d. $3 \cot 3\theta$
20. $\sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8}$ is equal to
 a. 1 b. $\frac{3}{2}$ c. 2 d. $\frac{1}{4}$
21. $\cos \frac{\pi}{11} + \cos \frac{3\pi}{11} + \cos \frac{5\pi}{11} + \cos \frac{7\pi}{11} + \cos \frac{9\pi}{11}$ equals
 a. 0 b. 1
 c. $1/2$ d. $-1/2$
22. The angle of elevation of a stationary cloud from a point 2500 m above a lake is 15° and the angle of depression of its reflection in the lake is 45° . The height of the cloud above the lake level is
 a. $2500\sqrt{3}$ m b. $2500\sqrt{2}$ m c. 1500 m d. $1500\sqrt{3}$ m
23. If $x = \cos 10^\circ \cos 20^\circ \cos 40^\circ$, then x is equal to
 a. $\frac{1}{4} \tan 10^\circ$ b. $\frac{1}{8} \cot 10^\circ$
 c. $\frac{1}{8} \operatorname{cosec} 10^\circ$ d. $\frac{1}{8} \sec 10^\circ$
24. The value of $\tan A + 2 \tan 2A + 4 \tan 4A + 8 \cot 8A$ is
 a. $\cot A$ b. $\tan A$ c. $\cos A$ d. $\sin A$
25. The value of $\sin 6^\circ \sin 42^\circ \sin 66^\circ \sin 78^\circ$ is
 a. $\frac{1}{13}$ b. $\frac{1}{14}$ c. $\frac{1}{15}$ d. $\frac{1}{16}$
26. $\cos 2\theta \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi)$ is equal to
 a. $\sin 2(\theta + \phi)$ b. $\cos 2(\theta + \phi)$
 c. $\sin 2(\theta - \phi)$ d. $\cos 2(\theta - \phi)$
27. The value of $\cos 12^\circ + \cos 84^\circ + \cos 156^\circ + \cos 132^\circ$ is
 a. $1/2$ b. 1 c. $-1/2$ d. $1/8$
28. If $\cos(\theta + \phi) = m \cos(\theta - \phi)$, then $\frac{1-m}{1+m} \cot \phi$ equals
 a. $\tan \theta$ b. $-\tan \theta$
 c. $2 \tan \theta$ d. None of these
29. The value of $\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8}$ is
 a. $\frac{1}{2}$ b. 2 c. 3 d. $\frac{3}{2}$
30. If $\cos x + \cos y - \cos(x+y) = 3/2$, then
 a. $x+y=0$ b. $x=2y$
 c. $x=y$ d. $\cos^2 \left(\frac{x-y}{2} \right) < 1$

31. The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$ can be written as

- a. $\sin A \cos A + 1$ b. $\sec A \operatorname{cosec} A + 1$
c. $\tan A + \cot A$ d. $\sec A + \operatorname{cosec} A$

32. If $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \leq \alpha$

$\beta \leq \frac{\pi}{4}$, then $\tan 2\alpha$ is equal to

- a. $\frac{25}{16}$ b. $\frac{56}{33}$ c. $\frac{19}{12}$ d. $\frac{20}{7}$

33. If A and B are acute positive angles satisfying the equations $3 \sin^2 A + 2 \sin^2 B = 1$ and $3 \sin 2A - 2 \sin 2B = 0$, then $A + 2B$ is equal to

- a. zero b. $\frac{\pi}{2}$ c. $\frac{\pi}{4}$ d. $\frac{\pi}{3}$

34. If the shadow of a pole of height $(\sqrt{3} + 1)$ m standing on the ground is found to be 2 m longer, when the elevation is 30° than when elevation was α , then α is equal to

- a. 15° b. 30° c. 45° d. 75°

35. The number of roots of the equation $3 \sin^2 x = 8 \cos x$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

- a. 1 b. 2 c. 3 d. 4

36. The number of distinct roots of

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0 \text{ in } \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \text{ is}$$

- a. 0 b. 1 c. 2 d. 3

37. If $a \cos 2\theta + b \sin 2\theta = c$ has α and β as its roots, then $\tan \alpha + \tan \beta$ is equal to

- a. $-\frac{2b}{a+c}$ b. $\frac{2b}{a+c}$
c. $\frac{3b}{a+c}$ d. None of these

38. If $2 \sin^2 \theta = 3 \cos \theta$, where $0 \leq \theta \leq 2\pi$, then find the value of θ .

- a. $\frac{2\pi}{3}, \frac{5\pi}{3}$ b. $\frac{\pi}{3}, \frac{5\pi}{3}$
c. $\frac{5\pi}{3}, \frac{4\pi}{3}$ d. None of these

39. If the angles of a triangle are in the ratio 4:1:1, then find the ratio of the longest side to the perimeter of the triangle.

- a. $2\sqrt{3} - 3$ b. $2\sqrt{3} + 3$ c. $3\sqrt{2} + 3$ d. $3\sqrt{2} + 2$

40. If $0 < \theta < 2\pi$, then the intervals of values of θ for which $2 \sin^2 \theta - 5 \sin \theta + 2 > 0$, is

- a. $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ b. $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$
c. $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ d. $\left(\frac{41\pi}{48}, \pi\right)$

41. The values of θ in $\left(0, \frac{\pi}{2}\right)$ satisfying

$$\begin{vmatrix} 1 + \sin^2 \theta & \cos^2 \theta & 4 \sin 4\theta \\ \sin^2 \theta & 1 + \cos^2 \theta & 4 \sin 4\theta \\ \sin^2 \theta & \cos^2 \theta & 1 + 4 \sin 4\theta \end{vmatrix} = 0, \text{ are}$$

a. $\frac{7\pi}{24}$ b. $\frac{5\pi}{24}$ c. $\frac{13\pi}{24}$ d. $\frac{\pi}{24}$

42. In $\triangle ABC$, $a^3 \cos(B - C) + b^3 \cos(C - A) + c^3 \cos(A - B)$ is equal to

- a. $3abc$ b. $2abc$ c. abc d. $5abc$

43. The number of distinct solutions of $\sec x + \tan x = \sqrt{3}$, where $0 \leq x \leq 3\pi$, is

- a. 1 b. 2 c. 3 d. 4

44. The general solution of $\sec 4x - \sec 2x = 2$ is

- a. $\frac{(2n+1)\pi}{10}, \frac{(2m+1)\pi}{2}; n, m \in I$
b. $\frac{(2n+1)\pi}{5}, \frac{(2m+1)\pi}{5}; n, m \in I$
c. $\frac{(2n+2)\pi}{10}, \frac{(2m+4)\pi}{2}; n, m \in I$
d. None of the above

45. If $\sin 3x = 4 \sin x \sin(y + x) \sin(y - x)$, where $0 < x < \pi$, then the number of solutions of this equation in each quadrant, is

- a. 10 b. 5 c. 1 d. 0

46. The general solution of

$$\sin x - 3 \sin 2x + \sin 3x = \cos x - 3 \cos 2x + \cos 3x \text{ is}$$

- a. $n\pi + \frac{\pi}{8}$ b. $\frac{n\pi}{2} + \frac{\pi}{8}$
c. $(-1)^n \frac{n\pi}{2} + \frac{\pi}{8}$ d. $2n\pi + \cos^{-1} \frac{3}{2}$

47. The value of x for $\sin x + \sqrt{3} \cos x = \sqrt{2}$, is

- a. $2n\pi - \frac{5\pi}{12}$ b. $2n\pi - \frac{6\pi}{12}$
c. $2n\pi - \frac{\pi}{12}$ d. $2n\pi + \frac{\pi}{12}$

48. The maximum and minimum values of $6 \sin x \cos x + 4 \cos 2x$ are respectively

- a. 5, 5 b. -5, 5 c. 5, -5 d. -4, 4

49. If $A = \sin^2 \theta + \cos^4 \theta$, then for all real values of θ ,

- a. $1 \leq A \leq 2$ b. $\frac{3}{4} \leq A \leq 1$
c. $\frac{13}{16} \leq A \leq 1$ d. $\frac{3}{4} \leq A \leq \frac{13}{16}$

50. The minimum value of the expression $\sin \alpha + \sin \beta + \sin \gamma$, where α, β, γ are the real numbers satisfying $\alpha + \beta + \gamma = \pi$, is
 a. positive b. zero c. negative d. -3
51. The maximum value of $(\sec^{-1} x)^2 + (\operatorname{cosec}^{-1} x)^2$ is
 a. $\frac{\pi^2}{2}$ b. $\frac{5\pi^2}{4}$
 c. π^2 d. None of these
52. If $x + \frac{1}{x} = 2$, then the principal value of $\sin^{-1} x$ is
 a. $\frac{\pi}{6}$ b. $\frac{\pi}{2}$ c. π d. $\frac{3\pi}{2}$
53. If $x = \sin(2 \tan^{-1} 2)$ and $y = \sin\left(\frac{1}{2} \tan^{-1} \frac{4}{3}\right)$, then
 a. $x > y$ and $y^2 = 1 - x$ b. $x < y$
 c. $x > y$ and $y^2 = x$ d. $y^2 = 1 + x$
54. The numerical value of $\tan\left[2 \tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right]$ is
 a. $\frac{7}{17}$ b. $-\frac{7}{17}$ c. $\frac{17}{7}$ d. $-\frac{17}{7}$
55. The root of the equation $\tan^{-1}\left(\frac{x-1}{x+1}\right) + \tan^{-1}\left(\frac{2x-1}{2x+1}\right) = \tan^{-1}\left(\frac{23}{36}\right)$ is
 a. $-\frac{3}{8}$ b. $-\frac{1}{2}$ c. $\frac{3}{4}$ d. $\frac{4}{3}$
56. If $x^2 + y^2 + z^2 = r^2$, then the value of $\tan^{-1} \frac{xy}{zr} + \tan^{-1} \frac{yz}{xr} + \tan^{-1} \frac{zx}{yr}$ is
 a. π b. $\frac{\pi}{2}$
 c. 0 d. None of these
57. The root of the equation $17x^2 + 17x \tan\left[2 \tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right] - 10 = 0$ is
 a. $\frac{10}{17}$ b. -1 c. $-\frac{7}{17}$ d. 1
58. In $\triangle ABC$, if $2s = a + b + c$, then the value of $\frac{s(s-a)}{bc} - \frac{(s-b)(s-c)}{bc}$ is
 a. $\sin A$ b. $\cos A$
 c. $\tan A$ d. None of these
59. If $\cot^{-1} \sqrt{\cos \alpha} - \tan^{-1} \sqrt{\cos \alpha} = x$, then $\sin x$ equals
 a. $\tan^2\left(\frac{\alpha}{2}\right)$ b. $\cot^2\left(\frac{\alpha}{2}\right)$ c. $\tan \alpha$ d. $\cot\left(\frac{\alpha}{2}\right)$
60. If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \dots\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \dots\right) = \frac{\pi}{2}$ where, $0 < |x| < \sqrt{2}$, then x is equal to
 a. $\frac{1}{2}$ b. 1 c. $-\frac{1}{2}$ d. -1
61. If $\sin^{-1}\left(\frac{4x}{x^2+4}\right) + 2 \tan^{-1}\left(-\frac{x}{2}\right)$ is independent of x , then the value of x is
 a. $-3 \leq x \leq 3$ b. $-2 \leq x \leq 2$
 c. $-1 \leq x \leq 1$ d. $0 \leq x \leq 1$
62. The angular elevation of a tower OP at a point A due South of it is 60° and at a point B due West of A , the elevation is 30° . If $AB = 3$ m, then the height of the tower is
 a. $3\sqrt{3}$ m b. $3\sqrt{14}$ m c. $\frac{3\sqrt{3}}{4}$ m d. $\frac{3\sqrt{6}}{4}$ m
63. The value of $\sin^{-1}\left\{\cos\left(\frac{43\pi}{5}\right)\right\}$ is
 a. $\frac{3\pi}{5}$ b. $-\frac{7\pi}{5}$ c. $\frac{\pi}{10}$ d. $-\frac{\pi}{10}$
64. If $3 \tan^{-1} x + \cot^{-1} x = \pi$, then x is equal to
 a. 0 b. 1 c. -1 d. $\frac{1}{2}$
65. If $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$, then the value of x is
 a. -2 b. -3 c. -1 d. 2
66. The value of $\tan\left\{\cos^{-1}\left(-\frac{2}{7}\right) - \frac{\pi}{2}\right\}$ is
 a. $\frac{2}{3\sqrt{5}}$ b. $\frac{2}{3}$ c. $\frac{1}{\sqrt{5}}$ d. $\frac{4}{5}$
67. If $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}\left(\frac{3}{5}\right)$, then the value of x is
 a. 3 b. 14 c. ± 4 d. 7
68. In $\triangle ABC$, if $\Delta = a^2 - (b-c)^2$, then the value of $\tan A$ is
 a. $\frac{8}{14}$ b. $\frac{8}{13}$ c. $\frac{8}{15}$ d. $\frac{8}{17}$
69. The value of $\sin^{-1} \frac{8}{17} + \sin^{-1} \frac{3}{5}$ is
 a. $\tan^{-1} \frac{77}{35}$ b. $\tan^{-1} \frac{77}{36}$ c. $\tan^{-1} \frac{77}{37}$ d. $\tan^{-1} \frac{77}{26}$
70. If $\alpha = \sin^{-1}\{\cos(\sin^{-1} x)\}$ and $\beta = \cos^{-1}[\sin(\cos^{-1} x)]$, then the value of $\tan \alpha \tan \beta$ is
 a. 1 b. 2 c. 3 d. 4
71. The solution of $\tan\left(\sin^{-1} \frac{3}{5} + \cot^{-1} \frac{3}{2}\right)$ is
 a. $\frac{17}{6}$ b. $\frac{17}{7}$ c. $\frac{17}{5}$ d. $\frac{17}{4}$
72. If $\sin\left(\sin^{-1} \frac{1}{5} + \cos^{-1} x\right) = 1$, then the value of x is
 a. $\frac{1}{4}$ b. $\frac{1}{3}$ c. $\frac{1}{5}$ d. $\frac{1}{2}$

73. The value of $\tan \left\{ \frac{1}{2} \sin^{-1} \left(\frac{2x}{1+x^2} \right) + \frac{1}{2} \cos^{-1} \left(\frac{1-y^2}{1+y^2} \right) \right\}$ is
 a. $\frac{x-y}{1-xy}$ b. $\frac{x-y}{1+xy}$ c. $\frac{x+y}{1+xy}$ d. $\frac{x+y}{1-xy}$
74. The sum of the radii of the circles, which are respectively inscribed and circumscribed about a polygon of n sides, whose side length is a , is
 a. $\frac{1}{2} a \tan \left(\frac{\pi}{2n} \right)$ b. $\frac{1}{2} a \cot \left(\frac{\pi}{2n} \right)$
 c. $\frac{1}{2} \cot \left(\frac{\pi}{3n} \right)$ d. $\frac{1}{2} \cot \left(\frac{\pi}{2n} \right)$
75. If $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$, then the value of x is
 a. $\frac{2\pi}{3}$ b. $\frac{\pi}{4}$ c. $\frac{3\pi}{4}$ d. $\frac{3\pi}{2}$
76. The real solutions of $\tan^{-1} \sqrt{x(x+1)} + \sin^{-1} \sqrt{x^2+x+1} = \frac{\pi}{2}$ is
 a. 2, 3 b. 1, 0 c. -1, 0 d. 3, 1
77. In $\triangle ABC$, if $r_1 = r_2 + r_3 + r$, then triangle is
 a. right angled triangle b. equilateral triangle
 c. isosceles triangle d. None of these
78. The value of $\cot \left(\operatorname{cosec}^{-1} \frac{5}{3} + \tan^{-1} \frac{2}{3} \right)$ is
 a. $\frac{5}{17}$ b. $\frac{6}{17}$
 c. $\frac{3}{17}$ d. $\frac{4}{17}$
79. If $\sin^{-1} \left(\frac{x}{5} \right) + \operatorname{cosec}^{-1} \left(\frac{5}{4} \right) = \frac{\pi}{2}$, then the value of x is
 a. 1 b. 3
 c. 4 d. 5
80. If $\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha$, then $4x^2 - 4xy \cos \alpha + y^2$ is equal to
 a. $-4 \sin^2 \alpha$ b. $4 \sin^2 \alpha$ c. 4 d. $2 \sin 2\alpha$
81. In $\triangle ABC$, if $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$, then the triangle is
 a. right angled b. obtuse angled
 c. equilateral d. isosceles
82. In $\triangle ABC$, if $\angle A : \angle B : \angle C = 3 : 5 : 4$, then $a + b + \sqrt{2}c$ is equal to
 a. $2b$ b. $2c$ c. $3b$ d. $2a$
83. In $\triangle ABC$, if $\angle A = 30^\circ$, $\angle B = 45^\circ$ and $a = 1$, then the values of b and c are respectively
 a. $\sqrt{2}, \frac{\sqrt{3}+1}{\sqrt{2}}$ b. $\sqrt{2}, \frac{\sqrt{3}-1}{\sqrt{2}}$
 c. $\sqrt{3}, \frac{\sqrt{3}-1}{\sqrt{2}}$ d. $\sqrt{2}, \frac{\sqrt{3}+2}{\sqrt{2}}$
84. If $\angle A = 75^\circ$ and $\angle B = 45^\circ$, then $b + c\sqrt{2}$ is equal to
 a. $2a$ b. $2a + 1$ c. $3a$ d. $2a - 1$
85. If a^2, b^2 and c^2 are in AP, then $\cot A, \cot B$ and $\cot C$ are in
 a. AP b. GP c. HP d. AGP
86. In $\triangle ABC$, $\left(\frac{b}{c} + \frac{c}{b} \right) \cos A + \left(\frac{a}{b} + \frac{b}{a} \right) \cos C + \left(\frac{a}{c} + \frac{c}{a} \right) \cos B$ is equal to
 a. 4 b. 5 c. 3 d. 2
87. In $\triangle ABC$, if the tangent of half the difference of two angles is one-third the tangent of half the sum of the angles. Then, the ratio of the sides opposite to the angles is
 a. 2 : 1 b. 1 : 2 c. 3 : 1 d. 1 : 1

BITSAT Archives

1. The number of roots of the equation $|\sin x \cos x| + \sqrt{2 + \tan^2 x + \cot^2 x} = \sqrt{3}$, where $x \in [0, 4\pi]$, is [2014]
 a. 1 b. 2
 c. 3 d. None of these
2. The greatest and least values of $(\sin^{-1} x)^2 + (\cos^{-1} x)^2$ are respectively [2014]
 a. $\frac{\pi^2}{4}$ and 0 b. $\frac{\pi}{2}$ and $-\frac{\pi}{2}$
 c. $\frac{5\pi^2}{4}$ and $\frac{\pi^2}{8}$ d. $\frac{\pi^2}{4}$ and $-\frac{\pi^2}{4}$
3. The number of solutions of $\cos x = |1 + \sin x|$, $0 \leq x \leq 3\pi$, is [2013]
 a. 1 b. 2 c. 3 d. 4
4. The value of the expression $\sin [\cot^{-1} \{ \cos(\tan^{-1} 1) \}]$ is [2013]
 a. 0 b. 1 c. $\frac{1}{\sqrt{3}}$ d. $\frac{\sqrt{2}}{\sqrt{3}}$
5. An object is observed from the points A, B and C lying in a horizontal straight line which passes directly underneath the object. The angular elevation at B is twice that at A and at C three times that at A . If $AB = a, BC = b$, then the height of the object is [2013]

- a. $\frac{3a}{2b} \sqrt{(a+b)(3b-a)}$ b. $\frac{3b}{2a} \sqrt{(a+b)(3a-b)}$
 c. $\frac{a}{2b} \sqrt{(a+b)(3b-a)}$ d. None of these
6. If $\tan 20^\circ = p$, then $\frac{\tan 160^\circ - \tan 110^\circ}{1 + \tan 160^\circ \tan 110^\circ}$ is equal to [2012]
 a. $\left(\frac{1-p^2}{2p}\right)$ b. $\left(\frac{2p}{1+p^2}\right)$ c. $\left(\frac{1+p}{2p}\right)$ d. $\left(\frac{1-p}{2p}\right)$
7. If the angles of a triangle are in AP and the least angle is 30° . Then, the greatest angle (in radian) is [2012]
 a. $\frac{7\pi}{12}$ b. $\frac{2\pi}{3}$ c. $\frac{5\pi}{6}$ d. $\frac{\pi}{2}$
8. If $A + B + C = 180^\circ$, then $\frac{\cot A + \cot B + \cot C}{\cot A \cot B \cot C}$ is equal to [2012]
 a. 1 b. $\cot A \cos B \cot C$
 c. -1 d. 0
9. In $\triangle ABC$, if $a = 2, b = 3$ and $\sin A = \frac{2}{3}$. Then, $\cos C$ is equal to [2012]
 a. $\frac{1}{2}$ b. $\frac{2}{3}$ c. $\frac{2}{\sqrt{13}}$ d. $\frac{1}{\sqrt{13}}$
10. If $4 \sin^{-1} x + \cos^{-1} x = \pi$, then x is equal to [2012]
 a. $\frac{1}{2}$ b. 2 c. 1 d. $\frac{1}{3}$
11. If $\tan \theta = \frac{1}{2}$ and $\tan \phi = \frac{1}{3}$, then the value of $\theta + \phi$ is [2011]
 a. $\frac{\pi}{6}$ b. π c. 0 d. $\frac{\pi}{4}$
12. If $\sin \theta = \frac{1}{2}$ and $\tan \theta = \frac{1}{\sqrt{3}}, \forall n \in I$, then most general values of θ is [2011]
 a. $2n\pi + \frac{\pi}{6}, \forall n \in I$ b. $2n\pi + \frac{\pi}{4}, \forall n \in I$
 c. $2n\pi + \frac{\pi}{3}, \forall n \in I$ d. $2n\pi + \frac{\pi}{3}, \forall n \in I$
13. The function $\sin x + \cos x$ is maximum, when x is equal to [2011]
 a. $\frac{\pi}{6}$ b. $\frac{\pi}{4}$ c. $\frac{\pi}{3}$ d. $\frac{\pi}{2}$
14. The principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is [2011]
 a. $-\frac{2\pi}{3}$ b. $-\frac{\pi}{3}$ c. $\frac{4\pi}{3}$ d. $\frac{5\pi}{3}$
15. If angles A, B and C are in AP, then $\frac{a+C}{b}$ is equal to [2011]
 a. $2 \sin\left(\frac{A-C}{2}\right)$ b. $2 \cos\left(\frac{A-C}{2}\right)$
 c. $\cos\left(\frac{A-C}{2}\right)$ d. $\sin\left(\frac{A-C}{2}\right)$
16. $\left(1 + \cos \frac{\pi}{8}\right)\left(1 + \cos \frac{3\pi}{8}\right)\left(1 + \cos \frac{5\pi}{8}\right)\left(1 + \cos \frac{7\pi}{8}\right)$ is equal to [2010]
 a. $\frac{1}{2}$ b. $\frac{1}{8}$
 c. $\cos \frac{\pi}{8}$ d. $\frac{1+\sqrt{2}}{2\sqrt{2}}$
17. The solution of the equation $\sec \theta - \operatorname{cosec} \theta = \frac{4}{3}$ is [2010]
 a. $\frac{1}{2} \left[n\pi + (-1)^n \sin^{-1} \frac{3}{4} \right]$ b. $\frac{n\pi}{2} + (-1)^n \sin^{-1} \frac{3}{4}$
 c. $n\pi + (-1)^n \sin^{-1} \frac{3}{4}$ d. None of these
18. The minimum value of $9 \tan^2 \theta + 4 \cot^2 \theta$ is [2010]
 a. 13 b. 9
 c. 6 d. 12
19. If $a \leq \sin^{-1} x + \cos^{-1} x + \tan^{-1} x \leq b$, then [2010]
 a. $a = 0, b = \pi$ b. $a = 0, b = \frac{\pi}{2}$
 c. $a = \frac{\pi}{2}, b = \pi$ d. None of these
20. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$ and $\cos^{-1} x - \cos^{-1} y = \frac{\pi}{3}$. Then, (x, y) is equal to [2010]
 a. (0, 1) b. $(1/2, 1)$
 c. $(1, 1/2)$ d. $(\sqrt{3}/2, 1)$
21. The sum of inradius and circumradius of incircle and circumcircle of a regular polygon of side n is [2010]
 a. $\frac{a}{4} \cot \frac{\pi}{2n}$ b. $a \cot \frac{\pi}{n}$
 c. $\frac{a}{2} \cot \frac{\pi}{2n}$ d. $a \cot \frac{\pi}{2n}$
22. In $\triangle ABC$, if $\angle A = \frac{\pi}{2}$, then $\cos^2 B + \cos^2 C$ equals [2009]
 a. -2 b. -1
 c. 1 d. 0
23. In $\triangle ABC$, if $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ and $a = 2$, then area of the triangle is [2009]
 a. 1 sq unit b. 2 sq units
 c. $\frac{\sqrt{3}}{2}$ sq units d. $\sqrt{3}$ sq units
24. A longer side of a parallelogram is 10 cm and the shorter is 6 cm. If the longer diagonal makes an angle 30° with the longer side, then the length of the longer diagonal (in cm) is [2009]
 a. $5\sqrt{3} + \sqrt{11}$
 b. $4\sqrt{3} + \sqrt{11}$
 c. $5\sqrt{3} + \sqrt{13}$
 d. $5\sqrt{3} - 11$

25. $\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ$ is equal to [2008]
 a. 2 b. $2 \sin 20^\circ \cdot \operatorname{cosec} 40^\circ$
 c. 4 d. $4 \sin 20^\circ \cdot \operatorname{cosec} 40^\circ$
26. If $\tan \theta + \tan\left(\theta + \frac{\pi}{3}\right) + \tan\left(\theta + \frac{2\pi}{3}\right) = 3$, then which of the following is equal to 1? [2008]
 a. $\tan 2\theta$ b. $\tan 3\theta$
 c. $\tan^2 \theta$ d. $\tan^3 \theta$
27. $\{x \in R: \cos 2x + 2 \cos^2 x = 2\}$ is equal to [2008]
 a. $\left\{2n\pi + \frac{\pi}{3}: n \in Z\right\}$ b. $\left\{n\pi \pm \frac{\pi}{6}: n \in Z\right\}$
 c. $\left\{n\pi + \frac{\pi}{3}: n \in Z\right\}$ d. $\left\{2n\pi - \frac{\pi}{3}: n \in Z\right\}$
28. If $\sin^{-1}\left(\frac{3}{x}\right) + \sin^{-1}\left(\frac{4}{x}\right) = \frac{\pi}{2}$, then x equal to [2008]
 a. 3 b. 5
 c. 7 d. 11
29. In $\triangle ABC$, if $\frac{1}{b+c} + \frac{1}{c+a} = \frac{3}{a+b+c}$, then C is equal to [2008]
 a. 90° b. 60°
 c. 45° d. 30°
30. In a triangle, if $r_1 = 2r_2 = 3r_3$ then $\frac{a}{b} + \frac{b}{c} + \frac{c}{a}$ is equal to [2008]
 a. $\frac{75}{60}$ b. $\frac{155}{60}$
 c. $\frac{176}{60}$ d. $\frac{191}{60}$
31. From the top of a hill h metre high, the angles of depressions of the top and the bottom of a pillar are α and β , respectively. The height (in metres) of the pillar is [2008]
 a. $\frac{h(\tan \beta - \tan \alpha)}{\tan \beta}$ b. $\frac{h(\tan \alpha - \tan \beta)}{\tan \alpha}$
 c. $\frac{h(\tan \beta + \tan \alpha)}{\tan \beta}$ d. $\frac{h(\tan \beta + \tan \alpha)}{\tan \alpha}$
32. The number of solutions of the equation $1 + \sin x \sin^2 \frac{x}{2} = 0$ in $[-\pi, \pi]$ is [2007]
 a. zero b. one
 c. two d. three
33. The solution set of the equation $\sin^{-1} x = 2 \tan^{-1} x$ is [2007]
 a. $\{1, 2\}$ b. $\{-1, 2\}$
 c. $\{-1, 1, 0\}$ d. $\left\{1, \frac{1}{2}, 0\right\}$
34. If $\sin A + \cos B = a$ and $\sin B + \cos A = b$, then $\sin(A+B)$ is equal to [2007]
 a. $\frac{a^2 + b^2}{2}$ b. $\frac{a^2 - b^2 + 2}{2}$
 c. $\frac{a^2 + b^2 - 2}{2}$ d. None of these
35. In $\triangle ABC$, $(a-b)^2 \cos^2 \frac{C}{2} + (a+b)^2 \sin^2 \frac{C}{2}$ is equal to [2007]
 a. a^2 b. b^2
 c. c^2 d. None of these
36. The horizontal distance between two towers is 60 m and the angle of depression of the top of the first tower as seen from the top to the second is 30° . If the height of the second tower is 150 m, then the height of the first tower is [2007]
 a. 90 m
 b. $(150 - 60\sqrt{3})$ m
 c. $(150 + 20\sqrt{3})$ m
 d. None of the above
37. If $12 \cot^2 \theta - 31 \operatorname{cosec} \theta + 32 = 0$, then the value of $\sin \theta$ is [2006]
 a. $\frac{3}{5}$ or 1 b. $\frac{2}{3}$ or $-\frac{2}{3}$
 c. $\frac{4}{5}$ or $\frac{3}{4}$ d. $\pm \frac{1}{2}$
38. The general solution of $\sin x - \cos x = \sqrt{2}$, for any integer n , is [2006]
 a. $n\pi$ b. $2n\pi + \frac{3\pi}{4}$
 c. $2n\pi$ d. $(2n+1)\pi$
39. In $\triangle ABC$, if the sides are $a=3$, $b=5$ and $c=4$, then $\sin \frac{B}{2} + \cos \frac{B}{2}$ is equal to [2006]
 a. $\sqrt{2}$ b. $\frac{\sqrt{3}+1}{2}$
 c. $\frac{\sqrt{3}-1}{2}$ d. 1
40. If $\sin^{-1}(1-x) - 2 \sin^{-1} x = \pi/2$, then x equals [2005]
 a. $\{0, -1/2\}$ b. $\{1/2, 0\}$
 c. $\{0\}$ d. $\{-1, 0\}$
41. If θ is an acute angle and $\sin \frac{\theta}{2} = \sqrt{\frac{x-1}{2x}}$, then $\tan \theta$ is equal to [2005]
 a. $x^2 - 1$ b. $\sqrt{x^2 - 1}$
 c. $\sqrt{x^2 + 1}$ d. $x^2 + 1$

Answer with Solutions

Practice Exercise

1. (b) Here, $l = 22$ cm and $r = 100$ cm

Using the formula $\theta = \frac{l}{r}$, we get

$$\begin{aligned}\theta &= \frac{22}{100} = \frac{11}{50} \text{ radian} \\ &= \left(\frac{11}{50} \times \frac{180}{\pi} \right)^\circ = \left(\frac{11 \times 18 \times 7}{5 \times 22} \right)^\circ = \left(\frac{63}{5} \right)^\circ = 12^\circ + \left(\frac{3}{5} \right)^\circ \\ &= 12^\circ + \frac{3}{5} \times 60' \quad [\because 1^\circ = 60'] \\ &= 12^\circ 36'\end{aligned}$$

2. (b) $\therefore$ First angle = 60°

$$\text{Second angle} = 60^\circ = 60 \times \frac{90}{100} = 54^\circ$$

$$\text{and third angle} = \frac{5\pi}{6} \text{ radian} = \frac{5 \times 180^\circ}{6} = 150^\circ$$

$$\begin{aligned}\therefore \text{Fourth angle} &= 360^\circ - (60^\circ + 54^\circ + 150^\circ) \\ &= 360^\circ - 264^\circ = 96^\circ\end{aligned}$$

3. (b) $\therefore 1 \text{ radian} = \frac{180^\circ}{\pi} = 57^\circ 30'$ (approx.)

[since, $\sin \theta$ is increasing from 0 to 90°]

$$\therefore \sin 1 > \sin 1^\circ$$

4. (a) By Pythagoras theorem,

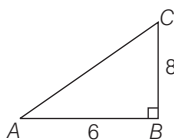
$$AC^2 = AB^2 + BC^2 = 6^2 + 8^2 = 36 + 64 = 100$$

$$\Rightarrow AC = 10$$

$$\therefore \sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{8}{10} = \frac{4}{5}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{6}{10} = \frac{3}{5}$$

$$\text{and } \tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} = \frac{8}{6} = \frac{4}{3}$$



5. (c) We have, $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$

$$\begin{aligned}&= (\tan 9^\circ + \tan 81^\circ) - (\tan 27^\circ + \tan 63^\circ) \\ &= \frac{1}{\cos 9^\circ \cos 81^\circ} - \frac{1}{\cos 27^\circ \cos 63^\circ} \\ &= \frac{1}{\sin 9^\circ \cos 9^\circ} - \frac{1}{\sin 27^\circ \cos 27^\circ} \\ &= \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ} = 2 \left\{ \frac{\sin 54^\circ - \sin 18^\circ}{\sin 18^\circ \sin 54^\circ} \right\} \\ &= 2 \left[\frac{2 \cos 36^\circ \sin 18^\circ}{\sin 18^\circ \cos 36^\circ} \right] = 4\end{aligned}$$

6. (d) $\log_3 \tan 1^\circ + \log_3 \tan 2^\circ + \dots + \log_3 \tan 89^\circ$

$$= \log_3 \tan 1^\circ \tan 2^\circ \dots \tan 89^\circ$$

$$= \log_3 (\tan 1^\circ \tan 2^\circ \dots \tan 45^\circ)$$

$$(\tan 46^\circ \tan 47^\circ \dots \tan 89^\circ)$$

$$\begin{aligned}&= \log_3 (\tan 1^\circ \tan 2^\circ \dots \tan 45^\circ) \times \cot 44^\circ \\ &\quad \times \cot 43^\circ \dots \cot 1^\circ \\ &= \log_3 (\tan 1^\circ \tan 2^\circ \dots \tan 44^\circ) \times \tan 45^\circ \\ &\quad \times \frac{1}{\tan 44^\circ \tan 43^\circ \dots \tan 1^\circ} \\ &= \log_3 (1) = 0\end{aligned}$$

7. (d) Since, the distance of the foot from each vertex is $h \cot \alpha$.

So the foot is at the circumcentre of triangle.

$$\therefore R = h \cot \alpha$$

$$\Rightarrow h = R \tan \alpha = \frac{a}{2 \sin A} \tan \alpha = \frac{1}{2} a \tan \alpha \cdot \operatorname{cosec} A$$

8. (b) Let $\sin A = x$

$$\text{Then, } \cos A = \tan B$$

$$\Rightarrow \sqrt{1 - \sin^2 A} = \tan B$$

$$\Rightarrow \sqrt{1 - x^2} = \tan B$$

$$\text{Now, } \cos B = \tan C \Rightarrow \frac{1}{\sqrt{1 + \tan^2 B}} = \tan C$$

$$\Rightarrow \frac{1}{\sqrt{2 - x^2}} = \tan C$$

$$\Rightarrow \cos C = \frac{1}{\sqrt{1 + \tan^2 C}} = \sqrt{\frac{2 - x^2}{3 - x^2}}$$

$$\text{Now, } \cos C = \tan A \Rightarrow \sqrt{\frac{2 - x^2}{3 - x^2}} = \frac{x}{1 - x^2}$$

$$\Rightarrow x^2 = \frac{(1 \pm \sqrt{5})^2}{4} \Rightarrow x = \frac{\sqrt{5} - 1}{2} = 2 \sin 18^\circ$$

9. (b) Given, $\sin 18^\circ = \frac{\sqrt{5} - 1}{4}$

$$\begin{aligned}\text{Then, } \cos 18^\circ &= \sqrt{1 - \left(\frac{\sqrt{5} - 1}{4} \right)^2} = \sqrt{\frac{16 - (5 + 1 - 2\sqrt{5})}{16}} \\ &= \frac{\sqrt{10 + 2\sqrt{5}}}{4}\end{aligned}$$

$$\therefore \sin 81^\circ = \sin(90^\circ - 9^\circ) = \cos 9^\circ$$

$$\begin{aligned}\sqrt{\frac{1 + \cos 18^\circ}{2}} &= \sqrt{\frac{1 + \frac{\sqrt{10 + 2\sqrt{5}}}{4}}{2}} = \sqrt{\frac{4 + \sqrt{10 + 2\sqrt{5}}}{8}} \\ &= \sqrt{\frac{8 + 2\sqrt{10 + 2\sqrt{5}}}{16}} = \sqrt{\frac{8 + 2\sqrt{(3 + \sqrt{5})(5 - \sqrt{5})}}{16}} \\ &\quad [\because 10 + 2\sqrt{5} = (3 + \sqrt{5})(5 - \sqrt{5})] \\ &= \sqrt{\frac{\sqrt{(3 + \sqrt{5})^2} + \sqrt{(5 - \sqrt{5})^2} + 2\sqrt{(3 + \sqrt{5})(5 - \sqrt{5})}}{16}} \\ &= \frac{\sqrt{3 + \sqrt{5}} + \sqrt{5 - \sqrt{5}}}{4}\end{aligned}$$

$$10. (c) \cos^2 \theta = \frac{(x+y)^2}{4xy}$$

$$\text{Since, } \cos^2 \theta \leq 1$$

$$\therefore \frac{(x+y)^2}{4xy} \leq 1$$

$$\Rightarrow (x+y)^2 \leq 4xy \Rightarrow (x-y)^2 \leq 0$$

$$\Rightarrow x=y=0 \Rightarrow x=y$$

$$11. (b) \therefore \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\therefore (1 - \tan A \tan B) = \frac{\tan A + \tan B}{\tan(A+B)}$$

$$\text{If } (A+B) = 45^\circ, \text{ then } \tan(A+B) = 1$$

$$\text{So, for such case, } 1 - \tan A \tan B = \tan A + \tan B$$

$$\Rightarrow \tan A + \tan B + \tan A \tan B = 1$$

$$\Rightarrow 1 + \tan A + \tan B + \tan A \tan B = 1 + 1$$

$$\Rightarrow (1 + \tan A)(1 + \tan B) = 2$$

$$\therefore (1 + \tan 1^\circ)(1 + \tan 44^\circ)(1 + \tan 2^\circ)(1 + \tan 43^\circ) \\ + \dots + (1 + \tan 22^\circ)(1 + \tan 23^\circ) = 2^{22}$$

$$12. (d) \tan 45^\circ = \tan(10^\circ + 35^\circ)$$

$$1 = \frac{\tan 10^\circ + \tan 35^\circ}{1 - \tan 10^\circ \tan 35^\circ}$$

$$\Rightarrow 1 - \tan 10^\circ \tan 35^\circ = \tan 10^\circ + \tan 35^\circ$$

$$\Rightarrow \tan 10^\circ + \tan 35^\circ + \tan 10^\circ \tan 35^\circ = 1$$

$$13. (c) \text{ Given, } \alpha + \beta = \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{2} - \beta$$

Taking tan on both sides, we get

$$\tan \alpha = \tan\left(\frac{\pi}{2} - \beta\right)$$

$$\Rightarrow \tan \alpha = \cot \beta \Rightarrow \tan \alpha \tan \beta = 1$$

$$\text{Again, } \beta + \gamma = \alpha$$

$$\Rightarrow \gamma = (\alpha - \beta)$$

$$\Rightarrow \tan \gamma = \tan(\alpha - \beta)$$

$$\Rightarrow \tan \gamma = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\Rightarrow \tan \gamma = \frac{\tan \alpha - \tan \beta}{1 + 1}$$

$$2 \tan \gamma = \tan \alpha - \tan \beta$$

$$\tan \alpha = \tan \beta + 2 \tan \gamma$$

$$14. (a) \text{ Given, } 2 \cos x + 2 \cos 3x = \cos y$$

$$\Rightarrow \cos x + \cos 3x = \frac{1}{2} \cos y \quad \dots(i)$$

$$\text{and } 2 \sin x + 2 \sin 3x = \sin y$$

$$\Rightarrow \sin x + \sin 3x = \frac{1}{2} \sin y \quad \dots(ii)$$

On squaring and adding Eqs. (i) and (ii), we get

$$\cos^2 x + \cos^2 3x + \sin^2 x + \sin^2 3x + 2 \cos 3x \cos x$$

$$+ 2 \sin 3x \sin x = \frac{1}{4} (\cos^2 y + \sin^2 y)$$

$$\Rightarrow \cos^2 x + \sin^2 x + \cos^2 3x + \sin^2 3x + 2(\cos 3x \cos x \\ + \sin 3x \sin x) = \frac{1}{4} (\cos^2 y + \sin^2 y)$$

$$\Rightarrow 1 + 1 + 2 \cos(3x - x) = \frac{1}{4}$$

$$\Rightarrow 2 \cos 2x = \frac{1}{4} - 2 = -\frac{7}{4}$$

$$\Rightarrow \cos 2x = -\frac{7}{4 \times 2}$$

$$\Rightarrow \cos 2x = -\frac{7}{8}$$

$$15. (d) \text{ We have, } \cos 5\theta = \cos(3\theta + 2\theta)$$

$$= \cos 3\theta \cos 2\theta - \sin 3\theta \sin 2\theta$$

$$= (4 \cos^3 \theta - 3 \cos \theta)(2 \cos^2 \theta - 1)$$

$$- (3 \sin \theta - 4 \sin^3 \theta) 2 \sin \theta \cos \theta$$

$$= 8 \cos^5 \theta - 10 \cos^3 \theta + 3 \cos \theta$$

$$- 2 \sin^2 \theta (3 - 4 \sin^2 \theta) \cos \theta$$

$$= 8 \cos^5 \theta - 10 \cos^3 \theta + 3 \cos \theta$$

$$- 2 \cos \theta (1 - \cos^2 \theta) [3 - 4 + 4 \cos^2 \theta]$$

$$= 8 \cos^5 \theta - 10 \cos^3 \theta + 3 \cos \theta$$

$$- 2 \cos \theta (4 \cos^2 \theta - 1)(1 - \cos^2 \theta)$$

$$= 8 \cos^5 \theta - 10 \cos^3 \theta + 3 \cos \theta$$

$$- 2 \cos \theta [4 \cos^2 \theta - 4 \cos^4 \theta - 1 + \cos^2 \theta]$$

$$= 16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta$$

$$= a \cos \theta + b \cos^3 \theta + c \cos^5 \theta + d \quad [\text{given}]$$

$$\therefore a = 5, b = -20, c = 16, d = 0$$

$$\text{Hence, } a + b + c + d = 5 - 20 + 16 + 0 = 1$$

$$16. (b) \frac{1+n}{1-n} = \frac{1 + \tan \frac{\alpha}{2} \tan \frac{\beta}{2}}{1 - \tan \frac{\alpha}{2} \tan \frac{\beta}{2}} \\ = \frac{\cos \frac{\alpha}{2} \cos \frac{\beta}{2} + \sin \frac{\alpha}{2} \sin \frac{\beta}{2}}{\cos \frac{\alpha}{2} \cos \frac{\beta}{2} - \sin \frac{\alpha}{2} \sin \frac{\beta}{2}} = \frac{\cos\left(\frac{\alpha - \beta}{2}\right)}{\cos\left(\frac{\alpha + \beta}{2}\right)}$$

$$\text{Also, } l^2 + m^2 = (\sin \alpha + \sin \beta)^2 + (\cos \alpha + \cos \beta)^2 \\ = 1 + 1 + 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta) \\ = 2 + 2 \cos(\alpha - \beta) = 2[1 + \cos(\alpha - \beta)] \\ = 2 \cdot 2 \cos^2\left(\frac{\alpha - \beta}{2}\right) = 4 \cos^2\left(\frac{\alpha - \beta}{2}\right)$$

$$\text{Also, } m = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\therefore \frac{l^2 + m^2}{2m} = \frac{4 \cos^2\left(\frac{\alpha - \beta}{2}\right)}{2 \cdot 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)} \\ = \frac{\cos\left(\frac{\alpha - \beta}{2}\right)}{\cos\left(\frac{\alpha + \beta}{2}\right)} = \frac{1+n}{1-n}$$

17. (b) $\sin n\theta = \sum_{r=0}^n b_r \sin^r \theta$, for every value of θ .

Putting $\theta = 0$, we get $b_0 = 0$

Putting $n = 1$, we get

$$\sin \theta = b_0 + b_1 \sin \theta = 0 + b_1 \sin \theta$$

$$\therefore b_1 = 1$$

Putting $n = 3$, we get

$$\sin 3\theta = b_0 + b_1 \sin \theta + b_2 \sin^2 \theta + b_3 \sin^3 \theta$$

$$\Rightarrow 3 \sin \theta - 4 \sin^3 \theta = b_1 \sin \theta + b_2 \sin^2 \theta + b_3 \sin^3 \theta$$

$$\Rightarrow 3 - 4 \sin^2 \theta = b_1 + b_2 \sin \theta + b_3 \sin^2 \theta$$

Putting $\theta = 0$, we get $b_1 = 3$

Similarly, for $n = 5$, $b_1 = 5$

Hence, $b_1 = n$ and $b_0 = 0$

18. (b) We have, $3 \left[\sin^4 \left(\frac{3\pi}{2} - \alpha \right) + \sin^4 (3\pi + \alpha) \right]$
 $- 2 \left[\sin^6 \left(\frac{\pi}{2} + \alpha \right) + \sin^6 (5\pi - \alpha) \right]$
 $= 3 [(-\cos \alpha)^4 + (-\sin \alpha)^4] - 2 [\cos^6 \alpha + \sin^6 \alpha]$
 $= 3 [(\cos^2 \alpha + \sin^2 \alpha)^2 - 2 \sin^2 \alpha \cos^2 \alpha]$
 $- 2 [(\cos^2 \alpha + \sin^2 \alpha)^3 - 3 \cos^2 \alpha \sin^2 \alpha (\cos^2 \alpha + \sin^2 \alpha)]$
 $= 3 - 6 \sin^2 \alpha \cos^2 \alpha - 2 + 6 \sin^2 \alpha \cos^2 \alpha = 3 - 2 = 1$

19. (d) Let $t = \tan \theta$

$$\therefore \cot \theta + \cot \left(\theta + \frac{\pi}{3} \right) + \cot \left(\theta + \frac{2\pi}{3} \right)$$

$$= \cot \theta + \frac{\cot \theta \cot \frac{\pi}{3} - 1}{\cot \theta + \cot \frac{\pi}{3}} + \frac{\cot \theta \cot \frac{2\pi}{3} - 1}{\cot \theta + \cot \frac{2\pi}{3}}$$

$$= \frac{1}{t} + \frac{1 - \sqrt{3}t}{t + \sqrt{3}} + \frac{1 + \sqrt{3}t}{t - \sqrt{3}} = \frac{3(1 - 3t^2)}{t(3 - t^2)} = \frac{3}{\tan 3\theta}$$

$$= 3 \cot 3\theta$$

20. (b) $\sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} = \sin^4 \frac{\pi}{8} + \cos^4 \frac{\pi}{8} = 1 - \frac{1}{2} \sin^2 \frac{\pi}{4} = \frac{3}{4}$

$$\text{Similarly, } \sin^4 \frac{7\pi}{8} + \sin^4 \frac{5\pi}{8} = \frac{3}{4}$$

$$\therefore \sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8} = \frac{3}{2}$$

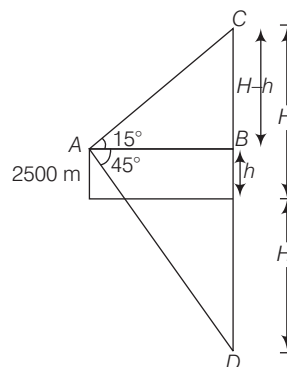
21. (c) On multiplying numerator and denominator by $2 \sin \frac{\pi}{11}$, we get

$$\frac{1}{2 \sin \frac{\pi}{11}} \left[\sin \frac{2\pi}{11} + \left(\sin \frac{4\pi}{11} - \sin \frac{2\pi}{11} \right) \right]$$

$$+ \dots + \left(\sin \frac{10\pi}{11} - \sin \frac{8\pi}{11} \right)$$

$$= \frac{\sin \frac{10\pi}{11}}{2 \sin \frac{\pi}{11}} = \frac{1}{2}$$

22. (a) In $\triangle ABC$, $\tan 15^\circ = \frac{H-h}{AB}$



$$\Rightarrow AB = (H-h) \cot 15^\circ \quad \dots(i)$$

In $\triangle ABD$,

$$\tan 45^\circ = \frac{H+h}{AB}$$

$$\Rightarrow AB = (H+h) \cot 45^\circ \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$(H-h) \cot 15^\circ = (H+h) \cot 45^\circ$$

$$\Rightarrow H(\cot 15^\circ - \cot 45^\circ) = h(\cot 15^\circ + \cot 45^\circ)$$

$$\Rightarrow H = \frac{h(\cot 15^\circ + 1)}{(\cot 15^\circ - 1)}$$

Given that, $h = 2500$ m and we know that,

$$\cot 15^\circ = 2 + \sqrt{3}$$

$$\therefore H = \frac{2500(2 + \sqrt{3} + 1)}{2 + \sqrt{3} - 1}$$

$$\Rightarrow H = 2500\sqrt{3} \text{ m}$$

23. (b) Given, $x = \cos 10^\circ \cos 20^\circ \cos 40^\circ$

$$\therefore 2x \sin 10^\circ = 2 \sin 10^\circ \cos 10^\circ \cos 20^\circ \cos 40^\circ$$

$$= \sin 20^\circ \cos 20^\circ \cos 40^\circ$$

$$= \frac{1}{2} (2 \sin 20^\circ \cos 20^\circ) \cos 40^\circ$$

$$= \frac{1}{2} (\sin 40^\circ) \cos 40^\circ = \frac{1}{4} (2 \sin 40^\circ \cos 40^\circ)$$

$$= \frac{1}{4} (\sin 80^\circ)$$

$$\therefore x = \frac{1}{8} \cdot \frac{\sin 80^\circ}{\sin 10^\circ} = \frac{\sin(90^\circ - 10^\circ)}{8 \sin 10^\circ} = \frac{1}{8} \cdot \frac{\cos 10^\circ}{\sin 10^\circ} = \frac{1}{8} \cot 10^\circ$$

24. (a) $\tan A + 2 \tan 2A + 4 \tan 4A + 8 \left(\frac{1 - \tan^2 4A}{2 \tan 4A} \right)$
- $$= \tan A + 2 \tan 2A + \left(\frac{4 \tan^2 4A + 4 - 4 \tan^2 4A}{\tan 4A} \right)$$
- $$= \tan A + 2 \tan 2A + 4 \cot 4A$$
- $$= \tan A + 2 \tan 2A + 4 \left(\frac{1 - \tan^2 2A}{2 \tan 2A} \right)$$
- $$= \tan A + \left[\frac{2 \tan^2 2A + 2 - 2 \tan^2 2A}{\tan 2A} \right]$$

$$= \tan A + 2 \cot 2A = \tan A + 2 \left(\frac{1 - \tan^2 A}{2 \tan A} \right)$$

$$= \frac{\tan^2 A + 1 - \tan^2 A}{\tan A} = \cot A$$

25. (d) $\sin 6^\circ \sin 42^\circ \sin 66^\circ \sin 78^\circ$

$$= \sin 6^\circ \cos 48^\circ \cos 24^\circ \cos 12^\circ$$

$$= \sin 6^\circ \frac{2^3 \sin 12^\circ \cos 12^\circ \cos 24^\circ \cos 48^\circ}{2^3 \sin 12^\circ}$$

$$= \sin 6^\circ \cdot \frac{\sin 96^\circ}{2^3 \sin 12^\circ} \quad [\because \sin 2A = 2 \sin A \cos A]$$

$$= \frac{2 \sin 6^\circ \cos 6^\circ}{2^4 \sin 12^\circ} = \frac{\sin 12^\circ}{2^4 \sin 12^\circ} = \frac{1}{16}$$

26. (b) $\cos 2\theta \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi)$

$$= \cos 2\theta \cos 2\phi + \sin(\theta - \phi + \theta + \phi) \sin\{\theta - \phi - (\theta + \phi)\}$$

$$[\because \sin^2 A - \sin^2 B = \sin(A + B) \sin(A - B)]$$

$$= \cos 2\theta \cos 2\phi + \sin 2\theta \sin(-2\phi)$$

$$= \cos 2\theta \cos 2\phi - \sin 2\theta \sin 2\phi$$

$$= \cos(2\theta + 2\phi) = \cos 2(\theta + \phi)$$

27. (c) $\cos 12^\circ + \cos 84^\circ + \cos 156^\circ + \cos 132^\circ$

$$= \cos 156^\circ + \cos 84^\circ + \cos 132^\circ + \cos 12^\circ$$

$$= 2 \cos \left(\frac{156^\circ + 84^\circ}{2} \right) \cos \left(\frac{156^\circ - 84^\circ}{2} \right)$$

$$+ 2 \cos \left(\frac{132^\circ + 12^\circ}{2} \right) \cos \left(\frac{132^\circ - 12^\circ}{2} \right)$$

$$= 2 \cos 120^\circ \cos 36^\circ + 2 \cos 72^\circ \cos 60^\circ$$

$$= 2 \left(-\frac{1}{2} \right) \cos 36^\circ + 2 \cos 72^\circ \times \frac{1}{2}$$

$$= -\cos 36^\circ + \cos 72^\circ = -\frac{\sqrt{5} + 1}{4} + \frac{\sqrt{5} - 1}{4} = -\frac{1}{2}$$

28. (a) We have, $\frac{1}{m} = \frac{\cos(\theta - \phi)}{\cos(\theta + \phi)}$

On applying componendo and dividendo rule, we get

$$\frac{1 - m}{1 + m} = \frac{\cos(\theta - \phi) - \cos(\theta + \phi)}{\cos(\theta - \phi) + \cos(\theta + \phi)}$$

$$= \frac{2 \sin \theta \sin \phi}{2 \cos \theta \cos(-\phi)} = \tan \theta \tan \phi$$

$$\therefore \left(\frac{1 - m}{1 + m} \right) \cot \phi = \tan \theta$$

29. (d) Hint $\cos \frac{7\pi}{8} = \cos \left(\pi - \frac{\pi}{8} \right)$ and

$$\cos \frac{5\pi}{8} = \cos \left(\pi - \frac{3\pi}{8} \right)$$

$$\Rightarrow \cos \frac{7\pi}{8} = -\cos \frac{\pi}{8} \text{ and } \cos \frac{5\pi}{8} = -\cos \frac{3\pi}{8}$$

$$\text{Now, } \cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{\pi}{8}$$

$$= 2 \cos^4 \frac{\pi}{8} + 2 \cos^4 \frac{3\pi}{8} = 2 \left[\cos^2 \frac{\pi}{8} \right]^2 + 2 \left[\cos^2 \frac{3\pi}{8} \right]^2$$

$$= \frac{2}{4} \left[1 + \cos \frac{\pi}{4} \right]^2 + \frac{2}{4} \left[1 + \cos \frac{3\pi}{4} \right]^2$$

30. (c) Given, $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$

$$\Rightarrow 2 \cos \left(\frac{x + y}{2} \right) \cos \left(\frac{x - y}{2} \right) - 2 \cos^2 \left(\frac{x + y}{2} \right) + 1 = \frac{3}{2}$$

$$\Rightarrow 2 \cos^2 \left(\frac{x + y}{2} \right) - 2 \cos \left(\frac{x + y}{2} \right) \cos \left(\frac{x - y}{2} \right) + \frac{1}{2} = 0$$

Now, $\cos \left(\frac{x + y}{2} \right)$ is always real, then discriminant ≥ 0

$$\Rightarrow 4 \cos^2 \left(\frac{x - y}{2} \right) - 4 \geq 0 \Rightarrow \cos^2 \left(\frac{x - y}{2} \right) \geq 1$$

$$\Rightarrow \cos^2 \left(\frac{x - y}{2} \right) = 1 \Rightarrow \frac{x - y}{2} = 0$$

$$\Rightarrow x = y$$

31. (b) Given expression is

$$\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A} = \frac{\sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A}$$

$$+ \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A - \sin A}$$

$$= \frac{1}{\sin A - \cos A} \left\{ \frac{\sin^3 A - \cos^3 A}{\cos A \sin A} \right\}$$

$$= \frac{\sin^2 A + \sin A \cos A + \cos^2 A}{\sin A \cos A}$$

$$= \frac{1 + \sin A \cos A}{\sin A \cos A} = 1 + \sec A \operatorname{cosec} A$$

32. (b) Given, $\cos(\alpha + \beta) = \frac{4}{5} \Rightarrow \alpha + \beta \in 1^{\text{st}} \text{ quadrant}$

and $\sin(\alpha - \beta) = \frac{5}{13} \Rightarrow \alpha - \beta \in 1^{\text{st}} \text{ quadrant}$

$$\Rightarrow 2\alpha = (\alpha + \beta) + (\alpha - \beta)$$

$$\therefore \tan 2\alpha = \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \tan(\alpha - \beta)} = \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}} = \frac{56}{33}$$

33. (b) We have, $3 \sin^2 A + 2 \sin^2 B = 1$

$$\Rightarrow \cos 2B = 3 \sin^2 A$$

$$\therefore \cos(A + 2B) = \cos A \cos 2B - \sin A \sin 2B$$

$$= \cos A \cdot 3 \sin^2 A - \sin A \cdot \frac{3}{2} \sin 2A = 0$$

$$[\because 3 \sin 2A - 2 \sin 2B = 0]$$

$$\Rightarrow A + 2B = \frac{\pi}{2}$$

34. (c) Let OP be a tower with height $(\sqrt{3} + 1)$ m and

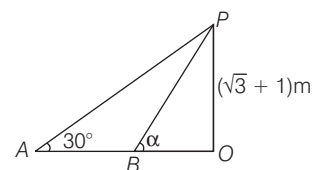
$$AB = 2 \text{ m,}$$

In $\triangle AOP$,

$$\tan 30^\circ = \frac{\sqrt{3} + 1}{OA} \Rightarrow OA = (\sqrt{3} + 1) \sqrt{3}$$

In $\triangle BOP$, $\tan \alpha = \frac{\sqrt{3} + 1}{OB}$

$$\Rightarrow OB = (\sqrt{3} + 1) \cot \alpha$$



Now, $OA - OB = (3 + \sqrt{3}) - (\sqrt{3} + 1) \cot \alpha$

$$\Rightarrow 2 = 3 + \sqrt{3} - (\sqrt{3} + 1) \cot \alpha \Rightarrow \cot \alpha = 1$$

$$\therefore \alpha = 45^\circ$$

35. (b) $\therefore 3 \cos^2 x + 8 \cos x - 3 = 0$

$$\therefore (3 \cos x - 1)(\cos x + 3) = 0$$

$$\Rightarrow \cos x = \frac{1}{3} \quad [\because \cos x \neq -3]$$

Hence, two solutions exist in a given interval.

36. (b) Expanding the given determinant, we get

$$\sin x (\sin^2 x - \cos^2 x) - \cos x (\cos x \sin x - \cos^2 x) + \cos x (\cos^2 x - \sin^2 x) = 0$$

$$\Rightarrow \sin^3 x - 3 \sin x \cos^2 x + 2 \cos^3 x = 0$$

$$\Rightarrow (\sin x - \cos x)^2 (\sin x + 2 \cos x) = 0$$

$$\Rightarrow \sin x + 2 \cos x = 0$$

$$\Rightarrow \tan x = -2, x \in \left[-\frac{\pi}{4}, \frac{\pi}{4} \right]$$

$$\Rightarrow \sin x - \cos x = 0 \Rightarrow x = \frac{\pi}{4}$$

37. (b) Given, $a \cos 2\theta + b \sin 2\theta = c$

$$\therefore a \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right) + b \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) = c$$

$$\Rightarrow a - a \tan^2 \theta + 2b \tan \theta = c (1 + \tan^2 \theta)$$

$$\Rightarrow \tan^2 \theta (a + c) - 2b \tan \theta + c - a = 0$$

Since, α and β are the roots of the above equation.

$$\therefore \tan \alpha + \tan \beta = \frac{2b}{a + c}$$

38. (b) Given, $2 \sin^2 \theta = 3 \cos \theta$

$$\therefore 2(1 - \cos^2 \theta) = 3 \cos \theta$$

$$\Rightarrow 2 \cos^2 \theta + 3 \cos \theta - 2 = 0$$

$$\Rightarrow (\cos \theta + 2)(2 \cos \theta - 1) = 0$$

$$\Rightarrow \cos \theta = -2, \cos \theta = \frac{1}{2}$$

But $\cos \theta \in [-1, 1]$

$$\therefore \cos \theta \neq -2$$

Now, $\cos \theta = \frac{1}{2}$

As $\theta \in [0, 2\pi]$

$$\therefore \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

39. (a) Given, $A : B : C = 4 : 1 : 1$

$$\therefore A + B + C = 180^\circ$$

$$\therefore A = 120^\circ, B = 30^\circ \text{ and } C = 30^\circ$$

Using sine rule,

$$a : b : c = \sin A : \sin B : \sin C = \frac{\sqrt{3}}{2} : \frac{1}{2} : \frac{1}{2} = \sqrt{3} : 1 : 1$$

$\therefore$ Ratio of longest side to the perimeter of triangle

$$= \frac{a}{a + b + c} = \frac{\sqrt{3}}{\sqrt{3} + 1 + 1} = 2\sqrt{3} - 3$$

40. (a) $2 \sin^2 \theta - 5 \sin \theta + 2 = (\sin \theta - 2)(2 \sin \theta - 1) > 0$

$$\Rightarrow \sin \theta < \frac{1}{2} \Rightarrow \theta \in \left(0, \frac{\pi}{6} \right) \cup \left(\frac{5\pi}{6}, 2\pi \right)$$

41. (a) Applying $C_1 \rightarrow C_1 + C_2$

$$\begin{vmatrix} 2 & \cos^2 \theta & 4 \sin 4\theta \\ 2 & 1 + \cos^2 \theta & 4 \sin 4\theta \\ 1 & \cos^2 \theta & 1 + 4 \sin 4\theta \end{vmatrix} = 0$$

Applying $R_2 \rightarrow R_2 - R_1$,

$$\begin{vmatrix} 2 & \cos^2 \theta & 4 \sin 4\theta \\ 0 & 1 & 0 \\ 1 & \cos^2 \theta & 1 + 4 \sin 4\theta \end{vmatrix} = 0$$

$$\Rightarrow 2 + 4 \sin 4\theta = 0$$

$$\Rightarrow \sin 4\theta = -\frac{1}{2} = \sin \left(-\frac{\pi}{6} \right)$$

$$\therefore 4\theta = n\pi + (-1)^n \left(-\frac{\pi}{6} \right)$$

When $n = 1 \Rightarrow \theta = \frac{7\pi}{24}$

When $n = 2 \Rightarrow \theta = \frac{11\pi}{24}$

42. (a) By sine rule, $a = 2R \sin A$

$$\therefore a^2 (2R \sin A) \cos (B - C) + b^2 (2R \sin B) \cos (C - A) + c^2 (2R \sin C) \cos (A - B)$$

$$= a^2 [2R \sin(B + C) \cos(B - C)] + b^2 [2R \sin(A + C) \cos(C - A)] + c^2 [2R \sin(A + B) \cos(A - B)]$$

$$[\because A = 180^\circ - (B + C)]$$

$$= a^2 R [\sin 2B + \sin 2C] + b^2 R [\sin 2A + \sin 2C] + c^2 R [\sin 2A + \sin 2B]$$

$$= a^2 R (2 \sin B \cdot \cos B + 2 \sin C \cdot \cos C)$$

$$+ b^2 R (2 \sin A \cdot \cos A + 2 \sin C \cdot \cos C)$$

$$+ c^2 R (2 \sin A \cdot \cos A + 2 \sin B \cdot \cos B)$$

$$= a^2 (b \cos B + c \cos C) + b^2 (c \cos C + a \cos A)$$

$$+ c^2 (a \cos A + b \cos B)$$

$$= ab (a \cos B + b \cos A) + bc (b \cos C + c \cos B)$$

$$+ ac (c \cos A + a \cos C)$$

$$= abc + abc + abc = 3abc$$

43. (b) Given, $\sec x + \tan x = \sqrt{3}$

$$\Rightarrow 1 + \sin x = \sqrt{3} \cos x \Rightarrow \sqrt{3} \cos x - \sin x = 1$$

On dividing both sides by $\sqrt{a^2 + b^2}$, i.e. $\sqrt{4} = 2$, we get

$$\Rightarrow \frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x = \frac{1}{2}$$

$$\Rightarrow \cos \frac{\pi}{6} \cos x - \sin \frac{\pi}{6} \sin x = \frac{1}{2}$$

$$\Rightarrow \cos \left(x + \frac{\pi}{6} \right) = \cos \left(\frac{\pi}{3} \right) \Rightarrow x + \frac{\pi}{6} = 2m\pi \pm \frac{\pi}{3}$$

$$\Rightarrow x = 2n\pi + \frac{\pi}{6}, 2n\pi - \frac{\pi}{2}$$

When $x = 2n\pi + \frac{\pi}{6}$, there are solutions for $n = 0, 1$

and when $x = 2n\pi - \frac{\pi}{2}$, it does not satisfy the equations.

$\therefore$ Total number of solutions is 2.

$$44. (a) \text{ Given, } \sec 4x - \sec 2x = 2 \Rightarrow \frac{1}{\cos 4x} - \frac{1}{\cos 2x} = 2$$

$$\Rightarrow \cos 2x - \cos 4x = 2 \cos 2x \cos 4x$$

$$[\because \cos 2x \neq 0, \cos 4x \neq 0]$$

$$\Rightarrow \cos 2x - \cos 4x = \cos(4x + 2x) + \cos(4x - 2x)$$

$$= \cos 6x + \cos 2x$$

$$\Rightarrow \cos 6x + \cos 4x = 0 \Rightarrow 2 \cos 5x \cos x = 0$$

$$\text{Either } \cos 5x = 0 \text{ or } \cos x = 0$$

$$\Rightarrow 5x = n\pi + \frac{\pi}{2} \text{ or } x = m\pi + \frac{\pi}{2} \quad [\because n, m \in I]$$

$$\Rightarrow x = \frac{(2n+1)\pi}{10} \text{ or } x = \frac{(2m+1)\pi}{2}$$

$$45. (c) \because 3 \sin x - 4 \sin^3 x = 4 \sin x (\sin^2 y - \sin^2 x)$$

$$\therefore 3 \sin x = 4 \sin x (\sin^2 y - \sin^2 x) + 4 \sin^3 x$$

$$= 4 \sin x (\sin^2 y - \sin^2 x + \sin^2 x) = 4 \sin x \sin^2 y$$

$$\Rightarrow \sin^2 y = \frac{3}{4} \Rightarrow \sin y = \pm \frac{\sqrt{3}}{2}$$

$$\Rightarrow x = n\pi \pm \frac{\pi}{3} \quad [\because n \in I]$$

So, the given equation has only one solution in each quadrant.

$$46. (b) \text{ We have,}$$

$$\sin x - 3 \sin 2x + \sin 3x = \cos x - 3 \cos 2x + \cos 3x$$

$$\Rightarrow \sin x + \sin 3x - 3 \sin 2x = \cos x + \cos 3x - 3 \cos 2x$$

$$\Rightarrow 2 \sin 2x \cos x - 3 \sin 2x - 2 \cos 2x \cos x + 3 \cos 2x = 0$$

$$\Rightarrow \sin 2x (2 \cos x - 3) - \cos 2x (2 \cos x - 3) = 0$$

$$\Rightarrow (\sin 2x - \cos 2x) (2 \cos x - 3) = 0$$

$$\Rightarrow \sin 2x = \cos 2x \quad \left[\because \cos x \neq \frac{3}{2} \right]$$

$$\Rightarrow 2x = 2n\pi \pm \left(\frac{\pi}{2} - 2x \right)$$

$$\therefore x = \frac{n\pi}{2} + \frac{\pi}{8} \quad [\because \text{neglect -ve sign}]$$

$$47. (c) \text{ Hint } \sqrt{3} \cos x + \sin x = \sqrt{2}$$

$$\Rightarrow \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos \left(x - \frac{\pi}{6} \right) = \cos \left(\frac{\pi}{4} \right)$$

$$48. (c) \because 6 \sin x \cos x + 4 \cos 2x = 3 \sin 2x + 4 \cos 2x$$

$$\therefore -\sqrt{3^2 + 4^2} \leq 3 \sin 2x + 4 \cos 2x \leq \sqrt{3^2 + 4^2}$$

$$\Rightarrow -5 \leq 3 \sin 2x + 4 \cos 2x \leq 5$$

Thus, maximum and minimum value of

$6 \sin x \cos x + 4 \cos 2x$ are 5 and -5, respectively.

$$49. (b) A = \sin^2 \theta + \cos^4 \theta$$

$$= 1 - \cos^2 \theta + \cos^4 \theta = 1 - \cos^2 \theta (1 - \cos^2 \theta)$$

$$= 1 - \cos^2 \theta \sin^2 \theta = 1 - \frac{4 \sin^2 \theta \cos^2 \theta}{4}$$

$$= 1 - \frac{(2 \sin \theta \cos \theta)^2}{4}$$

$$A = 1 - \frac{1}{4} \sin^2 2\theta$$

For maximum value of A ,

$$A = 1 - 0 \quad [\because \text{minimum value of } \sin^2 2\theta = 0]$$

$$\therefore A = 1$$

For minimum value of A ,

$$A = 1 - \frac{1}{4} = \frac{3}{4} \quad [\because \text{minimum value of } \sin^2 2\theta = 1]$$

$$\Rightarrow \frac{3}{4} \leq A \leq 1$$

$$50. (c) \sin x \geq -1, \text{ for any real } x, \text{ then } \sin \alpha + \sin \beta + \sin \gamma \geq -3, \text{ for any real } \alpha, \beta \text{ and } \gamma.$$

α, β, γ are real numbers (they may not be the angles of a triangle) and $\alpha + \beta + \gamma = \pi$.

Then, on taking $\alpha = -\frac{\pi}{2}, \beta = -\frac{\pi}{2}$ and $\gamma = 2\pi$, we get

$$\sin \alpha + \sin \beta + \sin \gamma = -2$$

Thus, minimum value of expression is negative.

$$51. (b) \text{ Let } I = (\sec^{-1} x)^2 + (\operatorname{cosec}^{-1} x)^2$$

$$= (\sec^{-1} x + \operatorname{cosec}^{-1} x)^2 - 2 \sec^{-1} x \operatorname{cosec}^{-1} x$$

$$= \frac{\pi^2}{4} - 2 \sec^{-1} x \left(\frac{\pi}{2} - \sec^{-1} x \right)$$

$$= \frac{\pi^2}{4} + 2 \left[(\sec^{-1} x)^2 - \frac{\pi}{2} (\sec^{-1} x) + \frac{\pi^2}{16} - \frac{\pi^2}{16} \right]$$

$$= \frac{\pi^2}{8} + 2 \left[\left(\sec^{-1} x - \frac{\pi}{4} \right)^2 \right]$$

$$\therefore I_{\max} = \frac{\pi^2}{8} + 2 \left[\frac{9\pi^2}{16} \right] = \frac{5\pi^2}{4}$$

$$52. (b) \text{ Given, } x + \frac{1}{x} = 2 \Rightarrow \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2 = 0$$

$$\Rightarrow \frac{x-1}{\sqrt{x}} = 0 \Rightarrow x = 1$$

So, the principal value of $\sin^{-1} x$ is $\frac{\pi}{2}$.

$$53. (a) 2 \tan^{-1} x = \sin^{-1} \frac{2x}{1+x^2}$$

$$\Rightarrow 2 \tan^{-1} 2 = \sin^{-1} \frac{4}{5} \therefore x = \frac{4}{5}$$

$$\text{RHS} = \sin \left(\frac{\theta}{2} \right)$$

$$\text{where, } \theta = \tan^{-1} \frac{4}{3} \Rightarrow \text{Then, } \tan \theta = \frac{4}{3}$$

$$\therefore \cos \theta = \frac{3}{5}$$

$$\Rightarrow 2 \sin^2 \frac{\theta}{2} = 1 - \cos \theta = \frac{2}{5}$$

$$\therefore \sin \theta = \frac{1}{\sqrt{5}} = y$$

$$\text{Clearly, } x > y \text{ and } 1 - x = 1 - \frac{4}{5} = \frac{1}{5} = y^2$$

$$\begin{aligned} 54. (b) \text{ Hint } \tan \left[2 \tan^{-1} \left(\frac{1}{5} \right) - \frac{\pi}{4} \right] &= \tan \left[2 \tan^{-1} \left(\frac{1}{5} \right) - \tan^{-1}(1) \right] \\ &= \tan \left[\tan^{-1} \frac{2 \times \frac{1}{5}}{1 - \left(\frac{1}{5} \right)^2} - \tan^{-1}(1) \right] \end{aligned}$$

$$55. (d) \tan^{-1} \left[\frac{\frac{x-1}{x+1} + \frac{2x-1}{2x+1}}{1 - \left(\frac{x-1}{x+1} \right) \left(\frac{2x-1}{2x+1} \right)} \right] = \tan^{-1} \left(\frac{23}{36} \right)$$

$$\Rightarrow \frac{2x^2 - 1}{3x} = \frac{23}{36} \Rightarrow 24x^2 - 12 - 23x = 0$$

$$\Rightarrow x = \frac{4}{3}, -\frac{3}{8}$$

But x cannot be negative.

$$\therefore x = \frac{4}{3}$$

$$56. (b) \therefore \tan^{-1} \frac{yz}{xr} + \tan^{-1} \frac{zx}{yr} = \tan^{-1} \frac{\frac{z}{r} \left(\frac{y}{x} + \frac{x}{y} \right)}{1 - \frac{z^2}{r^2}}$$

$$= \tan^{-1} \left(\frac{zr}{xy} \cdot \frac{x^2 + y^2}{x^2 + y^2} \right) = \cot^{-1} \left(\frac{xy}{zr} \right)$$

$$\therefore \tan^{-1} \frac{xy}{zr} + \cot^{-1} \frac{xy}{zr} = \frac{\pi}{2}$$

$$57. (d) \text{ Now, } \tan \left\{ 2 \tan^{-1} \left(\frac{1}{5} \right) \right\} = \frac{2 \times \frac{1}{5}}{1 - \frac{1}{25}} = \frac{5}{12}$$

Given equation can be rewritten as

$$17x^2 - 17x \tan \left\{ \frac{\pi}{4} - 2 \tan^{-1} \left(\frac{1}{5} \right) \right\} - 10 = 0$$

$$\therefore 17x^2 - 17x \cdot \frac{1 - \frac{5}{12}}{1 + \frac{5}{12}} - 10 = 0$$

$$\Rightarrow 17x^2 - 7x - 10 = 0 \Rightarrow (x-1)(17x+10) = 0$$

Hence, $x = 1$ is a root of the given equation.

$$\begin{aligned} 58. (b) \text{ We have, } \frac{s(s-a)}{bc} - \frac{(s-b)(s-c)}{bc} &= \cos^2 \frac{A}{2} - \sin^2 \frac{A}{2} \\ &= \cos \frac{2A}{2} = \cos A \end{aligned}$$

$$59. (a) \therefore x = \frac{\pi}{2} - 2 \tan^{-1} \sqrt{\cos \alpha}$$

$$\begin{aligned} \therefore \sin x &= \sin \left(\frac{\pi}{2} - 2 \tan^{-1} \sqrt{\cos \alpha} \right) = \cos (2 \tan^{-1} \sqrt{\cos \alpha}) \\ &= \cos \left(\tan^{-1} \frac{2 \sqrt{\cos \alpha}}{1 - \cos \alpha} \right) = \frac{1 - \cos \alpha}{1 + \cos \alpha} = \tan^2 \left(\frac{\alpha}{2} \right) \end{aligned}$$

$$60. (b) \text{ Here, } x - \frac{x^2}{2} + \frac{x^3}{4} - \dots = \frac{x}{1 + \frac{x}{2}} = \frac{2x}{2+x}$$

$$\text{and } x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \dots = \frac{x^2}{1 + \frac{x^2}{2}} = \frac{2x^2}{2+x^2}$$

$$\therefore \sin^{-1} \alpha + \cos^{-1} \alpha = \frac{\pi}{2}$$

$$\therefore \frac{2x}{2+x} = \frac{2x^2}{2+x^2}, x \neq 0$$

$$\Rightarrow 2+x^2 = 2x+x^2$$

$$\therefore x = 1$$

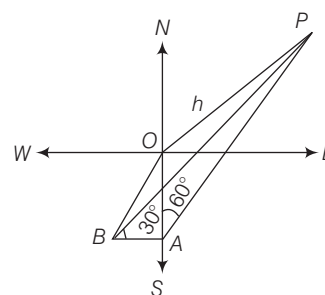
$$61. (b) \text{ Given, } \sin^{-1} \left(\frac{4x}{x^2+4} \right) + 2 \tan^{-1} \left(-\frac{x}{2} \right)$$

$$= \sin^{-1} \left(\frac{2 \times \frac{x}{2}}{\left(\frac{x}{2} \right)^2 + 1} \right) - 2 \tan^{-1} \frac{x}{2} = 2 \tan^{-1} \frac{x}{2} - 2 \tan^{-1} \frac{x}{2} = 0$$

$$\Rightarrow \left| \frac{x}{2} \right| \leq 1 \Rightarrow |x| \leq 2$$

$$\therefore -2 \leq x \leq 2$$

62. (d) Let OP be the tower of height h .



In $\triangle AOP$ and $\triangle BOP$,

$$\tan 60^\circ = \frac{h}{OA} \text{ and } \tan 30^\circ = \frac{h}{OB}$$

$$\Rightarrow OA = \frac{h}{\sqrt{3}}$$

$$\text{and } OB = h\sqrt{3}$$

$$\therefore OB^2 = OA^2 + AB^2$$

$$\Rightarrow 3h^2 = \frac{h^2}{3} + 9$$

$$\therefore h = \frac{3\sqrt{6}}{4} \text{ m}$$

$$\begin{aligned}
 63. (d) \text{ Given, } \sin^{-1}\left(\cos \frac{40\pi + 3\pi}{5}\right) &= \sin^{-1}\cos\left(8\pi + \frac{3\pi}{5}\right) \\
 &= \sin^{-1}\left(\cos \frac{3\pi}{5}\right) = \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)\right\} \\
 &= \sin^{-1}\left\{\sin\left(-\frac{\pi}{10}\right)\right\} = -\frac{\pi}{10}
 \end{aligned}$$

$$\begin{aligned}
 64. (b) \text{ Hint } 2 \tan^{-1} x + (\tan^{-1} x + \cot^{-1} x) &= \pi \\
 \Rightarrow 2 \tan^{-1} x + \frac{\pi}{2} &= \pi
 \end{aligned}$$

$$\begin{aligned}
 65. (c) \text{ Given, } (\tan^{-1} x)^2 + (\cot^{-1} x)^2 &= \frac{5\pi^2}{8} \\
 \Rightarrow (\tan^{-1} x + \cot^{-1} x)^2 - 2 \tan^{-1} x \cdot \cot^{-1} x &= \frac{5\pi^2}{8} \\
 \Rightarrow \left(\frac{\pi}{2}\right)^2 - 2 \tan^{-1} x \left(\frac{\pi}{2} - \tan^{-1} x\right) &= \frac{5\pi^2}{8} \\
 \Rightarrow \left[\because \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}\right] \\
 \Rightarrow \frac{\pi^2}{4} - 2 \cdot \frac{\pi}{2} \tan^{-1} x + 2 (\tan^{-1} x)^2 &= \frac{5\pi^2}{8} \\
 \Rightarrow 2 (\tan^{-1} x)^2 - \pi \tan^{-1} x - \frac{3\pi^2}{8} &= 0 \\
 \Rightarrow \tan^{-1} x = -\frac{\pi}{4}, \frac{3\pi}{4} \Rightarrow \tan^{-1} x = -\frac{\pi}{4} \Rightarrow x = -1 \\
 \left[\text{neglecting } \tan^{-1} x = \frac{3\pi}{4} \text{ as } \tan^{-1} x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]
 \end{aligned}$$

$$\begin{aligned}
 66. (a) \tan \left\{ \cos^{-1} \left(-\frac{2}{7} \right) - \frac{\pi}{2} \right\} &= \tan \left\{ \pi - \cos^{-1} \left(\frac{2}{7} \right) - \frac{\pi}{2} \right\} \\
 &= \tan \left\{ \frac{\pi}{2} - \cos^{-1} \left(\frac{2}{7} \right) \right\} = \tan \left\{ \sin^{-1} \left(\frac{2}{7} \right) \right\} \\
 &= \tan \left\{ \tan^{-1} \left(\frac{2}{3\sqrt{5}} \right) \right\} = \frac{2}{3\sqrt{5}}
 \end{aligned}$$

$$\begin{aligned}
 67. (c) \text{ Given, } \tan^{-1}(x+3) - \tan^{-1}(x-3) &= \sin^{-1}\left(\frac{3}{5}\right) \\
 \Rightarrow \tan^{-1} \left[\frac{(x+3) - (x-3)}{1 + (x+3)(x-3)} \right] &= \tan^{-1}\left(\frac{3}{5}\right) \\
 \Rightarrow \frac{6}{1+x^2-9} = \frac{3}{4} \Rightarrow x^2 = 16 \\
 \Rightarrow x = \pm 4
 \end{aligned}$$

$$\begin{aligned}
 68. (c) \text{ Given, } \Delta = a^2 - (b-c)^2 &= (a+b-c)(a-b+c) \\
 \Rightarrow \Delta &= [4(s-b)(s-c)] \\
 \Rightarrow s(s-a)(s-b)(s-c) &= 16(s-b)^2(s-c)^2 \\
 \Rightarrow \frac{(s-b)(s-c)}{s(s-a)} &= \frac{1}{16} \\
 \Rightarrow \tan \frac{A}{2} &= \frac{1}{4} \\
 \text{Now, } \tan A &= \frac{2 \tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}} = \frac{2 \cdot \left(\frac{1}{4}\right)}{1 - \frac{1}{16}} = \frac{8}{15}
 \end{aligned}$$

$$\begin{aligned}
 69. (b) \text{ Consider, } \sin^{-1}\left(\frac{8}{17}\right) + \sin^{-1}\left(\frac{3}{5}\right) \\
 &= \sin^{-1} \left[\frac{8}{17} \sqrt{1 - \left(\frac{3}{5}\right)^2} + \frac{3}{5} \sqrt{1 - \left(\frac{8}{17}\right)^2} \right] \\
 &\quad [\because \sin^{-1} x + \sin^{-1} y = \sin^{-1} (x \sqrt{1-y^2} + y \sqrt{1-x^2})] \\
 &= \sin^{-1} \left(\frac{8}{17} \times \frac{4}{5} + \frac{3}{5} \times \frac{15}{17} \right) = \sin^{-1} \left(\frac{77}{85} \right) \\
 &= \tan^{-1} \left[\frac{\frac{77}{85}}{\sqrt{1 - \left(\frac{77}{85}\right)^2}} \right] \quad \left[\because \sin^{-1} x = \tan^{-1} \frac{x}{\sqrt{1-x^2}} \right] \\
 &= \tan^{-1} \left[\frac{77}{85} \times \frac{85}{36} \right] = \tan^{-1} \frac{77}{36}
 \end{aligned}$$

$$\begin{aligned}
 70. (a) \text{ Given, } \alpha &= \sin^{-1}[\cos(\sin^{-1} x)] \\
 \text{and } \beta &= \cos^{-1}[\sin(\cos^{-1} x)] = \cos^{-1} \left[\sin \left(\frac{\pi}{2} - \sin^{-1} x \right) \right] \\
 \Rightarrow \beta &= \cos^{-1}[\cos(\sin^{-1} x)] \\
 \Rightarrow \alpha + \beta &= \frac{\pi}{2} \quad \left[\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \right] \\
 \Rightarrow \tan \alpha &= \cot \beta \Rightarrow \tan \alpha \cdot \tan \beta = 1
 \end{aligned}$$

$$\begin{aligned}
 71. (a) \tan \left(\sin^{-1} \frac{3}{5} + \cot^{-1} \frac{3}{2} \right) \\
 &= \tan \left[\tan^{-1} \frac{\frac{3}{5}}{\sqrt{1 - \left(\frac{3}{5}\right)^2}} + \tan^{-1} \frac{2}{3} \right] \\
 &\quad \left[\because \sin^{-1} x = \tan^{-1} \frac{x}{\sqrt{1-x^2}} \text{ and } \cot^{-1} \frac{x}{y} = \tan^{-1} \frac{y}{x} \right] \\
 &= \tan \left[\tan^{-1} \frac{3}{4} + \tan^{-1} \frac{2}{3} \right] = \tan \left[\tan^{-1} \frac{\left(\frac{3}{4} + \frac{2}{3}\right)}{1 - \frac{3}{4} \times \frac{2}{3}} \right] \\
 &\quad \left[\because \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right) \right] \\
 &= \tan \left[\tan^{-1} \frac{\frac{17}{12}}{\frac{1}{2}} \right] = \tan \left[\tan^{-1} \frac{17}{6} \right] = \frac{17}{6}
 \end{aligned}$$

$$\begin{aligned}
 72. (c) \text{ Given, } \sin \left(\sin^{-1} \frac{1}{5} + \cos^{-1} x \right) &= 1 \\
 \Rightarrow \sin^{-1} \frac{1}{5} + \cos^{-1} x &= \sin^{-1} 1 \\
 \Rightarrow \sin^{-1} \frac{1}{5} + \cos^{-1} x &= \frac{\pi}{2} \\
 \Rightarrow \sin^{-1} \frac{1}{5} &= \frac{\pi}{2} - \cos^{-1} x \\
 \Rightarrow \sin^{-1} \frac{1}{5} &= \sin^{-1} x \Rightarrow x = \frac{1}{5}
 \end{aligned}$$

73. (d) Given, $\tan \left\{ \frac{1}{2} \sin^{-1} \left(\frac{2x}{1+x^2} \right) + \frac{1}{2} \cos^{-1} \left(\frac{1-y^2}{1+y^2} \right) \right\} \dots (i)$

On putting $x = \tan \alpha$ and $y = \tan \beta$ in Eq. (i), we get

$$\begin{aligned} & \tan \left\{ \frac{1}{2} \sin^{-1} \left(\frac{2 \tan \alpha}{1 + \tan^2 \alpha} \right) + \frac{1}{2} \cos^{-1} \left(\frac{1 - \tan^2 \beta}{1 + \tan^2 \beta} \right) \right\} \\ &= \tan \left\{ \frac{1}{2} \sin^{-1} (\sin 2\alpha) + \frac{1}{2} \cos^{-1} (\cos 2\beta) \right\} \\ &= \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta} = \frac{x + y}{1 - xy} \end{aligned}$$

74. (b) Radius of the circumscribed circle,

$$R = \frac{a}{2} \operatorname{cosec} \frac{\pi}{n}$$

and radius of the inscribed circle, $r = \frac{1}{2} a \cot \left(\frac{\pi}{n} \right)$

$$\begin{aligned} \Rightarrow R + r &= \frac{a}{2 \sin \left(\frac{\pi}{n} \right)} + \frac{a \cos \left(\frac{\pi}{n} \right)}{2 \sin \left(\frac{\pi}{n} \right)} \\ &= \frac{a \left[1 + \cos \left(\frac{\pi}{n} \right) \right]}{2 \times 2 \sin \left(\frac{\pi}{2n} \right) \cdot \cos \left(\frac{\pi}{2n} \right)} = \frac{1}{2} a \cot \left(\frac{\pi}{2n} \right) \end{aligned}$$

75. (b) Hint $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$

$$\begin{aligned} \Rightarrow \tan^{-1} \left(\frac{2 \cos x}{1 - \cos^2 x} \right) &= \tan^{-1}(2 \operatorname{cosec} x) \\ \left[\because 2 \tan^{-1} x &= \tan^{-1} \left(\frac{2x}{1 - x^2} \right) \right] \end{aligned}$$

$$\Rightarrow \tan^{-1} \left(\frac{2 \cos x}{\sin^2 x} \right) = \tan^{-1} \left(\frac{2}{\sin x} \right)$$

76. (c) Hint $\tan^{-1} \sqrt{x(x+1)} + \sin^{-1} \sqrt{x^2 + x + 1} = \frac{\pi}{2}$

$$\Rightarrow \cos^{-1} \frac{1}{\sqrt{1 + (x^2 + x)}} + \sin^{-1} \sqrt{x^2 + x + 1} = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1} \frac{1}{\sqrt{1 + (x^2 + x)}} = \frac{\pi}{2} - \sin^{-1} \sqrt{x^2 + x + 1}$$

$$\Rightarrow \cos^{-1} \frac{1}{\sqrt{x^2 + x + 1}} = \cos^{-1} \sqrt{x^2 + x + 1}$$

77. (a) Given, $r_1 = r_2 + r_3 + r$

$$\Rightarrow r_1 - r = r_2 + r_3$$

$$\Rightarrow \frac{\Delta}{s-a} - \frac{\Delta}{s} = \frac{\Delta}{s-b} + \frac{\Delta}{s-c}$$

$$\Rightarrow \frac{s-s+a}{s(s-a)} = \frac{s-c+s-b}{(s-b)(s-c)}$$

$$\Rightarrow \frac{a}{s(s-a)} = \frac{a}{(s-b)(s-c)}$$

$$\Rightarrow s(a-b-c) + bc = 0$$

$$\Rightarrow \frac{(a+b+c)(a-b-c)}{2} + bc = 0$$

$$\Rightarrow a^2 - (b+c)^2 + 2bc = 0$$

$$\Rightarrow a^2 - b^2 - c^2 = 0$$

$$\Rightarrow b^2 + c^2 = a^2$$

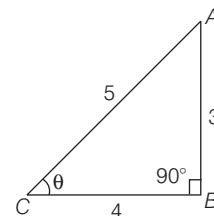
So, $\triangle ABC$ is a right angled triangle.

78. (b) Since, $\operatorname{cosec}^{-1} \left(\frac{5}{3} \right) = \tan^{-1} \left(\frac{3}{4} \right)$

$$\therefore \cot \left(\tan^{-1} \frac{3}{4} + \tan^{-1} \frac{2}{3} \right)$$

$$= \cot \tan^{-1} \left[\frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{1}{2}} \right] = \cot \tan^{-1} \left[\frac{\left(\frac{17}{12} \right)}{\left(\frac{1}{2} \right)} \right]$$

$$= \cot \left[\tan^{-1} \left(\frac{17}{6} \right) \right] = \frac{6}{17}$$



79. (b) Given, $\sin^{-1} \left(\frac{x}{5} \right) + \operatorname{cosec}^{-1} \left(\frac{5}{4} \right) = \frac{\pi}{2}$

$$\therefore \sin^{-1} \left(\frac{x}{5} \right) + \sin^{-1} \left(\frac{4}{5} \right) = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1} \left(\frac{x}{5} \right) = \frac{\pi}{2} - \sin^{-1} \left(\frac{4}{5} \right)$$

$$\Rightarrow \sin^{-1} \left(\frac{x}{5} \right) = \cos^{-1} \left(\frac{4}{5} \right)$$

$$\Rightarrow \sin^{-1} \left(\frac{x}{5} \right) = \sin^{-1} \left(\frac{3}{5} \right)$$

$$\therefore x = 3$$

80. (b) Given, $\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha$

$$\Rightarrow \cos^{-1} \left(\frac{xy}{2} + \sqrt{1-x^2} \sqrt{1-\frac{y^2}{4}} \right) = \alpha$$

$$\Rightarrow \frac{xy}{2} + \sqrt{1-x^2} \sqrt{1-\frac{y^2}{4}} = \cos \alpha$$

$$\Rightarrow 2\sqrt{1-x^2} \sqrt{1-\frac{y^2}{4}} = 2 \cos \alpha - xy$$

On squaring both sides, we get

$$\frac{4(1-x^2)(4-y^2)}{4} = 4 \cos^2 \alpha + x^2 y^2 - 4xy \cos \alpha$$

$$\Rightarrow 4 - 4x^2 - y^2 + x^2 y^2 = 4 \cos^2 \alpha + x^2 y^2 - 4xy \cos \alpha$$

$$\Rightarrow 4x^2 - 4xy \cos \alpha + y^2 = 4 \sin^2 \alpha$$

81. (c) Given, $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$

$$\Rightarrow \frac{\cos A}{k \sin A} = \frac{\cos B}{k \sin B} = \frac{\cos C}{k \sin C}$$

$$\Rightarrow \cot A = \cot B = \cot C$$

$$\Rightarrow A = B = C = 60^\circ$$

$\therefore \triangle ABC$ is equilateral.

82. (c)
- $\angle A : \angle B : \angle C = 3 : 5 : 4$

$$\angle A = \frac{180^\circ \times 3}{3+5+4} = \frac{180^\circ \times 3}{12} = 45^\circ$$

$$\angle B = \frac{180^\circ \times 5}{12} = 75^\circ$$

$$\angle C = \frac{180^\circ \times 4}{12} = 60^\circ$$

By sine rule, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$

$$\Rightarrow a = k \sin 45^\circ = \frac{k}{\sqrt{2}}$$

$$b = k \sin 75^\circ = k \frac{(\sqrt{3}+1)}{2\sqrt{2}}$$

$$c = k \sin 60^\circ = k \frac{\sqrt{3}}{2}$$

$$\therefore a + b + \sqrt{2}c = \frac{k}{\sqrt{2}} + \frac{\sqrt{3}+1}{2\sqrt{2}}k + \frac{\sqrt{3}}{2}k\sqrt{2}$$

$$= k \left[\frac{1}{\sqrt{2}} + \frac{\sqrt{3}+1}{2\sqrt{2}} + \frac{\sqrt{3} \times \sqrt{2}}{2} \right] = k \left[\frac{2 + \sqrt{3} + 1 + 2\sqrt{3}}{2\sqrt{2}} \right]$$

$$= k \left[\frac{3 + 3\sqrt{3}}{2\sqrt{2}} \right] = \frac{3(\sqrt{3}+1)k}{2\sqrt{2}} = 3b$$

83. (a) Given,
- $\angle A = 30^\circ$
- ,
- $\angle B = 45^\circ$
- ,
- $a = 1$

$$\therefore \angle C = 105^\circ$$

Using sine rule, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$\Rightarrow \frac{1}{\left(\frac{1}{2}\right)} = \frac{b}{\left(\frac{1}{\sqrt{2}}\right)} = \frac{c}{\sin 105^\circ}$$

$$\Rightarrow b = \sqrt{2}$$

and $c = 2 \sin 105^\circ$

$$\therefore c = 2 \left(\frac{\sqrt{3}+1}{2\sqrt{2}} \right) = \frac{\sqrt{3}+1}{\sqrt{2}}$$

84. (a) Given,
- $\angle A = 75^\circ$
- ,
- $\angle B = 45^\circ \Rightarrow \angle C = 60^\circ$

By sine rule, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$\Rightarrow \frac{a}{\sin 75^\circ} = \frac{b}{\sin 45^\circ} = \frac{c}{\sin 60^\circ}$$

$$\Rightarrow b + c\sqrt{2} = \frac{\sin 45^\circ}{\sin 75^\circ} a + \sqrt{2} \frac{\sin 60^\circ}{\sin 75^\circ} a$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}}} a + \sqrt{2} \frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}+1}{2\sqrt{2}}} a = \frac{2}{\sqrt{3}+1} a + \frac{2\sqrt{3}a}{\sqrt{3}+1} = 2a$$

$$85. (a) \cot A = \frac{\cos A}{\sin A} = \frac{b^2 + c^2 - a^2}{2bc \sin A} = \frac{b^2 + c^2 - a^2}{4\Delta}$$

Similarly, $\cot B = \frac{a^2 + c^2 - b^2}{4\Delta}$

and $\cot C = \frac{a^2 + b^2 - c^2}{4\Delta}$

Given that, a^2, b^2 and c^2 are in AP.

$$\Rightarrow -2a^2, -2b^2, -2c^2 \text{ are in AP.}$$

$$\Rightarrow (a^2 + b^2 + c^2) - 2a^2, (a^2 + b^2 + c^2) - 2b^2, (a^2 + b^2 + c^2) - 2c^2 \text{ are in AP.}$$

$$\Rightarrow b^2 + c^2 - a^2, c^2 + a^2 - b^2, a^2 + b^2 - c^2 \text{ are in AP.}$$

$$\Rightarrow \frac{b^2 + c^2 - a^2}{4\Delta}, \frac{c^2 + a^2 - b^2}{4\Delta}, \frac{a^2 + b^2 - c^2}{4\Delta} \text{ are in AP.}$$

86. (c) Using projection formulae,

$$a = b \cos C + c \cos B$$

$$b = a \cos C + c \cos A$$

$$c = a \cos B + b \cos A$$

$$\therefore \left(\frac{b}{c} + \frac{c}{b}\right) \cos A + \left(\frac{a}{b} + \frac{b}{a}\right) \cos C + \left(\frac{a}{c} + \frac{c}{a}\right) \cos B$$

$$= \left(\frac{b}{c} \cos A + \frac{a}{c} \cos B\right) + \left(\frac{c}{b} \cos A + \frac{a}{b} \cos C\right)$$

$$+ \left(\frac{b}{a} \cos C + \frac{c}{a} \cos B\right)$$

$$= \frac{c}{c} + \frac{b}{b} + \frac{a}{a} = 3$$

87. (a) According to the question,

$$\tan \left(\frac{A-B}{2} \right) = \frac{1}{3} \tan \left(\frac{A+B}{2} \right) \quad \dots(i)$$

Using Napier's rule,

$$\tan \left(\frac{A-B}{2} \right) = \frac{a-b}{a+b} \cdot \cot \left(\frac{C}{2} \right) \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{1}{3} \tan \left(\frac{A+B}{2} \right) = \frac{a-b}{a+b} \cdot \cot \left(\frac{C}{2} \right)$$

$$\Rightarrow \frac{1}{3} \cot \frac{C}{2} = \frac{a-b}{a+b} \cot \frac{C}{2}$$

$$\Rightarrow \frac{a-b}{a+b} = \frac{1}{3}$$

$$\Rightarrow \frac{2a}{-2b} = \frac{4}{-2}$$

[by componendo and dividendo]

$$\Rightarrow \frac{a}{b} = \frac{2}{1}$$

$$\therefore a : b = 2 : 1$$

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1. (d) We have,
- $|\sin x \cos x| + |\tan x + \cot x| = \sqrt{3}$

$$\Rightarrow |\sin x \cos x| + \frac{1}{|\sin x \cdot \cos x|} = \sqrt{3}$$

$$\text{But } |\sin x \cdot \cos x| + \frac{1}{|\sin x \cdot \cos x|} \geq 2$$

Hence, there is no solution.

2. (c) We have,
- $(\sin^{-1} x)^2 + (\cos^{-1} x)^2$

$$= (\sin^{-1} x + \cos^{-1} x)^2 - 2 \sin^{-1} x \cdot \cos^{-1} x$$

$$= \frac{\pi^2}{4} - 2 \sin^{-1} x \left(\frac{\pi}{2} - \sin^{-1} x \right) = \frac{\pi^2}{4} - \pi \sin^{-1} x + 2(\sin^{-1} x)^2$$

$$= 2 \left[(\sin^{-1} x)^2 - \frac{\pi}{2} \sin^{-1} x + \frac{\pi^2}{8} \right] = 2 \left[\left(\sin^{-1} x - \frac{\pi}{4} \right)^2 + \frac{\pi^2}{16} \right]$$

Thus, the least value is $2 \left(\frac{\pi^2}{16} \right)$ i.e. $\frac{\pi^2}{8}$ and the greatest

$$\text{value is } 2 \left[\left(\frac{-\pi}{2} - \frac{\pi}{4} \right)^2 + \frac{\pi^2}{16} \right] \text{ i.e. } \frac{5\pi^2}{4}.$$

3. (c) Clearly,
- $1 + \sin x \geq 0$

 $\therefore$ The given equation becomes

$$\cos x - \sin x = 1$$

$$\Rightarrow \cos x \cdot \frac{1}{\sqrt{2}} - \sin x \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \cos \left(x + \frac{\pi}{4} \right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow x + \frac{\pi}{4} = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{15\pi}{4} \Rightarrow x = 0, \frac{3\pi}{2}, 2\pi, \frac{7\pi}{2}$$

$$\therefore 0 \leq x \leq 3\pi \Rightarrow x = 0, \frac{3\pi}{2}, 2\pi$$

4. (d)
- $\sin \left[\cot^{-1} \left(\cos \frac{\pi}{4} \right) \right] = \sin \left[\cot^{-1} \frac{1}{\sqrt{2}} \right] = \sin \left[\sin^{-1} \sqrt{\frac{2}{3}} \right]$
-
- $$= \sqrt{\frac{2}{3}}$$

5. (c) Let
- $ED = h$
- ,
- $\angle EAB = \alpha$

$$\therefore \angle EBD = 2\alpha, \angle ECD = 3\alpha$$

Now,

$$\angle DBE = \angle EAB + \angle BEA$$

$$\Rightarrow 2\alpha = \alpha + \angle BEA$$

$$\Rightarrow \angle BEA = \alpha = \angle EAB$$

$$\Rightarrow AB = EB = a$$

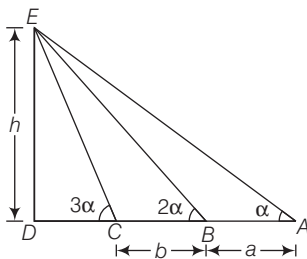
Similarly, $\angle BEC = \alpha$

$$\text{In } \triangle EBC, \frac{BC}{\sin \alpha} = \frac{EB}{\sin(180^\circ - 3\alpha)}$$

$$\Rightarrow \frac{b}{\sin \alpha} = \frac{a}{\sin 3\alpha} \Rightarrow \frac{a}{b} = \frac{\sin 3\alpha}{\sin \alpha}$$

$$\Rightarrow \frac{a}{b} = \frac{3 \sin \alpha - 4 \sin^3 \alpha}{\sin \alpha} = 3 - 4 \sin^2 \alpha$$

$$\Rightarrow 4 \sin^2 \alpha = 3 - \frac{a}{b} = \frac{3b - a}{b} \Rightarrow \sin \alpha = \sqrt{\frac{3b - a}{4b}}$$



$$\text{In } \triangle EBD, \sin 2\alpha = \frac{ED}{EB}$$

$$\Rightarrow \frac{ED}{EB} = a \cdot 2 \sin \alpha \cdot \cos \alpha$$

$$\Rightarrow h = 2a \sqrt{\frac{3b-a}{4b}} \cdot \sqrt{1 - \frac{3b-a}{4b}} = 2a \sqrt{\frac{3b-a}{4b}} \sqrt{\frac{b+a}{4b}} = \frac{a}{2b} \sqrt{(a+b)(3b-a)}$$

6. (a) Given that,
- $\tan 20^\circ = p$

$$\therefore \frac{\tan 160^\circ - \tan 110^\circ}{1 + \tan 160^\circ \tan 110^\circ} = \frac{\tan(180^\circ - 20^\circ) - \tan(90^\circ + 20^\circ)}{1 + \tan(180^\circ - 20^\circ) \tan(90^\circ + 20^\circ)} = \frac{-\tan 20^\circ + \cot 20^\circ}{1 + \tan 20^\circ \cot 20^\circ} = \frac{-p + \frac{1}{p}}{1 + 1} = \frac{1 - p^2}{2p}$$

7. (d) Since,
- $30^\circ = 30^\circ \times \frac{\pi}{180} \text{ rad} = \frac{\pi}{6}$

Let angles $a, a + d, a + 2d$ are in AP.

$$\text{Now, } 3a + 3d = \pi \quad [\because A + B + C = \pi]$$

$$\Rightarrow 3 \times \frac{\pi}{6} + 3d = \pi$$

$$\Rightarrow d = \frac{1}{3} \left(\pi - \frac{\pi}{2} \right) = \frac{\pi}{6}$$

$$\therefore \text{Greatest angle} = a + 2d = \frac{\pi}{6} + 2 \cdot \frac{\pi}{6} = \frac{\pi}{2}$$

8. (a) Since,
- $A + B + C = 180^\circ = \cot(A + B + C)$

$$\Rightarrow \frac{\sum \cot A \cot B - 1}{\cot A \cot B \cot C - \sum \cot A} = \frac{1}{0}$$

$$\Rightarrow \cot A \cot B \cot C - \sum \cot A = 0$$

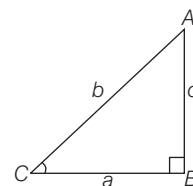
$$\Rightarrow \frac{\cot A + \cot B + \cot C}{\cot A \cot B \cot C} = 1$$

9. (b) Given,
- $a = 2, b = 3, \sin A = \frac{2}{3} \Rightarrow \sin^2 A = \frac{4}{9}$

$$\therefore \cos^2 A = 1 - \sin^2 A = \frac{5}{9}$$

$$\Rightarrow \cos A = \frac{\sqrt{5}}{3}$$

$$\therefore \cos C = \frac{2}{3}$$



10. (a) Given,
- $4 \sin^{-1} x + \cos^{-1} x = \pi$

$$\Rightarrow 4 \sin^{-1} x + \frac{\pi}{2} - \sin^{-1} x = \pi \Rightarrow 3 \sin^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow x = \sin \left(\frac{\pi}{6} \right) = \frac{1}{2}$$

11. (d)
- $\therefore \tan(\theta + \phi) = \frac{\tan \theta + \tan \phi}{1 - \tan \theta \cdot \tan \phi}$

$$= \left(\frac{1}{2} + \frac{1}{3} \right) / \left(1 - \frac{1}{2} \cdot \frac{1}{3} \right) = 1 \Rightarrow \theta + \phi = \frac{\pi}{4}$$

12. (a) $\because \sin \theta = \frac{1}{2} = \sin \frac{\pi}{6}$
 $\therefore \theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$ and $\tan \theta = \frac{1}{\sqrt{3}} = \tan \left(\frac{\pi}{6} \right)$
 $\Rightarrow \theta = \frac{\pi}{6}, \pi + \frac{\pi}{6}$
 $\therefore$ Common value of θ is $\frac{\pi}{6}$.

Hence, general value of θ is $2n\pi + \frac{\pi}{6}, \forall n \in I$.

13. (b) Let $y = \sin x + \cos x = \sqrt{2} \left[\sin \left(\frac{\pi}{4} + x \right) \right]$

Here, y will be maximum when $x = \frac{\pi}{4}$.

14. (b) $\sin^{-1} \left(-\frac{\sqrt{3}}{2} \right) = \sin^{-1} \left(\sin \left(-\frac{\pi}{3} \right) \right) = -\frac{\pi}{3}$

15. (b) Since, $A + B + C = \pi$

But $2B = A + C$

$\therefore 3B = \pi$

$\Rightarrow B = \frac{\pi}{3} \therefore \frac{a+c}{b} = \frac{\sin A + \sin C}{\sin B}$

$$= \frac{2 \sin \left(\frac{A+C}{2} \right) \cos \left(\frac{A-C}{2} \right)}{\sin \frac{\pi}{3}}$$

$$= \frac{2 \sin \frac{\pi}{3} \cos \left(\frac{A-C}{2} \right)}{\sin \frac{\pi}{3}} = 2 \cos \left(\frac{A-C}{2} \right)$$

16. (b) $\left(1 + \cos \frac{\pi}{8} \right) \left(1 + \cos \frac{3\pi}{8} \right)$
 $\left(1 + \cos \frac{5\pi}{8} \right) \left(1 + \cos \frac{7\pi}{8} \right)$

$$= \left(1 - \cos^2 \frac{\pi}{8} \right) \left(1 - \cos^2 \frac{3\pi}{8} \right) = \sin^2 \frac{\pi}{8} \sin^2 \frac{3\pi}{8}$$

$$= \frac{1}{4} \left[\cos \frac{\pi}{4} - \cos \frac{\pi}{2} \right]^2 = \frac{1}{8}$$

17. (a) $\sec \theta - \operatorname{cosec} \theta = \frac{4}{3}$

$$\Rightarrow 3(\sin \theta - \cos \theta) = 4 \sin \theta \cos \theta$$

On squaring both sides, we get

$$9(\sin - \cos \theta)^2 = (4 \sin \theta \cos \theta)^2$$

$$\Rightarrow 9(1 - \sin 2\theta) = 4 \sin^2 2\theta$$

$$\Rightarrow 4 \sin^2 2\theta + 9 \sin 2\theta - 9 = 0$$

$$\Rightarrow \sin 2\theta = \frac{-9 \pm \sqrt{81 + 144}}{8} = \frac{3}{4}, -3$$

But $\sin 2\theta \neq -3$

$$\therefore \sin 2\theta = \frac{3}{4}$$

$$\Rightarrow \theta = \frac{1}{2} \left[n\pi + (-1)^n \sin^{-1} \frac{3}{4} \right]$$

18. (d) $\because AM > GM$

$$\therefore \frac{9 \tan^2 \theta + 4 \cot^2 \theta}{2} > \sqrt{4 \cot^2 \theta \cdot 9 \tan^2 \theta}$$

$$\Rightarrow 9 \tan^2 \theta + 4 \cot^2 \theta > 12$$

$\therefore$ Minimum value is 12.

19. (a) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$... (i)

and $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$... (ii)

On adding Eqs. (i) and (ii), we get

$$\frac{\pi}{2} - \frac{\pi}{2} < \sin^{-1} x + \cos^{-1} x + \tan^{-1} x < \frac{\pi}{2} + \frac{\pi}{2}$$

$$\Rightarrow 0 < \sin^{-1} x + \cos^{-1} x + \tan^{-1} x < \pi$$

On comparing it with given condition, we get

$$a = 0, b = \pi$$

20. (b) We have, $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$... (i)

and $\cos^{-1} x - \cos^{-1} y = \frac{\pi}{3}$

$$\Rightarrow \left(\frac{\pi}{2} - \sin^{-1} x \right) - \left(\frac{\pi}{2} - \sin^{-1} y \right) = \frac{\pi}{3}$$

$$\Rightarrow -\sin^{-1} x + \sin^{-1} y = \frac{\pi}{3}$$
 ... (ii)

On adding Eqs. (i) and (ii), we get

$$\sin^{-1} y = \frac{\pi}{2} \Rightarrow y = 1$$

$$\Rightarrow \sin^{-1} x + \frac{\pi}{2} = \frac{2\pi}{3}$$
 [using Eq. (i)]

$$\Rightarrow \sin^{-1} x = \frac{2\pi}{3} - \frac{\pi}{2} = \frac{\pi}{6}$$

$$\therefore x = \frac{1}{2}$$

21. (c) Let r and R be the inradius and circumradius of incircle and circumcircle of the given regular polygon of side n .

Then, we know that

$$r + R = \frac{a}{2} \cot \frac{\pi}{n} + \frac{a}{2} \operatorname{cosec} \frac{\pi}{n} = \frac{a}{2} \left(\frac{1 + \cos \pi/n}{\sin \pi/n} \right)$$

$$= \frac{a}{2} \frac{2 \cos^2 \frac{\pi}{2n}}{2 \sin \frac{\pi}{2n} \cdot \cos \frac{\pi}{2n}} = \frac{a}{2} \cot \frac{\pi}{2n}$$

22. (c) We have, $\cos^2 B + \cos^2 C = \cos^2 B + \cos^2 \left(\frac{\pi}{2} - B \right)$
 $= \cos^2 B + \sin^2 B = 1$

23. (d) Using sine rule,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$
 ... (i)

and $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ [given] ... (ii)

We have, $\tan A = \tan B = \tan C$

$$\Rightarrow A = B = C = 60^\circ$$

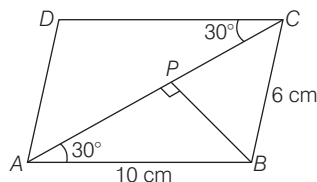
Hence, triangle is equilateral whose side is $a = 2$.

$$\therefore \text{Area of } \Delta = \frac{\sqrt{3}}{4} a^2 \Rightarrow \Delta = \frac{\sqrt{3}}{4} (2)^2 = \sqrt{3} \text{ sq units}$$

24. (a) Draw $BP \perp AC$, the longer diagonal.

From right larger diagonal of ΔAPB , we have

$$\frac{AP}{AB} = \cos 30^\circ \Rightarrow AP = AB \cos 30^\circ$$



$$\therefore AP = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3} \text{ cm}$$

Also, $\frac{BP}{AB} = \sin 30^\circ$

$$\therefore BP = AB \sin 30^\circ = 10 \times \frac{1}{2} = 5 \text{ cm}$$

From right longer diagonal of ΔBPC , we have

$$BP^2 + PC^2 = BC^2$$

$$\Rightarrow 5^2 + PC^2 = 6^2$$

$$\Rightarrow PC^2 = 6^2 - 5^2$$

$$\Rightarrow = (6-5)(6+5) = 11$$

$$\Rightarrow PC = \sqrt{11} \text{ cm}$$

$\therefore$ Longer diagonal,

$$AC = AP + PC = 5\sqrt{3} + \sqrt{11} \text{ cm}$$

25. (c) $\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ$

$$= \frac{\tan 60^\circ}{\sin 20^\circ} - \frac{1}{\cos 20^\circ} = \frac{\sin 60^\circ \cos 20^\circ - \sin 20^\circ \cos 60^\circ}{\cos 60^\circ \sin 20^\circ \cos 20^\circ}$$

$$= \frac{\sin 40^\circ}{\cos 60^\circ \sin 20^\circ \cos 20^\circ} = \frac{2 \sin 20^\circ \cos 20^\circ}{\frac{1}{2} (\sin 20^\circ \cos 20^\circ)} = 4$$

26. (b) Given, $\tan \theta + \tan \left(\theta + \frac{\pi}{3} \right) + \tan \left(\theta + \frac{2\pi}{3} \right) = 3$

$$\Rightarrow \tan \theta + \frac{\tan \theta + \sqrt{3}}{1 - \sqrt{3} \tan \theta} + \frac{\tan \theta - \sqrt{3}}{1 + \sqrt{3} \tan \theta} = 3$$

$$\Rightarrow \tan \theta + \frac{8 \tan \theta}{1 - 3 \tan^2 \theta} = 3 \Rightarrow \frac{9 \tan \theta - 3 \tan^3 \theta}{1 - 3 \tan^2 \theta} = 3$$

$$\Rightarrow 3 \tan 3\theta = 3 \Rightarrow \tan 3\theta = 1$$

27. (b) Given equation is

$$\cos 2x + 2 \cos^2 x = 2$$

$$\Rightarrow 2 \cos^2 x - 1 + 2 \cos^2 x = 2$$

$$\Rightarrow 4 \cos^2 x = 3$$

$$\Rightarrow \cos^2 x = \frac{3}{4}$$

$$\Rightarrow \cos x = \pm \frac{\sqrt{3}}{2}$$

$$x = n\pi \pm \frac{\pi}{6}, n \in \mathbb{Z}$$

28. (b) Given that, $\sin^{-1} \left(\frac{3}{x} \right) + \sin^{-1} \left(\frac{4}{x} \right) = \frac{\pi}{2}$

$$\therefore \sin^{-1} \left(\frac{3}{x} \right) = \frac{\pi}{2} - \sin^{-1} \left(\frac{4}{x} \right)$$

$$\Rightarrow \sin^{-1} \left(\frac{3}{x} \right) = \cos^{-1} \left(\frac{4}{x} \right)$$

$$\Rightarrow \sin^{-1} \left(\frac{3}{x} \right) = \sin^{-1} \left(\frac{\sqrt{x^2 - 16}}{x} \right)$$

$$\Rightarrow \frac{3}{x} = \left(\frac{\sqrt{x^2 - 16}}{x} \right)$$

$$\Rightarrow 9 = x^2 - 16 \Rightarrow x^2 = 25$$

$$\Rightarrow x = \pm 5 \Rightarrow x = 5$$

[$\because -5$ does not satisfy the given equation]

29. (b) Given that,

$$\frac{1}{b+c} + \frac{1}{c+a} = \frac{3}{a+b+c}$$

$$\Rightarrow 1 + \frac{b}{a+c} + 1 + \frac{a}{b+c} = 3$$

$$\Rightarrow b(b+c) + a(a+c) = (a+c)(b+c)$$

$$\Rightarrow b^2 + bc + a^2 + ac = ab + ac + bc + c^2$$

$$\Rightarrow a^2 + b^2 - c^2 = ab$$

We know that, $\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{ab}{2ab} = \frac{1}{2}$

$$\Rightarrow C = 60^\circ$$

30. (d) Given that, $r_1 = 2r_2 = 3r_3$

$$\therefore \frac{\Delta}{s-a} = \frac{2\Delta}{s-b} = \frac{3\Delta}{s-c}$$

Then, $s-a=k, s-b=2k, s-c=3k$

$$\Rightarrow 3s - (a+b+c) = 6k \Rightarrow s = 6k$$

$$\therefore \frac{a}{5} = \frac{b}{4} = \frac{c}{3} = k$$

Now, $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = \frac{5}{4} + \frac{4}{3} + \frac{3}{5}$

$$= \frac{75 + 80 + 36}{60} = \frac{191}{60}$$

31. (a) Let AB be a hill whose height is f metre and CD be a pillar of height h metre.

In ΔEDB ,

$$\tan \alpha = \frac{h-h'}{ED} \quad \dots (i)$$

and in ΔACB ,

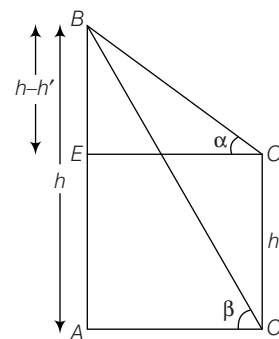
$$\tan \beta = \frac{h}{AC} = \frac{h}{ED} \quad \dots (ii)$$

Eliminate ED from Eqs. (i) and (ii), we get

$$\tan \alpha = \frac{h-h'}{h}$$

$$\Rightarrow h \frac{\tan \alpha}{\tan \alpha} = h - h'$$

$$\Rightarrow h' = \frac{h(\tan \beta - \tan \alpha)}{\tan \beta}$$



32. (a) Given that, $1 + \sin x \sin^2 \frac{x}{2} = 0$

$$\therefore 1 + \sin x \left(\frac{1 - \cos x}{2} \right) = 0$$

$$\Rightarrow 2 + \sin x - \sin x \cos x = 0$$

$$\Rightarrow \sin 2x - 2 \sin x = 4$$

Since, the maximum values of $\sin x$ and $\sin 2x$ are 1, which is not possible for any x in $[-\pi, \pi]$.

33. (c) We have,

$$\sin^{-1} x = 2 \tan^{-1} x$$

$$\therefore \sin^{-1} x = \sin^{-1} \frac{2x}{1+x^2} \Rightarrow x = \frac{2x}{1+x^2}$$

$$\Rightarrow x^3 - x = 0 \Rightarrow x(x+1)(x-1) = 0$$

$$\Rightarrow x \in \{-1, 1, 0\}$$

34. (c) Given that,

$$\sin A + \cos B = a \quad \dots(i)$$

$$\text{and} \quad \sin B + \cos A = b \quad \dots(ii)$$

On squaring and adding Eqs. (i) and (ii), we get

$$\sin^2 A + \cos^2 B + 2 \sin A \cos B + \sin^2 B + \cos^2 A + 2 \sin B \cos A = a^2 + b^2$$

$$\Rightarrow 2 \sin(A+B) + 2 = a^2 + b^2$$

$$\Rightarrow \sin(A+B) = \frac{a^2 + b^2 - 2}{2}$$

35. (c) $(a-b)^2 \cos^2 \frac{C}{2} + (a+b)^2 \sin^2 \frac{C}{2}$

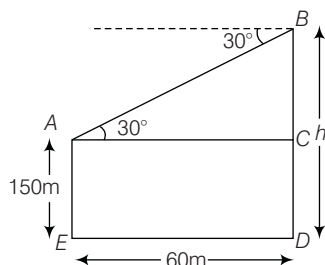
$$= (a^2 + b^2 - 2ab) \cos^2 \frac{C}{2} + (a^2 + b^2 + 2ab) \sin^2 \frac{C}{2}$$

$$= (a^2 + b^2) + 2ab \left(\sin^2 \frac{C}{2} - \cos^2 \frac{C}{2} \right)$$

$$= a^2 + b^2 - 2ab \cos C$$

$$= a^2 + b^2 - (a^2 + b^2 - c^2) = c^2$$

36. (c) In $\triangle ABC$, $\tan 30^\circ = \frac{BC}{AC}$



$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h-150}{60} \Rightarrow h-150 = \frac{60}{\sqrt{3}}$$

$$\Rightarrow h = (150 + 20\sqrt{3}) \text{ m}$$

37. (c) $12 \cot^2 \theta - 31 \operatorname{cosec} \theta + 32 = 0$

$$\Rightarrow 12 \cos^2 \theta - 31 \sin \theta + 32 \sin^2 \theta = 0$$

$$\Rightarrow 12(1 - \sin^2 \theta) - 31 \sin \theta + 32 \sin^2 \theta = 0$$

$$\Rightarrow 20 \sin^2 \theta - 31 \sin \theta + 12 = 0$$

which is a quadratic equation in $\sin \theta$.

$$\therefore \sin \theta = \frac{31 \pm \sqrt{31^2 - 4 \cdot 20 \cdot 12}}{2 - 20} = \frac{31 \pm \sqrt{961 - 960}}{40} = \frac{31 \pm 1}{40}$$

$$\Rightarrow \sin \theta = \frac{4}{5}, \frac{3}{4}$$

38. (b) Given that, $\sin x - \cos x = \sqrt{2}$

$$\Rightarrow \frac{1}{\sqrt{2}} \sin x - \frac{1}{\sqrt{2}} \cos x = 1$$

$$\Rightarrow \sin 45^\circ \sin x - \cos 45^\circ \cos x = 1$$

$$\Rightarrow \cos \left(x + \frac{\pi}{4} \right) = -1 \Rightarrow \cos \left(x + \frac{\pi}{4} \right) = \cos(\pi)$$

$$\Rightarrow x + \frac{\pi}{4} = 2n\pi + \pi \Rightarrow x = 2n\pi + \frac{3\pi}{4}$$

39. (a) We know that, $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$$\therefore \cos B = \frac{3^2 + 4^2 - 5^2}{2(3)(4)} = \frac{9 + 16 - 25}{2(3)(4)} = 0$$

$$\Rightarrow B = 90^\circ$$

$$\therefore \sin \frac{B}{2} + \cos \frac{B}{2} = \sin 45^\circ + \cos 45^\circ = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$$

40. (c) Given, $\sin^{-1}(1-x) + 2 \sin^{-1} x = \frac{\pi}{2}$

$$\Rightarrow \sin^{-1}(1-x) = \frac{\pi}{2} - 2 \sin^{-1} x$$

$$\Rightarrow (1-x) = \sin \left(\frac{\pi}{2} - 2 \sin^{-1} x \right)$$

$$\Rightarrow (1-x) = \cos(2 \sin^{-1} x)$$

$$\Rightarrow (1-x) = \cos[\cos^{-1}(1-2x^2)]$$

$$\Rightarrow (1-x) = 1 - 2x^2 \Rightarrow 2x^2 - x = 0$$

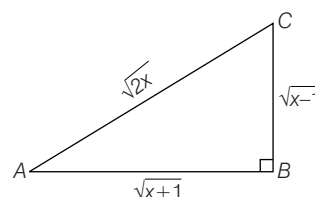
$$\Rightarrow x = 0, \frac{1}{2}$$

But $x = \frac{1}{2}$ does not satisfy the given equation.

So, $x = \{0\}$ is the answer.

41. (b) Given that,

$$\sin \frac{\theta}{2} = \sqrt{\frac{x-1}{2x}}$$



In $\triangle ABC$,

$$\tan \frac{\theta}{2} = \sqrt{\frac{x-1}{x+1}} \therefore \tan \theta = \frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}$$

$$= \frac{2 \sqrt{\frac{x-1}{x+1}}}{1 - \frac{x-1}{x+1}} = \frac{2 \sqrt{\frac{x-1}{x+1}}}{\frac{2}{x+1}} = \sqrt{x^2 - 1}$$

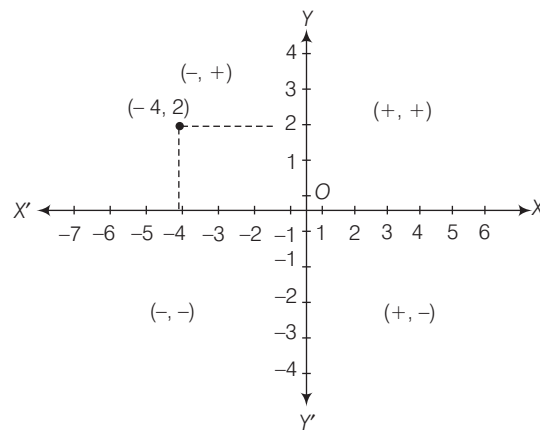
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Rectangular Coordinates and Straight Line

Cartesian System of Rectangular Coordinates in a Plane

The cartesian system of rectangular coordinates in the plane are specified in terms of X and Y -axes, which are known as **coordinate axes** as illustrated in the figure given alongside. The intersection of X and Y -axes is known as **origin**. The cartesian coordinates of a point in the plane are written as (x, y) .

Here, the point has coordinates $(-4, 2)$, as the point is four units to the left and two units up from the origin.



Distance Formula

When coordinates of two points are given in cartesian form.

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the two points.

Then,

$$PQ = d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

or

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Distance between the points $(0, 0)$ and (x, y) is $\sqrt{x^2 + y^2}$.

Distance formula can also be given as

$$\frac{x - x_1}{\cos \theta} = \frac{y - y_1}{\sin \theta} = r$$

where, r is the distance of the point (x, y) on the line from the point (x_1, y_1) and θ is the angle making the line with the positive direction of X -axis.

Applications of Distance Formulae

1. Collinearity of Three Given Points

The three given points A , B and C are collinear, i.e. lie on the same straight line, if any of three points (say B) lie on the straight line joining the other two points.

i.e. $AB + BC = AC$

2. Other Conditions for Collinearity

(i) Area of $\triangle ABC$ is zero. It means

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

(ii) Slope of AB = Slope of BC

Three points (x_1, y_1) , (x_2, y_2) and (x_3, y_3) are collinear,

$$\text{if } \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2}$$

3. Identifying the Types of Triangles

If A , B and C are vertices of triangle, then it would be

- (i) equilateral triangle, when $AB = BC = CA$.
- (ii) isosceles triangle, when any two sides are equal.
- (iii) right angled triangle, when sum of the square of any two sides is equal to square of the third side.

4. Position of Four Points

Let A , B , C and D be the four given points in a plane. By joining these points, following figures are formed:

- (i) Square, if $AB = BC = CD = DA$ and $AC = BD$.
- (ii) Rhombus, if $AB = BC = CD = DA$ and $AC \neq BD$.
- (iii) Parallelogram, if $AB = DC$, $BC = AD$, $AC \neq BD$.
- (iv) Rectangle, if $AB = DC$, $BC = DA$, $AC = BD$.

Conditions of Diagonals in Quadrilaterals

Quadrilateral	Diagonals	Angles between Diagonals
Parallelogram	Not equal	$\theta \neq \frac{\pi}{2}$
Rectangle	Equal	$\theta = \frac{\pi}{2}$
Rhombus	Not equal	$\theta = \frac{\pi}{2}$
Square	Equal	$\theta = \frac{\pi}{2}$

Section Formulae

Coordinates of a point which divide the line segment joining two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the ratio $m_1 : m_2$ are

$$(i) \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right) \quad [\text{internal division}]$$

$$(ii) \left(\frac{m_1 x_2 - m_2 x_1}{m_1 - m_2}, \frac{m_1 y_2 - m_2 y_1}{m_1 - m_2} \right) \quad [\text{external division}]$$

When m_1 and m_2 are of opposite signs, then division is **external**.

• Mid-point of the line joining (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.

• Coordinates of any point on one line segment which divide the line segment joining two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the ratio $\lambda : 1$, are given by

$$\left(\frac{x_1 + \lambda x_2}{\lambda + 1}, \frac{y_1 + \lambda y_2}{\lambda + 1} \right), (\lambda \neq -1)$$

• Lines formed by joining (x_1, y_1) and (x_2, y_2) is divided by

> X-axis in the ratio $-\frac{y_1}{y_2}$.

> Y-axis in the ratio $-\frac{x_1}{x_2}$.

If the ratio is positive, then the axis divides it internally and if ratio is negative, then the axis divides it externally.

• If the line $Ax + By + C = 0$ divides the line joining the points (x_1, y_1) and (x_2, y_2) in the ratio $\lambda : 1$, then

$$\lambda = - \left(\frac{Ax_1 + By_1 + C}{Ax_2 + By_2 + C} \right)$$

If λ is positive, then it divides internally and if λ is negative, then it divides externally.

Area of a Triangle

(a) Area of $\triangle ABC$, whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, is given by

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

(b) If one vertex (x_3, y_3) is at origin $(0, 0)$, then

$$\Delta = \frac{1}{2} |x_1 y_2 - x_2 y_1|$$

Centroid of $\triangle ABC$

Centroid of a $\triangle ABC$ whose vertices are (x_1, y_1) , (x_2, y_2) and (x_3, y_3) , is given by

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

Incentre of a ΔABC

Incentre of a ΔABC is given by

$$\left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c} \right)$$

Centres of a Inscribed Circle

Centres of inscribed circles are given by

$$\left(\frac{-ax_1 + bx_2 + cx_3}{-a + b + c}, \frac{-ay_1 + by_2 + cy_3}{-a + b + c} \right),$$

$$\left(\frac{ax_1 - bx_2 + cx_3}{a - b + c}, \frac{ay_1 - by_2 + cy_3}{a - b + c} \right)$$

and $\left(\frac{ax_1 + bx_2 - cx_3}{a + b - c}, \frac{ay_1 + by_2 - cy_3}{a + b - c} \right)$

respectively touching the sides BC , CA and AB internally; a, b, c being the sides BC, CA, AB of ΔABC .

Locus and its Equation

It is the path or curve traced by a moving point satisfying the given condition.

Equation to the Locus of a Point

The equation to the locus of a point is the algebraic relation which is satisfied by the coordinates of every point on the locus of the point.

Steps to Find the Locus of a Point

Step I Assumes the coordinates of the point, say (h, k) whose locus is to be find.

Step II Write the given condition involving (h, k) .

Step III Eliminate the variable(s), if any.

Step IV Replace $h \rightarrow x$ and $k \rightarrow y$. The equation so obtained is the locus of the point which moves under some definite conditions.

Translation of Axes

1. To Alter the Origin of Coordinates without Altering the Direction of the Axes

Let origin $O(0,0)$ be shifted to a point (a, b) by moving X and Y -axes parallel to themselves. If the coordinates of point P with reference to old axes are (x_1, y_1) , then coordinates of this point with respect to new axes will be $(x_1 - a, y_1 - b)$.

2. To Change the Direction of the Axes of Coordinates without Changing Origin

Let OX, OY be the old axes and OX', OY' be the new axes obtained by rotating the old OX and OY through an angle θ , then the coordinates of $P(x, y)$ with respect to new coordinate axes will be given by

	$x \downarrow$	$y \downarrow$
$x' \rightarrow$	$\cos \theta$	$\sin \theta$
$y' \rightarrow$	$-\sin \theta$	$\cos \theta$

(i) x, y are old coordinates and x', y' are new coordinates.

(ii) The axes rotation in anti-clockwise is positive and clockwise rotation of axes is negative.

3. To Change the Direction of the Axes of Coordinates by Changing the Origin

If $P(x, y)$ and the axes are shifted parallel to the original axis, so that new origin is (α, β) and then the axes are rotated about the new origin (α, β) by angle ϕ in the anti-clockwise (x', y') , then the coordinates of P will be given by

$$x = \alpha + x' \cos \phi - y' \sin \phi \quad \text{and} \quad y = \beta + x' \sin \phi + y' \cos \phi$$

Reflection (Image) of a Point

Let (x, y) be any point, then its image with respect to

X -axis is $(x, -y)$ Y -axis is $(-x, y)$ origin is $(-x, -y)$

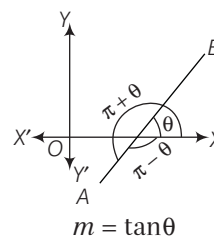
line $y = x$ is (y, x)

Straight Line

A straight line defined as the curve which is such that the segment joining any two points on it lies wholly on it.

Slope of a Line

The trigonometrical tangent of the angle that a line makes with the positive direction of X -axis in anti-clockwise sense, is called the slope or gradient of the line. The slope of a line is generally denoted by m . Thus, θ is positive or negative according as it is measured in anti-clockwise or clockwise direction.



Various Forms of Equations of a Line

The equation of a line in the general form can be written as $ax + by + c = 0$

1. Slope Intercept Form

The equation of a line with slope m and making an intercept c on Y -axis is

$$y = mx + c$$

2. General Form to Slope Intercept Form

The general form of the equation of a line is

$$Ax + By + C = 0 \Rightarrow By = -Ax - C$$

$$\Rightarrow y = \left(-\frac{A}{B}\right)x + \left(-\frac{C}{B}\right)$$

This is of the form $y = mx + c$, where $m = -\frac{A}{B}$ and $c = -\frac{C}{B}$.

Thus, for the straight line $Ax + By + C = 0$ having slope $m = -\frac{A}{B}$ and intercept on Y -axis $= -\frac{C}{B}$.

3. Point Slope Form

The equation of a line which passes through the point (x_1, y_1) and has the slope m , is $y - y_1 = m(x - x_1)$

4. Two Points Form

The equation of a line passing through two points (x_1, y_1)

and (x_2, y_2) is $(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

$$\text{or } \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

5. Intercept Form of a Line

The equation of a line which cuts-off intercepts a and b respectively from X and Y -axes, is $\frac{x}{a} + \frac{y}{b} = 1$.

6. General Equation of a Line to Intercept Form

The general equation of a line $Ax + By + C = 0$ is

$$-\frac{x}{\left(\frac{C}{A}\right)} + \frac{y}{\left(\frac{C}{B}\right)} = 1$$

7. Normal or Perpendicular Form

The equation of the straight line upon which the length of the perpendicular from the origin is p and this perpendicular makes an angle α with X -axis is

$$x \cos \alpha + y \sin \alpha = p, \text{ where } 0 \leq \alpha \leq \pi.$$

8. General Equation of a Line to the Normal Form

The general equation of a line is

$$Ax + By + C = 0.$$

Now, to reduce the general equation of a line to normal form, we first shift the constant term on the RHS and make it positive, if it is not so, then divide both sides by

$$\Rightarrow \frac{\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2}}{\sqrt{A^2 + B^2}} \Rightarrow \left(\frac{A}{\sqrt{A^2 + B^2}}\right)x + \left(\frac{B}{\sqrt{A^2 + B^2}}\right)y = \left(\frac{-C}{\sqrt{A^2 + B^2}}\right)$$

9. Distance Point Form

The equation of the straight line passing through (x_1, y_1) and making an angle θ with the positive direction of X -axis is

$$\frac{x - x_1}{\cos \theta} = \frac{y - y_1}{\sin \theta} = r$$

where, r is the distance of the point (x, y) on the line from the point (x_1, y_1) .

Angle between Two Lines

The angle θ between the lines having slopes m_1 and m_2 is given by

$$\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$$

1. Parallelism of Lines

If two lines of slopes m_1 and m_2 are parallel, then the angle θ between them is 0° .

$$\therefore \tan \theta = \tan 0^\circ = 0 \Rightarrow \frac{m_2 - m_1}{1 + m_1 m_2} = 0$$

$$\Rightarrow m_1 = m_2$$

Thus, when two lines are parallel, their slopes are equal.

2. Perpendicularity of Two Lines

If two lines of slopes m_1 and m_2 are perpendicular, then the angle θ between them is 90° .

$$\therefore \cot \theta = 0$$

$$\Rightarrow \frac{1 + m_1 m_2}{m_1 - m_2} = 0 \Rightarrow m_1 \cdot m_2 = -1$$

Thus, when two lines are perpendicular, the product of their slopes is -1 .

If m is the slope of a line, then the slope of a line perpendicular to it is $\left(-\frac{1}{m}\right)$.

Equation of a Line Parallel and Perpendicular to a Given Line

The equation of a line **parallel** to a given line $ax + by + c = 0$, is $ax + by + k = 0$, where k is a constant.

The equation of a line **perpendicular** to a given line $ax + by + c = 0$, is $bx - ay + k = 0$, where k is a constant.

Point of Intersection of Two Lines

Let the equations of two lines be

$$a_1x + b_1y + c_1 = 0 \quad \dots(i)$$

$$\text{and} \quad a_2x + b_2y + c_2 = 0 \quad \dots(ii)$$

Suppose these two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations

$$\therefore a_1x_1 + b_1y_1 + c_1 = 0$$

$$\text{and} \quad a_2x_1 + b_2y_1 + c_2 = 0$$

On solving these two equations by cross-multiplication method, we get

$$\frac{x_1}{b_1c_2 - b_2c_1} = \frac{y_1}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow x_1 = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

$$\text{and} \quad y_1 = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

which is the required intersection point.

Distance of a Point from a Line

The length of the perpendicular from a point (x_1, y_1) to a line

$$ax + by + c = 0 \text{ is } \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|.$$

(i) The length of the perpendicular from the origin to the line $ax + by + c = 0$ is $\frac{|c|}{\sqrt{a^2 + b^2}}$.

(ii) Distance between parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\frac{|c_2 - c_1|}{\sqrt{a^2 + b^2}}$.

Equation of Family of Lines Through the Intersection of Two Given Lines

The equation of the family of lines passing through the intersection of the lines

$$a_1x + b_1y + c_1 = 0$$

and

$$a_2x + b_2y + c_2 = 0, \text{ is}$$

$$(a_1x + b_1y + c_1) + \lambda(a_2x + b_2y + c_2) = 0$$

where, λ is a parameter.

Conditions for two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ to be coincident, parallel, perpendicular or intersecting, are given below:

$$(i) \text{ Coincident, if } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$(ii) \text{ Parallel, if } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$(iii) \text{ Perpendicular, if } a_1a_2 + b_1b_2 = 0$$

$$(iv) \text{ Intersecting, if } a_1b_2 - a_2b_1 \neq 0$$

$$\text{i.e.} \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Equations of Internal and External Bisectors of Angles between Two Lines

The equations of the bisectors of the angles between the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are given by

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

Conditions	Internal (acute) angle bisector	External (obtuse) angle bisector
$a_1a_2 + b_1b_2 > 0$	-	+
$a_1a_2 + b_1b_2 < 0$	+	-

Concurrent Lines

The three lines are concurrent, if they meet at a point.

Method 1 Find the point of intersection of any two lines and show that it satisfies the third also.

Method 2 The three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$, then they are concurrent if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Method 3 The condition for the lines $A = 0$, $B = 0$ and $C = 0$ to be concurrent is that three constants l , m , n (not all zeroes at the same time) can be obtained such that

$$lA + mB + nC = 0$$

Important Points

- The foot of perpendicular drawn from a point (x_1, y_1) to the line $ax + by + c = 0$ is

$$\left(\frac{x - x_1}{a}\right) = \left(\frac{y - y_1}{b}\right) = -\frac{ax_1 + by_1 + c}{a^2 + b^2}$$

- Let the equation of the line be $ax + by + c = 0$ and $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two given points. Then,

- If $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ both are of the same signs, then the points P and Q lie on the same side of the line $ax + by + c = 0$.
- If $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ both are of opposite signs, then the points P and Q lie on the opposite side of the line $ax + by + c = 0$.

- The position of a point (x_1, y_1) and (x_2, y_2) relative to the line $ax + by + c = 0$

- If $\frac{(ax_1 + by_1 + c)}{ax_2 + by_2 + c} > 0$, then points lie on the same side.
- If $\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c} < 0$, then the points lie on opposite side.

- The slope m of a line which is equally inclined with two lines of slopes m_1 and m_2 is given by

$$\frac{m - m_1}{1 + mm_1} = \frac{m_2 - m}{1 + mm_2}$$

- Let the image of a point (x_1, y_1) with respect to $ax + by + c = 0$ be (x_2, y_2) , then

$$\frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$$

- The image of the point $P(x_1, y_1)$ with respect to X -axis is $Q(x_1, -y_1)$.
- The image of the point $P(x_1, y_1)$ with respect to Y -axis is $Q(-x_1, y_1)$.
- The image of the point $P(x_1, y_1)$ with respect to mirror $y = x$ is $Q(y_1, x_1)$.
- The image of the point $P(x_1, y_1)$ with respect to the line mirror $y = x \tan \theta$ is

$$x = x_1 \cos 2\theta + y_1 \sin 2\theta;$$

$$y = x_1 \sin 2\theta - y_1 \cos 2\theta$$

- The image of the point $P(x_1, y_1)$ with respect to the origin is the point $(-x_1, -y_1)$.
- The length of perpendicular from a point (x_1, y_1) to a line $ax + by + c = 0$ is

$$\left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

Practice Exercise

- If the base of an equilateral triangle with side $2a$ lies along the Y -axis such that the mid-point of the base is at the origin. Then, the vertices of the triangle are
 - $(\sqrt{3}a, 0), (0, a), (0, -a)$
 - $(\sqrt{3}a, 0), (a, 0), (1, -a)$
 - $(\sqrt{3}a, 1), (a, 0), (-a, 0)$
 - $(\sqrt{3}a, 0), (0, a), (-a, 0)$
- If the equation of the base of an equilateral triangle is $x + y = 2$ and the vertex is $(2, -1)$, then the length of the side of the triangle is (in unit)
 - $\sqrt{2}$
 - $\sqrt{\frac{3}{2}}$
 - $\sqrt{\frac{1}{2}}$
 - $\sqrt{\frac{5}{6}}$
- The coordinates of the middle points of the sides of a triangle are $(4, 2)$, $(3, 3)$ and $(2, 2)$. The coordinates of its centroid are
 - $(3, 7/3)$
 - $(3, 3)$
 - $(4, 3)$
 - None
- If p is the length of perpendicular from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$ and a^2, p^2, b^2 are in an AP, then $a^4 + b^4$ is equal to
 - 1
 - 2
 - 3
 - 0
- If the line joining two points $A(2, 0)$ and $B(3, 1)$ is rotated about A in anti-clockwise direction through an angle of 15° , then equation of the line in new position will be
 - $y - \sqrt{2}x + 2\sqrt{3} = 0$
 - $y - \sqrt{3}x + \sqrt{3} = 0$
 - $y - \sqrt{2}x + \sqrt{3} = 0$
 - $y - \sqrt{3}x + 2\sqrt{3} = 0$
- The number of integer values of m for which the x -coordinates of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is
 - 2
 - 0
 - 4
 - 1
- If p is the length of perpendicular from origin to the line whose intercept on the axes are a and b , then $\frac{1}{a^2} + \frac{1}{b^2}$ is equal to
 - $1/p^3$
 - $1/P$
 - $1/p^2$
 - p
- If t_1, t_2 and t_3 are distinct, then the points $(t_1, 2at_1 + at_1^3)$, $(t_2, 2at_2 + at_2^3)$ and $(t_3, 2at_3 + at_3^3)$ are collinear, when
 - $t_1 t_2 t_3 = 1$
 - $t_1 + t_2 + t_3 = t_1 t_2 t_3$
 - $t_1 + t_2 + t_3 = 0$
 - $t_1 + t_2 + t_3 = 1$

9. If two vertices of a triangle are $(5, -1)$ and $(-2, 3)$ and if its orthocentre lies at the origin, then the coordinates of third vertex are
 a. $(4, 7)$ b. $(-4, -7)$ c. $(2, -3)$ d. $(5, -1)$
10. If the vertices of a triangle have integral coordinates, then the triangle is
 a. equilateral b. never equilateral
 c. always isosceles d. None of these
11. The area bounded by the curves $x + 2|y| = 1$ and $x = 0$ is
 a. $\frac{1}{4}$ b. $\frac{1}{2}$ c. 1 d. 2
12. If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in GP with same common ratio, then the points $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$
 a. lie on a straight line b. lie on an ellipse
 c. lie on a circle d. are vertices of a triangle
13. If the area of the triangle whose vertices are (b, c) , (c, a) and (a, b) is Δ , then the area of triangle whose vertices are $(ac - b^2, ab - c^2)$, $(ba - c^2, bc - a^2)$ and $(cb - a^2, ca - b^2)$, is
 a. Δ^2 b. $(a + b + c)^2 \Delta$
 c. $a \Delta + b \Delta^2$ d. None of these
14. If $\frac{x}{c} + \frac{y}{d} = 1$ is any line through the intersection of $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$, then
 a. $\frac{1}{c} + \frac{1}{d} = \frac{1}{a} + \frac{1}{b}$ b. $\frac{1}{d} + \frac{1}{a} = \frac{1}{b} + \frac{1}{c}$
 c. $\frac{1}{b} + \frac{1}{d} = \frac{1}{c} + \frac{1}{a}$ d. None of these
15. Let PS be the median of the triangle with vertices $P(2, 2)$, $Q(6, -1)$ and $R(7, 3)$. The equation of the line passing through $(1, -1)$ and parallel to PS , is
 a. $4x + 7y + 3 = 0$ b. $2x - 9y - 11 = 0$
 c. $4x - 7y - 11 = 0$ d. $2x + 9y + 7 = 0$
16. The x-coordinates of the incentre of the triangle that has the coordinates of mid-point of its sides as $(0, 1)$, $(1, 1)$ and $(1, 0)$, is
 a. $2 + \sqrt{2}$ b. $2 - \sqrt{2}$ c. $1 + \sqrt{2}$ d. $1 - \sqrt{2}$
17. The angle between the lines $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$ is
 a. 30° b. 60°
 c. 90° d. 45°
18. The tangent of an angle between the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{a} - \frac{y}{b} = 1$ is equal to
 a. $\frac{3ab}{a^2 - b^2}$ b. $\frac{ab}{a^2 - b^2}$ c. $\frac{2ab}{a^2 - b^2}$ d. $\frac{4ab}{a^2 - b^2}$
19. The locus of the point of intersection of the lines $x \sin \theta + (1 - \cos \theta)y = a \sin \theta$ and $x \sin \theta - (1 + \cos \theta)y + a \sin \theta = 0$ is
 a. $x^2 - y^2 = a^2$ b. $x^2 + y^2 = a^2$
 c. $y^2 = ax$ d. None of these
20. If $A(2, -3)$ and $B(-2, 1)$ are two vertices of a triangle and third vertex moves on the line $2x + 3y = 9$, then the locus of the centroid of the triangle is
 a. $2x - 3y = 1$ b. $x - y = 1$
 c. $2x + 3y = 1$ d. $2x + 3y = 3$
21. If the line $2x + y = k$ passes through the point which divides the line segment joining the points $(1, 1)$ and $(2, 4)$ in the ratio $3 : 2$, then k is equal to
 a. $\frac{29}{5}$ b. 5 c. 6 d. $\frac{11}{5}$
22. The distance between the lines $3x + 4y = 9$ and $6x + 8y = 15$ is (in units)
 a. $\frac{3}{10}$ b. $\frac{2}{9}$ c. $\frac{1}{4}$ d. $\frac{1}{3}$
23. The foot of the perpendicular from $(2, 3)$ upon the line $4x - 5y + 8 = 0$ is
 a. $(0, 0)$ b. $(1, 1)$
 c. $(\frac{41}{78}, \frac{128}{75})$ d. $(\frac{78}{41}, \frac{128}{41})$
24. A straight line L with negative slope passes through the point $(8, 2)$ and cuts the positive coordinate axes at points P and Q . As L varies, the absolute minimum value of $OP + OQ$ is (O is origin)
 a. 10 b. 18 c. 16 d. 112
25. If the equation of the sides of a triangle are $x + y = 2$, $y = x$ and $\sqrt{3}y + x = 0$, then which of the following is an exterior point of the triangle?
 a. Orthocentre b. Incentre
 c. Centroid d. None of these
26. Without change of axes, if the origin is shifted to (h, k) , then from the equation $x^2 + y^2 - 4x + 6y - 7 = 0$, the terms containing linear powers are missing. The point (h, k) is
 a. $(3, 2)$ b. $(-3, 2)$ c. $(2, -3)$ d. $(-2, -3)$
27. If the family of lines $a(2x + y + 4) + b(x - 2y - 3) = 0$. Then, the number of lines belonging to the family at a distance of $\sqrt{10}$ units from $P(2, -3)$ is
 a. 0 b. 1 c. 2 d. 4
28. Let $A(h, k)$, $B(1, 1)$ and $C(2, 1)$ be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1 sq unit, then the set of values which k can take, is given by
 a. $\{1, 3\}$ b. $\{0, 2\}$
 c. $\{-1, 3\}$ d. $\{-3, -2\}$

29. The value of k such that $3x^2 - 11xy + 10y^2 - 7x + 13y + k = 0$ may represent a pair of straight lines, is
 a. 3 b. 4 c. 6 d. 8
30. Let $A(2, -3)$ and $B(-2, 1)$ be the vertices of $\triangle ABC$. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line
 a. $2x + 3y = 9$ b. $2x - 3y = 7$
 c. $3x + 2y = 5$ d. $3x - 2y = 3$
31. Find the equations of the lines through the point of intersection of the lines $x - y + 1 = 0$ and $2x - 3y + 5 = 0$ and whose distance from the point $(3, 2)$ is $\frac{7}{5}$.
 a. $3x - 4y + 6 = 0$ and $4x - 3y + 1 = 0$
 b. $3x + 4y + 6 = 0$ and $4x + 3y + 1 = 0$
 c. $3x - 4y - 6 = 0$ and $4x + 3y + 1 = 0$
 d. None of the above
32. If the sides BC, CA, AB of $\triangle ABC$ are respectively $x + 2y = 1$, $3x + y + 5 = 0$, $x - y + 2 = 0$. Then, the altitude through B is
 a. $x - 3y + 1 = 0$ b. $x - 3y + 4 = 0$
 c. $3x - y + 4 = 0$ d. $x - y + 2 = 0$
33. The equation of the straight lines through $(-2, -7)$ and having intercept of length 3 units between the lines $4x + 3y = 12$ and $4x + 3y = 3$, is
 a. $7x - 24y - 182 = 0$
 b. $7x + 24y + 182 = 0$
 c. $7x + 24y - 182 = 0$
 d. None of the above
34. A variable straight line drawn through the point of intersection of the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$ meets the coordinate axes at A and B , the locus of the mid-point of AB is
 a. $2xy(a + b) = ab(x + y)$
 b. $2xy(a - b) = ab(x - y)$
 c. $2xy(a + b) = ab(x - y)$
 d. None of the above
35. If non-zero numbers a, b and c are in HP, then the straight line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ always passes through a fixed point
 a. $(1, -\frac{1}{2})$ b. $(1, -2)$
 c. $(-1, -2)$ d. $(-1, 2)$
36. The vertices of a triangle are $A(-1, -7)$, $B(5, 1)$ and $C(1, 4)$. Find the equation of the bisector of $\angle ABC$.
 a. $x + 7y - 2 = 0$ b. $x - 7y - 2 = 0$
 c. $x - 7y + 2 = 0$ d. None of these
37. If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}$, $x > 0$ and $y = 3x$, $x > 0$, then a belongs to
 a. $(\frac{1}{2}, 3)$ b. $(-3, -\frac{1}{2})$ c. $(0, \frac{1}{2})$ d. $(3, \infty)$
38. Two equal sides of an isosceles triangle are $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point $(1, -10)$. Find the equation of the third side.
 a. $x - 3y = -31$ b. $x - 3y = 31$
 c. $x + 3y = 31$ d. $x + 3y = -31$
39. Let $P(-1, 0)$, $Q(0, 0)$ and $R(3, 3\sqrt{3})$ be three points. Then, the equation of the bisector of $\angle PQR$ is
 a. $\frac{\sqrt{3}}{2}x + y = 0$ b. $x + \sqrt{3}y = 0$
 c. $\sqrt{3}x + y = 0$ d. $x + \frac{\sqrt{3}}{2}y = 0$
40. A straight line through the point $A(3, 4)$ is such that its intercept between the axes is bisected at A . Its equation is
 a. $4x + 3y = 24$ b. $3x + 4y = 25$
 c. $x + y = 7$ d. $3x - 4y + 7 = 0$
41. The straight lines $5x + 4y = 0$, $x + 2y - 10 = 0$ and $2x + y + 5 = 0$ are
 a. concurrent
 b. the sides of an equilateral triangle
 c. the sides of a right angled triangle
 d. None of the above
42. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lie in the fourth quadrant and is equidistant from the two axes, then
 a. $3bc - 2ad = 0$ b. $3bc + 2ad = 0$
 c. $2bc - 3ad = 0$ d. $2bc + 3ad = 0$
43. Let PQR be a right angled isosceles triangle, right angled at $P(2, 1)$. If the equation of the line QR is $2x + y = 3$, then the equation representing the pair of lines PQ and PR is
 a. $3x^2 - 3y^2 + 8xy + 20x - 10y + 25 = 0$
 b. $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$
 c. $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$
 d. $3x^2 - 3y^2 - 8xy - 10x - 15y - 20 = 0$

BITSAT Archives

- The value of k such that the lines $2x - 3y + k = 0$, $3x - 4y - 13 = 0$ and $8x - 11y - 33 = 0$ are concurrent, is [2014]
 - 20
 - 7
 - 7
 - 20
- Three straight lines $2x + 11y - 5 = 0$, $24x + 7y - 20 = 0$ and $4x - 3y - 2 = 0$ [2014]
 - form a triangle
 - are only concurrent
 - are concurrent with one line bisecting the angle between the other two
 - None of the above
- The equations of the lines through $(1, 1)$ and making an angle of 45° with the line $x + y = 0$, are [2013]
 - $x - 1 = 0, x - y = 0$
 - $x - y = 0, y - 1 = 0$
 - $x + y - 2 = 0, y - 1 = 0$
 - $x - 1 = 0, y - 1 = 0$
- The equation of the base BC of an equilateral $\triangle ABC$ is $x + y = 2$ and A is $(2, -1)$. The length of the side of the triangle is [2012]
 - $\sqrt{2}$
 - $\left(\frac{3}{2}\right)^{1/2}$
 - $\left(\frac{1}{2}\right)^{1/2}$
 - $\left(\frac{2}{3}\right)^{1/2}$
- The foot of the perpendicular from the point $(3, 4)$ on the line $3x - 4y + 5 = 0$ is [2012]
 - $\left(\frac{81}{25}, \frac{92}{25}\right)$
 - $\left(\frac{92}{25}, \frac{81}{25}\right)$
 - $\left(\frac{46}{26}, \frac{54}{24}\right)$
 - $\left(\frac{46}{26}, \frac{54}{26}\right)$
- The equation of the bisector of the acute angle between the lines $3x + 4y + 5 = 0$ and $12x - 5y - 7 = 0$, is [2011]
 - $21x + 77y + 100 = 0$
 - $99x - 27y + 30 = 0$
 - $99x + 27y + 30 = 0$
 - $21x - 77y - 100 = 0$
- The line $x + y = 4$ divides the line joining the points $(-1, 1)$ and $(5, 7)$ in the ratio [2011]
 - 2 : 1
 - 1 : 2
 - 1 : 2 externally
 - None of the above
- The condition that the straight line joining the origin to the points of intersection of the line $4x + 3y = 24$ with the circle $(x - 3)^2 + (y - 4)^2 = 25$ [2010]
 - are coincident
 - are perpendicular
 - make equal angle with X-axis
 - None of the above
- Two opposite vertices of a rectangle are $(1, 3)$ and $(5, 1)$. If the equation of a diagonal of this rectangle is $y = 2x + c$. Then, the value of c is [2010]
 - 1
 - 3
 - 4
 - 9
- The transformed equation of $3x^2 + 3y^2 + 2xy = 2$, when the coordinate axes are rotated through an angle of 45° , is [2008]
 - $x^2 + 2y^2 = 1$
 - $2x^2 + y^2 = 1$
 - $x^2 + y^2 = 1$
 - $x^2 + 3y^2 = 1$
- If l, m, n are in arithmetic progression, then the straight line $lx + my + n = 0$ will pass through the point [2008]
 - $(-1, 2)$
 - $(1, -2)$
 - $(1, 2)$
 - $(2, 1)$
- If a vertex of a triangle is $(1, 1)$ and the mid-points of two sides through the vertex are $(-1, 2)$ and $(3, 2)$, then the centroid of the triangle is [2007]
 - $\left(1, \frac{7}{3}\right)$
 - $\left(\frac{1}{3}, \frac{7}{3}\right)$
 - $\left(-\frac{1}{3}, \frac{7}{3}\right)$
 - $\left(-1, \frac{7}{3}\right)$
- The equation of the sides of a triangle are $x - 3y = 0$, $4x + 3y = 5$ and $3x + y = 0$. The line $3x - 4y = 0$ passes through [2007]
 - the incentre
 - the centroid
 - the orthocentre
 - the circumcentre
- If $(0, -1)$ and $(0, 3)$ are two opposite vertices of a square, then the other two vertices are [2006]
 - $(0, 1), (0, -3)$
 - $(3, -1), (0, 0)$
 - $(2, 1), (-2, 1)$
 - $(2, 2), (1, 1)$
- The equation to the line bisecting the joining of $(3, -4)$ and $(5, 2)$ having its intercepts on X-axis and Y-axis in the ratio 2 : 1, is [2006]
 - $x + y - 3 = 0$
 - $2x - y = 9$
 - $x + 2y = 2$
 - $2x + y = 7$
- The circumcentre of a triangle formed by the line $xy + 2x + 2y + 4 = 0$ and $x + y + 2 = 0$, is [2005]
 - $(-1, -1)$
 - $(0, -1)$
 - $(1, 1)$
 - $(-1, 0)$

Answer with Solutions

Practice Exercise

1. (a) Let BC be the base of a triangle which lies on Y -axis and third vertex be $A(h, 0)$ or $A'(-h, 0)$.

Since, $\triangle ABC$ is an equilateral triangle.

$$\therefore AB = BC$$

$$\Rightarrow AB^2 = BC^2$$

$$\Rightarrow (h-0)^2 + (0-a)^2 = 4a^2$$

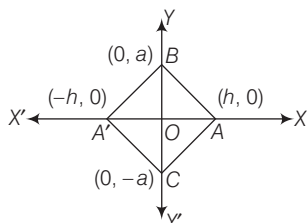
$$\left[\because \text{distance between two points } (x_1, y_1) \text{ and } (x_2, y_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \right]$$

$$\Rightarrow h^2 + a^2 = 4a^2 \Rightarrow h^2 = 3a^2$$

$$\Rightarrow h = \pm \sqrt{3}a \quad [\text{taking square root}]$$

Hence, the vertices of triangle are $(\sqrt{3}a, 0)$, $(0, a)$, $(0, -a)$

or $(-\sqrt{3}a, 0)$, $(0, a)$, $(0, -a)$.



2. (a) Let $\triangle ABC$ be an equilateral triangle.

Draw $AD \perp BC$. Then, length of $AD = \frac{|2-1-2|}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$

So, $\triangle ABC$ is an equilateral triangle.

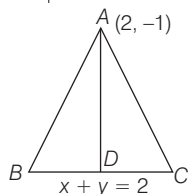
Hence, $\angle ABC = 60^\circ$

So, in $\triangle ABD$,

$$\sin 60^\circ = \frac{AD}{AB} \Rightarrow \frac{\frac{1}{\sqrt{2}}}{2} = \frac{1}{AB}$$

$$\Rightarrow AB = \frac{\sqrt{2}}{3} \text{ unit}$$

$$\Rightarrow AB = BC = CA = \frac{\sqrt{2}}{3} \text{ unit}$$

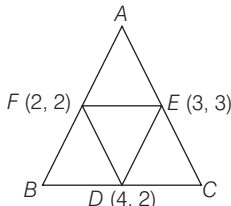


3. (a) Let D, E and F be the mid-points of BC, CA and AB .

$\therefore$ Centroid of $\triangle ABC$ = Centroid of $\triangle DEF$

$$= \left[\frac{1}{3}(2+4+3), \frac{1}{3}(2+2+3) \right]$$

$$= \left(3, \frac{7}{3} \right)$$



4. (d) Given equation is $\frac{x}{a} + \frac{y}{b} = 1$... (i)

At origin, perpendicular distance from line (i) is

$$p = \frac{|0+0-1|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \Rightarrow p = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} \quad \dots (ii)$$

Since, a^2, p^2 and b^2 are in an AP.

$$\therefore 2p^2 = a^2 + b^2$$

$$\text{Now, } \frac{1}{p^2} = \frac{a^2 + b^2}{a^2 b^2} \Rightarrow p^2 = \frac{a^2 b^2}{a^2 + b^2} \quad [\text{from Eq. (ii)}]$$

$$\Rightarrow \frac{a^2 + b^2}{2} = \frac{a^2 b^2}{a^2 + b^2}$$

$$\Rightarrow (a^2 + b^2)^2 = 2a^2 b^2$$

$$\Rightarrow a^4 + b^4 = 0$$

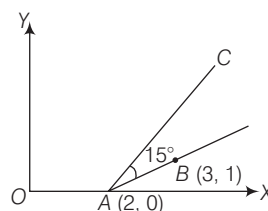
5. (d) The slope of the line AB is $\frac{1-0}{3-2} = 1$ or $\tan 45^\circ$. After

rotation of the line through 15° , the slope of the line AC in new position is $\tan 60^\circ = \sqrt{3}$.

Therefore, the equation of the new line AC is

$$y - 0 = \sqrt{3}(x - 2)$$

$$\Rightarrow y - \sqrt{3}x + 2\sqrt{3} = 0$$



6. (a) Given lines are

$$3x + 4y = 9$$

and

$$y = mx + 1$$

On solving both equations of lines, we get x -coordinates of the point of intersection as $x = \frac{5}{4m+3}$.

Since, x -coordinate is an integer.

$$\text{So, } 4m+3 = \pm 5$$

$$\text{or } 4m+3 = \pm 1$$

On solving these, we get

$$m = -1, -2$$

7. (c) Given equation of line in intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} - 1 = 0$$

Its distance from origin is

$$\left| \frac{0+0+(-1)}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \right| = p \quad \left[\because d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right| \right]$$

$$\Rightarrow p = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$$

$$\Rightarrow \frac{1}{p} = \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$$

On squaring both sides, we get

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

8. (c) Hint $\begin{vmatrix} t_1 & 2at_1 + at_1^3 & 1 \\ t_2 & 2at_2 + at_2^3 & 1 \\ t_3 & 2at_3 + at_3^3 & 1 \end{vmatrix} = 0 \Rightarrow a \begin{vmatrix} t_1 & 2t_1 + t_1^3 & 1 \\ t_2 & 2t_2 + t_2^3 & 1 \\ t_3 & 2t_3 + t_3^3 & 1 \end{vmatrix} = 0$

Applying $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$, we get

$$\begin{vmatrix} t_1 & 2t_1 + t_1^3 & 1 \\ t_2 - t_1 & 2(t_2 - t_1) + (t_2^3 - t_1^3) & 0 \\ t_3 - t_1 & 2(t_3 - t_1) + (t_3^3 - t_1^3) & 0 \end{vmatrix} = 0$$

9. (b) Let $O(0, 0)$ be the orthocentre, $A(x_1, y_1)$ be the third vertex and $B(-2, 3)$ $C(5, -1)$ be other two vertices of the given triangle.

Then, $OA \perp BC$

$$\Rightarrow \frac{y_1 - 0}{x_1 - 0} \times \frac{-1 - 3}{5 + 2} = -1$$

$$\Rightarrow 7x_1 = 4y_1 \quad \dots(i)$$

and $OB \perp AC$

$$\Rightarrow \frac{-3}{2} \times \frac{-1 - y_1}{5 - x_1} = -1$$

$$\Rightarrow 2x_1 - 3y_1 = 13 \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get $x_1 = -4, y_1 = -7$

Thus, the orthocentre is $(-4, -7)$.

10. (b) Area of the equilateral triangle with side $a = \frac{\sqrt{3}}{4} a^2$
[an irrational number]

Let $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) be vertices of an equilateral triangle. Then,

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \text{a rational number}$$

If x_1, y_1, x_2, y_2 and x_3, y_3 are integers.

An irrational number can't be equal to rational number.

Hence, a triangle with coordinates of the vertices as integers can't be an equilateral triangle.

11. (b) $x + 2|y| = 1$ and $x = 0$ gives $|y| = \frac{1}{2}$

$$\therefore y = \pm \frac{1}{2}$$

Required area = Area of $\triangle ABC$

where, AB is $x + 2y = 1$ and AC is $x - 2y = 1$.

$$\therefore \text{Required area} = \frac{1}{2} (BC)(OA) \\ = \frac{1}{2} (1)(1) = \frac{1}{2} \quad [\because BC = OA = 1]$$

12. (a) Let $x_1 = a, x_2 = ar, x_3 = ar^2$

$$\text{and } y_1 = b, y_2 = br, y_3 = br^2$$

$$\text{Now, } \frac{y_2 - y_1}{x_2 - x_1} = \frac{br - b}{ar - a} = \frac{b}{a}$$

$$\text{and } \frac{y_3 - y_2}{x_3 - x_2} = \frac{br^2 - br}{ar^2 - ar} = \frac{b}{a}$$

$\therefore$ Slope of PQ = Slope of QR

Hence, points P, Q, R are collinear.

13. (b) The area of the triangle formed by the second set of vertices is

$$\Delta = \frac{1}{2} \begin{vmatrix} b & c & 1 \\ c & a & 1 \\ a & b & 1 \end{vmatrix} \\ = \frac{1}{2} (a^2 + b^2 + c^2 - bc - ca - ab) \quad \dots(i)$$

and the area of the triangle formed by the second set of vertices is

$$\Delta' = \frac{1}{2} \begin{vmatrix} ac - b^2 & ab - c^2 & 1 \\ ba - c^2 & bc - a^2 & 1 \\ cb - a^2 & ca - b^2 & 1 \end{vmatrix} \\ = \frac{1}{2} \begin{vmatrix} (c-b)(a+b+c) & (a-c)(a+b+c) & 0 \\ (a-c)(a+b+c) & (b-a)(a+b+c) & 0 \\ bc - a^2 & ac - b^2 & 1 \end{vmatrix} \\ \text{[applying } R_1 \rightarrow R_1 - R_2 \text{ and } R_2 \rightarrow R_2 - R_3] \\ = \frac{1}{2} (a+b+c)^2 (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\therefore \Delta' = (a+b+c)^2 \Delta$$

14. (b) Any line through the intersection of

$$\frac{x}{a} + \frac{y}{b} = 1 \text{ and } \frac{x}{b} + \frac{y}{a} = 1 \text{ is}$$

$$\frac{x}{a} + \frac{y}{b} - 1 + k \left(\frac{x}{b} + \frac{y}{a} - 1 \right) = 0$$

$$\text{i.e. } \left(\frac{1}{a} + \frac{k}{b} \right) x + \left(\frac{1}{b} + \frac{k}{a} \right) y - (1+k) = 0$$

$$\text{If } \frac{x}{c} + \frac{y}{d} - 1 = 0, \text{ then } \frac{\frac{1}{a} + \frac{k}{b}}{\frac{1}{c}} = \frac{\frac{1}{b} + \frac{k}{a}}{\frac{1}{d}} = \frac{1+k}{1}$$

$$\therefore \frac{1}{a} + \frac{k}{b} = \frac{1}{c} + \frac{k}{d}$$

$$\Rightarrow \frac{1}{b} + \frac{k}{a} = \frac{1}{d} + \frac{k}{c}$$

$$\therefore \frac{1}{a} - \frac{1}{b} + k \left(\frac{1}{b} - \frac{1}{a} \right) = \frac{1}{c} - \frac{1}{d} + k \left(\frac{1}{d} - \frac{1}{c} \right)$$

$$\Rightarrow (1-k) \left(\frac{1}{a} - \frac{1}{b} \right) = (1-k) \left(\frac{1}{c} - \frac{1}{d} \right)$$

$$\Rightarrow \left(\frac{1}{a} - \frac{1}{b} \right) = \left(\frac{1}{c} - \frac{1}{d} \right)$$

$$\Rightarrow \frac{1}{a} + \frac{1}{d} = \frac{1}{b} + \frac{1}{c}$$

15. (d) Coordinates of S = Mid-point of line $QR = \left(\frac{13}{2}, 1 \right)$

$$\text{Hence, slope of line } PS = \frac{2-1}{2-\frac{13}{2}} = \frac{-2}{9}$$

Equation of line with slope $-2/9$ and passing through $(1, -1)$, is

$$\frac{y+1}{x-1} = \frac{-2}{9} \Rightarrow 9y+9 = -2x+2$$

$$\Rightarrow 2x+9y+7=0$$

16. (b) Given, mid-points of a triangle are $(0, 1)$, $(1, 1)$ and $(1, 0)$. Plot these points on a graph paper and make a triangle.

So, the sides of a triangle will be 2, 2 and $\sqrt{2^2 + 2^2}$, i.e. $2\sqrt{2}$.

$\therefore$ x-coordinate of incentre

$$= \frac{2 \cdot 0 + 2\sqrt{2} \cdot 0 + 2 \cdot 2}{2 + 2 + 2\sqrt{2}} = \frac{2}{2 + \sqrt{2}} \times \frac{2 - \sqrt{2}}{2 - \sqrt{2}} = 2 - \sqrt{2}$$

17. (a) **Hint** Given equations of lines are

$$\sqrt{3}x + y = 1 \Rightarrow y = -\sqrt{3}x + 1$$

and $x + \sqrt{3}y = 1$

$$\Rightarrow \sqrt{3}y = -x + 1 \Rightarrow y = \frac{-1}{\sqrt{3}}x + \frac{1}{\sqrt{3}}$$

Here, $m_1 = -\sqrt{3}$ and $m_2 = -\frac{1}{\sqrt{3}}$

18. (c) **Hint** Slope of given lines are

$$m_1 = -\frac{1/a}{1/b} = -\frac{b}{a} \text{ and } m_2 = \frac{-1/a}{-1/b} = \frac{b}{a}$$

19. (b) We have,

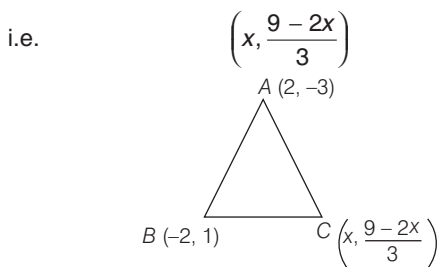
$$\frac{1 - \cos \theta}{\sin \theta} = \frac{a - x}{y} \quad \dots(i)$$

and $\frac{1 + \cos \theta}{\sin \theta} = \frac{a + x}{y} \quad \dots(ii)$

On multiplying Eqs. (i) and (ii), we get

$$\frac{1 - \cos^2 \theta}{\sin^2 \theta} = \frac{a^2 - x^2}{y^2} \Rightarrow x^2 + y^2 = a^2$$

20. (c) **Hint** The third vertex lies on $2x + 3y = 9$,



$\therefore$ Locus of centroid is

$$\left(\frac{2 - 2 + x}{3}, \frac{-3 + \frac{9 - 2x}{3} + 1}{3} \right) = (h, k)$$

21. (c) Given, line $L: 2x + y = k$ passes through point (say P) which divides a line segment (say AB) in the ratio 3 : 2, where $A(1, 1)$ and $B(2, 4)$. Using section formula, the coordinates of the point P which divides AB internally in the ratio 3 : 2, are

$$P\left(\frac{3 \times 2 + 2 \times 1}{3 + 2}, \frac{3 \times 4 + 2 \times 1}{3 + 2}\right) \equiv P\left(\frac{8}{5}, \frac{14}{5}\right)$$

Since, the line L passes through P . Hence, substituting the coordinates of $P\left(\frac{8}{5}, \frac{14}{5}\right)$ in the equation of

$L: 2x + y = k$, we get

$$2\left(\frac{8}{5}\right) + \left(\frac{14}{5}\right) = k \Rightarrow k = 6$$

22. (a) Given equations of lines $3x + 4y = 9$ and $6x + 8y = 15$ may be rewritten as $3x + 4y - 9 = 0$ and $3x + 4y - \frac{15}{2} = 0$.

Since, the slope of these lines are same, so they are parallel to each other.

Therefore, the distance between them is given by

$$d = \frac{\left| -9 + \frac{15}{2} \right|}{\sqrt{3^2 + 4^2}} = \frac{3}{10} \text{ unit}$$

23. (d) If foot is (h, k) , then

$$\frac{h - 2}{4} = \frac{k - 3}{-5} = \frac{-(8 - 15 + 8)}{16 + 25} = \frac{-1}{41}$$

$$\Rightarrow h = 2 - \frac{4}{41} = \frac{78}{41} \text{ and } k = 3 + \frac{5}{41} = \frac{128}{41}$$

So, the required point is $\left(\frac{78}{41}, \frac{128}{41}\right)$.

24. (b) The equation of the line L be $y - 2 = m(x - 8)$, $m < 0$.

Coordinates of P and Q are $\left(8 - \frac{2}{m}, 0\right)$ and $(0, 2 - 8m)$ respectively.

$$\text{So, } OP + OQ = 8 - \frac{2}{m} + 2 - 8m = 10 + \frac{2}{(-m)} + 8(-m)$$

$$\geq 10 + 2\sqrt{\frac{2}{(-m)} \times 8(-m)} \geq 18$$

[since, $AM \geq GM$ for two positive numbers]

So, absolute minimum value of $OP + OQ$ is 18.

25. (a) The lines $y = x$ and $\sqrt{3}y + x = 0$ are inclined at 45° and 150° , respectively. Thus, the angle between the two lines is an obtuse interior angle. Hence, orthocentre lies outside the given triangle.

26. (c) Let the new coordinates be $P(x', y')$.

After shifting origin to $P(x', y')$, i.e. $x = x' + h$ and $y = y' + k$
 $\therefore (x' + h)^2 + (y' + k)^2 - 4(x' + h) + 6(y' + k) - 7 = 0$
 $\Rightarrow (x')^2 + (y')^2 + 2(h - 2)x' + 2(k + 3)y' + (h^2 + k^2 - 4h + 6k - 7) = 0$

According to the question, $h - 2 = 0$ and $k + 3 = 0$

$$\Rightarrow (h, k) = (2, -3)$$

27. (b) **Hint** $P = \frac{a(4 - 3 + 4) + b(2 + 6 - 3)}{\sqrt{(2a + b)^2 + (a - 2b)^2}} = \sqrt{10}$

28. (c) **Hint** $AB = \sqrt{(1 - h)^2 + (1 - k)^2}$;

$$BC = \sqrt{(2 - 1)^2 + (1 - 1)^2} = 1$$

and $CA = \sqrt{(h - 2)^2 + (k - 1)^2}$

$$\begin{aligned} \text{Since, } AC^2 &= AB^2 + BC^2 = h^2 + 4 - 4h + k^2 + 1 - 2k \\ &= 1 + h^2 - 2h + k^2 + 1 - 2k + 1 \\ \Rightarrow h &= 1 \end{aligned} \quad \dots(i)$$

$$\therefore \text{Area of } \triangle ABC = \frac{1}{2} \times AB \times BC$$

29. (b) Given equation is

$$3x^2 - 11xy + 10y^2 - 7x + 13y + k = 0$$

On comparing with standard equation, we get

$$a = 3, b = 10, h = -\frac{11}{2}, g = \frac{-7}{2}, f = \frac{13}{2}, c = k$$

A second degree equation represents a pair of straight line, is given by

$$\begin{aligned} abc + 2fgh - af^2 - bg^2 - ch^2 &= 0 \\ \Rightarrow 3 \times 10 \times k + 2 \times \left(\frac{13}{2}\right) \left(\frac{-7}{2}\right) \left(\frac{-11}{2}\right) \\ &- 3 \left(\frac{13}{2}\right)^2 - 10 \left(\frac{-7}{2}\right)^2 - k \left(\frac{-11}{2}\right)^2 = 0 \\ \Rightarrow 30k + \frac{1001}{4} - \frac{507}{4} - \frac{490}{4} - \frac{121k}{4} &= 0 \\ \Rightarrow -\frac{k}{4} + 1 &= 0 \\ \Rightarrow k &= 4 \end{aligned}$$

30. (a) Let the third vertex be $C(x_1, y_1)$.

$$\text{The centroid is } \left(\frac{2-2+x_1}{3}, \frac{-3+1+y_1}{3} \right)$$

$$\text{i.e. } \left(\frac{x_1}{3}, \frac{y_1-2}{3} \right)$$

It lies on the line $2x + 3y = 1$

$$\therefore \frac{2}{3}(x_1) + 3 \left(\frac{y_1-2}{3} \right) = 1$$

Hence, the locus of C is $2x + 3y = 9$.

31. (a) **Hint** Equation of a line passing through the point of intersection of lines is

$$\begin{aligned} x - y + 1 + \lambda(2x - 3y + 5) &= 0 \\ \Rightarrow x(1 + 2\lambda) + y(-1 - 3\lambda) + 1 + 5\lambda &= 0 \end{aligned} \quad \dots(i)$$

Its distance from point $(3, 2) = \frac{7}{5}$

$$\Rightarrow \frac{|3(1 + 2\lambda) + 2(-1 - 3\lambda) + 1 + 5\lambda|}{\sqrt{(1 + 2\lambda)^2 + (-1 - 3\lambda)^2}} = \frac{7}{5}$$

32. (b) **Hint** Required equation of line is

$$x + 2y - 1 + \lambda(x - y + 2) = 0 \quad \dots(i)$$

It is perpendicular to $3x + y + 5 = 0$

$$\therefore 3(1 + \lambda) + 2 - \lambda = 0$$

33. (b) Let m be the slope of the line and angle θ , it makes with the parallel line.

$$\therefore \sin \theta = \frac{3}{5} \text{ or } \tan \theta = \frac{3}{4}$$

Hence, slope of the parallel lines is $-\frac{4}{3}$.

$$\begin{aligned} \therefore \left| \frac{m + \frac{4}{3}}{1 - \frac{4m}{3}} \right| &= \tan \theta = \frac{3}{4} \\ \Rightarrow \frac{3m + 4}{3 - 4m} &= \pm \frac{3}{4} \Rightarrow m = -\frac{7}{24} \end{aligned}$$

$$\text{Hence, the line is } y + 7 = -\frac{7}{24}(x + 2)$$

$$\text{or } 7x + 24y + 182 = 0$$

34. (a) **Hint** The intersection of given lines is $\frac{x}{a} + \frac{y}{b} - 1 + \lambda \left(\frac{x}{b} + \frac{y}{a} - 1 \right) = 0$ meets the coordinate axes at

$$A \left(\frac{1 + \lambda}{\frac{1}{a} + \frac{\lambda}{b}}, 0 \right) \text{ and } B \left(0, \frac{1 + \lambda}{\frac{1}{b} + \frac{\lambda}{a}} \right)$$

The mid-point of AB is given by

$$2x = \frac{1 + \lambda}{\frac{1}{a} + \frac{\lambda}{b}}, 2y = \frac{1 + \lambda}{\frac{1}{b} + \frac{\lambda}{a}}$$

35. (b) Since, a, b and c are in HP. Then, $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in an AP.

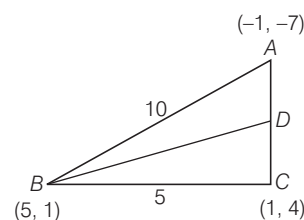
$$\therefore \frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Rightarrow \frac{1}{a} - \frac{2}{b} + \frac{1}{c} = 0$$

Hence, straight line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ is always passes through a fixed point $(1, -2)$.

36. (c) $BC = 5$ units, $BA = 10$ units

Let D divides AC in the ratio $2 : 1$

So, the coordinate of D is $\left(\frac{1}{3}, \frac{1}{3} \right)$.



$$\text{Slope of } BD = \frac{\frac{1}{3} - 1}{\frac{1}{3} - 5} = \frac{1}{7}$$

$\therefore$ Equation of the line joining B and D is

$$\frac{y - 1}{x - 5} = \frac{1}{7} \Rightarrow x - 7y + 2 = 0$$

37. (a) The point (a, a^2) lies in the sector bounded by the lines

$$x - 2y = 0 \text{ and } 3x - y = 0$$

$$\therefore (a - 2a^2)(3a - a^2) < 0 \Rightarrow \left[a - \frac{1}{2} \right] (a - 3) < 0$$

$$\text{Hence, } a \in \left(\frac{1}{2}, 3 \right)$$

38. (b) The third side is parallel to a bisector of the angle between the equal sides.

The bisectors are $7x - y + 3 = \pm 5(x + y - 3)$

$$\Rightarrow 2x - 6y + 18 = 0$$

$$\text{or } 12x + 4y - 12 = 0$$

$$\Rightarrow x - 3y + 9 = 0$$

$$\text{or } 3x + y - 3 = 0$$

Let the third side be $x - 3y = k$ or $3x + y = L$

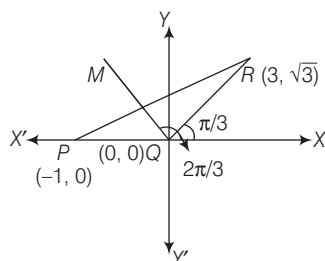
It passes through $(1, -10)$.

$$\therefore k = 31, L = -7$$

Hence, required lines are $x - 3y = 31$, $3x + y = -7$.

39. (c) Slope of $QR = \frac{3\sqrt{3} - 0}{3 - 0} = \sqrt{3} = \tan \theta \Rightarrow \theta = \frac{\pi}{3}$

$\therefore$ The angle between $\angle PQR$ is $\frac{2\pi}{3}$, so the line QM makes an angle $\frac{2\pi}{3}$ from positive direction of X -axis.



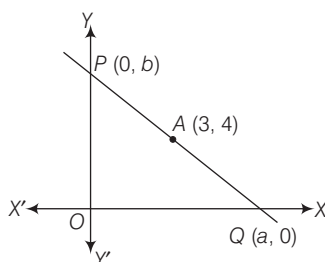
$$\text{Slope of the line } QM = \tan \frac{2\pi}{3} = -\sqrt{3}$$

Hence, equation of line QM is

$$y = -\sqrt{3}x \Rightarrow \sqrt{3}x + y = 0$$

40. (a) Since, A is the mid-point of line PQ .

$$\therefore 3 = \frac{a+0}{2} \Rightarrow a = 6 \text{ and } 4 = \frac{0+b}{2} \Rightarrow b = 8$$



Hence, the equation of line is $\frac{x}{6} + \frac{y}{8} = 1$ or $4x + 3y = 24$

$$41. (a) \text{ Hint } \Delta = \begin{vmatrix} 5 & 4 & 0 \\ 1 & 2 & -10 \\ 2 & 1 & 5 \end{vmatrix} = 0$$

42. (a) Given, lines are

$$4ax + 2ay + c = 0 \quad \dots(i)$$

$$\text{and } 5bx + 2by + d = 0 \quad \dots(ii)$$

Let the point of intersection be $(h, -h)$.

On substituting it in Eqs. (i) and (ii), we get

$$4ah - 2ah + c = 0$$

$$\text{and } 5bh - 2bh + d = 0$$

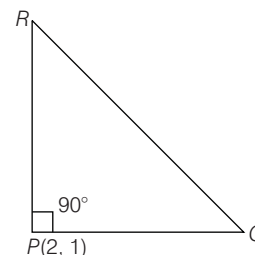
On eliminating h in the above two equations, we get

$$\frac{-c}{2a} = \frac{-d}{3b}$$

$$\text{Hence, } 3bc - 2ad = 0$$

43. (b) Let m be the slope of PR .

Slope of RQ , i.e. $2x + y = 3$ is -2 .



$$\therefore \tan 45^\circ = \left| \frac{m+2}{1+m(-2)} \right| = \left| \frac{m+2}{1-2m} \right|$$

$$\therefore m+2 = 1-2m$$

$$\text{or } -(m+2) = 2m-1$$

$$\Rightarrow 3m = -1 \text{ or } m = 3$$

$$\Rightarrow m = -\frac{1}{3} \text{ or } m = 3$$

$$\therefore \text{Equation of } PR \text{ is } y - 1 = -\frac{1}{3}(x - 2)$$

$$\text{i.e. } x + 3y - 5 = 0$$

and equation of PQ is

$$y - 1 = 3(x - 2)$$

$$\text{i.e. } 3x - y - 5 = 0$$

$\therefore$ Required pair of lines is given by

$$(x + 3y - 5)(3x - y - 5) = 0$$

$$\Rightarrow 3x^2 + 9xy - 15x - xy - 3y^2 + 5y - 5x - 15y + 25 = 0$$

$$\text{i.e. } 3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$$

BITSAT Archives

1. (b) Hint $\begin{vmatrix} 2 & -3 & k \\ 3 & -4 & -13 \\ 8 & -11 & -33 \end{vmatrix} = 0$

2. (c) For the two lines $24x + 7y - 20 = 0$ and $4x - 3y - 2 = 0$, the angle bisectors are given by

$$\frac{24x + 7y - 20}{25} = \pm \frac{4x - 3y - 2}{5}$$

Taking positive sign,

we get $24x + 7y - 20 = 4x - 3y - 2$

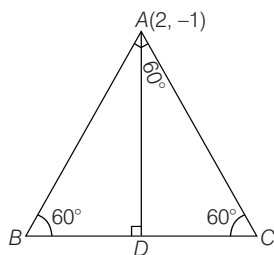
So, the given lines are concurrent with one line bisecting the angle between the other two.

3. (d) Hint Let m be the slope of required line.

$$\therefore \left| \frac{m - (-1)}{1 + m(-1)} \right| = 1 \Rightarrow \frac{m + 1}{1 - m} = \pm 1$$

4. (d) Length of perpendicular from $A(2, -1)$ to the line $x + y - 2 = 0$ is

$$AD = \left| \frac{2 - 1 - 2}{\sqrt{1 + 1}} \right| = \frac{1}{\sqrt{2}} \text{ unit}$$



In $\triangle ABD$, $\frac{AD}{AB} = \sin 60^\circ$

$$\Rightarrow \frac{1}{\sqrt{2} AB} = \frac{\sqrt{3}}{2}$$

$$\therefore AB = \frac{\sqrt{2}}{\sqrt{3}}$$

5. (a) Let M be the foot of perpendicular from $P(3, 4)$ on the line $3x - 4y + 5 = 0$. Then, M is the point of intersection of $3x - 4y + 5 = 0$ and line passing through $P(3, 4)$ and perpendicular to

$$3x - 4y + 5 = 0 \quad \dots(i)$$

Equation of the line perpendicular to

$$3x - 4y + 5 = 0 \text{ is } 4x + 3y + \lambda = 0$$

which passes through $(3, 4)$.

$$\Rightarrow 12 + 12 + \lambda = 0 \Rightarrow \lambda = -24$$

$$\therefore \text{Equation of line is } 4x + 3y - 24 = 0 \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$y = 92/25 \text{ and } x = 81/25$$

$$\therefore \text{Required point is } \left(\frac{81}{25}, \frac{92}{25} \right).$$

6. (c) Hint Given equations are

$$3x + 4y + 5 = 0$$

$$\text{and } 12x - 5y - 7 = 0$$

$$\therefore a_1 a_2 + b_1 b_2 = 3 \times 12 + 4 \times (-5) = 16 > 0$$

$\therefore$ For acute angle bisector,

$$\frac{a_1 x + b_1 y + c_1}{\sqrt{a_1^2 + b_1^2}} = - \frac{(a_2 x + b_2 y + c_2)}{\sqrt{a_2^2 + b_2^2}}$$

$$7. (c) \text{ Required ratio} = - \frac{(-1 + 1 - 4)}{(5 + 7 - 4)} = \frac{1}{2}$$

8. (b) Here, $a + b = 1 - 1 = 0$

Hence, pair of lines are perpendicular.

9. (c) The given diagonal does not pass through $(1, 3)$ and $(5, 1)$ as slope of the diagonal is 2 and that of the line joining $(1, 3)$ and $(5, 1)$ is

$$\frac{1 - 3}{5 - 1} = -\frac{1}{2}$$

The centre of the rectangle is mid-point of the join of $(1, 3)$ and $(5, 1)$ must lie on $y = 2x + c$.

$$\Rightarrow \left(\frac{1 + 5}{2}, \frac{3 + 1}{2} \right) = (3, 2) \text{ lies on } y = 2x + c$$

$$\Rightarrow 2 = 2 \times 3 + c$$

$$\Rightarrow c = -4$$

10. (b) Since, the axes are rotated through an angle 45° , so we replace (x, y) by

$$(x \cos 45^\circ - y \sin 45^\circ, x \sin 45^\circ + y \cos 45^\circ)$$

$$\text{i.e. } \left(\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}, \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} \right)$$

In the given equation,

$$3x^2 + 3y^2 + 2xy = 2$$

$$\therefore 3 \left(\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} \right)^2 + 3 \left(\frac{x + y}{\sqrt{2}} \right)^2 + 2 \left(\frac{x - y}{\sqrt{2}} \right) \left(\frac{x + y}{\sqrt{2}} \right) = 2$$

$$\Rightarrow \frac{3}{2} (x^2 + y^2 + 2xy) + \frac{3}{2} (x^2 + y^2 - 2xy) + \frac{2}{2} (x^2 - y^2) = 2$$

$$\Rightarrow 4x^2 + 2y^2 = 2$$

$$\Rightarrow 2x^2 + y^2 = 1$$

11. (b) Since, l, m, n are in an AP.

$$\therefore 2m = l + n$$

Given equation of line is

$$lx + my + n = 0$$

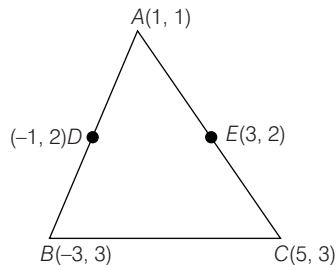
Now, assume that the point $(1, -2)$ satisfy the given equation.

$$\therefore l - 2m + n = 0$$

$$\Rightarrow 2m = l + n$$

$\Rightarrow l, m, n$ are in an AP.

12. (a) Let D and E be the mid-points of AB and AC . Then, coordinates of B and C are $(-3, 3)$ and $(5, 3)$ respectively.

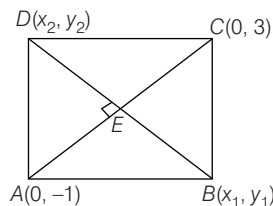


$$\begin{aligned}\text{Centroid of triangle} &= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right) \\ &= \left(\frac{1 - 3 + 5}{3}, \frac{1 + 3 + 3}{3} \right) = \left(1, \frac{7}{3} \right)\end{aligned}$$

13. (c) Since, two sides $x - 3y = 0$ and $3x + y = 0$ are perpendicular to each other. Therefore, its orthocentre is the point of intersection of $x - 3y = 0$ and $3x + y = 0$, i.e. $(0, 0)$. So, the line $3x - 4y = 0$ passes through the orthocentre of triangle.

14. (c) Let the points be $B(x_1, y_1)$ and $D(x_2, y_2)$. Then, the mid-point of

$$BD = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$



and mid-point of $AC = (0, 1)$

We know that, mid-point of both diagonals lie on the same point E .

$$\Rightarrow \frac{x_1 + x_2}{2} = 0 \text{ and } \frac{y_1 + y_2}{2} = 1$$

$$\Rightarrow x_1 + x_2 = 0 \quad \dots(i)$$

$$\text{and } y_1 + y_2 = 2 \quad \dots(ii)$$

$\therefore$ Slope of $BD \times$ Slope of $AC = -1$

$$\therefore \frac{(y_1 - y_2)}{(x_1 - x_2)} \times \frac{(3 + 1)}{(0 - 0)} = -1$$

$$\Rightarrow y_1 - y_2 = 0 \quad \dots(iii)$$

On solving Eqs. (ii) and (iii), we get

$$y_1 = 1, y_2 = 1$$

Now, slope of $AB \times$ slope of $BC = -1$

$$\Rightarrow \frac{(y_1 + 1)}{(x_1 - 0)} \times \frac{(y_1 - 3)}{(x_1 - 0)} = -1$$

$$\Rightarrow (y_1 + 1)(y_1 - 3) = -x_1^2$$

$$\Rightarrow 2(-2) = -x_1^2 \quad [\because y_1 = 1]$$

$$\Rightarrow x_1 = \pm 2$$

Hence, the required points are $(2, 1)$ and $(-2, 1)$.

15. (c) Let the points be $A(3, -4)$ and $B(5, 2)$ and mid-point of $AB = (4, -1)$

It is given that the bisecting line intercept the coordinate axes in the ratio $2 : 1$.

$\therefore$ Point of coordinate axes are $(2k, 0)$ and $(0, k)$. The equation of line passing through the above point is

$$y - 0 = \frac{k - 0}{0 - 2k}(x - 2k)$$

$$\text{or } y = -\frac{1}{2}(x - 2k) \quad \dots(i)$$

Since, it is passing through the mid-point of AB , i.e. $(4, -1)$.

$$\Rightarrow -1 = -\frac{1}{2}(4 - 2k)$$

$$\Rightarrow 2 = 4 - 2k$$

$$\Rightarrow k = 1$$

Putting the value of k in Eq. (i), we get

$$y = -\frac{1}{2}(x - 2)$$

$$\Rightarrow x + 2y = 2$$

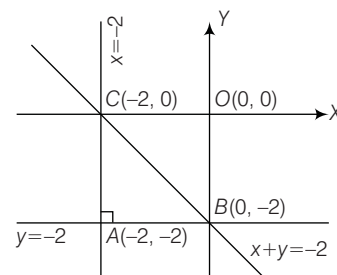
16. (a) Given equation of lines are

$$xy + 2x + 2y + 4 = 0$$

$$\text{or } (x + 2)(y + 2) = 0$$

$$\text{or } x + 2 = 0, y + 2 = 0 \quad \dots(i)$$

$$\text{and } x + y + 2 = 0 \quad \dots(ii)$$



These three lines makes $\triangle CAB$, right angled at A .

The circumcentre of a triangle is the mid-point of BC , i.e. $(-1, -1)$.

13

The Circle

Circle

The locus of a point which moves in such a way that its distance from a fixed point in the plane remains constant, is called a circle. This fixed point, is called the **centre** and the constant distance, is called the **radius** of the circle.

General Form of Equation of Circle

(i) The general form of the equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

Its centre is $(-g, -f)$ and radius is $\sqrt{g^2 + f^2 - c}$.

(ii) The general form of the equation of a circle can also be written as

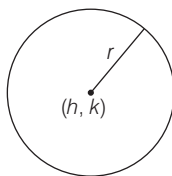
$$ax^2 + ay^2 + 2gx + 2fy + c = 0, a \neq 0$$

Its centre is $\left(\frac{-g}{a}, \frac{-f}{a}\right)$ and radius is $\sqrt{\frac{g^2}{a^2} + \frac{f^2}{a^2} - \frac{c}{a}}$.

- NOTE**
- The equation of a circle is a second degree equation in x and y .
 - It contains no term of xy .
 - Coefficient of x^2 = Coefficient of y^2
 - Coordinates of centre = $\left(\frac{-\text{Coefficient of } x}{2}, \frac{-\text{Coefficient of } y}{2}\right)$
 - A circle where radius is zero, is called point circle.

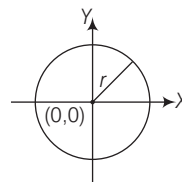
Different Types of Equations of Circle

(i) Circle having Centre (h, k) and Radius r



Equation of the circle is $(x - h)^2 + (y - k)^2 = r^2$.

(ii) Circle having Centre $(0, 0)$ and Radius r



Equation of the circle is $x^2 + y^2 = r^2$.

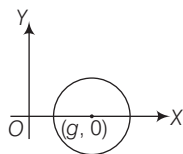
- (iii) Centre at X-axis y-coordinate of

centre $\equiv 0$, i.e. $f = 0$

Centre $\equiv (-g, 0)$

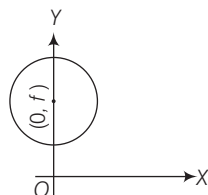
Equation of the circle is

$$x^2 + y^2 + 2gx + c = 0$$



- (iv) Centre at Y-axis x-coordinate of centre $\equiv 0$, i.e. $g = 0$

Centre $\equiv (0, -f)$

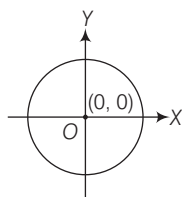


Equation of the circle is

$$x^2 + y^2 + 2fy + c = 0$$

- (v) Centre at Origin

Centre $\equiv (-g, -f) \equiv (0, 0)$



Equation of the circle is $x^2 + y^2 + c = 0$.

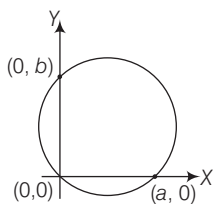
- (vi) Circle Passing through Origin

Equation of the circle is

$$x^2 + y^2 + 2gx + 2fy = 0$$

because $(0, 0)$ should satisfy the equation of the circle, i.e.

$$c = 0$$



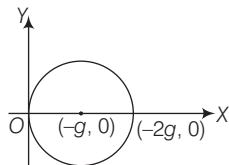
NOTE Equation of a circle passing through the origin which cuts X-axis at $(a, 0)$ and Y-axis at $(0, b)$ is $x^2 + y^2 - ax - by = 0$.

- (vii) Circle Touching Y-axis at Origin and Centred at X-axis

Equation of the circle is

$$x^2 + y^2 - 2gx = 0$$

$(0, 0)$ and $(2g, 0)$ are the end points of the diameter.

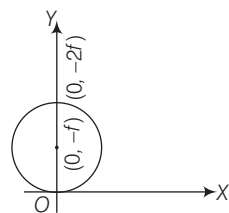


- (viii) Circle Touching X-axis at Origin and Centred at Y-axis

Equation of the circle is

$$x^2 + y^2 - 2fy = 0$$

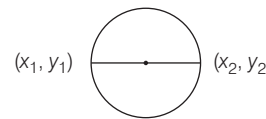
$(0, 0)$ and $(0, -2f)$ are the end points of the diameter.



- (ix) Equation of a Circle in

Diametric Form If (x_1, y_1) and (x_2, y_2) are the end points of one of the diameter, then the equation of the circle is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

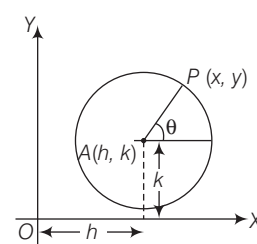


NOTE The diametric form of a circle can also be written as $x^2 + y^2 - x(x_1 + x_2) - y(y_1 + y_2) + x_1x_2 + y_1y_2 = 0$ or $x^2 + y^2 - x(\text{sum of abscissae}) - y(\text{sum of ordinates}) + \text{product of abscissae} + \text{product of ordinates} = 0$

- (x) Parametric Form of a Circle

Consider the circle $(x - h)^2 + (y - k)^2 = r^2$, centred at $A(h, k)$ and radius r .

Let $P \equiv (x, y)$, the coordinates of P can be expressed as $x = h + r \cos \theta$ and $y = k + r \sin \theta$.



These equations represent the coordinates of any point on the circle in terms of the parameter θ .

Position of a Point with Respect to a Circle

A point (x_1, y_1) lies outside, on or inside a circle

$$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$$

According as $S_1 >, =$ or < 0 respectively

where, $S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

NOTE The greatest and least distances of a point $A(x, y)$ from a circle with centre C and radius r are $|AC + r|$ and $|AC - r|$, respectively.

Tangent to a Circle

- (i) Let $P(x_1, y_1)$ be a point lying on the circle.

Then, equations of tangents in different forms of circles are given below:

Equation of the circle	Equation of the tangent at $P(x_1, y_1)$
(a) $x^2 + y^2 = r^2$	$T \equiv xx_1 + yy_1 - r^2 = 0$
(b) $(x - h)^2 + (y - k)^2 = r^2$	$T \equiv (x - h)(x_1 - h) + (y - k)(y_1 - k) - r^2 = 0$
(c) $x^2 + y^2 + 2gx + 2fy + c = 0$	$T \equiv xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

- (ii) **Condition for Tangency** A line L may be a Tangent to any given circle S , if the perpendicular distance L from the centre of S is equal to the radius of the circle S .

Let the line $y = mx + c$ be a tangent to $x^2 + y^2 = r^2$ with radius r , then $c = \pm r\sqrt{1+m^2}$.

The equation of tangents of slope m to the circle $x^2 + y^2 = r^2$ are $y = mx \pm r\sqrt{1+m^2}$ and coordinates of point of contact are $\left(\pm \frac{rm}{\sqrt{1+m^2}}, \mp \frac{r}{\sqrt{1+m^2}}\right)$.

(iii) **Length of Tangent** The length of the tangent from a given point (x_1, y_1) to a circle

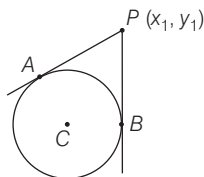
$x^2 + y^2 + 2gx + 2fy + c = 0$ is given as

$$l = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$

NOTE If a line touches the circle, then length of perpendicular from the centre upon the line is equal to the radius of the circle.

Pair of Tangents

If a point P lies outside a given circle, then a pair of tangents can be drawn from P to the circle as shown in figure.



The combined equation of the pair of tangents drawn from the point $P(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is

$$T^2 = SS_1$$

where, $S = x^2 + y^2 + 2gx + 2fy + c$,

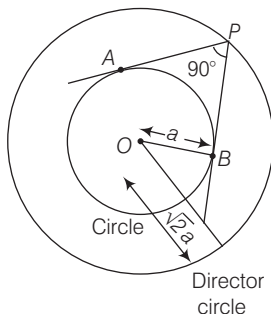
$$S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$$

and $T = xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c$

Director Circle

The locus of the point of intersection of two perpendicular tangents to a given circle, is known as director circle. If the equation of a circle is $x^2 + y^2 = a^2$. Then, equation of director circle is $x^2 + y^2 = 2a^2$.

NOTE The radius of the director circle is $\sqrt{2}$ times the radius of the given circle.



Equation of Normals

The normal at any point on a curve is a straight line which is perpendicular to the tangent to the curve at that point and it always passes through origin.

1. Point Form

The equation of normal to the circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ or $x^2 + y^2 = a^2$ at any point (x_1, y_1) is

$$\frac{x - x_1}{x_1 + g} = \frac{y - y_1}{y_1 + f} \quad \text{or} \quad \frac{x}{x_1} = \frac{y}{y_1}$$

2. Parametric Form

The equation of normal to the circle $x^2 + y^2 = a^2$ at point

$(a \cos \theta, a \sin \theta)$ is $\frac{x}{\cos \theta} = \frac{y}{\sin \theta}$.

Chord of Contact

The chord joining the points of contact of the two tangents from a point is called the chord of contact of tangents. The equation of the chord of contact of tangents drawn from a point (x_1, y_1) to the circle $x^2 + y^2 = r^2$ is

$$xx_1 + yy_1 = r^2 \text{ or } T = 0.$$

If AB is a chord of contact of tangents from C to the circle $x^2 + y^2 = r^2$ and M is the mid-point of AB as shown in figure.

Then,

$$(i) \text{ Coordinates of } M = \left(\frac{r^2 x_1}{x_1^2 + y_1^2}, \frac{r^2 y_1}{x_1^2 + y_1^2} \right)$$

$$(ii) AB = 2r \frac{\sqrt{x_1^2 + y_1^2 - r^2}}{\sqrt{x_1^2 + y_1^2}}$$

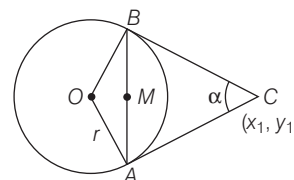
$$(iii) BC = \sqrt{x_1^2 + y_1^2 - r^2}$$

$$(iv) \text{ Area of quadrilateral } OACB = r \sqrt{x_1^2 + y_1^2 - r^2}$$

$$(v) \text{ Area of } \triangle ABC = \frac{r}{x_1^2 + y_1^2} (x_1^2 + y_1^2 - r^2)^{3/2}$$

$$(vi) \text{ Area of } \triangle OAB = \frac{r^3}{x_1^2 + y_1^2} \sqrt{x_1^2 + y_1^2 - r^2}$$

$$(vii) \text{ Angle between two tangents } \angle ACB \text{ is } 2 \tan^{-1} \frac{r}{\sqrt{S_1}}.$$



Chord Bisected at a Given Point

The equation of the chord of the circle $x^2 + y^2 = a^2$ bisected at the point (x_1, y_1) is given by

$$xx_1 + yy_1 - a^2 = x_1^2 + y_1^2 - a^2$$

or

$$T = S_1$$

Of all the chords which pass through a given point $M(a, b)$ inside the circle, the shortest chord is one whose middle point is (a, b) .

Equation of Common Chord of Two Circles

If $S = 0$ and $S' = 0$ are two intersecting circles, then equation of common chord is $S - S' = 0$.

Also, its length $= 2\sqrt{r_1^2 - p_1^2} = 2\sqrt{r_2^2 - p_2^2}$

where, p_1 and p_2 are the perpendicular distances of the centres from the common chord.

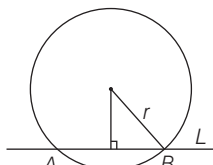
Intersection of a Line and a Circle

Consider the circle $x^2 + y^2 = r^2$ and the line $y = mx + c$.

(i) If $r > \left| \frac{c}{\sqrt{1+m^2}} \right|$, then the line intersects the circle at two distinct points.

(ii) If $r = \left| \frac{c}{\sqrt{1+m^2}} \right|$, then the line intersects the circle at only one point. It is called a tangent.

(iii) If $r < \left| \frac{c}{\sqrt{1+m^2}} \right|$, then the line does not intersect the circle.



NOTE The length of side of an equilateral triangle inscribed in the circle $x^2 + y^2 = a^2$ is $a\sqrt{3}$.

Family of Circles

The equation of a family of circles passing through the intersection of a circle $x^2 + y^2 + 2gx + 2fy + c = 0$ and line $L \equiv lx + my + n = 0$ is

$$x^2 + y^2 + 2gx + 2fy + c + \lambda(lx + my + n) = 0 \text{ or } S + \lambda L = 0$$

where, λ is any real number.

The equation of the family of circles passing through the point $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + \lambda \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (x - x_1)(x - x_2) + (y - y_1)(y - y_2) + \lambda L = 0$$

where, $L = 0$ represents the line passing through $A(x_1, y_1)$ and $B(x_2, y_2)$ and $\lambda \in R$.

The equation of the family of circles touching the circle $S = x^2 + y^2 + 2gx + 2fy + c = 0$ at point $P(x_1, y_1)$ is

$$x^2 + y^2 + 2gx + 2fy + c + \lambda\{xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c\} = 0$$

$$\text{or } S + \lambda L = 0$$

where, $L = 0$ is the equation of the tangent to $S = 0$ at (x_1, y_1) and $\lambda \in R$ the equation of the family of the circles passing through the intersection of the circles

$$S_1 = x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$$

$$\text{and } S_2 = x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0 \text{ is}$$

where $(\lambda \neq -1)$ is an arbitrary real number.

Angle of Intersection of Two Circles

The angle of intersection of two circles is defined as the angle between the tangents or angle between the normals to the two circles at their point of intersection is given by

$$\cos \theta = \frac{r_1^2 + r_2^2 - d^2}{2r_1 r_2}$$

where, d is distance between centres of the circles.

Orthogonal Circles

Two circles are said to intersect orthogonally, if their angle of intersection is a right angle.

$$(\text{Radius of 1st circle})^2 + (\text{Radius of 2nd circle})^2 = (\text{Distance between centres})^2$$

$$\Rightarrow 2(g_1g_2 + f_1f_2) = c_1 + c_2$$

which is the condition of orthogonality of two circles.

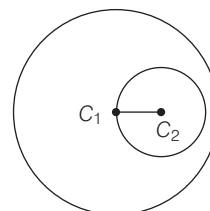
The circles having radii r_1 and r_2 intersect orthogonally.

$$\text{Length of their common chord} = \frac{2r_1r_2}{\sqrt{r_1^2 + r_2^2}}$$

Common Tangents of Two Circles

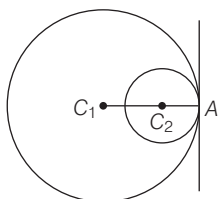
Let the centres and radii of two circles be C_1, C_2 and r_1, r_2 , respectively.

(i) When one circle contains other, no common tangent is possible.



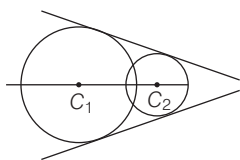
Condition $C_1C_2 < r_1 - r_2$.

- (ii) When two circles touch internally, one common tangent is possible.



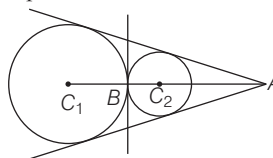
Condition $C_1C_2 = r_1 - r_2$

- (iii) When two circles intersect, two common tangents are possible.



Condition $|r_1 - r_2| < C_1C_2 < r_1 + r_2$

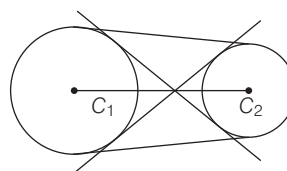
- (iv) When two circles touch externally, three common tangents are possible.



Condition $C_1C_2 = r_1 + r_2$

A divides C_1C_2 externally in the ratio $r_1:r_2$ and B divides C_1C_2 internally in the ratio $r_1:r_2$.

- (v) When two circles do not touch each other, four common tangents are possible.



Condition $C_1C_2 > r_1 + r_2$

Practice Exercise

- The points $(-5, 11)$, $(11, 19)$, $(18, -4)$ lie on a circle, centre of the circle is at
 - $(3, 4)$
 - $(4, 3)$
 - $(5, 4)$
 - None of these
- If the base of a triangle and the ratio of the lengths of the other two unequal sides are given, then the vertex lies on a/an.
 - straight line
 - circle
 - ellipse
 - parabola
- Find the equation of a circle concentric with the circle $x^2 + y^2 - 6x + 12y + 15 = 0$ and has double of its area.
 - $x^2 + y^2 - 6x + 12y - 15 = 0$
 - $x^2 + y^2 - 6x - 12y + 15 = 0$
 - $x^2 + y^2 - 6x + 12y + 15 = 0$
 - None of the above
- A circle of radius 5 units touches both the axes and lies in the first quadrant. If the circle makes one complete roll on the X-axis along the positive direction of X-axis, then its equation in the new position is
 - $x^2 + y^2 + 20\pi x - 10y + 100\pi^2 = 0$
 - $x^2 + y^2 + 20\pi x + 10y + 100\pi^2 = 0$
 - $x^2 + y^2 - 20\pi x - 10y + 100\pi^2 = 0$
 - None of the above
- If the circle $x^2 + y^2 + 6x + 5 + \lambda(x^2 + y^2 - 8x + 7) = 0$ is converted into a point, then the values of λ are
 - $1 \pm \frac{\sqrt{2}}{3}$
 - $2 \pm \frac{2\sqrt{2}}{3}$
 - $2 \pm \frac{4\sqrt{2}}{3}$
 - $1 \pm \frac{4\sqrt{2}}{3}$
- Find the equation of the circle which passes through the points $(2, 3)$ and $(4, 5)$ and the centre lies on the straight line $y - 4x + 3 = 0$.
 - $x^2 + y^2 - 4x - 10y + 25 = 0$
 - $x^2 + y^2 - 4x - 10y - 25 = 0$
 - $x^2 + y^2 - 4x + 10y - 25 = 0$
 - None of the above
- The equation of circle which passes through the point $(2, 0)$ and whose centre is the limit of the point of intersection of the lines $3x + 5y = 1$ and $(2 + c)x + 5c^2y = 1$ as $c \rightarrow 1$, is
 - $25(x^2 + y^2) - 20x + 2y + 60 = 0$
 - $25(x^2 + y^2) - 20x + 2y - 60 = 0$
 - $25(x^2 - y^2) - 20x - 2y - 60 = 0$
 - None of the above
- Consider a family of circles which are passing through the point $(-1, 1)$ and are tangent to X-axis. If (h, k) are the coordinates of the centre of the circles, then the set of values of k is given by the interval
 - $0 < k < \frac{1}{2}$
 - $k \geq \frac{1}{2}$
 - $-\frac{1}{2} \leq k \leq \frac{1}{2}$
 - $k \leq \frac{1}{2}$
- The cartesian equations of the curves $x = 7 + 4 \cos \alpha$ and $y = -3 + 4 \sin \alpha$ is
 - $x^2 + y^2 - 14x + 6y + 42 = 0$
 - $x^2 + y^2 - 6x + 14y + 21 = 0$
 - $x^2 + y^2 - 10x + 12y + 28 = 0$
 - None of the above

10. If the two conics $a_1x^2 + 2h_1xy + b_1y^2 = c_1$, $a_2x^2 + 2h_2xy + b_2y^2 = c_2$ intersect in 4 concyclic points, then
 a. $(a_1 - b_1)h_2 = (a_2 - b_2)h_1$ b. $(a_1 - b_1)h_1 = (a_2 - b_2)h_2$
 c. $(a_1 + b_1)h_2 = (a_2 + b_2)h_1$ d. $(a_1 + b_1)h_1 = (a_2 + b_2)h_2$
11. The circle passing through $(1, -2)$ and touching the X-axis at $(3, 0)$, also passes through the point
 a. $(-5, 2)$ b. $(2, -5)$
 c. $(5, -2)$ d. $(-2, 5)$
12. A circle touches the hypotenuse of a right angle triangle at its middle point and passes through the mid-point of the shorter side. If a and b ($a < b$) are the length of the sides, then the radius is
 a. $\frac{b}{a}\sqrt{a^2 + b^2}$ b. $\frac{b}{2a}\sqrt{a^2 - b^2}$
 c. $\frac{b}{4a}\sqrt{a^2 + b^2}$ d. None of these
13. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π sq units, then equation of the circle is
 a. $x^2 + y^2 + 2x - 2y - 62 = 0$
 b. $x^2 + y^2 - 2x + 2y - 62 = 0$
 c. $x^2 + y^2 - 2x + 2y - 47 = 0$
 d. $x^2 + y^2 + 2x - 2y - 47 = 0$
14. The equation of the locus of a point such that the sum of its distances from $(0, 2)$ and $(0, -2)$ is 6, is given by
 a. $\frac{x^2}{5} + \frac{y^2}{9} = 1$ b. $\frac{x^2}{9} + \frac{y^2}{5} = 1$
 c. $\frac{x^2}{5} - \frac{y^2}{9} = 1$ d. None of these
15. If the circle $x^2 + y^2 - 4x - 4y - 1 = 0$ has two points P and Q on it which are farthest and nearest respectively from the point $(6, 5)$, then
 a. $P \equiv \left(-\frac{22}{5}, 3\right)$ b. $Q \equiv \left(\frac{22}{5}, \frac{19}{5}\right)$
 c. $P \equiv \left(\frac{14}{3}, -\frac{11}{5}\right)$ d. $Q \equiv \left(-\frac{14}{3}, -4\right)$
16. Equation of the circle with centre on the Y-axis and passing through the origin and the point $(2, 3)$, is
 a. $x^2 + y^2 + 13y = 0$
 b. $3x^2 + 3y^2 + 13x + 3 = 0$
 c. $6x^2 + 6y^2 - 26y = 0$
 d. $x^2 + y^2 + 13x + 3 = 0$
17. Find the equation of a circle which touches both the axes and the line $3x - 4y + 8 = 0$ and lies in the third quadrant.
 a. $x^2 + y^2 + 4x + 4y - 4 = 0$
 b. $x^2 + y^2 - 4x - 4y + 4 = 0$
 c. $x^2 + y^2 + 4x + 4y + 4 = 0$
 d. None of the above
18. If a circle has centre $(3, -1)$ and cut-off an intercept of length 6 from the line $2x - 5y + 18 = 0$. Then, the equation of the circle is
 a. $x^2 + y^2 - 6x + 2y + 28 = 0$
 b. $x^2 + y^2 + 6x + 2y - 28 = 0$
 c. $x^2 + y^2 - 6x - 2y + 28 = 0$
 d. $x^2 + y^2 - 6x + 2y - 28 = 0$
19. The equation of a circle with origin as centre and passing through the vertices of an equilateral triangle whose median is of length $3a$, is
 a. $x^2 + y^2 = 9a^2$ b. $x^2 + y^2 = 16a^2$
 c. $x^2 + y^2 = 4a^2$ d. $x^2 + y^2 = a^2$
20. The centre of a circle passing through the points $(0, 0)$, $(1, 0)$ and touching the circle $x^2 + y^2 = 9$, is
 a. $\left(\frac{3}{2}, \frac{1}{2}\right)$ b. $\left(\frac{1}{2}, \frac{3}{2}\right)$
 c. $\left(\frac{1}{2}, \frac{1}{2}\right)$ d. $\left(\frac{1}{2}, -2^{1/2}\right)$
21. If the squares of the length of the tangents from a point P to the circles $x^2 + y^2 = a^2$, $x^2 + y^2 = b^2$ and $x^2 + y^2 = c^2$ are in AP, then
 a. a, b, c are in AP b. a, b, c are in GP
 c. a^2, b^2, c^2 are in AP d. a^2, b^2, c^2 are in GP
22. If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to a circle, then find the radius of the circle.
 a. $\frac{3}{4}$ b. $\frac{4}{3}$ c. $\frac{1}{4}$ d. $\frac{7}{4}$
23. What is the length of an equilateral triangle inscribed in the circle $x^2 + y^2 = \frac{4}{3}$?
 a. 2 units b. 5 units c. 3 units d. 7 units
24. The circle $x^2 + y^2 = 5$ has a tangent at the point $(1, -2)$. If this tangent touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$ also. Then, its point of contact is
 a. $(3, -1)$ b. $(-3, 0)$
 c. $(-1, -1)$ d. $(-2, 1)$
25. The point of contact of $4x + 5y + 6 = 0$ and $x^2 + y^2 - 2x - 4y - 8 = 0$ is
 a. $\left(\frac{2}{3}, \frac{2}{5}\right)$ b. $\left(\frac{2}{5}, \frac{5}{4}\right)$
 c. $(3, -2)$ d. None of these
26. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. If the tangents at the points $B(1, 7)$ and $D(4, -2)$ on the circle meet at C , then find the area of the quadrilateral $ABCD$.
 a. 78 b. 75
 c. 79 d. 85

27. The ΔPQR is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have coordinates $(3, 4)$ and $(-4, 3)$, respectively, then $\angle QPR$ is equal to
 a. $\frac{\pi}{2}$ b. $\frac{\pi}{3}$ c. $\frac{\pi}{4}$ d. $\frac{\pi}{6}$
28. If $(x - b)^2 + (y - a)^2 = c^2$ and $(x - a)^2 + (y - b)^2 = c^2$ are two circles touching each other, then
 a. $a = b \pm 2c$ b. $a = b \pm \sqrt{2}c$
 c. $b \pm c$ d. None of these
29. How many tangents can be drawn from the point $(3, -2)$ to the circle $x^2 + y^2 - 8x - 6y + 9 = 0$?
 a. 2 b. 1
 c. 0 d. None of these
30. If the line $3x - 4y - k = 0$, touches the circle $x^2 + y^2 - 4x - 8y - 5 = 0$ at (a, b) , then $k + a + b$ is equal to
 a. 20 b. 22 c. -30 d. -28
31. The locus of the point, the chord of contact of tangents from which to the circle $x^2 + y^2 = a^2$ subtends a right angle at the centre, is a circle of radius
 a. $2a$ b. $\frac{a}{2}$ c. $\sqrt{2}a$ d. a^2
32. The locus of the mid-point of the chords of a circle $x^2 + y^2 = 4$, which subtends a right angle at the centre, is
 a. $x + y = 2$ b. $x^2 + y^2 = 1$ c. $x^2 + y^2 = 2$ d. $x - y = 0$
33. If two distinct chords drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ (where $pq \neq 0$) are bisected by the X -axis, then
 a. $p^2 = q^2$ b. $p^2 = 8q^2$ c. $p^2 < 8q^2$ d. $p^2 > 8q^2$
34. The equation of the circle described on the common chord of the circles, $x^2 + y^2 - 12x + 2y - 10 = 0$ and $x^2 + y^2 - 8x + 5y - 37 = 0$ as a diameter, is
 a. $25(x^2 + y^2) - 348x + 14y - 74 = 0$
 b. $25(x^2 + y^2) - 348x + 140y - 74 = 0$
 c. $25(x^2 + y^2) - 300x + 14y + 70 = 0$
 d. None of the above
35. AB is chord of the circle $x^2 + y^2 = 25$. The tangents of A and B intersect at C . If $(2, 3)$ is the mid-point of AB , then area of the quadrilateral $OACB$ is
 a. $50\sqrt{\frac{13}{3}}$ b. $50\sqrt{\frac{3}{13}}$ c. $50\sqrt{3}$ d. $\frac{50}{\sqrt{3}}$
36. The equation of the smallest circle passing through the intersection of line $x + y = 2$ and the circle $x^2 + y^2 = 16$ is
 a. $x^2 + y^2 - 2x - 2y - 12 = 0$
 b. $x^2 + y^2 - 2x + 2y - 12 = 0$
 c. $x^2 + y^2 + 2x + 2y + 12 = 0$
 d. $x^2 + y^2 + 2x - 2y - 12 = 0$
37. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q . Then, the line $5x + by - a = 0$ passes through P and Q for
 a. exactly two values of a
 b. infinitely many values of a
 c. no value of a
 d. exactly one value of a
38. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points, if
 a. $-85 < m < -35$ b. $-35 < m < 15$
 c. $15 < m < 65$ d. $35 < m < 85$
39. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$, then there is a circle passing through P , Q and $(1, 1)$ and
 a. all values of p b. all except one value of p
 c. all except two values of p d. exactly one value of p
40. The circles $x^2 + y^2 - 10x + 16 = 0$ and $x^2 + y^2 = r^2$ intersect each other in two distinct points, if
 a. $r < 2$ b. $r > 8$ c. $2 < r < 8$ d. $2 \leq r \leq 8$
41. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is
 a. $2ax + 2by - (a^2 + b^2 + p^2) = 0$
 b. $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$
 c. $2ax + 2by - (a^2 + b^2 + 2b^2 + p^2) = 0$
 d. $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$
42. If $2x - 4y = 9$ and $6x - 12y + 7 = 0$ are common tangents to a circle, then radius of the circle is
 a. $\frac{\sqrt{3}}{5}$ b. $\frac{17}{6\sqrt{5}}$
 c. $\frac{\sqrt{2}}{3}$ d. $\frac{17}{3\sqrt{5}}$
43. Let C be the circle with centre $(1, 1)$ and radius 1. If T is the circle centred at $(0, y)$, passing through origin and touching the circle C externally, then the radius of T is equal to
 a. $\frac{1}{2}$ b. $\frac{1}{4}$
 c. $\frac{\sqrt{3}}{\sqrt{2}}$ d. $\frac{\sqrt{3}}{2}$
44. If two circles $x^2 + y^2 + 4x + 6y = 0$ and $x^2 + y^2 + 2g'x + 2f'y = 0$ touch each other, then
 a. $3g' = 2f'$
 b. $3f' = 2g'$
 c. $f' + g' = 6$
 d. $f' - g' = 1$

BITSAT Archives

- If the equation of circle which passes through the origin and cuts off intercepts 5 and 6 from the positive parts of the X-axis and Y-axis respectively, is $\left(x - \frac{5}{2}\right)^2 + (y - 3)^2 = \lambda$, then λ equals [2013]
 - $\frac{61}{4}$
 - $\frac{4}{6}$
 - $\frac{1}{4}$
 - 0
- The equation of the circle circumscribing the triangle formed by the lines $x + y = 6$, $2x + y = 4$ and $x + 2y = 5$ is [2012]
 - $x^2 + y^2 + 17x + 19y - 50 = 0$
 - $x^2 + y^2 - 17x - 19y - 50 = 0$
 - $x^2 + y^2 + 17x - 19y - 50 = 0$
 - $x^2 + y^2 - 17x - 19y + 50 = 0$
- The length of the tangent from (5, 1) to the circle $x^2 + y^2 + 6x - 4y - 3 = 0$ is [2012]
 - 7
 - 49
 - 63
 - 21
- The angle between the tangent drawn from origin to the circle $(x - 7)^2 + (y + 1)^2 = 25$ is [2010, 09]
 - $\frac{\pi}{3}$
 - $\frac{\pi}{6}$
 - $\frac{\pi}{2}$
 - $\frac{\pi}{8}$
- If the circle passes through (0, 0) (a, 0) and (0, b), then the coordinates of its centre are [2009]
 - $\left(\frac{b}{2}, \frac{a}{2}\right)$
 - $\left(\frac{a}{2}, \frac{b}{2}\right)$
 - (b, a)
 - (a, b)
- If the sum of the distance of a point P from two perpendicular lines in a plane, is 1, then the locus of P is a [2008]
 - rhombus
 - circle
 - straight line
 - pair of straight lines
- If the lines $2x - 3y = 5$ and $3x - 4y = 7$ are two diameters of a circle of radius 7, then the equation of the circle is [2008]
 - $x^2 + y^2 + 2x - 4y - 47 = 0$
 - $x^2 + y^2 = 49$
 - $x^2 + y^2 - 2x + 2y - 47 = 0$
 - $x^2 + y^2 = 17$
- The radius of the circle with the polar equation $r^2 - 8r(\sqrt{3} \cos \theta + \sin \theta) + 15 = 0$, is [2008]
 - 8
 - 7
 - 6
 - 5
- The centre of circle whose normals are $x^2 - 2xy - 3x + 6y = 0$, is [2007]
 - $\left(3, \frac{3}{2}\right)$
 - $\left(3, -\frac{3}{2}\right)$
 - $\left(\frac{3}{2}, 3\right)$
 - None of these
- The centres of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. The locus of any point in the set is [2007]
 - $4 \leq x^2 + y^2 \leq 64$
 - $x^2 + y^2 \leq 25$
 - $x^2 + y^2 \geq 25$
 - $3 \leq x^2 + y^2 \leq 9$
- If the two circles $x^2 + y^2 - 2x + 22y + 5 = 0$ and $x^2 + y^2 + 14x + 6y + k = 0$ intersect orthogonally, k is equal to [2006]
 - 47
 - 47
 - 49
 - 49
- The radius of the circle $x^2 + y^2 + 4x + 6y + 13 = 0$ is [2006]
 - $\sqrt{26}$
 - $\sqrt{13}$
 - $\sqrt{23}$
 - 0
- The centre of the circles $x = 2 + 3 \cos \theta$, $y = 3 \sin \theta - 1$ is [2006]
 - (3, 3)
 - (2, -1)
 - (-2, 1)
 - (-1, 2)
- The equation to the circle with centre (2, 1) and touching the line $3x + 4y = 5$ is [2006]
 - $x^2 + y^2 - 4x - 2y + 5 = 0$
 - $x^2 + y^2 - 4x - 2y - 5 = 0$
 - $x^2 + y^2 - 4x - 2y + 4 = 0$
 - $x^2 + y^2 - 4x - 2y - 4 = 0$
- The condition for a line $y = 2x + c$ to touch the circle $x^2 + y^2 = 16$ is [2006]
 - $c = 10$
 - $c^2 = 80$
 - $c = 12$
 - $c^2 = 64$
- Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points such that their abscissae x_1 and x_2 are the roots of the equation $x^2 + 2x - 3 = 0$ while the ordinates y_1 and y_2 are the roots of the equation $y^2 + 4y - 12 = 0$. Then, the centre of the circle with PQ as diameter is [2005]
 - (-1, -2)
 - (1, 2)
 - (1, -2)
 - (-1, 2)
- The number of common tangents to circles $x^2 + y^2 + 2x + 8y - 23 = 0$ and $x^2 + y^2 - 4x - 10y + 9 = 0$ is [2005]
 - 1
 - 3
 - 2
 - None of these
- If $\frac{x}{\alpha} + \frac{y}{\beta} = 1$ touches the circle $x^2 + y^2 = a^2$, then the point $\left(\frac{1}{\alpha}, \frac{1}{\beta}\right)$ lies on a/an [2005]
 - straight line
 - circle
 - parabola
 - ellipse

Answer with Solutions

Practice Exercise

1. (a) **Hint** Let the centre be (x, y) , $OA = OB$ (radius of same circle)

$$\Rightarrow \sqrt{(x+5)^2 + (y-11)^2} = \sqrt{(x-11)^2 + (y-19)^2}$$

and $OA = OC$ (radius)

$$\sqrt{(x+5)^2 + (y-11)^2} = \sqrt{(x-18)^2 + (y-14)^2}$$

2. (b) Let the base of the triangle be the line joining $B(0, 0)$ and $C(a, 0)$ and let the vertex be $A(h, k)$, where a is fixed. Also let

$$\frac{AB}{AC} = \lambda, \lambda \neq 1$$

$$\Rightarrow AB^2 = \lambda^2 AC^2$$

$$\Rightarrow h^2 + k^2 = \lambda^2 [(h-a)^2 + k^2]$$

$$\Rightarrow h^2(1-\lambda^2) + k^2(1-\lambda^2) + 2a\lambda^2 h - a^2\lambda^2 = 0$$

$$\text{So, } A(h, k) \text{ lies on } x^2 + y^2 + 2a \frac{\lambda^2}{1-\lambda^2} x - \frac{a^2\lambda^2}{1-\lambda^2} = 0$$

which is a circle.

3. (a) Centre of given circle $x^2 + y^2 - 6x + 12y + 15 = 0$ is $(3, -6)$.

$$\therefore \text{Radius} = \sqrt{(3)^2 + (-6)^2 - 15} = \sqrt{30}$$

$$\text{Area of circle} = \pi r^2 = \pi (\sqrt{30})^2 = 30\pi$$

Area of required circle = 2 (Area of given circle)

$$\Rightarrow \pi R^2 = 2 \times 30\pi = 60\pi$$

$$\Rightarrow R^2 = 60 \Rightarrow R = 2\sqrt{15}$$

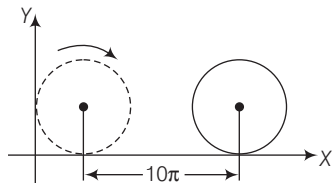
$\therefore$ Equation of required circle is

$$(x-3)^2 + (y+6)^2 = (2\sqrt{15})^2$$

$$\Rightarrow x^2 + 9 - 6x + y^2 + 36 + 12y = 60$$

$$\Rightarrow x^2 + y^2 - 6x + 12y - 15 = 0$$

4. (d) The x -coordinate of new position of the circle is $5 \times (\text{circumference of circle})$ i.e. $(5 + 10\pi)$.



Here, y -coordinate is 5 and radius is also 5.

Hence, the equation of the circle in new position is

$$(x-5-10\pi)^2 + (y-5)^2 = 5^2$$

$$\Rightarrow x^2 + y^2 - 10x - 10y - 20\pi x + 100\pi^2 + 100\pi = 0$$

5. (c) **Hint** We have,

$$x^2(1+\lambda) + y^2(1+\lambda) + (6-8\lambda)x + (5+7\lambda)y = 0$$

$$\Rightarrow x^2 + y^2 + \left(\frac{6-8\lambda}{1+\lambda}\right)x + \left(\frac{5+7\lambda}{1+\lambda}\right)y = 0$$

This will convert into a point circle, if

Radius of the circle = 0

$$\Rightarrow \sqrt{\left(\frac{3-4\lambda}{1+\lambda}\right)^2 + 0 - \left(\frac{5+7\lambda}{1+\lambda}\right)} = 0$$

6. (a) Let the equation of circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \dots(i)$$

Since, it passes through the points $(2, 3)$ and $(4, 5)$.

$$\therefore 2^2 + 3^2 + 2g(2) + 2f(3) + c = 0$$

$$\Rightarrow 4g + 6f + c + 13 = 0 \quad \dots(ii)$$

$$\text{and } 4^2 + 5^2 + 2g(4) + 2f(5) + c = 0$$

$$\Rightarrow 8g + 10f + c + 41 = 0 \quad \dots(iii)$$

On subtracting Eq. (ii) from Eq. (iii), we get

$$4g + 4f + 28 = 0$$

$$\Rightarrow g + f + 7 = 0 \quad \dots(iv)$$

Also, centre $(-g, -f)$ lies on line $y - 4x + 3 = 0$.

$$\therefore -f + 4g + 3 = 0 \quad \dots(v)$$

On solving Eqs. (iv) and (v), we get $g = -2, f = -5$

On putting $g = -2, f = -5$ in Eq. (ii), we get

$$4(-2) + 6(-5) + c + 13 = 0 \Rightarrow c = 25$$

On putting the values of g, f and c in Eq. (i), we get

$$x^2 + y^2 - 4x - 10y + 25 = 0$$

7. (b) Given lines are $3x + 5y = 1$... (i)

$$\text{and } (2+c)x + 5c^2y = 1 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$(1-c)x + 5(1-c^2)y = 0 \Rightarrow c = 1, x + 10y = 0$$

$$\therefore \text{Centre} = \left(\frac{2}{5}, -\frac{1}{25}\right)$$

$\therefore$ Equation of circle is

$$\left(x - \frac{2}{5}\right)^2 + \left(y + \frac{1}{25}\right)^2 = \left(2 - \frac{2}{5}\right)^2 + \frac{1}{25^2}$$

$$\Rightarrow 25(x^2 + y^2) - 20x + 2y - 60 = 0$$

8. (b) Equation of circle which touches X -axis and coordinates of centre are (h, k) , is $(x-h)^2 + (y-k)^2 = k^2$

Since, it passes through $(-1, 1)$, then

$$(-1-h)^2 + (1-k)^2 = k^2 \Rightarrow h^2 + 2h - 2k + 2 = 0$$

For real circles, $D \geq 0$

$$\Rightarrow (2)^2 - 4(-2k + 2) \geq 0 \Rightarrow k \geq 1/2$$

9. (a) **Hint** Given, $h = 7$, $k = -3$ and $r = 4$

$$\Rightarrow (x - 7)^2 + (y + 3)^2 = 16$$

10. (a)

11. (c) Let the equation of circle be

$$(x - 3)^2 + (y)^2 + \lambda y = 0$$

As it passes through $(1, -2)$.

$$\therefore (1 - 3)^2 + (-2)^2 + \lambda(-2) = 0$$

$$\Rightarrow 4 + 4 - 2\lambda = 0$$

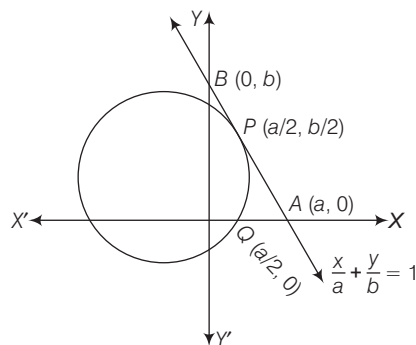
$$\Rightarrow \lambda = 4$$

$$\therefore \text{Equation of circle is } (x - 3)^2 + y^2 + 4y = 0$$

By hit and trial method, we see that point $(5, -2)$ satisfies equation of circle.

12. (c) The equation of the circle touching $\frac{x}{a} + \frac{y}{b} = 1$ at $P\left(\frac{a}{2}, \frac{b}{2}\right)$ is

$$\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 + \lambda\left(\frac{x}{a} + \frac{y}{b} - 1\right) = 0 \quad \dots(i)$$



Since, it passes through $Q\left(\frac{a}{2}, 0\right)$.

Therefore, $\lambda = \frac{b^2}{2}$ [from Eq. (i)]

On putting the value of λ in Eq. (i), we get

$$\begin{aligned} &\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 + \frac{b^2}{2}\left(\frac{x}{a} + \frac{y}{b} - 1\right) = 0 \\ \Rightarrow &x^2 + y^2 - \left(a - \frac{b^2}{2a}\right)x - \frac{by}{2} + \frac{a^2 - b^2}{4} = 0 \end{aligned}$$

Let r be the radius of this circle.

$$\text{Then, } r^2 = \frac{1}{4}\left(a - \frac{b^2}{2a}\right)^2 + \frac{b^2}{16} - \left(\frac{a^2 - b^2}{4}\right) = \frac{b^2(a^2 + b^2)}{16a^2}$$

$$\Rightarrow r = \frac{b}{4a} \sqrt{a^2 + b^2}$$

13. (c) **Hint** The intersection of two diameters is the centre of circle, which is $(1, -1)$.

Let r be the radius of circle, then

$$\text{Area of circle} = 49\pi$$

$$\Rightarrow \pi r^2 = 49\pi \Rightarrow r = 7 \text{ units}$$

14. (a) Let the moving point be $P(x, y)$ and $A(0, 2)$

and $B(0, -2)$.

Now,

$$AP + BP = \sqrt{(x - 0)^2 + (y - 2)^2} + \sqrt{(x - 0)^2 + (y + 2)^2} = 6$$

$$\Rightarrow \sqrt{x^2 + (y - 2)^2} = 6 - \sqrt{x^2 + (y + 2)^2}$$

Squaring both sides, we get

$$x^2 + (y - 2)^2 = 36 + x^2 + (y + 2)^2 - 12\sqrt{x^2 + (y + 2)^2}$$

$$\text{or } -8y - 36 = -12\sqrt{x^2 + (y + 2)^2}$$

$$2y + 9 = 3\sqrt{x^2 + (y + 2)^2}$$

Squaring again, we get

$$4y^2 + 36y + 81 = 9(x^2 + y^2 + 4y + 4)$$

$$\text{or } 9x^2 + 5y^2 = 45 \text{ or } \frac{x^2}{5} + \frac{y^2}{9} = 1$$

15. (b) Clearly, centre and radius of the given circle

$$x^2 + y^2 - 4x - 4y - 1 = 0$$

are $(2, 2)$ and $\sqrt{4 + 4 + 1} = 3$, respectively.

$$\therefore AQ = CA - CQ = 5 - 3 = 2$$

$$\therefore CQ : QA = 3 : 2, \text{ then } Q = \left(\frac{3 \times 6 + 2 \times 2}{3 + 2}, \frac{3 \times 5 + 2 \times 2}{3 + 2}\right)$$

$$\text{i.e. } Q = \left(\frac{22}{5}, \frac{19}{5}\right)$$

16. (c) **Hint** Let the centre of circle be $(0, k)$.

Let equation of circle be

$$x^2 + y^2 - 2(0)x - 2ky + c = 0$$

$$\Rightarrow x^2 + y^2 - 2ky + c = 0$$

17. (c) Let a be the radius of the circle and centre of circle be $(-a, -a)$.

Since, the perpendicular distance from centre to the tangent is equal to the radius of the circle.

$$\therefore a = \frac{3(-a) - 4(-a) + 8}{\sqrt{3^2 + (-4)^2}} \Rightarrow a = \frac{a + 8}{5}$$

$$\Rightarrow 4a = 8 \Rightarrow a = 2$$

$$\therefore \text{Equation of circle is } (x + 2)^2 + (y + 2)^2 = 2^2$$

$$\Rightarrow x^2 + y^2 + 4x + 4y + 4 = 0$$

18. (d) Let $C \equiv (3, -1)$ be the centre. Line AB has equation

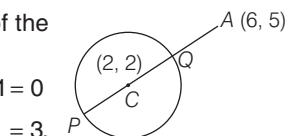
$$2x - 5y + 18 = 0$$

$$AB = 6$$

$$AL = 3$$

$CL =$ Length of the perpendicular from C on AB

$$= \frac{|2 \times 3 - 5 \times (-1) + 18|}{\sqrt{(2)^2 + (-5)^2}} = \sqrt{29}$$



$\therefore$ Radius of the circle, $AC = \sqrt{AL^2 + CL^2} = \sqrt{3^2 + 29} = \sqrt{38}$

Thus, equation of the required circle is

$$(x - 3)^2 + (y + 1)^2 = 38$$

$$\Rightarrow x^2 + y^2 - 6x + 2y - 28 = 0$$

19. (c) Let equation of circle be $x^2 + y^2 = r^2$... (i)

Since, in an equilateral triangle, the centroid coincides with the centre of the circumcircle.

Also, centroid divides the median in the ratio 2 : 1.

$$\therefore \text{Radius of circle, } r = \frac{2}{3}(3a) = 2a$$

On putting $r = 2a$ in Eq. (i), we get

$$x^2 + y^2 = (2a)^2 \Rightarrow x^2 + y^2 = 4a^2$$

20. (d) Since, the required circle touches the given circle and passes through its centre (0, 0).

$\therefore$ Its radius is half that of the given circle
 $= \frac{3}{2}$

Let this circle be $x^2 + y^2 + 2gx + 2fy = 0$

[$\because$ it passes through origin]

It also passes through (1, 0).

$$\therefore 1 + 2g = 0 \text{ i.e. } 2g = -1$$

$$\therefore \text{Centre of the circle} = \left(\frac{1}{2}, k\right) \quad [\text{say}]$$

$$\text{Radius} = \left(\frac{1}{4} + k^2\right)^{1/2}$$

$$\therefore \frac{9}{4} = \frac{1}{4} + k^2 \Rightarrow k^2 = \frac{8}{4} = 2$$

$$\Rightarrow k = \pm \sqrt{2}$$

$$\therefore \text{Centre of the circle is } \left(\frac{1}{2}, \pm \sqrt{2}\right).$$

21. (c) Let $P(x_1, y_1)$ be the given point and PT_1, PT_2, PT_3 be the lengths of the tangents from P to the circle $x^2 + y^2 = a^2$, $x^2 + y^2 = b^2$ and $x^2 + y^2 = c^2$, respectively. Then,

$$PT_1 = \sqrt{x_1^2 + y_1^2 - a^2}, PT_2 = \sqrt{x_1^2 + y_1^2 - b^2}$$

$$\text{and } PT_3 = \sqrt{x_1^2 + y_1^2 - c^2}$$

Now, PT_1^2, PT_2^2 and PT_3^2 are in A.P.

$$\Rightarrow 2PT_2^2 = PT_1^2 + PT_3^2$$

$$\Rightarrow 2(x_1^2 + y_1^2 - b^2) = (x_1^2 + y_1^2 - a^2) + (x_1^2 + y_1^2 - c^2)$$

$$\Rightarrow 2b^2 = a^2 + c^2$$

$\Rightarrow a^2, b^2, c^2$ are in A.P.

22. (a) Given lines are $3x - 4y + 4 = 0$

$$\text{and } 6x - 8y - 7 = 0 \text{ or } 3x - 4y - \frac{7}{2} = 0$$

are tangents to the circle.

Also, given lines are parallel.

$$\therefore \text{Radius} = \frac{1}{2} (\text{Distance between two parallel lines})$$

$$= \frac{1}{2} \frac{\left|4 + \frac{7}{2}\right|}{\sqrt{3^2 + 4^2}} = \frac{1}{2} \frac{\left|\frac{15}{2}\right|}{\sqrt{9 + 16}} = \frac{15}{4\sqrt{25}} = \frac{15}{4 \times 5} = \frac{3}{4}$$

23. (a) Given equation of circle is $x^2 + y^2 = \frac{4}{3}$.

$$\text{Radius of circle} = \frac{2}{\sqrt{3}}$$

$\therefore$ Length of equilateral triangle inscribed in circle

$$= \sqrt{3} \times \frac{2}{\sqrt{3}} = 2 \text{ units}$$

24. (a) The tangent at point (1, -2) to the circle $x^2 + y^2 = 5$ is

$$x - 2y - 5 = 0 \quad \dots (i)$$

Let this tangent touches the circle

$$x^2 + y^2 - 8x + 6y + 20 = 0 \text{ at } (x_1, y_1)$$

$\therefore$ Equation of tangent at (x_1, y_1) is

$$xx_1 + yy_1 - 4(x + x_1) + 3(y + y_1) + 20 = 0$$

$$\Rightarrow x(x_1 - 4) + y(y_1 + 3) - 4x_1 + 3y_1 + 20 = 0 \quad \dots (ii)$$

Now, Eqs. (i) and (ii) represent the same line.

$$\therefore \frac{x_1 - 4}{1} = \frac{y_1 + 3}{-2} = \frac{-4x_1 + 3y_1 + 20}{-5}$$

$$\Rightarrow -2x_1 + 8 = y_1 + 3$$

$$\Rightarrow 2x_1 + y_1 - 5 = 0$$

Only the point (3, -1) satisfies it.

Hence, the point of contact is (3, -1).

25. (b) Let the point of contact be (x_1, y_1) .

$\therefore$ The equation of tangent to the circle

$$x^2 + y^2 - 2x - 4y - 8 = 0 \text{ is}$$

$$xx_1 + yy_1 - (x + x_1) - 2(y + y_1) - 8 = 0$$

$$\text{or } x(x_1 - 1) + y(y_1 - 2) - (x_1 + 2y_1 + 8) = 0$$

But $4x + 5y + 6 = 0$, also represents the same line.

$$\therefore \frac{(x_1 - 1)}{4} = \frac{(y_1 - 2)}{5} = \frac{-(x_1 + 2y_1 + 8)}{6}$$

$$5x_1 - 4y_1 + 3 = 0 \quad \dots (i)$$

$$\text{and } 5x_1 + 16y_1 + 28 = 0 \quad \dots (ii)$$

On solving Eqs. (i) and (ii), we get

$$x_1 = \frac{2}{5} \text{ and } y_1 = \frac{5}{4}$$

So, the point of contact is $\left(\frac{2}{5}, \frac{5}{4}\right)$.

26. (b) The tangent at $B(1, 7)$ is $y = 7$

$$\text{and } D(4, -2) \text{ is } 3x - 4y - 20 = 0.$$

Then, meet at $C(16, 7)$.

$$\text{Now, } AB = 5, BC = 15$$

$$\text{Area of quadrilateral } ABCD = AB \cdot BC = 75$$

27. (c) QR subtend $\frac{\pi}{2}$ at the centre and QR subtends $\frac{\pi}{4}$ at P

or $\angle QPR = \frac{\pi}{4}$.

28. (b) |Radius of one circle $\pm$ Radius of other circle|
= Distance between centres

$$\Rightarrow |c \pm c| = \sqrt{(a-b)^2 + (a-b)^2}$$

Taking positive sign,

$$\pm(2c) = \sqrt{2}(a-b) \Rightarrow a-b = \pm\sqrt{2}c \Rightarrow a = b \pm \sqrt{2}c$$

29. (a) If $S \equiv x^2 + y^2 - 8x - 6y + 9 = 0$

Then, S at $(3, -2) = 9 + 4 - 24 + 12 + 9 > 0$

So, the point $(3, -2)$ lies outside the circle.

Hence, two tangents can be drawn to the circle from the point $(3, -2)$.

30. (a) Since, the given line touches the given circle, the length of the perpendicular from the centre $(2, 4)$ of the circle to the line $3x - 4y - k = 0$ is equal to the radius $\sqrt{4 + 16 + 5} = 5$ of the circle.

$$\therefore \frac{3 \times 2 - 4 \times 4 - k}{\sqrt{9 + 16}} = \pm 5$$

$$\Rightarrow k = 15 \quad [\because k > 0]$$

Now, equation of the tangent at (a, b) to the given circle is

$$xa + yb - 2(x + a) - 4(y + b) - 5 = 0$$

$$\Rightarrow (a-2)x + (b-4)y - (2a+4b+5) = 0$$

If it represents the given line $3x - 4y - k = 0$.

$$\text{Then, } \frac{a-2}{3} = \frac{b-4}{-4} = \frac{2a+4b+5}{k} = l \quad [\text{say}]$$

$$\Rightarrow a = 3l + 2, \quad b = 4 - 4l$$

$$\text{and } 2a + 4b + 5 = kl$$

$$\Rightarrow 2(3l+2) + 4(4-4l) + 5 = 15l \quad [\because k = 15]$$

$$\Rightarrow l = 1$$

$$\Rightarrow a = 5, b = 0$$

$$\therefore k + a + b = 15 + 5 + 0 = 20$$

31. (c) Let $P(h, k)$ be the point. Then, the chord of contact of tangents drawn from P to the circle $x^2 + y^2 = a^2$ is $hx + ky = a^2$.

The combined equation of the lines joining the (centre) origin to the points of intersection of the circle $x^2 + y^2 = a^2$ and the chord of the contact of tangents drawn from $P(h, k)$ is a homogeneous equation of second degree given by

$$x^2 + y^2 = a^2 \left(\frac{hx + ky}{a^2} \right)^2$$

$$\Rightarrow a^2(x^2 + y^2) = (hx + ky)^2$$

The lines given by above equation will be perpendicular if coefficient of x^2 + coefficient of $y^2 = 0$

$$\Rightarrow h^2 - a^2 + k^2 - a^2 = 0 \Rightarrow h^2 + k^2 = 2a^2$$

So, locus of (h, k) is $x^2 + y^2 = 2a^2$. Clearly, it is circle of radius $\sqrt{2}a$.

32. (c) Given circle is $x^2 + y^2 = 4$

Let MN be the one chord and P be the mid-point and the coordinate of P is (h, k) .

Then,

$$OM = ON = \text{radius}$$

$$\angle MON = 90^\circ$$

$\Rightarrow$

$$\angle MOP = \angle PON = 45^\circ$$

In $\triangle MOP$.

$$OP = OM \cos 45^\circ = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}$$

$\Rightarrow$

$$OP = \sqrt{2} \quad \dots(i)$$

Centre of circle O is $(0, 0)$ and P is (h, k) .

$$\therefore OP = \sqrt{(h-0)^2 + (k-0)^2}$$

$\Rightarrow$

$$OP = \sqrt{h^2 + k^2} \quad \dots(ii)$$

From Eqs. (i) and (ii),

$$\sqrt{h^2 + k^2} = \sqrt{2}$$

$\Rightarrow$

$$h^2 + k^2 = 2$$

$\Rightarrow$ Locus of (h, k) is $x^2 + y^2 = 2$.

33. (d) Let (t, m) be the other end of the chord drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$

$$\text{Their mid-point is } \left(\frac{t+p}{2}, \frac{m+q}{2} \right)$$

Since, mid-point lies on X -axis i.e. $y = 0$

$$\therefore m + q = 0 \quad \dots(i)$$

Also, (t, m) lies on the circle.

$$\therefore t^2 + m^2 - pt - qm = 0 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get $t^2 - pt + 2q^2 = 0$

which is quadratic in t such that,

Discriminant > 0

$$\Rightarrow p^2 - 8q^2 > 0 \Rightarrow p^2 > 8q^2$$

34. (a) $\therefore$ The equation of common chord of two given circles S_1 and S_2 is

$$S_1 - S_2 = 0 \quad \dots(i)$$

and the equation of circle passing through the end points of common chord is $S_1 + \lambda S_2 = 0$ $\dots(ii)$

$$\therefore \text{Common chord} = (x^2 + y^2 - 8x + 5y - 37) -$$

$$(x^2 + y^2 - 12x + 2y - 10) = 0$$

$\Rightarrow$

$$4x + 3y - 27 = 0 \quad \dots(iii)$$

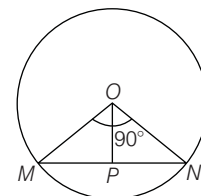
Therefore, the equation of circle passing through end points of $4x + 3y - 27 = 0$ is

$$(x^2 + y^2 - 8x + 5y - 37) + \lambda(4x + 3y - 27) = 0.$$

$$\Rightarrow x^2 + y^2 + (4\lambda - 8)x + (5 + 3\lambda)y - (37 + 27\lambda) = 0 \quad \dots(iv)$$

The centre of the circle (iv) is $\left[-\frac{(4\lambda - 8)}{2}, -\frac{(5 + 3\lambda)}{2} \right]$ and

it lies on the common chord.



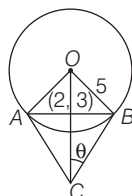
$$\begin{aligned}\therefore -4(2\lambda - 4) - 3\left(\frac{5 + 3\lambda}{2}\right) - 27 &= 0 \\ \Rightarrow -8\lambda + 16 - \frac{15}{2} - \frac{9\lambda}{2} - 27 &= 0 \\ \Rightarrow \frac{-25\lambda}{2} + \frac{17}{2} - 27 &= 0 \\ \Rightarrow \frac{-25\lambda}{2} - \frac{37}{2} = 0 \Rightarrow \lambda &= -\frac{37}{25}\end{aligned}$$

Therefore, the required circle is

$$\begin{aligned}x^2 + y^2 + \left[4 \times \left(-\frac{37}{25}\right) - 8\right]x + \left[5 + 3\left(-\frac{37}{25}\right)\right]y \\ - \left[37 + 27\left(-\frac{37}{25}\right)\right] = 0 \\ \Rightarrow x^2 + y^2 - \frac{348x}{25} + \frac{14y}{25} - \frac{74}{25} = 0 \\ \Rightarrow 25(x^2 + y^2) - 348x + 14y - 74 = 0\end{aligned}$$

35. (b) Area of quadrilateral $OACB$,

$$\begin{aligned}A = OB \cdot BC = 5^2 \cot \theta = 50 \sqrt{\frac{3}{13}} \\ \left[\because \sin \theta = \frac{\sqrt{13}}{5} \right]\end{aligned}$$



36. (a) Circle passing through the intersection of the given line and the circle has equation

$$(x^2 + y^2 - 16) + k(x + y - 2) = 0$$

Its centre is $\left(\frac{-k}{2}, \frac{-k}{2}\right)$.

The circle is the smallest, if the centre $\left(\frac{-k}{2}, \frac{-k}{2}\right)$ lies on the chord $x + y = 2$.

$$\therefore \frac{-k}{2} - \frac{k}{2} = 2 \Rightarrow \frac{-2k}{2} = 2 \Rightarrow k = -2$$

$\therefore$ Required equation of smallest circle is

$$(x^2 + y^2 - 16) - 2(x + y - 2) = 0$$

$$\text{i.e. } x^2 + y^2 - 2x - 2y - 12 = 0$$

37. (c) Let equation of circles be

$$S_1 \equiv x^2 + y^2 + 2ax + cy + a = 0$$

$$\text{and } S_2 \equiv x^2 + y^2 - 3ax + dy - 1 = 0$$

Chord through intersection points P and Q of the given circles is $S_1 - S_2 = 0$.

$$\therefore (x^2 + y^2 + 2ax + cy + a) - (x^2 + y^2 - 3ax + dy - 1) = 0$$

$$\Rightarrow 5ax + (c - d)y + a + 1 = 0$$

On comparing it with $5x + by - a = 0$, we get

$$\frac{5a}{5} = \frac{c - d}{b} = \frac{a + 1}{-a}$$

$$\Rightarrow a(-a) = a + 1 \Rightarrow a^2 + a + 1 = 0$$

which gives no real value of a .

Hence, the line passes through P and Q for no value of a .

38. (b) Since, the coordinates of the centre of the circle are $(2, 4)$.

$$\text{Also, } r^2 = 4 + 16 + 5 = 25$$

The line will intersect the circle at two distinct points, if the distance of $(2, 4)$ from $3x - 4y = m$ is less than radius of the circle.

$$\therefore \frac{|6 - 16 - m|}{5} < 5 \Rightarrow -25 < 10 + m < 25$$

$$\therefore -35 < m < 15$$

39. (c) Let $S \equiv x^2 + y^2 + 3x + 7y + 2p - 5 = 0$

$$\text{and } S' \equiv x^2 + y^2 + 2x + 2y - p^2 = 0$$

So, equation of the required circle is $S + \lambda S' = 0$.

As it passes through $(1, 1)$, the value of

$$\lambda = -(7 + 2p)/(6 - p^2)$$

Here, λ is not defined at $p = \pm \sqrt{6}$.

Hence, it is true for all except two values of p .

40. (c) Given circle is $x^2 + y^2 = r^2$

Its centre is $C_1(0, 0)$ and radius $r_1 = r$

$$\text{and } x^2 + y^2 - 10x + 16 = 0$$

Its centre, $C_2 = (5, 0)$

$$\text{and radius, } r_2 = \sqrt{(5)^2 - 16} = \sqrt{9} = 3$$

Distance between the centre C_1C_2 .

$$= \sqrt{(5 - 0)^2 + (0 - 0)^2} = 5$$

To intersect two circle at two distinct points

$$|r_1 - r_2| < C_1C_2 < |r_1 + r_2|$$

$$\Rightarrow |r_1 - r_2| < C_1C_2 \text{ and } C_1C_2 < |r_1 + r_2|$$

$$\Rightarrow r - 3 < 5 \text{ or } 5 < r + 3$$

$$\Rightarrow r < 8 \text{ or } 2 < r$$

$$\Rightarrow r < 8 \quad \dots(i)$$

$$\Rightarrow r > 2 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get $2 < r < 8$

41. (a) Let the equation of circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

It cuts the circle $x^2 + y^2 = p^2$ orthogonally.

$$\therefore 2g(0) + 2f(0) = c - p^2 \Rightarrow c = p^2$$

Also, it passes through (a, b) .

$$\therefore a^2 + b^2 + 2ga + 2fb + p^2 = 0$$

So, locus of $(-g, -f)$ is

$$a^2 + b^2 - 2ax - 2by + p^2 = 0$$

$$\Rightarrow 2ax + 2by - (a^2 + b^2 + p^2) = 0$$

42. (b) Distance between given parallel lines $2x - 4y - 9 = 0$

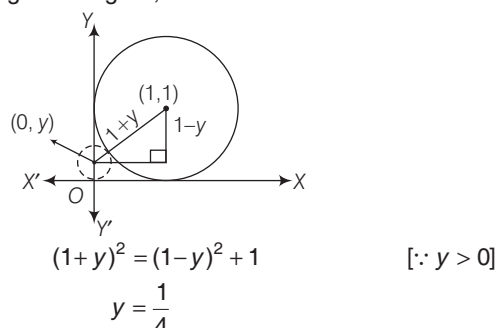
$$\text{and } 2x - 4y + \frac{7}{3} = 0 \text{ is}$$

$$d = \frac{\left| \frac{7}{3} + 9 \right|}{\sqrt{4 + 16}} \quad \left[\because d = \frac{c_2 - c_1}{\sqrt{a^2 + b^2}} \right]$$

$$= \frac{34}{3(2)\sqrt{5}} = \frac{17}{3\sqrt{5}}$$

$$\therefore \text{Radius of the circle} = \frac{1}{2} \left(\frac{17}{3\sqrt{5}} \right) = \frac{17}{6\sqrt{5}}$$

43. (b) According to the figure,



$\therefore$

$$y = \frac{1}{4}$$

BITSAT Archives

1. (a) From figure, we have

$$OP = 5, OQ = 6$$

and $OM = \frac{5}{2}, CM = 3$

In $\triangle OMC$, $OC^2 = OM^2 + MC^2$

$$\Rightarrow OC^2 = \left(\frac{5}{2}\right)^2 + (3)^2$$

$$\Rightarrow OC = \frac{\sqrt{61}}{2}$$

Thus, the required circle has its centre $\left(\frac{5}{2}, 3\right)$ and radius $\frac{\sqrt{61}}{2}$.

Hence, its equation is $\left(x - \frac{5}{2}\right)^2 + (y - 3)^2 = \frac{61}{4}$

$$\therefore \lambda = \frac{61}{4}$$

2. (d) Lines, $x + y = 6$, $2x + y = 4$ and $x + 2y = 5$ intersect at points $(-2, 8)$, $(7, -1)$ and $(1, 2)$.

Now, all these points lie on

$$x^2 + y^2 - 17x - 19y + 50 = 0$$

3. (a) Required length of tangent is $\sqrt{S_1}$

Where, $S_1 = 25 + 1 + 30 - 4 - 3 = 49$

$$\therefore \sqrt{S_1} = 7$$

4. (c) Let the equation of tangent drawn from $(0, 0)$ to the circle be $y = mx$. Then,

$$p = a \Rightarrow \frac{7m + 1}{\sqrt{m^2 + 1}} = 5$$

$$\Rightarrow 24m^2 + 14m - 24 = 0$$

$$\Rightarrow 12m^2 + 7m - 12 = 0$$

44. (a) The centre of the first circle is $(-2, -3)$ and radius $r_1 = \sqrt{13}$ and centre of second circle is $(-g', -f')$ and radius $r_2 = \sqrt{g'^2 + f'^2}$

$$\therefore C_1C_2 = \sqrt{(g' + 2)^2 + (f' + 3)^2}$$

$\therefore$ Circles touch each other, so $C_1C_2 = r_1 \pm r_2$

$$\text{i.e. } \sqrt{g'^2 + f'^2 + 4 + 9 + 4g' + 6f'} = \sqrt{13} \pm \sqrt{(g')^2 + (f')^2}$$

On squaring both sides, we get

$$(2g' + 3f')^2 = 13(g'^2 + f'^2)$$

$$\Rightarrow 12g'f' = 4f'^2 + 9g'^2$$

$$\Rightarrow (3g' - 2f')^2 = 0$$

$$\Rightarrow 3g' = 2f'$$

$$\Rightarrow m_1m_2 = \frac{-12}{12} = -1$$

$$\therefore \text{Required angle} = \frac{\pi}{2}$$

5. (b) Circle passing through the points $(0, 0)$, $(a, 0)$ and $(0, b)$ is

$$x^2 + y^2 - ax - by = 0 \text{ and its centre is } \left(\frac{a}{2}, \frac{b}{2}\right).$$

6. (a) If the sum of the distance of a point P from two perpendicular lines in a plane, is 1, then the locus of P is a rhombus.

7. (c) Since, the lines $2x - 3y = 5$ and $3x - 4y = 7$ are the diameters of a circle. Therefore, the point of intersection is the centre of the circle. On solving the given equations, we get $x = 1$ and $y = -1$ i.e. the centre of the circle.

$\therefore$ Required equation of circle is

$$(x - 1)^2 + (y + 1)^2 = 7^2$$

$$\Rightarrow x^2 + y^2 - 2x + 2y + 2 = 49$$

$$\Rightarrow x^2 + y^2 - 2x + 2y - 47 = 0$$

8. (b) Given polar equation of circle is

$$r^2 - 8r(\sqrt{3} \cos \theta + \sin \theta) + 15 = 0$$

$$\text{or } r^2 - 8(\sqrt{3}r \cos \theta + r \sin \theta) + 15 = 0$$

$$\text{where, } r \cos \theta = x \text{ and } y = r \sin \theta$$

It can be rewritten in cartesian form as

$$x^2 + y^2 - 8(\sqrt{3}x + y) + 15 = 0$$

$$\Rightarrow x^2 + y^2 - 8\sqrt{3}x - 8y + 15 = 0$$

Now, $\text{radius} = \sqrt{(4\sqrt{3})^2 + (4)^2 - 15}$
 $= \sqrt{48 + 16 - 15} = 7$

9. (a) Given equation can be rewritten as

$$x(x - 2y) - 3(x - 2y) = 0 \text{ or } (x - 3)(x - 2y) = 0$$

$$\begin{aligned} \text{or} \quad & x = 3 \quad \dots(i) \\ \text{and} \quad & x = 2y \quad \dots(ii) \end{aligned}$$

Since, normals always pass through the centre. Therefore, the point of intersection of two normals are the coordinates of the centre.

On solving Eqs. (i) and (ii), we get the required coordinates of centre are $\left(3, \frac{3}{2}\right)$.

10. (a) Let (h, k) be the centre of a circle, then equation of circle is $(x - h)^2 + (y - k)^2 = 9$

$$\text{This centre lies on } x^2 + y^2 = 25 \Rightarrow h^2 + k^2 = 25$$

$$\therefore 2 \leq \text{Distance between the centres of the two circles} \leq 8$$

$$\Rightarrow 2 \leq \sqrt{(h - 0)^2 + (k - 0)^2} \leq 8$$

$$\Rightarrow 2 \leq \sqrt{h^2 + k^2} \leq 8 \Rightarrow 4 \leq h^2 + k^2 \leq 64.$$

$$\therefore \text{Locus of } (h, k) \text{ is } 4 \leq x^2 + y^2 \leq 64.$$

11. (a) By using the condition that, if two circles intersect orthogonally, then

$$2(g_1g_2 + f_1f_2) = c_1 + c_2$$

$$\text{where, } g_1 = -1, f_1 = 11, c_1 = 5$$

$$\text{and } g_2 = 7, f_2 = 3, c_2 = k$$

$$\Rightarrow 2(-1 \cdot 7 + 11 \cdot 3) = 5 + k$$

$$\Rightarrow 2(26) = 5 + k \Rightarrow k = 47$$

12. (d) Given equation is $x^2 + y^2 + 4x + 6y + 13 = 0$

$$\text{or } (x^2 + 4x + 4) + (y^2 + 6y + 9) = 13 + 4 + 9 = 11$$

$$\text{or } (x + 2)^2 + (y + 3)^2 = 0$$

$$\therefore \text{Radius of circle} = 0$$

13. (b) Given parametric equations are

$$x = 2 + 3 \cos \theta, y = 3 \sin \theta - 1$$

$$\text{or } \cos \theta = \frac{x - 2}{3}, \sin \theta = \frac{y + 1}{3}$$

$$\text{Since, } \sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \left(\frac{x - 2}{3}\right)^2 + \left(\frac{y + 1}{3}\right)^2 = 1 \Rightarrow (x - 2)^2 + (y + 1)^2 = 3^2$$

$$\therefore \text{Centre of circle is } (2, -1).$$

14. (c) **Hint** Distance from centre $(2, 1)$ to the line $3x + 4y - 5 = 0$ is radius of circle

$$\Rightarrow \frac{|3(2) + 4(1) - 5|}{\sqrt{3^2 + 4^2}} = r$$

$$\Rightarrow r = 1$$

15. (b) If $y = mx + c$ touches the circle

$$x^2 + y^2 = a^2 \text{ then, } c^2 = a^2(1 + m^2)$$

Now, the line $y = 2x + c$ touches the circle

$$x^2 + y^2 = 16$$

$$\therefore c^2 = 16(1 + 4) = 16 \times 5 \Rightarrow c^2 = 80$$

16. (a) Given, x_1, x_2 are the roots of the equation

$$x^2 + 2x - 3 = 0$$

$$\Rightarrow x^2 + 3x - x - 3 = 0 \Rightarrow x(x + 3) - 1(x + 3) = 0$$

$$\Rightarrow (x - 1)(x + 3) = 0 \Rightarrow x_1 = -3, x_2 = 1$$

and y_1, y_2 are the roots of the equation

$$y^2 + 4y - 12 = 0$$

$$\Rightarrow y^2 + 6y - 2y - 12 = 0 \Rightarrow y(y + 6) - 2(y + 6) = 0$$

$$\Rightarrow y_1 = -6, y_2 = 2$$

$\therefore$ Points are $P(-3, -6)$ and $Q(1, 2)$.

Since, P and Q are the end points of a diameter.

$$\therefore \text{Centre} = \text{Mid-point of } PQ = \left(\frac{-3 + 1}{2}, \frac{-6 + 2}{2}\right) = (-1, -2)$$

17. (c) The centre and radius of the first circle

$$x^2 + y^2 + 2x + 8y - 23 = 0 \text{ are } C_1(-1, -4)$$

and $r_1 = \sqrt{40}$. Similarly, the centre and radius of second circle $x^2 + y^2 - 4x - 10y + 9 = 0$ are $C_2(2, 5)$ and $r_2 = \sqrt{20}$.

$$\text{Now, } C_1C_2 = \sqrt{(2 + 1)^2 + (5 + 4)^2}$$

$$= \sqrt{9 + 81} = \sqrt{90} \text{ and } r_1 + r_2 = \sqrt{40} + \sqrt{20}$$

$$\text{also, } r_1 - r_2 = \sqrt{40} - \sqrt{20}$$

$$\text{Here, } r_1 - r_2 < C_1C_2 < r_1 + r_2$$

$\therefore$ Two common tangents can be drawn.

18. (b) Since, the line $\frac{x}{\alpha} + \frac{y}{\beta} = 1$ touches the circle $x^2 + y^2 = a^2$.

$\therefore$ The perpendicular distance from centre $(0, 0)$ to the tangent = Radius of the circle

$$\Rightarrow \frac{|-1|}{\sqrt{\frac{1}{\alpha^2} + \frac{1}{\beta^2}}} = a$$

$$\Rightarrow \frac{1}{a^2} = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$\text{The locus of } \left(\frac{1}{\alpha}, \frac{1}{\beta}\right) \text{ is } \frac{1}{a^2} = \frac{1}{x^2} + \frac{1}{y^2}$$

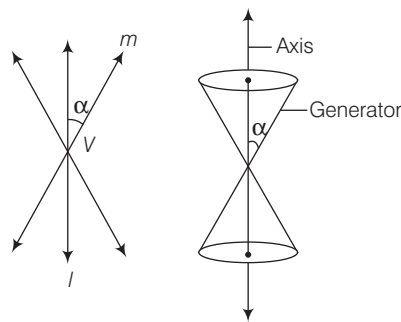
which represents a circle.

14

Conic Sections

Introduction

Let l be a fixed vertical line and m be another line intersecting it at fixed point V and inclined to it at a semi-vertical angle α . Suppose we rotate a line m around the line l in such a way that $\angle \alpha$ remain constant, then the surface generated is double napped right circular cone.

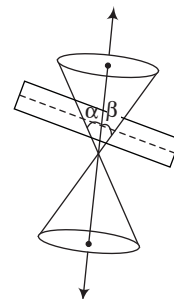


Section of a Cone

Let β be the angle made by the intersecting plane with vertical axis of the cone

When the plane cuts the nappe (other than the vertex) of the cone, we have following situations:

- (i) When $\beta = 90^\circ$, the section is a circle.
- (ii) When $\alpha < \beta < 90^\circ$, the section is an ellipse.
- (iii) When $\beta = \alpha$, the section is a parabola.
- (iv) When $0 \leq \beta < \alpha$, the plane cut through both the nappes and intersection is a hyperbola.



Conic Section as Locus

A conic is the locus of point which moves in a plane such that the ratio of its distance from fixed point **focus** and fixed line **directrix** is constant. This constant ratio is called eccentricity (e).

According to different values of e , conic section is named as

- (i) If $e = 1$, the conic is called a parabola.
- (ii) If $e < 1$, the conic is called an ellipse.
- (iii) If $e > 1$, the conic is called a hyperbola.
- (iv) If $e = 0$, the conic is called a circle.
- (v) If $e = \infty$, the conic is called a pair of straight lines.

Some Important Terms

- (i) **Focus** The fixed point is called the focus of the conic section or conic and denoted by S .
- (ii) **Directrix** The fixed straight line is called directrix of the conic.
- (iii) **Axis** The straight line perpendicular to the directrix and passing through the focus is said to be the axis.
- (iv) **Vertex** The point of intersection of the conic section and the axis is called vertex of the conic.
- (v) **Focal chord** Any chord passing through the focus is called focal chord.
- (vi) **Double ordinate** A straight line drawn perpendicular to the axis and terminated at both ends by the curve is a double ordinate.
- (vii) **Latusrectum** The double ordinate passing through the focus is called the latusrectum.

Equation of Conic Section

If the focus is (α, β) , directrix is $ax + by + c = 0$ and P is any point. Then, ratio of distance of point P from the fixed point (focus) to fixed line (directrix) is

$$\frac{PS}{PM} = e$$

$$\Rightarrow \sqrt{(x - \alpha)^2 + (y - \beta)^2} = e \frac{ax + by + c}{\sqrt{a^2 + b^2}}$$

$$\Rightarrow (x - \alpha)^2 + (y - \beta)^2 = \frac{e^2(ax + by + c)^2}{a^2 + b^2}$$

On simplifying above equation, we get second degree equation

$$ax^2 + by^2 + 2gx + 2fy + 2hxy + c = 0$$

Here discriminant,

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

Equations will represent different conics, they are given as

Case I When $\Delta = 0$

Condition	Nature of conic
$ab - h^2 = 0$	A pair of coincident lines
$ab - h^2 < 0$	Real and distinct pair of straight lines
$ab - h^2 > 0$	Lines are imaginary

Case II When $\Delta \neq 0$

Condition	Nature of conic
$h = 0, a = b, e = 0$	A circle
$ab - h^2 = 0, e = 1$	A parabola
$ab - h^2 > 0, e < 1$	An ellipse
$ab - h^2 < 0, e > 1$	A hyperbola
$ab - h^2 < 0, a + b = 0, e = \sqrt{2}$	A rectangular hyperbola

Parabola

A parabola is the locus of a point which moves in a plane such that its distance from a fixed point (focus) in the plane is always equal to its distance from fixed line in the same plane.

Mathematically, $\frac{|PS|}{|PM|} = e = 1$

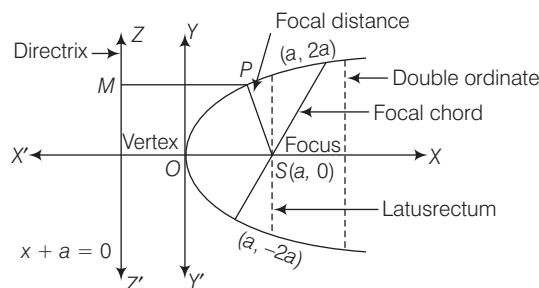
where, e is eccentricity.

Standard Equation of the Parabola

Let S be the focus and directrix be ZZ' , then standard form of parabola is $y^2 = 4ax$.

Other forms of parabola are

$$(i) y^2 = -4ax \quad (ii) x^2 = 4ay \quad (iii) x^2 = -4ay$$



Terminology of Parabolas

Terms	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Vertex	(0, 0)	(0, 0)	(0, 0)	(0, 0)
Focus	(a, 0)	(-a, 0)	(0, a)	(0, -a)
Directrix	$x = -a$	$x = a$	$y = -a$	$y = a$
Axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Latusrectum	$4a$	$4a$	$4a$	$4a$
Focal distance	$x + a$	$a - x$	$y + a$	$a - y$

- NOTE**
- If vertex of parabola is not origin, then its equation is $(y - k)^2 = 4a(x - h)$, where (h, k) are coordinates of vertex and axis is parallel to X-axis.
 - If vertex of parabola is (h, k) and axis is parallel to Y-axis, then equation of parabola is $(x - h)^2 = 4b(y - k)$.

Parametric Equations of Parabola

Expressing equation of parabola in some parameters, is called parametric equation of parabola. Here, we are giving some parametric forms of parabolic equation:

Parabola	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Parametric equations	$x = at^2, y = 2at$	$x = -at^2, y = 2at$	$x = 2at, y = at^2$	$x = -2at, y = at^2$

The parametric equations of parabola

$$(y - k)^2 = 4a(x - h) \text{ are } x = h + at^2, y = k + 2at.$$

Position of a Point w.r.t. Parabola

The point $P(x_1, y_1)$ lies outside, on or inside the parabola $y^2 = 4ax$, according as $y_1^2 - 4ax_1 >, =, < 0$

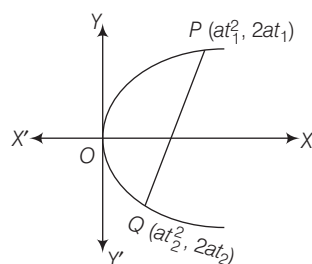
or $P_1 > 0, P_1 = 0, P_1 < 0,$

where, $P_1 = y_1^2 - 4ax_1$

Equation of Chord Joining any Two Points of the Parabola

If $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ are two given points, then equation of chord is

$$(y - 2at_1) = \frac{2at_2 - 2at_1}{at_2^2 - at_1^2} (x - at_1^2)$$



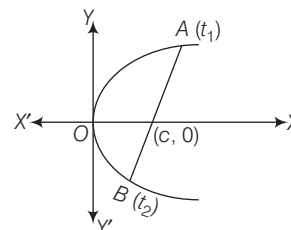
$$\Rightarrow y(t_1 + t_2) = 2x + 2at_1 t_2$$

which is the required equation.

Hence, slope of the chord = $\frac{2}{t_1 + t_2}$

- (i) If chord AB passes through the point $(c, 0)$, then

$$t_1 t_2 = -\frac{c}{a}.$$



- (ii) If chord AB is focal chord or passes through the focus, then $t_1 t_2 = -1$.

- (iii) Length of chord of the parabola intercepted by the line $y = mx + c$ is given by $l = \frac{4}{m^2} \sqrt{a(1+m^2)(a-mc)}$ and in parametric form, when $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ are its end points, is $\sqrt{(t_1 + t_2)^2 - 4t_1 t_2 \{(t_1 + t_2)^2 + 4\}}$.

- (iv) Length of the focal chord of parabola having t_1, t_2 as end points, is $a(t_2 - t_1)^2$.

Intersection of a Line and a Parabola

Let the parabola be $y^2 = 4ax$ and the given line be $y = mx + c$. Solving the line and parabola, we get

$$m^2 x^2 + 2(mc - 2a)x + c^2 = 0$$

which is quadratic in x .

Hence, the line intersects the parabola in 2 distinct points.

- NOTE**
- The two points are real and distinct, if $c > \frac{a}{m}$.
 - The two points are real and coincident, if $c = \frac{a}{m}$.
 - The two points are imaginary, if $c < \frac{a}{m}$.

Condition of Tangency

- (a) The straight line $y = mx + c$ touches parabola $y^2 = 4ax$, iff $c = \frac{a}{m}$.

Any tangent to the parabola $y^2 = 4ax$ is

$$y = mx + \frac{a}{m}$$

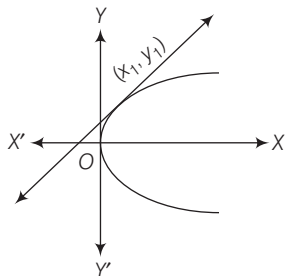
Points of contact is $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$.

- (b) The straight line $lx + my + n = 0$ touches $y^2 = 4ax$, if $nl = am^2$.

- (c) $x \cos \alpha + y \sin \alpha = p$ touches $y^2 = 4ax$, if $p \cos \alpha + a \sin^2 \alpha = 0$.

Equations of Tangent in Different Forms

If $y^2 = 4ax$ is parabola, then equation of tangent at the point (x_1, y_1) is $T = 0$, where $T = yy_1 - 2a(x + x_1)$.



(i) Cartesian Form

Equation of parabola	Equation of tangent
$y^2 = 4ax$	$yy_1 = 2a(x + x_1)$
$y^2 = -4ax$	$yy_1 = -2a(x + x_1)$
$x^2 = 4ay$	$xx_1 = 2a(y + y_1)$
$x^2 = -4ay$	$xx_1 = -2a(y + y_1)$

(ii) **Parametric Form** Equations of tangent in parametric form for different standard forms is given as follows:

Equation of parabola	Parametric coordinates	Tangent at t
$y^2 = 4ax$	$(at^2, 2at)$	$ty = x + at^2$
$y^2 = -4ax$	$(-at^2, 2at)$	$ty = -x + at^2$
$x^2 = 4ay$	$(2at, at^2)$	$xt = y + at^2$
$x^2 = -4ay$	$(2at, -at^2)$	$xt = -y + at^2$

(iii) **Slope Form** If a line $y = mx + c$ is tangent to the parabola, then following are equations of tangent line in slope form:

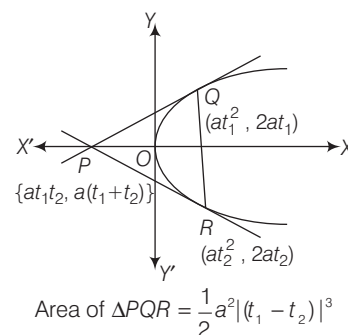
Equation of parabola	Point of contact	Equation of tangent line	Condition for tangency
$y^2 = 4ax$	$\left(\frac{a}{m^2}, \frac{2a}{m}\right)$	$y = mx + \frac{a}{m}$	$c = \frac{a}{m}$
$y^2 = -4ax$	$\left(\frac{-a}{m^2}, \frac{-2a}{m}\right)$	$y = mx - \frac{a}{m}$	$c = -\frac{a}{m}$
$x^2 = 4ay$	$(2am, am^2)$	$y = mx - am^2$	$c = -am^2$
$x^2 = -4ay$	$(-2am, -am^2)$	$y = mx + am^2$	$c = am^2$

NOTE

- If a line is tangent at a point $P(x_1, y_1)$, then equation of tangent line is $T = 0$.
- The equation of tangent line at (x_1, y_1) is obtained by replacing x^2 by xx_1 , y^2 by yy_1 , x by $\frac{x+x_1}{2}$, y by $\frac{y+y_1}{2}$ and xy by $\frac{xy_1 + x_1y}{2}$ and without changing constant (if any) in the equation of the curve.

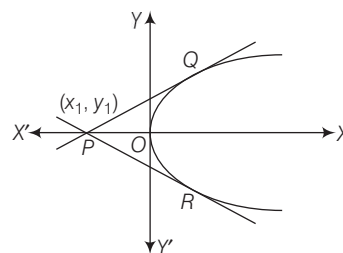
Some Important Points

- The point of intersection of tangent at two points $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ on the parabola $y^2 = 4ax$, is $\{(at_1t_2, a(t_1 + t_2))\}$ or (GM of x -coordinate, AM of y -coordinate).
- The locus of point of intersection of perpendicular tangents to a conic, is known as director circle. The director circle of parabola is directrix.
i.e. $x + a = 0$ for $y^2 = 4ax$
- The locus of point of intersection of tangent to a parabola, intersect at an angle α is $(x + a)^2 \tan^2 \alpha = y^2 - 4ax$.
- Area of triangle formed by two intersecting tangents is



Equation of Pair of Tangents from a Point to a Parabola

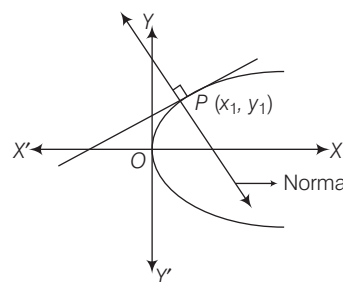
The combined equation of the pair of tangents drawn from a point to a parabola is $SS' = T^2$



where, $S = y^2 - 4ax$, $S' = y_1^2 - 4ax_1$
and $T = yy_1 - 2a(x + x_1)$

Equation of Normal in Different Forms

(i) **Cartesian Form** We know that, equation of tangent line is $yy_1 = 2a(x + x_1)$



$$\therefore \text{Slope of the normal} = -\frac{y_1}{2a}$$

$$\therefore \text{Equation of normal is } y - y_1 = \left(-\frac{y_1}{2a}\right)(x - x_1)$$

(ii) **Slope Form** Coordinates of point of contact, when m is slope of the tangent, is $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$. If m is slope of

the normal, then above coordinates can be written as $(am^2, -2am)$.

$$\therefore \text{Equation of the normal is } (y + 2am) = m(x - am^2)$$

$$\Rightarrow y = mx - 2am - am^3$$

(iii) **Parametric Form** Equation of normal in parametric form, where t is parameter at the point $(at^2, 2at)$ is

$$(y - 2at) = \frac{-2at}{2a}(x - at^2)$$

$$\Rightarrow y + tx = 2at + at^3$$

Equation of Normal for Different Parabolas

Forms $\rightarrow$	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Cartesian form	$y - y_1 = \frac{-y_1}{2a}(x - x_1)$	$y - y_1 = \frac{y_1}{2a}(x - x_1)$	$y - y_1 = \frac{-2a}{x_1}(x - x_1)$	$y - y_1 = \frac{2a}{x_1}(x - x_1)$
Parametric form	$y + tx = 2at + at^3$	$y - tx = 2at + at^3$	$x + ty = 2at + at^3$	$x - ty = 2at + at^3$
Slope form	$y = mx - 2am - am^3$	$y = mx + 2am + am^3$	$y = mx + 2a + \frac{a}{m^2}$	$y = mx - 2a - \frac{a}{m^2}$

Conditions of Normality and Points of Contact

Here, we are listing conditions of normality in slope form.

Equation of parabola	Point of contact	Condition of normality
$y^2 = 4ax$	$(am^2, -2am)$	$c = -2am - am^3$
$y^2 = -4ax$	$(-am^2, 2am)$	$c = 2am + am^3$
$x^2 = 4ay$	$\left(\frac{-2a}{m}, \frac{a}{m^2}\right)$	$c = 2a + \frac{a}{m^2}$
$x^2 = -4ay$	$\left(\frac{2a}{m}, -\frac{a}{m^2}\right)$	$c = -2a - \frac{a}{m^2}$

Some Important Points

- The point of intersection of normal at any two points $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ on the parabola $y^2 = 4ax$ is $R[2a + a(t_1^2 + t_2^2 + t_1 t_2), -at_1 t_2(t_1 + t_2)]$.
- If normal drawn at t_1 meets the parabola again at t_2 , then for parabola $y^2 = 4ax$, $t_2 = -t_1 - \frac{2}{t_1}$.

- The points on the curve at which the normal passes through a common point, are called conormal points. The conormal points, also called feet of the normals.
- Three normals can be drawn from a point to a parabola, in which one or all are real.
- The algebraic sum of the slope of three normals at conormal point is zero.
- The sum of the ordinates of the conormal points is zero.
- The centroid of the triangle formed by the conormal points lies on the axis of the parabola.
- If three normals are drawn from the point (h, k) to the parabola $y^2 = 4ax$, then $h > 2a$.

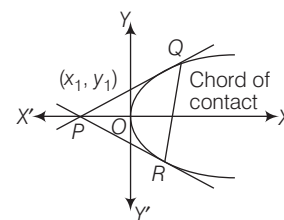
Equation of Chord of Contact of Tangents to a Parabola

Let PQ and PR be tangents to the parabola $y^2 = 4ax$ drawn from any point $P(x_1, y_1)$.

Then, QR is called chord of contact of the parabola $y^2 = 4ax$.

Equation of chord of contact to the parabola is $T = 0$, i.e. $yy_1 = 2a(x + x_1)$

Here, (x_1, y_1) is a point of intersection of two tangents.



Equation of Chord when Mid-point is Given

Equation of chord when its mid-point is given, is $T = S_1$, where $T = yy_1 - 2a(x + x_1)$ and $S_1 = y_1^2 - 4ax_1$.

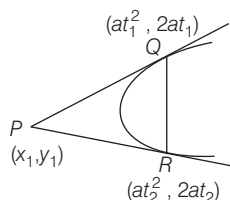
Here, (x_1, y_1) is the mid-point of the chord.

NOTE Equation of chord of contact to the parabola is $T = 0$ but the point (x_1, y_1) is in different positions in all cases.

Important Results of a Parabola

- The length of the side of an equilateral triangle inscribed in the parabola $y^2 = 4ax$ is $8a\sqrt{3}$ (one angular point is at the vertex).
- The equation of the parabola, whose axis is parallel to X-axis, is $x = Ay^2 + By + C$ and $y = Ax^2 + Bx + C$, is a parabola with its axis parallel to Y-axis.
- If the line $x \cos \alpha + y \sin \alpha = p$ touches the parabola $y^2 = 4ax$, then $p \cos \alpha + a \sin^2 \alpha = 0$ and the point of contact is $(a \tan^2 \alpha, -2a \tan \alpha)$.
- Area of a triangle formed by three points on a parabola is twice the area of the triangle formed by the tangents at these points.
- The angle of intersection of two parabolas $y^2 = 4ax$ and $x^2 = 4by$ is given by $\tan^{-1} \left| \frac{3a^{1/3}b^{1/3}}{2(a^{2/3} + b^{2/3})} \right|$.
- The equation of the common tangents to the parabola $y^2 = 4ax$ and $x^2 = 4by$ is $a^{1/3}x + b^{1/3}y + a^{2/3}b^{2/3} = 0$.

- If the normals at points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ on the parabola $y^2 = 4ax$ meet on the parabola, then $t_1 t_2 = 2$.
- If the normal at a point $P(at^2, 2at)$ to the parabola $y^2 = 4ax$ subtends a right angle at the vertex of the parabola, then $t^2 = 2$.
- If the tangents are drawn from the point (x_1, y_1) to the parabola $y^2 = 4ax$. Then, the length of their chord of contact is $\frac{1}{|a|} \sqrt{(y_1^2 - 4ax_1)(y_1^2 + 4a^2)}$.



- The area of triangle formed by the tangents drawn from (x_1, y_1) to $y^2 = 4ax$ and their chord of contact is $\frac{(y_1^2 - 4ax_1)^{3/2}}{2a}$.
- If one end of the focal chord of parabola is $(at^2, 2at)$, then other end will be $(a/t^2, -2a/t)$ and length of chord $= a(t + 1/t)^2$.
- If l_1 and l_2 are the lengths of segments of a focal chord of a parabola, then its latusrectum is $\frac{4l_1 l_2}{l_1 + l_2}$.

Diameter

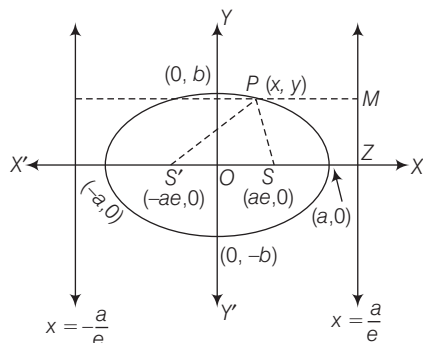
The locus of the mid-point of a system of parallel chords of a parabola is called its diameter.

Its equation is $y = \frac{2a}{m}$,

where, m is the slope of the chords.

Ellipse

An ellipse is the locus of a point which moves in a plane, so that the ratio of its distance from a fixed point (focus) and fixed line (directrix) is constant which is less than one. For an ellipse, $e < 1$.



Let S be the focus, ZM be the directrix of the ellipse and $P(x, y)$ be any point on the ellipse, then by definition, equation of ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where

$$b^2 = a^2(1 - e^2).$$

Since,

$$e < 1$$

$\Rightarrow$

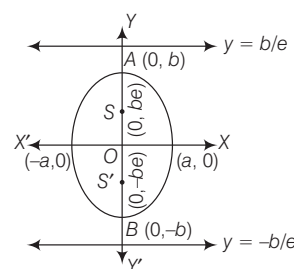
$$a^2(1 - e^2) < a^2$$

$\Rightarrow$

$$b^2 < a^2$$

The other form of equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

where, $a^2 = b^2(1 - e^2)$ i.e. $a < b$



Definition and Basic Terminology of an Ellipse

- Focal axis** Line containing two fixed points F_1 and F_2 , is called focal axis or major axis.
- Focal length** Distance between F_1 and F_2 , is called focal length or focal distance, i.e. $F_1 F_2 = 2ae$.
- Diameter** Any chord of the ellipse passing through its centre, is called diameter.
- Principal axis** Major and minor axes together, known as principal axis.
- Latusrectum** Line perpendicular to major axis and passing through focus, is called latusrectum.

Important Terms of an Ellipse

Important terms	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, b > a$
Centre	(0, 0)	(0, 0)
Foci	($\pm ae, 0$)	(0, $\pm be$)
Vertex	($\pm a, 0$)	(0, $\pm b$)
Major axis	$2a$	$2b$
Eccentricity	$e^2 = 1 - \frac{b^2}{a^2}$	$e^2 = 1 - \frac{a^2}{b^2}$
Equation of directrix	$x = \pm \frac{a}{e}$	$y = \pm \frac{b}{e}$
Length of latusrectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Ends of latusrectum	$(\pm ae, \pm \frac{b^2}{a})$	$(\pm \frac{a^2}{b}, \pm be)$

Parametric Form of an Ellipse

Let the equation of ellipse in standard form be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Then, the equations of an ellipse in the parametric form will be

$x = a \cos \phi$, $y = b \sin \phi$, where ϕ is eccentric angle.

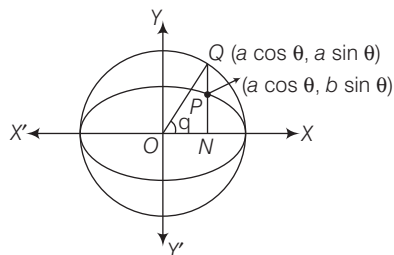
Auxiliary Circle

Circle described on the major axis of an ellipse as the diameter, is called an auxiliary circle.

Here, $x^2 + y^2 = a^2$ is the equation of auxiliary circle and

$(a \cos \theta, b \sin \theta)$, where

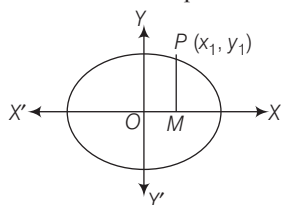
$\theta \in (0, 2\pi)$, is known as parametric coordinates of an ellipse and θ is an eccentric angle.



Position of a Point w.r.t. Ellipse

Let standard equation of an ellipse be $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$, then

the point $P(x_1, y_1)$ will lie outside the ellipse, if $E(P) > 0$ lie inside, if $E(P) < 0$ and on the ellipse, if $E(P) = 0$.



Condition for Intersection of a Line and an Ellipse

Let us consider a line $y = mx + c$ and an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

then the line will touch, intersect and do not intersect according as $a^2m^2 + b^2 = c^2$, $a^2m^2 + b^2 > c^2$ and $a^2m^2 + b^2 < c^2$, respectively.

Equations of Tangent in Different Forms

(i) **Point Form** The equation of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point (x_1, y_1) is $T = 0$.

i.e. $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

(ii) **Slope Form** If the line $y = mx + c$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Then, $c^2 = a^2m^2 + b^2$

Hence, the straight line $y = mx \pm \sqrt{a^2m^2 + b^2}$ is the equation of tangent in slope form and coordinates of point of contact are $\left(\frac{\pm a^2m}{\sqrt{a^2m^2 + b^2}}, \frac{\mp b^2}{\sqrt{a^2m^2 + b^2}} \right)$.

(iii) **Parametric Form** The equation of tangent in parametric form at the point $(a \cos \phi, b \sin \phi)$ is

$$\frac{x}{a} \cos \phi + \frac{y}{b} \sin \phi = 1$$

Condition for Tangency

(a) The straight line $y = mx + c$ is a tangent to an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ if } c^2 = a^2m^2 + b^2 \text{ at point}$$

$$\left(\frac{\pm a^2m}{\sqrt{a^2m^2 + b^2}}, \frac{\pm b^2}{\sqrt{a^2m^2 + b^2}} \right)$$

Thus, tangent is $y = mx + \sqrt{a^2m^2 + b^2}$

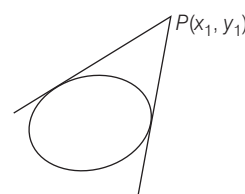
(b) $x \cos \alpha + y \sin \alpha = p$ touches $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if

$$p^2 = a^2 \cos^2 \alpha + b^2 \sin^2 \alpha$$

(c) $lx + my + n = 0$ touches $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if $n^2 = a^2l^2 + b^2m^2$

Equation of Pair of Tangents

The equation of pair of tangents PA and PB is $SS_1 = T^2$



where, $S = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1$

$$S_1 = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 \text{ and } T = \frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1$$

Director Circle

The director circle is the locus of points from which perpendicular tangents drawn to the ellipse. The equation of director circle is

$$x^2 + y^2 = a^2 + b^2$$

Equation of Normal in Different Forms

(i) **Point Form** The equation of the normal at (x_1, y_1) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$.

(ii) **Slope Form** If m is the slope of the normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Then, the equation of normal is

$y = mx \mp \frac{m(a^2 - b^2)}{\sqrt{a^2 + b^2 m^2}}$. The coordinates of point of contact is $\left(\pm \frac{a^2}{\sqrt{a^2 + b^2 m^2}}, \pm \frac{mb^2}{\sqrt{a^2 + b^2 m^2}} \right)$.

- (iii) **Parametric Form** The equation of the normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $(a \cos \phi, b \sin \phi)$ is $ax \sec \phi - by \operatorname{cosec} \phi = a^2 - b^2$

Chord of Contact

If PQ and PR are the tangents through the point $P(x_1, y_1)$ to the ellipse, then the equation of the chord of contact QR is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$ or $T = 0$ at (x_1, y_1) .

Equation of Chord with Mid-point

The equation of the chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose mid-point is (x_1, y_1) , is $T = S_1$

where, $T = \frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1$

$$\Rightarrow S_1 = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1$$

Equation of Chord Joining Two Points on an Ellipse

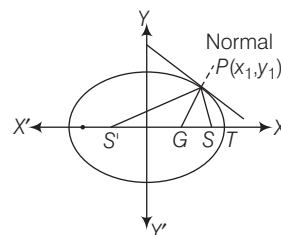
The equation of chord joining two points having eccentric angles θ and ϕ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, is

$$\frac{x}{a} \cos\left(\frac{\theta + \phi}{2}\right) + \frac{y}{b} \sin\left(\frac{\theta + \phi}{2}\right) = \cos\left(\frac{\theta - \phi}{2}\right)$$

Important Results of an Ellipse

- Sum of distance of the point $P(x, y)$ lie on the ellipse from two foci is equal to constant.
 - Product of length of perpendicular from either focus on a variable tangent to an ellipse is equal to square of semi-minor axis, i.e. b^2 .
 - Locus of the foot of perpendicular from the either foci on a variable tangent to an ellipse lie on its auxiliary circle.
- The straight line $lx + my + n = 0$ touches ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if $a^2 l^2 + b^2 m^2 = n^2$.
- The line $x \cos \alpha + y \sin \alpha = p$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if $a^2 \cos^2 \alpha + b^2 \sin^2 \alpha = p^2$ and that point of contact is $\left(\frac{a^2 \cos \alpha}{p}, \frac{b^2 \sin \alpha}{p} \right)$.

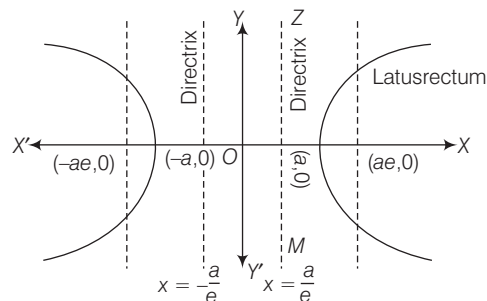
- The locus of the foot of the perpendicular drawn from centre upon any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $(x^2 + y^2)^2 = a^2 x^2 + b^2 y^2$ or $r^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta$ (in terms of polar coordinates).
- If $y = mx + c$ is the normal of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then condition of normality is $c^2 = \frac{m^2(a^2 - b^2)^2}{a^2 + b^2 m^2}$.
- The straight line $lx + my + n = 0$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \left(\frac{a^2 - b^2}{n} \right)^2$.
- If S is the focus and G is the point, where the normal at P meets the axis of an ellipse, then $SG = e \cdot SP$ and the tangent and normal at P bisect the external and internal angles between the focal distances of P .



Hyperbola

A hyperbola is the locus of a point which moves in a plane, so that the ratio of its distance from fixed point (focus) and fixed line (directrix) is constant and which is greater than 1.

Any second degree curve $f(x, y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a hyperbola, if $h^2 > ab$ or $e > 1$.



Standard Equation of the Hyperbola

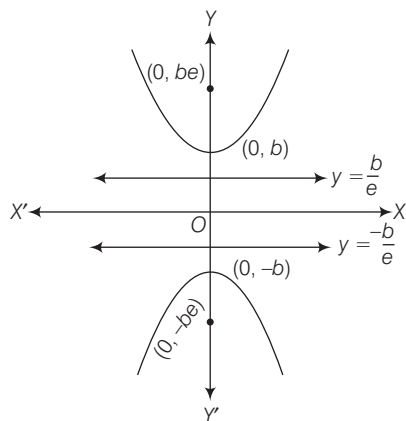
Let F_1 be the focus, ZM be the directrix and e be the eccentricity of the hyperbola, then $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $b^2 = a^2(e^2 - 1)$ is standard equation of hyperbola.

Conjugate Hyperbola

The hyperbola whose transverse and conjugate axes are respectively the conjugate and transverse axes of a given hyperbola, is called conjugate hyperbola of given hyperbola.

Equation of conjugate hyperbola is

$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Terms Related to Hyperbola

- (i) **Symmetry** Since, equation of hyperbola has only even powers of x and y , hence it is symmetric in X and Y -axes.
- (ii) **Transverse and conjugate axes** Transverse axis is one which lie along the line passing through the foci and perpendicular to the directrices and conjugate axis is one which is perpendicular to transverse axis and passes through the mid-point of foci.
- (iii) **Focal chord** A chord of hyperbola which passes through the focus, is called a focal chord of the hyperbola.
- (iv) **Directrix** A line which is perpendicular to the axis and it lies between centre and vertex, is known as directrix. The equation of directrix is $x = \pm \frac{a}{e}$.
- (v) **Double ordinates** Perpendicular line drawn to axis of hyperbola whose end points lie on hyperbola.
- (vi) **Latusrectum** The double ordinates passes through the focus, is called latusrectum.
 - (a) The vertex divides the join of focus and foot of the directrix externally or internally in the ratio $e : 1$.
 - (b) Domain and range of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $x \leq -a$ or $x \geq a$ and $y \in R$ respectively.

Important Terms of Hyperbola

Important terms	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
Centre	(0, 0)	(0, 0)
Length of transverse axis	$2a$	$2b$
Length of conjugate axis	$2b$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm be)$

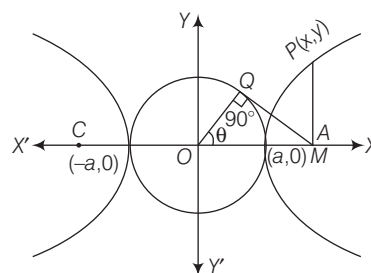
Important terms	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
Centre	(0, 0)	(0, 0)
Equation of directrices	$x = \pm \left(\frac{a}{e}\right)$	$y = \pm \left(\frac{b}{e}\right)$
Eccentricity	$e = \sqrt{\frac{a^2 + b^2}{a^2}}$	$e = \sqrt{\frac{a^2 + b^2}{b^2}}$
Length of latusrectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Parametric coordinates	$(a \sec \theta, b \tan \theta)$	$(b \sec \theta, a \tan \theta)$
Focal radii	$SP = ex_1 - a$ $S'P = ex_1 + a$	$SP = ey_1 - b$ $S'P = ey_1 + b$
Difference in focal radii = $(S'P - SP)$	$2a$	$2b$
Tangent at the vertices	$x = -a, x = a$	$y = -b, y = b$
Equation of transverse axis	$y = 0$	$x = 0$
Equations of conjugate axis	$x = 0$	$y = 0$

NOTE If the centre of the hyperbola is (h, k) and axes are parallel to the coordinate axes, then its equation is

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Auxiliary Circle

Let $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be the hyperbola, then the equation of the auxiliary circle is $x^2 + y^2 = a^2$.



Here, $\angle QOM = \theta$ is called eccentric angle and P, Q are corresponding points on hyperbola and auxiliary circle.

Parametric Equations of Hyperbola

The equations $x = a \sec \theta$ and $y = b \tan \theta$ are known as the parametric equations of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. The point $(a \sec \theta, b \tan \theta)$ will lie on the hyperbola for all values of θ .

Condition for Intersection of Line and Hyperbola

Let us consider a line $y = mx + c$

and a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

By above two equations, we have $\frac{x^2}{a^2} - \frac{(mx + c)^2}{b^2} = 1$.

This line will intersect, touch or do not intersect the hyperbola according to discriminant of above equation, i.e.

- (i) If $D > 0$, then line will intersect the hyperbola at two points.
- (ii) If $D = 0$, then $c^2 = a^2m^2 - b^2$ touches the parabola.
- (iii) If $D < 0$, then line is outside the hyperbola.

Position of a Point w.r.t. Hyperbola

Any point $P(x, y)$ lie on the hyperbola

$$H: \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$$

According to the following conditions:

- (i) Outside the hyperbola, $H(P) < 0$
- (ii) Inside the hyperbola, $H(P) > 0$
- (iii) On the hyperbola, $H(P) = 0$

Equation of Tangent in Different Forms

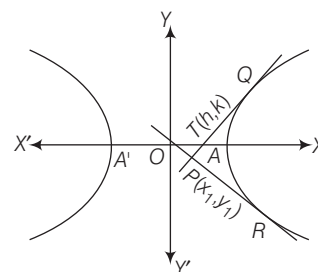
- (i) **Point Form** The equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at (x_1, y_1) is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$
- (ii) **Slope Form** The equation of tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $y = mx \pm \sqrt{a^2m^2 - b^2}$ and coordinates of the point of contact are $\left(\pm \frac{a^2m}{\sqrt{a^2m^2 - b^2}}, \pm \frac{b^2}{\sqrt{a^2m^2 - b^2}} \right)$.
- (iii) **Parametric Form** The equation of tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $(a \sec \theta, b \tan \theta)$ is $\frac{x}{a} \sec \theta - \frac{y}{b} \tan \theta = 1$

Condition for Tangency

- (a) The straight line $y = mx + c$ is a tangent to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, if $c^2 = a^2m^2 - b^2$
Hence, $y = mx \pm \sqrt{a^2m^2 - b^2}$ touches the hyperbola at $\left(\frac{\pm a^2m}{\sqrt{a^2m^2 - b^2}}, \frac{\mp b^2}{\sqrt{a^2m^2 - b^2}} \right)$.
- (b) $x \cos \alpha + y \sin \alpha = p$ touches hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, if $p^2 = a^2 \cos^2 \alpha - b^2 \sin^2 \alpha$.
- (c) $lx + my + n = 0$ touches hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, if $n^2 = a^2l^2 - b^2m^2$.

Equation of Pair of Tangents

If $P(x_1, y_1)$ is any point outside the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then pair of tangents PQ and PR can be drawn to it from P as shown alongside. The equation of tangents PQ and PR is $SS_1 = T^2$



where, $S = \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1$

$$\Rightarrow S_1 = \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1 \Rightarrow T = \frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1$$

Director Circle

The director circle is the locus of points from which perpendicular tangents is drawn to given hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, is $x^2 + y^2 = a^2 - b^2$.

Equation of Normal in Different Forms

- (i) **Cartesian Form** The equation of normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at (x_1, y_1) is $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$
- (ii) **Slope Form** The equation of the normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ in terms of slope is $y = mx \mp \frac{m(a^2 + b^2)}{\sqrt{a^2 - b^2m^2}}$
Point of contact is $\left(\pm \frac{a^2}{\sqrt{a^2 - m^2b^2}}, \mp \frac{mb^2}{\sqrt{a^2 - b^2m^2}} \right)$ and condition of normality is $c = \mp \frac{m(a^2 + b^2)}{\sqrt{a^2 - m^2b^2}}$ or $c^2 = \frac{m^2(a^2 + b^2)^2}{(a^2 - m^2b^2)}$
- (iii) **Parametric Form** The equation of the normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ in parametric form at $(a \sec \theta, b \tan \theta)$ is $ax \cos \theta + by \cot \theta = a^2 + b^2$

Equation of the Chord in Terms of Mid-point

The equation of the chord of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ whose mid-point is (x_2, y_2) , is $T = S_1$.

i.e. $\frac{xx_2}{a^2} - \frac{yy_2}{b^2} = \frac{x_2^2}{a^2} - \frac{y_2^2}{b^2}$

Chord of Contact

If PM and PN are the tangents passing through the point $P(x_1, y_1)$ to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then the equation of chord of contact MN is given by $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$ or $T = 0$.

Rectangular or Equilateral Hyperbola

A hyperbola for which $a = b$ is said to be rectangular or equilateral. Its equation is $x^2 - y^2 = a^2$.

Conjugate Hyperbola

$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ is known as conjugate hyperbola to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If e_1 and e_2 are their eccentricities, then

$$e_1^2 = 1 + \frac{b^2}{a^2} \quad \dots(i)$$

and

$$e_2^2 = 1 + \frac{a^2}{b^2} \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$$

Asymptote of Hyperbola

The combined equation of the asymptotes of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ is } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$

Some Important Results of Hyperbola

- (i) Difference of the focal distances, i.e. $PS' - PS = \text{constant} \Rightarrow 2a = \text{Length of transverse axis}$
- (ii) Eccentricity of the rectangular hyperbola $= \sqrt{2}$ and angle between asymptotes $= 90^\circ$.

Practice Exercise

- The curves described parametrically by $x = t^2 + t + 1$, $y = t^2 - t + 1$ represents
 - a pair of straight lines
 - an ellipse
 - a parabola
 - a hyperbola
- Consider a circle with its centre lying on the focus of the parabola $y^2 = 2px$ such that it touches the directrix of the parabola. Then, the point of intersection of the circle and the parabola is
 - $\left(\frac{p}{2}, p\right)$ or $\left(\frac{p}{2}, -p\right)$
 - $\left(\frac{p}{2}, -\frac{p}{2}\right)$
 - $\left(-\frac{p}{2}, p\right)$
 - $\left(-\frac{p}{2}, -\frac{p}{2}\right)$
- The equation of parabola having vertex $(0, 0)$ passing through $(2, 3)$ and axis is along X -axis, is
 - $x^2 = \frac{9}{2}y$
 - $y^2 = \frac{9}{2}x$
 - $y^2 = -\frac{9}{2}x$
 - $x^2 = -\frac{9}{2}y$
- If the points $(0, 4)$ and $(0, 2)$ are respectively the vertex and focus of a parabola, then the equation of the parabola is
 - $x^2 + 8y = 16$
 - $x^2 + 8y = 32$
 - $x^2 + 16y = 30$
 - $x^2 - 8y = 32$
- The equation of the parabola, whose axis is parallel to Y -axis and which passes through the points $(0, 4)$, $(1, 9)$ and $(-2, 6)$, is
 - $y = 2\left(x + \frac{3}{4}\right)^2 + \frac{23}{8}$
 - $y = 2\left(x + \frac{3}{2}\right)^2 - \frac{1}{2}$
 - $y = 2\left(x + \frac{3}{5}\right)^2 + \frac{1}{2}$
 - $y = 2\left(x + \frac{1}{2}\right)^2 - \frac{1}{2}$
- If PQ is a double ordinate of a parabola $y^2 = 4ax$. Then, the locus of its point of trisection is
 - $9y^2 = 8ax$
 - $9y^2 = 4ax$
 - $25y^2 = 16ax$
 - $3y^2 = 4ax$
- If a parabola has the origin as its focus and the line $x = 2$ as the directrix. Then, the vertex of the parabola is at
 - $(2, 0)$
 - $(0, 2)$
 - $(1, 0)$
 - $(0, 1)$
- If the tangent at the point $P(2, 4)$ to the parabola $y^2 = 8x$ meets the parabola $y^2 = 8x + 5$ at Q and R , then the mid-point of QR is
 - $(2, 4)$
 - $(4, 2)$
 - $(7, 9)$
 - None
- The equation of the common tangents touching the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above X -axis, is
 - $\sqrt{3}y = 3x + 1$
 - $\sqrt{3}y = -(x + 3)$
 - $\sqrt{3}y = x + 3$
 - $\sqrt{3}y = -(3x + 1)$

10. The equation(s) of the common tangent(s) to the parabolas $y^2 - 4x - 2y + 5 = 0$ and $y^2 = -4x$ is/are
 a. $x + y = 2, x - y = 1$ b. $x + y = 3, x - y = 2$
 c. $x + y = 1, 4x - 2y = 1$ d. $x + 2y = 1, x + y = -3$
11. The locus of the mid-point of the chords of the parabola $y^2 = 4ax$ which subtend a right angle at the vertex of the parabola, is
 a. $y^2 - 2ax + 8a^2 = 0$ b. $y^2 + 2ax + 8a^2 = 0$
 c. $y^2 - 2ax - 8a^2 = 0$ d. $y^2 - ax + 8a^2 = 0$
12. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$, is
 a. $\frac{1}{2}$ b. $\frac{3}{2}$ c. $\frac{1}{8}$ d. $\frac{2}{3}$
13. Tangents are drawn from $(-2, 0)$ to $y^2 = 8x$, radius of circle(s) that would touch these tangents and the corresponding chord of contact, can be equal to
 a. $4(\sqrt{2} + 1)$ b. $2(\sqrt{2} - 1)$ c. $8\sqrt{3}$ d. $9\sqrt{2}$
14. The number of distinct normals that can be drawn from $\left(\frac{11}{4}, \frac{1}{4}\right)$ to the parabola $y^2 = 4x$, is
 a. 3 b. 2 c. 1 d. 4
15. The locus of the mid-points of the chords of the parabola $y^2 = 4x$ which touch the parabola $x^2 = -8y$, is
 a. $y^3 - 2xy - 8 = 0$ b. $y^3 + 2xy - 8 = 0$
 c. $y^3 - 2xy + 8 = 0$ d. $y^3 - xy - 8 = 0$
16. If P is the point $(1, 0)$ and Q is the point on $y^2 = 8x$. Then, the locus of mid-point of PQ is equal to
 a. $x^2 - 4y + 2 = 0$ b. $x^2 + 4y + 2 = 0$
 c. $y^2 + 4x + 2 = 0$ d. $y^2 - 4x + 2 = 0$
17. If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then
 a. $d^2 + (2b + 3c)^2 = 0$ b. $d^2 + (3b + 2c)^2 = 0$
 c. $d^2 + (2b - 3c)^2 = 0$ d. $d^2 + (3b - 2c)^2 = 0$
18. The length of the chord of the parabola $x^2 = 4ay$ passing through the vertex and having slope $\tan \alpha$, is
 a. $4a \operatorname{cosec} \alpha \cot \alpha$ b. $4a \tan \alpha \sec \alpha$
 c. $4a \cos \alpha \cot \alpha$ d. $4a \sin \alpha \tan \alpha$
19. If P is a point on the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ whose foci are S and S' , then $PS + PS'$ is equal to
 a. 8 b. 7 c. 5 d. 10
20. The equation of the ellipse whose focus is $(1, -1)$, the directrix $x - y - 3 = 0$ and eccentricity $1/2$, is
 a. $7x^2 + 2xy + 7y^2 - 10x + 10y + 7 = 0$
 b. $7x^2 + 2xy + 7y^2 + 7 = 0$
 c. $7x^2 + 2xy + 7y^2 + 10x - 10y - 7 = 0$
 d. None of the above
21. The number of points outside the ellipse on major axis from which a normal (other than X-axis) can be drawn to the ellipse, is
 a. 0 b. 3
 c. 5 d. None of these
22. The eccentricity of the ellipse which meets the straight line $\frac{x}{7} + \frac{y}{2} = 1$ on X-axis and the straight line $\frac{x}{3} - \frac{y}{5} = 1$ on Y-axis and whose axes lie along the axes of coordinates, is
 a. $\frac{1}{\sqrt{2}}$ b. $\frac{2\sqrt{6}}{7}$ c. $\frac{\sqrt{3}}{7}$ d. $\frac{\sqrt{5}}{7}$
23. If tangent at any point P on the ellipse $7x^2 + 16y^2 = 12$ cuts the tangent at the end points of the major axis at the points A and B , then the circle with AB as diameter passes through a fixed point whose coordinates are
 a. $(\pm \sqrt{a^2 - b^2}, 0)$ b. $(\pm \sqrt{a^2 + b^2}, 0)$
 c. $(0, \pm \sqrt{a^2 - b^2})$ d. $(0, \pm \sqrt{a^2 + b^2})$
24. The minimum area of triangle formed by the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and coordinate axes, is
 a. ab b. $\frac{a^2 + b^2}{2}$
 c. $\frac{(a + b)^2}{2}$ d. $\frac{a^2 + ab + b^2}{3}$
25. If the angle between the lines joining the end points of minor axis of an ellipse with its foci is $\frac{\pi}{2}$, then the eccentricity of the ellipse is
 a. $1/2$ b. $1/\sqrt{2}$ c. $\sqrt{3}/2$ d. $1/2\sqrt{2}$
26. PQ is a chord of the ellipse through the centre. If the square of its length is the HM of the squares of major and minor axes, then find its inclination with X-axis.
 a. $\frac{\pi}{4}$ b. $\frac{\pi}{2}$
 c. $\frac{2\pi}{3}$ d. None of these
27. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it, is
 a. $(x^2 - y^2)^2 = 6x^2 + 2y^2$ b. $(x^2 - y^2)^2 = 6x^2 - 2y^2$
 c. $(x^2 + y^2)^2 = 6x^2 + 2y^2$ d. $(x^2 + y^2)^2 = 6x^2 - 2y^2$
28. Equation of tangent to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ which cut-off equal intercepts on the axis, is
 a. $y = x + \sqrt{3}$ b. $y = -x + \sqrt{3}$
 c. $y = -x - \sqrt{13}$ d. $y = -x - \sqrt{3}$
29. The equation of tangent to the ellipse $x^2 + 3y^2 = 3$ which is perpendicular to the line $4y = x - 5$, is
 a. $4x + y + 7 = 0$ b. $4x + y - 23 = 0$
 c. $4x + 2y + 5 = 0$ d. $4x + y + 23 = 0$

30. If the line $x \cos \alpha + y \sin \alpha = p$, is tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then the value of $a^2 \cos^2 \alpha + b^2 \sin^2 \alpha$ is
 a. p b. p^2
 c. $\frac{1}{p^2}$ d. None of these
31. The length of the common tangent to the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$ and the circle $x^2 + y^2 = 16$ intercepted by the coordinate axes, is
 a. 5 b. $2\sqrt{7}$ c. $\frac{7}{\sqrt{3}}$ d. $\frac{14}{\sqrt{3}}$
32. The distance of the centre of ellipse $x^2 + 2y^2 - 2 = 0$ to those tangents of the ellipse which are equally inclined from both the axes, is
 a. $\frac{3}{\sqrt{2}}$ b. $\sqrt{3/2}$ c. $\frac{\sqrt{2}}{3}$ d. $\frac{\sqrt{3}}{2}$
33. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point $(4, 0)$. Then, the equation of the ellipse is
 a. $x^2 + 12y^2 = 16$ b. $4x^2 + 48y^2 = 48$
 c. $4x^2 + 64y^2 = 48$ d. $x^2 + 16y^2 = 16$
34. If the chords of contact of tangents from two points (x_1, y_1) and (x_2, y_2) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are at right angles, then $\frac{x_1 x_2}{y_1 y_2}$ is equal to
 a. $\frac{a^2}{b^2}$ b. $-\frac{b^2}{a^2}$ c. $-\frac{a^4}{b^4}$ d. $-\frac{b^4}{a^4}$
35. If the line $3y = 3x + 1$ is a normal to the ellipse $\frac{x^2}{5} + \frac{y^2}{b^2} = 1$, then the length of the minor axis of the ellipse is
 a. 4 or $\frac{2}{3}\sqrt{55}$ b. 2 or $\frac{2}{5}\sqrt{55}$ c. 3 or $\sqrt{5}$ d. 11 or $\sqrt{13}$
36. If θ and ϕ are eccentric angles of the end of a pair of conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then $\theta - \phi$ is equal to
 a. $\pm \frac{\pi}{2}$ b. $\pm \pi$ c. 0 d. None
37. If the chords of contact of tangents from two points (x_1, y_1) and (x_2, y_2) to the ellipse $\frac{x^2}{(5)^2} + \frac{y^2}{(6)^2} = 1$ are at right angles, then $\frac{x_1 x_2}{y_1 y_2}$ is equal to
 a. $-\left(\frac{5}{6}\right)^4$ b. $\left(\frac{6}{5}\right)^4$ c. $-\left(\frac{6}{5}\right)^4$ d. $\left(\frac{5}{6}\right)^4$
38. The equation $16x^2 - 3y^2 - 32x + 12y - 44 = 0$ represents a hyperbola
 a. the length of whose transverse axis is $4\sqrt{3}$
 b. the length of whose conjugate axis is 4
 c. whose centre is $(-1, 2)$
 d. whose eccentricity is $\sqrt{19/3}$
39. Let $P(a \sec \theta, b \tan \theta)$ and $Q(a \sec \phi, b \tan \phi)$, where $\theta + \phi = \frac{\pi}{2}$ be two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If (h, k) is the point of intersection of the normals at P and Q , then k is equal to
 a. $\frac{a^2 + b^2}{a}$ b. $-\left(\frac{a^2 + b^2}{a}\right)$ c. $\frac{a^2 + b^2}{b}$ d. $-\left(\frac{a^2 + b^2}{b}\right)$
40. If the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, then the value of b^2 is
 a. 1 b. 5 c. 7 d. 9
41. If two points P and Q on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, whose centre C is such that CP is perpendicular to CQ , $a < b$. Then,
 a. $\frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{1}{b^2} - \frac{1}{a^2}$ b. $\frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{1}{a^2} - \frac{1}{b^2}$
 c. $\frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{1}{4a^2} - \frac{1}{b^2}$ d. $\frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{1}{a^2} - \frac{1}{2b^2}$
42. A hyperbola, having the transverse axis of length $2 \sin \theta$, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then, its equation is
 a. $x^2 \csc^2 \theta - y^2 \sec^2 \theta = 1$ b. $x^2 \sec^2 \theta - y^2 \csc^2 \theta = 1$
 c. $x^2 \sin^2 \theta - y^2 \cos^2 \theta = 1$ d. $x^2 \cos^2 \theta - y^2 \sin^2 \theta = 1$
43. If e_1 is the eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ and e_2 is the eccentricity of the hyperbola passing through the foci of the ellipse and $e_1 e_2 = 1$, then equation of the hyperbola is
 a. $\frac{x^2}{9} - \frac{y^2}{16} = 1$ b. $\frac{x^2}{16} - \frac{y^2}{9} = -1$
 c. $\frac{x^2}{9} - \frac{y^2}{25} = 1$ d. None of these
44. If m_1 and m_2 are slopes of tangents from a point $(1, 4)$ on the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$. Then, the point from which the tangents drawn on the hyperbola having slopes $|m_1|$ and $|m_2|$ and positive intercepts on Y-axis, is
 a. $(-7, 2)$ b. $(-7, -3)$ c. $(-7, -4)$ d. $(3, -7)$
45. The area of triangle formed by the lines $x - y = 0$, $x + y = 0$ and any tangent to the hyperbola $x^2 - y^2 = a^2$, is
 a. $2a^2$ b. $6a^2$ c. a^2 d. $4a^2$

46. If the line $2x + \sqrt{6}y = 2$ touches the hyperbola $x^2 - 2y^2 = 4$, then the point of contact is
 a. $(-2, \sqrt{6})$ b. $(-5, 2\sqrt{6})$ c. $\left(\frac{1}{2}, \frac{1}{\sqrt{6}}\right)$ d. $(4, -\sqrt{6})$
47. The common tangent to $9x^2 - 4y^2 = 36$ and $x^2 + y^2 = 3$ is
 a. $y - 2\sqrt{3}x - \sqrt{39} = 0$ b. $y + 2\sqrt{3}x + \sqrt{39} = 0$
 c. $y - 2\sqrt{3}x + \sqrt{39} = 0$ d. None of these
48. The locus of the points of intersection of perpendicular tangents to $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is
 a. $x^2 - y^2 = 7$ b. $x^2 - y^2 = 25$
 c. $x^2 + y^2 = 25$ d. $x^2 + y^2 = 7$
49. A tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ meets X -axis at P and Y -axis at Q . Lines PR and QR are drawn such that $OPRQ$ is a rectangle, where O is the origin. Then, R lies on
 a. $\frac{4}{x^2} + \frac{2}{y^2} = 1$ b. $\frac{2}{x^2} - \frac{4}{y^2} = 1$
 c. $\frac{2}{x^2} + \frac{4}{y^2} = 1$ d. $\frac{4}{x^2} - \frac{2}{y^2} = 1$
50. If P is a point on the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ and N is the foot of the perpendicular from P on the transverse axis. The tangent to the hyperbola at P meets the transverse axis at T . If O is the centre of the hyperbola, then $OT \cdot ON$ is equal to
 a. 9 b. 4 c. e^2 d. None
51. Tangents are drawn from points on the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ to the circle $x^2 + y^2 = 9$. The locus of the mid-point of the chord of contact is
 a. $x^2 + y^2 = \frac{x^2}{9} - \frac{y^2}{4}$ b. $(x^2 + y^2)^2 = \frac{x^2}{9} - \frac{y^2}{4}$
 c. $(x^2 + y^2)^2 = 81\left(\frac{x^2}{9} - \frac{y^2}{4}\right)$ d. $(x^2 + y^2)^2 = 9\left(\frac{x^2}{9} - \frac{y^2}{4}\right)$
52. The equation of a tangent to the hyperbola $3x^2 - y^2 = 3$ parallel to the line $y = 2x + 4$, is
 a. $y = 3x + 4$ b. $y = 2x + 1$ c. $y = 2x - 2$ d. $y = 3x + 5$
53. If chords of the hyperbola $x^2 - y^2 = a^2$ touch the parabola $y^2 = 4ax$. Then, the locus of their mid-point is
 a. $y^2(x - a) = 2x^2$ b. $y^2(x - a) = x^3$
 c. $y^2(x - a) = x^4$ d. $y^2(x + a) = x^3$
54. The product of the length of perpendiculars drawn from any point on the hyperbola $x^2 - 2y^2 - 2 = 0$ to its asymptotes, is
 a. $\frac{1}{2}$ b. $\frac{2}{3}$
 c. $\frac{3}{2}$ d. 2
55. If $x = 9$ is the chord of contact of tangents of $x^2 - y^2 = 9$, then the equation of the corresponding tangents is
 a. $9x^2 - 8y^2 + 18x + 9 = 0$ b. $9x^2 - 8y^2 - 18x + 9 = 0$
 c. $9x^2 - 8y^2 - 18x - 9 = 0$ d. $9x^2 - 8y^2 + 18x - 9 = 0$

BITSAT Archives

1. The locus of the points of intersection of the tangents at the extremities of the chords of the ellipse $x^2 + 2y^2 = 6$ which touches the ellipse $x^2 + 4y^2 = 4$, is
 a. $x^2 + y^2 = 4$ [2014]
 b. $x^2 + y^2 = 6$
 c. $x^2 + y^2 = 9$
 d. None of the above
2. If a variable chord PQ of the parabola $y^2 = 4ax$ subtends a right angle at the vertex, then the locus of the points of intersection of the normal at P and Q is
 a. a parabola [2014]
 b. a hyperbola
 c. a circle
 d. None of the above
3. The foci of the conic section $25x^2 + 16y^2 - 150x = 175$ are [2013]
 a. $(0, \pm 3)$ b. $(0, \pm 2)$ c. $(3, \pm 3)$ d. $(0, \pm 1)$
4. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and having its centre $(0, 3)$, is
 a. 4 b. $\frac{3}{7}$ c. $\sqrt{12}$ d. $\frac{7}{2}$ [2013]
5. If OAB is an equilateral triangle inscribed in the parabola $y^2 = 4ax$ with O as the vertex, then the length of the side of $\triangle OAB$ is [2013]
 a. $8a\sqrt{3}$ b. $4a\sqrt{3}$
 c. $2a\sqrt{3}$ d. $a\sqrt{3}$
6. If the length of the major axis of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is three times the length of minor axis, then its eccentricity is [2012]
 a. $\frac{1}{3}$ b. $\frac{1}{\sqrt{3}}$ c. $\sqrt{\frac{2}{3}}$ d. $\frac{2\sqrt{2}}{3}$

7. Let S and T be the foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and B be an end of the minor axis. If STB is an equilateral triangle, then eccentricity of the ellipse is [2012]
 a. $\frac{1}{4}$ b. $\frac{1}{3}$ c. $\frac{1}{2}$ d. $\frac{\sqrt{3}}{2}$
8. The difference of the focal distance of any point on the hyperbola is equal to its [2012]
 a. latusrectum
 b. eccentricity
 c. length of the transverse axis
 d. half the length of the transverse axis
9. If the focus of parabola is at $(0, -3)$ and its directrix is $y = 3$, then its equation is [2012]
 a. $x^2 = -12y$ b. $x^2 = 12y$
 c. $y^2 = -12x$ d. $y^2 = 12x$
10. The length of tangent from point $(5, 1)$ to the circle $x^2 + y^2 + 6x - 4y - 3 = 0$ is [2011]
 a. 81 b. 29 c. 7 d. 21
11. The length of the latusrectum of the parabola $169\{(x-1)^2 + (y-3)^2\} = (5x-12y+17)^2$ is [2011]
 a. $\frac{14}{13}$ b. $\frac{12}{13}$ c. $\frac{28}{13}$ d. None
12. If the centre, one of the foci and semi-major axis of an ellipse are $(0, 0)$, $(0, 3)$ and 5, then its equation is [2011]
 a. $\frac{x^2}{16} + \frac{y^2}{25} = 1$ b. $\frac{x^2}{25} + \frac{y^2}{16} = 1$
 c. $\frac{x^2}{9} + \frac{y^2}{25} = 1$ d. None of these
13. The radius of the director circle of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is [2011]
 a. $a - b$ b. $\sqrt{a - b}$
 c. $\sqrt{a^2 - b^2}$ d. $\sqrt{a^2 + b^2}$
14. The equation of the chord of $y^2 = 8x$ which is bisected at $(2, -3)$, is [2010]
 a. $4x + 3y = 1$ b. $4x - 3y = 1$
 c. $4x + 3y = 9$ d. None of these
15. All chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ which subtend a right angle at the origin, pass through the fix point [2009]
 a. $(1, 2)$ b. $(1, -2)$ c. $(-1, 2)$ d. $(-1, -2)$
16. The line $y = 2t^2$ meets the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ in real points, if [2009]
 a. $|t| \leq 1$ b. $|t| > 1$
 c. $|t| < 3$ d. $|t| < 4$
17. The distance between the foci of the hyperbola $x^2 - 3y^2 - 4x - 6y - 11 = 0$ is [2008]
 a. 4 b. 6 c. 8 d. 10
18. The length of the common chord of the ellipse $\frac{(x-1)^2}{9} + \frac{(y-2)^2}{4} = 1$ and the circle $(x-1)^2 + (y-2)^2 = 1$, is [2007]
 a. 0 b. $\sqrt{3}$ c. 4 d. 5
19. For hyperbola $\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1$, which of the following remains constant with change in α ? [2007]
 a. Abscissae of vertices b. Abscissae of foci
 c. Eccentricity d. Directrix
20. The equation of the parabola with its vertex at $(1, 1)$ and focus $(3, 1)$ is [2007]
 a. $(x-1)^2 = 8(y-1)$ b. $(y-1)^2 = 8(x-3)$
 c. $(y-1)^2 = 8(x-1)$ d. $(x-3)^2 = 8(y-1)$
21. The ends of the latusrectum of the conic $x^2 + 10x - 16y + 25 = 0$ are [2006]
 a. $(3, -4), (13, 4)$ b. $(-3, -4), (13, -4)$
 c. $(3, 4), (-13, 4)$ d. $(5, -8), (-5, 8)$
22. The equation to the hyperbola having its eccentricity 2 and the distance between its foci is 8, is [2006]
 a. $\frac{x^2}{12} - \frac{y^2}{4} = 1$ b. $\frac{x^2}{4} - \frac{y^2}{12} = 1$
 c. $\frac{x^2}{8} - \frac{y^2}{2} = 1$ d. $\frac{x^2}{16} - \frac{y^2}{9} = 1$
23. The sum of the focal distances of any point on the conic $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is [2006]
 a. 10 b. 9 c. 41 d. 18
24. The equation of a parabola which passes through the intersection of a straight line $x + y = 0$ and the circle $x^2 + y^2 + 4y = 0$ is [2005]
 a. $y^2 = 4x$ b. $y^2 = x$
 c. $y^2 = 2x$ d. None of these
25. The point $(4, -3)$ with respect to the ellipse $4x^2 + 5y^2 = 1$ [2005]
 a. lies on the curve b. is inside the curve
 c. is outside the curve d. is focus of the curve
26. The tangents from a point $(2\sqrt{2}, 1)$ to the hyperbola $16x^2 - 25y^2 = 400$ include an angle equal to [2005]
 a. $\pi/2$ b. $\pi/4$
 c. π d. $\pi/3$
27. The equation of a straight line drawn through the focus of the parabola $y^2 = -4x$ at an angle of 120° to X -axis, is [2005]
 a. $y + \sqrt{3}(x-1) = 0$ b. $y - \sqrt{3}(x-1) = 1$
 c. $y + \sqrt{3}(x+1) = 0$ d. $y - \sqrt{3}(x+1) = 0$

Answer with Solutions

Practice Exercise

1. (c) $x = t^2 + t + 1, y = t^2 - t + 1$

$$\Rightarrow x + y = 2(t^2 + 1) \text{ and } x - y = 2t$$

$$\therefore \frac{x+y}{2} = \left(\frac{x-y}{2}\right)^2 + 1$$

$$\Rightarrow x^2 + y^2 - 2xy - 2(x+y) + 4 = 0$$

which represent a parabola, since $h^2 = ab$.

$$[\because a = 1 = b \text{ and } h = -1]$$

2. (a) Here, centre = $\left(\frac{p}{2}, 0\right)$

Any circle with this centre is

$$\left(x - \frac{p}{2}\right)^2 + (y - 0)^2 = r^2$$

which touches directrix $x = -\frac{p}{2}$.

$$\therefore \frac{\left|\frac{p}{2} + \frac{p}{2}\right|}{1} = r \Rightarrow r^2 = p^2$$

$$\text{Now required circle is } \left(x - \frac{p}{2}\right)^2 + y^2 = p^2$$

$$\text{or } x^2 + y^2 - px = \frac{3p^2}{4} \quad \dots(i)$$

Also, parabola is $y^2 = 2px$.

On solving Eqs. (i) and (ii), we get

$$x^2 + 2px - px = \frac{3p^2}{4} \Rightarrow x^2 + px - \frac{3p^2}{4} = 0$$

$$\Rightarrow 4x^2 + 4px - 3p^2 = 0$$

$$\Rightarrow (2x - p)(2x + 3p) = 0$$

$$\Rightarrow x = \frac{p}{2} \text{ or } -\frac{3p}{2}$$

$$\text{When } x = \frac{p}{2}, y^2 = 2p \cdot \frac{p}{2} = p^2$$

$$\therefore y = p \text{ or } -p$$

$$\text{Now, required point is } \left(\frac{p}{2}, p\right) \text{ or } \left(\frac{p}{2}, -p\right).$$

3. (b) Given, vertex = (0, 0), point = (2, 3) and axis is along X-axis.

Since, point (2, 3) lies in first quadrant.

Hence, equation of parabola will be of the form $y^2 = 4ax$, which passes through (2, 3).

$$\text{i.e. } (3)^2 = 4a \times (2) \Rightarrow a = \frac{9}{8}$$

$\therefore$ Required equation of parabola is

$$y^2 = 4\left(\frac{9}{8}\right)x \Rightarrow y^2 = \frac{9}{2}x$$

4. (b) Given, vertex = (0, 4) and focus = (0, 2).

Let the equation of parabola be

$$(x - 0)^2 = -4a(y - 4) \quad \dots(i)$$

$\therefore a =$ Distance between focus and vertex

$$= \sqrt{0^2 + (2 - 4)^2} = 2$$

On putting $a = 2$ in Eq. (i), we get

$$x^2 = -4 \times 2(y - 4) \Rightarrow x^2 + 8y = 32$$

$$\Rightarrow x^2 + 8y - 32 = 0 \Rightarrow x^2 + 8y = 32$$

5. (a) The equation of the parabola whose axis is parallel to Y-axis, is

$$y = ax^2 + bx + c$$

which passes through the points (0, 4), (1, 9) and (-2, 6).

If $c = 4$, then $9 = a + b + c$ and $6 = 4a - 2b + c$

$$a + b = 5$$

$$\text{and } 4a - 2b = 2 \Rightarrow 2a - b = 1$$

$$\therefore 3a = 6 \Rightarrow a = 2 \text{ and } b = 3$$

Thus, required equation of the parabola is

$$y = 2x^2 + 3x + 4 = 2\left(x + \frac{3}{4}\right)^2 + \frac{23}{8}$$

6. (b) Let PQ be a double ordinate of a given parabola $y^2 = 4ax$.

Let $P(at^2, 2at)$, so that Q will be $(at^2, -2at)$.

Let A (α, β) or B (α', β') be the point of trisection of the double ordinate PQ, then

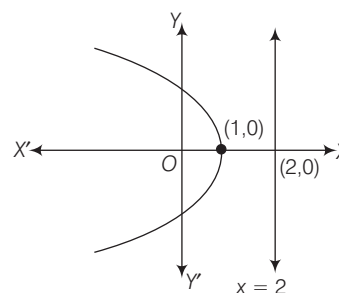
$$\alpha = at^2, \beta = \frac{2at}{3}, \alpha' = at^2, \beta' = -\frac{2at}{3}$$

On eliminating t from the above equations, we get

$$\beta^2 = \frac{4a}{9}\alpha, \beta'^2 = \frac{4a}{9}\alpha'$$

$\therefore$ Locus of (α, β) or (α', β') is given by $9y^2 = 4ax$.

7. (c) From the figure, it is clear that, vertex of the parabola is at (1, 0).



8. (a) The equation of the tangent to $y^2 = 8x$ at $(2, 4)$ is

$$4y = 4(x + 2) \text{ or } x - y + 2 = 0 \quad \dots(i)$$

Let (x_1, y_1) be the mid-point of chord QR .

Then, equation of QR is

$$yy_1 - 4(x + x_1) - 5 = y_1^2 - 8x_1 - 5$$

$$\Rightarrow 4x - yy_1 - 4x_1 + y_1^2 = 0 \quad \dots(ii)$$

Clearly, Eqs. (i) and (ii) represent the same line.

$$\text{So, } \frac{4}{1} = \frac{y_1}{1} = \frac{-4x_1 + y_1^2}{2}$$

$$\Rightarrow y_1 = 4 \text{ and } 8 = -4x_1 + y_1^2$$

$$\Rightarrow y_1 = 4 \text{ and } x_1 = 2$$

9. (c) Any tangent to the parabola $y^2 = 4ax$ is

$$y = mx + \frac{1}{m} \quad \dots(i)$$

It is a tangent to the circle $(x - 3)^2 + y^2 = 9$, if the length of the perpendicular from the centre $(3, 0)$ upon Eq. (i) is equal to radius of the circle, i.e. 3.

$$\therefore \frac{\left| 3m + \frac{1}{m} - 0 \right|}{\sqrt{1 + m^2}} = 3$$

$$\Rightarrow 9m^2 + \frac{1}{m^2} + 6 = 9(1 + m^2) \Rightarrow \frac{1}{m^2} = 3$$

$$\Rightarrow m = \pm \frac{1}{\sqrt{3}}, \text{ but } m > 0$$

$\therefore$ parabola is above X -axis

$\therefore$ Required line is

$$y = \frac{1}{\sqrt{3}}x + \sqrt{3} \Rightarrow \sqrt{3}y = x + 3$$

10. (c) The equation of tangent to $y^2 = -4x$ is

$$y = mx - \frac{1}{m} \quad \dots(i)$$

The parabola $y^2 - 4x - 2y + 5 = 0$ can be written as

$$y^2 - 2y + 1 = 4x - 4$$

$$\Rightarrow (y - 1)^2 = 4(x - 1)$$

So, the equation of tangent to $(y - 1)^2 = 4(x - 1)$ is

$$y - 1 = m(x - 1) + \frac{1}{m}$$

$$\Rightarrow y = mx - m + \frac{1}{m} + 1 \quad \dots(ii)$$

The equation of common tangent to the parabolas (i) and (ii) represent the same line.

$$\therefore -\frac{1}{m} = -m + \frac{1}{m} + 1$$

$$\Rightarrow m - \frac{2}{m} - 1 = 0 \Rightarrow m^2 - m - 2 = 0$$

$$\Rightarrow (m - 2)(m + 1) = 0 \Rightarrow m = 2, -1$$

$\therefore$ The required common tangents are

$$y = 2x - \frac{1}{2} \text{ and } y = -x + 1$$

$$\Rightarrow 4x - 2y - 1 = 0 \text{ and } x + y = 1$$

11. (a) Let $P(h, k)$ be the mid-point of a chord QR of the parabola $y^2 = 4ax$, then equation of chord QR is

$$T = S_1$$

$$\Rightarrow yk - 2a(x + h) = k^2 - 4ah$$

$$\Rightarrow yk - 2ax = k^2 - 2ah \quad \dots(i)$$

Let A be vertex of the parabola.

For combined equation of AQ and AR , use homogeneous form of $y^2 = 4ax$ with the help of Eq. (i).

$$\therefore y^2 = 4ax \left(\frac{yk - 2ax}{k^2 - 2ah} \right)$$

$$\Rightarrow y^2(k^2 - 2ah) - 4akxy + 8a^2x^2 = 0$$

$$\therefore \angle QAR = 90^\circ$$

$$\therefore \text{Coefficient of } x^2 + \text{Coefficient of } y^2 = 0$$

$$\therefore k^2 - 2ah + 8a^2 = 0$$

Hence, the locus of $P(h, k)$ is

$$y^2 - 2ax + 8a^2 = 0$$

12. (a) Equation of tangent at $A(t^2, 2t)$, $yt = x + t^2$ is tangent to $x^2 + 32y = 0$ at B .

$$\Rightarrow x^2 + 32\left(\frac{x}{t} + t\right) = 0$$

$$\Rightarrow x^2 + \frac{32}{t}x + 32t = 0$$

$$\Rightarrow \left(\frac{32}{t}\right)^2 - 4(32t) = 0$$

$$\Rightarrow 32\left(\frac{32}{t^2} - 4t\right) = 0$$

$$\Rightarrow t^3 = 8$$

$$\Rightarrow t = 2$$

$$\therefore \text{Slope of tangent is } \frac{1}{t} = \frac{1}{2}.$$

13. (a) Point P clearly lies on the directrix of $y^2 = 8x$.

Thus, slope of PA and PB are 1 and -1 respectively. Equation of $PA : y = x + 2$,

equation of $PB : y = -x - 2$ and equation of $AB : x = 2$.

Let the centre of the circle be $(h, 0)$ and radius be r .

$$\Rightarrow \frac{|h + 2|}{\sqrt{2}} = \frac{|h - 2|}{1} = r$$

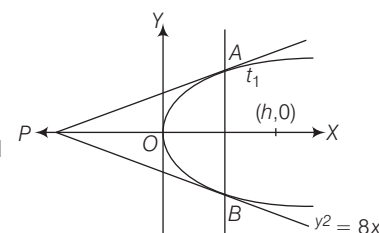
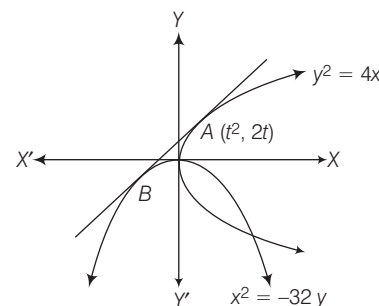
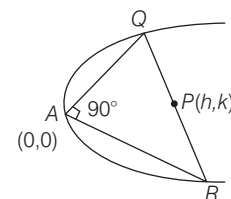
$$\Rightarrow h^2 + 4 + 4h = 2(h^2 + 4 - 4h)$$

$$\Rightarrow h^2 - 12h + 4 = 0$$

$$\Rightarrow h = \frac{12 \pm 8\sqrt{2}}{2}$$

$$\Rightarrow h = 6 \pm 4\sqrt{2}$$

$$\Rightarrow |h - 2| = 4(\sqrt{2} - 1), 4(\sqrt{2} + 1)$$



14. (a) The equation of a normal to $y^2 = 4x$ is $y = mx - 2m - m^3$.

If it passes through $\left(\frac{11}{4}, \frac{1}{4}\right)$, then $\frac{1}{4} = \frac{11m}{4} - 2m - m^3$.

$$\Rightarrow 1 = 11m - 8m - 4m^3$$

$$\Rightarrow 4m^3 - 3m + 1 = 0$$

$$\Rightarrow m = \frac{1}{2}, \frac{-1 \pm \sqrt{3}}{2}$$

Hence, three normals can be drawn from $(11/4, 1/4)$ to $y^2 = 4x$.

15. (a) Let the mid-point of a chord of the parabola $y^2 = 4x$ be (h, k)

$\therefore$ Equation of the chord is

$$\Rightarrow ky - 2(x + h) = k^2 - 4h$$

$$\Rightarrow ky - 2x = k^2 - 2h$$

$$\text{or } y = \frac{2}{k}x + \frac{k^2 - 2h}{k} \quad \dots(i)$$

Line (i) touches the parabola $x^2 = -8y$.

$$\therefore \frac{k^2 - 2h}{k} = 2\left(\frac{2}{k}\right)^2$$

$[\because \text{line } y = mx + c \text{ touches the parabola } x^2 = 4ay, \text{ if } c = -am^2]$

$$\Rightarrow k^3 - 2hk = 8$$

$\therefore$ Locus of (h, k) is $y^3 - 2xy - 8 = 0$.

16. (d) Since, the coordinates of P are $(1, 0)$.

Let any point Q on $y^2 = 8x$ be $(2t^2, 4t)$.

Again, let mid-point of PQ be (h, k) , then

$$h = \frac{2t^2 + 1}{2} \Rightarrow 2h = 2t^2 + 1 \quad \dots(i)$$

$$\text{and } k = \frac{4t + 0}{2} \Rightarrow t = \frac{k}{2} \quad \dots(ii)$$

On putting the value of t from Eq. (ii) in Eq. (i), we get

$$2h = \frac{2k^2}{4} + 1$$

$$\Rightarrow 4h = k^2 + 2$$

Hence, locus of (h, k) is $y^2 - 4x + 2 = 0$.

17. (a) Given, equations of parabolas are $y^2 = 4ax$ and $x^2 = 4ay$.

The points of intersection of parabolas are $A(0, 0)$ and $B(4a, 4a)$.

Also, given that line $2bx + 3cy + 4d = 0$ passes through the points A and B respectively.

$$\therefore d = 0 \quad \dots(i)$$

$$\text{and } 2b \cdot 4a + 3c \cdot 4a + 4d = 0$$

$$\Rightarrow 2ab + 3ac + d = 0$$

$$\Rightarrow a(2b + 3c) = 0 \quad [\because d = 0]$$

$$\Rightarrow 2b + 3c = 0 \quad \dots(ii)$$

On squaring and adding Eqs. (i) and (ii), we get

$$d^2 + (2b + 3c)^2 = 0$$

18. (b) Let A be the vertex and AP be a chord of $x^2 = 4ay$ such that slope of AP is $\tan \alpha$.

Let the coordinates of P be $(2at, at^2)$.

$$\text{Then, slope of } AP = \frac{at^2}{2at} = \frac{t}{2}$$

$$\Rightarrow \tan \alpha = \frac{t}{2} \Rightarrow t = 2 \tan \alpha$$

$$\begin{aligned} \text{Now, } AP &= \sqrt{(2at - 0)^2 + (at^2 - 0)^2} = at\sqrt{4 + t^2} \\ &= 2a \tan \alpha \sqrt{4 + 4 \tan^2 \alpha} = 4a \tan \alpha \sec \alpha \end{aligned}$$

19. (d) Given, equation of ellipse is $\frac{x^2}{16} + \frac{y^2}{25} = 1$

$$\text{Here, } a^2 = 16, b^2 = 25$$

$$\text{i.e. } b > a$$

$$\therefore PS + PS' = 2b \quad [\because \text{sum of focal radii} = 2b] \\ = 2 \times 5 = 10$$

20. (a) By definition of ellipse, $SP = e \cdot PM$, where P is any point, S is focus and M is point on directrix.

$$\Rightarrow \sqrt{(x-1)^2 + (y+1)^2} = \frac{1}{2} \cdot \frac{|x-y-3|}{\sqrt{1^2+1^2}}$$

$$\Rightarrow \sqrt{(x-1)^2 + (y+1)^2} = \frac{1}{2\sqrt{2}} |x-y-3|$$

$$\begin{aligned} \Rightarrow 8(x^2 + 1 - 2x + y^2 + 1 + 2y) \\ = x^2 + y^2 + 9 - 2xy + 6y - 6x \end{aligned}$$

$$\Rightarrow 7x^2 + 7y^2 + 2xy - 10x + 10y + 7 = 0$$

21. (a) Equation of any normal is

$$ax \sec \theta - by \operatorname{cosec} \theta = a^2 - b^2$$

If the normal intersects X -axis at $(\alpha, 0)$, then

$$\alpha = \frac{a^2 - b^2}{a \sec \theta} = \frac{a e^2}{\sec \theta}$$

$$[\because a = 1, e < 1 \text{ and } |\sec \theta| \geq 1 \Rightarrow |\alpha| < |a|]$$

$\Rightarrow (\alpha, 0)$ lies inside the ellipse.

Hence, no point lie outside the ellipse.

22. (b) Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

which passes through the points $(7, 0)$ and $(0, -5)$.

$$\text{So, } a^2 = 49 \text{ and } b^2 = 25 \quad [\because a^2 > b^2]$$

$$\therefore b^2 = a^2(1 - e^2) \Rightarrow 25 = 49(1 - e^2)$$

$$\Rightarrow e = \frac{2\sqrt{6}}{7}$$

23. (a) Equation of tangent at any point $P(a \cos \theta, b \sin \theta)$ is

$\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$. The equation of tangents at the end points of the major axis are $x = a, x = -a$.

$\therefore$ The intersection point of these tangents are

$$A = \left(a, b \tan \frac{\theta}{2}\right), B = \left(-a, b \cot \frac{\theta}{2}\right)$$

Equation of circle with AB as diameter is

$$(x-a)(x+a) + \left(y - b \tan \frac{\theta}{2}\right) \left(y - b \cot \frac{\theta}{2}\right) = 0$$

$$\Rightarrow x^2 - a^2 + y^2 - by \left(\tan \frac{\theta}{2} + \cot \frac{\theta}{2} \right) + b^2 = 0$$

$$\Rightarrow x^2 + y^2 - a^2 + b^2 - 2by \operatorname{cosec} \theta = 0$$

which is the equation of family of circles passing through the point of intersection of the circle

$$x^2 + y^2 - a^2 + b^2 = 0 \text{ and } y = 0$$

So, the fixed point is $(\pm\sqrt{a^2 - b^2}, 0)$.

24. (a) Equation of tangent $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$

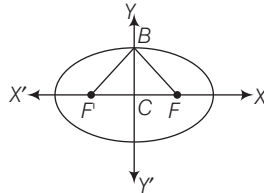
meets the axis at $A\left(\frac{a}{\cos \theta}, 0\right)$ and $B\left(0, \frac{b}{\sin \theta}\right)$.

$$\therefore \text{Area of } \triangle AOB = \frac{1}{2} OA \cdot OB = \frac{ab}{\sin 2\theta} \geq ab$$

25. (b) Let the equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$.

$$\therefore \angle FBF' = \frac{\pi}{2}$$

$$\Rightarrow \angle FBC = \frac{\pi}{4}$$



$$\therefore \angle CFB \text{ is also an angle of } \frac{\pi}{4}$$

$$\Rightarrow BC = CF \Rightarrow b = ae$$

$$\Rightarrow b^2 = a^2 e^2 \quad \dots(i)$$

$$\text{We know that, } b^2 = a^2(1 - e^2)$$

$$\therefore a^2 e^2 = a^2(1 - e^2) \quad [\text{from Eq. (i)}]$$

$$\Rightarrow e^2 = 1 - e^2$$

$$\Rightarrow 2e^2 = 1$$

$$\Rightarrow e = \frac{1}{\sqrt{2}}$$

26. (a) The straight line $x = r \cos \theta, y = r \sin \theta$ meets the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where r is given.

$$\therefore \frac{1}{r^2} = \frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2} \quad \dots(i)$$

$$\text{But } PQ^2 = 4r^2 = \text{HM of } 4a^2, 4b^2$$

$$\therefore \frac{1}{r^2} = \frac{1}{2a^2} + \frac{1}{2b^2} \quad \dots(ii)$$

From Eqs. (i) and (ii),

$$\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2} = \frac{1}{2a^2} + \frac{1}{2b^2}$$

which is possible when $\theta = \frac{\pi}{4}$.

27. (c) The equation of the line passing through $P(h, k)$ and perpendicular to line joining it to origin, is

$$y - k = \frac{-h}{k}(x - h)$$

$$\Rightarrow y = \frac{-h}{k}x + \frac{h^2 + k^2}{k}$$

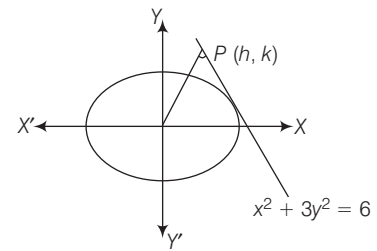
Since, it is tangent to ellipse.

$$\therefore c^2 = a^2 m^2 + b^2$$

$$\Rightarrow \left(\frac{h^2 + k^2}{k}\right)^2 = \frac{6h^2}{k^2} + 2$$

$$\Rightarrow (x^2 + y^2)^2 = 6x^2 + 2y^2$$

which is the required locus.



28. (c) Equation of tangent in terms of slope is

$$y = mx \pm \sqrt{(9m^2 + 4)}$$

$$\text{But } m = \pm 1$$

$$\therefore y = \pm x \pm \sqrt{13}$$

29. (a) Equation of the ellipse is $x^2 + 3y^2 = 3$

$$\Rightarrow \frac{x^2}{3} + \frac{y^2}{1} = 1 \Rightarrow \frac{x^2}{(\sqrt{3})^2} + \frac{y^2}{1^2} = 1$$

Equation of tangent in terms of slope is

$$y = mx \pm \sqrt{(3m^2 + 1)} \quad \dots(i)$$

$$\text{Here, } m = -4$$

[since, tangent is perpendicular to $4y = x - 5$]

From Eq. (i), we get

$$y = -4x \pm \sqrt{49} \Rightarrow y = -4x \pm 7$$

$$\Rightarrow 4x + y \mp 7 = 0$$

30. (b) Given equation of line is

$$y = -x \cot \alpha + p \operatorname{cosec} \alpha$$

$$\text{Here, } m = -\cot \alpha$$

The given line touches the ellipse, if

$$p \operatorname{cosec} \alpha = \pm \sqrt{a^2 m^2 + b^2} = \pm \sqrt{a^2 \cot^2 \alpha + b^2}$$

$$\Rightarrow p^2 \operatorname{cosec}^2 \alpha = a^2 \cot^2 \alpha + b^2$$

$$\Rightarrow p^2 = a^2 \frac{\cot^2 \alpha}{\operatorname{cosec}^2 \alpha} + b^2 \sin^2 \alpha$$

$$\Rightarrow p^2 = a^2 \cos^2 \alpha + b^2 \sin^2 \alpha$$

31. (d) The tangent to $\frac{x^2}{25} + \frac{y^2}{4} = 1$ is $\frac{x}{5} \cos \theta + \frac{y}{2} \sin \theta = 1$. If it is also tangent to the circle, then

$$16 = \frac{1}{\frac{\cos^2 \theta}{25} + \frac{\sin^2 \theta}{4}} = \frac{100}{4 + 21 \sin^2 \theta}$$

$$\Rightarrow \sin^2 \theta = \frac{3}{28}, \cos^2 \theta = \frac{25}{28}$$

If the tangents meet the axes at A and B, then

$$A = \left(\frac{5}{\cos \theta}, 0\right) \text{ and } B = \left(0, \frac{2}{\sin \theta}\right)$$

$$\therefore AB^2 = \frac{25}{\cos^2 \theta} + \frac{4}{\sin^2 \theta}$$

$$= 28 + \frac{4}{3} \cdot 28 = \frac{196}{3}$$

$$\Rightarrow AB = \frac{14}{\sqrt{3}}$$

32. (d) Given equation of ellipse is $\frac{x^2}{2} + \frac{y^2}{1} = 1$.

General equation of tangent to the ellipse of slope m is $y = mx \pm \sqrt{2m^2 + 1}$.

Since, this is equally inclined to axes, so $m = \pm 1$.

Then, tangents are $y = \pm x \pm \sqrt{2+1} = \pm x \pm \sqrt{3}$

$\therefore$ Distance of any tangent from origin = $\frac{|0 + 0 \pm \sqrt{3}|}{\sqrt{1^2 + 1^2}} = \frac{\sqrt{3}}{2}$

33. (a) Let the equation of the required ellipse be $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$.

But the ellipse passes through the point (2, 1).

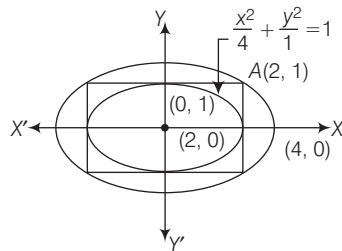
$$\Rightarrow \frac{1}{4} + \frac{1}{b^2} = 1$$

$$\Rightarrow \frac{1}{b^2} = \frac{3}{4} \Rightarrow b^2 = \frac{4}{3}$$

Hence, the equation is

$$\frac{x^2}{16} + \frac{3y^2}{4} = 1$$

$$\Rightarrow x^2 + 12y^2 = 16$$



34. (c) The equations of the chords of contact of tangents drawn from (x_1, y_1) and (x_2, y_2) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, are

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1 \quad \dots(i)$$

and

$$\frac{xx_2}{a^2} + \frac{yy_2}{b^2} = 1 \quad \dots(ii)$$

It is given that Eqs. (i) and (ii) are at right angles.

$$\therefore \frac{-b^2}{a^2} \cdot \frac{x_1}{y_1} \times \frac{-b^2}{a^2} \cdot \frac{x_2}{y_2} = -1 \Rightarrow \frac{x_1 x_2}{y_1 y_2} = -\frac{a^4}{b^4}$$

35. (a) Equation of normal with slope m to the given ellipse is

$$y = mx \pm \frac{m(5-b^2)}{\sqrt{b^2 m^2 + 5}}$$

$$\Rightarrow y = x \pm \frac{(5-b^2)}{\sqrt{b^2 + 5}} \quad [\text{as } m = 1] \dots(i)$$

Also, $y = x + \frac{1}{3}$ is the normal to ellipse. [given] ... (ii)

Eqs. (i) and (ii) both represent same line.

$$\therefore \pm \frac{(5-b^2)}{\sqrt{b^2 + 5}} = \frac{1}{3} \Rightarrow (5-b^2)^2 = \frac{1}{9} (5+b^2)$$

$$\Rightarrow 9b^4 - 91b^2 + 220 = 0 \Rightarrow b^2 = 4 \text{ or } \frac{55}{9}$$

So, length of minor axis is either 4 or $\frac{2}{3}\sqrt{55}$.

36. (a) Let $y = m_1 x$ and $y = m_2 x$ be a pair of conjugate diameters of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and let $P(a \cos \theta, b \sin \theta)$ and $Q(a \cos \phi, b \sin \phi)$ be ends of these two diameters. Then,

$$m_1 m_2 = -\frac{b^2}{a^2}$$

$$\Rightarrow \frac{b \sin \theta - 0}{a \cos \theta - 0} \times \frac{b \sin \phi - 0}{a \cos \phi - 0} = -\frac{b^2}{a^2}$$

$$\Rightarrow \sin \theta \sin \phi = -\cos \theta \cos \phi$$

$$\Rightarrow \cos(\theta - \phi) = 0 \Rightarrow \theta - \phi = \pm \frac{\pi}{2}$$

37. (a) Given chords of contact are

$$\frac{xx_1}{25} + \frac{yy_1}{36} = 1 \text{ and } \frac{xx_2}{25} + \frac{yy_2}{36} = 1$$

$\therefore$ Product of the slopes = -1

$$\therefore \left(-\frac{x_1/25}{y_1/36} \right) \left(-\frac{x_2/25}{y_2/36} \right) = -1$$

$$\Rightarrow \frac{x_1 x_2}{625} \cdot \frac{(36)^2}{y_1 y_2} = -1$$

$$\Rightarrow \frac{x_1 x_2}{y_1 y_2} = -\frac{(5)^4}{(6)^4} = -\left(\frac{5}{6}\right)^4$$

38. (d) $16(x^2 - 2x) - 3(y^2 - 4y) = 44$

$$\Rightarrow 16(x-1)^2 - 3(y-2)^2 = 48$$

$$\Rightarrow \frac{(x-1)^2}{3} - \frac{(y-2)^2}{16} = 1$$

which represents a hyperbola with eccentricity given by

$$e = \sqrt{1 + \left(\frac{\text{Conjugate axis}}{\text{Transverse axis}} \right)^2} = \sqrt{1 + \left(\frac{4}{\sqrt{3}} \right)^2} = \sqrt{\frac{19}{3}}$$

39. (d) Normals at θ, ϕ where $\phi = \frac{\pi}{2} - \theta$ pass through (h, k) .

$$\therefore ah \cos \theta + bk \cot \theta = a^2 + b^2 \quad \dots(i)$$

$$\text{and } ah \sin \theta + bk \tan \theta = a^2 + b^2 \quad \dots(ii)$$

Eliminating h from Eqs. (i) and (ii), we get $bk(\cot \theta \sin \theta - \tan \theta \cos \theta)$

$$= (a^2 + b^2)(\sin \theta - \cos \theta) \text{ or } k = -\frac{(a^2 + b^2)}{b}$$

40. (c) For the given ellipse, $a^2 = 16$.

$$\therefore e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$\Rightarrow e = \sqrt{1 - \frac{b^2}{16}} = \sqrt{\frac{16 - b^2}{16}}$$

So, the foci of the ellipse are $(\pm ae, 0)$, i.e. $(\pm \sqrt{16 - b^2}, 0)$.

For the hyperbola, $a^2 = \left(\frac{12}{5}\right)^2$, $b^2 = \left(\frac{9}{5}\right)^2$

The eccentricity e is given by

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{81}{144}} = \frac{15}{12} = \frac{5}{4}$$

Thus, the foci of the hyperbola are $(\pm ae, 0)$ or $(\pm 3, 0)$.

Since, the foci of the ellipse and hyperbola coincide.

$$\text{Therefore, } \sqrt{16 - b^2} = 3 \Rightarrow b^2 = 7$$

41. (b) Let $CP = r_1$ be inclined to transverse axis at an angle θ , so that P is $(r_1 \cos \theta, r_1 \sin \theta)$ and P lies on the hyperbola.

$$\Rightarrow r_1^2 \left(\frac{\cos^2 \theta}{a^2} - \frac{\sin^2 \theta}{b^2} \right) = 1$$

$$\Rightarrow \frac{1}{r_1^2} + \frac{1}{r_2^2} = \frac{\cos^2 \theta}{a^2} - \frac{\sin^2 \theta}{b^2} + \frac{\sin^2 \theta}{a^2} - \frac{\cos^2 \theta}{b^2}$$

$$\Rightarrow \frac{1}{r_1^2} + \frac{1}{r_2^2} = \frac{1}{a^2} - \frac{1}{b^2}$$

$$\Rightarrow \frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{1}{a^2} - \frac{1}{b^2}$$

42. (a) Here, $a = \sin \theta$

Since, foci of the ellipse are $(\pm 1, 0)$.

$$\therefore \pm 1 = \sqrt{a^2 + b^2} \Rightarrow b^2 = \cos^2 \theta$$

$$\text{Then, equation is } \frac{x^2}{\sin^2 \theta} - \frac{y^2}{\cos^2 \theta} = 1$$

$$\Rightarrow x^2 \operatorname{cosec}^2 \theta - y^2 \sec^2 \theta = 1$$

43. (b) Here, $e_1 = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

$$\therefore e_1 e_2 = 1 \Rightarrow e_2 = \frac{5}{3}$$

Since, foci of ellipse are $(0, \pm 3)$.

$$\text{Hence, equation of hyperbola is } \frac{x^2}{16} - \frac{y^2}{9} = -1.$$

44. (c) Any tangent to the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ of slope m is

$$y = mx \pm \sqrt{25m^2 - 16} \Rightarrow (4 - m)^2 = 25m^2 - 16$$

$$\Rightarrow 3m^2 + m - 4 = 0 \Rightarrow m_1 = 1, m_2 = -\frac{4}{3}$$

$$\Rightarrow |m_1| = 1, |m_2| = \frac{4}{3}$$

$$\therefore \text{ Tangents are } y = x + 3 \quad \dots(i)$$

$$\text{and } y = \frac{4}{3}x + \frac{16}{3} \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get $(-7, -4)$

45. (c) Given equation of tangent is $x \sec \theta - y \tan \theta = a$ and lines are $x - y = 0, x + y = 0$.

So, vertices of triangle are $(0, 0)$,

$$\left(\frac{a}{\sec \theta - \tan \theta}, \frac{a}{\sec \theta - \tan \theta} \right)$$

$$\text{and } \left(\frac{a}{\sec \theta + \tan \theta}, \frac{-a}{\sec \theta + \tan \theta} \right).$$

$$\text{Using } \Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = a^2$$

46. (d) As we know, equation of tangent at (x_1, y_1) is $xx_1 - 2yy_1 = 4$, which is same as $2x + \sqrt{6}y = 2$.

$$\therefore \frac{x_1}{2} = -\frac{2y_1}{\sqrt{6}} = \frac{4}{2} \Rightarrow x_1 = 4 \text{ and } y_1 = -\sqrt{6}$$

47. (a) Suppose the common tangent is $y = mx + c$ to

$$9x^2 - 4y^2 = 36 \text{ and } x^2 + y^2 = 3$$

$$\therefore c^2 = a^2 m^2 - b^2 = 4m^2 - 9 \quad \dots(i)$$

$$\text{and } c^2 = 3 + 3m^2 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$4m^2 - 9 = 3m^2 + 3 \Rightarrow m^2 = 12 \Rightarrow m = 2\sqrt{3}$$

$$\therefore c = \sqrt{3 + 3 \times 12} = \sqrt{39}$$

Hence, the common tangent is

$$y = 2\sqrt{3}x + \sqrt{39}$$

48. (d) The equation of tangent in slope form to the hyperbola

$$\frac{x^2}{16} - \frac{y^2}{9} = 1 \text{ is } y = mx + \sqrt{16m^2 - 9}.$$

Since, it passes through (h, k) .

$$\therefore k = mh + \sqrt{16m^2 - 9} \Rightarrow (k - mh)^2 = (16m^2 - 9^2)$$

$$\Rightarrow k^2 + m^2 h^2 - 2mkh - 16m^2 + 9^2 = 0$$

which is quadratic in m .

Let the slope of two tangents be m_1 and m_2 , then

$$m_1 m_2 = \frac{k^2 + 9}{h^2 - 16} \Rightarrow -1 = \frac{k^2 + 9}{h^2 - 16} \Rightarrow h^2 + k^2 = 7$$

So, the required locus is $x^2 + y^2 = 7$.

49. (d) Hint Here, $a^2 = 4, b^2 = 2 \Rightarrow a = 2, b = \sqrt{2}$

The equation of tangent is

$$\frac{x}{a} \sec \theta - \frac{y}{b} \tan \theta = 1$$

$$\Rightarrow \frac{x}{a} \sec \theta - \frac{y}{\sqrt{2}} \tan \theta = 1$$

So, the coordinates of P and Q are $P(2 \cos \theta, 0)$ and $Q(0, -\sqrt{2} \cot \theta)$, respectively.

Let coordinates of R be (h, k) .

$$\therefore h = 2 \cos \theta, k = -\sqrt{2} \cot \theta \Rightarrow \frac{k}{h} = \frac{-\sqrt{2}}{\sin \theta}$$

$$\Rightarrow \sin \theta = \frac{-\sqrt{2}h}{2k} \Rightarrow \sin^2 \theta = \frac{2h^2}{4k^2}$$

50. (a) The point on the hyperbola is $P(x_1, y_1)$, then N is $(x, 0)$.

$$\therefore \text{ Tangent at } (x_1, y_1) \text{ is } \frac{xx_1}{9} - \frac{yy_1}{4} = 1$$

$$\text{This meets X-axis at } T \left(\frac{9}{x_1}, 0 \right).$$

$$\therefore OT \cdot ON = \frac{9}{x_1} \cdot x_1 = 9$$

51. (c) Any point on the hyperbola is $(3 \sec \theta, 2 \tan \theta)$. The chord of contact to the circle is

$$3x \sec \theta + 2y \tan \theta = 9 \quad \dots(i)$$

If (x_1, y_1) is the mid-point of the chord, then its equation is

$$xx_1 + yy_1 = x_1^2 + y_1^2 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{3 \sec \theta}{x_1} = \frac{2 \tan \theta}{y_1} = \frac{9}{x_1^2 + y_1^2}$$

On eliminating θ , we get $\frac{1}{81}(x_1^2 + y_1^2)^2 = \frac{x_1^2}{9} - \frac{y_1^2}{4}$

Hence, the locus is $(x^2 + y^2)^2 = 81\left(\frac{x^2}{9} - \frac{y^2}{4}\right)$.

52. (b) Given equation of the hyperbola is $3x^2 - y^2 = 3$

$$\Rightarrow \frac{x^2}{1} - \frac{y^2}{3} = 1$$

Equation of tangent in terms of slope is

$$y = mx \pm \sqrt{(m^2 - 3)}$$

Given, $m = 2$

$$\therefore y = 2x \pm 1$$

53. (b) If (h, k) is the mid-point of the chord, then its equation by $T = S_1$ is

$$hx - ky = h^2 - k^2 \Rightarrow y = \frac{h}{k}x + \frac{k^2 - h^2}{k}$$

If it touches the parabola $y^2 = 4ax$, then

$$\frac{k^2 - h^2}{k} = \frac{a \cdot k}{h} \Rightarrow ak^2 = hk^2 - h^3 \Rightarrow ay^2 = xy^2 - x^3$$

So, required locus is $y^2(x - a) = x^3$.

54. (b) Given equation can be rewritten as

$$\frac{x^2}{2} - \frac{y^2}{1} = 1$$

Here, $a^2 = 2, b^2 = 1$

We know that, the length of perpendicular drawn from any

point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to the asymptotes is

$$\frac{a^2 b^2}{a^2 + b^2} = \frac{2(1)}{2+1} = \frac{2}{3}$$

55. (b) Let $P(x_1, y_1)$ be the point from which tangents are drawn.

The chord of contact is $S_1 = -8$.

$$\text{i.e. } xx_1 - yy_1 = 9$$

It is same as $x = 9$.

$$\therefore x_1 = 9$$

$$y_1 = 0$$

Equation of tangent is $T_1 = SS_1$.

$$\therefore (x - 9)^2 = -8(x^2 - y^2 - 9)$$

$$\Rightarrow 9x^2 - 8y^2 - 18x + 9 = 0$$

BITSAT Archives

1. (c) The given equation of second ellipse can be rewritten as

$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$

Equation of tangent to this ellipse is

$$\frac{x}{2} \cos \theta + y \sin \theta = 1 \quad \dots(i)$$

Equation of the first ellipse can be rewritten as

$$\frac{x^2}{6} + \frac{y^2}{3} = 1 \quad \dots(ii)$$

Let ellipse (i) meets the ellipse at P and Q and the tangents at P and Q to the second ellipse intersected at (h, k) , then Eq. (i) is the chord of contact of (h, k) with respect to the ellipse (ii) and thus its equation is

$$\frac{hx}{6} + \frac{ky}{3} = 1 \quad \dots(iii)$$

Since, Eqs. (i) and (iii) represent the same line.

$$\therefore \frac{h/6}{\cos \frac{\theta}{2}} = \frac{k/3}{\sin \theta} = 1$$

$$\Rightarrow h = 3 \cos \theta \text{ and } k = 3 \sin \theta$$

Hence, locus is $x^2 + y^2 = 9$.

2. (a) Let P be $(at_1^2, 2at_1)$ and Q be $(at_2^2, 2at_2)$.

Since, PQ subtends a right angle at the vertex $(0, 0)$.

$$\text{Hence, } t_1 t_2 = -4 \quad \dots(i)$$

If (h, k) is the point of intersection of normals at P and Q , then

$$h = 2a + a(t_1^2 + t_2^2 + t_1 t_2) \quad \dots(ii)$$

$$\text{and } k = -at_1 t_2(t_1 + t_2) \quad \dots(iii)$$

In order to find the locus of (h, k) , we have to eliminate t_1 and t_2 between Eqs. (i), (ii) and (iii).

$$\therefore k = 4a(t_1 + t_2) \quad \dots(iv)$$

[from Eqs. (i) and (iii)]

$$\text{and } h - 2a = a[(t_1 + t_2)^2 - t_1 t_2]$$

$$\Rightarrow h - 2a = a\left[\frac{k^2}{16a^2} + 4\right] \quad \text{[from Eq. (iv)]}$$

$$\Rightarrow h - 6a = \frac{k^2}{16a}$$

Hence, the required locus is $y^2 = 16a(x - 6a)$, which is a parabola.

$$3. (c) \text{ Hint } \frac{(x-3)^2}{16} + \frac{y^2}{25} = 1$$

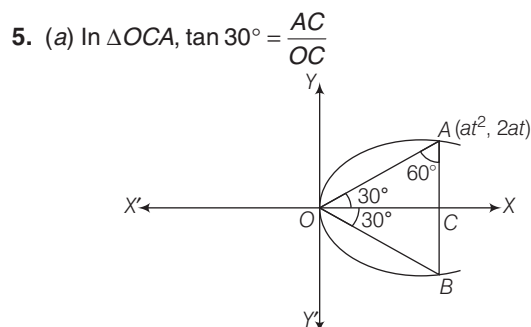
$$4. (a) \text{ Given, } \frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$\begin{aligned} \therefore e &= \sqrt{1 - \frac{9}{16}} \\ &= \sqrt{\frac{16-9}{16}} \\ &= \sqrt{\frac{7}{16}} \\ &= \frac{\sqrt{7}}{4} \end{aligned}$$

$\therefore$ Coordinates of foci are $(\pm \sqrt{7}, 0)$.

Since, centre of circle is $(0, 3)$ and passing through foci $(\pm \sqrt{7}, 0)$.

$$\begin{aligned} \therefore \text{Radius of the circle} &= \sqrt{(0 \pm \sqrt{7})^2 + (3 - 0)^2} \\ &= \sqrt{7 + 9} = 4 \text{ units} \end{aligned}$$



$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{2at}{at^2} \Rightarrow t = 2\sqrt{3}$$

Again in $\triangle OCA$,

$$\begin{aligned} OA &= \sqrt{OC^2 + AC^2} = \sqrt{(at^2)^2 + (2at)^2} \\ &= \sqrt{[(2\sqrt{3})^2]^2 a^2 + 4a^2 (2\sqrt{3})^2} \\ &= \sqrt{192a^2} = 8a\sqrt{3} \end{aligned}$$

6. (d) Length of minor axis = $2b$ and according to the given condition, length of major axis = $3(2b) = 6b$

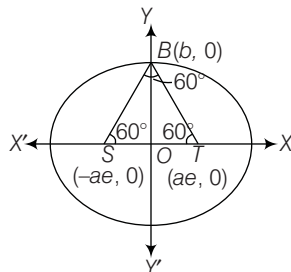
$$\therefore e = \sqrt{1 - \frac{b^2}{(3b)^2}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

7. (c) **Hint** In $\triangle BOT$,

$$\frac{b}{ae} = \tan 60^\circ$$

$$\Rightarrow b = ae\sqrt{3}$$

$$\therefore e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{a^2 e^2 3}{a^2}}$$



8. (c) The difference of the focal distance at any point on the hyperbola is same as length of the transverse axis, i.e. $2a$.

9. (a) **Hint** Let $P(x, y)$ be any point on the parabola.

$$\text{Then, } \sqrt{(x-0)^2 + (y+3)^2} = (y-3)$$

10. (c) Length of tangent = $\sqrt{S_1}$
- $$= \sqrt{5^2 + 1^2 + 30 - 4 - 3} = \sqrt{49} = 7 \text{ units}$$

11. (c) Given equation can be rewritten as

$$(x-1)^2 + (y-3)^2 = \left(\frac{5x-12y+17}{13}\right)^2$$

$$\Rightarrow SP = PM$$

Here, focus is $(1, 3)$, then directrix is

$$5x - 12y + 17 = 0$$

$\therefore$ The distance of the focus from the directrix

$$= \frac{|5 - 36 + 17|}{\sqrt{25 + 144}} = \frac{14}{13} = 2a$$

$$\therefore \text{Length of latusrectum} = 2 \times \frac{14}{13} = \frac{28}{13} \text{ units}$$

12. (a) **Hint** Given, centre = $(0, 0)$, focus = $(0, 3)$, $b = 5$

$$\Rightarrow be = 3 \Rightarrow e = \frac{3}{5}$$

$$\therefore a = b\sqrt{1 - e^2}$$

13. (c) The equation of director circle to the hyperbola is $x^2 + y^2 = a^2 - b^2$

$$\therefore \text{Radius} = \sqrt{a^2 - b^2}$$

14. (d) **Hint** The equations of the required chord is given by

$$S = T$$

15. (b) Given equation of the curve is

$$3x^2 - y^2 - 2x + 4y = 0 \quad \dots (i)$$

Let the equation of one of the chord be

$$y = mx + c \Rightarrow \frac{y - mx}{c} = 1 \quad \dots (ii)$$

On making Eq. (i) homogeneous, we get

$$3x^2 - y^2 + (-2x + 4y)\left(\frac{y - mx}{c}\right) = 0$$

$$\Rightarrow x^2(3c + 2m) + y^2(-c + 4) - 2xy - 4mxy = 0$$

which represents a pair of straight lines passing through origin.

Since, the angle subtended is a right angle.

$$\therefore 3c + 2m - c + 4 = 0$$

$$\Rightarrow c = -m - 2$$

Substituting the value of c in $y = mx + c$, we get

$$y = mx - m - 2$$

$$\Rightarrow y + 2 = m(x - 1)$$

$\therefore$ All such chords pass through a fixed point $(1, -2)$.

16. (a) Putting $y = 2t^2$ in the equation of the given ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, we get

$$\frac{x^2}{9} + \frac{4t^4}{4} = 1$$

$$\Rightarrow x^2 = 9(1 - t^4)$$

$$= 9(1 - t^2)(1 + t^2)$$

This will give real values of x , if $1 - t^2 \geq 0$

$$\text{i.e. } |t| \leq 1$$

17. (c) Given, equation of hyperbola is

$$x^2 - 3y^2 - 4x - 6y - 11 = 0 \text{ units}$$

$$\Rightarrow (x^2 - 4x + 4) - 3(y^2 + 2y + 1) - 11 = 4 - 3$$

$$\Rightarrow (x - 2)^2 - 3(y + 1)^2 = 12$$

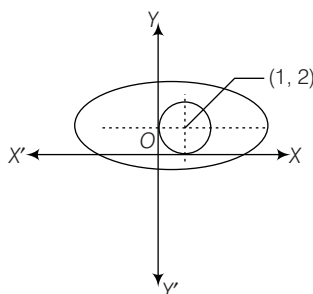
$$\Rightarrow \frac{(x - 2)^2}{12} - \frac{(y + 1)^2}{4} = 1$$

$$\text{Now, } e = \sqrt{1 + \frac{4}{12}} = \frac{2}{\sqrt{3}}$$

$\therefore$ Distance between foci

$$= 2ae = 2 \times \sqrt{12} \times \frac{2}{\sqrt{3}} = 8 \text{ units}$$

18. (a) It is clear from the figure that, the two curves do not intersect each other.



19. (b) **Hint** Here, $a^2 = \cos^2 \alpha$ and $b^2 = \sin^2 \alpha$

$$\therefore \sin^2 \alpha + \cos^2 \alpha = a^2 + b^2 \Rightarrow 1 = a^2 + b^2$$

$$\therefore e = \sqrt{\frac{a^2 + b^2}{a^2}}$$

20. (c) Given vertex of parabola $(h, k) \equiv (1, 1)$ and its focus $(a + h, k) \equiv (3, 1)$ or $a + h = 3 \Rightarrow a = 2$

Since, y -coordinate of vertex and focus are same, therefore axis of parabola is parallel to X -axis.

Thus, equation of parabola is

$$(y - k)^2 = 4a(x - h) \Rightarrow (y - 1)^2 = 8(x - 1)$$

21. (c) Given,

$$x^2 + 10x - 16y + 25 = 0 \Rightarrow (x + 5)^2 = 16y$$

$$\Rightarrow X^2 = 4AY$$

$$\text{where, } X = x + 5, A = 4, Y = y$$

The ends of the latusrectum are

$$(2A, A) \text{ and } (-2A, A)$$

$$\Rightarrow x + 5 = 2(4) \Rightarrow x = -8 - 5 = -13, y = 4$$

$$\text{and } x + 5 = -2(4) \Rightarrow x = -8 - 5 = -13, y = 4$$

$$\Rightarrow (3, 4) \text{ and } (-13, 4)$$

22. (b) **Hint** Given, $e = 2, 2ae = 8$

$$\Rightarrow ae = 4 \Rightarrow a = 2$$

$$\therefore b^2 = a^2(e^2 - 1)$$

23. (a) We know that, if P is any point on the curve, then Sum of focal distances = Length of major axis

$$\text{i.e. } SP + S'P = 2a = 2(5) \quad [\because a^2 = 5^2] \\ = 10$$

24. (c) Given equation of line is

$$x + y = 0 \quad \dots (i)$$

and equation of circle is

$$x^2 + y^2 + 4y = 0 \quad \dots (ii)$$

On solving Eqs. (i) and (ii), we get

$$x^2 + (-x)^2 + 4(-x) = 0 \Rightarrow 2x^2 - 4x = 0$$

$$\Rightarrow 2x(x - 2) = 0 \Rightarrow x = 0, 2$$

$$\text{and } y = 0, -2$$

Now, taking option (c),

$$y^2 = 2x$$

$$\text{At point } (0, 0) \Rightarrow 0 = 0$$

$$\text{and at point } (2, -2)$$

$$\Rightarrow (-2)^2 = 2(2) \Rightarrow 4 = 4$$

25. (c) Given equation of ellipse is $4x^2 + 5y^2 = 1$

$$\text{or } OS \equiv 4x^2 + 5y^2 - 1 = 0 \quad \dots (i)$$

At point $(4, -3)$,

$$S \equiv 4(4)^2 + 5(-3)^2 - 1 \\ \equiv 108 > 0$$

Therefore, the given point lies outside the ellipse.

26. (a) Equation of director circle of given hyperbola is

$$\frac{x^2}{25} - \frac{y^2}{16} = 1 \text{ is } x^2 + y^2 = 25 - 16$$

$$\Rightarrow x^2 + y^2 = 9 \quad \dots (i)$$

This circle passes through $(2\sqrt{2}, 1)$ and we know that director circle is the locus of point of intersection of perpendicular tangents drawn to hyperbola.

Thus, the angle between the tangents is $\pi/2$.

27. (c) Equation of parabola is

$$y^2 = -4x$$

$$\therefore \text{Focus is } (-1, 0).$$

The equation of line passing through $(-1, 0)$ is

$$y - 0 = m(x + 1) \quad \dots (i)$$

Since, the line makes an angle $\theta = 20^\circ$.

$$\therefore m = \tan \theta = \tan 120^\circ$$

$$\Rightarrow m = -\sqrt{3}$$

On putting the value of m in Eq. (i), we get

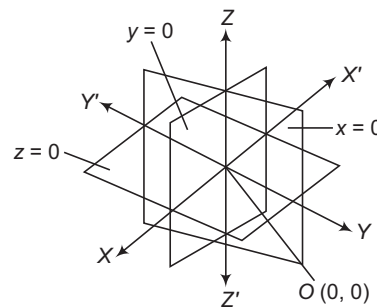
$$y = -\sqrt{3}(x + 1)$$

15

Three Dimensional Geometry

Coordinates Axes and Coordinate Planes

The three mutually perpendicular lines in a space which divides the space into eight parts are called **coordinate axes**. They determine three mutually perpendicular planes called **coordinate planes**. The coordinates of a point are the distances from the origin to the foot of the perpendiculars from the point on the respective coordinate axes. The coordinates of any point on X , Y and Z -axes will be as $(x,0,0)$ $(0,y,0)$ and $(0,0,z)$ respectively and the coordinates of any point will be as (x,y,z) .



Distance between Two Points

The distance between two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Section Formulae

There are two types of section formulae which are defined as follows :

1. For Internal Division

Formulae for internal division in cartesian and vector forms are given below:

Cartesian Form

Let two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in space and $R(x, y, z)$ be a point on the line joining points P and Q such that it divides the join of P and Q internally in the ratio $m : n$, then the coordinates of R are

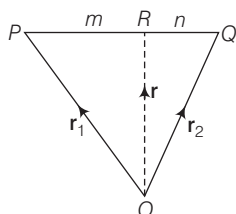
$$\left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right]$$

NOTE The coordinates of the mid-point of the line joining of $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$.

Vector Form

Let P and Q be two points with position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively and $OR = \mathbf{r}$. Then,

$$\mathbf{r} = \frac{m\mathbf{r}_2 + n\mathbf{r}_1}{m+n}$$



NOTE The position vector ($\mathbf{r}$) of the mid-point of the joining of P and Q , with position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, is $\mathbf{r} = \frac{\mathbf{r}_1 + \mathbf{r}_2}{2}$.

2. For External Division

Formulae for external division in cartesian and vector forms are given below:

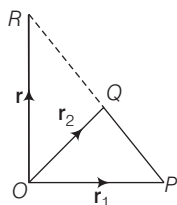
Cartesian Form

Let two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in space and R be a point on PQ produced dividing it externally in the ratio $m:n$ ($m \neq n$). Then, the coordinates of R are

$$\left[\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n} \right]$$

Vector Form

Let R be a point dividing PQ externally in the ratio $m:n$ and $OR = \mathbf{r}$, then



$$\mathbf{r} = \frac{m\mathbf{r}_2 - n\mathbf{r}_1}{m-n}$$

Some Important Facts

If $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ are the vertices of ΔABC , then

(i) Centroid of triangle = $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$

(ii) Area of $\Delta ABC = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix}$

(iii) If area of $\Delta ABC = 0$, then these points are collinear.

Direction Cosines and Ratios

If a vector makes angles α, β and γ with the positive directions of X, Y and Z -axes respectively, then $\cos \alpha, \cos \beta$ and $\cos \gamma$ are called the direction cosines and it is denoted by l, m, n i.e. $l = \cos \alpha, m = \cos \beta$ and $n = \cos \gamma$. If numbers a, b and c are proportional to l, m and n respectively, then a, b and c are called direction ratios.

Thus, a, b and c are the direction ratios of a vector, provided

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$$

where, $l = \pm \frac{a}{\sqrt{a^2 + b^2 + c^2}}, m = \pm \frac{b}{\sqrt{a^2 + b^2 + c^2}}$

and

$$n = \pm \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

Some Important Facts

(i) If a vector $\mathbf{r} = a\hat{i} + b\hat{j} + c\hat{k}$ having direction cosines l, m and n , then

$$l = \frac{a}{|\mathbf{r}|}, m = \frac{b}{|\mathbf{r}|}$$

and

$$n = \frac{c}{|\mathbf{r}|}$$

where, a, b and c are direction ratios of $\mathbf{r}$.

(ii) Direction ratios of the line joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $x_2 - x_1, y_2 - y_1, z_2 - z_1$ and its direction cosines are

$$\frac{x_2 - x_1}{|PQ|}, \frac{y_2 - y_1}{|PQ|}, \frac{z_2 - z_1}{|PQ|}$$

(iii) If $P(x, y, z)$ is a point in space, then

(a) $x = l|\mathbf{r}|, y = m|\mathbf{r}|, z = n|\mathbf{r}|$

(b) $l|\mathbf{r}|, m|\mathbf{r}|$ and $n|\mathbf{r}|$ are projections of $\mathbf{r}$ on OX, OY and OZ , respectively.

(c) $\mathbf{r} = |\mathbf{r}|(\hat{i}l + \hat{j}m + \hat{k}n)$

(iv) The projection of the line segment joining points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ to the line having direction cosines l, m, n , is

$$|(x_2 - x_1)l + (y_2 - y_1)m + (z_2 - z_1)n|$$

(v) The sum of squares of direction cosines is always unity i.e. $l^2 + m^2 + n^2 = 1$.

(vi) DC's of X, Y and Z -axes are $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$.

(vii) Direction cosines of a line are unique but direction ratio of a line are not unique and can be infinite.

(viii) DC's of a line which is equally inclined to the coordinate axes, are $\left(\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}} \right)$.

Straight Line in Space

A straight line is the locus of the intersection of any two planes.

Some Important Facts

- (i) **Equation of a line passing through a given point and having given direction cosines** The equation of line which passes through (x_1, y_1, z_1) and having direction cosines l, m, n , is given by

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n} = r \quad [\text{say}]$$

If we use direction ratios instead of direction cosines, then the equation of the line is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = \lambda$$

where, $a^2 + b^2 + c^2 \neq 1$

This is also called the symmetrical form of a line.

- (ii) **Vector equation of a line passing through a point and parallel to given vector** The equation of a line passing through a point with position vector $\mathbf{a}$ and parallel to vector $\mathbf{b}$, is $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$.
- (iii) **Equation of line passing through two given points**

- (a) **Cartesian Form** Direction ratios of AB

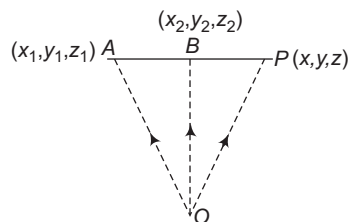
$$= (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

Direction ratios of AP

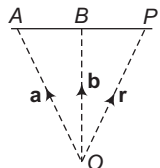
$$= (x - x_1, y - y_1, z - z_1)$$

Since, they are proportional.

$$\therefore \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$



- (b) **Vector Form** $\mathbf{r} = \mathbf{a} + \lambda (\mathbf{b} - \mathbf{a})$



- (iv) **Changing unsymmetrical form to symmetrical form**

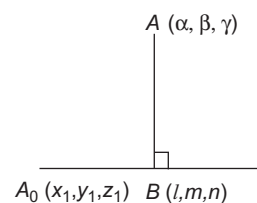
The unsymmetrical form of a line

$$ax + by + cz + d = 0 \quad \text{and} \quad a'x + b'y + c'z + d' = 0$$

can be changed to symmetrical form as

$$\frac{x - \frac{bd' - b'd}{ab' - a'b}}{bc' - b'c} = \frac{y - \frac{da' - d'a}{ab' - a'b}}{ca' - c'a} = \frac{z}{ab' - a'b}$$

- (v) **Foot of perpendicular from a point $A(\alpha, \beta, \gamma)$ to the line $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$, length and equation of perpendicular** If B is the foot of perpendicular, then B is $(lk + x_1, mk + y_1, nk + z_1)$. Find the direction ratios of AB and apply the condition of perpendicularity of AB and the given line. This will give the value of k and hence the point B which is foot of perpendicular. Its length is the distance AB and its equation is the line joining two known points A and B .



Angle between Two Intersecting Lines

The angle between two intersecting lines with cartesian equations and vector equations can be found as given below:

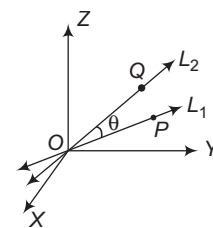
Cartesian Form

If θ is the angle between two lines whose direction cosines are (l_1, m_1, n_1) , (l_2, m_2, n_2) and (a_1, b_1, c_1) , (a_2, b_2, c_2) are direction ratios, then

$$\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2 = \pm \frac{\Sigma a_1 a_2}{[\Sigma a_1^2 \cdot \Sigma a_2^2]^{1/2}}$$

$$\sin^2 \theta = \left| \frac{m_1}{m_2} \frac{n_1}{n_2} \right|^2 + \left| \frac{n_1}{n_2} \frac{l_1}{l_2} \right|^2 + \left| \frac{l_1}{l_2} \frac{m_1}{m_2} \right|^2$$

$$\tan \theta = \pm \frac{[\Sigma (l_1 m_2 - l_2 m_1)^2]^{1/2}}{l_1 l_2 + m_1 m_2 + n_1 n_2}$$



The lines are parallel to each other if and only if

$$(i) \frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$$

$$(ii) \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

The lines are perpendicular to each other if and only if

$$(i) l_1 l_2 + m_1 m_2 + n_1 n_2 = 0 \quad \text{or} \quad \theta = 90^\circ$$

$$(ii) a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

Vector Form

If the vector equations of two lines

$$\mathbf{r} = \mathbf{a}_1 + \lambda \mathbf{b}_1 \quad \text{and} \quad \mathbf{r} = \mathbf{a}_2 + \mu \mathbf{b}_2, \text{ then}$$

$$\cos \theta = \frac{|\mathbf{b}_1 \cdot \mathbf{b}_2|}{|\mathbf{b}_1| |\mathbf{b}_2|}$$

Skew-lines

Two straight lines in a space which are neither parallel nor intersecting, are called skew-lines. Thus, skew-lines are those lines which do not lie in the same plane.

Shortest Distance between Two Lines

Cartesian Form

(i) Let the two skew-lines be

$$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$$

and

$$\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$$

$$\therefore d = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1n_2 - m_2n_1)^2 + (n_1l_2 - l_1n_2)^2 + (l_1m_2 - m_1l_2)^2}}$$

Vector Form

Shortest distance between the lines

$$\mathbf{r} = \mathbf{a}_1 + \lambda \mathbf{b}_1$$

and $\mathbf{r} = \mathbf{a}_2 + \mu \mathbf{b}_2$ is given by

$$d = \frac{|(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|}{|\mathbf{b}_1 \times \mathbf{b}_2|}$$

NOTE • The shortest distance between two parallel lines

$$\mathbf{r} = \mathbf{a}_1 + \lambda \mathbf{b} \text{ and } \mathbf{r} = \mathbf{a}_2 + \mu \mathbf{b} \text{ is } d = \frac{|(\mathbf{a}_2 - \mathbf{a}_1) \times \mathbf{b}|}{|\mathbf{b}|}$$

• Two lines $\mathbf{r}_1 = \mathbf{a}_1 + \lambda \mathbf{b}_1$ and $\mathbf{r}_2 = \mathbf{a}_2 + \mu \mathbf{b}_2$ will intersect provided $d = 0$ i.e. when $(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2) = 0$.

Plane

A plane is a surface such that all the points of a straight line joining any two points on the surface lie on it.

Equation of Plane in Different Forms

(i) General Equation of a Plane

Every equation of first degree in x, y, z represents a plane.

Thus, the general equation of a plane is

$$ax + by + cz + d = 0, \text{ where } a^2 + b^2 + c^2 \neq 0$$

NOTE (a) Equation of YZ-plane is $x = 0$.

(b) Equation of ZX-plane is $y = 0$.

(c) Equation of XY-plane is $z = 0$.

(ii) Normal Form

(a) **Cartesian Form** If l, m, n are the direction cosines of the normal to a given plane and p is the length of perpendicular from origin to the plane, then the equation of the plane is

$$lx + my + nz = p$$

(b) **Vector Form** If $\hat{\mathbf{n}}$ is a unit vector normal to given plane and p is the length of perpendicular from the origin to the plane, then the equation of the plane is

$$\mathbf{r} \cdot \hat{\mathbf{n}} = p$$

(iii) One-point Form

(a) **Cartesian Form** The equation of plane passing through one point (x_1, y_1, z_1) is

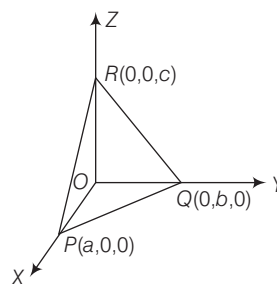
$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

(b) **Vector Form** If $\mathbf{a}$ is the position vector of the point through which plane passes, then the equation of plane is given by $(\mathbf{r} - \mathbf{a}) \cdot \hat{\mathbf{n}} = 0$.

(iv) Intercepts Form of the Plane

The equation of a plane whose intercepts are a, b and c on X, Y and Z -axes, respectively is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$



Equation of Planes in Particular Cases

1. The equation of plane parallel to the plane

$$ax + by + cz + d = 0 \text{ is}$$

$$ax + by + cz + k = 0 \quad [\text{only constant term is changed}]$$

2. Plane passes through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

$$\text{where, } a(x_2 - x_1) + b(y_2 - y_1) + c(z_2 - z_1) = 0$$

3. Plane passes through three non-collinear points (x_1, y_1, z_1) , (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

4. Plane $ax + by + cz + d = 0$ intersecting a line segment joining $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ divides in the ratio

$$\frac{ax_1 + by_1 + cz_1 + d}{ax_2 + by_2 + cz_2 + d}$$

- (i) If this ratio is positive, then A and B are on opposite sides of the plane.
 (ii) If this ratio is negative, then A and B are on the same side of the plane.
5. Any plane passing through the line of intersection of the planes $ax + by + cz + d = 0$ and $a_1x + b_1y + c_1z + d_1 = 0$ is
 $(ax + by + cz + d) + \lambda(a_1x + b_1y + c_1z + d_1) = 0$
6. Any plane containing the line $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ is
 $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$, where $al + bm + cn = 0$

Some Important Facts

- (i) A line parallel to plane $ax + by + cz + d = 0$ In this case, it will be perpendicular to normal.
 $\therefore al + bm + cn = 0$
- (ii) A line perpendicular to a plane $ax + by + cz + d = 0$
 In this case, it will be parallel to normal.
 $\therefore \frac{a}{l} = \frac{b}{m} = \frac{c}{n}$
- (iii) Line to lie in the plane $ax + by + cz + d = 0$
 The normal to the plane will be perpendicular to the line.
 $\therefore al + bm + cn = 0$
 Also, point (x_1, y_1, z_1) through which the line passes will also lie on the plane.
 $\therefore ax_1 + by_1 + cz_1 + d = 0$

Distance of a Point from a Plane

Distance of a point (x_1, y_1, z_1) from the plane $ax + by + cz + d = 0$ is $\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$.

Distance of the origin is $\frac{|d|}{\sqrt{a^2 + b^2 + c^2}}$.

Distance between Two Parallel Planes

The distance between two parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is $\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$.

Coplanar Line

A line which is in the same plane as another line. Any two intersecting lines must lie in the same plane and therefore will be coplanar.

Condition for Coplanarity of Two Lines

Cartesian Form

The lines $\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$

and $\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$

are coplanar, if $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$.

Vector Form

Two lines $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$ and $\mathbf{r} = \mathbf{c} + \mu \mathbf{d}$ are coplanar or intersecting, if $(\mathbf{a} - \mathbf{c}) \cdot (\mathbf{b} \times \mathbf{d}) = 0$

$$\Rightarrow [\mathbf{a} \ \mathbf{b} \ \mathbf{d}] = [\mathbf{c} \ \mathbf{b} \ \mathbf{d}]$$

Angle between Two Planes

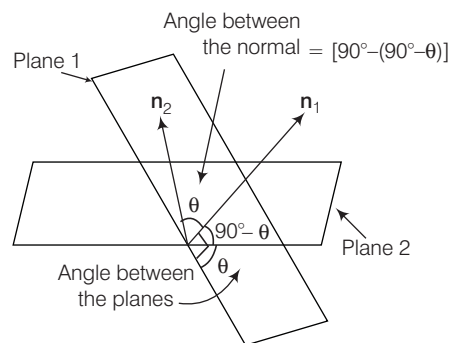
Cartesian Form

The angle between the planes

$$a_1x + b_1y + c_1z + d_1 = 0$$

and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos \theta = \pm \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{(\Sigma a_1^2)} \sqrt{(\Sigma a_2^2)}}$$



where, θ is acute angle between the planes.

Vector Form

If $\mathbf{n}_1$ and $\mathbf{n}_2$ are normals to the plane and θ is the angle between $\mathbf{r} \cdot \mathbf{n}_1 = d_1$ and $\mathbf{r} \cdot \mathbf{n}_2 = d_2$, then

$$\cos \theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{|\mathbf{n}_1| |\mathbf{n}_2|}$$

Angle between a Line and a Plane

Cartesian Form

If angle between the line $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ and the plane $a_1x + b_1y + c_1z + d = 0$ is θ , then $(90^\circ - \theta)$ is the angle between normal and the line.

$$\text{i.e. } \cos(90^\circ - \theta) = \left| \frac{aa_1 + bb_1 + cc_1}{\sqrt{a^2 + b^2 + c^2} \sqrt{a_1^2 + b_1^2 + c_1^2}} \right|$$

Vector Form

The angle between a line $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ and plane $\mathbf{r} \cdot \mathbf{n} = d$, is defined as the complement of the angle between the line and normal to the plane.

$$\text{i.e. } \sin\theta = \frac{\mathbf{n} \cdot \mathbf{b}}{|\mathbf{n}| |\mathbf{b}|}$$

Some Important Results

- **Foot of perpendicular from a point $A(\alpha, \beta, \gamma)$ to a given plane $ax + by + cz + d = 0$**

If AB is perpendicular from A to given plane, then it is parallel to normal so that its equation is

$$\frac{x-\alpha}{a} = \frac{y-\beta}{b} = \frac{z-\gamma}{c} = k$$

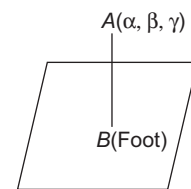
where, a, b, c are direction ratios of the normal.

Any point P on it is $(ak + \alpha, bk + \beta, ck + \gamma)$. It lies on the given plane and we find the value of k and hence the point B . $ax + by + cz + d = 0$

- Four non-coplanar points $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ and $D(x_4, y_4, z_4)$ forms a tetrahedron with vertices A, B, C and D , edges AB, AC, AD, BC, BD and CD ; faces ABC, ABD, ACD and BCD , then

$$(a) \text{ Centroid} = \left(\frac{x_1 + x_2 + x_3 + x_4}{4}, \frac{y_1 + y_2 + y_3 + y_4}{4}, \frac{z_1 + z_2 + z_3 + z_4}{4} \right)$$

$$(b) \text{ Volume} = \frac{1}{6} |\mathbf{AB} \times \mathbf{AC} \cdot \mathbf{AD}| = \frac{1}{6} \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{vmatrix}$$



Practice Exercise

- The XY -plane divides the line joining the points $(-1, 3, 4)$ and $(2, -5, 6)$
 - internally in the ratio $2 : 3$
 - externally in the ratio $2 : 3$
 - internally in the ratio $3 : 2$
 - externally in the ratio $3 : 2$
- If a line makes angles $\frac{\pi}{3}$ and $\frac{\pi}{4}$ with X -axis and Y -axis respectively, then the angle made by the line with Z -axis is
 - $\frac{\pi}{2}$
 - $\frac{\pi}{3}$
 - $\frac{\pi}{4}$
 - $\frac{5\pi}{12}$
- If $A(3, 2, 0)$, $B(5, 3, 2)$ and $C(-9, 6, -3)$ are three points forming a triangle and AD is bisector of $\angle BAC$, then AD meets BC at the point
 - $\left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$
 - $\left(-\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$
 - $\left(\frac{19}{8}, -\frac{57}{16}, \frac{17}{16}\right)$
 - None of these
- If $\triangle ABC$ is such that the mid-points of the sides BC, CA and AB are $(l, 0, 0)$, $(0, m, 0)$ and $(0, 0, n)$, respectively. Then, $\frac{AB^2 + BC^2 + CA^2}{l^2 + m^2 + n^2}$ is equal to
 - 2
 - 4
 - 8
 - 16
- If $P(x, y, z)$ is a point on the line segment joining $Q(2, 2, 4)$ and $R(3, 5, 6)$ such that the projection of OP on the axes are $\frac{13}{5}, \frac{19}{5}, \frac{26}{5}$ respectively, then P divides QR in the ratio.
 - $1 : 2$
 - $3 : 2$
 - $2 : 3$
 - $1 : 3$
- If the projections of a directed line segment on the coordinate axes are $12, 4, 3$, then the direction cosines of the line are
 - $\frac{12}{13}, \frac{-4}{13}, \frac{3}{13}$
 - $\frac{-12}{13}, \frac{-4}{13}, \frac{3}{13}$
 - $\frac{12}{13}, \frac{4}{13}, \frac{3}{13}$
 - None of these

7. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-p}{2} = \frac{z}{1}$ intersect, then p is equal to
 a. $\frac{9}{2}$ b. 10 c. 1 d. $\frac{12}{11}$
8. The foot of perpendicular from $(0, 2, 3)$ to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$, is
 a. $(-2, 3, 4)$ b. $(2, -1, 3)$ c. $(2, 3, -1)$ d. $(3, 2, -1)$
9. The perpendicular distance of $P(1, 2, 3)$ from the lines $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ is
 a. 7 b. 5 c. 8 d. 0
10. The length of perpendicular drawn from the point $(2, 3, 4)$ to line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$, is
 a. $\frac{3}{7}\sqrt{101}$ b. $\frac{2}{7}\sqrt{101}$ c. $\frac{2}{7}\sqrt{103}$ d. $\frac{3}{7}\sqrt{103}$
11. The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2}$, $z=2$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is equal to
 a. $\pi/2$ b. $\pi/3$
 c. $\pi/6$ d. None of these
12. The angle between the diagonal of a cube and an edge of the cube intersecting the diagonal, is
 a. $\cos^{-1}\left(\frac{1}{3}\right)$ b. $\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$
 c. $\tan^{-1}(\sqrt{2})$ d. None of these
13. The angle between the lines whose direction cosines are given by $l+m+n=0$ and $l^2+m^2-n^2=0$, is
 a. $\frac{\pi}{6}$ b. $\frac{\pi}{4}$ c. $\frac{\pi}{3}$ d. $\frac{\pi}{2}$
14. The angle between the lines whose direction cosines (l, m, n) satisfy the equations $l+m+n=0$ and $2lm+2ln-mn=0$, is
 a. 60° b. 90° c. 110° d. 120°
15. The shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$, is
 a. $\sqrt{29}$ units b. 29 units
 c. $\frac{29}{2}$ units d. $2\sqrt{29}$ units
16. If a plane meets the coordinate axes in A, B, C and (α, β, γ) is the centroid of $\triangle ABC$. Then, the equation of the plane is
 a. $\frac{x}{3\alpha} + \frac{y}{3\beta} + \frac{z}{3\gamma} = 1$ b. $\frac{3x}{\alpha} + \frac{3y}{\beta} + \frac{3z}{\gamma} = 1$
 c. $\alpha x + \beta y + \gamma z = 1$ d. None of these
17. If the plane $\frac{x}{2} + \frac{y}{3} + \frac{z}{6} = 1$ cuts the coordinate axes at points A, B and C . Then, find the area of $\triangle ABC$.
 a. $\sqrt{18}$ sq units b. 30 sq units
 c. $3\sqrt{14}$ sq units d. $13\sqrt{14}$ sq units
18. The distance of the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x-y+z=5$ from the point $(-1, -5, -10)$, is
 a. 10 b. 11 c. 12 d. 13
19. The intercepts made on the axes by the plane which bisects the line joining the points $(1, 2, 3)$ and $(-3, 4, 5)$ at right angle, are
 a. $\left(\frac{-9}{2}, 9, 9\right)$ b. $\left(\frac{9}{2}, -9, 9\right)$ c. $\left(19, -\frac{9}{2}, 9\right)$ d. $\left(9, \frac{9}{2}, 19\right)$
20. A line with positive direction cosines passes through the point $P(2, -1, 2)$ and makes equal angles with the coordinate axes. The line meets the plane $2x+y+z=9$ at point Q . The length of the line segment PQ equals
 a. $\sqrt{5}$ b. $3\sqrt{2}$ c. $\sqrt{3}$ d. $\sqrt{7}$
21. The projection of the line $\frac{x+1}{-1} = \frac{y}{2} = \frac{z-1}{3}$ on the plane $x-2y+z=6$ is the line of intersection of this plane with the plane
 a. $2x+y+2=0$ b. $3x+y+z=20$
 c. $2x+3y+8z=13$ d. $6x-y-2z=12$
22. The equation of the plane containing the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the point $(0, 7, -7)$, is
 a. $x+y+z=1$ b. $x+y+z=2$
 c. $x+y+z=0$ d. None of these
23. The equation of the plane through $(3, 1, -3)$ and $(1, -2, 2)$ and parallel to the line with DR's $1, 1, -2$, is
 a. $x-y+z+1=0$ b. $x+y-z+1=0$
 c. $x-y-z-1=0$ d. $x+y+z-1=0$
24. The equation of the plane through the line of intersection of the planes $x+y+z-1=0$ and $2x+y-3z+2=0$ passing through the point $(1, 1, 1)$, is
 a. $x-4z+3=0$ b. $x-y+z=1$
 c. $x+y+z=3$ d. $2x-y+z=2$
25. The coordinates of the point where the line passes through the points $(3, -4, -5)$ and $(2, -3, 1)$ crosses the plane passing through three points $(2, 2, 1)$, $(3, 0, 1)$ and $(4, -1, 0)$, is
 a. $(1, 2, 7)$ b. $(-1, 2, -7)$
 c. $(1, -2, 7)$ d. None of these
26. The plane passing through the point $(-2, -2, 2)$ and containing the line joining the points $(1, -1, 2)$ and

- (1, 1, 1) makes intercepts on the coordinate axes and the sum of whose length is
 a. 3 b. 6 c. 12 d. 20
27. The equation of the plane passing through (2, 1, 5) and parallel to the plane $3x - 4y + 5z = 4$, is
 a. $3x - 4y + 5z - 27 = 0$ b. $3x - 4y + 5z + 21 = 0$
 c. $3x - 4y + 5z + 26 = 0$ d. $3x - 4y + 5z + 17 = 0$
28. The equation of the line passing through the point (3, 0, 1) and parallel to the planes $x + 2y = 0$ and $3y - z = 0$, is
 a. $\frac{x-3}{-2} = \frac{y-0}{1} = \frac{z-1}{3}$ b. $\frac{x-3}{1} = \frac{y-0}{-2} = \frac{z-1}{3}$
 c. $\frac{x-3}{3} = \frac{y-0}{1} = \frac{z-1}{-2}$ d. None of these
29. The equation of the plane which contains two parallel lines $\frac{x+1}{3} = \frac{y-2}{2} = \frac{z}{1}$ and $\frac{x-3}{3} = \frac{y+4}{2} = \frac{z-1}{1}$, is
 a. $3x + 2y + z = 10$ b. $8x + y - 26z + 6 = 0$
 c. $4x + 6y + z = 50$ d. None of these
30. If the planes $x - cy - bz = 0$, $cx - y + az = 0$ and $bx + ay - z = 0$ pass through a straight line, then the value of $a^2 + b^2 + c^2 + 2abc$ is
 a. 2 b. 3 c. 0 d. 1
31. The equation of plane through the intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ and perpendicular to the plane $5x + 3y + 6z = 8$, is
 a. $51x + 15y - 50z - 173 = 0$ b. $51x - 15y + 50z + 173 = 0$
 c. $51x + 15y - 50z + 173 = 0$ d. $51x - 15y - 50z - 173 = 0$
32. The equation of a line $4x - 4y - z + 11 = 0 = x + 2y - z - 1$ can be form as
 a. $\frac{x}{2} = \frac{y-2}{1} = \frac{z-3}{4}$ b. $\frac{x-4}{2} = \frac{y-4}{1} = \frac{z-11}{5}$
 c. $\frac{x-2}{-2} = \frac{y}{3} = \frac{z+3}{5}$ d. $\frac{x+2}{3} = \frac{y+2}{4} = \frac{z-1}{4}$
33. Let the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$. Then, (α, β) equals
 a. (6, -17) b. (-6, 7) c. (5, -15) d. (-5, 15)
34. Find the distance of the plane $x + 2y - z = 2$ from the point (2, -1, 3) as measured in the direction with DR's (2, 2, 1).
 a. 2 b. -3 c. -2 d. 3
35. A plane is such that the foot of perpendicular drawn from the origin to it is (2, -1, 1). The distance of (1, 2, 3) from the plane, is
 a. $3/2$ b. $\sqrt{3/2}$ c. 2 d. 0
36. The distance of the point (1, 0, -3) from the plane $x - y - z = 9$ measured parallel to the line $\frac{x-2}{2} = \frac{y+2}{3} = \frac{z-6}{-6}$, is
 a. 6 b. 7 c. 17 d. 26
37. The equations of the planes through the intersection of the planes $x + 3y + 6 = 0$ and $3x - y - 4z = 0$, whose perpendicular distance from the origin is unity, are
 a. $x + y - 2z + 3 = 0$; $x - 2y - 2z - 3 = 0$
 b. $2x + y - 2z + 3 = 0$; $x - 2y - 2z - 3 = 0$
 c. $x - y + 2z + 3 = 0$; $x + 2y + 2z + 3 = 0$
 d. $2x - y + 2z - 3 = 0$; $x + 2y + 2z + 3 = 0$
38. A plane L passes through the point $(-1, -2, -1)$ and normal to it is perpendicular to the lines $\frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$ and $\frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$. The distance of the point (1, 1, 1) from the plane L is
 a. $\frac{12}{\sqrt{75}}$ b. $\frac{17}{\sqrt{75}}$ c. $\frac{13}{\sqrt{75}}$ d. $\frac{20}{\sqrt{75}}$
39. A plane passes through the point (1, -2, 3) and is parallel to the plane $2x - 2y + z - 9 = 0$. The distance of the point $(-1, 2, 0)$ from the plane, is
 a. 2 b. 3 c. 4 d. 5
40. The distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is
 a. $\frac{3}{2}$ b. $\frac{5}{2}$ c. $\frac{7}{2}$ d. $\frac{9}{2}$
41. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, then k can have
 a. any value b. exactly one value
 c. exactly two values d. exactly three values
42. The coordinates of the foot of the perpendicular drawn from the origin to plane $2x - 3y + 4z - 6 = 0$, are
 a. $\left(\frac{12}{29}, \frac{-18}{29}, \frac{24}{29}\right)$ b. $\left(\frac{13}{19}, \frac{-18}{29}, \frac{24}{29}\right)$
 c. $\left(\frac{-12}{29}, \frac{18}{29}, \frac{24}{29}\right)$ d. $\left(\frac{12}{19}, \frac{-18}{29}, \frac{-24}{29}\right)$
43. The image of the point P (1, 3, 4) in the plane $2x - y + z + 3 = 0$ is
 a. (3, 5, -2) b. (-3, 5, 2) c. (3, -5, 2) d. (3, 5, 2)
44. The image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is
 a. (-1, 0, 7) b. (-1, 0, -7) c. (1, 0, 7) d. (2, 0, 7)
45. The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line
 a. $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$ b. $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$
 c. $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$ d. $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$
46. The volume of the tetrahedron formed by coordinate planes and $2x + 3y + z = 6$, is
 a. 5 b. 4 c. 6 d. 0

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- If two lines, whose equations are $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{\lambda}$ and $\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-2}{3}$ lie in the same plane. Then, the value of $\sin^{-1} \sin \lambda$ is [2014]
a. 3 b. $\pi - 3$ c. 4 d. $\pi - 4$
- If the plane $x + y + z = 1$ is rotated through an angle 90° about its line of intersection with the plane $x - 2y + 3z = 0$, then the new position of the plane is [2014]
a. $x - 5y + 4z = 1$ b. $x - 5y + 4z = -1$ c. $x - 8y + 7z = 2$ d. $x - 8y + 7z = -2$
- The distance of the point $(1, -5, 9)$ from the plane $x + y + z = 5$ measured along a straight line $x = y = z$ is $2\sqrt{3}k$, then the value of k is [2014]
a. 5 b. 6 c. $\sqrt{3}$ d. 4
- The acute angle between the lines, whose direction cosines are given by $2l - m + 2n = 0$ and $lm + mn + nl = 0$, is [2013]
a. $\frac{\pi}{6}$ b. $\frac{\pi}{4}$ c. $\frac{\pi}{3}$ d. $\frac{\pi}{2}$
- Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive X -axis, then $\cos \alpha$ is equal to [2013]
a. $1/2$ b. 1 c. $1/\sqrt{2}$ d. $1/\sqrt{3}$
- A plane passes through the point $(1, -2, 3)$ and is parallel to the plane $2x - 2y + z = 0$. The distance of the point $(-1, 2, 0)$ from the plane is [2013]
a. 2 b. 3 c. 4 d. 5
- The line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the curve $xy = c^2, z = 0$, if c is equal to [2010]
a. ± 1 b. $\pm 1/3$ c. $\pm \sqrt{5}$ d. None
- The direction ratios of the line $x - y + z - 5 = 0 = x - 3y - 6$ are [2009]
a. 3, 1, -2 b. 2, -4, 1
c. $\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}$ d. $\frac{2}{\sqrt{41}}, \frac{-4}{\sqrt{41}}, \frac{1}{\sqrt{41}}$
- The intercepts of the plane $2x - 3y + 4z = 12$ on the coordinate axes are given by [2009]
a. 3, -2, 1.5 b. 6, -4, 3 c. 6, -4, -3 d. 2, -3, 4
- The point of intersection of the line $\frac{x-1}{3} = \frac{y+2}{4} = \frac{z-3}{-2}$ and plane $2x - y + 3z - 1 = 0$ is [2005]
a. (10, -10, 3) b. (10, 10, -3) c. (-10, 10, 3) d. None of these
- The equation of plane passing through a point $A(2, -1, 3)$ and parallel to the vectors $\mathbf{a} = (3, 0, -1)$ and $\mathbf{b} = (-3, 2, 2)$, is [2005]
a. $2x - 3y + 6z - 25 = 0$ b. $2x - 3y + 6z + 25 = 0$
c. $3x - 2y + 6z - 25 = 0$ d. $3x - 2y + 6z + 25 = 0$

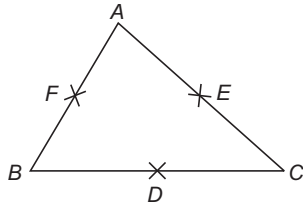
Answer with Solutions

Practice Exercise

- (b) Suppose XY -plane divides the line joining the given points in the ratio $\lambda : 1$. The coordinates of the point of division are $\left(\frac{2\lambda - 1}{\lambda + 1}, \frac{-5\lambda + 3}{\lambda + 1}, \frac{6\lambda + 4}{\lambda + 1}\right)$. This point lies on XY -plane.
 $\therefore \frac{6\lambda + 4}{\lambda + 1} = 0 \Rightarrow \lambda = -\frac{2}{3}$
- (b) Hint We have, $\alpha = \frac{\pi}{3}, \beta = \frac{\pi}{4}$
 $\therefore l = \cos \frac{\pi}{3} = \frac{1}{2}$
 $m = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$
 $\therefore l^2 + m^2 + n^2 = 1$

- (a) The bisector of $\angle BAC$ i.e. AD divides the side BC in the ratio $AB : AC$.
i.e. $\frac{BD}{CD} = \frac{AB}{AC}$
Now, $AB = \sqrt{(3-5)^2 + (2-3)^2 + (0-2)^2} = \sqrt{4+1+4} = 3$
 $AC = \sqrt{(-9-3)^2 + (6-2)^2 + (-3-0)^2} = \sqrt{144+16+9} = 13$
Let coordinates of D be (x, y, z) .
Then, $x = \frac{3 \times (-9) + 13 \times 5}{3+13} = \frac{38}{16} = \frac{19}{8}$
 $y = \frac{3 \times 6 + 13 \times 13}{3+13} = \frac{57}{16}$ and $z = \frac{-3 \times 3 + 13 \times 2}{3+13} = \frac{17}{16}$
 $\therefore D \equiv \left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$

4. (c) Given, mid-points of sides are $D(l, 0, 0)$, $E(0, m, 0)$ and $F(0, 0, n)$.



Also, $EF^2 = \frac{BC^2}{4}$ [by mid-point theorem]

$\Rightarrow BC^2 = 4(m^2 + n^2)$

Similarly, $AB^2 = 4(l^2 + m^2)$ and $CA^2 = 4(l^2 + n^2)$

$\therefore \frac{AB^2 + BC^2 + CA^2}{l^2 + m^2 + n^2} = 8$

5. (b) Since, $\mathbf{OP}$ has projections $\frac{13}{5}$, $\frac{19}{5}$ and $\frac{26}{5}$ on the coordinate axes.

$\therefore \mathbf{OP} = \frac{13}{5}\hat{i} + \frac{19}{5}\hat{j} + \frac{26}{5}\hat{k}$

Suppose P divides the join of Q and $R(3, 5, 6)$ in the ratio $\lambda : 1$. Then, the position vector of P is

$$\left(\frac{3\lambda + 2}{\lambda + 1}\right)\hat{i} + \left(\frac{5\lambda + 2}{\lambda + 1}\right)\hat{j} + \left(\frac{6\lambda + 4}{\lambda + 1}\right)\hat{k}$$

$\therefore \frac{13}{5}\hat{i} + \frac{19}{5}\hat{j} + \frac{26}{5}\hat{k} = \left(\frac{3\lambda + 2}{\lambda + 1}\right)\hat{i} + \left(\frac{5\lambda + 2}{\lambda + 1}\right)\hat{j} + \left(\frac{6\lambda + 4}{\lambda + 1}\right)\hat{k}$

$\Rightarrow \frac{3\lambda + 2}{\lambda + 1} = \frac{13}{5}, \frac{5\lambda + 2}{\lambda + 1} = \frac{19}{5}, \frac{6\lambda + 4}{\lambda + 1} = \frac{26}{5}$

$\Rightarrow 2\lambda = 3 \Rightarrow \lambda = 3/2$

Hence, P divides QR in the ratio $3 : 2$.

6. (c) **Hint** Let r be the magnitude of given line segments and l, m, n be the direction cosines.

So, projection on X -axis i.e. $|r| \cdot l = 12$... (i)

Projection on Y -axis i.e. $|r| \cdot m = 4$... (ii)

and projection on Z -axis i.e. $|r| \cdot n = 3$... (iii)

On squaring and adding Eqs. (i), (ii) and (iii), we get

$$|r|^2 (l^2 + m^2 + n^2) = 12^2 + 4^2 + 3^2$$

7. (a) Any point on the first line is $(2r_1 + 1, 3r_1 - 1, 4r_1 + 1)$ and on second line is $(r_2 + 3, 2r_2 + p, r_2)$.

The lines will intersect, when

$$2r_1 + 1 = r_2 + 3$$

$$3r_1 - 1 = 2r_2 + p$$

$\Rightarrow 2r_1 - r_2 = 2$... (i)

$4r_1 + 1 = r_2$ and $4r_1 - r_2 = -1$... (ii)

On solving Eqs. (i) and (ii), we get

$$r_1 = -\frac{3}{2} \text{ and } r_2 = -5$$

$\therefore p = 3r_1 - 2r_2 - 1 = -\frac{9}{2} + 10 - 1 = \frac{9}{2}$

8. (c) Let L be foot of perpendicular from $P(0, 2, 3)$ on the line

$$\frac{x - (-3)}{5} = \frac{y - 1}{2} = \frac{z - (-4)}{3} = t \quad \dots (i)$$

Any point on Eq. (i) is $L(-3 + 5t, 1 + 2t, -4 + 3t)$.

Then, DR's of PL are

$$(-3 + 5t - 0, 1 + 2t - 2, -4 + 3t - 3)$$

or $(5t - 3, 2t - 1, 3t - 7)$

Since, PL is perpendicular to Eq. (i).

$$\therefore 5(5t - 3) + 2(2t - 1) + 3(3t - 7) = 0 \Rightarrow t = 1$$

So, the coordinates of L are $(2, 3, -1)$.

9. (a) The point $A(6, 7, 7)$ lie on the line.

Let the perpendicular from P meet the line at L . Then,

$$AP^2 = (6 - 1)^2 + (7 - 2)^2 + (7 - 3)^2 = 66$$

Also, AL = Projection of AP on line

[since, actual DC's are

$$\left(\frac{3}{\sqrt{17}}, \frac{2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}\right)]$$

$$= (6 - 1) \cdot \frac{3}{\sqrt{17}} + (7 - 2) \cdot \frac{2}{\sqrt{17}} + (7 - 3) \left(\frac{-2}{\sqrt{17}}\right) = \sqrt{17}$$

$\therefore$ Perpendicular distance of P from the line is given by

$$d^2 = AP^2 - AL^2 \Rightarrow d^2 = 66 - 17 = 49 \Rightarrow d = 7$$

10. (a) Let P be the foot of the perpendicular drawn from $A(2, 3, 4)$ to the given line l .

Then, $\frac{x - 4}{-2} = \frac{y}{6} = \frac{z - 1}{-3}$

Now, any point on the line l is given by

$$x = 4 - 2\lambda, y = 6\lambda, z = 1 - 3\lambda$$

The coordinates of P are $(4 - 2\lambda, 6\lambda, 1 - 3\lambda)$.

The direction ratios of AP are

$$(4 - 2\lambda - 2, 6\lambda - 3, 1 - 3\lambda - 4)$$

i.e. $(2 - 2\lambda, 6\lambda - 3, -3 - 3\lambda)$

So, the direction ratios of l are $-2, 6$ and -3 .

Given, $AP \perp l$

$$\therefore -2(2 - 2\lambda) + 6(6\lambda - 3) - 3(-3 - 3\lambda) = 0$$

$$\Rightarrow \lambda = \frac{13}{49}$$

$$\therefore AP^2 = (4 - 2\lambda - 2)^2 + (6\lambda - 3)^2 + (1 - 3\lambda - 4)^2$$

$$= 22 - 26\lambda + 49\lambda^2$$

Put $\lambda = \frac{13}{49}$, we get

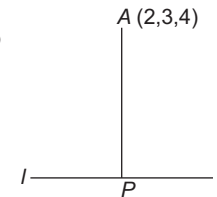
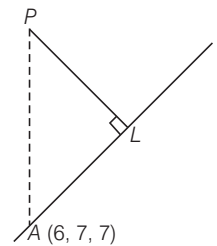
$$AP^2 = \frac{909}{49}$$

$$\Rightarrow AP = \frac{3}{7}\sqrt{101}$$

11. (a) Given lines are

$$\frac{x - 2}{3} = \frac{y + 1}{-2} = \frac{z - 2}{1} \quad \dots (i)$$

and $\frac{x - 1}{1} = \frac{2y + 3}{3} = \frac{z + 5}{2} \quad \dots (ii)$



Line (i) passes through $\mathbf{a}_1 = 2\hat{i} - \hat{j} + 2\hat{k}$ and parallel to $\mathbf{b}_1 = 3\hat{i} - 2\hat{j} + 0\hat{k}$ and line (ii) passes through

$$\mathbf{a}_2 = \hat{i} - \frac{3}{2}\hat{j} - 5\hat{k}$$

and parallel to $\mathbf{b}_2 = \hat{i} + \frac{3}{2}\hat{j} + 2\hat{k}$

Therefore, angle between lines (i) and (ii) is same as angle between $\mathbf{b}_1$ and $\mathbf{b}_2$.

$$\therefore \cos \theta = \frac{\mathbf{b}_1 \cdot \mathbf{b}_2}{|\mathbf{b}_1| |\mathbf{b}_2|}$$

$$\Rightarrow \cos \theta = \frac{(3\hat{i} - 2\hat{j} + 0\hat{k}) \cdot (\hat{i} + \frac{3}{2}\hat{j} + 2\hat{k})}{\sqrt{9+4+0} \sqrt{1+9/4+4}}$$

$$\Rightarrow \cos \theta = \frac{3-3}{\sqrt{13}\sqrt{\frac{29}{4}}} = 0 \Rightarrow \theta = \pi/2$$

12. (c) If three edges of the cube are along x, y and z , then diagonal has DR's $(1, 1, 1)$ and edge along X -axis has DR's $(1, 0, 0)$. The angle between them is

$$\cos^{-1} \frac{1}{\sqrt{3}} = \tan^{-1} \sqrt{2}$$

13. (c) $l^2 + m^2 - n^2 = 0 \Rightarrow 1 - 2n^2 = 0 \Rightarrow n = \pm \frac{1}{\sqrt{2}}$

Eliminating n from the given relations, we get $lm = 0$

So, the two lines having direction cosines are $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ and $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$.

Hence, the angle between them is $\cos^{-1} \frac{1}{2} = \frac{\pi}{3}$.

14. (d) Given, $l + m + n = 0$ and $mn - 2l(m + n) = 0$

On eliminating l , we get

$$mn + 2(m + n)^2 = 0 \Rightarrow 2m^2 + 5mn + 2n^2 = 0$$

$$\Rightarrow (m + 2n)(2m + n) = 0$$

Either $m + 2n = 0$ or $2m + n = 0$

$$\Rightarrow m = -2n \text{ or } n = -2m \quad \dots(i)$$

Let the two lines whose direction cosines satisfy the given relations have their direction cosines

$$(l_1, m_1, n_1) \text{ and } (l_2, m_2, n_2)$$

Then, from Eq. (i), $m_1 + 2n_1 = 0 \Rightarrow m_1 = -2n_1$

where, $l_1 + m_1 + n_1 = 0 \Rightarrow l_1 = -(m_1 + n_1)$

$$l_1 = n_1 \Rightarrow l_1 = \frac{m_1}{-2} = n_1$$

$$\Rightarrow \frac{l_1}{1} = \frac{m_1}{-2} = \frac{n_1}{1} = \frac{\sqrt{l_1^2 + m_1^2 + n_1^2}}{\sqrt{1+4+1}}$$

$$\Rightarrow l_1 = \frac{1}{\sqrt{6}}, m_1 = \frac{-2}{\sqrt{6}}, n_1 = \frac{1}{\sqrt{6}} \quad \dots(ii)$$

Again, from Eq. (i),

$$2m_2 + n_2 = 0 \text{ and } l_2 = -m_2 - n_2 = m_2$$

$$\therefore \frac{l_2}{1} = \frac{m_2}{1} = \frac{-n_2}{2} = \frac{\sqrt{l_2^2 + m_2^2 + n_2^2}}{\sqrt{1+1+4}} = \frac{1}{\sqrt{6}}$$

$$\Rightarrow l_2 = \frac{1}{\sqrt{6}}, m_2 = \frac{1}{\sqrt{6}}, n_2 = \frac{-2}{\sqrt{6}} \quad \dots(iii)$$

From Eqs. (ii) and (iii), we get

$$\begin{aligned} \cos \theta &= l_1 l_2 + m_1 m_2 + n_1 n_2 \\ &= \left(\frac{1}{\sqrt{6}}\right)\left(\frac{1}{\sqrt{6}}\right) + \left(\frac{-2}{\sqrt{6}}\right)\left(\frac{1}{\sqrt{6}}\right) + \left(\frac{1}{\sqrt{6}}\right)\left(\frac{-2}{\sqrt{6}}\right) \\ &= \frac{1}{6} - \frac{2}{6} - \frac{2}{6} = -\frac{1}{2} \end{aligned}$$

$\therefore$ The angle between the two lines is given by

$$\cos \theta = \left(-\frac{1}{2}\right) \Rightarrow \theta = 120^\circ$$

15. (d) **Hint** For line 1st, DR's is $(7, -6, 1)$ and it passes through $(-1, -1, -1)$, then equation of given lines (in vector form) is

$$\mathbf{r}_1 = -\hat{i} - \hat{j} - \hat{k} + \lambda(7\hat{i} - 6\hat{j} + \hat{k})$$

Similarly, $\mathbf{r}_2 = 3\hat{i} + 5\hat{j} + 7\hat{k} + \mu(\hat{i} - 2\hat{j} + \hat{k})$

Here, $\mathbf{a}_1 = -\hat{i} - \hat{j} - \hat{k}$, $\mathbf{b}_1 = 7\hat{i} - 6\hat{j} + \hat{k}$

and $\mathbf{a}_2 = 3\hat{i} + 5\hat{j} + 7\hat{k}$, $\mathbf{b}_2 = \hat{i} - 2\hat{j} + \hat{k}$

So, the shortest distance between the given lines

$$d = \frac{|(\mathbf{b}_1 \times \mathbf{b}_2) \cdot (\mathbf{a}_2 - \mathbf{a}_1)|}{|\mathbf{b}_1 \times \mathbf{b}_2|}$$

16. (a) The intercept form of the plane equation is

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$, where a, b and c are the intercepts made by the plane on the axes.

$$\therefore \alpha = \frac{0+0+a}{3}, \beta = \frac{0+0+b}{3}, \gamma = \frac{0+0+c}{3}$$

$$\Rightarrow a = 3\alpha, b = 3\beta \text{ and } c = 3\gamma$$

So, the equation of plane is $\frac{x}{3\alpha} + \frac{y}{3\beta} + \frac{z}{3\gamma} = 1$.

17. (c) Given, the plane meets axes at $A(2, 0, 0)$, $B(0, 3, 0)$ and $C(0, 0, 6)$.

$$\text{Then, area of } \triangle ABC = \frac{1}{2} |\mathbf{AB} \times \mathbf{AC}|$$

$$= \frac{1}{2} |(-2\hat{i} + 3\hat{j}) \times (-2\hat{i} + 6\hat{k})| = 3\sqrt{14} \text{ sq units}$$

18. (d) **Hint** Any point on the line is $3r + 2, 4r - 1, 12r + 2$.

If it lies on the plane $x - y + z = 5$, then

$$(3r + 2) - (4r - 1) + (12r + 2) = 5$$

$$\therefore r = 0$$

Hence, the point of intersection is $(2, -1, 2)$.

19. (a) Direction ratios of the line joining points $P(1, 2, 3)$ and $Q(-3, 4, 5)$ are $-4, 2, 2$ which are direction ratios of the normal to the plane.

Equation of plane is $-4x + 2y + 2z = k$.

This plane passes through $(-1, 3, 4)$.

$$\therefore -4(-1) + 2(3) + 2(4) = k \Rightarrow k = 18$$

Equation of plane is $2x - y - z = -9$

Thus, the intercepts are $\left(-\frac{9}{2}, 9, 9\right)$.

20. (c) Given equation of the line is

$$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-2}{1}$$

Now, any point on the line is given by

$$x = k + 2, y = k - 1, z = k + 2, \text{ where } k \text{ is constant}$$

Since, the point $Q(k + 2, k - 1, k + 2)$ lies on the plane

$2x + y + z = 9$, therefore we get

$$2(k + 2) + k - 1 + k + 2 = 9 \Rightarrow k = 1$$

The coordinates of Q are $(3, 0, 3)$.

$$\therefore PQ = \sqrt{(3-2)^2 + (0+1)^2 + (3-2)^2} = \sqrt{3}$$

21. (a) Equation of the plane through $(-1, 0, 1)$ is

$$a(x + 1) + b(y - 0) + c(z - 1) = 0 \quad \dots(i)$$

which is parallel to the given line and perpendicular to the given plane.

$$\therefore -a + 2b + 3c = 0 \quad \dots(ii)$$

$$\text{and } a - 2b + c = 0 \quad \dots(iii)$$

From Eqs. (ii) and (iii), we get

$$c = 0, a = 2b$$

On putting values of a and c in Eq. (i), we get

$$2b(x + 1) + by = 0$$

$$\therefore 2x + y + 2 = 0$$

22. (c) Any plane containing the line

$$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1} \text{ is}$$

$$a(x + 1) + b(y - 3) + c(z + 2) = 0 \quad \dots(i)$$

$$\text{where, } -3a + 2b + c = 0 \quad \dots(ii)$$

which passes through $(0, 7, -7)$.

$$\therefore a + 4b - 5c = 0 \quad \dots(iii)$$

From Eqs. (ii) and (iii), we get

$$\frac{a}{-14} = \frac{b}{-14} = \frac{c}{-14} \Rightarrow \frac{a}{1} = \frac{b}{1} = \frac{c}{1}$$

Hence, the required plane is

$$x + 1 + y - 3 + z + 2 = 0$$

$$\text{i.e. } x + y + z = 0$$

23. (d) Equation of plane passing through $(3, 1, -3)$ is

$$a(x - 3) + b(y - 1) + c(z + 3) = 0$$

It passes through $(1, -2, 2)$, then

$$-2a - 3b + 5c = 0$$

$$\text{Also, } a + b - 2c = 0$$

On solving, we get $a = 1, b = 1$ and $c = 1$

$\therefore$ Equation of plane is

$$(x - 3) \cdot 1 + (y - 1) \cdot 1 + (z + 3) \cdot 1 = 0$$

$$\Rightarrow x + y + z - 1 = 0$$

24. (a) **Hint** The required plane is

$$2x + y - 3z + 2 + \lambda(x + y + z - 1) = 0$$

25. (c) Given equation of plane through three points

$(2, 2, 1), (3, 0, 1)$ and $(4, -1, 0)$ is

$$[(\mathbf{r} - \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}})] \times (\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}) = 0$$

$$\text{i.e. } \mathbf{r} \cdot (2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 7 \text{ or } 2x + y + z - 7 = 0 \quad \dots(i)$$

Equation of line through $(3, -4, -5)$ and $(2, -3, 1)$ is

$$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} \quad \dots(ii)$$

Any point on line (ii) is $(-\lambda + 3, \lambda - 4, 6\lambda - 5)$. This point lies on plane (i).

$$\text{Therefore, } 2(-\lambda + 3) + (\lambda - 4) + (6\lambda - 5) - 7 = 0 \Rightarrow \lambda = 2$$

Hence, the required point is $(1, -2, 7)$.

26. (c) Required equation of plane is

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ -3 & -3 & 1 \\ 0 & -2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow x - 3y - 6z + 8 = 0$$

Since, the intercepts are $8, \frac{8}{3}, \frac{8}{6}$.

So, their sum is 12.

27. (a) Given equation of the plane passing through $(2, 1, 5)$

and parallel to the plane $3x - 4y + 5z = 4$ is

$$3x - 4y + 5z + k = 0$$

On substituting coordinates $(2, 1, 5)$, we get

$$3 \times 2 - 4 \times 1 + 5 \times 5 + k = 0$$

$$\Rightarrow k = -27$$

So, the equation of plane is $3x - 4y + 5z - 27 = 0$.

28. (a) Let a, b and c be the direction ratios of the required

line. Then, its equation is

$$\frac{x-3}{a} = \frac{y-0}{b} = \frac{z-1}{c} \quad \dots(i)$$

Since, Eq. (i) is parallel to the planes $x + 2y + 0z = 0$ and $0x + 3y - z = 0$. Therefore, normal to the plane is perpendicular to the line.

$$\therefore a(1) + b(2) + c(0) = 0 \text{ and } a(0) + b(3) + c(-1) = 0$$

On solving these two equations by cross-multiplication, we get

$$\frac{a}{(2)(-1) - (0)(3)} = \frac{b}{(0)(0) - (1)(-1)} = \frac{c}{(1)(3) - (0)(2)}$$

$$\Rightarrow \frac{a}{-2} = \frac{b}{1} = \frac{c}{3} = \lambda \quad [\text{say}]$$

$$\Rightarrow a = -2\lambda, b = \lambda \text{ and } c = 3\lambda$$

On substituting the values of a, b and c in Eq. (i), we get the equation of the required line as

$$\frac{x-3}{-2} = \frac{y-0}{1} = \frac{z-1}{3}$$

29. (b) Let the equation of the plane containing the first line be

$$a(x + 1) + b(y - 2) + cz = 0 \quad \dots(i)$$

$$\therefore 3a + 2b + c = 0$$

Also, other line will lie on the plane, if the point $(3, -4, 1)$ lies on the plane i.e. $a(3 + 1) + b(-4 - 2) + c = 0$

$$\Rightarrow 4a - 6b + c = 0 \quad \dots(ii)$$

On solving Eq. (ii), we get

$$\frac{a}{8} = \frac{b}{1} = \frac{c}{-26}$$

So, the required equation is $8x + y - 26z + 6 = 0$.

30. (a) Given, $x - cy - bz = 0$... (i)
 $cx - y + az = 0$... (ii)
 and $bx + ay - z = 0$... (iii)

Equation of plane passing through the line of intersection of planes (i) and (ii), is

$$(x - cy - bz) + \lambda(cx - y + az) = 0$$

$$\Rightarrow x(1 + \lambda c) - y(c + \lambda) + z(-b + a\lambda) = 0 \quad \dots (iv)$$

If planes (iii) and (iv) are same, then Eqs. (iii) and (iv) will be identical.

$$\therefore \frac{1 + c\lambda}{b} = \frac{-(c + \lambda)}{a} = \frac{-b + a\lambda}{-1}$$

$$\Rightarrow \lambda = \frac{-(a + bc)}{(ac + b)} \text{ and } \lambda = -\left(\frac{ab + c}{1 - a^2}\right)$$

$$\Rightarrow \frac{-(a + bc)}{(ac + b)} = \frac{-(ab + c)}{(1 - a^2)}$$

$$\Rightarrow (a + bc)(1 - a^2) = (ab + c)(ac + b)$$

$$\Rightarrow a - a^3 - a^2bc - ab^2 - ac^2 - a^2bc = 0$$

$$\Rightarrow -a^3 - a^2bc - ab^2 - ac^2 - a^2bc = -a$$

$$\Rightarrow a^2 + abc + b^2 + c^2 + abc = 1$$

$$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$$

31. (c) **Hint** Any plane through the intersection of the given planes P and Q is

$$P + \lambda Q = 0$$

$$\Rightarrow (x + 2y + 3z - 4) + \lambda(2x + y - z + 5) = 0$$

Use condition $5(1 + 2\lambda) + 3(2 + \lambda) + 6(3 - \lambda) = 0$

32. (a) Given equations are

$$4x - 4y - z + 11 = 0 \quad \dots (i)$$

and $x + 2y - z - 1 = 0 \quad \dots (ii)$

The direction ratios of normals to the planes (i), (ii) are $(4, -4, -1)$ and $(1, 2, -1)$, respectively.

Let direction ratios of line of intersection of planes be l, m, n .

As the line of intersection of the planes is perpendicular to the normals of the both planes.

$$\therefore 4l - 4m - n = 0 \text{ and } l + 2m - n = 0$$

$$\therefore \frac{l}{6} = \frac{m}{3} = \frac{n}{12} \Rightarrow \frac{l}{2} = \frac{m}{1} = \frac{n}{4}$$

If $x = 0$, then Eqs. (i) and (ii) will be

$$-4y - z + 11 = 0 \text{ and } 2y - z - 1 = 0$$

Then, $y = 2$ and $z = 3$

$\therefore$ Equation of line is

$$\frac{x}{2} = \frac{y - 2}{1} = \frac{z - 3}{4}$$

33. (b) Direction ratios of given line are $(3, -5, 2)$.

Direction ratios of normal to the plane $= (1, 3, -\alpha)$

Since, line is perpendicular to the normal.

$$\therefore 3(1) - 5(3) + 2(-\alpha) = 0$$

$$\Rightarrow 3 - 15 - 2\alpha = 0$$

$$\Rightarrow 2\alpha = -12 \Rightarrow \alpha = -6$$

Also, point $(2, 1, -2)$ lies on the plane

$$2 + 3 + 6(-2) + \beta = 0$$

$$\Rightarrow \beta = 7$$

$$\therefore (\alpha, \beta) = (-6, 7)$$

34. (a) Let the line through $(2, -1, 3)$ with DC's $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$, be

$$\frac{x - 2}{2/3} = \frac{y + 1}{2/3} = \frac{z - 3}{1/3} = r \quad [\text{say}]$$

$$\therefore x = 2 + \frac{2r}{3}, y = -1 + \frac{2r}{3}, z = 3 + \frac{r}{3}$$

Since, it lies on the plane $x + 2y - z = 2$.

$$\therefore 2 + \frac{2r}{3} - 2 + \frac{4r}{3} - 3 - \frac{r}{3} = 2 \Rightarrow r = 3$$

35. (b) Given plane is $2x - y + z = k$

It passes through $(2, -1, 1)$.

$$\therefore k = 6$$

The distance of $(1, 2, 3)$ from $2x - y + z = 6$ is

$$d = \frac{|2 - 2 + 3 - 6|}{\sqrt{2^2 + 1 + 1}} = \frac{3}{\sqrt{6}} = \frac{\sqrt{3}}{2}$$

36. (b) Given equation of plane is $x - y - z = 9$... (i)

and line AB is $\frac{x - 2}{2} = \frac{y + 2}{3} = \frac{z - 6}{-6}$... (ii)

Equation of line passing through $(1, 0, -3)$ are parallel to

$$\frac{x - 2}{2} = \frac{y + 2}{3} = \frac{z - 6}{-6} \text{ and } \frac{x - 1}{2} = \frac{y}{3} = \frac{z + 3}{-6} \quad \dots (iii)$$

Coordinate of any point on Eq. (iii) are

$$P(2k + 1, 3k, -6k - 3), \text{ where } k \text{ is constant.}$$

If P is intersection of Eqs. (i) and (iii), then it must lie on Eq. (i).

$$\text{i.e. } (2k + 1) - 3k - (-6k - 3) = 9 \Rightarrow k = 1$$

$$\therefore P \equiv (3, 3, -9)$$

Now, distance between $(1, 0, -3)$ and $(3, 3, -9)$

$$= \sqrt{(3 - 1)^2 + (3 - 0)^2 + (-9 + 3)^2} = 7$$

37. (b) Any plane through the intersection of given planes is

$$(x + 3y + 6) + \lambda(3x - y - 4z) = 0$$

$$\text{or } (1 + 3\lambda)x + (3 - \lambda)y - 4\lambda z + 6 = 0 \quad \dots (i)$$

Its perpendicular distance from $(0, 0, 0)$ is 1.

$$\therefore \frac{6}{[(1 + 3\lambda)^2 + (3 - \lambda)^2 + 16\lambda^2]^{1/2}} = 1$$

$$\Rightarrow 1 + 6\lambda + 9\lambda^2 + 9 + \lambda^2 - 6\lambda + 16\lambda^2 = 36$$

$$\Rightarrow 26\lambda^2 = 26 \Rightarrow \lambda = \pm 1$$

On putting the value of λ in Eq. (i), we get the required equations as

$$2x + y - 2z + 3 = 0 \text{ and } x - 2y - 2z - 3 = 0$$

38. (c) Since, the plane contains the first line, therefore it must be parallel to this line and direction ratios of the normal to plane be l, m, n , then

$$3l + m + 2n = 0$$

$$l + 2m + 3n = 0$$

$$\Rightarrow \frac{l}{-1} = \frac{m}{-7} = \frac{n}{5}$$

Equation of the plane L is

$$-1(x + 1) - 7(y + 2) + 5(z + 1) = 0$$

$$\Rightarrow x + 7y - 5z + 10 = 0$$

$$\text{Its distance from } (1, 1, 1) = \frac{|1 + 7 - 5 + 10|}{\sqrt{1^2 + 7^2 + (-5)^2}} = \frac{13}{\sqrt{75}} \text{ units}$$

39. (d) Let parallel plane be $2x - 2y + z + \lambda = 0$.

It passes through $(1, -2, 3)$.

$$\therefore \lambda = -9$$

The distance of $(-1, 2, 0)$ from the plane

$$2x - 2y + z - 9 = 0 \text{ is } \left| \frac{-2 - 4 - 9}{3} \right| = 5$$

40. (c) Given planes are

$$2x + y + 2z - 8 = 0 \text{ and } 2x + y + 2z + \frac{5}{2} = 0$$

Distance between two planes

$$\begin{aligned} &= \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2 + c^2}} = \frac{\left| -8 - \frac{5}{2} \right|}{\sqrt{(2)^2 + (1)^2 + (2)^2}} = \frac{\left| \frac{-21}{2} \right|}{\sqrt{9}} = \frac{21}{3} \\ &= \frac{21}{6} = \frac{7}{2} \end{aligned}$$

41. (c) Hint $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

42. (a) Let the coordinates of the foot of the perpendicular P from the origin to the plane be (x_1, y_1, z_1) .

Then, the direction ratios of the line OP are x_1, y_1, z_1 .

On writing the equation of the plane in the normal form, we get

$$\frac{2}{\sqrt{29}}x - \frac{3}{\sqrt{29}}y + \frac{4}{\sqrt{29}}z = \frac{6}{\sqrt{29}}$$

where, $\frac{2}{\sqrt{29}}, \frac{-3}{\sqrt{29}}, \frac{4}{\sqrt{29}}$ are the direction cosines of OP .

Since, direction cosines and direction ratios of a line are proportional.

$$\therefore \frac{x_1}{2} = \frac{y_1}{-3} = \frac{z_1}{4} = k$$

$$\text{i.e. } x_1 = \frac{2k}{\sqrt{29}}, y_1 = \frac{-3k}{\sqrt{29}}$$

$$\text{and } z_1 = \frac{4k}{\sqrt{29}}$$

On substituting these values in the equation of the plane, we get $k = \frac{6}{\sqrt{29}}$.

Thus, the foot of perpendicular is $\left(\frac{12}{29}, \frac{-18}{29}, \frac{24}{29} \right)$.

43. (b) Let the image of point $P(1, 3, 4)$ in the given plane be the point Q . The equation of the line through P and normal to the given plane is

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = r$$

Since, this line passes through Q , so let the coordinates of Q be $(2r+1, -r+3, r+4)$.

The coordinates of the mid-point of PQ are

$$\left(r+1, \frac{-r}{2} + 3, \frac{r}{2} + 4 \right)$$

This point lies on the given plane.

Therefore, $r = -2$.

Hence, the coordinates of Q are $(-3, 5, 2)$.

44. (c) Let $P(1, 6, 3)$ be the given point and L be the foot of perpendicular from P to the given line. The coordinates of a general point on the given line are

$$\frac{x-0}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

i.e.

$$x = \lambda$$

$$y = 2\lambda + 1$$

$$z = 3\lambda + 2$$

If the coordinates of L are $(\lambda, 2\lambda + 1, 3\lambda + 2)$, then the direction ratios of PL are $(\lambda - 1, 2\lambda - 5, 3\lambda - 1)$.

Since, the direction ratios of given line which is perpendicular to PL , are 1, 2 and 3. Therefore, $(\lambda - 1)1 + (2\lambda - 5)2 + (3\lambda - 1)3 = 0$, which gives $\lambda = 1$.

Hence, coordinates of L are $(1, 3, 5)$.

Let $Q(x_1, y_1, z_1)$ be the image of $P(1, 6, 3)$ in the given line. Then, L is the mid-point of PQ .

$$\text{Therefore, } \frac{x_1+1}{2} = 1, \frac{y_1+6}{2} = 3, \frac{z_1+3}{2} = 5$$

$$\Rightarrow x_1 = 1, y_1 = 0, z_1 = 7$$

Hence, the image of $(1, 6, 3)$ in the given line is $(1, 0, 7)$.

45. (b) Given equation of line is

$$\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5} \quad \dots(i)$$

and equation of plane is

$$2x - y + z + 3 = 0 \quad \dots(ii)$$

$$\begin{aligned} \therefore \frac{x-1}{2} &= \frac{y-3}{-1} = \frac{z-4}{1} \\ &= \frac{-2(2-3+4+3)}{4+1+1} \end{aligned}$$

$$\Rightarrow \frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = -2$$

Image of point $(1, 3, 4)$ on given line in the given plane is $(-3, 5, 2)$. Line is parallel to given plane.

Therefore, image is

$$\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$$

46. (c) Since, the vertices of the tetrahedron are $(0, 0, 0), (3, 0, 0), (0, 2, 0)$ and $(0, 0, 6)$.

$$\therefore \text{Volume of tetrahedron} = \frac{1}{6} \begin{vmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{vmatrix} = 6$$

BITSAT Archives

1. (d) Given lines are $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{\lambda}$... (i)

and $\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-2}{3}$... (ii)

These lines lie in the same plane. So, both are coplanar.

$$\therefore \begin{vmatrix} 2 & 3 & \lambda \\ 3 & 2 & 3 \\ 1 & -1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 2(-2+3) - 3(-3-3) + \lambda(-3-2) = 0$$

$$\Rightarrow 2 + 18 - 5\lambda = 0 \Rightarrow 5\lambda = 20 \Rightarrow \lambda = 4$$

$$\therefore \sin^{-1} \sin \lambda = \sin^{-1} \sin 4 = \sin^{-1} \sin(\pi - 4) = \pi - 4$$

2. (d) The new position of plane is

$$(x-2y+3z) + \lambda(x+y+z-1) = 0$$

$$\Rightarrow (1+\lambda)x + (\lambda-2)y + (3+\lambda)z - \lambda = 0$$

Given that this is perpendicular to

$$x + y + z = 1$$

$$\therefore (1+\lambda) \cdot 1 + (\lambda-2) \cdot 1 + (3+\lambda) \cdot 1 = 0$$

$$\Rightarrow 1 + \lambda + \lambda - 2 + 3 + \lambda = 0$$

$$\Rightarrow 3\lambda + 2 = 0 \Rightarrow \lambda = -\frac{2}{3}$$

Hence, the new position of the plane is

$$(x-2y+3z) - \frac{2}{3}(x+y+z-1) = 0$$

$$\Rightarrow 3x - 2x - 6y - 2y + 9z - 2z + 2 = 0$$

$$\Rightarrow x - 8y + 7z = -2$$

3. (a) Given equation of plane is $x + y + z = 5$.

The distance measured along the line $x = y = z$. Direction ratios of the given line is (1, 1, 1).

So, the equation of line PQ is

$$\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda \quad [\text{say}]$$

$$\Rightarrow x = \lambda + 1, y = \lambda + 5, z = \lambda + 9$$

lies on the plane $x + y + z = 5$

$$\therefore \lambda + 1 + \lambda + 5 + \lambda + 9 = 5 \Rightarrow \lambda = -10$$

The coordinate of Q is (-9, -15, -1) and the coordinate of P is (1, -5, 9).

$$PQ = \sqrt{(10)^2 + (10)^2 + (10)^2} = 10\sqrt{3}$$

$$\therefore 2\sqrt{3}k = 10\sqrt{3} \Rightarrow k = 5$$

4. (d) Given that, $2l - m + 2n = 0$... (i)

and $lm + mn + nl = 0$... (ii)

From Eq. (i), $m = 2(l + n)$

Put in Eq. (ii), we get

$$2l(l+n) + 2n(l+n) + nl = 0$$

$$\Rightarrow 2l^2 + 2nl + 2nl + 2n^2 + nl = 0$$

$$\Rightarrow 2l^2 + 5nl + 2n^2 = 0 \Rightarrow 2l^2 + 4nl + nl + 2n^2 = 0$$

$$\Rightarrow 2l(l+2n) + n(l+2n) = 0 \Rightarrow (l+2n)(n+2l) = 0$$

$$\Rightarrow l = -2n \text{ and } n = -2l$$

$$\text{If } l = -2n, \text{ then } m = 2(-2n+n) = -2n$$

$$\text{and if } n = -2l, \text{ then } m = 2(l-2l) = -2l$$

The DR's are 1, -2, -2 and -2, -2, 1.

$$\text{Now, } 1(-2) - (2)(-2) - 2(1) = -2 + 4 - 2 = 0$$

Hence, lines are perpendicular, so angle between them is $\pi/2$.

5. (d) The two normal vectors are $\mathbf{m} = 2\hat{i} + 3\hat{j} + \hat{k}$

and

$$\mathbf{n} = \hat{i} + 3\hat{j} + 2\hat{k}$$

$$\text{The line } L \text{ is along, } \mathbf{m} \times \mathbf{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 3 & 2 \end{vmatrix}$$

$$= (6-3)\hat{i} - (4-1)\hat{j} + (6-3)\hat{k}$$

$$= 3\hat{i} - 3\hat{j} + 3\hat{k} = 3(\hat{i} - \hat{j} + \hat{k})$$

Now, the direction cosines of X-axis are (1, 0, 0).

$$\therefore \cos \alpha = \frac{3(\hat{i} - \hat{j} + \hat{k}) \cdot \hat{i}}{\sqrt{3^2(1^2 + 1^2 + 1^2)}\sqrt{1}} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \cos \alpha = \frac{1}{\sqrt{3}}$$

6. (d) Let the parallel plane to $2x - 2y + z = 0$ be

$$2x - 2y + z + \lambda = 0$$

It passes through (1, -2, 3).

$$\therefore 2 + 4 + 3 + \lambda = 0 \Rightarrow \lambda = -9$$

The distance of (-1, 2, 0) from the plane

$$2x - 2y + z - 9 = 0 \text{ is}$$

$$\left| \frac{-2 - 4 - 9}{\sqrt{4 + 4 + 1}} \right| = \left| \frac{-15}{3} \right| = 5$$

7. (c) At the point on the line, where it intersects the curve, we have $z = 0$, so that

$$\frac{x-2}{3} = \frac{y+1}{2} = \frac{0-1}{-1}$$

$$\Rightarrow \frac{x-2}{3} = 1 \text{ and } \frac{y+1}{2} = 1 \Rightarrow x = 5 \text{ and } y = 1$$

Putting these values of x and y in $xy = c^2$, we get

$$c^2 = 5 \Rightarrow c = \pm \sqrt{5}$$

8. (a) If l, m, n are the direction cosines of the line, then

$$1 \cdot l - 1 \cdot m + 1 \cdot n = 0 \text{ and } 1 \cdot l - 3 \cdot m + 0 \cdot n = 0$$

$$\therefore \frac{l}{0+3} = \frac{m}{1-0} = \frac{n}{-3+1}$$

Hence, the direction ratios of the line are 3, 1, -2.

9. (b) Plane can be rewritten as $\frac{x}{6} + \frac{y}{-4} + \frac{z}{3} = 1$.

$\therefore$ Intercepts are 6, -4, 3.

10. (b) Given equation of line is $\frac{x-1}{3} = \frac{y+2}{4} = \frac{z-3}{-2} = k$ [say]

Any point on the line is $(3k+1, 4k-2, -2k+3)$.

If the given line intersect the plane $2x - y + 3z = 0$, then any point on the line lies in the plane.

$$\therefore 2(3k+1) - (4k-2) + 3(-2k+3) - 1 = 0$$

$$\Rightarrow -4k + 12 = 0 \Rightarrow k = 3$$

$$\therefore \text{Point is } (9+1, 12-2, -6+3) \text{ i.e. } (10, 10, -3).$$

11. (a)

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Limits, Continuity and Differentiability

Limits

Let $y = f(x)$ be a function of x . If at $x = a$, $f(x)$ takes indeterminate form, then we consider the values of the function which are very near to a . If these values tend to a definite unique number as x tends to a , then the unique number, so obtained called the limit of $f(x)$ at $x = a$ and we write it as $\lim_{x \rightarrow a} f(x)$.

NOTE If a function $f(x)$ takes any of the following form, when $x = a$, then we say that $f(x)$ is indeterminate at $x = a$.
 $0/0, \infty - \infty, \infty/\infty, 0 \times \infty, 0^0, \infty^0, 1^\infty$

Left Hand and Right Hand Limits

Consider the values of the functions at the points which are very near to a on the left of a . If these values tend to definite unique number as x tends to a from left, then the unique number, so obtained is called the **left hand limit** of $f(x)$ at $x = a$ and symbolically we write it as $f(a - 0) = \lim_{x \rightarrow a^-} f(x) = \lim_{h \rightarrow 0} f(a - h)$

Similarly, **right hand limit** can be expressed as

$$f(a + 0) = \lim_{x \rightarrow a^+} f(x) = \lim_{h \rightarrow 0} f(a + h)$$

Existence of Limit

$\lim_{x \rightarrow a} f(x)$ exists when

- (i) $\lim_{x \rightarrow a^-} f(x)$ and $\lim_{x \rightarrow a^+} f(x)$ exist i.e. LHL and RHL both exist.
- (ii) $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$ i.e. LHL = RHL

Fundamental Theorems on Limits

If $\lim_{x \rightarrow a} f(x) = l$ and $\lim_{x \rightarrow a} g(x) = m$ (where, l and m are real numbers), then

- (i) $\lim_{x \rightarrow a} \{f(x) + g(x)\} = l + m$ [sum rule]
- (ii) $\lim_{x \rightarrow a} \{f(x) - g(x)\} = l - m$ [difference rule]
- (iii) $\lim_{x \rightarrow a} \{f(x) \cdot g(x)\} = l \cdot m$ [product rule]
- (iv) $\lim_{x \rightarrow a} k \cdot f(x) = k \cdot l$ [constant multiple rule]
- (v) $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{l}{m}, m \neq 0$ [quotient rule]
- (vi) If $\lim_{x \rightarrow a} f(x) = \infty$ or $-\infty$, then $\lim_{x \rightarrow a} \frac{1}{f(x)} = 0$.
- (vii) $\lim_{x \rightarrow a} \log\{f(x)\} = \log\{\lim_{x \rightarrow a} f(x)\}$
- (viii) If $f(x) \leq g(x), \forall x$, then $\lim_{x \rightarrow a} f(x) \leq \lim_{x \rightarrow a} g(x)$.
- (ix) $\lim_{x \rightarrow a} \{f(x)\}^{g(x)} = \{\lim_{x \rightarrow a} f(x)\}^{\lim_{x \rightarrow a} g(x)}$
- (x) $\lim_{x \rightarrow a} f\{g(x)\} = f\{\lim_{x \rightarrow a} g(x)\} = f(m)$ provided f is continuous at $\lim_{x \rightarrow a} g(x) = m$.

Some Important Results

1. Trigonometric Limits

- (i) $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 = \lim_{x \rightarrow 0} \frac{x}{\sin x}$ (ii) $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 = \lim_{x \rightarrow 0} \frac{x}{\tan x}$
- (iii) $\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} = 1 = \lim_{x \rightarrow 0} \frac{x}{\sin^{-1} x}$
- (iv) $\lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x} = 1 = \lim_{x \rightarrow 0} \frac{x}{\tan^{-1} x}$ (v) $\lim_{x \rightarrow 0} \frac{\sin x^\circ}{x} = \frac{\pi}{180^\circ}$
- (vi) $\lim_{x \rightarrow 0} \cos x = 1$ (vii) $\lim_{x \rightarrow a} \frac{\sin(x-a)}{x-a} = 1$
- (viii) $\lim_{x \rightarrow a} \frac{\tan(x-a)}{x-a} = 1$ (ix) $\lim_{x \rightarrow a} \sin^{-1} x = \sin^{-1} a, |a| \leq 1$
- (x) $\lim_{x \rightarrow a} \cos^{-1} x = \cos^{-1} a, |a| \leq 1$
- (xi) $\lim_{x \rightarrow a} \tan^{-1} x = \tan^{-1} a, -\infty < a < \infty$
- (xii) $\lim_{x \rightarrow \infty} \frac{\sin x}{x} = \lim_{x \rightarrow \infty} \frac{\cos x}{x} = 0$ (xiii) $\lim_{x \rightarrow \infty} \frac{\sin 1/x}{1/x} = 1$

2. Logarithmic Limits

We use the series $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$, where $-1 < x \leq 1$ and expansion is true only, if base is e .

- (i) $\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1$
- (ii) $\lim_{x \rightarrow 0} \frac{\log(1-x)}{x} = -1$
- (iii) $\lim_{x \rightarrow 0} \frac{\log_a(1+x)}{x} = \log_a e; a > 0, \neq 1$

3. Exponential Limits

We use the series $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$.

- (i) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$
- (ii) $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a$
- (iii) $\lim_{x \rightarrow 0} \frac{e^{\lambda x} - 1}{x} = \lambda; \lambda \neq 0$

4. Based on the Form 1^∞

To evaluate the exponential form 1^∞ , we use following results :

- (i) If $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$, then

$$\lim_{x \rightarrow a} \{1 + f(x)\}^{1/g(x)} = e^{\lim_{x \rightarrow a} \frac{f(x)}{g(x)}}$$
- (ii) If $\lim_{x \rightarrow a} f(x) = 1$ and $\lim_{x \rightarrow a} g(x) = \infty$, then

$$\lim_{x \rightarrow a} \{f(x)\}^{g(x)} = e^{\lim_{x \rightarrow a} \{f(x) - 1\} g(x)}$$

Some Useful Expansions

- (i) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$
- (ii) $a^x = 1 + \frac{x \log a}{1!} + \frac{(x \log a)^2}{2!} + \dots$
- (iii) $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots, -1 < x < 1$
- (iv) $\frac{x^n - a^n}{x - a} = x^{n-1} + x^{n-2} \cdot a + x^{n-3} \cdot a^2 + \dots + a^{n-1}$
- (v) $\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$
- (vi) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots, x \in R$
- (vii) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots, x \in R$
- (viii) $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \dots, |x| < \frac{\pi}{2}$

L' Hospital's Rule

This result is applicable to only two indeterminate forms $\left(\frac{0}{0} \text{ or } \frac{\infty}{\infty}\right)$. This result states that, if $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ reduces to $\frac{0}{0}$ or $\frac{\infty}{\infty}$. Then, differentiate numerator and denominator till this form is removed.

i.e. $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$, provided the later limit exists.

Continuity

If the graph of a function has no break or gap, then it is called a continuous function. A function which is not continuous is called a discontinuous function.

Consider the following graphs :

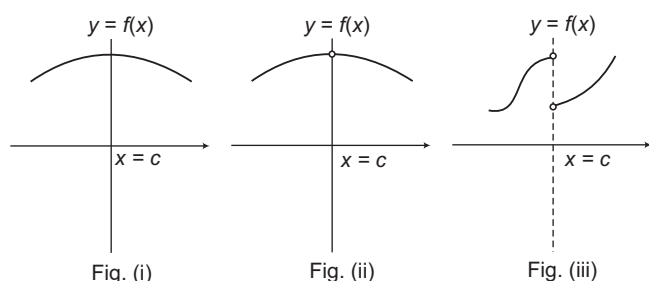


Fig. (i) shows that the function has no break, so it is continuous. Fig. (ii) and Fig. (iii) show a break at $x = c$, therefore it can be termed as discontinuous at $x = c$.

Continuity of a Function at a Point

A function $f(x)$ is said to be continuous at a point $x = a$ of its domain if and only if it satisfies the following conditions:

- (i) $f(a)$ exists (a lies in the domain of f)
- (ii) $\lim_{x \rightarrow a} f(x)$ exists i.e. $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$
or $RHL = LHL$
- (iii) $\lim_{x \rightarrow a} f(x) = f(a)$

Continuity in an Open Interval

A function $f(x)$ is said to be continuous in an open interval (a, b) , if it is continuous at each and every point of (a, b) .

Continuity in a Closed Interval

A function $f(x)$ is said to be continuous in a closed interval $[a, b]$, if

- (i) it is continuous in (a, b) .
- (ii) the value of the function at b is equal to left hand limit at b i.e. $f(b) = \lim_{x \rightarrow b^-} f(x)$.
- (iii) the value of the function at a is equal to right hand limit at a i.e. $f(a) = \lim_{x \rightarrow a^+} f(x)$.

Results on Continuous Functions

Let $f(x)$ and $g(x)$ be two continuous functions at $x = a$, then

- (i) $f(x) \pm g(x)$ is continuous at $x = a$.
- (ii) $f(x) \cdot g(x)$ is continuous at $x = a$.
- (iii) $f(x)/g(x)$ is continuous at $x = a$, provided $g(a) \neq 0$.

- (iv) the function $f(x)$ is said to be everywhere continuous, if it is continuous on the entire real line i.e. $(-\infty, \infty)$.
- (v) the integral function of a continuous function is a continuous function.
- (vi) the composition of function f and g , $(f \circ g)(x)$ is continuous at $x = a$.
- (vii) the $f(x)$ is a continuous function defined on $[a, b]$ such that $f(a)$ and $f(b)$ are of opposite signs, then there exists atleast one value of x for which $f(x)$ vanishes i.e. $f(a) > 0, f(b) < 0 \Rightarrow \exists c \in (a, b)$ such that $f(c) = 0$.

Some Important Points

- If $f(x)$ is continuous at $x = c$ and $g(x)$ is discontinuous at $x = a$, then
 - (i) $f + g$ and $f - g$ are discontinuous.
 - (ii) $f \cdot g$ may be continuous.
- If f and g are discontinuous at $x = c$, then $f + g, f - g$ and $f \cdot g$ may still be continuous.
- The function f will be discontinuous at $x = c$ in any of the following cases:
 - (i) $\lim_{x \rightarrow c^-} f(x)$ and $\lim_{x \rightarrow c^+} f(x)$ exist but are not equal.
 - (ii) $\lim_{x \rightarrow c^-} f(x)$ and $\lim_{x \rightarrow c^+} f(x)$ exist and are equal but not equal to $f(c)$.
 - (iii) $f(c)$ is not defined.
 - (iv) Atleast one of the limits does not exist.

Continuity of Some Common Functions

S.No.	Function $f(x)$	Intervals in which $f(x)$ is continuous
(i)	The constant function i.e. $f(x) = c$	$(-\infty, \infty)$
(ii)	The identity function i.e. $f(x) = x$	$(-\infty, \infty)$
(iii)	The polynomial function i.e. $f(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n$	$(-\infty, \infty)$
(iv)	$\sin x$	$(-\infty, \infty)$
(v)	$\cos x$	$(-\infty, \infty)$
(vi)	$ x - a $	$(-\infty, \infty)$
(vii)	e^x	$(-\infty, \infty)$
(viii)	x^n, n is an integer ≥ 0	$(-\infty, \infty)$
(ix)	x^{-n}, n is a positive integer	$(-\infty, \infty) - \{0\}$
(x)	$\frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials in x	$R - \{x : q(x) = 0\}$
(xi)	$\tan x$	$(-\infty, \infty) - \left\{(2n+1)\frac{\pi}{2} : n \in \mathbb{Z}\right\}$
(xii)	$\cot x$	$(-\infty, \infty) - \{n\pi : n \in \mathbb{Z}\}$
(xiii)	$\sec x$	$(-\infty, \infty) - \left\{(2n+1)\frac{\pi}{2} : n \in \mathbb{Z}\right\}$
(xiv)	$\operatorname{cosec} x$	$(-\infty, \infty) - \{n\pi : n \in \mathbb{Z}\}$
(xv)	$\log_e x$	$(0, \infty)$

Differentiability

Let f be a real valued function defined on an open interval (a, b) and $c \in (a, b)$. Then, $f(x)$ is said to be differentiable or derivable at $x = c$, if $\lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$ exists finitely.

This limit is called the derivative or differential coefficient of the function $f(x)$ at $x = c$ and is denoted by $f'(c)$ or $D f(c)$ or $\left. \frac{d}{dx} f(x) \right|_{x=c}$. If we vary the point c over all numbers for

which the limit, $\lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$ exists, then we obtain f' or $f'(x)$ and it is called derivative of f .

Let $f(x)$ be a function and c be a point in the domain of f . Then, the right hand derivative (RHD) denoted by $Rf'(c)$ is defined as

$$\begin{aligned} \text{RHD} = Rf'(c) &= \lim_{x \rightarrow c^+} \frac{f(x) - f(c)}{x - c} \\ \text{RHD} &= \lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h} \end{aligned}$$

and the left hand derivative (LHD) denoted by $Lf'(c)$ is defined as

$$\begin{aligned} \text{LHD} = Lf'(c) &= \lim_{x \rightarrow c^-} \frac{f(x) - f(c)}{x - c} \\ \text{LHD} &= \lim_{h \rightarrow 0} \frac{f(c-h) - f(c)}{-h} \end{aligned}$$

Another Form of Differentiability

A function $f(x)$ is said to be differentiable at a point c in its domain, if

Left hand derivative = Right hand derivative

or

$$\text{LHD} = \text{RHD}$$

i.e.

$$Lf'(c) = Rf'(c)$$

$$\Rightarrow \lim_{x \rightarrow c^-} \frac{f(x) - f(c)}{x - c} = \lim_{x \rightarrow c^+} \frac{f(x) - f(c)}{x - c}$$

If $Lf'(c) \neq Rf'(c)$, then function $f(x)$ is not differentiable at $x = c$.

NOTE

- Every differentiable function is continuous.
- But the converse of this statement is not true, i.e. every continuous function may not be differentiable.
- Geometrically, $f(x)$ is differentiable at a point P , if the curve does not have P as a corner point, i.e. the function is not differentiable at those points on which function has jumps (or holes) and sharp edges.

Differentiability in an Interval

A function $y = f(x)$ defined on an **open interval** (a, b) is said to be differentiable in an open interval (a, b) , if it is differentiable at each point of (a, b) .

A function $y = f(x)$ defined on a **closed interval** $[a, b]$ is said to be differentiable in closed interval $[a, b]$, if it is differentiable at each point of an open interval (a, b)

and $\lim_{x \rightarrow a^+} \frac{f(x) - f(a)}{x - a}$ and $\lim_{x \rightarrow b^-} \frac{f(x) - f(b)}{x - b}$ both exist. A

function f is said to be differentiable, if it is differentiable at every point of its domain.

Some Standard Results on Differentiability

- Every polynomial function, every exponential function $a^x (a > 0)$ and every constant function are differentiable at each $x \in R$.
- The logarithmic functions, trigonometrical functions and inverse trigonometrical functions are always differentiable in their domains.
- The sum, difference, product and quotient of two differentiable functions is differentiable.
- The composite of differentiable function is a differentiable function.
- Absolute functions are always continuous throughout but not differentiable at their critical points.

Practice Exercise

- The value of $\lim_{x \rightarrow 2} \frac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}}$ is
 a. 16 b. 8 c. 4 d. 2
- Evaluate $\lim_{x \rightarrow 0} \frac{\sin(3x+a) - 3\sin(2x+a) + 3\sin(x+a) - \sin a}{x^3}$.
 a. 0 b. $\cos a$ c. $-\cos a$ d. $\sin a$
- If $f(x) = \frac{\sin[x]}{[x]}$, $[x] \neq 0$, where $[x]$ denotes the greatest integer less than or equal to x , then $\lim_{x \rightarrow 0} f(x)$ equals
 a. -1 b. 0 c. 1 d. None
- $\lim_{x \rightarrow 2} \left(\frac{\sqrt{1 - \{\cos 2(x-2)\}}}{x-2} \right)$ is equal to
 a. $\sqrt{2}$ b. $-\sqrt{2}$
 c. $\frac{1}{\sqrt{2}}$ d. Does not exist
- The value of $\lim_{x \rightarrow 0} \frac{1}{x} \left[\tan^{-1} \left(\frac{x+1}{2x+1} \right) - \frac{\pi}{4} \right]$ is
 a. 1 b. $-\frac{1}{2}$ c. 2 d. 0
- $\lim_{x \rightarrow \infty} \left[\sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x} \right]$ is equal to
 a. 0 b. $\frac{1}{2}$ c. $\log 2$ d. e^4
- $\lim_{x \rightarrow 1} [x-1]$, where $[]$ denotes the greatest integer function, is equal to
 a. 1 b. 2
 c. 0 d. Does not exist
- The value of $\lim_{x \rightarrow \pi} \frac{1 + \cos^3 x}{\sin^2 x}$ is
 a. $\frac{1}{3}$ b. $\frac{3}{2}$ c. $-\frac{1}{4}$ d. $-\frac{3}{2}$
- The value of $\lim_{x \rightarrow \infty} \frac{(x+2)! + (x+1)!}{(x+2)! - (x+1)!}$ is
 a. 1 b. 2 c. 3 d. 4
- $\lim_{a \rightarrow \infty} \left[\frac{1}{1-a^4} + \frac{8}{1-a^4} + \dots + \frac{a^3}{1-a^4} \right]$ is equal to
 a. $\frac{1}{3}$ b. $\frac{1}{4}$ c. $-\frac{1}{3}$ d. $-\frac{1}{4}$
- If $f(x) = \begin{cases} x+2, & x \leq -1 \\ cx^2, & x > -1 \end{cases}$, then find c , if $\lim_{x \rightarrow -1} f(x)$ exists.
 a. -1 b. 1 c. 0 d. 2
- The value of $\lim_{x \rightarrow \pi/4} \frac{2 - \cot x - \cot^3 x}{1 - \cot^3 x}$ is
 a. $\frac{4}{3}$ b. 1
 c. 0 d. None of these
- Find the value of $\lim_{x \rightarrow 0} \frac{\sin x - 2 \sin 3x + \sin 5x}{x}$.
 a. -1 b. 0
 c. 1 d. 3
- If $f(x) = \begin{cases} x^2 - 1, & 0 < x < 2 \\ 2x + 3, & 2 \leq x < 3 \end{cases}$, then quadratic equation whose roots are $\lim_{x \rightarrow 2^-} f(x)$ and $\lim_{x \rightarrow 2^+} f(x)$, is
 a. $x^2 - 6x + 9 = 0$ b. $x^2 - 7x + 8 = 0$
 c. $x^2 - 14x + 49 = 0$ d. $x^2 - 10x + 21 = 0$
- $\lim_{x \rightarrow 0} \frac{e^{x^2} - \cos x}{x^2}$ is equal to
 a. $\frac{3}{2}$ b. $\frac{1}{2}$
 c. $\frac{2}{3}$ d. None of these
- The value of $\lim_{x \rightarrow 0} \frac{(1+x)^{1/x} - e}{x}$ is
 a. 1 b. $\frac{e}{2}$ c. $-\frac{e}{2}$ d. $\frac{2}{e}$
- The values of a and b such that $\lim_{x \rightarrow 0} \frac{x(1+a \cos x) - b \sin x}{x^3} = 1$, are
 a. $\frac{5}{3}, \frac{3}{2}$ b. $\frac{5}{2}, -\frac{3}{2}$
 c. $-\frac{5}{2}, -\frac{3}{2}$ d. None of these
- $\lim_{n \rightarrow \infty} \sin[\pi\sqrt{n^2+1}]$ is equal to
 a. ∞ b. 0
 c. Does not exist d. None of these
- The value of $\lim_{x \rightarrow \infty} \left(\frac{3x-4}{3x+2} \right)^{\frac{x+1}{3}}$ is
 a. $e^{-1/3}$ b. $e^{-2/3}$ c. e^{-1} d. e^{-2}
- If $\lim_{x \rightarrow 1} \frac{x^4 - 1}{x - 1} = \lim_{x \rightarrow k} \frac{x^3 - k^3}{x^2 - k^2}$, then the value of k is
 a. $-\frac{8}{3}$ b. $\frac{5}{3}$
 c. $-\frac{5}{3}$ d. None of these

21. $\lim_{x \rightarrow \infty} \left(\frac{x+1}{x+2} \right)^{2x+1}$ is equal to
 a. e^3 b. e^{-3} c. e^{-2} d. e^2
22. $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$ is equal to
 a. $-\frac{1}{5}$ b. $-\frac{1}{6}$
 c. -1 d. -6
23. $\lim_{x \rightarrow 0} \left[\frac{1^x + 2^x + 3^x + \dots + n^x}{n} \right]^{\frac{a}{x}}$ is equal to
 a. $(n!)^{a/n}$ b. $(n!)^a$
 c. $\frac{1}{2a(n!)}$ d. None of these
24. The polynomial of least degree, such that
 $\lim_{x \rightarrow 0} \left[1 + \frac{x^2 + f(x)}{x^2} \right]^{1/x} = e^2$, is
 a. $2x^3 + x^2$ b. $2x^3 - x^2$
 c. $2x^2 + 3x^3$ d. None of these
25. $\lim_{x \rightarrow 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3}$ is equal to
 a. $\frac{1}{2}$ b. 2
 c. $-\frac{1}{2}$ d. None of these
26. The value of p for which the function

$$f(x) = \begin{cases} \frac{(4^x - 1)^3}{\sin\left(\frac{x}{p}\right) \log\left(1 + \frac{x^2}{3}\right)}, & x \neq 0 \\ 12(\log 4)^3, & x = 0 \end{cases}$$
 may be continuous at $x = 0$, is
 a. 1 b. 2
 c. 3 d. None of these
27. The value of $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{2/x}$,
 where $(a, b, c > 0)$ is
 a. $(abc)^3$ b. abc
 c. $(abc)^{1/3}$ d. None of these
28. The values of a, b and c , such that
 $\lim_{x \rightarrow 0} \frac{ae^x - b \cos x + ce^{-x}}{x \sin x} = 2$, are
 a. $a = 1, b = -2, c = 1$ b. $a = 1, b = 2, c = -1$
 c. $a = 1, b = 2, c = 1$ d. $a = -1, b = 2, c = 1$
29. If $\lim_{n \rightarrow \infty} \left(an - \frac{1+n^2}{1+n} \right) = b$, where a is finite number, then
 a. $a = 2$ b. $a = 0$ c. $b = 1$ d. $b = -1$
30. The value of $f(0)$, so that the function
 $f(x) = \frac{(27 - 2x)^{1/3} - 3}{9 - 3(243 + 5x)^{1/5}}$ ($x \neq 0$) is continuous, is given
 by
 a. $2/3$ b. 6 c. 2 d. 4
31. If $f(x) = \begin{cases} |x| \cos\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$, then $f(x)$ is
 a. discontinuous at $x = 0$ b. continuous at $x = 0$
 c. Does not exist d. None of these
32. For what value of k , the function

$$f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & \text{if } -1 \leq x < 0 \\ \frac{2x+1}{x-2}, & \text{if } 0 \leq x \leq 1 \end{cases}$$
 is continuous at $x = 0$?
 a. $\frac{1}{2}$ b. 1 c. $-\frac{3}{2}$ d. $-\frac{1}{2}$
33. If $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then
 a. $f(0+0) = 1$
 b. $f(0-0) = 1$
 c. $f(x)$ is continuous at $x = 0$
 d. None of the above
34. Let $f(x) = -1 + |x - 2|$ and $g(x) = 1 - |x|$, then the set of all points where $f \circ g$ is discontinuous, is
 a. $\{0, 2\}$ b. $\{0, 1, 2\}$
 c. $\{0\}$ d. an empty set
35. If $f(x) = \begin{cases} |x| + 3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \geq 3 \end{cases}$, then $f(x)$ is
 a. continuous at $x = -3$ and discontinuous at $x = 3$
 b. continuous at $x = -3, 3$
 c. discontinuous at $x = -3, 3$
 d. continuous at $x = 3$ and discontinuous at $x = -3$
36. If $f(x) = \frac{3x + 4 \tan x}{x}$ is to be defined to make continuous at $x = 0$, then the defined function should be
 a. $f(x) = \begin{cases} \frac{3x + 4 \tan x}{x}, & x \neq 0 \\ 7, & x = 0 \end{cases}$
 b. $f(x) = \begin{cases} \frac{3x + 4 \tan x}{x}, & x \neq 0 \\ 6, & x = 0 \end{cases}$
 c. $f(x) = \begin{cases} \frac{3x + 4 \tan x}{x}, & x \neq 0 \\ 7, & x = 0 \end{cases}$
 d. None of the above

37. The value of $f(0)$, so that the function $f(x) = \frac{2x - \sin^{-1}x}{2x + \tan^{-1}x}$ is continuous at each point in its domain, is

a. $\frac{1}{3}$ b. $-\frac{1}{3}$ c. $\frac{2}{3}$ d. $-\frac{2}{3}$

38. For what value of k , function $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$

is continuous at $x = \frac{\pi}{2}$?

a. 1 b. 3
c. R d. 6

39. If $f(x) = \begin{cases} |x - a| \sin \frac{1}{x - a}, & \text{if } x \neq a \\ 0, & \text{if } x = a \end{cases}$, then $f(x)$ is

a. continuous at $x = a$
b. discontinuous at $x = a$
c. discontinuous for all $x \in R$
d. None of the above

40. The values of p and q for which the function

$$f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}}, & x > 0 \end{cases}$$

is continuous for all x in R , are

a. $p = \frac{5}{2}, q = \frac{1}{2}$ b. $p = -\frac{3}{2}, q = \frac{1}{2}$
c. $p = \frac{1}{2}, q = \frac{3}{2}$ d. $p = \frac{1}{2}, q = -\frac{3}{2}$

41. If $f(x) = \begin{cases} x^k \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$,

then

a. $k \in (-\infty, 0)$ b. $k \in (1, \infty)$
c. $k \in (-1, \infty)$ d. None of these

42. If $f(x) = \frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{x}$, then the value of f at

$x = 0$, so that f is continuous everywhere, is

a. $1/4$ b. -1 c. 1 d. 2

43. If the function $f(x)$ defined by

$$f(x) = \begin{cases} \left(x^2 + e^{\frac{1}{2-x}}\right)^{-1}, & x \neq 2 \\ c, & x = 2 \end{cases}$$

is continuous from right at $x = 2$, then c is equal to

a. $\frac{1}{2}$ b. $\frac{1}{3}$ c. $\frac{1}{4}$ d. $\frac{1}{5}$

44. If $f(x) = \frac{(3^x - 1)^2}{\sin x \cdot \log_e(1+x)}$, $x \neq 0$, is continuous at $x = 0$, then $f(0)$ is

a. $\log_e 3$ b. $2 \log_e 3$ c. $(\log_e 3)^2$ d. None

45. If a function $f: R \rightarrow R$ satisfy the equation $f(x+y) = f(x) + f(y)$, $\forall x, y$ and the function $f(x)$ is continuous at $x = 0$, then

a. $f(x)$ is continuous for all positive real values of x
b. $f(x)$ is continuous for all x
c. $f(x) = 0$ for all x
d. None of the above

46. The values of a and b such that the function f defined by

$$f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, & \text{if } x < 4 \\ a+b, & \text{if } x = 4 \\ \frac{x-4}{|x-4|} + b, & \text{if } x > 4 \end{cases}$$

is continuous function at $x = 4$, are

a. $a = 1$ and $b = -1$ b. $a = -1$ and $b = 1$
c. $a = 0$ and $b = -1$ d. $a = 1$ and $b = 0$

47. If $f(x) = \frac{\sqrt{2} \cos x - 1}{\cot x - 1}$, $x \neq \frac{\pi}{4}$, then the value of $f\left(\frac{\pi}{4}\right)$, so

that $f(x)$ becomes continuous at $x = \frac{\pi}{4}$, is

a. $\frac{1}{2}$ b. $-\frac{1}{2}$ c. 1 d. 9

48. If $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2}, & \text{if } x < 0 \\ a, & \text{if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}, & \text{if } x > 0 \end{cases}$, then what value of a ,

f is continuous at $x = 0$?

a. 2 b. 4 c. 6 d. 8

49. If the function

$$f(x) = \begin{cases} x + a^2 \sqrt{2} \sin x, & 0 \leq x < \frac{\pi}{4} \\ x \cot x + b, & \frac{\pi}{4} \leq x < \frac{\pi}{2} \\ b \sin 2x - a \cos 2x, & \frac{\pi}{2} \leq x \leq \pi \end{cases}$$

is continuous in the interval $[0, \pi]$, then the values of a and b are respectively

a. $(0, 0)$ b. $\left(0, \frac{1}{2}\right)$ c. $(0, 1)$ d. $(-1, 1)$

50. If $f(x) = \frac{a \sin x + \sin 2x}{x^3}$, $x \neq 0$ and $f(x)$ is continuous at

$x = 0$, then

a. $a = 2$ b. $f(0) = 1$ c. $f(0) = -1$ d. $a = 1$

51. If $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$, then the value of $\lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{h^3 + 3h}$ is
 a. $\frac{53}{3}$ b. $\frac{22}{3}$ c. 13 d. $\frac{22}{13}$
52. If $f'(2) = 6$ and $f'(1) = 4$, then $\lim_{h \rightarrow 0} \frac{f(2h + 2 + h^2) - f(2)}{f(h - h^2 + 1) - f(1)}$ is equal to
 a. 3 b. $-3/2$
 c. $3/2$ d. Does not exist
53. If $f(x) = \begin{cases} e^x, & x \leq 0 \\ |1-x|, & x > 0 \end{cases}$, then
 a. $f(x)$ is differentiable at $x = 0$
 b. $f(x)$ is continuous at $x = 0, 1$
 c. $f(x)$ is differentiable at $x = 1$
 d. None of the above
54. The function defined by $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \frac{1}{4}x^2 - \frac{3}{2}x + \frac{13}{4}, & x < 1 \end{cases}$ is
 a. continuous at $x = 1$ b. continuous at $x = 3$
 c. differentiable at $x = 1$ d. All of these
55. If $\lim_{x \rightarrow 0} \frac{\log(a+x) - \log a}{x} + k \lim_{x \rightarrow 0} \frac{\log x - 1}{x - e} = 1$, then
 a. $k = e \left(1 - \frac{1}{a}\right)$ b. $k = e(1+a)$
 c. $k = e(2-a)$ d. equality is not possible
56. If $f(x) = \begin{cases} -\left(\frac{1}{|x|} + \frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$, then $f(x)$ is
 a. continuous as well as differentiable for all x
 b. continuous for all x but not differentiable at $x = 0$
 c. neither differentiable nor continuous at $x = 0$
 d. discontinuous everywhere
57. Consider the greatest integer function, defined by $f(x) = [x]$, $0 \leq x < 2$. Then,
 a. f is derivable at $x = 1$ b. f is not derivable at $x = 1$
 c. f is derivable at $x = 2$ d. None of these
58. Let $f(x) = \lambda + \mu|x| + \nu|x|^2$, where λ, μ, ν are real constants. Then, $f'(0)$ exists, if
 a. $\mu = 0$ b. $\nu = 0$ c. $\lambda = 0$ d. $\mu = \nu$
59. Consider a function $f: R \rightarrow R$ which satisfies the equation $f(x+y) = f(x) \cdot f(y)$, $\forall x, y \in R$, $f(x) \neq 0$. Suppose that the function is differentiable at $x = 0$ and $f'(0) = 2$. Then, $f'(x)$ is equal to
 a. $f(x)$ b. $2f(x)$ c. $\frac{1}{2}f(x)$ d. None
60. Consider the function $f(x)$, defined by

$$f(x) = \begin{cases} x, & x \leq 1 \\ x^2 + bx + c, & x > 1 \end{cases}$$

 Let $f'(x)$ exists finitely, $\forall x \in R$. Then,
 a. $b = -1, c \in R$
 b. $c = 1, b \in R$
 c. $b = 1, c = -1$
 d. $b = -1, c = 1$
61. Consider the function $f(x) = |\log_e x|$, $\forall x > 0$. Then,
 a. LHD does not exist at $x = 1$
 b. RHD does not exist at $x = 1$
 c. f is differentiable at $x = 1$
 d. f is not differentiable at $x = 1$
62. Consider the function $f(x)$ defined by
 $f(x) = |x-2| + |x| + |x+2|$. Then,
 a. f is derivable at $x = 0, 2$
 b. f is derivable at $x = -2, 0$
 c. f is derivable at $x = -2, 2$
 d. f is not derivable at $x = -2, 0, 2$

BITSAT Archives

1. $\lim_{x \rightarrow 0} \frac{(1+x)^8 - 1}{(1+x)^2 - 1}$ is equal to [2014]
 a. 8 b. 6 c. 4 d. 2
2. $\lim_{x \rightarrow \tan^{-1} 3} \frac{\tan^2 x - 2 \tan x - 3}{\tan^2 x - 4 \tan x + 3}$ is equal to [2014]
 a. 1 b. 2 c. 0 d. 3
3. $\lim_{x \rightarrow -\infty} \frac{x^4 \cdot \sin\left(\frac{1}{x}\right) + x^2}{1 + |x|^3}$ is equal to [2014]
 a. 0 b. -1 c. 2 d. 1
4. $\lim_{x \rightarrow 1} \frac{x^m - 1}{x^n - 1}$ is equal to [2014]
 a. $\frac{n}{m}$ b. $\frac{m}{n}$
 c. $\frac{2m}{n}$ d. $\frac{2n}{m}$
5. If $f(x) = \begin{cases} \left[\tan\left(\frac{\pi}{4} + x\right)\right]^{1/x}, & x \neq 0 \\ k, & x = 0 \end{cases}$
 For what value of k , $f(x)$ is continuous at $x = 0$? [2014]
 a. 1 b. 0 c. e d. e^2

6. The value of $\lim_{x \rightarrow 0} \left(\frac{1+5x^2}{1+3x^2} \right)^{1/x^2}$ is [2013]
 a. e^2 b. e c. $1/e$ d. $1/e^2$
7. $\lim_{x \rightarrow 0} \left(\frac{(2+x)\sin(2+x) - 2\sin 2}{x} \right)$ is equal to [2012]
 a. $\sin 2$ b. $\cos 2$ c. 1 d. $2\cos 2 + \sin 2$
8. If $f(x) = \frac{3x + \tan^2 x}{x}$ is continuous at $x = 0$, then $f(0)$ is equal to [2012]
 a. 3 b. 2 c. 4 d. 0
9. If $f(x) = \frac{\log(1+ax) - \log(1-bx)}{x}$ for $x \neq 0$ and $f(0) = k$ and $f(x)$ is continuous at $x = 0$, then k is equal to [2012]
 a. $a+b$ b. $a-b$ c. a d. b
10. $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ is equal to [2011]
 a. 2 b. -1 c. 1 d. 0
11. $\lim_{x \rightarrow 0} \frac{\cos(\sin x) - 1}{x^2}$ is equal to [2011]
 a. 1 b. -1 c. $1/2$ d. $-1/2$
12. In order that the function $f(x) = (x+1)^{1/x}$ is continuous at $x = 0$, $f(0)$ must be defined as [2011]
 a. $f(0) = 0$ b. $f(0) = e$ c. $f(0) = 1/e$ d. $f(0) = 1$
13. The function $f(x) = |x|$ at $x = 0$ is [2011]
 a. continuous but non-differentiable
 b. discontinuous and differentiable
 c. discontinuous and non-differentiable
 d. continuous and differentiable
14. $\lim_{x \rightarrow 0} (\operatorname{cosec} x)^{1/\log x}$ is equal to [2010]
 a. 0 b. 1
 c. $1/e$ d. None of these
15. The value of $\lim_{x \rightarrow \infty} \left(\frac{x+6}{x+1} \right)^{x+4}$ is [2010]
 a. e b. e^2 c. e^4 d. e^5
16. The set of points, where the function $f(x) = x|x|$ is differentiable, is [2010]
 a. $(-\infty, \infty)$ b. $(-\infty, 0) \cup (0, \infty)$
 c. $(0, \infty)$ d. $[0, \infty)$
17. $\lim_{x \rightarrow 2} \frac{\sqrt{1+\sqrt{2+x}} - \sqrt{3}}{x-2}$ is equal to [2009]
 a. $\frac{1}{8\sqrt{3}}$ b. $\frac{1}{\sqrt{3}}$ c. $8\sqrt{3}$ d. $\sqrt{3}$
18. $\lim_{x \rightarrow 1} (1-x) \tan\left(\frac{\pi x}{2}\right)$ is equal to [2009]
 a. $\frac{\pi}{2}$ b. $\frac{2}{\pi}$ c. $-\frac{\pi}{2}$ d. $-\frac{2}{\pi}$
19. If $f: R \rightarrow R$ is defined by $f(x) = [x-3] + |x-4|$ for $x \in R$, then $\lim_{x \rightarrow 3^-} f(x)$ is equal to [2008]
 a. -2 b. -1 c. 0 d. 1
20. If $f: R \rightarrow R$ is defined by [2008]

$$f(x) = \begin{cases} \frac{\cos 3x - \cos x}{x^2}, & \text{for } x \neq 0 \\ \lambda, & \text{for } x = 0 \end{cases}$$
 and if f is continuous at $x = 0$, then λ is equal to
 a. -2 b. -4 c. -6 d. -8
21. If $f(2) = 4$ and $f'(2) = 1$, then $\lim_{x \rightarrow 2} \frac{x f(2) - 2f(x)}{x-2}$ is equal to [2008]
 a. -2 b. 1 c. 2 d. 3
22. If $\lim_{x \rightarrow \infty} \left[\frac{x^3+1}{x^2+1} - (ax+b) \right] = 2$, then [2007]
 a. $a = 1$ and $b = 1$ b. $a = 1$ and $b = -1$
 c. $a = 1$ and $b = -2$ d. $a = 1$ and $b = 2$
23. If $f(x) = \begin{cases} \frac{1-\cos x}{x}, & x \neq 0 \\ x, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is [2007]
 a. 0 b. $\frac{1}{2}$ c. $\frac{1}{4}$ d. $-\frac{1}{2}$
24. $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$ is equal to [2006]
 a. $1/2$ b. $-1/2$ c. 0 d. 1
25. If $f(x) = \begin{cases} \frac{\sin 5x}{x^2+2x}, & x \neq 0 \\ k + \frac{1}{2}, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is [2006]
 a. 1 b. -2
 c. 2 d. $\frac{1}{2}$
26. The value of the constant α and β such that $\lim_{x \rightarrow \infty} \left(\frac{x^2+1}{x+1} - \alpha x - \beta \right) = 0$ are respectively [2005]
 a. (1, 1) b. (-1, 1) c. (1, -1) d. (0, 1)
27. $\lim_{\theta \rightarrow 0} \frac{4\theta(\tan \theta - 2\theta \tan \theta)}{(1 - \cos 2\theta)}$ is equal to [2005]
 a. $1\sqrt{2}$ b. $1/2$
 c. 1 d. 2
28. If $f(x) = \begin{cases} 1, & x < 0 \\ 1 + \sin x, & 0 \leq x \leq \pi/2 \end{cases}$, then what is the value of $f'(x)$ at $x = 0$? [2005]
 a. 1 b. -1
 c. ∞ d. Does not exist

Answer with Solutions

Practice Exercise

$$\begin{aligned}
 1. (b) \lim_{x \rightarrow 2} \frac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}} &= \lim_{x \rightarrow 2} \frac{(2^x)^2 - 6 \cdot 2^x + 8}{\sqrt{2^x} - 2} \\
 &= \lim_{x \rightarrow 2} \frac{(2^x - 4)(2^x - 2)(\sqrt{2^x} + 2)}{(\sqrt{2^x} - 2)(\sqrt{2^x} + 2)} \\
 &= \lim_{x \rightarrow 2} \frac{(2^x - 4)(2^x - 2)(\sqrt{2^x} + 2)}{(2^x - 4)} \\
 &= \lim_{x \rightarrow 2} (2^x - 2)(\sqrt{2^x} + 2) \\
 &= (4 - 2)(2 + 2) = 8 \\
 2. (c) \lim_{x \rightarrow 0} \frac{2 \sin\left(\frac{3x}{2}\right) \cos\left(\frac{3x + 2a}{2}\right) - 3\left(2 \sin \frac{x}{2} \cos \frac{3x + 2a}{2}\right)}{x^3} \\
 &= \lim_{x \rightarrow 0} \frac{2 \cos\left(\frac{3x + 2a}{2}\right) \left[3 \sin \frac{x}{2} - 4 \sin^3 \frac{x}{2} - 3 \sin \frac{x}{2}\right]}{x^3} \\
 &= \lim_{x \rightarrow 0} 2 \cos\left(\frac{3x + 2a}{2}\right) \left\{ \frac{-4(x/2)^3}{x^3} \right\} \quad [\because \lim_{\theta \rightarrow 0} \sin \theta = \theta] \\
 &= 2 \cos a \left(-\frac{4}{8}\right) = -\cos a
 \end{aligned}$$

$$\begin{aligned}
 3. (d) \text{LHL} &= \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} \frac{\sin(0 - h)}{(0 - h)} \\
 &= \lim_{h \rightarrow 0} \frac{\sin(-1)}{(-1)} = \sin 1 \\
 \text{RHL} &= \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} \frac{\sin(0 + h)}{(0 + h)} = 0 \quad [\text{given}]
 \end{aligned}$$

Thus, LHL $\neq$ RHL

Hence, limit does not exist.

$$\begin{aligned}
 4. (d) \lim_{x \rightarrow 2} \frac{\sqrt{1 - \cos 2(x - 2)}}{x - 2} &= \lim_{x \rightarrow 2} \frac{\sqrt{2 \sin^2(x - 2)}}{(x - 2)} \\
 &= \frac{\sqrt{2} |\sin(x - 2)|}{(x - 2)} = \lim_{x \rightarrow 2} \frac{\sqrt{2} |\sin(x - 2)|}{(x - 2)}
 \end{aligned}$$

RHL at $x = 2$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{\sqrt{2} |\sin(2 + h - 2)|}{(2 + h) - 2} = \lim_{h \rightarrow 0} \frac{\sqrt{2} |\sin h|}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{2} \sin h}{h} = \sqrt{2}
 \end{aligned}$$

LHL at $x = 2$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{\sqrt{2} |\sin(2 - h - 2)|}{(2 - h) - 2} = \lim_{h \rightarrow 0} \frac{\sqrt{2} |\sin(-h)|}{-h} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{2} \sin h}{-h} = -\sqrt{2}
 \end{aligned}$$

So, limit does not exist.

5. (b)

$$\begin{aligned}
 6. (b) \lim_{x \rightarrow \infty} \left[\sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x} \right] &= \lim_{x \rightarrow \infty} \frac{x + \sqrt{x + \sqrt{x}} - x}{\sqrt{x + \sqrt{x + \sqrt{x}}} + \sqrt{x}} \\
 &= \lim_{x \rightarrow \infty} \frac{\sqrt{x + \sqrt{x}}}{\sqrt{x + \sqrt{x + \sqrt{x}}} + \sqrt{x}} = \lim_{x \rightarrow \infty} \frac{\sqrt{1 + x^{-1/2}}}{\sqrt{1 + \sqrt{x^{-1} + x^{-3/2}}} + 1} = \frac{1}{2}
 \end{aligned}$$

7. (d) Since, $\lim_{x \rightarrow 1^+} [x - 1] = 0$ and $\lim_{x \rightarrow 1^-} [x - 1] = -1$

So, $\lim_{x \rightarrow 1} [x - 1]$ does not exist.

$$\begin{aligned}
 8. (b) \lim_{x \rightarrow \pi} \frac{1 + \cos^3 x}{\sin^2 x} &= \lim_{x \rightarrow \pi} \frac{(1 + \cos x)(1 + \cos^2 x - \cos x)}{(1 + \cos x)(1 - \cos x)} \\
 &= \lim_{x \rightarrow \pi} \frac{1 + \cos^2 x - \cos x}{1 - \cos x} = \frac{1 + (-1)^2 - (-1)}{1 - (-1)} = \frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 9. (a) \lim_{x \rightarrow \infty} \frac{(x + 2)! + (x + 1)!}{(x + 2)! - (x + 1)!} &= \lim_{x \rightarrow \infty} \frac{(x + 2)(x + 1)! + (x + 1)!}{(x + 2)(x + 1)! - (x + 1)!} \\
 &= \lim_{x \rightarrow \infty} \frac{x + 2 + 1}{x + 2 - 1} = \lim_{x \rightarrow \infty} \frac{x + 3}{x + 1} = \lim_{x \rightarrow \infty} \frac{1 + \frac{3}{x}}{1 + \frac{1}{x}} \\
 &\quad \left[\text{as } x \rightarrow \infty, \frac{1}{x} \rightarrow 0 \right]
 \end{aligned}$$

$$= \frac{1}{1} = 1$$

$$\begin{aligned}
 10. (d) \lim_{a \rightarrow \infty} \left[\frac{1}{1 - a^4} + \frac{8}{1 - a^4} + \frac{27}{1 - a^4} + \dots + \frac{a^3}{1 - a^4} \right] \\
 &= \lim_{a \rightarrow \infty} \left[\frac{1^3}{1 - a^4} + \frac{2^3}{1 - a^4} + \frac{3^3}{1 - a^4} + \dots + \frac{a^3}{1 - a^4} \right] \\
 &= \lim_{a \rightarrow \infty} [1^3 + 2^3 + 3^3 + \dots + a^3] \times \frac{1}{1 - a^4} \\
 &= \lim_{a \rightarrow \infty} \left[\frac{a(a + 1)}{2} \right]^2 \times \frac{1}{1 - a^4} = \lim_{a \rightarrow \infty} \left[\frac{a^4 + a^2 + 2a^3}{4} \right] \times \frac{1}{1 - a^4} \\
 &= \lim_{a \rightarrow \infty} \frac{a^4 \left(1 + \frac{1}{a^2} + \frac{2}{a} \right)}{4a^4 \left(\frac{1}{a^4} - 1 \right)} = -\frac{1}{4} \quad \left[\text{as } a \rightarrow \infty, \frac{1}{a} \rightarrow 0 \right]
 \end{aligned}$$

$$11. (b) \text{ Given, } f(x) = \begin{cases} x + 2, & x \leq -1 \\ cx^2, & x > -1 \end{cases}$$

Since, at $x = -1$, limit exists.

$$\therefore \text{RHL} = \text{LHL} \Rightarrow \lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^-} f(x)$$

$$\Rightarrow \lim_{h \rightarrow 0} f(-1 + h) = \lim_{h \rightarrow 0} f(-1 - h)$$

$$\Rightarrow \lim_{h \rightarrow 0} c(-1 + h)^2 = \lim_{h \rightarrow 0} (-1 - h + 2)$$

$$\Rightarrow c(-1 + 0)^2 = 1 - 0$$

$$\Rightarrow c = 1$$

$$\begin{aligned}
 12. (a) \lim_{x \rightarrow \pi/4} \frac{2 - \cot x - \cot^3 x}{1 - \cot^3 x} &= \lim_{x \rightarrow \pi/4} \frac{1 + (1 - \cot x) - \cot^3 x}{1 - \cot^3 x} \\
 &= \lim_{x \rightarrow \pi/4} \frac{(1 - \cot x)(1 + \cot^2 x + \cot x) + (1 - \cot x)}{(1 - \cot x)(1 + \cot^2 x + \cot x)} \\
 &= \lim_{x \rightarrow \pi/4} \frac{1 + \cot^2 x + \cot x + 1}{1 + \cot^2 x + \cot x} = \frac{4}{3}
 \end{aligned}$$

$$\begin{aligned}
 13. (b) \text{ Given, } \lim_{x \rightarrow 0} \frac{\sin x - 2 \sin 3x + \sin 5x}{x} \\
 &= \lim_{x \rightarrow 0} \frac{(\sin 5x + \sin x) - 2 \sin 3x}{x} \\
 &= \lim_{x \rightarrow 0} \frac{2 \sin \left(\frac{5x+x}{2} \right) \cdot \cos \left(\frac{5x-x}{2} \right) - 2 \sin 3x}{x} \\
 &= \lim_{x \rightarrow 0} \left[\because \sin C + \sin D = 2 \sin \left(\frac{C+D}{2} \right) \cdot \cos \left(\frac{C-D}{2} \right) \right] \\
 &= \lim_{x \rightarrow 0} \frac{2 \sin 3x \cdot \cos 2x - 2 \sin 3x}{x} \\
 &= \lim_{x \rightarrow 0} \frac{2 \sin 3x (\cos 2x - 1)}{x} \\
 &= \lim_{x \rightarrow 0} 2 \left(\frac{\sin 3x}{3x} \right) \times 3 \left[\frac{-(1 - \cos 2x)}{1} \right] \\
 &= -2 \times 1 \times 3 (1 - \cos 0) = -6 (1 - 1) = -6 \times 0 = 0
 \end{aligned}$$

$$\begin{aligned}
 14. (d) \text{ Given, } f(x) &= \begin{cases} x^2 - 1, & 0 < x < 2 \\ 2x + 3, & 2 \leq x < 3 \end{cases} \\
 \text{RHL} = \lim_{x \rightarrow 2^+} f(x) &= \lim_{h \rightarrow 0} f(2+h) = \lim_{h \rightarrow 0} 2(2+h) + 3 \\
 &= 2(2+0) + 3 = 4 + 3 = 7 = \alpha \text{ (say) } [\because f(x) = 2x + 3] \\
 \text{LHL} = \lim_{x \rightarrow 2^-} f(x) &= \lim_{h \rightarrow 0} f(2-h) \\
 &= \lim_{h \rightarrow 0} (2-h)^2 - 1 = (2-0)^2 - 1 \\
 &= 4 - 1 = 3 = \beta \quad (\text{say}) [\because f(x) = x^2 - 1]
 \end{aligned}$$

Since, quadratic equation has roots α and β , so the equation is

$$\begin{aligned}
 x^2 - (\text{Sum of roots})x + \text{Product of roots} &= 0 \\
 \Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta &= 0 \Rightarrow x^2 - (7+3)x + 7 \times 3 = 0 \\
 \Rightarrow x^2 - 10x + 21 &= 0
 \end{aligned}$$

$$\begin{aligned}
 15. (a) \lim_{x \rightarrow 0} \frac{e^{x^2} - \cos x}{x^2} &= \lim_{x \rightarrow 0} \left[\frac{e^{x^2} - 1}{x^2} + \frac{1 - \cos x}{x^2} \right] \\
 &= \lim_{x \rightarrow 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = 1 + \frac{1}{2} = \frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 16. (c) \text{ We know that, } (1+x)^{1/x} &= e^{\frac{\log(1+x)}{x}} = e^{\frac{1}{x} \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \right)} \\
 &= e^{1 - \frac{x}{2} + \frac{x^2}{3} - \dots} = e \cdot e^{-\frac{x}{2} + \frac{x^2}{3} - \dots} \\
 \therefore \lim_{x \rightarrow 0} \frac{(1+x)^{1/x} - e}{x} &= \lim_{x \rightarrow 0} \frac{e \cdot e^{-\frac{x}{2} + \frac{x^2}{3} - \dots} - e}{x} \\
 &= e \lim_{x \rightarrow 0} \frac{[e^{-x/2 + x^2/3 - \dots} - 1]}{(-2)(-x/2)} = \frac{-1}{2} e
 \end{aligned}$$

$$\begin{aligned}
 17. (c) \lim_{x \rightarrow 0} \frac{x(1 + a \cos x) - b \sin x}{x^3} &= 1 \\
 &= \lim_{x \rightarrow 0} \frac{x \left[1 + a \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \right) \right] - b \left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right]}{x^3} \\
 &= \lim_{x \rightarrow 0} \frac{(1+a-b) + x^2 \left(\frac{b}{3!} - \frac{a}{2!} \right) + x^4 \left(\frac{a}{4!} - \frac{b}{5!} \right) + \dots}{x^3} \quad \dots(i) \\
 &= \lim_{x \rightarrow 0} \frac{(1+a-b) + x^2 \left(\frac{b}{3!} - \frac{a}{2!} \right) + x^4 \left(\frac{a}{4!} - \frac{b}{5!} \right) + \dots}{x^2}
 \end{aligned}$$

If $1+a-b \neq 0$, then LHS $\rightarrow \infty$ as $x \rightarrow 0$ while RHS = 1.

Therefore, $1+a-b=0$

Now, from Eq. (i), we get

$$\begin{aligned}
 \lim_{x \rightarrow 0} \frac{x^2 \left(\frac{b}{3!} - \frac{a}{2!} \right) + x^4 \left(\frac{a}{4!} - \frac{b}{5!} \right)}{x^2} &= 1 \\
 \Rightarrow \frac{b}{3!} - \frac{a}{2!} &= 1 \Rightarrow b - 3a = 6
 \end{aligned}$$

On solving $1+a-b=0$ and $b-3a=6$, we get

$$a = -\frac{5}{2}, b = -\frac{3}{2}$$

$$\begin{aligned}
 18. (b) \lim_{n \rightarrow \infty} \sin \left\{ n\pi \left(1 + \frac{1}{n^2} \right)^{1/2} \right\} \\
 &= \lim_{n \rightarrow \infty} \sin \left\{ n\pi \left(1 + \frac{1}{2n^2} - \frac{1}{8n^4} + \dots \right) \right\} \\
 &= \lim_{n \rightarrow \infty} \sin \left\{ n\pi + \frac{\pi}{2n} - \frac{\pi}{8n^3} + \dots \right\} \\
 &= \lim_{n \rightarrow \infty} (-1)^n \sin \pi \left(\frac{1}{2n} - \frac{1}{8n^3} + \dots \right) = 0
 \end{aligned}$$

$$\begin{aligned}
 19. (b) \lim_{x \rightarrow \infty} \left(\frac{3x-4}{3x+2} \right)^{\frac{x+1}{3}} &= \lim_{x \rightarrow \infty} \left(\frac{3x+2-6}{3x+2} \right)^{\frac{x+1}{3}} \\
 &= \lim_{x \rightarrow \infty} \left(1 - \frac{6}{3x+2} \right)^{\frac{x+1}{3}} \\
 &= \lim_{x \rightarrow \infty} \left[\left(1 - \frac{6}{3x+2} \right)^{\frac{3x+2}{-6}} \right]^{\frac{-6}{3x+2} \cdot \frac{x+1}{3}} \\
 &= \lim_{x \rightarrow \infty} e^{\frac{-2(x+1)}{3x+2}} = e^{-2/3} \quad \left[\because \lim_{x \rightarrow \infty} \frac{-2(x+1)}{3x+2} = \frac{-2}{3} \right]
 \end{aligned}$$

$$\begin{aligned}
 20. (d) \lim_{x \rightarrow 1} \frac{x^4 - 1^4}{x - 1} &= \lim_{x \rightarrow 1} \frac{x^3 - k^3}{x^2 - k^2} \\
 \Rightarrow 4 \cdot (1)^{4-1} &= \frac{3}{2} (k)^{3-2} \Rightarrow 4 = \frac{3}{2} k \Rightarrow k = \frac{8}{3}
 \end{aligned}$$

$$\begin{aligned}
 21. (c) \lim_{x \rightarrow \infty} \left(\frac{x+1}{x+2} \right)^{2x+1} &= \lim_{x \rightarrow \infty} \left(1 - \frac{1}{x+2} \right)^{2x+1} \\
 &= \lim_{x \rightarrow \infty} \left[\left\{ 1 + \left(\frac{-1}{x+2} \right) \right\}^{-(x+2)} \right]^{\left(\frac{2x+1}{x+2} \right)}
 \end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow \infty} \left[\left\{ 1 + \left(\frac{-1}{x+2} \right) \right\}^{-(x+2)} \right]^{\left(\frac{2+1/x}{1+2/x} \right)} \\
 &= e^{-2} \left[\because \lim_{x \rightarrow \infty} \left\{ 1 + \frac{1}{f(x)} \right\}^{f(x)} = e \text{ and } \frac{1}{x} \rightarrow 0 \right] \\
 &\quad \left[\frac{0}{0} \text{ form} \right]
 \end{aligned}$$

22. (b) $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$

$$\begin{aligned}
 &= \lim_{x \rightarrow 0} \frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right) - x}{x^3} \\
 &= \lim_{x \rightarrow 0} \frac{\left(-\frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right)}{x^3} = -\frac{1}{6}
 \end{aligned}$$

23. (a) Let $\lim_{x \rightarrow 0} f(x) = \frac{1^0 + 2^0 + 3^0 + \dots + n^0}{n} = 1$

$$\therefore \lim_{x \rightarrow 0} [f(x)]^{g(x)} = e^{\lim_{x \rightarrow 0} g(x) \{f(x) - 1\}}$$

$$\text{So, } \lim_{x \rightarrow 0} \left[\frac{1^x + 2^x + 3^x + \dots + n^x}{n} \right]^{a/x}$$

$$= e^{\lim_{x \rightarrow 0} \frac{a}{x} \left[\frac{1^x + 2^x + 3^x + \dots + n^x}{n} - 1 \right]}$$

$$= e^{\lim_{x \rightarrow 0} \left\{ \frac{(1^x + 2^x + 3^x + \dots + n^x) - n}{x} \right\} \frac{a}{n}}$$

$$= e^{\frac{a}{n} \lim_{x \rightarrow 0} \left\{ \frac{1^x - 1}{x} + \frac{2^x - 1}{x} + \frac{3^x - 1}{x} + \dots + \frac{n^x - 1}{x} \right\}}$$

$$= e^{\frac{a}{n} [\log 1 + \log 2 + \dots + \log n]}$$

$$= e^{\frac{a}{n} \log n!} = e^{\log (n!)^{a/n}} = (n!)^{a/n}$$

24. (b)

25. (a) $\lim_{x \rightarrow 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3}$ $\left[\frac{0}{0} \text{ form} \right]$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{1-x^2}} - \frac{1}{1+x^2}}{3x^2} \quad [\text{applying L'Hospital's rule}]$$

$$= \lim_{x \rightarrow 0} \frac{1+x^2 - \sqrt{1-x^2}}{3x^2(1+x^2)\sqrt{1-x^2}}$$

$$= \lim_{x \rightarrow 0} \frac{2x - \frac{1(-2x)x}{2\sqrt{1-x^2}}}{\left[6x(1+x^2)\sqrt{1-x^2} + 3x^2 \cdot 2x\sqrt{1-x^2} + 3x^2(1+x^2) \cdot \frac{1(-2x)}{2\sqrt{1-x^2}} \right]}$$

$$= \lim_{x \rightarrow 0} \frac{2x + \frac{x}{\sqrt{1-x^2}}}{6x(1+x^2)\sqrt{1-x^2} + 6x^3\sqrt{1-x^2} - \frac{3x^3(1+x^2)}{\sqrt{1-x^2}}}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 0} \frac{2 + \frac{1}{\sqrt{1-x^2}}}{6(1+x^2)\sqrt{1-x^2} - \frac{3x^2(1+x^2)}{\sqrt{1-x^2}} + 6x^2\sqrt{1-x^2}} \\
 &= \frac{2+1}{6(1+0)\sqrt{1-0+0+0}} = \frac{3}{6} = \frac{1}{2}
 \end{aligned}$$

26. (d) $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{(4^x - 1)^3}{\sin\left(\frac{x}{p}\right) \log\left(1 + \frac{x^2}{3}\right)}$

$$= \lim_{x \rightarrow 0} \frac{4^x - 1}{x} \cdot \frac{\left(\frac{x}{p}\right) p(4^x - 1)^2}{\sin\left(\frac{x}{p}\right) \log\left(1 + \frac{x^2}{3}\right)}$$

$$= (\log 4) \cdot p \lim_{x \rightarrow 0} \left(\frac{4^x - 1}{x} \right)^2 \cdot \frac{x^2}{\log\left(1 + \frac{x^2}{3}\right)} \quad \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

$$= p(\log 4)^3 \cdot \lim_{x \rightarrow 0} \frac{x^2}{\log\left(1 + \frac{x^2}{3}\right)} \quad \left[\frac{0}{0} \text{ form} \right]$$

$$= p(\log 4)^3 \cdot \lim_{x \rightarrow 0} \frac{2x}{\frac{1}{1 + \frac{x^2}{3}} \cdot \frac{2x}{3}} \quad [\text{using L'Hospital's rule}]$$

$$= p(\log 4)^3 \lim_{x \rightarrow 0} (3 + x^2) = 3p(\log 4)^3$$

$$= 12(\log 4)^3 \quad [\text{for continuity at } x = 0]$$

$$\therefore p = 4$$

27. (d) Let $y = \lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{2/x}$

$$\Rightarrow \log y = \lim_{x \rightarrow 0} \frac{2}{x} \log \left(\frac{a^x + b^x + c^x}{3} \right)$$

$$= 2 \lim_{x \rightarrow 0} \frac{\log(a^x + b^x + c^x) - \log 3}{x}$$

$$= 2 \lim_{x \rightarrow 0} \frac{a^x \log a + b^x \log b + c^x \log c}{a^x + b^x + c^x}$$

$$[\text{apply L'Hospital's rule}]$$

$$\Rightarrow \log y = \log(abc)^{2/3} \Rightarrow y = (abc)^{2/3}$$

28. (c) $\lim_{x \rightarrow 0} \frac{ae^x - b \cos x + ce^{-x}}{x \sin x}$

$$\Rightarrow ae^x - b \cos x + c \cdot e^{-x} = 0 \text{ for } x = 0$$

[as denominator is zero and limit of function exists at $x = 0$, hence numerator should be zero at $x = 0$]

$$\therefore a - b + c = 0 \quad \dots(i)$$

$$\therefore 2 = \lim_{x \rightarrow 0} \frac{ae^x - b \cos x + ce^{-x}}{x \sin x} = \lim_{x \rightarrow 0} \frac{ae^x + b \sin x - ce^{-x}}{\sin x + x \cos x}$$

$$= ae^0 + b \sin 0 - ce^{-0} = 0$$

$$\Rightarrow a - c = 0 \quad \dots(ii)$$

Also, $\lim_{x \rightarrow 0} \frac{ae^x + b \cos x + ce^{-x}}{2 \cos x - x \sin x}$ [using L' Hospital's rule]

$$\Rightarrow \frac{a+b+c}{2} = 2$$

$$\Rightarrow a+b+c=4 \quad \dots(iii)$$

From Eqs. (i), (ii) and (iii), we get

$$a=1, b=2, c=1$$

$$29. (c) \lim_{n \rightarrow \infty} \frac{an(1+n) - (1+n^2)}{1+n} = \lim_{n \rightarrow \infty} \frac{(a-1)n^2 + an - 1}{n+1} = \infty,$$

if $a-1 \neq 0$ and if $a-1=0$, then $\lim_{n \rightarrow \infty} \frac{an-1}{n+1} = a=b$

$$\Rightarrow a=b=1$$

30. (c) **Hint** Since, $f(x)$ is continuous at $x=0$.

$$\begin{aligned} \text{Therefore, } f(0) &= \lim_{x \rightarrow 0} f(x) \\ &= \lim_{x \rightarrow 0} \frac{(27-2x)^{1/3} - 3}{9 - 3(243+5x)^{1/5}} \quad \left[\frac{0}{0} \text{ form} \right] \end{aligned}$$

[applying L' Hospital's rule]

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{3}(27-2x)^{-2/3}(-2)}{\frac{-3}{5}(243+5x)^{-4/5} \times (5)}$$

31. (b) We want to check the continuity at $x=0$.

$$\begin{aligned} \therefore \text{LHL} &= \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} |-h| \cos\left(\frac{1}{-h}\right) \\ &= \lim_{h \rightarrow 0} h \cos\left(\frac{1}{h}\right) = 0 \times \cos \frac{1}{0} = 0 \end{aligned}$$

[$\because -1 \leq \cos x \leq 1, \forall x \in \mathbb{R}$]

$$\begin{aligned} \text{RHL} &= \lim_{x \rightarrow 0} f(x) = \lim_{h \rightarrow 0} |0+h| \cos\left(\frac{1}{h}\right) \\ &= \lim_{h \rightarrow 0} |h| \cos\left(\frac{1}{h}\right) = \lim_{h \rightarrow 0} h \cos \frac{1}{h} = 0 \times \cos \frac{1}{0} = 0 \end{aligned}$$

and $f(0) = 0$

Thus, $\text{LHL} = \text{RHL} = f(0) = 0$

Hence, the function is continuous at $x=0$.

32. (d) Given that, the function is continuous.

$$\text{At } x=0, \text{ LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h)$$

$$\begin{aligned} &= \lim_{h \rightarrow 0^-} \frac{\sqrt{1-kh} - \sqrt{1+kh}}{-h} \\ &= \lim_{h \rightarrow 0} \frac{(\sqrt{1-kh} - \sqrt{1+kh})}{-h} \times \frac{(\sqrt{1-kh} + \sqrt{1+kh})}{(\sqrt{1-kh} + \sqrt{1+kh})} \\ &= \lim_{h \rightarrow 0} \frac{(1-kh) - (1+kh)}{-h(\sqrt{1-kh} + \sqrt{1+kh})} = \frac{2k}{2} = k \end{aligned}$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{2h+1}{h-2} = \frac{-1}{2}$$

Since, $f(x)$ is continuous, so $k = -\frac{1}{2}$.

$$33. (c) \lim_{x \rightarrow 0^+} f(x) = x^2 \sin \frac{1}{x}, -1 \leq \sin \frac{1}{x} \leq 1 \text{ as } x \rightarrow 0$$

$$\therefore \lim_{x \rightarrow 0^+} f(x) = 0, \lim_{x \rightarrow 0^-} f(x) = 0$$

Hence, $f(x)$ is continuous at $x=0$.

$$34. (d) g(x) = \begin{cases} 1+x, & x < 0 \\ 1-x, & x \geq 0 \end{cases}$$

$$\begin{aligned} \therefore f\{g(x)\} &= \begin{cases} 1+|x-1|, & x < 0 \\ 1+|-x-1|, & x \geq 0 \end{cases} \\ &= \begin{cases} 1+1-x, & x < 0 \\ 1+x+1, & x \geq 0 \end{cases} = \begin{cases} 2-x, & x < 0 \\ 2+x, & x \geq 0 \end{cases} \end{aligned}$$

It is a polynomial function, so it is continuous in everywhere except at $x=0$.

$$\text{Now, LHL} = \lim_{x \rightarrow 0^-} 2-x = 2$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} 2+x = 2$$

$$\text{Also, } f(0) = 2+0 = 2$$

Hence, it is continuous everywhere.

35. (a) Polynomial function is always continuous. So, we have to check the continuity only at $x=-3$ and 3 .

$$\text{Here, } f(x) = \begin{cases} |x|+3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x+2, & \text{if } x \geq 3 \end{cases}$$

For $x < -3$, $f(x) = |x|+3$; $-3 < x < 3$, $f(x) = -2x$ and $x > 3$, $f(x) = 6x+2$ is a polynomial function, so it is continuous in a given interval. So, we have to check the continuity only at $x=-3$ and 3 .

$$\text{At } x=-3, \text{ LHL} = \lim_{x \rightarrow -3^-} f(x) = \lim_{h \rightarrow 0} (|x|+3)$$

$$\therefore = \lim_{h \rightarrow 0} [|-3-h|+3] = \lim_{h \rightarrow 0} (6+h) = 6$$

$$\text{RHL} = \lim_{h \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0^+} (-2x)$$

$$\therefore = \lim_{h \rightarrow 0} -2(-3+h) = \lim_{h \rightarrow 0} (6-2h) = 6$$

$$\text{Also, } f(-3) = |-3|+3 = 6 \quad [\because f(x) = |x|+3]$$

$$\therefore \text{LHL} = \text{RHL} = f(-3)$$

Thus, $f(x)$ is continuous at $x=-3$.

$$\text{At } x=3, \text{ LHL} = \lim_{x \rightarrow 3^-} f(x) = \lim_{h \rightarrow 0} (-2x)$$

$$= \lim_{h \rightarrow 0} -2(3-h) = \lim_{h \rightarrow 0} (-6+2h) = -6$$

$$\text{RHL} = \lim_{x \rightarrow 3^+} f(x) = \lim_{h \rightarrow 0} (6x+2)$$

$$= \lim_{h \rightarrow 0} [6(3+h)+2] = \lim_{h \rightarrow 0} (18+6h+2)$$

$$= \lim_{h \rightarrow 0} (20+6h) = 20 \therefore \text{LHL} \neq \text{RHL}$$

Thus, $f(x)$ is discontinuous at $x=3$.

$$\begin{aligned} 36. (a) \therefore \lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \frac{3x+4 \tan x}{x} \\ &= \lim_{x \rightarrow 0} \left[3 + 4 \cdot \frac{\sin x}{x} \cdot \frac{1}{\cos x} \right] = \lim_{x \rightarrow 0} [3+4] = 7 \end{aligned}$$

Since, $f(x)$ is not defined at $x=0$, so it cannot be continuous at $x=0$.

$\therefore$ Defined function should be

$$f(x) = \begin{cases} \frac{3x+4 \tan x}{x}, & x \neq 0 \\ 7, & x = 0 \end{cases}$$

37. (a) Since, x , $\sin^{-1} x$, $\tan^{-1} x$ are continuous functions, so the function f is clearly continuous at each point in its domain except possibly at $x = 0$.

So, for f to be continuous at $x = 0$,

$$f(0) = \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{2 - \frac{\sin^{-1} x}{x}}{2 + \frac{\tan^{-1} x}{x}} = \frac{1}{3}$$

$$\left[\because \lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} = 1, \lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x} = 1 \right]$$

38. (d) Here, $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$

$$\therefore \text{LHL} = \lim_{x \rightarrow \frac{\pi}{2}^-} f(x) = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{k \cos x}{\pi - 2x}$$

On putting $x = \frac{\pi}{2} - h$, as $x \rightarrow \frac{\pi}{2}^-$, when $h \rightarrow 0$, we get

$$\text{LHL} = \lim_{h \rightarrow 0} \frac{k \cos \left(\frac{\pi}{2} - h \right)}{\pi - 2 \left(\frac{\pi}{2} - h \right)} = \lim_{h \rightarrow 0} \frac{k \sin h}{2h}$$

$$= \lim_{h \rightarrow 0} \frac{k}{2} \times \frac{\sin h}{h} = \frac{k}{2} \times 1 = \frac{k}{2} \quad \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

and $\text{RHL} = \lim_{x \rightarrow \frac{\pi}{2}^+} f(x) = \lim_{x \rightarrow \frac{\pi}{2}^+} \frac{k \cos x}{\pi - 2x}$

On putting $x = \frac{\pi}{2} + h$, as $x \rightarrow \left(\frac{\pi}{2} \right)^+$, when $h \rightarrow 0$, we get

$$\therefore \text{RHL} = \lim_{h \rightarrow 0} \frac{k \cos \left(\frac{\pi}{2} + h \right)}{\pi - 2 \left(\frac{\pi}{2} + h \right)} = \lim_{h \rightarrow 0} \frac{-k \sin h}{-2h}$$

$$= \lim_{h \rightarrow 0} \frac{k}{2} \times \frac{\sin h}{h} = \frac{k}{2} \times 1 = \frac{k}{2} \quad \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

Also, $f\left(\frac{\pi}{2}\right) = 3$

Since, $f(x)$ is continuous at $x = \frac{\pi}{2}$.

$$\therefore \text{LHL} = \text{RHL} = f\left(\frac{\pi}{2}\right) \Rightarrow \frac{k}{2} = 3 \quad \therefore k = 6$$

39. (a) **Hint** For this, we have to prove that $\text{LHL} = \text{RHL}$.

$$\therefore \text{LHL} = \lim_{x \rightarrow a^-} f(x) = \lim_{h \rightarrow 0} f(a - h)$$

$$= \lim_{h \rightarrow 0} |a - h - a| \sin \left[\frac{1}{(a - h - a)} \right]$$

$$[\because -1 \leq \sin x \leq 1, \forall x \in R]$$

$$\text{RHL} = \lim_{x \rightarrow a^+} f(x) = \lim_{h \rightarrow 0} f(a + h)$$

$$= \lim_{h \rightarrow 0} |a + h - a| \sin \left[\frac{1}{(a + h - a)} \right]$$

40. (b) **Hint** Since, $f(x)$ is continuous for $x \in R$.

So, the function is continuous at $x = 0$.

RHL at $x = 0$,

$$= \lim_{h \rightarrow 0} \frac{\sqrt{h+h^2} - \sqrt{h}}{h^{3/2}} = \lim_{h \rightarrow 0} \frac{\sqrt{h} \{\sqrt{h+1} - 1\}}{h\sqrt{h}}$$

$$= \lim_{h \rightarrow 0} \left(\frac{\sqrt{h+1} - 1}{h} \times \frac{\sqrt{h+1} + 1}{\sqrt{h+1} + 1} \right)$$

LHL at $x = 0$,

$$= \lim_{h \rightarrow 0} \frac{\sin(p+1)(-h) + \sin(-h)}{-h}$$

$$= \lim_{h \rightarrow 0} \left[\frac{\sin(p+1)h}{h} + \frac{\sin h}{h} \right]$$

$$= (p+1) + 1 = (p+2)$$

41. (d) If $f(x)$ is continuous at $x = 0$, then

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0) = 0$$

$$\text{So, } \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} (-1)^k h^k \sin \left(-\frac{1}{h} \right)$$

$$= - \lim_{h \rightarrow 0} (-1)^k (h)^k \cdot \sin \left(\frac{1}{h} \right) = 0$$

[for $k > 0$ and $-1 \leq \sin x \leq 1$]

and $\lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h)$

$$= \lim_{h \rightarrow 0} h^k \sin \left(\frac{1}{h} \right) = 0$$

[for $k > 0$ and $-1 \leq \sin x \leq 1$]

So, $f(x)$ is continuous at $x = 0$, if $k > 0$.

Hence, $k \in (0, \infty)$.

42. (c) **Hint** Since it is clear that f is continuous everywhere except possibly at $x = 0$, so at $x = 0$ for f to be continuous,

$$f(0) = \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{(1 + \sin x)^{1/2} - (1 - \sin x)^{1/2}}{x}$$

$$= \lim_{x \rightarrow 0} \frac{(1 + \sin x) - (1 - \sin x)}{x} \cdot \frac{1}{(1 + \sin x)^{1/2} + (1 - \sin x)^{1/2}}$$

43. (c) Since, $f(x)$ is right continuous at $x = 2$.

$$\therefore \lim_{x \rightarrow 2^+} f(x) = f(2) \Rightarrow \lim_{h \rightarrow 0} f(2 + h) = c$$

$$\Rightarrow \lim_{h \rightarrow 0} \left\{ (2 + h)^2 + e^{\frac{1}{2 - (2 + h)}} \right\}^{-1} = c$$

$$\Rightarrow \lim_{h \rightarrow 0} (4 + h^2 + 4h + e^{-1/h})^{-1} = c$$

$$\Rightarrow (4 + 0)^{-1} = c \Rightarrow c = \frac{1}{4}$$

44. (c) Since, $f(x)$ is continuous at $x = 0$.

$$\therefore \lim_{x \rightarrow 0} f(x) = f(0) \Rightarrow \lim_{x \rightarrow 0} \frac{(3^x - 1)^2}{\sin x \cdot \log_e(1 + x)} = f(0)$$

$$\Rightarrow f(0) = \lim_{x \rightarrow 0} \frac{\left(\frac{3^x - 1}{x} \right)^2}{\frac{\sin x}{x} \cdot \log_e(1 + x)} = (\log_e 3)^2$$

45. (b) $\therefore \lim_{x \rightarrow 0} f(x) = f(0)$

Let a be any point.

Now, at $x = a$, $\lim_{x \rightarrow a} f(x) = \lim_{h \rightarrow 0} f(a + h) = \lim_{h \rightarrow 0} f(a) + \lim_{h \rightarrow 0} f(h)$

$$= f(a) + f(0) = f(a + 0) = f(a)$$

$\therefore f(x)$ is continuous at $x = a$, where a is any arbitrary point.

Hence, $f(x)$ is continuous for all x .

46. (a) Here, $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, & \text{if } x < 4 \\ a + b, & \text{if } x = 4 \\ \frac{x-4}{|x-4|} + b, & \text{if } x > 4 \end{cases}$

Given that, the function is continuous at $x = 4$.

$$\text{LHL} = \text{RHL} = f(4) \quad \dots(i)$$

Now, $f(4) = a + b \quad \dots(ii)$

$$\text{LHL} = \lim_{x \rightarrow 4^-} f(x) = \lim_{h \rightarrow 0} f(4 - h) = \lim_{h \rightarrow 0} \frac{4 - h - 4}{|4 - h - 4|} + a$$

$$= \lim_{h \rightarrow 0} \frac{-h}{h} + a = a - 1 \quad \dots(iii)$$

$$\text{RHL} = \lim_{x \rightarrow 4^+} f(x) = \lim_{h \rightarrow 0} f(4 + h) = \lim_{h \rightarrow 0} \frac{4 + h - 4}{|4 + h - 4|} + b$$

$$= \lim_{h \rightarrow 0} \frac{h}{h} + b = b + 1 \quad \dots(iv)$$

From Eqs. (i), (ii) and (iii), we get $a + b = a - 1 \Rightarrow b = -1$

From Eqs. (ii) and (iv), $a + b = b + 1 \Rightarrow a = 1$

Thus, $a = 1$ and $b = -1$

47. (a) Given, $f(x) = \frac{\sqrt{2} \cos x - 1}{\cot x - 1}, x \neq \frac{\pi}{4}$

$$\begin{aligned} \text{Therefore, } \lim_{x \rightarrow \pi/4} f(x) &= \lim_{x \rightarrow \pi/4} \frac{\sqrt{2} \cos x - 1}{\cot x - 1} \\ &= \lim_{x \rightarrow \pi/4} \frac{(\sqrt{2} \cos x - 1) \sin x}{\cos x - \sin x} \end{aligned}$$

$$= \lim_{x \rightarrow \pi/4} \frac{(\sqrt{2} \cos x - 1)}{(\sqrt{2} \cos x + 1)} \cdot \frac{(\sqrt{2} \cos x + 1)}{(\cos x - \sin x)} \cdot \frac{(\cos x + \sin x)}{(\cos x + \sin x)} \cdot \sin x$$

$$= \lim_{x \rightarrow \pi/4} \frac{\cos 2x}{\cos 2x} \cdot \left(\frac{\cos x + \sin x}{\sqrt{2} \cos x + 1} \right) \cdot (\sin x)$$

$$= \lim_{x \rightarrow \pi/4} \frac{(\cos x + \sin x)}{\sqrt{2} \cos x + 1} \sin x = \frac{1}{\sqrt{2} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right)} = \frac{1}{2}$$

$$\text{Thus, } \lim_{x \rightarrow \pi/4} f(x) = \frac{1}{2}$$

If we define $f\left(\frac{\pi}{4}\right) = \frac{1}{2}$, then $f(x)$ will become continuous at

$$x = \frac{\pi}{4}.$$

Hence, for f to be continuous at $x = \frac{\pi}{4}$, $f\left(\frac{\pi}{4}\right) = \frac{1}{2}$.

48. (d)

49. (a) At $x = \frac{\pi}{4}$,

$$\lim_{x \rightarrow \pi/4^-} f(x) = \lim_{x \rightarrow \pi/4^-} (x + a^2 \sqrt{2} \sin x) = a^2 + \frac{\pi}{4}$$

and $\lim_{x \rightarrow \pi/4^+} f(x) = \lim_{x \rightarrow \pi/4^+} (x \cot x + b) = b + \frac{\pi}{4}$

Since, $f(x)$ is continuous at $x = \frac{\pi}{4}$.

$$\therefore a^2 + \frac{\pi}{4} = b + \frac{\pi}{4} \Rightarrow a^2 = b \quad \dots(i)$$

Again at $x = \frac{\pi}{2}$,

$$\lim_{x \rightarrow \pi/2^-} f(x) = \lim_{x \rightarrow \pi/2^-} (b \sin 2x - a \cos 2x) = a$$

and $\lim_{x \rightarrow \pi/2^+} f(x) = \lim_{x \rightarrow \pi/2^+} (x \cot x + b) = b$

Since, $f(x)$ is continuous at $x = \frac{\pi}{2}$.

$$\therefore a = b \quad \dots(ii)$$

From Eqs. (i) and (ii), we get $(a, b) = (0, 0)$ and $(1, 1)$.

50. (c) Let $g(x) = a \sin x + \sin 2x$ and $h(x) = x^3$

$$\therefore g(0) = 0 \text{ and } h(0) = 0$$

$$\therefore \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{a \cos x + 2 \cos 2x}{3x^2}$$

[using L' Hospital's rule]

Again, $\lim_{x \rightarrow 0} x^2 = 0$ and $\lim_{x \rightarrow 0} f(x)$ exist, hence numerator should also be equal to zero.

$$\therefore a + 2 = 0 \Rightarrow a = -2$$

$$\begin{aligned} \text{So, } \lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \frac{-2 \cos x + 2 \cos 2x}{3x^2} \\ &= \lim_{x \rightarrow 0} \frac{2 \sin x - 4 \sin 2x}{6x} \\ &= \frac{2}{6} - \frac{4}{3} = -1 \quad [\text{using L'Hospital's rule}] \end{aligned}$$

$$\therefore a = -2 \text{ and } f(0) = -1$$

51. (a) $\lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{h^3 + 3h} = \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h} \cdot \frac{-1}{h^2 + 3}$

$$= f'(1) \cdot \left(\frac{-1}{3} \right) = \frac{53}{3}$$

52. (a) $\lim_{h \rightarrow 0} \frac{f(2h + 2 + h^2) - f(2)}{f(h - h^2 + 1) - f(1)} = \lim_{h \rightarrow 0} \frac{f'(2h + 2 + h^2)(2 + 2h)}{f'(h - h^2 + 1)(1 - 2h)}$

$$= \frac{f'(2) \times 2}{f'(1) \times 1} = \frac{6 \times 2}{4 \times 1} = 3$$

53. (b) $f(x) = \begin{cases} e^x, & x \leq 0 \\ 1 - x, & 0 < x \leq 1 \\ x - 1, & x > 1 \end{cases}$

$$Rf'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{1 - h - 1}{h} = -1$$

$$Lf'(0) = \lim_{h \rightarrow 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h \rightarrow 0} \frac{e^{-h} - 1}{-h} = 1$$

So, it is not differentiable at $x = 0$.

Similarly, it is not differentiable at $x = 1$ but it is continuous at $x = 0$ and 1 .

54. (c) Since, $|x - 3| = x - 3$, if $x \geq 3$ and $|x - 3| = -x + 3$, if $x < 3$

$\therefore$ The given function can be defined as

$$f(x) = \begin{cases} \frac{1}{4}x^2 - \frac{3}{2}x + \frac{13}{4}, & x < 1 \\ 3 - x, & 1 \leq x < 3 \\ x - 3, & x \geq 3 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} \frac{x}{2} - \frac{3}{2}, & x < 1 \\ -1, & 1 \leq x < 3 \\ 1, & x \geq 3 \end{cases}$$

At $x = 1$,

$$\text{LHD} = f'(1) = -1 \text{ and } \text{RHD} = f'(1) = -1$$

55. (a) Let $f(x) = \log x \Rightarrow f'(x) = \frac{1}{x}$

Therefore, given function $= f'(a) + kf'(e) = 1$

$$\Rightarrow \frac{1}{a} + \frac{k}{e} = 1$$

$$\therefore k = e \left(\frac{a-1}{a} \right)$$

$$k = e \left(1 - \frac{1}{a} \right)$$

56. (b) $\text{RHL} = \lim_{h \rightarrow 0} (0+h) e^{-2/h} = \lim_{h \rightarrow 0} \frac{h}{e^{2/h}} = 0$

$$\text{LHL} = \lim_{h \rightarrow 0} (0-h) e^{-\left(\frac{1}{h} - \frac{1}{h}\right)} = 0$$

Hence, $f(x)$ is continuous at $x = 0$.

$$\text{Now, } Rf'(x) = \lim_{h \rightarrow 0} \frac{(0+h) e^{-\left(\frac{1}{h} + \frac{1}{h}\right)} - 0}{h} = \lim_{h \rightarrow 0} e^{-2/h} = \infty$$

$$\text{and } Lf'(x) = \lim_{h \rightarrow 0} \frac{(0-h) e^{-\left(\frac{1}{h} - \frac{1}{h}\right)} - 0}{-h} = \lim_{h \rightarrow 0} e^{-0} = 1$$

$$\therefore Lf'(x) \neq Rf'(x)$$

Hence, $f(x)$ not differentiable at $x = 0$.

57. (b) The given function is

$$f(x) = \begin{cases} 0, & 0 \leq x < 1 \\ 1, & 1 \leq x < 2 \end{cases}$$

$$\text{Now, } \text{LHD} = \lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1^-} \frac{0 - 1}{x - 1}$$

which does not exist.

$\therefore f$ is not derivable at $x = 1$.

58. (a) Here, the function can be written as

$$f(x) = \begin{cases} \lambda - \mu x + vx^2, & x < 0 \\ \lambda + \mu x + vx^2, & x \geq 0 \end{cases}$$

$$\text{Now, } \text{LHD} = \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0^-} \frac{\lambda - \mu x + vx^2 - \lambda}{x - 0} = \lim_{x \rightarrow 0^-} \frac{(-\mu + vx)x}{x} = \lim_{x \rightarrow 0^-} -\mu + vx = -\mu$$

$$\text{Also, } \text{RHD} = \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0^+} \frac{\lambda + \mu x + vx^2 - \lambda}{x} = \lim_{x \rightarrow 0^+} \frac{(\mu + vx)x}{x} = \lim_{x \rightarrow 0^+} \mu + vx = \mu$$

Thus, $f'(0)$ exists, if $\text{LHD} = \text{RHD}$.

$$\Rightarrow -\mu = \mu \Rightarrow \mu = 0$$

59. (b) Here, we are given that

$$f(x+y) = f(x) \cdot f(y)$$

$$\text{and } f'(0) = \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h} = 2$$

$$\text{Now, } f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x+0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x) \cdot f(h) - f(x) \cdot f(0)}{h}$$

[by given condition]

$$= f(x) \cdot \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h}$$

$$= f(x) \times f'(0) = 2f(x) \quad [\because f'(0) = 2]$$

60. (d) The given function is $f(x) = \begin{cases} x, & x \leq 1 \\ x^2 + bx + c, & x > 1 \end{cases}$

Since, $f'(x)$ exists finitely, $\forall x \in R$.

$\therefore f(x)$ is differentiable at $x = 1$.

$$\Rightarrow \text{LHD} = \text{RHD}$$

$$\Rightarrow 2x + b = 1$$

$$\Rightarrow 2 + b = 1$$

[put $x = 1$]

$$\Rightarrow b = -1$$

Also, f is continuous at $x = 1$ as it is differentiable at $x = 1$.

$$\Rightarrow \text{LHL} = \text{RHL} \Rightarrow 1 = 1 + b + c \Rightarrow c = 1$$

61. (d) The given function is $f(x) = |\log_e x|$

$$= \begin{cases} -\log_e x, & \text{for } 0 < x < 1 \\ \log_e x, & \text{for } x \geq 1 \end{cases}$$

Since, f is differentiable, $\forall x > 1$ and $x < 1$

So, check its differentiability at $x = 1$.

$$\begin{aligned} \text{Now, } \text{LHD} &= \lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x - 1} \\ &= \lim_{x \rightarrow 1^-} \frac{-\log x - \log 1}{x - 1} = \lim_{x \rightarrow 1^-} \frac{-\log x}{x - 1} \\ &= -\lim_{h \rightarrow 0} \frac{\log(1-h)}{-h} = -1 \end{aligned}$$

[using L' Hospital's rule]

$$\begin{aligned}\text{Now, RHD} &= \lim_{x \rightarrow 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1^+} \frac{\log x - \log 1}{x - 1} \\ &= \lim_{x \rightarrow 1^+} \frac{\log x}{x - 1} \\ &= \lim_{h \rightarrow 0} \frac{\log(1+h)}{h} = 1 \quad [\text{using L' Hospital's rule}]\end{aligned}$$

$$\Rightarrow \text{RHD} = 1 \Rightarrow \text{LHD} \neq \text{RHD at } x = 1$$

$\therefore f$ is not differentiable at $x = 1$.

62. (d) The given function is $f(x) = |x - 2| + |x| + |x + 2|$

$$= \begin{cases} -(x-2) - x - (x+2) = -3x, & x < -2 \\ -(x-2) - x + (x+2) = -x + 4, & -2 \leq x < 0 \\ -(x-2) + x + (x+2) = x + 4, & 0 \leq x < 2 \\ (x-2) + x + (x+2) = 3x, & x \geq 2 \end{cases}$$

Now, at $x = -2$,

$$\begin{aligned}\text{RHD} &= \lim_{x \rightarrow -2^+} \frac{f(x) - f(-2)}{x + 2} \\ &= \lim_{x \rightarrow -2^+} \frac{-x + 4 - 6}{x + 2} = \lim_{x \rightarrow -2^+} \frac{-(x+2)}{(x+2)} = -1\end{aligned}$$

$$\text{LHD} = \lim_{x \rightarrow -2^-} \frac{f(x) - f(-2)}{x + 2} = \lim_{x \rightarrow -2^-} \frac{-3x - 6}{x + 2} = -3$$

$\therefore \text{LHD} \neq \text{RHD}$

$\therefore f$ is not derivable at $x = -2$.

At $x = 0$,

$$\text{RHD} = \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0^+} \frac{x + 4 - 4}{x} = 1$$

$$\text{LHD} = \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0^-} \frac{-x + 4 - 4}{x} = -1$$

$\therefore \text{RHD} \neq \text{LHD}$

$\therefore f$ is not derivable at $x = 0$.

At $x = 2$,

$$\text{RHD} = \lim_{x \rightarrow 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x \rightarrow 2^+} \frac{3x - 6}{x - 2} = 3$$

$$\text{LHD} = \lim_{x \rightarrow 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x \rightarrow 2^-} \frac{x + 4 - 6}{x - 2} = 1$$

$\therefore \text{RHD} \neq \text{LHD}$

$\therefore f$ is not derivable at $x = 2$.

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$$\begin{aligned}1. (c) \lim_{x \rightarrow 0} \frac{(1+x)^8 - 1}{(1+x)^2 - 1} \\ = \lim_{x \rightarrow 0} \frac{[(1+x)^4 + 1][(1+x)^2 + 1][(1+x)^2 - 1]}{(1+x)^2 - 1} = 2 \times 2 = 4\end{aligned}$$

$$\begin{aligned}2. (b) \lim_{x \rightarrow \tan^{-1} 3} \frac{\tan^2 x - 2 \tan x - 3}{\tan^2 x - 4 \tan x + 3} &= \lim_{\tan x \rightarrow 3} \frac{(\tan x - 3)(\tan x + 1)}{(\tan x - 3)(\tan x - 1)} \\ &= \lim_{\tan x \rightarrow 3} \frac{\tan x + 1}{\tan x - 1} = \frac{3 + 1}{3 - 1} = \frac{4}{2} = 2\end{aligned}$$

$$\begin{aligned}3. (b) \text{ We have, } \lim_{x \rightarrow -\infty} \frac{x^4 \sin\left(\frac{1}{x}\right) + x^2}{1 + |x|^3} \\ = \lim_{y \rightarrow \infty} \frac{y^4 \sin\left(\frac{-1}{y}\right) + y^2}{1 + |y|^3}, \text{ where } y = -x \\ = \lim_{y \rightarrow \infty} \frac{-y^4 \sin\left(\frac{1}{y}\right) + y^2}{1 + y^3} \\ = \lim_{y \rightarrow \infty} \frac{-y \sin\left(\frac{1}{y}\right) + \frac{1}{y}}{\frac{1}{y^3} + 1} \\ = \frac{-1 + 0}{0 + 1} = -1\end{aligned}$$

$$\begin{aligned}4. (b) \lim_{x \rightarrow 1} \frac{x^m - 1}{x^n - 1} &= \lim_{x \rightarrow 1} \frac{mx^{m-1}}{nx^{n-1}} \quad [\text{using L'Hospital's rule}] \\ &= \frac{m}{n}\end{aligned}$$

$$\begin{aligned}5. (d) \lim_{x \rightarrow 0} \left[\tan\left(\frac{\pi}{4} + x\right) \right]^{1/x} &= \lim_{x \rightarrow 0} \left[\frac{1 + \tan x}{1 - \tan x} \right]^{1/x} \\ &= \lim_{x \rightarrow 0} \left[(1 + \tan x)^{\frac{1}{\tan x}} \right]^{\frac{\tan x}{x}} \times \lim_{x \rightarrow 0} \left[(1 - \tan x)^{-\frac{1}{\tan x}} \right]^{\frac{\tan x}{x}} \\ &= e \cdot e = e^2\end{aligned}$$

$$\begin{aligned}6. (a) \lim_{x \rightarrow 0} \left(\frac{1 + 5x^2}{1 + 3x^2} \right)^{1/x^2} &= \lim_{x \rightarrow 0} \left(1 + \frac{2x^2}{1 + 3x^2} \right)^{1/x^2} \\ &= e^{\lim_{x \rightarrow 0} \left(\frac{2x^2}{1 + 3x^2} \right) \times \frac{1}{x^2}} = e^{\lim_{x \rightarrow 0} \left(\frac{2}{1 + 3x^2} \right)} = e^2\end{aligned}$$

$$\begin{aligned}7. (d) \lim_{x \rightarrow 0} \frac{(2+x) \sin(2+x) - 2 \sin 2}{x} &\quad \left[\frac{0}{0} \text{ form} \right] \\ = \lim_{x \rightarrow 0} \frac{\sin(2+x) + (2+x) \cos(2+x)}{1} \\ &= \sin 2 + 2 \cos 2 \quad [\text{using L' Hospital's rule}]\end{aligned}$$

$$\begin{aligned}8. (a) \lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \frac{3x + \tan^2 x}{x} \quad \left[\frac{0}{0} \text{ form} \right] \\ &= \lim_{x \rightarrow 0} \frac{3 + 2 \tan x \sec^2 x}{1} = 3 \quad [\text{using L' Hospital's rule}]\end{aligned}$$

Since, $f(x)$ is continuous at $x = 0$.

$$\therefore f(0) = 3$$

$$9. (a) \text{ Given, } f(x) = \frac{\log(1+ax) - \log(1-bx)}{x}$$

$f(x)$ is continuous at $x = 0$ and $f(0) = k$.

$$\therefore \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{\log(1+ax) - \log(1-bx)}{x} \quad \left[\frac{0}{0} \text{ form} \right]$$

$$= \lim_{x \rightarrow 0} \left(\frac{1}{1+ax} \cdot a + \frac{b}{1-bx} \right) = a + b$$

$$\therefore a + b = f(0) = k$$

$$10. (c) \lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{\cos x}{1} = \cos 0 = 1$$

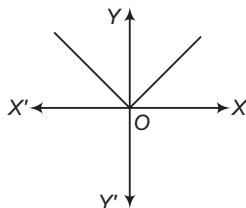
$$11. (d) \lim_{x \rightarrow 0} \frac{\cos(\sin x) - 1}{x^2} = \lim_{x \rightarrow 0} \frac{-2 \sin^2\left(\frac{\sin x}{2}\right)}{x^2}$$

$$= -2 \lim_{x \rightarrow 0} \frac{\sin^2\left(\frac{\sin x}{2}\right) \cdot \left(\frac{\sin x}{2}\right)^2}{\left(\frac{\sin x}{2}\right)^2 \times x^2} = -2 \cdot (1)^2 \cdot \frac{1}{4} = -\frac{1}{2}$$

$$12. (b) \lim_{x \rightarrow 0} f(x) = f(0) = \lim_{x \rightarrow 0} (1+x)^{1/x} = e$$

$$13. (a) \text{ Given, } f(x) = |x|$$

It is clear from the graph that, $f(x)$ is continuous but non-differentiable at $x = 0$.



$$14. (c) \text{ Let } p = \lim_{x \rightarrow 0} (\operatorname{cosec} x)^{1/\log x} \quad [\infty^0 \text{ form}]$$

$$\Rightarrow \log p = \lim_{x \rightarrow 0} \frac{1}{\log x} (\log \operatorname{cosec} x)$$

$$= \lim_{x \rightarrow 0} \frac{\log (\operatorname{cosec} x)}{\log x} \quad \left[\frac{\infty}{\infty} \text{ form} \right]$$

$$\therefore \log p = \lim_{x \rightarrow 0} \left(\frac{-x \operatorname{cosec} x \cot x}{\operatorname{cosec} x} \right)$$

$$= \lim_{x \rightarrow 0} \left(-\frac{x}{\tan x} \right) = -1 \quad [\text{using L' Hospital's rule}]$$

$$\therefore p = e^{-1} = \frac{1}{e}$$

$$15. (d) \lim_{x \rightarrow \infty} \left(\frac{x+6}{x+1} \right)^{x+4} = \lim_{x \rightarrow \infty} \left(1 + \frac{5}{x+1} \right)^{x+4}$$

$$= \lim_{x \rightarrow \infty} \left\{ \left(1 + \frac{5}{x+1} \right)^{x+1} \right\}^{\frac{x+4}{x+1}} = e^{\lim_{x \rightarrow \infty} \frac{5(x+4)}{(x+1)}} = e^5$$

$$16. (a) \text{ We have, } f(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$

Clearly, $f(x)$ is differentiable for all $x > 0$ and for all $x < 0$.

So, we check the differentiability at $x = 0$.

Now, RHD at $x = 0$,

$$\text{RHD} = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(0+h)^2 - 0}{h} = \lim_{h \rightarrow 0} \frac{h^2}{h} = \lim_{h \rightarrow 0} h = 0$$

$\therefore$ LHD at $x = 0$,

$$\text{LHD} = \lim_{h \rightarrow 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h \rightarrow 0} \frac{-(0-h)^2 - 0}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{-h^2}{-h} = \lim_{h \rightarrow 0} h = 0$$

$\therefore$ (LHD at $x = 0$) = (RHD at $x = 0$)

So, $f(x)$ is differentiable for all x i.e. the set of all points where, $f(x)$ is differentiable in $(-\infty, \infty)$ i.e. $\mathbb{R}$.

$$17. (a) \lim_{x \rightarrow 2} \frac{\sqrt{1+\sqrt{2+x}} - \sqrt{3}}{x-2}$$

$$= \lim_{x \rightarrow 2} \frac{(1+\sqrt{2+x}-3)}{(x-2)(\sqrt{1+\sqrt{2+x}} + \sqrt{3})}$$

$$= \lim_{x \rightarrow 2} \frac{x-2}{(x-2)(\sqrt{1+\sqrt{2+x}} + \sqrt{3})(\sqrt{2+x}+2)}$$

$$= \lim_{x \rightarrow 2} \frac{1}{(\sqrt{1+\sqrt{2+x}} + \sqrt{3})(\sqrt{2+x}+2)}$$

$$= \frac{1}{(\sqrt{1+2} + \sqrt{3})(\sqrt{2+2}+2)} = \frac{1}{8\sqrt{3}}$$

$$18. (b) \lim_{x \rightarrow 1} (1-x) \tan\left(\frac{\pi x}{2}\right) \quad [0 \times \infty \text{ form}]$$

$$= \lim_{x \rightarrow 1} \frac{(1-x)}{\cot\left(\frac{\pi x}{2}\right)}$$

$$= \lim_{x \rightarrow 1} \frac{-1}{-\operatorname{cosec}^2\left(\frac{\pi x}{2}\right) \frac{\pi}{2}} \quad [\text{using L'Hospital's rule}]$$

$$= \frac{2}{\pi} \lim_{x \rightarrow 1} \sin^2\left(\frac{\pi x}{2}\right) = \frac{2}{\pi}$$

$$19. (c) \text{ Given that, } f(x) = [x-3] + |x-4|$$

$$\therefore \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} ([x-3] + |x-4|)$$

$$= \lim_{h \rightarrow 0} ([3-h-3] + |3-h+4|)$$

$$= \lim_{h \rightarrow 0} ([-h] + 1+h) - 1 + 1 + 0 = 0$$

$$20. (b) \text{ Given that, } f(x) = \begin{cases} \frac{\cos 3x - \cos x}{x^2}, & \text{for } x \neq 0 \\ \lambda, & \text{for } x = 0 \end{cases}$$

$$\text{Now, LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{\cos 3x - \cos x}{x^2}$$

$$= \lim_{h \rightarrow 0} \frac{\cos 3(0-h) - \cos(0-h)}{(0-h)^2}$$

$$= \lim_{h \rightarrow 0} \frac{\cos 3h - \cos h}{h^2}$$

$$= \lim_{h \rightarrow 0} \frac{-3 \sin 3h + \sin h}{2h} \quad [\text{using L'Hospital's rule}]$$

$$= \lim_{h \rightarrow 0} \frac{-9 \cos 3h + \cos h}{2} \quad [\text{using L'Hospital's rule}]$$

$$= \frac{-9 + 1}{2} = -4$$

Since, $f(x)$ is continuous at $x = 0$

$$\therefore \lim_{x \rightarrow 0^-} f(x) = f(0) \Rightarrow -4 = \lambda \Rightarrow \lambda = -4$$

21. (c) Given that, $f(2) = 4$ and $f'(2) = 1$

$$\therefore \lim_{x \rightarrow 2} \frac{xf(2) - 2f(x)}{x - 2}$$

$$= \lim_{x \rightarrow 2} \frac{xf(2) - 2f(2) + 2f(2) - 2f(x)}{x - 2}$$

$$= \lim_{x \rightarrow 2} f(2) - 2 \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} = f(2) - 2f'(2) = 4 - 2(1) = 2$$

22. (c) Given that, $\lim_{x \rightarrow \infty} \left[\frac{x^3 + 1}{x^2 + 1} - (ax + b) \right] = 2$

$$\Rightarrow \lim_{x \rightarrow \infty} \left[\frac{x^3(1-a) - bx^2 - ax + (1-b)}{x^2 + 1} \right] = 2$$

$$\Rightarrow \lim_{x \rightarrow \infty} \left[\frac{x(1-a) - b - \frac{a}{x} + \frac{(1-b)}{x^2}}{1 + \frac{1}{x^2}} \right] = 2$$

This limit will exist, if

$$1 - a = 0 \text{ and } b = -2$$

$$\Rightarrow a = 1 \text{ and } b = -2$$

23. (a) Given that, $f(x) = \begin{cases} \frac{1 - \cos x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$

$$\text{Now, } \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin^2 x / 2}{4(x/2)^2} \cdot x = 0 \text{ and } f(0) = k$$

Since, function is continuous at $x = 0$.

$$\therefore \lim_{x \rightarrow 0} f(x) = f(0) \Rightarrow k = 0$$

24. (a) $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3} \quad \left[\frac{0}{0} \text{ form} \right]$

$$= \lim_{x \rightarrow 0} \frac{\sec^2 x - \cos x}{3x^2} \quad [\text{using L'Hospital's rule}]$$

$$= \lim_{x \rightarrow 0} \frac{2 \sec^2 x \tan x + \sin x}{6x} \quad [\text{using L'Hospital's rule}]$$

$$\left[\frac{0}{0} \text{ form} \right]$$

$$= \lim_{x \rightarrow 0} \frac{2(\sec^2 x \sec^2 x + 2 \sec x \times \sec x \tan x \tan x) + \cos x}{6}$$

[using L'Hospital's rule]

$$= \frac{2[1 \cdot 1 + 2(0)] + 1}{6} = \frac{3}{6} = \frac{1}{2}$$

$$25. (c) f(x) = \begin{cases} \frac{\sin 5x}{x^2 + 2x}, & x \neq 0 \\ k + \frac{1}{2}, & x = 0 \end{cases} \quad \text{LHL } f(0^-) = \lim_{h \rightarrow 0} f(0 - h)$$

$$= \lim_{h \rightarrow 0} \frac{\sin 5(0 - h)}{(0 - h)^2 + 2(0 - h)} = \lim_{h \rightarrow 0} \frac{\sin(-5h)}{h^2 - 2h}$$

$$= - \lim_{h \rightarrow 0} \frac{\sin 5h}{\frac{1}{5}(h - 2)} = \frac{1}{\frac{1}{5}(-2)} = \frac{5}{2}$$

Since, it is continuous at $x = 0$

$$\therefore \text{LHL} = f(0) \Rightarrow \frac{5}{2} = k + \frac{1}{2} \Rightarrow k = 2$$

26. (c) Given that, $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 1}{x + 1} - \alpha x - \beta \right) = 0$

$$\Rightarrow \lim_{x \rightarrow \infty} \left(\frac{x^2 + 1 - \alpha(x^2 + x) - \beta(x + 1)}{x + 1} \right) = 0$$

Using L'Hospital's rule, we get

$$\lim_{x \rightarrow \infty} \left(\frac{2x - \alpha(2x + 1) - \beta(1)}{1} \right) = 0$$

If this limit is zero, then the function

$$2x - \alpha(2x + 1) - \beta = 0 \text{ or } x(2 - 2\alpha) - (\alpha + \beta) = 0$$

Equating the coefficient of x and constant terms, we get

$$2 - 2\alpha = 0 \text{ and } \alpha + \beta = 0$$

$$\Rightarrow \alpha = 1 \text{ and } \beta = -1$$

27. (d) $\lim_{\theta \rightarrow 0} \frac{40(\tan \theta - 2\theta \tan \theta)}{1 - \cos 2\theta}$

$$= \lim_{\theta \rightarrow 0} \frac{4(\theta \tan \theta - 2\theta^2 \tan \theta)}{1 - \cos 2\theta} \quad [\text{using L'Hospital's rule}]$$

$$= \lim_{\theta \rightarrow 0} \frac{4(\theta \sec^2 \theta + \tan \theta - 4\theta \tan \theta - 2\theta^2 \sec^2 \theta)}{2 \sin 2\theta}$$

Again using L'Hospital's rule,

$$\left[\frac{4(\sec^2 \theta + 2\theta \sec^2 \theta \tan \theta + \sec^2 \theta - 4 \tan \theta - 4\theta \sec^2 \theta - 4\theta \sec^2 \theta - 4\theta^2 \sec^2 \theta \tan \theta)}{4 \cos 2\theta} \right]$$

$$= \frac{4(1 + 0 + 1)}{4} = 2$$

28. (d) Given that, $f(x) = \begin{cases} 1, & x < 0 \\ 1 + \sin x, & 0 \leq x \leq \pi/2 \end{cases}$

At $x = 0$,

$$\text{LHD} = \lim_{h \rightarrow 0} \frac{f(0 - h) - f(0)}{-h} = \lim_{h \rightarrow 0} \frac{1 - 1}{-h} = 0$$

$$\text{RHD} = \lim_{h \rightarrow 0} \frac{f(0 + h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{1 + \sin(0 + h) - 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1$$

$\therefore \text{LHD} \neq \text{RHD}$

$\therefore f'(x)$ does not exist at $x = 0$.

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Differential Coefficients

Derivative (Differential Coefficient)

The rate of change of a quantity y with respect to another quantity x is called the derivative or differential coefficient of y with respect to x . The process of finding derivative of a function is called **differentiation**.

Some Standard Differentiations

1. Differentiation of Algebraic Functions

$$(i) \frac{d}{dx} x^n = nx^{n-1}$$

$$(ii) \frac{d}{dx} (\sqrt{x}) = \frac{1}{2\sqrt{x}}$$

$$(iii) \frac{d}{dx} \left(\frac{1}{x^n} \right) = -\frac{n}{x^{n+1}}$$

2. Differentiation of Trigonometric Functions

$$(i) \frac{d}{dx} (\sin x) = \cos x$$

$$(ii) \frac{d}{dx} (\cos x) = -\sin x$$

$$(iii) \frac{d}{dx} (\tan x) = \sec^2 x$$

$$(iv) \frac{d}{dx} (\sec x) = \sec x \tan x$$

$$(v) \frac{d}{dx} (\cot x) = -\operatorname{cosec}^2 x$$

$$(vi) \frac{d}{dx} (\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

3. Differentiation of Logarithmic and Exponential Functions

$$(i) \frac{d}{dx} (\log x) = \frac{1}{x}, \text{ for } x > 0$$

$$(ii) \frac{d}{dx} (e^x) = e^x$$

$$(iii) \frac{d}{dx} (a^x) = a^x \log a, \text{ for } a > 0 \quad (iv) \frac{d}{dx} (\log_a x) = \frac{1}{x \log a}, \text{ for } x > 0, a > 0, a \neq 1$$

4. Differentiation of Inverse Trigonometric Functions

Sometimes the given function can be deduced with the help of inverse trigonometrical substitution and then to find the differential coefficient is very easy.

- (i) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}, \text{ for } -1 < x < 1$
- (ii) $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}, \text{ for } -1 < x < 1$
- (iii) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}, \text{ for } |x| > 1$
- (iv) $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = -\frac{1}{|x|\sqrt{x^2-1}}, \text{ for } |x| > 1$
- (v) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}, \text{ for } x \in R$
- (vi) $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}, \text{ for } x \in R$

5. Differentiation using Substitution

In order to find differential coefficients of complicated expression involving inverse trigonometric functions, some substitutions are very helpful, which are listed below:

S. No.	Function	Substitution
(i)	$\sqrt{a^2 - x^2}$	$x = a \sin \theta \text{ or } a \cos \theta$
(ii)	$\sqrt{x^2 - a^2}$	$x = a \sec \theta \text{ or } a \operatorname{cosec} \theta$
(iii)	$\sqrt{\frac{a^2 - x^2}{a^2 + x^2}}$	$x^2 = a^2 \cos 2\theta$
(iv)	$\sqrt{\frac{x}{a+x}}$	$x = a \tan^2 \theta$
(v)	$\sqrt{(x-a)(x-b)}$	$x = a \sec^2 \theta - b \tan^2 \theta$
(vi)	$\sqrt{x^2 + a^2}$	$x = a \tan \theta \text{ or } a \cot \theta$
(vii)	$\sqrt{\frac{a-x}{a+x}}$	$x = a \cos 2\theta$
(viii)	$\sqrt{ax - x^2}$	$x = a \sin^2 \theta$
(ix)	$\sqrt{\frac{x}{a-x}}$	$x = a \sin^2 \theta$
(x)	$\sqrt{(x-a)(b-x)}$	$x = a \cos^2 \theta + b \sin^2 \theta$

Differentiation of Implicit Functions

If the relation between the variables x and y are given by an equation containing both the variables and this equation is not immediately solvable for y , then y is called an implicit function of x . Implicit functions are given by $f(x, y) = 0$. To find the differentiation of implicit function, we differentiate each term w.r.t. x considering y as a function of x and then collect the terms of dy/dx together on left hand side and remaining terms on right hand side and find dy/dx .

Algebra of Differentiation

Algebra of differentiation is defined in following ways:

1. Differentiation of the Sum of Two Functions

Let $f(x)$ and $g(x)$ be two real valued functions. Then,
 $[f(x) + g(x)]' = f'(x) + g'(x)$

2. Differentiation of the Difference of Two Functions

Let $f(x)$ and $g(x)$ be two real valued functions. Then,
 $[f(x) - g(x)]' = f'(x) - g'(x)$

3. Differentiation of the Product of Two Functions

Let $f(x)$ and $g(x)$ be two real valued functions. Then,
 $[f(x) \cdot g(x)]' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

NOTE If three functions are given, then

$$\frac{d}{dx}[f(x) \cdot g(x) \cdot h(x)] = f(x) \cdot g(x) \cdot h'(x) + f(x) \cdot g'(x) \cdot h(x) + f'(x) \cdot g(x) \cdot h(x) = \frac{(fg)'h + (gh)'f + (hf)'g}{2}$$

4. Differentiation of the Quotient of Two Functions

Let $f(x)$ and $g(x)$ be two real valued functions. Then,

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x) \cdot g(x) - g'(x) \cdot f(x)}{[g(x)]^2}, \text{ provided } g(x) \neq 0$$

while applying the quotient rule, think twice and check whether the given function could be simplified prior to differentiation.

5. Chain/Composite Rule

If $y = f\{g(x)\}$, then $\frac{dy}{dx} = f'\{g(x)\}g'(x)$

Derivative of Function in Parametric Form

A relation expressed between two variables x and y in the form $x = f(t)$, $y = g(t)$ is said to be parametric form, where t is a parameter. The derivative dy/dx of such function is given by

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{g'(t)}{f'(t)}$$

provided $f'(t) \neq 0$.

Logarithmic Differentiation

When a function consists of product or quotient of number of functions, then we take the logarithm and then differentiate.

The functions which can be evaluated by using this method are of following types

$$(i) y = \{f_1(x)\}^{\{f_2(x)\}} \quad (ii) y = f_1(x), f_2(x), f_3(x), \dots$$

$$(iii) y = \frac{f_1(x) \cdot f_2(x) \cdot f_3(x) \dots}{\phi_1(x) \cdot \phi_2(x) \cdot \phi_3(x) \dots}$$

NOTE If $y = \{f(x)\}^{g(x)}$, then $\frac{dy}{dx} = \text{Differentiation of } \{f(x)\}^{g(x)} \text{ w.r.t. } x$
 [taking $g(x)$ as a constant] + Differentiation of $\{f(x)\}^{g(x)}$ w.r.t. x
 [taking $f(x)$ as a constant]

$$\frac{dy}{dx} = g(x) \cdot \{f(x)\}^{g(x)-1} \cdot \frac{d}{dx} f(x) + \{f(x)\}^{g(x)} \cdot \log f(x) \cdot \frac{d}{dx} g(x)$$

Differentiation of a Function of x with Respect to Another Function of x

If $y = f(x)$ and $z = g(x)$

$$\Rightarrow \frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{f'(x)}{g'(x)}$$

Differentiation of a Determinant

$$\text{If } y = \begin{vmatrix} p & q & r \\ u & v & w \\ l & m & n \end{vmatrix}, \text{ then}$$

$$\frac{dy}{dx} = \begin{vmatrix} \frac{dp}{dx} & \frac{dq}{dx} & \frac{dr}{dx} \\ u & v & w \\ l & m & n \end{vmatrix} + \begin{vmatrix} p & q & r \\ \frac{du}{dx} & \frac{dv}{dx} & \frac{dw}{dx} \\ l & m & n \end{vmatrix} + \begin{vmatrix} p & q & r \\ u & v & w \\ \frac{dl}{dx} & \frac{dm}{dx} & \frac{dn}{dx} \end{vmatrix}$$

Derivative of Order Two

For given $y = f(x)$, the process of finding its higher derivatives is called successive differentiation. The derivative of order two is denoted by $\frac{d^2y}{dx^2}$ or y_2 or y'' and is obtained by differentiating dy/dx again w.r.t. x or differentiating $y = f(x)$ twice w.r.t. x .

NOTE • The derivative of order three is obtained by differentiating $\frac{d^2y}{dx^2}$ again w.r.t. x and it is denoted by $\frac{d^3y}{dx^3}$ or y_3 or y''' .
 • In order to find second order derivative of parametric function, we can also use the following formulae

$$\text{Let } x = \phi(t), y = \psi(t)$$

$$\begin{aligned} \text{Then, } \frac{d^2y}{dx^2} &= \frac{d}{dx} \left(\frac{dy}{dx} \right) \\ &= \frac{d}{dx} \left(\frac{\psi'(t)}{\phi'(t)} \right) \\ &= \frac{\frac{d}{dt} \left(\frac{\psi'(t)}{\phi'(t)} \right)}{\frac{dx}{dt}} \end{aligned}$$

Practice Exercise

- If $f(x)$ is an odd differentiable function defined on $(-\infty, \infty)$ such that $f'(3) = 2$, then $f'(-3)$ equals
 a. 0 b. 1 c. 2 d. 4
- If $x^2 + y^2 = t - \frac{1}{t}$, $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then $x^3y \frac{dy}{dx}$ is equal to
 a. 1 b. 2 c. 3 d. 4
- If $f(x) = |\cos x|$, then $f'\left(\frac{3\pi}{4}\right)$ is equal to
 a. $\frac{1}{\sqrt{2}}$ b. $\sqrt{2}$ c. $\frac{1}{2}$ d. $2\sqrt{2}$
- If $f(x) = |\cos x - \sin x|$, then $f'\left(\frac{\pi}{2}\right)$ is equal to
 a. 1 b. -1 c. 0 d. None
- If $f'(x) = \phi(x)$ and $\phi'(x) = f(x)$ for all x . Also, $f(3) = 5$ and $f'(3) = 4$. Then, the value of $[f(3)]^2 - [\phi(3)]^2$ is
 a. 0 b. 9
 c. 41 d. None of these
- If $f(x) = x^n$, then the value of $f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + \dots + \frac{(-1)^n f^n(1)}{n!}$ is
 a. 2^n b. 0
 c. 2^{n-1} d. None of these
- Let $f(x) = x^2 + x g'(1) + g''(2)$ and $g(x) = x^2 + x f'(2) + f''(3)$, then
 a. $f'(1) = 4 - f'(2)$ b. $g'(2) = 8 - g'(1)$
 c. $g''(2) + f''(3) = 4$ d. None of these

8. If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx}$ is equal to
 a. $\frac{2^x + 2^y}{2^x - 2^y}$ b. $\frac{2^x + 2^y}{1 + 2^{x+y}}$
 c. $2^{x-y} \left(\frac{2^y - 1}{1 - 2^x} \right)$ d. $\frac{2^{x+y} - 2^x}{2^y}$
9. The solution set of $f'(x) > g'(x)$, where $f(x) = \frac{1}{2}(5)^{2x+1}$ and $g(x) = 5^x + 4x \log_e 5$, is
 a. $(1, \infty)$ b. $(0, 1)$
 c. $(\infty, 0)$ d. $(0, \infty)$
10. If $y = \sec^{-1} \left(\frac{\sqrt{x} + 1}{\sqrt{x} - 1} \right) + \sin^{-1} \left(\frac{\sqrt{x} - 1}{\sqrt{x} + 1} \right)$, then $\frac{dy}{dx}$ is equal to
 a. 0 b. $\frac{1}{\sqrt{x} + 1}$ c. 1 d. 3
11. If $y = \tan^{-1} \left(\frac{4x}{1 + 5x^2} \right) + \tan^{-1} \left(\frac{2 + 3x}{3 - 2x} \right)$, then $\frac{dy}{dx}$ is equal to
 a. $\frac{1}{1 + x^2}$ b. $\frac{5}{1 + 25x^2}$ c. 1 d. 2
12. Let $g(x)$ be the inverse of the function $f(x)$ and $f'(x) = \frac{1}{1 + x^3}$, then $g'(x)$ is equal to
 a. $\frac{1}{1 + [g(x)]^3}$ b. $\frac{1}{1 + [f(x)]^3}$
 c. $1 + [g(x)]^3$ d. $1 + [f(x)]^3$
13. If $f(x) = e^x g(x)$, $g(0) = 2$, $g'(0) = 1$, then $f'(0)$ equals
 a. 1 b. 3 c. 2 d. 0
14. If $y = \log x \cdot e^{(\tan x + x^2)}$, then $\frac{dy}{dx}$ is equal to
 a. $e^{(\tan x + x^2)} \left[\frac{1}{x} + (\sec^2 x + x) \log x \right]$
 b. $e^{(\tan x + x^2)} \left[\frac{1}{x} + (\sec^2 x - x) \log x \right]$
 c. $e^{(\tan x + x^2)} \left[\frac{1}{x} + (\sec^2 x + 2x) \log x \right]$
 d. $e^{(\tan x + x^2)} \left[\frac{1}{x} + (\sec^2 x - 2x) \log x \right]$
15. If $f(x) = \cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x \cdot \cos 16x$, then $f' \left(\frac{\pi}{4} \right)$ is equal to
 a. $\sqrt{2}$ b. $\frac{1}{\sqrt{2}}$ c. 1 d. $\frac{\sqrt{3}}{2}$
16. The derivative of $y = (1-x)(2-x) \dots (n-x)$ at $x = 1$ is
 a. 0 b. $(-1)(n-1)!$
 c. $n! - 1$ d. $(-1)^{n-1}(n-1)!$
17. If f and g are differentiable functions satisfying $g'(a) = 2, g(a) = b$ and $f \circ g = I$ (identity function). Then, $f'(b)$ is equal to
 a. $\frac{1}{2}$ b. 2 c. $\frac{2}{3}$ d. None
18. If $f(x) = \sqrt{1 + \cos^2(x^2)}$, then $f' \left(\frac{\sqrt{\pi}}{2} \right)$ is equal to
 a. $\sqrt{\pi}/6$ b. $-\sqrt{\pi}/6$ c. $1/\sqrt{6}$ d. $\pi/\sqrt{6}$
19. If $y = (1-x)(1+x^2)(1+x^4) \dots (1+x^{2^n})$, then $\frac{dy}{dx}$ at $x = 0$ is equal to
 a. -1 b. $\frac{1}{(1+x)^2}$ c. $\frac{x}{(1+x^2)}$ d. $\frac{x}{(1-x)^2}$
20. If $f(x) = \frac{x^2 - x}{x^2 + 2x}$, where $x \neq 0, -2$, then $\frac{d}{dx} [f^{-1}(x)]$ (wherever it is defined) is equal to
 a. $\frac{-1}{(1-x)^2}$ b. $\frac{3}{(1-x)^2}$ c. $\frac{1}{(1-x)^2}$ d. $\frac{-3}{(1-x)^2}$
21. If $y = \sin[\cos(x^2)]$, then $\frac{dy}{dx}$ is equal to
 a. $-2x \sin x^2 \cdot \cos(\cos x^2)$ b. $-2x \sin x^2 \cdot \cos x^2$
 c. $2x^2 \sin x^2 \cdot \cos x^2$ d. None of these
22. If $F(x) = f(x) \cdot g(x)$ and $f'(x) \cdot g'(x) = c$, then
 a. $F' = c \left[\frac{f}{f'} + \frac{g}{g'} \right]$ b. $F' = c \left[\frac{f}{f'} - \frac{g}{g'} \right]$
 c. $\frac{F''}{F} = \frac{f''}{f} + \frac{g''}{g} + \frac{2c}{fg}$ d. None of these
23. If $y \cos x + x \cos y = \pi$, then $\frac{dy}{dx}$ is equal to
 a. $\frac{y \sin x - \cos y}{\cos x - x \sin y}$ b. $\frac{\cos y - y \sin x}{x \sin y + \cos x}$
 c. $\frac{y \sin x + \cos y}{\cos x - x \sin y}$ d. $\frac{y \sin x + \cos y}{\cos x + x \sin y}$
24. If $f: (-1, 1) \rightarrow \mathbb{R}$ is a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$, then $g'(0)$ is equal to
 a. 4 b. -4 c. 0 d. -2
25. If $y = \frac{a^{\cos^{-1} x}}{1 + a^{\cos^{-1} x}}$ and $z = a^{\cos^{-1} x}$, then $\frac{dy}{dz}$ is equal to
 a. $-\frac{1}{1 + a^{\cos^{-1} x}}$ b. $\frac{1}{1 + a^{\cos^{-1} x}}$
 c. $\frac{1}{(1 + a^{\cos^{-1} x})^2}$ d. None of these
26. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then $\frac{dy}{dx}$ is equal to
 a. $\sqrt{\frac{1-x^2}{1-y^2}}$ b. $\sqrt{\frac{1-y^2}{1-x^2}}$ c. $\sqrt{\frac{x^2-1}{1-y^2}}$ d. $\sqrt{\frac{y^2-1}{1-x^2}}$

27. If $y = \sin^{-1}(x\sqrt{1-x} + \sqrt{x}\sqrt{1-x^2})$, then $\frac{dy}{dx}$ is equal to

- a. $\frac{-2x}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{x-x^2}}$ b. $\frac{-1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x-x^2}}$
 c. $\frac{1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{x-x^2}}$ d. None of these

28. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to

- a. $\frac{1}{\sqrt{2}}$ b. $\frac{1}{2}$ c. 1 d. $\sqrt{2}$

29. If $y = \sin^{-1} x + \sin^{-1} \sqrt{1-x^2}$; $-1 \leq x \leq 1$, then $\frac{dy}{dx}$ is

- a. 0 b. $\frac{\pi}{2}$ c. 1 d. 4

30. If $x = \frac{2t}{1+t^2}$ and $y = \frac{1-t^2}{1+t^2}$, then $\frac{dy}{dx}$ is equal to

- a. $\frac{2t}{t^2+1}$ b. $\frac{2t}{t^2-1}$ c. $\frac{2t}{1-t^2}$ d. None

31. If $x = a \cos t \sqrt{\cos 2t}$ and $y = a \sin t \sqrt{\cos 2t}$

(where, $a > 0$), then $\left| \frac{1 + \left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}} \right|^{\frac{3}{2}}$ at $\frac{\pi}{6}$ is given by

- a. $\frac{a}{3}$ b. $a\sqrt{2}$ c. $\frac{\sqrt{2}}{3a}$ d. $\frac{\sqrt{2}a}{3}$

32. If $x = a(\theta + \sin \theta)$ and $y = a(1 - \cos \theta)$, then $\frac{dy}{dx}$ is equal to

- a. $\tan \frac{\theta}{2}$ b. $\cot \frac{\theta}{2}$ c. $\tan \theta$ d. $\cot \theta$

33. If $x = e^{-t^2}$ and $y = \tan^{-1}(2t + 1)$, then $\frac{dy}{dx}$ equals

- a. $\frac{e^{t^2}}{2t(t^2 + 2t + 1)}$ b. $\frac{-e^{t^2}}{2t(2t^2 + 2t + 1)}$
 c. $\frac{e^{-t^2}}{2t(t^2 + 2t + 1)}$ d. None of these

34. If $\sqrt{(x^2 + y^2)} = ae^{\tan^{-1}(y/x)}$, $a > 0$, then $y''(0)$ equals

- a. $\frac{a}{2}e^{\pi/2}$ b. $ae^{\pi/2}$ c. $\frac{-2}{a}e^{-\pi/2}$ d. $\frac{a}{2}e^{-\pi/2}$

35. The expression of $\frac{dy}{dx}$ of the function $y = a^{x^{a^x}}$ equals

- a. $\frac{y^2}{x(1-y \log x)}$ b. $\frac{y^2 \log y}{x(1-y \log x)}$
 c. $\frac{y^2 \log y}{x(1-y \log x \log y)}$ d. $\frac{y^2 \log y}{x(1+y \log x \log y)}$

36. Derivative of $(\sin x)^x + \sin^{-1} \sqrt{x}$ with respect to x is

- a. $(x \cot x + \log \sin x) + \frac{1}{2\sqrt{x-x^2}}$
 b. $(x \cot x + \log \sin x) + \frac{1}{\sqrt{x-x^2}}$
 c. $(\sin x)(x \cot x + \log x) + \frac{1}{\sqrt{x-x^2}}$
 d. $(\sin x)^x(x \cot x + \log \sin x) + \frac{1}{2\sqrt{x-x^2}}$

37. If $x^m y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is equal to

- a. $\frac{x+y}{xy}$ b. xy c. $\frac{x}{y}$ d. $\frac{y}{x}$

38. Let $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots}}}$, then $\frac{dy}{dx}$ is equal to

- a. $\frac{1}{2y-1}$ b. $\frac{y}{2x+y}$ c. $\frac{1}{\sqrt{1+4x}}$ d. All

39. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then, $y'(1)$ equals

- a. -1 b. 1 c. $\log 2$ d. $-\log 2$

40. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to

$\tan^{-1}\left(2x \frac{\sqrt{1-x^2}}{1-2x^2}\right)$ at $x = 0$ is

- a. $1/8$ b. $1/4$ c. $1/2$ d. 1

41. If $f(x) = \begin{vmatrix} \sec \theta & \tan^2 \theta & 1 \\ \theta \sec x & \tan x & x \\ 1 & \tan x - \tan \theta & 0 \end{vmatrix}$, then $f'(\theta)$ is

- a. zero b. -1
 c. independent of θ d. None of these

42. If $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ P & P^2 & P^3 \end{vmatrix}$, where P is a constant.

Then, $\frac{d^3}{dx^3} \{f(x)\}$ at $x = 0$ is equal to

- a. P b. $P + P^2$
 c. $P + P^3$ d. independent of P

43. If $y = ae^{-kt} \cos(pt + c)$ and $\frac{d^2y}{dt^2} + 2k \frac{dy}{dt} + n^2y = 0$, then n^2 equals

- a. $p^2 - k^2$ b. p^2 c. k^2 d. $p^2 + k^2$

44. If $\sqrt{x+y} + \sqrt{y-x} = c$, then $\frac{d^2y}{dx^2}$ equals

- a. $2/c$ b. $-2/c^2$ c. $2/c^2$ d. None

BITSAT Archives

1. If x is measured in degree, then $\frac{d}{dx}(\cos x)$ is equal to [2012]
 - a. $-\sin x$
 - b. $-\frac{180}{\pi} \sin x$
 - c. $-\frac{\pi}{180} \sin x$
 - d. $\sin x$
2. $\left(\frac{d}{dx}\right)[\log(\sec x - \tan x)]$ is equal to [2012]
 - a. $-\sec x$
 - b. $\sec x + \tan x$
 - c. $\sec x$
 - d. $\sec x - \tan x$
3. If $x = \cos^3 \theta$ and $y = \sin^3 \theta$, then $1 + \left(\frac{dy}{dx}\right)^2$ is equal to [2012]
 - a. $\tan^2 \theta$
 - b. $\cot^2 \theta$
 - c. $\sec^2 \theta$
 - d. $\operatorname{cosec}^2 \theta$
4. If $x = at^2$, $y = 2at$, then $\frac{d^2y}{dx^2}$ is equal to [2012]
 - a. $-\frac{1}{t^2}$
 - b. $-\frac{1}{2at^3}$
 - c. $\frac{1}{t^2}$
 - d. $-\frac{a}{2t^3}$
5. If $\Delta_1 = \begin{vmatrix} x & a & b \\ b & x & a \\ a & b & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ are the given determinants, then [2012]
 - a. $\Delta_1 = 3(\Delta_2)^2$
 - b. $\left(\frac{d}{dx}\right)(\Delta_1) = 3\Delta_2$
 - c. $\left(\frac{d}{dx}\right)(\Delta_1) = 3(\Delta_2)^2$
 - d. $\Delta_1 = 3(\Delta_2)^{3/2}$
6. $\frac{d}{dx}(x^x)$ is equal to [2011]
 - a. $x^x \log(e/x)$
 - b. $x^x \log ex$
 - c. $\log ex$
 - d. $x^x \log x$
7. If $y^2 = P(x)$ is a cubic polynomial, then $2 \frac{d}{dx} \left(y^3 \frac{d^2y}{dx^2} \right)$ is equal to [2010]
 - a. $P'''(x) P'(x)$
 - b. $P''(x) P'''(x)$
 - c. $P(x) P'''(x)$
 - d. constant
8. If $f(x) = (1-x)^n$, then the value of $f(0) + f'(0) + \frac{f''(0)}{2!} + \dots + \frac{f^n(0)}{n!}$ is equal to [2009]
 - a. 2^n
 - b. 0
 - c. 2^{n-1}
 - d. 1
9. If $f(x) = \log_x(\ln x)$, then $f'(x)$ at $x = e$ is [2009]
 - a. $1/e$
 - b. e
 - c. $-1/e$
 - d. 0
10. Let $f(x) = 2^{2x-1}$ and $\phi(x) = -2^x + 2x \log 2$. If $f'(x) > \phi'(x)$, then [2009]
 - a. $0 < x < 1$
 - b. $0 \leq x < 1$
 - c. $x > 0$
 - d. $x \geq 0$
11. If $y = \sin^{-1}\left(\frac{5x + 12\sqrt{1-x^2}}{13}\right)$, then $\frac{dy}{dx}$ is equal to [2009]
 - a. $\frac{1}{\sqrt{1-x^2}}$
 - b. $-\frac{1}{\sqrt{1-x^2}}$
 - c. $\frac{3}{\sqrt{1-x^2}}$
 - d. $\sqrt{1-x^2}$
12. If $x = a \left\{ \cos \theta + \log \tan\left(\frac{\theta}{2}\right) \right\}$ and $y = a \sin \theta$, then $\frac{dy}{dx}$ is equal to [2008]
 - a. $\cot \theta$
 - b. $\tan \theta$
 - c. $\sin \theta$
 - d. $\cos \theta$
13. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is [2007]
 - a. -1
 - b. 1
 - c. 2
 - d. 4
14. If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$, then $\frac{dy}{dx}$ is equal to [2007]
 - a. $\frac{x}{y}$
 - b. $-\frac{x}{y}$
 - c. $\frac{y}{x}$
 - d. $-\frac{y}{x}$
15. If $y = \tan^{-1} \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}$, then $\frac{dy}{dx}$ is equal to [2006]
 - a. $\frac{x^2}{\sqrt{1-x^4}}$
 - b. $\frac{x^2}{\sqrt{1+x^4}}$
 - c. $\frac{x}{\sqrt{1+x^4}}$
 - d. $\frac{x}{\sqrt{1-x^4}}$
16. If $x = \sin t$, $y = \cos pt$, then [2006]
 - a. $(1-x^2)y_2 + xy_1 + p^2y = 0$
 - b. $(1-x^2)y_2 + xy_1 - p^2y = 0$
 - c. $(1+x^2)y_2 - xy_1 + p^2y = 0$
 - d. $(1-x^2)y_2 - xy_1 + p^2y = 0$
17. If $r = [2\phi + \cos^2(2\phi + \pi/4)]^{1/2}$, then what is the value of the derivative of $dr/d\phi$ at $\phi = \pi/4$? [2005]
 - a. $2\left(\frac{1}{\pi+1}\right)^{1/2}$
 - b. $2\left(\frac{2}{\pi+1}\right)^2$
 - c. $\left(\frac{2}{\pi+1}\right)^{1/2}$
 - d. $2\left(\frac{2}{\pi+1}\right)^{1/2}$

Answer with Solutions

Practice Exercise

1. (c) Since, $f(x)$ is odd.

$$\begin{aligned}\therefore f(-x) &= -f(x) \\ \therefore f'(-x)(-1) &= -f'(x) \\ \Rightarrow f'(-x) &= f'(x) \Rightarrow f'(-3) = f'(3) = 2\end{aligned}$$

2. (a) $x^4 + y^4 = \left(t - \frac{1}{t}\right)^2 + 2 = (x^2 + y^2)^2 + 2$

$$\Rightarrow x^2 y^2 = -1 \Rightarrow y^2 = -\frac{1}{x^2}$$

On differentiating, we get

$$2y \frac{dy}{dx} = \frac{2}{x^3} \Rightarrow x^3 y \frac{dy}{dx} = 1$$

3. (a) When $\frac{\pi}{2} < x < \pi$, $\cos x < 0$

So that, $|\cos x| = -\cos x$ i.e. $f(x) = -\cos x$, $f'(x) = \sin x$

Hence, $f'\left(\frac{3\pi}{4}\right) = \sin\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}}$

4. (a) When $0 < x < \frac{\pi}{4}$, $\cos x > \sin x$

$$\therefore \cos x - \sin x > 0$$

Also, when $\frac{\pi}{4} < x < \pi$, $\cos x < \sin x$

When $\frac{\pi}{4} < x < \pi$, $\cos x - \sin x < 0$

$$\therefore |\cos x - \sin x| = -(\cos x - \sin x), \text{ when } \frac{\pi}{4} < x < \pi$$

$$\Rightarrow f'(x) = \sin x + \cos x$$

$$\Rightarrow f'\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} + \cos \frac{\pi}{2} = 1 + 0 = 1$$

5. (b) $\frac{d}{dx} \{[f(x)]^2 - [\phi(x)]^2\} = 2[f(x) \cdot f'(x) - \phi(x) \cdot \phi'(x)]$

$$= 2[f(x) \cdot \phi(x) - \phi(x) \cdot f(x)]$$

$$= 0 \quad [\because f'(x) = \phi(x) \text{ and } \phi'(x) = f(x)]$$

$$\Rightarrow [f(x)]^2 - [\phi(x)]^2 = \text{Constant}$$

$$\therefore [f(3)]^2 - [\phi(3)]^2 = [f(3)]^2 - [f'(3)]^2 = 25 - 16 = 9$$

6. (b) We have, $f(x) = x^n \Rightarrow f(1) = 1 = {}^nC_0$

$$\frac{f'(1)}{1!} = \frac{n}{1!} = {}^nC_1$$

$$\frac{f''(1)}{2!} = \frac{n(n-1)}{2!} = {}^nC_2$$

$$\frac{f'''(1)}{3!} = \frac{n(n-1)(n-2)}{3!} = {}^nC_3$$

$$\vdots$$

$$\frac{f^n(1)}{n!} = \frac{n!}{n!} = {}^nC_n$$

$$\begin{aligned}\therefore f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + \dots + \frac{(-1)^n f^n(1)}{n!} \\ = {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \dots + (-1)^n {}^nC_n = (1-1)^n = 0\end{aligned}$$

7. (c) Given that,

$$f(x) = x^2 + xg'(1) + g''(2)$$

and

$$g(x) = x^2 + xg'(2) + f''(3)$$

Now,

$$f'(x) = 2x + g'(1); f''(x) = 2$$

Also,

$$g'(x) = 2x + f'(2)$$

$$g''(x) = 2$$

Now, at $x = 1$,

$$f'(1) = 2 + g'(1)$$

and

$$g'(1) = 2 + f'(2)$$

$\Rightarrow$

$$f'(1) = 4 + f'(2)$$

Also, at $x = 2$,

$$f'(2) = 4 + g'(1) \text{ and } g'(2) = 4 + f'(2)$$

$\therefore$

$$g'(2) = 4 + 4 + g'(1) = 8 + g'(1)$$

$$\Rightarrow g'(2) = 8 + g'(1) \text{ and } g''(2) + f''(3) = 2 + 2 = 4$$

8. (c) $2^x + 2^y = 2^{x+y}$

[given]

$$\Rightarrow 2^x \log 2 + 2^y \log 2 \frac{dy}{dx} = 2^{x+y} \log 2 \left[1 + \frac{dy}{dx}\right]$$

$$\Rightarrow 2^x + 2^y \frac{dy}{dx} = 2^{x+y} + 2^{x+y} \frac{dy}{dx}$$

$$\Rightarrow (2^y - 2^{x+y}) \frac{dy}{dx} = 2^{x+y} - 2^x$$

$$\Rightarrow 2^y(1 - 2^x) \frac{dy}{dx} = 2^x(2^y - 1)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2^x(2^y - 1)}{2^y(1 - 2^x)} = 2^{x-y} \frac{(2^y - 1)}{(1 - 2^x)}$$

9. (d) Hint Since, $f'(x) > g'(x)$

$$\Rightarrow \left(\frac{1}{2}\right) 5^{2x+1} \log_e 5 \times 2 > 5^x \log_e 5 + 4 \log_e 5$$

$$\Rightarrow 5^{2x} \cdot 5 > 5^x + 4$$

$$\Rightarrow 5 \cdot 5^{2x} - 5^x - 4 > 0$$

$$\Rightarrow (5^x - 1)(5 \cdot 5^x + 4) > 0$$

$$\therefore 5^x > 1$$

$$\Rightarrow x > 0$$

10. (a) $\therefore y = \sec^{-1} \left[\frac{\sqrt{x} + 1}{\sqrt{x} - 1} \right] + \sin^{-1} \left[\frac{\sqrt{x} - 1}{\sqrt{x} + 1} \right]$

$$= \cos^{-1} \left[\frac{\sqrt{x} - 1}{\sqrt{x} + 1} \right] + \sin^{-1} \left[\frac{\sqrt{x} - 1}{\sqrt{x} + 1} \right] = \frac{\pi}{2}$$

$$\Rightarrow \frac{dy}{dx} = 0 \quad \left[\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \right]$$

11. (b) $\tan^{-1} \frac{4x}{1+5x^2} = \tan^{-1} \left(\frac{5x-x}{1+5x \cdot x} \right) = \tan^{-1} 5x - \tan^{-1} x$

$$\text{and } \tan^{-1} \left(\frac{2+3x}{3-2x} \right) = \tan^{-1} \left(\frac{\frac{2}{3} + x}{1 - \frac{2}{3}x} \right) = \tan^{-1} \frac{2}{3} + \tan^{-1} x$$

$$\therefore y = \tan^{-1} 5x + \tan^{-1} \frac{2}{3} \Rightarrow \frac{dy}{dx} = \frac{5}{1+25x^2}$$

12. (c) Since,
- $g(x)$
- is the inverse
- $f(x)$
- .

$$\begin{aligned} \therefore f(x) = y &\Rightarrow g(y) = x \\ \Rightarrow g'[f(x)] &= \frac{1}{f'(x)}, \forall x \Rightarrow g'[f(x)] = 1 + x^3, \forall x \\ \Rightarrow g'(y) &= 1 + [g(y)]^3 \\ &\quad [\text{using } f(x) = y \Leftrightarrow x = g(y)] \\ \Rightarrow g'(x) &= 1 + [g(x)]^3 \end{aligned}$$

13. (b)
- $f(x) = e^x g(x)$

$$\begin{aligned} \Rightarrow f'(x) &= e^x g(x) + e^x g'(x) \\ \Rightarrow f'(0) &= g(0) + g'(0) = 2 + 1 = 3 \end{aligned}$$

14. (c) Given,
- $y = \log x \cdot e^{(\tan x + x^2)}$

$$\begin{aligned} \therefore \frac{dy}{dx} &= e^{(\tan x + x^2)} \cdot \frac{1}{x} + \log x \cdot e^{(\tan x + x^2)} (\sec^2 x + 2x) \\ &= e^{(\tan x + x^2)} \left[\frac{1}{x} + (\sec^2 x + 2x) \log x \right] \end{aligned}$$

15. (a)
- $f(x) = \frac{2 \sin x \cdot \cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x \cdot \cos 16x}{2 \sin x}$

$$\begin{aligned} &= \frac{\sin 2x \cos 2x \cos 4x \cdot \cos 8x \cdot \cos 16x}{2 \sin x} = \frac{\sin 32x}{2^5 \sin x} \\ \therefore f'(x) &= \frac{1}{32} \cdot \frac{32 \cos 32x \cdot \sin x - \cos x \cdot \sin 32x}{\sin^2 x} \\ \Rightarrow f'\left(\frac{\pi}{4}\right) &= \frac{32 \times \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \times 0}{32 \left[\frac{1}{\sqrt{2}}\right]^2} = \sqrt{2} \end{aligned}$$

16. (b)
- $\frac{dy}{dx} = -[(2-x)(3-x) \dots (n-x) + (1-x)(3-x) \dots (n-x)$

$$\begin{aligned} &\quad + \dots + (1-x)(2-x) \dots (n-1-x)] \\ \Rightarrow \left(\frac{dy}{dx}\right)_{x=1} &= -[(n-1)! + 0 + \dots + 0] = (-1)(n-1)! \end{aligned}$$

17. (a) Since,
- $f \circ g = I \Rightarrow f \circ g(x) = x, \forall x$

$$\begin{aligned} \Rightarrow f'[g(x)] g'(x) &= 1, \forall x \\ \Rightarrow f'[g(a)] &= \frac{1}{g'(a)} = \frac{1}{2} \\ \Rightarrow f'(b) &= \frac{1}{2} \quad [\because g(a) = b] \end{aligned}$$

18. (b)
- $f(x) = \sqrt{1 + \cos^2(x^2)}$

$$\begin{aligned} \Rightarrow f'(x) &= \frac{1}{2\sqrt{1 + \cos^2(x^2)}} (2 \cos x^2) \cdot (-\sin x^2) \cdot (2x) \\ \Rightarrow f'(x) &= \frac{-x \sin 2x^2}{\sqrt{1 + \cos^2(x^2)}} \end{aligned}$$

$$\text{At } x = \frac{\sqrt{\pi}}{2}, f'\left(\frac{\sqrt{\pi}}{2}\right) = \frac{-\frac{\sqrt{\pi}}{2} \cdot \sin \frac{2\pi}{4}}{\sqrt{1 + \cos^2 \frac{\pi}{4}}} = \frac{-\frac{\sqrt{\pi}}{2} \cdot 1}{\sqrt{\frac{3}{2}}}$$

$$\therefore f'\left(\frac{\sqrt{\pi}}{2}\right) = -\sqrt{\frac{\pi}{6}}$$

19. (a) Given,
- $y = (1-x)(1+x^2)(1+x^4) \dots (1+x^{2^n})$

$$\begin{aligned} \text{or } y &= \frac{(1-x^2)(1+x^2) \dots (1+x^{2^n})}{(1+x)} = \frac{1-x^{4n}}{(1+x)} \\ \therefore \frac{dy}{dx} &= \frac{(1+x) \cdot (0 - 4n \cdot x^{4n-1}) - (1-x^{4n}) \cdot 1}{(1+x)^2} \\ \therefore \left(\frac{dy}{dx}\right)_{x=0} &= -1 \end{aligned}$$

20. (b) Let
- $y = \frac{x^2 - x}{x^2 + 2x}$

$$\begin{aligned} \Rightarrow x &= \frac{2y+1}{-y+1}; x \neq 0 \Rightarrow f^{-1}(x) = \frac{2x+1}{-x+1} \\ \therefore \frac{d}{dx} \{f^{-1}(x)\} &= \frac{(-x+1) \cdot 2 - (2x+1)(-1)}{(-x+1)^2} = \frac{3}{(-x+1)^2} \end{aligned}$$

21. (a) Here,
- $y = \sin[\cos(x^2)]$

$$\text{So, } \frac{dy}{dx} = \frac{d}{dx} [\sin \{\cos(x^2)\}]$$

On applying chain rule, we get

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{d \cos x^2} \sin \{\cos(x^2)\} \times \frac{d}{dx^2} \cos(x^2) \times \frac{d}{dx} x^2 \\ &= \cos(\cos x^2) \times (-\sin x^2) \times 2x \\ &= -2x \sin x^2 \cdot \cos(\cos x^2) \end{aligned}$$

22. (a) It is given that,
- $F(x) = f(x) \cdot g(x)$

On differentiating both sides w.r.t. x , we get

$$\begin{aligned} F'(x) &= f'(x) \cdot g(x) + f(x) \cdot g'(x) \\ &= f'(x) \cdot g'(x) \left[\frac{f(x)}{f'(x)} + \frac{g(x)}{g'(x)} \right] \end{aligned}$$

$$\Rightarrow F' = c \left[\frac{f}{f'} + \frac{g}{g'} \right]$$

23. (a) It is given that,
- $y \cos x + x \sin y = \pi$

On differentiating both sides w.r.t. x , we get

$$\begin{aligned} \frac{dy}{dx} \cos x - y \sin x + \cos y - x \sin y \frac{dy}{dx} &= 0 \\ \Rightarrow (\cos x - x \sin y) \frac{dy}{dx} &= y \sin x - \cos y \\ \Rightarrow \frac{dy}{dx} &= \frac{y \sin x - \cos y}{\cos x - x \sin y} \end{aligned}$$

24. (b) We have,
- $f: (-1, 1) \rightarrow R$

$$f(0) = -1, f'(0) = 1$$

$$g(x) = [f(2f(x) + 2)]^2$$

$$\begin{aligned} \Rightarrow g'(x) &= 2[f(2f(x) + 2)] \times f'(2f(x) + 2) \times 2f'(x) \\ \Rightarrow g'(0) &= 2[f(2f(0) + 2)] \times f'(2f(0) + 2) \times 2f'(0) \\ &= 2[f(0)] \times f'(0) \times 2f'(0) \\ &= 2 \times (-1) \times 1 \times 2 \times 1 = -4 \end{aligned}$$

25. (c)
- $y = \frac{a^{\cos^{-1} x}}{1 + a^{\cos^{-1} x}}, z = a^{\cos^{-1} x} \Rightarrow y = \frac{z}{1+z}$

$$\Rightarrow \frac{dy}{dz} = \frac{(1+z)1 - z(1)}{(1+z)^2} = \frac{1}{(1+z)^2} = \frac{1}{(1 + a^{\cos^{-1} x})^2}$$

26. (b) On putting $x = \sin \theta$ and $y = \sin \phi$, we get

Given equation becomes

$$\begin{aligned} \cos \theta + \cos \phi &= a(\sin \theta - \sin \phi) \\ \Rightarrow 2 \cos \left(\frac{\theta + \phi}{2} \right) \cos \left(\frac{\theta - \phi}{2} \right) &= a \left\{ 2 \cos \left(\frac{\theta + \phi}{2} \right) \sin \left(\frac{\theta - \phi}{2} \right) \right\} \\ \Rightarrow \frac{\theta - \phi}{2} &= \cot^{-1} a \Rightarrow \theta - \phi = 2 \cot^{-1} a \\ \Rightarrow \sin^{-1} x - \sin^{-1} y &= 2 \cot^{-1} a \\ \Rightarrow \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} &= 0 \quad \therefore \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}} \end{aligned}$$

27. (c) On putting $x = \sin A$ and $\sqrt{x} = \sin B$

$$\begin{aligned} y &= \sin^{-1}(\sin A \sqrt{1-\sin^2 B} + \sin B \sqrt{1-\sin^2 A}) \\ &= \sin^{-1}(\sin A \cos B + \sin B \cos A) \\ &= \sin^{-1}[\sin(A+B)] = A+B = \sin^{-1} x + \sin^{-1} \sqrt{x} \\ \Rightarrow \frac{dy}{dx} &= \frac{1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{x-x^2}} \end{aligned}$$

28. (a) **Hint** Given, $y = \sec(\tan^{-1} x)$

Let $\tan^{-1} x = \theta \Rightarrow x = \tan \theta$

$$\therefore y = \sec \theta = \sqrt{1+x^2}$$

29. (a) **Hint** Here, $y = \sin^{-1} x + \sin^{-1} \sqrt{1-x^2}$

Put $x = \sin \theta \Rightarrow \theta = \sin^{-1} x$

$$\Rightarrow y = \sin^{-1}(\sin \theta) + \sin^{-1} \sqrt{1-\sin^2 \theta} = \theta + \sin^{-1} \sqrt{\cos^2 \theta}$$

30. (b) We have, $x = \frac{2t}{1+t^2}$, $y = \frac{1-t^2}{1+t^2}$

Put $t = \tan \theta$

$$\therefore x = \frac{2 \tan \theta}{1 + \tan^2 \theta} = \sin 2\theta \text{ and } y = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \cos 2\theta$$

$$\begin{aligned} \therefore \frac{dy}{dx} &= \frac{dy/d\theta}{dx/d\theta} = \frac{-2 \sin 2\theta}{2 \cos 2\theta} = -\tan 2\theta \\ &= \frac{-2 \tan \theta}{1 - \tan^2 \theta} = \frac{-2t}{1 - t^2} = \frac{2t}{t^2 - 1} \end{aligned}$$

31. (d) We have,

$$\frac{dx}{dt} = a \left[-\sin t \sqrt{\cos 2t} - \frac{\cos t \cdot \sin 2t}{\sqrt{\cos 2t}} \right] = \frac{-a \sin 3t}{\sqrt{\cos 2t}}$$

$$\text{and } \frac{dy}{dt} = a \left[\cos t \sqrt{\cos 2t} - \frac{\sin t \cdot \sin 2t}{\sqrt{\cos 2t}} \right] = \frac{a \cos 3t}{\sqrt{\cos 2t}}$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = -\cot 3t$$

$$\begin{aligned} \Rightarrow \frac{d^2 y}{dx^2} &= 3 \operatorname{cosec}^2 3t \cdot \frac{dt}{dx} \\ &= \frac{-3 \operatorname{cosec}^2 3t \cdot \sqrt{\cos 2t}}{a \sin 3t} \\ &= -\left(\frac{3}{a}\right) \operatorname{cosec}^3 3t \cdot \sqrt{\cos 2t} \end{aligned}$$

$$\begin{aligned} \therefore \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2} \bigg/ \frac{d^2 y}{dx^2} &= (1 + \cot^2 3t)^{3/2} \bigg/ \left(\frac{-3}{a} \right) \operatorname{cosec}^3 3t \sqrt{\cos 2t} \\ \Rightarrow \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2} \bigg/ \frac{d^2 y}{dx^2} \bigg|_{t=\frac{\pi}{6}} &= \frac{a}{3 \sqrt{\cos \frac{\pi}{3}}} = \frac{\sqrt{2}a}{3} \end{aligned}$$

32. (a) **Hint** $\frac{dx}{d\theta} = a(1 + \cos \theta)$ and $\frac{dy}{d\theta} = a(\sin \theta)$

33. (b) **Hint** $x = e^{-t^2}$ and $y = \tan^{-1}(2t + 1)$

$$\begin{aligned} \Rightarrow \frac{dx}{dt} &= -e^{-t^2} \times 2t \\ \text{and } \frac{dy}{dt} &= \frac{1}{1 + (2t + 1)^2} \quad (2) \\ \Rightarrow \frac{dx}{dt} &= -2t e^{-t^2} \text{ and } \frac{dy}{dt} = \frac{1}{4t^2 + 1 + 4t + 1} \quad (2) \\ \Rightarrow \frac{dx}{dt} &= -2t e^{-t^2} \text{ and } \frac{dy}{dt} = \frac{1}{4t^2 + 4t + 2} \times 2 \end{aligned}$$

34. (c) Given that, $\sqrt{x^2 + y^2} = ae^{\tan^{-1}(y/x)}$... (i)

Taking log on both sides, we get

$$\begin{aligned} \frac{1}{2} \log(x^2 + y^2) &= \log a + \tan^{-1} \left(\frac{y}{x} \right) \log_e e \quad [\because \log_e e = 1] \\ \Rightarrow \frac{1}{2} \times \frac{1}{(x^2 + y^2)} \left(2x + 2y \frac{dy}{dx} \right) &= 0 + \frac{1}{1 + \left(\frac{y}{x} \right)^2} \cdot \frac{d}{dx} \left(\frac{y}{x} \right) \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{x + y \frac{dy}{dx}}{x^2 + y^2} &= \frac{x^2}{x^2 + y^2} \cdot \frac{\left(x \frac{dy}{dx} - y \times 1 \right)}{x^2} \\ \Rightarrow x + y \frac{dy}{dx} &= x \frac{dy}{dx} - y \\ \Rightarrow \frac{dy}{dx} &= \frac{x + y}{x - y} \quad \dots (ii) \end{aligned}$$

$$\frac{d^2 y}{dx^2} = \frac{(x - y) \left(1 + \frac{dy}{dx} \right) - (x + y) \left(1 - \frac{dy}{dx} \right)}{(x - y)^2} \quad \dots (iii)$$

From Eq. (i) at $x = 0$, we get

$$y = ae^{\tan^{-1} \infty} = ae^{\pi/2} \Rightarrow y = ae^{\pi/2} \quad \dots (iv)$$

Now, from Eq. (ii) at $x = 0$, we get

$$\frac{dy}{dx} = \frac{0 + ae^{\pi/2}}{0 - ae^{\pi/2}} = -1$$

$$\Rightarrow \frac{dy}{dx} = -1$$

Now, from Eq. (iii), at $x = 0$, we get

$$\begin{aligned} \frac{d^2 y}{dx^2} &= \frac{(0 - ae^{\pi/2})(1 - 1) - (0 + ae^{\pi/2}) \times (1 + 1)}{(0 - ae^{\pi/2})^2} \\ &= \frac{0 - 2ae^{\pi/2}}{(0 - ae^{\pi/2})^2} = \frac{-2}{ae^{\pi/2}} = \frac{-2}{a} e^{-\pi/2} \end{aligned}$$

35. (c) **Hint** Given, $y = a^{x^{a^{x^{\dots}}}}$

$$y = (a^{x^y}) \Rightarrow \log y = \log a^{x^y} \Rightarrow \log y = x^y \log a$$

$$\Rightarrow \log(\log y) = y \log x + \log(\log a)$$

36. (d) Let $y = (\sin x)^x + \sin^{-1} \sqrt{x}$

$$\text{and } u = (\sin x)^x, v = \sin^{-1} \sqrt{x} \therefore y = u + v$$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

Now,

$$u = (\sin x)^x$$

$$\log u = x \log(\sin x)$$

$$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = x \frac{d}{dx} \log(\sin x) + \log(\sin x) \frac{d}{dx} (x)$$

$$= \frac{x}{\sin x} \cos x + \log(\sin x)$$

$$\frac{du}{dx} = u [x \cot x + \log \sin x]$$

$$= (\sin x)^x [x \cot x + \log \sin x]$$

Again,

$$v = \sin^{-1} \sqrt{x}$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \frac{d}{dx} (x^{1/2})$$

$$= \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2} x^{-1/2} \quad [\text{using chain rule}]$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{x-x^2}}$$

$$\therefore \frac{dy}{dx} = (\sin x)^x (x \cot x + \log \sin x) + \frac{1}{2\sqrt{x-x^2}}$$

37. (d) **Hint** Taking log on the given equation, we get

$$m \log x + n \log y = (m+n) \log(x+y)$$

38. (d) It is given that, $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots}}}$

$$\Rightarrow y = \sqrt{x+y} \Rightarrow y^2 = x+y \quad \dots(i)$$

On differentiating both sides w.r.t. x , we get

$$2y \frac{dy}{dx} = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2y-1}$$

Now, from Eq.(i), we get

$$y^2 = x+y$$

$$\Rightarrow y^2 - y - x = 0$$

$$\Rightarrow y = \frac{1 \pm \sqrt{1+4x}}{2} \Rightarrow y = \frac{1 + \sqrt{1+4x}}{2} \quad [\text{as } y > 0]$$

Now, again differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1+4x}} \times \frac{4}{2} = \frac{1}{\sqrt{1+4x}}$$

$$\text{Also, } y^2 = x+y \Rightarrow y = 1 + \frac{x}{y} \quad \dots(ii)$$

On differentiating Eq. (ii) w.r.t. x , we get

$$\frac{dy}{dx} = \frac{y-x}{y^2} \frac{dy}{dx} = \frac{1}{y} - \frac{x}{y^2} \frac{dy}{dx}$$

$$\Rightarrow \left(1 + \frac{x}{y^2}\right) \frac{dy}{dx} = \frac{1}{y}$$

$$\Rightarrow (y^2 + x) \frac{dy}{dx} = y$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{y^2 + x} = \frac{y}{x + x + y} \quad [\text{from Eq. (i)}]$$

$$\therefore \frac{dy}{dx} = \frac{y}{2x + y}$$

39. (a) **Hint** Given, $x^{2x} - 2x^x \cot y - 1 = 0 \quad \dots(i)$

$$\text{Now, at } x = 1, 1 - 2 \cot y - 1 = 0$$

$$\Rightarrow \cot y = 0 \Rightarrow y = \frac{\pi}{2}$$

On differentiating Eq. (i) w.r.t. x , we get

$$2x^{2x} (1 + \log x) - 2[x^x (-\operatorname{cosec}^2 y) \frac{dy}{dx} + \cot y x^x (1 + \log x)] = 0$$

40. (b) **Hint** Suppose $y = \tan^{-1} \left[\frac{\sqrt{1+x^2}-1}{x} \right] \quad \dots(i)$

$$\text{and } z = \tan^{-1} \left[\frac{2x\sqrt{1-x^2}}{1-2x^2} \right] \quad \dots(ii)$$

Putting $x = \tan \theta$ in Eq. (i) and $x = \sin \theta$ in Eq. (ii), we get

$$y = \tan^{-1} \left[\frac{\sec \theta - 1}{\tan \theta} \right] = \tan^{-1} \left[\tan \frac{\theta}{2} \right] = \frac{1}{2} \tan^{-1} x$$

$$z = \tan^{-1} \left[\frac{2 \sin \theta \cos \theta}{\cos 2\theta} \right]$$

$$= 2\theta = 2 \sin^{-1} x$$

$$41. (b) \text{ We have, } f(x) = \begin{vmatrix} \sec \theta & \tan^2 \theta & 1 \\ \theta \sec x & \tan x & x \\ 1 & \tan x - \tan \theta & 0 \end{vmatrix}$$

$$\Rightarrow f'(x) = \begin{vmatrix} 0 & 0 & 0 \\ \theta \sec x & \tan x & x \\ 1 & \tan x - \tan \theta & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} \sec \theta & \tan^2 \theta & 1 \\ \theta \sec x \tan x & \sec^2 x & 1 \\ 1 & \tan x - \tan \theta & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} \sec \theta & \tan^2 \theta & 1 \\ \theta \sec x & \tan x & x \\ 0 & \sec^2 x & 0 \end{vmatrix}$$

$$\therefore f'(\theta) = 0 + \begin{vmatrix} \sec \theta & \tan^2 \theta & 1 \\ \theta \sec \theta \tan \theta & \sec^2 \theta & 1 \\ 1 & 0 & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} \sec \theta & \tan^2 \theta & 1 \\ \theta \sec \theta & \tan \theta & \theta \\ 0 & \sec^2 \theta & 0 \end{vmatrix}$$

$$= \tan^2 \theta - \sec^2 \theta + \sec^2 \theta (\theta \sec \theta - \theta \sec \theta) = -1$$

$$42. (d) f'''(x) = \begin{vmatrix} \frac{d^3}{dx^3}(x^3) & \frac{d^3}{dx^3}(\sin x) & \frac{d^3}{dx^3}(\cos x) \\ 6 & -1 & 0 \\ P & P^2 & P^3 \end{vmatrix}$$

$$= \begin{vmatrix} 6 & -\cos x & \sin x \\ 6 & -1 & 0 \\ P & P^2 & P^3 \end{vmatrix} \therefore f'''(0) = \begin{vmatrix} 6 & -1 & 0 \\ 6 & -1 & 0 \\ P & P^2 & P^3 \end{vmatrix} = 0$$

which is independent of P .

$$43. (d) \frac{dy}{dt} = -kae^{-kt} \cos(pt + c) - pa e^{-kt} \sin(pt + c)$$

$$\Rightarrow \frac{dy}{dt} = -ky - pa e^{-kt} \sin(pt + c) \quad \dots (i)$$

Now, $\frac{d^2y}{dt^2} + k \frac{dy}{dt} = pa[-ke^{-kt} \sin(pt + c) + pe^{-kt} \cos(pt + c)]$

$$\Rightarrow \frac{d^2y}{dt^2} + k \frac{dy}{dt} = k \left(-ky - \frac{dy}{dt} \right) - p^2 y \quad [\text{from Eq. (i)}]$$

$$\Rightarrow \frac{d^2y}{dt^2} + 2k \frac{dy}{dt} + (p^2 + k^2)y = 0 \Rightarrow n^2 = p^2 + k^2$$

BITSAT Archives

$$1. (c) \frac{d}{dx}(\cos x) = -\frac{\pi}{180} \sin x$$

$$2. (a) \frac{d}{dx}[\log(\sec x - \tan x)]$$

$$= \frac{1}{\sec x - \tan x} [\sec x \tan x - \sec^2 x]$$

$$= \frac{\sec x [\tan x - \sec x]}{\sec x - \tan x} = -\sec x$$

$$3. (c) \text{ Given, } x = \cos^3 \theta, y = \sin^3 \theta$$

On differentiating w.r.t. θ respectively, we get

$$\frac{dx}{d\theta} = -3 \cos^2 \theta \sin \theta$$

and

$$\frac{dy}{d\theta} = 3 \sin^2 \theta \cos \theta$$

Now,

$$\frac{dy}{dx} = -\frac{3 \sin^2 \theta \cos \theta}{3 \cos^2 \theta \sin \theta} = -\tan \theta$$

$$\therefore 1 + \left(\frac{dy}{dx} \right)^2 = 1 + \tan^2 \theta = \sec^2 \theta$$

$$4. (b) \text{ Hint } x = at^2, y = 2at$$

On differentiating w.r.t. t respectively, we get

$$\frac{dx}{dt} = 2at, \frac{dy}{dt} = 2a \therefore \frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$$

$$5. (b) \text{ Given, } \Delta_1 = \begin{vmatrix} x & a & b \\ b & x & a \\ a & b & x \end{vmatrix}, \Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$$

$$\therefore \frac{d}{dx}(\Delta_1) = \begin{vmatrix} 1 & 0 & 0 \\ b & x & a \\ a & b & x \end{vmatrix} + \begin{vmatrix} x & a & b \\ 0 & 1 & 0 \\ a & b & x \end{vmatrix} + \begin{vmatrix} x & a & b \\ b & x & a \\ 0 & 0 & 1 \end{vmatrix}$$

$$44. (c) \text{ If the given relation is } l + m = c, \text{ then } l^2 - m^2 = 2x$$

$$\text{Dividing, } l - m = \frac{2x}{c}$$

$$\text{Adding, } 2\sqrt{x+y} = c + \frac{2x}{c}$$

$$\text{Squaring, } 4(x+y) = c^2 + \frac{4x^2}{c^2} + 4x$$

$$\Rightarrow 4y = c^2 + \frac{4x^2}{c^2}$$

Differentiate with respect to y , we get

$$\frac{4dy}{dx} = 0 + \frac{8x}{c^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x}{c^2}$$

Again differentiate, we get

$$\frac{d^2y}{dx^2} = \frac{2}{c^2}$$

$$= \begin{vmatrix} x & a \\ b & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & a \\ b & x \end{vmatrix} = 3\Delta_2$$

$$\therefore \frac{d}{dx}(\Delta_1) = 3\Delta_2$$

$$6. (b) \text{ Hint Let } y = x^x$$

$$\Rightarrow \log y = x \log x$$

$$7. (c) P'(x) = 2yy', P''(x) = 2yy'' + 2y'^2$$

$$\Rightarrow P'''(x) = 2yy''' + 6y'y''$$

$$\therefore 2 \frac{d}{dx}(y^3 y'') = 2[y^3 y''' + 3y^2 y' y'']$$

$$= y^2 [2yy''' + 6y'y''] = P(x) P'''(x)$$

$$8. (b) \text{ We have, } f(x) = (1-x)^n, f'(x) = -n(1-x)^{n-1}$$

$$f''(x) = n(n-1)(1-x)^{n-2}$$

$$f'''(x) = -n(n-1)(n-2)(1-x)^{n-3}$$

$$\vdots$$

$$f^n(x) = (-1)^n n(n-1)(n-2) \dots 1$$

$$\Rightarrow f(0) = 1, f'(0) = -n, f''(0) = n(n-1),$$

$$f'''(0) = -n(n-1)(n-2), \dots, f^n(0) = (-1)^n n!$$

$$\therefore f(0) + f'(0) + \frac{f''(0)}{2!} + \dots + \frac{f^n(0)}{n!}$$

$$= 1 - n + \frac{n(n-1)}{2!} - \dots + \frac{(-1)^n n!}{n!}$$

$$= {}^nC_0 - {}^nC_1 + {}^nC_2 - \dots + (-1)^n {}^nC_n$$

$$= (1-1)^n = 0$$

$$9. (a) \text{ Hint } f(x) = \log_x (\log x) = \frac{\log (\log x)}{\log (x)}$$

10. (c) Since, $f'(x) > \phi'(x)$

$$\begin{aligned} \therefore 2^{2x-1} \log 2 &> -2^x \log 2 + 2 \log 2 \\ \Rightarrow 2^{2x} &> -2^x + 2 \\ \Rightarrow 2^{2x} + 2^x - 2 &> 0 \\ \Rightarrow (2^x - 1)(2^x + 2) &> 0 \\ \Rightarrow 2^x - 1 &> 0 & [\because 2^x + 2 > 0, \forall x] \\ \Rightarrow 2^x &> 1 \\ \therefore x &> 0 \end{aligned}$$

11. (a) Given, $y = \sin^{-1} \left(\frac{5x + 12\sqrt{1-x^2}}{13} \right)$

Putting $x = \sin \theta$, $5 = r \cos \alpha$ and $12 = r \sin \alpha$, we get

$$\begin{aligned} y &= \sin^{-1} \left(\frac{r \cos \alpha \sin \theta + r \sin \alpha \cos \theta}{13} \right) \\ &= \sin^{-1} \left(\frac{13 \sin(\theta + \alpha)}{13} \right) \quad [\because r = \sqrt{25 + 144} = 13] \\ &= \theta + \alpha = \sin^{-1} x + \tan^{-1} \frac{12}{5} \quad \left[\because \tan \alpha = \frac{12}{5} \right] \end{aligned}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

12. (b) Given that, $x = a \left(\cos \theta + \log \tan \left(\frac{\theta}{2} \right) \right)$ and $y = a \sin \theta$

On differentiating w.r.t. θ respectively, we get

$$\begin{aligned} \frac{dx}{d\theta} &= a \left(-\sin \theta + \frac{1}{\tan \left(\frac{\theta}{2} \right)} \cdot \sec^2 \frac{\theta}{2} \cdot \frac{1}{2} \right) \\ &= a \left(-\sin \theta + \frac{1}{\sin \theta} \right) = \frac{a \cos^2 \theta}{\sin \theta} \end{aligned}$$

and $\frac{dy}{d\theta} = a \cos \theta$

$$\begin{aligned} \therefore \frac{dy}{dx} &= \frac{dy/d\theta}{dx/d\theta} \\ &= \frac{a \cos \theta}{a \cos^2 \theta / \sin \theta} = \tan \theta \end{aligned}$$

13. (b) Hint Let $p = \sin^{-1} \frac{2x}{1+x^2} = 2 \tan^{-1} x$

and $q = \cos^{-1} \frac{1-x^2}{1+x^2} = 2 \tan^{-1} x$

14. (b) Hint Given that, $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$

$$\therefore \sin^{-1} x = \cos^{-1} y \Rightarrow y = \sqrt{1-x^2}$$

15. (d) $y = \tan^{-1} \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}$

Put $x^2 = \cos 2\theta$

$$\begin{aligned} \therefore y &= \tan^{-1} \frac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}} \\ &= \tan^{-1} \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} = \tan^{-1} \tan \left(\frac{\pi}{4} - \theta \right) \end{aligned}$$

$$\Rightarrow y = \frac{\pi}{4} - \theta = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x^2$$

On differentiating both sides, we get

$$\frac{dy}{dx} = 0 - \frac{1}{2} \left(-\frac{(2x)}{\sqrt{1-x^4}} \right) = \frac{x}{\sqrt{1-x^4}}$$

16. (d) Hint Given that, $x = \sin t$, $y = \cos pt$

$$\Rightarrow \frac{dx}{dt} = \cos t, \frac{dy}{dt} = -p \sin pt$$

$$\therefore \frac{dy}{dx} = -\frac{p \sin pt}{\cos t}$$

$$\Rightarrow y_1 = \frac{-p \sqrt{1-y^2}}{\sqrt{1-x^2}}$$

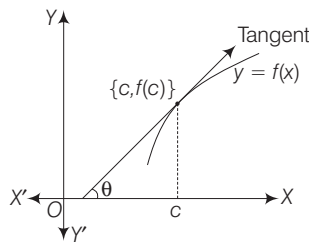
17. (d) Hint $\frac{dr}{d\phi} = \frac{\left[2 - 2 \cos \left(2\phi + \frac{\pi}{4} \right) \sin \left(2\phi + \frac{\pi}{4} \right) \cdot 2 \right]}{2 \sqrt{2\phi + \cos^2 \left(2\phi + \frac{\pi}{4} \right)}}$
- $$= \frac{\left[1 - \sin \left(4\phi + \frac{\pi}{2} \right) \right]}{\sqrt{2\phi + \cos^2 \left(2\phi + \frac{\pi}{4} \right)}}$$

18

Applications of Derivatives

Geometrical Interpretation of Derivative

Geometrically derivative of a function at a point $x = c$ is the slope of the tangent to the curve $y = f(x)$ at the point $\{c, f(c)\}$. Thus, slope of tangent $= f'(c) = \tan \theta$



Rate of Change of Quantities

The derivative $\frac{dy}{dx}$ represents the rate of change of variable y with respect to x . So, the rate of change of any physical quantity at any time is obtained by differentiating the physical quantity with respect to time.

If two variables are varying with respect to another variable t , i.e. $y = f(t)$, $x = g(t)$. Then, rate of

change of y with respect to x is $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ provided $\frac{dx}{dt} \neq 0$ or $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$ [by chain rule of derivative]

Thus, the rate of change of y with respect to x can be calculated using the rate of change of y and that of x both with respect to t .

NOTE If $y = f(x)$ and δy is an increment in y , corresponding to an increment δx in x , then we have $\delta y = \left(\frac{dy}{dx}\right) \delta x$.

Tangent and Normal of a Curve

1. If a tangent is drawn to the curve $y = f(x)$ at a point $P(x_1, y_1)$ and this tangent makes an angle ψ with positive x -direction, then

- (i) The slope of the tangent is $\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = \tan \psi$

- (ii) Equation of tangent is $y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} (x - x_1)$

- (iii) Length of tangent $PA = y \operatorname{cosec} \psi = \left| y \frac{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}{\left(\frac{dy}{dx}\right)} \right|$

- (iv) Length of subtangent $AC = y \cot \psi = \left| \frac{y}{dy/dx} \right|$

2. The normal to a curve at a point $P(x_1, y_1)$ is a line perpendicular to tangent at P and passing through P , then

- (i) The slope of the normal is $-\frac{1}{\frac{dy}{dx}}$

- (ii) Equation of normal is $y - y_1 = -\frac{1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}} (x - x_1)$

- (iii) Length of normal $PB = y \sec \psi = \left| y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \right|$

- (iv) Length of subnormal $BC = y \tan \psi = \left| y \left(\frac{dy}{dx}\right) \right|$

Some Important Points

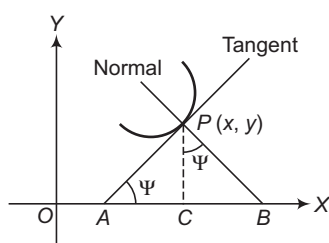
- (i) If the tangent at P is parallel to X -axis, then $\theta = 0$.

$$\Rightarrow \tan \theta = 0 \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 0$$

- (ii) If the tangent at P is perpendicular to X -axis or parallel to Y -axis, then

$$\theta = \frac{\pi}{2} \text{ and } \cot \theta = 0$$

$$\Rightarrow \frac{1}{\tan \theta} = 0 \Rightarrow \left(\frac{dx}{dy}\right)_{(x_1, y_1)} = 0$$



- (iii) If equation of the curve is in parametric form i.e. $x = f(t)$ and $y = g(t)$.

$$\text{Then, } \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{g'(t)}{f'(t)}$$

- (a) Equation of tangent is $y - g(t) = \frac{g'(t)}{f'(t)} \{x - f(t)\}$

- (b) Equation of normal is $y - g(t) = \frac{-f'(t)}{g'(t)} \{x - f(t)\}$

- (iv) If the tangent at any point on the curve is equally inclined to both the axes, then

$$\frac{dy}{dx} = \pm 1$$

If the tangent at any point makes an equal intercept on the coordinate axes.

$$\text{Then, } \frac{dy}{dx} = -1$$

Angle of Intersection of Two Curves

The angle of intersection of two curves is defined to be the angle between the tangents to the two curves at their point of intersection. Thus, the angle between the tangents of the two curves $y = f_1(x)$ and $y = f_2(x)$ is given by

$$\tan \phi = \left| \frac{\left(\frac{dy}{dx}\right)_{I(x_1, y_1)} - \left(\frac{dy}{dx}\right)_{II(x_1, y_1)}}{1 + \left(\frac{dy}{dx}\right)_{I(x_1, y_1)} \left(\frac{dy}{dx}\right)_{II(x_1, y_1)}} \right| \text{ or } \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

- NOTE**
- Curves intersect orthogonally, if $m_1 m_2 = -1$
 - Curves touch each other, if $m_1 = m_2$.

Monotonic Function

A function f is said to be monotonic in an interval, if it is either increasing or decreasing in that interval.

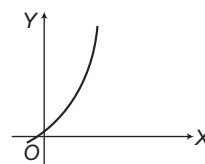
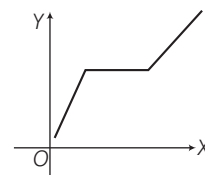
Increasing and Decreasing Functions (Monotonicity)

Let $y = f(x)$ be a given function with its domain D . Let $D_1 \subseteq D$.

Increasing Function $f(x)$ is said to be increasing function in D_1 , $\forall x_1, x_2 \in D_1$, if $x_1 > x_2 \Rightarrow f(x_1) \geq f(x_2)$

Strictly Increasing Function $f(x)$ is said to be strictly increasing function in D_1 , $\forall x_1, x_2 \in D_1$, if $x_1 < x_2$, $f(x_1) < f(x_2)$

$$\text{or } x_1 > x_2 \Rightarrow Nf(x_1) > f(x_2)$$

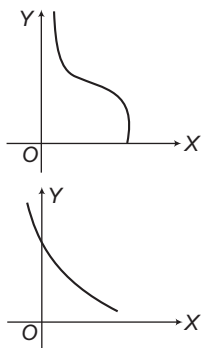


Decreasing Function $f(x)$ is said to be decreasing function in $D_1, \forall x_1, x_2 \in D_1$,

if $x_1 < x_2$
 $\Rightarrow f(x_1) \geq f(x_2)$

Strictly Decreasing Function $f(x)$ is said to strictly decreasing function in $D_1, \forall x_1, x_2 \in D_1$,

if $x_1 < x_2, f(x_1) > f(x_2)$
 or $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$



Some Important Points

Let $f(x)$ be a function that is continuous in $[a, b]$ and differentiable in (a, b) . Then,

- $f(x)$ is an increasing function in $[a, b]$, if $f'(x) \geq 0$ in (a, b) .
- $f(x)$ is a strictly increasing function in $[a, b]$, if $f'(x) > 0$ in (a, b) .
- $f(x)$ is a decreasing function in $[a, b]$, if $f'(x) \leq 0$ in (a, b) .
- $f(x)$ is a strictly decreasing function in $[a, b]$, if $f'(x) < 0$ in (a, b) .
- If $f(x)$ is a strictly increasing function on an interval $[a, b]$, then f^{-1} exists and it is also a strictly increasing function.
- If $f(x)$ is strictly increasing function on an interval $[a, b]$ such that it is continuous, then f^{-1} is continuous on $[f(a), f(b)]$.

NOTE The points for which $f'(x) = 0$ or $f'(x)$ does not exist are called critical points.

Maxima and Minima of Function

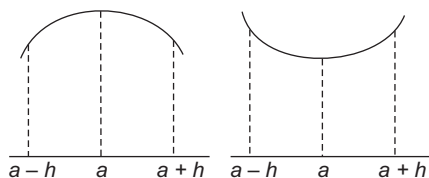
Let $y = f(x)$ be a function defined at $x = a$ and also in the vicinity of the point $x = a$. Then, $f(x)$ is said to have a local maximum at $x = a$, if the value of the function at $x = a$ is greater than the value of the function at the neighbouring points of $x = a$. Mathematically,

$$f(a) > f(a-h) \text{ and } f(a) > f(a+h), \text{ where } h > 0 \text{ (very small quantity)}$$

Similarly, $f(x)$ is said to have a local minimum at $x = a$, if the value of the function at $x = a$ is less than the value of the function at the neighbouring points of $x = a$. Mathematically,

$$f(a) < f(a-h) \text{ and } f(a) < f(a+h), \text{ where } h > 0.$$

A local maximum or a local minimum is also called a local extremum.



Method to Find Local Maxima or Local Minima

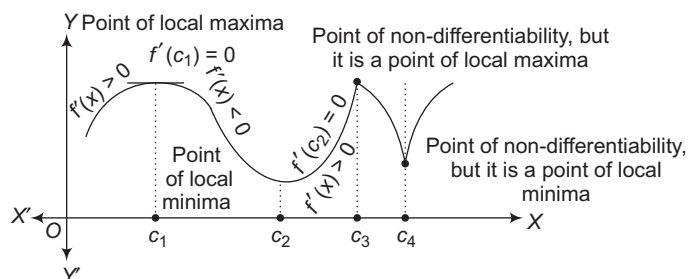
There are three tests under which local maxima and local minima can be found out. These three tests are first derivative test, second derivative test and n th derivative test.

The tests are given below:

First Derivative Test

Let f be a function defined on an open interval I and f be continuous at a critical point c in I . Then,

- If $f'(x)$ changes sign from positive to negative as x increases through c , i.e. if $f'(x) > 0$ at every point sufficiently close to and to the left of c and $f'(x) < 0$ at every point sufficiently close to and to the right of c , then c is a point of **local maxima**.
- If $f'(x)$ changes sign from negative to positive as x increases through point c , i.e. if $f'(x) < 0$ at every point sufficiently close to and to the left of c and $f'(x) > 0$ at every point sufficiently close to and to the right of c , then c is a point of **local minima**.
- If $f'(x)$ does not change sign as x increases through c , then c is neither a point of local maxima nor a point of local minima. Infact, such a point is called **point of inflection**.



If c is a point of local maxima of f , then $f(c)$ is a local maximum value of f . Similarly, if t is a point of local minima of f , then $f(t)$ is a local minimum value of f .

Second or Higher Order Derivative Test

- Find $f'(x)$ and equate it to zero. Solve $f'(x) = 0$ and let its roots be $x = a_1, a_2, \dots$.
- Find $f''(x)$ and at $x = a_1$.
 - If $f''(a_1)$ is positive, then $f(x)$ is minimum at $x = a_1$.
 - If $f''(a_1)$ is negative, then $f(x)$ is maximum at $x = a_1$.
- (i) If at $x = a_1, f''(a_1) = 0$, then find $f'''(x)$. If $f'''(a_1) \neq 0$, then $f(x)$ is neither maximum nor minimum at $x = a$.
 - If $f'''(a_1) = 0$, then find $f^{iv}(x)$.

(iii) If $f^{iv}(x)$ is positive (minimum value) and $f^{iv}(x)$ is negative (maximum value).

4. If at $x = a_1, f^{iv}(a_1) = 0$, then find $f^v(x)$ and proceed similarly.

n th Derivative Test

It says that if, $f'(a) = f''(a) = f'''(a) = \dots = f^n(a) = 0$ and $f^{n+1}(a) \neq 0$ (all derivatives of the function up to order n vanishes and $(n+1)$ th order derivative does not vanish at $x = a$, then $f(x)$ would have a local maximum or local minimum at $x = a$.

If n is even and $f^n(a) < 0$, then $x = a$ is a point of local maximum. If n is even and $f^n(a) > 0$, then $x = a$ is a point of local minimum. If n is odd, then $x = a$ is a point of local maximum nor a point of local minimum.

Concept of Global Maximum/Minimum

Let $y = f(x)$ be a continuous function with domain D . Let $[a, b] \subseteq D$. Global maximum/minimum of $f(x)$ in $[a, b]$ is basically the greatest/least value of $f(x)$ in $[a, b]$.

Global maximum and minimum in $[a, b]$ would always occur at critical points of $f(x)$ within $[a, b]$ or at the end points of the interval.

Method of Global Maximum/Minimum in $[a, b]$

In order to find the global maximum/minimum of $f(x)$ in $[a, b]$, find out all the critical points of $f(x)$ in $[a, b]$, i.e. all points at which $f'(x) = 0$.

Let $c_1, c_2, c_3, \dots, c_n$ be the points at which $f'(x) = 0$ and $f(c_1), f(c_2), \dots, f(c_n)$ be the values of function at these points, say

$$M_1 = \max\{f(a), f(c_1), f(c_2), \dots, f(c_n), f(b)\}$$

and $M_2 = \min\{f(a), f(c_1), f(c_2), \dots, f(c_n), f(b)\}$

Then, M_1 is the greatest value of $f(x)$ in $[a, b]$ and M_2 is the least value of $f(x)$ in $[a, b]$.

Method of Global Maximum/Minimum in $[a, b]$

Method for obtaining the greatest and least values of $f(x)$ in (a, b) is almost same as the method used for obtaining the greatest and least values in $[a, b]$.

(i) Let $y = f(x)$ be a function and $c_1, c_2, \dots, c_n$ be different critical points of the function in (a, b) .

$$\text{Let } M_1 = \max\{f(c_1), f(c_2), \dots, f(c_n)\}$$

$$\text{and } M_2 = \min\{f(c_1), f(c_2), \dots, f(c_n)\}$$

Now, if $\lim_{x \rightarrow a+0 \text{ or } x \rightarrow b-0} f(x) > M_1$ or $M_2 > \lim_{x \rightarrow a+0 \text{ (and } x \rightarrow b-0)} f(x)$

$f(x)$ would not have global maximum/minimum in (a, b) . This means that the limiting values at the end points are greater than M_1 or less than M_2 , then global maximum/minimum does not exist.

(ii) If $M_1 > \lim_{x \rightarrow a+0 \text{ (and } x \rightarrow b-0)} f(x)$ and $M_2 < \lim_{x \rightarrow a+0 \text{ (and } x \rightarrow b-0)} f(x)$,

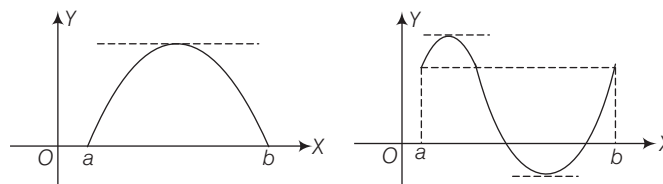
then M_1 and M_2 would respectively be the global maximum/minimum of $f(x)$ in (a, b) .

Rolle's Theorem

Let f be a real-valued function defined in the closed interval $[a, b]$, such that

- $f(x)$ is continuous in the closed interval $[a, b]$.
- $f(x)$ is differentiable in the open interval (a, b) .
- $f(a) = f(b)$, then there is some point c in the open interval (a, b) , such that $f'(c) = 0$.

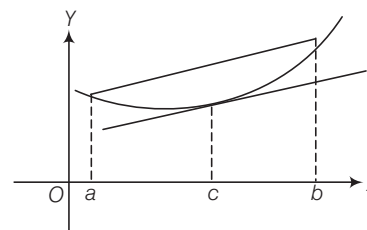
Geometrically Under the assumptions of Rolle's theorem, the graph of $f(x)$ starts at point $(a, 0)$ and ends at point $(b, 0)$ as shown in figures.



The conclusion is that there is atleast one point c between a and b , such that the tangent to the graph at $\{c, f(c)\}$ is parallel to the X -axis.

Lagrange's Mean Value Theorem

Let f be a real function, continuous on the closed interval $[a, b]$ and differentiable in the open interval (a, b) . Then, there is atleast one point c in the open interval (a, b) , such that



$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Geometrically Any chord of the curve $y = f(x)$, there is a point on the graph, where the tangent is parallel to this chord.

Practice Exercise

- If the surface area of a sphere of radius r is increasing uniformly at the rate $8 \text{ cm}^2/\text{s}$, then the rate of change of its volume is
 - constant
 - proportional to $\sqrt{r}$
 - proportional to r^2
 - proportional to r
- A kite is moving horizontally at a height of 151.5 m. If the speed of kite is 10 m/s, how fast is the string being let out, when the kite is 250 m away from the boy who is flying the kite, if the height of boy is 1.5 m?
 - 8 m/s
 - 12 m/s
 - 16 m/s
 - 19 m/s
- If the tangent at $(1, 1)$ on $y^2 = x(2-x)^2$ meets the curve again at P , is
 - $(4, 4)$
 - $(-1, 2)$
 - $(\frac{9}{4}, \frac{3}{8})$
 - None
- Line joining the points $(0, 3)$ and $(5, -2)$ is a tangent to the curve $y = \frac{ax}{1+x}$, then
 - $a = 1 \pm \sqrt{3}$
 - $a \in \phi$
 - $a = -1 \pm \sqrt{3}$
 - $a = -2 \pm 2\sqrt{3}$
- If m is the slope of a tangent to the curve $e^y = 1+x^2$, then
 - $|m| \leq 1$
 - $m > -1$
 - $m > 1$
 - $|m| > 1$
- The point on the curve $3y = 6x - 5x^3$ the normal at which it passes through the origin, is
 - $(\frac{1}{3}, 1)$
 - $(\frac{1}{3}, 1)$
 - $(2, -\frac{28}{3})$
 - $(-1, -\frac{1}{3})$
- If the normal to the curve $y^2 = 5x - 1$ at the point $(1, -2)$ is of the form $ax - 5y + b = 0$, then a and b are
 - 4, -14
 - 4, 14
 - 4, 14
 - 4, 2
- Coordinates of a point of the curve $y = x \log x$ at which the normal is parallel to the line $2x - 2y = 3$ are
 - $(0, 0)$
 - (e, e)
 - $(e^2, 2e^2)$
 - $(e^{-2}, -2e^{-2})$
- If the line $ax + by + c = 0$ is normal to curve $xy + 5 = 0$, then
 - $a + b = 0$
 - $a > 0$
 - $a < 0, b < 0$
 - $a = -2b$
- The length of subnormal to the curve $y = \frac{x}{1-x^2}$ at the point having abscissa $\sqrt{2}$ is
 - $5\sqrt{2}$
 - $3\sqrt{3}$
 - $\sqrt{3}$
 - $3\sqrt{2}$
- If the curve $y = a^x$ and $y = b^x$ intersect at angle α , then $\tan \alpha$ is equal to
 - $\frac{a-b}{1+ab}$
 - $\frac{\log a - \log b}{1 + \log a \log b}$
 - $\frac{a+b}{1-ab}$
 - $\frac{\log a + \log b}{1 - \log a \log b}$
- The parabolas $y^2 = 4ax$ and $x^2 = 4by$ intersect orthogonally at point $P(x_1, y_1)$, where $x_1, y_1 \neq 0$, then
 - $b = a^2$
 - $b = a^3$
 - $b^3 = a^2$
 - None of these
- If the curves $\frac{x^2}{\alpha} + \frac{y^2}{4} = 1$ and $y^3 = 16x$ intersect at right angles, then a value of α is
 - 2
 - $\frac{4}{3}$
 - $\frac{1}{2}$
 - $\frac{3}{4}$
- The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 = 2$
 - touch each other
 - cut at right angle
 - cut at an angle $\frac{\pi}{3}$
 - cut at an angle $\frac{\pi}{4}$
- If $a < 0$, the function $f(x) = e^{ax} + e^{-ax}$ is monotonically decreasing for all values of x , where
 - $x > 0$
 - $x < 0$
 - $x > 1$
 - $x < 1$
- The value of k in order that $f(x) = \sin x - \cos x - kx + b$ decreases for all real values, is given by
 - $k < 1$
 - $k > 1$
 - $k > \sqrt{2}$
 - $k < \sqrt{2}$
- The set of all x for which $\log(1+x) \leq x$ is
 - $(0, \infty)$
 - $(-1, \infty)$
 - $(-1, 0)$
 - None of these
- The function $f(x) = \sin^4 x + \cos^4 x$ increases, if
 - $0 < x < \frac{\pi}{8}$
 - $\frac{\pi}{8} < x < \frac{3\pi}{8}$
 - $\frac{3\pi}{8} < x < \frac{5\pi}{8}$
 - $\frac{5\pi}{8} < x < \frac{3\pi}{x}$
- The value of a for which the function $f(x) = (4a - 3)(x + \log 5) + 2(a - 7) \cot \frac{x}{2} \sin^2 \frac{x}{2}$ does not possess critical points is
 - $(-\infty, \frac{4}{3})$
 - $(-\infty, -1)$
 - $[1, \infty)$
 - $[2, \infty)$
- The function $f(x) = \cos x - 2px$ is monotonically decreasing for
 - $p < \frac{1}{2}$
 - $p > \frac{1}{2}$
 - $p < 2$
 - $p > 2$

21. The values of a for which the function $(a+2)x^3 - 3ax^2 + 9ax - 1 = 0$ decreases monotonically throughout for all real x , are
 a. $a < -2$ b. $a > -2$
 c. $-3 < a < 0$ d. $-\infty < a \leq -3$
22. The function $\frac{a \sin x + b \cos x}{c \sin x + d \cos x}$ is decreasing, if
 a. $ad - bc > 0$ b. $ad - bc < 0$
 c. $ab - cd > 0$ d. $0 \leq x \leq -2$
23. The length of a longest interval in which the function $3 \sin x - 4 \sin^3 x$ is increasing, is
 a. $\frac{\pi}{3}$ b. $\frac{\pi}{2}$ c. $\frac{3\pi}{2}$ d. π
24. If $f(x) = xe^{x(1-x)}$, then $f(x)$ is
 a. increasing in $\left[-\frac{1}{2}, 1\right]$
 b. decreasing in R
 c. increasing in R
 d. decreasing in $\left[-\frac{1}{2}, 1\right]$
25. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have?
 a. 5 b. 7 c. 1 d. 3
26. The function $f(x) = 2x + \cot^{-1} x + \log_e (\sqrt{1+x^2} - x)$ is increasing in
 a. $x \in (2, 4)$ b. R c. $[2, 6]$ d. $[-3, 5]$
27. The function $f(x) = \tan x - 4x$ is strictly decreasing on
 a. $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$ b. $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$ c. $\left[-\frac{\pi}{3}, \frac{\pi}{2}\right]$ d. $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$
28. The number of solutions of the equation $x^3 + 2x^2 + 5x + 2 \cos x = 0$ in $[0, 2\pi]$ is
 a. 0 b. 1 c. 2 d. 3
29. Let $f(x) = -2x^3 + 21x^2 - 60x + 41$, then
 a. $f(x)$ is decreasing in $(-\infty, 1)$
 b. $f(x)$ is decreasing in $(-\infty, 2)$
 c. $f(x)$ is increasing in $(-\infty, 1)$
 d. $f(x)$ is increasing in $(-\infty, 2)$
30. Divide 20 into two parts such that the product of one part and the cube of the other is maximum. The two parts are
 a. (10, 10) b. (5, 15) c. (13, 7) d. None
31. The minimum value of $4e^{2x} + 9e^{-2x}$ is
 a. 11 b. 12 c. 10 d. 14
32. The maximum value of $f(x) = x + \sin 2x$, $x \in [0, 2\pi]$ is
 a. $\frac{\pi}{2}$ b. 2π c. $\frac{3\pi}{4}$ d. $\frac{3\pi}{2}$
33. The value of a for which the function $f(x) = a \sin x + \frac{1}{3} \sin 3x$ has an extremum at $x = \frac{\pi}{3}$, is
 a. 1 b. -1 c. 0 d. 2
34. The difference between the greatest and least values of the function $f(x) = \sin 2x - x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is
 a. π b. 2π
 c. 3π d. $\frac{\pi}{2}$
35. The function $f(x) = x^2 + \frac{\lambda}{x}$ has a
 a. minimum at $x = 2$, if $\lambda = 16$
 b. minimum at $x = 2$, if $\lambda = 15$
 c. maximum for all real values of λ
 d. maximum at $x = 2$, if $\lambda = 4$
36. If the function $f(x) = \frac{ax+b}{(x-1)(x-4)}$ has a local maxima at $(2, -1)$, then
 a. $b = 1, a = 0$ b. $a = 1, b = 0$
 c. $b = -1, a = 0$ d. $a = -1, b = 0$
37. If $y = a \log x + bx^2 + x$ has its extremum value at $x = 1$ and $x = 2$, then (a, b) is equal to
 a. $\left(1, \frac{1}{2}\right)$ b. $\left(\frac{1}{2}, 2\right)$ c. $\left(2, \frac{-1}{2}\right)$ d. $\left(\frac{-2}{3}, \frac{-1}{6}\right)$
38. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$ attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a is equal to
 a. 3 b. 1 c. 2 d. $\frac{1}{2}$
39. If the function $y = a \log |x| + bx^2 + x$ has its extreme values at $x = -1$ and $x = +2$, then
 a. $a = 2, b = 1$ b. $a = 2, b = -\frac{1}{2}$
 c. $b = -\frac{1}{2}, a = 3$ d. None of these
40. Let $\begin{cases} x+2 & -1 \leq x < 0 \\ 1 & x = 0 \\ \frac{x}{2} & 0 < x \leq 1 \end{cases}$
 Then, on $[-1, 1]$, this function has
 a. a minimum
 b. a maximum
 c. either a maximum or a minimum
 d. neither a maximum nor a minimum
41. If the function $f(x) = x^3 - 6x^2 + ax + b$ defined on $[1, 3]$ satisfies the Rolle's theorem for $c = \frac{2\sqrt{3}+1}{\sqrt{3}}$, then
 a. $a = 11, b = 6$ b. $a = -11, b = 6$
 c. $a = 11, b \in R$ d. None of these

42. If $3(a+2c) = 4(b+3d)$, then the equation $ax^3 + bx^2 + cx + d = 0$ will have
 a. no real solution
 b. atleast one real root in $(-1, 0)$
 c. atleast one real root in $(0, 1)$
 d. None of the above
43. The function $f(x) = x(x+3)e^{-(1/2)x}$ satisfies all the conditions of Rolle's theorem in $[-3, 0]$. The value of c is
 a. 0
 b. -1
 c. -2
 d. -3
44. If $a + b + c = 0$, then the equation $3ax^2 + 2bx + c = 0$ has
 a. atleast one real root in $(0, 1)$
 b. one root in $(-1, 0)$ and other in $(1, 2)$
 c. both imaginary roots
 d. two coincident roots
45. A value of c for which the conclusion of mean value theorem holds for the function $f(x) = \log_e x$ on the interval $[1, 3]$, is
 a. $2\log_3 e$ b. $\frac{1}{2}\log_e 3$ c. $\log_3 e$ d. $\log_e 3$
46. $x(x-2)(x-4)$, $1 \leq x \leq 4$ will satisfy mean value theorem at
 a. 1 b. 2 c. 3 d. 4
47. In the mean value theorem, $f(b) - f(a) = (b-a)f'(c)$, if $a = 4$, $b = 9$ and $f(x) = \sqrt{x}$, then the value of c is
 a. 8.00 b. 5.25 c. 4.00 d. 6.25
48. The function $f(x) = (x-3)^2$ satisfies all the conditions of mean value theorem in $[3, 4]$. A point on $y = (x-3)^2$, where the tangent is parallel to the chord joining $(3, 0)$ and $(4, 1)$ is
 a. $(\frac{7}{2}, \frac{1}{2})$ b. $(\frac{7}{2}, \frac{1}{4})$ c. $(1, 4)$ d. $(4, 1)$

BITSAT Archives

1. The value of a , so that the sum of squares of the roots of the equation $x^2 - (a-2)x - a + 1 = 0$ assume the least value, is [2014]
 a. 2 b. 0
 c. 3 d. 1
2. Function $f: (-\infty, -1] \rightarrow (0, e^5]$ defined by $f(x) = e^{x^3 - 3x + 2}$ is [2013]
 a. many-one and onto b. many-one and into
 c. one-one and onto d. one-one and into
3. If $f(x) = (\tan^{-1} x)^2 + \frac{2}{\sqrt{x^2 + 1}}$, then $f(x)$ is increasing in [2013]
 a. $(0, \infty)$ b. $(-\infty, 0)$
 c. $(-\infty, -5)$ d. None of these
4. The maximum value of $f(x) = \frac{\log x}{x}$ is [2013]
 a. 1 b. $\frac{2}{e}$ c. e d. $\frac{1}{e}$
5. What are the values of c for which Rolle's theorem for the function $f(x) = x^3 - 3x^2 + 2x$ in the interval $[0, 2]$ is verified? [2013]
 a. $c = \pm 1$ b. $c = 1 \pm \frac{1}{\sqrt{3}}$
 c. $c = \pm 2$ d. None of these
6. If the rate of change in the circumference of a circle of 0.3 cm/s, then the rate of change in the area of the circle when the radius is 5 cm, is [2012]
 a. 1.5 sq cm/s b. 0.5 sq cm/s
 c. 5 sq cm/s d. 3 sq cm/s
7. If $y = x^3 - ax^2 + 48x + 7$ is an increasing function for all real values of x , then a lies in [2012]
 a. $(-14, 14)$ b. $(-12, 12)$
 c. $(-16, 16)$ d. $(-21, 21)$
8. Rolle's theorem is not applicable for the function $f(x) = |x|$ in the interval $[-1, 1]$ because [2012]
 a. $f'(1)$ does not exist
 b. $f'(-1)$ does not exist
 c. $f(x)$ is discontinuous at $x = 0$
 d. $f'(0)$ does not exist
9. The function $f(x) = x^{1/x}$ is [2011]
 a. increasing in $(1, \infty)$
 b. decreasing in $(1, \infty)$
 c. increasing in $(1, e)$, decreasing in (e, ∞)
 d. decreasing in $(1, e)$, increasing in (e, ∞)
10. The point $(0, 5)$ is closer to the curve $x^2 = 2y$ at [2011]
 a. $(2\sqrt{2}, 0)$ b. $(0, 0)$
 c. $(2, 2)$ d. None of these
11. The intersection angle of the curve $xy = a^2$ and $x^2 - y^2 = a^2$ is [2010]
 a. $\frac{\pi}{3}$ b. $\frac{\pi}{6}$ c. $\frac{\pi}{2}$ d. $\frac{5\pi}{6}$
12. On which of the following intervals is the function $f(x) = 2x^2 - \log|x|$, $x \neq 0$ increasing? [2010]
 a. $(\frac{1}{2}, \infty)$ b. $(-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, \infty)$
 c. $(-\infty, -\frac{1}{2}) \cup (0, \frac{1}{2})$ d. $(-\frac{1}{2}, 0) \cup (\frac{1}{2}, \infty)$

13. The greatest value of $f(x) = (x+1)^{1/3} - (x-1)^{1/3}$ on $[0, 1]$ is [2010]
 a. 1 b. 2 c. 3 d. $\frac{1}{3}$
14. If $f'(x) = (x-a)^{2m} (x-b)^{2n+1}$, where $m, n \in \mathbb{N}$, then [2010]
 a. $x = b$ is a point of minimum
 b. $x = b$ is a point of maximum
 c. $x = b$ is a point of inflexion
 d. None of the above
15. The equation of the normal to the curve $y^4 = ax^3$ at (a, a) is [2008]
 a. $x + 2y = 3a$ b. $3x - 4y + a = 0$
 c. $4x + 3y = 7a$ d. $4x - 3y = 0$
16. The length of the subtangent at $(2, 2)$ to the curve $x^5 = 2y^4$ is [2008]
 a. $\frac{5}{2}$ b. $\frac{8}{5}$ c. $\frac{2}{5}$ d. $\frac{5}{8}$
17. The radius of a cylinder is increasing at the rate of 3 m/s and its altitude is decreasing at the rate of 4 m/s. The rate of change of volume when radius is 4 m and altitude is 6 m, is [2007]
 a. 80π cu m/s b. 144π cu m/s
 c. 80 cu m/s d. 64 cu m/s
18. The minimum value of $2x + 3y$, when $xy = 6$ is [2007]
 a. 9 b. 12 c. 8 d. 6
19. If ST and SN are the lengths of the subtangent and the subnormal at the point $\theta = \frac{\pi}{2}$ on the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, $a \neq 1$, then [2006]
 a. $ST = SN$ b. $ST = 2SN$
 c. $ST^2 = aSN^3$ d. $ST^3 = aSN$
20. If θ is the acute angle of intersection at a real point of intersection of the circle $x^2 + y^2 = 5$ and the parabola $y^2 = 4x$, the $\tan\theta$ is equal to [2006]
 a. 1 b. $\sqrt{3}$ c. 3 d. $\frac{1}{\sqrt{3}}$
21. A population $p(t)$ of 1000 bacteria introduced into nutrient medium grows according to the relation $p(t) = 1000 + \frac{1000t}{100 + t^2}$. The maximum size of the this bacterial population is [2006]
 a. 1100 b. 1250 c. 1050 d. 5250
22. Given function $f(x) = \left(\frac{e^{2x} - 1}{e^{2x} + 1} \right)$ is [2005]
 a. increasing b. decreasing
 c. even d. None of these

Answer with Solutions

Practice Exercise

1. (d) Surface area, $s = 4\pi r^2$

$$\frac{ds}{dt} = 8\pi r \cdot \frac{dr}{dt} \Rightarrow 8\pi r \cdot \frac{dr}{dt} = 8 \quad \left[\because \frac{ds}{dt} = 8 \text{ cm}^2 / \text{s} \right]$$

$$\Rightarrow \frac{dr}{dt} = \frac{1}{\pi r}$$

$$\text{Now, } V = \frac{4\pi r^3}{3}$$

$$\Rightarrow \frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt} = 4\pi r^2 \cdot \frac{1}{\pi r} = 4r$$

$$\therefore \frac{dV}{dt} \propto r$$

2. (a) Let AB be the position of boy who is flying the kite and C be the position of the kite at any time t .

Let $BD = x$ and $AC = y$, then $AE = x$

Given, $AB = 1.5 \text{ m}$,
 $CD = 151.5 \text{ m}$
 $\therefore CE = 150 \text{ m}$

Given, $\frac{dx}{dt} = 10 \text{ m/s}$

Here, we have to find $\frac{dy}{dt}$, when

$y = 250 \text{ m}$

Now, from $\triangle CAE$,

$$y^2 = x^2 + 150^2$$

On differentiating, we get

$$2y \frac{dy}{dt} = 2x \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{x}{y} \cdot \frac{dx}{dt} = \frac{x}{y} \cdot 10 \quad \dots (i)$$

$$\text{In } \triangle ACE, x = \sqrt{250^2 - 150^2} = 200 \text{ m}$$

$$[\because y = 250]$$

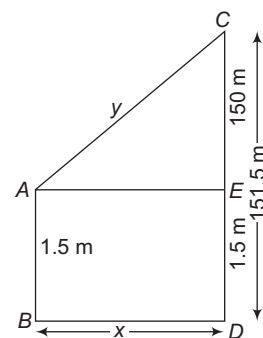
$$\text{From Eq. (i), } \frac{dy}{dt} = \frac{200}{250} \times 10 = 8 \text{ m/s}$$

3. (c) $y^2 = x(2-x)^2$

$$\Rightarrow y^2 = x^3 - 4x^2 + 4x \quad \dots (i)$$

$$\Rightarrow 2y \frac{dy}{dx} = 3x^2 - 8x + 4$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2 - 8x + 4}{2y}$$



$$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = \frac{3-8+4}{2} = -\frac{1}{2}$$

$\therefore$ The equation of the tangent at (1, 1) is

$$y - 1 = -\frac{1}{2}(x - 1)$$

$$\Rightarrow x + 2y - 3 = 0 \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we get $x = \frac{9}{2}$ and $y = \frac{3}{8}$

Hence, the coordinates of P are $\left(\frac{9}{4}, \frac{3}{8}\right)$.

4. (b) Equation of line joining the points (0, 3) and (5, -2) is

$y = 3 - x$. If this line is tangent to $y = \frac{ax}{(x+1)}$, then

$(3-x)(x+1) = ax$ should have equal roots. Thus,
 $(a-2)^2 + 12 = 0 \Rightarrow$ No value of $a \Rightarrow a \in \phi$.

5. (a) We have, $e^y = 1 + x^2$

$$\Rightarrow e^y \frac{dy}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{2x}{1+x^2}$$

$$\Rightarrow m = \frac{2x}{1+x^2}, |m| = \frac{2|x|}{1+x^2}$$

$$\therefore |x|^2 + 1 - 2|x| \geq 0 \Rightarrow (|x| - 1)^2 \geq 0$$

$$\Rightarrow |x|^2 + 1 \geq 2|x| \Rightarrow 1 \geq \frac{2|x|}{1+|x|^2}$$

$$\therefore |m| \leq 1$$

6. (a) Let the required point be (x_1, y_1) .

$$\text{Now, } 3y = 6x - 5x^3 \Rightarrow 3 \frac{dy}{dx} = 6 - 15x^2$$

$$\Rightarrow \frac{dy}{dx} = 2 - 5x^2 \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 2 - 5x_1^2$$

The equation of the normal at (x_1, y_1) is

$$y - y_1 = -\frac{1}{2 - 5x_1^2}(x - x_1)$$

$$0 - y_1 = -\frac{1}{2 - 5x_1^2}(0 - x_1)$$

$$\Rightarrow -y_1 = \frac{-x_1}{2 - 5x_1^3} \quad \dots(i)$$

Since, (x_1, y_1) lies on the given curve. Therefore,

$$3y_1 = 6x_1 - 5x_1^3 \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we get

$$x_1 = 1 \text{ and } y_1 = \frac{1}{3}$$

Hence, the required point is $\left(1, \frac{1}{3}\right)$.

7. (a) We have,

$$y^2 = 5x - 1 \quad \dots(i)$$

$$\text{At } (1, -2), \frac{dy}{dx} = \left(\frac{5}{2y}\right)_{(1, -2)} = \frac{-5}{4}$$

$\therefore$ Equation of normal at the point (1, -2) is

$$[y - (-2)] \left(\frac{-5}{4}\right) + x - 1 = 0$$

$$\therefore 4x - 5y - 14 = 0 \quad \dots(ii)$$

As the normal is of the form

$$ax - 5y + b = 0$$

On comparing this with Eq. (ii), we get

$$a = 4 \text{ and } b = -14$$

8. (a) Hint $y = x \log x \Rightarrow \frac{dy}{dx} = 1 + \log x$

$$\text{The slope of the normal} = -\frac{1}{(dy/dx)} = \frac{-1}{1 + \log x}$$

The slope of the line $2x - 2y = 3$ is 1.

$$\therefore \frac{-1}{1 + \log x} = 1$$

9. (c) The curve, $xy = -5 < 0$

$$\Rightarrow x \frac{dy}{dx} + y = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{x} > 0 \quad [\because xy = -5 < 0]$$

$$\text{Slope of normal} = \frac{-1}{\frac{dy}{dx}} = \frac{x}{y} < 0 \quad [\because xy = -5 < 0]$$

Hence, slope of normal will be negative.

The line $ax + by + c = 0$

$$\Rightarrow by = -ax - c \Rightarrow y = \frac{-a}{b}x - \frac{c}{b}$$

Slope of normal $\frac{-a}{b}$ is negative.

$$\Rightarrow \frac{-a}{b} < 0 \Rightarrow \frac{a}{b} > 0$$

$$\Rightarrow a > 0, b > 0 \text{ or } a < 0, b < 0$$

10. (d) Given, $y = \frac{x}{1-x^2}$

At $x = \sqrt{2}$, $y = -\sqrt{2}$ point is $(\sqrt{2}, -\sqrt{2})$.

$$\therefore \frac{dy}{dx} = \frac{1+x^2}{(1-x^2)^2}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\text{at } (\sqrt{2}, -\sqrt{2})} = \frac{1+2}{(1-2)^2} = 3 = m \quad [\text{say}]$$

$$\text{Equation of normal, } y + \sqrt{2} = -\frac{1}{3}(x - \sqrt{2})$$

It intersects X-axis at $N(-2\sqrt{2}, 0)$.

Length of subnormal, $MN = 3\sqrt{2}$

11. (b) Clearly, the point of intersection of curves is (0, 1).

Now, slope of tangent of first curve,

$$m_1 = \frac{dy}{dx} = a^x \log a \Rightarrow \left(\frac{dy}{dx}\right)_{(0,1)} = m_1 = \log a$$

Slope of tangent of second curve,

$$m_2 = \frac{dy}{dx} = b^x \log b$$

$$\Rightarrow m_2 = \left(\frac{dy}{dx} \right)_{(0,1)} = \log b$$

$$\therefore \tan \alpha = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\log a - \log b}{1 + \log a \log b}$$

12. (d)

13. (b) Slope of the curve $\frac{x^2}{\alpha} + \frac{y^2}{4} = 1$ is $m_1 = \frac{-4x}{\alpha y}$.

Now, slope of the curve $y^3 = 16x$ is $m_2 = \frac{16}{3y^2}$.

Now, apply the condition of perpendicularity of two curves, i.e.

$$m_1 \cdot m_2 = -1$$

and get $\alpha = \frac{4}{3}$ with the help of equation of curves.

14. (b) Differentiating first equation of the curve, we have

$$3x^2 - 3y^2 - 6xy \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 - y^2}{2xy} = m_1 \quad [\text{say}]$$

and differentiating second equation of the curve gives

$$6xy + 3x^2 \frac{dy}{dx} - 3y^2 \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2xy}{x^2 - y^2} = m_2 \quad [\text{say}]$$

$$\text{Since, } m_1 \cdot m_2 = -1$$

So, the two curves cut at right angle.

15. (b) Hint $f'(x) = ae^{ax} - ae^{-ax} = a[e^{ax} - e^{-ax}]$

Since, $f(x)$ is decreasing for all values of x .

$$\therefore f'(x) < 0$$

16. (c) $f(x) = \sin x - \cos x - kx + b$

$$f'(x) = \cos x + \sin x - k$$

For decreasing function, put $f'(x) < 0$

$$\text{So, } \cos x + \sin x - k < 0$$

$\therefore$ Maximum value of $\cos x + \sin x$ is $\sqrt{2}$.

$$\Rightarrow \sqrt{2} - k < 0 \Rightarrow k > \sqrt{2}$$

17. (b) Let $f(x) = \log(1+x) - x$

Clearly, $f(x)$ is defined for $x > -1$

$$\text{Now, } f(x) = \log(1+x) - x \Rightarrow f'(x) = \frac{1}{1+x} - 1 = \frac{-x}{1+x}$$

$$\Rightarrow f'(x) < 0 \text{ for } x > 0$$

$$\text{and } f'(x) > 0 \text{ for } -1 < x < 0$$

$\Rightarrow f(x)$ is decreasing on $[0, \infty)$ and increasing on $(-1, 0)$

$$\Rightarrow f(x) \leq f(0) \text{ for } 0 \leq x < \infty$$

$$\text{and } f(x) \leq f(0) \text{ for } -1 < x < 0$$

$$\Rightarrow f(x) \leq f(0) \text{ for } -1 < x < \infty$$

$$\Rightarrow \log(1+x) - x \leq 0 \text{ for } -1 < x < \infty$$

$$\Rightarrow \log(1+x) \leq x \text{ for } x \in (-1, \infty)$$

18. (b) $y = f(x) = \sin^4 x + \cos^4 x$

$$\therefore \frac{dy}{dx} = 4 \sin^3 x \cos x - 4 \cos^3 x \sin x$$

$$= 4 \sin x \cos x (\sin^2 x - \cos^2 x) = -\sin 4x$$

If $4x$ lies between π and 2π , $\sin 4x$ is negative, i.e. if x lies between $\frac{\pi}{4}$ and $\frac{\pi}{2}$, $\sin 4x$ is negative, so that $\frac{dy}{dx} > 0$.

$$\therefore f(x) \text{ increases, if } \frac{\pi}{4} < x < \frac{\pi}{2}$$

$$\therefore f(x) \text{ increases, if } \frac{\pi}{8} < x < \frac{3\pi}{8}$$

19. (d) We have,

$$f(x) = (4a-3)(x + \log 5) + 2(a-7) \cot \frac{x}{2} \sin^2 \frac{x}{2}$$

$$= (4a-3)(x + \log 5) + (a-7) \sin x$$

$$\therefore f'(x) = (4a-3) + (a-7) \cos x$$

If $f(x)$ does not have critical points, then

$f'(x) = 0$ does not have any solution in R .

$$\text{Now, } f'(x) = 0 \Rightarrow \cos x = \frac{4a-3}{7-a}$$

$$\left| \frac{4a-3}{7-a} \right| \leq 1 \Rightarrow -1 \leq \frac{4a-3}{7-a} \leq 1$$

$$\Rightarrow a-7 \leq 4a-3 \leq 7-a \Rightarrow a \geq -\frac{4}{3} \text{ or } a \leq 2$$

Thus, $f'(x) = 0$ has solutions in R , if

$$a \geq -\frac{4}{3} \text{ or } a \leq 2$$

So, $f'(x) = 0$ is not solution in R , if

$$a < -\frac{4}{3} \text{ or } a > 2$$

$$\text{i.e. } a \in \left(-\infty, -\frac{4}{3} \right) \cup (2, \infty)$$

20. (b) $f(x)$ will be monotonically decreasing, if $f'(x) < 0$.

$$\Rightarrow f'(x) = -\sin x - 2p < 0$$

$$\Rightarrow \frac{1}{2} \sin x + p > 0$$

$$\Rightarrow p > \frac{1}{2} \quad [\because -1 \leq \sin x \leq 1]$$

21. (d) Let $f(x) = (a+2)x^3 - 3ax^2 + 9ax - 1$ decreases monotonically for all $x \in R$, then $f'(x) \leq 0$ for all $x \in R$.

$$\Rightarrow 3(a+2)x^2 - 6ax + 9a \leq 0, \forall x \in R$$

$$\Rightarrow (a+2)x^2 - 2ax + 3a \leq 0, \forall x \in R$$

$$\Rightarrow a+2 < 0 \text{ and discriminant } \leq 0$$

$$\Rightarrow a < -2, -8a^2 - 24a \leq 0$$

$$\Rightarrow a < -2 \text{ and } a(a+3) \geq 0$$

$$\Rightarrow a < -2, a \leq -3 \text{ or } a \geq 0$$

$$\Rightarrow a \leq -3$$

$$\therefore -\infty < a \leq -3$$

22. (b) Let $y = \frac{a \sin x + b \cos x}{c \sin x + d \cos x}$

The function will be decreasing when $\frac{dy}{dx} < 0$.

$$\therefore \frac{(c \sin x + d \cos x)(a \cos x - b \sin x) - (a \sin x + b \cos x)(c \cos x - d \sin x)}{(c \sin x + d \cos x)^2} < 0$$

$$\begin{aligned} &\Rightarrow ac \sin x \cos x - bc \sin^2 x + ad \cos^2 x - bd \sin x \cos x \\ &- ac \sin x \cos x + ad \sin^2 x - bc \cos^2 x + bd \sin x \cos x < 0 \\ &\Rightarrow ad (\sin^2 x + \cos^2 x) - bc (\sin^2 x + \cos^2 x) < 0 \\ &\therefore (ad - bc) < 0 \end{aligned}$$

23. (a) Let $f(x) = \sin 3x$

Thus, $\sin x$ is increasing in $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ is of length π .

Hence, the length of largest interval in which $f(x) = \sin 3x$ is increasing, is $\frac{\pi}{3}$.

24. (a) $f'(x) = e^{x(1-x)} + xe^{x(1-x)}(1-2x)$
 $= e^{x(1-x)}(1+x-2x^2) = -e^{x(1-x)}(x-1)(2x+1)$

Hence, $f(x)$ is increasing in $\left[-\frac{1}{2}, 1\right]$.

25. (c) Let $f(x) = x^7 + 14x^5 + 16x^3 + 30x - 560$

$$\Rightarrow f'(x) = 7x^6 + 70x^4 + 48x^2 + 30 > 0, \forall x \in R$$

Since, $f(x)$ is increasing.

So, $f(x) = 0$ has only one solution.

26. (b) Given that, $f(x) = 2x + \cot^{-1} x + \log_e(\sqrt{1+x^2} - x)$

$$\begin{aligned} \Rightarrow f'(x) &= 2 - \frac{1}{1+x^2} + \frac{1}{\sqrt{1+x^2}-x} \left(\frac{2x}{2\sqrt{1+x^2}} - 1 \right) \\ &= 2 - \frac{1}{1+x^2} + \frac{x - \sqrt{1+x^2}}{(\sqrt{1+x^2}-x) \cdot \sqrt{1+x^2}} \\ &= 2 - \frac{1}{1+x^2} - \frac{1}{\sqrt{1+x^2}} = \frac{2x^2+1}{1+x^2} - \frac{1}{\sqrt{1+x^2}} \\ &= \frac{(2x^2+1) - \sqrt{1+x^2}}{(1+x^2)} \end{aligned}$$

So, $f'(x) \geq 0$ for all real values of x .

Hence, $f(x) = 2x + \cot^{-1} x + \log_e(\sqrt{1+x^2} - x)$ is increasing in R .

27. (a) Given, $f(x) = \tan x - 4x \Rightarrow f'(x) = \sec^2 x - 4$

When $-\frac{\pi}{3} < x < \frac{\pi}{3}$, then $1 < \sec x < 2$

Therefore, $1 < \sec^2 x < 4 \Rightarrow -3 < (\sec^2 x - 4) < 0$

Thus, for $-\frac{\pi}{3} < x < \frac{\pi}{3}$, $f'(x) < 0$

Hence, f is strictly decreasing on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$.

28. (a) Given, $f(x) = x^3 + 2x^2 + 5x + 2 \cos x$

$$\Rightarrow f'(x) = 3x^2 + 4x + 5 - 2 \sin x$$

$$\text{Least value of } 3x^2 + 4x + 5, \left(\frac{-D}{4a}\right) = -\frac{(4)^2 - 4(3)(5)}{4(3)} = \frac{11}{3}$$

and $-2 \sin x \in [-2, 2] \therefore f'(x) > 0, \forall x \in R$
 $\Rightarrow f(x)$ is strictly increasing function, also $f(0) = 2$.

Thus, $f(x) \neq 0, \forall x \in R$

So, number of root is zero.

29. (b) Given, $f(x) = -2x^3 + 21x^2 - 60x + 41$... (i)

On differentiating Eq. (i) w.r.t. x , we get

$$f'(x) = -6x^2 + 42x - 60$$

$$= -6(x^2 - 7x + 10) = -6(x-2)(x-5)$$

If $x < 2$, $f'(x) < 0$, i.e. $f(x)$ is decreasing, then $f'(x) < 0$ for $x < 2$.

30. (b) Let one part be x and other part be $(20-x)$.

$$\text{So, } P = (20-x)x^3 = 20x^3 - x^4$$

$$\Rightarrow \frac{dP}{dx} = 60x^2 - 4x^3 = 4x^2(15-x)$$

$$\Rightarrow \frac{d^2P}{dx^2} = 120x - 12x^2 = 12x(10-x)$$

For maximum or minimum value of P , put $\frac{dP}{dx} = 0$

$$\Rightarrow 4x^2(15-x) = 0 \Rightarrow x = 0 \text{ and } x = 15$$

$\therefore x = 0$ is not possible.

$$\begin{aligned} \left(\frac{d^2P}{dx^2}\right)_{\text{at } x=15} &= 12 \times 15(10-15) = 12 \times 15 \times (-5) \\ &= \text{Negative value} \end{aligned}$$

$\Rightarrow$ At $x = 15$, P is maximum.

Other part is $20 - 15 = 5$

$\therefore$ Two part of 20 is (5, 15).

31. (b) Let $f(x) = 4e^{2x} + 9e^{-2x}$

$$\therefore f'(x) = 8e^{2x} - 18e^{-2x}$$

Put $f'(x) = 0$

$$\Rightarrow 8e^{2x} - 18e^{-2x} = 0$$

$$\Rightarrow e^{2x} = 3/2$$

$$\Rightarrow x = \log(3/2)^{1/2}$$

$$\text{Again, } f''(x) = 16e^{2x} + 36e^{-2x} > 0$$

$$\begin{aligned} \text{Now, } f(\log(3/2)^{1/2}) &= 4e^{2[\log(3/2)]^{1/2}} + 9e^{-2[\log(3/2)]^{1/2}} \\ &= 4 \times \frac{3}{2} + 9 \times \frac{2}{3} = 6 + 6 = 12 \end{aligned}$$

Hence, the minimum value is 12.

32. (b) Given, $f(x) = x + \sin 2x$

$$\Rightarrow f'(x) = 1 + 2 \cos 2x$$

$$\Rightarrow f'(x) = 0 \Rightarrow 1 + 2 \cos 2x = 0$$

$$\Rightarrow \cos 2x = -\frac{1}{2} \Rightarrow 2x = 2m\pi \pm \frac{2\pi}{3}$$

$$\Rightarrow x = m\pi \pm \frac{\pi}{3} \Rightarrow x = \frac{4\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}$$

$$\text{Find } f(0), f\left(\frac{2\pi}{3}\right), f\left(\frac{4\pi}{3}\right), f\left(\frac{5\pi}{3}\right), f(2\pi)$$

$$\Rightarrow f(0) = 0, f\left(\frac{2\pi}{3}\right) = \frac{2\pi}{3} - 0.8$$

$$\Rightarrow f\left(\frac{4\pi}{3}\right) = \frac{4\pi}{3} + 0.8, f\left(\frac{5\pi}{3}\right) = \frac{5\pi}{3} - 0.8$$

$$\Rightarrow f(2\pi) = 2\pi + 0 = 2\pi$$

$\therefore$ Maximum value of $f(x) = 2\pi$

33. (d) Hint Now, $f'(x) = a \cos x + \frac{1}{3}(3 \cos 3x)$

put $f'\left(\frac{\pi}{3}\right) = 0$

34. (a)

35. (a) Given, $f(x) = x^2 + \frac{\lambda}{x} \Rightarrow f'(x) = 2x - \frac{\lambda}{x^2}$

$\therefore f'(x) = 0$

$2x - \frac{\lambda}{x^2} = 0 \Rightarrow x = \left(\frac{\lambda}{2}\right)^{1/3}$

If $\lambda = 16$, then $x = 2$

Now, $f''(x) = 2 + \frac{2\lambda}{x^3}$

If $\lambda = 16$, $f''(x) > 0$, i.e. $f(x)$ has a minimum at $x = 2$.

Also, $f''\left\{\left(\frac{\lambda}{2}\right)^{1/3}\right\} = 2 + \frac{2\lambda}{\lambda} = 2 + 4 > 0$

Hence, $f(x)$ has maximum for no real value of λ .

So, $f''(x) > 0$ for all real values of λ .

36. (b) Clearly, $f(2) = -1$

$\Rightarrow -1 = \frac{2a+b}{(2-1)(2-4)} \Rightarrow 2a+b=2$

Now, $f'(x) = \frac{4a+5b-2bx-ax^2}{(x-1)^2(x-4)^2}$, $f'(2) = 0$

$\Rightarrow b=0 \Rightarrow a=1$

$\Rightarrow f'(x) = -\frac{(x-2)(x+2)}{(x-1)^2(x-4)^2}$

Clearly, for $x > 2$, $f'(x) < 0$ and for $x < 2$, $f'(x) > 0$. Thus, $x = 2$ is indeed the point of local maxima for $y = f(x)$.

37. (d) Hint $\frac{dy}{dx} = \frac{a}{x} + 2bx + 1$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = a + 2b + 1 = 0$

$\Rightarrow a = -2b - 1$

and $\left(\frac{dy}{dx}\right)_{x=2} = \frac{a}{2} + 4b + 1 = 0$

38. (c) $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$

$\Rightarrow f'(x) = 6x^2 - 18ax + 12a^2$

$\Rightarrow f''(x) = 12x - 18a$

For maximum and minimum, put $f'(x) = 0$.

$\therefore 6x^2 - 18ax + 12a^2 = 0$

$\Rightarrow x^2 - 3ax + 2a^2 = 0$

$\Rightarrow x = a \text{ or } x = 2a$

At $x = a$ maximum and at $x = 2a$ minimum.

$\therefore p^2 = q$

$\therefore a^2 = 2a \Rightarrow a = 2 \text{ or } a = 0$

But $a > 0$, therefore $a = 2$.

39. (b) Given, $f(x) \equiv y = a \log|x| + bx^2 + x$... (i)

On differentiating Eq.(i) w.r.t. x , we get

$$\frac{dy}{dx} = \frac{a}{x} + 2bx + 1$$

For extremum values, $\frac{dy}{dx} = 0$

$\therefore \frac{a}{-1} + 2b(-1) + 1 = 0$

$\Rightarrow a + 2b - 1 = 0$... (ii)

and $\frac{a}{2} + 2b(2) + 1 = 0$

$\Rightarrow a + 8b + 2 = 0$... (iii)

From Eqs. (ii) and (iii), we get $a = 2$ and $b = -\frac{1}{2}$

40. (d) $f'(x) = 1$, if $-1 \leq x < 0$

$$L'f'(0) = \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x \rightarrow 0^-} \frac{x + 2 - 1}{x} = \lim_{x \rightarrow 0^-} \left(1 + \frac{1}{x}\right)$$

which does not exist.

$\therefore f'(0)$ does not exist.

Again $f'(x) = \frac{1}{2}$ when $0 < x \leq 1$

In all the cases, $f'(x) \neq 0$ for any $x \in [-1, 1]$

$\therefore f(x)$ has neither a maximum nor a minimum.

41. (c) Since, $f(x)$ satisfies conditions of Rolle's theorem on $[1, 3]$.

$\therefore f(1) = f(3)$

$\therefore 1 - 6 + a + b = 27 - 54 + 3a + b$

$\Rightarrow 2a = 22 \text{ or } a = 11$

Since, $f(1) = f(3)$ is independent of b .

$\therefore a = 11 \text{ and } b \in R$

42. (b) Let $f(x) = \frac{ax^4}{4} + \frac{bx^3}{3} + \frac{cx^2}{2} + dx$

which is continuous and differentiable.

Then, $f(0) = 0$ and $f(-1) = \frac{a}{4} - \frac{b}{3} + \frac{c}{2} - d$

$$= \frac{1}{4}(a + 2c) - \frac{1}{3}(b + 3d) = 0$$

So, by Rolle's theorem, there exists atleast one real root of $f'(x) = 0$ in $(-1, 0)$.

43. (c) To determine c in Rolle's theorem, $f'(c) = 0$.

Here, $f'(x) = (x^2 + 3x)e^{-(1/2)x} \left(-\frac{1}{2}\right) + (2x + 3)e^{-(1/2)x}$

$$= e^{-(1/2)x} \left\{-\frac{1}{2}(x^2 + 3x) + 2x + 3\right\}$$

$$= -\frac{1}{2}e^{-(x/2)}(x^2 - x - 6)$$

$\therefore f'(c) = 0 \Rightarrow c^2 - c - 6 = 0 \Rightarrow c = 3, -2$

$\therefore c = 3 \notin [-3, 0]$

$\therefore c = -2$

44. (a) Let $f(x) = ax^3 + bx^2 + cx$ in $[0, 1]$

Apply Rolle's theorem on $f(x)$ in $[0, 1]$

$\therefore f(x)$ has atleast one real root in $(0, 1)$

45. (a) Using mean value theorem,

$$f'(c) = \frac{f(3) - f(1)}{3 - 1} \Rightarrow \frac{1}{c} = \frac{\log_e 3 - \log_e 1}{3 - 1}$$

$$\therefore c = \frac{2}{\log_e 3} = 2 \log_3 e$$

46. (c) **Hint** $f(x) = x^3 - 6x^2 + 8x \Rightarrow f'(x) = 3x^2 - 12x + 8$

Since, polynomial function is everywhere continuous and differentiable, therefore $f(x)$ is continuous on $(1, 4)$ and differentiable on $(1, 4)$. Both conditions of mean value theorem are satisfied, therefore there exists atleast one number $c \in (1, 4)$ such that

$$f'(c) = \frac{f(4) - f(1)}{4 - 1}$$

47. (d) $\therefore f(x) = \sqrt{x}$

$$\therefore f(a) = \sqrt{4} = 2$$

$$f(b) = \sqrt{9} = 3, f'(x) = \frac{1}{2\sqrt{x}}$$

$$\text{Also, } f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{3 - 2}{9 - 4} = \frac{1}{5}$$

$$\therefore \frac{1}{2\sqrt{c}} = \frac{1}{5}$$

$$\Rightarrow c = \frac{25}{4} = 6.25$$

48. (b) **Hint** Let the point be (x_1, y_1) .

$$\text{Therefore, } y_1 = (x_1 - 3)^2 \quad \dots(i)$$

Now, slope of the tangent at (x_1, y_1) is $2(x_1 - 3)$ but it is equal to 1.

$$\text{Therefore, } 2(x_1 - 3) = 1$$

BITSAT Archives

1. (d) Let α and β be the roots of the equation

$$x^2 - (a - 2)x - a + 1 = 0$$

$$\therefore \alpha + \beta = a - 2 \text{ and } \alpha\beta = -(a - 1)$$

$$\begin{aligned} \text{Let } s &= \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta \\ &= (a - 2)^2 + 2(a - 1) \\ &= a^2 - 4a + 4 + 2a - 2 = a^2 - 2a + 2 \end{aligned}$$

$$\text{Now, } \frac{ds}{da} = 2a - 2$$

For maximum and minimum, $ds / da = 0$

$$\Rightarrow 2a - 2 = 0 \Rightarrow a = 1$$

$$\text{Also, } \frac{d^2s}{da^2} = 2 > 0$$

Hence, at $a = 1$, s will have minimum value.

2. (d) We have, $f(x) = e^{x^3 - 3x + 2}$

$$\text{Let } h(x) = x^3 - 3x + 2$$

$$\therefore h'(x) = 3x^2 - 3 = 3(x^2 - 1)$$

$$\Rightarrow h'(x) \geq 0 \text{ for } x \in (-\infty, -1]$$

$\therefore f(x)$ is an increasing function.

$\therefore f(x)$ is one-one.

$$\text{Now, range of } f(x) = (0, e^4]$$

$$\text{But codomain of } f(x) = (0, e^5]$$

$\therefore f(x)$ is an into function.

3. (a) $f(x) = (\tan^{-1} x)^2 + \frac{2}{\sqrt{x^2 + 1}}$

$$\Rightarrow f'(x) = \frac{2}{1 + x^2} \left[\tan^{-1} x - \frac{x}{\sqrt{1 + x^2}} \right]$$

$$\text{Let } g(x) = \tan^{-1} x - \frac{x}{\sqrt{x^2 + 1}}$$

$$\Rightarrow g'(x) = \frac{1}{1 + x^2} \left[1 - \frac{1}{\sqrt{x^2 + 1}} \right] > 0, \forall x \in R$$

$\Rightarrow g(x)$ is increasing, $\forall x \in R$.

$$\text{But } g(0) = 0$$

$$\Rightarrow g(x) > 0 \text{ for } x > 0$$

$$\text{So, } f'(x) > 0 \text{ for } x > 0$$

Hence, $f(x)$ is increasing in $(0, \infty)$.

4. (d) For maximum value, find $f'(x)$.

$$f'(x) = \frac{1}{x^2} (1 - \log x)$$

$$f'(x) > 0 \text{ for } x < e \text{ and } f'(x) < 0 \text{ for } x > e$$

$\Rightarrow f(x)$ is increasing for $x < e$ and decreasing for $x > e$

$\Rightarrow x = e$ is the point of local maxima.

$$\therefore \text{Maximum value of } f(x) = \frac{1}{e}$$

5. (b) Here, we observe that

(i) $f(x)$ is a polynomial, so it is continuous in the interval $[0, 2]$.

(ii) $f'(x) = 3x^2 - 6x + 2$ exists for all $x \in (0, 2)$.

So, $f(x)$ is differentiable for all $x \in (0, 2)$

$$(iii) f(0) = 0, f(2) = 2^3 - 3(2)^2 + 2(2) = 0$$

$$\therefore f(0) = f(2)$$

Thus, all the three conditions of Rolle's theorem are satisfied.

So, there must exist $c \in (0, 2)$ such that $f'(c) = 0$

$$\Rightarrow f'(c) = 3c^2 - 6c + 2 = 0$$

$$\Rightarrow c = 1 \pm \frac{1}{\sqrt{3}} \Rightarrow c \in (0, 2)$$

6. (a) Circumference of circle, $C = 2\pi r$

$$\Rightarrow \frac{dC}{dt} = 2\pi \frac{dr}{dt}$$

$$\Rightarrow \frac{0.3}{2\pi} = \frac{dr}{dt} \quad \left[\because \frac{dC}{dt} = 0.3 \text{ cm/s, given} \right]$$

$$\text{Now, } A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\Rightarrow \frac{dA}{dt} = r \times 0.3$$

$$\Rightarrow \left[\frac{dA}{dt} \right]_{r=5} = 5 \times 0.3 = 1.5 \text{ sq cm/s}$$

7. (b) $\frac{dy}{dx} = 3x^2 - 2ax + 48 > 0$ [$\because y$ is an increasing function]

$$\therefore \text{Discriminant, } D < 0$$

$$\Rightarrow 4a^2 - 4 \times 3 \times 48 < 0$$

$$\Rightarrow a^2 - 144 < 0$$

$$\Rightarrow a \in (-12, 12)$$

8. (d) Rolle's theorem is not applicable for the function

$$f(x) = |x| \text{ in } [-1, 1]$$

$$\because f'(0) \text{ does not exist.}$$

9. (c) Let $y = x^{1/x}$

$$\Rightarrow \log y = \frac{\log x}{x} \Rightarrow \frac{1}{y} \frac{dy}{dx} = \frac{1}{x^2} - \frac{\log_e x}{x^2}$$

$$\Rightarrow \frac{dy}{dx} = x^{1/x} \left(\frac{1 - \log_e x}{x^2} \right)$$

$$\text{For } 1 < x < \infty, x^{1/x} > 0$$

$$\text{and } \frac{1 - \log_e x}{x^2} > 0 \text{ in } (1, e) \text{ and } \frac{1 - \log_e x}{x^2} < 0 \text{ in } (e, \infty)$$

$$\text{Hence, } f(x) \text{ is increasing in } (1, e) \text{ and decreasing in } (e, \infty).$$

10. (d) Let any point on the curve be (h, k) .

$$\text{Then, } h^2 = 2k$$

$$\therefore \text{Distance, } D = \sqrt{h^2 + (k - 5)^2}$$

$$\Rightarrow D = \sqrt{2k + (k - 5)^2}$$

$$\Rightarrow \frac{dD}{dk} = \frac{1}{2\sqrt{2k + (k - 5)^2}} \times \{2 + 2(k - 5)\} = 0$$

$$\text{Then, point will be } (\pm 2\sqrt{2}, 4).$$

11. (c) $xy = a^2$

$$\therefore x \frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

$$\therefore \left(\frac{dy}{dx} \right)_1 = -\frac{y}{x}$$

$$x^2 - y^2 = 2a^2 \Rightarrow 2x - 2y \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{y}$$

$$\therefore \left(\frac{dy}{dx} \right)_2 = \frac{x}{y}$$

$$\Rightarrow \left(\frac{dy}{dx} \right)_1 \times \left(\frac{dy}{dx} \right)_2 = -\frac{y}{x} \times \frac{x}{y} = -1$$

$$\therefore \text{Intersection angle} = \frac{\pi}{2}$$

12. (d) $f(x) = 2x^2 - \log|x|$

$$= \begin{cases} 2x^2 - \log x, & x > 0 \\ 2x^2 - \log(-x), & x < 0 \end{cases}$$

$$\Rightarrow f'(x) = 4x - \frac{1}{x}, \forall x \neq 0$$

$$\text{For } f(x) \text{ to be increasing,}$$

$$f'(x) > 0 \Rightarrow 4x - \frac{1}{x} > 0$$

$$\Rightarrow \frac{4x^2 - 1}{x} > 0 \Rightarrow \frac{(2x - 1)(2x + 1)}{x} > 0$$

$$\Rightarrow x \in \left(-\frac{1}{2}, 0 \right) \cup \left(\frac{1}{2}, \infty \right)$$

13. (b) We have, $f(x) = (x + 1)^{1/3} - (x - 1)^{1/3}$

$$\therefore f'(x) = \frac{1}{3} \left[\frac{1}{(x + 1)^{2/3}} - \frac{1}{(x - 1)^{2/3}} \right]$$

$$= \frac{(x - 1)^{2/3} - (x + 1)^{2/3}}{3(x^2 - 1)^{2/3}}$$

$$\text{Clearly, } f'(x) \text{ does exist at } x = \pm 1.$$

$$\therefore f'(x) = 0 \Rightarrow (x - 1)^{2/3} = (x + 1)^{2/3}$$

$$\Rightarrow x = 0$$

$$\text{Clearly, } f'(x) \neq 0 \text{ for any other value of } x \in [0, 1]. \text{ The value of } f(x) \text{ at } x = 0 \text{ is } 2.$$

$$\text{Hence, the greatest value of } f(x) \text{ is } 2.$$

14. (a) We have, $f'(x) = (x - a)^{2n} (x - b)^{2m+1}$

$$\therefore f'(x) = 0 \Rightarrow x = a, b$$

$$\text{Now, for } x = b - h$$

$$f'(x) = (b - h - a)^{2n} (-h)^{2m+1} < 0$$

$$\text{and for } n = b + h,$$

$$f'(x) = (b + h - a)^{2n} h^{2m+1} > 0$$

$$\text{Thus, as } x \text{ passes through } b, f'(x) \text{ changes sign from negative to positive. Hence, } x = b \text{ is a point of minimum.}$$

15. (c) Hint $4y^3 \frac{dy}{dx} = 3ax^2$

$$\Rightarrow \left(\frac{dy}{dx} \right)_{(a, a)} = \frac{3a^3}{4a^3} = \frac{3}{4}$$

16. (b) Hint Length of subtangent $= \frac{y}{\frac{dy}{dx}} = \frac{2}{\frac{5}{4}} = \frac{8}{5}$

17. (a) Let h and r be the height and radius of cylinder.

$$\text{Given that, } \frac{dr}{dt} = 3 \text{ m/s} \Rightarrow \frac{dh}{dt} = -4 \text{ m/s}$$

$$\text{Also, } V = \pi r^2 h$$

$$\text{On differentiating w.r.t. } t, \text{ we get}$$

$$\frac{dV}{dt} = \pi \left[r^2 \frac{dh}{dt} + h \cdot 2r \frac{dr}{dt} \right]$$

$$\text{At } r = 4 \text{ m and } h = 6 \text{ m,}$$

$$\frac{dV}{dt} = \pi [-64 + 144]$$

$$= 80\pi \text{ cu m/s}$$

18. (b) **Hint** $f'(x) = 2 - \frac{18}{x^2}$

For maximum or minima, put $f'(x) = 0$

$$\Rightarrow x = \pm 3 \text{ and } f''(3) = \frac{36}{3^3} > 0, \text{ minimum}$$

19. (a) **Hint**

$$\Rightarrow \frac{dx}{d\theta} = a(1 + \cos\theta) \text{ and } \frac{dy}{d\theta} = a \sin\theta$$

$$\frac{dy}{dx} = \tan \frac{\theta}{2}$$

$$\text{Now, length of subtangent} = \left| \frac{y}{dy/dx} \right|_{\theta=\frac{\pi}{2}}$$

$$\text{and length of subnormal} = \left| y \frac{dy}{dx} \right|_{\theta=\frac{\pi}{2}}$$

20. (c) **Hint** The point of intersection of give curves are (1, 2) and (1, -2).

$$\text{Now, } 2x + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$$

$$\therefore m_1 = \left(\frac{dy}{dx} \right)_{(1,2)} = -\frac{1}{2}$$

$$\text{And } 2y \frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = \frac{2}{y}, m_2 = \left(\frac{dy}{dx} \right)_{(1,2)} = \frac{2}{2} = 1$$

21. (c) $p(t) = 1000 + \frac{1000t}{100 + t^2}$... (i)

On differentiating both sides w.r.t. t , we get

$$\begin{aligned} p'(t) &= 0 + \frac{(100 + t^2)(1000) - 1000t(2t)}{(100 + t^2)^2} \\ &= 1000 \frac{(100 - t^2)}{(100 + t^2)^2} \end{aligned} \quad \dots (ii)$$

For maxima or minima, put $p'(t) = 0$

$$\Rightarrow 100 - t^2 = 0$$

$$\Rightarrow t = \pm 10$$

Again differentiating Eq. (ii) w.r.t. x , we get

$$\begin{aligned} p''(t) &= 1000 \times \left[\frac{(100 + t^2)^2(-2t) - (100 - t^2)2(100 + t^2)2t}{(100 + t^2)^4} \right] \\ &= 1000t \frac{[(100 + t^2)(-2) - (100 - t^2)(4)]}{(100 + t^2)^3} \\ &= -1000t \frac{[600 - 2t^2]}{(100 + t^2)^3} \end{aligned}$$

At $t = 10$, $p''(t) < 0$

$\therefore$ The maximum value is

$$\begin{aligned} p(10) &= 1000 + \frac{10000}{100 + 100} \\ &= 1000 + \frac{10000}{200} \\ &= 1000 + 50 \\ &= 1050 \end{aligned}$$

22. (a) Given that, $f(x) = \frac{e^{2x} - 1}{e^{2x} + 1}$

On differentiating w.r.t. x , we get

$$\begin{aligned} f'(x) &= \frac{2(e^{2x} + 1)(e^{2x}) - 2(e^{2x} - 1)(e^{2x})}{(e^{2x} + 1)^2} \\ &= \frac{2(e^{2x} + e^{2x})}{(e^{2x} + 1)} \\ &= \frac{4e^{2x}}{(e^{2x} + 1)^2} > 0 \end{aligned}$$

$\therefore$ Function is increasing.

19

Indefinite Integral

Introduction

Integration is the inverse process of differentiation, i.e. the process of finding $f(x)$, when its derivative $f'(x)$ is given. If $f(x)$ is a differentiable function such that $f'(x) = g(x)$, then integration of $g(x)$ w.r.t. x is $f(x) + C$. Symbolically, it is written as

$$\int g(x)dx = f(x) + C$$

where, $f(x)$ is an anti-derivative of $g(x)$, $g(x)$ is an integrand and C is an arbitrary constant known as the constant of integration.

Properties of Integral

- (i) $\int cf(x) dx = c \int f(x) dx$, where c is any constant.
- (ii) $\int [f_1(x) \pm f_2(x) \pm f_3(x) \pm f_4(x) \pm \dots] dx = \int f_1(x) dx \pm \int f_2(x) dx \pm \int f_3(x) dx \pm \int f_4(x) dx \pm \dots$
- (iii) If $\int f(x) dx = g(x) + C$, then $\int f(ax \pm b) dx = \frac{1}{a} g(ax \pm b) + C$

Fundamental Integration Formulae

1. Algebraic Formulae

- (i) $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
- (ii) $\int (ax + b)^n dx = \frac{1}{a} \cdot \frac{(ax + b)^{n+1}}{n+1} + C, n \neq -1$
- (iii) $\int \frac{1}{x} dx = \log |x| + C$
- (iv) $\int \frac{1}{ax + b} dx = \frac{1}{a} (\log |ax + b|) + C$
- (v) $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$
- (vi) $\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$

- (vii) $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + C$
- (viii) $\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1} \frac{x}{a} + C$
- (ix) $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$
- (x) $\int \frac{-1}{a^2 + x^2} dx = \frac{1}{a} \cot^{-1} \frac{x}{a} + C$
- (xi) $\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1} \left(\frac{x}{a} \right) + C$
- (xii) $\int \frac{-1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \operatorname{cosec}^{-1} \left(\frac{x}{a} \right) + C$
- (xiii) $\int \frac{1}{\sqrt{x^2 - a^2}} dx = \log |x + \sqrt{x^2 - a^2}| + C$
- (xiv) $\int \frac{1}{\sqrt{x^2 + a^2}} dx = \log |x + \sqrt{x^2 + a^2}| + C$
- (xv) $\int \sqrt{a^2 - x^2} dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{1}{2} a^2 \sin^{-1} \left(\frac{x}{a} \right) + C$
- (xvi) $\int \sqrt{x^2 - a^2} dx = \frac{1}{2} x \sqrt{x^2 - a^2} - \frac{1}{2} a^2 \log |x + \sqrt{x^2 - a^2}| + C$
- (xvii) $\int \sqrt{x^2 + a^2} dx = \frac{1}{2} x \sqrt{x^2 + a^2} + \frac{1}{2} a^2 \log |x + \sqrt{x^2 + a^2}| + C$

2. Trigonometric Formulae

- (i) $\int \sin x dx = -\cos x + C$
- (ii) $\int \cos x = \sin x + C$
- (iii) $\int \tan x dx = -\log |\cos x| + C = \log |\sec x| + C$
- (iv) $\int \cot x dx = \log |\sin x| + C = -\log |\operatorname{cosec} x| + C$
- (v) $\int \sec x dx = \log |\sec x + \tan x| + C = \log \left| \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right| + C$
- (vi) $\int \operatorname{cosec} x dx = \log |\operatorname{cosec} x - \cot x| + C = \log \left| \tan \frac{x}{2} \right| + C$
- (vii) $\int \sec^2 x dx = \tan x + C$
- (viii) $\int \operatorname{cosec}^2 x dx = -\cot x + C$
- (ix) $\int \sec x \tan x dx = \sec x + C$
- (x) $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + C$

3. Exponential Formulae

- (i) $\int e^x dx = e^x + C$ (ii) $\int e^{(ax+b)} dx = \frac{1}{a} \cdot e^{(ax+b)} + C$
- (iii) $\int a^x dx = \frac{a^x}{\log_e a} + C$ (iv) $\int a^{(bx+c)} dx = \frac{1}{b} \cdot \frac{a^{(bx+c)}}{\log_e a} + C$

Different Methods of Integration

Mainly, there are three methods of integration which are given below:

1. Integration by Substitution

(i) Direct Substitution

If $\int f(x) dx = g(x) + C$, then to solve $I = \int f\{h(x)\} \cdot h'(x) dx$ we put $h(x) = t \Rightarrow h'(x) dx = dt$
So, $I = \int f(t) dt = g(t) + C \Rightarrow g\{h(x)\} + C$

Some Standard Integral

- $\int f(x)' \cdot f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + C$
- $\int \frac{f'(x)}{f(x)} dx = \log |f(x)| + C$
- $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + C$

(ii) Standard Substitution

Some standard integrands or a part of it have standard substitution. Some standard substitutions are given below:

S.No.	Integrand form	Substitution
(i)	$\sqrt{a^2 - x^2}, \frac{1}{\sqrt{a^2 - x^2}}, a^2 - x^2$	$x = a \sin \theta$ or $x = a \cos \theta$
(ii)	$\sqrt{x^2 + a^2}, \frac{1}{\sqrt{x^2 + a^2}}, x^2 + a^2$	$x = a \tan \theta$ or $x = a \cot \theta$
(iii)	$\sqrt{x^2 - a^2}, \frac{1}{\sqrt{x^2 - a^2}}, x^2 - a^2$	$x = a \sec \theta$ or $x = a \operatorname{cosec} \theta$
(iv)	$\frac{x}{\sqrt{a+x}}, \frac{\sqrt{a+x}}{x}, \sqrt{x(a+x)}, \frac{1}{\sqrt{x(a+x)}}$	$x = a \tan^2 \theta$ or $x = a \cot^2 \theta$
(v)	$\frac{x}{\sqrt{a-x}}, \frac{\sqrt{a-x}}{x}, \sqrt{x(a-x)}, \frac{1}{\sqrt{x(a-x)}}$	$x = a \sin^2 \theta$ or $x = a \cos^2 \theta$
(vi)	$\frac{x}{\sqrt{x-a}}, \frac{\sqrt{x-a}}{x}, \sqrt{x(x-a)}, \frac{1}{\sqrt{x(x-a)}}$	$x = a \sec^2 \theta$ or $x = a \operatorname{cosec}^2 \theta$
(vii)	$\frac{\sqrt{a-x}}{\sqrt{a+x}}, \frac{\sqrt{a+x}}{\sqrt{a-x}}$	$x = a \cos 2\theta$
(viii)	$\frac{\sqrt{x-\alpha}}{\sqrt{\beta-x}}, \sqrt{(x-\alpha)(\beta-x)}, (\beta > \alpha)$	$x = \alpha \cos^2 \theta + \beta \sin^2 \theta$

(iii) Indirect Substitution

If integrand $f(x)$ can be rewritten as product of two functions, i.e. $f(x) = f_1(x) \cdot f_2(x)$, where $f_2(x)$ is a function of integral of $f_1(x)$, then put integral of $f_1(x) = t$.

2. Integration by Parts

If integrand $f(x)$ can be expressed as product of two functions, i.e. $f(x) = f_1(x) \cdot f_2(x)$, then we use the following formula:

$$\int f_1(x) \cdot f_2(x) dx = f_1(x) \int f_2(x) dx - \int \{f_1'(x) [\int f_2(x) dx]\} dx$$

where, $f_1(x)$ and $f_2(x)$ are known as first and second functions respectively.

i.e. The integral of the product of two functions

$$= (\text{First function}) \times (\text{Integral of the second function})$$

$$- \text{Integral of } \{(\text{Differentiation of first function}) \times (\text{Integral of second function})\}$$

Some Important Points

- We do not put constant of integration in 1st integral. We put this only, once at the end.
- Order of $f_1(x)$ and $f_2(x)$ is normally, decided by the rule ILATE, where,
 I = Inverse function
 L = Logarithmic function
 A = Algebraic function
 T = Trigonometric function
 E = Exponential function
- Normally, we have to suppose 2nd function as that function, which can be integrated easily.

3. Integration by Partial Fractions

If integrand is a rational function, i.e. of the form $\frac{f(x)}{g(x)}$,

where $f(x)$ and $g(x)$ are the polynomial functions of x and $g(x) \neq 0$, then use method of partial fractions, if degree of $f(x)$ is less than the degree of $g(x)$. Otherwise, first reduced to the proper rational function by long division process. The following table indicates the types of simpler partial fractions that are associated with various kinds of rational functions.

Form of the rational function	Form of the partial fraction
$\frac{px + q}{(x - a)(x - b)}$	$\frac{A}{x - a} + \frac{B}{x - b}$
$\frac{px + q}{(x - a)^2}$	$\frac{A}{x - a} + \frac{B}{(x - a)^2}$
$\frac{px^2 + qx + r}{(x - a)(x - b)(x - c)}$	$\frac{A}{x - a} + \frac{B}{x - b} + \frac{C}{x - c}$
$\frac{px^2 + qx + r}{(x - a)^2(x - b)}$	$\frac{A}{x - a} + \frac{B}{(x - a)^2} + \frac{C}{x - b}$
$\frac{px^2 + qx + r}{(x - a)(x^2 + bx + c)}$	$\frac{A}{x - a} + \frac{Bx + C}{x^2 + bx + c}$, where $x^2 + bx + c$ cannot be factorised further.

Here A , B and C are the real constants and these can be determined by reducing both sides of the equation as identity in polynomial form and by comparing the coefficients of like powers.

Integration of Logarithmic Functions

$$(i) \int f(x) dx = \int f(x) \cdot 1 dx$$

Now, integrate taking $f(x)$ as a first and 1 as a second function.

$$(ii) \int \frac{f(x)}{[g(x)]^n} dx = \int \frac{f(x)}{g'(x)} \cdot \frac{g'(x)}{[g(x)]^n} dx$$

Now, integrate by taking $\frac{f(x)}{g'(x)}$ as first function and

$\frac{g'(x)}{[g(x)]^n}$ as a second function.

Here, $f(x)$ and $g(x)$ both are logarithmic functions.

Integration of the Form $\int e^x \{g(x) + g'(x)\} dx$

If integrand is of the form $e^x \{g(x) + g'(x)\}$, then

$$\int e^x \cdot [g(x) + g'(x)] dx = e^x g(x) + C$$

Integration of Different Types of Functions

$$1. \int \frac{dx}{ax^2 + bx + c} \text{ we write, } \int \frac{dx}{ax^2 + bx + c} = \frac{1}{a} \int \frac{dx}{x^2 + \frac{b}{a}x + \frac{c}{a}}$$

$$= \frac{1}{a} \int \frac{dx}{\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4a^2}}, \text{ which is of the form}$$

$$\int \frac{dx}{X^2 - A^2}, \int \frac{dx}{X^2 + A^2} \text{ or } \int \frac{dx}{A^2 - X^2}$$

$$2. \int \frac{dx}{\sqrt{ax^2 + bx + c}} \text{ or } \int \sqrt{ax^2 + bx + c} dx$$

This can be reduced to one of the form of

$$\int \frac{dx}{\sqrt{A^2 - X^2}}, \int \frac{dx}{\sqrt{X^2 - A^2}}, \int \frac{dx}{\sqrt{X^2 + A^2}}$$

$$\text{or } \int \sqrt{A^2 - X^2} dx, \int \sqrt{X^2 - A^2} dx, \int \sqrt{A^2 + X^2} dx$$

$$3. \int \frac{(px + q)}{ax^2 + bx + c} dx, \int \frac{(px + q)}{\sqrt{ax^2 + bx + c}} dx, \int (px + q) \sqrt{ax^2 + bx + c} dx$$

Put $px + q = A \{ \text{differentiation of } (ax^2 + bx + c) \} + B$. Find A and B by comparing the coefficients of like powers of x on the two sides and split the integral into sum of two integrals.

$$4. \int \frac{ax^2 + bx + c}{(px^2 + qx + r)} dx, \int \frac{ax^2 + bx + c}{\sqrt{px^2 + qx + r}} dx,$$

$$\int (ax^2 + bx + c) \sqrt{px^2 + qx + r} dx$$

$$\text{Substitute } ax^2 + bx + c = \lambda (px^2 + qx + r) + \mu \left\{ \frac{d}{dx} (px^2 + qx + r) \right\} + \gamma$$

Find λ, μ and γ and reduce the integral integration of three independent functions.

$$5. \int e^{ax} \sin(bx + c) dx = \frac{e^{ax}}{a^2 + b^2} \{ a \sin(bx + c) - b \cos(bx + c) \} + C$$

$$6. \int e^{ax} \cos(bx + c) dx = \frac{e^{ax}}{a^2 + b^2} \{ a \cos(bx + c) + b \sin(bx + c) \} + C$$

NOTE • $\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + C$

• $\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + C$

$$7. \int \frac{dx}{a + b \cos^2 x}, \int \frac{dx}{a + b \sin^2 x}, \int \frac{dx}{a \sin^2 x + b \cos^2 x},$$

$$\int \frac{dx}{(a \sin x + b \cos x)^2}, \int \frac{dx}{a + b \sin^2 x + c \cos^2 x}$$

(i) Divide both the numerator and denominator by $\cos^2 x$.

(ii) Replace $\sec^2 x$ by $1 + \tan^2 x$ in the denominator, if any.

(iii) Put $\tan x = t \Rightarrow \sec^2 x dx = dt$. Further integrate it.

$$8. \int \frac{1}{a \sin x + b \cos x} dx, \int \frac{1}{a + b \sin x} dx,$$

$$\int \frac{1}{a + b \cos x} dx, \int \frac{1}{a \sin x + b \cos x + c} dx$$

$$(i) \text{ Put } \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

(ii) Replace $1 + \tan^2 \frac{x}{2} = \sec^2 \frac{x}{2}$ in numerator.

(iii) Put $\tan \frac{x}{2} = t$, then $\sec^2 \frac{x}{2} dx = 2 dt$.

(iv) Hence, integral reduces to the form $\int \frac{dt}{at^2 + bt + c}$

$$9. \int \frac{a \sin x + b \cos x}{p \sin x + q \cos x} dx$$

(i) Consider numerator = $A \times$ (Derivative of denominator) + B (denominator)

(ii) Find A and B by comparing coefficients of $\sin x$ and $\cos x$.

$$10. \int \frac{p \cos x + q \sin x + r}{a \cos x + b \sin x + c} dx. \text{ Express numerator as, } \lambda$$

$$(\text{denominator}) + \mu (\text{differentiation of denominator}) + \gamma$$

Find λ, μ and γ by comparing coefficients of $\sin x$, $\cos x$ and constant term and split the integral into sum of three integrals.

$$11. \int \frac{x^2 + 1}{x^4 + kx^2 + 1} dx, \int \frac{x^2 - 1}{x^4 + kx^2 + 1} dx, \int \frac{dx}{x^4 + kx^2 + 1}$$

Here, we divide the numerator and denominator by x^2 and put $x + \frac{1}{x} = t$ or $x - \frac{1}{x} = t$ as required.

NOTE In case of $\int \frac{dx}{x^4 + kx^2 + 1}$, we first write it as

$$\int \frac{dx}{x^4 + kx^2 + 1} = \frac{1}{2} \int \frac{(x^2 + 1) - (x^2 - 1)}{x^4 + kx^2 + 1} dx$$

$$12. \int \frac{x^2 + a^2}{x^4 + kx^2 + a^4} dx, \int \frac{x^2 - a^2}{x^4 + kx^2 + a^4} dx,$$

Here, we divide numerator and denominator by x^2 and put $x - \frac{a^2}{x} = t$ or $x + \frac{a^2}{x} = t$, respectively.

$$13. \int \frac{dx}{(ax + b)\sqrt{cx + d}} \text{ Here, put } cx + d = t^2$$

$$14. \int \frac{dx}{(ax^2 + bx + c)\sqrt{px + q}} \text{ Here, put } px + q = t^2$$

$$15. \int \frac{dx}{(ax + b)\sqrt{px^2 + qx + r}} \text{ Here, put } ax + b = \frac{1}{t}$$

$$16. \int \frac{x}{(ax^2 + b)\sqrt{px^2 + q}} dx \text{ Here, put } px^2 + q = t^2$$

$$17. \int \frac{dx}{(ax^2 + b)\sqrt{px^2 + q}}$$

Here, first put $x = 1/t$, to obtain the similar expression as given in previous form and then put the expression inside the square root as y^2 .

18. To evaluate integrals of the form $\int \sin mx \cdot \cos nx dx$, $\int \sin mx \cdot \sin nx dx$, $\int \cos mx \cdot \cos nx dx$ and $\int \cos mx \cdot \sin nx dx$, we use the trigonometrical identities.

Practice Exercise

- If $\int \frac{f(x)}{\log \sin x} dx = \log \log \sin x$, then $f(x)$ is equal to
 a. $\sin x$ b. $\cos x$ c. $\log \sin x$ d. $\cot x$
- If $\int f(x) dx = f(x)$, then $\int \{f(x)\}^2 dx$ is equal to
 a. $\frac{1}{2}\{f(x)\}^2$ b. $\{f(x)\}^3$
 c. $\frac{\{f(x)\}^3}{3}$ d. $\{f(x)\}^2$
- If $x = f''(t) \cos t + f'(t) \sin t$, $y = -f''(t) \sin t + f'(t) \cos t$, then $\int \left[\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \right]^{1/2} dt$ equals
 a. $f'(t) + f''(t) + C$ b. $f''(t) + f'''(t) + C$
 c. $f(t) + f''(t) + C$ d. $f'''(t) - f''(t) + C$
- If $f(x) = \begin{vmatrix} 0 & x^2 - \sin x & \cos x - 2 \\ \sin x - x^2 & 0 & 1 - 2x \\ 2 - \cos x & 2x - 1 & 0 \end{vmatrix}$, then $\int f(x) dx$ is equal to
 a. $\frac{x^3}{3} - x^2 \sin x + \sin 2x + C$
 b. $\frac{x^3}{3} - x^2 \sin x - \cos 2x + C$
 c. $\frac{x^3}{3} - x^2 \cos x - \cos 2x + C$
 d. None of the above
- $\int \frac{\sin^8 x - \cos^8 x}{1 - 2 \sin^2 x \cos^2 x} dx$ is equal to
 a. $\sin 2x + C$ b. $-\frac{1}{2} \sin 2x + C$
 c. $\frac{1}{2} \sin 2x + C$ d. $-\sin 2x + C$
- $\int \tan(x - \alpha) \cdot \tan(x + \alpha) \cdot \tan 2x dx$ is equal to
 a. $\ln \left| \frac{\sqrt{\sec 2x} \cdot \sec(x - \alpha)}{\sec(x - \alpha)} \right| + C$
 b. $\ln \left| \frac{\sqrt{\sec 2x}}{\sec(x - \alpha) \cdot \sec(x + \alpha)} \right| + C$
 c. $\ln \left| \frac{\sqrt{\sec 2x} \cdot \sec(x - \alpha)}{\sec(x + \alpha)} \right| + C$
 d. $\ln \left| \frac{\sec 2x}{\sec(x - \alpha) \cdot \sec(x + \alpha)} \right| + C$
- $\int \frac{x^2 + \cos^2 x}{x^2 + 1} \cdot \operatorname{cosec}^2 x dx$ is equal to
 a. $\cot x + \cot^{-1} x + C$ b. $-e^{\ln \tan^{-1} x} - \cot x + C$
 c. $C - \cot x + \cot^{-1} x$ d. $-\tan^{-1} x - \frac{\operatorname{cosec} x}{\sec x} + C$
- $\int e^{3 \log x} (x^4 + 1)^{-1} dx$ is equal to
 a. $\log(x^4 + 1) + C$ b. $\frac{1}{4} \log(x^4 + 1) + C$
 c. $-\log(x^4 + 1) + C$ d. None of these
- If $\int \frac{2^x}{\sqrt{1 - 4^x}} dx = k \sin^{-1}(2^x) + C$, then k is equal to
 a. $\log 2$ b. $\frac{1}{2} \log 2$ c. $\frac{1}{2}$ d. $\frac{1}{\log 2}$
- $\int \frac{(\sin \theta + \cos \theta)}{\sqrt{\sin 2\theta}} d\theta$ is equal to
 a. $\log |\cos \theta - \sin \theta + \sqrt{\sin 2\theta}| + C$
 b. $\log |\sin \theta - \cos \theta + \sqrt{\sin 2\theta}| + C$
 c. $\sin^{-1}(\sin \theta - \cos \theta) + C$
 d. $\sin^{-1}(\sin \theta + \cos \theta) + C$
- $\int \frac{\ln(x+1) - \ln x}{x(x+1)} dx$ is equal to
 a. $-\frac{1}{2} \left[\ln \left(\frac{x+1}{x} \right) \right]^2 + C$ b. $-\ln(x+1)^2 - (\ln x)^2 + C$
 c. $-\ln \left[\ln \frac{x+1}{x} \right] + C$ d. $-\ln \left(\frac{x+1}{x} \right) + C$
- $\int \frac{(x - x^5)^{1/5}}{x^6} dx$ is equal to
 a. $\frac{5}{24} \left(\frac{1}{x^4} - 1 \right)^{6/5} + C$ b. $\frac{5}{24} \left(1 - \frac{1}{x^4} \right)^{6/5} + C$
 c. $\frac{-5}{24} \left(\frac{1}{x^4} - 1 \right)^{6/5} + C$ d. $\frac{5}{12} \left(1 + \frac{1}{x^4} \right)^{6/5} + C$
- $\int \frac{x^3}{(1+x^2)^{1/3}} dx$ is equal to
 a. $\frac{20}{3} (1+x^2)^{2/3} (2x^2 - 3) + C$
 b. $\frac{3}{20} (1+x^2)^{2/3} (2x^2 - 3) + C$
 c. $\frac{3}{20} (1+x^2)^{2/3} (2x^2 + 3) + C$
 d. $\frac{3}{20} (1+x^2)^{3/2} (2x^2 - 3) + C$

14. $\int \frac{\sqrt{1+\sqrt{x}}}{x} dx$ is equal to

a. $2\sqrt{1+\sqrt{x}} - 2\ln \left| \frac{\sqrt{1+\sqrt{x}}+1}{\sqrt{1+\sqrt{x}}-1} \right| + C$

b. $4\sqrt{1+\sqrt{x}} - 2\ln \left| \frac{\sqrt{1+\sqrt{x}}+1}{\sqrt{1+\sqrt{x}}-1} \right| + C$

c. $4\sqrt{1+\sqrt{x}} + 2\ln \left| \frac{\sqrt{1+\sqrt{x}}-1}{\sqrt{1+\sqrt{x}}+1} \right| + C$

d. None of the above

15. $\int \frac{x + \sqrt[3]{x^2} + \sqrt[6]{x}}{x(1 + \sqrt[3]{x})} dx$ is equal to

a. $\frac{3}{2}x^{2/3} + 6\tan^{-1}x^{1/6} + C$ b. $\frac{3}{2}x^{2/3} - 6\tan^{-1}x^{1/6} + C$

c. $-\frac{3}{2}x^{2/3} + 6\tan^{-1}x^{1/6} + C$ d. $\frac{1}{2}x^{2/3} - 6\tan^{-1}x^{1/6} + C$

16. $\int \frac{\tan x}{\sqrt{\sin^4 x + \cos^4 x}} dx$ is equal to

a. $\ln(\tan^2 x + \sqrt{1 + \tan^2 x}) + C$

b. $\sec x + C$

c. $\sqrt{1 - \tan^2 x} + C$

d. None of the above

17. If $x \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$, then $\int \frac{\sin x - \cos x}{\sqrt{1 - \sin 2x}} e^{\sin x} \cos x dx$ is equal to

a. $e^{\sin x} + C$

b. $e^{\sin x - \cos x} + C$

c. $e^{\sin x + \cos x} + C$

d. $e^{\cos x - \sin x} + C$

18. Integral of $f(x) = \sqrt{1+x^2}$ with respect to x^2 is

a. $\frac{2}{3} \frac{(1+x^2)^{3/2}}{x} + C$

b. $\frac{2}{3} x(1+x^2)^{3/2} + C$

c. $\frac{2}{3} (1+x^2)^{3/2} + C$

d. None of these

19. If $\int f(x) \sin x \cos x dx = \frac{1}{2(b^2 - a^2)} \log \{f(x)\} + C$, then $f(x)$ is equal to

a. $\frac{1}{a^2 \sin^2 x + b^2 \cos^2 x}$

b. $\frac{1}{a^2 \sin^2 x - b^2 \cos^2 x}$

c. $\frac{1}{a^2 \cos^2 x + b^2 \sin^2 x}$

d. $\frac{1}{a^2 \cos^2 x - b^2 \sin^2 x}$

20. Let $x^2 \neq n\pi - 1, n \in \mathbb{N}$, then

$\int x \sqrt{\frac{2 \sin(x^2 + 1) - \sin 2(x^2 + 1)}{2 \sin(x^2 + 1) + \sin 2(x^2 + 1)}} dx$ equals

a. $\log \left| \frac{1}{2} \sec(x^2 + 1) \right| + C$ b. $\log \left| \sec \left(\frac{1}{2}(x^2 + 1) \right) \right| + C$

c. $\frac{1}{2} \log \left| \sec(x^2 + 1) \right| + C$ d. None of these

21. If $\int \frac{xe^x}{\sqrt{1+e^x}} dx = f(x)\sqrt{1+e^x} - 2 \log g(x) + C$, then $g(x)$

is equal to

a. $x - 1$ b. $\frac{\sqrt{1+e^x} - 1}{\sqrt{1+e^x} + 1}$

c. $\frac{\sqrt{1+e^x} + 1}{\sqrt{1+e^x} - 1}$ d. $2(x - 2)$

22. $\int \frac{\sqrt{(x^2 + 1)} [\log(x^2 + 1) - 2 \log x]}{x^4} dx$ is equal to

a. $\frac{1}{3} \left(1 + \frac{1}{x^2}\right)^{1/2} \left[\log \left(1 + \frac{1}{x^2}\right) + \frac{2}{3} \right] + C$

b. $-\frac{1}{3} \left(1 + \frac{1}{x^2}\right)^{3/2} \left[\log \left(1 + \frac{1}{x^2}\right) - \frac{2}{3} \right] + C$

c. $\frac{2}{3} \left(1 + \frac{1}{x^2}\right)^{3/2} \left[\log \left(1 + \frac{1}{x^2}\right) + \frac{2}{3} \right] + C$

d. None of the above

23. If $\int \frac{e^{4x} - 1}{e^{2x}} \log \left(\frac{e^{2x} + 1}{e^{2x} - 1} \right) dx = \frac{t^2}{2} \log t - \frac{t^2}{4} - \frac{u^2}{2} \log u + \frac{u^2}{4} + C$, then

a. $t = u = e^x + e^{-x}$

b. $t = e^x - e^{-x}, u = e^x + e^{-x}$

c. $t = e^x + e^{-x}, u = e^x - e^{-x}$

d. None of the above

24. If an anti-derivative of $f(x)$ is e^x and that of $g(x)$ is $\cos x$, then $\int f(x) \cos x dx + \int g(x) e^x dx$ is equal to

a. $f(x) \cdot g(x) + C$

b. $f(x) + g(x) + C$

c. $e^x \cos x + C$

d. $f(x) - g(x) + C$

25. $\int \cos 2\theta \log \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) d\theta$ is equal to

a. $(\cos \theta - \sin \theta)^2 \log \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) + C$

b. $(\cos \theta + \sin \theta)^2 \log \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) + C$

c. $\frac{(\cos \theta - \sin \theta)^2}{2} \log \left(\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} \right) + C$

d. $\frac{1}{2} \sin 2\theta \log \tan \left(\frac{\pi}{4} + \theta \right) - \frac{1}{2} \log \sec 2\theta + C$

26. $\int \frac{x^2 + 1}{x(x^2 - 1)} dx$ is equal to

a. $\log \left(\frac{x^2 - 1}{x} \right) + C$ b. $-\log \left(\frac{x^2 - 1}{x} \right) + C$

c. $\log \left(\frac{x}{x^2 + 1} \right) + C$ d. $-\log \left(\frac{x}{x^2 + 1} \right) + C$

27. $\int \frac{dx}{1+x+x^2+x^3}$ is equal to
 a. $\log \sqrt{1+x} - \frac{1}{2} \log \sqrt{1+x^2} + \frac{1}{2} \tan^{-1} x + C$
 b. $\log \sqrt{1+x} - \log \sqrt{1+x^2} + \tan^{-1} x + C$
 c. $\log \sqrt{1+x^2} - \log \sqrt{1+x} + \frac{1}{2} \tan^{-1} x + C$
 d. $\log \sqrt{1+x} + \tan^{-1} x + \log \sqrt{1+x^2} + C$
28. If $\int \frac{dx}{x+x^7} = p(x)$, then $\int \frac{x^6}{x+x^7} dx$ is equal to
 a. $\log |x| - p(x) + C$ b. $\log |x| + p(x) + C$
 c. $x - p(x) + C$ d. $x + p(x) + C$
29. $\int \frac{(x+3)e^x}{(x+4)^2} dx$ is equal to
 a. $\frac{1}{(x+4)^2} + C$ b. $\frac{e^x}{(x+4)^2} + C$
 c. $\frac{e^x}{x+4} + C$ d. $\frac{e^x}{x+3} + C$
30. $\int e^{\tan^{-1} x} \left(\frac{1+x+x^2}{1+x^2} \right) dx$ is equal to
 a. $x e^{\tan^{-1} x} + C$ b. $x^2 e^{\tan^{-1} x} + C$
 c. $\frac{1}{x} e^{\tan^{-1} x} + C$ d. None of these
31. If $\int \left[\ln(\ln x) + \frac{1}{(\ln x)^2} \right] dx = x[f(x) - g(x)] + C$, then
 a. $f(x) = \ln(\ln x), g(x) = \frac{1}{\ln(x)}$ b. $f(x) = \ln x, g(x) = \frac{1}{\ln x}$
 c. $f(x) = \frac{1}{\ln x}, g(x) = \ln(\ln x)$ d. $f(x) = \ln x, g(x) = \ln(\ln x)$
32. $\int \frac{x^2-1}{x^4+x^2+1} dx$ is equal to
 a. $\frac{1}{2} \log \left(\frac{x^2+x+1}{x^2-x+1} \right) + C$ b. $\frac{1}{2} \log \left(\frac{x^2-x-1}{x^2+x+1} \right) + C$
 c. $\log \left(\frac{x^2-x+1}{x^2+x+1} \right) + C$ d. $\frac{1}{2} \log \left(\frac{x^2-x+1}{x^2+x+1} \right) + C$
33. If $\int \frac{\cos 8x + 1}{\cot 2x - \tan 2x} dx = A \cos 8x + k$, where k is an arbitrary constant, then A is equal to
 a. $-\frac{1}{16}$ b. $\frac{1}{16}$ c. $\frac{1}{8}$ d. $-\frac{1}{8}$

BITSAT Archives

1. If $\int f(x) \cos x dx = \frac{1}{2} f^2(x) + C$, then $f(x)$ can be [2013]
 a. x b. 1 c. $\cos x$ d. $\sin x$
2. If $\int \frac{4}{\sin^4 x + \cos^4 x} dx$ [2013]
 $= a \tan^{-1} \left(\frac{\tan x - \frac{1}{\tan x}}{b} \right) + C$, then find the values of a and b , respectively.
 a. $2\sqrt{2}, \sqrt{2}$ b. $\sqrt{2}, 2$ c. $\sqrt{3}, \sqrt{2}$ d. $\sqrt{2}, 4$
3. $\int \frac{2}{(e^x + e^{-x})^2} dx$ is equal to [2012]
 a. $-\frac{e^{-x}}{(e^x + e^{-x})} + C$ b. $-\frac{1}{(e^x + e^{-x})} + C$
 c. $\frac{1}{(e^x + 1)^2} + C$ d. $\frac{1}{(e^x - e^{-x})^2} + C$
4. $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$ is equal to [2011]
 a. $2 \tan^{-1}(\tan^2 x) + C$ b. $\tan^{-1}(x \tan^2 x) + C$
 c. $\tan^{-1}(\tan^2 x) + C$ d. None of these
5. If $\int \frac{1}{1+\sin x} dx = \tan \left(\frac{x}{2} + a \right) + b$, then [2010]
 a. $x e^{\tan^{-1} x} + C$ b. $x^2 e^{\tan^{-1} x} + C$
 c. $\frac{1}{x} e^{\tan^{-1} x} + C$ d. None of these
6. $\int \frac{d(\cos \theta)}{\sqrt{1 - \cos^2 \theta}}$ is equal to [2009]
 a. $\cos^{-1} \theta + C$ b. $\theta + C$
 c. $\sin^{-1} \theta + C$ d. $\sin^{-1}(\cos \theta) + C$
7. If $2f(x) = f'(x)$ and $f(0) = 3$, then $f(2)$ equals [2009]
 a. $4e^3$ b. $3e^4$
 c. $2e^3$ d. $3e^2$
8. If $\int \frac{dx}{x^4 + x^3} = \frac{a}{x^2} + \frac{b}{x} + \log \left| \frac{x}{x+1} \right| + C$, then [2009]
 a. $a = \frac{1}{2}, b = 1$ b. $a = -\frac{1}{2}, b = 1$
 c. $a = 1, b = -\frac{1}{2}$ d. $a = 1, b = 1$
9. $\int e^x \left(\frac{1-x}{1+x} \right)^2 dx$ is equal to [2009]
 a. $e^x \left(\frac{1-x}{1+x^2} \right) + C$ b. $e^x \left(\frac{x-1}{1+x^2} \right) + C$
 c. $e^x \left(\frac{1}{1+x^2} \right) + C$ d. $e^x \left(\frac{1}{1-x^2} \right) + C$

10. If $\int e^x(1+x) \cdot \sec^2(xe^x) dx = f(x) + C$, then $f(x)$ is equal to [2008]

a. $\cos(xe^x)$ b. $\sin(xe^x)$ c. $2 \tan^{-1}(x)$ d. $\tan(xe^x)$

11. If $\int e^x \left(\frac{1 - \sin x}{1 - \cos x} \right) dx = f(x) + C$, then $f(x)$ is equal to [2008]

a. $e^x \cot\left(\frac{x}{2}\right)$ b. $e^{-x} \cot\left(\frac{x}{2}\right)$ c. $-e^x \cot\left(\frac{x}{2}\right)$ d. $-e^{-x} \cot\left(\frac{x}{2}\right)$

12. $\int \frac{1 + \tan^2 x}{1 - \tan^2 x} dx$ is equal to [2007]

a. $\log\left(\frac{1 - \tan x}{1 + \tan x}\right) + C$ b. $\log\left(\frac{1 + \tan x}{1 - \tan x}\right) + C$
 c. $\frac{1}{2} \log\left(\frac{1 - \tan x}{1 + \tan x}\right) + C$ d. $\frac{1}{2} \log\left(\frac{1 + \tan x}{1 - \tan x}\right) + C$

13. $\int \frac{e^x(1 + \sin x)}{1 + \cos x} dx$ is equal to [2007]

a. $e^x \tan\left(\frac{x}{2}\right) + C$ b. $e^x \tan x + C$
 c. $e^x \left(\frac{1 + \sin x}{1 - \cos x} \right) + C$ d. $C - e^x \cot\left(\frac{x}{2}\right)$

14. $\int \frac{\sin(2x)}{1 + \cos^2 x} dx$ is equal to [2006]

a. $-\frac{1}{2} \log(1 + \cos^2 x) + C$
 b. $2 \log(1 + \cos^2 x) + C$
 c. $\frac{1}{2} \log(1 + \cos 2x) + C$
 d. $C - \log(1 + \cos^2 x)$

Answer with Solutions

Practice Exercise

1. (a) $\int \frac{f(x)}{\log \sin x} dx = \log \log \sin x$

On differentiating both sides w.r.t. x , we get

$$\frac{f(x)}{\log \sin x} = \frac{\cot x}{\log \sin x}$$

$$\therefore f(x) = \cot x$$

2. (a) We have, $\int f(x) dx = f(x)$

$$\Rightarrow \frac{d}{dx} \{f(x)\} = f(x) \Rightarrow \frac{1}{f(x)} d\{f(x)\} = dx$$

$$\Rightarrow \log \{f(x)\} = x + \log C \Rightarrow f(x) = Ce^x$$

$$\Rightarrow \{f(x)\}^2 = C^2 e^{2x}$$

$$\therefore \int \{f(x)\}^2 dx = \int C^2 e^{2x} = \frac{C^2 e^{2x}}{2} = \frac{1}{2} \{f(x)\}^2$$

3. (c) $\frac{dx}{dt} = f'''(t) \cos t - f''(t) \sin t + f'(t) \sin t + f(t) \cos t$

$$= [f'''(t) + f'(t)] \cos t$$

$$\frac{dy}{dt} = -f'''(t) \sin t - f''(t) \cos t + f''(t) \cos t - f'(t) \sin t$$

$$= -[f'''(t) + f'(t)] \sin t$$

$$\therefore \int \left[\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \right]^{1/2} dt = f''(t) + f(t) + C$$

4. (a) We have, $f(x) = \begin{vmatrix} 0 & x^2 - \sin x & \cos x - 2 \\ \sin x - x^2 & 0 & 1 - 2x \\ 2 - \cos x & 2x - 1 & 0 \end{vmatrix}$

$$\Rightarrow f(x) = \begin{vmatrix} 0 & \sin x - x^2 & 2 - \cos x \\ x^2 - \sin x & 0 & 2x - 1 \\ \cos x - 2 & 1 - 2x & 0 \end{vmatrix}$$

[interchanging rows and columns]

$$\Rightarrow f(x) = (-1)^3 \begin{vmatrix} 0 & x^2 - \sin x & \cos x - 2 \\ \sin x - x^2 & 0 & 1 - 2x \\ 2 - \cos x & 2x - 1 & 0 \end{vmatrix}$$

[taking (-1) common from each column]

$$\Rightarrow f(x) = -f(x)$$

$$\Rightarrow f(x) = 0$$

$$\therefore \int f(x) dx = 0$$

5. (b) $\int \frac{(\sin^4 x + \cos^4 x)(\sin^4 x - \cos^4 x)}{(\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x} dx$
 $= \int (\sin^4 x - \cos^4 x) dx = \int (\sin^2 x - \cos^2 x) dx$
 $= \int -\cos 2x dx$
 $= -\frac{\sin 2x}{2} + C$

6. (b) $\tan 2x = \tan [(x - \alpha) + (x + \alpha)]$

$$= \frac{\tan(x - \alpha) + \tan(x + \alpha)}{1 - \tan(x - \alpha) \cdot \tan(x + \alpha)}$$

$$\Rightarrow \tan(x - \alpha) \cdot \tan(x + \alpha) \cdot \tan 2x$$

$$= \tan 2x - \tan(x - \alpha) - \tan(x + \alpha)$$

$$\Rightarrow \int \tan(x - \alpha) \cdot \tan(x + \alpha) \cdot \tan 2x dx$$

$$= \int [\tan 2x - \tan(x - \alpha) - \tan(x + \alpha)] dx$$

$$= \frac{1}{2} \ln |\sec 2x| - \ln |\sec(x - \alpha)| - \ln |\sec(x + \alpha)| + C$$

$$= \ln \left| \frac{\sqrt{\sec 2x}}{\sec(x - \alpha) \cdot \sec(x + \alpha)} \right| + C$$

7. (c) Hint $I = \int \frac{x^2 + 1 + \cos^2 x - 1}{x^2 + 1} \operatorname{cosec}^2 x dx$

$$= \int \left(1 - \frac{\sin^2 x}{x^2 + 1} \right) \cdot \operatorname{cosec}^2 x dx = \int \left(\operatorname{cosec}^2 x - \frac{1}{x^2 + 1} \right) dx$$

$$8. (b) \text{ Hint } I = \int e^{3 \log x} (x^4 + 1)^{-1} dx = \int e^{\log x^3} \cdot \frac{dx}{x^4 + 1}$$

$$= \int \frac{x^3}{x^4 + 1} dx$$

$$\text{Put } (x^4 + 1) = t \Rightarrow 4x^3 dx = dt \Rightarrow x^3 dx = \frac{dt}{4}$$

$$9. (d) \text{ Let } I = \int \frac{2^x}{\sqrt{1-4^x}} dx$$

$$\text{Put } 2^x = t \Rightarrow 2^x \log 2 dx = dt$$

$$\therefore I = \frac{1}{\log 2} \sin^{-1} t + C = \frac{1}{\log 2} \sin^{-1} (2^x) + C$$

$$\text{Hence, } k = \frac{1}{\log 2}$$

$$10. (c) \text{ Hint Let } I = \int \frac{\sin \theta + \cos \theta}{\sqrt{1-(1-2 \sin \theta \cos \theta)}} d\theta$$

$$= \int \frac{\sin \theta + \cos \theta}{\sqrt{1-(\sin \theta - \cos \theta)^2}} d\theta$$

$$\text{Put } \sin \theta - \cos \theta = t$$

$$11. (a) \text{ Let } I = \int \frac{\ln(x+1) - \ln x}{x(x+1)} dx$$

$$= - \int [\ln(x+1) - \ln(x)] \cdot \frac{1}{-x(x+1)} dx$$

$$\text{Put } \ln(x+1) - \ln(x) = t \Rightarrow \frac{-1}{x(x+1)} dx = dt$$

$$\therefore I = - \int t dt = \frac{-t^2}{2} + C = \frac{-1}{2} [\ln(x+1) - \ln(x)]^2 + C$$

$$= \frac{-1}{2} \left[\ln \left(\frac{x+1}{x} \right) \right]^2 + C$$

$$12. (c) \text{ Hint } I = \int \frac{(x - x^5)^{1/5}}{x^6} dx = \int \frac{\left(\frac{1}{x^4} - 1 \right)^{1/5}}{x^5} dx$$

$$\text{Put } \frac{1}{x^4} - 1 = t \Rightarrow \frac{dx}{x^5} = \frac{-1}{4} dt$$

$$13. (b) \text{ Hint Let } I = \int \frac{x^3}{(1+x^2)^{1/3}} dx = \int \frac{x^2 \cdot x}{(1+x^2)^{1/3}} dx$$

$$\text{Put } 1+x^2 = t^3 \Rightarrow 2x dx = 3t^2 dt$$

$$\therefore I = \frac{3}{2} \int (t^3 - 1)t dt$$

$$14. (c) \text{ Hint Put } 1 + \sqrt{x} = z^2 \Rightarrow dx = 4z\sqrt{x} dz$$

$$\therefore I = \int \frac{z}{x} (4z\sqrt{x}) dz = 4 \int \frac{z^2}{z^2-1} dz = 4 \int \left(1 + \frac{1}{z^2-1} \right) dz$$

$$15. (a) \text{ Let } I = \int \frac{x + \sqrt[3]{x^2} + \sqrt[6]{x}}{x(1 + \sqrt[3]{x})} dx$$

$$\text{Put } x = t^6 \Rightarrow dx = 6t^5 dt$$

$$\therefore I = \int \frac{t^6 + t^4 + t}{t^6(1+t^2)} 6t^5 dt = 6 \int \left(t^3 + \frac{1}{t^2+1} \right) dt$$

$$= \frac{3}{2} x^{2/3} + 6 \tan^{-1} x^{1/6} + C$$

$$16. (d) \text{ Hint } I = \int \frac{\tan x}{\sqrt{\sin^4 x + \cos^4 x}} dx = \int \frac{\tan x \cdot \sec^2 x}{\sqrt{1 + \tan^4 x}} dx$$

$$\text{Put } \tan^2 x = t \Rightarrow 2 \tan x \sec^2 x dx = dt$$

$$17. (a) \int \frac{\sin x - \cos x}{\sin x - \cos x} e^{\sin x} \cdot \cos x dx$$

$$= \int e^{\sin x} \cdot \cos x dx = e^{\sin x} + C$$

$$18. (c) \int \sqrt{1+x^2} d(x^2) = \int (1+x^2)^{1/2} (2x) dx$$

$$= \frac{(1+x^2)^{3/2}}{3/2} + C = \frac{2}{3} (1+x^2)^{3/2} + C$$

$$19. (a) \int f(x) \sin x \cos x dx = \frac{1}{2(b^2 - a^2)} \log f(x) + C$$

$$\Rightarrow f(x) \sin x \cos x = \frac{1}{2(b^2 - a^2)} \cdot \frac{1}{f(x)} f'(x)$$

$$\Rightarrow 2(b^2 - a^2) \sin x \cos x = \frac{f'(x)}{\{f(x)\}^2}$$

$$\Rightarrow \int (2b^2 \sin x \cos x - 2a^2 \sin x \cos x) dx = \int \frac{f'(x)}{\{f(x)\}^2}$$

$$\Rightarrow -b^2 \cos^2 x - a^2 \sin^2 x = -\frac{1}{f(x)}$$

$$\Rightarrow f(x) = \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x}$$

$$20. (b) \int x \sqrt{\frac{2 \sin(x^2+1) - \sin 2(x^2+1)}{2 \sin(x^2+1) + \sin 2(x^2+1)}} dx$$

$$= \int x \sqrt{\frac{2 \sin(x^2+1) - 2 \sin(x^2+1) \cos(x^2+1)}{2 \sin(x^2+1) + 2 \sin(x^2+1) \cos(x^2+1)}} dx$$

$$= \int x \sqrt{\frac{1 - \cos(x^2+1)}{1 + \cos(x^2+1)}} dx$$

$$= \int x \sqrt{\frac{2 \sin^2 \left(\frac{x^2+1}{2} \right)}{2 \cos^2 \left(\frac{x^2+1}{2} \right)}} dx = \int x \cdot \tan \left(\frac{x^2+1}{2} \right) dx$$

$$= \int \tan z dz \quad \left[\text{put } \frac{x^2+1}{2} = z \Rightarrow x dx = dz \right]$$

$$= \log |\sec z| + C = \log \left| \sec \left(\frac{x^2+1}{2} \right) \right| + C$$

$$21. (b) \text{ Let } I = \int x \frac{e^x}{\sqrt{1+e^x}} dx$$

$$\text{Put } 1 + e^x = t^2 \Rightarrow e^x = 2t dt \Rightarrow x = \log(t^2 - 1)$$

$$\therefore I = 2 \int \log(t^2 - 1) dt$$

$$= 2 \left[t \log(t^2 - 1) - 2 \int \frac{t^2}{t^2 - 1} dt \right]$$

$$= 2 \left[t \log(t^2 - 1) - 2 \int 1 + \left(\frac{1}{t^2 - 1} \right) dt \right]$$

$$= 2 \left[t \log(t^2 - 1) - 2t - \log \left(\frac{t-1}{t+1} \right) + C \right]$$

$$= 2x \sqrt{1+e^x} - 4\sqrt{1+e^x} - 2 \log \left(\frac{\sqrt{1+e^x} - 1}{\sqrt{1+e^x} + 1} \right) + C$$

$$\text{Hence, } f(x) = 2x - 4 = 2(x - 2) \text{ and } g(x) = \frac{\sqrt{1+e^x} - 1}{\sqrt{1+e^x} + 1}$$

$$22. (b) \text{ Let } I = \int \frac{\sqrt{x^2+1} [\log(x^2+1) - 2 \log x]}{x^4} dx$$

$$= \int \sqrt{1+\frac{1}{x^2}} \log \left(1+\frac{1}{x^2} \right) \cdot \frac{1}{x^3} dx$$

$$\text{Put } 1+\frac{1}{x^2} = t \Rightarrow -\frac{2}{x^3} dx = dt$$

$$\therefore I = -\frac{1}{2} \int \sqrt{t} \log t dt = -\frac{1}{2} \left[\frac{2 \log t \cdot t^{3/2}}{3} - \int \frac{1}{t} \cdot \frac{t^{3/2}}{3/2} dt \right]$$

$$= -\frac{1}{2} \left[\frac{2}{3} (\log t) t^{3/2} - \frac{2}{3} \int t^{1/2} dt \right]$$

$$= -\frac{1}{3} (\log t) t^{3/2} + \frac{1}{3} \frac{t^{3/2}}{3/2} + C$$

$$= -\frac{1}{3} \left(1+\frac{1}{x^2} \right)^{3/2} \left[\log \left(1+\frac{1}{x^2} \right) - \frac{2}{3} \right] + C$$

$$23. (c) \int \frac{e^{4x}-1}{e^{2x}} \cdot \log \left(\frac{e^{2x}+1}{e^{2x}-1} \right) dx$$

$$= \int (e^{2x} - e^{-2x}) \log(e^x + e^{-x}) dx$$

$$- \int (e^{2x} - e^{-2x}) \log(e^x - e^{-x}) dx$$

$$= \int (e^x + e^{-x}) \log(e^x + e^{-x}) (e^x - e^{-x}) dx$$

$$- \int (e^x - e^{-x}) \log(e^x - e^{-x}) \cdot (e^x + e^{-x}) dx$$

$$= \int t \log t dt - \int u \log u du$$

$$\text{where, } t = e^x + e^{-x} \text{ and } u = e^x - e^{-x}$$

$$= \left[\log t \cdot \frac{t^2}{2} - \int \frac{1}{t} \cdot \frac{t^2}{2} dt \right] - \left[\log u \cdot \frac{u^2}{2} - \int \frac{1}{u} \cdot \frac{u^2}{2} du \right]$$

$$= \frac{t^2}{2} \log t - \frac{t^2}{4} - \frac{u^2}{2} \log u + \frac{u^2}{4} + C$$

$$24. (c) \int f(x) \cos x dx + \int g(x) e^x dx$$

$$= \frac{e^x}{2} (\cos x + \sin x) - \frac{e^x}{2} (\sin x - \cos x) + C$$

$$= \frac{e^x}{2} (2 \cos x) + C = e^x \cos x + C$$

$$25. (d) \text{ Since, } \log \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) = \log \tan \left(\frac{\pi}{4} + \theta \right)$$

$$\text{and } \int \sec \theta d\theta = \log \tan \left(\frac{\pi}{4} + \frac{\theta}{2} \right)$$

$$\text{or } \int \sec 2\theta d\theta = \frac{1}{2} \log \tan \left(\frac{\pi}{4} + \theta \right)$$

$$\Rightarrow 2 \sec 2\theta = \frac{d}{d\theta} \log \tan \left(\frac{\pi}{4} + \theta \right)$$

$$\therefore I = \frac{1}{2} \sin 2\theta \log \tan \left(\frac{\pi}{4} + \theta \right) - \int \tan 2\theta d\theta$$

$$= \frac{1}{2} \sin 2\theta \log \tan \left(\frac{\pi}{4} + \theta \right) - \frac{1}{2} \log \sec 2\theta + C$$

$$26. (a) \text{ Hint } \int \frac{x^2+1}{x(x^2-1)} dx = \int \frac{x^2+1}{x(x-1)(x+1)} dx$$

$$= \int \left(-\frac{1}{x} + \frac{1}{x-1} + \frac{1}{x+1} \right) dx$$

$$27. (a) \int \frac{dx}{(1+x)(1+x^2)} = \frac{1}{2} \int \frac{1}{1+x^2} dx + \frac{1}{2} \int \frac{1}{1+x} dx$$

$$- \frac{1}{2} \int \frac{x}{1+x^2} dx$$

$$= \frac{1}{2} \tan^{-1} x + \frac{1}{2} \log(1+x) - \frac{1}{4} \log(1+x^2) + C$$

$$= \frac{1}{2} \tan^{-1} x + \log \sqrt{1+x} - \frac{1}{2} \log \sqrt{1+x^2} + C$$

$$28. (a) \text{ Let } I = \int \frac{x^6}{x+x^7} dx = \int \frac{x^6}{x(1+x^6)} dx = \int \frac{(1+x^6)-1}{x(1+x^6)} dx$$

$$\Rightarrow I = \int \frac{dx}{x} - \int \frac{dx}{x+x^7} = \log|x| - p(x) + C$$

$$29. (c) \text{ Let } I = \int \frac{(x+3)e^x}{(x+4)^2} dx = \int \frac{(x+4-1)e^x}{(x+4)^2} dx$$

$$= \int e^x \left(\frac{1}{x+4} - \frac{1}{(x+4)^2} \right) dx = \frac{e^x}{x+4} + C$$

$$30. (a) \text{ Let } I = \int e^{\tan^{-1} x} \left(\frac{1+x+x^2}{1+x^2} \right) dx$$

$$\text{Put } \tan^{-1} x = t \Rightarrow \frac{dx}{1+x^2} = dt$$

$$\therefore I = \int e^t (\tan t + \sec^2 t) dt = e^t \tan t + C = x e^{\tan^{-1} x} + C$$

$$31. (a) \int \left\{ \ln(\ln x) + \frac{1}{(\ln x)^2} \right\} dx$$

$$\text{Put } \log x = t \Rightarrow \int \left(\ln t + \frac{1}{t^2} \right) \cdot e^t dt$$

$$= \int e^t \left[\left(\ln t - \frac{1}{t} \right) + \frac{d}{dt} \left(\ln t - \frac{1}{t} \right) \right] dt$$

$$= x \left(\ln(\ln x) - \frac{1}{\ln x} \right) + C$$

$$\therefore f(x) = \ln[\ln x] \text{ and } g(x) = \frac{1}{\ln(x)}$$

$$32. (d) \text{ Hint } I = \int \frac{\left(1 - \frac{1}{x^2} \right)}{\left(x + \frac{1}{x} \right)^2 - 1} dx$$

$$\text{Put } x + \frac{1}{x} = t \Rightarrow \left(1 - \frac{1}{x^2} \right) dx = dt$$

$$33. (a) \text{ LHS} = \int \frac{2 \cos^2 4x}{\cos^2 2x - \sin^2 2x} dx$$

$$= \int \frac{2 \cos^2 4x \cos 2x \sin 2x}{\cos 4x} dx = \int \cos 4x \sin 4x dx$$

$$= \frac{1}{2} \int \sin 8x dx = \frac{-1}{2} \cdot \frac{\cos 8x}{8} + k$$

$$\therefore A = -\frac{1}{16}$$

BITSAT Archives

1. (d) Given that, $\int f(x) \cos x \, dx = \frac{1}{2} f^2(x) + C$

On differentiating both sides w.r.t. x , we get

$$f(x) \cos x = \frac{1}{2} \cdot 2f(x) \cdot f'(x)$$

$$\Rightarrow \cos x = f'(x) \Rightarrow \cos x = \frac{d}{dx} \{f(x)\}$$

Now, integrating both sides, we get

$$\int \cos x \, dx = f(x)$$

$$\therefore f(x) = \sin x + C$$

2. (a) Let $I = \int \frac{4}{\sin^4 x + \cos^4 x} \, dx = \int \frac{4}{\cos^4 x (\tan^4 x + 1)} \, dx$
 $= \int \frac{4 \sec^4 x}{1 + \tan^4 x} \, dx = 4 \int \frac{\sec^2 x (1 + \tan^2 x)}{1 + \tan^4 x} \, dx$

Put $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$$\therefore I = 4 \int \frac{1+t^2}{1+t^4} \, dt = 4 \int \frac{1+1/t^2}{t^2+1/t^2} \, dt \Rightarrow I = 4 \int \frac{1+1/t^2}{\left(t-\frac{1}{t}\right)^2 + 2} \, dt$$

Now, put $t - \frac{1}{t} = z \Rightarrow \left(1 + \frac{1}{t^2}\right) dt = dz$

$$\therefore I = 4 \int \frac{dz}{z^2 + (\sqrt{2})^2} = \frac{4}{\sqrt{2}} \tan^{-1} \left(\frac{z}{\sqrt{2}} \right) + C$$

$$\Rightarrow I = 2\sqrt{2} \tan^{-1} \left(\frac{\tan x - \frac{1}{\tan x}}{\sqrt{2}} \right) + C$$

$$\therefore a = 2\sqrt{2} \text{ and } b = \sqrt{2}$$

3. (a) Hint $I = \int \frac{2}{(e^x + e^{-x})^2} \, dx$
 $= \int \frac{2}{e^{2x} + e^{-2x} + 2} \, dx = \int \frac{2e^{2x}}{e^{4x} + 2e^{2x} + 1} \, dx$

Put $e^{2x} = t \Rightarrow 2e^{2x} \, dx = dt$

4. (d) Hint $I = \int \frac{\sin 2x}{(\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x} \, dx$
 $= \int \frac{\sin 2x}{1 - \frac{1}{2} (\sin 2x)^2} \, dx = \int \frac{\sin 2x}{2(1 + \cos^2 2x)} \, dx$

Put $\cos 2x = t \Rightarrow -2 \sin 2x \, dx = dt$

5. (a) Hint $\int \frac{1}{1 + \sin x} \, dx = \int \frac{1}{1 + \cos \left(\frac{\pi}{2} - x \right)} \, dx$
 $= \frac{1}{2} \int \sec^2 \left(\frac{\pi}{4} - \frac{x}{2} \right) \, dx$

where $b = \text{constant}$

$$\therefore a = -\frac{\pi}{4}, b \in R$$

6. (d) $\int \frac{d(\cos \theta)}{\sqrt{1 - \cos^2 \theta}} = \sin^{-1}(\cos \theta) + C$

7. (b) We have, $2f(x) = f'(x) \Rightarrow \frac{f'(x)}{f(x)} = 2$

On integrating, we get

$$\log f(x) = 2x + C_1 \Rightarrow f(x) = e^{2x + C_1} = e^{C_1} e^{2x} = C e^{2x}$$

where, $C = e^{C_1}$

Putting $x = 0$, $f(0) = 3$, we get $C = 3$

$$\therefore f(x) = 3e^{2x} \Rightarrow f(2) = 3e^4$$

8. (b) Hint $\int \frac{dx}{x^4 + x^3} = \int \frac{(x+1) - x}{x^3(x+1)} \, dx$
 $= \int \left(\frac{1}{x^3} - \frac{1}{x^2(x+1)} \right) \, dx = \int \left(\frac{1}{x^3} - \frac{1}{x^2} + \frac{1}{x} - \frac{1}{x+1} \right) \, dx$

9. (c) $\int e^x \left(\frac{1-x}{1+x^2} \right)^2 \, dx = \int e^x \cdot \frac{1+x^2-2x}{(1+x^2)^2} \, dx$
 $= \int e^x \left[\frac{1}{1+x^2} - \frac{2x}{(1+x^2)^2} \right] \, dx$
 $= \int e^x \left[\frac{1}{1+x^2} + \frac{d}{dx} \left(\frac{1}{1+x^2} \right) \right] \, dx = \frac{e^x}{1+x^2} + C$

10. (d) Given that, $\int e^x (1+x) \cdot \sec^2(xe^x) \, dx = f(x) + C$

Put $xe^x = t$ in LHS $\Rightarrow (e^x + xe^x) \, dx = dt$

$$\therefore \text{LHS} = \int \sec^2 t \, dt = \tan t + C$$

$$\Rightarrow \tan(xe^x) + C = f(x) + C \Rightarrow f(x) = \tan(xe^x)$$

11. (c) $\int e^x \left(\frac{1 - \sin x}{1 - \cos x} \right) \, dx = \int e^x \left(\frac{1 - 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \sin^2 \frac{x}{2}} \right) \, dx$
 $= \frac{1}{2} \int e^x \left(\text{cosec}^2 \frac{x}{2} \right) \, dx - \int e^x \cot \frac{x}{2} \, dx$
 $= \frac{1}{2} \left[-e^x \cot \frac{x}{2} \cdot 2 + \int e^x \cot \frac{x}{2} \cdot 2 \, dx \right] - \int e^x \cot \frac{x}{2} \, dx + C$
 $= -e^x \cot \frac{x}{2} + C \Rightarrow f(x) = -e^x \cot \frac{x}{2}$

12. (d) Hint $I = \int \frac{1 + \tan^2 x}{1 - \tan^2 x} \, dx = \int \frac{\sec^2 x}{1 - \tan^2 x} \, dx$

Put $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

13. (a) Let $I = \int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) \, dx = \int e^x \left(\frac{1 + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 x/2} \right) \, dx$
 $= \int \frac{1}{2} e^x \sec^2 \frac{x}{2} \, dx + \int e^x \tan \frac{x}{2} \, dx$
 $= \frac{1}{2} \left[2e^x \tan \frac{x}{2} - \int 2e^x \tan \frac{x}{2} \, dx + \int e^x \tan \frac{x}{2} \, dx \right]$
 $= e^x \tan \frac{x}{2} - \int e^x \tan \frac{x}{2} \, dx + \int e^x \tan \frac{x}{2} \, dx + C$
 $= e^x \tan \frac{x}{2} + C$

14. (d) Hint $I = \int \frac{2 \sin x \cos x}{1 + \cos^2 x} \, dx$

Put $1 + \cos^2 x = t \Rightarrow -2 \cos x \sin x \, dx = dt$

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Definite Integrals and Its Applications

Definite Integral

Let $f(x)$ be the primitive or anti-derivative of a function $g(x)$ define on $[a, b]$, i.e. $\frac{d}{dx} f(x) = g(x)$, then definite integral of $g(x)$ over $[a, b]$ is denoted by $\int_a^b g(x) dx$ and defined as $[f(b) - f(a)]$.

$$\therefore \int_a^b g(x) dx = f(b) - f(a)$$

Here, a and b are called the limits of integration, where a is called the lower limit and b is called the upper limit.

NOTE

- In definite integral, there is no need to keep the constant of integration.
- $\int_a^b f(x) dx$ is read as the integral from a to b .

Fundamental Theorem of Calculus

The fundamental theorem of Calculus is a theorem that links the concept of the derivative of a function with the concept of the integral.

First Fundamental Theorem of Calculus

Let f be a continuous real valued function defined on a closed interval $[a, b]$. If F is the function defined, for all x in $[a, b]$, then $F(x) = \int_a^x f(t) dt$

NOTE

- F is continuous on $[a, b]$, differentiable on the open interval (a, b) and $F'(x) = f(x)$ for all x in (a, b) .
- The integral function of integrable function is continuous.

Second Fundamental Theorem of Calculus

Let f and g be real valued functions defined on a closed interval $[a, b]$ such that the derivative of g is f i.e. f and g are functions such that for all x in $[a, b]$,

$$f(x) = g'(x)$$

Then,

$$\int_a^b f(x) dx = g(b) - g(a)$$

Properties of Definite Integral

Here, we will study some fundamental properties of definite integrals, which are very useful in evaluating integrals.

Property 1

$$\int_a^b f(x) dx = \int_a^b f(t) dt = \int_a^b f(u) du$$

i.e. integration is independent of change in variable.

Property 2

$$\int_a^b f(x) dx = - \int_b^a f(x) dx$$

i.e. if the limits of definite integral are interchanged, then its value is changed by negative sign only.

Property 3

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx, \text{ where } a < c < b$$

The above property can be generalised into the following form

$$\int_a^b f(x) dx = \int_a^{c_1} f(x) dx + \int_{c_1}^{c_2} f(x) dx + \int_{c_2}^{c_3} f(x) dx + \dots + \int_{c_n}^b f(x) dx$$

where, $a < c_1 < c_2 < \dots < c_n < b$

Property 4

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx \text{ or } \int_0^a f(x) dx = \int_0^a f(a-x) dx$$

Property 5

$$\int_{-a}^a f(x) dx = \int_0^a f(x) dx + \int_0^a f(-x) dx$$

$$\text{or } \int_{-a}^a f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(-x) = f(x) \\ 0, & \text{if } f(-x) = -f(x) \end{cases}$$

NOTE A function $f(x)$ is said to be an even function, if $f(x) = f(-x)$ and a function $f(x)$ is said to be an odd function, if $f(-x) = -f(x)$

Property 6

$$\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a-x) dx$$

$$\Rightarrow \int_0^{2a} f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}$$

Property 7

If $f(x)$ is a periodic function with period T , then

$$\int_0^{nT} f(x) dx = n \int_0^T f(x) dx.$$

Property 8

If $f(x)$ is a periodic function with period T , then $\int_a^{a+T} f(x) dx$ will be independent of a , i.e. $\int_a^{a+T} f(x) dx = \int_0^T f(x) dx$.

Inequalities in Definite Integrals

1. If $f(x) \geq 0$ in the interval $[\alpha, \beta]$, then $\int_{\alpha}^{\beta} f(x) dx \geq 0$.
2. If f is continuous on $[\alpha, \beta]$, then $\left| \int_{\alpha}^{\beta} f(x) dx \right| \leq \int_{\alpha}^{\beta} |f(x)| dx$
3. If f is continuous on $[\alpha, \beta]$ and $l \leq f(x) \leq M, \forall x \in [\alpha, \beta]$, then $l(\beta - \alpha) \leq \int_{\alpha}^{\beta} f(x) dx \leq M(\beta - \alpha)$
4. If $f(x) \leq g(x)$ on $[\alpha, \beta]$, then $\int_{\alpha}^{\beta} f(x) dx \leq \int_{\alpha}^{\beta} g(x) dx$.
5. If $f_1(x), f(x)$ and $f_2(x)$ are continuous functions in $[\alpha, \beta]$ such that $f_1(x) \leq f(x) \leq f_2(x), \forall x \in [\alpha, \beta]$, then $\int_{\alpha}^{\beta} f_1(x) dx \leq \int_{\alpha}^{\beta} f(x) dx \leq \int_{\alpha}^{\beta} f_2(x) dx$

Leibnitz's Theorem

If functions $\phi(x)$ and $\psi(x)$ are defined on $[\alpha, \beta]$ and differentiable at a point (α, β) and $f(t)$ is continuous on $[\phi(\alpha), \phi(\beta)]$, then

$$\frac{d}{dx} \left[\int_{\phi(x)}^{\psi(x)} f(t) dt \right] = \left\{ \frac{d}{dt} \psi(x) \right\} f\{\psi(x)\} - \left\{ \frac{d}{dx} \{\phi(x)\} \right\} f\{\phi(x)\}$$

Integral as a Limit of Sum

If $f(x)$ is a continuous function on $[a, b]$, then

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h \sum_{r=1}^n f(a+rh), \text{ where } nh = b-a.$$

Since, $h = \frac{b-a}{n}$, where $n \rightarrow \infty, h \rightarrow 0$

$$\therefore \int_a^b f(x) dx = \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{r=1}^n f \left[a + r \left\{ \frac{(b-a)}{n} \right\} \right]$$

On putting $a = 0, b = 1$, we get

$$\int_0^1 f(x) dx = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n f \left(\frac{r}{n} \right)$$

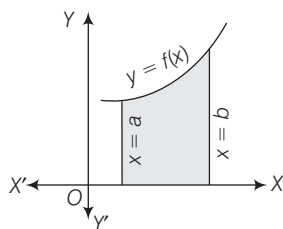
Method to Express the Infinite Series as Definite Integral

- (i) Express the given series in the form $\sum \frac{1}{n} f \left(\frac{r}{n} \right)$.
- (ii) Then, the limit is its sum when $n \rightarrow \infty$, i.e. $\lim_{n \rightarrow \infty} \sum \frac{1}{n} f \left(\frac{r}{n} \right)$.
- (iii) Replace $\frac{r}{n}$ by x and $\frac{1}{n}$ by dx and $\lim_{n \rightarrow \infty} \sum$ by the sign of $\int$.
- (iv) The lower and the upper limits of integration are limiting values of $\frac{r}{n}$ for the first and the last terms of r , respectively.

Determining Areas of the Regions Bounded by Simple Curves in Standard Form

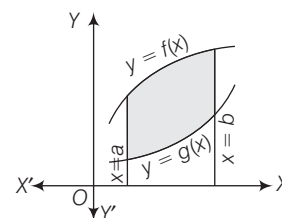
- (i) The area bounded by the continuous curve $y = f(x)$, X-axis and lines $x = a, x = b$ ($b > a$) is given by the formula

$$A = \int_a^b f(x) dx = \int_a^b y dx$$



- (ii) The area bounded by the straight lines $x = a, x = b$ ($a < b$) and the curves $y = f(x)$ and $y = g(x)$, provided $f(x) \geq g(x)$ and $a \leq x \leq b$, is given by

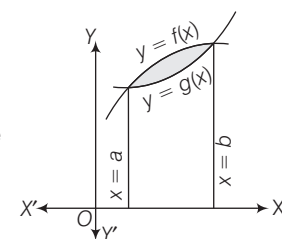
$$A = \int_a^b [f(x) - g(x)] dx$$



- (iii) If the curves $y = f(x)$ and $y = g(x)$ intersect, then the bounded area is given by

$$A = \int_a^b [f(x) - g(x)] dx$$

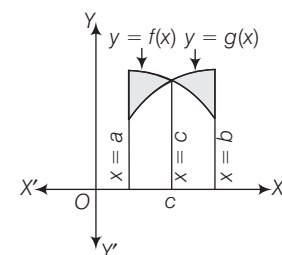
where, a and b are the roots of the equation $f(x) = g(x)$.



- (iv) The area bounded by $y = f(x)$ and $y = g(x)$, where $a \leq x \leq b$ intersecting at $x = c \in (a, b)$, is given by

$$A = \int_a^b |f(x) - g(x)| dx$$

$$\text{or } A = \int_a^c [f(x) - g(x)] dx + \int_c^b [g(x) - f(x)] dx$$

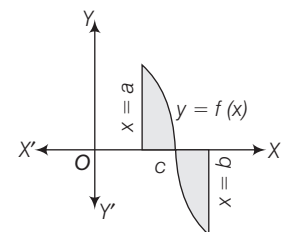


- (v) The area bounded by the curve $y = f(x)$ and the lines $x = a, x = b$ on crossing the X-axis at c (where, $b > a$), is given by

$$A = \left| \int_a^c f(x) dx \right| + \left| \int_c^b f(x) dx \right|$$

$$\text{or } A = \int_a^c f(x) dx - \int_c^b f(x) dx$$

$$\left[\because \int_a^c f(x) dx > 0 \text{ and } \int_c^b f(x) dx < 0 \right]$$



NOTE • If some part of the curve lies below X-axis, then its area is negative and since area cannot be negative, so we consider the magnitude only by taking its modulus.

Practice Exercise

- $\int_0^{\pi/2} \frac{\tan^2 x}{1 + \tan^2 x} dx$ is equal to
a. ∞ b. 0 c. $\frac{\pi}{4}$ d. $\frac{\pi}{2}$
- Let $\frac{d}{dx} f(x) = \frac{e^{\sin x}}{x}$, $x > 0$. If $\int_1^4 \frac{2e^{\sin x^2}}{x} dx = F(k) - F(1)$ then, one of the possible values of k is
a. 15 b. 16 c. 10 d. None of these
- If $I_{1(n)} = \int_0^{\pi/2} \frac{\sin(2n-1)x}{\sin x} dx$ and $I_{2(n)} = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin^2 x} dx$, $n \in N$, then
a. $I_{2(n+1)} - I_{2(n)} = I_{1(n)}$ b. $I_{2(n+1)} - I_{2(n)} = I_{1(n+1)}$
c. $I_{2(n+1)} + I_{1(n)} = I_{2(n)}$ d. $I_{2(n+1)} + I_{1(n+1)} = I_{2(n)}$
- $\int_0^{\pi/4} [\sqrt{\tan x} + \sqrt{\cot x}] dx$ is equal to
a. $\sqrt{2}\pi$ b. $\frac{\pi}{2}$ c. $\frac{\pi}{\sqrt{2}}$ d. 2π
- $\int_0^1 \sin\left(2 \tan^{-1} \sqrt{\frac{1+x}{1-x}}\right) dx$ is equal to
a. $\pi/6$ b. $\pi/4$ c. $\pi/2$ d. π
- $\int_0^{\pi/4} \frac{\sin^2 x \cdot \cos^2 x}{(\sin^3 x + \cos^3 x)^2} dx$ is equal to
a. $\frac{1}{6}$ b. $\frac{1}{12}$ c. $\frac{1}{4}$ d. $\frac{1}{24}$
- The value of $\int_0^{\pi/4} \frac{\sin x + \cos x}{\cos^2 x + \sin^4 x} dx$ is
a. $\frac{1}{2} \log\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)$ b. $\frac{1}{2\sqrt{3}} \log\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right) + \frac{\pi}{4}$
c. $\frac{1}{2} \log\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right) + \frac{\pi}{4}$ d. $\frac{1}{\sqrt{3}} \log\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right) + \frac{\pi}{4}$
- The value of $\int_0^{2\pi} e^x \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$ is
a. $\frac{\sqrt{2}}{5} (e^{2\pi} + 1)$ b. $-\frac{\sqrt{2}}{5} (e^{2\pi} + 1)$
c. $\frac{-\sqrt{2}}{5} (e^{2\pi} - 1)$ d. $\frac{\sqrt{2}}{5} (e^{2\pi} - 1)$
- If $\phi'(x) = \frac{\log \sin x}{x}$, $x \neq n\pi$, $n \in I$ and $\int_1^3 \frac{3 \log(\sin x^3)}{x} dx = \phi(k) - \phi(1)$, then the possible values of k is
a. 27 b. 18 c. 9 d. 36
- The value of $\int_3^5 e^{f[\phi(x)]} \cdot f'[\phi(x)] \cdot \phi'(x) dx$, where $\phi(3) = \phi(5)$, is
a. 1 b. 0 c. 3 d. 4
- $\int_0^{\pi/2} \frac{dx}{\sin\left(x - \frac{\pi}{3}\right) \sin\left(x - \frac{\pi}{6}\right)}$ is equal to
a. $4 \log \sqrt{3}$ b. $-4 \log \sqrt{3}$ c. $2 \log \sqrt{3}$ d. $-2 \log \sqrt{3}$
- $\int_{-2}^2 \frac{dx}{1+|x-1|}$ is equal to
a. $\log 2$ b. $2 \log 2$ c. $3 \log 2$ d. $4 \log 2$
- The value of $\int_{\pi/6}^{5\pi/6} \sqrt{4 - 4 \sin^2 t} dt$ is
a. 0 b. 2 c. 1 d. None of these
- The integral $\int_0^{1.5} [x^2] dx$, where $[x^2]$ denotes the greatest integer less than or equal to x , is equal to
a. $2 + \sqrt{2}$ b. $2 - \sqrt{2}$ c. 1.5 d. None
- Let a, b and c be non-zero real numbers such that $\int_0^3 (3ax^2 + 2bx + c) dx = \int_1^3 (3ax^2 + 2bx + c) dx$, then
a. $a + b + c = 3$ b. $a + b + c = 1$
c. $a + b + c = 0$ d. $a + b + c = 2$
- The value of $\sum_{k=1}^n \int_0^1 f(k-1+x) dx$ is
a. $\int_0^1 f(x) dx$ b. $\int_0^2 f(x) dx$
c. $\int_0^n f(x) dx$ d. $n \int_0^1 f(x) dx$
- If $\int_{-2}^x |2t| dt = f(x)$, then for any $x \geq 0$, $f(x)$ is equal to
a. $4 + x^2$ b. $4 - x^2$
c. $\frac{1}{2}(4 + x^2)$ d. $\frac{1}{4}(4 - x^2)$
- The correct evaluation of $\int_0^{\pi/2} \left| \sin\left(x - \frac{\pi}{4}\right) \right| dx$ is
a. $2 + \sqrt{2}$ b. $2 - \sqrt{2}$ c. $-2 + \sqrt{2}$ d. 0
- $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx$ is equal to
a. 1 b. 0 c. -1 d. None
- The value of $\int_{7\pi/4}^{7\pi/3} \sqrt{\tan^2 x} dx$ is
a. $\log 2\sqrt{2}$ b. $\log 2$
c. $2 \log 2$ d. $\log \sqrt{2}$

21. The value of $\int_1^3 |(x-1)(x-2)(x-3)| dx$ is
 a. $\frac{1}{3}$ b. $\frac{1}{2}$ c. $\frac{9}{4}$ d. $\frac{9}{5}$
22. $\int_0^1 |\sin 2\pi x| dx$ is equal to
 a. 0 b. $-\frac{1}{\pi}$
 c. $\frac{1}{\pi}$ d. $\frac{2}{\pi}$
23. The value of $\int_0^{\pi/2} \frac{\psi(x)}{\psi(x) + \psi\left(\frac{\pi}{2} - x\right)} dx$ is
 a. $\pi/4$ b. $\pi/2$ c. π d. None
24. The value of $\int_0^1 \cot^{-1}(1-x+x^2) dx$ is
 a. $\pi - \log 2$ b. $\pi/2 - \log 2$
 c. $\pi + \log 2$ d. $\pi/2 + \log 2$
25. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, a > 0$, is
 a. π b. $a\pi$ c. $\pi/2$ d. 2π
26. If f and g are continuous functions in $[0, 1]$ satisfying $f(x) = f(a-x)$ and $g(x) + g(a-x) = a$, then $\int_0^a f(x) \cdot g(x) dx$ is equal to
 a. $\frac{a}{2}$ b. $\frac{a}{2} \int_0^a f(x) dx$
 c. $\int_0^a f(x) dx$ d. $a \int_0^a f(x) dx$
27. The value of $\int_0^{\pi} e^{\cos^2 x} \cos^5 3x dx$ is
 a. 1 b. -1 c. 0 d. None
28. The value of $\int_0^{\pi} \frac{\sin 2kx}{\sin x} dx$, where $k \in I$, is
 a. $\frac{\pi}{2}$ b. π c. $\frac{3\pi}{2}$ d. 0
29. $\int_{-1/2}^{1/2} (\cos x) \left[\log \left(\frac{1-x}{1+x} \right) \right] dx$ is equal to
 a. 0 b. 1
 c. $e^{1/2}$ d. $2e^{1/2}$
30. The value of $\int_{-1}^1 \frac{\sin x - x^2}{3-|x|} dx$ is
 a. 0 b. $2 \int_0^1 \frac{\sin x}{3-|x|} dx$
 c. $2 \int_0^1 \frac{-x^2}{3-|x|} dx$ d. $2 \int_0^1 \frac{\sin x - x^2}{3-|x|} dx$
31. $\int_{-1}^1 \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1} dx$ is equal to
 a. $\ln 2$ b. $2 \ln 2$ c. $\frac{1}{2} \ln 2$ d. $4 \ln 2$
32. $\int_{-\pi/2}^{\pi/2} \sin^{10} x (6x^9 - 25x^7 + 4x^3 - 2x) dx$ is equal to
 a. π b. 0 c. 25 d. $\frac{\pi}{2}$
33. If f and g are two continuous functions, then the value of $\int_{-\pi/4}^{\pi/4} \{f(x) + f(-x)\} \{g(x) - g(-x)\} dx$ is
 a. $\frac{\pi}{4}$ b. 0 c. $-\frac{\pi}{4}$ d. π
34. If $P = \int_0^{3\pi} f(\cos^2 x) dx$ and $Q = \int_0^{\pi} f(\cos^2 x) dx$, then
 a. $P - Q = 0$ b. $P - 2Q = 0$
 c. $P - 3Q = 0$ d. $P - 5Q = 0$
35. If $f(2-x) = f(2+x)$ and $f(4-x) = f(4+x)$ for all x and $f(x)$ is a function for which $\int_0^2 f(x) dx = 5$, then $\int_0^{50} f(x) dx$ is equal to
 a. 125 b. $\int_{-4}^{46} f(x) dx$ c. $\int_1^{51} f(x) dx$ d. $\int_2^{52} f(x) dx$
36. If $I_1 = \int_1^2 \frac{dx}{\sqrt{1+x^2}}$ and $I_2 = \int_1^2 \frac{dx}{x}$, then
 a. $I_1 > I_2$ b. $I_2 > I_1$ c. $I_1 = I_2$ d. $I_1 > 2I_2$
37. If $\phi(x) = \int_{1/x}^{\sqrt{x}} \sin(t^2) dt$, then $\phi'(1)$ is equal to
 a. $\sin 1$ b. $2 \sin 1$ c. $3/2 \sin 1$ d. None
38. Let $f(x) = \int_1^x \sqrt{2-t^2} dt$, then the real roots of the equation $x^2 - f'(x) = 0$ are
 a. ± 1 b. $\pm 1/\sqrt{2}$ c. $\pm 1/2$ d. 0 and 1
39. If $f(x) = \frac{1}{x^2} \int_4^x [4t^2 - 2f'(t)] dt$, then $f'(4)$ is equal to
 a. 32 b. $\frac{32}{3}$ c. $\frac{32}{9}$ d. None
40. If $f(x)$ is differentiable and $\int_0^{t^2} x f(x) dx = \frac{2}{5} t^5$, then $f\left(\frac{4}{25}\right)$ is equal to
 a. $\frac{2}{5}$ b. $-\frac{5}{2}$ c. 1 d. $\frac{5}{2}$
41. $\lim_{n \rightarrow \infty} \frac{(r)^{1/n}}{n}$ equals
 a. e b. e^{-1} c. 1 d. None
42. The value of $\lim_{n \rightarrow \infty} \left(\frac{n}{(n+1)\sqrt{2n+1}} + \frac{n}{(n+2)\sqrt{2(2n+2)}} + \frac{n}{(n+3)\sqrt{3(2n+3)}} + \dots + \frac{1}{2n\sqrt{3}} \right)$ is
 a. $\frac{\pi}{3}$ b. $\frac{\pi}{2}$
 c. $\frac{\pi}{4}$ d. None of these

43. $\lim_{n \rightarrow \infty} \left[\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{n+n} \right]$ is equal to
 a. $3 \log 2$ b. $\log 2$ c. $2 \log 2$ d. $4 \log 2$
44. $\lim_{n \rightarrow \infty} \left[\frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + \dots + \frac{1}{8n} \right]$
 is equal to
 a. $\frac{3}{8}$ b. $\frac{1}{4}$ c. $\frac{1}{8}$ d. $\frac{1}{2}$
45. The value of $\lim_{n \rightarrow 0} \left[\frac{\sqrt{n}}{(3+4\sqrt{n})^2} + \frac{\sqrt{n}}{\sqrt{2}(3\sqrt{2}+4\sqrt{n})^2} + \frac{\sqrt{n}}{\sqrt{3}(3\sqrt{3}+4\sqrt{n})^2} + \dots + \frac{1}{49n} \right]$ is
 a. 1 b. $\frac{1}{14}$ c. $\frac{1}{7}$ d. $\frac{2}{7}$
46. The area of the region lying between the line $x - y + 2 = 0$ and the curve $x = \sqrt{y}$, is
 a. 9 b. $\frac{9}{2}$ c. $\frac{10}{3}$ d. None
47. The area bounded by the curve $y = x \sin x$ and X -axis between $x = 0$ and $x = 2\pi$, is
 a. 2π b. 3π c. 4π d. None
48. The area bounded by the parabola $y^2 = x$, straight line $y = 4$ and Y -axis, is
 a. $\frac{16}{3}$ sq units b. $\frac{64}{3}$ sq units
 c. $7\sqrt{2}$ sq units d. None of these
49. The area enclosed by the curves $y = x^3$ and $y = \sqrt{x}$ is
 a. $\frac{5}{3}$ sq units b. $\frac{5}{4}$ sq units
 c. $\frac{5}{12}$ sq unit d. $\frac{12}{5}$ sq units
50. The area of smaller segment cut-off from the circle $x^2 + y^2 = 9$ by $x = 1$ is
 a. $\frac{1}{2}(9 \sec^{-1} 3 - \sqrt{x})$ b. $9 \sec^{-1} 3 - \sqrt{8}$
 c. $\sqrt{8} - 9 \sec^{-1} 3$ d. None of these
51. Sketch the graph of $y = |x + 3|$ and the area bounded by $\int_{-6}^0 (x+3) dx$ is
 a. 9 sq units b. $\frac{9}{2}$ sq units
 c. 3 sq units d. 11 sq units
52. If $y = f(x)$ makes positive intercepts of 2 and 1 unit on x and y -coordinates axes and encloses an area of $\frac{3}{4}$ sq unit with the axes, then $\int_0^2 x f'(x) dx$ equals
 a. $\frac{3}{2}$ b. 1 c. $\frac{5}{4}$ d. $-\frac{3}{4}$
53. The area in the first quadrant between $x^2 + y^2 = \pi^2$ and $y = \sin x$, is
 a. $\frac{\pi^3 - 8}{4}$ b. $\frac{\pi^3}{4}$ c. $\frac{\pi^3 - 16}{4}$ d. $\frac{\pi^3 - 8}{2}$
54. The area of the region $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$, is
 a. $\frac{\pi^2}{5}$ b. $\frac{\pi^2}{2}$
 c. $\frac{\pi^2}{3}$ d. $\frac{\pi}{4} - \frac{1}{2}$
55. The area bounded by the curve $y = \ln(x)$ and the lines $y = 0$, $y = \ln(3)$ and $x = 0$, is equal to
 a. 3 b. $3 \ln(3) - 2$ c. $3 \ln(3) + 2$ d. 2
56. The area under the curve $y = |\cos x - \sin x|$, $0 \leq x \leq \frac{\pi}{2}$ and above X -axis, is
 a. $2\sqrt{2}$ b. $2\sqrt{2} - 2$ c. $2\sqrt{2} + 2$ d. 0
57. The area of the region enclosed by the curves $y = x$, $x = e$, $y = \frac{1}{x}$ and the positive X -axis, is
 a. 1 sq unit b. $\frac{3}{2}$ sq units
 c. $\frac{5}{2}$ sq units d. $\frac{1}{2}$ sq unit
58. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$, is
 a. $(4\sqrt{2} - 2)$ sq units b. $(4\sqrt{2} + 2)$ sq units
 c. $(4\sqrt{2} - 1)$ sq units d. $(4\sqrt{2} + 1)$ sq units
59. The area bounded by the curves $y^2 = 4x$ and $x^2 = 4y$, is
 a. 0 b. $\frac{32}{3}$ sq units
 c. $\frac{16}{3}$ sq units d. $\frac{8}{3}$ sq units
60. The area bounded by $y = xe^{|x|}$ and lines $|x| = 1$, $y = 0$, is
 a. 1 sq unit b. 2 sq units
 c. 3 sq units d. None of these
61. The area bounded by the curves $y^2 = x^3$ and $|y| = 2x$ is
 a. $\frac{5}{16}$ sq unit b. $\frac{16}{5}$ sq units
 c. $\frac{3}{8}$ sq unit d. None of these
62. The area bounded by the curves $y = x|x|$, X -axis and the coordinates $x = -1$, $x = 1$, is given by
 a. 0 b. $\frac{1}{3}$
 c. $\frac{2}{3}$ d. $\frac{4}{3}$

BITSAT Archives

1. The value of $x > 1$ satisfying the equation $\int_1^x t \log t dt = \frac{1}{4}$, is [2014]
 a. $\sqrt{e}$ b. $e^{3/2}$ c. e^2 d. $2e - 1$
2. If $\int_0^{25} e^{x-[x]} dx = k(e - 1)$, then the value of k is [2014]
 a. 12 b. 25 c. 23 d. 24
3. The area bounded by the curves $y = -\sqrt{-x}$ and $x = -\sqrt{-y}$, where $x, y \leq 0$, is [2014]
 a. $1/3$ b. $1/4$ c. $1/5$ d. $1/2$
4. The value of $\int_0^{10\pi} ([\sec^{-1} x] + [\cot^{-1} x]) dx$, where $[\]$ denotes the greatest integer function, is [2013]
 a. $10\pi - \tan^{-1} x$ b. $8\pi - \sec 1$
 c. $10\pi - \sec 1$ d. $10\pi + \sec 1$
5. The area of the figure bounded by two branches of the curve $(y - x)^2 = x^3$ and straight line $x = 1$, is [2013]
 a. $\frac{4}{5}$ sq unit b. $\frac{4}{7}$ sq unit c. $\frac{4}{9}$ sq unit d. $\frac{4}{11}$ sq unit
6. $\int_0^{\pi/2} \frac{\sin^n \theta}{\sin^n \theta + \cos^n \theta} d\theta$ is equal to [2012]
 a. 1 b. 0 c. $\frac{\pi}{2}$ d. $\frac{\pi}{4}$
7. $\int_0^{\pi} \cos^{101} x dx$ is equal to [2012]
 a. $\frac{\pi}{4}$ b. $\frac{1}{102}$ c. $\left(\frac{\pi}{3}\right)^{101}$ d. 0
8. $\lim_{n \rightarrow \infty} \left[\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{6n} \right]$ equals [2012]
 a. $\log 2$ b. $\log(1 + \sqrt{5})$ c. $\log 6$ d. 0
9. If $\frac{d}{dx} \{\phi(x)\} = f(x)$, then $\int_1^2 f(x) dx$ is equal to [2011]
 a. $f(1) - f(2)$ b. $\phi(1) - \phi(2)$ c. $f(2) - f(1)$ d. $\phi(2) - \phi(1)$
10. $\int_0^2 |1 - x| dx$ is equal to [2011]
 a. 0 b. 1 c. $\frac{3}{2}$ d. $\frac{1}{2}$
11. The area bounded by X-axis and the curve $y = \sin x$ and $x = 0, x = \pi$, is [2011]
 a. 1 sq unit b. 2 sq units c. 0 d. 4 sq units
12. $\int_0^1 |5x - 3| dx$ is equal to [2010]
 a. $\frac{10}{13}$ b. $\frac{31}{10}$
 c. $\frac{13}{10}$ d. None of these
13. The area of the region bounded by $y = |x - 1|$ and $y = 1$ is [2010]
 a. 1 b. 2 c. $1/2$ d. None
14. $\int_{-1}^2 |x|^3 dx$ is equal to [2009]
 a. $\frac{5}{4}$ b. $\frac{17}{4}$ c. $\frac{15}{4}$ d. $\frac{4}{5}$
15. $\int_{-\pi/2}^{\pi/2} \sin|x| dx$ is equal to [2008]
 a. 0 b. 1 c. 2 d. π
16. The area (in sq units) of the region bounded by the curve $2x = y^2 - 1$ and $x = 0$ is [2008]
 a. $1/3$ b. $2/3$ c. 1 d. 2
17. $\int_0^8 |x - 5| dx$ is equal to [2007]
 a. 17 b. 9 c. 12 d. 18
18. If $I_1 = \int_0^1 2^x dx, I_2 = \int_0^1 2^{x^3} dx, I_3 = \int_1^2 2^{x^2} dx$ and $I_4 = \int_1^2 2^{x^3} dx$, then [2007]
 a. $I_3 > I_4$ b. $I_3 = I_4$ c. $I_1 > I_2$ d. $I_2 > I_1$
19. The area of the region satisfying $x \leq 2, y \leq |x|$ and $x \geq 0$ is [2007]
 a. 4 sq units b. 1 sq unit c. 2 sq units d. None
20. $\int_0^{\pi/4} \log(1 + \tan x) dx$ is equal to [2006]
 a. $\frac{\pi}{8} \log_e 2$ b. $\frac{\pi}{4} \log_2 e$ c. $\frac{\pi}{4} \log_e 2$ d. $\frac{\pi}{8} \log_e \left(\frac{1}{2}\right)$
21. The area bounded by the parabola $y^2 = 4ax$ and the lines $x = a$ and $x = 4a$, is [2006]
 a. $\frac{35a^2}{3}$ b. $\frac{4a^2}{3}$ c. $\frac{7a^2}{3}$ d. $\frac{56a^2}{3}$
22. If $\int_{\log 2}^x \frac{du}{(e^u - 1)^{1/2}} = \frac{\pi}{6}$, then e^x is equal to [2005]
 a. 1 b. 2 c. 4 d. -1
23. $\int_0^{2\pi} \left\{ |\sin x| - \left| \frac{1}{2} \sin x \right| \right\} dx$ equals [2005]
 a. n b. $2n$ c. $-2n$ d. None
24. The value of $\left[\int_0^{\sin^2 \theta} \sin^{-1} \sqrt{\phi} d\phi + \int_0^{\cos^2 \theta} \cos^{-1} \sqrt{\phi} d\phi \right]$ is [2005]
 a. π b. $\pi/2$ c. $\pi/3$ d. $\pi/4$
25. The area bounded by the curves $y = x^2$ and $y = 2 - x^2$ is [2005]
 a. $8/3$ sq units b. $3/8$ sq unit
 c. $3/2$ sq units d. None of these

Answer with Solutions

Practice Exercise

$$\begin{aligned} 1. (c) \text{ Let } I &= \int_0^{\pi/2} \frac{\tan^2 x}{1 + \tan^2 x} dx = \int_0^{\pi/2} \tan^2 x \cos^2 x dx \\ &= \int_0^{\pi/2} \sin^2 x dx = \int_0^{\pi/2} \frac{1 - \cos 2x}{2} dx \\ &= \left[\frac{1}{2}x - \frac{\sin 2x}{4} \right]_0^{\pi/2} = \frac{\pi}{4} \end{aligned}$$

$$\begin{aligned} 2. (b) \text{ Let } I &= \int_1^4 \frac{2e^{\sin x^2}}{x} dx = F(k) - F(1) = [F(x)]_1^k \\ \text{Put } x^2 &= t \Rightarrow 2x dx = dt \\ \therefore I &= \int_1^{16} \frac{e^{\sin t}}{t} dt = [F(t)]_1^{16} \Rightarrow k = 16 \end{aligned}$$

$$\begin{aligned} 3. (b) I_{2(n)} - I_{2(n-1)} &= \int_0^{\pi/2} \frac{[\sin^2 nx - \sin^2(n-1)x]}{\sin^2 x} dx \\ &= \int_0^{\pi/2} \frac{\sin(2n-1)x \cdot \sin x}{\sin^2 x} dx \\ &= \int_0^{\pi/2} \frac{\sin(2n-1)x}{\sin x} dx = I_{1(n)} \end{aligned}$$

Put $n = n + 1$, we get

$$\begin{aligned} I_{2(n+1)} - I_{2(n)} &= I_{1(n+1)} \\ 4. (c) \text{ Hint } I &= \int_0^{\pi/4} [\sqrt{\tan x} + \sqrt{\cot x}] dx \\ &= \int_0^{\pi/4} \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} dx \\ &= \sqrt{2} \int_0^{\pi/4} \frac{\sin x + \cos x}{\sqrt{1 - (\sin x - \cos x)^2}} dx \end{aligned}$$

Put $\sin x - \cos x = t \Rightarrow (\cos x + \sin x) dx = dt$

$$5. (b) \text{ Let } I = \int_0^1 \sin \left[2 \tan^{-1} \sqrt{\frac{1+x}{1-x}} \right] dx$$

Put $x = \cos \theta$

$$\begin{aligned} \Rightarrow \sin \left[2 \tan^{-1} \sqrt{\frac{1+\cos \theta}{1-\cos \theta}} \right] \\ = \sin \left[2 \tan^{-1} \left(\cot \frac{\theta}{2} \right) \right] &= \sin \left[2 \tan^{-1} \left\{ \tan \left(\frac{\pi}{2} - \frac{\theta}{2} \right) \right\} \right] \\ = \sin \left[2 \left(\frac{\pi}{2} - \frac{\theta}{2} \right) \right] &= \sin(\pi - \theta) = \sin \theta = \sqrt{1 - \cos^2 \theta} = \sqrt{1 - x^2} \\ \therefore I &= \int_0^1 \sqrt{1 - x^2} dx = \left[\frac{1}{2} x \sqrt{1 - x^2} \right]_0^1 + \frac{1}{2} [\sin^{-1} x]_0^1 = \frac{\pi}{4} \end{aligned}$$

$$6. (a) \text{ Let } I = \int_0^{\pi/4} \frac{\sin^2 x \cdot \cos^2 x}{(\sin^3 x + \cos^3 x)^2} dx = \int_0^{\pi/4} \frac{\tan^2 x \cdot \sec^2 x}{(1 + \tan^3 x)^2} dx$$

Put $1 + \tan^3 x = t$

$$\Rightarrow 3 \tan^2 x \cdot \sec^2 x dx = dt \Rightarrow \tan^2 x \cdot \sec^2 x dx = \frac{dt}{3}$$

When $x = 0$, then $t = 1$ and when $x = \frac{\pi}{4}$, then $t = 2$

$$\therefore I = \int_1^2 \frac{dt}{3t^2} = -\frac{1}{3} \left(\frac{1}{2} - 1 \right) = \frac{1}{6}$$

$$\begin{aligned} 7. (d) \text{ Hint } I &= \int_0^{\pi/4} \frac{\sin x + \cos x}{\cos^2 x + \sin^4 x} dx \\ &= \int_0^{\pi/4} \frac{4(\sin x + \cos x)}{2(1 + \cos 2x) + (1 - \cos 2x)^2} dx \\ &= 4 \int_0^{\pi/4} \frac{\sin x + \cos x}{3 + \cos^2 2x} dx \\ &= 4 \int_0^{\pi/4} \frac{(\sin x + \cos x)}{4 - \sin^2 2x} dx \end{aligned}$$

Put $\sin x - \cos x = t$

$\Rightarrow (\cos x + \sin x) dx = dt$ and $1 - \sin 2x = t^2$

When $x = 0$, then $t = -1$ and when $x = \frac{\pi}{4}$, then $t = 0$

$$\begin{aligned} \therefore I &= 4 \int_{-1}^0 \frac{dt}{4 - (1 - t^2)^2} = 4 \int_{-1}^0 \frac{dt}{(3 - t^2)(1 + t^2)} \\ &= 4 \int_{-1}^0 \frac{dt}{(3 - t^2)(1 + t^2)} \\ &= 4 \int_{-1}^0 \left[\frac{1}{4(3 - t^2)} + \frac{1}{4(1 + t^2)} \right] dt \end{aligned}$$

$$\begin{aligned} 8. (b) \text{ Let } I &= \int_0^{2\pi} e^x \sin \left(\frac{\pi}{4} + \frac{x}{2} \right) dx \\ &= \left[\sin \left(\frac{\pi}{4} + \frac{x}{2} \right) e^x \right]_0^{2\pi} - \int_0^{2\pi} \frac{e^x}{2} \cos \left(\frac{\pi}{4} + \frac{x}{2} \right) dx \\ &= \sin \left(\pi + \frac{\pi}{4} \right) e^{2\pi} - \sin \frac{\pi}{4} - \frac{1}{2} \left[\left\{ e^x \cos \left(\frac{\pi}{4} + \frac{x}{2} \right) \right\}_0^{2\pi} \right. \\ &\quad \left. - \frac{1}{2} \int_0^{2\pi} \sin \left(\frac{\pi}{4} + \frac{x}{2} \right) e^x dx \right] \end{aligned}$$

After substituting the values, we get

$$I = \frac{-\sqrt{2}}{5} (e^{2\pi} + 1)$$

$$\begin{aligned} 9. (a) \text{ Let } I &= \int_1^3 \frac{3 \log(\sin x^3)}{x} dx = \int_1^{27} \frac{\log \sin t}{t} dt \\ &\quad [\text{put } x^3 = t \Rightarrow 3x^2 dx = dt] \\ \Rightarrow \int_1^{27} \phi'(t) dt &= [\phi(t)]_1^{27} = \phi(27) - \phi(1) \end{aligned}$$

$$\therefore k = 27$$

$$10. (b) \text{ Let } I = \int_3^5 e^{f[\phi(x)]} f'[\phi(x)] \cdot \phi'(x) dx$$

Put $\phi(x) = t \Rightarrow \phi'(x) dx = dt$

$$\therefore I = [e^{f(t)}]_{\phi(3)}^{\phi(5)} = e^{f[\phi(5)]} - e^{f[\phi(3)]} = 0 \quad [\because \phi(5) = \phi(3)]$$

$$\begin{aligned} 11. (b) \text{ Let } I &= \int_0^{\pi/2} \frac{dx}{\sin \left(x - \frac{\pi}{3} \right) \sin \left(x - \frac{\pi}{6} \right)} \\ &= 2 \int_0^{\pi/2} \frac{\sin \left(\frac{\pi}{3} - \frac{\pi}{6} \right)}{\sin \left(x - \frac{\pi}{3} \right) \sin \left(x - \frac{\pi}{6} \right)} dx \end{aligned}$$

$$\begin{aligned}
 &= 2 \int_0^{\pi/2} \frac{\sin \left[\left(x - \frac{\pi}{6} \right) - \left(x - \frac{\pi}{3} \right) \right]}{\sin \left(x - \frac{\pi}{3} \right) \sin \left(x - \frac{\pi}{6} \right)} dx \\
 &= 2 \left[\log \frac{\sin \left(x - \frac{\pi}{3} \right)}{\sin \left(x - \frac{\pi}{6} \right)} \right]_0^{\pi/2} = -4 \log \sqrt{3}
 \end{aligned}$$

12. (c) Hint $f(x) = \frac{1}{1 + |x - 1|}$

$$f(x) = \begin{cases} \frac{1}{1 + x - 1} = \frac{1}{x}, & \text{if } x > 1 \\ \frac{1}{1 - x + 1} = \frac{1}{2 - x}, & \text{if } x < 1 \end{cases}$$

Therefore, $\int_{-2}^2 \frac{dx}{1 + |x - 1|} = \int_{-2}^1 \frac{1}{2 - x} dx + \int_1^2 \frac{1}{x} dx$

13. (b) Hint $I = \int_{\pi/6}^{5\pi/6} \sqrt{4 - 4 \sin^2 t} dt = 2 \int_{\pi/6}^{5\pi/6} |\cos t| dt$

$$\begin{aligned}
 &= 2 \left[\int_{\pi/6}^{\pi/2} |\cos t| dt + \int_{\pi/2}^{5\pi/6} |\cos t| dt \right] \\
 &= 2 \left[\int_{\pi/6}^{\pi/2} \cos t dt - \int_{\pi/2}^{5\pi/6} \cos t dt \right]
 \end{aligned}$$

14. (b) We have, $\int_0^{1.5} [x^2] dx$

The value of x^2 at $x = 0$ and $x = 1.5$ are 0 and 2.25.

Hence, $x^2 = 0, 1, 2$

$$\Rightarrow 0 < x < 1 \Rightarrow 0 < x^2 < 1 \Rightarrow [x^2] = 0$$

$$\text{Also, } 1 < x < \sqrt{2} \Rightarrow 1 < x^2 < 2 \Rightarrow [x^2] = 1$$

$$\text{and } \sqrt{2} < x^2 < 1.5 \Rightarrow 2 < x^2 < 2.25 \Rightarrow [x^2] = 2$$

$$\begin{aligned}
 \therefore \int_0^{1.5} [x^2] dx &= \int_0^1 [x^2] dx + \int_1^{\sqrt{2}} [x^2] dx + \int_{\sqrt{2}}^{1.5} [x^2] dx \\
 &= \int_0^1 0 dx + \int_1^{\sqrt{2}} 1 dx + \int_{\sqrt{2}}^{1.5} 2 dx \\
 &= 0 + [x]_1^{\sqrt{2}} + 2[x]_{\sqrt{2}}^{1.5} \\
 &= (\sqrt{2} - 1) + 2(1.5 - \sqrt{2}) \\
 &= \sqrt{2} - 1 + 3 - 2\sqrt{2} \\
 &= 2 - \sqrt{2}
 \end{aligned}$$

15. (c) $\int_0^3 (3ax^2 + 2bx + c) dx = \int_1^3 (3ax^2 + 2bx + c) dx$

$$\begin{aligned}
 &\Rightarrow \int_0^1 (3ax^2 + 2bx + c) dx + \int_1^3 (3ax^2 + 2bx + c) dx \\
 &= \int_1^3 (3ax^2 + 2bx + c) dx \\
 &\Rightarrow \int_0^1 (3ax^2 + 2bx + c) dx = 0 \Rightarrow \left[\frac{3ax^3}{3} + \frac{2bx^2}{2} + cx \right]_0^1 = 0 \\
 &\therefore a + b + c = 0
 \end{aligned}$$

16. (c) Let $I = \int_0^1 f(k - 1 + x) dx$

$$\Rightarrow I = \int_{k-1}^k f(t) dt, \text{ where, } t = k - 1 + x$$

$$\Rightarrow I = \int_{k-1}^k f(x) dx$$

$$\begin{aligned}
 \therefore \sum_{k=1}^n \int_{k-1}^k f(x) dx &= \int_0^1 f(x) dx + \int_1^2 f(x) dx \\
 &\quad + \dots + \int_{n-1}^n f(x) dx \\
 &= \int_0^n f(x) dx
 \end{aligned}$$

17. (a) $\int_{-2}^x |2t| dt = f(x) = \int_{-2}^0 |2t| dt + \int_0^x 2t dt$

$$\begin{aligned}
 &= -2 \left[\frac{t^2}{2} \right]_{-2}^0 + 2 \left[\frac{t^2}{2} \right]_0^x \\
 &= -2[0 - 2] + 2 \left[\frac{x^2}{2} - 0 \right] \\
 &= 4 + x^2
 \end{aligned}$$

18. (b) Hint $I = \int_0^{\pi/2} \left| \sin \left(x - \frac{\pi}{4} \right) \right| dx$

$$= - \int_0^{\pi/4} \sin \left(x - \frac{\pi}{4} \right) dx + \int_{\pi/4}^{\pi/2} \sin \left(x - \frac{\pi}{4} \right) dx$$

19. (a) Let $I = \int_{-\pi/2}^{\pi/2} \frac{\cos x}{1 + e^x} dx$

$$= \int_{-\pi/2}^0 \frac{\cos x}{1 + e^x} dx + \int_0^{\pi/2} \frac{\cos x}{1 + e^x} dx \quad \dots(i)$$

On putting $x = -x$ in 1st integral, we get

$$\int_{-\pi/2}^0 \frac{\cos x}{1 + e^x} dx = \int_0^{\pi/2} \frac{e^x \cos x}{1 + e^x} dx$$

$$\begin{aligned}
 \therefore I &= \int_0^{\pi/2} \frac{e^x \cos x}{1 + e^x} dx + \int_0^{\pi/2} \frac{\cos x}{1 + e^x} dx \\
 &= \int_0^{\pi/2} \frac{(1 + e^x) \cos x}{(1 + e^x)} dx \\
 &= \int_0^{\pi/2} \cos x dx \\
 &= [\sin x]_0^{\pi/2} = 1
 \end{aligned}$$

20. (a) Let $I = \int_{7\pi/4}^{7\pi/3} \sqrt{\tan^2 x} dx = \int_{7\pi/4}^{2\pi} |\tan x| dx + \int_{2\pi}^{7\pi/3} |\tan x| dx$

$$\begin{aligned}
 &= - \int_{7\pi/4}^{2\pi} \tan x dx + \int_{2\pi}^{7\pi/3} \tan x dx \\
 &= [\log \cos x]_{7\pi/4}^{2\pi} - [\log \cos x]_{2\pi}^{7\pi/3} \\
 &= \left(\log \cos 2\pi - \log \cos \frac{7\pi}{4} \right) - \left(\log \cos \frac{7\pi}{3} - \log \cos 2\pi \right) \\
 &= \left[0 - \log \frac{1}{\sqrt{2}} \right] - [-\log 2 - 0] \\
 &= \log \sqrt{2} + \log 2 = \frac{3}{2} \log 2 = \log 2\sqrt{2}
 \end{aligned}$$

21. (b) Hint $I = \int_1^3 |(x - 1)(x - 2)(x - 3)| dx$

$$\begin{aligned}
 &= \int_1^2 |(x - 1)(x - 2)(x - 3)| dx \\
 &\quad + \int_2^3 |(x - 1)(x - 2)(x - 3)| dx \\
 &= \int_1^2 (x^3 - 6x^2 + 11x - 6) dx \\
 &\quad - \int_2^3 (x^3 - 6x^2 + 11x - 6) dx
 \end{aligned}$$

22. (d) Since, $\sin 2\pi x$ is positive, when $0 < x \leq \frac{1}{2}$ and

negative, when $\frac{1}{2} < x < 1$.

$$\therefore |\sin 2\pi x| = \begin{cases} \sin 2\pi x, & 0 < x \leq \frac{1}{2} \\ -\sin 2\pi x, & \frac{1}{2} < x < 1 \end{cases}$$

$$\Rightarrow \int_0^1 |\sin 2\pi x| dx = \int_0^{1/2} \sin 2\pi x dx + \int_{1/2}^1 -\sin 2\pi x dx = \frac{2}{\pi}$$

23. (a) Let $I = \int_0^{\pi/2} \frac{\psi(x)}{\psi(x) + \psi\left(\frac{\pi}{2} - x\right)} dx \quad \dots(i)$

$$\therefore I = \int_0^{\pi/2} \frac{\psi\left(\frac{\pi}{2} - x\right)}{\psi\left(\frac{\pi}{2} - x\right) + \psi\left(\frac{\pi}{2} - \frac{\pi}{2} + x\right)} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\psi\left(\frac{\pi}{2} - x\right)}{\psi\left(\frac{\pi}{2} - x\right) + \psi(x)} dx \quad \dots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

24. (b) Let $I = \int_0^1 \cot^{-1}(1-x+x^2) dx$

$$= \int_0^1 \tan^{-1}\left(\frac{1}{1-x(1-x)}\right) dx = \int_0^1 \tan^{-1}\left(\frac{x+(1-x)}{1-x(1-x)}\right) dx$$

$$= \int_0^1 \tan^{-1} x dx - \int_0^1 \tan^{-1}(1-x) dx$$

$$= \int_0^1 \tan^{-1} x dx - \int_0^1 \tan^{-1}[1-(1-x)] dx$$

$$= \int_0^1 \tan^{-1} x dx + \int_0^1 \tan^{-1} x dx = 2 \int_0^1 \tan^{-1} x dx = \frac{\pi}{2} - \log 2$$

25. (c) Hint $I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx \quad \dots(i)$

$$\Rightarrow I = \int_{-\pi}^{\pi} \frac{\cos^2(0-x)}{1+a^{-x}} dx = \int_{-\pi}^{\pi} \frac{a^x \cos^2 x}{1+a^x} dx \quad \dots(ii)$$

Now, adding Eqs. (i) and (ii)

26. (b) $\therefore I = \int_0^a f(x) \cdot g(x) dx$

$$\therefore I = \int_0^a f(a-x) g(a-x) dx = \int_0^a f(x) \{a-g(x)\} dx$$

$$= a \int_0^a f(x) dx - \int_0^a f(x) \cdot g(x) dx = a \int_0^a f(x) dx - I$$

$$\therefore I = \frac{a}{2} \int_0^a f(x) dx$$

27. (c) Let $I = \int_0^{\pi} e^{\cos^2 x} \cos^5 3x dx$

$$\Rightarrow I = \int_0^{\pi} e^{\cos^2(\pi-x)} \cos^5 3(\pi-x) dx$$

$$\left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right]$$

$$\Rightarrow I = - \int_0^{\pi} e^{\cos^2 x} \cos^5 3x dx = -I \Rightarrow 2I = 0$$

$$\therefore I = 0$$

28. (d) Let $I = \int_0^{\pi} \frac{\sin 2kx}{\sin x} dx = \int_0^{\pi} \frac{\sin 2k(\pi-x)}{\sin(\pi-x)} dx$

$$\Rightarrow I = \int_0^{\pi} \frac{\sin(2k\pi - 2kx)}{\sin x} dx \Rightarrow I = - \int_0^{\pi} \frac{\sin 2kx}{\sin x} dx$$

$$\Rightarrow I = -I \Rightarrow 2I = 0 \Rightarrow I = 0$$

29. (a) Let $I = \int_{-1/2}^{1/2} \cos x \left[\log \left(\frac{1-x}{1+x} \right) \right] dx \quad \dots(i)$

$$\therefore I = \int_{-1/2}^{1/2} \cos(-x) \left[\log \left(\frac{1+x}{1-x} \right) \right] dx \quad [\text{put } x = -x]$$

$$\Rightarrow I = - \int_{-1/2}^{1/2} \cos x \left[\log \left(\frac{1-x}{1+x} \right) \right] dx \quad \dots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_{-1/2}^{1/2} \cos x \left[\log \left(\frac{1-x}{1+x} \right) \right] dx - \int_{-1/2}^{1/2} \cos x \left[\log \left(\frac{1-x}{1+x} \right) \right] dx$$

$$\Rightarrow 2I = 0 \Rightarrow I = 0$$

30. (c) Let $I = \int_{-1}^1 \frac{\sin x - x^2}{3-|x|} dx$

$$\Rightarrow I = \int_{-1}^1 \frac{\sin x}{3-|x|} dx - \int_{-1}^1 \frac{x^2}{3-|x|} dx$$

Here, $f(x) = \frac{\sin x}{3-|x|}$ is an odd function.

But $f(x) = \frac{x^2}{3-|x|}$ is an even function.

$$\therefore I = - \int_{-1}^1 \frac{x^2}{3-|x|} dx = 2 \int_0^1 \frac{-x^2}{3-|x|} dx$$

31. (b) $\int_{-1}^1 \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1} dx$

$$= \int_{-1}^1 \frac{x^3}{x^2 + 2|x| + 1} dx + \int_{-1}^1 \frac{|x| + 1}{x^2 + 2|x| + 1} dx$$

$$= 0 + 2 \int_0^1 \frac{|x| + 1}{(|x| + 1)^2} dx = 2 \int_0^1 \frac{x + 1}{(x + 1)^2} dx$$

$$= 2 \int_0^1 \frac{1}{1+x} dx = 2 \ln 2 \quad [\because x > 0, \text{ from } 0 \text{ to } 1]$$

32. (b) $f(x) = \sin^{10} x (6x^9 - 25x^7 + 4x^3 - 2x)$

$$\therefore f(-x) = -f(x) \text{ is odd function.}$$

$$\therefore \int_{-\pi/2}^{\pi/2} \sin^{10} x (6x^9 - 25x^7 + 4x^3 - 2x) dx = 0$$

33. (b) Since, $f(x) + f(-x)$ is an even function and $g(x) - g(-x)$ is an odd function.

So, $\{f(x) + f(-x)\} \{g(x) - g(-x)\}$ is an odd function.

$$\therefore \int_{-\pi/4}^{\pi/4} [f(x) + f(-x)] [g(x) - g(-x)] dx = 0$$

34. (c) Given, $P = \int_0^{3\pi} f(\cos^2 x) dx$

and $Q = \int_0^{\pi} f(\cos^2 x) dx$

Also, $P = 3 \int_0^{\pi} f(\cos^2 x) dx = 3Q$

$$\therefore P - 3Q = 0$$

35. (a) Given, $f(2-x) = f(2+x)$, $f(4-x) = f(4+x)$

$$\begin{aligned} f(4+x) &= f(4-x) = f(2+2-x) \\ &= f\{2-(2-x)\} = f(x) \\ \therefore \int_0^{50} f(x) dx &= \int_0^{48} f(x) dx + \int_{48}^{50} f(x) dx \\ &= 12 \int_0^4 f(x) dx + \int_0^2 f(x) dx \\ &= 12 \left[\int_0^2 f(x) dx + \int_0^2 f(4-x) dx \right] + 5 \\ &= 24 \int_0^2 f(x) dx + 5 = 24(5) + 5 = 125 \end{aligned}$$

36. (b) We have, $(1+x^2) > x^2, \forall x$

$$\Rightarrow \sqrt{1+x^2} > x, \forall x \in (1, 2) \Rightarrow \frac{1}{\sqrt{1+x^2}} < \frac{1}{x}, \forall x \in (1, 2)$$

$$\therefore \int_1^2 \frac{dx}{\sqrt{1+x^2}} < \int_1^2 \frac{dx}{x} \Rightarrow I_1 < I_2 \Rightarrow I_2 > I_1$$

37. (c) By Leibnitz's rule, we have

$$\begin{aligned} \phi'(x) &= \frac{1}{2\sqrt{x}} \sin x - \left[-\frac{1}{x^2} \right] \sin \frac{1}{x^2} \\ \therefore \phi'(1) &= \frac{1}{2} \sin 1 + \sin 1 = \frac{3}{2} \sin 1 \end{aligned}$$

38. (a) $f(x) = \int_1^x \sqrt{2-t^2} dt$

$$\begin{aligned} \Rightarrow f'(x) &= \sqrt{2-x^2} \cdot 1 - \sqrt{2-1} \cdot 0 = \sqrt{2-x^2} \\ \therefore x^2 &= f'(x) = \sqrt{2-x^2} \\ \text{or } x^4 + x^2 - 2 &= 0 \text{ or } (x^2+2)(x^2-1) = 0 \\ \therefore x &= \pm 1 \end{aligned}$$

39. (c) We have, $f(x) = \frac{1}{x^2} \int_4^x [4t^2 - 2f'(t)] dt$

$$\begin{aligned} \therefore f'(x) &= \frac{1}{x^2} [4x^2 - 2f'(x)] - \frac{2}{x^3} \int_4^x [4t^2 - 2f'(t)] dt \\ \Rightarrow f'(4) &= \frac{1}{16} [64 - 2f'(4)] - 0 \therefore f'(4) = \frac{32}{9} \end{aligned}$$

40. (a) Using Newton-Leibnitz's formula, we get

$$\begin{aligned} t^2 \{f(t^2)\} \left\{ \frac{d}{dt}(t^2) \right\} - 0 \cdot f(0) \left\{ \frac{d}{dt}(0) \right\} &= 2t^4 \\ \Rightarrow t^2 \{f(t^2)\} 2t &= 2t^4 \Rightarrow f(t^2) = t \\ \Rightarrow f\left(\frac{4}{25}\right) &= \pm \frac{2}{5} \quad \left[\text{put } t = \pm \frac{2}{5} \right] \\ \therefore f\left(\frac{4}{25}\right) &= \frac{2}{5} \quad [\text{neglecting negative sign}] \end{aligned}$$

41. (b) Let $A = \lim_{n \rightarrow \infty} \frac{(r)^{1/n}}{n}$

$$\begin{aligned} \log A &= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \log \frac{r}{n} = \int_0^1 \log x dx = [x \log x - x]_0^1 \\ \therefore A &= e^{-1} \end{aligned}$$

42. (a) $S = \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{n}{(n+r)\sqrt{r(2n+r)}}$
- $$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n\left(1+\frac{r}{n}\right)\sqrt{\frac{r}{n}\left(2+\frac{r}{n}\right)}}$$

$$\begin{aligned} &= \int_0^1 \frac{dx}{(1+x)\sqrt{2x+x^2}} = \int_0^1 \frac{dx}{(1+x)\sqrt{(1+x)^2-1}} \\ &= [\sec^{-1}(1+x)]_0^1 = \sec^{-1} 2 - \sec^{-1} 1 = \frac{\pi}{3} \end{aligned}$$

43. (b) $\lim_{n \rightarrow \infty} \left[\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n} \right] = \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{r+n}$
- $$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} \cdot \frac{1}{1+\frac{r}{n}} = \int_0^1 \frac{1}{1+x} dx$$
- $$= [\log(1+x)]_0^1 = \log 2$$

44. (a) $\lim_{n \rightarrow \infty} \left[\frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + \dots + \frac{1}{8n} \right]$

$$= \lim_{n \rightarrow \infty} \sum_{r=0}^n \frac{n^2}{(n+r)^3} = \lim_{n \rightarrow \infty} \sum_{r=0}^n \frac{1}{n} \cdot \frac{1}{\left(1+\frac{r}{n}\right)^3}$$

$$= \int_0^1 \frac{dx}{(1+x)^3} = \left[-\frac{1}{2(1+x)^2} \right]_0^1 = -\frac{1}{2} \left(\frac{1}{4} - 1 \right) = \frac{3}{8}$$

45. (b) $\lim_{n \rightarrow \infty} \left[\frac{\sqrt{n}}{(3+4\sqrt{n})^2} + \frac{\sqrt{n}}{\sqrt{2}(3\sqrt{2}+4\sqrt{n})^2} + \frac{\sqrt{n}}{\sqrt{3}(3\sqrt{3}+4\sqrt{n})^2} + \dots + \frac{1}{49n} \right]$

$$= \lim_{n \rightarrow \infty} \left[\frac{\sqrt{n}}{(3\sqrt{1}+4\sqrt{n})^2} + \frac{\sqrt{n}}{\sqrt{2}(3\sqrt{2}+4\sqrt{n})^2} + \dots + \frac{\sqrt{n}}{\sqrt{n}(3\sqrt{n}+4\sqrt{n})^2} \right]$$

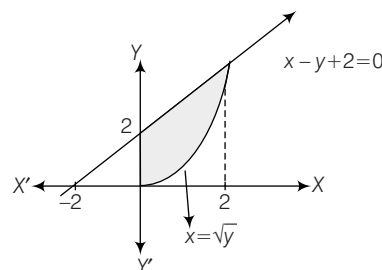
$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{\sqrt{n}}{\sqrt{r}(3\sqrt{r}+4\sqrt{n})^2} = \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{\sqrt{\frac{r}{n}} \left(3\sqrt{\frac{r}{n}} + 4 \right)^2} \cdot \frac{1}{n}$$

$$= \int_0^1 \frac{dx}{\sqrt{x}(3\sqrt{x}+4)^2}$$

Put $3\sqrt{x}+4=t$

$$= \frac{2}{3} \int_4^7 \frac{dt}{t^2} = \frac{2}{3} \left[-\frac{1}{t} \right]_4^7 = \frac{2}{3} \left(\frac{1}{4} - \frac{1}{7} \right) = \frac{1}{14}$$

46. (c) Required area = $\int_0^2 (y_1 - y_2) dx$



$$\begin{aligned} \therefore \int_0^2 [(x+2) - x^2] dx &= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_0^2 \\ &= 2 + 4 - \frac{8}{3} = \frac{10}{3} \end{aligned}$$

47. (c) $y = x \sin x$ is positive for $0 < x < \pi$ and negative for $\pi < x < 2\pi$.

$$\therefore \text{Required area} = \int_0^{\pi} x \sin x \, dx + \left| \int_{\pi}^{2\pi} x \sin x \, dx \right|$$

$$= \pi + 3\pi = 4\pi$$

48. (b) $y = 4$ meet the parabola

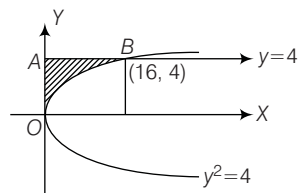
$$y^2 = x \text{ at } B.$$

$$\text{At } 16 = x$$

$$\Rightarrow B \text{ is } (16, 4)$$

$$\therefore \text{Required area} = \int_0^4 y^2 \, dy$$

$$= \left[\frac{y^3}{3} \right]_0^4 = \frac{64}{3} \text{ sq units}$$

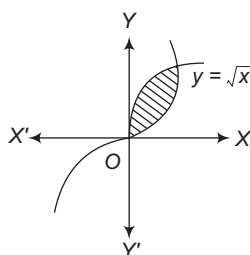


49. (c) Since, the intersection of two curves $y = x^3$ and $y = \sqrt{x}$ are $x = 0$ and $x = 1$.

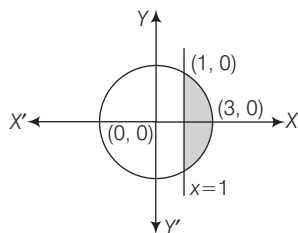
$$\therefore A = \left| \int_0^1 (x^3 - \sqrt{x}) \, dx \right|$$

$$= \left| \left[\frac{x^4}{4} - \frac{2x^{3/2}}{3} \right]_0^1 \right| = \left| \left[\frac{1}{4} - \frac{2}{3} \right] \right|$$

$$= \frac{5}{12} \text{ sq unit}$$



50. (b) **Hint** Required area, $A = 2 \int_1^3 \sqrt{9 - x^2} \, dx$

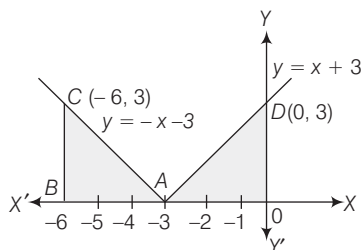


$$= 2 \cdot \frac{1}{2} \left[x \sqrt{9 - x^2} + 9 \sin^{-1} \frac{x}{3} \right]_1^3$$

51. (a) **Hint** Required area

$$= \int_{-6}^{-3} |x+3| \, dx + \int_{-3}^0 |x+3| \, dx$$

$$= \int_{-6}^{-3} (-x-3) \, dx + \int_{-3}^0 (x+3) \, dx$$



52. (d) Clearly, $y = f(x)$ passes through $(2, 0)$ and $(0, 1)$.

$$\therefore 0 = f(2) \text{ and } 1 = f(0)$$

Also, $\int_0^2 f(x) \, dx = \frac{3}{4}$ [given]

Now, $\int_0^2 x f'(x) \, dx = [x f(x)]_0^2 - \int_0^2 f(x) \, dx$

$$\Rightarrow \int_0^2 x f'(x) \, dx = [2f(2) - 0f(0)] - \frac{3}{4}$$

$$\Rightarrow \int_0^2 x f'(x) \, dx = 2 \times 0 - 0 \times 1 - \frac{3}{4} = -\frac{3}{4}$$

53. (a) Area of the circle in 1st quadrant is $\frac{\pi(\pi^2)}{4} = \frac{\pi^3}{4}$.

Also, area of curve $y = \sin x$ and X-axis is 2 sq units.

$$\therefore \text{Required area, } A = \frac{\pi^3}{4} - 2 = \frac{\pi^3 - 8}{4}$$

54. (d) Given curves, $x^2 + y^2 = 1$,

$$x + y = 1 \text{ meet when}$$

$$x^2 + (1-x)^2 = 1$$

$$\Rightarrow x^2 + 1 + x^2 - 2x = 1$$

$$\Rightarrow 2x^2 - 2x = 0$$

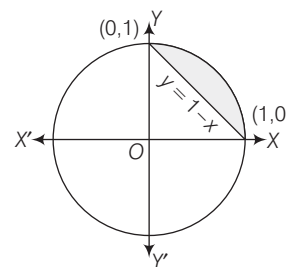
$$\Rightarrow 2x(x-1) = 0$$

$$\Rightarrow x = 0, x = 1 \Rightarrow y = 1, y = 0$$

$$\therefore A = \int_0^1 [\sqrt{1-x^2} - (1-x)] \, dx$$

$$= \left[\frac{x \sqrt{1-x^2}}{2} + \frac{1}{2} \sin^{-1} x - x + \frac{x^2}{2} \right]_0^1 = \frac{1}{2} \cdot \frac{\pi}{2} - 1 + \frac{1}{2}$$

$$= \frac{\pi}{4} - \frac{1}{2}$$



55. (d) Required area $= \int_0^{\log 3} x \, dy = \int_0^{\log 3} e^y \, dy$

$$= [e^y]_0^{\log 3} = [e^{\log 3} - e^0] = 3 - 1 = 2$$

56. (b) Required area $= \int_0^{\pi/2} |\cos x - \sin x| \, dx$

$$= \int_0^{\pi/4} (\cos x - \sin x) \, dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x) \, dx$$

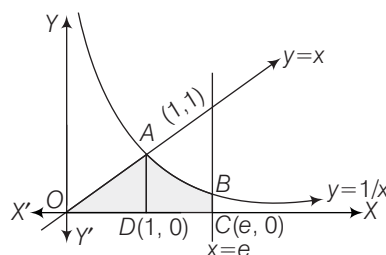
$$= [\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2}$$

$$= (\sqrt{2} - 1) - (1 - \sqrt{2}) = 2\sqrt{2} - 2$$

57. (b) Given, $y = x$, $x = e$ and $y = \frac{1}{x}$, $x \geq 0$

Since, $y = x$ and $x \geq 0 \Rightarrow y \geq 0$

$\therefore$ Area to be calculated in 1st quadrant is shown in figure.



$$\therefore \text{Required area} = \text{Area of } \triangle ODA + \text{Area of } DABCD$$

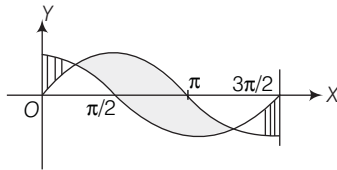
$$= \frac{1}{2} (1 \times 1) + \int_1^e \frac{1}{x} \, dx$$

$$= \frac{1}{2} + [\log |x|]_1^e = \frac{1}{2} + [\log |x|]_1^e$$

$$= \frac{1}{2} + 1 = \frac{3}{2} \text{ sq units}$$

58. (a) Required area

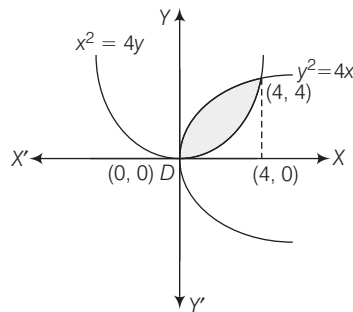
$$= \int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx + \int_{5\pi/4}^{3\pi/2} (\cos x - \sin x) dx$$



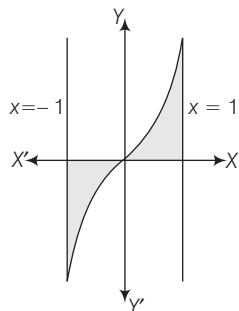
$$= [\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{5\pi/4} + [\sin x + \cos x]_{5\pi/4}^{3\pi/2} = (4\sqrt{2} - 2) \text{ sq units}$$

59. (c) **Hint** Area bounded between curves

$$= \int_0^4 \left(\sqrt{4x} - \frac{x^2}{4} \right) dx$$



60. (b) Required area $= 2 \int_0^1 x e^x dx$



$$= 2[xe^x - e^x]_0^1 = 2 \text{ sq units}$$

61. (d) $y^2 = x^3$ and $|y| = 2x$ are symmetric about Y-axis.

BITSAT Archives

1. (a) **Hint** Consider, $I = \int_1^x t \log t dt$

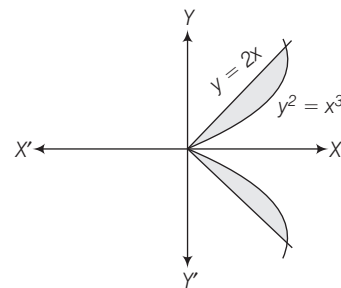
$$= \left[\log t \cdot \frac{t^2}{2} \right]_1^x - \int_1^x \frac{1}{t} \cdot \frac{t^2}{2} dt$$

$$\Rightarrow \frac{1}{4} = \frac{x^2}{2} \log x - \frac{1}{2} \left[\frac{t^2}{2} \right]_1^x$$

2. (b) We know that, $x - [x]$ is periodic function with period one.

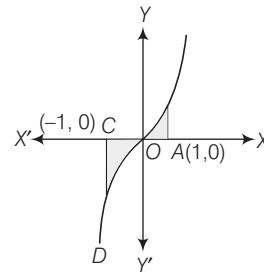
$\Rightarrow$
 $\Rightarrow$

$$4x^2 = x^3 \\ x = 0, 4$$



$$\begin{aligned} \therefore \text{Required area} &= 2 \int_0^4 (2x - x^{3/2}) dx \\ &= 2 \left[x^2 - \frac{x^{5/2} \times 2}{5} \right]_0^4 \\ &= 2 \times \frac{16}{5} \\ &= \frac{32}{5} \\ &= 6 \frac{2}{5} \text{ sq units} \end{aligned}$$

62. (c) **Hint** Given, $y = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$



$$\begin{aligned} \text{Required area} &= 2 \int_0^1 x|x| dx \\ &= 2 \int_0^1 x^2 dx \end{aligned}$$

$\therefore e^{x-[x]}$ has period one.

[since, $f(x)$ is periodic with period T , then $\int_0^{nT} f(x) dx = n \int_0^T f(x) dx$]

$$\begin{aligned} \therefore \int_0^{25 \times 1} e^{x-[x]} dx &= 25 \int_0^1 e^{x-[x]} dx \\ &= 25 \int_0^1 e^{x-0} dx = 25[e^x]_0^1 = 25[e - 1] \end{aligned}$$

$$\begin{aligned} \therefore 25(e - 1) &= k(e - 1) \\ \text{Hence, } k &= 25 \end{aligned}$$

3. (a) Given that, $y = -\sqrt{-x}$
 $\Rightarrow y^2 = -x$, where x and y
 both are negative.

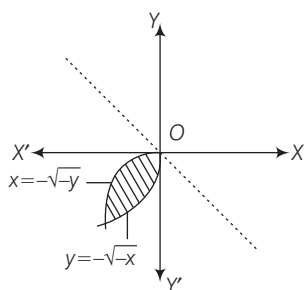
Now, $x = -\sqrt{-y}$

$\Rightarrow x^2 = -y$, where x and y
 both are negative.

$\therefore$ Required area

$$= \left| \int_{-1}^0 -\sqrt{-x} \, dx - \int_{-1}^0 -x^2 \, dx \right|$$

$$= \frac{1}{3}$$



4. (c) Given that, $I = \int_0^{10\pi} ([\sec^{-1} x] + [\cot^{-1} x]) \, dx$

$$= \int_0^{\sec^{-1} 1} ([\sec^{-1} x] + [\cot^{-1} x]) \, dx + \int_{\sec^{-1} 1}^{10\pi} ([\sec^{-1} x] + [\cot^{-1} x]) \, dx$$

$$= \int_0^{\sec^{-1} 1} (0 + 0) \, dx + \int_{\sec^{-1} 1}^{10\pi} (1 + 0) \, dx$$

$$= 0 + [x]_{\sec^{-1} 1}^{10\pi} = 10\pi - \sec^{-1} 1$$

5. (a) Given curve is

$$(y - x)^2 = x^3$$

$$\Rightarrow y - x = \pm x\sqrt{x}$$

$$\Rightarrow y = x + x\sqrt{x} \quad \dots(i)$$

$$\Rightarrow y = x - x\sqrt{x} \quad \dots(ii)$$

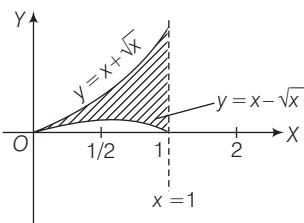
and $x = 1$

From the figure, required area

$$= \int_0^1 \{(x + x\sqrt{x}) - (x - x\sqrt{x})\} \, dx$$

$$= \int_0^1 2x\sqrt{x} \, dx = 2 \int_0^1 x^{3/2} \, dx$$

$$= 2 \left[\frac{x^{5/2}}{5/2} \right]_0^1 = \frac{4}{5} [1 - 0] = \frac{4}{5} \text{ sq unit}$$



6. (d) Hint $I = \int_0^{\pi/2} \frac{\sin^n \theta}{\sin^n \theta + \cos^n \theta} \, d\theta \quad \dots(i)$
- $$= \int_0^{\pi/2} \frac{\sin^n(\pi/2 - \theta)}{\sin^n(\pi/2 - \theta) + \cos^n(\pi/2 - \theta)} \, d\theta$$
- $$= \int_0^{\pi/2} \frac{\cos^n \theta}{\cos^n \theta + \sin^n \theta} \, d\theta \quad \dots(ii)$$

Now, add Eqs. (i) and (ii).

7. (d) Hint $I = \int_0^\pi \cos^{101} x \, dx$

$$\Rightarrow I = \int_0^\pi [\cos(\pi - x)]^{101} \, dx \Rightarrow I = \int_0^\pi -\cos^{101} x \, dx$$

8. (c) $\lim_{n \rightarrow \infty} \left[\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+5n} \right]$

$$= \lim_{n \rightarrow \infty} \sum_{r=0}^{5n} \frac{1}{n+r} = \lim_{n \rightarrow \infty} \left[\frac{1}{n} \sum_{r=0}^{5n} \frac{n}{n+r} \right]$$

$$= \lim_{n \rightarrow \infty} \left[\frac{1}{n} \sum_{r=0}^{5n} \frac{1}{1+(r/n)} \right]$$

$$= \int_0^5 \frac{1}{1+x} \, dx = [\log(1+x)]_0^5 = \log 6 - \log 1 = \log 6$$

9. (d) $\int_1^2 f(x) \, dx = \int_1^2 \frac{d}{dx} \{\phi(x)\} \, dx = [\phi(x)]_1^2 = \phi(2) - \phi(1)$

10. (b) Hint $\int_0^2 |1-x| \, dx = \int_0^1 (1-x) \, dx + \int_1^2 (x-1) \, dx$

11. (b) Required area $= \int_0^\pi \sin x \, dx$

$$= [-\cos x]_0^\pi = [1 + 1] = 2 \text{ sq units}$$

12. (c) Hint Let $I = \int_0^1 |5x-3| \, dx$

$$\text{Now, } |5x-3| = \begin{cases} 5x-3, & x \geq \frac{3}{5} \\ -(5x-3), & x < \frac{3}{5} \end{cases}$$

$$\therefore I = \int_0^{3/5} -(5x-3) \, dx + \int_{3/5}^1 (5x-3) \, dx$$

13. (a) Required area

$$= \int_0^1 (x_1 - x_2) \, dx$$

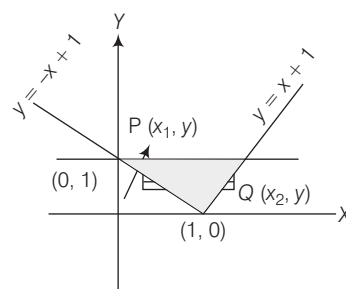
$$= \int_0^1 \{(-x+1) - (x-1)\} \, dx$$

$$= \int_0^1 (-2x+2) \, dx$$

$$= [-x^2 + 2x]_0^1$$

$$= -1 + 2 - 0 + 0$$

$$= 1$$

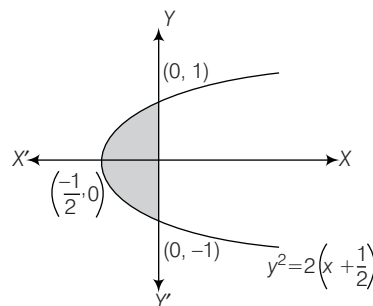


14. (b) Hint $\int_{-1}^2 |x|^3 \, dx = \int_{-1}^0 -x^3 \, dx + \int_0^2 x^3 \, dx$

15. (c) Let $I = \int_{-\pi/2}^{\pi/2} \sin |x| \, dx$

$$= 2 \int_0^{\pi/2} \sin x \, dx = 2[-\cos x]_0^{\pi/2} = 2$$

16. (b) Hint



$$\therefore \text{Required area} = \int_{-1}^0 x \, dy$$

$$= 2 \int_0^1 \frac{y^2 - 1}{2} \, dy$$

17. (a) Let $I = \int_0^8 |x-5| \, dx$

$$= \int_0^5 -(x-5) \, dx + \int_5^8 (x-5) \, dx = 25 - 8 = 17$$

18. (c) Given that, $I_1 = \int_0^1 2^{x^2} \, dx$, $I_2 = \int_0^1 2^{x^3} \, dx$, $I_3 = \int_1^2 2^{x^2} \, dx$

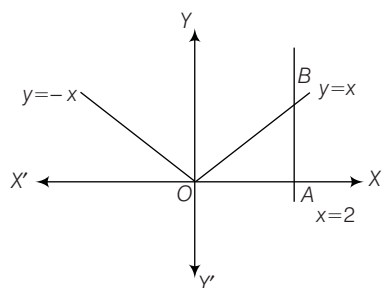
$$\text{and } I_4 = \int_1^2 2^{x^3} \, dx$$

$$\therefore 2^{x^3} < 2^{x^2}, 0 < x < 1$$

$$\text{and } 2^{x^3} > 2^{x^2}, x > 1$$

$$\therefore I_4 > I_3 \text{ and } I_2 < I_1$$

19. (c) Required area = Area of shaded region OAB



$$= \int_0^2 y dx = \int_0^2 x dx = \left[\frac{x^2}{2} \right]_0^2 = 2 \text{ sq units}$$

Alternate Solution

Required area = Area of $\triangle OAB$

$$= \frac{1}{2} \times 2 \times 2$$

$$= 2 \text{ sq units}$$

20. (a) Let $I = \int_0^{\pi/4} \log(1 + \tan x) dx$... (i)

$$\Rightarrow I = \int_0^{\pi/4} \log \left[1 + \tan \left(\frac{\pi}{4} - x \right) \right] dx$$

$$\left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right]$$

$$\Rightarrow I = \int_0^{\pi/4} \log \left[1 + \frac{1 - \tan x}{1 + \tan x} \right] dx$$

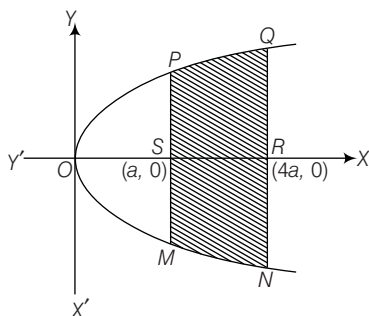
$$= \int_0^{\pi/4} \log \left[\frac{2}{1 + \tan x} \right] dx$$

$$= \int_0^{\pi/4} \log 2 dx - \int_0^{\pi/4} \log(1 + \tan x) dx$$

$$\Rightarrow I = \log 2 [x]_0^{\pi/4} - I \quad [\text{from Eq. (i)}]$$

$$\Rightarrow 2I = \frac{\pi}{4} \log_e 2 \Rightarrow I = \frac{\pi}{8} \log_e 2$$

21. (d) Required area = Area of curve $PSMRQP$



$$= 2 \times \text{Area of curve } PSRQP$$

$$= 2 \int_a^{4a} \sqrt{4ax} dx$$

$$= 4\sqrt{a} \left[\frac{x^{3/2}}{3/2} \right]_a^{4a} = \frac{8}{3} \sqrt{a} [(4a)^{3/2} - a^{3/2}]$$

$$= \frac{8}{3} \sqrt{a} (8a^{3/2} - a^{3/2}) = \frac{56}{3} a^2$$

22. (c) Hint $I = \int_{\log 2}^x \frac{du}{(e^u - 1)^{1/2}}$ or $I = \int_{\log 2}^x \frac{e^u}{e^u (e^u - 1)^{1/2}} du$

$$\text{Put } e^u - 1 = t^2 \Rightarrow e^u du = 2t dt$$

$$\therefore I = \int_1^{\sqrt{e^x - 1}} \frac{2t}{(t^2 + 1)t} dt = 2 \int_1^{\sqrt{e^x - 1}} \frac{dt}{(1 + t^2)}$$

$$= [\tan^{-1} t]_1^{\sqrt{e^x - 1}} = 2 \tan^{-1} \sqrt{e^x - 1} - \tan^{-1} 1$$

23. (b) Let $I = \int_0^{2\pi} \left\{ |\sin x| - \frac{1}{2} \sin x \right\} dx$
- $$= \int_0^{2\pi} \left\{ |\sin x| - \frac{1}{2} \sin x \right\} dx = \int_0^{2\pi} \frac{1}{2} |\sin x| dx$$
- $$= \frac{1}{2} \left[\int_0^{2\pi} |\sin x| dx + \int_{2\pi}^{4\pi} |\sin x| dx \right.$$
- $$\left. + \dots + \int_{2(n-1)\pi}^{2n\pi} |\sin x| dx \right]$$

$$\text{Now, } I_1 = \int_0^{2\pi} |\sin x| dx$$

$$I_1 = \int_0^{\pi} \sin x dx - \int_{\pi}^{2\pi} \sin x dx$$

$$= [-\cos x]_0^{\pi} + [\cos x]_{\pi}^{2\pi} = -[1 - 1] + [1 + 1]$$

$$= 2 + 2 = 4$$

$$\therefore I = \frac{1}{2} [4 + 4 + 4 + \dots n \text{ times}] = \frac{1}{2} (4n) = 2n$$

24. (d) Let $f(\theta) = \left[\int_0^{\sin^2 \theta} \sin^{-1} \sqrt{\phi} d\phi + \int_0^{\cos^2 \theta} \cos^{-1} \sqrt{\phi} d\phi \right]$

$$\Rightarrow f'(\theta) = \frac{d}{d\theta} \sin^2 \theta [\sin^{-1} \sqrt{\sin^2 \theta}]$$

$$+ \frac{d}{d\theta} \cos^2 \theta [\cos^{-1} \sqrt{\cos^2 \theta}]$$

$$= (2 \sin \theta \cos \theta) \theta - (2 \sin \theta \cos \theta) \theta = 0$$

$$\therefore f(\theta) = \text{constant} = a \quad [\text{say}]$$

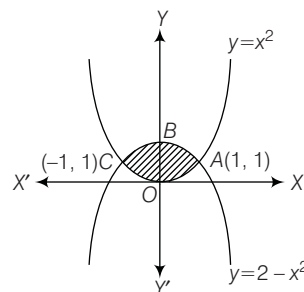
$$\therefore f\left(\frac{\pi}{4}\right) = a$$

$$\Rightarrow \int_0^{1/2} \sin^{-1} \sqrt{\phi} d\phi + \int_0^{1/2} \cos^{-1} \sqrt{\phi} d\phi = a$$

$$\Rightarrow \int_0^{1/2} (\sin^{-1} \sqrt{\phi} + \cos^{-1} \sqrt{\phi}) d\phi = a$$

$$\Rightarrow \frac{\pi}{2} [\phi]_0^{1/2} = a \Rightarrow \frac{\pi}{4} = a$$

25. (a) Hint



$$\therefore \text{Required area} = \text{Area of curve } OABCO$$

$$= 2 \times \text{Area of curve } OABO$$

$$= 2 \int_0^1 y dx$$

$$= 2 \int_0^1 [(2 - x^2) - (x^2)] dx$$

21

Differential Equations

Differential Equation

An equation that involves independent variables, dependent variables, derivatives of the dependent variables w.r.t. independent variables and constant, is called a differential equation.

Types of Differential Equations

Ordinary Differential Equation Ordinary differential equations are those equations which contain only one independent variable.

In particular, an equation involving only one independent variable (say x), dependent variable (say y) and the differential coefficients $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \dots$, is called ordinary differential equation.

Partial Differential Equation Partial differential equations are those equations which contain two or more independent variables.

Order and Degree of a Differential Equation

The **order** of a differential equation is the order of the highest derivative occurring in the differential equation.

By the **degree** of differential equation, when it is polynomial equation in derivatives, then the greatest exponent (positive integral index) of the highest order derivative, occurring in the differential equation.

- The order and degree of a differential equation is always a positive integer.
- The degree of the differential equation is found only when the equation is polynomial in derivatives.

Solution of Differential Equation

A primitive or solution of a differential equation is a functional relation between x and y which is free from derivatives and this relation on substitution satisfies the differential equation.

There are two types of solutions of a differential equation

1. General Solution

A general solution of a differential equation is a relation between the variables (not involving the derivatives) which contains the same number of the arbitrary constants as the order of the differential equation.

2. Particular Solution

Particular solution of the differential equation obtained from the general solution by assigning particular values to the arbitrary constant in the general solution.

Formation of Differential Equation whose General Solution is Given

Formulating differential equation that represent the family of given curves, means finding a differential equation whose solution is the given equation.

Suppose an equation contains n arbitrary constants, then differentiate the given equation n times to obtain n equations. The equation so obtained by eliminating arbitrary constant is the differential equation of order n for the family of given curves.

Solution of Ordinary Differential Equation of the First Order and First Degree

An ordinary differential equation of the first order and first degree is of the form

$$M + N \frac{dy}{dx} = 0 \text{ or } Mdx + Ndy = 0$$

where, M and N are functions of x and y or constant. The general solution of such equation will contain only one arbitrary constant.

Differential equation of first degree and first order can be classified into following types

Variables Separable Differential Equation

If the differential equation can be put in the form $f(x)dx + \phi(y)dy = 0$ will be termed as variable separable. Such equation will be solved by integrating and adding arbitrary constant in side.

Hence, complete general solution of $f(x)dx + \phi(y)dy = 0$ will be $\int f(x)dx + \int \phi(y)dy = C$

where, C is an arbitrary constant or constant of integration.

Reducible to Variable Separable Form

Differential equations of the form $\frac{dy}{dx} = f(ax + by + c)$ can be reduced to variable separable form by the substitution

$$\begin{aligned} \therefore \quad & ax + by + c = z \\ & a + b \frac{dy}{dx} = \frac{dz}{dx} \\ \Rightarrow \quad & \left(\frac{dz}{dx} - a \right) \frac{1}{b} = f(z) \\ \Rightarrow \quad & \frac{dz}{dx} = a + bf(z) \end{aligned}$$

Homogeneous Differential Equation of First Order and First Degree

A differential equation of the form

$$\frac{dy}{dx} = \frac{f(x, y)}{\phi(x, y)}$$

where, $f(x, y)$ and $\phi(x, y)$ are both homogeneous functions of x and y of the same degree, is called a homogeneous differential equation.

The solution of this equation is obtained by substituting, $y = vx$ or $x = vy$ according as $f(x, y)$ and $\phi(x, y)$ is converted into function of $\left(\frac{y}{x}\right)$ or $\left(\frac{x}{y}\right)$.

This substitution reduces the given equation to the variable separable form.

Reducible to Variable Separable Form

(i) Consider the differential equation of the form

$$\frac{dy}{dx} = \frac{ax + by + c}{a'x + b'y + c'}, \text{ where } \frac{a}{a'} \neq \frac{b}{b'}$$

This equation is not homogeneous. In order to reduced this equation to the homogeneous form substitute $x = X + h$, $y = Y + k$, where h and k , are constants which are to be determined.

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dY}{dX}$$

and the above equation becomes

$$\frac{dY}{dX} = \frac{a(X + h) + b(Y + k) + c}{a'(X + h) + b'(Y + k) + c'} \quad \dots(i)$$

$$\frac{dY}{dX} = \frac{(aX + bY) + ah + bk + c}{(a'X + b'Y) + a'h + b'k + c'} \quad \dots(ii)$$

Now, h and k will be chosen such that

$$\begin{aligned} ah + bk + c &= 0 \\ a'h + b'k + c' &= 0 \\ \frac{h}{bc' - b'c} &= \frac{k}{ca' - c'a} \\ &= \frac{1}{ab' - a'b} \end{aligned}$$

For these values of h and k in Eq. (ii) reduces to

$$\frac{dY}{dX} = \frac{aX + bY}{a'X + b'Y}$$

which is a homogeneous equation in X, Y and can be solved by the substitution $Y = vX$.

$$\frac{dY}{dX} = v + X \cdot \frac{dv}{dX}$$

Replacing, X and Y in the solution so obtained by $x - h$ and $y - k$, respectively, we can obtain the required solution in term of x and y .

(ii) If $\frac{dy}{dx} = \frac{ax + by + c}{a'x + b'y + c'}$ and $\frac{a}{a'} = \frac{b}{b'} = m$ [say]

then, $\frac{dy}{dx} = \frac{m(a'x + b'y) + c}{a'x + b'y + c'}$

where, m is any number.

In such case substitute $a'x + b'y = v$

So that, $a' + b' \frac{dy}{dx} = \frac{dv}{dx}$

Transform the differential equation of the form

$$\frac{1}{b'} \left(\frac{dv}{dx} - a' \right) = \frac{mv + c}{v + c'}, \text{ we get}$$

$$\frac{dv}{dx} = a' + b' \left(\frac{mv + c}{v + c'} \right)$$

which is a differential equation in variable separable form and it can easily be solved.

Linear Differential Equation of First Order

A differential equation of the form

$$\frac{dy}{dx} + Py = Q$$

where, P and Q are either constants or functions of x (and not of y) is said to be linear differential equation of first order.

To obtain the solution of this equation, we first obtain an integrating factor (IF) as

$$\text{IF} = e^{\int P dx}$$

The solution of the given equation is given by

$$y \cdot (\text{IF}) = \int Q(\text{IF}) dx + C$$

- NOTE**
- If the equation is given by $\frac{dx}{dy} + Px = Q$, where P and Q are functions of y (not of x), then $\text{IF} = e^{\int P dy}$ and solution will be $x \cdot (\text{IF}) = \int Q(\text{IF}) dy + C$.
 - Before solving linear differential equation, make the coefficient of $\frac{dy}{dx}$ or $\frac{dx}{dy}$ equal to 1.

Extended Form of Linear Equations

(i) Bernoulli's Equation

An equation of the form $\frac{dy}{dx} + Py = Qy^n$, where P and Q are functions of x alone or constant and n is constant, other than 0 and 1, is called a Bernoulli's equation.

This equation can be reduced to the linear form by dividing y^n , we get

$$y^{-n} \frac{dy}{dx} + P \cdot y^{-n+1} = Q$$

Now, put $y^{-n+1} = v$, so that

$$(-n+1)y^{-n} \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dv}{dx} + (1-n)Pv = (1-n)Q$$

which is a linear differential equation.

(ii) If the equation of the form

$$\frac{dy}{dx} + P\phi(y) = Q\psi(y)$$

where, P and Q are functions of x alone or constant. Dividing the given equation by $\psi(y)$, we get

$$\frac{1}{\psi(y)} \cdot \frac{dy}{dx} + \frac{\phi(y)}{\psi(y)} P = Q$$

Now, put $\frac{\phi(y)}{\psi(y)} = v$ so that $\frac{d}{dx} \left[\frac{\theta(y)}{\psi(y)} \right] = \frac{dv}{dx}$

or $\frac{dy}{dx} = k \cdot \frac{1}{\psi(y)} \cdot \frac{dy}{dx}$, where k is constant.

We get $\frac{dv}{dx} + kPv = kQ$

which is a linear differential equation.

Practice Exercise

1. If m and n are the order and degree of the differential

equation $\left(\frac{d^2y}{dx^2}\right)^5 + \frac{4\left(\frac{d^2y}{dx^2}\right)^3}{\frac{d^3y}{dx^3}} + \frac{d^3y}{dx^3} = x^2 - 1$, then

- a. $m = 3, n = 3$ b. $m = 3, n = 2$
c. $m = 3, n = 5$ d. $m = 3, n = 1$

2. The degree of the differential equation

$$\frac{d^2y}{dx^2} + 3\left[\frac{dy}{dx}\right]^2 = x^2 \log\left[\frac{d^2y}{dx^2}\right], \text{ is}$$

- a. 1 b. 2
c. 3 d. None of these

3. The order and degree of the differential equation

$$\sqrt{\frac{d^2y}{dx^2}} = 3\sqrt{\frac{dy}{dx}} + 5 \text{ are respectively}$$

- a. 2 and 3 b. 3 and 2
c. 2 and 1 d. 2 and 2

4. The order and degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^{3/2} - \sqrt{\left(\frac{dy}{dx}\right)} - 4 = 0 \text{ are respectively}$$

- a. 2 and 6 b. 3 and 6
c. 1 and 4 d. 2 and 4

5. The differential equation whose solution is $(x-h)^2 + (y-k)^2 = a^2$, is given by

a. $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)$ b. $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$
c. $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$ d. None of these

6. The differential equation of all parabolas with axis parallel to Y-axis, is

- a. $y_2 = 2y_1 + x$ b. $y_3 = 2y_1$
c. $y_2^3 = y_1$ d. None of these

7. The degree of the differential equation of all circles of radius r , having centre on Y-axis and passing through the origin, is

- a. 1 b. 2 c. 3 d. 4

8. The order of the differential equation whose general solution is given by

$$y = (c_1 + c_2) \cos(x + c_3) - c_4 e^{x+c_5}, \quad \text{where}$$

c_1, c_2, c_3, c_4, c_5 are arbitrary constants, is

- a. 5 b. 4 c. 3 d. 2

9. The degree of the differential equation satisfying

$$(\sqrt{1+x^2} + \sqrt{1+y^2} = A(x\sqrt{1+y^2} - y\sqrt{1+x^2})) \text{ is}$$

- a. 2 b. 3 c. 4 d. None

10. The differential equation of all straight lines which are at a constant distance p from the origin is

- a. $(y - xy_1)^2 = p^2(1-x)^2$ b. $(y - xy_1)^2 = p^2(1+x^2)$
c. $(y - xy_1)^2 = p^2(1+y_1^2)$ d. $(y + xy_1)^2 = p^2(1+y_1^2)$

11. The differential equation corresponding to the family of curves $y = e^x (a \cos x + b \sin x)$, where a and b are arbitrary constants, is

- a. $2y_2 + y_1 - 3y = 0$ b. $y_2 - 2y_1 + 2y = 0$
c. $2y_2 + 2y_1 - y = 0$ d. $2y_2 - y_1 + 2y = 0$

12. The number of straight lines which satisfy the differential equation $\frac{dy}{dx} + x\left(\frac{dy}{dx}\right)^2 - y = 0$, is

- a. 3 b. 2 c. 4 d. None

13. If the slope of a curve at any point is the reciprocal of twice the ordinate at the point and it passes through the point (4, 3). Then, the equation of the curve is

- a. $x^2 = y + 5$ b. $y^2 = x - 5$ c. $y^2 = x + 5$ d. $x^2 = y - 5$

14. The solution of $\frac{dy}{dx} = \frac{ax+h}{by+k}$ represents a parabola when

- a. $a = 0, b = 0$ b. $a + 1, b = 2$ c. $a = 0, b \neq 0$ d. $a = 2, b = 1$

15. The solution of the differential equation $2x \frac{dy}{dx} - y = 3$ represents

- a. a straight line b. a circle
c. a parabola d. an ellipse

16. If $M \frac{dV}{dt} = F - kV$ and $V = 0$ when $t = 0$, then V is given by

- a. $V = \frac{k}{F}(1 - e^{-kt/M})$ b. $V = \frac{F}{k}(1 + e^{-kt/M})$
c. $V = \frac{F}{k}(1 - e^{-kt/M})$ d. None of these

17. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines family of circles with

- a. variable radii and a fixed centre at (0, 1)
b. variable radii and a fixed centre at (0, -1)
c. fixed radius 1 and variable centres along the X-axis
d. fixed radius 1 and variable centres along the Y-axis

18. The equation of a curve passing through $(2, 7/2)$ and having gradient $1 - \frac{1}{x^2}$ at (x, y) , is
 a. $y = x^2 + x + 1$ b. $xy = x^2 + x + 1$
 c. $xy = x + 1$ d. None of these
19. The solution of the differential equation $(x^2 - yx^2) \frac{dy}{dx} + y^2 + xy^2 = 0$, is
 a. $\log\left(\frac{x}{y}\right) = \frac{1}{x} + \frac{1}{y} + C$ b. $\log\left(\frac{y}{x}\right) = \frac{1}{x} + \frac{1}{y} + C$
 c. $\log(xy) = \frac{1}{x} + \frac{1}{y} + C$ d. $\log(xy) + \frac{1}{x} + \frac{1}{y} = C$
20. The solution of $\frac{dy}{dx} = \frac{x \log x^2 + x}{\sin y + y \cos y}$, is
 a. $y \sin y = x^2 \log x + C$ b. $y \sin y = x^2 + C$
 c. $y \sin y = x^2 + \log x + C$ d. $y \sin y = x \log x + C$
21. A function $y = f(x)$ has a second order derivative $f''(x) = 6(x - 1)$. If its graph passes through the point $(2, 1)$ and at the point, the tangent to the graph is $y = 3x - 5$, then the function is
 a. $(x + 1)^3$ b. $(x - 1)^3$
 c. $(x + 1)^2$ d. $(x - 1)^2$
22. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is
 a. $2 \log 18$ b. $\log 9$ c. $\frac{1}{2} \log 18$ d. $\log 18$
23. If $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, then at $x = \log_e 2$ the value of y is equal to
 a. 5 b. 13 c. -2 d. 7
24. The solution of $\frac{dy}{dx} = \cos(x + y) + \sin(x + y)$, is
 a. $\log\left[1 + \tan\left(\frac{x+y}{2}\right)\right] + C = 0$
 b. $\log\left[1 + \tan\left(\frac{x+y}{2}\right)\right] = x + C$
 c. $\log\left[1 - \tan\left(\frac{x+y}{2}\right)\right] = x + C$
 d. None of the above
25. The solution of differential equation $(x + y)^2 \frac{dy}{dx} = a^2$, is
 a. $y = a \tan^{-1}\left(\frac{x+y}{a}\right) + C$ b. $y = a \tan^{-1}\left(\frac{x+y}{2}\right) + C$
 c. $y = a \tan^{-1}(x + y) + C$ d. $y = \tan^{-1}\left(\frac{x+y}{a}\right) + C$
26. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi(y/x)}{\phi'(y/x)}$ is
 a. $x \phi(y/x) = C$ b. $\phi(y/x) = Cx$
 c. $y \phi(y/x) = C$ d. $\phi(y/x) = Cy$
27. If $x \frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is
 a. $\log \frac{x}{y} = Cy$ b. $\log \frac{x}{y} = Cx$ c. $\log \frac{y}{x} = Cy$ d. $\log \frac{y}{x} = Cx$
28. If a curve passes through the point $\left(1, \frac{\pi}{4}\right)$ and its slope $\frac{dy}{dx}$ at any point (x, y) is given by $\frac{dy}{dx} = \frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$, then the equation of the curve is
 a. $y = \tan^{-1}\left\{\log\left(\frac{e}{x}\right)\right\}$ b. $y = x \tan^{-1}\left\{\log\left(\frac{x}{e}\right)\right\}$
 c. $y = x \tan^{-1}\left\{\log\left(\frac{e}{x}\right)\right\}$ d. None of these
29. The solution of the differential equation $(x - y) dy - (x + y) dx = 0$, is
 a. $\tan^{-1} \frac{x}{y} + \frac{1}{2} \log(x^2 + y^2) = C$
 b. $\tan^{-1} \frac{y}{x} - \frac{1}{2} \log(x^2 + y^2) = C$
 c. $\cot^{-1}\left(\frac{y}{x}\right) + \frac{1}{2} \log(x^2 + y^2) = C$
 d. None of the above
30. The general solution of $y^2 dx + (x^2 - xy + y^2) dy = 0$, is
 a. $\tan^{-1}\left(\frac{x}{y}\right) + \log y + C = 0$
 b. $2 \tan^{-1}\left(\frac{x}{y}\right) + \log x + C = 0$
 c. $\log(y + \sqrt{x^2 + y^2}) + \log y + C = 0$
 d. $\sin^{-1}\left(\frac{x}{y}\right) + \log y + C = 0$
31. The solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$, is
 a. $ay^2 = e^{x^2/y^2}$ b. $ay = ex^{x/y}$
 c. $y = e^{x^2} + e^{y^2} + C$ d. $y = e^{x^2} + y^2 + C$
32. The solution of differential equation $y dx + (2\sqrt{xy} - x) dy = 0$, is
 a. $\log|y| - \sqrt{\frac{x}{y}} = C$ b. $\log|y| + \sqrt{\frac{x}{y}} = C$
 c. $\log|y| + 2\sqrt{\frac{x}{y}} = C$ d. None of these

33. The solution of the differential equation $x^2 \frac{dy}{dx} = x^2 + xy + y^2$, is
- $\tan^{-1}\left(\frac{y}{x}\right) = 2 \log x + C$
 - $\tan^{-1}\left(\frac{y}{x}\right) = 3 \log x + C$
 - $\tan^{-1}\left(\frac{y}{x}\right) = \log x + C$
 - $\tan^{-1}\left(\frac{y}{x}\right) = 4 \log x + C$
34. Which of the following is the integrating factor of $x \log x \frac{dy}{dx} + y = 2 \log x$?
- x
 - e^x
 - $\log x$
 - $\log(\log x)$
35. The solution of the differential equation $(1 + y^2)dx + (x - e^{-\tan^{-1} y})dy = 0$ is
- $ye^{\tan^{-1} x} = \tan^{-1} x + C$
 - $xe^{\tan^{-1} y} = \tan^{-1} y + C$
 - $y = \tan^{-1} x \cdot e^{\tan^{-1} x} + C$
 - $y = xe^{\tan^{-1} x} + C$
36. The solution of the differential equation is $\cos^2 x \frac{dy}{dx} - (\tan 2x)y = \cos^4 x$, $|x| < \frac{\pi}{4}$, where $y\left(\frac{\pi}{6}\right) = \frac{3\sqrt{3}}{8}$ is
- $\frac{1}{2} \cdot \frac{\cos 2x}{1 - \tan^2 x}$
 - $\frac{1}{2} \cdot \frac{\sin 2x}{1 - \tan^2 x}$
 - $\frac{\sin 2x}{1 + \tan^2 x}$
 - None of these
37. The solution of the equation $x \frac{dy}{dx} + 2y = x^2 \log_e x$ is
- $16yx^2 = x^4 \log(x^4/e) + C$
 - $yx^2 = \frac{1}{4}x^4 \log x - \frac{1}{16}x^4 + C$
 - $16yx^2 = 4x^4 \log x - x^4 + C$
 - All of the above
38. If $\phi(x)$ is a differentiable function, then the solution of the differential equation $dy + \{y\phi'(x) - \phi(x) \cdot \phi'(x)\} dx = 0$ is
- $y = \{\phi(x) - 1\} + Ce^{-\phi(x)}$
 - $y\phi(x) = \{\phi(x)\}^2 + C$
 - $ye^{\phi(x)} = \phi(x)e^{\phi(x)} + C$
 - None of these
39. The solution of the differential equation $\frac{dy}{dx} + \frac{3x^2}{1+x^3}y = \frac{\sin^2 x}{1+x^3}$, is
- $y(1+x^3) = x + \frac{1}{2} \sin 2x + C$
 - $y(1+x^3) = Cx + \frac{1}{2} \sin 2x$
 - $y(1+x^3) = Cx - \frac{1}{2} \sin 2x$
 - $y(1+x^3) = \frac{x}{2} - \frac{1}{4} \sin 2x + C$
40. Consider the differential equation $y^2 dx + \left(x - \frac{1}{y}\right) dy = 0$. If $y(1) = 1$, then x is given by
- $1 - \frac{1}{y} + \frac{e^{1/y}}{e}$
 - $4 - \frac{2}{y} - \frac{e^{1/y}}{e}$
 - $3 - \frac{1}{y} + \frac{e^{1/y}}{e}$
 - $1 + \frac{1}{y} - \frac{e^{1/y}}{e}$
41. The solution of differential equation $\frac{dy}{dx} - 3y \cot x = \sin 2x$, where $y = 2$ and $x = \frac{\pi}{2}$ is
- $y = 4 \sin^3 x + 2 \sin^2 x$
 - $y = 4 \sin^3 x - 2 \sin^2 x$
 - $y = 2 \sin^3 x + 4 \sin^2 x$
 - $y = 2 \sin^3 x - 4 \sin^2 x$
42. The solution of differential equation $\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right] \frac{dx}{dy} = 1$, ($x \neq 0$) is
- $y e^{2\sqrt{x}} = 2\sqrt{x} + C$
 - $y e^{2\sqrt{x}} = 3\sqrt{x} + C$
 - $2y e^{2\sqrt{x}} = 3\sqrt{x} + C$
 - $y e^{\sqrt{x}} = 2\sqrt{x} + C$
43. The solution of differential equation $(x^2 - 1) \frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}$ is
- $y(x^2 - 1) = \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| + C$
 - $y(x^2 + 1) = \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| + C$
 - $y(x^2 + 1) = \frac{1}{3} \log \left| \frac{x-1}{x+1} \right| + C$
 - $y(x^2 - 1) = \frac{1}{3} \log \left| \frac{x-1}{x+1} \right| + C$
44. The solution of the differential equation $x \cos x \left(\frac{dy}{dx}\right) + y(x \sin x + \cos x) = 1$ is
- $xy = \sin x + C \cos x$
 - $xy + \cos x + C \sin x = 0$
 - $xy + \sec x + C \sin x = 0$
 - None of these
45. The solution of differential equation $\cos x dy = y(\sin x - y) dx$, where $0 < x < \frac{\pi}{2}$, is
- $\sec x = (\tan x + C)y$
 - $y \sec x = \tan x + C$
 - $y \tan x = \sec x + C$
 - $\tan x = (\sec x + C)y$
46. The solution of differential equation $(xy^5 + 2y) dx - x dy = 0$, is
- $9x^8 + 4x^9 y^4 = 9y^4 C$
 - $9x^8 - 4x^9 y^4 - 9y^4 C = 0$
 - $x^8(9 + 4y^4) = 10y^4 C$
 - None of the above

47. The solution of the differential equation $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$ is
 a. $e^y = (e^x + 1) + Ce^{-e^x}$ b. $e^y = (e^x - 1) + C$
 c. $e^y = (e^x - 1) + Ce^{-e^x}$ d. None of these
48. If $y = f(x)$ passing through (1, 2) satisfies the differential equation $y(1 + xy)dx - xdy = 0$, then
 a. $f(x) = \frac{2x}{2 - x^2}$ b. $f(x) = \frac{x + 1}{x^2 + 1}$
 c. $f(x) = \frac{x - 1}{4 - x^2}$ d. $f(x) = \frac{4x}{1 - x^2}$
49. If $x dy = y(dx + ydy)$, $y(1) = 1$ and $y(x) > 0$. Then, $y(-3)$ is equal to
 a. 3 b. 2
 c. 1 d. 0
50. The solution of the differential equation $x dx + y dy + \frac{x dy - y dx}{x^2 + y^2} = 0$, is
 a. $y = x \tan\left(\frac{x^2 + y^2 + C}{2}\right)$ b. $x = y \tan\left(\frac{x^2 + y^2 + C}{2}\right)$
 c. $y = x \tan\left(\frac{C - x^2 - y^2}{2}\right)$ d. None of these

BITSAT Archives

1. The form of the differential equation of the central conics $ax^2 + by^2 = 1$ is [2014]
 a. $x = y \frac{dy}{dx}$
 b. $x \left(\frac{dy}{dx}\right)^2 + xy \frac{d^2y}{dx^2} - y \frac{dy}{dx} = 0$
 c. $x + y \frac{d^2y}{dx^2} = 0$
 d. None of the above
2. The solution of the differential equation $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots = \frac{dx - dy}{dx + dy}$ is [2014]
 a. $2ye^{2x} = Ce^{2x} + 1$ b. $2ye^{2x} = Ce^{2x} - 1$
 c. $ye^{2x} = Ce^{2x} + 2$ d. None of these
3. The solution of the differential equation $x = 1 + xy \frac{dy}{dx} + \frac{x^2 y^2}{2!} \left(\frac{dy}{dx}\right)^2 + \frac{x^3 y^3}{3!} \left(\frac{dy}{dx}\right)^3 + \dots$ is [2014]
 a. $y = \log x + C$
 b. $y^2 = (\log x)^2 + C$
 c. $y = \log x + xy$
 d. $xy = x^y + C$
4. The general solution of the differential equation $\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$ is [2013]
 a. $\log \tan\left(\frac{y}{2}\right) = C - 2 \sin x$
 b. $\log \tan\left(\frac{y}{4}\right) = C - 2 \sin\left(\frac{x}{2}\right)$
 c. $\log \tan\left(\frac{y}{2} + \frac{\pi}{4}\right) = C - 2 \sin x$
 d. None of the above
5. By eliminating the arbitrary constants A and B from $y = Ax^2 + Bx$, we get the differential equation [2012]
 a. $\frac{d^3y}{dx^3} = 0$ b. $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$
 c. $\frac{d^2y}{dx^2} = 0$ d. $x^2 \frac{d^2y}{dx^2} + y = 0$
6. The order and degree of the differential equation $\sqrt{\frac{dy}{dx}} - 4 \frac{dy}{dx} - 7x = 0$ are [2011]
 a. 1 and $\frac{1}{2}$ b. 2 and 1 c. 1 and 1 d. 1 and 2
7. The differential equation of all non-vertical lines in a plane is [2010]
 a. $\frac{d^2y}{dx^2} = 0$ b. $\frac{d^2x}{dy^2} = 0$
 c. $\frac{dy}{dx} = 0$ d. $\frac{dx}{dy} = 0$
8. The solution of $\frac{dy}{dx} = \frac{ax + g}{by + f}$ represents a circle, when [2009]
 a. $a = b$ b. $a = -b$ c. $a = -2b$ d. $a = 2b$
9. The equation of the curve satisfying the equation $(xy - x^2) \frac{dy}{dx} = y^2$ and passing through the point $(-1, 1)$, is [2009]
 a. $y = (\log y - 1)x$ b. $y = (\log y + 1)x$
 c. $x = (\log x - 1)y$ d. $x = (\log x - 1)y$
10. The solution of the differential equation $\frac{dy}{dx} = \frac{xy + y}{xy + x}$ is [2008]
 a. $x + y - \log\left(\frac{Cy}{x}\right)$ b. $x + y = \log(Cxy)$
 c. $x - y - \log\left(\frac{Cx}{y}\right)$ d. $y - x = \log\left(\frac{Cx}{y}\right)$

11. The solution of the differential equation $xy^2 dy - (x^3 + y^3) dx = 0$ is [2008]
 a. $y^3 = 3x^3 + C$ b. $y^3 = 3x^3 \log(Cx)$
 c. $y^3 = 3x^3 + \log(Cx)$ d. $y^3 + 3x^3 = \log(Cx)$
12. The solution of the differential equation $\frac{dy}{dx} - y \tan x = e^x \sec x$ is [2008]
 a. $y = e^x \cos x + C$ b. $y \cos x = e^x + C$
 c. $y = e^x \sin x + C$ d. $y \sin x = e^x + C$
13. The solution of the differential equation $\frac{dy}{dx} + \frac{2yx}{1+x^2} = \frac{1}{(1+x^2)^2}$ is [2007]
 a. $y(1+x)^2 = C + \tan^{-1} x$ b. $\frac{y}{1+x^2} = C + \tan^{-1} x$
 c. $y \log(1+x^2) = C + \tan^{-1} x$ d. $y(1+x^2) = C + \sin^{-1} x$
14. The differential equation representing a family of circles touching Y-axis at the origin, is [2006]
 a. $x^2 + y^2 - 2xy \frac{dy}{dx} = 0$ b. $x^2 + y^2 + 2xy \frac{dy}{dx} = 0$
 c. $x^2 - y^2 - 2xy \frac{dy}{dx} = 0$ d. $x^2 - y^2 + 2xy \frac{dy}{dx} = 0$
15. The general solution of the differential equation $(2x - y + 1)dx + (2y - x + 1)dy = 0$ is [2006]
 a. $x^2 + y^2 + xy - x + y = C$ b. $x^2 + y^2 - xy + x + y = C$
 c. $x^2 - y^2 + 2xy - x + y = C$ d. $x^2 - y^2 - 2xy + x - y = C$
16. The degree of the differential equation $y(x) = 1 + \frac{dy}{dx} + \frac{1}{1 \cdot 2} \left(\frac{dy}{dx}\right)^2 + \frac{1}{1 \cdot 2 \cdot 3} \left(\frac{dy}{dx}\right)^3 + \dots$ is [2005]
 a. 2 b. 3
 c. 1 d. None of these
17. The solution of $x^2 + y^2 - 2xy \frac{dy}{dx} = 0$ is [2005]
 a. $x^2 - y^2 = Cx$ b. $x^2 + y^2 = Cx$
 c. $2(x^2 - y^2) = Cx$ d. None of these

Answer with Solutions

Practice Exercise

1. (b) Clearly, order = 3

i.e. order of the highest derivative is $\frac{d^3 y}{dx^3}$.

The given differential equation can be written as

$$\left(\frac{d^2 y}{dx^2}\right)^5 \frac{d^3 y}{dx^3} + 4 \left(\frac{d^2 y}{dx^2}\right)^3 + \left(\frac{d^3 y}{dx^3}\right)^2 = (x^2 - 1) \frac{d^3 y}{dx^3}$$

Clearly, degree = 2 $\therefore m = 3, n = 2$

2. (d) Since, the equation is not a polynomial in all differential coefficients, so its degree is not defined.

3. (a) Given, $\sqrt{\frac{d^2 y}{dx^2}} = \sqrt[3]{\frac{dy}{dx} + 5} \Rightarrow \left(\frac{d^2 y}{dx^2}\right)^3 = \left(\frac{dy}{dx} + 5\right)^2$

Order is 2 and degree is 3.

4. (a) Given, $\left(\frac{d^2 y}{dx^2}\right)^{3/2} = \left(\frac{dy}{dx}\right)^{1/2} + 4$

On squaring both sides, we get

$$\begin{aligned} \left(\frac{d^2 y}{dx^2}\right)^3 &= \frac{dy}{dx} + 8 \left(\frac{dy}{dx}\right)^{1/2} + 16 \\ \Rightarrow \left[\left(\frac{d^2 y}{dx^2}\right)^3 - \frac{dy}{dx} - 16\right]^2 &= 64 \frac{dy}{dx} \\ \Rightarrow \left(\frac{d^2 y}{dx^2}\right)^6 - 32 \left(\frac{d^2 y}{dx^2}\right)^3 - 2 \left(\frac{dy}{dx}\right) \left(\frac{d^2 y}{dx^2}\right)^3 \\ &\quad + \left(\frac{dy}{dx}\right)^2 - 32 \frac{dy}{dx} + 256 = 0 \end{aligned}$$

So, it is clear that order is 2 and degree is 6.

5. (b) On differentiating the given equation, we get

$$2(x - h) + 2(y - k) \frac{dy}{dx} = 0$$

$$\text{or } (x - h) + (y - k) \frac{dy}{dx} = 0 \quad \dots(i)$$

Again differentiating, we get

$$1 + (y - k) \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0 \quad \dots(ii)$$

From Eq. (ii), we get

$$(y - k) = \frac{\left[-1 + \left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2 y}{dx^2}}$$

From Eq. (i), we get

$$\begin{aligned} (x - h) &= -(y - k) \frac{dy}{dx} \\ \Rightarrow (x - h) &= \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right] \frac{dy}{dx}}{\frac{d^2 y}{dx^2}} \end{aligned}$$

On substituting values of $(x - h)$ and $(y - k)$ in the given relation, we get the required relation.

$$\text{i.e. } \left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = a^2 \left(\frac{d^2 y}{dx^2}\right)^2$$

6. (d) The equation of family of parabola with axis parallel to Y-axis is $(x - a)^2 = A(y - b)$

Here, a, b and A are arbitrary constants.

On differentiating with respect to x , we get

$$2(x - a) = A \frac{dy}{dx}$$

Again differentiating, $2 = A \frac{d^2y}{dx^2}$

Now, differentiating, $\frac{d^3y}{dx^3} = 0$

7. (a) **Hint** Let the centre of circle be $(0, k)$, then the radius of circle is equal to k , i.e. $r = k$ as the circle passes through origin.

Thus, equation of circle is

$$(x - 0)^2 + (y - k)^2 = k^2 \Rightarrow x^2 + y^2 - 2ky = 0$$

$$\Rightarrow k = \frac{x^2 + y^2}{2y} \quad \dots(i)$$

$$2x + 2y \frac{dy}{dx} - 2k \frac{dy}{dx} = 0 \Rightarrow x + (y - k) \frac{dy}{dx} = 0$$

8. (c) We have, $y = (c_1 + c_2) \cos(x + c_3) - c_4 e^{x + c_5}$
 $\Rightarrow y = (c_1 \cos c_3 + c_2 \cos c_3) \cos x$
 $\quad - (c_1 \sin c_3 + c_2 \sin c_3) \sin x - c_4 e^{c_5} e^x$
 $\Rightarrow y = A \cos x - B \sin x + C e^x$
 where, $A = c_1 \cos c_3 + c_2 \cos c_3$
 $B = c_1 \sin c_3 + c_2 \sin c_3$ and $C = -c_4 e^{c_5}$

It is an equation containing three arbitrary constants.

So, the order of the differential equation is 3.

9. (d)

10. (c) We know that, $x \cos \alpha + y \sin \alpha = p \quad \dots(i)$

On differentiating Eq. (i) w.r.t. x , we get

$$\cos \alpha + \sin \alpha \frac{dy}{dx} = 0 \Rightarrow \tan \alpha = -\frac{1}{y_1} \quad \left[\because \frac{dy}{dx} = y_1 \right]$$

$$\therefore \sin \alpha = \frac{1}{\sqrt{1 + y_1^2}}, \cos \alpha = \frac{-y_1}{\sqrt{1 + y_1^2}} \quad \dots(ii)$$

$$\text{From Eqs. (i) and (ii), we get } \frac{-xy_1}{\sqrt{1 + y_1^2}} + \frac{y}{\sqrt{1 + y_1^2}} = p$$

$$\Rightarrow (y - xy_1)^2 = p^2(1 + y_1^2)$$

11. (b) Given, $y = e^x(a \cos x + b \sin x)$

On differentiating both sides w.r.t. x , we get

$$y_1 = e^x(-a \sin x + b \cos x) + e^x(a \cos x + b \sin x)$$

$$\Rightarrow y_1 = e^x(-a \sin x + b \cos x) + y$$

Again, differentiating both sides w.r.t. x , we get

$$y_2 = e^x(-a \cos x - b \sin x) + e^x(-a \sin x + b \cos x) + y_1$$

$$\Rightarrow y_2 - 2y_1 + 2y = 0$$

12. (b) **Hint** Let $y = mx + c$ be the general equation of the straight line.

On differentiating both sides w.r.t. x , we get $\frac{dy}{dx} = m$

On substituting it in given equation

$$\frac{dy}{dx} + x \left(\frac{dy}{dx} \right)^2 - y = 0$$

13. (c) Slope $= \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2y} \Rightarrow 2y dy = dx$

On integrating both sides, we get $y^2 = x + C$
 which passes through $(4, 3)$.

$$9 = 4 + C \Rightarrow C = 5$$

So, the equation of the curve is $y^2 = x + 5$.

14. (c) $\frac{dy}{dx} = \frac{ax + h}{by + k} \Rightarrow (by + k)dy = (ax + h)dx$

On integrating, we get $by^2 + ky = \frac{a}{2}x^2 + hx + C$

For this to represent a parabola, one of the term either x^2 or y^2 is zero, therefore either $a = 0, b \neq 0$ or $a \neq 0, b = 0$.

15. (c) **Hint** $2x \frac{dy}{dx} = y + 3 \Rightarrow 2 \frac{dy}{y + 3} = \frac{dx}{x}$

16. (c) $\frac{dV}{F - kV} = \frac{1}{M} dt$

On integrating, we get

$$\int \frac{dV}{F - kV} = \frac{1}{M} \int dt$$

$$\Rightarrow -\frac{1}{k} \log(F - kV) = \frac{t}{M} + C \quad \dots(i)$$

$$\text{When } t = 0, V = 0 \Rightarrow -\frac{1}{k} \log F = C$$

On substituting it in Eq. (i), we get

$$\frac{1}{k} [(\log F - \log(F - kV))] = \frac{t}{M}$$

$$\Rightarrow \log \left(\frac{F}{F - kV} \right) = \frac{kt}{M} \Rightarrow \frac{F - kV}{F} = e^{-kt/M}$$

$$\Rightarrow \frac{kV}{F} = (1 - e^{-kt/M}) \Rightarrow V = \frac{F}{k} (1 - e^{-kt/M})$$

17. (c) $\therefore \int \frac{y}{\sqrt{1 - y^2}} dy = \int dx$

$$\Rightarrow -\sqrt{1 - y^2} = x + C \Rightarrow (x + C)^2 + y^2 = 1$$

Hence, the centre is $(-C, 0)$ and radius is 1.

18. (b) **Hint** We have, $\frac{dy}{dx} = 1 - \frac{1}{x^2} \Rightarrow y = x + \frac{1}{x} + C$

19. (a) **Hint** $(x^2 - yx^2) \frac{dy}{dx} + y^2 + xy^2 = 0$

$$\Rightarrow \frac{1 - y}{y^2} dy + \frac{1 + x}{x^2} dx = 0$$

$$\Rightarrow \left(\frac{1}{y^2} - \frac{1}{y} \right) dy + \left(\frac{1}{x^2} + \frac{1}{x} \right) dx = 0$$

20. (a) $\frac{dy}{dx} = \frac{x \log x^2 + x}{\sin y + y \cos y}$

On separating the variables and integrating, we get

$$\int (\sin y + y \cos y) dy = \int (x \log x^2 + x) dx$$

$$\Rightarrow -\cos y + y \sin y + \cos y = \frac{x^2}{2} \log x^2 - \int \frac{x^2}{2} \cdot \frac{1}{x^2} \cdot 2x dx$$

$$\quad + \int x dx + C$$

$$y \sin y = \frac{x^2}{2} 2 \log x - \int x dx + \int x dx + C$$

$$\therefore y \sin y = x^2 \log x + C$$

21. (b)

22. (a) **Hint** Given, $p'(t) = \frac{dp(t)}{dt} = 0.5 p(t) - 450$

$$\Rightarrow \frac{2dp(t)}{p(t) - 900} = dt \Rightarrow \int \frac{2dp(t)}{p(t) - 900} = \int dt$$

$$\Rightarrow 2 \log |p(t) - 900| = t + C$$

To find the value of C , substitute $t = 0$,

23. (a) Here, $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$

$$\Rightarrow \int \frac{dy}{y+3} = \int dx \Rightarrow \log |y+3| = x + C$$

$$\text{But } y(0) = 2 \Rightarrow \log |2+3| = 0 + C$$

$$\therefore C = \log_e 5$$

$$\Rightarrow \log |y+3| = x + \log_e 5$$

$$\text{When } x = \log_e 2$$

$$\Rightarrow \log |y+3| = \log_e 2 + \log_e 5 = \log_e 10$$

$$\therefore y+3 = 10 \Rightarrow y = 7$$

24. (b) Put $x + y = v$ and $1 + \frac{dy}{dx} = \frac{dv}{dx}$

Therefore, the differential equation reduces to

$$\frac{dv}{dx} = (1 + \cos v) + \sin v = 2 \cos^2 \frac{v}{2} + 2 \sin \frac{v}{2} \cos \frac{v}{2}$$

$$= 2 \cos^2 \frac{v}{2} \left(1 + \tan \frac{v}{2} \right) \Rightarrow \int \frac{\sec^2 \left(\frac{v}{2} \right)}{2 \left[1 + \tan \left(\frac{v}{2} \right) \right]} dv = \int dx$$

$$\therefore \log \left[1 + \tan \left(\frac{x+y}{2} \right) \right] = x + C$$

25. (a) **Hint** Put $x + y = t \Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx}$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{dt}{dx} - 1 \right)$$

$$\text{Then, given equation reduces to } t^2 \left(\frac{dt}{dx} - 1 \right) = a^2$$

$$\Rightarrow t^2 \frac{dt}{dx} = a^2 + t^2 \Rightarrow \int dx = \int \frac{t^2}{a^2 + t^2} dt$$

$$\Rightarrow \int dx = \int \frac{t^2 + a^2 - a^2}{a^2 + t^2} dt$$

26. (b) Let $\frac{y}{x} = v \therefore \frac{dy}{dx} = v + x \frac{dv}{dx}$

$$\text{Then, } \frac{dy}{dx} = \frac{y}{x} + \frac{\phi(y/x)}{\phi'(y/x)}$$

$$\text{reduces to } v + x \frac{dv}{dx} = v + \frac{\phi(v)}{\phi'(v)} \Rightarrow \frac{\phi'(v)}{\phi(v)} dv = \frac{dx}{x}$$

On integrating, we get

$$\log \phi(v) = \log x + \log C \Rightarrow \phi(v) = Cx \Rightarrow \phi\left(\frac{y}{x}\right) = Cx$$

27. (a) $\frac{dy}{dx} = \frac{y}{x} \left(\log \frac{y}{x} + 1 \right)$

$$\text{Put } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\therefore v + x \frac{dv}{dx} = v \log v + v \Rightarrow \frac{dv}{v \log v} = \frac{dx}{x}$$

On integrating, we get

$$\log(\log v) = \log x + \log C = \log Cx$$

$$\therefore \log \frac{y}{x} = Cx$$

28. (c) **Hint** Put $y = vx$

$$\text{Given equation reduces to } \sec^2 v dx = \frac{-1}{x} dx$$

29. (b) **Hint** $\frac{dy}{dx} = \frac{x+y}{x-y}$

...(i)

$$\text{Put } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx} \text{ Eq. (i) reduces to}$$

$$\Rightarrow \frac{1+v}{1+v^2} dv = \frac{dx}{x}$$

$$\Rightarrow \int \frac{1}{1+v^2} dv - \int \frac{v}{1+v^2} dx = \log |x| + C$$

30. (a) **Hint** $\frac{dx}{dy} + \left(\frac{x}{y} \right)^2 - \left(\frac{x}{y} \right) + 1 = 0$

$$\text{Put } v = \frac{x}{y} \Rightarrow x = vy \Rightarrow \frac{dx}{dy} = v + y \frac{dv}{dy}$$

$$\therefore \frac{dv}{v^2 + 1} + \frac{dy}{y} = 0$$

31. (a) **Hint** Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

$$\text{Given equation reduces to } \left(\frac{1}{v^3} + \frac{1}{v} \right) dv = -\frac{dx}{x}$$

32. (b) Given, $y dx + (2\sqrt{xy} - x) dy = 0$

This is homogeneous equation.

To simplify it, put $y = vx$.

On differentiating both sides w.r.t. x , we get

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x - 2\sqrt{xy}} \Rightarrow v + x \frac{dv}{dx} = \frac{v}{1 - 2\sqrt{v}}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{2v^{3/2}}{1 - 2\sqrt{v}}$$

$$\Rightarrow \left(\frac{1 - 2\sqrt{v}}{2v^{3/2}} \right) dv = \frac{dx}{x}$$

On integrating both sides w.r.t. x , we get

$$-v^{-1/2} - \log |v| = \log x + \log C$$

$$\Rightarrow \left(\frac{y}{x} \right)^{-1/2} + \log \left| \frac{y}{x} \right| = -\log x - \log C$$

$$\Rightarrow \sqrt{\frac{x}{y}} + \log |y| = -\log C \Rightarrow \sqrt{\frac{x}{y}} + \log |y| = C$$

33. (c) Hint $\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$... (i)

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

Eq. (i) reduces to $\frac{1}{1+v^2} dv = \frac{1}{x} dx$

34. (c) Given equation is $x \log x \cdot \frac{dy}{dx} + y = 2 \log x$

$\Rightarrow \frac{dy}{dx} + y \cdot \frac{1}{x \log x} = \frac{2}{x}$

This is of the form $\frac{dy}{dx} + Py = Q$

Integrating factor = $e^{\int P dx} = e^{\int \frac{1}{x \log x} dx}$

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$\therefore IF = e^{\int \frac{1}{t} dt} = e^{\log t} = e^{\log \log x} = \log x$

35. (b) Given differential equation is

$(1 + y^2)dx + (x - e^{-\tan^{-1} y})dy = 0$

$\Rightarrow \frac{dx}{dy} = \frac{e^{-\tan^{-1} y} - x}{1 + y^2}$

$\Rightarrow \frac{dx}{dy} + \frac{1}{1 + y^2} x = \frac{e^{-\tan^{-1} y}}{1 + y^2}$

which becomes a linear equation of the form $\frac{dx}{dy} + Rx = S$

where, $R = \frac{1}{1 + y^2}, S = \frac{e^{-\tan^{-1} y}}{1 + y^2}$

$IF = e^{\int R dy} = e^{\int \frac{1}{1 + y^2} dy} = e^{\tan^{-1} y}$

Solution of this differential equation is

$x \times IF = \int S \times IF dy + C$

$\Rightarrow x \times e^{\tan^{-1} y} = \int \frac{e^{-\tan^{-1} y}}{1 + y^2} \times e^{\tan^{-1} y} dy + C$

$\Rightarrow x e^{\tan^{-1} y} = \int \frac{e^0}{1 + y^2} dy + C$

$\Rightarrow x e^{\tan^{-1} y} = \int \frac{1}{1 + y^2} dy + C$

$\Rightarrow x e^{\tan^{-1} y} = \tan^{-1} y + C$

36. (b)

37. (d) Hint $\frac{dy}{dx} + \frac{2}{x} y = x \log x$

$\therefore IF = e^{\int \frac{2}{x} dx} = e^{2 \log x} = x^2$

Hence, required solution is

$y(IF) = \int (x \log x) x^2 dx + C$

$\Rightarrow x^2 y = \int \log x \cdot x^3 dx + C$

$\Rightarrow x^2 y = \log x \cdot x^4 - \frac{x^4}{16} + C$ [integration by parts]

38. (a) We have, $dy + \{y\phi'(x) - \phi(x) \cdot \phi'(x)\} dx = 0$

$\Rightarrow \frac{dy}{dx} + \phi'(x) \cdot y = \phi(x) \cdot \phi'(x)$... (i)

$\therefore IF = e^{\int \phi'(x) dx} = e^{\phi(x)}$

$\therefore$ Solution is $y e^{\phi(x)} = \int \phi(x) \cdot \phi'(x) e^{\phi(x)} dx$

$\Rightarrow y e^{\phi(x)} = \int e^{\phi(x)} \cdot \phi(x) \cdot \phi'(x) dx$

$\Rightarrow y e^{\phi(x)} = \int \phi(x) \cdot e^{\phi(x)} \cdot \phi'(x) dx$

$\Rightarrow y \cdot e^{\phi(x)} = \phi(x) \cdot e^{\phi(x)} - \phi(x) e^{\phi(x)} + C$

$\therefore y = \{\phi(x) - 1\} + C e^{-\phi(x)}$

39. (d) $\frac{dy}{dx} + \frac{3x^2}{1 + x^3} y = \frac{\sin^2 x}{1 + x^3}$

Here, $P = \frac{3x^2}{1 + x^3}$

$\therefore IF = e^{\int P dx} = e^{\log(1 + x^3)} = 1 + x^3$

Thus, solution is $y(1 + x^3) = \int \frac{\sin^2 x}{1 + x^3} (1 + x^3) dx$

$\Rightarrow y(1 + x^3) = \int \frac{1 - \cos 2x}{2} dx$

$\therefore y(1 + x^3) = \frac{1}{2} x - \frac{\sin 2x}{4} + C$

40. (d) Hint $\frac{dx}{dy} + \frac{1}{y^2} \cdot x = \frac{1}{y^3}$ which is a linear differential equation in x.

$\therefore IF = e^{\int \frac{1}{y^2} dy} = e^{-1/y}$

$\therefore$ Complete solution is

$x \cdot e^{-1/y} = \int \frac{1}{y^3} \cdot e^{-1/y} dy + C$

$\Rightarrow x \cdot e^{-1/y} = \int \frac{1}{y} \cdot \frac{1}{y^2} \cdot e^{-1/y} dy + C$

Put $-\frac{1}{y} = t \Rightarrow \frac{1}{y^2} dy = dt$

41. (b) Hint Here, $P = -3 \cot x, Q = \sin 2x$

$\therefore IF = e^{-3 \int \cot x dx} = e^{-3 \log |\sin x|} = e^{\log |\sin x|^{-3}}$

$\Rightarrow IF = \frac{1}{\sin^3 x}$

$\therefore y \times IF = \int Q \times IF dx + C$

$\Rightarrow y \times \frac{1}{\sin^3 x} = \int \frac{1}{\sin^3 x} \sin 2x dx + C$

$\Rightarrow y \times \frac{1}{\sin^3 x} = 2 \int \frac{\sin x \cos x}{\sin^3 x} dx + C$

$\Rightarrow \frac{y}{\sin^3 x} = 2 \int \cot x \operatorname{cosec} x dx + C$

42. (a) Hint $\frac{dy}{dx} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} \Rightarrow \frac{dy}{dx} + \frac{1}{\sqrt{x}} y = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

Here, $P = \frac{1}{\sqrt{x}}, Q = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

$$\begin{aligned} \therefore \quad \text{IF} &= e^{\int \frac{1}{\sqrt{x}} dx} \Rightarrow \text{IF} = e^{2x^{1/2}} = e^{2\sqrt{x}} \\ \therefore \quad y \cdot \text{IF} &= \int Q \times \text{IF} dx + C \\ \therefore \quad ye^{2\sqrt{x}} &= \int e^{2\sqrt{x}} \times \frac{e^{-2\sqrt{x}}}{\sqrt{x}} dx + C \end{aligned}$$

43. (a) The given differential equation is

$$(x^2 - 1) \frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1} \Rightarrow \frac{dy}{dx} + \frac{2x}{x^2 - 1} y = \frac{1}{(x^2 - 1)^2}$$

This is a linear differential equation of the form

$$\frac{dy}{dx} + Py = Q$$

$$\text{Here, } P = \frac{2x}{x^2 - 1} \text{ and } Q = \frac{1}{(x^2 - 1)^2}$$

$$\therefore \text{IF} = e^{\int P dx} = e^{\int \frac{2x}{x^2 - 1} dx} = e^{\log(x^2 - 1)} = (x^2 - 1)$$

The general solution of the given differential equation is

$$y \cdot \text{IF} = \int Q \times \text{IF} dx + C$$

$$\Rightarrow y(x^2 - 1) = \int \frac{1}{x^2 - 1} dx + C \Rightarrow y(x^2 - 1) = \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| + C$$

which is the required solution.

$$44. (a) \text{ Given, } x \cos x \left(\frac{dy}{dx} \right) + y(x \sin x + \cos x) = 1$$

$$\text{We can write, } \frac{dy}{dx} + y \left(\tan x + \frac{1}{x} \right) = \left(\frac{1}{x \cos x} \right)$$

which is a linear differential equation.

$$\therefore \text{IF} = e^{\int \left(\tan x + \frac{1}{x} \right) dx} = e^{\log \sec x + \log x} = e^{\log(x \sec x)} = x \sec x$$

Now, required solution

$$y(\text{IF}) = \int \text{IF} \cdot \frac{1}{x \cos x} dx + C$$

$$\Rightarrow y(x \sec x) = \int x \sec x \cdot \frac{1}{x \cos x} dx + C$$

$$\Rightarrow xy \sec x = \int \sec^2 x dx + C$$

$$\Rightarrow xy \sec x = \tan x + C$$

On multiplying by $\cos x$, we get

$$xy = \sin x + C \cos x$$

$$45. (a) \text{ Since, } \cos x dy = y \sin x dx - y^2 dx$$

$$\Rightarrow \frac{1}{y^2} \frac{dy}{dx} - \frac{1}{y} \tan x = -\sec x$$

$$\text{Put } -\frac{1}{y} = z \Rightarrow \frac{1}{y^2} \frac{dy}{dx} = \frac{dz}{dx}$$

$$\therefore \frac{dz}{dx} + (\tan x) z = -\sec x$$

This is a linear differential equation. Therefore,

$$\text{IF} = e^{\int \tan x dx} = e^{\log \sec x} = \sec x$$

Hence, the solution is

$$z \cdot (\sec x) = \int -\sec x \cdot \sec x dx + C_1$$

$$\Rightarrow -\frac{1}{y} \sec x = -\tan x + C_1$$

$$\therefore \sec x = y(\tan x + C)$$

$$46. (a)$$

$$47. (c) \text{ Given, } \frac{dy}{dx} = \frac{e^x}{e^y} (e^x - e^y) \Rightarrow e^y \frac{dy}{dx} + e^x \cdot e^y = e^x \cdot e^x$$

$$\text{Let } e^y = t \Rightarrow e^y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\text{Then, given equation reduces to } \frac{dt}{dx} + e^x t = e^{2x}$$

$$\text{Here, } P = e^x \text{ and } Q = e^{2x}$$

$$\therefore \text{IF} = e^{\int P dx} = e^{\int e^x dx} = e^{e^x}$$

Required solution is

$$t \cdot e^{e^x} = \int e^{2x} \cdot e^{e^x} dx + C$$

$$\Rightarrow e^y = (e^x - 1) + Ce^{-e^x}$$

$$48. (a)$$

$$49. (a)$$

$$50. (c)$$

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$$1. (b) \text{ We have, } ax^2 + by^2 = 1$$

On differentiating both sides w.r.t. x , we get

$$2ax + 2by \frac{dy}{dx} = 0$$

$$\Rightarrow ax + by \frac{dy}{dx} = 0 \quad \dots(i)$$

$$\Rightarrow \frac{-a}{b} = \frac{y}{x} \frac{dy}{dx} \quad \dots(ii)$$

Again, differentiating Eq. (i) w.r.t. x , we get

$$a + b \left(\frac{dy}{dx} \right)^2 + by \frac{d^2y}{dx^2} = 0$$

$$\Rightarrow -\frac{a}{b} = \left(\frac{dy}{dx} \right)^2 + y \frac{d^2y}{dx^2} \quad \dots(iii)$$

From Eqs. (ii) and (iii), we get

$$\frac{y}{x} \frac{dy}{dx} = \left(\frac{dy}{dx} \right)^2 + y \frac{d^2y}{dx^2} \Rightarrow y \left(\frac{dy}{dx} \right) = x \left(\frac{dy}{dx} \right)^2 + xy \frac{d^2y}{dx^2}$$

$$2. (b) \text{ We have, } \frac{x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots}{1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots} = \frac{dx - dy}{dx + dy}$$

On applying componendo and dividendo, we get

$$\begin{aligned} \frac{\left(x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots \right) + \left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots \right)}{\left(x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots \right) - \left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots \right)} &= \frac{(dx - dy) + (dx + dy)}{(dx - dy) - (dx + dy)} \end{aligned}$$

$$\Rightarrow \frac{\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\dots\right)}{-\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\dots\right)} = \frac{2dx}{-2dy}$$

$$\Rightarrow \frac{e^x}{-e^{-x}} = -\frac{dx}{dy} \Rightarrow \frac{dy}{dx} = \frac{e^{-x}}{e^x}$$

$$\Rightarrow \frac{dy}{dx} = e^{-2x} \Rightarrow dy = e^{-2x} dx$$

On integrating both sides, we get

$$y = \frac{e^{-2x}}{(-2)} + C_1 \Rightarrow 2y = -e^{-2x} + 2C_1$$

$$\Rightarrow 2y = -e^{-2x} + C \quad [\text{where, } C = 2C_1]$$

$$\Rightarrow 2ye^{2x} = -1 + e^{2x} C \Rightarrow 2ye^{2x} = Ce^{2x} - 1$$

3. (b) **Hint** The given equation is reduced to $x = e^{xy\left(\frac{dy}{dx}\right)}$

$$\Rightarrow \log x = xy \frac{dy}{dx} \Rightarrow y dy = \frac{\log x}{x} dx$$

$$\Rightarrow \int y dy = \int \frac{1}{x} \log x dx \quad [\text{where, } C = 2C']$$

4. (b) **Hint** $\frac{dy}{dx} = \sin\left(\frac{x-y}{2}\right) - \sin\left(\frac{x+y}{2}\right)$

$$\Rightarrow \frac{dy}{dx} = -2 \cos\left(\frac{x}{2}\right) \sin\left(\frac{y}{2}\right)$$

$$\Rightarrow \frac{dy}{\sin\left(\frac{y}{2}\right)} = -2 \cos\left(\frac{x}{2}\right) dx$$

$$\Rightarrow \frac{1}{2} \int \operatorname{cosec}\left(\frac{y}{2}\right) dy = -\int \cos\left(\frac{x}{2}\right) dx$$

$$\Rightarrow \frac{1}{2} \left[\frac{\log\left\{\operatorname{cosec}\left(\frac{y}{2}\right) - \cot\left(\frac{y}{2}\right)\right\}}{\frac{1}{2}} \right] = -\frac{\sin\left(\frac{x}{2}\right)}{\frac{1}{2}} + C$$

5. (b) **Hint** Given, $y = Ax^2 + Bx$... (i)

$$\frac{dy}{dx} = 2Ax + B$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2A$$

$$\therefore \frac{dy}{dx} - \frac{d^2y}{dx^2} x = B$$

6. (d) Given, $\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$

On squaring, we get

$$\frac{dy}{dx} = 16\left(\frac{dy}{dx}\right)^2 + 49x^2 + 56x \frac{dy}{dx}$$

Here, order is 1 and degree is 2.

7. (a) **Hint** The general equation of all non-vertical lines in a plane is

$$ax + by = 1$$

where, $b \neq 0$

Now, differentiate it twice.

8. (b) We have, $\frac{dy}{dx} = \frac{ax+g}{by+f}$

$$\Rightarrow (by+f)dy = (ax+g)dx$$

On integrating, we get

$$\frac{by^2}{2} + fy = \frac{ax^2}{2} + gx + C$$

$$\Rightarrow ax^2 - by^2 + 2gx - 2fy + c = 0$$

which represents a circle, if $a = -b$.

9. (a) We have, $(xy - x^2) \frac{dy}{dx} = y^2$

$$\Rightarrow y^2 \frac{dx}{dy} = xy - x^2 \Rightarrow \frac{1}{x^2} \frac{dx}{dy} - \frac{1}{x} \cdot \frac{1}{y} = -\frac{1}{y^2}$$

Put $\frac{1}{x} = v \Rightarrow -\frac{1}{x^2} \frac{dx}{dy} = \frac{dv}{dy}$

$$\therefore \frac{dv}{dy} + \frac{v}{y} = \frac{1}{y^2}, \text{ which is linear.}$$

$$\therefore \text{IF} = e^{\int \frac{1}{y} dy} = e^{\log y} = y$$

$$\therefore \text{The solution is } vy = \int \frac{1}{y^2} \cdot y dy + C$$

$$\Rightarrow \frac{y}{x} = \log y + C \Rightarrow y = x(\log y + C)$$

which passes through the point $(-1, 1)$.

$$\therefore 1 = -1(\log 1 + C)$$

i.e. $C = -1$

Thus, the equation of the curve is $y = x(\log y - 1)$

10. (d) **Hint** $\frac{(1+y)}{y} dy = \frac{(1+x)}{x} dx$

$$\Rightarrow \int \left(\frac{1}{y} + 1\right) dy = \int \left(\frac{1}{x} + 1\right) dx$$

11. (b) **Hint** $\frac{dy}{dx} = \frac{x^3 + y^3}{xy^2}$... (i)

It is a homogeneous differential equation.

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

Eq. (i) reduces to

$$\Rightarrow v^2 dv = \frac{dx}{x}$$

12. (b) Given linear differential equation is

$$\frac{dy}{dx} - y \tan x = e^x \sec x$$

$$\therefore \text{IF} = e^{\int -\tan x dx} = e^{-\log \sec x} = \frac{1}{\sec x}$$

$\therefore$ Complete solution is

$$y \cdot \frac{1}{\sec x} = \int e^x \sec x \cdot \frac{1}{\sec x} dx + C$$

$$\Rightarrow \frac{y}{\sec x} = e^x + C$$

$$\Rightarrow y \cos x = e^x + C$$

13. (a) **Hint** Here, $P = \frac{2x}{1+x^2}$ and $Q = \frac{1}{(1+x^2)^2}$

$$\text{Now, IF} = e^{\int P dx} = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = 1+x^2$$

$\therefore$ Solution of differential equation is

$$y(1+x^2) = \int \frac{1}{(1+x^2)^2} (1+x^2) dx + C$$

14. (d) Since, the circle touches the Y-axis, therefore the centre lies on the X-axis. Let the centre be $(h, 0)$.

$\therefore$ Radius of circle = h

The equation of circle is given by

$$(x-h)^2 + (y-0)^2 = h^2$$

$$\Rightarrow x^2 + y^2 - 2hx = 0 \quad \dots(i)$$

On differentiating both sides w.r.t. x , we get

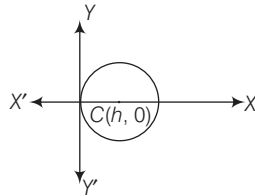
$$2x + 2y \frac{dy}{dx} - 2h = 0 \Rightarrow h = x + y \frac{dy}{dx}$$

Putting the value of h in Eq. (i)

$$x^2 + y^2 - 2x \left(x + y \frac{dy}{dx} \right) = 0$$

$$\Rightarrow -x^2 + y^2 - 2xy \frac{dy}{dx} = 0 \Rightarrow x^2 - y^2 + 2xy \frac{dy}{dx} = 0$$

which is the required differential equation.



15. (b) Given differential equation is

$$(2x - y + 1)dx + (2y - x + 1)dy = 0$$

$$\Rightarrow 2x dx + 2y dy - (y dx + x dy) + dx + dy = 0$$

$$\Rightarrow (2x dx + 2y dy) - d(xy) + dx + dy = 0$$

On integrating both sides, we get

$$x^2 + y^2 - xy + x + y = C$$

16. (c) Given that,

$$y(x) = 1 + \frac{dy}{dx} + \frac{1}{1 \cdot 2} \left(\frac{dy}{dx} \right)^2 + \frac{1}{1 \cdot 2 \cdot 3} \left(\frac{dy}{dx} \right)^3 + \dots$$

$$\text{or } y(x) = 1 + \frac{1}{1!} \left(\frac{dy}{dx} \right) + \frac{1}{2!} \left(\frac{dy}{dx} \right)^2 + \frac{1}{3!} \left(\frac{dy}{dx} \right)^3 + \dots$$

$$\Rightarrow y(x) = e^{dy/dx}$$

Taking log on both sides, we get

$$\log y(x) = \frac{dy}{dx}$$

$\therefore$ The degree of this equation is 1.

17. (a) **Hint** This is a homogeneous equation.

Put $y = vx$

$$\text{and } \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Then, Eq. (i) reduces to

$$\Rightarrow -\frac{2v}{1-v^2} dv = -\frac{dx}{x}$$

22

Probability

Probability

Probability is a measure or estimation of how likely it is that something will happen or that a statement is true. The higher degree of probability, the more likely the event is to happen or in a longer series of samples the greater the number of times such event is expected to happen.

Or

Mathematical measurement of the possibility of an event is called probability.

Important Terminology

Here, we are introducing some important terminology, which are used in probability.

- (i) **Experiment** An activity which gives results in some well-defined outcomes, is called an experiment.
- (ii) **Random Experiment** The experiment about which no confirm prediction is possible, is called the random experiment.
- (iii) **Sample Space** The set of all possible outcomes of an experiment, is called the sample space, which is denoted by S .
- (iv) **Event** Every subset of sample space is defined as the event. It is denoted by E .

Types of Events

According to possibility of outcomes and sample points, events are defined as follows:

- (i) **Null Event or Impossible Event** It has no element and is denoted by ϕ .
- (ii) **Sure Event** Since, $S \subseteq S$, so S is an event which is called sure event.
- (iii) **Simple Event or Elementary Event** It has only one sample point.
- (iv) **Mixed Event or Composite Event or Compound Event** This event has two or more than two sample points or elements.
- (v) **Equally Likely Events** Outcomes are said to be equally likely when we have no reason to believe that one is more likely to occur than the other.

(vi) **Mutually Exclusive Events** Events E_1 and E_2 are mutually exclusive, if $E_1 \cap E_2 = \phi$.

Events $E_1, E_2, E_3, \dots, E_n$ are said to be mutually exclusive events, if

$$E_1 \cap E_2 \cap \dots \cap E_n = \phi$$

(vii) **Mutually Exclusive and Exhaustive Events** Events $E_1, E_2, E_3, \dots, E_n$ are mutually exclusive and exhaustive events, if

$$E_i \cap E_j = \phi, \text{ where } i \neq j \text{ and } i, j = 1, 2, 3, \dots, n$$

$$\text{and } E_1 \cup E_2 \cup E_3 \cup \dots \cup E_n = S$$

(viii) **Complement of an Event E** It is denoted by E' or E^c or $\bar{E}$ and it is the set of all sample points of sample space other than sample points of event E .

Probability of an Event

Let E be an event and $P(E)$ denotes the probability of occurrence of the event E . Then, we have

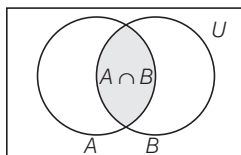
$$\begin{aligned} P(E) &= \frac{n(E)}{n(S)} \\ &= \frac{\text{Number of cases favourable to event } E}{\text{Total number of cases in the sample space}} \end{aligned}$$

It must be clearly understood that, all the cases considered in the above definition should be equally likely.

- NOTE** (i) $P(S) = 1$; $P(\phi) = 0$ (ii) $0 \leq P(E) \leq 1$
 (iii) If $A \subseteq B$, then $P(A) \leq P(B)$ (iv) $P(A) + P(\bar{A}) = 1$
 (v) If $P(A) = \frac{m}{n}$, then
 (a) Probability of odds in favour of $A = \frac{P(A)}{P(\bar{A})} = \frac{m}{n-m}$
 (b) Probability of odds against $A = \frac{P(\bar{A})}{P(A)} = \frac{n-m}{m}$

Addition Theorems on Probability

If $P(A+B)$ or $P(A \cup B)$ = Probability of occurrence of atleast one event A or B and $P(AB)$



or $P(A \cap B)$ = Probability of happening of events A and B together, then

(i) **When Events are Not Mutually Exclusive** If A and B are two events which are not mutually exclusive, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\text{or } P(A+B) = P(A) + P(B) - P(AB)$$

For any three events A, B, C ,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$\text{or } P(A+B+C) = P(A) + P(B) + P(C) - P(AB) - P(BC) - P(CA) + P(ABC)$$

(ii) **When Events are Mutually Exclusive** If A and B are mutually exclusive events, then

$$n(A \cap B) = 0 \Rightarrow P(A \cap B) = 0$$

$$\therefore P(A \cup B) = P(A) + P(B)$$

For any three events A, B, C which are mutually exclusive,

$$\begin{aligned} P(A \cap B) &= P(B \cap C) = P(C \cap A) \\ &= P(A \cap B \cap C) = 0 \end{aligned}$$

$$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

The probability of happening of anyone of several mutually exclusive events is equal to the sum of their probabilities, i.e. if $A_1, A_2, \dots, A_n$ are mutually exclusive events, then

$$P(A_1 + A_2 + \dots + A_n) = P(A_1) + P(A_2) + \dots + P(A_n)$$

$$\text{i.e. } P(\sum A_i) = \sum P(A_i)$$

(iii) **Some Other Theorems**

(a) Let A and B be two events associated with a random experiment, then

$$P(\bar{A} \cap B) = P(B) - P(A \cap B)$$

$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

(b) If $B \subset A$, then $P(A \cap \bar{B}) = P(A) - P(B)$

$$P(B) \leq P(A)$$

Similarly, if $A \subset B$, then $P(\bar{A} \cap B) = P(B) - P(A)$

$$P(A) \leq P(B)$$

NOTE Probability of occurrence of neither A nor B is

$$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$$

(iv) **Generalisation of the Addition Theorem**

If $A_1, A_2, \dots, A_n$ are n events associated with a random experiment, then

$$\begin{aligned} P\left(\bigcup_{i=1}^n A_i\right) &= \sum_{i=1}^n P(A_i) - \sum_{\substack{i,j=1 \\ i \neq j}}^n P(A_i \cap A_j) \\ &\quad + \sum_{\substack{i,j,k=1 \\ i \neq j \neq k}}^n P(A_i \cap A_j \cap A_k) \\ &\quad + \dots + (-1)^{n-1} P(A_1 \cap A_2 \cap \dots \cap A_n) \end{aligned}$$

If all the events $A_i; i=1, 2, \dots, n$ are mutually

exclusive, then $P\left(\bigcup_{i=1}^n A_i\right) = \sum_{i=1}^n P(A_i)$.

- NOTE** • $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$
 • $P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$
 • $P(A) = P(A \cap B) + P(A \cap \bar{B})$
 • $P(B) = P(B \cap A) + P(B \cap \bar{A})$

$$\begin{aligned}
P(\text{exactly one of } E_1, E_2 \text{ occurs}) &= P(E_1 \cap E_2') + P(E_1' \cap E_2) \\
&= P(E_1) - P(E_1 \cap E_2) + P(E_2) - P(E_1 \cap E_2) \\
&= P(E_1) + P(E_2) - 2P(E_1 \cap E_2)
\end{aligned}$$

- $P(\text{neither } E_1 \text{ nor } E_2) = P(E_1' \cap E_2') = 1 - P(E_1 \cup E_2)$
- $P(E_1' \cup E_2') = 1 - P(E_1 \cap E_2)$

Notations Used in Probability

Following are the important notations used in probability:

Verbal description of events	Equivalent set theoretic notation
Only A occurs	$A \cap \bar{B} \cap \bar{C}$
Both A and B but not C occur	$A \cap B \cap \bar{C}$
All the three events occur	$A \cap B \cap C$
Atleast one occurs	$A \cup B \cup C$
Atleast two occur	$(A \cap B) \cup (B \cap C) \cup (A \cap C)$
One and no more occurs	$(A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)$
Exactly two of A, B and C occur	$(A \cap B \cap \bar{C}) \cup (\bar{A} \cap B \cap C) \cup (A \cap \bar{B} \cap C)$
None occurs	$\bar{A} \cap \bar{B} \cap \bar{C} = \overline{A \cup B \cup C}$
Not more than two occur	$(A \cap B) \cup (B \cap C) \cup (A \cap C) - (A \cap B \cap C)$
Exactly one of A and B occurs	$(A \cap \bar{B}) \cup (\bar{A} \cap B)$

Multiplication Theorems on Probability

If A and B are two events associated with a random experiment, then

$$P(A \cap B) = P(A) \cdot P\left(\frac{B}{A}\right), \text{ if } P(A) \neq 0$$

or
$$P(A \cap B) = P(B) \cdot P\left(\frac{A}{B}\right), \text{ if } P(B) \neq 0$$

Extension of Multiplication Theorem

If $A_1, A_2, \dots, A_n$ are n events related to a random experiment, then

$$\begin{aligned}
P(A_1 \cap A_2 \cap A_3 \cap \dots \cap A_n) &= P(A_1) \cdot P\left(\frac{A_2}{A_1}\right) \\
&\quad \cdot P\left(\frac{A_3}{A_1 \cap A_2}\right) \dots P\left(\frac{A_n}{A_1 \cap A_2 \cap \dots \cap A_{n-1}}\right)
\end{aligned}$$

Conditional Probability

Let A and B be two events associated with a random experiment. Then, the probability of occurrence of A under

the condition that B has already occurred and $P(B) \neq 0$, is called the conditional probability and it is denoted by $P\left(\frac{A}{B}\right)$. Thus, $P\left(\frac{A}{B}\right)$ = Probability of occurrence of A , given

that B has already happened $= \frac{P(A \cap B)}{P(B)} = \frac{n(A \cap B)}{n(B)}$

Similarly, $P\left(\frac{B}{A}\right)$ = Probability of occurrence of B , given that A has already happened

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{n(A \cap B)}{n(A)}$$

Properties of Conditional Probability

Let A and B be two events of a sample space S of an experiment, then

(i) $P\left(\frac{S}{A}\right) = P\left(\frac{A}{A}\right) = 1$

(ii) $P\left(\frac{A'}{B}\right) = 1 - P\left(\frac{A}{B}\right)$, where A' is the complement of A .

Total Probability Theorem

If an event A can occur with one of the n mutually exclusive and exhaustive events $B_1, B_2, \dots, B_n$ and the probabilities

$P\left(\frac{A}{B_1}\right), P\left(\frac{A}{B_2}\right), \dots, P\left(\frac{A}{B_n}\right)$ are known, then

$$P(A) = \sum_{i=1}^n P(B_i) \cdot P\left(\frac{A}{B_i}\right)$$

Baye's Theorem

If an event A can occur with one of the n mutually exclusive and exhaustive events $B_1, B_2, \dots, B_n$ and the probabilities

$P\left(\frac{A}{B_i}\right)$ are known, then

$$P\left(\frac{B_i}{A}\right) = \frac{P(B_i) \cdot P\left(\frac{A}{B_i}\right)}{\sum_{i=1}^n P(B_i) \cdot P\left(\frac{A}{B_i}\right)}$$

Independent Events

Two events are said to be independent, if the occurrence of one does not depend upon the other. If $E_1, E_2, \dots, E_n$ are independent events, then

$$P(E_1 \cap E_2 \cap E_3 \cap \dots \cap E_n) = P(E_1) \cdot P(E_2) \dots P(E_n)$$

If E and F are independent events, then the pairs E and $\bar{F}$, $\bar{E}$ and F , $\bar{E}$ and $\bar{F}$ are also independent.

Properties of Independent Events

If A and B are two independent events, then

- A' and B are also independent events.
- A and B' are also independent events.
- A' and B' are also independent events.

Important Results

If n letters corresponding to n envelopes are placed in the envelopes at random, then

- Probability that all letters are in right envelopes = $\frac{1}{n!}$
- Probability that all letters are not in right envelopes = $1 - \frac{1}{n!}$
- Probability that no letter is in right envelopes = $\frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots + (-1)^n \frac{1}{n!}$
- Probability that exactly r letters are in right envelopes = $\frac{1}{r!} \left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots + (-1)^{n-r} \frac{1}{(n-r)!} \right)$

Probability Distribution

A random variable is a real valued function whose domain is the sample space of a random experiment. A random variable is usually denoted by the capital letters $X, Y, Z, \dots$ and so on.

The random variable may be of two types as given below.

1. Discrete Random Variable

A random variable which can take only finite or countably infinite number of values is called a discrete random variable.

2. Continuous Random Variable

A random variable which can take any value between two given limits is called a continuous random variable.

Probability Distribution of a Random Variable

If the values of a random variable together with the corresponding probabilities are given, then this description is called a probability distribution of the random variable.

If a random variable X takes values, $x_1, x_2, x_3, \dots, x_n$ with respective probabilities $p_1, p_2, p_3, \dots, p_n$, then

X	x_1	x_2	x_3	$\dots$	x_n
$P(X)$	p_1	p_2	p_3	$\dots$	p_n

is known as the probability distribution of X .

Mean

If X is a discrete random variable which assumes values $x_1, x_2, x_3, \dots, x_n$ with respective probabilities $p_1, p_2, p_3, \dots, p_n$, then the mean $\bar{X}$ of X is defined as

$$\bar{X} = p_1 x_1 + p_2 x_2 + \dots + p_i x_i \quad \text{or} \quad \bar{X} = \sum_{i=1}^n p_i x_i$$

The mean of a random variable X is also known as its mathematical expectation and it is denoted by $E(X)$.

Variance

If X is a discrete random variable which assumes values $x_1, x_2, x_3, \dots, x_n$ with the respective probabilities $p_1, p_2, \dots, p_n$, then variance of X is defined as

$$\text{Var}(X) = \sum_{i=1}^n p_i x_i^2 - \left(\sum_{i=1}^n p_i x_i \right)^2$$

- NOTE**
- The mean of a random variable X is also known as its mathematical expectation or expected value and is denoted by $E(X)$.
 - The variance and standard deviation of a random variable are always non-negative.

Bernoulli Trials

Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions:

- There should be a finite number of trials.
- The trials should be independent of each other.
- Each trial has exactly two outcomes i.e. success or failure.
- The probability of success (or failure) remains the same in each trial.

Binomial Distribution

The probability of r successes in n independent Bernoulli trials is denoted by $P(X=r)$ and is given by

$$P(X=r) = {}^nC_r p^r q^{n-r}$$

where, p = Probability of success

and q = Probability of failure and $p + q = 1$

- Mean = np
- Variance = npq
- Mean is always greater than variance.

Practice Exercise

- A single letter is selected at random from the word 'PROBABILITY'. The probability that it is a vowel, is
 a. $\frac{3}{11}$ b. $\frac{4}{11}$ c. $\frac{2}{11}$ d. 0
- A five digit numbers is formed by the digits 1, 2, 3, 4, 5 without repetition. Find the probability that the number formed is divisible by 4.
 a. $\frac{1}{5}$ b. $\frac{6}{5}$ c. $\frac{4}{5}$ d. None
- Three identical dice are rolled. The probability that the same number will appear on each of them is
 a. $\frac{1}{6}$ b. $\frac{1}{36}$ c. $\frac{1}{18}$ d. $\frac{3}{28}$
- A determinant is chosen at random from the set of all determinants of order 2 with elements 0 and 1 only. The probability that value of the determinant chosen is positive, is
 a. $\frac{1}{8}$ b. $\frac{3}{16}$
 c. $\frac{1}{4}$ d. None of these
- In shuffling a pack of playing cards, four cards are incidently dropped. The probability that the missing cards should be one from each unit is
 a. $\frac{1}{256}$ b. $\frac{2}{20825}$
 c. $\frac{2197}{20825}$ d. None of these
- If three distinct numbers are chosen randomly from the first 100 natural numbers, then the probability that all three of them are divisible by both 2 and 3, is
 a. $\frac{4}{55}$ b. $\frac{4}{35}$ c. $\frac{4}{33}$ d. $\frac{4}{1155}$
- A four digit numbers is formed with the digits from 1, 2, 3, 4, 5, without repetition. The probability that it is divisible by 3, is
 a. $\frac{1}{3}$ b. $\frac{1}{4}$ c. $\frac{1}{5}$ d. $\frac{1}{6}$
- A and B stand in a ring along with 10 other persons. If the arrangement is at random, then the probability that there are exactly 3 persons between A and B, is
 a. $\frac{1}{11}$ b. $\frac{2}{11}$ c. $\frac{3}{11}$ d. $\frac{1}{12}$
- One function is selected from all the functions $F: S \rightarrow S$, where $S = \{1, 2, 3, 4, 5, 6\}$. The probability that it is onto function, is
 a. $\frac{5}{324}$ b. $\frac{7}{324}$ c. $\frac{5}{162}$ d. $\frac{5}{81}$
- There are 2 teams with n persons in each. The probability of selecting 2 persons from one team and 1 person from the other team is $\frac{6}{7}$, then n is equal to
 a. 3 b. 4 c. 5 d. 6
- If $x_1, x_2, \dots, x_{50}$ are fifty real numbers such that $x_r < x_{r+1}$ for $r = 1, 2, 3, \dots, 49$. Five numbers out of these are picked up at random. The probability that the five numbers have x_{20} as the middle number, is
 a. $\frac{{}^{20}C_2 \times {}^{30}C_2}{{}^{50}C_5}$ b. $\frac{{}^{30}C_2 \times {}^{19}C_2}{{}^{50}C_5}$
 c. $\frac{{}^{19}C_2 \times {}^{31}C_2}{{}^{50}C_5}$ d. None of these
- Two numbers are selected randomly from the set $S = \{1, 2, 3, 4, 5, 6\}$ without replacement one-by-one. The probability that minimum of the two numbers is less than 4, is
 a. $\frac{1}{15}$ b. $\frac{14}{15}$ c. $\frac{1}{5}$ d. $\frac{4}{5}$
- If two events A and B are mutually exclusive events such that $P(A) = \frac{3x+1}{3}$ and $P(B) = \frac{1-x}{4}$, then the set of possible values of x lies in the interval
 a. $[0, 1]$ b. $\left[\frac{1}{3}, \frac{2}{3}\right]$ c. $\left[-\frac{1}{3}, \frac{5}{9}\right]$ d. $\left[-\frac{7}{9}, \frac{4}{9}\right]$
- An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours, is
 a. $\frac{1}{3}$ b. $\frac{2}{7}$ c. $\frac{1}{21}$ d. $\frac{2}{23}$
- Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house, is
 a. $\frac{7}{9}$ b. $\frac{8}{9}$ c. $\frac{1}{9}$ d. $\frac{2}{9}$
- If a number of two digits is formed with the digits 2, 3, 5, 7, 9 without repetition of digits, then the probability that the number formed 35, is
 a. $\frac{1}{10}$ b. $\frac{1}{20}$ c. $\frac{1}{30}$ d. None
- If the papers of 4 students can be checked by anyone of the 7 teachers, then the probability that all the 4 papers are checked by exactly 2 teachers, is equal to
 a. $\frac{12}{49}$ b. $\frac{6}{49}$ c. $\frac{9}{49}$ d. $\frac{15}{49}$

18. If the word 'UNIVERSITY' is arranged randomly, then the probability that both 'I' are not together, is
 a. $\frac{3}{5}$ b. $\frac{2}{5}$ c. $\frac{4}{5}$ d. $\frac{3}{5}$
19. If 6 objects are distributed at random among 6 persons, then the probability that atleast one person does not get any object, is
 a. $\frac{313}{324}$ b. $\frac{315}{322}$ c. $\frac{317}{324}$ d. $\frac{319}{324}$
20. The probability of not getting a sum of 7 in a single throw with a pair of dice, is
 a. $\frac{1}{6}$ b. $\frac{2}{3}$ c. $\frac{1}{3}$ d. $\frac{5}{6}$
21. The events E_1 and E_2 have probabilities 0.25 and 0.50 respectively. If the probability that both E_1 and E_2 occur simultaneously is 0.14. Then, the probability that neither E_1 nor E_2 occurs is
 a. 0.39 b. 0.25
 d. 0.11 d. None of these
22. Four persons are selected at random from a group of 3 men, 2 women and 4 children. What is the chance that exactly two of them are children?
 a. 9/21 b. 10/23 c. 11/24 d. 10/21
23. A coin is tossed 3 times by 2 persons. What is the probability that both get equal number of heads?
 a. $\frac{3}{8}$ b. $\frac{1}{9}$ c. $\frac{5}{16}$ d. None
24. A bag contains 30 balls numbered from 1 to 30. One ball is drawn at random. Find the probability that the number of the ball drawn will be a multiple of 3 or 7.
 a. $\frac{14}{30}$ b. $\frac{40}{900}$ c. $\frac{13}{30}$ d. None
25. A probability that a leap year selected at random contains either 53 Sunday or 53 Monday, is
 a. $\frac{2}{7}$ b. $\frac{4}{7}$ c. $\frac{3}{7}$ d. $\frac{1}{7}$
26. For the three events A, B and C , P (exactly one of the events A or B occurs) = P (exactly one of the events B or C occurs) = P (exactly one of the events C or A occurs) = p and P (all the three events occur simultaneously) = p^2 where $0 < p < 1/2$. Then, the probability of atleast one of the three events A, B and C occurring, is
 a. $\frac{3p + 2p^2}{2}$ b. $\frac{p + 3p^2}{4}$ c. $\frac{p + 3p^2}{2}$ d. $\frac{3p + 2p^2}{4}$
27. A, B, C are events such that
 $P_r(A) = 0.3, P_r(B) = 0.4, P_r(C) = 0.8$
 $P_r(AB) = 0.08, P_r(AC) = 0.28$ and $P_r(ABC) = 0.09$.
 If $P_r(A \cup B \cup C) \geq 0.75$, then $P_r(BC)$ lies in the interval
 a. $0.23 \leq x \leq 0.48$ b. $0.25 \leq x \leq 0.30$
 c. $0.49 \leq x \leq 0.74$ d. None of these
28. If A and B are two events, then the probability that exactly one of them occurs, is given by
 a. $P(A) + P(B) - 2P(A \cap B)$
 b. $P(A \cap B') - P(A' \cap B)$
 c. $P(A \cup B) + P(A \cap B)$
 d. $P(A') + P(B') + 2P(A' \cap B')$
29. Probability that in the toss of two dice, we obtain an even sum or a sum less than 5, is
 a. $\frac{1}{2}$ b. $\frac{1}{6}$ c. $\frac{2}{3}$ d. $\frac{5}{9}$
30. A number is chosen at random among the first 120 natural numbers. The probability of the number chosen being a multiple of 5 or 15, is
 a. $\frac{1}{8}$ b. $\frac{1}{5}$ c. $\frac{1}{24}$ d. $\frac{1}{6}$
31. For any two events A and B , if $P(A \cup B) = P(A \cap B)$, then
 a. $P(A) = P(B)$ b. $P(A) > P(B)$
 c. $P(A) < P(B)$ d. None of these
32. For any two independent events E_1 and E_2 , $P\{(E_1 \cup E_2) \cap (\bar{E}_1 \cap \bar{E}_2)\}$ is
 a. $< \frac{1}{4}$ b. $> \frac{1}{4}$ c. $\geq \frac{1}{2}$ d. None
33. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is
 a. $\frac{4}{5}$ b. $\frac{3}{5}$ c. $\frac{1}{5}$ d. $\frac{2}{5}$
34. A bag contains 3 red and 3 white balls. Two balls are drawn one-by-one. The probability that they are of different colours, is
 a. $\frac{3}{10}$ b. $\frac{2}{5}$ c. $\frac{3}{5}$ d. None
35. Let X be a set containing n elements. Two subsets A and B of X are chosen at random. The probability that $A \cup B = X$ is
 a. $\frac{2^n C_n}{2^{2n}}$ b. $\frac{1}{2^n} C_n$
 c. $1 \cdot 3 \cdot 5 \dots (2n-1) 2^n \cdot n!$ d. $(3/4)^n$
36. In a hurdle race, a runner has probability p of jumping over a specific hurdle. In 5 trials, if the runner succeeded 3 times, then the conditional probability that the runner had succeeded in the first trial, is
 a. $\frac{3}{5}$ b. $\frac{2}{5}$ c. $\frac{1}{5}$ d. None
37. A committee of 4 students is selected at random from a group consisting of 8 boys and 4 girls. If there is atleast one girl in the committee, then the probability that there are exactly 2 girls in the committee, is
 a. $\frac{7}{99}$ b. $\frac{13}{99}$ c. $\frac{14}{99}$ d. None

38. If one Indian and four American men and their wives are to be seated randomly around a circular table. Then, the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife, is
 a. $\frac{1}{2}$ b. $\frac{1}{3}$ c. $\frac{2}{5}$ d. $\frac{1}{5}$
39. If the events A and B are such that $P(A) = \frac{1}{4}$, $P(A/B) = \frac{1}{2}$ and $P(B/A) = \frac{2}{3}$. Then, $P(B)$ is equal to
 a. $\frac{1}{2}$ b. $\frac{1}{6}$ c. $\frac{1}{3}$ d. $\frac{2}{3}$
40. If $P(A) = \frac{2}{5}$, $P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$, then $P\left(\frac{A}{B}\right) \cdot P\left(\frac{B'}{A'}\right)$ is equal to
 a. $\frac{5}{9}$ b. $\frac{5}{7}$ c. $\frac{25}{42}$ d. 1
41. If A and B are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P\left(\frac{A}{B}\right) = \frac{1}{4}$, then $P(A' \cap B')$ is equal to
 a. $\frac{1}{12}$ b. $\frac{3}{4}$ c. $\frac{1}{4}$ d. $\frac{3}{16}$
42. If $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$, then $P\left(\frac{B}{A}\right) + P\left(\frac{A}{B}\right)$ is equal to
 a. $\frac{1}{4}$ b. $\frac{1}{3}$ c. $\frac{5}{12}$ d. $\frac{7}{12}$
43. If two events A and B are such that $0 < P(A)$, $P(B) < 1$ then $P\left(\frac{A}{B}\right) + P\left(\frac{\bar{A}}{\bar{B}}\right)$ is equal to
 a. 1 b. $\frac{3}{2}$
 c. $\frac{1}{2}$ d. None of these
44. A purse contains 4 copper coins and 3 silver coins. The second purse contains 6 copper coins and 2 silver coins. A coin is taken out from any purse. The probability that it is a copper coin, is
 a. $\frac{4}{7}$ b. $\frac{37}{56}$ c. $\frac{3}{7}$ d. $\frac{3}{4}$
45. Bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. The probability that it was drawn from the bag B is
 a. $\frac{5}{9}$ b. $\frac{4}{9}$
 c. $\frac{25}{52}$ d. None of these
46. The chances of defective screws in three boxes A, B and C are $\frac{1}{5}, \frac{1}{6}$ and $\frac{1}{7}$, respectively. A box is selected at random and a screw drawn from it at random is found to be defective. Then, the probability that it come from box A , is
 a. $\frac{16}{29}$ b. $\frac{1}{15}$ c. $\frac{27}{59}$ d. $\frac{42}{107}$
47. In three throws of two dice, the probability of throwing doublets not more than twice is
 a. $\frac{1}{6}$ b. $\frac{5}{72}$ c. $\frac{215}{216}$ d. None
48. A and B are two independent events. The probability that both A and B occur is $\frac{1}{6}$ and the probability that neither of them occurs is $\frac{1}{3}$. The probability of occurrence of A is
 a. $\frac{1}{2}, \frac{1}{3}$ b. $\frac{1}{3}, \frac{1}{4}$ c. $\frac{1}{2}, \frac{1}{4}$ d. None
49. The probability that a 50 year old man will be alive at 60 is 0.83 and the probability that a 45 year old woman will be alive at 55 is 0.87. The probability that atleast one of the will be alive ten years hence, is
 a. 0.221 b. 0.779 c. 0.669 d. 0.758
50. The probability that any of the men A_1, A_2, A_3, A_4 is alive after 95 years of age is $\frac{1}{2}$. The probability that A_1 will die at the age of 95 and will be the first to die is
 a. $\frac{15}{16}$ b. $\frac{15}{64}$ c. $\frac{1}{4}$ d. $\frac{8}{15}$
51. A man throws a fair coin a number of times and gets 2 points for each head he throws and 1 point for each tail he throws. The probability that he gets exactly 6 points is
 a. $\frac{21}{32}$ b. $\frac{23}{32}$ c. $\frac{41}{64}$ d. $\frac{43}{64}$
52. There are 5 balls numbered 1 to 5 and 5 boxes numbered 1 to 5. The balls are kept in the boxes, one in each box. The probability that exactly 2 balls are kept in the corresponding numbered boxes and the remaining 3 balls in the wrong boxes, is
 a. $\frac{1}{5}$ b. $\frac{1}{6}$ c. $\frac{1}{10}$ d. $\frac{1}{12}$
53. A draws two cards at random from a pack of 52 cards. After returning them to the pack and shuffling it, B draws two cards at random. The probability that their draws contain exactly one common card is
 a. $\frac{25}{546}$ b. $\frac{50}{663}$ c. $\frac{25}{663}$ d. $\frac{24}{563}$
54. Let A, B and C try to hit a target simultaneously but independently. Their respective probabilities of hitting the targets are $\frac{3}{4}, \frac{1}{2}$ and $\frac{5}{8}$. The probability that the target is hit by A or B but not by C , is
 a. $\frac{21}{64}$ b. $\frac{7}{8}$ c. $\frac{7}{32}$ d. $\frac{9}{64}$

55. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where

$\overline{A}$ stands for complement of event A . Then, events A and B are

- mutually exclusive and independent
 - independent but not equally likely
 - equally likely but not independent
 - equally likely and mutually exclusive
56. A bag contains 17 tickets numbered 1 to 17. A ticket is drawn and replaced, then one more ticket is drawn and replaced. Probability that first number drawn is even and second is odd, is
- 82/289
 - 72/289
 - 64/289
 - 63/280
57. Three letters are to be sent to different persons and addresses on the three envelopes are also written. Without looking at the addresses, the probability that the letters go into the right envelope is equal to
- $\frac{1}{27}$
 - $\frac{1}{9}$
 - $\frac{4}{27}$
 - $\frac{1}{6}$
58. A fair coin is tossed repeatedly. If tail appears on first four tosses, then the probability of head appearing on fifth toss is equal to
- $\frac{1}{2}$
 - $\frac{1}{32}$
 - $\frac{31}{32}$
 - $\frac{1}{5}$
59. A die is thrown and a card is selected at random from a deck of 52 playing cards. The probability of getting an even number on the die and a spade card is
- $\frac{1}{2}$
 - $\frac{1}{4}$
 - $\frac{1}{8}$
 - $\frac{3}{4}$
60. If A and B are two independent events such that $P(\overline{A}) = \frac{7}{10}$, $P(\overline{B}) = \alpha$ and $P(A \cup B) = \frac{4}{5}$, then the value of α is
- 1
 - 5/7
 - 2/7
 - 1/4
61. In a throw of a die, the probability of getting one in even number of throw is
- $\frac{5}{36}$
 - $\frac{5}{11}$
 - $\frac{6}{11}$
 - $\frac{1}{6}$
62. Two aeroplanes I and II, bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only, if the first misses the target. The probability that the target is hit by the second plane, is
- 0.06
 - 0.14
 - 0.32
 - 0.7
63. A discrete random variable X has the following probability distribution
- | X | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|--------|-----|------|------|------|-------|--------|------------|
| $P(X)$ | C | $2C$ | $2C$ | $3C$ | C^2 | $2C^2$ | $7C^2 + C$ |
- The value of C and the mean of the distribution are
- 1/10 and 3.66
 - 1/20 and 2.66
 - 1/15 and 1.33
 - None of these

64. A random variable X has the probability distribution

X	1	2	3	4	5	6	7	8
$P(X)$	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events $E = \{X \text{ is a prime number}\}$ and $F = \{X < 4\}$, $P(E \cup F)$ is equal to

- 0.87
 - 0.77
 - 0.35
 - 0.50
65. If 10% of bolts produced by a machine are defective, then the probability that out of a sample selected at random, of 7 bolts, not more than 1 is defective, is
- $(1.6)^6(0.9)^4$
 - $(0.9)^6(1.6)$
 - $(1.6)^4(0.9)^4$
 - None
66. If the mean and variance of a binomial variate X are α and 1 respectively, then the probability that X takes a value greater than one equal to
- 13/16
 - 12/16
 - 9/16
 - 11/16
67. A fair coin is tossed n times. If X is the number of times head occur and $P(X = 4)$, $P(X = 5)$ and $P(X = 6)$ are in AP, then n is equal to
- 13
 - 7
 - 11
 - 12
68. If the mean and variance of a binomial distribution are 4 and 3, respectively. Then, the probability of getting exactly six successes in this distribution is
- ${}^{16}C_6 \left(\frac{1}{4}\right)^{10} \left(\frac{3}{4}\right)^6$
 - ${}^{16}C_6 \left(\frac{1}{4}\right)^6 \left(\frac{3}{4}\right)^{10}$
 - ${}^{12}C_6 \left(\frac{1}{4}\right)^{10} \left(\frac{3}{4}\right)^6$
 - None of these
69. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is
- $\frac{17}{3^5}$
 - $\frac{13}{3^5}$
 - $\frac{11}{3^5}$
 - $\frac{10}{3^5}$
70. A die is thrown 20 times. Getting a number greater than 4 is considered a success. Then, mean and variance of the number of successes are
- 6.6, 4.4
 - 6.5, 4.5
 - 6.3, 4.3
 - 6.2, 4.2
71. A fair coin is tossed at a fixed number of times. If the probability of getting 7 heads is equal to getting 9 heads, then the probability of getting 2 heads is
- $\frac{15}{2^8}$
 - $\frac{2}{15}$
 - $\frac{15}{2^{13}}$
 - $\frac{4}{9}$
72. A die is thrown 100 times, getting an even number is considered a success. The variance of the number of successes is
- 10
 - 25
 - 18
 - 10
73. A coin is tossed 7 times. Each time a man calls head. The probability that he wins the toss atleast 4 occasions is
- $\frac{1}{4}$
 - $\frac{5}{8}$
 - $\frac{1}{2}$
 - $\frac{1}{6}$

BITSAT Archives

- Five persons A, B, C, D and E are in queue of a shop. The probability that A and E are always together, is [2014]
 - $\frac{1}{4}$
 - $\frac{2}{3}$
 - $\frac{2}{5}$
 - $\frac{3}{5}$
- The probability of simultaneous occurrence of atleast one of two events A and B is p . If the probability that exactly one of A, B occurs is q , then $P(A') + P(B')$ is equal to [2014]
 - $2 - 2p + q$
 - $2 + 2p - q$
 - $3 - 3p + q$
 - $2 - 4p + q$
- If A and B are two events such that $P(A) = 0.6$, $P(B) = 0.2$ and $P\left(\frac{A}{B}\right) = 0.5$, then $P\left(\frac{A'}{B'}\right)$ is equal to [2013]
 - $\frac{1}{10}$
 - $\frac{3}{10}$
 - $\frac{3}{8}$
 - $\frac{6}{7}$
- A bag contains 3 white and 5 black balls. One ball is drawn at random. Then, the probability that it is white, is [2011]
 - $\frac{1}{8}$
 - $\frac{3}{8}$
 - $\frac{5}{8}$
 - $\frac{3}{5}$
- For any two events A and B , if $P(A \cup B) = 5/6$, $P(A \cap B) = 1/3$, $P(B) = 1/2$, then $P(A)$ is [2011]
 - $1/2$
 - $2/3$
 - $1/3$
 - None of these
- If $\frac{1+3p}{3}$, $\frac{1-p}{4}$ and $\frac{1-2p}{2}$ are mutually exclusive events. Then, range of p is [2010]
 - $\frac{1}{3} \leq p \leq \frac{1}{2}$
 - $\frac{1}{4} \leq p \leq \frac{1}{2}$
 - $\frac{1}{3} \leq p \leq \frac{2}{3}$
 - $\frac{1}{3} \leq p \leq \frac{2}{5}$
- If A and B are two events such that $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$, then [2010]
 - $P(A \cup B) \geq 3/4$
 - $P(A' \cap B) \leq 1/4$
 - $\frac{3}{8} \leq P(A \cap B) \leq \frac{5}{8}$
 - All of these
- A die is tossed twice. The probability of having a number greater than 4 on each toss is [2009]
 - $\frac{1}{3}$
 - $\frac{1}{9}$
 - $\frac{2}{3}$
 - $\frac{1}{12}$
- The probability that a leap year selected at random will contain either 53 Thursday or 53 Friday, is [2009]
 - $3/7$
 - $2/7$
 - $5/7$
 - $1/7$
- An unbiased die is tossed until a number greater than 4 appears. The probability that an even number of tosses is needed, is [2009]
 - $\frac{1}{2}$
 - $\frac{2}{5}$
 - $\frac{1}{5}$
 - $\frac{2}{3}$
- The mean and variance of a binomial variable X are 2 and 1, respectively. The probability that X takes values greater than 1, is [2009]
 - $\frac{5}{16}$
 - $\frac{8}{16}$
 - $\frac{11}{16}$
 - $\frac{1}{16}$
- If A and B are independent events of a random experiment such that $P(A \cap B) = \frac{1}{6}$ and $P(\bar{A} \cap \bar{B}) = \frac{1}{3}$, then $P(A)$ is equal to [2008]
 - $\frac{1}{4}$
 - $\frac{1}{3}$
 - $\frac{5}{7}$
 - $\frac{2}{3}$
- For $k = 1, 2, 3$, the box B_k contains k red balls and $(k+1)$ white balls. Let $P(B_1) = \frac{1}{2}$, $P(B_2) = \frac{1}{3}$ and $P(B_3) = \frac{1}{6}$. A box is selected at random and a ball is drawn from it. If a red ball is drawn, then the probability that it has come from box B_2 , is [2008]
 - $\frac{35}{78}$
 - $\frac{14}{39}$
 - $\frac{10}{13}$
 - $\frac{12}{13}$
- The probability that the same number appear on throwing three dice simultaneously, is [2007]
 - $\frac{1}{36}$
 - $\frac{5}{36}$
 - $\frac{1}{6}$
 - $\frac{4}{13}$
- A coin is tossed n times. If the probability of getting head atleast once is greater than 0.8, then the least value of n is [2007]
 - 2
 - 3
 - 5
 - 4
- Let A and B be two events and $P(A') = 0.3$, $P(B) = 0.4$, $P(A \cap B') = 0.5$, then $P(A \cup B')$ is [2008]
 - 0.5
 - 0.8
 - 1
 - 0.1

Answer with Solutions

Practice Exercise

1. (b) Total number of words = 11

Number of vowels (O, A, I, I) = 4

$$\therefore \text{Required probability} = \frac{4}{11}$$

2. (a) Total number of five digit numbers = $5! = 120$

Now, a number will be divisible by 4, if the last two digits are divisible by 4. Therefore, the last two digits can be 12, 24, 32, 52, i.e. they can be filled in 4 ways.

The remaining three places can be filled in $3!$ ways = 6

Hence, favourable number of ways = $4 \times 6 = 24$

$$\therefore \text{Required probability} = \frac{24}{120} = \frac{1}{5}$$

3. (b) Exhaustive number of cases = $6^3 = 216$

The same number can appear on each of the dice in the following ways (1, 1, 1), (2, 2, 2), (3, 3, 3) ... (6, 6, 6)

So, favourable number of cases = 6

$$\text{Hence, required probability} = \frac{6}{216} = \frac{1}{36}$$

4. (b) Since, each of the four places in a determinant of order 2 can be filled in two ways either by 0 or by 1.

Hence, total number of ways = $2^4 = 16$

Further, the value of the determinant will be positive in the following 3 cases:

$$\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \text{ or } \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \text{ or } \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\text{So, required probability} = \frac{3}{16}$$

5. (c) **Hint** Number of possible ways by which 4 cards can be dropped are ${}^{52}C_4$.

One card from each suit means that there is one diamond, one club, one spade and one heart cards.

Hence, number of favourable ways

$$= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = ({}^{13}C_1)^4$$

6. (d) Here, $n(S) = {}^{100}C_3$

Let E = All three of them are divisible by both 2 and 3.

$\Rightarrow$ Divisible by 6 i.e. {6, 12, 18, ..., 96}

Thus, out of 16, we have to select 3.

i.e. $n(E) = {}^{16}C_3$

$$\therefore \text{Required probability} = \frac{{}^{16}C_3}{{}^{100}C_3} = \frac{4}{1155}$$

7. (c) Four digit numbers = ${}^5P_4 = 120$

It is divisible by 3 in 4! ways = 24

$$\therefore \text{Required probability} = \frac{24}{120} = \frac{1}{5}$$

8. (b) Number of ways of arranging 3 persons between A and B = $2 \times ({}^{10}P_3) \times 7! = 2 \times 10!$

Total number of arrangement of 12 persons is 11!.

$$\therefore \text{Required probability} = \frac{2 \times 10!}{11!} = \frac{2}{11}$$

9. (a) Total number of functions is 6^6 and number of onto function = 6!

$$\therefore \text{Required probability} = \frac{6!}{6^6} = \frac{5}{324}$$

10. (b) Required probability = $\frac{2({}^nC_2) \cdot ({}^nC_1)}{({}^{2n}C_3)} = \frac{6}{7}$

$$\Rightarrow \frac{3n}{2(2n-1)} = \frac{6}{7} \therefore n = 4$$

11. (b) Here, $n(S) = {}^{50}C_5$, $n(E) = {}^{30}C_2 \times {}^{19}C_2$

$$\therefore P(E) = \frac{{}^{30}C_2 \times {}^{19}C_2}{{}^{50}C_5}$$

12. (d) Here, two numbers are selected from {1, 2, 3, 4, 5, 6}.

$\Rightarrow n(S) = 6 \times 5$ [as one-by-one without replacement]

Also, the minimum of the two numbers is less than 4.

$$\therefore n(E) = 6 \times 4$$

We can select one from (1, 2, 3, 4) and other from (1, 2, 3, 4, 5, 6).

$$\therefore \text{Required probability} = \frac{n(E)}{n(S)} = \frac{24}{30} = \frac{4}{5}$$

13. (c) $\therefore 0 \leq \frac{3x+1}{3} \leq 1, 0 \leq \frac{1-x}{4} \leq 1$ and $0 \leq \frac{3x+1}{3} + \frac{1-x}{4} \leq 1$

$$\therefore 0 \leq 3x+1 \leq 3, 0 \leq 1-x \leq 4, 0 \leq \frac{12x+4+3-3x}{12} \leq 1$$

$$\Rightarrow -1 \leq 3x \leq 2, 0 \leq 1-x \leq 4, 0 \leq 9x+7 \leq 12$$

$$\Rightarrow -\frac{1}{3} \leq x \leq \frac{2}{3}, -1 \leq -x \leq 3, -7 \leq 9x \leq 5$$

$$\Rightarrow -\frac{1}{3} \leq x \leq \frac{2}{3}, -3 \leq x \leq 1, -\frac{7}{9} \leq x \leq \frac{5}{9}$$

$$\therefore -\frac{1}{3} \leq x \leq \frac{5}{9}$$

14. (b) Total number of cases = ${}^9C_3 = 84$

Number of favourable cases = ${}^3C_1 \cdot {}^4C_1 \cdot {}^2C_1 = 24$

$$\therefore p = \frac{24}{84} = \frac{2}{7}$$

15. (c) All the three persons have three options to apply for a house.

$\therefore$ Total number of cases = 3^3

Now, favourable cases = 3 (either all has applied for house 1 or 2 or 3)

$$\therefore \text{Required probability} = \frac{3}{3^3} = \frac{1}{9}$$

16. (b) Required probability $= \frac{1}{{}^5P_2} = \frac{1}{20}$
17. (b) Total ways in which papers can be checked is equal to 7^4 . Now, two teachers who have to check all the papers can be selected in 7C_2 ways and papers can be checked by them in $(2^4 - 2)$ favourable ways.

$$\text{Thus, required probability} = \frac{{}^7C_2 \cdot (2^4 - 2)}{7^4} = \frac{6}{49}$$

18. (c) Total number of ways $= \frac{10!}{2!}$
 Favourable number of ways for I come together is $9!$.
 Thus, probability that I come together

$$= \frac{9! \times 2!}{10!} = \frac{2}{10} = \frac{1}{5}$$

 Hence, required probability $= 1 - \frac{1}{5} = \frac{4}{5}$

19. (d) Number of ways of distributing 6 objects to 6 persons $= 6^6$

Number of ways of distributing 1 object to each person $= 6!$

$$\therefore \text{Required probability} = 1 - \frac{6!}{6^6} = 1 - \frac{5!}{6^5} = \frac{319}{324}$$

20. (d) Favourable cases are (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1).

$$\text{Probability of getting a sum } 7 = \frac{6}{36} = \frac{1}{6}$$

$$\text{Required probability} = 1 - \frac{1}{6} = \frac{5}{6}$$

21. (a) We have, $P(E_1) = 0.25$, $P(E_2) = 0.50$, $P(E_1 \cap E_2) = 0.14$

$$\begin{aligned} \therefore \text{Required probability} &= P(\overline{E_1} \cap \overline{E_2}) = P(\overline{E_1 \cup E_2}) \\ &= 1 - P(E_1 \cup E_2) = 1 - \{P(E_1) + P(E_2) - P(E_1 \cap E_2)\} \\ &= 1 - \{0.25 + 0.50 - 0.14\} = 0.39 \end{aligned}$$

22. (d) There are $(3M + 2W + 4C) = 9$ persons

Number of ways of selecting 4 persons

$$= {}^9C_4 = \frac{9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4} = 126 \text{ numbers}$$

Number of ways of selecting 4 persons, in which 2 are children, are

$$(i) \quad 2C \text{ and } 2M \Rightarrow {}^4C_2 \times {}^3C_2 = 6 \times 3 = 18 \text{ number}$$

$$(ii) \quad 2C \text{ and } 2W \Rightarrow {}^4C_2 \times {}^2C_2 = 6 \times 1 = 6 \text{ number}$$

$$(iii) \quad 2C \text{ and } 1M \text{ and } 1W = {}^4C_2 \times {}^3C_1 \times {}^2C_1 \\ = 6 \times 3 \times 2 = 36$$

$$\text{Number of favourable cases} = 18 + 6 + 36 = 60$$

$$\therefore \text{Required probability} = \frac{60}{126} = \frac{10}{21}$$

23. (c) Probability of getting no head in three throws

$$= \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

Probability of getting 1 head in three throws

$$= {}^3C_1 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right) = \frac{3}{8}$$

Probability of getting 2 head in three throws

$$= {}^3C_2 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^2 = \frac{3}{8}$$

Probability of getting 3 head in three throws

$$= {}^3C_3 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

Therefore, the required probability,

$$P = \frac{1}{8} \cdot \frac{1}{8} + \frac{3}{8} \cdot \frac{3}{8} + \frac{3}{8} \cdot \frac{3}{8} + \frac{1}{8} \cdot \frac{1}{8} = \frac{20}{64} = \frac{5}{16}$$

24. (c) The probability of the number being a multiple of 3 is

$$P(3, 6, 9, 12, 15, 18, 21, 24, 27, 30) = \frac{10}{30}$$

The probability of the number being a multiple of 7 is

$$P(7, 14, 21, 28) = \frac{4}{30}$$

Here, these two events are not mutually exclusive, it means $P(21) = \frac{1}{30}$.

The probability of getting a number which is multiple of 3 or 7 is

$$= \frac{10}{30} + \frac{4}{30} - \frac{1}{30} = \frac{13}{30}$$

25. (c) A leap year consists of 366 days comprising of 52 weeks and 2 days. There are 7 possibilities for these 2 extra days (i) Sunday, Monday, (ii) Monday, Tuesday (iii) Tuesday, Wednesday (iv) Wednesday, Thursday (v) Thursday, Friday (vi) Friday, Saturday, (vii) Saturday, Sunday.

Let us consider two events.

A : The leap year contains 53 Sunday

B : The leap year contains 53 Monday

Then, we have

$$P(A) = \frac{2}{7}, P(B) = \frac{2}{7}, P(A \cap B) = \frac{1}{7}$$

So, required probability $= P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B) = \frac{2}{7} + \frac{2}{7} - \frac{1}{7} = \frac{3}{7}$$

26. (a) We know that, if

P (exactly one of A or B occurs)

$$= P(A) + P(B) - 2P(A \cap B)$$

Therefore, $P(A) + P(B) - 2P(A \cap C) = p$... (i)

Similarly, $P(B) + P(C) - 2P(C \cap A) = p$... (ii)

and $P(C) + P(A) - 2P(C \cap A) = p$... (iii)

On adding Eqs. (i), (ii) and (iii), we get

$$2[P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)] = 3p$$

$$\Rightarrow P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) = \frac{3p}{2} \dots (iv)$$

We are also given that

$$P(A \cap B \cap C) = p^2 \quad \dots(v)$$

Now, $P(\text{atleast one } A, B \text{ and } C)$

$$\begin{aligned} &= P(A) + P(B) + P(C) \\ &\quad - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C) \\ &= \frac{3p}{2} + p^2 \quad [\text{from Eqs. (iv) and (v)}] \\ &= \frac{3p + 2p^2}{2} \end{aligned}$$

27. (a) Let $P_r(B \cap C) = x$. Then, we have

$$\begin{aligned} &P_r(A \cup B \cup C) \geq 0.75 \\ \Rightarrow &P_r(A) + P_r(B) + P_r(C) - P_r(B \cap C) - P_r(C \cap A) \\ &\quad - P_r(A \cap B) + P_r(A \cap B \cap C) \geq 0.75 \\ \Rightarrow &0.3 + 0.4 + 0.8 - x - 0.28 - 0.08 + 0.09 \geq 0.75 \\ \Rightarrow &0.48 - x \geq 0 \Rightarrow 0.48 \geq x \\ \Rightarrow &x \leq 0.48 \quad \dots(i) \end{aligned}$$

$$\text{Similarly, } P_r(A \cup B \cup C) \leq 1 \Rightarrow 0.23 - x \leq 0$$

$$\Rightarrow 0.23 \leq x \quad \dots(ii)$$

From Eqs. (i) and (ii), we get $0.23 \leq x \leq 0.48$

28. (a) $P(\text{Exactly one of the events})$

$$= P(A) + P(B) - 2P(A \cap B)$$

29. (d) Let A be the event of obtaining an even sum and B be the event of obtaining a sum less than five.

Then, we have to find $P(A \cup B)$.

Since, A, B are not mutually exclusive,

$$\begin{aligned} \therefore P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{18}{36} + \frac{6}{36} - \frac{4}{36} = \frac{5}{9} \end{aligned}$$

[since, there are 18 ways to get an even sum and 6 ways to get a sum less than 5 i.e. (1, 3), (3, 1), (2, 2), (1, 2), (2, 1), (1, 1) and 4 ways to get an even sum less than 5 i.e. (1, 3), (3, 1), (2, 2), (1, 1)]

30. (b) In first 120 natural numbers, total number of multiples of 5, $n(A) = 24$ and total number of multiples of 15,

$$n(B) = 8 \text{ and } n(A \cap B) = 8.$$

$$\begin{aligned} \therefore n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 24 + 8 - 8 = 24 \end{aligned}$$

$$\therefore \text{Required probability} = \frac{24}{120} = \frac{1}{5}$$

31. (a) Given, $P(A \cup B) = P(A \cap B)$

This is possible only when $P(A) = P(B)$.

32. (a) Since, $\overline{E_1} \cap \overline{E_2} = \overline{E_1 \cup E_2}$

$$\text{and } (E_1 \cup E_2) \cap (\overline{E_1} \cap \overline{E_2}) = \phi$$

$$P\{(E_1 \cup E_2) \cap (\overline{E_1} \cap \overline{E_2})\} = P(\phi) = 0 < 1/4$$

33. (d) The probability that Mr. A selected the losing horse

$$= \frac{4}{5} \times \frac{3}{4} = \frac{3}{5}$$

The probability that Mr. A selected the winning horse

$$= 1 - \frac{3}{5} = \frac{2}{5}$$

34. (c) Let A be an event that drawn ball is red and B be an event that drawn ball is white.

Then, AB and BA are two disjoint cases of the given event.

$$\begin{aligned} \therefore P(AB + BA) &= P(AB) + P(BA) \\ &= P(A) \cdot P\left(\frac{B}{A}\right) + P(B) \cdot P\left(\frac{A}{B}\right) = \frac{3}{6} \cdot \frac{3}{5} + \frac{3}{6} \cdot \frac{3}{5} = \frac{3}{5} \end{aligned}$$

35. (d) We know that, the number of subset of a set containing n elements is 2^n . Therefore, the number of ways of choosing A and B is ${}^{2^n}C_1 \times {}^{2^n}C_1 = 2^n \times 2^n = 4^n$

When the set A consisting of r elements is chosen from X , then the probability of choosing the set B from amongst the remaining $(n - r)$ elements is $2^{n-r} / 2^n$.

Hence, the probability that A and B have no common element is given by

$$\sum_{r=0}^n \frac{{}^nC_r \cdot 2^{n-r}}{2^n} = \frac{1}{4^n} \sum {}^nC_r 2^{n-r} = \frac{1}{4^n} (1 + 2)^n = (3/4)^n$$

[by binomial theorem]

36. (a)

37. (d) Let A denote the event that atleast one girl will be chosen and B the event that exactly 2 girls will be chosen. We require $P(B/A)$.

Since, A denotes the event that atleast one girl will be chosen and A denotes that no girl is chosen i.e. 4 boys are chosen.

$$\text{Then, } P(A') = \frac{{}^8C_4}{{}^{12}C_4} = \frac{70}{495} = \frac{14}{99} \text{ and } P(A) = 1 - \frac{14}{99} = \frac{85}{99}$$

$$\begin{aligned} \text{Now, } P(A \cap B) &= P(2 \text{ boys and } 2 \text{ girls}) \\ &= \frac{{}^8C_2 \cdot {}^2C_2}{{}^{12}C_4} = \frac{6 \times 28}{495} = \frac{56}{165} \end{aligned}$$

$$\text{Thus, } P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{56}{165} \times \frac{99}{85} = \frac{168}{425}$$

38. (c) Let E be the event when each American man is seated adjacent to his wife and A be the event when Indian man is seated adjacent of his wife.

$$\text{Now, } n(A \cap E) = (4!) \times (2!)^5$$

Event when each American man is seated adjacent to his wife.

$$\text{Again, } n(E) = (5!) \times (2!)^4$$

$$\therefore P\left(\frac{A}{E}\right) = \frac{n(A \cap E)}{n(E)} = \frac{(4!) \times (2!)^5}{(5!) \times (2!)^4} = \frac{2}{5}$$

39. (c) Given that, $P(A) = \frac{1}{4}$, $P\left(\frac{A}{B}\right) = \frac{1}{2}$ and $P\left(\frac{B}{A}\right) = \frac{2}{3}$

$$\text{We know that, } P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$\text{and } P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$$

$$\therefore P(B) = \frac{P\left(\frac{B}{A}\right) \cdot P(A)}{P\left(\frac{A}{B}\right)} = \frac{\left(\frac{2}{3}\right) \left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)} = \frac{1}{3}$$

40. (a) Hint $P(A) = \frac{2}{5}$, $P(B) = \frac{3}{10}$, $P(A \cap B) = \frac{1}{5}$, then

$$P(\bar{A}) = 1 - P(A) = \frac{3}{5}, P(\bar{B}) = \frac{7}{10}$$

$$\therefore P(A' \cap B') = 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A \cap B) \\ = 1 - \frac{2}{5} - \frac{3}{10} + \frac{1}{5} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\text{Now, } P\left(\frac{A}{B}\right) \cdot P\left(\frac{B'}{A'}\right) = \frac{P(A \cap B)}{P(B)} \cdot \frac{P(A' \cap B')}{P(A')}$$

41. (c) Hint $P(A' \cap B') = 1 - P(A \cup B)$
 $= 1 - P(A) - P(B) + P(A \cap B)$
 $= 1 - P(A) - P(B) + P\left(\frac{A}{B}\right) \cdot P(B)$

42. (d) $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $\Rightarrow \frac{3}{5} = \frac{3}{10} + \frac{2}{5} - P(A \cap B) \Rightarrow P(A \cap B) = \frac{1}{10}$
 $\therefore P\left(\frac{B}{A}\right) + P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(A)} + \frac{P(A \cap B)}{P(B)}$
 $= \frac{1}{10} \left[\frac{10}{3} + \frac{5}{2} \right] = \frac{1}{10} \left[\frac{20 + 15}{6} \right] = \frac{35}{60} = \frac{7}{12}$

43. (a) $P\left(\frac{A}{B}\right) + P\left(\frac{\bar{A}}{\bar{B}}\right) = \frac{P(A \cap B)}{P(B)} + \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$
 $= \frac{P(A \cap B) + P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \frac{P(\bar{B})}{P(\bar{B})} = 1$

44. (b) Required probability $= \frac{1}{2} \times \frac{4}{7} + \frac{1}{2} \times \frac{6}{8}$
 $= \frac{2}{7} + \frac{3}{8} = \frac{16 + 21}{56} = \frac{37}{56}$

45. (c) Let E_1 be the event that the ball is drawn from bag A, E_2 be the event that ball is drawn from bag B and E be the event that the ball drawn is red. We have to find $P(E_2/E)$.
 As both the bags are equally likely to be selected, we have

$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$\text{Also, } P(E/E_1) = \frac{3}{5} \text{ and } P(E/E_2) = \frac{5}{9}$$

Hence, by Baye's theorem,

$$P(E_2/E) = \frac{P(E_2) \cdot P(E/E_2)}{P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2)}$$

$$= \frac{\frac{1}{2} \cdot \frac{5}{9}}{\frac{1}{2} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{5}{9}} = \frac{25}{52}$$

46. (d) Here, $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

$$\therefore P\left(\frac{A}{E_1}\right) = \frac{1}{5}, P\left(\frac{A}{E_2}\right) = \frac{1}{6}, P\left(\frac{A}{E_3}\right) = \frac{1}{7}$$

$$\therefore P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{3} \times \frac{1}{5}}{\frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{7}} = \frac{42}{107}$$

47. (c) Total number of possible cases in a throw of two dice is $6 \times 6 = 36$.

Favourable case $[(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)] = 6$

$$\text{Probability of getting double} = \frac{6}{36} = \frac{1}{6}$$

Probability of getting all three double

$$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$$

$\therefore$ Probability of throwing doublets not more than twice

$$= 1 - \frac{1}{216} = \frac{215}{216}$$

48. (a) Let $P(A) = p_1$ and $P(B) = p_2$

$$\text{Then, } P(AB) = p_1 p_2 = \frac{1}{6} \quad \dots(i)$$

$$\text{Also, } q_1 q_2 = \frac{1}{3} \text{ or } (1 - p_1)(1 - p_2) = \frac{1}{3}$$

$$\text{or } 1 - (p_1 + p_2) + p_1 p_2 = \frac{1}{3}$$

$$\text{or } p_1 + p_2 = 1 + \frac{1}{6} - \frac{1}{3} = \frac{7}{6} - \frac{1}{3}$$

$$\Rightarrow p_1 + p_2 = \frac{7 - 2}{6} = \frac{5}{6} \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$p_1 = \frac{1}{2} \text{ or } \frac{1}{3}$$

49. (b) Probability of the man living upto 60 $= p_1 = 0.83$

Probability of the man not living upto 60

$$= q_1 = 1 - 0.83 = 0.17$$

Probability of the woman living upto 55

$$= p_2 = 0.87$$

Probability of the woman not living upto 55

$$= q_2 = 1 - 0.87 = 0.13$$

Probability that none of them lives for 10 yr

$$= q_1 q_2 = 0.17 \times 0.13 = 0.221$$

Probability that atleast one of them lives for next 10 yr

$$= 1 - 0.221 = 0.779$$

50. (b)

51. (d) $P(HHH) + P(HHTT) + P(HTTTT) + P(TTTTTT)$

$$= \frac{1}{2^3} + {}^4C_2 \frac{1}{2^4} + {}^5C_1 \frac{1}{2^5} + \frac{1}{2^6}$$

$$= \frac{43}{64}$$

52. (b) The number of ways of choosing the 2 correct boxes is $({}^5C_2)$.

The number of ways of choosing 3 wrong boxes is

$$3! \left(-\frac{1}{3!} + \frac{1}{2!} \right)$$

$$\therefore \text{Required probability} = ({}^5C_2) \cdot 3! \left(-\frac{1}{3!} + \frac{1}{2!} \right) \div 5!$$

$$= \frac{10 \times 2}{120} = \frac{1}{6}$$

53. (b) The probability of both drawing the common cards x ,
 $P(X) = (\text{Probability of } A \text{ drawing the card } x \text{ and any other card } y) \times (\text{Probability of } B \text{ drawing the card } x \text{ and a card other than } y)$

$$= \frac{{}^{51}C_1}{{}^{52}C_2} \times \frac{{}^{50}C_1}{{}^{52}C_2}, \forall x, \text{ where } x \text{ has 52 values.}$$

$\therefore$ Required probability $= \sum P(X)$

$$= 52 \times \frac{51 \times 50 \times 4}{52 \times 51 \times 52 \times 51} = \frac{50}{663}$$

54. (a) $P(A \cup B \cap \bar{C}) = P(A \cup B) \times P(\bar{C})$
 $= [P(A) + P(B) - P(A \cap B)] \times P(\bar{C})$
 $= \left(\frac{3}{4} + \frac{1}{2} - \frac{3}{8} \right) \times \frac{3}{8} \left[\because P(A \cap B) = P(A) \cdot P(B) = \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8} \right]$
 $= \frac{7}{8} \cdot \frac{3}{8} = \frac{21}{64}$

55. (b) Since, $1 - P(A \cup B) = \frac{1}{6} \Rightarrow P(\bar{A}) - P(B) + \frac{1}{4} = \frac{1}{6}$

$$\Rightarrow P(B) = \frac{1}{3} \text{ and } P(A) = \frac{3}{4}$$

$$\text{Now, } P(A \cap B) = \frac{1}{4} = P(A) P(B)$$

Hence, the events A and B are independent events but not equally likely.

56. (b) Let A be the event that first drawn number is even and B be the event that the second drawn number is odd.

$$\therefore \text{Total number of even numbers} = \left(\frac{17}{2} \right) = 8$$

$$\Rightarrow P(A) = \frac{8}{17} \text{ and } P(B) = \frac{9}{17}$$

$$\Rightarrow P(A \cap B) = P(A) \times P(B) = \frac{8}{17} \times \frac{9}{17} = \frac{72}{289}$$

57. (d) Total number of ways of placing 3 letters in three envelopes $= 3!$

Out of these ways, only one way is correct.

$$\text{Hence, the required probability} = \frac{1}{3!} = \frac{1}{6}$$

58. (a) Appearance of head on fifth toss does not depend on the outcomes of first four tosses.

$$\text{Hence, } P(\text{head on 5th toss}) = \frac{1}{2}$$

59. (c) Let S be the sample space, then

$$n(S) = 6 \times 52 \Rightarrow n(S) = 312$$

Let E be the required event.

$$\text{i.e. } n(E) = 3 \times 13$$

$$\therefore \text{Required probability} = \frac{n(E)}{n(S)} = \frac{3 \times 13}{6 \times 52} = \frac{1}{8}$$

60. (c) $\therefore P(A \cup B) = 1 - P(\bar{A}) \cdot P(\bar{B}) = \frac{4}{5}$

$$\Rightarrow 1 - \frac{7\alpha}{10} = \frac{4}{5}$$

$$\therefore \frac{7\alpha}{10} = \frac{2}{10} \Rightarrow \alpha = \frac{2}{7}$$

61. (b) Required probability

$$= \left(\frac{5}{6} \right) \left(\frac{1}{6} \right) + \left(\frac{5}{6} \right)^3 \left(\frac{1}{6} \right) + \left(\frac{5}{6} \right)^5 \left(\frac{1}{6} \right) + \dots$$

$$= \frac{(5/6) \cdot (1/6)}{1 - \left(\frac{5}{6} \right)^2} = \frac{5}{36 - 25} = \frac{5}{11}$$

62. (c) Hint Let the events be $A =$ 1st aeroplane hit the target

$B =$ 2nd aeroplane hit the target

and their corresponding probabilities are

$$P(A) = 0.3 \text{ and } P(B) = 0.2$$

$$\Rightarrow P(\bar{A}) = 0.7 \text{ and } P(\bar{B}) = 0.8$$

$\therefore$ Required probability

$$= P(\bar{A})P(B) + P(\bar{A})P(\bar{B})P(\bar{A})P(B) + \dots$$

63. (a) Since, $\sum p_i = 1$, we have

$$C + 2C + 2C + 3C + C^2 + 2C^2 + 7C^2 + C = 1$$

$$\text{i.e. } 10C^2 + 9C - 1 = 0$$

$$\text{i.e. } (10C - 1)(C + 1) = 0$$

$$\Rightarrow C = \frac{1}{10}, C = -1$$

Therefore, the permissible value of $C = \frac{1}{10}$.

$$\text{Mean} = \sum_{i=1}^n x_i p_i = \sum_{i=1}^7 x_i p_i$$

$$= 1 \times \frac{1}{10} + 2 \times \frac{2}{10} + 3 \times \frac{2}{10} + 4 \times \frac{3}{10} + 5 \left(\frac{1}{10} \right)^2$$

$$+ 6 \times 2 \left(\frac{1}{10} \right)^2 + 7 \left[7 \left(\frac{1}{10} \right)^2 + \frac{1}{10} \right]$$

$$= \frac{1}{10} + \frac{4}{10} + \frac{6}{10} + \frac{12}{10} + \frac{5}{100} + \frac{12}{100} + \frac{49}{100} + \frac{7}{10} = 3.66$$

64. (b) Given, $E = \{X \text{ is a prime number}\} = \{2, 3, 5, 7\}$

$$\therefore P(E) = P(X = 2) + P(X = 3) + P(X = 5) + P(X = 7)$$

$$\Rightarrow P(E) = 0.23 + 0.12 + 0.20 + 0.07 = 0.62$$

$$\text{and } F = \{X < 4\} = \{1, 2, 3\}$$

$$\Rightarrow P(F) = P(X = 1) + P(X = 2) + P(X = 3)$$

$$\Rightarrow P(F) = 0.15 + 0.23 + 0.12 = 0.5$$

$$\text{and } E \cap F = \{X \text{ is prime number as well as } < 4\} = \{2, 3\}$$

$$P(E \cap F) = P(X = 2) + P(X = 3)$$

$$= 0.23 + 0.12 = 0.35$$

$\therefore$ Required probability, $P(E \cup F)$

$$= P(E) + P(F) - P(E \cap F)$$

$$= 0.62 + 0.5 - 0.35 = 0.77$$

65. (b) $p = \frac{10}{100} = \frac{1}{10}, q = \frac{9}{10}, p = 0.1, q = 0.9, n = 7$

$\therefore$ Required probability $= P(0) + P(1)$

$$= {}^nC_0(q)^7 + {}^nC_1 q^6 p = {}^7C_0 (0.9)^7 + {}^7C_1 (0.9)^6 (0.1)$$

$$= (0.9)^7 + 7 \times (0.9)^6 \times 0.1 = (0.6)^6 [0.9 + 0.7] = 1.6 \times (0.9)^6$$

66. (d)

67. (b) Since, ${}^nC_4 \frac{1}{2^n}$, ${}^nC_5 \frac{1}{2^n}$ and ${}^nC_6 \frac{1}{2^n}$ are in AP.

Also, nC_4 , nC_5 and nC_6 are in an AP.

$$\therefore 2 \cdot {}^nC_5 = {}^nC_4 + {}^nC_6$$

Dividing by nC_5 both sides, we get

$$\begin{aligned} 2 &= \frac{{}^nC_4}{{}^nC_5} + \frac{{}^nC_6}{{}^nC_5} = \frac{5}{n-4} + \frac{n-5}{6} \\ &= \frac{n^2 - 9n + 50}{6(n-4)} \end{aligned}$$

$$\Rightarrow n^2 - 21n + 98 = 0$$

$$\Rightarrow n = 7, 14$$

68. (b) Here, $npq = 3$ and $np = 4$

$$\therefore \frac{npq}{np} = \frac{3}{4} \Rightarrow q = \frac{3}{4}, p = \frac{1}{4} \text{ and } n = 16$$

$$\therefore \text{Probability of exactly six successes} = {}^{16}C_6 \left(\frac{1}{4}\right)^6 \left(\frac{3}{4}\right)^{10}$$

69. (c) $\therefore$ Probability of guessing a correct answer, $p = \frac{1}{3}$

and probability of guessing a wrong answer, $q = \frac{2}{3}$

$\therefore$ The probability of guessing a 4 or more correct answer

$$\begin{aligned} &= {}^5C_4 \left(\frac{1}{3}\right)^4 \cdot \frac{2}{3} + {}^5C_5 \left(\frac{1}{3}\right)^5 \\ &= 5 \cdot \frac{2}{3^5} + \frac{1}{3^5} = \frac{11}{3^5} \end{aligned}$$

70. (a) Here, $p = \frac{2}{6} = \frac{1}{3}$, $q = 1 - \frac{1}{3} = \frac{2}{3}$

$$\therefore \text{Mean} = np = 20 \times \frac{1}{3} = 6.6$$

$$\text{Variance} = npq = 20 \times \frac{1}{3} \times \frac{2}{3} = \frac{40}{9} = 4.4$$

71. (c) $\therefore P(X=7) = P(X=9)$

$$\Rightarrow {}^nC_7 \cdot \left(\frac{1}{2}\right)^7 \cdot \left(\frac{1}{2}\right)^{n-7} = {}^nC_9 \cdot \left(\frac{1}{2}\right)^{n-9} \cdot \left(\frac{1}{2}\right)^9$$

$$\Rightarrow {}^nC_7 = {}^nC_9 \Rightarrow n = 16 \quad [\because {}^nC_x = {}^nC_y \Rightarrow x + y = n]$$

$$\text{Now, } P(X=2) = {}^{16}C_2 \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^{14} = \frac{{}^{16}C_2}{2^{16}} = \frac{16 \cdot 15}{2^{17}} = \frac{15}{2^{13}}$$

72. (b) Let E = Event of getting an even number = $\{2, 4, 6\}$

$$\text{i.e. } n(E) = 3$$

$$\therefore \text{Probability of success, } p = \frac{3}{6} = \frac{1}{2}$$

$$\text{And probability of failure, } q = \frac{1}{2}$$

$$\therefore \text{Variance} = npq = 100 \times \frac{1}{2} \times \frac{1}{2} = 25$$

73. (c) The man has to win atleast 4 times.

$\therefore$ Required probability

$$\begin{aligned} &= {}^7C_4 \left(\frac{1}{2}\right)^4 \cdot \left(\frac{1}{2}\right)^3 + {}^7C_5 \cdot \left(\frac{1}{2}\right)^5 \cdot \left(\frac{1}{2}\right)^2 + {}^7C_6 \left(\frac{1}{2}\right)^6 \cdot \frac{1}{2} + {}^7C_7 \cdot \left(\frac{1}{2}\right)^7 \\ &= ({}^7C_4 + {}^7C_5 + {}^7C_6 + {}^7C_7) \times \frac{1}{2^7} = \frac{64}{2^7} = \frac{1}{2} \end{aligned}$$

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1. (c) $\therefore$ Total number of ways = 5!

and favourable number of ways = 2.4!

$$\therefore \text{Required probability} = \frac{2.4!}{5!} = \frac{2}{5}$$

2. (a) Since, P (exactly one of A, B occurs) = q .

$$\therefore P(A \cup B) - P(A \cap B) = q$$

$$\Rightarrow p - P(A \cap B) = q \Rightarrow P(A \cap B) = p - q$$

$$\Rightarrow 1 - P(A' \cup B') = p - q \Rightarrow P(A' \cup B') = 1 - p + q$$

$$\Rightarrow P(A') + P(B') - P(A' \cap B') = 1 + q - p$$

$$\begin{aligned} \Rightarrow P(A') + P(B') &= (1 - p + q) + [1 - P(A \cup B)] \\ &= (1 - p + q) + (1 - p) = 2 - 2p + q \end{aligned}$$

3. (c) $\therefore P(A \cap B) = P(A/B) \cdot P(B) = 0.5 \times 0.2 = 0.1$

$$\begin{aligned} \therefore P(A'/B') &= \frac{P(A' \cap B')}{P(B')} = \frac{P(A \cup B) - P(A \cap B)}{P(B')} \\ &= \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{1 - P(A) - P(B) + P(A \cap B)}{1 - 0.2} \\ &= \frac{1 - 0.6 - 0.2 + 0.1}{0.8} = \frac{3}{8} \end{aligned}$$

4. (b) Required probability = $\frac{{}^3C_1}{{}^8C_1} = \frac{3}{8}$

5. (b) Hint Given, $P(A \cup B) = \frac{5}{6}$

$$P(A \cap B) = \frac{1}{3}, P(B) = \frac{1}{2}$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

6. (a) Since, the probability lies between 0 and 1.

$$\therefore 0 \leq \frac{1+3p}{3} \leq 1, 0 \leq \frac{1-p}{4} \leq 1, 0 \leq \frac{1-2p}{2} \leq 1$$

$$\Rightarrow 0 \leq 1+3p \leq 3, 0 \leq 1-p \leq 4, 0 \leq 1-2p \leq 2$$

$$\Rightarrow -\frac{1}{3} \leq p \leq \frac{2}{3}, -3 \leq p \leq 1, -\frac{1}{2} \leq p \leq \frac{1}{2} \quad \dots(i)$$

Again, the events are mutually exclusive.

$$\therefore 0 \leq \frac{1+3p}{3} + \frac{1-p}{4} + \frac{1-2p}{2} \leq 1$$

$$\Rightarrow 0 \leq 13-3p \leq 12 \Rightarrow \frac{1}{3} \leq p \leq \frac{13}{5} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\max \left\{ -\frac{1}{3}, -3, -\frac{1}{2}, \frac{1}{3} \right\} \leq p \leq \min \left\{ \frac{2}{3}, 1, \frac{1}{2}, \frac{13}{5} \right\}$$

$$\Rightarrow \frac{1}{3} \leq p \leq \frac{1}{2}$$

7. (b) $A \subseteq A \cup B \Rightarrow P(A) \leq P(A \cup B) \Rightarrow P(A \cup B) \geq 3/4$

$$\text{Also, } P(A \cap B) = P(A) + P(B) - P(A \cup B) \\ \geq P(A) + P(B) - 1 = \frac{3}{4} + \frac{5}{8} - 1 = \frac{3}{8}$$

$$\text{Now, } A \cap B \subseteq B \Rightarrow P(A \cap B) \leq P(B) = \frac{5}{8}$$

$$\therefore \frac{3}{8} \leq P(A \cap B) \leq \frac{5}{8} \quad \dots(i)$$

$$\text{Next, } P(A \cap B') = P(A) - P(A \cap B)$$

$$\Rightarrow \frac{3}{4} - \frac{5}{8} \leq P(A \cap B') \leq \frac{3}{4} - \frac{3}{8} \Rightarrow \frac{1}{8} \leq P(A \cap B') \leq \frac{3}{8}$$

$$\therefore P(A' \cap B) = P(B) - P(A \cap B)$$

$$\therefore P(A \cap B) = P(B) - P(A' \cap B) \quad [\text{using Eq. (i)}]$$

$$\Rightarrow \frac{3}{8} \leq P(B) - P(A' \cap B) \leq \frac{5}{8} \Rightarrow 0 \leq P(A' \cap B) \leq \frac{1}{4}$$

8. (b) Required probability $= P(A) \cdot P(B) = \frac{2}{6} \cdot \frac{2}{6} = \frac{1}{9}$

9. (a) A leap year consists of 366 days, i.e. 52 full weeks and two extra days. These two extra days can be anyone of the following possible outcomes ; (i) Monday and Tuesday, (ii) Tuesday and Wednesday, (iii) Wednesday and Thursday, (iv) Thursday and Friday, (v) Friday and Saturday (vi) Saturday and Sunday (vii) Sunday and Monday.

Let A and B be the event that a leap year contains 53 Thursday and 53 Friday, respectively. Then,

$$P(A) = \frac{2}{7}, P(B) = \frac{2}{7} \text{ and } P(A \cap B) = \frac{1}{7}$$

$\therefore$ Required probability is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \\ = \frac{2}{7} + \frac{2}{7} - \frac{1}{7} = \frac{3}{7}$$

10. (b) p = Probability of success (s) $= \frac{2}{6} = \frac{1}{3}$

$$q = \text{Probability of failure (f)} = 1 - \frac{1}{3} = \frac{2}{3}$$

Probability that success occurs in even number of tosses

$$= p(fs) + p(fffs) + p(fffffs) + \dots$$

$$= qp + q^3p + q^5p + \dots = \frac{qp}{1 - q^2}$$

$$= \frac{\frac{2}{3} \times \frac{1}{3}}{1 - \left(\frac{2}{3}\right)^2} = \frac{\frac{2}{9}}{1 - \frac{4}{9}} = \frac{2}{9} \times \frac{9}{5} = \frac{2}{5}$$

11. (c)

12. (b) Given that, $P(A \cap B) = \frac{1}{6}$ and $P(\bar{A} \cap \bar{B}) = \frac{1}{3}$

Since, A and B are independent.

$$\therefore P(A) \cdot P(B) = \frac{1}{6} \text{ and } P(\bar{A}) \cdot P(\bar{B}) = \frac{1}{3}$$

$$\Rightarrow [1 - P(A)][1 - P(B)] = \frac{1}{3}$$

$$\Rightarrow 1 - [P(A) + P(B)] + P(A)P(B) = \frac{1}{3}$$

$$\Rightarrow 1 + \frac{1}{6} - \frac{1}{3} = P(A) + P(B) \Rightarrow P(A) + P(B) = \frac{5}{6}$$

$$\Rightarrow P(A) = \frac{1}{3}, P(B) = \frac{1}{2}$$

13. (b) In a box, $B_1 = 1R, 2W$, $B_2 = 2R, 3W$ and $B_3 = 3R, 4W$

$$\text{Also given that, } P(B_1) = \frac{1}{2}, P(B_2) = \frac{1}{3} \text{ and } P(B_3) = \frac{1}{6}$$

$$\therefore P\left(\frac{B_2}{R}\right) = \frac{P(B_2) \cdot P\left(\frac{R}{B_2}\right)}{P(B_1) \cdot P\left(\frac{R}{B_1}\right) + P(B_2) \cdot P\left(\frac{R}{B_2}\right) + P(B_3) \cdot P\left(\frac{R}{B_3}\right)} \\ = \frac{\frac{1}{3} \times \frac{2}{5}}{\frac{1}{2} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{5} + \frac{1}{6} \times \frac{3}{7}} = \frac{\frac{2}{15}}{\frac{1}{6} + \frac{2}{15} + \frac{1}{14}} \\ = \frac{\frac{2}{15}}{\frac{2/15}{35 + 28 + 15}} = \frac{2}{15} \times \frac{210}{78} = \frac{14}{39}$$

14. (a) Total number of favourable cases = 6

$$\text{Total number of cases} = 216$$

$$\therefore \text{ Required probability} = \frac{6}{216} = \frac{1}{36}$$

15. (b) Let X be the number of heads getting in n tossed. Therefore, X follows binomial distribution with parameters

$$n, p = \frac{1}{2}, q = \frac{1}{2}$$

Given that, $P(X \geq 1) \geq 0.8$

$$\therefore 1 - P(X = 0) \geq 0.8 \Rightarrow P(X = 0) \leq 0.2$$

$$\Rightarrow {}^nC_0 \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^0 \leq 0.2 \Rightarrow \frac{1}{2^n} \leq \frac{1}{5} \Rightarrow 2^n \geq 5$$

$\therefore$ The least value of n is 3.

16. (b) Given that, $P(A') = 0.3$, $P(B) = 0.4$ and $P(A \cap B') = 0.5$

$$P(B') = 1 - P(B) = 1 - 0.4 = 0.6$$

$$P(A) = 1 - P(A') = 1 - 0.3 = 0.7$$

$$P(A \cup B') = P(A) + P(B') - P(A \cap B') \\ = 0.7 + 0.6 - 0.5 = 0.8$$

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Vector Algebra

Introduction

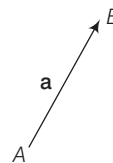
Vectors represent one of the most important mathematical system, which is used to handle different types of problems in Geometry, Mechanics and other branches of applied Mathematics, Physics and Engineering.

Representation of a Vector

A vector is represented by a line segment. e.g. $\mathbf{a} = \overrightarrow{AB}$

Here, A is called the initial point and B is called the terminal point.

Magnitude or modulus of $\mathbf{a}$ is $|\mathbf{a}| = |\overrightarrow{AB}| = AB$.



Types of Vector

Vectors can be defined into following types:

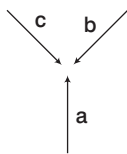
- (i) **Zero or Null Vector** A vector whose magnitude is zero and has arbitrary direction, is known as zero or null vector.
- (ii) **Unit Vector** A vector whose modulus is unity, is known as unit vector.
A unit vector in the direction of a vector $\mathbf{a}$ is denoted by $\hat{\mathbf{a}}$, thus $|\hat{\mathbf{a}}| = 1$.

$$\hat{\mathbf{a}} = \frac{\mathbf{a}}{|\mathbf{a}|}$$

- (iii) **Like and Unlike Vectors** The vectors which have same direction, are called like vectors and which have opposite direction, are called unlike vectors.
- (iv) **Collinear or Parallel Vectors** Vectors having the same or parallel support, are called collinear vectors.
- (v) **Coinitial Vectors** Two or more vectors having the same initial point, are called coinital vectors.
- (vi) **Coplanar Vectors** A system of vectors is said to be coplanar, if their support is parallel to the same plane.

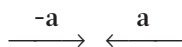
- (vii) **Coterminous Vectors** Vectors which have same terminal points, are called coterminous vectors. e.g.

Here, $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are coterminous vectors.



- (viii) **Negative of a Vector** A vector is said to be negative of a given vector, if its magnitude is the same as that of the given vector but direction is opposite.

e.g.

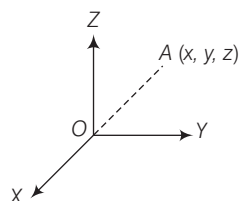


- (ix) **Reciprocal of a Vector** A vector having same direction as that of a given vector $\mathbf{a}$ but magnitude equal to the reciprocal of the given vector, is known as the reciprocal of $\mathbf{a}$ and it is denoted by $\mathbf{a}^{-1}$.

$$|\mathbf{a}^{-1}| = \frac{1}{|\mathbf{a}|}$$

- (x) **Localised and Free Vectors** Those vectors which have not fixed initial point, are called free vectors and a vector which is drawn parallel to a given vector through a specified point in space, is called localised vector.

- (xi) **Position Vectors** The vector $\mathbf{OA}$ which represents the position of the point A with respect to a fixed point O , are called position vectors.



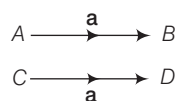
- (xii) **Equal Vectors** Two vectors $\mathbf{a}$ and $\mathbf{b}$ are said to be equal, if

(a) $|\mathbf{a}| = |\mathbf{b}|$

(b) they have the same or parallel support

(c) they have same direction

e.g.



- (xiii) **Orthogonal Vectors** Two vectors are called orthogonal, if the angle between them is a right angle.

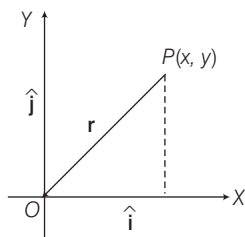
Components of a Vector in Two and Three Dimensional System

- (i) Any vector $\mathbf{r}$ can be expressed as a linear combination of two unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ at right angle. i.e.

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

The vectors $x\hat{\mathbf{i}}$ and $y\hat{\mathbf{j}}$ are vector components of vector $\mathbf{r}$. The scalars x and y are called the scalar components of $\mathbf{r}$ in the direction of X -axis and Y -axis.

$$|\mathbf{r}| = \sqrt{x^2 + y^2} \Rightarrow \theta = \tan^{-1} \frac{y}{x}$$



- (ii) The position vector of $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

The vectors $x\hat{\mathbf{i}}, y\hat{\mathbf{j}}$ and $z\hat{\mathbf{k}}$ are vector components of $\mathbf{r}$. The scalars x, y and z are scalar components of $\mathbf{r}$ in the direction of X -axis, Y -axis and Z -axis.

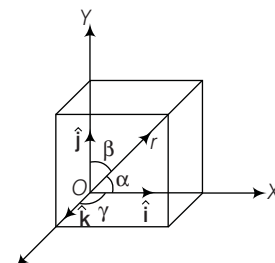
$$|\mathbf{r}| = \sqrt{x^2 + y^2 + z^2}$$

Direction cosines of $\mathbf{r}$ are $\cos\alpha, \cos\beta$ and $\cos\gamma$ such

$$\text{that } \cos\alpha = l = \frac{x}{|\mathbf{r}|}$$

$$\cos\beta = m = \frac{y}{|\mathbf{r}|}$$

$$\cos\gamma = n = \frac{z}{|\mathbf{r}|}$$



Addition of Vectors

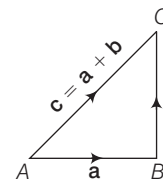
The addition of two vectors $\mathbf{a}$ and $\mathbf{b}$ is denoted by $\mathbf{a} + \mathbf{b}$ and it is known as resultant of $\mathbf{a}$ and $\mathbf{b}$.

There are following three methods of addition of vectors:

(i) Triangle Law of Addition

If two vectors $\mathbf{a}$ and $\mathbf{b}$ lie along the two sides of a triangle in consecutive order (as shown in the figure), then third side represents the sum (resultant) $\mathbf{a} + \mathbf{b}$ i.e.

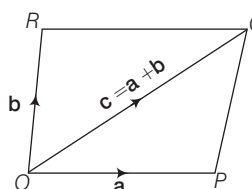
$$\mathbf{c} = \mathbf{a} + \mathbf{b}$$



(ii) Parallelogram Law of Addition

If two vectors are represented by two adjacent sides of a parallelogram, then their sum is represented by the diagonal of the parallelogram.

$$\mathbf{OQ} = \mathbf{OP} + \mathbf{PQ} \Rightarrow \mathbf{c} = \mathbf{a} + \mathbf{b}$$



(iii) Addition in Component Form

If the vectors are defined in terms of $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$, i.e. if

$$\mathbf{a} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}, \quad \mathbf{b} = b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}}$$

Then, $\mathbf{a} + \mathbf{b} = (a_1 + b_1)\hat{\mathbf{i}} + (a_2 + b_2)\hat{\mathbf{j}} + (a_3 + b_3)\hat{\mathbf{k}}$

Properties of Vector Addition

Following are the properties of vector addition:

- **Closure property** The sum of two vectors is always a vector.

- **Commutativity** For any two vectors $\mathbf{a}$ and $\mathbf{b}$, we have

$$\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$$

- **Associativity** For any three vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$, we have

$$\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$$

- **Additive identity** For any vector $\mathbf{a}$, we have

$$\mathbf{0} + \mathbf{a} = \mathbf{a} + \mathbf{0}$$

- **Additive inverse** For every vector $\mathbf{a}$, $(-\mathbf{a})$ is the additive inverse of the vector $\mathbf{a}$.
i.e. $\mathbf{a} + (-\mathbf{a}) = (-\mathbf{a}) + \mathbf{a} = \mathbf{0}$

Subtraction of Vectors

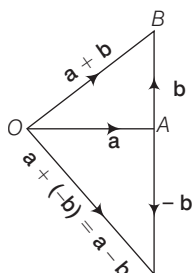
If $\mathbf{a}$ and $\mathbf{b}$ are two vectors, then

$$\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$$

e.g. If $\mathbf{a} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$

and $\mathbf{b} = b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}}$

Then, $\mathbf{a} - \mathbf{b} = (a_1 - b_1)\hat{\mathbf{i}} + (a_2 - b_2)\hat{\mathbf{j}} + (a_3 - b_3)\hat{\mathbf{k}}$



Multiplication of a Vector by a Scalar

If $\mathbf{a}$ is a vector and m is a scalar, then $m\mathbf{a}$ is a vector whose magnitude is m times the magnitude of $\mathbf{a}$.

Properties of Scalar Multiplication

Following are the properties of scalar multiplication:

$$m(-\mathbf{a}) = -m\mathbf{a}$$

$$(-m)(-\mathbf{a}) = m\mathbf{a}$$

$$m(n\mathbf{a}) = (mn)\mathbf{a} = n(m\mathbf{a})$$

$$(m+n)\mathbf{a} = m\mathbf{a} + n\mathbf{a}$$

$$m(\mathbf{a} + \mathbf{b}) = m\mathbf{a} + m\mathbf{b}$$

Linearly Independent Vectors

The vectors $\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n$ in a plane are said to be linearly independent, iff

$$x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \dots + x_n\mathbf{a}_n = \mathbf{0}$$

$$\Rightarrow x_1 = x_2 = \dots = x_n = 0$$

Linearly independent vectors are precisely non-collinear vectors.

Linearly Dependent Vectors

The vectors $\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n$ are said to be linearly dependent, iff there exists scalars $x_1, x_2, \dots, x_n$ not all zero such that

$$x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 + \dots + x_n\mathbf{a}_n = \mathbf{0}$$

Linearly dependent vectors in a plane are precisely collinear or parallel vectors.

- NOTE**
- (i) Two collinear vectors are linearly dependent.
 - (ii) Two non-zero and non-collinear vectors are linearly independent.
 - (iii) Any three non-coplanar vectors are linearly independent.
 - (iv) Any three coplanar vectors are linearly dependent.
 - (v) Any four vectors in three-dimensional space are linearly dependent.

Test of Collinearity

- (i) Two vectors $\mathbf{a}$ and $\mathbf{b}$ are collinear or parallel or linearly dependent, iff $\mathbf{a} = k\mathbf{b}$ for some scalar k .
- (ii) Three vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are collinear, if there exists scalars x, y, z such that $x\mathbf{a} + y\mathbf{b} + z\mathbf{c} = \mathbf{0}$, where $x + y + z = 0$.

Also, the points A, B, C are collinear, if $\mathbf{AB} = k\mathbf{BC}$ for some scalar k .

Test of Coplanarity

- (i) A vector $\mathbf{c}$ is coplanar with the vectors $\mathbf{a}$ and $\mathbf{b}$, if $\mathbf{c} = \lambda\mathbf{a} + \mu\mathbf{b}$ for some scalars λ and μ .
- (ii) Three vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$, are coplanar or linearly dependent, iff there are scalars x, y, z (not all zero) such that $x\mathbf{a} + y\mathbf{b} + z\mathbf{c} = \mathbf{0}$.
- (iii) Four vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ are coplanar, iff there exist scalars x, y, z, u not all zero such that $x\mathbf{a} + y\mathbf{b} + z\mathbf{c} + u\mathbf{d} = \mathbf{0}$, where, $x + y + z + u = 0$

Section Formula

Let $\mathbf{a}$ and $\mathbf{b}$ be two vectors represented by OA and OB and the point P divides AB in the ratio $m:n$.

If P divides AB in the ratio $m:n$ internally, then

$$\mathbf{r} = \frac{m\mathbf{b} + n\mathbf{a}}{m + n}$$

If P divides AB in the ratio $m:n$ externally, then

$$\mathbf{r} = \frac{m\mathbf{b} - n\mathbf{a}}{m - n}$$

Mid-point Formula

If $C(c)$ is the mid-point of AB , then

$$c = \frac{a+b}{2}$$

Centroid of a Triangle

$$\text{Centroid of } \triangle ABC = \frac{a+b+c}{3}$$

where, a, b and c are the position vectors of the vertices with respect to origin O .

Product of Two Vectors

There are two types of product of two vectors:

1. Scalar or Dot Product of Two Vectors

The scalar product of two vectors a and b is expressed as

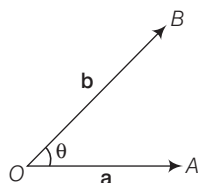
$$a \cdot b = |a| |b| \cos \theta$$

where, $0 \leq \theta \leq \pi$

$$\therefore a \cdot b \leq |a| |b|$$

NOTE • If $a \cdot b > 0$, then the angle between a and b is acute.

• If $a \cdot b < 0$, then the angle between a and b is obtuse.



Geometrical Interpretation

OL is the projection of vector b in the direction of vector a .

$$OL = b \cos \theta \quad [\because |a| = a \text{ and } |b| = b]$$

$$a \cdot b = a(b \cos \theta) = (ab) \cos \theta = b(a \cos \theta)$$

$$\Rightarrow \cos \theta = \frac{a \cdot b}{ab}$$

Projection of b in the direction of OA

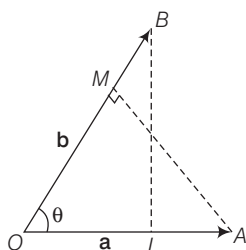
$$= OL = \frac{a \cdot b}{|a|}$$

$$OL = \frac{a \cdot b}{|a|} \hat{a} = \frac{a \cdot b}{|a|^2} a$$

Projection of a in the direction of

$$OB = OM = \frac{a \cdot b}{b}$$

$$OM = \frac{a \cdot b}{|b|^2} b$$



Properties of Scalar Product

Following are the properties of scalar product:

- $a \cdot b = b \cdot a$ [commutativity]

- $a \cdot (b + c) = a \cdot b + a \cdot c$ [distributivity]
- $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$
- $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{i} = 0, \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{j} = 0, \hat{k} \cdot \hat{i} = \hat{i} \cdot \hat{k} = 0$
- For any two vectors a and b
 - (a) $|a + b| = |a| + |b| \Rightarrow a \parallel b$
 - (b) $|a + b|^2 = |a|^2 + |b|^2 \Rightarrow a \perp b$
 - (c) $|a + b| = |a - b| \Rightarrow a \perp b$
- If dot product of two vectors is zero, then atleast one of the vectors is a zero vector or they are perpendicular.

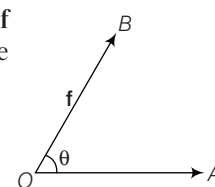
Application of Dot Product

Let a particle be placed at O and a force f represented by OB be acting on the particle at O .

Then,

$$\text{Work done} = (\text{Force}) \cdot (\text{Displacement})$$

$$\text{i.e. } W = f \cdot d$$



2. Vector Product of Two Vectors

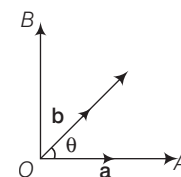
The vector product of two non-null and non-parallel vectors a and b is expressed as

$$a \times b = ab \sin \theta \hat{n}$$

where, $|a| = a, |b| = b,$

where θ is the angle between a, b and $\hat{n}$ is a unit vector perpendicular to the plane of a and b such that a, b and $\hat{n}$ form a right handed system.

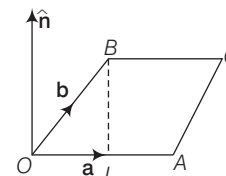
$$|a \times b| = |a| |b| \sin \theta$$



Geometrical Interpretation of Vector Product

Modulus of $a \times b$ is the area of the parallelogram whose adjacent sides are represented by a and b .

$$|a \times b| = \text{Area of parallelogram } OACB.$$



Properties of Vector Product

Following are the properties of vector product:

- $a \times b \neq b \times a$
 - where, $a \times b = -(b \times a)$
- $a \times b = 0$
 - $\Leftrightarrow a \parallel b$ or collinear and $a = 0$ or $b = 0$
- $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$
- $\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$
- $\hat{j} \times \hat{i} = -\hat{k}, \hat{k} \times \hat{j} = -\hat{i}, \hat{i} \times \hat{k} = -\hat{j}$
- Lagrange's identity, $|a \times b|^2 = |a|^2 |b|^2 - (a \cdot b)^2$
- $(ma) \times b = m(a \times b) = a \times (mb)$
- $a \times (b + c) = a \times b + a \times c$

Vector Product in Terms of Components

If $\mathbf{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\mathbf{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Then,
$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Angle between Two Vectors

If θ is the angle between two vectors $\mathbf{a}$ and $\mathbf{b}$, then $\sin\theta = \frac{|\mathbf{a} \times \mathbf{b}|}{ab}$, if

$$\mathbf{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \mathbf{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\sin^2\theta = \frac{(a_2b_3 - a_3b_2)^2 + (a_1b_3 - a_3b_1)^2 + (a_1b_2 - a_2b_1)^2}{(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)}$$

Vector Normal to the Plane of Two Given Vectors

The vectors of magnitude λ normal to the plane of $\mathbf{a}$ and $\mathbf{b}$ are

$$\pm \frac{\lambda(\mathbf{a} \times \mathbf{b})}{|\mathbf{a} \times \mathbf{b}|}$$

Condition for Vectors to be Parallel

If $\mathbf{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\mathbf{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are parallel, then

$$\mathbf{a} \times \mathbf{b} = 0 \text{ or } \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

Condition for Three Points A, B, C to be Collinear

Determine $\mathbf{AB}$ and $\mathbf{BC}$ and show that

$$\mathbf{AB} \times \mathbf{BC} = 0 \text{ or } \mathbf{AB} = k\mathbf{BC}$$

where, k is any scalar.

Area of Parallelogram and Triangle

- The area of a parallelogram with adjacent sides $\mathbf{a}$ and $\mathbf{b}$ is $|\mathbf{a} \times \mathbf{b}|$.
- The area of a parallelogram with diagonals $\mathbf{a}$ and $\mathbf{b}$ is $\frac{1}{2}|\mathbf{a} \times \mathbf{b}|$.
- The area of a plane quadrilateral $ABCD$ is $\frac{1}{2}|\mathbf{AC} \times \mathbf{BD}|$, where AC and BD are diagonals.
- The area of a triangle with adjacent sides $\mathbf{a}$ and $\mathbf{b}$ is $\frac{1}{2}|\mathbf{a} \times \mathbf{b}|$.

(v) The area of a $\triangle ABC$ is $\frac{1}{2}|\mathbf{AB} \times \mathbf{AC}|$.

(vi) If $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are position vectors of vertices of $\triangle ABC$, then area = $\frac{1}{2}|(\mathbf{a} \times \mathbf{b}) + (\mathbf{b} \times \mathbf{c}) + (\mathbf{c} \times \mathbf{a})|$.

If $(\mathbf{a} \times \mathbf{b}) + (\mathbf{b} \times \mathbf{c}) + (\mathbf{c} \times \mathbf{a}) = 0$,

then three points with position vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are collinear.

Moment

The moment or torque of a force F acting at A about any point O is given by

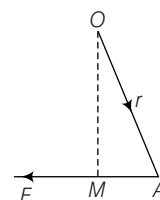
$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

where, r is the position vector of A with respect to point O , i.e. $r = \mathbf{OA}$.

The magnitude of the moment is $M = |\mathbf{M}|$.

i.e. $|\mathbf{M}| = |\mathbf{OA} \times \mathbf{F}|$

$$= F(OA \sin\theta) = FOM$$



Scalar Triple Product

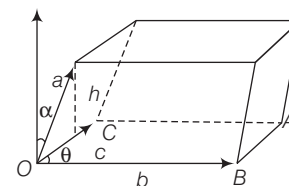
The scalar triple product of three vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ is defined as

$$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = |\mathbf{a}||\mathbf{b}||\mathbf{c}|\sin\theta\cos\phi$$

where, θ is the angle between $\mathbf{a}$ and $\mathbf{b}$ and ϕ is the angle between $\mathbf{a} \times \mathbf{b}$ and $\mathbf{c}$. It is also defined as $[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]$.

Geometrical Interpretation of a Scalar Triple Product

The scalar triple product $[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]$ represents the volume of the parallelepiped whose coterminous edges $\mathbf{a}, \mathbf{b}, \mathbf{c}$ form a right handed system of vectors.



Properties of Scalar Triple Product

Following are the properties of scalar triple product :

- If $\mathbf{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \mathbf{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\mathbf{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$.
- Then, $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$
- $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$
- $[\mathbf{a} \ \mathbf{b} \ \mathbf{c}] = [\mathbf{b} \ \mathbf{c} \ \mathbf{a}] = [\mathbf{c} \ \mathbf{a} \ \mathbf{b}]$
- $[\mathbf{a} \ \mathbf{b} \ \mathbf{c}] = -[\mathbf{b} \ \mathbf{a} \ \mathbf{c}]$
- $[k\mathbf{a} \ \mathbf{b} \ \mathbf{c}] = k[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]$
- $[\mathbf{a} + \mathbf{b} \ \mathbf{c} \ \mathbf{d}] = [\mathbf{a} \ \mathbf{c} \ \mathbf{d}] + [\mathbf{b} \ \mathbf{c} \ \mathbf{d}]$
- If $[\mathbf{a} \ \mathbf{b} \ \mathbf{c}] = 0$, then $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are coplanar.

Some Important Points

- (i) For points with position vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and $\mathbf{d}$ will be coplanar, if

$$[\mathbf{d} \ \mathbf{b} \ \mathbf{c}] + [\mathbf{d} \ \mathbf{c} \ \mathbf{a}] + [\mathbf{d} \ \mathbf{a} \ \mathbf{b}] = [\mathbf{a} \ \mathbf{b} \ \mathbf{c}]$$
- (ii) The volume of the tetrahedron whose three adjacent (coterminal edges) are represented by the vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$, is $\frac{1}{6}[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]$.
- (iii) The volume of the triangular prism whose adjacent sides are represented by the vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$, is $\frac{1}{2}[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]$.
- (iv) If $[\mathbf{a} \ \mathbf{b} \ \mathbf{c}] = 0 \Leftrightarrow \mathbf{a} \cdot \mathbf{b}$ and $\mathbf{c}$ are coplanar and $[\mathbf{a} \ \mathbf{b} \ \mathbf{c}] = 0$, if any two of the three vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are collinear or equal.
- (v) Let $\mathbf{p} = x_1\mathbf{a} + x_2\mathbf{b} + x_3\mathbf{c}$
 $\mathbf{q} = y_1\mathbf{a} + y_2\mathbf{b} + y_3\mathbf{c}$
 $\mathbf{r} = z_1\mathbf{a} + z_2\mathbf{b} + z_3\mathbf{c}$

$$\text{Then, } [\mathbf{p} \ \mathbf{q} \ \mathbf{r}] = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} [\mathbf{a} \ \mathbf{b} \ \mathbf{c}]$$

Vector Triple Product

Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be any three vectors, then the expression $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ is a vector and is called a vector triple product.

Geometrical Interpretation of $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$

Consider the expression $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ which itself is a vector, since it is a cross product of two vectors $\mathbf{a}$ and $(\mathbf{b} \times \mathbf{c})$. Now, $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ is a vector perpendicular to the plane containing $\mathbf{a}$ and $(\mathbf{b} \times \mathbf{c})$ but $\mathbf{b} \times \mathbf{c}$ is a vector perpendicular to the plane $\mathbf{b}$ and $\mathbf{c}$, therefore $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ is a vector lies in the plane of $\mathbf{b}$ and $\mathbf{c}$ and perpendicular to $\mathbf{a}$.

Hence, we can express $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ in terms of $\mathbf{b}$ and $\mathbf{c}$.

i.e. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = x\mathbf{b} + y\mathbf{c}$, where x and y are scalars.

- $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$
- $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{b} \cdot \mathbf{c})\mathbf{a}$
- $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} \neq \mathbf{a} \times (\mathbf{b} \times \mathbf{c})$

Application of Vectors to Geometry

Geometrical Bisector of an Angle

- (a) If $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ are unit vectors along the sides of an angle, then $\hat{\mathbf{a}} + \hat{\mathbf{b}}$ and $\hat{\mathbf{a}} - \hat{\mathbf{b}}$ are respectively the vectors along the internal and external bisectors of angle.
- (b) The bisectors of the angles between the lines $\mathbf{r} = x\mathbf{a}$ and $\mathbf{r} = y\mathbf{b}$, is given by

$$\mathbf{r} = k \left(\frac{\mathbf{a}}{|\mathbf{a}|} + \frac{\mathbf{b}}{|\mathbf{b}|} \right), (k \in \mathbb{R})$$

Practice Exercise

1. If $|\alpha + \beta| = |\alpha - \beta|$, then
 - a. α is parallel to β
 - b. α is perpendicular to β
 - c. $|\alpha| = |\beta|$
 - d. None of the above
2. If $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ are two unit vectors and θ is the angle between them, then $\hat{\mathbf{a}} + \hat{\mathbf{b}}$ is a unit vector, if
 - a. $\theta = \frac{\pi}{3}$
 - b. $\theta = \frac{\pi}{4}$
 - c. $\theta = \frac{\pi}{2}$
 - d. $\theta = \frac{2\pi}{3}$
3. If the vector $-\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ bisects the angle between the vector $\mathbf{c}$ and the vector $3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$, then the unit vector in the direction of $\mathbf{c}$, is
 - a. $\frac{1}{15}(11\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$
 - b. $-\frac{1}{15}(11\hat{\mathbf{i}} - 10\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$
 - c. $-\frac{1}{15}(11\hat{\mathbf{i}} + 10\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$
 - d. $-\frac{1}{15}(11\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$
4. In a trapezoid, the vector $\mathbf{BC} = \lambda \mathbf{AD}$, we will then find that $\mathbf{P} = \mathbf{AC} + \mathbf{AD}$ is collinear with $\mathbf{AD}$. If $\mathbf{P} = \mu \mathbf{AD}$, then
 - a. $\mu = \lambda + 1$
 - b. $\lambda = \mu + 1$
 - c. $\lambda + \mu = 1$
 - d. $\mu = 2 + \lambda$
5. Let $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ be three non-zero vectors, no two of which are collinear and the vector $\mathbf{a} + \mathbf{b}$ is collinear with $\mathbf{c}$ while $\mathbf{b} + \mathbf{c}$ is collinear with $\mathbf{a}$, then $\mathbf{a} + \mathbf{b} + \mathbf{c}$ is equal to
 - a. $\mathbf{a}$
 - b. $\mathbf{b}$
 - c. $\mathbf{c}$
 - d. None

6. A vector $\mathbf{a}$ has component $2p$ and 1 with respect to a rectangular cartesian system. The system is rotated through a certain angle about the origin in the counter clockwise sense. If with respect to new system, $\mathbf{a}$ has components $p+1$ and 1 , then
 a. $p=0$ b. $p=1$ or $p=-1/3$
 c. $p=-1$ or $p=1/3$ d. $p=1$ or $p=-1$
7. If the sum of two unit vectors is a unit vector, then the magnitude of their difference is
 a. $\sqrt{2}$ b. $\sqrt{3}$ c. $\sqrt{5}$ d. $\sqrt{7}$
8. If the position vectors of $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{S}$ are $2\hat{i} + \hat{j}$, $\hat{i} - 3\hat{j}$, $3\hat{i} + 2\hat{j}$ and $\hat{i} + \mu\hat{j}$ respectively and $\mathbf{PQ} \parallel \mathbf{RS}$, then the value of μ is
 a. -7 b. 7 c. -6 d. None
9. If $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ are unit vectors inclined at an angle α , $\alpha \in [0, \pi]$ to each other and $|\hat{\mathbf{a}} + \hat{\mathbf{b}}| < 1$. Then, α belongs to
 a. $\left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$ b. $\left(\frac{2\pi}{3}, \pi\right)$ c. $\left(0, \frac{\pi}{3}\right)$ d. $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$
10. Let $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ be three non-zero vectors which are pairwise non-collinear. If $\mathbf{a} + 3\mathbf{b}$ is collinear with $\mathbf{c}$ and $\mathbf{b} + 2\mathbf{c}$ is collinear with $\mathbf{a}$, then $\mathbf{a} + 3\mathbf{b} + 6\mathbf{c}$ is equal to
 a. $\mathbf{a} + \mathbf{c}$ b. $\mathbf{a}$ c. $\mathbf{c}$ d. 0
11. If the vectors $\mathbf{AB} = 3\hat{i} + 4\hat{k}$ and $\mathbf{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of $\triangle ABC$, then the length of the median through A is (in unit)
 a. $\sqrt{18}$ b. $\sqrt{72}$ c. $\sqrt{33}$ d. $\sqrt{45}$
12. If $\mathbf{a}$ and $\mathbf{b}$ are non-collinear vectors, then the value of α for which the vectors $\mathbf{u} = (\alpha - 2)\mathbf{a} + \mathbf{b}$ and $\mathbf{v} = (2 + 3\alpha)\mathbf{a} - 3\mathbf{b}$ are collinear, is
 a. $\frac{3}{2}$ b. $\frac{2}{3}$ c. $\frac{-3}{2}$ d. $\frac{-2}{3}$
13. If $|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = 1$ and $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{c} = \mathbf{c} \cdot \mathbf{a} = \cos \theta$, then the maximum value of θ is
 a. $\frac{\pi}{3}$ b. $\frac{\pi}{2}$ c. $\frac{2\pi}{3}$ d. $\frac{2\pi}{5}$
14. If the resolved parts of the force vector $5\hat{i} + 4\hat{j} + 2\hat{k}$ along and perpendicular to the vector $3\hat{i} + 4\hat{j} - 5\hat{k}$ are α and β respectively. Then, the value of α is
 a. $\frac{21}{50}(3\hat{i} + 4\hat{j} - 5\hat{k})$ b. $\frac{21}{50}(3\hat{i} - 4\hat{j} + 5\hat{k})$
 c. $\frac{11}{50}(2\hat{i} - 4\hat{j} + 3\hat{k})$ d. $\frac{1}{50}(187\hat{i} + 116\hat{j} + 205\hat{k})$
15. Let $|\mathbf{a}| = 2\sqrt{2}$, $|\mathbf{b}| = 3$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{4}$. If a parallelogram is constructed with adjacent sides $2\mathbf{a} - 3\mathbf{b}$ and $\mathbf{a} + \mathbf{b}$, then its longer diagonal is of length
 a. 10 b. 8 c. $2\sqrt{26}$ d. 6
16. Let $\mathbf{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\mathbf{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\mathbf{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector in the plane of $\mathbf{b}$ and $\mathbf{c}$, whose projection on $\mathbf{a}$ is of magnitude $\sqrt{(2/3)}$, is
 a. $2\hat{i} + 3\hat{j} - 3\hat{k}$ b. $2\hat{i} + 3\hat{j} + 3\hat{k}$
 c. $-2\hat{i} - \hat{j} + 5\hat{k}$ d. $2\hat{i} + \hat{j} + 5\hat{k}$
17. The vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are equal in length and taken pairwise, they make equal angles. If $\mathbf{a} = \hat{i} + \hat{j}$, $\mathbf{b} = \hat{j} + \hat{k}$ and $\mathbf{c}$ makes an obtuse angle with X -axis, then $\mathbf{c}$ is equal to
 a. $-\hat{i} + 4\hat{j} - \hat{k}$ b. $\hat{i} + \hat{k}$
 c. $\frac{1}{3}(-\hat{i} + 4\hat{j} - \hat{k})$ d. $\frac{\hat{i} - 4\hat{j} + \hat{k}}{3}$
18. If $\mathbf{a}$ and $\mathbf{b}$ are two non-collinear vectors such that $|\mathbf{a}| = 3$, $|\mathbf{b}| = 4$ and $\mathbf{a} - \mathbf{b} = \hat{i} + 2\hat{j} + 3\hat{k}$, then the value of $\left\{ \frac{\mathbf{a}}{|\mathbf{a}|^2} - \frac{\mathbf{b}}{|\mathbf{b}|^2} \right\}^2$ is equal to
 a. $\frac{1}{24}$ b. $\frac{5}{72}$ c. $\frac{7}{72}$ d. $\frac{7}{48}$
19. If the points $P(\mathbf{a} + \mathbf{b} - \mathbf{c})$, $Q(2\mathbf{a} + 3\mathbf{b})$ and $R(\mathbf{b} + t\mathbf{c})$ are collinear, where $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are three coplanar vectors, then the value of t is
 a. -2 b. $-1/2$ c. $1/2$ d. 2
20. If the three vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ with magnitude 3 , 4 and 5 respectively and $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$, then the value of $\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}$ is
 a. -23 b. -25
 c. 30 d. 26
21. The angle between the vectors $\mathbf{a} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\mathbf{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$ is
 a. $\cos^{-1} \frac{3}{11}$ b. $\cos^{-1} \frac{2}{11}$ c. $\cos^{-1} \frac{4}{21}$ d. $\cos^{-1} \frac{3}{22}$
22. If $\mathbf{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\mathbf{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\mathbf{c} = 3\hat{i} + \hat{j}$, then p such that $\mathbf{a} + p\mathbf{b}$ is at right angle to $\mathbf{c}$, will be
 a. 7 b. 9 c. 3 d. 5
23. If three vectors $\mathbf{a} = \hat{i} + \hat{j} - \hat{k}$, $\mathbf{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\mathbf{c} = -\hat{i} + 2\hat{j} - \hat{k}$, then the unit vector perpendicular to both $\mathbf{a} + \mathbf{b}$ and $\mathbf{b} + \mathbf{c}$, is
 a. $\frac{\hat{i}}{\sqrt{3}}$ b. $\hat{k}$ c. $\frac{\hat{k}}{\sqrt{3}}$ d. $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$
24. The vector $\mathbf{c}$ directed along the internal bisector of the angle between the vectors $\mathbf{a} = 7\hat{i} - 4\hat{j} - 4\hat{k}$ and $\mathbf{b} = -2\hat{i} - \hat{j} + 2\hat{k}$ with $|\mathbf{c}| = 5\sqrt{6}$, is
 a. $\frac{2}{3}(\hat{i} - 7\hat{j} + 2\hat{k})$ b. $\frac{5}{3}(\hat{i} - 7\hat{j} + 2\hat{k})$
 c. $\frac{7}{3}(\hat{i} - 7\hat{j} - \hat{k})$ d. $\frac{2}{3}(\hat{i} + 7\hat{j} - 2\hat{k})$

25. If a unit vector $\hat{d}$ is equally inclined at an angle α with the vectors $\mathbf{a} = \cos \theta \cdot \hat{i} + \sin \theta \cdot \hat{j}$, $\mathbf{b} = -\sin \theta \cdot \hat{i} + \cos \theta \cdot \hat{j}$ and $\mathbf{c} = \hat{k}$. Then, α is equal to
- a. $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ b. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
 c. $\cos^{-1}\left(\frac{1}{3}\right)$ d. $\frac{\pi}{2}$
26. Vectors $\mathbf{a}$ and $\mathbf{b}$ are such that $|\mathbf{a}| = 1$, $|\mathbf{b}| = 4$ and $\mathbf{a} \cdot \mathbf{b} = 2$. If $\mathbf{c} = 2\mathbf{a} \times \mathbf{b} - 3\mathbf{b}$, then the angle between $\mathbf{b}$ and $\mathbf{c}$ is
- a. $\frac{\pi}{6}$ b. $\frac{5\pi}{6}$ c. $\frac{\pi}{3}$ d. $\frac{2\pi}{3}$
27. If the vectors $\mathbf{c}, \mathbf{a} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\mathbf{b} = \hat{j}$ are such that $\mathbf{a}, \mathbf{c}$ and $\mathbf{b}$ form a right handed system, then $\mathbf{c}$ is equal to
- a. $z\hat{i} - x\hat{k}$ b. 0 c. $y\hat{i}$ d. $-z\hat{i} + x\hat{k}$
28. If the vectors $\mathbf{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\mathbf{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\mathbf{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then (λ, μ) is equal to
- a. $(-3, 2)$ b. $(2, -3)$ c. $(-2, 3)$ d. $(3, -2)$
29. If the non-zero vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are related by $\mathbf{a} = 8\mathbf{b}$ and $\mathbf{c} = -7\mathbf{b}$. Then, the angle between $\mathbf{a}$ and $\mathbf{c}$ is
- a. π b. 0 c. $\frac{\pi}{4}$ d. $\frac{\pi}{2}$
30. Let $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ be two unit vectors. If the vectors $\mathbf{c} = \hat{\mathbf{a}} + 2\hat{\mathbf{b}}$ and $\mathbf{d} = 5\hat{\mathbf{a}} - 4\hat{\mathbf{b}}$ are perpendicular to each other, then the angle between $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ is
- a. $\frac{\pi}{6}$ b. $\frac{\pi}{2}$ c. $\frac{\pi}{3}$ d. $\frac{\pi}{4}$
31. Let $\mathbf{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\mathbf{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\mathbf{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector of the type $\mathbf{b} + \lambda\mathbf{c}$ for some scalar λ , whose projection on $\mathbf{a}$ is of magnitude $\frac{\sqrt{2}}{3}$, is
- a. $2\hat{i} + \hat{j} + 5\hat{k}$ b. $2\hat{i} + 3\hat{j} - 3\hat{k}$
 c. $2\hat{i} - \hat{j} + 5\hat{k}$ d. $2\hat{i} + 3\hat{j} + 3\hat{k}$
32. The vector $(\hat{i} \times \mathbf{a} \cdot \mathbf{b})\hat{i} + (\hat{j} \times \mathbf{a} \cdot \mathbf{b})\hat{j} + (\hat{k} \times \mathbf{a} \cdot \mathbf{b})\hat{k}$ is equal to
- a. $\mathbf{b} \times \mathbf{a}$ b. $\mathbf{a}$ c. $\mathbf{a} \times \mathbf{b}$ d. $\mathbf{b}$
33. If $\hat{\mathbf{a}}, \hat{\mathbf{b}}$ and $\hat{\mathbf{c}}$ are unit vectors satisfying $\hat{\mathbf{a}} - \sqrt{3}\hat{\mathbf{b}} + \hat{\mathbf{c}} = 0$, then the angle between the vectors $\hat{\mathbf{a}}$ and $\hat{\mathbf{c}}$ is
- a. $\frac{\pi}{4}$ b. $\frac{\pi}{3}$ c. $\frac{\pi}{6}$ d. $\frac{\pi}{2}$
34. If a vector $\mathbf{r}$ of magnitude $3\sqrt{6}$ is directed along the bisector of the angle between the vectors $\mathbf{a} = 7\hat{i} - 4\hat{j} - 4\hat{k}$ and $\mathbf{b} = -2\hat{i} - \hat{j} + 2\hat{k}$, then $\mathbf{r}$ is equal to
- a. $\hat{i} - 7\hat{j} + 2\hat{k}$ b. $\hat{i} + 7\hat{j} - 2\hat{k}$
 c. $\hat{i} + 7\hat{j} + 2\hat{k}$ d. $\hat{i} - 7\hat{j} - 2\hat{k}$
35. There are three vectors $\mathbf{a} = \hat{i} + \hat{j}$, $\mathbf{b} = \hat{j} + \hat{k}$ and $\mathbf{c} = x\hat{\mathbf{a}} + y\hat{\mathbf{b}}$. If the vectors $\hat{i} - 2\hat{j} + \hat{k}$, $3\hat{i} + 2\hat{j} - \hat{k}$ and $\mathbf{c}$ are coplanar, then $\frac{x}{y}$ is equal to
- a. -2 b. -3 c. $\frac{2}{3}$ d. -1
36. If $\mathbf{b} = 4\hat{i} + 3\hat{j}$ and $\mathbf{c}$ are two vectors perpendicular to each other in the XY-plane. Then, a vector in the same plane having projections 1 and 2 along $\mathbf{b}$ and $\mathbf{c}$ respectively, is
- a. $\hat{i} + 2\hat{j}$ b. $2\hat{i} - \hat{j}$ c. $2\hat{i} + \hat{j}$ d. None
37. The value of λ for which the points $P(1, 0, 3)$, $Q(-1, 3, 4)$, $R(1, 2, 1)$ and $S(\lambda, 2, 5)$ are coplanar, is
- a. 3 b. -2 c. 5 d. -1
38. If $\hat{\mathbf{a}} = 2\hat{i} + \hat{j} - \hat{k}$ and $\mathbf{b} = \hat{j} + \hat{k}$, then the vector $\mathbf{c}$ such that $\mathbf{a} \cdot \mathbf{c} = 4$ and $\mathbf{a} \times \mathbf{c} = \mathbf{b}$, is
- a. $\hat{i} + \hat{j} - \hat{k}$ b. $3\hat{i} - \hat{j} + \hat{k}$ c. $\hat{i} + 3\hat{j} + \hat{k}$ d. $2\hat{i} - \hat{j} - \hat{k}$
39. Let $\mathbf{a} = 2\hat{i} + \hat{j} + \hat{k}$, $\mathbf{b} = \hat{i} + 2\hat{j} - \hat{k}$ and a unit vector $\hat{\mathbf{c}}$ be coplanar. If $\hat{\mathbf{c}}$ is perpendicular to $\mathbf{a}$, then $\hat{\mathbf{c}}$ is equal to
- a. $(-\hat{j} + \hat{k})$ b. $\pm \frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$
 c. $\pm \frac{1}{\sqrt{2}}(\hat{j} + \hat{k})$ d. None of these
40. If $\mathbf{a} = \frac{1}{\sqrt{10}}(3\hat{i} + \hat{k})$ and $\mathbf{b} = \frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k})$, then the value of $(2\mathbf{a} - \mathbf{b}) \cdot [(\mathbf{a} \times \mathbf{b}) \times (\mathbf{a} + 2\mathbf{b})]$ is
- a. -3 b. 5 c. 3 d. -5
41. If $\mathbf{a} = \hat{j} - \hat{k}$ and $\mathbf{c} = \hat{i} - \hat{j} - \hat{k}$. Then, the vector $\mathbf{b}$ satisfying $\mathbf{a} \times \mathbf{b} + \mathbf{c} = 0$ and $\mathbf{a} \cdot \mathbf{b} = 3$, is
- a. $-\hat{i} + \hat{j} - 2\hat{k}$ b. $2\hat{i} - \hat{j} + 2\hat{k}$
 c. $\hat{i} - \hat{j} - 2\hat{k}$ d. $\hat{i} + \hat{j} - 2\hat{k}$
42. If vectors $\mathbf{a}$ and $\mathbf{b}$ are not perpendicular and $\mathbf{c}, \mathbf{d}$ are two vectors satisfying $\mathbf{b} \times \mathbf{c} = \mathbf{b} \times \mathbf{d}$ and $\mathbf{a} \cdot \mathbf{d} = 0$. Then, the vector $\mathbf{d}$ is equal to
- a. $\mathbf{c} + \left(\frac{\mathbf{a} \cdot \mathbf{c}}{\mathbf{a} \cdot \mathbf{b}}\right)\mathbf{b}$ b. $\mathbf{b} + \left(\frac{\mathbf{b} \cdot \mathbf{c}}{\mathbf{a} \cdot \mathbf{b}}\right)\mathbf{c}$
 c. $\mathbf{c} - \left(\frac{\mathbf{a} \cdot \mathbf{c}}{\mathbf{a} \cdot \mathbf{b}}\right)\mathbf{b}$ d. $\mathbf{b} - \left(\frac{\mathbf{b} \cdot \mathbf{c}}{\mathbf{a} \cdot \mathbf{b}}\right)\mathbf{c}$
43. If the vectors $p\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + q\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + r\hat{k}$ ($p \neq q \neq r \neq 1$) are coplanar, then the value of $pqr - (p + q + r)$ is
- a. -2 b. 2
 c. 0 d. -1
44. If $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ are three non-coplanar vectors, then $(\mathbf{u} + \mathbf{v} - \mathbf{w}) \cdot (\mathbf{u} - \mathbf{v}) \times (\mathbf{v} - \mathbf{w})$ is equal to
- a. 0 b. $\mathbf{u} \cdot \mathbf{v} \times \mathbf{w}$
 c. $\mathbf{u} \cdot \mathbf{w} \times \mathbf{v}$ d. $3\mathbf{u} \cdot \mathbf{v} \times \mathbf{w}$

45. Let $\mathbf{b}$ and $\mathbf{c}$ be non-collinear vectors. If $\mathbf{a}$ is a vector such that $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = 4$ and $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (x^2 - 2x + 6)\mathbf{b} + \sin y \cdot \mathbf{c}$, then (x, y) lies on the line
 a. $x + y = 0$ b. $x - y = 0$ c. $x = 1$ d. $y = \pi$
46. Let $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ be two non-collinear unit vectors. If $\mathbf{u} = \hat{\mathbf{a}} - (\hat{\mathbf{a}} \cdot \hat{\mathbf{b}})\hat{\mathbf{b}}$ and $\mathbf{v} = \hat{\mathbf{a}} \times \hat{\mathbf{b}}$, then $|\mathbf{v}|$ is equal to
 a. $|\mathbf{u}|$ b. $|\mathbf{u}| + |\mathbf{v} \cdot \hat{\mathbf{a}}|$
 c. $2|\mathbf{v}|$ d. $|\mathbf{u}| + \mathbf{u} \cdot (\hat{\mathbf{a}} + \hat{\mathbf{b}})$
47. The value of $[(\mathbf{a} - \mathbf{b}) \cdot (\mathbf{b} - \mathbf{c}) \times (\mathbf{c} - \mathbf{a})]$ is
 a. 0 b. $2[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]$ c. $3[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]$ d. None
48. A vector $\mathbf{c}$ of magnitude $20\sqrt{6}$ directed along the bisector of the angle between $\mathbf{a} = 7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ and $\mathbf{b} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, is
 a. $\pm \frac{5}{3}(2\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + \hat{\mathbf{k}})$ b. $\pm \frac{3}{5}(\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$
 c. $\pm \frac{5}{3}(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 7\hat{\mathbf{k}})$ d. $\pm \frac{5}{3}(\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$
49. If $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ are the unit vectors and $\mathbf{a}$ is a vector such that $\mathbf{a} \times \mathbf{r} = \hat{\mathbf{j}}$, then $\mathbf{a} \cdot \mathbf{r}$ is
 a. -1 b. 0 c. 1 d. None
50. If $\mathbf{a}$ makes an acute angle with $\mathbf{b}$ and $\mathbf{r} \times \mathbf{b} = \mathbf{c} \times \mathbf{b}$, then $\mathbf{r}$ is equal to
 a. $\mathbf{a} \times \hat{\mathbf{i}} - \mathbf{b}$ b. $\mathbf{c} \times \mathbf{a}$ c. $\mathbf{c} - \frac{\mathbf{c} \cdot \mathbf{a}}{\mathbf{b} \cdot \mathbf{a}} \cdot \mathbf{b}$ d. $\mathbf{c} + \frac{\mathbf{c} \cdot \mathbf{a}}{\mathbf{b} \cdot \mathbf{a}} \cdot \mathbf{b}$
51. The vectors $(a, a + 1, a + 2)$, $(a + 3, a + 4, a + 5)$, $(a + 6, a + 7, a + 8)$ are coplanar for
 a. $\forall a \in \mathbb{R}$ b. $\forall a \notin \mathbb{R}$
 c. $a = \sqrt{-3}$ d. None of these
52. If $\mathbf{a} = \lambda\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$, $\mathbf{b} = 2\hat{\mathbf{i}} + \lambda\hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\mathbf{c} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $[\mathbf{a} \ \mathbf{b} \ \mathbf{c}] = 6$, then λ is equal to
 a. -8 or 3 b. -9 or 3 c. -3 or +9 d. 8 or 5
53. If $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are coplanar vectors, then the scalar triple product $[2\mathbf{a} - \mathbf{b}, 2\mathbf{b} - \mathbf{c}, 2\mathbf{c} - \mathbf{a}]$ is
 a. 2 b. -3 c. 0 d. None
54. If $\mathbf{a}$ is perpendicular to $\mathbf{b}$ and $\mathbf{c}$, $a = 2, b = 3, c = 4$ and angle between $\mathbf{b}$ and $\mathbf{c}$ is $\frac{2\pi}{3}$, then $[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]$ is equal to
 a. $7\sqrt{3}$ b. $9\sqrt{3}$ c. $12\sqrt{3}$ d. $5\sqrt{3}$
55. If $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are non-zero and non-collinear vectors, then $[\mathbf{a} \ \mathbf{b} \ \mathbf{i}] + [\mathbf{a} \ \mathbf{b} \ \mathbf{j}] + [\mathbf{a} \ \mathbf{b} \ \mathbf{k}] =$
 a. $\mathbf{a} + \mathbf{b}$ b. $\mathbf{a} \times \mathbf{b}$ c. $\mathbf{a} - \mathbf{b}$ d. $\mathbf{b} \times \mathbf{a}$
56. Let $\mathbf{a} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$. If $\mathbf{c}$ is a vector such that $\mathbf{a} \cdot \mathbf{c} = |\mathbf{c}|$, $|\mathbf{c} - \mathbf{a}| = 2\sqrt{2}$ and the angle between $\mathbf{a} \times \mathbf{b}$ and $\mathbf{c}$ is 30° , then $|(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}|$ is equal to
 a. $\frac{1}{2}$ b. $\frac{3\sqrt{3}}{2}$
 c. 3 d. $\frac{3}{2}$
57. Let the vectors $\mathbf{a} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$. If $\mathbf{c}$ is a vector such that $\mathbf{a} \cdot \mathbf{c} = |\mathbf{c}|$ and $|\mathbf{c} - \mathbf{a}| = 2\sqrt{2}$, angle between $\mathbf{a} \times \mathbf{b}$ and $\mathbf{c}$ is 45° , then $|(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}|$ is
 a. 3 b. $\frac{\sqrt{3}}{2}$ c. $\frac{3\sqrt{2}}{2}$ d. $\frac{2\sqrt{2}}{3}$
58. If the three vectors $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\mathbf{c} = \hat{\mathbf{i}}$ and $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \alpha \mathbf{a} + \beta \mathbf{b}$, then the value of $\alpha + \beta$ is
 a. 2 b. 3 c. 0 d. None

BITSAT Archives

1. The unit vector perpendicular to the vectors $\hat{\mathbf{i}} - \hat{\mathbf{j}}$ and $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ forming a right handed system, is [2014]
 a. $\hat{\mathbf{k}}$ b. $-\hat{\mathbf{k}}$ c. $\frac{\hat{\mathbf{i}} - \hat{\mathbf{j}}}{\sqrt{2}}$ d. $\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}$
2. Let $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ be non-zero vectors such that no two are collinear and $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \frac{1}{3}|\mathbf{b}||\mathbf{c}|\mathbf{a}$. If θ is the acute angle between the vectors $\mathbf{b}$ and $\mathbf{c}$, then $\sin \theta$ is equal to [2013]
 a. $\frac{2\sqrt{2}}{3}$ b. $\frac{\sqrt{2}}{3}$ c. $\frac{2}{3}$ d. $\frac{1}{3}$
3. If $|\mathbf{a}| = 2, |\mathbf{b}| = 5$ and $|\mathbf{a} \times \mathbf{b}| = 8$, then $|\mathbf{a} \cdot \mathbf{b}|$ is equal to [2013]
 a. 3 b. 4 c. 5 d. 6
4. The work done by the force $4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ in moving a particle along a straight line from the point $(3, 2, -1)$ to $(2, -1, 4)$ is [2012]
 a. 0 unit b. 4 units c. 15 units d. 19 units
5. If $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0$, then the correct statement is [2011]
 a. out of $\mathbf{a}, \mathbf{b}, \mathbf{c}$, any two vectors are parallel
 b. $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are coplanar
 c. any two are equal among $\mathbf{a}, \mathbf{b}, \mathbf{c}$
 d. atleast one statement is correct
6. If $2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + \lambda\hat{\mathbf{k}}$ are perpendicular to each other, then λ is equal to [2011]
 a. -3 b. -2 c. -1 d. 0
7. If $\mathbf{a} \cdot \hat{\mathbf{i}} = 4$, then $(\mathbf{a} \times \hat{\mathbf{j}}) \cdot (2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$ is equal to [2010]
 a. 12 b. 2 c. 0 d. -12

8. The vector $\mathbf{a}$ is equal to [2010]
 a. $(\mathbf{a} \cdot \hat{\mathbf{i}})\hat{\mathbf{i}} + (\mathbf{a} \cdot \hat{\mathbf{j}})\hat{\mathbf{j}} + (\mathbf{a} \cdot \hat{\mathbf{k}})\hat{\mathbf{k}}$ b. $(\mathbf{a} \cdot \hat{\mathbf{j}})\hat{\mathbf{i}} + (\mathbf{a} \cdot \hat{\mathbf{i}})\hat{\mathbf{j}} + (\mathbf{a} \cdot \hat{\mathbf{i}})\hat{\mathbf{k}}$
 c. $(\mathbf{a} \cdot \hat{\mathbf{k}})\hat{\mathbf{i}} + (\mathbf{a} \cdot \hat{\mathbf{i}})\hat{\mathbf{j}} + (\mathbf{a} \cdot \hat{\mathbf{j}})\hat{\mathbf{k}}$ d. $(\mathbf{a} \cdot \mathbf{a})(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$
9. If $\mathbf{r} \cdot \mathbf{a} = 0$, $\mathbf{r} \cdot \mathbf{b} = 0$, and $\mathbf{r} \cdot \mathbf{c} = 0$ for some non-zero vector $\mathbf{r}$. Then, the value of $[\mathbf{a} \mathbf{b} \mathbf{c}]$ is [2009]
 a. 0 b. $1/2$ c. 1 d. 2
10. Let $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ be three vectors such that $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{b} \cdot (\mathbf{c} + \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} + \mathbf{b}) = 0$ and $|\mathbf{a}| = 1$, $|\mathbf{b}| = 4$, $|\mathbf{c}| = 8$, then $|\mathbf{a} + \mathbf{b} + \mathbf{c}|$ is equal to [2009]
 a. 13 b. 81 c. 9 d. 5
11. The position vector of P and Q are respectively $\mathbf{a}$ and $\mathbf{b}$. If R is a point on PQ such that $\mathbf{PR} = 5 \mathbf{PQ}$, then the position vector of R is [2008]
 a. $5\mathbf{b} - 4\mathbf{a}$ b. $5\mathbf{b} + 4\mathbf{a}$ c. $4\mathbf{b} - 5\mathbf{a}$ d. $4\mathbf{b} + 5\mathbf{a}$
12. If the position vectors of A , B and C are respectively $2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$, then $\cos^2 A$ is equal to [2008]
 a. 0 b. $\frac{6}{41}$ c. $\frac{35}{41}$ d. 1
13. Let $\mathbf{a}$ be a unit vector, $\mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\mathbf{c} = \hat{\mathbf{i}} + 3\hat{\mathbf{k}}$. Then, the maximum value of $[\mathbf{a} \mathbf{b} \mathbf{c}]$, is [2008]
 a. -1 b. $\sqrt{10} + \sqrt{6}$ c. $\sqrt{10} - \sqrt{6}$ d. $\sqrt{59}$
14. The unit vector which is orthogonal to the vector $3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ and is coplanar with the vectors $2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, is [2007]
 a. $\frac{2\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{41}}$ b. $\frac{2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}}{\sqrt{13}}$
 c. $\frac{3\hat{\mathbf{j}} - \hat{\mathbf{k}}}{\sqrt{10}}$ d. $\frac{4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 3\hat{\mathbf{k}}}{\sqrt{34}}$
15. Let $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ be three non-coplanar vectors and $\mathbf{p}$, $\mathbf{q}$, $\mathbf{r}$ be vectors defined by the relations

$$\mathbf{p} = \frac{\mathbf{b} \times \mathbf{c}}{[\mathbf{a} \mathbf{b} \mathbf{c}]}, \mathbf{q} = \frac{\mathbf{c} \times \mathbf{a}}{[\mathbf{a} \mathbf{b} \mathbf{c}]}, \mathbf{r} = \frac{\mathbf{a} \times \mathbf{b}}{[\mathbf{a} \mathbf{b} \mathbf{c}]}$$
 Then, the value of the expression $(\mathbf{a} + \mathbf{b}) \cdot \mathbf{p} + (\mathbf{b} + \mathbf{c}) \cdot \mathbf{q} + (\mathbf{c} + \mathbf{a}) \cdot \mathbf{r}$ is [2007]
 a. 0 b. 1 c. 2 d. 3
16. If $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$ and $\hat{\mathbf{c}}$ are mutually perpendicular unit vectors, then $|\hat{\mathbf{a}} + \hat{\mathbf{b}} + \hat{\mathbf{c}}|$ is equal to [2007]
 a. 3 b. $\sqrt{3}$ c. $\frac{\sqrt{a^2 + b^2 + c^2}}{3}$ d. 1
17. The projection of the vector $2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ on the vector $\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ is [2006]
 a. $-\frac{3}{\sqrt{14}}$ b. $\frac{3}{\sqrt{14}}$ c. $-\frac{\sqrt{3}}{2}$ d. $\frac{3}{\sqrt{2}}$
18. If $\mathbf{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\mathbf{b} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, then the angle between the vectors $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ is [2005]
 a. 60° b. 90° c. 45° d. 55°
19. If the vectors $\alpha\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\hat{\mathbf{i}} + \beta\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \lambda\hat{\mathbf{k}}$ ($\alpha, \beta, \gamma \neq 1$) are coplanar, then the value of $\frac{1}{1-\alpha} + \frac{1}{1-\beta} + \frac{1}{1-\gamma}$ is [2005]
 a. -1 b. 0 c. 1 d. $1/2$
20. If a vector α lie in the plane of β and γ , then which is correct? [2005]
 a. $[\alpha \beta \gamma] = 0$ b. $[\alpha \beta \gamma] = 1$ c. $[\alpha \beta \gamma] = 3$ d. $[\beta \gamma \alpha] = 1$
21. If $\alpha = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\beta = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ and $\gamma = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, then what is the value of $(\vec{\alpha} \times \vec{\beta}) \cdot (\vec{\alpha} \times \vec{\gamma})$? [2005]
 a. 47 b. 74
 c. -74 d. None of these

Answer with Solutions

Practice Exercise

1. (b) $\because |\alpha + \beta| = |\alpha - \beta|$
 $\therefore |\alpha + \beta|^2 = |\alpha - \beta|^2$
 $\Rightarrow (\alpha + \beta) \cdot (\alpha + \beta) = (\alpha - \beta) \cdot (\alpha - \beta)$
 $\Rightarrow \alpha \cdot \alpha + \alpha \cdot \beta + \beta \cdot \alpha + \beta \cdot \beta = \alpha \cdot \alpha - \alpha \cdot \beta - \beta \cdot \alpha + \beta \cdot \beta$
 $\Rightarrow \alpha^2 + 2\alpha \cdot \beta + \beta^2 = \alpha^2 - 2\alpha \cdot \beta + \beta^2$ [$\because \alpha \cdot \beta = \beta \cdot \alpha$]
 $\Rightarrow 4\alpha \cdot \beta = 0$ or $\alpha \cdot \beta = 0$
 Hence, $\vec{\alpha}$ and $\vec{\beta}$ are perpendicular to each other.
2. (d) $(\mathbf{a} + \mathbf{b})^2 = 1 + 1 + 2(1 \cdot 1 \cos \theta) = 1$
 $\therefore \cos \theta = -1/2$
 $\therefore \theta = \frac{2\pi}{3}$
3. (d) Let $x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ be the unit vector along $\mathbf{c}$. Since, $-\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ bisects the angle between $\mathbf{c}$ and $3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$.
 $\therefore \lambda(-\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) = (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) + \frac{3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}}{5}$
 $\Rightarrow x + \frac{3}{5} = -\lambda, y + \frac{4}{5} = \lambda$ and $z = -\lambda$
 Now, $x^2 + y^2 + z^2 = 1 \Rightarrow \left(-\lambda - \frac{3}{5}\right)^2 + \left(\lambda + \frac{4}{5}\right)^2 + \lambda^2 = 1$
 But $\lambda \neq 0$ because $\lambda = 0$ implies that given vectors are parallel.
 $\therefore \lambda = \frac{2}{15} \Rightarrow x = \frac{-11}{15}, y = \frac{-10}{15}$ and $z = \frac{-2}{15}$
 Hence, $x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}} = \frac{-1}{15}(11\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

4. (a) We have, $\mathbf{P} = \mathbf{AC} + \mathbf{BD} = \mathbf{AC} + \mathbf{BC} + \mathbf{CD}$
 $= \mathbf{AC} + \lambda \mathbf{AD} + \mathbf{CD} = \lambda \mathbf{AD} + (\mathbf{AC} + \mathbf{CD})$
 $= \lambda \mathbf{AD} + \mathbf{AD} = (\lambda + 1) \mathbf{AD}$

Therefore, $\mathbf{P} = \mu \mathbf{AD} = \mu = \lambda + 1$

5. (d) Given that, $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non-zero and non-collinear vectors and vector $(\mathbf{a} + \mathbf{b})$ is collinear with $\mathbf{c}$ and vector $(\mathbf{b} + \mathbf{a})$ collinear with $\mathbf{a}$.

$\Rightarrow (\mathbf{a} + \mathbf{b}) = x \mathbf{c} \quad \dots(i)$

$(\mathbf{b} + \mathbf{c}) = y \mathbf{a} \quad \dots(ii)$

where, x and y are scalars.

From Eq. (i), $(\mathbf{a} + \mathbf{b}) + \mathbf{c} = x \mathbf{c} + \mathbf{c}$

$\Rightarrow (\mathbf{a} + \mathbf{b} + \mathbf{c}) = (x + 1) \mathbf{c} \quad \dots(iii)$

From Eq. (ii),

$\mathbf{a} + (\mathbf{b} + \mathbf{c}) = \mathbf{a} + y \mathbf{a}$

$\Rightarrow (\mathbf{a} + \mathbf{b} + \mathbf{c}) = (y + 1) \mathbf{a} \quad \dots(iv)$

From Eqs. (iii) and (iv),

$(x + 1) \mathbf{c} = (y + 1) \mathbf{a} \Rightarrow \mathbf{c} = \left(\frac{y + 1}{x + 1} \right) \mathbf{a}$

So, $\mathbf{c}$ and $\mathbf{a}$ are collinear which contradicts the given condition that no two of $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are collinear.

6. (b) Clearly, $\mathbf{a} = 2p\hat{\mathbf{i}} + \hat{\mathbf{j}}$

On rotation, let $p + 1$ and 1

$\therefore \mathbf{b} = (p + 1)\hat{\mathbf{i}} + \hat{\mathbf{j}}$

Now, under rotation about origin,

$|\mathbf{a}| = |\mathbf{b}| \Rightarrow |\mathbf{a}|^2 = |\mathbf{b}|^2$

$\Rightarrow 4p^2 + 1 = (p + 1)^2 + 1 \Rightarrow 4p^2 = p^2 + 2p + 1$

$\Rightarrow 3p^2 - 2p - 1 = 0 \Rightarrow (p - 1)(3p + 1) = 0$

$\Rightarrow p = 1 \text{ or } -\frac{1}{3}$

7. (b)

8. (c) $\mathbf{PQ} = (\hat{\mathbf{i}} - 3\hat{\mathbf{j}}) - (2\hat{\mathbf{i}} + \hat{\mathbf{j}}) = -\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$

$\mathbf{RS} = -2\hat{\mathbf{i}} + (\mu - 2)\hat{\mathbf{j}}$

$\mathbf{PQ} \parallel \mathbf{RS} \Rightarrow \mathbf{PQ} = \lambda \mathbf{RS}$

$\Rightarrow -\hat{\mathbf{i}} - 4\hat{\mathbf{j}} = [\lambda\{-2\hat{\mathbf{i}} + (\mu - 2)\hat{\mathbf{j}}\}]$

$\Rightarrow \lambda = 1/2 \text{ and } \mu = -6$

9. (b) Since, $|\hat{\mathbf{a}} + \hat{\mathbf{b}}|^2 < 1$

$\Rightarrow 2 + 2 \cos \alpha < 1 \Rightarrow 4 \cos^2 \frac{\alpha}{2} < 1$

$\Rightarrow \cos \frac{\alpha}{2} < \frac{1}{2} \Rightarrow \alpha \in \left(\frac{2\pi}{3}, \pi \right)$

10. (d) As $\mathbf{a} + 3\mathbf{b}$ is collinear with $\mathbf{c}$.

$\therefore \mathbf{a} + 3\mathbf{b} = \lambda \mathbf{c} \quad \dots(i)$

Also, $\mathbf{b} + 2\mathbf{c}$ is collinear with $\mathbf{a}$.

$\Rightarrow \mathbf{b} + 2\mathbf{c} = \mu \mathbf{a} \quad \dots(ii)$

From Eq. (i), $\mathbf{a} + 3\mathbf{b} + 6\mathbf{c} = (\lambda + 6) \mathbf{c} \quad \dots(iii)$

From Eq. (ii), $\mathbf{a} + 3\mathbf{b} + 6\mathbf{c} = (1 + 3\mu) \mathbf{a} \quad \dots(iv)$

From Eqs. (iii) and (iv), we get

$(\lambda + 6) \mathbf{c} = (1 + 3\mu) \mathbf{a}$

Since, $\mathbf{a}$ is not collinear with $\mathbf{c}$.

$\therefore \lambda + 6 = 1 + 3\mu = 0$

From Eq. (iv), $\mathbf{a} + 3\mathbf{b} + 6\mathbf{c} = 0$

11. (c)

12. (b) Since, vectors $\mathbf{u}$ and $\mathbf{v}$ are collinear.

$\therefore \mathbf{u} = \lambda \mathbf{v}$

$\Rightarrow \{(\alpha - 2)\mathbf{a} + \mathbf{b}\} = \lambda\{(2 + 3\alpha)\mathbf{a} - 3\mathbf{b}\}$

On comparing, we get

$(2 + 3\alpha)\lambda = \alpha - 2 \quad \dots(i)$

and $1 = -3\lambda \Rightarrow \lambda = -\frac{1}{3}$

$\therefore (2 + 3\alpha)\left(-\frac{1}{3}\right) = \alpha - 2 \Rightarrow \alpha = \frac{2}{3}$

13. (c) $[\mathbf{a} \ \mathbf{b} \ \mathbf{c}] = \begin{vmatrix} \mathbf{a} \cdot \mathbf{a} & \mathbf{a} \cdot \mathbf{b} & \mathbf{a} \cdot \mathbf{c} \\ \mathbf{b} \cdot \mathbf{a} & \mathbf{b} \cdot \mathbf{b} & \mathbf{b} \cdot \mathbf{c} \\ \mathbf{c} \cdot \mathbf{a} & \mathbf{c} \cdot \mathbf{b} & \mathbf{c} \cdot \mathbf{c} \end{vmatrix} = \begin{vmatrix} 1 & \cos \theta & \cos \theta \\ \cos \theta & 1 & \cos \theta \\ \cos \theta & \cos \theta & 1 \end{vmatrix}$

$= 1 - 3 \cos^2 \theta + 2 \cos^3 \theta = (1 - \cos \theta)^2 (1 + 2 \cos \theta)$

$\Rightarrow 1 + 2 \cos \theta \geq 0 \Rightarrow \theta \leq \frac{2\pi}{3}$

14. (a) Unit vector along $3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}} = \frac{3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}}{5\sqrt{2}}$

$\therefore \alpha = \left\{ (5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \cdot \frac{3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}}{5\sqrt{2}} \right\} \left(\frac{3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}}{5\sqrt{2}} \right)$
 $= \frac{21}{50} (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}})$

15. (c) Here, $\mathbf{a} \cdot \mathbf{b} = 2\sqrt{2} \cdot 3 \cdot \frac{1}{\sqrt{2}} = 6$

The diagonals are $2\mathbf{a} - 3\mathbf{b} \pm (\mathbf{a} + \mathbf{b})$.

$\therefore$ Length of diagonals are

$|3\mathbf{a} - 2\mathbf{b}|^2 = 9 \cdot 8 + 4 \cdot 9 - 12 \cdot 6 = 36$

and $|\mathbf{a} - 4\mathbf{b}|^2 = 8 + 16 \cdot 9 - 8 \cdot 6 = 104$

So, the length of the longer diagonal is $\sqrt{104}$, i.e. $2\sqrt{26}$ units.

16. (a) Hint Any vector $\mathbf{r}$ in the plane of $\mathbf{b}$ and $\mathbf{c}$ is

$\mathbf{r} = \mathbf{b} + \lambda \mathbf{c}$

or $\mathbf{r} = (1 + \lambda)\hat{\mathbf{i}} + (2 + \lambda)\hat{\mathbf{j}} - (1 + 2\lambda)\hat{\mathbf{k}} \quad \dots(i)$

$\therefore$ Projection of $\mathbf{r}$ on $\mathbf{a}$ is $\sqrt{\frac{2}{3}}$.

$\therefore \frac{\mathbf{r} \cdot \mathbf{a}}{|\mathbf{a}|} = \sqrt{\frac{2}{3}}$

17. (c) Let $\mathbf{c} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

Then, $|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = x^2 + y^2 + z^2 = 2 \quad \dots(i)$

Since, $\mathbf{c}$ makes an obtuse angle with X-axis.

$\therefore \mathbf{c} \cdot \hat{\mathbf{i}} < 0$

$\Rightarrow x < 0 \quad \dots(ii)$

It is given that $\mathbf{a}, \mathbf{b}, \mathbf{c}$ taken pairwise make equal angles.

$\therefore \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{\mathbf{a} \cdot \mathbf{c}}{|\mathbf{a}||\mathbf{c}|} = \frac{\mathbf{b} \cdot \mathbf{c}}{|\mathbf{b}||\mathbf{c}|}$

$$\Rightarrow \frac{1}{2} = \frac{x+y}{2} = \frac{y+z}{2} \Rightarrow x+y=1=y+z$$

$$\Rightarrow x = z \text{ and } y = 1 - z$$

Putting $x = z$ and $y = 1 - z$ in Eq. (i), we get

$$z^2 + (1-z)^2 + z^2 = 0$$

$$\Rightarrow 3z^2 - 2z - 1 = 0 \Rightarrow (3z+1)(z-1) = 0$$

$$z = 1, -1/3$$

$$\text{Since, } x = z \text{ and } < 0$$

$$\therefore x = z = -\frac{1}{3}$$

$$\text{Hence, } \mathbf{c} = \frac{1}{3}(-\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}})$$

18. (c) we know that

$$\left\{ \frac{\mathbf{a}}{|\mathbf{a}|^2} - \frac{\mathbf{b}}{|\mathbf{b}|^2} \right\}^2 = \left\{ \frac{\mathbf{a}-\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} \right\}^2$$

$$= \frac{|\mathbf{a}-\mathbf{b}|^2}{|\mathbf{a}|^2|\mathbf{b}|^2} = \frac{1+4+9}{9 \times 16} = \frac{7}{72}$$

19. (d) Given that, points P, Q and R are collinear, so there exist three scalars x, y and z such that

$$x(\mathbf{a} + \mathbf{b} - \mathbf{c}) + y(2\mathbf{a} + 3\mathbf{b}) + z(\mathbf{b} + t\mathbf{c}) = 0$$

On comparing the coefficients of $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$, we get

$$x + 2y + 0 = 0 \quad \dots(i)$$

$$x + 3y + z = 0 \quad \dots(ii)$$

$$-x + tz = 0 \quad \dots(iii)$$

On solving Eqs. (i) and (ii) by cross-multiplication method

$$\frac{x}{2-0} = \frac{-y}{1-0} = \frac{z}{3-2} = \lambda \quad [\text{say}]$$

$$\Rightarrow x = 2\lambda, y = -2\lambda, z = \lambda$$

These values of x, y and z will satisfy Eq. (iii).

$$\therefore -2\lambda + t\lambda = 0 \Rightarrow \lambda(t-2) = 0$$

$$\therefore \lambda \neq 0$$

$$\therefore t-2=0 \Rightarrow t=2$$

20. (b) Given, $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$

On squaring both sides, we get

$$|\mathbf{a} + \mathbf{b} + \mathbf{c}|^2 = 0$$

$$\Rightarrow \mathbf{a}^2 + \mathbf{b}^2 + \mathbf{c}^2 + 2(\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}) = 0$$

$$\Rightarrow 2(\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}) = -(9 + 16 + 25)$$

$$\therefore \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a} = -25$$

21. (c) The angle between $\mathbf{a}$ and $\mathbf{b}$ is θ .

$$\text{So, } \cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{ab} = \frac{(2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (6\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})}{3 \cdot 7} = \frac{4}{21}$$

$$\therefore \theta = \cos^{-1} \frac{4}{21}$$

22. (d) Given, $\mathbf{a} + p\mathbf{b}$ is at right angle to $\mathbf{c}$.

$$\therefore (\mathbf{a} + p\mathbf{b}) \cdot \mathbf{c} = 0$$

$$\text{Also, } \mathbf{a} \cdot \mathbf{c} = 5 \text{ and } \mathbf{b} \cdot \mathbf{c} = -1$$

$$\Rightarrow 5 - p = 0$$

$$\therefore p = 5$$

$$23. (b) (\mathbf{a} + \mathbf{b}) \times (\mathbf{b} + \mathbf{c}) = 3\hat{\mathbf{j}} \times (-2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) = 6\hat{\mathbf{k}}$$

So, required unit vector is $\hat{\mathbf{k}}$.

$$24. (b) \text{ Let } \mathbf{a} = 7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}} \text{ and } \mathbf{b} = -2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$

$$\text{Now, required vector } \mathbf{c} = \alpha \left(\frac{\mathbf{a}}{|\mathbf{a}|} + \frac{\mathbf{b}}{|\mathbf{b}|} \right)$$

$$= \alpha \left(\frac{7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}}{9} + \frac{-2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}}{3} \right) = \frac{\alpha}{9} (\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$$

$$\Rightarrow |\mathbf{c}|^2 = \frac{\alpha^2}{81} \times 54 = 150 \Rightarrow \alpha = \pm 15$$

$$\therefore \mathbf{c} = \pm \frac{5}{3} (\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$$

$$25. (b) \text{ Let } \hat{\mathbf{d}} \cdot \mathbf{a} = \hat{\mathbf{d}} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{c} = \cos \alpha$$

$$\Rightarrow \hat{\mathbf{d}} \cdot (\mathbf{a} - \mathbf{k}) = 0$$

Since, $\hat{\mathbf{d}}$ is parallel to $(\mathbf{a} - \mathbf{k}) \times (\mathbf{b} - \mathbf{k})$.

$$\therefore \mathbf{d} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \cos \theta & \sin \theta & -1 \\ -\sin \theta & \cos \theta & -1 \end{vmatrix}$$

$$\Rightarrow \hat{\mathbf{d}} = \frac{(\cos \theta - \sin \theta) \hat{\mathbf{i}} + (\cos \theta + \sin \theta) \hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{3}}$$

$$\cos \alpha = \hat{\mathbf{d}} \cdot \mathbf{k} = \frac{1}{\sqrt{3}} \Rightarrow \alpha = \cos^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

$$26. (b) \text{ Here, } |\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2 = 16 - 4 = 12$$

$$\text{and } |\mathbf{c}|^2 = (2\mathbf{a} \times \mathbf{b} - 3\mathbf{b}) \cdot (2\mathbf{a} \times \mathbf{b} - 3\mathbf{b})$$

$$= 4|\mathbf{a} \times \mathbf{b}|^2 + 9|\mathbf{b}|^2 = 4 \cdot 12 + 9 \cdot 16$$

$$= 192 \Rightarrow |\mathbf{c}| = 8\sqrt{3}$$

$$\text{Now, } \mathbf{b} \cdot \mathbf{c} = \mathbf{b} \cdot (2\mathbf{a} \times \mathbf{b} - 3\mathbf{b}) = -3|\mathbf{b}|^2 = -48$$

$$\therefore \cos \theta = \frac{\mathbf{b} \cdot \mathbf{c}}{|\mathbf{b}||\mathbf{c}|} = -\frac{48}{4 \cdot 8\sqrt{3}} = -\frac{\sqrt{3}}{2}$$

$$\therefore \theta = \frac{5\pi}{6}$$

$$27. (a) \text{ Here, } \mathbf{c} = \mathbf{b} \times \mathbf{a} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 1 & 0 \\ x & y & z \end{vmatrix} = z\hat{\mathbf{i}} - x\hat{\mathbf{k}}$$

28. (a) Since, the given vectors are mutually authogonal.

$$\therefore \mathbf{a} \cdot \mathbf{b} = 2 - 4 + 2 = 0$$

$$\mathbf{a} \cdot \mathbf{c} = \lambda - 1 + 2\mu = 0 \quad \dots(i)$$

$$\text{and } \mathbf{b} \cdot \mathbf{c} = 2\lambda + 4 + \mu = 0 \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$\mu = 2 \text{ and } \lambda = -3$$

$$\text{Hence, } (\lambda, \mu) = (-3, 2)$$

29. (a) Since, $\mathbf{a} = 8\mathbf{b}$ and $\mathbf{c} = -7\mathbf{b}$

So, $\mathbf{a}$ is parallel to $\mathbf{b}$ and $\mathbf{c}$ is anti-parallel to $\mathbf{b}$.

Since, $\mathbf{a}$ and $\mathbf{c}$ are anti-parallel.

So, the angle between $\mathbf{a}$ and $\mathbf{c}$ is π .

30. (c) Given that, $\mathbf{a}$ and $\mathbf{b}$ are unit vectors, i.e. $|\hat{\mathbf{a}}| = |\hat{\mathbf{b}}| = 1$.

$$\mathbf{c} = \hat{\mathbf{a}} + 2\hat{\mathbf{b}}, \mathbf{d} = 5\hat{\mathbf{a}} - 4\hat{\mathbf{b}}$$

$\mathbf{c}$ and $\mathbf{d}$ are perpendicular to each other, i.e. $\mathbf{c} \cdot \mathbf{d} = 0$.

$$\text{Now, } \mathbf{c} \cdot \mathbf{d} = 0 \Rightarrow (\hat{\mathbf{a}} + 2\hat{\mathbf{b}}) \cdot (5\hat{\mathbf{a}} - 4\hat{\mathbf{b}}) = 0$$

$$\Rightarrow 5\hat{\mathbf{a}} \cdot \hat{\mathbf{a}} - 4\hat{\mathbf{a}} \cdot \hat{\mathbf{b}} + 10\hat{\mathbf{b}} \cdot \hat{\mathbf{a}} - 8\hat{\mathbf{b}} \cdot \hat{\mathbf{b}} = 0$$

$$\Rightarrow 6\hat{\mathbf{a}} \cdot \hat{\mathbf{b}} = 3 \Rightarrow \hat{\mathbf{a}} \cdot \hat{\mathbf{b}} = \frac{1}{2}$$

So, the angle between $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ is $\frac{\pi}{3}$.

31. (b) Given, $\mathbf{a} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}, \mathbf{b} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\mathbf{c} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$$\text{Now, } \mathbf{b} + \lambda\mathbf{c} = (1 + \lambda)\hat{\mathbf{i}} + (2 + \lambda)\hat{\mathbf{j}} + (-1 - 2\lambda)\hat{\mathbf{k}}$$

$$\therefore \text{Projection of } (\mathbf{b} + \lambda\mathbf{c}) \text{ on } \mathbf{a} = \frac{(\mathbf{b} + \lambda\mathbf{c}) \cdot \mathbf{a}}{|\mathbf{a}|} = \sqrt{\frac{2}{3}} \quad [\text{given}]$$

$$\Rightarrow \frac{2(1 - \lambda) - (2 + \lambda) + (-1 - 2\lambda)}{\sqrt{4 + 1 + 1}} = \sqrt{\frac{2}{3}}$$

$$\Rightarrow \frac{-\lambda - 1}{\sqrt{6}} = \frac{\sqrt{2}}{\sqrt{3}} \Rightarrow \lambda + 1 = 2 \Rightarrow \lambda = 1$$

$$\therefore \mathbf{b} + \lambda\mathbf{c} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$$

32. (c)

33. (b) $\sqrt{3}\hat{\mathbf{b}} = (\hat{\mathbf{a}} + \hat{\mathbf{c}}) \Rightarrow 3|\hat{\mathbf{b}}|^2 = |\hat{\mathbf{a}}|^2 + |\hat{\mathbf{c}}|^2 + 2\hat{\mathbf{a}} \cdot \hat{\mathbf{c}}$

$$\Rightarrow 3(1) = 1 + 1 + 2\hat{\mathbf{a}} \cdot \hat{\mathbf{c}} \Rightarrow 2\hat{\mathbf{a}} \cdot \hat{\mathbf{c}} = 1$$

$$\Rightarrow |\hat{\mathbf{a}}| |\hat{\mathbf{c}}| \cos \theta = \frac{1}{2} \Rightarrow \cos \theta = \frac{1}{2}$$

$$\therefore \theta = \frac{\pi}{3}$$

34. (a) **Hint** The required vector $\mathbf{r} = \lambda(\hat{\mathbf{a}} + \hat{\mathbf{b}})$, where λ is a scalar.

$$\Rightarrow \mathbf{r} = \lambda \left(\frac{1}{9}(7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}) + \frac{1}{3}(-2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \right)$$

$$\Rightarrow \mathbf{r} = \frac{\lambda}{9}(\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$$

$$\text{But } |\mathbf{r}|^2 = 54$$

35. (b) **Hint** Let $P_1 = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}, P_2 = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$

$$P_3 = \mathbf{c} = x\mathbf{a} + y\mathbf{b} = x(\hat{\mathbf{i}} + \hat{\mathbf{j}}) + y(\hat{\mathbf{j}} + \hat{\mathbf{k}})$$

$$= x\hat{\mathbf{i}} + (x + y)\hat{\mathbf{j}} + y\hat{\mathbf{k}} = \mathbf{c}$$

Since, P_1, P_2, P_3 are coplanar.

$$\therefore \begin{vmatrix} 1 & -2 & 1 \\ 3 & 2 & -1 \\ x & x+y & y \end{vmatrix} = 0$$

36. (b) Let $\mathbf{c} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$

$$\text{Then, } \mathbf{b} \perp \mathbf{c} \Rightarrow \mathbf{b} \cdot \mathbf{c} = 0 \Rightarrow 4x + 3y = 0$$

$$\Rightarrow \frac{x}{3} = \frac{y}{-4} = \lambda$$

$$\Rightarrow x = 3\lambda, y = -4\lambda$$

$$\therefore \mathbf{c} = \lambda(3\hat{\mathbf{i}} - 4\hat{\mathbf{j}})$$

Let the required vector be $\alpha = p\hat{\mathbf{i}} + q\hat{\mathbf{j}}$. Then, the projections of α on $\mathbf{b}$ and $\mathbf{c}$ are $\frac{\alpha \cdot \mathbf{b}}{|\mathbf{b}|}$ and $\frac{\alpha \cdot \mathbf{c}}{|\mathbf{c}|}$, respectively.

$$\therefore \alpha \cdot \frac{\mathbf{b}}{|\mathbf{b}|} = 1 \text{ and } \frac{\alpha \cdot \mathbf{c}}{|\mathbf{c}|} = 2 \quad [\text{given}]$$

$$\Rightarrow 4p + 3q = 5 \text{ and } 3p - 4q = 10$$

$$\Rightarrow p = 2, q = -1$$

$$\therefore \vec{\alpha} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}}$$

37. (d) Since, points P, Q, R and S are coplanar.

$$\therefore [\mathbf{PQ} \ \mathbf{PR} \ \mathbf{PS}] = 0$$

$$\Rightarrow \begin{vmatrix} -2 & 3 & 1 \\ 0 & 2 & -2 \\ \lambda - 1 & 2 & 2 \end{vmatrix} = 0 \Rightarrow \lambda = -1$$

38. (a) **Hint** Given $\mathbf{a} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}, \mathbf{b} = \hat{\mathbf{j}} + \hat{\mathbf{k}}$

$$\text{Let } \mathbf{c} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

$$\text{Since, } \mathbf{a} \cdot \mathbf{c} = 4 \Rightarrow 2x + y - z = 4$$

$$\text{and } \mathbf{a} \times \mathbf{c} = \mathbf{b}$$

$$\therefore \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 1 & -1 \\ x & y & z \end{vmatrix} = \hat{\mathbf{j}} + \hat{\mathbf{k}}$$

39. (b) $\mathbf{a} \times (\mathbf{a} \times \mathbf{b}) = (\mathbf{a} \cdot \mathbf{b})\mathbf{a} - (\mathbf{a} \cdot \mathbf{a})\mathbf{b}$

$$= 3(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - 6(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}) = -9\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$$

$$\therefore \text{Required unit vector} = \frac{\mathbf{a} \times (\mathbf{a} \times \mathbf{b})}{|\mathbf{a} \times (\mathbf{a} \times \mathbf{b})|} = \pm \frac{1}{\sqrt{2}}(-\hat{\mathbf{j}} + \hat{\mathbf{k}})$$

40. (d) Given, $\mathbf{a} = \frac{1}{\sqrt{10}}(3\hat{\mathbf{i}} + \hat{\mathbf{k}})$ and $\mathbf{b} = \frac{1}{7}(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}})$

$$\begin{aligned} \therefore (2\mathbf{a} - \mathbf{b}) \cdot \{(\mathbf{a} \times \mathbf{b}) \times (\mathbf{a} + 2\mathbf{b})\} \\ = (2\mathbf{a} - \mathbf{b}) \cdot \{(\mathbf{a} \times \mathbf{b}) \times \mathbf{a} + (\mathbf{a} \times \mathbf{b}) \times 2\mathbf{b}\} \\ = (2\mathbf{a} - \mathbf{b}) \cdot \{(\mathbf{a} \cdot \mathbf{a})\mathbf{b} + (\mathbf{b} \cdot \mathbf{a})\mathbf{a} + 2(\mathbf{a} \cdot \mathbf{b})\mathbf{b} - 2(\mathbf{b} \cdot \mathbf{b})\mathbf{a}\} \\ = (2\mathbf{a} - \mathbf{b}) \cdot \{1(\mathbf{b}) - (0)\mathbf{a} + 2(0)\mathbf{b} - 2(1)\mathbf{a}\} \\ [\because \mathbf{a} \cdot \mathbf{b} = 0 \text{ and } \mathbf{a} \cdot \mathbf{a} = \mathbf{b} \cdot \mathbf{b} = 1] \\ = (2\mathbf{a} - \mathbf{b}) \cdot (\mathbf{b} - 2\mathbf{a}) = -(4|\mathbf{a}|^2 + 4\mathbf{a} \cdot \mathbf{b} + |\mathbf{b}|^2) \\ = -(4 + 0 + 1) = -5 \end{aligned}$$

41. (a) We have, $\mathbf{a} \times \mathbf{b} + \mathbf{c} = 0$

$$\Rightarrow \mathbf{a} \times (\mathbf{a} \times \mathbf{b}) + \mathbf{a} \times \mathbf{c} = 0$$

$$\Rightarrow (\mathbf{a} \cdot \mathbf{b})\mathbf{a} - (\mathbf{a} \cdot \mathbf{a})\mathbf{b} + \mathbf{a} \times \mathbf{c} = 0$$

$$\Rightarrow 3\mathbf{a} - 2\mathbf{b} + \mathbf{a} \times \mathbf{c} = 0$$

$$\Rightarrow 2\mathbf{b} = 3\mathbf{a} + \mathbf{a} \times \mathbf{c}$$

$$\Rightarrow 2\mathbf{b} = 3\hat{\mathbf{j}} - 3\hat{\mathbf{k}} - 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}} \\ = -2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$$

$$\therefore \mathbf{b} = -\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$$

42. (c) Given, $\mathbf{a} \cdot \mathbf{b} \neq 0, \mathbf{a} \cdot \mathbf{d} = 0$... (i)

$$\text{and } \mathbf{b} \times \mathbf{c} = \mathbf{b} \times \mathbf{d} \Rightarrow \mathbf{b} \times (\mathbf{c} - \mathbf{d}) = 0$$

$$\therefore \mathbf{b} \parallel (\mathbf{c} - \mathbf{d}) \Rightarrow \mathbf{c} - \mathbf{d} = \lambda \mathbf{b}$$

$$\Rightarrow \mathbf{d} = \mathbf{c} - \lambda \mathbf{b} \quad \dots (ii)$$

Taking dot product with $\mathbf{a}$, we get

$$\begin{aligned} \mathbf{a} \cdot \mathbf{d} &= \mathbf{a} \cdot \mathbf{c} - \lambda \mathbf{a} \cdot \mathbf{b} \\ \Rightarrow 0 &= \mathbf{a} \cdot \mathbf{c} - \lambda (\mathbf{a} \cdot \mathbf{b}) \quad [\text{from Eq. (i)}] \\ \Rightarrow \lambda &= \frac{\mathbf{a} \cdot \mathbf{c}}{\mathbf{a} \cdot \mathbf{b}} \quad \dots(\text{iii}) \\ \therefore \mathbf{d} &= \mathbf{c} - \frac{(\mathbf{a} \cdot \mathbf{c})}{(\mathbf{a} \cdot \mathbf{b})} \mathbf{b} \end{aligned}$$

43. (a) **Hint** Given, $\mathbf{a} = p\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} + q\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{c} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + r\hat{\mathbf{k}}$ are coplanar and $p \neq q \neq r \neq 1$.

Since, $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are coplanar.

$$\therefore [\mathbf{a} \ \mathbf{b} \ \mathbf{c}] = 0$$

44. (b) Here, $(\mathbf{u} - \mathbf{v}) \times (\mathbf{v} - \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{v} \times \mathbf{w} + \mathbf{w} \times \mathbf{u}$

$$\begin{aligned} \therefore (\mathbf{u} + \mathbf{v} - \mathbf{w}) \cdot (\mathbf{u} - \mathbf{v}) \times (\mathbf{v} - \mathbf{w}) \\ &= (\mathbf{u} + \mathbf{v} - \mathbf{w}) \cdot (\mathbf{u} \times \mathbf{v} + \mathbf{v} \times \mathbf{w} + \mathbf{w} \times \mathbf{u}) \\ &= \mathbf{u} \cdot \mathbf{v} \times \mathbf{w} + \mathbf{v} \cdot \mathbf{w} \times \mathbf{u} - \mathbf{w} \cdot \mathbf{u} \times \mathbf{v} \\ &= \mathbf{u} \cdot \mathbf{v} \times \mathbf{w} \end{aligned}$$

45. (c) By expanding $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$, we get

$$\mathbf{a} \cdot \mathbf{c} = x^2 - 2x + 6, \mathbf{a} \cdot \mathbf{b} = -\sin y$$

$$\text{Given, } \mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = 4$$

$$\Rightarrow x^2 - 2x + 2 = \sin y$$

$$\Rightarrow \sin y = x^2 - 2x + 2 = (x - 1)^2 + 1 \geq 1$$

$$\text{But } \sin y \leq 1$$

So, both sides are equal only for $x = 1$.

46. (a) $\mathbf{v} = \mathbf{a} \times \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \sin \theta \hat{\mathbf{n}} = \sin \theta \hat{\mathbf{n}}$

where, θ is the angle between $\mathbf{a}$ and $\mathbf{b}$. $[\because |\mathbf{a}| = 1 = |\mathbf{b}|]$

$$\therefore |\mathbf{v}| = \sin \theta$$

$$\text{Now, } \mathbf{u} = \mathbf{a} - (\mathbf{a} \cdot \mathbf{b}) \mathbf{b} = \mathbf{a} - (\cos \theta) \mathbf{b}$$

$$\begin{aligned} \Rightarrow |\mathbf{u}|^2 &= [\mathbf{a} - (\cos \theta) \mathbf{b}] \cdot [\mathbf{a} - (\cos \theta) \mathbf{b}] \\ &= \mathbf{a}^2 + \cos^2 \theta \mathbf{b}^2 - 2 \cos \theta \mathbf{a} \cdot \mathbf{b} \\ &= 1 + \cos^2 \theta - 2 \cos^2 \theta = 1 - \cos^2 \theta = \sin^2 \theta \end{aligned}$$

$$\therefore |\mathbf{u}| = \sin \theta, \text{ thus } |\mathbf{v}| = |\mathbf{u}|$$

47. (a) The given value

$$\begin{aligned} (\mathbf{a} - \mathbf{b}) \cdot (\mathbf{b} \times \mathbf{c} - \mathbf{b} \times \mathbf{a} - \mathbf{c} \times \mathbf{c} + \mathbf{c} \times \mathbf{a}) \\ &= (\mathbf{a} - \mathbf{b}) \cdot (\mathbf{b} \times \mathbf{c} + \mathbf{a} \times \mathbf{b} + \mathbf{c} \times \mathbf{a}) \\ &= \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) + 0 + 0 - \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) \\ &= [\mathbf{a} \ \mathbf{b} \ \mathbf{c}] - [\mathbf{b} \ \mathbf{c} \ \mathbf{a}] = 0 \end{aligned}$$

48. (d) Let $\mathbf{a} = 7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ and $\mathbf{b} = -2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ be the position vectors of points A and B respectively. Then the bisector of $\angle AOB$ divides AB in the ratio $OA : OB$, i.e. $9 : 3$ or $3 : 1$. Therefore, the vector lying along the bisector

$$\begin{aligned} &= \frac{3(-2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) + (7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}})}{3 + 1} \\ &= \frac{1}{4} (\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \end{aligned}$$

Hence, the required vector

$$\begin{aligned} &= \pm 20\sqrt{6} \cdot \frac{1}{4} \left(\frac{\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 2\hat{\mathbf{k}}}{\sqrt{54}} \right) \\ &= \pm \frac{5}{3} (\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \end{aligned}$$

49. (d) Given, $\mathbf{a} \times \mathbf{r} = \hat{\mathbf{j}}$

Since, $\hat{\mathbf{j}}$ is perpendicular to both $\mathbf{a}$ and $\mathbf{r}$.

$$\therefore \mathbf{a} \cdot \hat{\mathbf{j}} = 0 \text{ and } \mathbf{r} \cdot \hat{\mathbf{j}} = 0$$

If the angle between $\mathbf{a}$ and $\mathbf{r}$ is θ , then $\mathbf{a} \cdot \mathbf{r} = |\mathbf{a}| |\mathbf{r}| \cos \theta$

$$\begin{aligned} &= ar \cos \theta = \sqrt{a^2 r^2 \cos^2 \theta} = \sqrt{a^2 r^2 (1 - \sin^2 \theta)} \\ &= \sqrt{a^2 r^2 \cdot (\mathbf{a} \times \mathbf{r})^2} = \sqrt{a^2 r^2 - 1} \end{aligned}$$

50. (c) Given, $\mathbf{r} \times \mathbf{b} = \mathbf{c} \times \mathbf{b}$ and $\mathbf{r} \cdot \mathbf{a} = 0$

$$\Rightarrow \mathbf{r} \times \mathbf{b} - \mathbf{c} \times \mathbf{b} = 0 \Rightarrow (\mathbf{r} - \mathbf{c}) \times \mathbf{b} = 0$$

$\Rightarrow (\mathbf{r} - \mathbf{c})$ and $\mathbf{b}$ are parallel vectors.

$$\text{Therefore, } (\mathbf{r} - \mathbf{c}) = \lambda \mathbf{b}$$

$$\Rightarrow \mathbf{r} = \mathbf{c} + \lambda \mathbf{b} \quad \dots(\text{i})$$

$$\text{We have, } \mathbf{r} \cdot \mathbf{a} = 0$$

$$\Rightarrow (\mathbf{c} + \lambda \mathbf{b}) \cdot \mathbf{a} = 0 \Rightarrow \mathbf{c} \cdot \mathbf{a} + \lambda \mathbf{b} \cdot \mathbf{a} = 0$$

$$\lambda = -\frac{\mathbf{c} \cdot \mathbf{a}}{\mathbf{b} \cdot \mathbf{a}}$$

On putting λ in Eq. (i), we get $\mathbf{r} = \mathbf{c} - \frac{\mathbf{c} \cdot \mathbf{a}}{\mathbf{b} \cdot \mathbf{a}} \mathbf{b}$

51. (a) Vectors $(a, a+1, a+2)$, $(a+3, a+4, a+5)$ and $(a+6, a+7, a+8)$ are coplanar.

$$\therefore \begin{vmatrix} a & a+1 & a+2 \\ a+3 & a+4 & a+5 \\ a+6 & a+7 & a+8 \end{vmatrix} = 0$$

On applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we get

$$\begin{vmatrix} a & 1 & 2 \\ a+3 & 1 & 2 \\ a+6 & 1 & 2 \end{vmatrix} = 0$$

Vectors are coplanar, $\forall a \in R$.

$$52. (a) [\mathbf{a} \ \mathbf{b} \ \mathbf{c}] = \begin{vmatrix} \lambda & 2 & -3 \\ 2 & \lambda & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

On applying $C_1 \rightarrow C_1 - C_3$, we get

$$6 = \lambda^2 + 5\lambda - 18 \Rightarrow \lambda^2 + 5\lambda - 24 = 0$$

$$\therefore \lambda = -8 \text{ or } 3$$

53. (c) Given $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are coplanar vectors.

So, $2\mathbf{a} - \mathbf{b}$, $2\mathbf{b} - \mathbf{c}$, $2\mathbf{c} - \mathbf{a}$ are also coplanar vectors.

$$\therefore [2\mathbf{a} - \mathbf{b} \ 2\mathbf{b} - \mathbf{c} \ 2\mathbf{c} - \mathbf{a}] = 0$$

54. (c) $[\mathbf{a} \ \mathbf{b} \ \mathbf{c}] = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{a} \cdot (bc \sin \theta \hat{\mathbf{n}})$

$$\begin{aligned} &= \mathbf{a} \cdot \left(3 \times 4 \sin \frac{2\pi}{3} \hat{\mathbf{n}} \right) = \mathbf{a} \cdot \left(12 \times \frac{\sqrt{3}}{2} \hat{\mathbf{n}} \right) \\ &= 6\sqrt{3} |\mathbf{a}| |\hat{\mathbf{n}}| \\ &= 6\sqrt{3} \times 2 \times 1 \\ &= 12\sqrt{3} \end{aligned}$$

55. (b) we have,

$$\begin{aligned} &[\mathbf{a} \ \mathbf{b} \ \hat{\mathbf{i}}] + [\mathbf{a} \ \mathbf{b} \ \hat{\mathbf{j}}] + [\mathbf{a} \ \mathbf{b} \ \hat{\mathbf{k}}] \\ &= \{(\mathbf{a} \times \mathbf{b}) \cdot \hat{\mathbf{i}}\} \hat{\mathbf{j}} + \{(\mathbf{a} \times \mathbf{b}) \cdot \hat{\mathbf{j}}\} \hat{\mathbf{i}} + \{(\mathbf{a} \times \mathbf{b}) \cdot \hat{\mathbf{k}}\} \hat{\mathbf{k}} \\ &= \mathbf{a} \times \mathbf{b} \end{aligned}$$

56. (d) $|\mathbf{c} - \mathbf{a}| = 2\sqrt{2} \Rightarrow |\mathbf{c}|^2 + |\mathbf{a}|^2 - 2\mathbf{a} \cdot \mathbf{c} = 8$

$$\Rightarrow |\mathbf{c}|^2 + (\sqrt{9})^2 - 2|\mathbf{c}| = 8$$

$$\Rightarrow |\mathbf{c}|^2 - 2|\mathbf{c}| + 1 = 0 \Rightarrow (|\mathbf{c}| - 1)^2 = 0$$

$$\Rightarrow |\mathbf{c}| = 1$$

Now, $\mathbf{a} \times \mathbf{b} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

$$\therefore |\mathbf{a} \times \mathbf{b}| = \sqrt{4 + 4 + 1} = 3$$

Hence, $[(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}] = |\mathbf{a} \times \mathbf{b}| |\mathbf{c}| \sin 30^\circ = 3 \times 1 \times \frac{1}{2} = \frac{3}{2}$

57. (c) $|\mathbf{a}|^2 = 9 \Rightarrow |\mathbf{b}|^2 = 2$

$$\therefore |\mathbf{c} - \mathbf{a}|^2 = |\mathbf{c}|^2 + |\mathbf{a}|^2 - 2\mathbf{c} \cdot \mathbf{a} = 8$$

$$\Rightarrow |\mathbf{c}|^2 + 9 - 2|\mathbf{c}| = 8$$

$$\Rightarrow |\mathbf{c}| = 1$$

Now, $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = 3$$

$$\therefore |(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}| = |\mathbf{a} \times \mathbf{b}| |\mathbf{c}| \sin 45^\circ = 3(1) \left(\frac{1}{\sqrt{2}} \right) = \frac{3\sqrt{2}}{2}$$

58. (c) $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \alpha \mathbf{a} + \beta \mathbf{b}$

$$\therefore (\mathbf{a} \cdot \mathbf{c}) \mathbf{b} - (\mathbf{b} \cdot \mathbf{c}) \mathbf{a} = \alpha \mathbf{a} + \beta \mathbf{b}$$

Here, $\alpha = -\mathbf{b} \cdot \mathbf{c}, \beta = \mathbf{a} \cdot \mathbf{c}$

Therefore, $\alpha + \beta = \mathbf{a} \cdot \mathbf{c} - \mathbf{b} \cdot \mathbf{c} = (\mathbf{a} - \mathbf{b}) \cdot \mathbf{c} = \{(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - (\hat{\mathbf{i}} + \hat{\mathbf{j}})\} \cdot \hat{\mathbf{i}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{i}} = 0$

BITSAT Archives

1. (a) Required unit vector $= \frac{(\hat{\mathbf{i}} - \hat{\mathbf{j}}) \times (\hat{\mathbf{i}} + \hat{\mathbf{j}})}{|(\hat{\mathbf{i}} - \hat{\mathbf{j}}) \times (\hat{\mathbf{i}} + \hat{\mathbf{j}})|} = \frac{\hat{\mathbf{k}} + \hat{\mathbf{k}}}{2} = \frac{2\hat{\mathbf{k}}}{2} = \hat{\mathbf{k}}$

2. (a) We have, $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \frac{1}{3} |\mathbf{b}| |\mathbf{c}| \mathbf{a}$

$$\Rightarrow (\mathbf{a} \cdot \mathbf{c}) \mathbf{b} - (\mathbf{b} \cdot \mathbf{c}) \mathbf{a} = \frac{1}{3} |\mathbf{b}| |\mathbf{c}| \mathbf{a}$$

$$\Rightarrow (\mathbf{a} \cdot \mathbf{c}) \mathbf{b} - \{(\mathbf{b} \cdot \mathbf{c}) + \frac{1}{3} |\mathbf{b}| |\mathbf{c}|\} \mathbf{a} = 0$$

$$\Rightarrow (\mathbf{a} \cdot \mathbf{c}) = 0 \text{ and } \mathbf{b} \cdot \mathbf{c} + \frac{1}{3} |\mathbf{b}| |\mathbf{c}| = 0$$

$$\Rightarrow |\mathbf{b}| |\mathbf{c}| \cos \theta + \frac{1}{3} |\mathbf{b}| |\mathbf{c}| = 0 \quad [\because \theta \text{ is the angle between } \mathbf{b} \text{ and } \mathbf{c}]$$

$$\Rightarrow \cos \theta = -\frac{1}{3} \therefore \sin \theta = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

3. (d) Since, $\sin \theta = \frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}| |\mathbf{b}|} = \frac{8}{10} = \frac{4}{5}$

$$\therefore \cos \theta = \pm \frac{3}{5}$$

Now, $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta = 2 \times 5 \times \left(\pm \frac{3}{5} \right) = \pm 6 \Rightarrow |\mathbf{a} \cdot \mathbf{b}| = 6$

4. (c)

5. (b) Since, scalar triple product is zero, then $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are coplanar.

6. (b) Since, given vectors are perpendicular.

$$\therefore (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + \lambda\hat{\mathbf{k}}) = 0$$

$$\Rightarrow 2 - 4 - \lambda = 0 \Rightarrow \lambda = -2$$

7. (d) $(\mathbf{a} \times \hat{\mathbf{j}}) \cdot (2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) = \mathbf{a} \cdot \{\hat{\mathbf{j}} \times (2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})\} = \mathbf{a} \cdot (-3\hat{\mathbf{i}}) = -3(\mathbf{a} \cdot \hat{\mathbf{i}}) = -12 \quad [\because \mathbf{a} \cdot \hat{\mathbf{i}} = 4]$

8. (a) Let $\mathbf{a} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$, then

$$\mathbf{a} \cdot \hat{\mathbf{i}} = a_1, \mathbf{a} \cdot \hat{\mathbf{j}} = a_2, \mathbf{a} \cdot \hat{\mathbf{k}} = a_3$$

$$\therefore \mathbf{a} = (\mathbf{a} \cdot \hat{\mathbf{i}}) \hat{\mathbf{i}} + (\mathbf{a} \cdot \hat{\mathbf{j}}) \hat{\mathbf{j}} + (\mathbf{a} \cdot \hat{\mathbf{k}}) \hat{\mathbf{k}}$$

9. (a) The given condition mean that $\mathbf{r}$ is perpendicular to all three vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.

This is possible only if they are coplanar.

$$\therefore [\mathbf{a} \ \mathbf{b} \ \mathbf{c}] = 0$$

10. (c) Adding vectors

$$\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{b} \cdot (\mathbf{c} + \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} + \mathbf{b}) = 0, \text{ we get}$$

$$2(\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}) = 0$$

Now, $|\mathbf{a} + \mathbf{b} + \mathbf{c}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2(\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}) = 1 + 16 + 64 = 81$

$$\therefore |\mathbf{a} + \mathbf{b} + \mathbf{c}| = 9$$

11. (a) Given, $\mathbf{PR} = 5 \mathbf{PQ}$

It means R divides PQ externally in the ratio $5 : 4$.

$$\therefore \text{Position vector of } R = \frac{5\mathbf{b} - 4\mathbf{a}}{5 - 4} = 5\mathbf{b} - 4\mathbf{a}$$

12. (c) Hint Let $\mathbf{OA} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{OB} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

and $\mathbf{OC} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

$$\therefore a = |\mathbf{OA}| = \sqrt{6}, b = |\mathbf{OB}| = \sqrt{35}$$

and $c = |\mathbf{OC}| = \sqrt{41}$

Hence, $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

13. (d) Given, $\mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\mathbf{c} = \hat{\mathbf{i}} + 3\hat{\mathbf{k}}$

Now, $\mathbf{b} \times \mathbf{c} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 1 & -1 \\ 1 & 0 & 3 \end{vmatrix} = \hat{\mathbf{i}}(3 - 0) - \hat{\mathbf{j}}(6 + 1) + \hat{\mathbf{k}}(0 - 1) = 3\hat{\mathbf{i}} - 7\hat{\mathbf{j}} - \hat{\mathbf{k}}$

Now, $|\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}| = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = |\mathbf{a}| |\mathbf{b} \times \mathbf{c}| \cos \theta = 1(\sqrt{3^2 + 7^2 + 1^2}) \cos \theta = \sqrt{59} \cos \theta$

$$[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]_{\max} = \sqrt{59} \cdot 1 \quad [\because \text{maximum value of } \cos \theta \text{ is } 1]$$

Hence, maximum value is $\sqrt{59}$.

14. (c) As we know, a vector coplanar to $\mathbf{a}$, $\mathbf{b}$ and orthogonal to $\mathbf{c}$ is $\lambda \{(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}\}$.

$\therefore$ A vector coplanar to $(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$, $(\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$ and orthogonal to $(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$

$$= \lambda [(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \times (\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})] \times (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}}]$$

$$= \lambda (-21\hat{\mathbf{j}} + 7\hat{\mathbf{k}})$$

$$\therefore \text{Required unit vector} = \pm \frac{(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}}{|(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}|}$$

$$= \pm \frac{-21\hat{\mathbf{j}} + 7\hat{\mathbf{k}}}{\sqrt{(-21)^2 + (7)^2}} = \pm \frac{(3\hat{\mathbf{j}} - \hat{\mathbf{k}})}{\sqrt{10}}$$

15. (d) Given that, $\mathbf{p} = \frac{\mathbf{b} \times \mathbf{c}}{[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]}$, $\mathbf{q} = \frac{\mathbf{c} \times \mathbf{a}}{[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]}$ and $\mathbf{r} = \frac{\mathbf{a} \times \mathbf{b}}{[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]}$

$$\therefore \mathbf{a} \cdot \mathbf{p} = \frac{\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})}{[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]} = \frac{\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})}{[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]} = 1$$

$$\text{and } \mathbf{a} \cdot \mathbf{q} = \mathbf{a} \cdot \frac{\mathbf{c} \times \mathbf{a}}{[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]} = \frac{\mathbf{a} \cdot (\mathbf{c} \times \mathbf{a})}{[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]} = 0$$

$$\text{Similarly, } \mathbf{b} \cdot \mathbf{q} = \mathbf{c} \cdot \mathbf{r} = 1$$

$$\text{and } \mathbf{a} \cdot \mathbf{r} = \mathbf{b} \cdot \mathbf{p} = \mathbf{c} \cdot \mathbf{q} = \mathbf{c} \cdot \mathbf{p} = \mathbf{b} \cdot \mathbf{r} = 0$$

$$\therefore (\mathbf{a} + \mathbf{b}) \cdot \mathbf{p} + (\mathbf{b} + \mathbf{c}) \cdot \mathbf{q} + (\mathbf{c} + \mathbf{a}) \cdot \mathbf{r}$$

$$= \mathbf{a} \cdot \mathbf{p} + \mathbf{b} \cdot \mathbf{p} + \mathbf{b} \cdot \mathbf{q} + \mathbf{c} \cdot \mathbf{q} + \mathbf{c} \cdot \mathbf{r} + \mathbf{a} \cdot \mathbf{r}$$

$$= 1 + 1 + 1 = 3$$

16. (b) Since, $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are mutually perpendicular to each other then

$$\therefore \mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{c} = \mathbf{c} \cdot \mathbf{a} = 0$$

$$\text{Now, } |\mathbf{a} + \mathbf{b} + \mathbf{c}|^2 = (\mathbf{a} + \mathbf{b} + \mathbf{c}) \cdot (\mathbf{a} + \mathbf{b} + \mathbf{c}) \quad [\because |\mathbf{a}|^2 = \mathbf{a} \cdot \mathbf{a}]$$

$$= |\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2(\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a})$$

$$= 1 + 1 + 2(0 + 0 + 0)$$

$$[\because \mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{c} = \mathbf{c} \cdot \mathbf{a} = 0]$$

$$\Rightarrow |\mathbf{a} + \mathbf{b} + \mathbf{c}|^2 = 3 \therefore |\mathbf{a} + \mathbf{b} + \mathbf{c}| = \sqrt{3}$$

17. (c) Let $\mathbf{a} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\mathbf{b} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$$\text{Projection of } \mathbf{a} \text{ on } \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$$

$$= \frac{(2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}})}{\sqrt{1^2 + (-2)^2 + 1^2}}$$

$$= \frac{2 - 2 - 3}{\sqrt{6}} = -\frac{3}{\sqrt{6}} = -\frac{\sqrt{3}}{2}$$

18. (b) Given, $\mathbf{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\mathbf{b} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$$\text{Now, } \mathbf{a} + \mathbf{b} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}} + 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}} = 4\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$$

$$\text{and } \mathbf{a} - \mathbf{b} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) - (3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$$

$$= -2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$$

Let θ be the angle between $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$.

$$\therefore \cos \theta = \frac{(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b})}{|\mathbf{a} + \mathbf{b}| |\mathbf{a} - \mathbf{b}|}$$

$$= \frac{(4\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (-2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}})}{|4\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}| |-2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}|}$$

$$= \frac{-8 + 3 + 5}{\sqrt{16 + 1 + 1} \sqrt{4 + 9 + 25}} = 0 \Rightarrow \theta = 90^\circ$$

19. (c) Hint Since, the given vectors $\alpha \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\hat{\mathbf{i}} + \beta \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \lambda \hat{\mathbf{k}}$ are coplanar.

$$\therefore \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \beta & 1 \\ 1 & 1 & \gamma \end{vmatrix} = 0$$

$$\text{Applying } C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_2$$

$$\Rightarrow \begin{vmatrix} \alpha & 1-\alpha & 0 \\ 1 & \beta-1 & 1-\beta \\ 1 & 0 & \gamma-1 \end{vmatrix} = 0$$

$$\Rightarrow (1-\alpha)(1-\beta)(1-\gamma) \begin{vmatrix} \alpha & 1 & 0 \\ 1-\alpha & 1 & 0 \\ 1-\beta & -1 & 1 \\ 1-\gamma & 0 & -1 \end{vmatrix} = 0$$

20. (a) Since, α lies in the plane of β and γ .

It means that, all three vectors are coplanar.

$$\therefore [\alpha \ \beta \ \gamma] = 0$$

$$\text{21. (c) Hint } (\vec{\alpha} \times \vec{\beta}) = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & -1 \\ -1 & 2 & -4 \end{vmatrix}$$

$$\text{and } (\alpha \times \gamma) = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

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Statistics

Introduction

Statistics is concerned with scientific method for collecting, organising, summarising, presenting and analysing data as well as with drawing valid conclusions and making reasonable decision on the basis of such analysis.

Measure of Central Tendency

An average or a central value of the statistical (series) data is the value of the variable which describes the characteristics of entire distribution.

There are five types of central tendency

1. Arithmetic Mean

The sum of all the observations divided by the number of observations, is called arithmetic mean and it is denoted by $\bar{x}$.

(i) Mean of Ungrouped or Individual Data

If $x_1, x_2, x_3, \dots, x_n$ are n observations, then mean by

(a) Direct method

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} \Rightarrow \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

(b) Shortcut method

$$\bar{x} = A + \frac{1}{n} \sum_{i=1}^n d_i$$

where, A = Assumed mean and $d_i = x_i - A$

(ii) Mean of Grouped or Continuous Data

If $x_1, x_2, \dots, x_n$ are n observations whose corresponding frequencies are $f_1, f_2, \dots, f_n$, then formula will be

(a) **Direct Method**

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \dots + f_nx_n}{f_1 + f_2 + \dots + f_n}$$

$$\bar{x} = \frac{1}{N} \sum_{i=1}^n f_i x_i, \text{ where } N = \sum_{i=1}^n f_i$$

(b) **Shortcut Method** If $d_i = x_i - A$, where A is assumed mean, then

$$\text{Mean } (\bar{x}) = A + \frac{1}{N} \sum_{i=1}^n f_i d_i$$

where, $\sum f_i = N$

(c) **Step Deviation Method** If $u_i = \frac{x_i - A}{h}$, where A is assumed mean, then

$$\text{Mean } (\bar{x}) = A + h \frac{1}{N} \sum_{i=1}^n f_i u_i$$

NOTE • The algebraic sum of the deviations of all the variates from their arithmetic mean is zero.

i.e. $\sum(x - \bar{x}) = 0$ or $\sum d_i = 0$

• The sum of the squares of the deviations of a set of values is minimum when taken about mean.

i.e. $\sum(x - \bar{x})^2 \leq \sum(x - A)^2$

(iii) **Weighted Arithmetic Mean**

Let $w_1, w_2, w_3, \dots, w_n$ be the weights assigned to the values $x_1, x_2, x_3, \dots, x_n$ respectively, then the weighted arithmetic mean is defined as

$$\text{Weighted arithmetic mean} = \frac{w_1x_1 + w_2x_2 + \dots + w_nx_n}{w_1 + w_2 + \dots + w_n}$$

(iv) **Combined Mean**

For two sets of observations,

$$\bar{x}_{12} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

where,

$\bar{x}_{12}$ = Combined mean of both the sets

n_1 = Number of first set of observations

n_2 = Number of second set of observations

$\bar{x}_1$ = Mean of the first set of observations

$\bar{x}_2$ = Mean of the second set of observations

2. Geometric Mean

The n th root of the product of the values, is called geometric mean.

(i) **For Ungrouped Data**

The geometric mean of n observations $x_1, x_2, \dots, x_n$, is defined as (when none of them being zero)

$$G = (x_1 \cdot x_2 \cdot x_3 \cdot \dots \cdot x_n)^{1/n}$$

$$\Rightarrow \log G = \frac{1}{n} \sum_{i=1}^n \log x_i$$

$$G = \text{antilog} \left(\frac{\log x_1 + \log x_2 + \dots + \log x_n}{n} \right)$$

(ii) **For Grouped Data**

Let $x_1, x_2, \dots, x_n$ be n observations and whose corresponding frequencies are $f_1, f_2, \dots, f_n$, then geometric mean is given by

$$GM = (x_1^{f_1} \cdot x_2^{f_2} \cdot \dots \cdot x_n^{f_n})^{\frac{1}{N}}$$

where,

$$N = \sum_{i=1}^n f_i$$

We can also write,

$$GM = \text{antilog} \left(\frac{\sum_{i=1}^n f_i \log x_i}{N} \right)$$

3. Harmonic Mean

The harmonic mean of any series is the reciprocal of the arithmetic mean of the reciprocals of the observations.

(i) **For Ungrouped Data**

If $x_1, x_2, \dots, x_n$ be n observations, then harmonic mean is given by

$$HM = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}}$$

$$HM = \frac{n}{\sum_{i=1}^n \frac{1}{x_i}}$$

(ii) **For Grouped Data**

If $x_1, x_2, \dots, x_n$ be n observations, whose corresponding frequencies are $f_1, f_2, \dots, f_n$, then harmonic mean is given by

$$HM = \frac{\sum_{i=1}^n f_i}{\sum_{i=1}^n \frac{f_i}{x_i}}$$

NOTE It is known that $AM \geq GM \geq HM$, when $x_1, x_2, \dots, x_n > 0$

4. Median

The median is the middle most point in a rank ordered set of measures.

(i) Median for Ungrouped or Individual Data

Suppose n observations are arranged in ascending or descending order.

(a) If n is an odd number, then

$$\text{Median} = \text{Value of } \left(\frac{n+1}{2}\right)\text{th term}$$

(b) If n is an even number, then

$$\text{Median} = \frac{\text{Value of } \left(\frac{n}{2}\right)\text{th term} + \text{Value of } \left(\frac{n}{2} + 1\right)\text{th term}}{2}$$

(ii) Median for Discrete Frequency Data

By arranging the data in ascending or descending order and by finding the cumulative frequency, find $\frac{N}{2}$, where

$$N = \sum f_i.$$

Now, see the cumulative frequency just greater than $\frac{N}{2}$. The corresponding value of x is median.

(iii) Median for Continuous or Grouped Data

In this case, the class corresponding to the cumulative frequency just greater than $\frac{N}{2}$, is called the median class

and the value of median is obtained by the following formula

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c}{f} \right) \times h$$

where,

l = Lower limit of median class

f = Frequency of median class

h = size of median class

c = Cumulative frequency of class before median class

NOTE Cumulative frequency is defined as a consecutive sum of frequencies.

5. Mode

Mode is the value of the variate which occurs most frequently.

Mode can be calculated in following ways :

(i) Mode for Individual Data

The value which is repeated maximum number of times is the mode of the given series.

(ii) Mode for Discrete Data

The mode is the value of the variate corresponding to the maximum frequency.

(iii) Mode for Continuous Data

The class which has maximum frequency, is called modal class or group. The mode is given by the formula

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_2 - f_0} \times h$$

where, l = Lower limit of the modal class
(the class having maximum frequency)

f_1 = Frequency of the modal class

f_0 = Frequency of the class preceding the modal class

f_2 = Frequency of the class succeeding the modal class

h = Width of the modal class

NOTE Let AM, GM and HM be the arithmetic mean, geometric mean and harmonic mean respectively, then the following relations are given below

$$> (AM)(HM) = (GM)^2$$

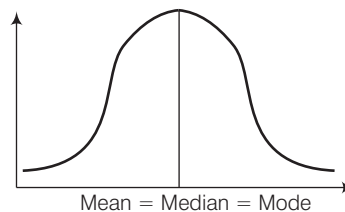
$$> \text{Mode} = 3\text{Median} - 2\text{Mean} \quad [\text{Empirical formula}]$$

Measure of Skewness

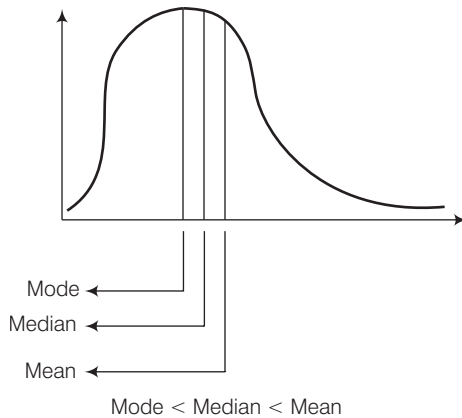
Skewness describes the degree to which the data deviates from symmetry. If the distribution of the data is not symmetrical, it is said to be asymmetrical or skewed.

Types of Distribution**1. Symmetrical Distribution/Bell Shaped Distribution**

In this distribution, mean, median and mode are all located at one point.

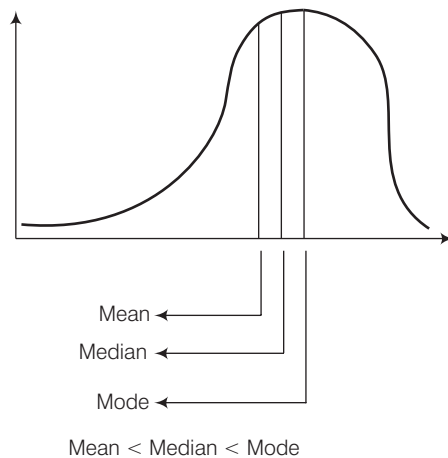
**2. Positively Skewed Distribution**

In this distribution, observations are mostly concentrated towards the smaller values and there are some extremely high values.



3. Negatively Skewed Distribution

In this distribution, observations are mostly concentrated towards the larger values and there are some extremely low values.



The following is the important method of measuring skewness.

Karl Pearson's Coefficient of Skewness

The method is most frequently used for measuring skewness. The formula for measuring coefficient of skewness is as follows:

$$Sk_p = \frac{\text{Mean} - \text{Mode}}{\sigma}$$

or

$$Sk_p = \frac{3(\text{Mean} - \text{Median})}{\sigma}$$

where, Sk_p = Pearsonian (or Karl Pearson's) coefficient of skewness.

The value of this coefficient will be zero, if it is a symmetrical distribution. If the value is greater than zero, then it is positively skewed and if the value is less than zero, then it is negatively skewed distribution.

Measures of Dispersion

Dispersion means 'scatteredness'. Dispersion measures the degree of scatteredness of the variable about a central value.

There are following measures of dispersion

1. Range

The difference between the maximum and the minimum observation is called range.

i.e. $\text{Range} = L - S$

$$\text{Coefficient of range} = \frac{L - S}{L + S}$$

where, L = Maximum observation and S = Minimum observation

2. Quartile Deviation

It is one-half of the difference between the third and the first quartile.

i.e. $QD = \frac{Q_3 - Q_1}{2}$

and coefficient of $QD = \frac{Q_3 - Q_1}{Q_3 + Q_1}$

where, Q_1 = Value of $\frac{n}{4}$ th term and Q_3 = Value of $\frac{3n}{4}$ th term

3. Mean Deviation (MD)

The mean (mode or median) of the absolute deviations of the values of the variable from a measure of their average is called mean deviation (MD).

(i) MD for Ungrouped Data

If $x_1, x_2, \dots, x_n$ are n observations, then $MD = \frac{\sum_{i=1}^n |x_i - z|}{n}$

where, z = mean or mode or median

(ii) MD for Grouped Data

If corresponding frequencies of each observation are $f_1, f_2, \dots, f_n$, then

$$MD = \frac{\sum_{i=1}^n f_i |x_i - z|}{N}, \text{ where } \sum_{i=1}^n f_i = N$$

$$\text{Coefficient of MD} = \frac{\text{Mean deviation}}{\text{Corresponding average}} \times 100\%$$

Standard deviation

Standard deviation is also known as root mean square deviation because it is the square root of the means of the square deviation from the arithmetic mean. It is denoted by σ .

(i) For Ungrouped Data

$$(a) \text{ SD } = \sigma = \sqrt{\frac{\sum d^2}{n}}, \text{ where } d = x - \bar{x}$$

when we take deviation from the assumed mean A , then

$$(b) \text{ SD } = \sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}, \text{ where } d = x - A$$

(ii) For Grouped Data (Discrete and Continuous Series)

$$(a) \sigma = \sqrt{\frac{\sum f d^2}{\sum f}}, \text{ where } d = x - \bar{x}$$

$$(b) \sigma = \sqrt{\frac{\sum f d^2}{\sum f} - \left(\frac{\sum f d}{\sum f}\right)^2}$$

where, $d = x - A$, A is assumed mean

$$(c) \sigma = i \sqrt{\frac{\sum f d'^2}{\sum f} - \left(\frac{\sum f d'}{\sum f}\right)^2}$$

where, $d' = \frac{x - A}{i}$, i = width of the class

- NOTE**
- We use formula (a), only when we get the mean as an integer, otherwise calculations become difficult.
 - Quartile deviation = $\frac{2}{3}\sigma$
and Mean deviation = $\frac{4}{5}\sigma$

4. Variance

Variance is the arithmetic mean of the squares of deviations about mean $\bar{x}$ and denoted as $\text{var}(X)$ or σ^2 .

So, variance is defined as the square of standard deviation.

$$\therefore \text{Variance} = (\text{Standard deviation})^2$$

Combined Standard Deviation

Let n_1 and n_2 be the sizes of two series. Their means and standard deviations are $\bar{x}_1, \bar{x}_2$ and σ_1, σ_2 , respectively.

$$\text{Let combined mean be } \bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} = \bar{x}_{12}$$

$\therefore$ For two series, combined standard deviation is

$$\sigma_{12} \text{ or } \sigma = \sqrt{\frac{n_1 \sigma_1^2 + n_2 \sigma_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$$

$$\sigma_{12} \text{ or } \sigma = \sqrt{\frac{n_1 (\sigma_1^2 + d_1^2) + n_2 (\sigma_2^2 + d_2^2)}{n_1 + n_2}}$$

where,

n_1, n_2 = Number of observations of series

$\bar{x}_1$ and $\bar{x}_2$ = Mean value of respective series

σ_{12} = Standard deviations of σ_1 and σ_2

$$d_1 = (\bar{x}_1 - \bar{x}_{12}), \quad d_2 = (\bar{x}_2 - \bar{x}_{12}).$$

Effects of Average and Dispersion on Change of Origin and Scale

	Change of origin	Change of scale
Mean	Dependent	Dependent
Median	Dependent	Dependent
Mode	Dependent	Dependent
Standard deviation	Not dependent	Dependent
Variance	Not dependent	Dependent

Coefficient of Variation

Less the coefficient of variation, more consistent is the data.

$$\text{Thus, } CV = \frac{\sigma}{\bar{x}} \times 100$$

The distribution having smaller value of the coefficient of variation has less variability around the central value than the distribution having greater value of the coefficient of variation.

For two distributions having equal means, the distribution having greater standard deviation (or variance) is said to be more variable or dispersed than other.

Practice Exercise

- A student obtain 75%, 80% and 85% marks in three subjects. If the marks of another subject are added, then his average cannot be less than
a. 60% b. 65% c. 80% d. 90%
- If a variable takes the values $0, 1, 2, \dots, n$ with frequencies proportional to the binomial coefficients ${}^nC_0, {}^nC_1, \dots, {}^nC_n$, then mean of the distribution is
a. $\frac{n}{2}$ b. $\frac{n(n+1)}{2}$
c. $\frac{n(n-1)}{2}$ d. None of these
- The mean height of 25 male workers in a factory is 161 cm and mean height of 35 female workers in the same factory is 158 cm. The combined mean height of 60 workers in the factory is
a. 159.25 b. 159.5 c. 159.75 d. 158.75
- In an English learning classes, there are 8 men, 7 women and 5 children whose mean ages separately are respectively 24, 20 and 6 yr. The mean age of English learning classes candidate is
a. 18.0 b. 18.1 c. 18.2 d. 18.3
- The average income of male employees in a financial sector company A was ₹ 520 and that of females was ₹ 420. The mean income of all the employees was ₹ 500. The percentage of male employees is
a. 50% b. 80% c. 40% d. 20%
- If g_1 and g_2 are the geometric means of two series of n_1 and n_2 items. Then, the GM of the series obtained on combining is
a. $[(g_1)^{n_1} (g_2)^{n_2}]^{\frac{1}{n_1+n_2}}$ b. $(g_1 g_2)^{\frac{n_1}{n_1+n_2}}$
c. $(g_1 g_2)^{\frac{n_2}{n_1+n_2}}$ d. $(g_1 g_2)^{\frac{n_1 n_2}{n_1+n_2}}$
- The mean and median of 100 items are 50 and 52, respectively. The value of largest item is 100. It was later found that, it is 110 not 100. The true mean and median are
a. 50.10, 51.5 b. 50.10, 52
c. 50, 52 d. None of these
- If the difference between mean and mode is 63, then the difference between mean and median is
a. 189 b. 21 c. 31.5 d. 48.5
- If a distribution has negative skewness, then in what order (lowest to highest) will the averages be?
a. Mean, mode, median b. Mean, median, mode
c. Mode, median, mean d. Median, mode, mean
- Which average is the most sensitive to extreme values?
a. Mean b. Median c. Mode d. None
- If the median and the range of four numbers $\{x, y, 2x+y, x-y\}$, where $0 < y < x < 2y$, are 10 and 28 respectively, then the mean of the numbers is
a. 18 b. 10 c. 5 d. 14
- The quartile deviation of daily wages (in ₹) of 7 persons given 12, 7, 15, 10, 17, 19 and 25 is
a. 14.5 b. 5 c. 9 d. 4.5
- If the mean deviations about the median of the numbers $a, 2a, \dots, 5a$ is 50, then $|a|$ is equal to
a. 3 b. 4 c. 5 d. 2
- If the mean deviation of number $1, 1+d, 1+2d, \dots, 1+100d$ from their mean is 255, then d is equal to
a. 10.0 b. 20.0 c. 10.1 d. 20.2
- The mean and SD of the marks of 200 candidates were found to be 40 and 15, respectively. Later, it was discovered that a score 40 was wrongly read as 50. The correct mean and SD are respectively
a. 14.98, 39.95 b. 39.95, 14.98
c. 39.95, 224.5 d. None of these
- If quartile deviation of a sample is 20, then the most likely value of SD is
a. 30 b. 12 c. 18 d. 13
- If $\sum_{i=1}^{18} (x_i - 8) = 9$ and $\sum_{i=1}^{18} (x_i - 8)^2 = 45$, then the standard deviation of $x_1, x_2, \dots, x_{18}$ is
a. $\frac{4}{9}$ b. $\frac{9}{4}$
c. $\frac{3}{2}$ d. None of these
- In a series of $2n$ observations, half of them equal a and remaining half equal $-a$. If the standard deviation of the observations is 2, then $|a|$ is equal to
a. $\frac{1}{n}$ b. $\sqrt{2}$ c. 2 d. $\frac{\sqrt{2}}{n}$
- If the variance of the observations $x_1, x_2, \dots, x_n$ is σ^2 , then the variance of $\alpha x_1, \alpha x_2, \dots, \alpha x_n$, $\alpha \neq 0$, is
a. σ^2 b. $\alpha \sigma^2$ c. $\alpha^2 \sigma^2$ d. $\frac{\sigma^2}{\alpha^2}$
- The mean of five observations is 4 and their variance is 5.2. If three of them are 1, 2, 6, then other two are
a. 4, 7 b. 2, 9 c. 5, 6 d. 2, 10

21. Mean of 5 observations is 7. If four of these observations are 6, 7, 8, 10 and one is missing, then the variance of all the five observations is
 a. 4 b. 6 c. 8 d. 2
22. The variance of first 50 even natural numbers is
 a. $\frac{833}{4}$ b. 833 c. 437 d. $\frac{437}{4}$
23. The mean of the numbers $a, b, 8, 5, 10$ is 6 and the variance is 6.80. Then, which one of the following gives possible values of a and b ?
 a. $a = 3, b = 4$ b. $a = 0, b = 7$
 c. $a = 5, b = 2$ d. $a = 1, b = 6$
24. Let $x_1, x_2, \dots, x_n$ be n observations, such that $\sum x_i^2 = 400$ and $\sum x_i = 80$. Then, the possible values of n among the following is
 a. 12 b. 9 c. 16 d. 15
25. One set containing five numbers has mean 8 and variance 24 and the second set containing three numbers has mean 8 and variance 24. Then, the variance of the combined set is
 a. 42 b. 24 c. 20 d. 25
26. The first of two samples has 100 items with mean 15 and SD = 3. If the whole group has 250 items with mean 15.6 and SD = $\sqrt{13.44}$, then SD of the second group is
 a. 4 b. 5 c. 6 d. 3.52
27. A sample of 90 values has standard deviation 3 and their mean is 55. A second sample of 110 values has mean 60 and its standard deviation is 2. The combined variance is equal to
 a. 12.44 b. 13.24 c. 16.42 d. 13.65
28. The standard deviation of 25 numbers is 40. If each of the numbers which is greater than the standard deviation is increased by 5, then the new standard deviation will be
 a. 40 b. 45
 c. $40 + (21/25)$ d. None of these
29. If the standard deviation of the observations $-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$ is $\sqrt{10}$. The standard deviation of the observations 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25 will be
 a. $\sqrt{10} + 20$ b. $\sqrt{10} + 10$
 c. $\sqrt{10}$ d. None of these
30. The marks of some students were listed out of 75. The SD of marks was found to be 9. Subsequently, the marks were raised to a maximum of 100 and variance of new marks was calculated. The new variance is
 a. 81 b. 122 c. 144 d. None
31. The SD of a variate x is σ . The SD of the variate $\frac{ax+b}{c}$, where a, b and c are constants, is
 a. $\left(\frac{a}{c}\right)\sigma$ b. $\left|\frac{a}{c}\right|\sigma$
 c. $\left(\frac{a^2}{c^2}\right)\sigma$ d. None of these
32. The SD of 15 items is 6 of each item is decreased by 1, then standard deviation will be
 a. 5 b. 7 c. $\frac{91}{15}$ d. 6
33. The standard deviation of a variable x is 10. Then, the standard deviation of $50 + 5x$ is
 a. 50 b. 550 c. 10 d. 500
34. Suppose population A has 100 observations 101, 102, ..., 200 and another population B has 100 observations 151, 152, ..., 250. If V_A and V_B represent the variances of the two populations respectively, then $\frac{V_A}{V_B}$ is
 a. $\frac{9}{4}$ b. $\frac{4}{9}$
 c. $\frac{2}{3}$ d. 1
35. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given?
 a. Mean b. Median c. Mode d. Variance
36. If the standard deviation of $y_1, y_2, \dots, y_n$ is 3.5, then the standard deviation of $-2y_1 - 3, -2y_2 - 3, \dots, -2y_n - 3$ is
 a. -7 b. 9 c. 7 d. 2.45
37. The best statistical measure used for comparing two series is
 a. mean deviation b. range
 c. coefficient of variation d. None of these
38. Coefficient of variation of two distributions are 50 and 60 and their arithmetic means are 30 and 25, respectively. Difference of their standard deviation is
 a. 0 b. 1
 c. 1.5 d. 2.5
39. The sum of squares of deviations for 10 observations taken from mean 50 is 250. The coefficient of variation is
 a. 10% b. 40%
 c. 50% d. None of these

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- The mean of n terms is $\bar{x}$. If the first term is increased by 1, second by 2 and so on, then the new mean is [2014]
 - $\bar{x} + n$
 - $\bar{x} + \frac{n}{2}$
 - $\bar{x} + \frac{n+1}{2}$
 - None of these
- A sample of 35 observations has the mean 80 and standard deviation as 4. A second sample of 65 observations from the same population has mean 70 and standard deviation 3. Then, the standard deviation of the combined sample is [2014]
 - 5.85
 - 5.58
 - 34.2
 - None of these
- The mean life of a sample of 60 bulbs was 650 h and the standard deviation was 8 h. A second sample of 80 bulbs has a mean life of 660 h and standard deviation 7 h. Find the overall standard deviation. [2013]
 - 8.97
 - 8.98
 - 8.94
 - None of these
- If x_1 and x_2 are the means of two distributions such that $x_1 < x_2$ and $\bar{x}$ is the mean of the combined distribution, then [2010]
 - $\bar{x} < \bar{x}_1$
 - $\bar{x} > \bar{x}_2$
 - $\bar{x} = \frac{\bar{x}_1 + \bar{x}_2}{2}$
 - $\bar{x}_1 < \bar{x} < \bar{x}_2$

Answer with Solutions

Practice Exercise

- (a) Marks obtained from 3 subjects out of 300
 $= 75 + 80 + 85 = 240$
 If the marks of another subject is added, then the marks will be ≥ 240 out of 400.
 $\therefore$ Minimum average marks $= \frac{240}{4} = 60\%$
- (a) Here, $N = \sum_{i=1}^n f_i = k ({}^nC_0 + {}^nC_1 + \dots + {}^nC_n)$
 $= k(1+1)^n = k2^n$
 where, k is a constant of proportionality.
 and $\sum f_i x_i = k(1 \cdot {}^nC_1 + 2 \cdot {}^nC_2 + \dots + n \cdot {}^nC_n)$
 $= kn \left[1 + (n-1) + \frac{(n-1)(n-2)}{2!} + \dots + 1 \right] = kn2^{n-1}$
 $\therefore$ Mean, $\bar{x} = \frac{1}{2^n} (n \cdot 2^{n-1}) = \frac{n}{2}$
- (a) Combined mean height $= \frac{25 \times 161 + 35 \times 158}{60} = 159.25$
- (b) **Hint** We have, $\bar{x}_1 = 24$, $n_1 = 8$, $\bar{x}_2 = 20$, $n_2 = 7$ and $\bar{x}_3 = 6$, $n_3 = 5$
 We know that, $A = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2 + n_3 \bar{x}_3}{n_1 + n_2 + n_3}$
- (b) **Hint** We can write from given data in the question,
 $\bar{x}_1 = 520$, $\bar{x}_2 = 420$ and $\bar{x} = 500$
 $\therefore \bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$... (i)
 Percentage of males employees in the company
 $= \frac{n_1}{n_1 + n_2} \times 100$
- (a) Suppose $g_1 = (x_1 x_2 \dots x_{n_1})^{1/n_1}$ and $g_2 = (y_1 y_2 \dots y_{n_2})^{1/n_2}$
 Clearly, g is the GM of the combined series.
 $\frac{1}{n} \ln g = \frac{1}{n} \ln [x_1 x_2 \dots x_{n_1} \cdot y_1 y_2 \dots y_{n_2}]$
 $= \frac{1}{n} [\ln(x_1 x_2 \dots x_{n_1}) + \ln(y_1 y_2 \dots y_{n_2})]$
 $= \frac{1}{n} [n_1 \ln g_1 + n_2 \ln g_2] = \frac{n_1}{n} \ln g_1 + \frac{n_2}{n} \ln g_2$
- (b) Given, mean $\bar{x} = \frac{\sum x_i}{n}$
 $\Rightarrow \sum x_i = 100 \times 50 = 5000$
 $\therefore$ Corrected mean $= \frac{5000 - 100 + 110}{100} = 50.10$
 But median remain same i.e. 52.
- (b) **Hint** Mode = 3 Median - 2 Mean
 $\Rightarrow$ Mean - Mode = 3 (Mean - Median)
- (b) In negatively skew distribution,
 Mean < Median < Mode
- (a) Mean is the most sensitive average to extreme values.
- (d) **Hint** Firstly, four numbers according to the condition $0 < y < x < 2y$, i.e. $x - y, y, x, 2x + y$
 $\text{Median} = \frac{2\text{nd term} + 3\text{rd term}}{2} = 10$
 $\Rightarrow y + x = 20$... (i)
 $\text{Range} = (2x + y) - (x - y) = 28$
 $\Rightarrow x + 2y = 28$... (ii)
- (d) Firstly, arrange the data in ascending order.
 i.e. 7, 10, 12, 15, 17, 19, 25.
 $\therefore Q_1 = \text{Size of } \left(\frac{n+1}{4} \right)\text{th item}$
 $= \text{Size of 2nd item} = 10$

$$Q_3 = \text{Size of } \left[\frac{3(n+1)}{4} \right] \text{th item}$$

$$= \text{Size of 6th item} = 19$$

$$\therefore \text{Quartile deviation} = \frac{Q_3 - Q_1}{2} = \frac{19 - 10}{2} = 4.5$$

$$13. (b) \text{ Median of } a, 2a, 3a, 4a, \dots, 50a \text{ is } \frac{25a + 26a}{2} = (25.5)a$$

$$\text{Mean deviation about median} = \frac{\sum_{i=1}^n |x_i - \text{Median}|}{n}$$

$$\Rightarrow 50 = \frac{1}{50} \{2|a| \cdot (0.5 + 1.5 + 2.5 + \dots + 24.5)\}$$

$$\Rightarrow 2500 = 2|a| \cdot \frac{25}{2} (2 \times 0.5 + 24 \times 1) = 2|a| \cdot \frac{25}{2} (25)$$

$$\therefore |a| = 4$$

$$14. (c) \bar{x} = \frac{\text{Sum of quantities}}{n} = \frac{\frac{n}{2}(a+l)}{n}$$

$$= \frac{1}{2} [1 + 1 + 100d] = 1 + 50d$$

$$\therefore \text{MD} = \frac{1}{n} \sum |x_i - \bar{x}| \Rightarrow 255 = \frac{1}{101}$$

$$[50d + 49d + 48d + \dots + d + 0 + d + \dots + 50d]$$

$$\Rightarrow 255 = \frac{2d}{101} \left[\frac{50 \times 51}{2} \right]$$

$$\therefore d = \frac{255 \times 101}{50 \times 51} = 10.1$$

$$15. (b) \text{ We know, } \bar{x} = \frac{\sum x}{n} \text{ or } \sum x = n\bar{x}$$

$$\therefore \text{Incorrect } \sum x = 200 \times 40 = 8000$$

$$\text{Corrected } \sum x = 8000 - 50 + 40 = 7990$$

$$\text{Corrected mean} = \frac{7990}{200} = 39.95$$

$$\text{We know that, } \sigma^2 = \frac{\sum x^2}{n} - \bar{x}^2 \text{ or } \sum x^2 = n(\sigma^2 + \bar{x}^2)$$

$$\text{Incorrect } \sum x^2 = 200(15^2 + 40^2) = 365000$$

$$\text{Corrected } \sum x^2 = 365000 - 50^2 + 40^2 = 364100$$

$$\therefore \text{Corrected } \sigma^2 = \frac{364100}{200} - (39.95)^2 = 224.5$$

$$\text{Hence, corrected } \sigma = \sqrt{224.5} = 14.98$$

$$16. (a) \therefore \text{Quartile deviation} = \frac{2}{3} \times \text{Standard deviation}$$

$$\therefore \text{Standard deviation} = \frac{20 \times 3}{2} = 30$$

$$17. (c) \text{ Let } d_i = x_i - 8$$

$$\therefore \sigma_x^2 = \sigma_d^2 = \frac{1}{18} \sum d_i^2 - \left(\frac{1}{8} \sum d_i \right)^2$$

$$= \frac{1}{18} \times 45 - \left(\frac{9}{18} \right)^2$$

$$= \frac{5}{2} - \frac{1}{4} = \frac{9}{4} \Rightarrow \sigma_x = \frac{3}{2}$$

$$18. (c) \text{ In the } 2n \text{ observations, half of them equal to } a \text{ and remaining half equal to } -a. \text{ Then, the mean of total } 2n \text{ observations is equal to zero.}$$

$$\therefore \text{SD} = \sqrt{\frac{\sum (x - \bar{x})^2}{N}} \Rightarrow 2 = \sqrt{\frac{\sum x^2}{2n}}$$

$$\Rightarrow 4 = \frac{\sum x^2}{2n} = \frac{2na^2}{2n} \Rightarrow a^2 = 4$$

$$\therefore |a| = 2$$

$$19. (c) \text{ Variance} = \frac{1}{n} \sum (x - \bar{x})^2 = \sigma^2$$

$$\text{New variance} = \frac{1}{n} \sum (\alpha x - \alpha \bar{x})^2 = \alpha^2 \frac{1}{n} \sum (x - \bar{x})^2 = \alpha^2 \sigma^2$$

$$20. (a) \text{ Let the two observations be } x_1 \text{ and } x_2.$$

$$\text{Then, } \frac{1 + 2 + 6 + x_1 + x_2}{5} = 4$$

$$\Rightarrow x_1 + x_2 = 11 \quad \dots (i)$$

$$\text{and } \frac{\left[(1-4)^2 + (2-4)^2 + (6-4)^2 + (x_1-4)^2 + (x_2-4)^2 \right]}{5} = 5.2$$

$$\Rightarrow (x_1 - 4)^2 + (x_2 - 4)^2 = 26 - 9 - 4 - 4 = 9 \quad \dots (ii)$$

On solving Eqs. (i) and (ii), we get

$$x_1 = 4 \text{ and } x_2 = 7$$

$$21. (a) \text{ Mean, } 7 = \frac{6 + 7 + 8 + 10 + x}{5} \Rightarrow x = 4$$

Variance

$$= \frac{(6-7)^2 + (7-7)^2 + (8-7)^2 + (10-7)^2 + (4-7)^2}{5}$$

$$= \frac{1^2 + 0 + 1^2 + 3^2 + 3^2}{5} = \frac{20}{5} = 4$$

$$22. (b) \sigma^2 = \left(\frac{\sum x_i^2}{n} \right) - \bar{x}^2$$

$$\bar{x} = \frac{\sum_{r=1}^{50} 2r}{50} = 51, \sigma^2 = \frac{\sum_{r=1}^{50} 4r^2}{50} - (51)^2 = 833$$

$$23. (a) \text{ According to the given condition,}$$

$$6.80 = \frac{\left[(6-a)^2 + (6-b)^2 + (6-8)^2 + (6-5)^2 + (6-10)^2 \right]}{5}$$

$$\Rightarrow 34 = (6-a)^2 + (6-b)^2 + 4 + 1 + 16$$

$$\Rightarrow (6-a)^2 + (6-b)^2 = 13 = 9 + 4$$

$$\Rightarrow (6-a)^2 + (6-b)^2 = 3^2 + 2^2$$

$$\therefore a = 3, b = 4$$

$$24. (c) \text{ Given that, } \sum x_i^2 = 400 \text{ and } \sum x_i = 80$$

$$\therefore \sigma^2 \geq 0$$

$$\Rightarrow \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n} \right)^2 \geq 0 \Rightarrow \frac{400}{n} - \frac{6400}{n^2} \geq 0$$

$$\therefore n \geq 16$$

25. (b) Given that, $\bar{x}_1 = 8, n_1 = 5, \sigma_1^2 = 24; \bar{x}_2 = 8, n_2 = 3, \sigma_2^2 = 24$

Mean of combined set,

$$\bar{x}_{12} = \frac{\bar{x}_1 \times n_1 + \bar{x}_2 \times n_2}{n_1 + n_2} \Rightarrow \bar{x}_{12} = \frac{8 \times 5 + 8 \times 3}{5 + 3} = 8$$

Combined variance,

$$\sigma_{12}^2 = \frac{n_1 \sigma_1^2 + n_2 \sigma_2^2 + d_1^2 + d_2^2}{n_1 + n_2}$$

Here, $d_1 = \bar{x}_1 - \bar{x}_{12}$ and $d_2 = \bar{x}_2 - \bar{x}_{12}$

$$\begin{aligned} \sigma_{12}^2 &= \frac{5 \times 24 + 3 \times 24 + (8 - 8)^2 + (8 - 8)^2}{5 + 3} \\ &= \frac{120 + 72}{8} = \frac{192}{8} = 24 \end{aligned}$$

Hence, the variance of combined set is 24.

26. (a) We know that, $\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$,

where, $d_1 = m_1 - a, d_2 = m_2 - a$, a being the mean of the whole group.

$$\therefore 15.6 = \frac{100 \times 15 + 150 \times m_2}{250} \Rightarrow m_2 = 16$$

$$\text{Thus, } 13.44 = \frac{\left[\begin{array}{l} (100 \times 9 + 150 \times \sigma^2) \\ + 100 \times (0.6)^2 + 150 \times (0.4)^2 \end{array} \right]}{250}$$

$$\Rightarrow 33.60 = (900 + 150 \sigma^2 + 36 + 24)$$

$$\therefore \sigma^2 = 16 \Rightarrow \sigma = 4$$

27. (a) We have, $n_1 = 90, \bar{x}_1 = 55$ and $\sigma_1 = 3$

and $n_2 = 110, \bar{x}_2 = 60, \sigma_2 = 2$

$$\text{Now, } \bar{x} = \frac{90 \times 55 + 110 \times 60}{200} = 57.75$$

$$d_1 = 55 - 57.75 = -2.75$$

$$\text{and } d_2 = 60 - 57.75 = 2.25$$

$$\begin{aligned} \therefore \sigma^2 &= \frac{90(9 + 7.5625) + 110(4 + 5.0625)}{200} \\ &= 12.44 \text{ (approx.)} \end{aligned}$$

28. (a) Given that, $M = 25$

$$\text{SD } (\sigma) = 40$$

If each number is increased by 5, then mean also increased by 5, therefore for the value of

$$\sigma^2 = \frac{1}{M} \sum (x_i - \bar{x})^2 \text{ will not change.}$$

Now, SD will also be 40.

29. (c) The new observations are obtained by adding 20 to each previous observation.

Hence, the standard deviation of new observations will be same, i.e. $\sqrt{10}$.

30. (c) Given, $\sigma = 9$

Let a student obtains x marks out of 75. Then, his marks out of 100 are $\frac{4x}{3}$. Each observation is multiplied by $\frac{4}{3}$.

$$\therefore \text{New SD, } \sigma = \frac{4}{3} \times 9 = 12$$

Hence, variance is $\sigma^2 = 144$.

31. (b) Suppose $y = \frac{ax+b}{c}$ and $y = \frac{a}{c}x + \frac{b}{c}$

$$\text{Now, } y = Ax + B, \text{ where } A = \frac{a}{c}, B = \frac{b}{c}$$

$$\bar{y} = A\bar{x} + B \Rightarrow y - \bar{y} = A(x - \bar{x}) \quad [\because B = y - Ax]$$

$$\Rightarrow (y - \bar{y})^2 = A^2(x - \bar{x})^2 \Rightarrow \sum (y - \bar{y})^2 = A^2 \sum (x - \bar{x})^2$$

$$\Rightarrow n \cdot \sigma_y^2 = A^2 \cdot n \sigma_x^2 \Rightarrow \sigma_y^2 = A^2 \sigma_x^2$$

$$\Rightarrow \sigma_y = |A| \sigma_x \therefore \sigma_y = \left| \frac{a}{c} \right| \sigma_x$$

32. (d) We know that, standard deviation is independent of change of origin.

Hence, if each item is decreased by 1, then standard deviation will be same, i.e. 6.

33. (a) Given, the standard deviation (SD) of the variable x is 10.

$$\therefore \text{Standard deviation of } 50 + 5x = 5x = 50 \quad [\because x = 10]$$

34. (d) Since, variance is independent of change of origin.

Therefore, variance of observations 101, 102, ..., 200 is same as variance of 151, 152, ..., 250.

$$\therefore V_A = V_B \Rightarrow \frac{V_A}{V_B} = 1$$

35. (d) If initially all marks were x_i , then

$$\sigma_2^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

Now, each is increased by 10.

$$\therefore \sigma_2^2 = \frac{\sum [(x_i + 10) - (\bar{x} + 10)]^2}{n} = \sigma_1^2$$

So, variance will not change whereas mean, median and mode will increase by 10.

36. (c) Let new observations be $x_1, x_2, \dots, x_n$ such that

$$x_i = -2y_i - 3 \Rightarrow y_i = -\frac{1}{2}x_i - \frac{3}{2}$$

$$\text{Thus, } \sigma_y = |-1/2| \sigma_x \Rightarrow \sigma_x = 2\sigma_y = 2 \times 3.5 = 7$$

37. (c) The best statistical measure used for comparing two series is coefficient of variation.

38. (a) Given, coefficient of variation, $C_1 = 50$

and coefficient of variation, $C_2 = 60$

$$\text{We have, } \bar{x}_1 = 30 \text{ and } \bar{x}_2 = 25$$

$$\therefore C = \frac{\sigma}{\bar{x}} \times 100$$

$$\therefore 50 = \frac{\sigma_1}{30} \times 100 \Rightarrow \sigma_1 = 15$$

$$\text{and } 60 = \frac{\sigma_2}{25} \times 100 \Rightarrow \sigma_2 = 15$$

$$\therefore \text{Required difference, } \sigma_1 - \sigma_2 = 15 - 15 = 0$$

39. (a) Clearly, we have

$$\text{Coefficient of variation} = \frac{\sigma}{\bar{x}} \times 100 = \frac{\sigma}{50} \times 100 \quad [\because \bar{x} = 50]$$

$$\text{Now, } \sigma = \sqrt{\frac{\sum (x_i - 50)^2}{n}} = \sqrt{\frac{250}{10}} = \sqrt{25} = 5$$

$$\therefore \text{Coefficient of variation} = \frac{5}{50} \times 100 = 10\%$$

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1. (c) **Hint** Let the observation be $x_1, x_2, x_3, \dots, x_n$.

$$\text{Now, mean } (\bar{x}) = \frac{x_1 + x_2 + \dots + x_n}{n}$$

When first term is increased by 1, second by 2 and so on, then observations will be

$$(x_1 + 1), (x_1 + 2), (x_1 + 3), \dots, (x_n + n)$$

Then, new mean

$$\begin{aligned} (\bar{x}_1) &= \frac{(x_1 + 1) + (x_2 + 2) + \dots + (x_n + n)}{n} \\ &= \frac{(x_1 + x_2 + \dots + x_n) + (1 + 2 + 3 + \dots + n)}{n} \end{aligned}$$

2. (a) Mean of the combined sample

$$\begin{aligned} \bar{x} &= \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} \\ &= \frac{35 \times 80 + 65 \times 70}{35 + 65} \\ &= 73.5 \end{aligned}$$

Standard deviation of the combined sample is given by

$$\sigma^2 = \frac{n_1 (\sigma_1^2 + d_1^2) + n_2 (\sigma_2^2 + d_2^2)}{n_1 + n_2}$$

$$\text{where, } d_1^2 = (\bar{x}_1 - \bar{x})^2 = (80 - 73.5)^2 = 42.25$$

$$d_2^2 = (\bar{x}_2 - \bar{x})^2 = (70 - 73.5)^2 = 12.25$$

$$\therefore \sigma^2 = \frac{35(4^2 + 42.25) + 65(3^2 + 12.25)}{35 + 65}$$

$$\Rightarrow \sigma^2 = 34.2 \Rightarrow \sigma = 5.85$$

3. (c) **Hint** Given, $n_1 = 60, \bar{x}_1 = 650, \sigma_1 = 8,$
 $n_2 = 80, \bar{x}_2 = 660, \sigma_2 = 7$

$$\therefore \text{ Combined SD} = \sqrt{\frac{n_1 \sigma_1^2 + n_2 \sigma_2^2}{n_1 + n_2} + \frac{n_1 n_2 (\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}}$$

4. (d) Let n_1 and n_2 be the number of observations in two groups having means $\bar{x}_1$ and $\bar{x}_2$, respectively.

$$\text{Then, } \bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

$$\begin{aligned} \text{Now, } \bar{x} - \bar{x}_1 &= \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} - \bar{x}_1 \\ &= \frac{n_2 (\bar{x}_2 - \bar{x}_1)}{n_1 + n_2} > 0 \quad [\because \bar{x}_2 > \bar{x}_1] \end{aligned}$$

$$\Rightarrow \bar{x} > \bar{x}_1 \quad \dots(i)$$

$$\text{And } \bar{x} - \bar{x}_2 = \frac{n_1 (\bar{x}_1 - \bar{x}_2)}{n_1 + n_2} < 0 \quad [\because \bar{x}_2 > \bar{x}_1]$$

$$\Rightarrow \bar{x} < \bar{x}_2 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\bar{x}_1 < \bar{x} < \bar{x}_2$$

25

Linear Programming

Linear Programming

It is an important optimisation (maximisation or minimisation) technique used in decision making business and everyday life for obtaining the maximum or minimum values as required of a linear expression by satisfying certain number of given linear restrictions.

Linear Programming Problem (LPP)

The linear programming problem in general calls for optimising a linear function of variables called the objective function subject to a set of linear equations and/or linear inequations called the constraints or restrictions.

Objective Function

The function which is to be optimised (maximised/minimised) is called an objective function.

Constraints

The system of linear inequations or equations under which the objective function is to be optimised is called constraints.

Non-negative Restrictions

All the variables considered for making decisions assume non-negative values.

Mathematical Description of a General Linear Programming Problem

A general LPP can be stated as
(max/min) $Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$ (Objective function)
subject to constraints

$$\left\{ \begin{array}{cccccc} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n & (\leq = \geq) & b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n & (\leq = \geq) & b_2 \\ \vdots & & \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n & (\leq = \geq) & b_m \end{array} \right\}$$

and the non-negative restrictions

$$x_1, x_2, \dots, x_n \geq 0$$

where, all $a_{11}, a_{12}, \dots, a_{mn}; b_1, b_2, \dots, b_m; c_1, c_2, \dots, c_n$ are constants and $x_1, x_2, \dots, x_n$ are variables.

Slack and Surplus Variables

The positive variables which are added to left hand sides of the constraints to convert them into equalities, are called the slack variables. The positive variables which are subtracted from the left hand sides of the constraints to convert them into equalities, are called the surplus variables.

Important Definitions and Results

- **Solution of an LPP** A set of values of the variables $x_1, x_2, \dots, x_n$ satisfying the constraints of an LPP is called a solution of the LPP.
- **Feasible Solution of an LPP** A set of values of the variables $x_1, x_2, \dots, x_n$ satisfying the constraints and non-negative restrictions of an LPP is called a feasible solution of the LPP.
- **Optimal Solution of an LPP** A feasible solution of an LPP is said to be optimal or optimum, if it also optimises the objective function of the problem.
- **Graphical Solution of an LPP** The solution of an LPP obtained by graphical method i.e. by drawing the graphs corresponding to the constraints and the non-negative restrictions is called the graphical solution of an LPP.
- **Unbounded Solution** If the value of the objective function can be increased or decreased indefinitely, such solutions are called unbounded solutions.
- **Fundamental Extreme Point Theorem** An optimum solution of an LPP, if it exists, occurs at one of the extreme points (i.e. corner points) of the convex polygon of the set of all feasible solutions.

Graphical Method of Solving Linear Programming Problem

The graphical method is suitable for solving linear programming problems containing two variables only. This method of solving linear programming problem is referred as corner point method. The procedure of this method is as follows :

1. Find the feasible region of the LPP and determine its corner points (vertices) either by inspection or by solving the two equations of the lines intersecting at that point.

- Find the value of objective function $Z = ax + by$ at each corner point. Let M and m respectively denote the largest and the smallest values of these points.

- (i) When the feasible region is bounded, M and m are the maximum and minimum values of Z .

- (ii) When the feasible region is unbounded, then

- (a) M is the maximum value of Z , if the open half plane determined by $ax + by > M$ has no point in common with the feasible region. Otherwise, Z has no maximum value.

- (b) m is the minimum value of Z , if the open half plane determined by $ax + by < m$ has no point in common with the feasible region. Otherwise, Z has no minimum value.

NOTE A half plane (if a line divides XY -plane into two parts, then each part is known as half plane) in XY -plane is called an open half plane, if the line separating the half plane is not included in the half plane.

Method to Solve LPP

To solve a Linear Programming Problem (LPP) we use the following working steps :

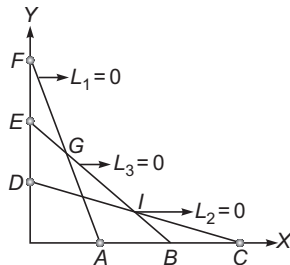
- Step I** Firstly, write the given LPP in mathematical form by using mathematical formulation (if not given in mathematical form).
- Step II** Consider all constraints as linear equations.
- Step III** Draw the graph of each linear equation obtained in Step II and find their intersection points (if any).
- Step IV** Shade the common region of all the linear inequalities i.e. find feasible region and check that feasible region is bounded or unbounded.
- Step V** Now, find the corner points of feasible region and calculate the value of objective function at each corner point.
- Step VI**
- (i) If region is bounded, then maximum (say M) or minimum (say m) value out of these values obtained in Step V, is the required maximum or minimum value of the objective function.
 - (ii) If region is unbounded, no need to go next step. Then, maximum (say M) or minimum (say m) value out of these values obtained in Step V may or may not be required maximum or minimum value of the objective function. Then, we go to next step.
- Step VII** Suppose the given objective function is $ax + by$, then draw the graph of inequality $ax + by > M$ or $ax + by < m$. If open half plane obtained by these inequalities has no point in common with the feasible region obtained in Step IV, then M or m is the required maximum or minimum value. Otherwise, objective function has no maximum or no minimum value.

Practice Exercise

1. A wholesale merchant wants to start the business of cereal with ₹ 24000. Wheat is ₹ 400 per quintal and rice is ₹ 600 per quintal. He has capacity to store 200 quintal cereal. He earns the profit ₹ 25 per quintal on wheat and ₹ 40 per quintal on rice. If he stores x quintal rice and y quintal wheat, then for maximum profit, the objective function is

- a. $25x + 40y$ b. $40x + 25y$
c. $400x + 600y$ d. $\frac{400}{40}x + \frac{600}{25}y$

2. The feasible region for the following constraints $L_1 \leq 0$, $L_2 \geq 0$, $L_3 = 0$, $x \geq 0$, $y \geq 0$ in the diagram shown is



- a. area DHF
b. area AHC
c. line segment EG
d. line segment GI

3. The linear programming problem
Maximise $Z = x_1 + x_2$
Subject to constraints

$$\begin{aligned} x_1 + 2x_2 &\leq 2000 \\ x_1 + x_2 &\leq 1500 \\ x_2 &\leq 600 \\ x_1 &\geq 0, \text{ has} \end{aligned}$$

- a. no feasible solution
b. unique optimal solution
c. a finite number of optimal solutions
d. infinite number of optimal solutions
4. The objective function $Z = 30x + 20y$, subject to constraints $x + y \leq 8$, $x + 2y \geq 4$, $6x + 4y \geq 12$, $x \geq 0$, $y \geq 0$, has
- a. unique solution
b. infinitely many solutions
c. minimum at (4, 0)
d. minimum 60 at point (0, 3)

5. If the objective function $Z = 4x + 2y$, subject to constraints $4x + 2y \geq 46$, $x + 3y \leq 24$ and x and y are greater than or equal to zero, then the maximum value of Z is

- a. 46 b. 96
c. 52 d. None of these

6. The maximum and minimum values of $Z = 5x + 2y$, subject to the constraints $2x + 3y \geq 3$, $x - 2y \geq 2$, $6x + 4y \leq 24$, $-3x + 2y \leq 3$ and $x, y \geq 0$, are respectively

- a. $\frac{18}{7}, \frac{2}{7}$ b. $19, \frac{63}{13}$ c. 19, 63 d. 19, 13

7. The maximum value of $Z = 10x + 6y$ subject to constraints $x \geq 0$, $y \geq 0$, $x + y \leq 12$, $2x + y \leq 20$, is
- a. 72 b. 80 c. 104 d. 110

8. The maximum value of $Z = 3x + 5y$, under the constraints $x + 2y \leq 2000$, $x + y \leq 1500$, $y \leq 600$ and $x, y \geq 0$, is

- a. 5000 b. 5500
c. 6000 d. None of these

9. If x_1 and x_2 are optimal solution of LPP, then which of the following is also an optimal solution?

- a. $x = kx_1 + (1 - k)x_2$; $k \in R$
b. $x = kx_1 + (1 - k)x_2$; $0 \leq k \leq 1$
c. $x = kx_1 + (1 + k)x_2$; $0 \leq k \leq 1$
d. $x = kx_1 + (1 + k)x_2$; $k \in R$

10. The maximum value of $Z = 9x + 13y$ subject to constraints $2x + 3y \leq 18$, $2x + y \leq 10$, $x \geq 0$, $y \geq 0$, is
- a. 130 b. 81 c. 79 d. 99

11. Which of the following is a convex set?

- a. $\{(x, y) : x^2 + y^2 \geq 4\}$ b. $\{(x, y) : 3x^2 + 2y^2 \leq 6\}$
c. $\{x, y : 9 \leq x^2 + y^2 \leq 25\}$ d. None of these

12. The maximum value of $Z = 5x + 3y$, subject to the constraints $3x + 5y \leq 15$, $5x + 2y \leq 10$; $x, y \geq 0$, is

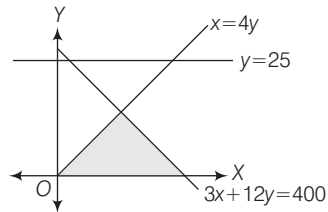
- a. $\frac{235}{19}$ b. $\frac{325}{19}$ c. $\frac{523}{19}$ d. $\frac{532}{19}$

13. The minimum value of $Z = 2x_1 + 3x_2$, subject to the constraints $2x_1 + 7x_2 \geq 22$, $x_1 + x_2 \geq 6$, $5x_1 + x_2 \geq 10$ and $x_1, x_2 \geq 0$, is

- a. 14 b. 20 c. 10 d. 16

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1. A furniture dealer deals in only two items namely tables and chairs. He has ₹ 5000 to invest and space to store at most 60 pieces. A table cost him ₹ 250 and a chair ₹ 60. He can sell a table at a profit of ₹ 15. Assume that he can sell all the items that he produced. The number of constraints in the problem are [2014]
 - a. 2
 - b. 3
 - c. 4
 - d. 5
2. The shaded region given below represents the constraints (other than $x \geq 0, y \geq 0$) [2013]



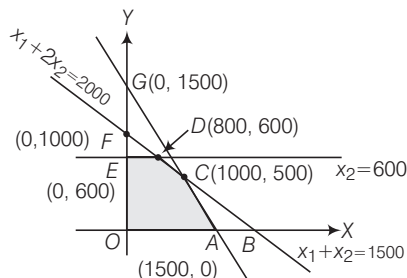
- a. $3x + 4y \leq 400, y \leq 25, x \leq 4y$
 - b. $3x + 12y \geq 400, y \leq 25, x \geq 4y$
 - c. $3x + 12y \leq 400, y \leq 25, x \geq 4y$
 - d. None of these
3. The optimal value of the objective function is attained at the point [2009]
 - a. given by intersection of inequations with axes only
 - b. given by intersection of inequations with X-axis only
 - c. given by corner point of the feasible region
 - d. None of the above

Answer with Solutions

Practice Exercise

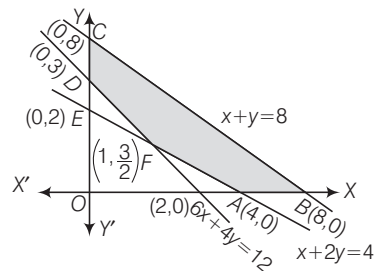
1. (b) For maximum profit, $Z = 40x + 25y$
2. (c) In the given figure, the feasible region for given constraints is the line segment EG.
3. (d) Given constraints are $x_1 + 2x_2 \leq 2000, x_1 + x_2 \leq 1500, x_2 \leq 600$ and $x_1 \geq 0$. The feasible region is OACDEO.

Given, $Z = x_1 + x_2$



At $O(0, 0)$, $Z = 0 + 0 = 0$
 At $A(1500, 0)$, $Z = 1500 + 0 = 1500$
 At $C(1000, 500)$, $Z = 1000 + 500 = 1500$
 At $D(800, 600)$, $Z = 800 + 600 = 1400$
 At $E(0, 600)$, $Z = 0 + 600 = 600$
 Here, Z is maximum on the segment AC.
 Hence, there are infinite optimal solutions.

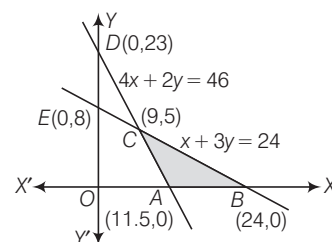
4. (d) Feasible region is ABCDFA and $Z = 30x + 20y$.



Now, at $A(4, 0)$, $Z = 30 \times 4 + 0 = 120$
 At $B(8, 0)$, $Z = 30 \times 8 + 0 = 240$
 At $C(0, 8)$, $Z = 0 + 20 \times 8 = 160$
 At $D(0, 3)$, $Z = 0 + 20 \times 3 = 60$
 At $F\left(1, \frac{3}{2}\right)$, $Z = 30 \times 1 + 20 \times \frac{3}{2} = 60$

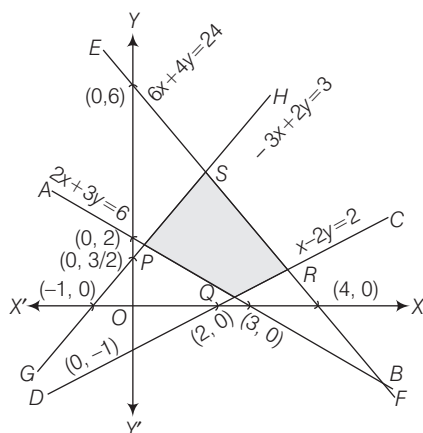
Hence, minimum value is 60 at $(0, 3)$.

5. (b) Feasible region is ABCA.



and $Z = 4x + 2y$
 At point $C(9, 5)$, $Z = 4 \times 9 + 2 \times 5 = 46$
 At point $A(11.5, 0)$ $Z = 4 \times 11.5 + 2 \times 0 = 46$
 and at point $B(24, 0)$, $Z = 4 \times 24 + 0 = 96$
 $\therefore$ Maximum value of $Z = 96$

6. (b) Draw a graph of the following straight lines in the first quadrant.



$AB: 2x + 3y = 6$, two points on AB are $(3, 0)$ and $(0, 2)$
 $CD: x - 2y = 2$, two points on CD are $(2, 0)$ and $(0, 1)$
 $EF: 6x + 4y = 24$, two points on EF are $(4, 0)$ and $(0, 6)$
 $GH: -3x + 2y = 3$, two points on GH are $(-1, 0)$ and $(0, 3/2)$

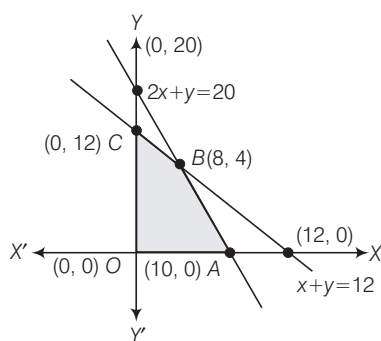
Hint The intersection points of given lines are

$$Q = \left(\frac{18}{7}, \frac{2}{7}\right), P = \left(\frac{3}{13}, \frac{24}{13}\right), R = \left(\frac{7}{2}, \frac{3}{4}\right) \text{ and } S = \left(\frac{3}{2}, \frac{15}{4}\right)$$

7. (c) **Hint** Given constraints are

$$x \geq 0, y \geq 0, x + y \leq 12, 2x + y \leq 20$$

The feasible region is $OABCO$.



Corner points are $O(0, 0)$, $A(10, 0)$, $B(8, 4)$ and $C(0, 12)$

8. (b) **Hint** Draw the graph of the straight lines

$$x + 2y = 2000 \quad \dots(i)$$

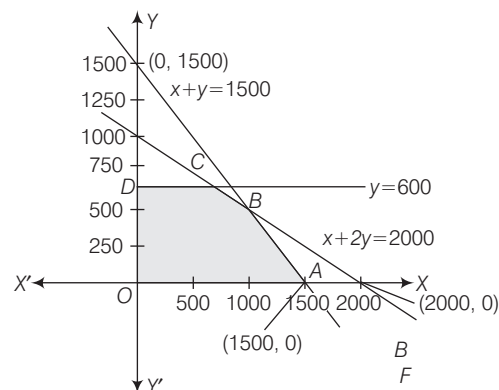
$$x + y = 1500 \quad \dots(ii)$$

and $y = 600 \quad \dots(iii)$

In the first quadrant and obtain the feasible region as $OABCD$.

Solving Eqs. (i), (ii) and (iii) in pairs, we get

$O(0, 0)$, $A(1500, 0)$, $B(1000, 500)$, $C(800, 600)$, $D(0, 600)$



9. (b) In case, there are infinite number of optimal solutions, then every points x_1 and x_2 respectively the two solutions, is also solution. Here, x is the point dividing the line joining x_1 and x_2 internally in the ratio $k : 1 - k$ ($0 \leq k \leq 1, k \in R$)

10. (c) The feasible region is $OABCO$.

$$\text{At } O(0, 0), Z = 0$$

$$\text{At } A(5, 0), Z = 45$$

$$\text{At } B(3, 4), Z = 27 + 52 = 79$$

$$\text{At } C(0, 6), Z = 78$$

$\therefore$ Maximum value of Z is 79.

11. (b) $\{(x, y) : \{3x^2 + 2y^2 \leq 6\} = \left\{(xy) : \frac{x^2}{2} + \frac{y^2}{3} \leq 1\right\}$

= Set of points on and inside the ellipse

$$\frac{x^2}{2} + \frac{y^2}{3} = 1$$

12. (a) Feasible region is $ABCOA$.

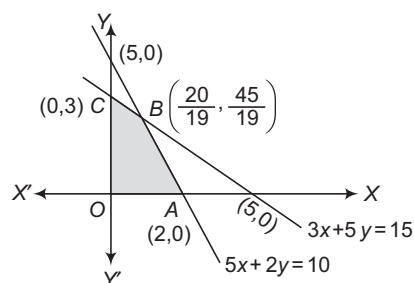
$$\text{Given, } Z = 5x + 3y$$

$$\text{At point } A(2, 0)$$

$$Z = 5 \times 2 + 0 = 10$$

$$\text{At point } B\left(\frac{20}{19}, \frac{45}{19}\right),$$

$$Z = \frac{5 \times 20}{19} + \frac{3 \times 45}{19} = \frac{235}{19}$$

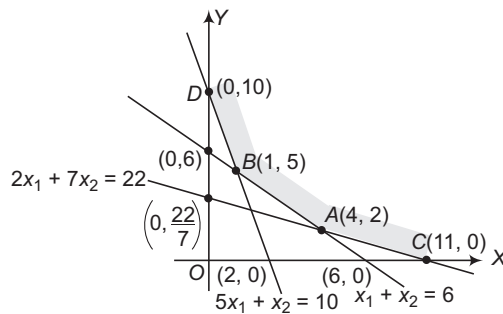


$$\text{At point } C(0, 3)$$

$$Z = 5 \times 0 + 3 \times 3 = 9$$

Hence, maximum value of Z is $\frac{235}{19}$.

13. (a) Given, $Z = 2x_1 + 3x_2$



At $D(0, 10)$, $Z = 2(0) + 3(10) = 30$

At $B(1, 5)$, $Z = 2(1) + 3(5) = 17$

At $A(4, 2)$, $Z = 2(4) + 3(2) = 14$

At $C(11, 0)$, $Z = 2(11) + 3(0) = 22$

Hence, the minimum value of Z is 14.

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1. (c) Let x tables and y chairs be purchased for maximum profit.

Then,

$$\begin{aligned} x + y &\leq 60 \\ 5x + \frac{6y}{5} &\leq 100 \\ x \geq 0, y &\geq 0 \end{aligned}$$

So, number of constraints are four.

2. (c) Consider a point $(2, 0)$ on X -axis.

Substituting $x = 2, y = 0$ in

$$3x + 12y = 6 < 400$$

Hence, one constraints is

$$3x + 12y \leq 400.$$

Again, substituting $x = 2, y = 0$ in

$$x - 4y = 2 - 0 > 0$$

$\therefore x - 4y \geq 0$ is other constraints.

and also the third constraint from the figure is $y \leq 25$.

3. (c) The optimal value of the objective function is attained at the points given by corner points of the feasible region.

**ENGLISH
PROFICIENCY**

1

Grammar

Tense and Agreement

Tense can be Past, Present or Future. These can further be divided into the following segments

1. **Simple** The action is mentioned simply. Nothing is said about whether the action is complete or not.
2. **Continuous** The action is incomplete or going on at the time of speaking.
3. **Perfect** The action is finished or complete with respect to a certain point of time.
4. **Perfect Continuous** The action is going on continuously over a long period of time and is yet to be finished.

Simple Present

(*He, She, It*, singulars – $V_1 + s, es$)

(*We, They, You*, I, plurals – V_1)

To express a habitual action.

e.g. He *goes* for a walk in the morning.

To express universal truth.

e.g. The Sun *rises* in the East.

In exclamatory sentences beginning with '*here*' and '*there*', to express what is actually taking place in the present.

e.g. Here he *comes*!

To indicate a future event that is part of a plan or an arrangement.

e.g. PM *comes* to the town next month.

To introduce quotations.

e.g. Gita *says*, give your best and do not worry for the result.

Present Continuous

(Is/Am/Are + V₁ + ing)

To express an action going on at the time of speaking.

e.g. I am *studying* in the class.

To express a temporary action which may not be actually happening at the time of speaking. (These days)

e.g. I am *preparing* for the competition.

It also represents future action or a definite arrangement in the near future.

e.g. I am *going* to Mumbai tomorrow.

When the reference is to a particular obstinate habit, the present continuous is used instead of present simple. An adverb, like always constantly, etc is also used.

e.g. It is no use scolding him, he *always does* / *is always doing* what is forbidden.

The following verbs are normally used in the present simple instead of present continuous.

- (a) **Verbs of Appearance** Look, appear, seem.
- (b) **Verbs of Emotion** Want, wish, desire, feel, like, love, hate, hope, prefer, etc.
- (c) **Verbs of Perception** See, hear, smell, taste, notice, recognise, etc.
- (d) **Verbs of Possession** Belong to, consist of, contain, own, etc.
- (e) **Verbs of Thinking** Agree, believe, consider, forget, imagine, know, mind, remember, etc.

Present Perfect (Has / Have + V₃)

It is used to indicate completed activities in the immediate past. It is a mixture of present and past. It always implies a strong connection of past with the present. e.g. He *has* just gone out.

The most important point is that it is used with the past actions whose time is not given and not definite.

e.g. He *has* come here.

Present Perfect Continuous

(Have been/Has been + V₁ + ing)

To express an action which began at some time in the past and is still continuing.

e.g. He *has been playing* since 8 o' clock.

Simple Past (V₂)

To indicate an action completed in the past at a definite time.

e.g. I *did* this yesterday.

Denoted by *last, ago, yesterday, back, before, formerly, any fixed time, etc.*

e.g. We *heard* a terrifying news *last* night.

To indicate past habits, indicated generally by the words like *often, seldom, never, normally, generally, always, frequently, rarely, daily, used to, etc.*

e.g. As a kid, I *often went* to school on foot.

After *it is time*. Which is a phrasal expression.

e.g. It is time Indian cricket team ~~*starts*~~ / *started* winning tense matches.

Past Continuous (Was / Were + V₁ + ing)

To denote an action going on at some time in the past.

e.g. When I went to his house, he *was* playing.
I *was* studying yesterday.

For persistent habits in the past.

e.g. He *was* always *mooching* around.

Past Perfect (Had + V₃)

To describe an action completed before a certain moment in the past.

e.g. I met him in New Delhi in 2000. I *had* seen him last five years before.

Past perfect should be used only when we wish to say that one action completed before the other started. It should never be used at all in any other sense.

e.g. I *had* gone to Mumbai when he came to meet me.

Past Perfect Continuous

(Had been + V₁ + ing)

To express an action that began before a certain point of time in the past and continued upto that time.

e.g. He *had been studying* for two hours when his girlfriend came.

If there is past tense in the Principal clause, it must be followed by a past tense in the Dependent clause. In an Indirect narration, the Past Simple in the Dependent clause is changed to Past Perfect, if the Principal clause is in the Past tense.

e.g. He told me that he *intended* to start a business. He told me that he *had intended* to start a business.

The exception to the above rule is if some universal, habitual or generally recognised fact is mentioned in the Dependent clause, the Present tense must be retained in all conditions.

e.g. He told me that the Earth *moves* round the Sun.

Simple Future

To express an action that is still to take place.

e.g. I *shall* go for the preparation when I *receive* the call letter.

Future Continuous

(Shall be / Will be + V_1 + ing)

To express an action as going on at some time in the future.

e.g. I *shall* be earning when I am 21.

Future Perfect

(Shall have / Will have + V_3)

To indicate the completion of an action by a certain future time.

e.g. We *shall* have completed our syllabus by next month.

Future Perfect Continuous

(Will have been / Shall have been + V_1 + ing)

To indicate an action which is in progress over a period of time and will be in progress at a certain time in future.

e.g. Time *will have been* clocking for ages in the coming moments.

Subject-Verb Agreement

1. If two subjects together express one idea, one being added to the other for the sake of emphasis or clarification, the verb is singular. No plurality is left to exist in such a case.
e.g. Slow and steady wins the race.
2. When the plural noun denotes some specific quantity, distance, time or amount considered as a whole, the verb *is* generally singular.
e.g. Six miles *is* not a long distance for me.
3. Two or more singular subjects connected by *either-or*, *neither-nor*, take a verb in singular (third person singular verb).
e.g. *Either* Vivek *or* Vimal *is* absent today.
4. When the subjects joined by *or* or *nor* are of different numbers, the verb must be plural and the plural subject must be placed next to the verb.
e.g. *Either* Amit *or* his parents *are* coming to the party.
5. Any noun qualified by *each* or *every* is followed by a singular verb. Even if two nouns so qualified are connected by *and*, the verb must still be singular.
e.g. *Each* one of these boys *has* the potential to get selected.

6. Verb is used according to the first with subject as *and not, with, as* well as, in addition to, along with, beside, like, together with.
e.g. Rahul *and not* his friend was absent.
7. When two nouns or pronouns are joined by *not only...but also*, the verb agrees with the second noun or pronoun.
e.g. *Not only* the officer *but also* the soldiers were awarded.
8. If the subject is *the number of*, the singular verb is used and the noun is plural.
e.g. The number of one-dayers played these days *has* / ~~have~~ led to the deterioration of the game.
9. *A great many* is always followed by plural verb and a plural noun.
e.g. A great many students *have* passed this year.
10. Many *a* is always followed by a singular verb and a singular noun.
e.g. Many *a* soldier *has* got medal this year.
11. A singular or a plural verb is used with such nouns as *pains, a lot of, means, variety, plenty, rest, wages*, a large number of according to the sense in which they are used.
e.g. A large number of girls were absent on account of bad weather.
12. A verb should agree with its subject and not with the complement. But in the case of sentence beginning with *The*, the verb is according to the predicate/complement.
e.g. Our only guide was *the* stars.
The stars were our only guide.
13. In a compound sentence, both auxiliary verbs and principal verbs should be mentioned separately if they differ in number, form or voice. In such cases, one verb cannot act for both the clauses.
He has not married and will not marry in near future.
14. Use of *shall* and *will*.

To express simple future action *shall* is used in the first person, and *will* in the second and third person.

e.g. I *shall* come.

You *will* come. He *will* come.

Shall is used in the second and third person to express *command, promise, threat, determination*.

Will is used in the first person to express *Willingness, Promise, Threat, Determination*.

e.g. You *shall* not steal. (Command)

Modals

What are Modals?

The finites which express the mode or manner of the actions denoted in the **principal verb** are termed as Modals. This class of helping verbs not only assists in forming questions and negatives, but expresses a wide range of meanings also.

These helping verbs are, 'can', 'could', 'will', 'would', 'shall', 'should', 'may', 'might', 'must', 'ought to', 'have to', 'has to' and 'had to'. These are also known as **modal auxiliaries**.

They express the degree of certainty of the action in the sentence or the attitude or opinion of the writer/speaker concerning the action. 'need', 'dare' and 'used to' are called **semi-modals**.

Functions of Modals

Would (past form of 'will')

- (i) It expresses the past form of will
e.g. He informed me that he *would* dance in my marriage function.
- (ii) To express past habit
e.g. He *would* drink a cup of coffee in the morning.
- (iii) For request
e.g. *Would* you please give me your bike?
- (iv) To express wish
e.g. I *would* that you were rich!
- (v) To express an imaginary condition
e.g. I *would* have a big house if I earned ₹ 10 crore.

Should (past form of 'shall')

- (i) To express duty or obligation
e.g. I *should* help my friends. (*duty*)
- (ii) To express opinion
e.g. They *should* be on the way to Jaipur.
- (iii) To give or take advice or suggestion.
e.g. We *should* go to the temple.
- (iv) After 'lest' when someone expresses fear.
e.g. Walk carefully *lest* you should fall.

Can

- (i) To express an ability, capability, capacity or power
e.g. (a) I *can* lift 60 kg.
(b) She *can* pass the MBA examination.
(c) *Can* you understand the English language?
(d) I *can* grant you leave.
- (ii) To show possibility
e.g. (a) She *can* fall on the road.
(b) You *can* go there.

- (iii) To take or give permission
e.g. (a) *Can* I sit here?
(b) You *can* park your scooter.

Could (past form of 'can')

- (i) To express ability/talent in the past
e.g. (a) They *could* win the game last month.
(b) She *could* sing beautifully.
- (ii) To express polite request
e.g. (a) *Could* I have your pen?

May

- (i) To show possibility or probability
e.g. (a) You *may* attend the school today.
(b) She *may* win the match.
- (ii) To give or take permission
e.g. (a) *May* I come in?
(b) Yes, you *may* come in.
- (iii) To wish or pray
e.g. (a) *May* you get well soon!
(b) *May* God bless you with a child!
- (iv) To show a purpose
e.g. My friend is joining a new company, so that he *may* achieve his target.

Might (past form of 'may')

- (i) To express less possibility
e.g. He *might* help us.
- (ii) For permission
e.g. *Might* I begin to reveal the truth.
- (iii) To express guess
e.g. That *might* be Rohit.

Must

- (i) To express necessity or obligation
e.g. (a) You *must* take part in the competition.
(b) We *must* love our motherland.
- (ii) To indicate assumption or conclusion
e.g. (a) They *must* be here in the evening.
(b) Mr AK Gupta *must* be a good teacher.
- (iii) In case of prohibition
e.g. You *must not* bunk school.

Ought to

- (i) To express moral obligation / duty
e.g. (a) You *ought to* respect your parents.
(b) You *ought to* serve the nation.
- (ii) To give advice
e.g. You *ought to* study hard to achieve success.

Need

As a modal auxiliary verb in negative terms, it indicates *absence of obligation*. It expresses the *speaker's authority or advice* and is used for the present and the future.

e.g. You **needn't** type this letter.

- (i) The interrogative is formed by inversion.
e.g. *Need* I speak to him?
- (ii) Its past is **had to** in the affirmative sentence, **need not have** in the negative and **need have** in the interrogative.
e.g. (a) *Need* I have gone to him?
(Interrogative in the past)

- (b) I had *needed to* go to him. (Affirmative in the past)
- (c) You *needn't have* gone to meet him.
(Negative in the past)

Dare

As a modal auxiliary, 'dare' refers to being bold and courageous. The negative is formed by **dare not** and the interrogative by inversion.

- e.g. (a) I *dare not* disturb them.
(b) *Dare* we talk to them?

Used to

- (i) A discontinued habit or a past situation which is no more in the present.
e.g. He used to drink daily. (Now he does not drink)
- (ii) Something existing in the past.
e.g. This *used to* be a dense jungle before.

Prepositions

Words used with nouns/pronouns to show their relationship.

1. **In** is used for bigger places (towns, cities, countries) while **at** is used for smaller places.
e.g. I live *at* Shastri Nagar *in* Meerut.
2. **In / Into** **In** is used in speaking of things at rest. **Into** is used in speaking of things in motion.
e.g. (a) He is shopping *in* the market.
(b) He jumped *into* the well.
3. **On** denotes position, **upon** denotes movement.
e.g. (a) The cat is *on* the table.
(b) The cat pounced *upon* the mouse.
4. **With** denotes the instrument and **by** denotes the agent.
e.g. (a) The letter was written *by* him *with* his pen.
(b) The music was generated *by* / *with* a guitar.
5. **Ago** refers to past time while **before** denotes precedence between two events.
e.g. (a) Long *ago*, there was a king named Rama.
(b) Ram existed *before* Mahabharata was fought.
6. **Above** and **below** merely denote position while **over** and **under** also carry a sense of covering or movement.
e.g. (a) We live *below* the roof.
(b) Sky is *above* us.
(c) Train is running *under* the bridge.
(d) The train is standing *below* the bridge.
(e) The bird is flying *over* the pond.

Under is used before a noun to indicate that a person or thing is being affected by something or is going through a particular process.

e.g. I'm rarely under pressure and my co-workers are always nice to me.

Under can mean junior in ranks.

e.g. He is *under* me.

If something happens *under* a particular person or government, it happens when that person or government is in power.

e.g. There will be no new taxes *under* his leadership.

If someone does something *under* a particular name, he uses that name instead of his real name.

e.g. The patient was registered *under* a false name.

Beneath has the same meaning as *under*, but it is better to use it for abstract meanings.

e.g. *Beneath* the festive mood, there is an underlying apprehension.

7. Difference between *On time*, *In time* and *In good time*.

On time signifies absolutely right time, neither before nor after.

e.g. The flight is *on time*.

In time means you are not late for it.

e.g. I arrived just *in time* for my flight.

In good time means with comfortable margin.

e.g. I arrived at the airport *in good time*.

8. Difference between *at the beginning/at the end* and *in the beginning/in the end*.
- At the beginning* means literally at the beginning.
e.g. India scored fast *at the beginning* of the match.
- At the end* means literally at the end.
e.g. *At the end* of the book, you'll find the bibliography.
- In the beginning* (or *at first*) means in the early stage. It implies that later there was a change.
e.g. Sachin was nervous *in the beginning*, later he settled down.
- In the end* (or *at last*) means eventually/after sometime.
e.g. At first he was scared, but *in the end* he started enjoying.
9. No preposition is placed after the following verbs when they are used in active voice. *Order, request, reach, attack, resemble, emphasis, accompany, discuss, investigate, comprise, enter (come into), flee* (a place), *join, affect, board, etc.*
- e.g. (a) Our forces attacked ~~on~~ the enemy fort.
(b) We reached ~~at~~ the station on time.
(c) We ordered ~~for~~ a cup of tea.
(d) He resembles ~~to~~ his father.
10. Omit *to* after verb of communication such as *advise, tell, ask, beg, command, encourage, request, inform, order*.
- e.g. (a) I advised ~~to~~ him to study hard.
(b) I commanded ~~to~~ him to leave.
11. *Till* is used in particular time while *until* is used for indefinite time. But whenever *till* and *until* are being used as the first word of a sentence, only *until* is used.
- e.g. (a) We shall work *until* we fell down.
(b) We shall work *till* 15 hrs.
(c) ~~Till~~ / *Until* 30, he was a bachelor.
12. *Till* is used for time while *upto* for place.
- e.g. (a) We shall work *till* 5 pm.
(b) We walked ~~till~~ / *upto* the station.
13. The same preposition should not be used with two words unless it is appropriate to each of them.
- e.g. (a) It is different and inferior *to* the other.
(b) It is different *from* and inferior *to* the other.
14. *Since* and *from* are used before a noun or phrase denoting some point of time, but whereas *since* is preceded by a verb in some perfect tense, *from* is used with other tenses except the perfect tense. *For* refers to a period of time, not to a point of time and should not be replaced by *since* or *from*.
- e.g. (a) I haven't taken anything *since* yesterday.
(b) I started my work *from* 1st January.
(c) I have been practicing *for* ten days.
15. Regarding the phrases of time, *morning, afternoon* and *evening* are preceded by the preposition *in* whereas *dawn, daybreak, noon, midday* and *midnight* are preceded by the preposition *at*. Besides, when these time phrases are qualified by *last* or *next*, they are not preceded by any preposition.
- e.g. (a) I like to roam around *in* the evening.
(b) I'll see you *at* night.
(c) The Sun is hottest *at* midday.
(d) I met him last evening.
16. *Across*, something that lies between two points. Through, from one side of an opening to the other.
e.g. Walk *across* a road and pass *through* a tunnel.
17. *Between/Among* *Between* is used while referring to two persons/things whereas *Among* is used for more than two.
- e.g. (a) *Between* the two of you, who is stronger?
(b) The sweets are to be distributed *among* ten friends.
18. *Beside/Besides* *Beside* means 'by the side of', whereas *Besides* means 'in addition to'.
- e.g. (a) *Besides* eating he is also watching TV.
(b) You were sitting *beside* him.
19. *From/Between* *From* is normally used with 'to/till', whereas *Between* is used with 'and'.
- e.g. (a) He works *from* nine *to* six (or nine *till* six).
(b) The meeting was scheduled to be held *between* two pm *and* three pm.
20. *Within/In* *Within* means before the end of time, whereas *In* means at the end of time.
- e.g. (a) He will return *in* five minutes.
(b) He will return *within* five minutes.

Conjunctions

Conjunctions are special kinds of connectors. They help you to join two clauses together. The conjunctions **as**, **since**, **because**, **for** and **in case** are used to give reasons. They introduce the explanation that answers the question 'why'?

The conjunctions **so that**, **in order that** and **in case** and the connectors **so as to** and **in order to** are used to state a purpose. They introduce the answer to the question what for? The conjunctions **if**, **unless** and **as long as** are used to introduce conditions.

Read the sentences in Column 'A', some words are in bold. Their role is analysed in the sentences as indicated in Column 'B'.

S No.	Column A	Column B
1.	Rabindranath Tagore won the Nobel Prize for Literature for his collection of poems, 'Geetanjali.' Furthermore , he continued writing inspite of his busy schedule.	Adds information (and as well as, both, furthermore, moreover, additionally but also).
2.	If one has a positive attitude, one can achieve success. Similarly , if one works hard unconditionally, success will never elude one.	Introduces a similar idea (in the same way).
3.	"Today's generation is really obstinate, Granny said, "Even when you scold them for their mistakes, still they do not change their ways".	Gives a different/opposite idea (However, nonetheless, whereas, nevertheless, but),
4.	Most people in their young days do not understand the value of money. Consequently , they have to regret when they reach old age.	Introduces the result (so, thus, hence),
5.	It is important to exercise so that we may not be prone to diseases.	Expresses purpose (that, therefore, in order that),
6.	I understand your situation, but I would appreciate it if you complete the project on time.	Indicates difference/contrast (while, whereas, but, still, yet).
7.	I like you because of your sincerity and integrity.	Explains 'why' because, since, as).
8.	He is very tense about his new job. Until he is settled there, he will have no peace of mind.	Explains the sequence of events (when, while, after, before, as soon as, no sooner,, than, till, until, whenever, since).
9.	It is said that beauty lies in the eyes of the beholder, but I feel she is prettier than Sita.	To express comparisons (as, ...as, so ...as, than.)
10.	Although it is too late to prepare for the exams, I am hopeful of securing good marks.	When one has to depict concession (although, though, even though, but however, yet, despite, inspite of),
11.	You do not have many choices. Either you promise to solve the matter or else fight.	To give an alternative (either...or, neither...nor, or)

Voice

Voice Mode of making statements is called *voice*.

Rules for change in voice

Changes of Persons

Subjective Case	Objective Case	Possessive Case
I	my	me
we	our	us
you	your	you
he	his	him
she	her	her
they	their	them

Rules for Formation of Passive Voice

Present Indefinite

Active Subject + first form of verb + s/es or first form of verb + Object

Passive Object + is/ am/ are + third form of verb + preposition +Subject

Present Continuous

Active Subject + is/am/are + first form of verb + ing + Object

Passive Object + is/am/are+ being + third form of verb + preposition + Subject

Present Perfect**Active** Subject + has/have + third form of verb + Object**Passive** Object + has/have + been + third form of verb + preposition + Subject**Past Indefinite****Active** Subject + second form of verb + Object**Passive** Object + was/were + third form of verb + preposition + Subject**Past Continuous****Active** Subject + was/were + first form of verb + ing + Object**Passive** Object + was/were + being + third form of verb + preposition + Subject**Past Perfect****Active** Subject + had + third form of verb + Object**Passive** Object + had + been + third form of verb + preposition + Subject**Future Indefinite****Active** Subject + will/shall + first form of verb + object**Passive** Object + will/shall + be + third form of verb + preposition + Subject**Future Perfect****Active** Subject + shall have/will have + third form of verb + Object**Passive** Object + shall have been/will have been + third form of verb + preposition + Subject**Change of Passive Voice with Different Modals****Rule 1. Active** Subject + modal auxiliary + first form of verb + Object**Passive** Object + modal auxiliary + be + third form of verb + preposition + Subject**Rule 2. Active** Subject + modal auxiliary + have + third form of verb + Object**Passive** Object + modal + have + been + third form of verb + preposition + Subject**Rule 3. Active** Subject + ought + to + first form of verb + Object**Passive** Object + ought + to + be + third form of verb + Subject**Some Specific Conditions****Passive Voice of Verbs with Two Objects***Read the following*1. **Active** She taught me English.**Passive** I was taught English by her.*Or*

English was taught to me by her.

NOTE Here we can observe that any one of the two objects can be chosen as the subject of the passive verb.**Passive Voice of Sentences with Infinitive with 'To'***Rules to Change Active Voice into Passive Voice*

Object + helping verb + to be + third form of verb by + Subject

e.g. **Active** She is to cook the food.**Passive** The food is to be cooked by her.**Passive Voice of Imperative Sentences***Study the following rules to make the passive form of imperative sentences***Rule 1.** Let + object + be + third form of verb**Rule 2.** Object + should + be + third form of verb**Rule 3.** You are ordered/requested/ advised + to + first form of verb +**Rule 4.** Let + object + be + third form of verb + by +**Question Tags**

Question tags are not a complete question in itself. These are a form of question attached with a statement. This acts as a confirmation to that of the statements. Question tags are also used in everyday life conversations in order to seek the attention of the listener.

Types of Question Tags(a) **Positive Question Tags** Tags are said to be positive if it is attached to a negative statement.

e.g. (i) She is not working is she?
 ↓ ↓
 Negative statement Positive tag

(b) **Negative Question Tags** Tags are said to be negative if it is attached to a positive statement.

e.g. (i) We are reading aren't we?
 ↓ ↓
 Positive statement Negative tag

NOTE *Statement Positive Tag Negative*
Statement Negative Tag Positive

Structure of Tags

- (a) Structure of a positive tag
 Auxiliary + Subject + ?
- (b) Structure of a negative tag
 Auxiliary + not + Subject + ?

Rules Regarding the Formation of Tags

Rule 1. Subject of a tag can never be a noun. It is always a pronoun.

e.g. (i) Ram is doing maths, isn't he ?
 ↓ ↓
 Noun Pronoun

(ii) Parul has not eaten, has she ?
 ↓ ↓
 Noun Pronoun

Rule 2. Question tag never starts with a capital letter.

Rule 3. The auxiliary that is to be used in the formation of tag must be in the contracted form. This holds good only in the case of a negative tag.

Practice Exercise

Based on Tense and Agreement

Directions (Q. Nos. 1-23) *Some of the following sentences are grammatically incorrect and some are correct. Find out which part of a sentence has an error and mark that part. If there is no error "e" part as your answer.*

- If you had seen yesterday's cricket I am sure you would have enjoyed seeing our team bat. No error
 a b c d e
- As the meeting was about to end he insisted to ask several questions. No error
 a b c d e
- When he will come I will make sure I meet him. No error
 a b c d e
- I began relating several details connecting with the accident unmindful of boring the audience. No error
 a b c d e
- Weather permitted there will be a garden party at Government House tomorrow. No error
 a b c d e
- The traveller being weary he sat by Woodside to rest. No error
 a b c d e

- He did not and could not have understood the full facts of the case. No error
 a b c d e
- I am opposed to the plan of action not because it is ill conceived but that it seems impracticable. No error
 a b c d e
- He as well as you is tired of this long and troublesome affair. No error
 a b c d e
- There are many important details to attend to before this book gets printed. No error
 a b c d e
- Along the northern frontier of India is seen the Himalayas mighty in their splendour. No error
 a b c d e
- The recommendations of the committee that the age should be lowered down immediately was not accepted. No error
 a b c d e
- He is overworked and that seems to have seriously effected his health. No error
 a b c d e

14. Each of the students whom I have chosen to take part
a b
in the discussion have indicated that he will be happy
c d
to do so. No error
e
15. Honestly speaking I like him not because he is
a b
handsome and charming but that he is exceedingly kind.
c d
No error
e
16. The father with the son were mysteriously missing
a b c
from the house. No error
d e
17. Our country need a number of self sacrificing
a b c
and devoted political leaders. No error
d e
18. The constant shouting of slogans do not
a b
solve the problems of the country. No error
c d e
19. It is not advisable to take heavy luggages [2005]
a b
while on journey these days. No error
c d e
20. Mr Bose accompanied by his wife and children [2005]
a b
were present there. No error
c d e
21. You must pay respect to those who has [2005]
a b c
respect for you. No error
d e
22. Whenever is the matter I shall [2006]
a b
do this work because I have
c
to expose my working capacity at any cost. No error
d e
23. She is so lazy as she can't do this work properly and
a b c
cannot cooperate us in your scheme. No error [2006]
d e

Based on Modals

Directions (Q. Nos. 1-12) *Complete the sentences with suitable alternative.*

1. She told me that she English fluently and was very happy.

a. could speak b. could have spoken
c. can speak d. could have speak
2. He returned the money to the police though he it.

a. could keep b. should keep
c. could have kept d. can keep
3. If we request her she to college.

a. must give us lift b. might give us lift
c. can give us lift d. could give us lift
4. It is possible Marlowe plays for Shakespeare.

a. may write b. might have written
c. might write d. may have written
5. The police presume that Neena there at the time of murder.

a. might have been
b. can have been
c. should have been
d. would have been
6. Don't worry. You left your purse in staff room.

a. can have
b. should have
c. may have
d. could not
7. You him that gambling would ruin him.

a. should warn
b. should have warned
c. must warn
d. may warn
8. Before you started you your purse.

a. must check b. should have checked
c. should check d. must have check
9. She work hard if she wants to top the merit list.

a. must have b. must
c. must not d. must have been
10. She alone as it was raining heavily.

a. must not leave b. must not have left
c. should not leave d. may not have left
11. Principal to a student : You with bad girls.

a. must not mix b. cannot mix
c. may not mix d. could not mix
12. She alone as: it is raining heavily.

a. must not leave b. must not have left
c. should not have left d. may not have left

Based on Preposition

Directions (Q. Nos. 1-20) *Fill up the blanks with suitable prepositions from the alternatives given under each sentence.*

- His professional ability proves that he is cut for this job.
a. out b. up c. down d. for
- The history of Hindu religion dates ancient times.
a. from b. since c. for d. of
- Now government servants have the day every Saturday.
a. out b. off c. since d. in
- Nothing can deter him pursuing his aim of life.
a. for b. from c. in d. at
- After the death of his father, the responsibility has devolved him.
a. on b. at c. from d. in
- Normally, he stays until 11 p.m. these days.
a. up b. out c. on d. at
- No one believes him because he is false heart.
a. to b. at c. of d. on
- The officials have the habit of fawning ministers.
a. above b. at c. on d. up
- The problem of communal harmony cannot be glossed by government.
a. at b. on c. over d. for
- Kanishka was initiated Buddhism by Buddhist monks.
a. to b. into c. in d. on
- Consequent upon heavy loss he is worse these days.
a. upon b. off c. over d. in
- Co-operation between friends stems mutual consideration.
a. in b. out c. from d. up
- Many Russians name their children Indians.
a. after b. to c. for d. on
- Indians have pinned their hopes the emergence of some superman.
a. on b. in c. at d. of
- She could not muster courage to stand against the maltreatment.
a. to b. up c. about d. on
- Strangely her name did not occur me on the second meeting.
a. to b. on c. about d. into
- The whole town was plunged sorrow after the massacre of the students.
a. in b. into c. to d. from

- Are you really desirous visiting Japan? (2012)
a. of b. in c. to d. about
- So many servants attended him during his illness. (2006)
a. with b. on c. for d. to
- At least he yielded the temptation. (2006)
a. on b. off c. for d. to

Based on Conjunction

Directions (Q.Nos. 1-20) *Some of the following sentences have used conjunctions incorrectly and some correctly. Find out which part of a sentence is that and mark that part. If there is no error mark, "e" part as your answer.*

- Although they listen to me (a)/ but their actions (b)/ prove otherwise. (c)/ No error (d)
- Most of the girls are doing (a)/ their post graduation because (b)/ they may get good husbands. (c)/ No error (d)
- Such was his pronunciation (a)/ as (b)/ I could not understand him. (c)/ No error (d)
- I am interested (a)/ in such books (b)/ that are interesting. (c)/ No error (d)
- Each member of the alliance (a)/ agrees to take such action (b)/ that it deems necessary. (c)/ No error (d)
- She looked at him (a)/ in such distress (b)/ as he had to look away. (c)/ No error (d)
- This film is interesting (a)/ and the previous one (b)/ was boring. (c)/ No error (d)
- It is difficult to know (a)/ whether (b)/ you are selected or not. (c)/ No error (d)
- He has no chance (a)/ than to start (b)/ his own business. (c)/ No error (d)
- They had hardly finished (a)/ their meals that at once (b)/ they resumed their duty. (c)/ No error (d)
- I don't know whether (a)/ Raj is equally (b)/ good as Vimal. (c)/ No error (d)
- He (a)/ will return (b)/ on either Monday or Tuesday. (c)/ No error (d)
- He has no other business (a)/ but to play (b)/ with computers. (c)/ No error (d)
- Hardly had I reached the airport (a)/ where I learned about (b)/ the powerful bomb explosion. (c)/ No error (d)
- My book has been missing (a)/ from my room (b)/ till yesterday. (c)/ No error (d)
- The manager of the bank was busy; (a)/ so he asked them to come and see him (b)/ between two to three in the afternoon. (c)/ No error (d)

17. No sooner did the Sun rise (a)/ when we took a hasty breakfast (b)/ and resumed the journey. (c)/ No error (d)
18. Because he is physically strong (a)/ therefore he was selected (b)/ for the school boxing team. (c)/ No error (d)
19. Arjun asked him (a)/ that which was the way (b)/ to the post office. (c)/ No error (d)
20. Unless you do not listen to his advice (a)/ I am not going (b)/ to help you. (c)/ No error (d)

Based on Voice

Directions (Q. Nos. 1-19) *Match the sentence given in either Active or Passive voice with its correct Active or Passive form, from the options given below.*

1. Have the box broken.
 - a. Have the broken box.
 - b. Break the box.
 - c. Get someone to break the box.
 - d. They have broken the box.
2. His pocket has been picked.
 - a. They have his pocket picked.
 - b. Picking has been done to his pocket.
 - c. Picked has been his pocket.
 - d. Someone has picked his pocket.
3. The French surrendered Quebec to the English in 1759.
 - a. Quebec was surrendered by the French to the English in 1759.
 - b. Quebec was surrendered to the English in 1759 by the French.
 - c. The English were surrendered Quebec in 1759 by the French.
 - d. Quebec was surrendered in 1759 by the French to the English.
4. Rain disrupted the last day's play between India and Sri Lanka.
 - a. The last day's play of India and Sri Lanka was disrupted by rain.
 - b. India and Sri Lanka's play of the last day was disrupted by rain.
 - c. The last day's play between India and Sri Lanka was disrupted by rain.
 - d. The last day's play between India and Sri Lanka were disrupted by rain.
5. My uncle promised me a present.
 - a. A present was promised by my uncle to me.
 - b. I was promised a present by my uncle.
 - c. I had been promised a present by my uncle.
 - d. I was promised by my uncle a present.
6. They are building a house next door to our school.
 - a. Next door to our school a house is being built by them.
 - b. Next door to our school is being built house by them.
 - c. A house next door to our school is being built by them.
 - d. A house is being built by them next door to our school.
7. One should keep one's promise.
 - a. One's promise should be kept by us.
 - b. One's promise has to be kept.
 - c. A promise should be keeping.
 - d. A promise should be kept.
8. Who is creating this mess?
 - a. Who has created this mess?
 - b. By whom has this mess been created?
 - c. By whom this mess is being created?
 - d. By whom is this mess being created?
9. Do you imitate others?
 - a. Are others imitated by you?
 - b. Are others being imitated by you?
 - c. Were others being imitated by you?
 - d. Have others been imitated by you?
10. I saw him conducting the rehearsal.
 - a. He was seen conducting the rehearsal.
 - b. I saw the rehearsal to be conducted by him.
 - c. He was seen by me to conduct the rehearsal.
 - d. I saw the rehearsal being conducted by him.
11. Someone gave her a bull dog.
 - a. She was given a bull dog.
 - b. A bull dog was given to her.
 - c. She has been given a bull dog.
 - d. She is being given a bull dog by someone.
12. A stone struck me on the head.
 - a. I was struck by a stone on the head.
 - b. My head was struck by a stone.
 - c. I had been struck by a stone on the head.
 - d. I was struck on the head by a stone.
13. The Romans expected to conquer Carthage.
 - a. Carthage was expected to be conquered by the Romans.
 - b. The Romans were expected to conquer Carthage.
 - c. It was expected by the Romans that they would conquer Carthage.
 - d. The Romans expected to have conquered Carthage.
14. The clown was being laughed at by them.
 - a. They were laughing at the clown.
 - b. They were laughing on the clown.
 - c. They laughed at the clown.
 - d. The clown was laughed at by them.
15. I saw him leaving the house.
 - a. He had been seen leaving the house.
 - b. He was seen to be leaving the house.
 - c. Leaving the house he was seen by me.
 - d. He was seen leaving the house by me.
16. A lion may be helped even by a little mouse.
 - a. A little mouse may even help a lion.
 - b. Even a little mouse may help a lion.
 - c. A little mouse can even help a lion.
 - d. Even a little mouse ought to help a lion.

17. Who taught her such things?
 - a. Who was she taught such things by?
 - b. She was taught such things by who?
 - c. By whom she was taught such things?
 - d. By whom was she taught such things?
18. The noise of the traffic kept me awake.
 - a. I remained awake by the noise of the traffic.
 - b. I was kept waking by the noise of the traffic.
 - c. I was kept awake by the noise of the traffic.
 - d. The traffic kept me awake by the noise.
19. He was arrested on a charge of theft, but for lack of evidence he was released.
 - a. He was arrested on a charge of theft, but was released for lack of evidence.
 - b. The police arrested him on a charge of theft, but for lack of evidence he was released.
 - c. The police arrested him on charge of theft, but for lack of evidence released him.
 - d. None of the above
20. Please help me.
 - a. You were requested to help me.
 - b. You are being requested to help me.
 - c. You are requested to help me.
 - d. You have been requested to help me.
21. The residents celebrated the Independence Day.
 - a. The Independence Day is celebrated by the residents.
 - b. The Independence Day was celebrated by the residents.
 - c. The Independence Day has been celebrated by the residents.
 - d. Celebration of Independence Day was done by the residents.
22. People claim to have seen the suspect in several cities.
 - a. The suspect is being seen in several cities. [2007]
 - b. The suspect has been the people in several cities.
 - c. The suspect is claimed to have been seen in several cities.
 - d. The suspect was seen by people in several cities.
23. The teacher punished the boys who had not done their homework. [2007]
 - a. The boys who had not done their homework had been punished by their teacher.
 - b. The boys were punished by their teacher who had not done their homework.
 - c. The boys who had not done their homework were punished by the teacher.
 - d. The boys who had not done their homework were being punished by the teacher.

Based on Question Tags

Directionhs (Q. Nos. 1-5) *Choose the correct options from the questions given below*

1. Do as I say, ?
 - a. didn't you
 - b. won't you
 - c. needn't you
 - d. shan't you

2. Now you can make question tags, ?
 - a. can you
 - b. won't you
 - c. can't you
 - d. will you
3. I am working hard on this book, ?
 - a. don't I
 - b. won't I
 - c. aren't I
 - d. can't I
4. He was asleep, ?
 - a. was he
 - b. wasn't he
 - c. didn't he
 - d. did he
5. Let us start now, ?
 - a. shall we
 - b. should we
 - c. could we
 - d. would we

Miscellaneous

Directions (Q. Nos. 1-18) *In the following questions you will find sentences, part of which are bold. Compare the bold part of each sentence with the expressions a., b. and c. given below. Choose the expression which is an improvement upon the bold part. If none of the three expressions improves the sentence, then your answer is (d).*

1. I am tired as **I am working** since 7 o'clock in the morning.
 - a. I was working
 - b. I had been working
 - c. I have been working
 - d. No improvement
2. A highly improved variety of seeds **is** available to the farmer these days.
 - a. are
 - b. will be
 - c. has been
 - d. No improvement
3. The boy **told his teacher** to explain the passage.
 - a. asked his teacher
 - b. said to his teacher
 - c. invited his teacher
 - d. No improvement
4. **What struck one most** was their kindness.
 - a. What stroke one most
 - b. What struck one more
 - c. What stroke one more
 - d. No improvement
5. You should **avoid to make** such trivial mistakes.
 - a. avoid making
 - b. avoid to have made
 - c. avoid make
 - d. No improvement
6. If you come across my umbrella anywhere, bring it to me, **can you?**
 - a. isn't it
 - b. don't you
 - c. will you
 - d. No improvement
7. The passengers **are waiting** for the bus outside the bus stand at the main gate for long.
 - a. were waited
 - b. were waiting
 - c. have been waiting
 - d. No improvement
8. He has **fallen out** with the girl he wanted to marry.
 - a. fallen off
 - b. fallen in love
 - c. fallen for
 - d. No improvement
9. They are social insects, **living in communities**, regulated by definite laws, each member of society bearing a well-defined and separate part in the work of a colony.
 - a. who are living in communities
 - b. living among a community
 - c. who lives with a community
 - d. No improvement

10. If the room had been brighter, I would have been able to read for a while before bed time.
a. if the room was brighter b. if rooms are brighter
c. had the room brighter d. No improvement
11. The reason why he wrote the letter was because he could not contact him over the phone.
a. why he wrote the letter was since
b. for which he wrote the letter because
c. why he wrote the letter was that
d. No improvement
12. The right-to-work implies the obligation on the part of the government to give a job to **all the unemployed**.
a. any of the unemployed b. every unemployed
c. each of the unemployed d. No improvement
13. Most donors would seriously **protest any** effort to infer from such limited data.
a. protest against
b. protest at
c. protest to
d. No improvement
14. I hope you vividly remember the premier of the film when **I, my wife and you** were present in the hall.
a. my wife, I and you b. you, my wife and I
c. my wife, you and I d. No improvement
15. With a thundering roar the huge rocket **soared up** from the launching pad.
a. flew up b. went upwards
c. took on d. No improvement
16. My **opinion for** the film is that it will bag the national award.
a. opinion to b. opinion about
c. opinion on d. No improvement
17. He **sent a word** to me that he would be coming late.
a. sent word b. had sent a word
c. sent words d. No improvement
18. Young men and women should get **habituated** to reading and writing about current affairs.
a. used b. prepared
c. trained d. No improvement

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1. They **requested** me to follow them. [2014]
a. ordered b. urged
b. asked d. No improvement
2. She did not **believed** me. [2014]
a. believing b. believe to
c. believe d. No improvement
3. I am fine, what about **you**? [2014]
a. your b. your's
c. yours d. No improvement
4. It was not possible to **drag** any conclusion so he left the case. [2013]
a. fetch b. find
c. draw d. No improvement
5. I am **looking after** my pen which is missing. [2013]
a. looking for b. looking in
c. looking back d. No improvement
6. **Mind** your language he shouted. [2013]
a. change b. inspect
c. hold d. No improvement
7. He **declined** all the allegations against him. [2012]
a. spurned b. refused
c. refuted d. No improvement
8. It is time we **leave**. [2012]
a. left b. have to leave
c. would have d. No improvement
9. We spent an hour discussing **about his character**.
a. on his character b. of his character [2011]
c. his character d. No improvement
10. She is quite without **affection** and has no false pride. [2011]
a. admiration b. affliction
c. affectation d. No improvement
11. The monograph which was published 3 years ago, **would suggest** that by 2001 there will be 73 million TV. sets in India. [2007]
a. has been suggesting b. had suggested
c. would have suggested d. No improvement
12. The greatest thing in style is to have a **use** of metaphor. [2007]
a. command
b. knowledge
c. need
d. No improvement

Answer with Solutions

Based on Tense and Agreement

1. (e) No error
2. (c) 'on asking' for 'to ask'
3. (a) Say 'comes' for 'will come'
4. (b) Say 'connected with'
5. (a) Say 'Weather permitting'
6. (b) Drop 'he'
7. (a) Add 'understand' after 'did not'
8. (c) Say 'but because'
9. (e) No error
10. (c) to be attended
11. (c) Say 'are seen'
12. (d) Say 'were not'
13. (d) Use 'affected'
14. (c) Say 'has'
15. (c) Say 'because'
16. (b) Say 'was'
17. (a) Say 'needs'
18. (b) Say 'does not'
19. (b) Use 'luggage' in place of 'luggages'.
20. (c) Use 'was' in place of 'were'.
21. (c) Use 'have' in place of 'has'.
22. (a) Use 'whatever' for 'whenever'
23. (b) Use 'that' for 'as'.

Based on Modals

1. (a) could speak
2. (b) should keep
3. (b) might give us lift
4. (b) might have written
5. (a) might have been
6. (c) may have
7. (b) should have warned
8. (a) must check
9. (b) must
10. (b) must not have left
11. (a) must not mix
12. (a) must not leave

Based on Preposition

1. (a) out
2. (a) from

3. (b) off
4. (b) from
5. (a) on
6. (a) up
7. (c) of
8. (c) on
9. (c) over
10. (b) into
11. (b) off
12. (c) from
13. (a) after
14. (a) on
15. (b) up
16. (a) to
17. (b) into
18. (a) of
19. (b) on
20. (d) to

Based on Conjunction

1. (b) Replace 'but' by 'yet'.
2. (b) Replace 'because' by 'so that'.
3. (b) Replace 'as' by 'that'.
4. (c) Replace 'that' by 'as'.
5. (c) Replace 'that' by 'as'.
6. (c) Replace 'as' by 'that'.
7. (b) Replace 'and' by 'but'.
8. (d) No error.
9. (b) Replace 'than' by 'but'.
10. (b) Replace 'that' by 'when'.
11. (b) Replace 'equally' by 'as'.
12. (c) Replace 'on either Monday or Tuesday' by 'either on Monday or on Tuesday'.
13. (b) Replace 'but to play' by 'than playing'.
14. (b) Replace 'where' by 'when'.
15. (c) Replace 'till' by 'since'.
16. (c) Replace 'to' by 'and'.
17. (b) Replace 'when' by 'than'.
18. (b) Delete 'therefore'.
19. (b) Delete 'that'.
20. (a) Delete 'do not'.

Based on Voice

1. (c) Get someone to break the box.
2. (d) Some one has picked his pocket.
3. (a) Quebec was surrendered by the French to the English in 1759.
4. (c) The last day's play between India and Srilanka was disrupted by rain.
5. (b) I was promised a present by my uncle.
6. (d) A house is being built by them next door to our school.
7. (d) A promise should be kept.
8. (a) Who has created this mess?
9. (a) Are others imitated by you?
10. (d) I saw the rehearsal being conducted by him.
11. (b) A bull dog was given to her.
12. (d) I was struck on the head by a stone.
13. (c) It was expected by the Romans that they would conquer Carthage.
14. (a) They were laughing at the clown.
15. (d) He was seen leaving the house by me.
16. (b) Even a little mouse may help a lion.
17. (d) By whom was she taught such things?
18. (c) I was kept awake by the noise of the traffic.
19. (c) The police arrested him on a charge of theft, but for lack of evidence released him.
20. (c) You are requested to help me.
21. (b) The Independence Day was celebrated by the residents.
22. (c) The suspect is claimed to have been seen in several cities.
23. (c) The boys who had not done their homework were punished by the teacher.

Based on Question Tags

1. (b) won't you
2. (c) can't you
3. (c) Aren't I
4. (b) wasn't he
5. (a) shall we

Miscellaneous

1. (c) I have been working
2. (d) No improvement
3. (a) asked his teacher
4. (d) No improvement
5. (a) avoid making
6. (c) will you
7. (c) have been waiting
8. (d) No improvement
9. (d) No improvement
10. (d) No improvement
11. (c) Why he wrote the letter was that
12. (c) each of the unemployed
13. (a) protest against
14. (b) you, my wife and I
15. (b) went upwards
16. (b) opinion about
17. (a) sent word
18. (a) used

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1. (a) ordered
2. (c) believe
3. (c) yours
4. (c) draw
5. (a) looking for
6. (d) No improvement
7. (c) refuted
8. (a) left
9. (c) his character
10. (c) affectation
11. (b) had suggested
12. (b) knowledge

2

Vocabulary

Vocabulary is a broad concept in itself. One can enhance one's language skills by acquiring a good hold over **vocabulary**. To score high in the exam, a candidate should improve his/her vocabulary skills. It helps you in solving questions of Comprehension, Cloze Test, Antonyms and Synonyms etc.

Here we illustrate some steps to enrich vocabulary.

Step I Identify the Word

Whenever we come across a new word in a sentence while reading a text book, newspaper or a magazine, we should look up its meaning. This is the best way to enhance vocabulary skills.

Step II Identify the Antonym of that Word

Along with the meaning of a word, its antonyms (words with opposite meaning) should also be taken into consideration. A good knowledge of words and their antonyms is very beneficial from the examination point of view.

Step III Find Synonym of that Word

The knowledge of words that are similar or closer in meaning to one another is very useful. It makes a student efficient enough to have a strong sense of the language.

Step IV Form a Proper Sentence

This is one of the most important parts in vocabulary building. It serves to stimulate memory by recalling the words as and when needed, apart from making the proper sense and the use of words clear.

To understand these steps, we consider an example. Suppose we take a word 'Antique'.

Step I Antique means something that is old and has not been renewed for long e.g. Antique building (Old building).

Step II Opposite of Antique is modern. It also Modern can be replaced by, Renovated and Recent etc.

Step III Synonyms are the similar words in meaning. For synonyms, Antique can be replaced by Traditional and Ancient etc.

Step IV To be more familiar with the word, we should use them in sentence form. For 'Antique' a proper sentence is "People love to see the Antique items."

Following are the topics that we learn in vocabulary.

1. Antonyms
2. Synonyms
3. One word substitutions

Practice Exercise

Antonyms

Directions (Q. Nos. 1-38) *The following sentences consist of a word or a phrase which is written in italicised letters. Each of them is followed by four words or phrases. Select the word or the phrase which is closest to the opposite in meaning of the italicised word or phrase.*

1. Self-reliance has been *adopted* as an important objective of economic planning in modern India.
a. refused b. forsaken c. denied d. discarded
2. His *appointment* was confirmed last month.
a. disappointment b. suspension
c. dismissal d. discharge
3. He has a passion for *indigenous* goods.
a. native b. foreign c. silly d. cheap
4. He made an *exhaustive* list of the items his secretary should attend to.
a. short b. incomplete
c. interesting d. meticulous
5. He appears to be a *phony* person.
a. beautiful b. unread c. genuine d. ugly
6. He was in a *dejected* mood.
a. jubilant b. rejected c. irritable d. romantic
7. Philosophers say that the world is *an illusion*.
a. a fact b. a reality
c. an actuality d. a truth
8. There was a marked *deterioration* in his condition.
a. improvement b. revision
c. reformation d. amendment
9. The Principal hardly managed to give the papers a *cursory glance*.
a. curious b. thorough c. through d. superficial
10. The attack on the freedom of the press is a *retrograde* step.
a. progressive b. stubborn
c. punitive d. aggressive
11. We received a *cordial* welcome from our host.
a. indifferent b. distrustful
c. cold d. official
12. We should not *belittle* the value of small things.
a. extol b. praise c. inflate d. expand
13. It used to be said that travel *broadens* one's outlook.
a. narrows b. shrinks c. contracts d. restricts
14. The criminal was *detained* by the police.
a. deterred b. released
c. dismissed d. protected
15. It is *obligatory* for a common citizen to follow the rules.
a. advisable b. unnecessary
c. superfluous d. optional
16. His *urbane* attitude won him many friends.
a. indifferent b. violent
c. rustic d. rude
17. The leader might have had some *covert* reason for the change of his political affiliations.
a. unjustifiable b. obvious
c. inexplicable d. flimsy
18. The room was filled with a *delicious* odour.
a. bitter b. repulsive
c. strange d. unpalatable
19. That man is known for his *elegance*.
a. awkwardness b. indelicacy
c. clumsiness d. savagery
20. Unsettled conditions in the land led to the *exodus* of hundreds of its citizens.
a. invasion
b. intrusion
c. immigration
d. expulsion
21. In the interest of one's own reputation one should avoid *ostentation* while entertaining friends.
a. miserliness b. simplicity
c. purity d. innocence
22. Every thing about him especially his talkative nature proclaims his *effeminacy*.
a. aggressiveness b. attractiveness
c. manliness d. boorishness
23. The leader was *pragmatic* in his approach to the problems facing the country.
a. indefinite b. vague
c. idealistic d. optimistic
24. She used to *disparage* her neighbours every now and then.
a. please b. praise
c. belittle d. denigrate
25. The plantation workers were on a *collision* course before the labour officer intervened.
a. retaliatory b. perfunctory
c. conciliatory d. circuitous
26. Every self-respecting circus needs at least one *dwarf*.
a. pigmy b. undersized
c. giant d. human

27. Our knowledge of the past is still largely a matter of *conjecture*.
a. certainty b. guess
c. position d. form
28. In all places, and at all times, there is a *profusion* of talents.
a. plenty b. scarcity c. aversion d. generosity
29. It was *altercation* throughout and there was no discussion.
a. consonance b. alternative
c. inconsistency d. resonance
30. Unlike the other candidates, his manner was entirely *languid*.
a. energetic b. lazy c. liquid d. slow
31. People who are actually running the system often take a *myopic* view of the situation.
a. farsighted b. visionary c. blind d. glassy
32. That was a *dauntless* action.
a. cowardly b. secret c. subtle d. devious
33. The momentum of the movement *slackened* in course of time.
a. stopped b. quickened
c. multiplied d. recovered
34. He is a man of *mellow* temper.
a. fickle b. hot c. irrational d. excitable
35. His was a *delicate* constitution.
a. fit b. strong
c. rugged d. ungainly
36. Let us not *aggravate* the sufferings of the poor.
a. advocate b. appreciate
c. alleviate d. abbreviate
37. *Misogynists* are not applauded in society.
a. Philanthropists b. Philologists
c. Philogynists d. Philanderers
38. Her *vivacity* attracted everybody.
a. lethargy b. truthfulness
c. beauty d. apathy
3. Buoyed by these visions of the future of his son, the artist *soldiered* on.
a. continued bravely b. stopped further
c. struggled hard d. fought on
4. Last fortnight the rains came to the parched soils of India after the merciless drought had already *sapped* the vitality and livelihood of millions of peasants.
a. decreased b. demolished
c. destroyed d. drained away
5. The boy gave a *vivid* description of all that happened.
a. brilliant b. fresh
c. explanatory d. picturesque
6. We don't know how we are to *recompense* you for the trouble you have taken.
a. reward b. help c. praise d. thank
7. John Milton wrote 'Paradise Lost' to *vindicate* the ways of God to man.
a. explain b. support c. justify d. criticise
8. When youngsters do not have good role-models to *emulate* they start searching for them among sportsmen or filmstars.
a. inhabit b. imitate c. mollify d. modify
9. The eyewitness testimony was *incontrovertible*.
a. debatable b. unquestionable
c. unacceptable d. disputable
10. The invasion forces had no artillery and were completely *annihilated*.
a. reduced b. destroyed
c. dismembered d. split
11. The novel was so interesting that I was *oblivious* of my surroundings.
a. indifferent b. watchful c. unmindful d. precarious
12. Elsie is seventy years old and a *fanatical* gardener.
a. crazy b. fantastic
c. excessively enthusiastic d. obsessed
13. Before I could *make out anything* he had spoken again.
a. find out b. apprehend c. explain d. reveal
14. It is amazing how such a *motley* group could get along so well.
a. heterogenous b. assorted
c. promiscuous d. confused
15. His speech was nothing but a string of *platitudes*.
a. grand statements b. stereo-typed statements
c. noble sentiments d. humorous anecdotes
16. He wrote a *scathing* review of the prize-winning novel.
a. biased b. scornful
c. unbalanced d. subjective
17. The opposition criticised the ruling party for the *deteriorating* law and order situation in the state.
a. disrupting b. worsening c. crumbling d. eroding
18. The flat has been *refurbished* recently.
a. white-washed b. painted
c. renovated d. repaired

Synonyms

Directions (Q. Nos. 1-37) *In the following sentences given below, a word or phrase is written in italicised letter. For each italicised word four words/phrases are listed below each sentence. Choose the word nearest in meaning to the italicised word.*

1. A million cinemas a year bring the same stale *blatant*.
a. adventure b. nonsense
c. thriller d. romance
2. There must be lively discussion of Indian authors if we are to *foster* our knowledge.
a. promote b. cherish
c. nourish d. nurture

- ## One Word Substitution

Directions (Q. Nos. 1-10) *In each of the following questions, out of the four alternatives choose the one which can be substituted for the given words/sentences.*

1. Music sung or played at night below a person's window.
 - a. Serenade
 - b. Sonnet
 - c. Lyric
 - d. Primo
2. A government by the nobles.
 - a. Democracy
 - b. Bureaucracy
 - c. Autocracy
 - d. Aristocracy
3. Anything written in a letter after it is signed.
 - a. Posterity
 - b. Post dication
 - c. Postscript
 - d. Corrigendum
4. Bringing about gentle and painless death from incurable disease.
 - a. Suicide
 - b. Euphoria
 - c. Gallows
 - d. Euthanasia
5. A man of lax moral.
 - a. Ruffian
 - b. Licentious
 - c. Pirate
 - d. Vagabond
6. Large scale departure of people.
 - a. Migration
 - b. Emigration
 - c. Immigration
 - d. Exodus
7. Lottery in which an article is assigned by lot to one of those buying tickets.
 - a. Auction
 - b. Raffle
 - c. Audit
 - d. Transit
8. To send an unwanted person out of the country.
 - a. Exclude
 - b. Ostracise
 - c. Deport
 - d. Expatriate
9. Voluntarily giving up throne by king in favour of his son.
 - a. Abdication
 - b. Resurrection
 - c. Accession
 - d. Renunciation
10. To examine one's own thoughts and feelings.
 - a. Meditation
 - b. Retrospection
 - c. Reflection
 - d. Introspection

BITSAT Archives**Antonyms**

- | | | | | | |
|--|--------------------------------|--------|---|---------------------------------|--------|
| 1. Recompense
a. Emolument
c. Payment | b. Reward
d. Penalty | [2014] | 16. The doctor advised us to give him <i>wholesome</i> nutrition.
a. sickly
c. depressing | b. stupendous
d. fragmentary | [2007] |
| 2. Impede
a. Block
c. Push | b. Delay
d. Freeze | [2014] | 17. He is good fellow; but what I dislike is his <i>reckless</i> handling of things.
a. intelligent
c. soft | b. cautious
d. brilliant | [2007] |
| 3. Auspicious
a. Prosperous
c. Improper | b. Unfavourable
d. New | [2014] | 18. Ingratitude
a. Stimulation
c. Sympathy | b. Reward
d. Thankfulness | [2006] |
| 4. Fidelity
a. Faith
c. Allegiance | b. Devotedness
d. Treachery | [2013] | Synonyms | | |
| 5. Infrangible
a. Complicated
c. Weird | b. Breakable
d. Software | [2013] | 1. Augment
a. Increase
c. Save | b. Decrease
d. Mention | [2014] |
| 6. Progeny
a. Kid
c. Friend | b. Parent
d. Enemy | [2013] | 2. Sagacious
a. Shameless
c. Powerless | b. Wise
d. Foolish | [2013] |
| 7. In toto
a. Bluntly
c. Entirely | b. Partially
d. Strongly | [2012] | 3. Remedial
a. Corrective
c. General | b. Proficient
d. Optional | [2013] |
| 8. Protean
a. Amateur
c. Unchanging | b. Catholic
d. Rapid | [2012] | 4. Reticent
a. Confident
c. Truthful | b. Sad
d. Secretive | [2013] |
| 9. Fragrance
a. Aroma
c. Smell | b. Perfume
d. Stink | [2011] | 5. Flabbergasted
a. Scared
c. Dumbfounded | b. Embarrassed
d. Humiliated | [2012] |
| 10. Eternal
a. Momentary
c. Everlasting | b. Continual
d. Endless | [2011] | 6. Eternal
a. Innumerable
c. Prolonged | b. Unmeasurable
d. Perpetual | [2012] |
| 11. Reprimand
a. Reward
c. Encourage | b. Appreciate
d. Praise | [2009] | 7. Enlightenment
a. Lightness
c. Twilight | b. Insight
d. Proficiency | [2011] |
| 12. Impertinent
a. Polite
c. Unpleasant | b. Indifferent
d. Stubborn | [2009] | 8. Sublimity
a. Despression
c. Subversion | b. Reduction
d. Proficiency | [2011] |
| 13. Equivocal
a. Mistaken
c. Clear | b. Quaint
d. Universal | [2009] | 9. Paramour
a. Lever
c. Friend | b. Companion
d. Rival | [2010] |
| 14. Infallible
a. Erring
b. Untrustworthy
c. Dubious
d. Unreliable | | [2008] | 10. Refectory
a. Dining Room
c. Living Room | b. Parlour
d. Restaurant | [2010] |
| 15. Exalt
a. Depreciate
c. Glorify | b. Ennoble
d. Simplify | [2008] | 11. Denouement
a. Outcome
c. Action | b. Eschew
d. Character | [2009] |

12. Gauche [2009]
 a. Vain b. Rich
 c. Polished d. Tactless
13. Accolade [2009]
 a. Honour
 b. Appreciation
 c. Greeting
 d. Gift
14. Ephemeral [2008]
 a. Uneral b. Mythical
 c. Short-living d. Artificial
15. Stubborn [2008]
 a. Easy b. Obstinate
 c. Willing d. Pliable
16. Prognosis [2008]
 a. Identification b. Preface
 c. Forecast d. Scheme
17. Many of his acquaintances avoid him because he is so *garrulous*. [2007]
 a. proud
 b. unreasonable
 c. talkative
 d. quarrelsome
18. Connoisseur [2006]
 a. Lover of art b. Interpreter
 c. Delinquent d. Ignorant
19. Lethal [2006]
 a. Unlawful b. Sluggish
 c. Deadly d. Smooth
20. Picturesque [2006]
 a. Photogenic b. Ugly
 c. Simple d. Stimulating
21. Renounce [2005]
 a. Reform b. Revoke
 c. Retain d. Resign

One Word Substitution

1. Despite being in the career of singing for the last 10 years, he has not been able to earn fame on account of his practice of borrowing ideas and words from others and using them as his own. [2010]
 a. Adaptation b. Pantomime
 c. Imitation d. Plagiarism
2. Every person is not allowed to enter the place where public, government or historical records are kept. [2010]
 a. Scullery b. Pantry
 c. Archives d. Coffe
3. The advertisement assured the public that the medicine would give back to the users, their youthful vigour and appearance. [2010]
 a. Rejuvenate b. Restore
 c. Replenish d. Render
4. Design made by putting together coloured pieces of glass or stones. [2009]
 a. Oleograph b. Mosaic
 c. Tracery d. Relief
5. The doctrine that human soul passes from one body to another at the time of death. [2009]
 a. Metamorphosis b. Transition
 c. Transmigration d. Extrapolation
6. A place adjoining kitchen, for washing dishes etc. [2008]
 a. Cellar b. Wardrobe
 c. Scullery d. Pantry
7. Elderly woman in charge of a girl on social occasions. [2009]
 a. Spinster b. Matron
 c. Chaperon d. Chandler
8. Land so surrounded by water as to be almost an island. [2009]
 a. Archipelago b. Isthmus
 c. Peninsula d. Lagoon

Answers

Antonyms

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (d) | 2. (c) | 3. (b) | 4. (b) | 5. (c) | 6. (a) | 7. (b) | 8. (a) | 9. (b) | 10. (a) |
| 11. (a) | 12. (a) | 13. (a) | 14. (b) | 15. (d) | 16. (d) | 17. (b) | 18. (d) | 19. (a) | 20. (c) |
| 21. (b) | 22. (c) | 23. (c) | 24. (b) | 25. (c) | 26. (c) | 27. (a) | 28. (b) | 29. (a) | 30. (a) |
| 31. (a) | 32. (a) | 33. (b) | 34. (d) | 35. (c) | 36. (c) | 37. (c) | 38. (a) | | |

Synonyms

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (b) | 2. (d) | 3. (a) | 4. (d) | 5. (d) | 6. (a) | 7. (c) | 8. (b) | 9. (b) | 10. (b) |
| 11. (c) | 12. (c) | 13. (b) | 14. (b) | 15. (b) | 16. (b) | 17. (b) | 18. (c) | 19. (c) | 20. (b) |
| 21. (c) | 22. (b) | 23. (a) | 24. (a) | 25. (b) | 26. (b) | 27. (b) | 28. (b) | 29. (d) | 30. (b) |
| 31. (c) | 32. (b) | 33. (a) | 34. (a) | 35. (d) | 36. (b) | 37. (c) | | | |

One Word Substitution

- | | | | | | | | | | |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------|
| 1. (a) | 2. (d) | 3. (c) | 4. (d) | 5. (b) | 6. (d) | 7. (b) | 8. (c) | 9. (a) | 10. (d) |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------|

BITSAT Archives

Antonyms

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|--------|---------|
| 1. (d) | 2. (c) | 3. (b) | 4. (d) | 5. (b) | 6. (b) | 7. (b) | 8. (a) | 9. (d) | 10. (a) |
| 11. (d) | 12. (a) | 13. (c) | 14. (a) | 15. (a) | 16. (a) | 17. (b) | 18. (d) | | |

Synonyms

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (a) | 2. (b) | 3. (a) | 4. (d) | 5. (c) | 6. (d) | 7. (b) | 8. (d) | 9. (a) | 10. (a) |
| 11. (a) | 12. (d) | 13. (a) | 14. (c) | 15. (b) | 16. (c) | 17. (c) | 18. (b) | 19. (b) | 20. (a) |
| 21. (d) | | | | | | | | | |

One Word Substitution

- | | | | | | | | |
|--------|--------|--------|--------|--------|--------|--------|--------|
| 1. (d) | 2. (c) | 3. (a) | 4. (b) | 5. (a) | 6. (d) | 7. (c) | 8. (b) |
|--------|--------|--------|--------|--------|--------|--------|--------|

3

Comprehension Ability

Comprehension involves a thorough understanding of the given passage consisting of one or more paragraphs. It forms a very good exercise for brilliant and intelligent reading and judicious selection. It is meant to test the intelligence of candidates and their ability to comprehend a given piece of text.

Tips for Attempting Comprehension

1. **Skim** once as rapidly as possible to determine the main idea before you look at the questions. Do not worry about words you do not know, at this stage.
2. **Underline the words** that you do not understand to facilitate a complete understanding of the passage. This will enable you to solve the vocabulary questions quicker.
3. **Keep forging ahead** and don't get bogged down if there is a word or sentence you do not understand. Sometimes you can sense the meaning of the word/sentence from the context. It is also possible that the word/sentence may not be the basis of any questions.
4. **Read each question** carefully and be certain that you understand exactly what is being asked.
5. **Answer** strictly on the basis of the passage and avoid selecting general answers.
6. **Note transition** from one idea to the next and examine the relationships among the different ideas or part of the passage.
7. **Concentrate** on the vocabulary items and puzzle out the meanings of the words you do not know in the context. Generally, options are tricky or some options are closely related. Therefore, analyse the questions and options carefully before answering.
8. **Select the answer** on the basis of the information provided in the passage. You are not expected to rely on outside knowledge of a particular topic. Avoid selecting answer which are not based on the passage.

Practice Exercise

Directions (Passages 1-10) *Read the passages given below and answer the questions that follow*

Passage 1

White cement is the basic raw material for producing cement tiles and cement paint which are used extensively in building construction. The main consumers of white cement are, therefore, cement tile and cement paint manufacturing units. These consumers, mostly in the small scale sector, are today facing a major crisis because of a significant increase in the price of white, cement during a short period.

The present annual licensed production capacity, of white and grey cement in the country is approximately 3.5 lakh tonnes. The average demand is 2-2.5 lakh tonnes. This means that there is idle capacity to the tune of one lakh tonnes or more. The price rise is, therefore not a phenomenon arising out of inadequate production capacity, but evidently because of artificial scarcity created by the manufacturers in their self-interest.

The main reason for the continuing spurt in cement price is its decontrol. As it is, there is stiff competition in the cement paint and tile manufacturing business. Any further price revision at this stage is bound to have a severe adverse impact on the market conditions. The government should take adequate steps to ensure that suitable controls are brought in. Else, it should allow import of cement.

1. Why is the price of cement going up?
 - a. Because the Government is controlling the quota
 - b. Because of export of white cement
 - c. Because of the large usage of white cement
 - d. None of the above
2. Which of the following statements is false according to the passage?
 - a. Price rise in white cement would increase the price of cement paint
 - b. White cement is a controlled product
 - c. Increase in price of white cement is not because of production problem
 - d. Price rise in white cement would upset cement tile market
3. What is the crisis being faced by the cement tile manufacturers as described in the passage?
 - a. White cement is priced very low
 - b. White cement is not of good quality
 - c. White cement usage is high
 - d. White cement prices are very high
4. Which of the following words has the same meaning as the word 'idle' as used in the passage?
 - a. Lazy
 - b. Clumsy
 - c. Large
 - d. Excess

5. Which of the following is correct according to the passage?
 - a. White cement is not used in the manufacture of paint
 - b. White cement forms a minor part of tile manufacturing
 - c. White cement is very important for tile manufacturing
 - d. White cement is used only for making paints
6. Which of the following words has the same meaning as the word 'artificial' as used in the passage?
 - a. Unnatural
 - b. Prolonged
 - c. Practical
 - d. Deliberate
7. What is the author's suggestion to bring down prices?
 - a. Government should control the white cement market
 - b. Production capacity should be increased
 - c. Usage of white cement in other areas should be reduced
 - d. Competition among manufacturers of cement paints and tiles should be reduced
8. Which of the following words has the opposite meaning as the word 'basic' as used in the passage?
 - a. Vital
 - b. Unimportant
 - c. Acidic
 - d. Last
9. Which of the following is correct according to the passage?
 - a. Cement production capacity is more than usage
 - b. Cement production capacity is less than usage
 - c. Cement production capacity is equal to usage
 - d. Cement production capacity is twice than usage
10. What can be presumed from the passage regarding foreign trade in cement?
 - a. The country is exporting cement at present
 - b. The country is importing cement at present
 - c. The country is exporting and importing cement
 - d. The country is neither exporting nor importing cement

Passage 2

The forces that generate conditions conducive to crime and riots are stronger in urban communities than in rural areas. Urban living is more anonymous living. It often releases the individual from community restraints more common in tradition-oriented societies. But more freedom from constraints and controls also provides greater freedom to deviate. And living in the more impersonalised, formally controlled urban society means that regulatory orders of conduct are often directed by distant bureaucrats. The police are strangers executing these prescriptions on an anonymous set of subjects. Minor offences in small town or village are often handled without resort to official police action. As disputable as such action may seem to be, it results in fewer recorded violations of the law compared to the

big cities. Although perhaps causing some decision-difficulties for the police in small town, formal and objective law enforcement is not always acceptable to villagers.

Urban areas with mass population, greater wealth, more commercial establishments and more products of our technology also provide more frequent opportunities for theft. Victims are impersonalised, property is insured, consumer goods in more abundance are vividly displayed and more portable. The crime rate increases despite formal moral education given in schools.

11. According to the passage, all of the following contribute to higher crime rates in urban areas except
 - a. vivid display of consumer goods
 - b. higher standard of living
 - c. urban impersonalised living
 - d. inadequate police force
12. Which of the following is a characteristic of an urban setting?
 - a. Unreported minor crimes
 - b. Deviation from freedom
 - c. Less forceful social control
 - d. Minimal opportunities of crime due to better law enforcement
13. The author's view of 'Traditional Societies' is best expressed by which of the following?
 - a. They provide less freedom for the individual in many circumstances
 - b. They have lower crime rates because of the moral teachings in schools
 - c. They provide inadequate freedom for personal movements and travel
 - d. They do not have adequate modern technology
14. People live under more social control in
 - a. formally controlled urban societies
 - b. the presence of the police authorities
 - c. an anonymous form of living
 - d. None of the above
15. It can be inferred from the passage that urban crime can be controlled by
 - a. greater emphasis on moral education
 - b. enforcement of law by distant bureaucrats
 - c. vivid display of expensive consumer goods
 - d. None of the above
16. Which of the following statements is true in the context of the passage?
 - a. The display of consumer goods is the main cause of crime
 - b. Lack of personal contacts increases crimes in urban areas
 - c. Small communities have more minor crimes than in urban centres
 - d. Urban crimes areas cannot be prevented
17. Which of the following statements is not true in the context of the passage?
 - a. Moral education imparted in schools is ineffective in checking crime rate
 - b. There is less freedom in the current society than in a traditional society

- c. Urban areas are thickly populated and commercialised.
- d. Anonymous living in urban areas may lead to a freedom to deviate from rules

18. According to the passage, the crime in small towns
 - a. is less frequently reported or dealt with officially
 - b. is brought well under control by distant bureaucrats
 - c. leads to an impersonalised style of living
 - d. is often dealt with objective law enforcement

Passage 3

A pioneering scheme has been started recently in Southampton of England's South coast to educate tourists who have been convicted of drunken driving.

The penalty for drunken driving might be the loss of a driving licence and a heavy fine. But under the new scheme, convicted drivers do not pay the fine. Instead they have to attend eight training; sessions one a week organised by the local authority probation service.

Designed to demonstrate the damage alcohol can do, the scheme was devised by senior probation officer John Cook. He said about a quarter of the people who came to him had a drink problem, but had not realised how much they were drinking. One way of getting the message across was to make the drivers pour out their usual ration of alcohol and then measure it. Almost everyone pours out not a single measure, but a double at least an example of how easy it is to have more than just one drink and to encourage other people to do the same.

The instructors on the course are giving clinical evidence of the effects of alcohol on the body and brain. The sober truth is that drinking badly affects driving skills, although the drinker might like to believe otherwise.

19. The Southampton scheme requires convicted drivers
 - a. to pay a heavy fine
 - b. to attend eight driving sessions-one a week
 - c. to undergo a probation service
 - d. to surrender their driving licence
20. John Cook-devised the scheme
 - a. as a demonstration technique for driving
 - b. to demonstrate the harmful effects of alcohol
 - c. to show that Southampton was concerned about drivers
 - d. to prove that alcohol does influence driving
21. The problem with a quarter of the people who went to John Cook was that they
 - a. did not want to stop drinking
 - b. were unaware of the fact that they could get drunk
 - c. would not admit that they had a drinking problem
 - d. did not know how much they were drinking
22. Most drivers start off with at least
 - a. a double measure
 - b. a single measure
 - c. a little less than a single measure
 - d. two doubles

23. The truth is that alcohol
- does not affect the body but only the brain
 - affects only the brain
 - affects the body and the brain
 - has no effect on the body or the brain

Passage 4

I consider myself something of an authority on apologies, because a quick temper has provided me with plenty of opportunities to make them. In one of my earliest memories, my mother is telling me don't look at the ground when you say, 'I'm sorry'. Hold your head up and look the person in the eye, so he'll know you mean it.

My mother thus, conveyed the first principle of successful apology : it must be direct. You must never pretend to be doing something else. You do not leaf through a sheaf of correspondence while apologising to a subordinate after blaming her for a mistake that turned out to be your fault.

You do not apologise to a hostess, whose guest of honour you insulted, by sending flowers the next day without mentioning your bad behaviour.

24. According to the author's mother, the best way to apologise is
- to say 'sorry' while looking at the ground
 - to look the person in the eye, even if you don't say 'sorry'
 - while saying 'I'm sorry' look the person in the eye
 - to hold your head up and avoid looking at the person
25. The successful way of apology is that it
- should not be mixed up with other activities
 - could be made carefully even when you are busy
 - should be hinted at indirectly to the person concerned
 - could be made seriously while doing other activities
26. You should apologise for your bad behaviour at a party by sending
- flowers to the guest of your hostess
 - flowers to your hostess at your leisure
 - flowers to your hostess with a note of apology
 - only a note of apology the next day
27. The phrase 'leaf through a book' means to .
- copy from a book
 - turn over the pages of a book quickly
 - read a book seriously
 - turnover a new leaf
28. The author considers himself to be an authority on apologies because
- his mother trained him how to apologise
 - he has read many books on how to apologise
 - he has apologised many a time
 - he has been offered apologies in many ways

Passage 5

Education is for life, not merely for a livelihood. So long as we are unmindful of this truth, the quality of our educational curriculum as well as that of our teachers and students is likely to remain inadequate. It is not enough for a society to have experts. It needs human beings who can think, feel and act generously, the kind of people who cannot be replaced by computers and robots. The great fault of our present age is its emphasis on efficiency at the cost of humanity.

29. When the author says that education is for life rather than for a livelihood, he means to say that
- education is a generally useful thing in life
 - the primary purpose of education is to train people to be proper human beings
 - education is useful for earning a living
 - education can enable one to become rich
30. The author says that is not enough for a society to have experts because
- experts are egocentric
 - experts do not have an overall view of things because of their being highly specialised
 - along with efficiency, educated people need to be sympathetic and sensitive to the needs of society
 - experts are highly eccentric people
31. The author is critical of the present educational system because it
- overemphasises efficiency
 - neglects social sciences
 - is science-oriented
 - is obsessed with computers
32. Proper human beings cannot be replaced by computers and robots because
- computers and robots are non-human
 - computers and robots react mechanically
 - only human beings can have, specialised knowledge
 - only proper human beings can think creatively and act generously
33. The main idea of the author in this passage is that
- educated people should be sympathetic towards human suffering
 - education should emphasise improvement in quality of life rather than efficiency
 - specialisation improves efficiency
 - to attain efficiency is the only goal of education

Passage 6

Grandchildren at university now how time passes!

I wonder if they or anyone else would be interested in what student life was like in the '30s ? Well, here goes....

The academic part of a student's life doesn't change all that much through the generations, but the style of social life

has changed. (For better or worse — who are we to say ?) The 'English Lit. more properly, Edinburgh University English Literature Society, met on Tuesday evening in the Non-Soc Hall, which is roundabout where the student shop is now. It was called the Non Soc Hall 'because the meetings there were of Non-Associated Societies the Associated Societies being high-and-mighty affairs with a long history, such as the Dialectic, Philomathic and Diagnostic, which, in those days, certainly didn't admit women.

34. The author of the passage feels nostalgic because
 - a. he doesn't belong to the present generation
 - b. he represents the past generation
 - c. he is conscious of time passing swiftly
 - d. things have changed since he was a student
35. The author observes that the academic life of a university
 - a. changes faster than the social life
 - b. does not change as fast as the social life
 - c. does not change because of a generation gap
 - d. changes abruptly after a long gap
36. When the author was a student at Edinburgh, English Literature Society used to meet
 - a. where it meets even now
 - b. where the Dialectic Society used to meet
 - c. near the place the student shop is located now
 - d. where the Diagnostic Society meets now
37. The English Literature Society was considered to be a Non-Associated Society because
 - a. its activities were deemed to be insignificant
 - b. it refused to be associated with any other society
 - c. it was an exclusive society of Department of English
 - d. it indulged in unconventional literary movements
38. The membership of the Associated Societies was
 - a. open to all students
 - b. restricted to students of History
 - c. restricted to students of Psychology
 - d. open to only men

Passage 7

The private sector has today reached a stage of development and expertise where it can make a massive contribution to the country's further economic development. The majority of India's entrepreneurs, large and small, are patriotic men who do not ask for special favours or large profits, nor do they seek monopolies or any concentration of wealth and power. What they want are the opportunities to exercise their initiative, their skills and their resources for the benefit of their stakeholders and above all, to be allowed to get on with the job. Yet, mistrust of the private sector, particularly in large-scale industry, has been such as seriously to impede economic growth in the sixties and it now threatens to make it impossible for it to meet its targets in the current plan.

If we, in private enterprise, are to play our legitimate role in the decade to come, it is clear to me that we must prove to Government, to Parliament and to the public in general that we deserve to be trusted. There is also the opposition from socialists in our country who, though opposed to violence and regimentation of communism, accept its economic policies in the mistaken belief that private enterprise is incompatible with the achievements of socialist goals and that even if it were, Indian businessmen and industrialists do not believe in those goals or are unwilling to make the necessary sacrifices.

39. The majority of Indian entrepreneurs want
 - a. to monopolise the trade
 - b. to concentrate wealth and power
 - c. to exploit the workers
 - d. the opportunity to use their talent and resources for the benefit of their stakeholders
40. The socialists oppose the private sector because
 - a. it exploits the workers
 - b. they think that entrepreneurs cannot achieve the socialist goals
 - c. they believe in Marxist ideology
 - d. they think that private sector should not be trusted
41. What impeded the economic growth in the sixties?
 - a. The non-availability of the raw materials
 - b. Strikes and lock-outs in the industries
 - c. The faulty planning by the Government
 - d. The mistrust of the private sector by the Government
42. The socialists think that Indian businessmen should not be trusted because
 - a. the businessmen do not believe in socialist goals and are unwilling to make any sacrifices
 - b. they only calculate their own profit
 - c. they exploit the economically weaker section of the society
 - d. they do not think about the general good of the society
43. Private sector can contribute to the economy because of
 - a. market driven economy
 - b. globalisation
 - c. disinvestment policy
 - d. the expertise it has acquired

Passage 8

One of the great delusions of modern times is the prevailing belief that we are more civilised than our ancestors. At no other period in the history of mankind has the word 'civilisation' been so often misused and misunderstood. It is not in the abundance of material alone that civilisation lies. Neither does it consist of the advance of scientific knowledge, however amazing that may be. The inventions and discoveries of science have undoubtedly brought unheard of blessings and comforts to us. But they are not civilisation. What we have failed to see is that civilisation is a special way of human behaviour, of thought and feeling. The essential quality of civilisation lies in the spirit of man, in his

attitude to himself, to his fellowmen and to life as a whole. Looking everywhere around us today, you will find that man has made no progress since the dawn of history, in his attitude towards his fellowmen. On all the highest pillars in cities of the world, you will find the figure of a conqueror or a general soldier. We know far more about our killers and destroyers than about our saviours and saints. The heroes of war are the heroes of the world.

44. What have scientific inventions given to mankind?
 - a. Wealth
 - b. Civilisation
 - c. Blessings and comforts
 - d. Destructive weapons
45. Does the writer think that we are more civilised than our ancestors?
 - a. Yes
 - b. No
 - c. There has been no perceptible change
 - d. The change, if any, cannot be determined
46. What is civilisation according to the author?
 - a. Possession of goods
 - b. Inventions and discoveries
 - c. Way of human behaviour, thought and feeling
 - d. Another form of affluence
47. Who are the heroes of the world?

a. Warriors	b. Saints
c. Prophets	d. Teachers
48. Who should be our heroes?
 - a. Saints and saviours
 - b. Conquerors
 - c. Great men of history
 - d. Politicians
49. A vital quality of civilisation depends upon
 - a. competitive spirit of man
 - b. attitude of man towards his fellowmen
 - c. man's desire to conquer nature
 - d. management of resources

Passage 9

In place of old pleasures demanding intelligence and personal initiative, we have vast organisations that provide us with ready-made distractions, which demand from pleasure-seekers, no personal participation and no intellectual effort of any sort. To the interminable democracies of the world, a million cinemas bring the same stale bladerdash. There have always been fourth-rate, writers and dramatists; but their works in the past, quickly died without getting beyond the boundaries of the city or the country in which they appeared. Today, the inventions of the scenario-writer go out from Los Angeles across the whole world.

Countless audiences soak passively in the tepid bath of nonsense. No mental effort is demanded of them, no participation; they need only sit and keep their eyes open.

50. According to this passage, most cinemas
 - a. deal with the familiar uninteresting family dramas
 - b. tend to be didactic and moralistic
 - c. bring the images of the same old adventurous activities
 - d. bring nothing..but some dull, nonsensical talk
51. The author says that the fourth-rate writers and dramatists
 - a. never existed, in old times
 - b. wrote novels and plays which are not of permanent value
 - c. are mushrooming in modern times unlike, in the past
 - d. succeed easily in the modern times
52. According to this passage, the modern audience
 - a. is cosmopolitan in its outlook
 - b. is very discriminating
 - c. never falls asleep in a cinema hall
 - d. lacks individual initiative and refuses to exercise the mind
53. To indulge in their favourite pastime, our ancestors
 - a. exercised some intelligence and individual initiative
 - b. enjoyed nonsensical works
 - c. never moved out of their familiar environment
 - d. formed recreational clubs
54. The author thinks that the bane of modern times is that
 - a. physical exercise has been given a go-by
 - b. the trash travels very fast
 - c. people are taking active interest in mindless pleasures
 - d. the cheap ready-made distractions have become more and more pervasive

Passage 10

Those of us who already possess knives and use them as a matter of course at our meals can hardly understand the longing of an infant to be given the freedom of so perilous an instrument. Man has been defined as a tool-using animal, and there is no another tool that appeals to the imagination so strongly as a knife. It is through long months and year a forbidden thing and all the more fascinating on that count. There is no glory in using a spoon. There is no honour in holding a fork in the right hand and in taking up on it little squares of meat that have been cut with a knife by some more privileged, hand. Fork and spoon are little more than an extension of the fingers and a spoon, at least, is so safe that it can be left in the hands of an infant in the cradle. But a knife is a danger against which constant warning is necessary-something out of reach and waiting as a prize at the end of a long avenue of years.

55. The 'privileged hand' in this passage refers to
 - a. the spoon
 - b. expert hand
 - c. the adults
 - d. the children

56. To the child, there is no glory in handling a spoon because
- the child does not find the spoon attractive
 - for the child it is nothing but the extension of fingers
 - it is an extremely safe instrument
 - it is not as exciting as a fork
57. Adults cannot understand why a child finds knives so attractive because they
- use knives routinely at the dining table
 - have forgotten their childhood experiences
 - use the knife to cut tough vegetables
 - wield power at the time of meals
58. The child
- longs to be tool-using animal
 - desires the freedom to use the perilous instrument
 - wants to use the knife because he wants to become a soldier
 - dreams of crime and adventure
59. The knife fascinates a child all the more because it is
- out of his reach
 - a precious prize
 - a strong weapon to defend oneself with
 - a very attractive toy

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India is a country which has been subjected to foreign invasions since the dawn of Indian history. The fertile plains of India have been attracting avaricious tribals from all over the world. Long back the Aryans from Central Asia invaded India and settled down permanently in this beautiful land where food and fodder were available in plenty. After a chain of invasions from the bordering countries through land routes, the European nations, including the British, finally came to India to exploit her rich resources. The Englishmen came to India as traders, but stealthily became her masters. India became the 'brightest Jewel' in the British diadem. They proclaimed to civilise her and started to exploit her. Neither the imperialist might, nor the treachery of some of her sons, nor the treachery of some of her sons, nor the nerve-racking exploitation could curb the indomitable urge for freedom of the people who bid defiance to time. They fought and fought heroically; they never submitted.

Their struggle for independence is an inspiring and exhilarating story. It is a story not only of firm determination

and will, but also of sacrifice and suffering, a story of heroism and courage that happens in all revolutionaries. [2006]

- Why did the Aryans settle permanently in India?
 - Because here food and fodder were available in plenty
 - Because they invaded India
 - Because here food was available in plenty
 - Because here fodder was available in plenty
- British came to India
 - to exploit her rich resources
 - to become her master
 - to civilize her
 - None of the above
- The Indians indomitable urge for freedom could be curbed neither
 - by the nerve-racking exploitation of the British
 - by the treachery of some of her sons
 - by the imperialist might
 - All of the above

Answers

Practice Exercise

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (d) | 2. (b) | 3. (d) | 4. (d) | 5. (c) | 6. (c) | 7. (a) | 8. (b) | 9. (a) | 10. (d) |
| 11. (d) | 12. (c) | 13. (a) | 14. (d) | 15. (d) | 16. (b) | 17. (b) | 18. (a) | 19. (b) | 20. (d) |
| 21. (d) | 22. (a) | 23. (c) | 24. (c) | 25. (a) | 26. (c) | 27. (b) | 28. (c) | 29. (a) | 30. (c) |
| 31. (a) | 32. (d) | 33. (b) | 34. (b) | 35. (d) | 36. (c) | 37. (c) | 38. (d) | 39. (d) | 40. (b) |
| 41. (d) | 42. (a) | 43. (d) | 44. (c) | 45. (b) | 46. (c) | 47. (a) | 48. (a) | 49. (b) | 50. (d) |
| 51. (b) | 52. (d) | 53. (d) | 54. (d) | 55. (c) | 56. (b) | 57. (a) | 58. (a) | 59. (a) | |

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1. (a) 2. (a) 3. (d)

4

Rearrangement

One of the questions set in the objective test in English relates to rearrangement. A student is required to rearrange the jumbled parts of a sentence/paragraph so as to form a proper sentence. In fact, there are no hard and fast rules for this kind of test. Only sound knowledge of English language and constant practice may enable a student to solve such questions.

The questions based on rearrangement are usually of three types.

In the **first type** of questions, some parts of a sentence are jumbled up. You are required to rearrange these parts which are labelled P, Q, R, and S so as to produce the correct sentence.

In the **second type** of questions, a paragraph consisting of six sentences is given. The first and the last sentences are given and the middle four sentences are removed and jumbled up. You are required to find the proper order of these four sentences so as to form a coherent paragraph.

In the **third type** of questions, some sentences are given. These sentences form a coherent paragraph when they are arranged in proper order. You are required to rearrange these sentences to form a coherent paragraph and then answer the given questions.

Some Useful Tips

1. First, look for the part which may be the beginning part of the sentence. The part must have a word that could act as the subject of a sentence.
2. Usually, the subject of a sentence is either a noun or a pronoun.
3. Sometimes a sentence can also start with an adverb, a gerund or a conjunction. Keep an eye on such words also.
4. Try to find out the tense of the given sentence. It will also help you in arranging the parts in a proper order.
5. While choosing the proper order of sentences in a paragraph, try to find out the logical pairs.
6. Sometimes a chronological order can also be set. It will make your work very easy.
7. You can also select the correct option from among the given alternatives by eliminating the wrong options.

Practice Exercise

Directions (Q. Nos. 1-19) *Some parts of each of the following sentences, have been jumbled up. You are required to choose the correct sequence to rearrange these parts which are labelled P, Q, R, S so as to produce the correct sentence.*

1. The only time when this dynamic cricketer
P

was on the 1979 tour of England failed with
Q R

the bat the tag of an all-rounder did not fit him
S

The proper sequence should be

- a. S P R Q b. P R S Q c. S R P Q d. S Q P R

2. Kapil left in an aeroplane after reading a sailing magazine
P Q

had decided to build his own boat nine years earlier
R S

The proper sequence should be

- a. P R Q S b. R S Q P c. R Q P S d. P S R Q

3. When it becomes an honour of a lifetime
P

in recognition of their great performance illustrious
Q R

personalities win an award
S

The proper sequence should be

- a. R P Q S b. P Q R S c. Q R S P d. S Q R P

4. The newspaper reported that according to one estimate
P

and many of them assume epidemic proportions most
Q

illness in India are related to water-borne diseases
R S

The proper sequence should be

- a. P R Q S b. P R S Q c. R Q S P d. R Q P S

5. The zoologist says that having been bitten by them
P

once trembled to make a second attempt at catching
Q R

the snakes the snake-charmers
S

The proper sequence should be

- a. P R Q S b. S Q P R
c. R P Q S d. P S Q R

6. If you had told me I would not have given it to you
P

that you had already bought this book as a birthday
Q R S

present

The proper sequence should be

- a. Q R P S b. P Q R S c. S P Q R d. S R Q P

7. All precautionary measures were taken to prevent the capture of booths during the election by the
P Q

Government by the terrorists
R S

The proper sequence should be

- a. S P R Q b. Q S P R c. R P S Q d. R Q S P

8. The Government has assured the people that in the Kargil operation who have sacrificed their lives will be
P Q

properly looked after the families of all those
R S

soldiers.

The proper sequence should be

- a. P S R Q b. S Q P R c. S R Q P d. S P R Q

9. All through collecting and storing up food the long
P

warm summer days an ant was very busy for the
Q R S

winter.

The proper sequence should be

- a. S R Q P b. Q P S R c. Q R P S d. R S Q P

10. Standing carelessly with his sword, the fat major
P

held horizontally behind him and with his legs far apart
Q R

looked after the receding horseman and laughed
S

The proper sequence should be

- a. P Q S R b. Q P R S c. P Q R S d. P R S Q

11. Since life has now become intensely competitive
P

not only by the grades he obtains but also by his skill
Q

of observation and logical reasoning we judge a
R

person's intellectual worth
S

The proper sequence should be

- a. S Q R P b. P S R Q
c. P Q R S d. P S Q R

12. By the time known to everyone we appeared on the
P Q
scene he had already become an institution
R
who had ever stayed at Anand Bhawan
S

The proper sequence should be

- a. Q P R S b. P S Q R c. Q R P S d. R S Q P

13. I can guarantee as a medical representative
P
without any doubt for the post for which he has
Q R
applied that this candidate will be successful
S

The proper sequence should be

- a. S P R Q b. S R Q P c. P S R Q d. Q S P R

14. Most of our small farmers do not get a fair return
P
on the sale of their farm-products on their investments
Q
in back-breaking labour and high cost of seeds
R
and fertilizers
S

The proper sequence should be

- a. P Q R S b. P Q S R c. P R Q S d. P R S Q

15. The guide said that nowhere in the world
P
a fairer building you will find than the Taj Mahal
Q R S

The proper sequence should be

- a. R Q P S b. P R Q S c. P S Q R d. R Q S P

16. The students knew that to control unruly students
P
in the college administration our new Principal took a
Q R
number of strong measures
S

The proper sequence should be

- a. R S P Q b. R S Q P c. P Q R S d. Q R P S

17. According to the standard big-bang theory, in a moment
P
of infinite temperature the universe came into existence
Q
some ten to fifteen billion years ago and density.
R S

The proper sequence should be

- a. P S R Q b. Q P S R
c. R P Q S d. Q S P R

18. Nearly to NTPC's environmental activities bloom
P
around NTPC's power plants and stand lush green
Q R
testimony one and a half crore trees.
S

The proper sequence should be

- a. P R Q S b. S Q R P c. P Q R S d. S R Q P

19. Animal welfare experts to demolish illegal structures
P
have condemned by the Forest Department the use
Q R S
of elephants.

The proper sequence should be

- a. Q S R P b. R P Q S c. Q P R S d. R S Q P

Directions (Q. Nos. 20-28) *In the following items, each paragraph consists of six sentences. The first and the sixth sentences are given in the beginning and labelled S₁ and S₆ respectively. The middle four sentences in each have been removed and jumbled up. These are labelled P, Q, R and S. You are required to find the proper order for the four sentences and mark accordingly.*

20. S₁ : Yes, a mirage can be photographed.

S₆ : That kind of mirage obviously cannot be photographed.

P : That is hallucination.

Q : Anything reflected by the lens of the human eye will naturally be picked up by the photographic lens.

R : Sometimes, however, a person imagines he is seeing something but actually he is not.

S : The image, of course, will be hazy and shimmering because of refraction of light.

- a. S P R Q b. Q R P S c. P S Q R d. Q S R P

21. S₁ : One of the greatest curses of the world is the belief in the necessity of poverty.

S₆ : We have been poor in the very midst of abundance, simply because of our own blighting limiting thought.

P : There need not be poor person on the planet.

Q : But there was no poverty, no want, no lack, in the creator's plan for man.

R : Most people have a strong conviction that some must necessarily be poor, that they were made to be poor.

S : The Earth is full of resources which we have scarcely yet touched.

- a. S P Q R b. R S P Q
c. R Q P S d. P Q S R

22. S_1 : There is nothing strange in the fact that so many foreign students should wish to learn English.
 S_6 : This key will open to him whatever is valuable in the literature of the world.
 P : If any valuable book is written in another language an English translation of it is sure to be speedily published.
 Q : Anyone who masters the English tongue acquires a key.
 R : Most books found to be generally useful are written in English.
 S : The English speaking want no monopoly of knowledge.
 a. P S Q R b. S Q R P c. R P S Q d. P Q S R
23. S_1 : Much of our adult behaviour and our attitudes are determined by our upbringing.
 S_6 : Psychologists have studied these forces in depth.
 P : But the process does not stop here.
 Q : In particular by the effects of that small part of society which is our family.
 R : As we grow we are constantly and increasingly affected by new forces such as the social pressure of our friends and the larger world of society.
 S : The family and our early life have profound effect on our later life.
 a. S R P Q b. P R S Q c. Q P S R d. Q S P R
24. S_1 : Many people, who are hit on the head and suffer from brain injury, temporarily lose memory of things happening to them a few hour or few days before the accident.
 S_6 : It is as if the memories of this time do not have time to become established in the permanent long memory.
 P : The first ones to come back are usually older memories.
 Q : After a while the memories begin to return.
 R : Then other items appear.
 S : But there is always a short period of a few minutes just before the accident, which remains a complete blank.
 a. P Q R S b. S R Q P c. Q P R S d. Q R P S
25. S_1 : Chanakya, by his foresight and strength of character, built up the Mauryan Empire.
 S_6 : It is not true as some people said that Sardar Patel was not particular about the means as long as he achieved his end.
 P : They were men of iron.
 Q : Therefore, they were impervious to criticism and had implicit confidence in themselves.
 R : Both of them were guided by stern realism and would tolerate no deviation from the objectives which they pursued relentlessly.
 S : Similarly, Patel refashioned a compact India out of the fragments left by the British.
 a. S Q R P b. S R Q P c. S R P Q d. P Q S R
26. S_1 : The spiritual training of the boys was a much more difficult training.
 S_6 : And I held that this was an essential part of the training of the young and that all training without culture of the spirit was of no use, and might be even harmful.
 P : I had realised that the training of the spirit was a thing in itself.
 Q : Of course, I believed that every student should be acquainted with the elements of his own religion and have a general knowledge of his own scriptures.
 R : I relied little on religious books for spiritual training.
 S : To develop the spirit is to build character and to enable one to work towards a knowledge of God and self-realisation.
 a. P Q S R b. R Q S P c. R Q P S d. P R S Q
27. S_1 : The word 'victory' generally reminds us of wars and conquerors like Alexander and Napoleon.
 S_6 : The victories of peace are more lasting and useful to humanity than the victories of war.
 P : Thousands of people are killed and hundreds of towns and cities are destroyed.
 Q : But the victory associated with a war comes after so much of destruction.
 R : In this category, we may include the achievements of scientists and scholars, statesmen and social reformers and the like.
 S : However, there is another type of victory which is creative and constructive.
 a. P R Q S b. S P Q R c. Q P S R d. R P Q S
28. S_1 : The Arabs who are not in cities live in the desert all the year round.
 S_6 : An Arab loves his horse almost as much as he loves his wife and children.
 P : They have the finest horses in the world.
 Q : So they can easily move from one place to another.
 R : These desert Arabs eat ripe, sweet figs and also the dates that grow upon the palm trees.
 S : They live in tents that can be put up and taken down easily.
 a. R S P Q b. S Q R P
 c. P Q R S d. Q R S P

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1. 1. Today we live in modern technology era. [2014]
P : We have a lot of problems now.
Q : We want to get everything in one day.
R : Ancient time was quite pleasant.
S : We had no problems then.
6. Perhaps greed is the main cause for this.
a. P Q R S b. P R S Q c. S R Q P d. R P Q S
2. 1. He is a common man. [2014]
P : Yesterday our city saw a brutal crime.
Q : Police is trying to arrest innocent persons.
R : The criminals are well known.
S : Police as well as whole system in corrupt.
6. Police will arrest him as he is an easy target because of being a common man.
a. P R S Q b. P Q S R
c. P Q R S d. P S Q R
3. 1. He is a famous doctor. [2013]
P : Once I had to consult with him.
Q : I never believed him.
R : He suggested me a proper remedy.
S : I become completely fine.
6. Now I also admit this fact.
a. P Q R S b. Q P S R c. Q P R S d. R Q S P
4. 1. We don't know the plan of Ram. [2013]
P : He cares for his friends.
Q : He is a complete person.
R : We want some help and advice.
S : As we are in a trouble.
6. We hope he will do his best for us.
a. P R S Q b. Q P R S
c. P Q R S d. P S R Q
5. 1. It is not my problem. [2013]
P : All residents of this society are careless.
Q : I am unable to convince anyone.
R : They don't want to do some good.
S : Every one seems to be unwise here,
6. We all have to suffer one day.
a. P R S Q b. P R Q S
c. P Q R S d. P S R Q
6. 1. Early to bed, early to rise, makes a man healthy, wealthy and wise. [2012]
P : But for the morning tea, I had to wait for someone to get up before me.
Q : This saying inspired me to rise early
R : That day I was the first to get up
S : One day I got up early in the morning
6. Then I realised that it was a waste of time to get up early and wait for the morning tea.
a. Q S R P b. Q P R S
c. P Q R S d. S P Q R
7. 1. A wood-cutter was cutting a tree on a river bank. [2012]
P : He knelt down and prayed.
Q : His axe slipped and fell into the water.
R : God Mercury appeared before him and asked about the matter.
S : He could not get it back as the river was very deep.
6. He dived into the water and came up with an axe of wood.
a. R P Q S b. R P S Q c. Q S R P d. Q S P R
8. 1. A dog stole a piece of meat from a butcher's shop. [2012]
P : He barked in anger.
Q : He ran to the jungle with the piece of meat.
R : He saw his reflection.
S : He crossed a river on the way.
6. He lost his piece of meat.
a. Q P S R b. Q S R P
c. Q P R S d. S R P Q
9. 1. Making ourselves [2011]
P : our language
Q : part of growing into
R : masters of
S : is an important
6. full manhood or womanhood
a. P S R Q b. S Q P R
c. R P S Q d. P R S Q
10. 1. The very first battle they fought [2011]
P : and they had to fall back
Q : cross the border
R : was lost
S : letting the enemy
6. an enter the country
a. R Q S P b. R P S Q
c. Q R P S d. Q P R S
11. 1. A nation [2011]
P : the material assets it possesses
Q : is not made by
R : and collective determination
S : but by the will
6. of the people
a. P Q R S b. Q P S R
c. R S P Q d. S R P Q
12. 1. When the Governor [2011]
P : the bell had rung
Q : justice should be immediately
R : he ordered that
S : found out why
6. done to the horse
a. R S P Q b. P Q S R
c. S P R Q d. S Q R P

13. 1. When you ponder over [2011]
 P : that the only hope
 Q : you will realise
 R : of world peace lies
 S : the question deeply
 6. in the United Nations
 a. Q R S P b. S P Q R c. S Q P R d. R S P Q

Directions (Q. Nos. 14 to 18) *Rearrange the sentences labelled A to E to form a coherent paragraph and then answer the questions.*

- A. The large crowd, these myriad activities and the bright lights combine to transform the beach into a photographers' delight.
 B. Sand and spray are all pervasive on a beach.
 C. They love to construct fragile sand castles and splash about in the shallow waters.
 D. Even the elderly seem to forget their ageing worries here.
 E. These make beaches a favourite haunt of children.
14. Which sentence should come First in the paragraph?
 a. A b. B
 c. C d. D [2010]
15. Which sentence should come Second in the paragraph? [2010]
 a. D b. E
 c. A d. B
16. Which sentence should come Last in the paragraph?
 a. D b. A
 c. E d. C [2010]
17. Which sentence should come Third in the paragraph?
 a. C b. D
 c. E d. A [2010]
18. Freedom, is the restricted kind in the sense/(P), the rich and poor woman/(Q), that a wide gulf separates/(R), which a modern woman enjoys/(S). [2009]
 a. P S R Q b. S R Q P
 c. R Q P S d. S P R Q

19. In life,
 (P) some rules are
 (Q) as in business
 (R) they seem almost instinctive
 (S) learnt so early that
 a. R S P Q b. Q P S R [2009]
 c. R P S Q d. Q S P R
20. The Bible, [2008]
 (P) has in many respects
 (Q) the sacred books of all Christians
 (R) among all the books of the world
 (S) a unique character and position
 a. Q P S R b. Q R P S c. R P Q S d. R Q P S
21. The ultimate hope [2008]
 (P) will force the nations
 (Q) that the destructive nature of weapons
 (R) to give up war
 (S) has not been fulfilled
 a. P Q R S b. P R Q S c. Q P R S d. R S Q P
22. It was [2008]
 (P) in keeping with my mood
 (Q) a soft summer evening
 (R) as I walked sedately
 (S) in the direction of the new house
 a. Q P R S b. Q R P S c. S Q P R d. S R P Q
23. (P) I decided to call on him [2005]
 (Q) at the earliest opportunity
 (R) having heard of the palmist
 (S) before I came into town
 a. R S P Q b. P Q R S c. S Q P R d. Q P R S
24. (P) When a chemical substance [2005]
 (Q) the food poisoning occurred
 (R) in the food preparations
 (S) was mistaken for salt and used
 a. R Q P S b. S R Q P
 c. Q P S R d. P S R Q

Answers

Practice Exercise

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (d) | 2. (a) | 3. (d) | 4. (b) | 5. (d) | 6. (a) | 7. (c) | 8. (b) | 9. (c) | 10. (c) |
| 11. (d) | 12. (c) | 13. (d) | 14. (d) | 15. (b) | 16. (b) | 17. (b) | 18. (b) | 19. (a) | 20. (d) |
| 21. (c) | 22. (c) | 23. (d) | 24. (c) | 25. (c) | 26. (c) | 27. (c) | 28. (b) | | |

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- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (b) | 2. (a) | 3. (c) | 4. (b) | 5. (a) | 6. (a) | 7. (d) | 8. (b) | 9. (c) | 10. (b) |
| 11. (b) | 12. (c) | 13. (c) | 14. (a) | 15. (d) | 16. (a) | 17. (c) | 18. (d) | 19. (b) | 20. (a) |
| 21. (c) | 22. (a) | 23. (a) | 24. (c) | | | | | | |

5

Fill in the Blanks

Question based on Fill in the blanks or Sentence Completion are designed to test the comprehension ability as well as the knowledge of the meaning of the words required to be filled in. The candidate is asked to select the best of the alternatives given below a sentence.

A word of caution may be put in here for the benefit of the students. Before attempting this section they must go through the plethora of words to guide them in choice of selecting the best of the alternatives.

These questions are usually of two types.

In the **first type** of questions, a sentence is given with a blank space. Below the sentence, four alternatives are suggested for the blank space. You are required to select the alternative which makes the sentence meaningfully complete.

In the **second type** of questions, a sentence is given with two blank spaces. Below the sentence, four pairs of words are suggested for the blank spaces. You are required to select the alternative which has the correct pair of words required to be filled in.

Some Useful Tips

1. First read the sentence and try to find out its meaning.
2. Go through all the given alternatives before selecting the answer.
3. Select the word which makes the sentence meaningful and grammatically correct.
4. Try to find out which part of speech is required to fill in the blank. It will make your choice very easy.
5. Sometimes more than one option may seem to be correct. In such case, choose your answer in the context of the sentence. The answer must provide a proper sense and must belong to the required grammatical class or part of speech.
6. In case of double blanks, be careful not to choose the alternative which has only one correct word. Make sure that the alternative must have the words that can fill both the blank spaces appropriately.
7. Eliminating the wrong options will also help you in selecting the right answer.

Practice Exercise

1. The pilot was injured; he died within half an hour.
a. seriously b. fatally
c. fatefully d. vitally
2. His..... directions misled us; we did not know which road to take.
a. complex b. obscure
c. mingled d. vague
3. He was very friendly with the press and it really him even for what he did not achieve.
a. praised b. lionised
c. appreciated d. highlighted
4. These essays are intellectually and represent various levels of complexity.
a. revealing b. modern
c. superior d. demanding
5. The soldiers were instructed to ... restraint and handle the situation peacefully.
a. exercise b. control
c. prevent d. enforce
6. Since, one cannot read every book, one should be content with making a selection.
a. normal b. standard
c. moderate d. judicious
7. There has been a lack of efficiency in all the crucial areas of the working of Public Sector Undertakings.
a. positive b. surprising
c. conspicuous d. stimulative
8. The Managing Director treated the employees to a lunch at an expensive hotel.
a. precious b. thriving
c. stupendous d. sumptuous
9. Satish was with a natural talent for music.
a. given b. found
c. endowed d. entrusted
10. She always insisted on the need to between ends and means.
a. analyse b. define
c. distribute d. distinguish
11. If greater security measures had been taken the tragedy might have been
a. removed
b. repeated
c. restrained
d. averted
12. A politician's speech must be read the lines, it may have a double
a. behind; edge b. between; meaning
c. among; ends d. beyond; face
13. Those who live in houses should not throw at others.
a. big, abuse b. own, challenges
c. stone, bricks d. glass, stones
14. I write a letter to you tentatively the dates of the programme.
a. involving b. urging
c. guiding d. indicating
15. Contemporary economic development differs..... from the Industrial Revolution of the 19th century.
a. naturally b. usually
c. literally d. markedly
16. Ravi had to drop his plan of going to picnic as he had certain to meet during that period.
a. preparations b. observations
c. urgencies d. commitments
17. It was hot that day and the cable suffered the brunt of the heat.
a. treacherously
b. acceptably
c. unfailingly
d. unbelievably
18. eye-witness, the news reporter gave a graphic description of how fire broke out.
a. Reporting b. Observing
c. Seeing d. Quoting
19. He gave me an additional of ₹ 100 a month by of the commissioner.
a. allowance; permission
b. pay; allowance
c. payment; agreement
d. salary; admittance
20. There is no glory in war the blood it....
a. considering; sheds b. comparing; spills
c. worth; costs d. thinking; demands
21. The Director has me to a speech for him.
a. notified; do
b. demanded; prepare
c. entrusted; write
d. commissioned; draft
22. I will come tomorrow I hear to the
a. unless; contrary
b. until; contradictory
c. except; oppose
d. despite; otherwise
23. The children are so , their parents are thinking of sending them to a school.
a. small; high
b. naughty; neighbouring
c. misruled; lodging
d. unruly; boarding

24. Only people are after death.
 a. vicious; condemned
 b. virile; forgotten
 c. virtuous; remembered
 d. virulent; glorified
25. The Secretary the society's funds, he was dismissed.
 a. robbed, thus
 b. pirated, therefore
 c. misplaced, and
 d. embezzelled, therefore
26. His life consists of..... of drinking punctuated by periods of drunken sleep.
 a. barrels
 b. bouts
 c. bowls
 d. pints
27. He lost the match easily because he had played a five set match in the earlier round.
 a. sensational
 b. gruelling
 c. wonderful
 d. controversial

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1. Our company signed a profitable last month. [2014]
a. issue b. agenda
c. deal d. paper
2. When Indians from the South move North, they find certain aspects of life quite from their own. [2012]
a. strange b. separate
c. different d. divergent
3. The sky is overcast, we the storm will soon burst. [2012]
a. expect b. hope
c. trust d. suspect
4. We must our students on subjects like health and sanitation besides the usual subjects. [2010]
a. learn b. teach
c. insist d. educate
5. The judge used his power and let him off with a reprimand. [2007]
a. residuary b. official
c. legal d. discretionary
6. It is difficult to believe what he tells us because his account of any event is always full of and of all sorts. [2009]
a. discrepancies b. differences
c. discretions d. distinctions
7. The bank clerk tried to money from his friend's account. [2009]
a. empower b. embellish
c. embroil d. embezzle
8. Eight scientists have the national awards for outstanding contribution and dedication to the profession. [2008]
a. bestowed b. picked
c. bagged d. conferred
9. In a little published deal, Pepsi Cola has ... the entire soft drink market in Afghanistan. [2008]
a. conquered b. swallowed
c. captured d. occupied
10. Mr Shyam Lal has gone to his native village with the of starting an adult school. [2007]
a. suggestion b. presumption
c. opinion d. intention
11. The twins are so alike that I cannot one from the other.
a. discern b. tell
c. say d. notice
12. We must to authority.
a. bend b. surrender
c. subdue d. submit

Answers

Practice Exercise

1. (b) 2. (d) 3. (b) 4. (a) 5. (a) 6. (d) 7. (c) 8. (d) 9. (c) 10. (d)
11. (d) 12. (b) 13. (d) 14. (d) 15. (d) 16. (d) 17. (d) 18. (d) 19. (a) 20. (a)
21. (c) 22. (a) 23. (d) 24. (c) 25. (d) 26. (b) 27. (b)

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1. (c) 2. (c) 3. (a) 4. (d) 5. (d) 6. (a) 7. (d) 8. (c) 9. (c) 10. (d)
11. (b) 12. (d)

**LOGICAL
REASONING**

1

VERBAL

Analogy

Analogy means similarity or correspondence i.e. having similar features. The questions based on analogy, a particular relationship is given and another similar relationship has to be identified from the alternatives provided. Questions based on analogy are set-up to test a candidate's overall knowledge, power of reasoning and ability to think.

These types of questions cover every types of relationships that one can think. There are many ways of establishing a relationship like quantity and unit, worker and tools, cause and effect, word-synonym, word-antonym, country and capital, state and capital, country and currency, animal and the young ones (kid), male and female, animals and their resting places, games and places of playing, occupation their working place and their work.

Here, some relationships are given, which are useful for solving questions based on analogy

Country	Capital	Currency
Bangladesh	Dhaka	Taka
Iraq	Baghdad	Dinar
China	Beijing	Yuan
India	New Delhi	Rupee
UK	London	Pound
Japan	Tokyo	Yen
USA	Washington DC	Dollar
Russia	Moscow	Rouble
Greece	Athens	Euro

Occupation	Working Place	Working
Lawyer	Court	Legal Practice
Servant	House	Service
Beautician	Parlour	Make up
Mechanic	Garage	Repairing
Waiter	Restaurant	Serving
Teacher	School	Teaching
Chef	Kitchen	Cooking
Sailor	Ship	Sailing

States of India	Capital
Sikkim	Gangtok
West Bengal	Kolkata
Goa	Panaji
Tamil Nadu	Chennai
Odisha	Bhubaneswar
Bihar	Patna

Male	Female
Dog	Bitch
Horse	Mare
Bull	Cow
Cock	Hen
Son	Daughter
Brother	Sister

Practice Exercise

1. Major is related Lieutenant in the same way as Squadron Leader is related to?.....
 a. Group Captain b. Flying Attendant
 c. Flying Officer d. Pilot Officer
 e. Driver
2. Kilogram is related to Quintal in the same way as Paisa is related to ?
 a. Rupee b. Coin
 c. Wealth d. Money
 e. Dollar
3. Roentgen is related to X-rays in the same way as Becquerel is related to ?
 a. Uranium b. Radioactivity
 c. Fission d. Superconductivity
 e. None of these
4. Jade is related to Green in the same way as Garnet is related to?.....
 a. Blue b. Orange
 c. Red d. Yellow
 e. White
5. Victory is related to Happiness in the same way as Failure is related to?.....
 a. Defeat b. Anger
 c. Frustration d. Sandness
6. Food is to Fad as Religion is to?.....
 a. Crucification b. Notion
 c. Superstition d. Mythology
 e. Psychology
7. Glutton is related to Eat in the same way as Garrulous is related to?.....
 a. Talk b. Walk
 c. Laugh d. Travel
 e. Run
8. Vitro is related to Glass in the same way as Ligno is related to?.....
 a. Marble b. Metal
 c. Rock d. Wood
 e. Gold
9. Buffalo is to Leather as Llama is to?.....
 a. Wool b. Meat
 c. Silk d. Fur
 e. Linen
10. Cell is related to Tissue in the same way as Tissue is related to?.....
 a. Object b. Organ
 c. Limb d. None of these

11. Prairies is to North America as Downs is to?.....
 a. Europe b. Australia
 c. Africa d. India
 e. Nepal

Directions (Q. Nos. 12-14) *In each of the following questions, there is certain relationship between two given words on one side of (::) and one word is given on another side of (::) while another word is to be selected from the given alternatives having the same relationship with the word, as the words of the given pair bear. Choose the correct alternative.*

12. Cougar : South America :: Okapi : ?
 a. India b. Central Africa
 c. North America d. Pakistan
 e. Australia
13. Bow : Arrow :: Pistol : ?
 a. Bullet b. Gun
 c. Shoot d. Rifle
 e. Revolver
14. Cobbler : Leather :: Carpenter : ?
 a. Furniture b. Wood
 c. Hammer d. Chair
 e. Wool

Directions (Q. Nos.15-19) *The following questions consists of two words each that have a certain relationship to each other, followed by four lettered pairs of words. Select the lettered pair that has the same relationship as the original pair of words.*

15. War : Destruction
 a. Fire : Burn b. Court : Justice
 c. Water : Drown d. Food : Hunger
16. Banyan tree : Proproots
 a. Potato : Tuber b. Climbers : Tendrils
 c. Ginger : Stem d. Spinach : Root
17. Vaseline : Petrol
 a. Tea : Leaves b. Plant : Tree
 c. Butter : Ghee d. Cream : Milk
18. Preamble : Constitution
 a. Word : Dictionary b. Contents : Magazine
 c. Explanation : Poetry d. Preface : Book
19. Garbage : Squalor
 a. Poor : Hunger
 b. Diamond : Magmticence
 c. Colour : Brush
 d. Dirtiness : Cleanliness

Directions (Q. Nos. 31-42) Find the related words analogy from the given alternatives.

31. Godan : Premchand : : Guide : ?
a. RK Narayan
b. KR Narayan
c. Premchand
d. MK Gandhi
32. WHO : Geneva : : IMF : ?
a. New York
b. Paris
c. Geneva
d. Washington DC
33. Distance : Scalar quantity : : Velocity : ?
a. Scalar quantity
b. Vector quantity
c. Speed $\times$ Time
d. Graph
34. Bihu : Asom : : Garba : ?
a. Uttar Pradesh
b. Rajasthan
c. Punjab
d. Gujarat

35. National tree : Banyan :: National song : ?

- Jan Gan Man
- Vande Matram
- Inqulab Zindabad
- Sare Jaha Se Acha

36. Raj Ghat : Mahatma Gandhi :: Shanti Van : ?
a. Indira Gandhi
b. Subhash Chandra Bose
c. Rajiv Gandhi
d. Jawahar Lal Nehru
37. Japan : Yen :: UK : ?
a. Rial b. Dollar c. Pound d. Yuan
38. Bihar : Patna :: Nagaland : ?
a. Kohima b. Gangtok
c. Lucknow d. Chandigarh
39. Red : Violence :: White : ?
a. Lotus b. Peace
c. Colour d. Cleanliness
40. Wish : Desire :: Refuse : ?
a. Accept b. Rebuff
c. Abuse d. Deny
41. Botany : Plants :: Entomology : ?
a. Snakes b. Germs
c. Birds d. Insects
42. virology : virus :: Semantics : ?
a. Compilers b. Bacteria
c. Greeks d. Language

Answer with Solutions

1. (c) Major and Squadron Leader are equivalent ranks in Army and Air Force respectively and so are Lieutenant and Flying Officer.
2. (a) Second is bigger unit than the first, though both are used to measure the same quantity, 100 kilograms = 1 quintal and 100 paise = 1 rupee.
3. (b) Roentgen discovered X-rays. Similarly, Becquerel discovered radioactivity.
4. (c) Jade is a green precious stone and garnet is a red precious stone.
5. (c) Second is the result of the first.
6. (c) Second is the name given to wrong notions about the first.
7. (a) Glutton is one who eats too much. Garrulous is one who talks too much.
8. (d) Vitro means 'related to glass' and Ligno means 'related to wood'.
9. (a) Second is a product obtained from the first.
10. (b) Tissue is made up of cell and organ is made up of Tissue.
11. (b) The grasslands of North America are known as Prairies and those of Australia are called Downs.
12. (b) As, Cougar is found in South America. Similarly, Okapi is found in Central Africa.
13. (a) As, Arrow is released from Bow. Similarly, Bullet is released from Pistol.
14. (b) As, Cobbler uses Leather to make shoes. Similarly, Carpenter uses Wood to make furniture.
15. (a) First causes the second.
16. (b) Second provides support to the first.
17. (d) First is extracted from the second.
18. (d) Preamble is the introduction to the Constitution. Similarly, preface is the introduction to a book.
19. (b) Second is the property of first.
20. (d) Nurse receives instructions from the doctor and follower receives the instructions from leader.
21. (c) The fire Burns due to presence of Oxygen and the fire Extinguishes due to Carbon dioxide.
22. (d) Pollution is the result of Smoke. Similarly, Destruction is the result of War.
23. (b) As, Tailor makes Dress. Similarly, Carpenter makes Furniture.
24. (c) Scalpel is used by surgeon during operation, in the same way, sculptor uses his chisel for crating sculptures.
25. (a) The Hoofs of an animal such as a Horse are the hard lower parts of its feet. In the same way, Foothills are the lower parts of Mountains.
26. (b) Sailor is judge the direction through Compass same as Doctor is judge the breathing and heart beats through Stethoscope.
27. (b) Cytology is the science of study Cells. In the same way, Entomology is the science of study of Insects.
28. (b) As, Chair is made of Wood. Similarly, Mirror is made of Glass.
29. (c) As, Nuts are covered with Bolts. Similarly, naked is covered with cloths.
30. (c) Book is written by Author. Similarly, Symphony is composed by composer.
31. (a) Godan is written by Premchand. In the same manner, Guide is the book written by RK narayan.
32. (d) The headquarter of WHO is in Geneva, Similarly, the headquarter of IMF is in Washington DC.
33. (b) Distance is a scalar quantity as it has no direction. In the same way, velocity is a vector quantity as it has unit as well as direction both.
34. (d) Bihu is the dance from of Asom. In the same way, dance from of Gujarat is Garba.
35. (b) National tree of India is Banyan. In the same way, National Song of India is Vande Matram.
36. (d) Raj Ghat belongs to Mahatma Gandhi. In the same way, Shanti van belongs to Jawahar Lal Nehru.
37. (c) Yen is the currency of Japan. In the same way, Pound is the currency of UK.
38. (a) Patna is the capital of Bihar. In the same way, Kohima is the capital of Nagaland.
39. (b) Red is the symbol of violence. In the same way, white is the symbol of peace.
40. (d) Wish is the higher intensity of desire. In the same way, Refuse is the higher intensity of deny.
41. (d) Botany is the study of plants. In the same way, Entomology is the study of Insects.
42. (d) Virology is the scientific study of virus. In the same way, semantics is the study of meanings of words and phrasemes in a language.

2

Classification

Classification means 'to assort the items' of a given group on the basis of a certain common quality they possess and then spot the stranger or 'odd one out'.

These questions are based on words, letters and numerals. In these types of problems, we consider the defining quality of particular things. In these questions, four or five elements are given, out of which one does not belong to the group. You are required to find the 'odd one'.

Classification problems are divided into the following types

1. Number Classification

In number classification, there is a similarity among the numbers or groups of numbers.

The number classification is generally based on similarities like

- Square and square root of a number
- Cube and cube root of a number
- Even and odd number
- Prime number
- Divisibility test of a number etc.

2. Alphabet Classification

In alphabet classification, there is a similarity or pattern among the alphabets in the group and the candidate has to identify the pattern and select the one which does not follow the common pattern in the group.

The relationship among the alphabets is based on the following

- Position of alphabets
- Vowel and consonant
- Reverse order of alphabets
- Addition or multiplication of position etc.

3. Word Classification

In word classification, the candidate is given a group of words which are related to one another in a particular manner. The candidate has to identify the odd word that does not belong to the group.

The relationship among the word classification is generally based on the following

- Synonym of word
- Antonym of word etc.

Practice Exercise

Directions (Q. Nos. 1-8) *In each of the following questions some groups of letters are given, all of which, except one, share a common feature while one is different. Choose the odd one out.*

- | | |
|------------|---------|
| 1. a. BEH | b. CFI |
| c. DGJ | d. EHL |
| e. FIL | |
| 2. a. AOT | b. CPA |
| c. REB | d. TIW |
| e. QUD | |
| 3. a. BDW | b. DFU |
| c. FHS | d. GIQ |
| e. IKP | |
| 4. a. RSXY | b. NOUV |
| c. MNST | d. DEJK |
| 5. a. ACZX | b. BDYW |
| c. EGVF | d. GTR |
| e. CEUS | |
| 6. a. BCYZ | b. EFVW |
| c. HIUV | d. JKQR |
| e. ABZA | |
| 7. a. APoQ | b. DxeM |
| c. SFiK | d. oWjB |
| e. CQuL | |
| 8. a. xXYA | b. iIMP |
| c. hHIR | d. DBCE |

[2005]

Directions (Q. Nos. 9-19) *In the following questions, three / four out of the four / five alternatives are same in a certain way and so form a group. Find the odd one that does not belong to the group.*

- | | |
|-----------------|----------------|
| 9. a. Yen | b. Lira |
| c. Dollar | d. Ounce |
| 10. a. Litchi | b. Papaya |
| c. Jackfruit | d. Guava |
| 11. a. Peak | b. Mountain |
| c. Valley | d. Hillock |
| e. Mound | |
| 12. a. Jumping | b. Running |
| c. Sprinting | d. Exercising |
| e. Jogging | |
| 13. a. Mother | b. Friend |
| c. Brother | d. Sister |
| e. Father | |
| 14. a. Geometry | b. Algebra |
| c. Trigonometry | d. Mathematics |
| e. Arithmetic | |

[2006]

- | | |
|----------------|-------------|
| 15. a. Poland | b. Greece |
| c. Spain | d. Italy |
| e. Korea | |
| 16. a. Calf | b. Cub |
| c. Piglet | d. Duckling |
| e. Hireling | |
| 17. a. Sketch | b. Diagram |
| c. Poster | d. Chart |
| e. Paper | |
| 18. a. Metre | b. Furlong |
| c. Yard | d. Mile |
| e. Acre | |
| 19. a. Cheetah | b. Lion |
| c. Bear | d. Tiger |

Directions (Q. Nos. 20-25) *In each of the following questions, five pairs of words are given, out of which the words in four pairs bear a certain common relationship. Choose the pair in which the words are differently related.*

- | | |
|---------------------------|-----------------------|
| 20. a. Shoe : Leather | b. Iron : Axe |
| c. Table : Wood | d. Jewellery : Gold |
| e. Shirt : fabric | |
| 21. a. Mason : Wall | b. Cobbler : Shoe |
| c. Farmer : Crop | d. Chef : Cook |
| e. Choreographer : Ballet | |
| 22. a. Daring : Timid | b. Beautiful : Pretty |
| c. Clear : Vague | d. Youth : Adult |
| e. Native : Alien | |
| 23. a. See : Eye | b. Hear : Ears |
| c. Smell : Nose | d. Touch : Skin |
| e. Tongue : Taste | |
| 24. a. Bottle : Wine | b. Cup : Tea |
| c. Pitcher : Water | d. Ball : Bat |
| e. Inkpot : Ink | |
| 25. a. Stallion : Colt | b. Horse : Mare |
| c. Dog : Bitch | d. Drake : Duck |
| e. Drone : Bee | |

Directions (Q. Nos. 26-30) *In each of the following questions, five numbers are given. Out of these, four are alike in a certain way but the rest one is different. Choose the one which is different from the rest four.*

- | | | | |
|-----------|-------|-------|-------|
| 26. a. 17 | b. 27 | c. 29 | d. 37 |
| e. 59 | | | |
| 27. a. 6 | b. 12 | c. 18 | d. 9 |
| e. 7 | | | |
| 28. a. 15 | b. 21 | c. 24 | d. 28 |
| e. 30 | | | |

29. a. 43 b. 53
 c. 63 d. 73
 e. 83
30. a. 488 b. 929 [2007]
 c. 776 d. 667
 e. 596

Directions (Q. Nos. 31-35) *Choose the odd numeral pair group in each of the following questions.*

31. a. 34 – 43 b. 55 – 62
 c. 62 – 71 d. 83 – 92

32. a. 2 – 8 b. 3 – 27
 c. 4 – 32 d. 5 – 125
33. a. 13 – 21 b. 19 – 27
 c. 15 – 23 d. 16 – 24
34. a. 2 : 4 b. 4 : 8
 c. 6 : 18 d. 8 : 32
35. a. 3 : 12
 b. 4 : 20
 c. 6 : 42
 d. 7 : 63

Answer with Solutions

- (d) In all the groups, there is a gap of two letters between first and second as well as between second and third letters.
- (b) In all other groups, the middle letter is a vowel.
- (d) In all other groups, the first two letters are alternate letters of the alphabet, while second and third letters occupy the same positions from the beginning and the end of the alphabet respectively.
- (b) In all other groups, the first second and third letters are respectively moved one, five and one step forward to obtain second, third and fourth letters respectively.
- (e) In all other groups, first and second as well as fourth and third letters are alternate. Also, the first and third letters occupy the same positions from the beginning and end of the alphabet, respectively.
- (c) In all other groups, first and second as well as third and fourth letters are consecutive and the third letter occupies the same position from Z backward as the first occupies from A onward.
- (d) In all other groups, the third letter is small and a vowel.
- (d) Except (d), all other groups contains only small letter.
- (d) Except Ounce, others are currencies of different countries.
- (c) Jackfruit is a vegetable while others are fruits.
- (c) Except Valley, all other are the elevated features.
- (d) Except Exercising, all others are different forms of exercising.
- (b) Except Friend, all others are denote blood relations.
- (d) All except Mathematics are branches of Mathematics.
- (e) All except Korea are European countries, while Korea is an Asian country.
- (e) All except Hireling are young ones of animals.
- (e) All others can be drawn on paper.
- (e) All except Acre, are units of measuring distance, while acre is a unit of area.
- (c) All except Bear, belong to the cat family.
- (b) In other pairs, first is a product made from the second.
- (d) In other pairs, second is prepared by the first.
- (b) In all other pairs, the two words are antonyms of each other.
- (e) In other pairs, first denotes the function performed by the second.
- (d) In other pairs, first is used to hold the second.
- (a) In other pairs, second is the female of the first.
- (b) Each of the numbers except 27, is a prime number.
- (e) 7 is the only prime number in the group.
- (d) Each of the numbers except 28, is divisible by 3.
- (c) Each of the numbers except 63, is a prime number.
- (d) Except 667, sum of digits is 20.
- (b) In all other pairs, second number is 9 more than the first.
- (c) In all other pairs, second number is the cube of the first.
- (d) All other pairs consist of odd numbers only.
- (a) In all other pairs, $\frac{(1\text{st number})^2}{2} = 2\text{nd number}$.
- (d) In all other pairs, $(1\text{st number}) \times (1\text{st number} + 1) = 2\text{nd number}$.

3

Series Completion

Series is a sequential order of letters, numbers or both arranged in such a way that each term in the series is obtained according to some specific rules. These rules can be based on mathematical operations, place of letters in alphabetical order etc.

In questions on series, a specified sequence/order of letters, numbers or a combination of both is given where one of the term (letter/number/letter and number) of the series is missing either at the end of the series or in between the series.

The candidate is required to identify the pattern involved in the formation of series and accordingly find the missing term to complete the series. Also, there may be some questions where one of the term in the series is incorrect and the candidate is required to find out that term of the series by identifying the pattern involved in the formation of series.

There is no set pattern and each question may follow a different pattern or sequential arrangement of letters or digits, which you have to detect using your common sense and reasoning ability.

Types of series covered in this chapter are as follows

1. Number Series

In number series, the numbers are arranged in a particular pattern based on some mathematical or logical operations.

The pattern of number series is based on following

- Square and square roots of a number
- Cube and cube roots of a number
- Arithmetic operations (like addition, subtraction, division, multiplication)
- Prime number
- Even and odd numbers
- A combination of above relations

The questions are asked in two formats

1. Find the missing term
2. Find the wrong term

Basic Approach to Number series

The best way/method of approaching the number series question is as follows.

- (i) Firstly, analyse the difference between terms.
- (ii) If the difference is constant, it is a constant difference series.
- (iii) If the difference is increasing or decreasing fastly, then try the square series approach.
- (iv) If the increase is very much and it is not a square series, then try the cube series approach.
- (v) If there is no constant increasing or decreasing difference, then try product series approach.
- (vi) If the number obtained are same, then it is a product series.

2. Alphabet Series

In alphabet series, letters of english alphabet are arranged in a particular pattern like reverse order of letters, position of letters, skipping of letters etc.

The following problem can be solved by remembering the position of the letters in the alphabetical order from forward and backward direction.

Forward	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
Letters	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Backward	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

3. Alpha-numeric Series

In alpha-numeric series, the pattern of series is based on the combination of both numbers and letters.

These kind of problems use both mathematical operation and position of letters in the alphabet in both forward, backward or reverse order.

4. Continuous Pattern Series

Continuous pattern series consist of a series of small letters that follow a certain pattern like repetition of letters.

Method to Solve Continuous Series

- Firstly, count the number of blanks and given letters.
- Divide the whole sum of blanks and letters by a multiple.
- Note down the pattern common to all groups separately.

Practice Exercise

Directions (Q. Nos. 1-7) *In each of the following questions, various terms of alphabet series are given with one or more terms missing as shown by (?). Choose the missing terms out of the given alternatives.*

1. A, C, F, J, ?
a. K b. O c. M d. N
e. L
2. A, Z, D, Y, G, X, ?, ?
a. W, J b. J, W c. W, K d. K, W
3. NZ, OY, PX, QW, RV, ?
a. FS b. SU c. UF d. TU
e. SW
4. AZY, EXW, IVU, ?
a. MTS b. MQR c. NRQ d. LST
e. JWV
5. DKM, FJP, HIS, JHV, ?
a. LGY b. HGY
c. IGZ d. IGY
6. bc, cde, de, efg, fg, ?
a. ghi b. fgh
c. hij d. ijk
e. gh
7. CFL, EIK, GLJ, IOI, ?
a. KRH b. KRJ
c. JRU d. KOV

Directions (Q. Nos. 8-12) *These questions are based on the letter series. In each of these series, some letters are missing. Select the correct alternative.*

8. ab_d__b_dm__x__
a. xmaxabda b. xmaxabdm
c. mxaxabdm d. mxaxbadm
9. mc_m_a_ca_ca_c_mc
a. acmma b. camcam
c. aaacmm d. acmmc
10. b_ab_b_aab_b
a. abbb b. abba
c. baaa d. aaba
11. _bcab_cabc_abca_b
a. abca b. aabc
c. bbca d. abac
12. __r_ttp__s__tp__s__
a. pqrssqtpqtrp b. pqstrsqptrst
c. pqsqrqrtpq d. None of these

Directions (Q. Nos. 13-19) *In each of the following questions, number series is given with one of the terms missing. Choose the correct alternative that will continue the same pattern and replace the questions (?) in the given series.*

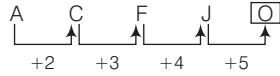
13. 21, 23, 27, 33, ?
a. 37 b. 41 c. 43 d. 49
14. 68, 67, 64, 59, ?
a. 52 b. 53 c. 54 d. 56
15. 1, 8, 27, ?, 125, 216 [2014]
a. 52 b. 58
c. 64 d. 65
16. 4, 7, 13, ?, 34, 49, ? [2010]
a. 20 b. 22
c. 24 d. 21
17. 5, 11, 24, ?, 106, 217, ? [2010]
a. 50 b. 50 c. 49 d. 51
18. 24, 6, 18, 9, 36, 9, 24, ? [2013]
a. 24 b. 12
c. 8 d. 6
e. 26
19. 83, 82, 81,, 69, 60, 33 [2007]
a. 73 b. 80
c. 77 d. None of these

Directions (Q. Nos. 20-24) *In each of the following questions, one of the terms in the number series is wrong. Find out the wrong term.*

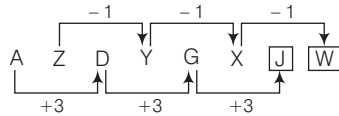
20. 160, 118, 83, 65, 34, 20
a. 83 b. 118 c. 34 d. 65
e. 34
21. 1, 9, 25, 50, 81 [2012]
a. 1 b. 25
c. 50 d. 81
22. 5, 10, 17, 24, 37 [2011]
a. 10 b. 24
c. 17 d. 37
23. 1788, 892, 444, 220, 112, 52, 24 [2010]
a. 52 b. 112
c. 220 d. 444
24. 2, 3, 5, 8, 12, 18, 23
a. 13 b. 15 c. 16 d. 18
e. 23

Answer with Solutions

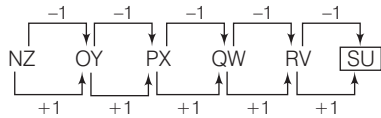
1. (b) The pattern of the series is as follows



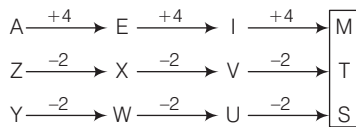
2. (b) The pattern of the series is as follows



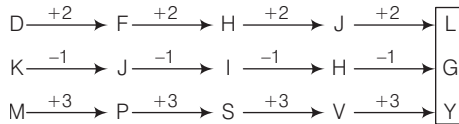
3. (b) The pattern of the series is as follows



4. (a) The pattern of the series is as follows



5. (a) The pattern of the series is as follows



6. (a) As, bc, cde, de, efg, fg, ghi

In this series, last letter of a term is started to new term and next two letter is included in the term.

7. (a)

8. (b) Here, repeated block is 'abxdm'.

Now, abxdm abxdm abxdm

9. (a) m c a / m c a / m c a / m c a / m c a / mc

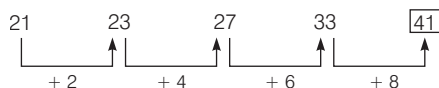
10. (d) ba/ab/ab/ba/ab/ab

11. (a) abc/abb/cab/cca/bca/ab

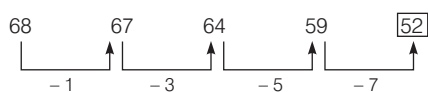
12. (c) The last letter of each previous block is moved to the beginning of the successive block.

Now, pqrst/tpqrs/stpq/rstopq

13. (b) The pattern of the series is as follows



14. (a) The pattern of the series is as follows



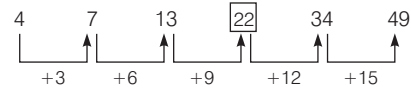
15. (c) From the given series,

$$1^3 \longrightarrow 1, \quad 2^3 \longrightarrow 8$$

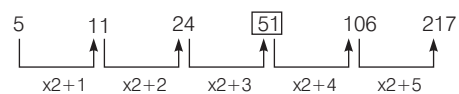
$$3^3 \longrightarrow 27, \quad 4^3 \longrightarrow 64$$

$$5^3 \longrightarrow 125, \quad 6^3 \longrightarrow 216$$

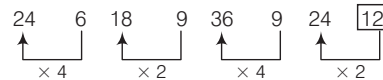
16. (b) The pattern of the series is as follows



17. (d) The pattern of the series is as follows



18. (b) The pattern of the series is



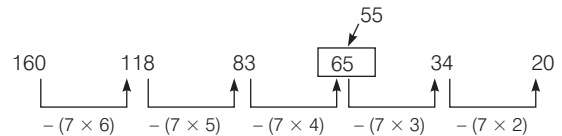
19. (c) Series is written in reverse order with a difference of

$$1^2, 1^3, 2^2, 2^3, 3^2, 3^3$$

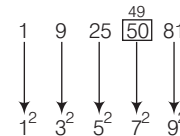
i.e.

$$1, 1, 4, 8, 9, 27$$

20. (d) The pattern of the series is as follows

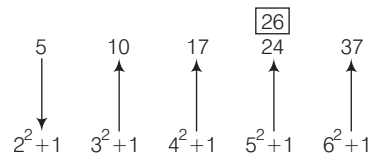


21. (c) The pattern of the series is



Hence, 50 is the wrong number.

22. (c) The pattern of the series is



Hence, 24 is the wrong number.

23. (b)

24. (d) The pattern of the series is as follows



4

Logical Deduction

It is a combination of an information based passage followed by some conclusions based on that, which are to be checked according to a given set of directions. This section deals with the questions in which unusual patterns of conclusions are inferred on the basis of a passage. Such conclusions are unusual as they are different from other type of conclusions which are based on the statements (as in case of syllogism). Infact, other type of conclusions are definite i.e. they are either definitely true or definitely false but here in the passage based conclusion, probability factor is also taken into consideration.

Directions (Examples 1-5) *In each question below is given a passage followed by several inferences. You have to examine each inference separately in the context of the passage and decide upon its degree of truth or falsity.*

Mark your answer as

- (a) If the inference is 'definitely true' i.e., it directly follows from the facts given in the passage;
- (b) If the inference is 'probably true' though not definitely true in the light of the facts given;
- (c) If you think the data are inadequate i.e., from the facts given you cannot say whether the inference is likely to be true or false;
- (d) If you think the inference is 'probably false' though not definitely false in the light of the facts given and;
- (e) If you think the inference is 'definitely false' i.e., it contradicts the given facts.

Passage

The railroad was not the first institution to impose regularity on society or to draw attention to the importance of precise time keeping. For as long as merchants have set out their wares at day break and communal festivities have been celebrated, people have been in rough agreement with their neighbours as to the time of day. The value of tradition is today more apparent than ever. Were it not for public acceptance of a single yardstick of time, social life would be unbearably chaotic. The massive daily transfer of goods. Services and information would proceed in fits and starts, the very fabric of modern society would begin to unravel.

1. An accepted way of measuring time is essential for the smooth functioning of society.
2. In modern society we must make more time for our neighbour's.
3. Society judges people by the time at which they conduct certain activities.

Sol. (Examples 1-3)

1. (a) Statement is a major objective in making the decision one of the goals sought by the decision maker.
2. (c) Statement is a minor factor in making the decision, a less important element bearing on or affecting a major factor rather than a major objective directly.
3. (c) This statement is minor factor in making the decision.

Practice Exercise

Passage 1

The smaller pesticide formulation units in India operate under heavy constraints such as obsolete technology, small scale of operation and dependence on large units for raw materials. In view of the loss of expensive material by the smaller units it is important to either eliminate or reduce losses to the extent possible through innovative and sustainable waste minimisation techniques. Operating profit margins of the units are very low and small adverse conditions land these companies in trouble. Maximum losses suffered by these units are through poor house keeping, sub-optimal operating practices, and lack of proper opportunities for recycling waste.

1. Smaller units should be operationally self-sufficient so as to minimise losses.
2. Recycling or wastes through modern techniques can set off large part of the losses incurred by the smaller units.
3. Pesticide units should necessarily be on a large scale to make them economically viable.
4. Waste management process in India needs modernisation.
5. Lack of funds compels smaller units to ignore house keeping.

Passage 2

The basic thrust of the Government's policy is to provide price incentives to farmers to make them produce more food. But is a price-incentive system always efficient in ensuring incremental yields ? Our contention is that this incentive works only in persuading farmers to shift cultivation from one crop to another depending on which crop is more profitable at the given prices. But it would not be a sufficient condition in ensuring incremental output of all crops which is what is required.

6. This passage is taken from an article written by an expert on agricultural finance.
7. The author is advocating for more yield of various crops.
8. The Government is not ready to increase the procurement price of crops.
9. According to the passage, the farmers are not income-conscious.
10. Recently there was an agitation by farmers for increase in procurement price of crops.

Passage 3

The water resources of our country are very much underutilised. The main reason of this underutilisation is the lack of capital and technology. A large portion of our water resources is wasted due to floods and unwise use of water for irrigation as well as domestic purposes. We can make full use of our water resources by building dams on rivers and by adoption policy of awareness among people not to waste water.

11. Our country has large areas to be irrigated and much water is wasted.
12. Building of dams is a essential step in the conservation of water resources.
13. Occurrence of floods adds to the water resources.
14. Some people do not use water resources in a judicious way.
15. The country does not have enough funds to develop water resources.

Passage 4

A recent survey shows that India has the lowest death rate for blood cancer. China, Thailand and Myanmar (countries that have taste for spices) also have low rates. Higher rates are found in U.S.A. where spices are not used. The typical American food remains chicken rolls, butter and beef.

16. Americans are unorthodox in their food habits.
17. Americans dislike spices.
18. Spices prevent blood cancer.
19. Spices promote forms of cancer other than blood cancer.
20. Chicken rolls, butter and beef promote cancer.

Passage 5

More than a decade of erosion in budgetary support from the Union Government has seriously affected Indian Railways capacity to finance its plan expenditures. The situation has come to a pass where the railways must now think of innovative ways to get longer mileage from its investments. Significantly the resource crunch has had grievous impact on the railways.

As a result, it will not be in a position to acquire necessary equipments and this will seriously affect the railways capacity to serve the needs of the economy in future.

21. Railways had so far believed in traditional ways in generating income.

- 22. Government has shifted its priority from railways to other areas.
- 23. The Union Government has reduced drastically the budgetary support to railways during the last decade.
- 24. The fiscal position of railways in the earlier plan period was better than the current plan period.
- 25. During the current plan period, the railways will not be able to expand its network.

Passage 6

Urban services have not expanded fast enough to cope with urban expansion. Low investment allocations have tended to be underspent. Both public (e.g., water and sewage) and private (e.g., low-income area housing) infrastructure quality has declined. This impact of the environment in which children live and the supporting services available to them when they fall ill, seems clear. The decline in average food availability and the rise in absolute poverty point in the same unsatisfactory direction.

- 26. There is nothing to boast about urban services.
- 27. The public transport system is in the hands of private sector.
- 28. Birth rate is higher in urban areas as compared to rural areas.
- 29. Low-cost urban housing is one of the priorities.
- 30. The environment around plays an important role on the health status.

- 31. Though adequate provisions of funds were made but they remained unspent.

Passage 7

The World Health Organisation has called for an improved surveillance to combat dengue and says the outbreak can be controlled in two weeks if all necessary steps are taken to stop the mosquitoes from breeding and break the transmission cycle. Dengue is already the most widespread mosquito-borne disease among humans. In the past 15 years, outbreaks in South and South-east Asia have been rapidly rising mostly due to falling environmental and public health standards during urbanisation. WHO reports that several forms of the disease such as haemorrhagic fever (DHF) and shock syndrome (DSS) are putting more than 2.5 million people at risk worldwide each year. Importantly, 95% of the DHF cases are among children less than 15 years. Therefore, the disease has major impacts on public health and future generations.

- 32. If rate of urbanisation in South Asia is controlled, outbreaks of all diseases may be reduced.
- 33. World Health Organisation has not collected data of outbreak of dengue in the past.
- 34. There was no outbreak of dengue in the European countries in the recent past.
- 35. Over the last decade South Asian countries have not successfully stepped up mechanism to combat dengue.
- 36. DSS type dengue seems mostly to be affecting the adults.

Answer with Solutions

1. (c) The given fact is neither mentioned in nor can be derived from the given passage.
2. (a) It is mentioned in the passage that maximum losses suffered by the smaller units are because of lack of proper opportunities for recycling waste. This clearly implies the given fact.
3. (b) From the first sentence of the passage, it is quite probable that operating on a large scale might make the pesticide units economically viable.
4. (a) The fact directly follows from the sentences 'In view of the loss..... waste minimisation techniques' in the passage.
5. (b) It is mentioned in the passage that the profit margins of small units are low and so small adverse conditions land them in trouble. Thus, it is quite probable that lack of funds compels these units to ignore house keeping.
6. (a) Since the passage analyses the merits and demerits of the Government's new fiscal policy relating to agriculture, the given conclusion follows.
7. (a) It is mentioned that the new policy will lead to a shift in cultivation and would not fulfill its real objective, which is to make farmers produce more food.
The last sentence of the passage also confirms the fact.
8. (e) According to the passage, the Government is providing price incentives to farmers. This contradicts the fact given in the question.
9. (d) According to the passage, the farmers would shift cultivation from the present crop to the more profitable one. This means that they are income-conscious.
10. (c) The fact is neither mentioned in nor can be derived from the passage.
11. (c) No mention of the land to be irrigated in India is there in the passage.
12. (a) It is given in the passage that we can make full use of our water resources by building of dams.
13. (b) It is mentioned that much of our water resources are wasted due to floods.
14. (d) It is mentioned that wastage of water takes place due to their unwise use for domestic purposes.
15. (a) It is given in the passage that underutilisation of the water resources of our country is due to lack of capital or funds.
16. (d) It is mentioned that Americans have almost the same diet, with no intake of spices. This means that probably they cannot change diet easily and are orthodox in food habits.
17. (c) Nothing about the reason for Americans not using spices in their diet is mentioned in the passage.
18. (a) The fact that blood cancer is more prominent in America, where spices are not used while the rate is much lower in Asian countries where spices are used, makes the given conclusion valid.
19. (c) Other forms of cancer have not been talked about in the passage.
20. (a) It is mentioned that blood cancer is more prominent in America, where the typical diet comprises of chicken rolls, butter and beef.
21. (a) It is mentioned in the passage that railways now need to find 'innovative ways' to get bigger returns for their investments. This clearly implies the given fact.
22. (b) According to the passage, budgetary support to the railways from the Union Government has declined. Perhaps this is because the Government has shifted its priority to some other areas.
23. (a) The fact directly follows from the first sentence of the passage.
24. (a) The fact that railways have now fallen in need of bigger returns from their investment, implies the given fact.
25. (a) The given fact directly follows from the last sentence of the passage.
26. (a) The first sentence of the passage verifies the given conclusion.
27. (c) Nothing about the public transport system is mentioned in the passage.
28. (c) The passage deals with deteriorating living conditions in urban areas. Nothing about the birth rate is mentioned.
29. (b) Since the passage condemns the decline in quality of low-income area housing, it is probable that it is a vital aspect of urban services.
30. (a) The conclusion directly follows from the statement. 'The impact of the environment seems clear' in the passage.
31. (e) The phrase low investment allocations in the passage implies that the funds provided were not adequate.
32. (b) It is mentioned in the passage that outbreaks in South and South-east Asia have been rapidly rising mostly due to falling environmental and public health standards during urbanisation. Thus, the fact in the question is quite probable.
33. (e) The passage talks of outbreak of dengue during the past 15 years. This contradicts the fact given in the question.
34. (c) The passage mentions the outbreak of dengue in Asian countries only and not the European countries.
35. (a) It is mentioned in the passage that cases of outbreak of dengue are rapidly rising in South Asian countries since the last 15 years. This means that adequate steps to combat dengue have not been taken.
36. (c) Nothing about the effect of DSS type dengue is mentioned in the passage.

5

Logic Chart

Logic charts is one of the major areas in most of the competitive exams. Any data like daily production, financial performance or productivity will have to be presented in a concise manner. Study and manipulation of such data leads us to an important area called logic charts.

Data can be organized in a number of ways so that larger volume of data can be presented in a more compact and precise form. Data thus presented has to be deciphered correctly by the user of data. Numerical data can be presented in one or more of the following ways.

- (i) Data Table
- (ii) Pie chart
- (iii) Bar chart
- (iv) Line Graph

Data can also be presented by using a combination of two or more of the above forms.

Let us understand each of the above forms of data representaion with an example.

Data Table

A Data Table is a chart of facts and figures represented in **horizontal rows** and **vertical columns**. These facts and figures can be of imports, exports, income of employees in a factory students applying for and qualifying a certain field of study.

The amount of data that can be presented on data table is much higher than that which can be presented on any other type of graph or chart.

e.g. The data table given below shows profit of 3 companies from year 2010 to 2012.

Column	Year	2010 (Profit%)	2011 (Profit%)	2012 (Profit%)	Row
	Company				
	SATYAM	30	20	15	1
	TCS	15	20	10	2
	L&T	15	10	35	3
		1	2	3	

Directions (Examples. 1-3) *Study the given table carefully and answer the questions that follow.*

Percentage of marks obtained by five students in five different subjects in a school

Subject Student	English (100)	Science (125)	Mathematics (150)	Social Studies (75)	Hindi (50)
Rahul	67	84	70	64	90
Veena	59	72	74	88	84
Soham	66	90	84	80	76
Shreya	71	66	80	66	86
Varun	63	76	88	68	72

Figures in brackets indicate maximum marks for a particular subject.

Ex. 1 What is Varun's overall percentage in the examination?

Sol. Total marks of Varun

= Sum of marks in English + Science + Mathematics
+ Social Studies + Hindi

= 63% of 100 + 76% of 125 + 88% of 150 + 68% of 75
+ 72% of 50

$= 63 + \frac{76 \times 125}{100} + \frac{88 \times 150}{100} + \frac{68 \times 75}{100} + \frac{72 \times 50}{100}$

$= 63 + 95 + 132 + 51 + 36 = 377$

$\therefore \text{Required percentage} = \frac{\text{Total marks of Varun}}{\text{Total marks}} \times 100\%$

$= \frac{377}{500} \times 100\% = 75.4\%$

Ex. 2 If in order to pass the exam, a minimum of 95 marks are needed in Science, how many students pass in the exam?

Sol. Let pass percentage in Science be x .

Then, $x\%$ of 125 = 95

$\Rightarrow \frac{125 \times x}{100} = 95 \Rightarrow x = \frac{95 \times 100}{125} = 76$

Only three students will pass. (Rahul, Soham and Varun)

Ex.3 What is the respective ratio of total marks obtained by Veena and Shreya together in Mathematics to the marks obtained by Rahul in the same subject?

Sol. Total marks obtained by Veena and Shreya together in Mathematics

= 74% of 150 + 80% of 150

$= \frac{74 \times 150}{100} + \frac{80 \times 150}{100} = 111 + 120 = 231$

Marks obtained by Rahul in Mathematics = 70% of 150

$= 150 \times \frac{70}{100} = 105$

$\therefore \text{Ratio} = \frac{231}{105} = \frac{11}{5} = 11:5$

Pie Chart

Pie chart is a circular chart divided into sectors in which the arc length, its central angle and area are proportional to the quantities that it represents.

Pie chart is called so because of its shape. Each slice of pie is allowed to each category and shows the portion of the entire pie.

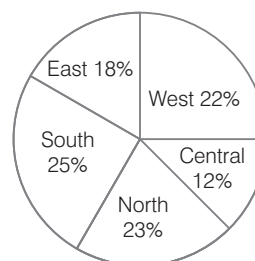
In the questions of pie chart, the total quantity is distributed over a total angle of 360° or 100%. Here, the data can be plotted with respect to only one parameter.

Uses of pie charts are restricted to represent limited type of information.

Pie chart is also useful for representing proportions or percentages of various elements with respect to the total quantity.

The following pie chart gives the distribution of the population in different geographical zones.

Distribution of population in geographical zones



From the above pie chart, we can calculate the following

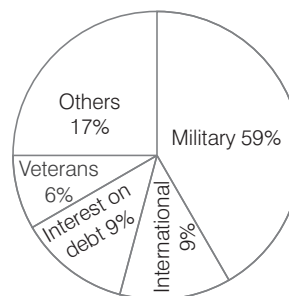
Population in any zone when the total population is given.

Population of any zone as a percentage of that of another zone.

Percentage increase in the total population, given that percentage increase in the population of one or more zones.

Direction (Example. 4) *Refer to the following pie chart and answer the questions that follow.*

National budget expenditure in the year 2012
(Percentage distribution)



Ex.4 In year 2012, if India had a total expenditure of ₹ 120 billion, then how many billions did it spend on interest on debt?

Sol. Total expenditure = 120 billion

∴ Expenditure of interest on debt = 9% of 120

$$= \frac{9}{100} \times 120$$

$$= ₹ 10.8 \text{ billion}$$

Bar Chart

A bar chart is a chart with rectangular bars with lengths proportional to the values that they represent. Bar charts are diagrammatic representation of discrete data.

A bar is a thick line whose width is shown merely for attention. In this method of data representation, the data is plotted on the *X* and *Y*-axes as bars.

One of the axes (normally the *X*-axis) of the bar diagram represents a discrete variable, while the other axis represents the scale for continuous variable.

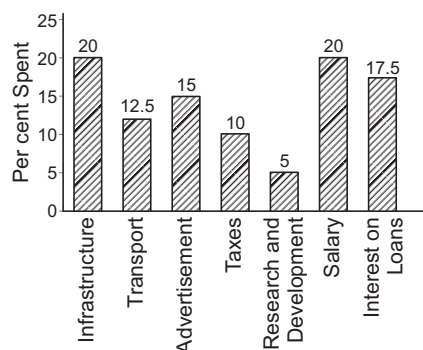
Each bar diagram has a title indicating the subject matter represented in the diagram.

Types of Bar Chart

Following are the main bar charts

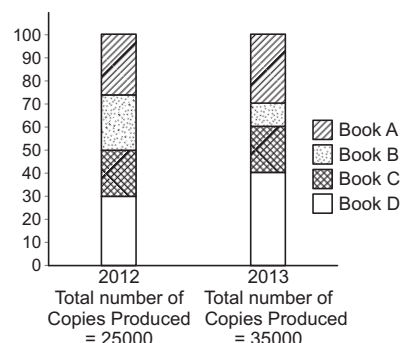
1. **Simple bar chart** A simple bar chart relates to only one variable.

For example Given below chart shows the percentage distribution of total expenditure of a company



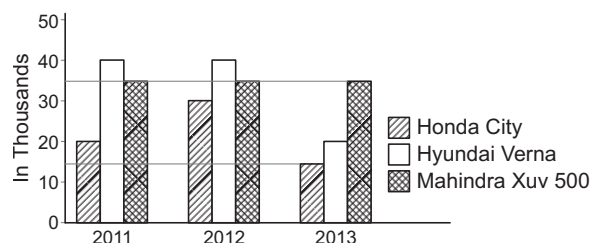
2. **Sub-divided bar chart** A sub-divided bar chart is used to represent various parts of total magnitude of a given variable.

For example Given below chart shows percentage distribution of sales of 4 books in two different year.



3. **Multiple bar chart** In this type, two or more bars are constructed adjoining each other to represent either different components of a complete data or to show multiple variables.

For example Given below chart show the production of 3 types of car in three different year.



Line Graph

A line graph (cartesian graph) indicates the variation of a quantity with respect to the two parameters calibrated (plotted) on *X* and *Y*-axes, respectively.

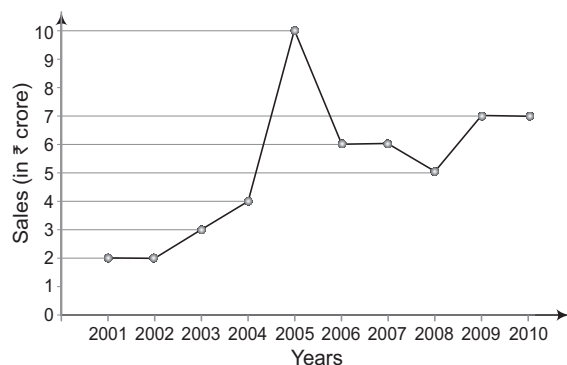
A line graph shows the quantitative information or a relationship between two changing quantities (variables) with a **line or curve** that connects a series of successive data points.

Types of Line Graph

Different types of line graph are discussed below

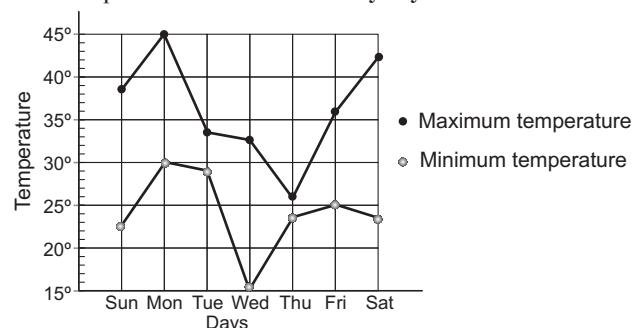
1. **Single Line Graph** Used for single variable representation.

For example The following single line graph represents the yearly sales figure of a company in the years 2001-2010.



2. Multiple Line Graph Used for more than one variable representation.

For example The following multiple line graph represents the maximum and minimum temperature recorded everyday in a certain week.



Practice Exercise

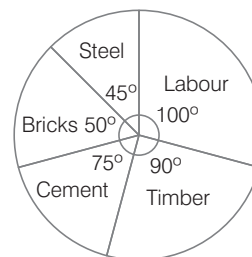
Directions (Q. Nos. 1-4) Study the following table and answer the questions based on it.

The table given below depicts the export of a commodity through four ports in the year 1998 and 1999.

Port	Export in 1998 (in ₹ crore)	Export in 1999 (in ₹ crore)
A	57	61
B	148	160
C	229	234
D	146	150

- The percentage increase in the export of the commodity from the year 1998 to 1999 was the highest from which port?
a. A b. B c. C d. D
- What was the change in the aggregate export of the commodity in the year 1999 as compared to the year 1998?
a. Nearly 4.3% increase
b. Nearly 4.3% decrease
c. Nearly 0.04% increase
d. Nearly 0.04% decrease
- What was the average increase in the export of the commodity from the ports in the year 1999 as compared the year 1998?
a. ₹ 82500000 b. ₹ 80000000
c. ₹ 75000000 d. ₹ 62500000
- The percentage increase in the export of the commodity from the year 1998 to 1999 was the lowest from which port?
a. A b. B
c. C d. D

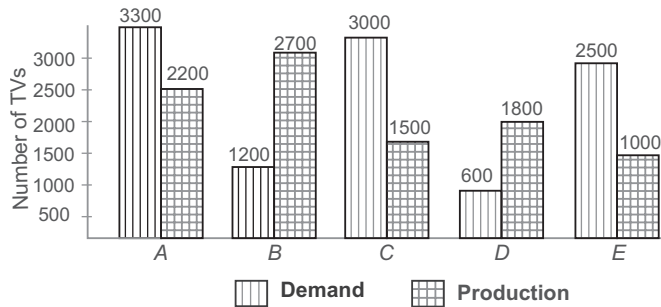
Directions (Q. Nos. 5-9) The following pie chart represents a total expenditure of ₹ 540000 on different items in constructing a flat in a town. Study the pie chart and answer the questions.



- The expenditure (in ₹) on bricks is
a. 75000 b. 67500
c. 150000 d. 70000
- The expenditure on bricks is less than the expenditure on timber (in ₹) by
a. 10000 b. 12500
c. 60000 d. 65000
- The percentage of the total expenditure spent on steel and cement is
a. 33.23% b. 25%
c. $33\frac{1}{3}\%$ d. 30%
- Which is the item of maximum expenditure?
a. Cement b. Steel
c. Timber d. Labour
- The expenditure (in ₹) on cement is
a. 75000 b. 90000
c. 135000 d. 112500

Directions (Q. Nos. 10-13) Study the following graph and answer the questions that follow.

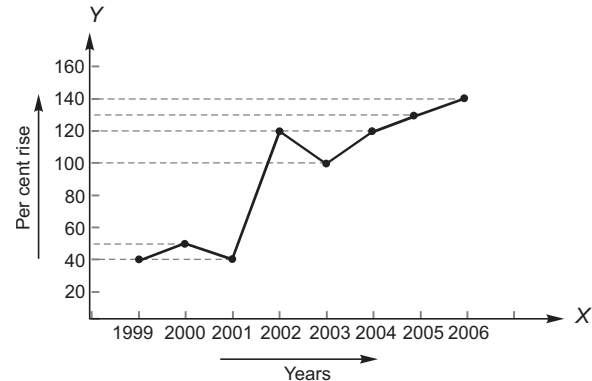
Demand and production of colour TVs of five companies for Jan 2006



10. What is the ratio of the number of companies having more demand than production to the number of companies having more production than demand?
 - a. 2 : 3
 - b. 4 : 1
 - c. 2 : 2
 - d. 3 : 2
11. What is the difference between average demand and average production of the five companies taken together?
 - a. 1400
 - b. 400
 - c. 280
 - d. 138
12. Demand of company D is approximately what per cent of demand of company E?
 - a. 12%
 - b. 20%
 - c. 24%
 - d. 30%
13. What is the ratio of average demand to average production of companies B and D?
 - a. 1 : 5
 - b. 2 : 5
 - c. 3 : 5
 - d. 4 : 5

Directions (Q. Nos. 14-18) Study the following graph carefully and answer the questions that follow.

The per cent rise in production from the years 1999 to 2006.



14. For how many years, the per cent rise was more than 100%?
 - a. One
 - b. Two
 - c. Three
 - d. Five
 - e. None of these
15. What was the average of per cent rise over the given years?
 - a. 110%
 - b. 105%
 - c. 100.5%
 - d. 92.5%
 - e. None of these
16. For how many years, the per cent rise was lower than the average of the per cent rise over the given years?
 - a. Two
 - b. One
 - c. Five
 - d. Three
 - e. None of these
17. For which of the given years, the per cent rise (from the previous year) was the least? [Years 2001 and 2003 are not to be considered.]
 - a. 2000
 - b. 2004
 - c. 2006
 - d. Cannot be determined
 - e. None of these
18. If the production in year 1998 was 1000 units, then how much was the production in year 2002?
 - a. 35280 units
 - b. 64680 units
 - c. 46200 units
 - d. Cannot be determined
 - e. None of these

Answer with Solutions

1. (b) Percentage increase from port A

$$= \frac{61 - 57}{57} \times 100\% = 7.01\%$$

Percentage increase from port B

$$= \frac{160 - 148}{148} \times 100\% = 8.10\%$$

Percentage increase from port C

$$= \frac{234 - 229}{229} \times 100\% = 2.18\%$$

Percentage increase from port D

$$= \frac{150 - 146}{146} \times 100 = 2.74\%$$

So, the highest increment is from port B.

2. (a) Total export in year 1998

$$= 57 + 148 + 229 + 146 = 580 \text{ crore}$$

Total export in year 1999

$$= 61 + 160 + 234 + 150 = 605 \text{ crore}$$

$$\text{Required percentage} = \frac{605 - 580}{580} \times 100\%$$

$$= 4.3\% \text{ increase}$$

3. (d) Total increase = 605 - 580 = 25 crore

$$\therefore \text{Average increase} = \frac{25}{4} \text{ crore}$$

$$= 62500000$$

4. (c) It is clear from solution 1 that lowest increment is from port C.

5. (a) Angle made by the expenditure on bricks

$$= 360^\circ - (45^\circ + 100^\circ + 90^\circ + 75^\circ)$$

$$= 360^\circ - 310^\circ = 50^\circ$$

Thus, expenditure on bricks

$$= \frac{50^\circ}{360^\circ} \times 540000 = ₹ 75000$$

6. (c) Required value

$$= \frac{(90^\circ - 50^\circ)}{360^\circ} \times 540000 = ₹ 60000$$

7. (c) Expenditure on steel and cement

$$= 75^\circ + 45^\circ = 120^\circ$$

$\therefore$ Required percentage

$$= \frac{120^\circ}{360^\circ} \times 100\% = \frac{100}{3}\% = 33\frac{1}{3}\%$$

8. (d) From the given pie chart, it is clear that 'Labour' has maximum expenditure.

9. (d) Expenditure on cement

$$= \frac{75^\circ}{360^\circ} \times 540000 = ₹ 112500$$

10. (d) Companies which have more demand than production = A, C and E, i.e. 3 companies.

Companies which have more production

than demand = B, D i.e. 2 companies

$\therefore$ Required ratio = 3 : 2

11. (c) Average demand

$$= \frac{\text{Sum of all demands}}{\text{Number of all demands}} \\ = \frac{3300 + 1200 + 3000 + 600 + 2500}{5} \\ = \frac{10600}{5} = 2120$$

Similarly, average production

$$= \frac{2200 + 2700 + 1500 + 1800 + 1000}{5} = \frac{9200}{5} = 1840$$

$$\therefore \text{Required difference} = 2120 - 1840 \\ = 280$$

12. (c) Required percentage = $\frac{600}{2500} \times 100\%$
= 24%

13. (b) Average demand of B and D

$$= \frac{1200 + 600}{2} = \frac{1800}{2} = 900$$

Average production of B and D

$$= \frac{2700 + 1800}{2} = \frac{4500}{2} = 2250$$

$$\therefore \text{Required ratio} = \frac{900}{2250} = 2 : 5$$

14. (e) For 4 yr, i.e. 2002, 2004, 2005 and 2006, the percentage rise was more than 100%.

15. (d) Required average

$$= \frac{(40 + 50 + 40 + 120 + 100) + (120 + 130 + 140)}{8} = \frac{740}{8} = 92.5\%$$

16. (d) The relevant years were 1999, 2000 and 2001.

17. (c) In the year 2000,

$$= \frac{50 - 40}{40} \times 100\% = 25\%$$

$$\text{In the year 2004, } \frac{120 - 100}{100} \times 100\% = 20\%$$

$$\text{In the year 2006, } \frac{140 - 120}{120} \times 100\% = 16.67\%$$

Clearly, per cent rise was the least for the year 2006.

18. (e) Production in year 2002

$$= 1000 \times \frac{140}{100} \times \frac{150}{100} \times \frac{140}{100} \times \frac{220}{100} = 6468$$

6

NON-VERBAL

Completion of Figure

Figure completion is a way or process to find out a missing part of a incomplete figure.

In this chapter, we deal with the questions in which a part of figure is missing, generally 1/4th part of figure and a candidate is asked to find the missing part from the option figure. These type of questions are designed to test the candidate's ability to correlate missing visual parts.

Here, you are required to understand the general structure of the figure, then choose the correct alternative.

Different types of questions covered in this chapter are as follows

1. Rotational Symmetry through an Angle of 90°

In this type of questions, the question figure looks the same if it is rotated by 90° in clockwise (CW) or anti-clockwise (ACW) direction.

2. Diagonally Opposite Parts are Similar

In this type of questions, the diagonally opposite parts have similar design and if the figure is rotated through 180° in CW or ACW direction the figure remains the same.

3. Adjacent Parts are Laterally/Vertically Inverted Images

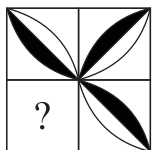
In this type of questions, the adjacent parts are laterally/vertically inverted images of each other.

4. Based on Certain Design and Pattern

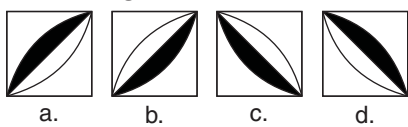
In this type of questions, they do not follow any of the patterns discussed in the previous types. These questions contain a certain design or pattern with one of its parts missing. A candidate has to complete the design accordingly.

Practice Exercise

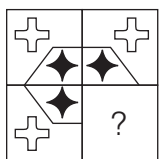
1. Problem Figure



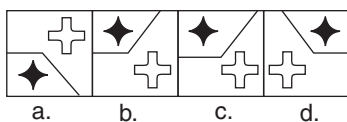
Answer Figures



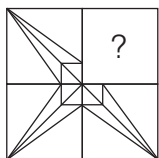
2. Problem Figure



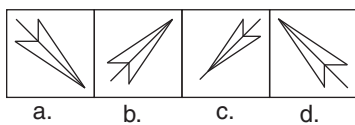
Answer Figures



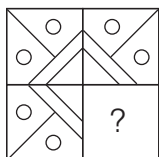
3. Problem Figure



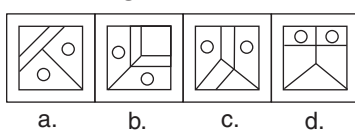
Answer Figures



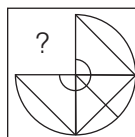
4. Problem Figure



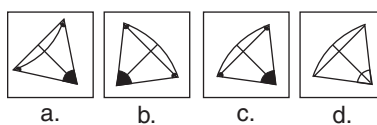
Answer Figures



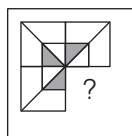
5. Problem Figure



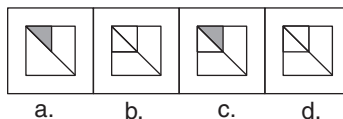
Answer Figures



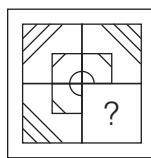
6. Problem Figure



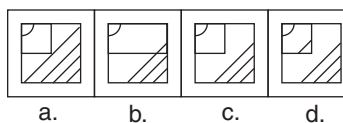
Answer Figures



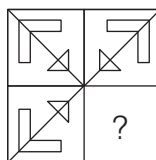
7. Problem Figure



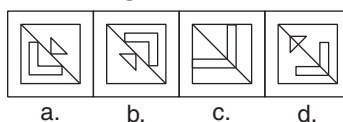
Answer Figures



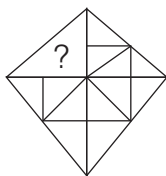
8. Problem Figure



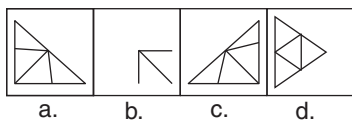
Answer Figures



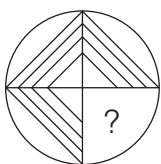
9. Problem Figure



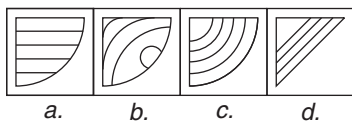
Answer Figures



10. Problem Figure



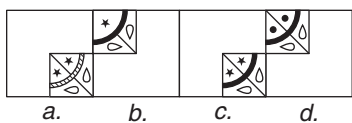
Answer Figures



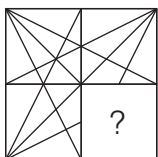
11. Problem Figure



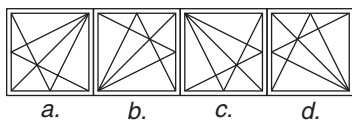
Answer Figures



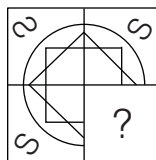
12. Problem Figure



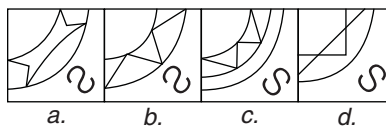
Answer Figures



13. Problem Figure



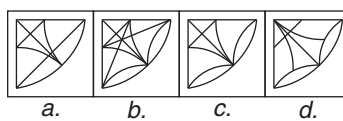
Answer Figures



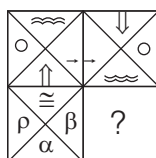
14. Problem Figure



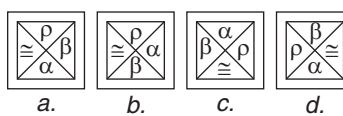
Answer Figures



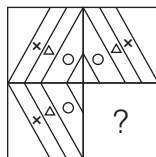
15. Problem Figure



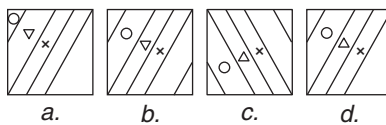
Answer Figures



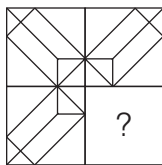
16. Problem Figures



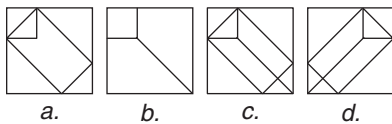
Answer Figures



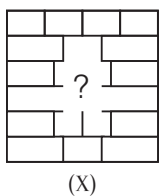
17. Problem Figure



Answer Figures

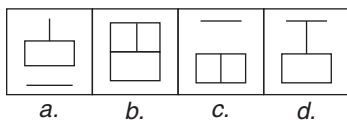


18. Problem Figure

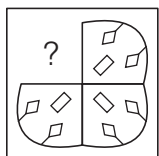


(X)

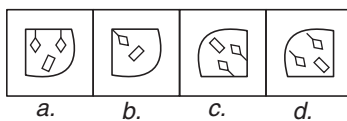
Answer Figures



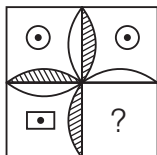
19. Problem Figure



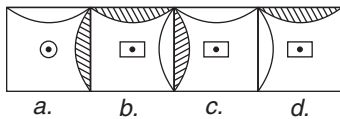
Answer Figures



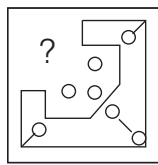
20. Problem Figure



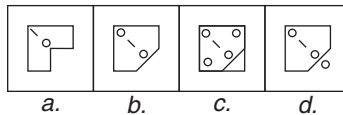
Answer Figures



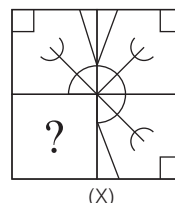
21. Problem Figure



Answer Figures

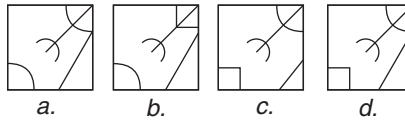


22. Problem Figure

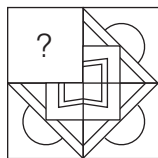


(X)

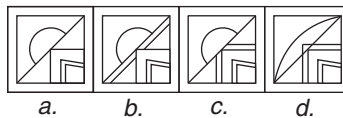
Answer Figures



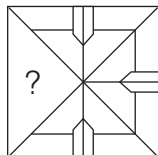
23. Problem Figure



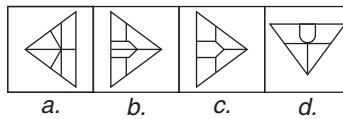
Answer Figures

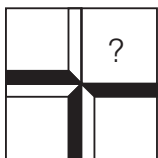
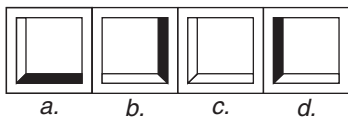
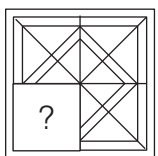
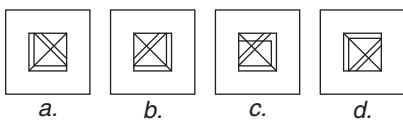


24. Problem Figure



Answer Figures

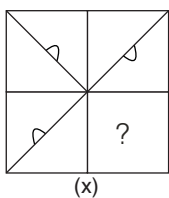
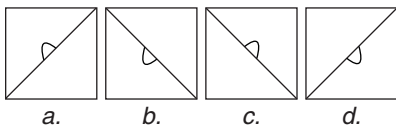


25. Problem Figure

Answer Figures

26. Problem Figure

Answer Figures


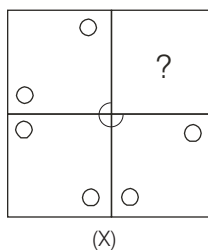
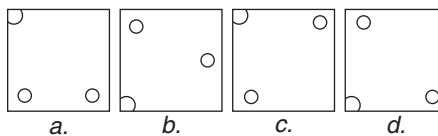
Directions (Q. Nos. 27-30) In each of the following problems, select a figure from the given four alternative, which when placed in the blank space of problem figure (X) would complete the pattern.

27. Problem Figure

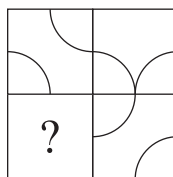
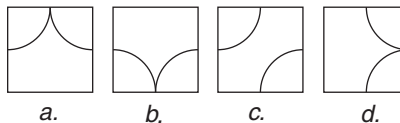
[2011]


Answer Figures

28. Problem Figure

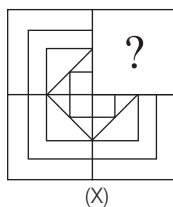
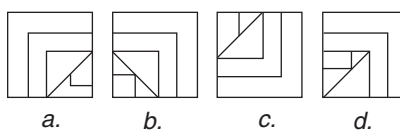
[2010]


Answer Figures

29. Problem Figure

[2010]


Answer Figures

30. Problem Figure

[2009 & 2007]


Answer Figures


Answer with Solutions

1. (b) The missing figure which will complete the figure pattern is given by option (b) i.e.



2. (b) The missing figure which will complete the figure pattern is given by option (b) i.e.



3. (b) The missing figure which will complete the figure pattern is given by option (b) i.e.



4. (a) The missing figure which will complete the figure pattern is given by option (a) i.e.



5. (d) The missing figure which will complete the figure pattern is given by option (d) i.e.



6. (c) The missing figure which will complete the figure pattern is given by option (c) i.e.



7. (d) The missing figure which will complete the figure pattern is given by option (d) i.e.



8. (d) The missing figure which will complete the figure pattern is given by option (d) i.e.



9. (b) The missing figure which will complete the figure pattern is given by option (b) i.e.



10. (d) The missing figure which will complete the figure pattern is given by option (d) i.e.



11. (c) The missing figure which will complete the figure pattern is given by option (c) i.e.



12. (d) The missing figure which will complete the figure pattern is given by option (d) i.e.



13. (d) The missing figure which will complete the figure pattern is given by option (d) i.e.



14. (c) The missing figure which will complete the figure pattern is given by option (c) i.e.



15. (c) The missing figure which will complete the figure pattern is given by option (c) i.e.



16. (d) The missing figure which will complete the figure pattern is given by option (d) i.e.



17. (c) The missing figure which will complete the figure pattern is given by option (c) i.e.



18. (d) The missing figure which will complete the figure pattern is given by option (d) i.e.



19. (d) The missing figure which will complete the figure pattern is given by option (d) i.e.



20. (d) The missing figure which will complete the figure pattern is given by option (d) i.e.



21. (d) The missing figure which will complete the figure pattern is given by option (d) i.e.



22. (d) The missing figure which will complete the figure pattern is given by option (d) i.e.



23. (b) The missing figure which will complete the figure pattern is given by option (b) i.e.



24. (b) The missing figure which will complete the figure pattern is given by option (b) i.e.



25. (d) The missing figure which will complete the figure pattern is given by option (d) i.e.



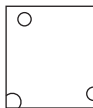
26. (a) The missing figure which will complete the figure pattern is given by option (a) i.e.



27. (b) The missing figure which will complete the figure pattern is given by option (b) i.e.



28. (d) The missing figure which will complete the figure pattern is given by option (d) i.e.



29. (d) The missing figure which will complete the figure pattern is given by option (d) i.e.



30. (b) The missing figure which will complete the figure pattern is given by option (b) i.e.



7

Figure Formation

Introduction

The process of formation of a defined geometrical figure with the use of pieces of different designs is known as formation of figure.

Figure formation deals with the problems related to the formation of geometrical figures like triangle, square, circle, etc., by joining some pieces out of a group of pieces of different designs. Besides this, some questions are asked in which the question figure contains some figures which can be joined together to obtain one of the alternative answer figures and one has to find out the correct alternative. Also, there are questions in which one question figure is given and we have to find out the correct answer figure in which all the pieces which are required to form the question figure are present.

In such questions, a question figure is given followed by four/five answer choices. The candidate is required to select the correct figure out of the given answer choice figures.

Different types of questions covered in this chapter are as follows

1. Based on Arrangement of the Figures

In this type of questions, a candidate is required to select a figure, out of the answer choice figures, that can be formed by arranging the pieces given in the question figure.

2. Based on Signs and Symbols

Such problems are just the reverse of type I. In this type of questions, question figures are to be formed with the pieces of one of the figures given in the answer choices and the candidate is required to find out the correct answer figure. Sometimes, pieces of boundary lines of the question figure are also given in the answer figure, then in this case these pieces are also counted for the formation of question figure. These questions require deep analysis of the question and answer figures.

3. Based on Formation of Triangle

In this type of questions, a triangle (equilateral, isosceles etc.) is to be formed by joining three figures out of a group of figures of different designs. A candidate is required to find out the correct combination of figures from the given alternative combinations.

To solve these type of questions, there is no fixed rule or method. These questions requires a deep analysis of various

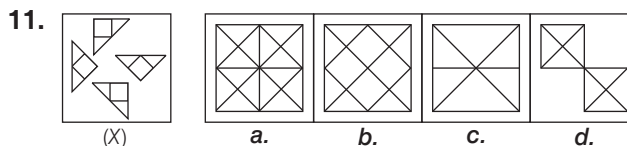
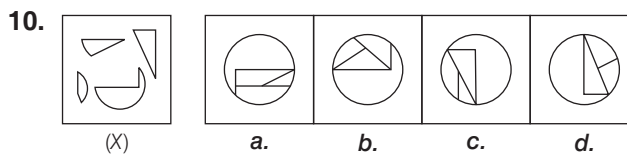
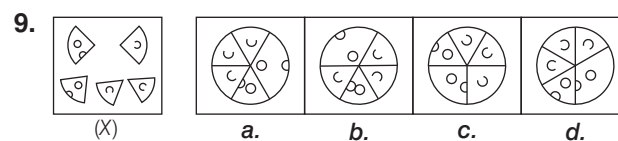
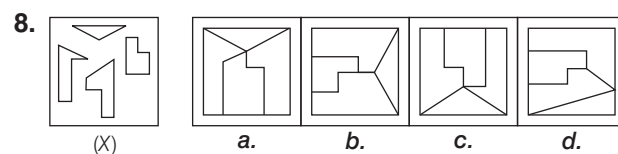
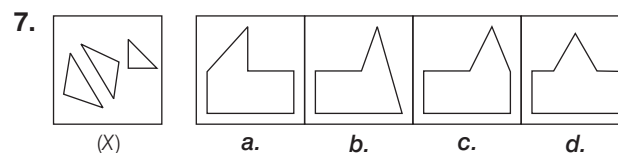
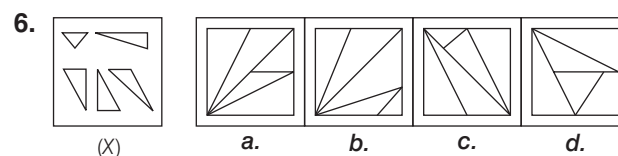
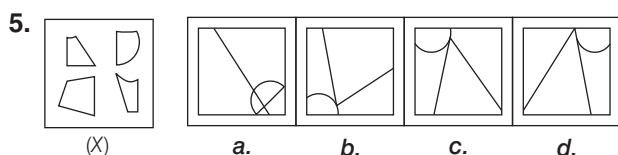
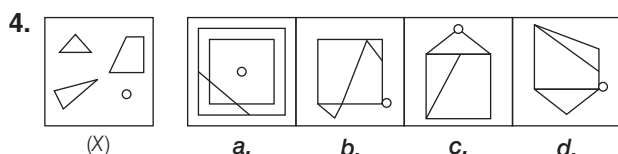
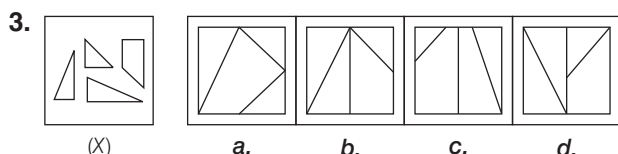
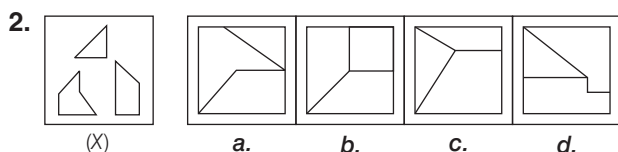
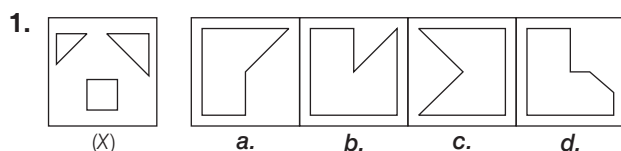
element (designs) present to form a figure and spatial analysis of the figure to be formed.

4. Square Formation Using a Set of Figures

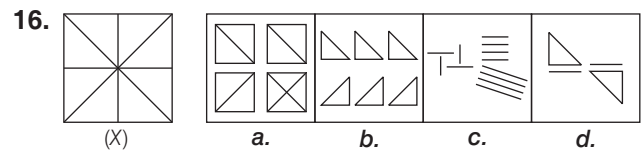
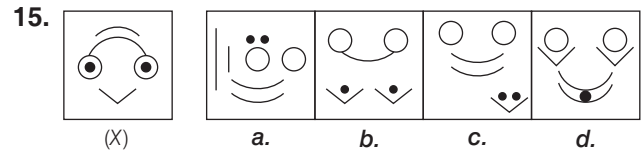
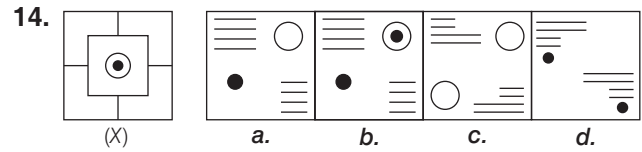
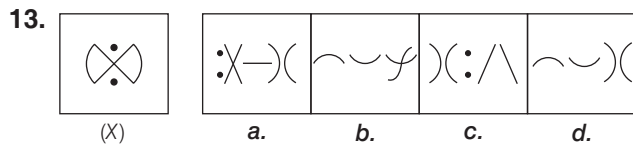
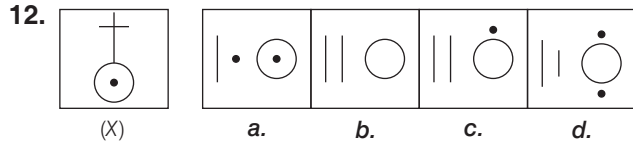
In this type of questions, a square is to be formed by joining three figures out of a group of figures of different designs. A candidate is required to find out the correct combination of figures from the given sets of figures.

Practice Exercise

Directions (Q. Nos. 1-11) In each of the following questions, find out which of the figures (a), (b), (c) and (d) can be formed from the pieces given in figure (X).



Directions (Q. Nos. 12-16) *In each of the following questions, find out which of the alternatives (a), (b), (c) and (d) will exactly make up the key figure (X).*



Answers

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|--------|--------|--------|---------|
| 1. (a) | 2. (a) | 3. (b) | 4. (c) | 5. (b) | 6. (c) | 7. (b) | 8. (c) | 9. (c) | 10. (c) |
| 11. (b) | 12. (c) | 13. (c) | 14. (a) | 15. (c) | 16. (c) | | | | |

8

Paper Folding and Cutting

1. Paper Folding

Paper folding is a process in which a piece of paper (with certain embedded design pattern) is folded along a certain dotted line and it is asked to select the correct folded pattern.

Questions based on 'Paper Folding' comprise of a question figure representing a transparent sheet and four answer figures. The transparent sheet contains some figural pattern and one/more dotted lines. These dotted lines indicate the axis (axes) along which the paper is folded. A candidate is asked to choose an option from the given set of options, which would resemble the design that appears on the transparent sheet after folding.

2. Paper Cutting

Paper cutting is a process in which a piece of paper (of any shape) is folded along a certain direction and then cuts are made into it.

In paper cutting, questions based on one or more question figures are given showing the manner in which a piece of paper is folded, where a dotted line is the reference line along which the paper has to be folded and the arrow indicates the direction of the fold. Then, the paper is punched (cut) from a particular section. Such cuts may be based on varying patterns. These patterns appear on the paper when the paper is unfolded after cutting. The candidates are required to see the question figure(s) carefully and then identify the unfolded paper having correct cutting pattern out of the four answer choices given in the form of figures.

Types of Questions

1. Selecting Unfolded Pattern of a Folded Punched Piece of Paper

In this type of questions, the order in which a paper is folded and its punching pattern after folding is depicted through a series of question figures. The objective is to select from among the four answer figures, the one which correctly shows the shape of the paper after it is unfolded.

2. Selecting Folded and Punched Pattern Based on Unfolded Pattern

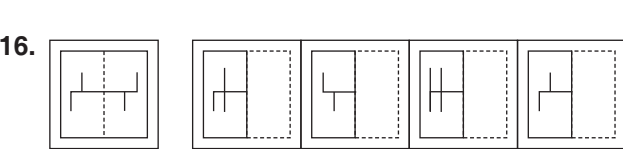
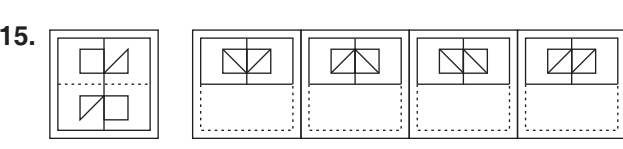
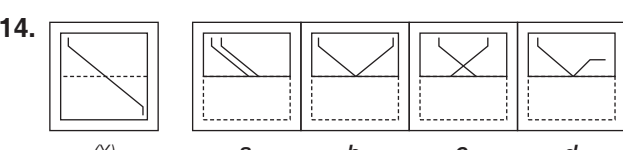
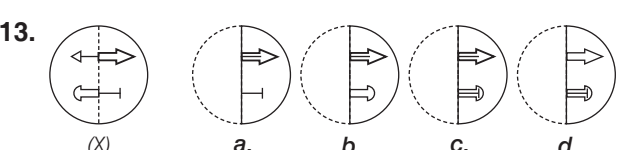
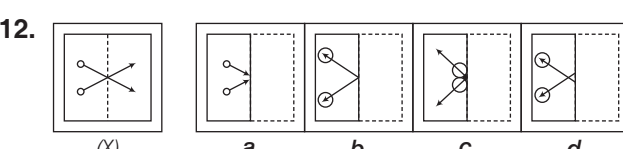
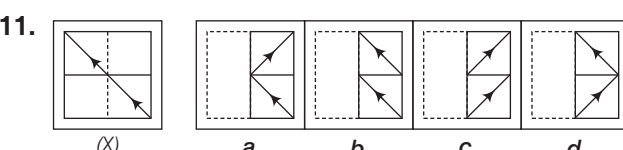
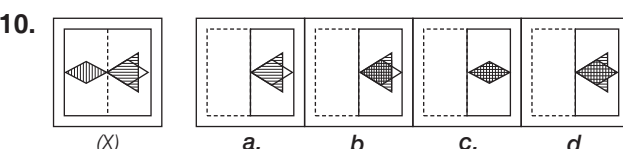
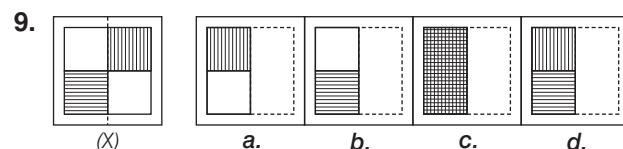
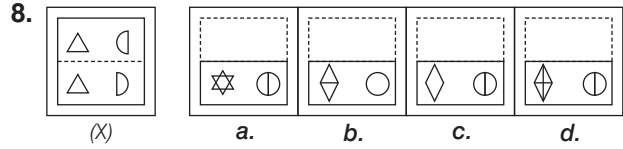
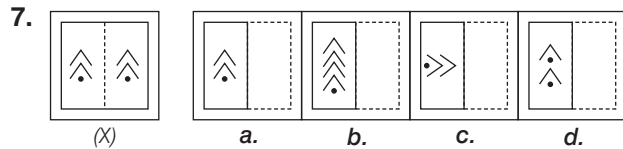
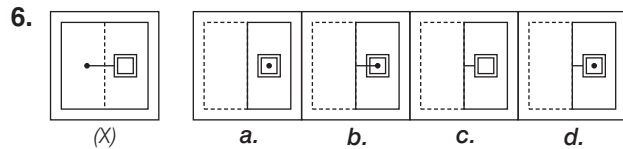
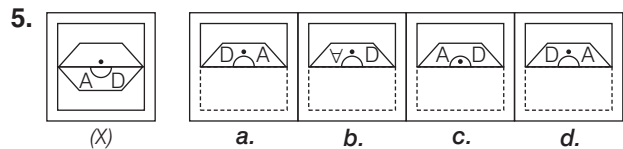
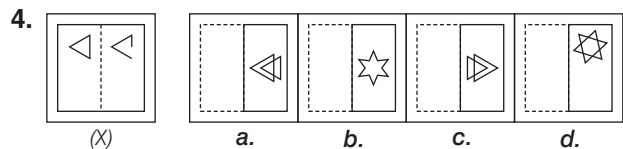
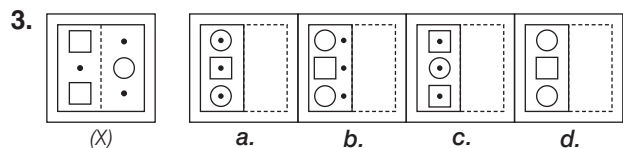
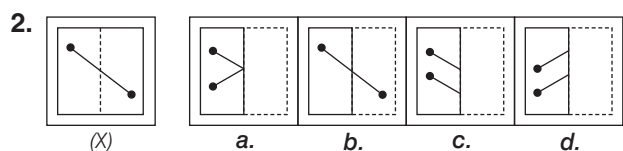
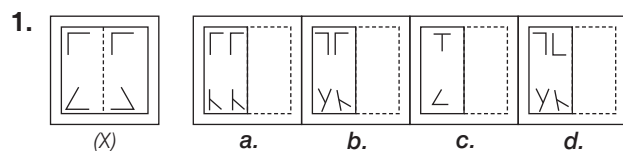
In such type of questions, a question figure is given which represents an unfolded piece of paper with several punches (cuts) made into it along with it, four alternatives representing differently folded and punched paper patterns are also given. The candidates are required to analyse the answer figures to find out the correct folded and punched paper pattern for the given unfolded pattern.

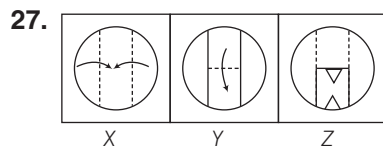
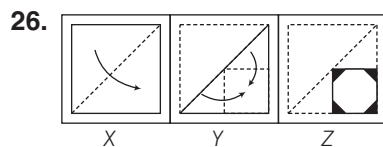
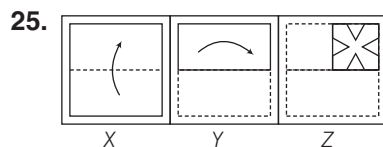
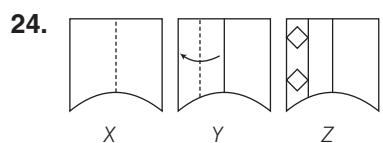
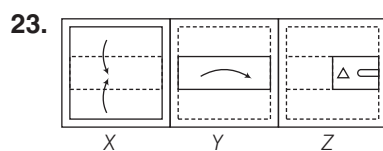
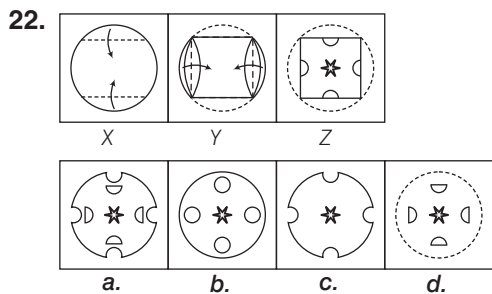
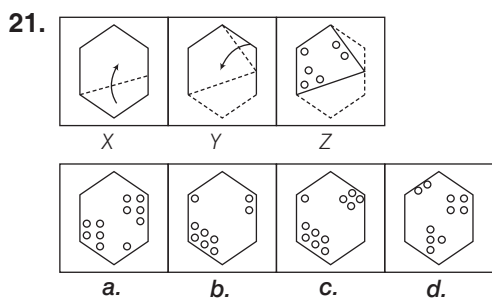
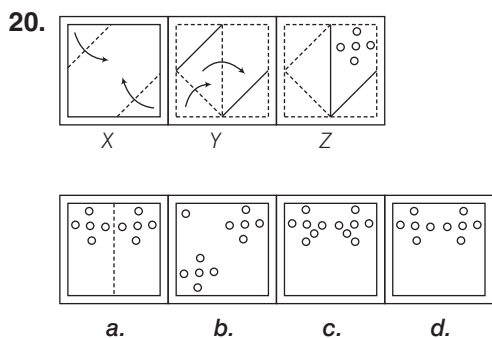
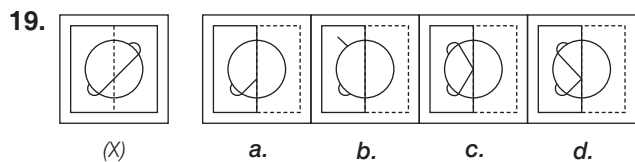
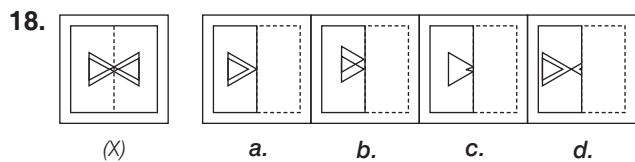
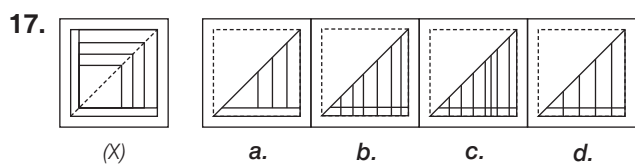
Practice Exercise

Directions (Q. Nos. 1-19) In each of the following questions, a figure marked as transparent sheet is given and followed by four answer figures, one out of these four options resembles the figure which is obtained by folding transparent sheet along the dotted line. Find the answer from these figures.

Transparent sheet

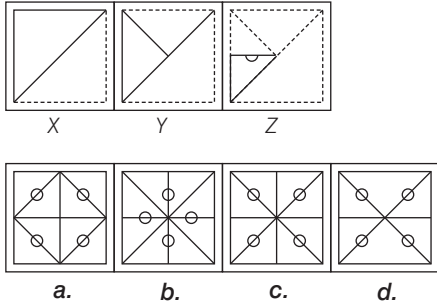
Answer figures



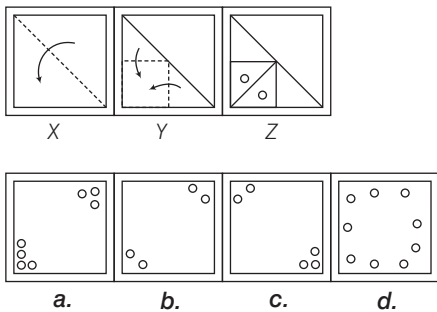


Directions (Q. Nos. 28-34) In each of the following questions, a set of three figures (X), (Y) and (Z) have been given, showing a sequence in which a paper is folded and finally cut at a particular section. Below these figures a set of answer figures marked (a), (b), (c) and (d) showing the design which the paper actually acquires when it is unfolded are also given. You have to select the answer figure which is closest to the unfolded piece of paper.

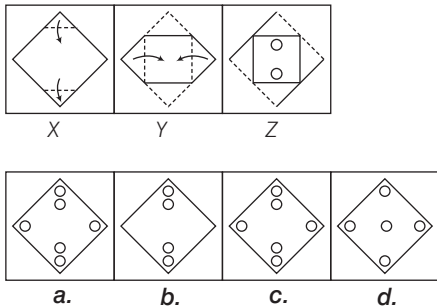
28. [2014]



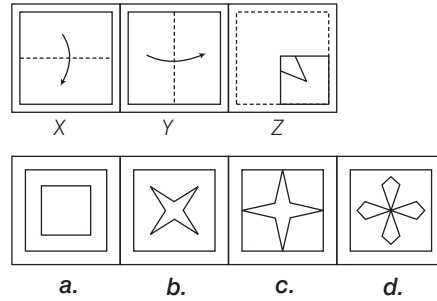
29. [2012]



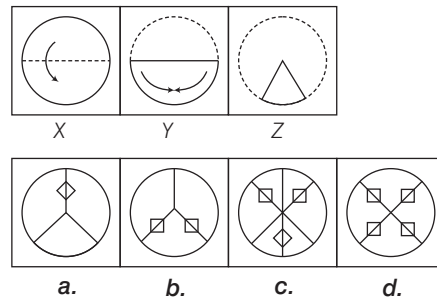
30. [2010]



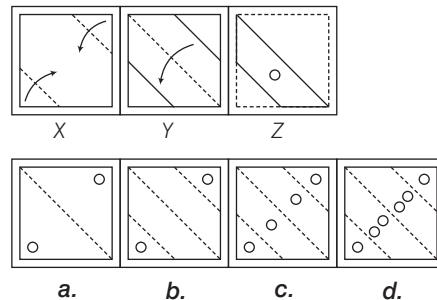
31. [2010]



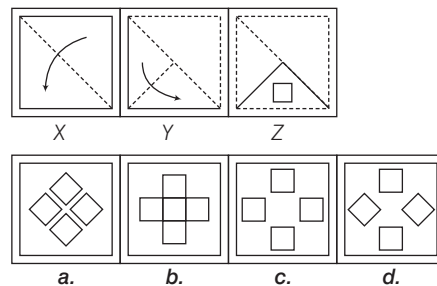
32. [2010]



33. [2007]



34. [2006]



Answers

Practice Exercise

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (c) | 2. (a) | 3. (c) | 4. (d) | 5. (b) | 6. (b) | 7. (a) | 8. (a) | 9. (d) | 10. (d) |
| 11. (a) | 12. (b) | 13. (c) | 14. (b) | 15. (a) | 16. (c) | 17. (a) | 18. (b) | 19. (d) | 20. (a) |
| 21. (c) | 22. (b) | 23. (d) | 24. (c) | 25. (b) | 26. (c) | 27. (d) | 28. (b) | 29. (d) | 30. (b) |

9

Figure Matrix

Figure matrix is a 2×2 or 3×3 representation of figures following a certain pattern in matrix form.

In this chapter, we deal with questions which have either 2×2 or 3×3 matrix. These matrix are formed by a group of figures. Corresponding rows or corresponding columns follow a certain pattern. You are required to analyse each of the sets to find out the common pattern and on the basis of that, the missing figure is to be identified from a set of alternatives, so as to complete the matrix.

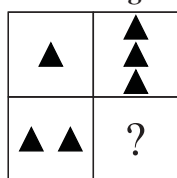
The common pattern can be on the basis of number of elements, orientation of figure, addition or deletion of elements etc.

For clear understanding of this chapter, few example are given below

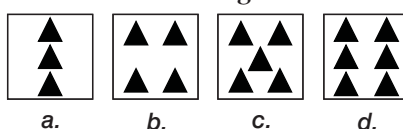
Directions (Examples 1-2) *In each of the following questions, find out the answer figure which completes the problem figure matrix.*

Ex. 1

Problem Figure



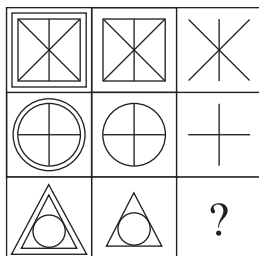
Answer Figures



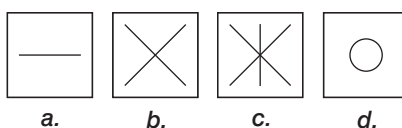
Sol. (d) The second part of each row is thrice the first part. So, the figure given in option (d) will replace the question mark.

Ex. 2

Problem Figure



Answer Figures

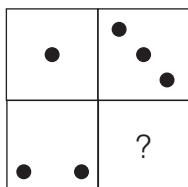


Sol. (d) In each row, the second figure is obtained by removing the outer most element of the first figure and the third figure is obtained by removing the outer most element of the second figure.

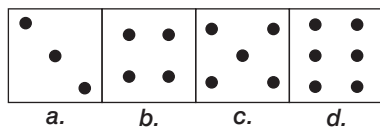
Practice Exercise

Directions (Q. Nos. 1-24) *In each of the following questions, find out which of the answer figures (a), (b), (c) and (d) completes the figure matrix?*

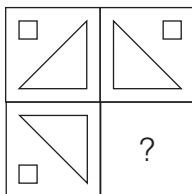
1. Problem Figure



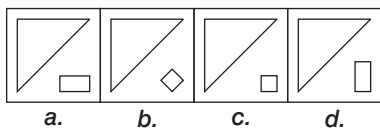
Answer Figures



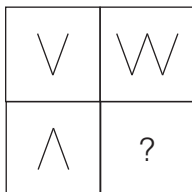
2. Problem Figure



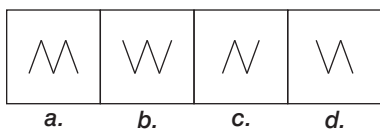
Answer Figures



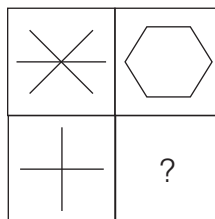
3. Problem Figure



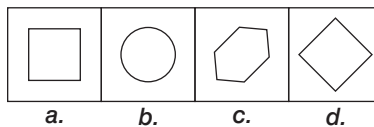
Answer Figures



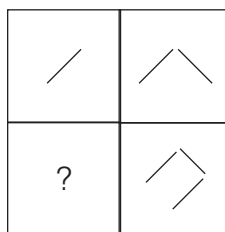
4. Problem Figure



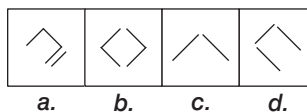
Answer Figures



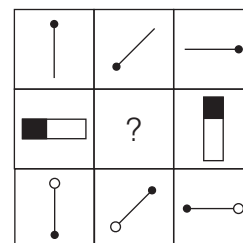
5. Problem Figure



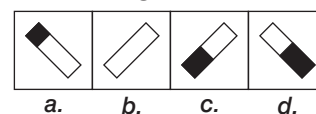
Answer Figures

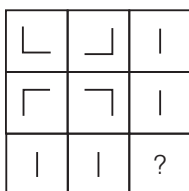
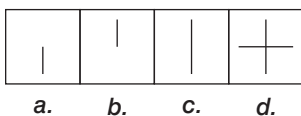
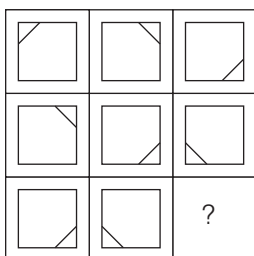
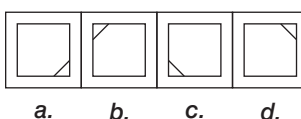
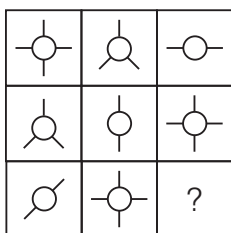
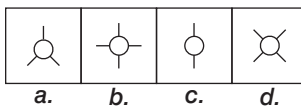
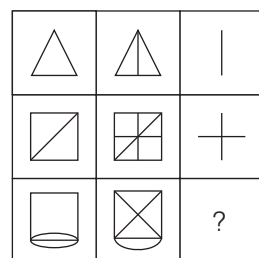
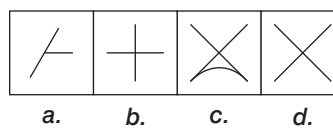
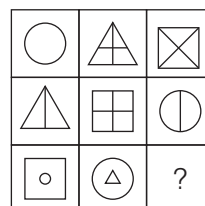
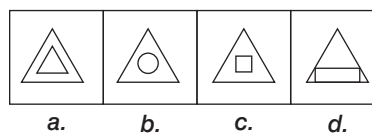
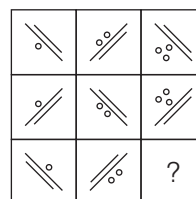
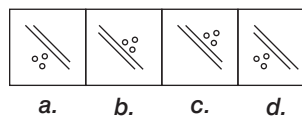
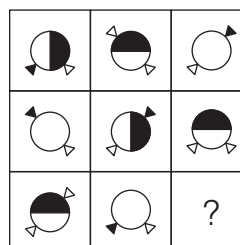
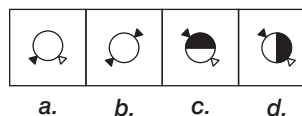


6. Problem Figure

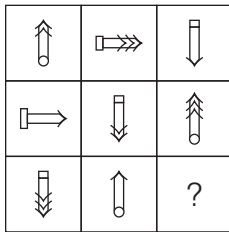


Answer Figures

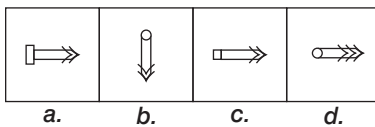


7. Problem Figure

Answer Figures

8. Problem Figure

Answer Figures

9. Problem Figure

Answer Figures

10. Problem Figure

Answer Figures

11. Problem Figure

Answer Figures

12. Problem Figure

Answer Figures

13. Problem Figure

Answer Figures


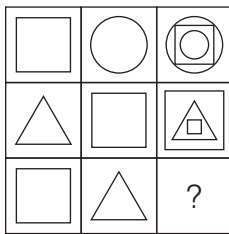
14. Problem Figure



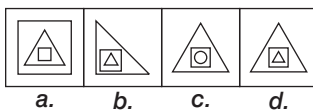
Answer Figures



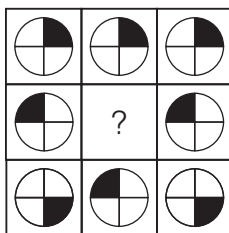
15. Problem Figure



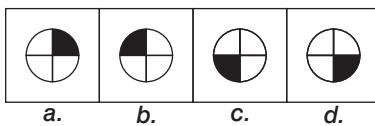
Answer Figures



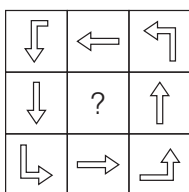
16. Problem Figure



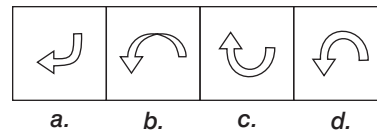
Answer Figures



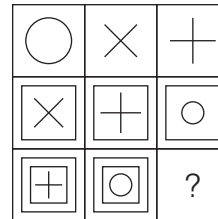
17. Problem Figure



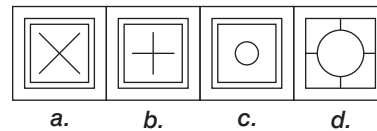
Answer Figures



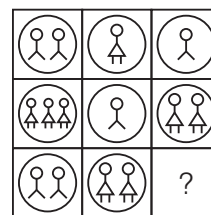
18. Problem Figure



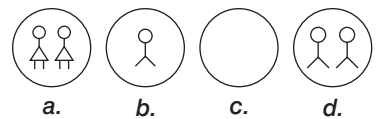
Answer Figures



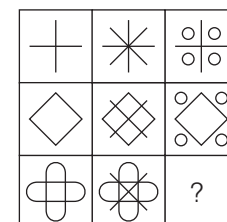
19. Problem Figure



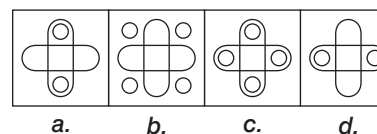
Answer Figures



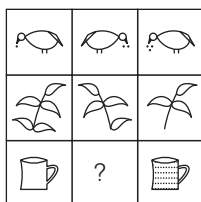
20. Problem Figure



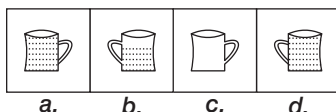
Answer Figures



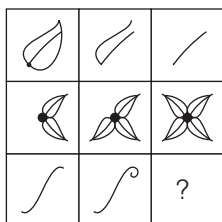
21. Problem Figure



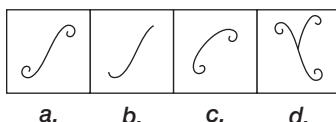
Answer Figures



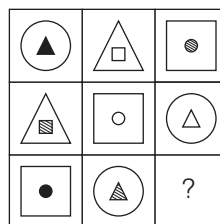
22. Problem Figure



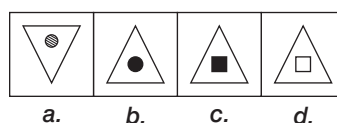
Answer Figures



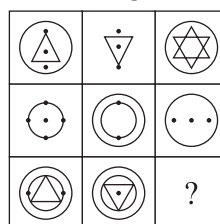
23. Problem Figure



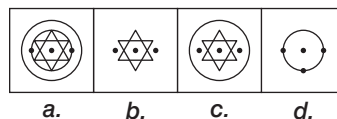
Answer Figures



24. Problem Figure

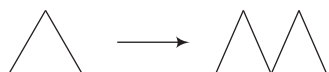


Answer Figures



Answer with Solutions

- (d) In each row of figure, the number of dots in the second figure is thrice the number of dots in the first figure.
- (c) Columnwise, second element is the mirror image of first element.
- (a) The pattern is as follows



Hence, option (a) is the correct answer.

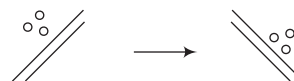
- (d) The pattern is as follows



Hence, option (d) is the correct answer.

- (b) In clockwise direction, every figure is increasing with one line. So, it will be 4 lines in figure (b).
- (d) Each row, the second figure is obtained by rotating the first figure 135° anti-clockwise.

- (c) In each row, the third figure is a collection of the common elements.
- (b) Figures in each row are rotating clockwise 90° .
- (a) In figure, each row consists of a circle and two segments or three line segments or four line segments. So, in third row only the figure with three line segments is missing.
- (d) The third figure in each row comprises of parts which are not common in the first two figures.
- (c) There is a triangle after circle and square after triangle. In the last row, we have to find the figure inside the triangle. In the next segment of circle, there is triangle and a square will be inside the triangle.
- (b) The pattern is as follows



Hence, option (b) is the correct answer.

13. (d) Clearly, there are three types of shadings of circles, one is unshaded, another has its right half shaded with vertical lines and yet another and has its upper half shaded with horizontal line. Also, two of the figures in each row have one triangle shaded.
14. (a) Clearly, there are three types of arrows—one arrow is with a single head and without any base, other arrow with double head having a circle at its base, last arrow is with triple head having a rectangle at its base. Also, in each row, the arrows point to three directions—upwards, downwards and towards the right.
15. (d) Clearly, in each row, the second figure forms the innermost which is also at the outermost position and first figure at middle position.
16. (d) In first and third set, the black portion of middle image is diagonally opposite to that of first image. Hence, second image in second set must be option (d).
17. (d) If we use figure (d) from the alternatives, the given matrix is complete.
18. (a) Each column has one '+', one 'o' and one 'x'. First figure of each column is without boundary and second one is in the square. Third figure is in two squares. As, there are '+' and 'o' in third column.



So, figure 'x' will be surrounded by two squares.

19. (c) Clearly, in each row, the number of elements in the third figure is equal to the difference in the number of elements in the first and second figures. Also, the third figure has the same types of elements as that of first figure.
20. (b) In each row, the second figure is obtained from the first figure by adding two mutually perpendicular line segments at the centre and the third figure is obtained from the first figure by adding four circles outside the main figure.
21. (b) In each row, the figure are getting laterally inverted in each step. The number of components or the quantities are either increasing or decreasing from left to right sequentially.
22. (a) The number of components in each row either increases or decreases from left to right. In the third row, it increases.
23. (d) In each row, there are three types of outer elements (circle, triangle and square), three types of inner elements (circle, triangle and square) and three types of shadings in the inner elements (black, white and lines).
24. (b) The third figure in each row comprises of parts which are not common to the first two figures.

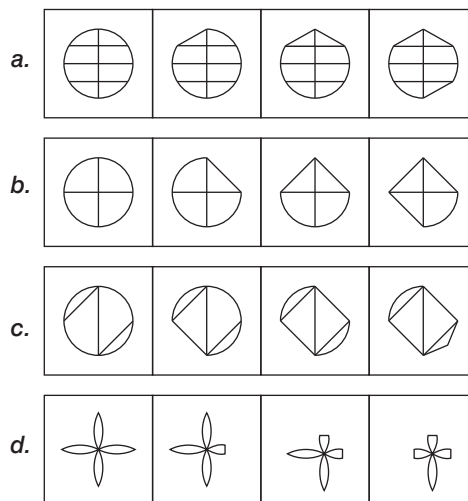
10

Rule Detection

In this chapter, we will solve problems in which, a particular rule is given and it is required to select from the given sets of figures, a set of figures which obeys the rule and thus forms a series.

e.g. Which one of the given sets of figures follows the following rule?

Rule Sectors get converted to triangles one by one.



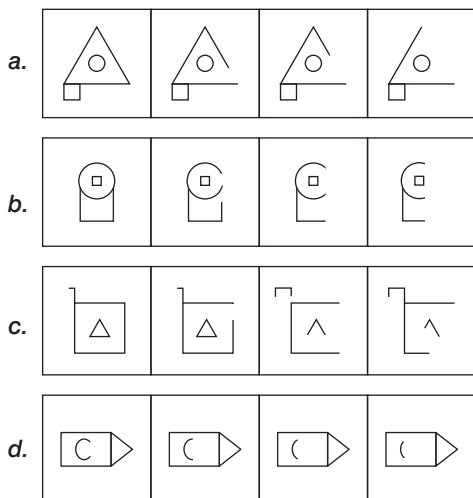
Sol (b) The region of the circle enclosed between two radii is called a sector. In the first figure of set (b), the circle is divided into sectors and in the subsequent figures of set (b), the arcs are replaced one by one, by straight lines so that the sectors are converted one by one into triangles. Hence, set (b) is the answer.

Practice Exercise

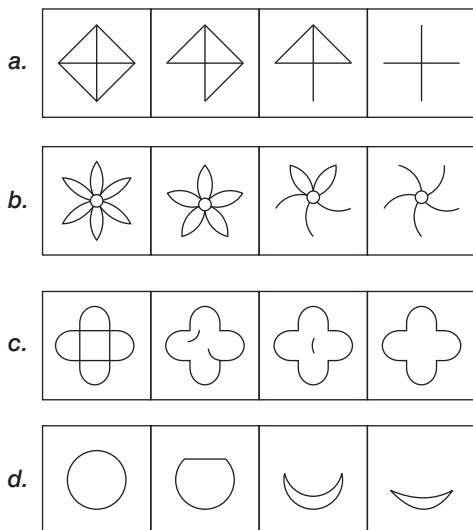
Directions (Q. Nos. 1-12) *In each of the following questions, choose the set of figures which follows the given rule. (X).*

- 1. Rule** Closed figures become more and more open and open figures become more and more closed.

[2007]

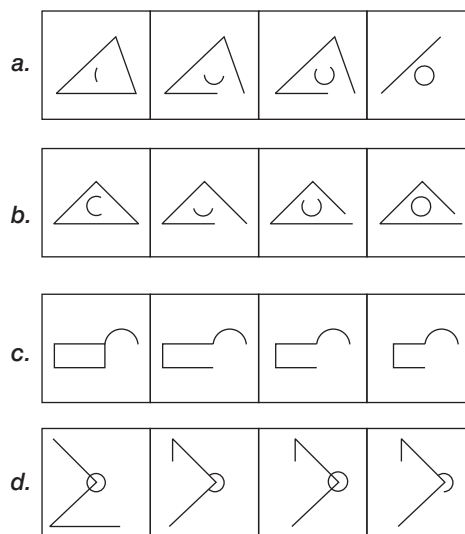


- 2. Rule** Closed figure becomes more and more open.

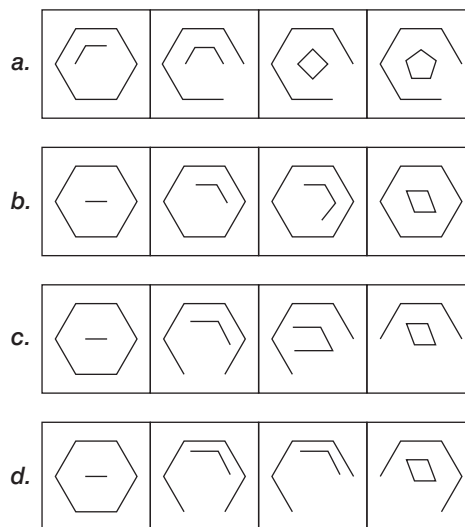


- 3. Rule** Closed figures become more and more open and open figures become more and more closed.

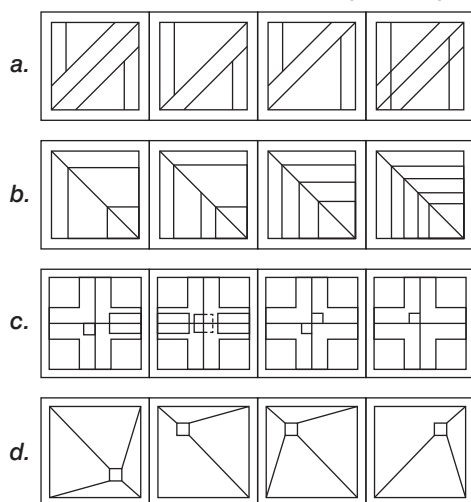
[2005]



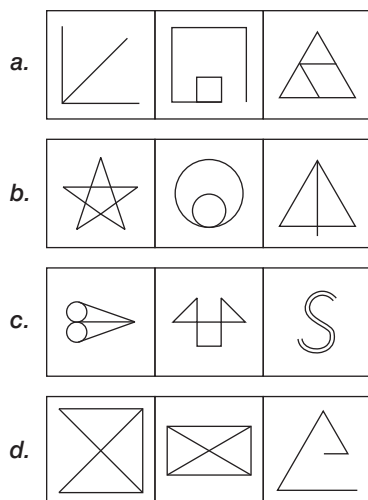
- 4. Rule** Closed figures losing their sides and open figures gaining their sides.



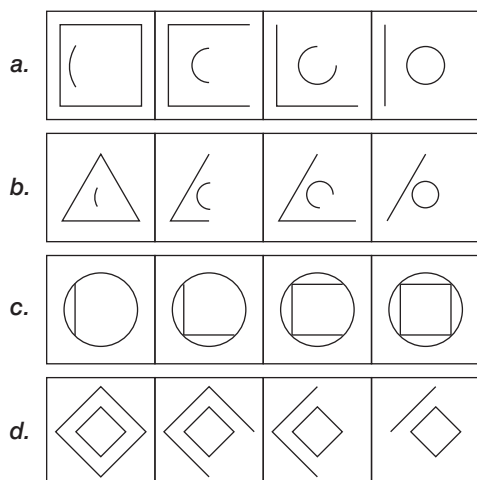
5. Rule The series becomes complex as proceeds.



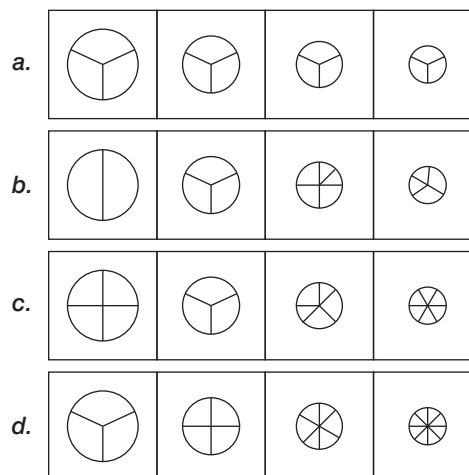
6. Rule Any figure can be traced by a single unbroken line unbound retracing. [2008]



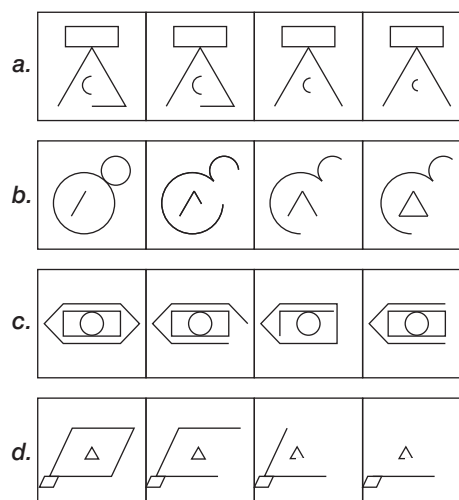
7. Rule Closed figures become more and more open and open figures become more and more closed.



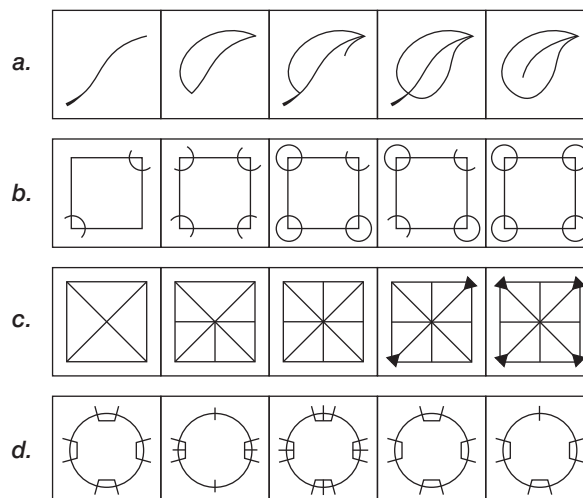
8. Rule As the circle decreases in size, its sectors increase in number.



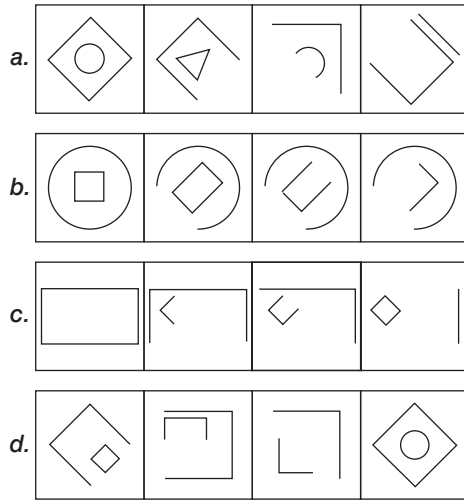
9. Rule Closed figures become more and more open and open figures become more and more closed.



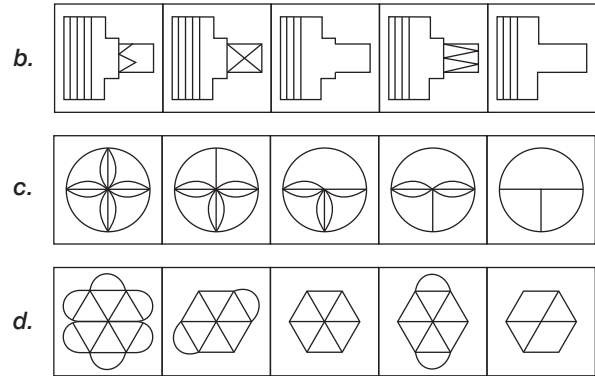
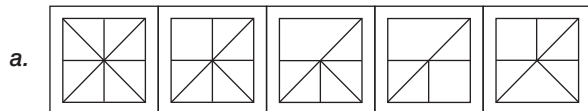
10. Rule The series becomes complex it proceeds.



11. **Rule** Closed figures gradually become open and open figures gradually become closed.

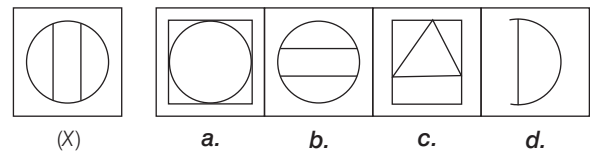


12. **Rule** The series becomes simpler as it proceeds.



13. Which of the figures (a), (b), (c) and (d) will be the answer figure if the following rule is applied to figure (X)?

Rule The curves should become straight lines and the straight lines should become curves.



Answerss

1. (c) 2. (a) 3. (a) 4. (c) 5. (b) 6. (b) 7. (a) 8. (d) 9. (b) 10. (c)
11. (c) 12. (c) 13. (a)



PRACTICE SETS

Practice Set 1

BITSAT

Instructions

- There are 150 questions in all. The number of questions in each part is as follows Subject
- All questions are multiple choice questions with four options, only one is correct.
- Each correct answer fetches 3 marks while incorrect answer has a penalty of 1 mark.

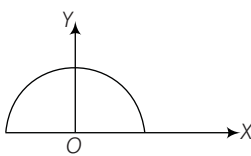
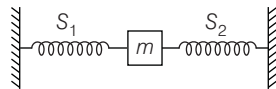
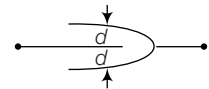
No. of Questions

Part I Physics	1-40
Part II Chemistry	41-80
Part III (a) English Proficiency	81-95
(b) Logical Reasoning	96-105
Part IV Mathematics	106-150

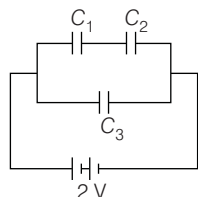
PART I

Physics

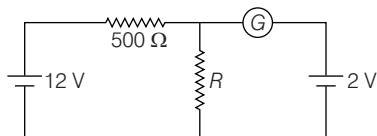
- In the relation, $p = \frac{\alpha}{\beta} e^{\left(-\frac{\alpha z}{k\theta}\right)}$, where p is pressure, z is distance, k is Boltzmann constant and θ is temperature. The dimensional formula of β will be
 - $[M^0 L^2 T^0]$
 - $[ML^2 T]$
 - $[M^0 L^2 T^{-1}]$
 - $[ML^0 T^{-1}]$
- A cube has a side of length 1.2×10^{-2} m. Calculate its volume.
 - $1.7 \times 10^{-6} \text{ m}^3$
 - $1.73 \times 10^{-6} \text{ m}^3$
 - $1.70 \times 10^{-6} \text{ m}^3$
 - $1.732 \times 10^{-6} \text{ m}^3$
- Maximum height reached by projectile is 4 m. The horizontal range is 12 m. Velocity of projection is
 - $4\sqrt{\frac{g}{2}}$
 - $5\sqrt{\frac{g}{2}}$
 - $\frac{1}{4}\sqrt{\frac{g}{2}}$
 - $\frac{1}{5}\sqrt{\frac{g}{2}}$
- A body A starts from rest with an acceleration a_1 . After two seconds, another body B starts from rest with an acceleration a_2 . If they travel equal distances in 5th second after the start of A , then the ratio of $a_1 : a_2$ is equal to
 - 5 : 9
 - 1 : 3
 - 3 : 1
 - 9 : 1
- A smooth block is released at rest on a 45° incline and then slides distance d . The time taken to slide is n times as much to slide on rough incline than on a smooth incline. The coefficient of friction is
 - $1 - \frac{1}{n^2}$
 - $\sqrt{1 - \frac{1}{n^2}}$
 - $\frac{1}{n^2} - 1$
 - $\sqrt{\frac{1}{n^2} + 1}$
- A long horizontal rod has a bead which can slide along its length and initially placed at a distance L from one end of the rod. The rod is set in angular motion about A with constant angular acceleration α . If the coefficient of friction between the rod and the bead is μ and the gravity is neglected, then time after which the bead starts slipping is
 - $\sqrt{\mu/\alpha}$
 - $\mu/\sqrt{\alpha}$
 - $\frac{1}{\sqrt{\mu\alpha}}$
 - $\frac{1}{\mu\sqrt{\alpha}}$

7. A particle tied to string describes a vertical circular motion of radius r continually. If it has a velocity $\sqrt{3gr}$ at the highest point, then the ratio of the respective tensions in the string holding it at the highest and lowest points is
 a. 4 : 3 b. 5 : 3
 c. 1 : 4 d. 4 : 1
8. A shell of mass 20 kg at rest explodes into two fragments whose masses are in the ratio 2 : 3. The smaller fragment moves with a velocity of 6 ms^{-1} . The kinetic energy of the larger fragment is
 a. 96 J b. 216 J
 c. 144 J d. 360 J
9. A wire of length l and mass m is bent in the form of a semicircle. The gravitational field intensity at the centre of semicircle is

 a. $\frac{Gm}{\pi l}$ along X-axis b. $\frac{Gm}{\pi l}$ along Y-axis
 c. $\frac{2\pi Gm}{l^2}$ along Y-axis d. $\frac{2\pi Gm}{l^2}$ along X-axis
10. A spherical liquid drop of radius R is divided into eight equal droplets. If surface tension is T , then work done in the process will be
 a. $2\pi R^2 T$ b. $3\pi R^2 T$
 c. $4\pi R^2 T$ d. $2\pi R T^2$
11. A body of density D_1 and mass M is moving downward in glycerine of density D_2 . What is the viscous force acting on it ?
 a. $Mg D_1$ b. $Mg D_2$
 c. $Mg \left(1 - \frac{D_2}{D_1}\right)$ d. $Mg \left(1 + \frac{D_2}{D_1}\right)$
12. A 2 kg copper block is heated to 500°C and then it is placed on a large block of ice at 0°C . If the specific heat capacity of copper is $400 \text{ J/kg}^\circ\text{C}$ and latent heat of fusion of water is $3.5 \times 10^5 \text{ J/kg}$, the amount of ice that can melt is
 a. $\frac{7}{3} \text{ kg}$ b. $\frac{7}{5} \text{ kg}$
 c. $\frac{7}{8} \text{ kg}$ d. $\frac{8}{7} \text{ kg}$
13. A same force is acting on two wires made of same material. One wire has length l and diameter d . The other wire has length $\left(\frac{l}{2}\right)$ and diameter $2d$. If the extensions in the two wires are l_1 and l_2 such that $l_1 + l_2 = 1 \text{ cm}$, then the values of l_1 and l_2 are
 a. 0.80 cm, 0.20 cm b. 0.89 cm, 0.11 cm
 c. 0.90 cm, 0.10 cm d. 0.95 cm, 0.05 cm
14. A temperature of equal masses of three different liquids A, B and C are $12^\circ\text{C}, 19^\circ\text{C}$ and 28°C , respectively. The temperature when A and B are mixed is 16°C , when B and C are mixed is 23°C . What is the temperature when A and C are mixed ?
 a. 26.02°C b. 26.60°C
 c. 20.26°C d. 21.62°C
15. 1 mm^3 of a gas is compressed at 1 atmospheric pressure and temperature 27°C to 627°C . What is the pressure under adiabatic condition? (γ for gas = 1.5)
 a. $27 \times 10^5 \text{ Nm}^{-2}$
 b. $36 \times 10^5 \text{ Nm}^{-2}$
 c. $56 \times 10^5 \text{ Nm}^{-2}$
 d. $80 \times 10^5 \text{ Nm}^{-2}$
16. If one mole of a monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with one mole of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$, the value of γ for the mixture is
 a. 3.7 b. 1.63
 c. 1.50 d. 1.40
17. In the given diagram, S_1 and S_2 are identical springs. The frequency of oscillation of mass m is f . If one spring is removed, the frequency will be

 a. $\frac{f}{\sqrt{2}}$ b. $\frac{f}{2}$
 c. $\sqrt{2}f$ d. $2f$
18. A capacitor of capacitance $1 \mu\text{F}$ is filled with two dielectrics of dielectric constants 4 and 6. The new capacitance is
 a. $10 \mu\text{F}$ b. $5 \mu\text{F}$
 c. $4 \mu\text{F}$ d. $7 \mu\text{F}$
19. A small oil drop of mass 10^{-6} kg is hanging at rest between two plates separated by 1 mm having a potential difference of 500 V. The charge on the drop is
 a. $2 \times 10^{-9} \text{ C}$ b. $2 \times 10^{-11} \text{ C}$
 c. $2 \times 10^{-6} \text{ C}$ d. $2 \times 10^{-8} \text{ C}$
20. Three plates of area A each are connected as shown in figure. The effective capacitance becomes :

 a. $\frac{\epsilon_0 A}{d}$ b. $\frac{3\epsilon_0 A}{d}$
 c. $\frac{1.5 \epsilon_0 A}{d}$ d. $\frac{2 \epsilon_0 A}{d}$

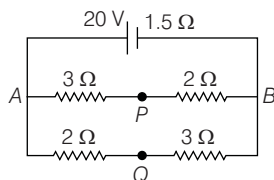
21. Two capacitors $C_1 = 2\mu\text{F}$ and $C_2 = 6\mu\text{F}$ in series are connected in parallel to a third capacitor $C_3 = 4\mu\text{F}$. This arrangement is then connected to a battery of emf 2V as shown in figure. How much energy is lost by the battery in charging the capacitors?



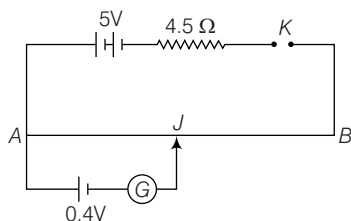
- a. $22 \times 10^{-16}\text{J}$
 b. $11 \times 10^{-6}\text{J}$
 c. $\frac{32}{3} \times 10^{-6}\text{J}$
 d. $\frac{16}{3} \times 10^{-6}\text{J}$
22. In a circuit, galvanometer G shows zero deflection. If the batteries A and B have negligible resistance, the value of R will be



- a. 1000Ω
 b. 500Ω
 c. 200Ω
 d. 100Ω
23. In the given figure, the internal resistance of the battery is 1.5Ω and V_P and V_Q are potential at P and Q respectively. What is the potential difference between the points P and Q ?

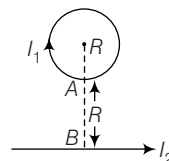


- a. Zero
 b. 4V ($V_P > V_Q$)
 c. 4V ($V_Q > V_P$)
 d. 2.5V ($V_Q > V_P$)
24. In the given figure, the potentiometer wire AB has a resistance of 5Ω and length 10 m. The balancing length AJ for the emf of 0.4V is

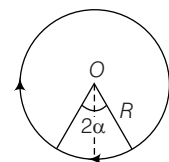


- a. 0.4m
 b. 0.8m
 c. 1.52m
 d. 4m
25. A milliammeter of range 10mA has a coil of resistance 1Ω . To use it as a voltmeter of range 10V , the resistance that must be connected in series with it is
- a. 9Ω
 b. 99Ω
 c. 999Ω
 d. 1000Ω

26. In the diagram I_1, I_2 are the strength of the currents in the loop and straight conductor respectively. $OA = OB = R$. The net magnetic field at the centre O is zero. Then, the ratio of the currents in the loop and the straight conductor is



- a. π
 b. 2π
 c. $1/\pi$
 d. $1/2\pi$
27. The magnetic field intensity due to a thin wire carrying current I in the figure shown, is

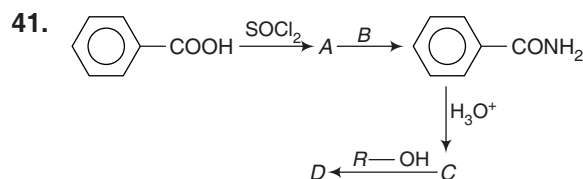


- a. $\frac{\mu_0 I}{2\pi R}(\pi - \alpha + \tan \alpha)$
 b. $\frac{\mu_0 I}{2\pi R}(\pi - \alpha)$
 c. $\frac{\mu_0 I}{2\pi R}(\pi + \alpha)$
 d. $\frac{\mu_0 I}{2\pi R}(\pi + \alpha - \tan \alpha)$
28. Electromagnets are made of soft iron because soft iron has
- a. low susceptibility and high retentivity
 b. high permeability and low retentivity
 c. low permeability and high coercivity
 d. low permeability and high retentivity
29. A magnetic dipole of moment 0.72Am^2 is placed horizontally with the north pole pointing south. The magnetic field of earth is $18\mu\text{T}$. The neutral points is at
- a. 0.1m on axial line
 b. 0.2m on equatorial line
 c. 0.2m on axial line
 d. 0.16m on equatorial line
30. Two coils have a mutual inductance 0.005H . The current changes in the first coil according to equation $I = I_0 \sin \omega t$, where $I_0 = 10\text{A}$ and $\omega = 100\pi\text{rad/s}$. The maximum value of emf in the second coil is
- a. 2π
 b. 5π
 c. 6π
 d. 12π
31. In a series LCR circuit, voltage across R is 100V and $R = 1\text{k}\Omega$ with $C = 2\mu\text{F}$. The resonant frequency ω is 200rad/s . At resonance the voltage across L is
- a. $4 \times 10^{-3}\text{V}$
 b. $2.5 \times 10^{-2}\text{V}$
 c. 40V
 d. 250V
32. A thin lens of glass ($n = 1.5$) of focal length 10cm is immersed in water ($n' = 1.33$). The new focal length is
- a. 12cm
 b. 20cm
 c. 40cm
 d. 48cm
33. Time taken by sunlight to pass through a window of thickness 4mm and refractive index $3/2$ is
- a. $2 \times 10^{-4}\text{s}$
 b. $2 \times 10^8\text{s}$
 c. $2 \times 10^{-11}\text{s}$
 d. $2 \times 10^{11}\text{s}$

34. The size of image of an object, which is at infinity, as formed by a thin convex lens of focal length 30 cm is 2 cm. If a concave lens of focal length 20 cm is placed between the convex lens and the image at a distance of 26 cm from the convex lens, calculate the new size of the image.
 a. 1.25 cm b. 2.5 cm c. 1.05 cm d. 2 cm
35. A slit width a is illuminated by red light of wavelength 6500 Å. If the first minimum falls at 30° , the value of a is
 a. 6.5×10^{-4} mm b. 1.3×10^{-6} m
 c. 2.6×10^{-4} m d. 1.3×10^{-4} m
36. The energy (E) of a photon is equal to the kinetic energy of proton. Let λ_1 be the de-Broglie wavelength of the proton and λ_2 be the wavelength of the photon. The ratio of λ_1 and λ_2 is proportional to
 a. E^{-2} b. E^{-1} c. $E^{1/2}$ d. E^0
37. The transition from the state $n=4$ to $n=3$ in a hydrogen atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from
 a. $3 \rightarrow 2$ b. $4 \rightarrow 2$
 c. $5 \rightarrow 4$ d. $2 \rightarrow 1$
38. In a transistor the base is
 a. an insulator
 b. a conductor of low resistance
 c. a conductor of high resistance
 d. an extrinsic semi-conductor
39. In a common emitter transistor, the base current $I_b = 2 \mu\text{A}$, $\alpha = 0.9$, then I_c is equal to
 a. $18 \mu\text{A}$ b. $20 \mu\text{A}$
 c. $22 \mu\text{A}$ d. $24 \mu\text{A}$
40. The antenna of an AM broadcast transmitter modulated by 50% is 11A. Find the carrier current.
 a. 9.25 A b. 10A
 c. 10.35 A d. 5.5 A

PART II

Chemistry



The product D has functional group

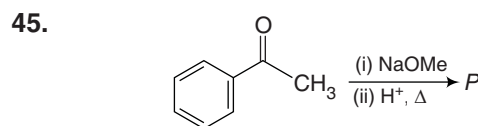
- a. amide b. alcoholic c. ether d. ester
42. Choose the correct statement from the following.
 a. BF_3 and NF_3 are covalent compounds and both are polar in nature
 b. BF_3 and NF_3 are covalent compounds but BF_3 is a non-polar, while NF_3 is polar compound
 c. BF_3 and NF_3 are covalent compounds but BF_3 is polar while NF_3 is a non-polar compound
 d. BF_3 and NF_3 are covalent compounds and both are non-polar in nature

43. The freezing point depression of a 1.00×10^{-3} m solution of $\text{K}_x[\text{Fe}(\text{CN})_6]$ is 7.10×10^{-3} K. Calculate the value of x . ($K_f = 1.86 \text{ K kg mol}^{-1}$ for H_2O)

a. 2 b. 3 c. 4 d. 1

44. Given below are several electronic configuration that may be correct for oxygen atom. Which of the following represents the ground state configuration for oxygen?

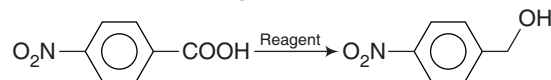
a. $\uparrow\downarrow \uparrow\downarrow \uparrow\downarrow \uparrow \text{ — } \downarrow$
 b. $\uparrow\downarrow \uparrow\downarrow \downarrow \downarrow \downarrow\downarrow \text{ — }$
 c. $\uparrow\downarrow \uparrow\downarrow \uparrow\downarrow \uparrow\downarrow \text{ — — }$
 d. $\uparrow\downarrow \uparrow\downarrow \uparrow\downarrow \uparrow \uparrow \text{ — }$
 1s 2s 2p 3s



The product P is

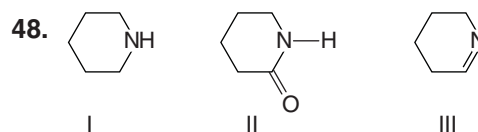
- a. acetone b. α, β -unsaturated ketone
 c. α, β -unsaturated acid d. None of these
46. The decreasing order of boiling points of the following is
 I. butanal II. butan-1-ol
 III. pentane IV. diethyl ether
 a. $\text{IV} > \text{III} > \text{II} > \text{I}$ b. $\text{IV} > \text{II} > \text{III} > \text{I}$
 c. $\text{II} > \text{I} > \text{IV} > \text{III}$ d. $\text{I} > \text{IV} > \text{II} > \text{III}$

47. Consider the following reaction,



The reagent is

- a. LAH b. $\text{B}_2\text{H}_6/\text{THF}$
 c. Both a and b d. None of these



The correct decreasing order of basic strengths of I, II and III is

- a. $\text{I} > \text{III} > \text{II}$ b. $\text{II} > \text{III} > \text{I}$
 c. $\text{I} > \text{II} > \text{III}$ d. $\text{II} > \text{I} > \text{III}$

49. Which of the following is/are correct?

- I. XeF_6 has lone pair of electron.
 - II. XeF_4 has two lone pairs of electrons with sp^3d^2 hybridisation.
 - III. XeOF_2 has dsp^2 -hybridisation.
 - IV. XeO_3 has square planar geometry.
- a. Only I
 - b. I and II
 - c. III and IV
 - d. Only IV

50. Which one undergoes fastest nucleophilic substitution reaction?

- a. Benzyl chloride
- b. Allyl chloride
- c. Formaldehyde
- d. Acetaldehyde

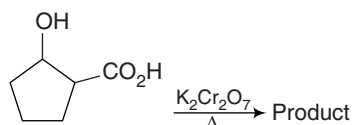
51. Match the following columns and choose the correct code given below.

Column I	Column II
P. $\text{H}_2(\text{g}) + \text{I}_2(\text{g}) \rightleftharpoons 2\text{HI}(\text{g})$	(i) $K_p = K_c(RT)$
Q. $\text{N}_2(\text{g}) + 3\text{H}_2(\text{g}) \rightleftharpoons 2\text{NH}_3(\text{g})$	(ii) $K_p = K_c(RT)^2$
R. $\text{PCl}_5(\text{g}) \rightleftharpoons \text{PCl}_3(\text{g}) + \text{Cl}_2(\text{g})$	(iii) $K_p = K_c(RT)^{-2}$
S. $\text{NH}_4\text{Cl}(\text{s}) \rightleftharpoons \text{NH}_3(\text{g}) + \text{HCl}(\text{g})$	(iv) $K_p = K_c$

Codes

- | | | | | | | | |
|----------|-------|------|-------|---------|-------|------|-------|
| P | Q | R | S | P | Q | R | S |
| a. (i) | (iii) | (iv) | (iii) | b. (iv) | (iii) | (i) | (ii) |
| c. (iii) | (iv) | (ii) | (i) | d. (i) | (iv) | (ii) | (iii) |

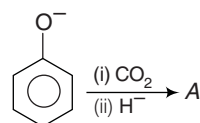
52.



What is the product of the following reaction?

- | | |
|----|----|
| a. | b. |
| c. | d. |

53.



Which one of the following statements is true about the reaction?

- a. Para isomer is major, if PhOK is used
- b. Ortho isomer is major, if PhONa is used
- c. Product formed is further used for preparation of drug aspirin
- d. All of the above

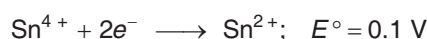
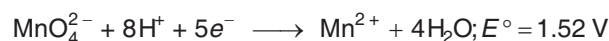
54. A mixture of solid SrSO_4 and solid BaSO_4 is shaken up with water until saturated equilibrium is established. Given that,

$$K_{\text{sp}}(\text{SrSO}_4) = 7.5 \times 10^{-7}, K_{\text{sp}}(\text{BaSO}_4) = 1.5 \times 10^{-9}$$

Then, the concentration of SO_4^{2-} is

- a. $8.52 \times 10^{-3} \text{ M}$
- b. $8.67 \times 10^{-4} \text{ M}$
- c. $1.73 \times 10^{-6} \text{ M}$
- d. $1.82 \times 10^{-4} \text{ M}$

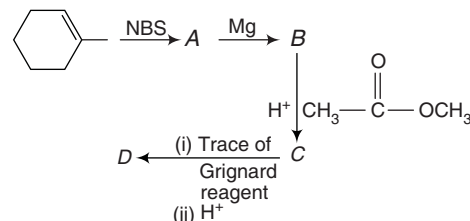
55. The value of E° of some elements are as



On the basis of values of E° , which of the following reactions will be spontaneous?

- a. $\text{Sn}^{4+} + 2\text{Fe}^{2+} \longrightarrow \text{Sn}^{2+} + 2\text{Fe}^{3+}$
- b. $2\text{Fe}^{2+} + \text{I}_2 \longrightarrow 2\text{Fe}^{3+} + 2\text{I}^-$
- c. $\text{Sn}^{4+} + 2\text{I}^- \longrightarrow \text{Sn}^{2+} + \text{I}_2$
- d. $\text{Sn}^{2+} + \text{I}_2 \longrightarrow \text{Sn}^{4+} + 2\text{I}^-$

56.



What would be the product D in the above reaction sequence?

- a. 1° alcohol
- b. 2° alcohol
- c. 3° alcohol
- d. cyclohexene

57. The monomer of the polymer nylon-6, 10 is

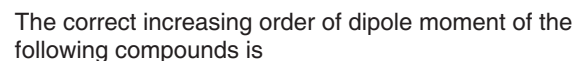
- a. hexamethylene and sebacic acid
- b. hexamethylene and adipic acid
- c. caprolactum
- d. None of the above

58. A dye absorbs light ray of wavelength (λ) 4530 Å and re-emits radiation of wavelength 5080 Å. If we assume that 47% of energy absorbed is emitted, then the ratio of quanta emitted and absorbed is

- a. 0.231
- b. 0.527
- c. 0.412
- d. 0.612

59. On the basis of electronic configuration, if the ion X^{3-} has 14 electrons, then the number of electrons present in X^{2+} is

- a. 16
- b. 14
- c. 12
- d. 18



- The correct IUPAC name of compound X is

- 62.** Standard heat of formation (ΔH_f°) of PCl_5 and PCl_3 are x_1 and x_2 respectively. The ΔH° for the reaction $\text{PCl}_3(g) + \text{Cl}_2(g) \longrightarrow \text{PCl}_5(g)$ is

- 63.** A unit cell of sodium chloride has four formula units. Calculate the density of sodium chloride, if the edge length of the unit cell is 0.564 nm.

- 64.** Which of the following reactions occurs at the anode during the recharging of lead storage battery?

- $\text{PbSO}_4 + 2e^- \longrightarrow \text{Pb} + \text{SO}_4^{2-}$
- $\text{Pb} + \text{SO}_4^{2-} \longrightarrow \text{PbSO}_4 + 2e^-$
- $\text{Pb} + \text{PbO}_2 + \text{H}_2\text{SO}_4 \longrightarrow 2\text{PbSO}_4 + 2\text{H}_2\text{O}$
- $\text{PbSO}_4 + 2\text{H}_2\text{O} \longrightarrow \text{Pb} + \text{PbO}_2 + 2\text{H}_2\text{SO}_4 + 2e^-$

- 65.** Which of the following acids on heating loses a molecule of H_2O to form an α,β -unsaturated acid?

- a. HOCH_2COOH b. $\text{CH}_3\text{CHOHCH}_2\text{COOH}$
c. $\text{CH}_3\text{CHOHCOOH}$ d. $\text{HOCH}_2\text{CH}_2\text{CH}_2\text{COOH}$

- 66.** For a reaction,



Order of reaction w.r.t NO and H₂ respectively, are

- a. 1, 2 b. 1, 1 c. 1, 3 d. 2, 2

- 67.** The number of electrons exchanged in the reaction given below



68. $\text{HC}\equiv\text{CH} \xrightarrow[\text{NaNH}_2]{1 \text{ mol}} \text{A} \xrightarrow{\text{C}_2\text{H}_5\text{Br}} \text{B} \xrightarrow{\text{NaNH}_2} \text{C} \xrightarrow{\text{CH}_3\text{I}} \text{D}$
 Ethyne

The product D of the following reaction is

- a. pent-2-yne b. pent-2-ene
c. but-2-yne d. but-1-yne

- 69.** The separation of a racemic mixture into the pure enantiomers is known as

- a. resolution b. racemisation
c. isomerisation d. equilibration



- a. $2\text{CH}_3\text{CH}_2\text{OH}$, no reaction, $2\text{CH}_3\text{CH}_2\text{OH}$
 b. $2\text{CH}_3\text{CH}_2\text{OH}$, no reaction, no reaction
 c. $2\text{CH}_3\text{CH}_2\text{OH}$, $2\text{CH}_3\text{CH}_2\text{OH}$, $2\text{CH}_3\text{CH}_2\text{OH}$
 d. no reaction, no reaction, $2\text{CH}_3\text{CH}_2\text{OH}$

- 71.** Consider the following reaction,



- Formation of (I) and (II) proceeds via S_N1
- Formation of (I) proceeds via S_N1 while (II)
- Formation of (I) and (II) proceed via S_N2
- Formation of (I) proceed via, S_N2 while (II) via S_N1

- 72.** The equivalent mass of which of the following (underlined> is 8 g equiv^{-1} ?

- a. $\text{H}_3\text{PO}_4 + \text{Ca}(\text{OH})_2 \longrightarrow \text{CaHPO}_4 + 2\text{H}_2\text{O}$
 b. $\text{H}_3\text{PO}_4 + 3\text{NaOH} \longrightarrow \text{Na}_3\text{PO}_4 + 3\text{H}_2\text{O}$
 c. $\text{H}_3\text{PO}_4 + \text{NaOH} \longrightarrow \text{NaH}_2\text{PO}_4 + \text{H}_2\text{O}$
 d. $2\text{H}_2 + \text{O}_2 \longrightarrow 2\text{H}_2\text{O}$

- 73.** Analysis of chlorophyll shows that it contains 2.68 per cent magnesium. How many atoms of magnesium does 1.00 g of chlorophyll contain?

- a. 8.12×10^{21} b. 5.92×10^{21}
c. 6.72×10^{20} d. 5.21×10^{20}

- 74.** The artificial sweetener used in soft drinks is

- a. fructose
- b. cellulose
- c. glucose
- d. aspartame

75. Silver chloride dissolves in a solution of ammonia but not in water because
 a. ammonia is a stronger base than water
 b. ammonia is a better solvent than water
 c. silver ion forms a complex ion with ammonia
 d. the dipole moment of water molecule is higher than that of ammonia molecule
76. When NaCl or KCl is heated with conc. H_2SO_4 and solid $\text{K}_2\text{Cr}_2\text{O}_7$, the product is
 a. chromous chloride b. chromic chloride
 c. chromic sulphate d. chromyl chloride
77. The IUPAC name of the complex $\text{fac} - [\text{Co}(\text{CH}_3\text{NH}_2)_3\text{Cl}_3]$ is
 a. *fac*-trichlorotris(methylamine)cobalt(V)
 b. *fac*-trichlorotris(methylamine)cobalt(III)
 c. *tris*(methylamine)trichlorocobalt(III)
 d. *tris*(methylamine)trichlorocobalt(V)
78. Out of TiF_6^{2-} , CoF_6^{3-} , Cu_2Cl_2 and NiCl_4^{2-} , the colourless species are (Given atomic number of Ti = 22, Co = 27, Cu = 29 and Ni = 28)
 a. CoF_6^{3-} and NiCl_4^{2-} b. Cu_2Cl_2 and NiCl_4^{2-}
 c. TiF_6^{2-} and CoF_6^{3-} d. TiF_6^{2-} and Cu_2Cl_2
79. NH_3 has a net dipole moment but boron trifluoride (BF_3) has zero dipole moment, because
 a. F is more electronegative than H
 b. B is less electronegative than N
 c. NH_3 is pyramidal, while BF_3 is trigonal planar
 d. BF_3 is pyramidal, while NH_3 is planar
80. The antibiotic streptomycin is specific against which of the following disease?
 a. Typhoid b. Malaria
 c. AIDS d. Tuberculosis

PART III

a. English Proficiency

Directions (Q. Nos. 81 to 84) *Read the following passage carefully and choose the most suitable option from the given ones.*

An old man steel-rimmed spectacles and very dusty clothes sat by the side of the road. There was a pontoon bridge across the river and carts, truck and men, women and children were crossing it. The mule-drawn cart staggered up the steep bank from the bridge with soldiers helping to push against the spokes of the wheels. The truck wound up the away heading out of it all. The peasants plodded along in the ankle-deep dust. But the old man sat there without moving.

81. The term 'pontoon bridge' means
 a. a temporary bridge constructed with the help of ropes
 b. a bridge made by soldiers during a war
 c. a bridge supported by floating flat-bottomed boats
 d. a bridge made with wooden planks
82. The soldiers were 'helping to push against the spokes of the wheels' because
 a. they wanted to stop the carts
 b. the spokes of the wheels were broken
 c. the mules refused to draw the carts
 d. there was a steep elevation
83. The mule-drawn carts staggered up because
 a. there were too many mule-carts
 b. the mules were undisciplined
 c. it was a steep uphill journey
 d. the carts were blocked by the peasants
84. The reference to the old man in the beginning and the end of the passage indicates that
 a. the writer want to compare between the bridge and motionless old man
 b. the description that takes place between the first sentence and the last sentence is irrelevant

- c. there is an unnecessary repetition in the reference to the old man
 d. the figure of the old man is brought under a sharp focus

Directions (Q. Nos. 85 to 87) *In the following sentences, there is an error. Spot the error and indicate your answer.*

85. Order has been issued (a)/for his transfer to another district (b)/but he has not received them so far. (c)/No error (d)
86. Although she has studied (a)/English for almost a year (b)/she is yet to learn the alphabets. (c)/No error (d)
87. There are two scores of books (a)/which are lying (b)/unused in the library. (c)/No error (d)

Directions (Q. Nos. 88 to 90) *Find the word from the options that is synonym to the word written in capital letters.*

88. We should not allow ADVERSITY to discourage us.
 a. Poverty b. Darkness
 c. Time of trouble d. Unfriendly criticism
89. He is very OBLIGING by nature.
 a. Helpful b. Nice c. Thankful d. Compelling
90. She thought that RUNNING INTO her old friend was a strokes of fate.
 a. Meeting by chance b. Colliding
 c. Travelling with d. Quarrelling with

Directions (Q. Nos. 91 to 93) *Find the word from the options that is antonym to the word written in capital letters.*

91. The COMPLAINANT was not supportive of providing all facts in the court.
 a. Defendant b. Advocate
 c. Indulgence d. Servant

92. Living in a SOLITARY place brings in some kind of satisfaction.

- a. Limited b. Exotic
c. Healthy d. Populous

93. They spent a DISTURBED night after hearing the tragic news.

- a. Restless b. Sleepless
c. Mournful d. Peaceful

Directions (Q. Nos. 94 and 95) Find one word for the set of words given below.

94. One who knows everything.

- a. Pedantic b. Omniscient
c. Knowledgeous d. Educated

95. Aversion from water

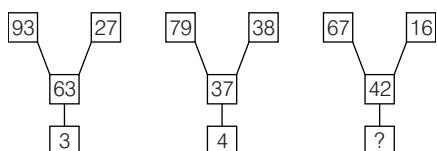
- a. Hydrophobic b. Aquatic
c. Hydrophilic d. Exaquatic

b. Logical Reasoning

96. In a certain code, SOBER is written as RNADQ. How LOTUS can be written in that code?

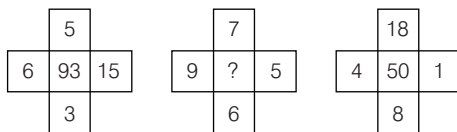
- a. KNSTR b. MPUWT
c. KMSTR d. LMSRT

97. Find the missing number in the following figure.



- a. 5 b. 19 c. 27 d. 9

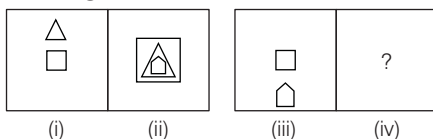
98. Find the missing number in the following figure.



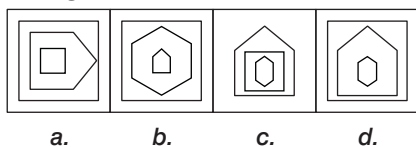
- a. 5 b. 19 c. 27 d. 89

99. Find the figure will that replace the question mark (?).

Problem Figures

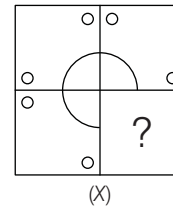


Answer Figures

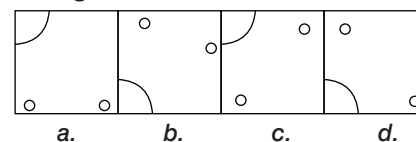


100. Select a figure from the given four alternatives which when placed in the missing portion (?) of problem figure (X) would complete the pattern.

Problem Figure



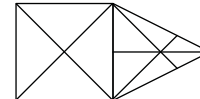
Answer Figures



101. Mohit is 14th from the right end in a row of 40 boys. What is his position from the left end ?

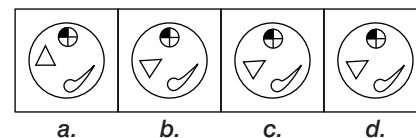
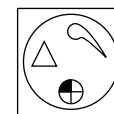
- a. 24th
b. 25th
c. 27th
d. None of these

102. The number of triangles in the figure is



- a. 24 b. 21
c. 14 d. 11

103. Which of the following is the water image of the given figure?



Directions (Q.Nos. 104 and 105) Choose the odd one out.

104. a. Home
b. City
c. Town
d. Village
105. a. Parrot
b. Owl
c. Eagle
d. Hawk

PART IV

Mathematics

106. The sum of order and degree of the differential equation $y_3^{2/3} + 2 + 3y_2 + y_1 = 0$ is
 a. 2 b. 3 c. 4 d. 5
107. The solution of the inequation $\left(\frac{1}{3}\right)^{\frac{|x+2|}{2-|x|}} > 9$ is
 a. (2, 4) b. (4, 6) c. (2, 6) d. None
108. If $z^2 + z|z| + |z|^2 = 0$, then the locus of z is
 a. a circle b. an ellipse
 c. a pair of straight lines d. None of these
109. If α, β, γ are the roots of $x^3 - x^2 - 1 = 0$, then the value of $\frac{(1+\alpha)}{(1-\alpha)} + \frac{(1+\beta)}{(1-\beta)} + \frac{(1+\gamma)}{(1-\gamma)}$ is
 a. -2 b. -3 c. -4 d. -5
110. If AM, GM and HM of the first and last terms of the series 100, 101, 102, ..., $n-1, n$ are the terms of the series itself, then the value of n ($100 < n \leq 500$) is
 a. 200 b. 300 c. 400 d. 500
111. A train time table must be compiled for various days of the week, so that two trains twice a day depart for three days, one train daily for two days, and three trains once a day for two days. How many different time table can be compiled?
 a. 120 b. 210 c. 133 d. 72
112. If $C_0, C_1, C_2, \dots, C_n$ are the binomial coefficients, then $2 \times C_1 + 2^3 \times C_3 + 2^5 \times C_5 + \dots$ equals
 a. $\frac{3^n + (-1)^n}{2}$ b. $\frac{3^n - (-1)^n}{2}$ c. $\frac{3^n + 1}{2}$ d. $\frac{3^n - 1}{2}$
113. In $\triangle ABC$, if

$$\begin{vmatrix} 1 & 1 & 1 \\ \cot \frac{A}{2} & \cot \frac{B}{2} & \cot \frac{C}{2} \\ \tan \frac{B}{2} + \tan \frac{C}{2} & \tan \frac{C}{2} + \tan \frac{A}{2} & \tan \frac{A}{2} + \tan \frac{B}{2} \end{vmatrix} = 0,$$
 then the triangle must be
 a. equilateral b. isosceles
 c. obtuse d. None of these
114. If A and B are square matrices of the same order and A is non-singular, then for a positive integer n , $(A^{-1}BA)^n$ is equal to
 a. $A^{-n}B^nA^n$ b. $A^nB^nA^{-n}$
 c. $A^{-1}B^nA$ d. $n(A^{-1}BA)$
115. Consider $f(x) = x^3 + ax^2 + bx + c$. Parameters a, b and c are chosen, respectively, by throwing a die three times. Then, the probability that $f(x)$ is an increasing function, is
 a. $\frac{5}{36}$ b. $\frac{8}{36}$ c. $\frac{4}{9}$ d. $\frac{1}{3}$
116. If $[x]$ and $\{x\}$ represent integral and fractional parts of x , then the expression $[x] + \sum_{r=1}^{2000} \frac{\{x+r\}}{2000}$ is equal to
 a. $\frac{2001}{2}x$ b. $x + 2001$ c. x d. $[x] + \frac{2001}{2}$
117. The value of $\lim_{x \rightarrow 0^+} x^m (\log x)^n, m, n \in N$, is
 a. 0 b. m/n c. mn d. n/m
118. If $f(x) = \begin{cases} \frac{1-|x|}{1+x}, & x \neq 1 \\ 1, & x = -1 \end{cases}$, where $[]$ represents the greatest integer function, then $f([2x])$ is
 a. continuous at $x = -1$
 b. continuous at $x = 0$
 c. discontinuous at $x = \frac{1}{2}, 1$
 d. All of the above
119. If $f(x) = (1-x)^n$, then the value of $f(0) + f'(0) + \frac{f''(0)}{2!} + \dots + \frac{f^n(0)}{n!}$ is
 a. 2^n b. 0 c. 2^{n-1} d. None
120. Let $F(x) = f(x)g(x)h(x)$ for all real x , where $f(x), g(x)$ and $h(x)$ are differentiable functions. At some point x_0 , if $F'(x_0) = 21f(x_0), f'(x_0) = 4f(x_0), g'(x_0) = -7g(x_0)$ and $h'(x_0) = \lambda h(x_0)$, then λ is equal to
 a. 12 b. -12 c. 24 d. -24
121. If $y = 4x - 5$ is a tangent to the curve $y^2 = px^3 + q$ at (2, 3), then
 a. $p = 2, q = -7$ b. $p = -2, q = 7$
 c. $p = -2, q = -7$ d. $p = 2, q = 7$
122. If $f(x) = x^\alpha \log x$ and $f(0) = 0$, then the value of α for which Rolle's theorem can be applied in $[0, 1]$ is
 a. -2 b. -1 c. 0 d. 1/2
123. The minimum value of $27^{\cos 2x} \cdot 81^{\sin 2x}$ is
 a. 1/243 b. -5
 c. 1/5 d. 1/3
124. If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$
 a. $f(x)$ is strictly increasing function
 b. $f(x)$ has a local maxima
 c. $f(x)$ is a strictly decreasing function
 d. $f(x)$ is unbounded

125. $(1+x)^n \leq 1+x^n$, where
 a. $n > 1$ b. $0 \leq n \leq 1$ and $x > 0$
 c. $n > 1$ and $x > 0$ d. $x < 0$
126. If $y = f(x)$ makes positive intercepts of 2 units and 1 unit on x and y -coordinate axes and encloses an area of $3/4$ sq unit with the axes, then $\int_0^2 x f'(x) dx$ is
 a. $\frac{3}{2}$ b. 1 c. $\frac{5}{4}$ d. $-\frac{3}{4}$
127. $\int \sqrt{x-3} \{\sin^{-1}(\ln x) + \cos^{-1}(\ln x)\} dx$ is equal to
 a. $\frac{\pi}{3}(x-3)^{3/2} + C$ b. 0
 c. Does not exist d. None of these
128. Let f be a positive function. If $I_1 = \int_{1-k}^k x f\{x(1-x)\} dx$, $I_2 = \int_{1-k}^k f\{x(1-x)\} dx$, where $2k-1=0$, then $\frac{I_1}{I_2}$ is
 a. 2 b. k c. $1/2$ d. 1
129. The equation of the line which is parallel to the line common to the pair of lines given by $6x^2 - xy - 12y^2 = 0$ and $15x^2 + 14xy - 8y^2 = 0$ and at a distance of 7 units from it, is
 a. $3x - 4y = 35$ b. $5x - 2y = 7$
 c. $3x + 4y = 35$ d. $2x - 3y = 7$
130. If $ax + by = 1$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $a^2 - b^2$ is equal to
 a. $1/a^2 e^2$ b. $a^2 e^2$
 c. $b^2 e^2$ d. None
131. The normal at a variable point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ of eccentricity e meets the axes of the ellipse at Q and R . Then, the locus of the mid-point of QR is conic with eccentricity e' such that
 a. e' is independent of e b. $e' = 1$
 c. $e' = e$ d. $e' = \frac{1}{e}$
132. Let $y = f(x)$ be a parabola having its axis parallel to the Y -axis, which is touched by the line $y = x$ at $x = 1$, then
 a. $2f(0) = 1 - f'(0)$ b. $f(0) + f'(0) + f''(0) = 1$
 c. $f'(1) = 1$ d. $f'(0) = f'(1)$
133. In $\triangle ABC$, right-angled at A , on the leg AC as diameter, a semi-circle is described. If a chord joins A with the point of intersection D of the hypotenuse and semi-circle, then the length of AC is equal to
 a. $\frac{AB \cdot AD}{\sqrt{AB^2 + AD^2}}$ b. $\frac{AB \cdot AD}{AB^2 + AD^2}$
 c. $\sqrt{AB \cdot AD}$ d. $\frac{AB \cdot AD}{\sqrt{AB^2 - AD^2}}$
134. If the point $(a^2, a+1)$ lies in the angle between the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ containing the origin, then the value of a is
 a. $a \in (3, 0) \cap \left(1, \frac{1}{3}\right)$ b. $a \in (5, 0) \cap (0, 5)$
 c. $a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$ d. None of these
135. If $\mathbf{a} = -\hat{i} + \hat{j} + \hat{k}$, $\mathbf{b} = 2\hat{i} + 0\hat{j} + \hat{k}$, then a vector $\mathbf{x}$ satisfying the conditions that it is coplanar with $\mathbf{a}$ and $\mathbf{b}$; perpendicular to $\mathbf{b}$ and $\mathbf{a} \cdot \mathbf{x} = 7$, is
 a. $-3\hat{i} + 5\hat{j} + 6\hat{k}$ b. $\frac{1}{2}(-3\hat{i} + 5\hat{j} + 6\hat{k})$
 c. $\frac{1}{2}(3\hat{i} + 5\hat{j} - 6\hat{k})$ d. $3\hat{i} - 5\hat{j} + 6\hat{k}$
136. Let $\mathbf{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\mathbf{b} = 3\hat{i} + 3\hat{j} - \hat{k}$ and $\mathbf{c} = d\hat{i} + \hat{j} + (2d-1)\hat{k}$. If $\mathbf{c}$ is parallel to the plane of the vectors $\mathbf{a}$ and $\mathbf{b}$, then $11d$ is equal to
 a. 2 b. 1 c. -1 d. 0
137. If the vectors $\mathbf{a} = (2, \log_3 x, a)$ and $\mathbf{b} = (-3, a \log_3 x, \log_3 x)$ are inclined at an acute angle, then
 a. $a = 0$ b. $a < 0$ c. $a > 0$ d. None
138. The coordinates of the foot of the perpendicular drawn from the point $A(1, 0, 3)$ to the join of the points $B(4, 7, 1)$ and $C(3, 5, 3)$ are
 a. $(5/3, 7/3, 17/3)$ b. $(5, 7, 17)$
 c. $(5/7, -7/3, 17/3)$ d. $(-5/3, 7/3, -17/3)$
139. The solution set of the inequality $\log_3(x+2)(x+4) + \log_{1/3}(x+2) < (1/2)\log_{\sqrt{3}} 7$ is
 a. $(-2, -1)$ b. $(-2, 3)$ c. $(-1, 3)$ d. $(3, \infty)$
140. Let $A = \sin^8 \theta + \cos^{14} \theta$, then for every real θ ,
 a. $A \geq 1$ b. $0 < A \leq 1$
 c. $\frac{1}{2} < A \leq \frac{3}{2}$ d. None of these
141. If $y = (1 + \tan A)(1 - \tan B)$, where $A - B = \frac{\pi}{4}$, then $(y+1)^{y+1}$, is equal to
 a. 9 b. 4 c. 27 d. None
142. If $\frac{1}{6} \sin \theta, \cos \theta, \tan \theta$ are in GP, then θ is equal to $(n \in \mathbb{Z})$
 a. $2n\pi \pm \frac{\pi}{3}$ b. $2n\pi \pm \frac{\pi}{6}$
 c. $n\pi + (-1)^n \frac{\pi}{3}$ d. $n\pi + \frac{\pi}{3}$
143. $\sin^{-1}(\sin 5) > x^2 - 4x$ holds, if
 a. $x = 2 - \sqrt{9 - 2\pi}$
 b. $x = 2 + \sqrt{9 - 2\pi}$
 c. $x > 2 + \sqrt{9 - 2\pi}$
 d. $x \in (2 - \sqrt{9 - 2\pi}, 2 + \sqrt{9 - 2\pi})$

144. In $\triangle ABC$, if $a^2 + b^2 + c^2 = ac + ab\sqrt{3}$, then the triangle is
 a. equilateral b. isosceles
 c. right angled d. None of these
145. Let $f(x) = x^2$ and $g(x) = 2^x$. Then, the solution set of the equation $fog(x) = gof(x)$ is
 a. R b. $\{0\}$
 c. $\{0, 2\}$ d. None
146. The negation of the proposition "If a number is divisible by 15, then it is divisible by 5 or 3", is
 a. If a number is divisible by 15, then it is not divisible by 5 and 3
 b. A number is divisible by 15 and it is not divisible by 5 and 3
 c. A number is divisible by 15 and it is not divisible by 5 or 3
 d. A number is not divisible by 15 or it is not divisible by 5 and 3
147. If X is a binomial variate with parameters n and p , where $0 < p < 1$ such that $\frac{p(X=r)}{p(X=n-r)}$ is independent of n and r , then p equals
 a. $1/2$ b. $1/3$
 c. $1/4$ d. None of these
148. For $n \in N$, $x^{n+1} + (x+1)^{2n-1}$ is divisible by
 a. x b. $x+1$
 c. $x^2 + x + 1$ d. $x^2 - x + 1$
149. If mode of a data exceeds its mean by 12, then mode exceeds the median by
 a. 4 b. 8 c. 6 d. 10
150. In a series of $2n$ observations, half of them equal a and remaining half equal $-a$. If the standard deviation of the observations is 2, then $|a|$ equals
 a. $\frac{\sqrt{2}}{n}$ b. $\sqrt{2}$ c. 2 d. $\frac{1}{n}$

Answers

Physics

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (a) | 2. (a) | 3. (b) | 4. (a) | 5. (a) | 6. (a) | 7. (c) | 8. (a) | 9. (c) | 10. (c) |
| 11. (c) | 12. (d) | 13. (b) | 14. (c) | 15. (a) | 16. (c) | 17. (a) | 18. (b) | 19. (b) | 20. (d) |
| 21. (b) | 22. (d) | 23. (d) | 24. (c) | 25. (c) | 26. (d) | 27. (a) | 28. (b) | 29. (c) | 30. (b) |
| 31. (d) | 32. (c) | 33. (c) | 34. (b) | 35. (b) | 36. (c) | 37. (c) | 38. (b) | 39. (a) | 40. (c) |

Chemistry

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 41. (d) | 42. (b) | 43. (b) | 44. (d) | 45. (b) | 46. (b) | 47. (b) | 48. (a) | 49. (b) | 50. (a) |
| 51. (b) | 52. (a) | 53. (d) | 54. (b) | 55. (d) | 56. (c) | 57. (a) | 58. (b) | 59. (c) | 60. (b) |
| 61. (b) | 62. (b) | 63. (a) | 64. (a) | 65. (b) | 66. (b) | 67. (c) | 68. (a) | 69. (a) | 70. (a) |
| 71. (c) | 72. (d) | 73. (c) | 74. (d) | 75. (c) | 76. (d) | 77. (b) | 78. (d) | 79. (c) | 80. (d) |

English Proficiency

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 81. (c) | 82. (d) | 83. (c) | 84. (d) | 85. (a) | 86. (a) | 87. (a) | 88. (c) | 89. (b) | 90. (a) |
| 91. (a) | 92. (d) | 93. (d) | 94. (b) | 95. (a) | | | | | |

Logical Reasoning

- | | | | | | | | | | |
|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| 96. (a) | 97. (d) | 98. (d) | 99. (c) | 100. (c) | 101. (c) | 102. (a) | 103. (c) | 104. (a) | 105. (a) |
|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|

Mathematics

- | | | | | | | | | | |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 106. (d) | 107. (c) | 108. (c) | 109. (d) | 110. (c) | 111. (b) | 112. (b) | 113. (b) | 114. (c) | 115. (c) |
| 116. (c) | 117. (a) | 118. (d) | 119. (b) | 120. (c) | 121. (a) | 122. (c) | 123. (a) | 124. (a) | 125. (b) |
| 126. (d) | 127. (c) | 128. (c) | 129. (c) | 130. (a) | 131. (c) | 132. (a) | 133. (d) | 134. (c) | 135. (b) |
| 136. (c) | 137. (d) | 138. (a) | 139. (b) | 140. (b) | 141. (c) | 142. (a) | 143. (d) | 144. (c) | 145. (c) |
| 146. (b) | 147. (a) | 148. (c) | 149. (b) | 150. (c) | | | | | |

Practice Set 2

BITSAT

Instructions

- There are 150 questions in all. The number of questions in each part is as follows Subject
- All questions are multiple choice questions with four options, only one is correct.
- Each correct answer fetches 3 marks while incorrect answer has a penalty of 1 mark.

No. of Questions

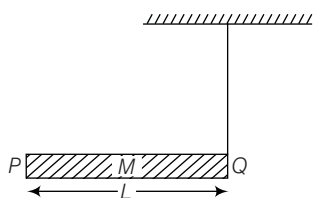
Part I Physics	1-40
Part II Chemistry	41-80
Part III (a) English Proficiency	81-95
(b) Logical Reasoning	96-105
Part IV Mathematics	106-150

PART I

Physics

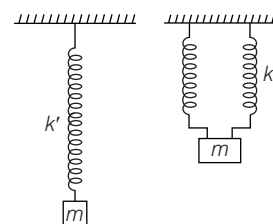
- Frequency f of oscillations of a mass m suspended from a spring of force constant k is given by $f = cm^x k^y$ where, c is a dimensionless constant. The values of x and y are
 - $x = -\frac{1}{2}, y = \frac{1}{2}$
 - $x = \frac{1}{2}, y = -\frac{1}{2}$
 - $x = -\frac{1}{2}, y = -\frac{1}{2}$
 - $x = \frac{1}{2}, y = \frac{1}{2}$
- Given $X = (Gh/c^3)^{1/2}$, where G, h and c are gravitational constant, Planck's constant and the velocity of light, respectively. Dimensions of X are the same as those of
 - mass
 - time
 - length
 - acceleration
- A projectile can have same range R for two angles of projection. If t_1 and t_2 be the time of flights in the two cases, then the product of time of flight is proportional to
 - $\frac{1}{R}$
 - $\frac{1}{R^2}$
 - R
 - R^2
- A body of mass m is accelerated uniformly from rest to a speed v in a time T . The instantaneous power delivered to the body as function of time is given by
 - $\frac{1}{2} \frac{mv^2}{T^2} t^2$
 - $\frac{1}{2} \frac{mv^2}{T^2} t$
 - $\frac{mv^2}{T^2} t^2$
 - $\frac{mv^2}{T^2} t$
- A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension, the string can bear is 324 N. The maximum possible value of angular velocity of ball (in rad/s) is
 - 9
 - 18
 - 27
 - 36
- A sphere of mass 1 kg is connected to a spring of spring constant 5.0 Nm^{-1} as shown in figure. A force of 0.5 N is applied on the sphere along X-axis. What is the velocity of the sphere when it is displaced through a distance of 10 cm along X-axis ?
 - 0.11 ms^{-1}
 - 0.22 ms^{-1}
 - 0.11 cms^{-1}
 - 0.22 cms^{-1}

7. Two identical balls A and B , collide head on elastically. If velocities of A and B , before collision are $+0.5 \text{ ms}^{-1}$ and -0.3 ms^{-1} respectively, then their velocities after the collision, are respectively
- -0.5 ms^{-1} and $+0.3 \text{ ms}^{-1}$
 - $+0.5 \text{ ms}^{-1}$ and -0.3 ms^{-1}
 - $+0.3 \text{ ms}^{-1}$ and -0.5 ms^{-1}
 - -0.3 ms^{-1} and $+0.5 \text{ ms}^{-1}$
8. A rod PQ of mass M and length L is hinged at end P . The rod is kept horizontal by a massless string tied to point Q as shown in figure. When string is cut, the initial angular acceleration of the rod is

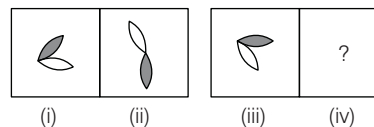


- $\frac{3g}{2L}$
 - $\frac{g}{L}$
 - $\frac{2g}{L}$
 - $\frac{2g}{3L}$
9. A satellite is moving with a constant speed v in a circular orbit about the earth. An object of mass m is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is
- $\frac{1}{2}mv^2$
 - mv^2
 - $\frac{3}{2}mv^2$
 - $2mv^2$
10. A solid sphere of volume (V) and density (ρ) floats at the interface of two immiscible liquids of densities ρ_1 and ρ_2 , respectively. If $\rho_1 < \rho < \rho_2$, then the ratio of the volume of the parts of the sphere in upper and lower liquids is
- $\frac{\rho + \rho_1}{\rho + \rho_2}$
 - $\frac{\rho - \rho_1}{\rho_2 - \rho}$
 - $\frac{\rho_2 - \rho}{\rho - \rho_1}$
 - $\frac{\rho + \rho_2}{\rho + \rho_1}$
11. Two rods of same material having same length and area. If heat ΔQ flows through them for 12 min when they are jointed side by side and now both rods are joined in, then the same amount of heat ΔQ will flow in
- 24 min
 - 18 min
 - 6 min
 - 3 min
12. A hole of area 1 mm^2 opens in the pipe near the lower end of a large water storage tank and a stream of water shoots from it. If the top of the water in the tank is 20 m above the point of the leak, then the amount of water escapes in 1 s is
- 87.5 cm^3
 - 43.1 cm^3
 - 27.5 cm^3
 - 19.8 cm^3
13. The pressure of a medium is changed from $1.01 \times 10^5 \text{ Pa}$ to 1.165 Pa and change in volume is 10% keeping temperature constant. The bulk modulus of the medium is
- $204.8 \times 10^5 \text{ Pa}$
 - $102.4 \times 10^5 \text{ Pa}$
 - $51.2 \times 10^5 \text{ Pa}$
 - $1.55 \times 10^5 \text{ Pa}$

14. A vessel contains 1 mole of O_2 gas (molar mass = 32) at a temperature T . The pressure of the gas is p . An identical vessel containing one mole of helium as (molar mass = 4) at a temperature $2T$ has a pressure of
- $\frac{p}{8}$
 - p
 - $2p$
 - $8p$
15. A refrigerator works between 2°C and 27°C . To keep the temperature of the refrigerated space constant, 660 calories of heat are to be removed every second. The power required is
- 60 W
 - 55 W
 - 252 W
 - 231 W
16. In an isothermal process, the pressure of a gas is decreased by 20%. The percentage change in the volume
- increase by 20%
 - decrease by 20%
 - increase by 25%
 - decrease by 25%
17. An object suspended from a spring exhibits oscillations of period T . Now, the spring is cut in two halves and the same object is suspended with two halves as shown in figure. The new period of oscillation will become

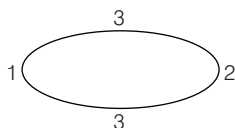


- $\frac{T}{2\sqrt{2}}$
 - $\frac{T}{2}$
 - $\frac{T}{\sqrt{2}}$
 - T
18. Four particles each having charge q are placed on the four vertices of a regular pentagon as shown in figure. The distance of each corner from the centre is a . The electric field at the centre of the pentagon is



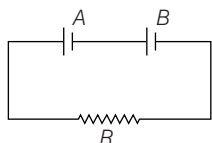
- $\frac{1}{4\pi\epsilon_0} \frac{q}{a^2}$ along OE
 - $\frac{1}{4\pi\epsilon_0} \frac{q}{a^2}$ along EO
 - $\frac{1}{4\pi\epsilon_0} \frac{q}{a^2}$
 - None of these
19. Two equal charges q are kept fixed at a and $+a$ along X -axis. A particles of mass m and charge $\left(\frac{q}{2}\right)$ is brought to the origin and given a small displacement along the X -axis, then
- the particle executes oscillatory motion
 - the particle remains stationary
 - the particle executes, SHM along X -axis
 - the particle executes SHM along Y -axis

20. Consider a non-spherical conductor shown in the figure which is given a certain amount of positive charge. The charge distributes itself on the surface such that the charge densities are σ_1 , σ_2 and σ_3 at regions 1, 2 and 3 respectively.

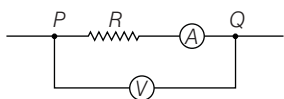


Then,

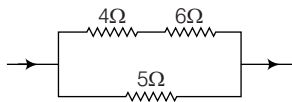
- a. $\sigma_1 > \sigma_2 > \sigma_3$ b. $\sigma_2 > \sigma_3 > \sigma_1$
 c. $\sigma_3 > \sigma_1 > \sigma_2$ d. $\sigma_2 > \sigma_1 > \sigma_3$
21. Two batteries A and B each of emf $2V$ are connected in series to an external resistance $R = 1\Omega$. If the internal resistance of battery A is 1.9Ω and that of B is 0.9Ω . What is the potential difference between the terminals of battery A?



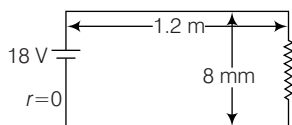
- a. $2V$ b. $3.8V$
 c. Zero d. None of these
22. 32 cells each of emf $3V$ are connected in series and kept in a box. Externally, the combination shows $84V$. The number of cells reversed in the connection is
- a. 0 b. 2 c. 8 d. 16
23. In the circuit, the voltmeter resistance is 10000Ω and the ammeter resistance is 2Ω . The voltmeter reads $12V$ and ammeter reads $0.1A$. The value of R is



- a. 118Ω b. 122Ω c. 10022Ω d. 10018Ω
24. In the circuit shown in figure, the heat produced in 5Ω resistor due to a current flowing in it is 10 cal s^{-1} . The heat produced in 4Ω resistor is

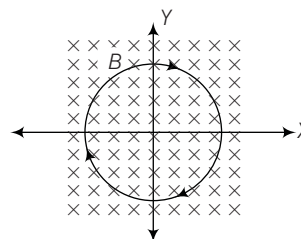


- a. 1 cal/s b. 2 cal/s c. 3 cal/s d. 4 cal/s
25. Force per unit length between parallel wires in the circuit is $3.6 \times 10^{-3} \text{ N/m}$. Resistance of the circuit is

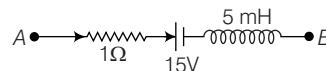


- a. 3Ω b. 1.5Ω c. 4.5Ω d. 6Ω

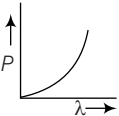
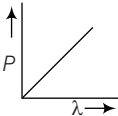
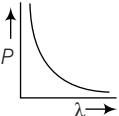
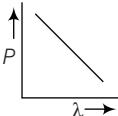
26. A conducting loop carrying a current I is placed in xy -plane. A uniform magnetic field pointing into Z -direction is switched on. The loop will have a tendency to



- a. contract
 b. expand
 c. move towards +ve X -axis
 d. move towards -ve X -axis
27. The magnetic susceptibility of a paramagnetic material at -73°C is 0.0075 , its value at -173°C will be
- a. 0.0045 b. 0.0030
 c. 0.015 d. 0.0075
28. Two similar poles of 60 Am and 240 Am are 12 cm apart. The magnetic field is zero at a position on the line joining them at
- a. 4 cm from 60 Am b. 6 cm from 240 Am
 c. 3 cm from 60 Am d. 8 cm from 240 Am
29. A conducting circular loop is placed in a uniform magnetic field 0.04 T with its plane perpendicular to the magnetic field. The radius of the loop starts shrinking at 2 mm/s . The induced emf in the loop when the radius is 2 cm is
- a. $4.8 \pi \mu\text{V}$ b. $0.8 \pi \mu\text{V}$
 c. $1.6 \pi \mu\text{V}$ d. $3.2 \pi \mu\text{V}$
30. An AC voltage, $e = 200 \sin 100t$ is applied to a series combination of $R = 30\Omega$ and an inductor of 400 mH . The power factor of the circuit is
- a. 0.01 b. 0.2 c. 0.5 d. 0.6
31. The network shown in figure is a part of a circuit. What is the potential difference $V_B - V_A$, when current I is $5A$ and is decreasing at a rate of 10^3 As^{-1} ?

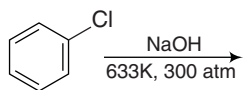
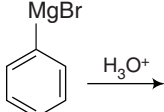
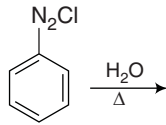
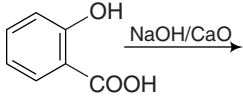


- a. $10V$ b. $5V$
 c. $15V$ d. $20V$
32. In a two slit experiment, with monochromatic light, fringes are obtained on a screen placed at some distance from the slits. If the screen is moved by $5 \times 10^{-2} \text{ m}$ towards the slits, the change in fringe width is 10^{-2} m . Then, the wavelength of light used when distance between slits is 0.3 mm will be
- a. $4000 \text{ \AA}$ b. $6000 \text{ \AA}$
 c. $5000 \text{ \AA}$ d. $7000 \text{ \AA}$

33. The intensity of the central fringe due to the interference of light from two similar slits is I . When one of the slit is closed, the intensity is I_0 . What is the relation between I and I_0 ?
 a. $I = 4I_0$ b. $I = 2I_0$
 c. $I = I_0$ d. $I = I_0/2$
34. Which one of the following showing correct relationship between power of a lens and wavelength of incident light?
- a.  b. 
- c.  d. 
35. The surface of a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photo electrons was found to be 1.68 eV. The work function of the metal is
 a. 1.41 eV b. 1.51 eV c. 1.68 eV d. 3.09 eV
36. If the wavelength of the first line of the Balmer series of hydrogen is 6561 Å, the wavelength of the second line of the series should be
 a. 1312 Å b. 3280 Å c. 4860 Å d. 2187 Å
37. Half-life of radioactive substance is 140 days. Initially, the mass of radioactive substance is 16g. Calculate the time for this substance when it reduces to 1g.
 a. 140 days b. 280 days
 c. 420 days d. 560 days
38. When a hydrogen atom emits a photon in going from $n = 5$ to $n = 1$, its recoil speed is almost
 a. 10^{-4} ms^{-1} b. $8 \times 10^2 \text{ ms}^{-1}$
 c. $2 \times 10^{-2} \text{ ms}^{-1}$ d. 4 ms^{-1}
39. In the ratio of the concentration of electrons and of holes in a semiconductor is 7/5 and the ratio of current is 7/4, then what is the ratio of their drift velocities ?
 a. $\frac{4}{7}$ b. $\frac{5}{8}$ c. $\frac{4}{5}$ d. $\frac{5}{4}$
40. An 80 MHz carrier is modulated by 400 Hz sine wave. The carrier voltage is 5V and the frequency deviation is 20 kHz. Find modulation index.
 a. 25 b. 50 c. 400 d. 5

PART II

Chemistry

41. On heating 1.763 g hydrated $\text{BaCl}_2 \cdot x\text{H}_2\text{O}$, 1.505 g anhydrous salt is obtained. The number of water molecules in $\text{BaCl}_2 \cdot x\text{H}_2\text{O}$ is
 a. 3 b. 2 c. 5 d. 1
42. The electron represented by the following sets of quantum numbers:
 I. $n = 4, l = 0, m_l = 0, m_s = +1/2$
 II. $n = 3, l = 1, m_l = 1, m_s = -1/2$
 III. $n = 3, l = 2, m_l = 0, m_s = +1/2$
 IV. $n = 3, l = 0, m_l = 0, m_s = -1/2$
 The correct increasing order of energy is
 a. IV > I > II > III b. III > I > II > IV
 c. II > I > IV > III d. I > IV > III > II
43. The phenol is not obtained in which of the following?
- a.  b. 
- c.  d. 
44. Determine the enthalpy change that occurs in the following isomerisation reaction,
 Propene (g) $\longrightarrow$ Cyclopropane (g)
 If BE (C—C) = 348 kJ/mol and BE (C=C) = 615 kJ/mol
 a. 81 kJ/mol b. -81 kJ/mol
 c. 75 kJ/mol d. -75 kJ/mol
45. In the sequence of reaction,
 $(\text{CH}_3)_2\text{CHOH} \xrightarrow[\text{Oxidation [O]}]{\text{Mild}} \text{X} \xrightarrow[\text{(ii) Hydrolysis}]{\text{(i) CH}_3\text{MgI}} \text{Y}$, Y is
 a. tertiary butyl alcohol b. n-butyl alcohol
 c. isobutylene d. isobutyl alcohol
46. The (—COOH) group is not present in which of the following?
 a. Benzoic acid b. Picric acid
 c. Aspirin d. All have (—COOH) group
47. Ethanamine is treated with nitrous acid at ordinary temperature, then the product will be
 a. only ethanol
 b. acetic acid, ethane and H_2O
 c. ethanol, ethene, N_2 and H_2O
 d. ethanol, acetic acid, N_2 and H_2O

48. Choose the correct statement from the following.

- H₂S and SO₂ both act as oxidising and reducing agent
- H₂S acts only as reducing agent while SO₂ acts both as oxidising and reducing agent
- H₂S acts only as oxidising agent while SO₂ acts as reducing agent
- SO₂ acts only as oxidising agent while H₂S acts both as oxidising and reducing agent

49. The weight of CaO required to remove hardness of 10⁶ L of water containing 1.62 g of Ca(HCO₃)₂ in 1L, is (M_W of Ca(HCO₃)₂ = 162 and M_W of CaO = 56)

- 5.6×10^2 mg
- 5.6×10^5 g
- 4.2×10^4 mg
- 4.2×10^4 g

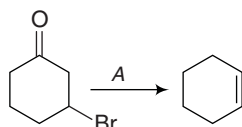
50. For concentration of sulphide ores, froth floatation process is used which is based on

- specific gravity of ore particles
- electrical properties of ore particles
- wetting properties of ore particles
- magnetic properties of ore particles

51. Which of the following will show optical activity?

- trans*-[Co(NH₃)₄Cl₂]⁺
- trans*-[Co(NH₃)₂(en)₂]³⁺
- [Cr(H₂O)₆]³⁺
- cis*-[Co(NH₃)₂(en)₂]³⁺

52.



The reagent A is

- NaBH₄ + PtCl₄
- (i) N₂H₄ / H⁺ (ii) C₂H₅O⁻Na⁺
- Zn – Hg / HCl
- LAH + AlCl₃

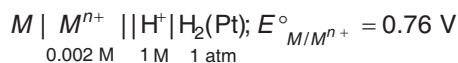
53. The geometry of [Ni(CN)₄]²⁻, [NiCl₄]²⁻ and [Ni(CO)₄] are

- [Ni(CN)₄]²⁻ is square planar and [Ni(CN)₄]²⁻, [Ni(CO)₄] are tetrahedral
- [Ni(CN)₄]²⁻ is square planar, [NiCl₄]²⁻ and [Ni(CO)₄] are tetrahedral
- [Ni(CO)₄] is square planar, [Ni(CN)₄]²⁻ and [NiCl₄]²⁻ are tetrahedral
- None of the above

54. The volume of 3.7 g of a gas at 25°C and volume of 0.184 g of H₂ gas at 17°C are same at same pressure. The molecular mass of gas is

- 52
- 41
- 45
- 55

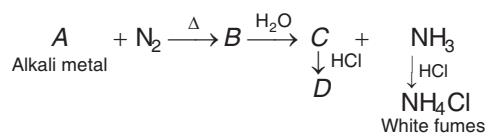
55. The emf of the following cell is 0.81 V.



Then, the value of n for metal ion is

- 1
- 4
- 2
- 3

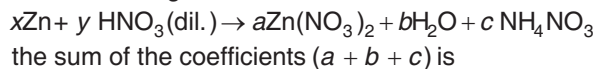
56. Consider the reaction given below,



The alkali metal A is

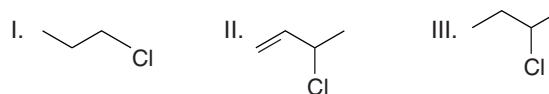
- K
- Li
- Na
- Cs

57. In the following reaction,



- 6
- 8
- 4
- 5

58. Arrange the following three chlorides in decreasing order of reactivity towards S_N1 reaction.



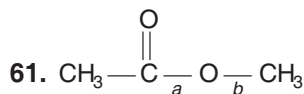
- II > III > I
- III > II > I
- I > II > III
- II > I > III

59. The % CaSO₄ in the sample containing 0.455 g sample of plaster of Paris $\left(CaSO_4 \cdot \frac{1}{2} H_2O \right)$, when treated with excess of Ba(NO₃)₂ gave 0.617 g of anhy, BaSO₄ is

- 87%
- 92%
- 80%
- 75%

60. Choose the incorrect statement from the following.

- Chlorine is less selective and more reactive
- Vinyl free radical is more stable than allyl free radical
- Bromine is more selective and less reactive
- Benzyl free radical is more stable than allyl free radical



The relation between bond lengths a and b is

- $a = b$
- $b < a$
- $b > a$
- impossible to predict

62. The maximum number of emission lines when the excited electron of a H-atom in $n=6$ drops to the ground state is

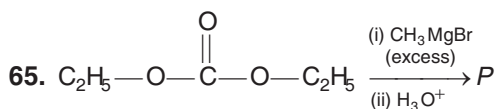
- 14
- 36
- 15
- 10

63. Choose the correct option from the following in case of van der Waals' adsorption.

- High temperature, low pressure
- High temperature, high pressure
- Low temperature, low pressure
- Low temperature, high pressure

64. If the solubility of calcium carbonate in water is 0.0305 g/L, then its solubility product is

a. 8.40×10^{-6} b. 9.30×10^{-8}
c. 7.21×10^{-8} d. 9.21×10^{-7}



The product P is

a. $\text{CH}_3\text{—}\overset{\text{O}}{\parallel}\text{C—CH}_3$ b. $\text{CH}_3\text{—CH}_2\text{—CH}_3$
c. $\text{CH}_3\text{—}\overset{\text{O}}{\parallel}\text{C—O—C}_2\text{H}_5$ d. $\text{CH}_3\text{—}\overset{\text{OH}}{\underset{\text{CH}_3}{\text{C}}}\text{—CH}_3$

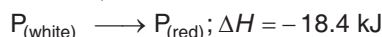
66. On applying pressure to the equilibrium,



Which phenomenon will happen?

a. More ice will be formed
b. More water will be formed
c. Water will evaporate
d. Equilibrium will not be disturbed

67. From the reaction,



which of the following statements is correct?

a. Red P is readily formed from white P
b. White P is readily formed from red P
c. White P cannot be converted to red P
d. White P can be converted into red P and red P is more stable

68. From which of the following mixtures, buffer solution can be prepared?

a. $\text{NaH}_2\text{PO}_4 + \text{Na}_2\text{HPO}_4$ b. $\text{NH}_4\text{OH} + \text{NH}_3$
c. $\text{HCl} + \text{NaCl}$ d. $\text{CH}_3\text{COOH} + \text{NaCl}$

69. Which of the following has regular tetrahedral structure?

a. SF_4 b. XeF_4
c. $[\text{Ni}(\text{CN})_4]^{2-}$ d. BF_4^-

70. If the reaction rate at a given temperature becomes slower, then

a. the entropy changes
b. the initial concentration of the reactants remains constant
c. the free energy of activation is lower
d. the free energy of activation is higher

71. The number of sigma and pi-bonds in peroxydisulphuric acid respectively are

a. 10 and 4 b. 11 and 4 c. 13 and 2 d. 11 and 2

72. Calculate the resonance energy of benzene, if the enthalpy of hydrogenation of benzene is ΔH_1 and for cyclohexene, it is ΔH_2 .

a. $\Delta H_1 - 3\Delta H_2$ b. $3\Delta H_2 - \Delta H_1$
c. $3\Delta H_1 - \Delta H_2$ d. $3\Delta H_1 + \Delta H_2$

73. Which one of the following order is not in accordance with the property stated against it?

a. $\text{F}_2 > \text{Cl}_2 > \text{Br}_2 > \text{I}_2$: Oxidising power
b. $\text{F}_2 > \text{Cl}_2 > \text{Br}_2 > \text{I}_2$: Electronegativity
c. $\text{F}_2 > \text{Cl}_2 > \text{Br}_2 > \text{I}_2$: Bond dissociation energy
d. $\text{HI} > \text{HBr} > \text{HCl} > \text{HF}$: Acidic property in water

74. In corundum, oxide ions are arranged in hcp array and the aluminium ions occupy two-thirds of octahedral voids. The formula of corundum is

a. Al_2O_3 b. Al_3O_2
c. Al_2O_6 d. Al_2O_4

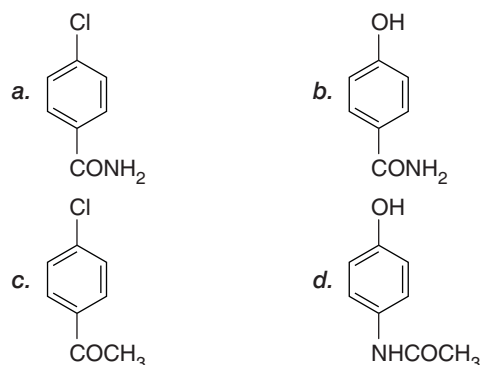
75. The freezing point depression of 0.1 molar solution of acetic acid in benzene is 0.256 K. K_f for benzene is $5.12 \text{ K kg mol}^{-1}$. What conclusion can you find about the molecular state of acetic acid in benzene?

a. Benzene is doubly associated
b. Acetic acid is doubly associated
c. Both are equally associated
d. None of the above

76. Which of the following is not an addition homopolymer?

a. PVC b. Teflon
c. SBR d. Natural rubber

77. Choose the correct structure of the drug paracetamol.



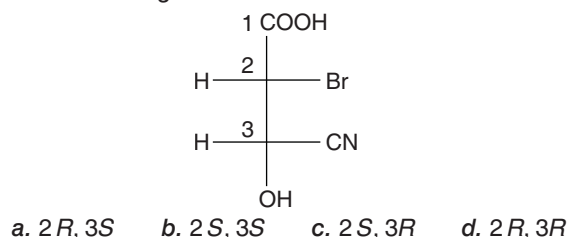
78. The same osazone is formed by which of the following pairs?

a. Glucose and fructose b. Glucose and galactose
c. Glucose and arabinose d. Lactose and maltose

79. The rate of change of rms speed of a gas is twice the rate of change of absolute temperature. Then, the rms speed of Ne is (Atomic mass of Ne = 20)

a. 344 ms^{-1} b. 302 ms^{-1} c. 381 ms^{-1} d. 312 ms^{-1}

80. The absolute configurations of the centres (1 and 2) in the molecule given below are



PART III

a. English Proficiency

Directions (Q. Nos. 81 to 84) *Read the passage and choose the most suitable option from the given options.*

When vegetation sprouts in the desert, it is a good sign but when the ice in the Arctic and Antarctic begins to turn green, there is something terribly wrong. Reports say that an iceberg, approximately the size of New York city, has broken off from the icy continent. An Argentine team discovered huge cracks in the polar ice caps. These developments can have serious implications. If polar ice fields melt, our coastal cities might be submerged and sea levels across the world could rise between 3.65 m and 6.09 m in different parts of the Earth. This is probably due to global warming.

81. What can be considered as a 'Good sign' in the desert ?
 a. When shrubs and trees grow there
 b. When it shows in the desert
 c. When ice caps melt and there is water
 d. When there is vegetation found in the desert
82. The melting of ice fields in the Arctic and Antarctic regions is dangerous because
 a. the ice will turn green and poisonous
 b. it will cause huge floods which will destroy coastal regions
 c. it will create global warming and will badly affect our climate
 d. huge cracks will develop all over the world
83. Polar ice caps develop huge cracks because
 a. of the movement of the Earth
 b. of the breaking off of Earth
 c. of the crowding of cities like New York
 d. of rising temperatures
84. What do you think about the intention of the author?
 a. To describe strange phenomena in nature
 b. To report findings of research teams working in the polar regions
 c. To make us aware of the dangers of global warming
 d. To compare development in deserts and Arctic regions caused by global warming

Directions (Q. Nos. 85 to 88) *Rearrange the jumbled up parts of the sentence to make it meaningful and then choose the most suitable option.*

85. It seemed to him
 P : like seeing one's reflection
 Q : an endless quest
 R : two mirrors
 S : while standing between
 The proper sequence should be
 a. PRSQ b. SPQR
 c. RSPQ d. QPSR

86. A series of shocks
 P : is known as earthquake
 Q : which can be recognised through seismic waves.
 R : that result from sudden Earth movements or tremors
 S : causing widespread destruction of life and property
 The proper sequence should be
 a. PQRS b. RPQS c. RSPQ d. RQSP

87. P : in this world
 Q : a man has
 R : it is possible that the best friend
 S : may turn against him
 The proper sequence should be
 a. QPRS b. PQRS c. RQPS d. QRSP

88. P : in estimating the size of the earth but they were
 Q : hampered by the lack of instruments of precision
 R : ancient astronomers
 S : used method which were theoretically valid
 The proper sequence should be
 a. RPQS b. PRQS
 c. RSQP d. RPSQ

Directions (Q. Nos. 89 to 91) *Spot the error in the given sentence and indicate your answer.*

89. Children are prone (a)/ to making mischiefs (b)/ if they have nothing to do. (c)/ No error (d)
90. Sheeps are economically useful(a)/ and so they are reared (b)/ in the hills. (c)/ No error (d)
91. I have not gone through (a)/ the letter and so I am not aware (b)/ of its content. (c)/ No error (d)

Directions (Q. Nos. 92 and 93) *Find the synonym of the word written in capital letters.*

92. The convict's **INGENUOUS** explanation brought tears in every eye.
 a. Candid b. Secret c. Insincere d. Consistent
93. The **ENORMITY** of the population problem is irksome.
 a. Intensity b. Vastness c. Cruelty d. Fragility

Directions (Q. Nos. 94 and 95) *Find the antonym of the word written in capital letters.*

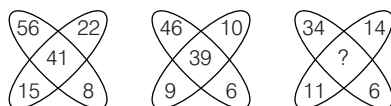
94. We have **CREATED** a beautiful new house from out of the old ruin.
 a. Destroyed b. Built
 c. Constructed d. Planned
95. A large number designer clothes are **REVOLTING**.
 a. Inviting b. Beautiful
 c. Fashionable d. Delightful

b. Logical Reasoning

96. Choose the odd one out.

- a. BDFH b. MOQS
c. SUWY d. TUZE

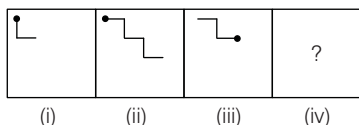
97. Find the missing number.



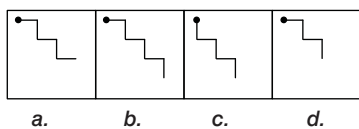
- a. 12 b. 25 c. 48 d. 52

98. Find the figure that would replace the question mark (?)

Problem Figures



Answer Figures



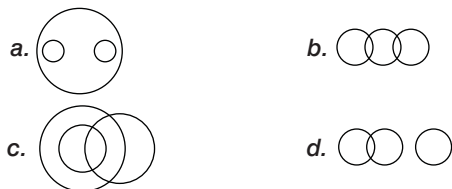
99. If KASHMIR written as 8142753, how RIMSHAK can be written in that code?

- a. 3574218 b. 3571842
c. 3521478 d. 3574812

100. If 'PAPER' is written as 'OZODQ' how 'PENCIL' can be written in that code?

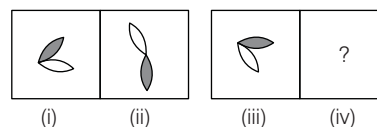
- a. QFODJM b. OFOBHM c. ODMDJM d. ODMBHK

101. Which one of the following diagram best depicts the relationship among elephants, wolves, and animals?

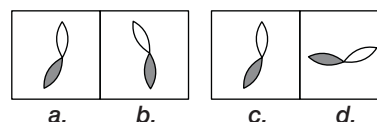


102. Which figure from the given alternatives will replace the question mark (?)

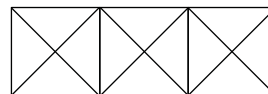
Problem Figures



Answer Figures



103. How many different triangles are there in the figures shown below ?



- a. 28 b. 24 c. 20 d. 16

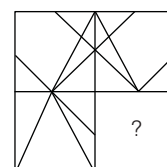
104. Find the wrong number in the series.

1, 9, 36, 81, 99, 121

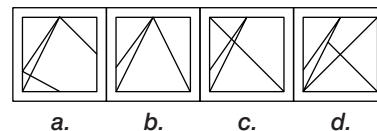
- a. 1 b. 121
c. 99 d. 36

105. In the following question, select a figure from amongst the four alternatives, which when placed in the blank space of figure (X) would complete the pattern.

Problem Figure



Answer Figures



Mathematics

- 106.** The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is

 - $\frac{x}{e^x}$
 - $\frac{e^x}{x}$
 - xe^x
 - e^x

107. Let α and β be the roots of $x^2 - 6x - 2 = 0$ with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is

 - 1
 - 2
 - 3
 - 4

108. Let $a_1, a_2, a_3, \dots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^p a_i$, $1 \leq p \leq 100$. For any integer n with $1 \leq n \leq 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not depend on n , then a_2 equals

 - 6
 - 5
 - 7
 - 4

109. If r, s, t are prime numbers and p, q are the positive integers such that the LCM of p, q is $r^2 t^4 s^2$, then the number of ordered pair (p, q) is

 - 252
 - 254
 - 225
 - 224

110. The expression $[x + (x^3 - 1)^{1/2}]^5 = [x - (x^3 - 1)^{1/2}]^5$ is a polynomial of degree

 - 5
 - 6
 - 7
 - 8

111. Let a, b and c be three real numbers satisfying

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}.$$

If the point (a, b, c) with reference to (E) lies on the plane $2x + y + z = 1$, then the value of $7a + b + c$ is

 - 0
 - 12
 - 7
 - 6

112. The determinant $\begin{vmatrix} xp + y & x & y \\ yp + z & y & z \\ 0 & xp + y & yp + z \end{vmatrix} = 0$, if

 - x, y, z are in AP
 - x, y, z are in GP
 - x, y, z are in HP
 - xv, vz, zx are in AP

113. A bag contains n balls out of which some balls are white. If probability that a bag contains exactly i white ball is proportional to i^2 . A ball is drawn at random from the bag and found to be white, then find the probability that bag contains exactly 2 white balls.

 - $\frac{\left[\frac{6}{n(n+1)} \right] \left(\frac{2^3}{n} \right)}{\frac{2}{3(n+1)}}$
 - $\frac{2}{\left[\frac{6}{n(n+1)} \right] \left(\frac{2^3}{n} \right)}$
 - $\frac{\left(\frac{6}{n} \right) \left[\frac{2^3}{n(n+1)} \right]}{\frac{2}{3n}}$
 - None of these

114. The domain of definition of $f(x) = \cos^{-1}(x + [x])$ is

 - $[0, 1]$
 - $R - Z$
 - $(0, \infty)$
 - None

115. Let $f : R \rightarrow R$ be given by $f(x + y) = f(x) - f(y) + 2xy + 1$ for all $x, y \in R$. If $f(x)$ is everywhere differentiable and $f'(0) = 1$, then $f'(x)$ is equal to

 - $2x + 1$
 - $2x - 1$
 - $x + 1$
 - $x - 1$

116. $\frac{d^2y}{dx^2}$ is equal to

 - $\left(\frac{d^2y}{dx^2} \right)^{-1}$
 - $-\left(\frac{d^2y}{dx^2} \right) \left(\frac{dy}{dx} \right)^{-3}$
 - $\left(\frac{d^2y}{dx^2} \right) \left(\frac{dy}{dx} \right)^{-2}$
 - $-\left(\frac{d^2y}{dx^2} \right) \left(\frac{dy}{dx} \right)^{-3}$

117. The equation of a curve is $y = f(x)$. The tangents at $(1, f(1))$, $(2, f(2))$ and $(3, f(3))$ makes angles $\frac{\pi}{6}, \frac{\pi}{3}$ and $\frac{\pi}{4}$ respectively with the positive direction of the X-axis. Then, the value of $\int_2^3 f'(x)f''(x) dx + \int_1^3 f'''(x) dx$ is equal to

 - $-\frac{1}{\sqrt{3}}$
 - $\frac{1}{\sqrt{3}}$
 - 0
 - 1

118. If $f(x) = \int_x^{x^2} \frac{1}{(\log t)^2} dt$; $x \neq 0, x \neq 1$, then $f(x)$ is monotonically

 - increasing on $(2, \infty)$
 - increasing on $(1, 2)$
 - decreasing on $(2, \infty)$
 - decreasing on $(0, 3)$

119. If $\int \log(\sqrt{1-x} + \sqrt{1+x}) dx = x f(x + B \sin^{-1} x + C)$, then
 a. $f(x) = \log(\sqrt{1-x} + \sqrt{1+x})$
 b. $A = 1/3$
 c. $B = 2/3$
 d. $B = -1/2$
120. If $I = \int_0^1 \frac{1}{1+x^{\pi/2}} dx$, then
 a. $\ln 2 < I < \frac{\pi}{4}$
 b. $I < \ln^2$
 c. $I > \frac{\pi}{4}$
 d. None of these
121. The area between the curve $y = 2x^4 - x^2$, the X-axis and the ordinates of two minima of the curve is
 a. $7/120$ sq unit
 b. $9/120$ sq unit
 c. $11/120$ sq unit
 d. $13/120$ sq unit
122. Let $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}$ and $\mathbf{r}$ be a variable such that $\mathbf{r} \cdot \hat{i}$, $\mathbf{r} \cdot \hat{j}$ and $\mathbf{r} \cdot \hat{k}$ are positive integers. If $\mathbf{r} \cdot \mathbf{a} \leq 12$, then the total number of such vectors is
 a. ${}^{12}C_3$
 b. ${}^{12}C_4$
 c. ${}^{12}C_9 - 1$
 d. ${}^{12}C_8$
123. Given, $|\mathbf{a}| = |\mathbf{b}| = 1$ and $|\mathbf{a} + \mathbf{b}| = \sqrt{3}$. If $\mathbf{c}$ is a vector such that $\mathbf{c} - \mathbf{a} - 2\mathbf{b} = 3(\mathbf{a} \times \mathbf{b})$, then $\mathbf{c} \cdot \mathbf{b}$ is equal to
 a. $-\frac{1}{2}$
 b. $\frac{1}{2}$
 c. $\frac{3}{2}$
 d. $\frac{5}{2}$
124. If the plane $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$ cuts the coordinate axes at A, B, C , then the area of $\triangle ABC$ is
 a. $\sqrt{29}$ sq units
 b. $\sqrt{41}$ sq units
 c. $\sqrt{61}$ sq units
 d. $\sqrt{71}$ sq units
125. If d_1, d_2, d_3 denote the distance of the plane $2x - 3y + 4z + 2 = 0$ from the planes $2x - 3y + 4z + 6 = 0$, $4x - 6y + 8z + 3 = 0$ and $2x - 3y + 4z - 6 = 0$ respectively, then
 a. $d_1 + 8d_2 + d_3 = 0$
 b. $d_1 + 16d_2 = 0$
 c. $8d_2 = d_1$
 d. $d_1 - 2d_2 + 3d_3 = \sqrt{29}$
126. The median of the series 8, 12, 15, 7, x, 19, 22 lies in the interval
 a. $[12, 15]$
 b. $[7, 15]$
 c. $[15, 17]$
 d. $[9, 12]$
127. The variable X takes two values x_1 and x_2 with frequencies f_1 and f_2 respectively. If σ denotes the standard deviation of X , then σ^2 is equal to
 a. $\frac{f_1^2 x_1^2 + f_2^2 x_2^2}{f_1 + f_2} - \left(\frac{f_1 x_1 + f_2 x_2}{f_1 + f_2} \right)^2$
 b. $\frac{f_1 f_2}{(f_1 + f_2)^2} (x_1 - x_2)^2$
 c. $\frac{f_1 f_2}{(f_1 - f_2)^2} (x_1 - x_2)^2$
 d. None of the above
128. If $S = \{x \in N : 2 + \log_2 \sqrt{x+1} > 1 - \log_{1/2} \sqrt{4-x^2}\}$, then
 a. $S = \{1\}$
 b. $S = Z$
 c. $S = N$
 d. None
129. The area of the parallelogram contained by the lines $4y - 3x - a = 0$, $3y - 4x + a = 0$, $4y - 3x - 3a = 0$ and $3y - 4x + 2a = 0$ is
 a. $\frac{2}{7} a^2$ sq units
 b. $\frac{3}{8} a^2$ sq units
 c. $2a^2$ sq units
 d. $\frac{5}{4} a^2$ sq units
130. The two circles which pass through $(0, a)$ and $(0, -a)$ touch the line $y = mx + c$, will intersect each other at right angle, if
 a. $a^2 = c^2(2m+1)$
 b. $a^2 = c^2(2+m^2)$
 c. $c^2 = a^2(2+m^2)$
 d. $c^2 = a^2(2m+1)$
131. Two mutually perpendicular tangents of the parabola $y^2 = 4ax$ meet the axis at P_1 and P_2 . If S is the focus of the parabola, then $\frac{1}{SP_1} + \frac{1}{SP_2}$ is equal to
 a. $\frac{4}{a}$
 b. $\frac{2}{a}$
 c. $\frac{1}{a}$
 d. $\frac{1}{4a}$
132. Tangents are drawn to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, ($a > b$) and the circle $x^2 + y^2 = a^2$ at the points, where a common ordinate cuts them (on the same side of X-axis). Then, the greatest acute angle between these tangents is given by
 a. $\tan^{-1}\left(\frac{a-b}{2\sqrt{ab}}\right)$
 b. $\tan^{-1}\left(\frac{a+b}{2\sqrt{ab}}\right)$
 c. $\tan^{-1}\left(\frac{2ab}{\sqrt{a-b}}\right)$
 d. $\tan^{-1}\left(\frac{2ab}{\sqrt{a+b}}\right)$
133. The chords of contact of a point p with respect to a hyperbola and its auxiliary circle are at right angle. Then, the point P lies on
 a. conjugate hyperbola
 b. one of the directrix
 c. one of the asymptotes
 d. None of these
134. Through a point A on the X-axis, a straight line is drawn parallel to the Y-axis, so as to meet the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ at B and C . If $AB = BC$, then
 a. $h^2 = 4ab$
 b. $8h^2 = 9ab$
 c. $9h^2 = 8ab$
 d. $4h^2 = ab$
135. If a and b are positive quantities such that $a > b$, then the minimum value of $a \sec \theta - b \tan \theta$ is
 a. $2ab$
 b. $\sqrt{a^2 - b^2}$
 c. $a - b$
 d. $\sqrt{a^2 + b^2}$

- 136.** Let $x = \sin 1^\circ$, then the value of expression
 $\frac{1}{\cos 0^\circ \cdot \cos 1^\circ} + \frac{1}{\cos 1^\circ \cdot \cos 2^\circ} + \frac{1}{\cos 2^\circ \cdot \cos 3^\circ}$
 $+ \dots + \frac{1}{\cos 44^\circ \cdot \cos 45^\circ}$ is equal to
 a. x b. $1/x$
 c. $\sqrt{2}/x$ d. $x/\sqrt{2}$

137. If $\tan x = n \tan y, n \in R^+$, then the maximum value of $\sec^2(x - y)$ is equal to
 a. $\frac{(n+1)^2}{2n}$ b. $\frac{(n+1)^2}{n}$
 c. $\frac{(n+1)^2}{2}$ d. $\frac{(n+1)^2}{4n}$

138. $\frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta - \cos \theta} - \frac{\cos \theta}{\sqrt{1 + \cot^2 \theta}} - 2 \tan \theta \cot \theta = -1$, if
 a. $\theta \in \left(0, \frac{\pi}{2}\right)$ b. $\theta \in \left(\frac{\pi}{2}, \pi\right)$
 c. $\theta \in \left(\pi, \frac{3\pi}{2}\right)$ d. $\theta \in \left(\frac{3\pi}{2}, 2\pi\right)$

139. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, then
 $x^4 + y^4 + z^4 + 4x^2y^2z^2 = k (x^2y^2 + y^2z^2 + z^2x^2)$,
 where k is equal to
 a. 1 b. 2 c. 3 d. 4

140. Two medians drawn from the acute angles of a right angled triangle intersect at an angle $\pi/6$. If the length of the hypotenuse of the triangle is 3 units, then the area of triangle (in sq units) is
 a. $\sqrt{3}$ b. 3 c. $\sqrt{2}$ d. 9

141. The minimum value of the sum of real numbers $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$ and a^{10} with $a > 0$, is
 a. 7 b. 8 c. 6 d. 10

142. A man walks a distance of 3 units from the origin towards the North-East ($N 45^\circ E$) direction. From there, he walks a distance of 4 units towards the North-West ($N 45^\circ W$) direction to reach a point P . Then, the position of P in the argand plane is
 a. $3e^{i\pi/4} + 4i$ b. $(3 - 4i)e^{i\pi/4}$
 c. $(4 + 3i)e^{i\pi/4}$ d. $(3 + 4i)e^{i\pi/4}$

143. If S is the set of all real x such that $\frac{2x-1}{2x^3+3x^2+x}$ is positive, then S contains
 a. $(-\infty, +3/2)$ b. $(-3/2, -1/4)$
 c. $(-1/4, 1/2)$ d. $(1/2, 3)$

144. For any three sets A, B and C , the set $(A \cup B \cup C) \cap (A \cap B' \cap C') \cap C'$ is equal to
 a. $B \cap C'$ b. $B' \cap C'$
 c. $B \cap C$ d. $A \cap B \cap C$

145. Let w denotes words in the English dictionary. Define the relation R by $R = \{(x, y) \in w \times w : \text{the words } x \text{ and } y \text{ have atleast one letter in common}\}$ then, R is
 a. not reflexive, symmetric and transitive
 b. not transitive, reflexive and symmetric
 c. not symmetric, reflexive and transitive
 d. reflexive, transitive and symmetric

146. Let $f : [2, \infty) \rightarrow x$ be defined by $f(x) = 4x - x^2$, then f is invertible, if x is equal to
 a. $[2, \infty)$ b. $(-\infty, 2]$
 c. $(-\infty, 4]$ d. $[4, \infty)$

147. Which of the following is always true?
 a. $(p \rightarrow q) \equiv (\sim q \rightarrow \sim p)$ b. $\sim(p \vee q) \equiv (\sim p \vee \sim q)$
 c. $\sim(p \rightarrow q) \equiv (p \vee \sim q)$ d. $\sim(p \wedge q) \equiv (\sim p \wedge \sim q)$

148. If $49^n + 16n + \lambda$ is divisible by 64 for all $n \in N$, then the least negative integral value of λ is
 a. -2 b. -1
 c. -3 d. -4

149. Between any two real roots of the equation $e^x \sin x - 1 = 0$, the equation $e^x \cos x + 1 = 0$ has
 a. atleast one root
 b. no root
 c. atmost one root
 d. exactly one root

150. If the mean and variance of a binomial variable x are 2 and 1 respectively, then the probability that x takes a value greater than or equal to 1 is
 a. $\frac{2}{3}$ b. $\frac{4}{5}$
 c. $\frac{7}{8}$ d. $\frac{15}{16}$

Answers

Physics

1. (a)	2. (c)	3. (c)	4. (d)	5. (d)	6. (b)	7. (d)	8. (a)	9. (b)	10. (c)
11. (d)	12. (d)	13. (d)	14. (c)	15. (c)	16. (c)	17. (b)	18. (a)	19. (c)	20. (d)
21. (c)	22. (b)	23. (a)	24. (b)	25. (b)	26. (b)	27. (c)	28. (a)	29. (d)	30. (d)
31. (c)	32. (b)	33. (a)	34. (c)	35. (a)	36. (c)	37. (d)	38. (d)	39. (d)	40. (b)

Chemistry

41. (b)	42. (b)	43. (b)	44. (b)	45. (a)	46. (b)	47. (c)	48. (b)	49. (b)	50. (c)
51. (d)	52. (b)	53. (b)	54. (b)	55. (d)	56. (d)	57. (b)	58. (a)	59. (c)	60. (b)
61. (c)	62. (c)	63. (d)	64. (b)	65. (d)	66. (b)	67. (d)	68. (a)	69. (d)	70. (d)
71. (b)	72. (b)	73. (c)	74. (a)	75. (b)	76. (c)	77. (d)	78. (a)	79. (d)	80. (a)

English Proficiency

81. (d)	82. (b)	83. (d)	84. (c)	85. (d)	86. (d)	87. (c)	88. (c)	89. (b)	90. (a)
91. (c)	92. (a)	93. (b)	94. (a)	95. (a)					

Logical Reasoning

96. (d)	97. (b)	98. (b)	99. (a)	100. (d)	101. (a)	102. (a)	103. (a)	104. (c)	105. (b)
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Mathematics

106. (b)	107. (c)	108. (a)	109. (c)	110. (c)	111. (d)	112. (b)	113. (a)	114. (a)	115. (b)
116. (b)	117. (a)	118. (a)	119. (a)	120. (a)	121. (a)	122. (a)	123. (d)	124. (c)	125. (c)
126. (a)	127. (b)	128. (a)	129. (a)	130. (c)	131. (c)	132. (a)	133. (c)	134. (b)	135. (b)
136. (b)	137. (d)	138. (b)	139. (b)	140. (a)	141. (b)	142. (d)	143. (d)	144. (a)	145. (b)
146. (c)	147. (a)	148. (b)	149. (a)	150. (d)					

Practice Set 3

BITSAT

Instructions

- There are 150 questions in all. The number of questions in each part is as follows Subject
- All questions are multiple choice questions with four options, only one is correct.
- Each correct answer fetches 3 marks while incorrect answer has a penalty of 1 mark.

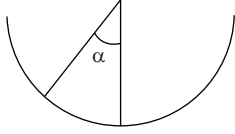
No. of Questions

Part I Physics	1-40
Part II Chemistry	41-80
Part III (a) English Proficiency	81-95
(b) Logical Reasoning	96-105
Part IV Mathematics	106-150

PART I

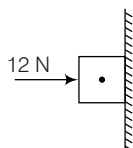
Physics

- The velocity of water waves may depend on their wavelength λ , the density of water ρ and the acceleration due to gravity g . The method of dimensional analysis gives the relation between these quantities as
 - $v^2 = k\lambda^{-1}g^{-1}\rho^{-1}$
 - $v^2 = k\lambda g$
 - $v^2 = k\lambda\rho g$
 - $v^2 = k\lambda^3g^{-1}\rho^{-1}$
- A particle located at $x = 0$ and $t = 0$ starts moving along the positive x -direction with a velocity v that varies as $v = \alpha\sqrt{x}$, the displacement of the particle varies with time as
 - $x \propto \sqrt{t}$
 - $x \propto t$
 - $x \propto t^2$
 - $x \propto \frac{1}{t^2}$
- From an elevated point P , a stone is projected vertically upwards. When the stone reaches a distance h below P , its velocity is double of its velocity at a height h above P . The greatest height attained by the stone from the point of projection P is
 - $\frac{3}{5}h$
 - $\frac{5}{3}h$
 - $\frac{7}{5}h$
 - $\frac{5}{7}h$
- A train accelerates from rest at a constant rate α for distance x_1 and time t_1 . After that it retards to rest at constant rate β for distance x_2 and time t_2 . Then, it is found that
 - $\frac{x_1}{x_2} = \frac{d}{\beta} = \frac{t_1}{t_2}$
 - $\frac{x_1}{x_2} = \frac{\beta}{\alpha} = \frac{t_1}{t_2}$
 - $\frac{x_1}{x_2} = \frac{\beta}{\alpha} = \frac{t_2}{t_1}$
 - $\frac{x_1}{x_2} = \frac{\beta}{\alpha} = \frac{t_2}{t_1}$
- An ant crawls up a hemispherical surface very slowly as shown in figure. The coefficient of friction between the ant and the surface is $1/3$. If the line joining the centre of hemispherical surface to ant makes an angle α with the vertical, the maximum possible value of α is



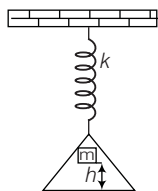
- $\sec^{-1}(3)$
 - $\cot^{-1}(3)$
 - $\tan^{-1}(3)$
 - $\operatorname{cosec}^{-1}(3)$

6. A block of weight 5 N is pushed against a vertical wall by a force 12 N. The coefficient of friction between the wall and block is 0.6. The magnitude of the force exerted by wall on the block is



- a. 12 N b. 5 N c. 7.2 N d. 13 N

7. A vertical spring with force constant k is fixed on a table. A ball of mass m at a height h above the free upper end of spring falls vertically on the springs, so that the spring is compressed by a distance d . The net work done in this process is

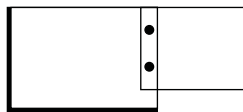


- a. $mg(h+d) + \frac{1}{2}kd^2$ b. $mg(h+d) - \frac{1}{2}kd^2$
c. $mg(h-d) - \frac{1}{2}kd^2$ d. $mg(h-d) + \frac{1}{2}kd^2$

8. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular velocity is ω . Its centre of mass rises to a maximum height of

- a. $\frac{l^2\omega^2}{6g}$ b. $\frac{l^2\omega^2}{3g}$ c. $\frac{l^2\omega^2}{2g}$ d. $\frac{l\omega}{6g}$

9. Two long metallic strips are joined together by two rivets each of radius 0.1 cm (see figure). Each rivet can withstand a maximum shearing stress of $4 \times 10^8 \text{ Nm}^{-2}$. The maximum tangential force which a strip can exert is



- a. 185 N b. 1885 N c. 855 N d. 1555 N

10. Two rods P and Q of same length and same diameter having thermal conductivity ratio 2:3 joined end to end. If temperature at one end of P is 100°C and at one end of Q is 0°C , then temperature of interface is



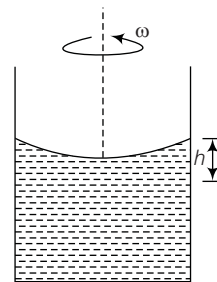
- a. 60°C b. 70°C
c. 50°C d. 40°C

11. A square wire frame of size L is dipped in a liquid. On taking out, a membrane is formed. If the surface tension of liquid is T , force acting on the frame will be
- a. $2TL$ b. $4TL$ c. $8TL$ d. $10TL$

12. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T , density of liquid is ρ and L is its latent heat of vaporisation.

- a. $\sqrt{T/\rho L}$ b. $T/\rho L$ c. $2T/\rho L$ d. $\rho L/T$

13. A liquid is kept in a cylindrical vessel which is rotated along its axis. The liquid rises at the sides (figure). If the radius of the vessel is 0.05 m and the speed of rotation is 2 rad/s, then find the difference in the height of liquid at centre and sides.



- a. 20 cm b. 4 cm c. 2 cm d. 0.2 cm

14. An ideal gas is expanding such that $pT^2 = \text{constant}$. The coefficient of volume expansion of the gas is

- a. $\frac{1}{T}$ b. $\frac{2}{T}$ c. $\frac{3}{T}$ d. $\frac{4}{T}$

15. The ratio of the speed of sound in nitrogen gas to that in helium gas at 300 K is

- a. $\sqrt{\frac{1}{7}}$ b. $\sqrt{\frac{2}{7}}$ c. $\frac{\sqrt{3}}{5}$ d. $\frac{\sqrt{6}}{5}$

16. A point mass oscillates along X -axis according to the law $x = x_0 \cos\left(\omega t - \frac{\pi}{4}\right)$. If the acceleration of the

particle is written as $a = A \cos(\omega t + \delta)$, then

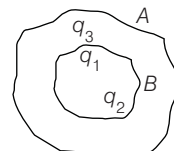
- a. $A = x_0\omega^2$, $\delta = \pi/4$ b. $A = x_0\omega^2$, $\delta = -\pi/4$
c. $A = x_0\omega^2$, $\delta = 3\pi/4$ d. $A = x_0$, $\delta = \pi/4$

17. The fundamental frequency of a closed end pipe is 480 Hz. What is the fundamental frequency when $\left(\frac{1}{4}\right)^{\text{th}}$ length is filled with water?

- a. 120 Hz b. 240 Hz c. 640 Hz d. 960 Hz

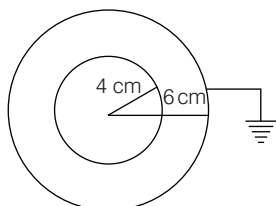
18. The electric flux for Gaussian surface A that enclose the charged particles in free space is

(given, $q_1 = -14 \text{ nC}$, $q_2 = 78.85 \text{ nC}$, $q_3 = -56 \text{ nC}$)

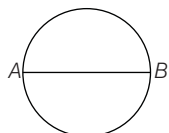


- a. $10^3 \text{ CN}^{-1}\text{m}^{-2}$ b. $10^3 \text{ Nm}^2 \text{ C}^{-1}$
c. $6.32 \times 10^3 \text{ Nm}^2 \text{ C}^{-1}$ d. $6.32 \times 10^3 \text{ CN}^{-1}\text{m}^{-2}$

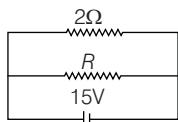
19. A sphere of 4 cm radius is suspended with in a hollow sphere of 6 cm radius. The inner sphere is charged to a potential of 3 esu. When the outer sphere is earthed, then the charge on the inner sphere is



- a. 54 esu b. 0.25 esu c. 30 esu d. 36 esu
20. Six identical capacitors are joined in parallel, charged to a potential difference of 10 V, separated and then connected in series, i.e. the positive plate of one is connected to negative plate of other. Then, potential difference between free plates is
- a. 10 V b. 30 V c. 60 V d. $\frac{10}{5}$ V
21. Kirchhoff's first law ($\Sigma i = 0$) and second law ($\Sigma iR = \Sigma E$), where symbols have their usual meanings, are respectively on
- a. conservation of charge, conservation of energy
b. conservation of charge, conservation of momentum
c. conservation of energy, conservation of charge
d. conservation of momentum, conservation of energy
22. In the given figure, find the resistance between A and B. Both the circle and the diameter are made of uniform wire of resistance $1 \times 10^{-4} \Omega/\text{m}$. The length AB is 2 m.

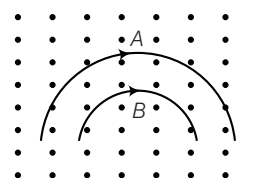


- a. $\frac{2}{3} \times 10^{-4} \Omega$
b. $2\pi \times 10^{-4} \Omega$
c. $14.56 \times 10^{-4} \Omega$
d. $0.88 \times 10^{-4} \Omega$
23. The resistance of series combination of 2 resistors is S. When they are joined in parallel, the total resistance is P. If $S = nP$, then minimum possible value of n is
- a. 4 b. 3
c. 2 d. 1
24. Find R, if power dissipated in the circuit is 150 W.



- a. 2 Ω b. 6 Ω
c. 7 Ω d. 5 Ω

25. The oscillating frequency of a cyclotron is 10 MHz. If the radius of its dees is 0.5 m, the kinetic energy of a proton, which is accelerated by the cyclotron is
- a. 10.2 MeV b. 2.55 MeV
c. 20.4 MeV d. 5.1 MeV
26. Two particles A and B of masses m_A and m_B respectively and having the same charge are moving in a plane. A uniform magnetic field exists perpendicular to this plane. The speeds of particles are v_A and v_B respectively and the trajectories are as shown in figure, then



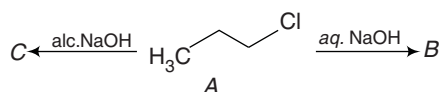
- a. $m_A v_A < m_B v_B$
b. $m_A v_A > m_B v_B$
c. $m_A < m_B$ and $v_A < v_B$
d. $m_A = m_B$ and $v_A = v_B$
27. A magnetic needle suspended by a silk thread is vibrating in the earth's magnetic field. If the temperature of the needle is increased by 500°C , then
- a. the time period decreases
b. the time period remains unchanged
c. the time period increases
d. the needle stops vibrating
28. A circular disc of radius 0.2 m is placed in a uniform magnetic field of induction $\left(\frac{1}{\pi}\right) \text{ Wb/m}^2$ in such a way that its axis makes an angle of 60° with $\mathbf{B}$. The magnetic flux linked with the disc is
- a. 0.01 Wb b. 0.02 Wb
c. 0.06 Wb d. 0.08 Wb
29. A wire of length 50 cm moves with velocity of 300 m/min perpendicular to a magnetic field. If the emf induced in the wire is 2V, magnitude of field in tesla is
- a. 2 b. 5
c. 0.4 d. 0.8
30. In an LCR series AC circuit, the voltage across each of the components L, C and R is 50 V. The voltage across LC combination will be
- a. 50 V b. $50\sqrt{2}$ V c. 100 V d. 0 V
31. The refractive index of air is 1.0003. The thickness of air column which has one more wavelength of yellow light ($\lambda = 600 \text{ nm}$), then the same thickness of vacuum is
- a. 0.02 mm b. 0.2 mm
c. 2 mm d. 2 cm

32. In an interference experiment, third bright fringe is obtained at a point on the screen with a light of 700 nm. What should be the wavelength of the light source in order to obtain 5th bright fringe at the same point ?
a. 420 nm b. 750 nm c. 630 nm d. 500 nm
33. Two coherent light beams of intensity I and $4I$ are superposed. The maximum and minimum possible intensities in the resulting beam are
a. $5I$ and $3I$ b. $9I$ and I c. $9I$ and $3I$ d. $5I$ and I
34. Two convex lenses of focal lengths 0.3 m and 0.05 m are used to make a telescope. Distance kept between the two in order to obtain image at infinity is
a. 0.25 m b. 0.3 m c. 0.35 m d. 0.20 m
35. The number of photons of wavelength 660 nm emitted per second by an electric bulb of 60 W is
(Take, 6.6×10^{-34} Js)
a. 2×10^{20} b. 2×10^{-20}
c. 3×10^{20} d. 1.5×10^{20}
36. When ${}_{88}\text{Ra}^{236}$ decays in a series by emission of 3 α -particles and one β -particle, isotope X formed as
a. ${}_{83}\text{X}^{224}$ b. ${}_{84}\text{X}^{218}$ c. ${}_{84}\text{X}^{220}$ d. ${}_{82}\text{X}^{223}$
37. The radioactivity of a sample is A_1 at time t_1 and A_2 at time t_2 . If the mean life of the specimen is T , then the number of atoms that have disintegrated in the time interval of $(t_2 - t_1)$ is
a. $(A_1 - A_2)$ b. $\frac{(A_1 - A_2)}{T}$
c. $(A_1 - A_2)T$ d. $(A_1 t_1 - A_2 t_2)$
38. The radioactive isotope X with a half-life of 10^9 year decays to Y which is stable. A sample of rocks was found to contain both the elements X and Y in the ratio 1 : 7. What is the age of the rocks ?
a. 2×10^9 yr b. 3×10^9 yr
c. 6×10^9 yr d. 7×10^9 yr
39. A common emitter amplifier has a voltage gain of 50, an input impedance of 100Ω and an output impedance of 200Ω . The power gain of the amplifier is
a. 500 b. 1000
c. 1250 d. 100
40. A 600 W carrier is modulated to a depth of 75% by a 400 Hz sine wave. Find the total power of antenna.
a. 769 W b. 796 W c. 679 W d. 637.5 W

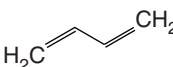
PART II

Chemistry

41. Choose the correct statement from the following reaction.



- a. B is obtained by elimination reaction
b. B is an isomer of ethyl methyl ether, while C is the dehydrated compound of B
c. C is obtained by substitution reaction
d. The molecular formula of B is C_3H_6 and that of C is $\text{C}_3\text{H}_8\text{O}$
42. An inorganic halide P reacts with water to form two acids Q and R , P also reacts with NaOH to form two salts S and T which remain in solution. The solution gives white precipitate with both AgNO_3 and BaCl_2 solutions respectively. P is a useful reagent. What is P in the reaction?
a. AgCl b. NaCl c. SO_2Cl_2 d. HCl
43. The vapour density of completely dissociated NH_4Cl would be
a. half of that of ammonium chloride
b. double that of ammonium chloride
c. slightly less than half of that of ammonium chloride
d. determined by the amount of solid ammonium chloride used
44. A binary solid (A^+B^-) has a zinc blende structure with B^- ions constituting the lattice and A^+ ions occupying 25% tetrahedral holes, then the formula of solid is
a. AB b. A_2B c. AB_2 d. AB_4
45. Which of the following reactions will not give propane?
a. $\text{H}_3\text{C}-\text{CH}=\text{CH}_2 \xrightarrow[\text{CH}_3\text{COOH}]{\text{B}_2\text{H}_6, \text{ ether}}$
b. $\text{H}_3\text{C}-\text{CH}(\text{Cl})-\text{CH}_3 \xrightarrow{\text{P+HI}}$
c. $\text{H}_3\text{C}-\text{C}(=\text{O})-\text{CH}_3 \xrightarrow[\text{H}_2\text{O}]{\text{CH}_3\text{MgX}}$
d. $\text{H}_3\text{C}-\text{CH}_2-\text{CH}_2-\text{Cl} \xrightarrow[\text{H}_2\text{O}]{\text{Mg, ether}}$
46. The molecular formula of certain compound has X_4O_6 . If 10 g of X_4O_6 has 5.72 g X, then the atomic mass of X is
a. 42 b. 30
c. 24 d. 32
47. 3.15 g oxalic acid $[(\text{COOH})_2 \cdot x\text{H}_2\text{O}]$ was dissolved to make 500 mL solution. On titration, 33.36 mL of this solution were neutralised by 50 mL $N/15$ NaOH. The value of x will be
a. 3 b. 2
c. 1 d. 4

48. Which of the following represent the given mode of hybridisation $sp-sp-sp^2-sp^2$ from right to
- $H_2C=CH-C\equiv CH$
 - $H_2C=C=C=CH_2$
 - 
 - $HC\equiv C-CH_2-C\equiv CH$
49. Which of the following statements is incorrect?
- The degree of dissociation of a weak electrolyte decreases as its concentration decreases
 - The freezing point of water is depressed by the addition of glucose
 - If two liquids that form an ideal solution, are mixed, the change in entropy is positive
 - Energy is released when a substance dissolves in water provided that the hydration energy of the substance is more than its lattice energy
50. The hardness of water sample which contains 0.001 mole of $MgSO_4$ dissolved per litre of water, is
- 150 ppm
 - 100 ppm
 - 120 ppm
 - 180 ppm
51. Density of a gas is found to be 5.46 g/dm^3 at 27°C and 2 bar pressure. At STP, its density will be
- 3 g/cm^3
 - 0.3 g/dm^3
 - 3 g/dm^3
 - 4 g/dm^3
52. On the basis of intermolecular forces, predict the correct decreasing order of boiling points of the compounds.
- $CH_3OH > H_2 > CH_4$
 - $CH_3OH > CH_4 > H_2$
 - $CH_4 > CH_3OH > H_2$
 - $H_2 > CH_4 > CH_3OH$
53. Which of the following options does not have isoelectronic species?
- CN^- , N_2 , C_2^{2-}
 - SO_3^{2-} , CO_3^{2-} , NO_3^-
 - BO_3^{3-} , CO_3^{2-} , NO_3^-
 - PO_4^{3-} , SO_4^{2-} , ClO_4^-
54. Actinoids exhibit more number of oxidation states than lanthanoids. The main reason for this is
- more active nature of the actinoids
 - more energy difference between $5f$ and $6d$ -orbitals than between $4f$ and $5d$ -orbitals
 - lesser energy difference between $5f$ and $6d$ -orbitals than that between $4f$ and $5d$ -orbitals
 - greater metallic character of the lanthanoids than that of the corresponding actinoids
55. Among the elements Li, K, Ca, S and Kr, which one is expected to have the lowest first ionisation enthalpy and which have the highest first ionisation enthalpy? The increasing order of first ionisation enthalpy of the elements Li, K, Ca, S and Kr, is
- $Li < K < Ca < S < Kr$
 - $K < Li < Ca < S < Kr$
 - $Kr < Ca < S < Li < K$
 - $Ca < Li < S < K < Kr$
56. Toluene is nitrated and the resulting product is reduced with tin and HCl. The product so obtained is diazotised and then heated with cuprous bromide. The final product is
- mixture of *o*- and *p*-dibromobenzene
 - mixture of *o*- and *p*-bromotoluene
 - mixture of *o*- and *p*-bromoaniline
 - mixture of *o*- and *m*-bromotoluene
57. In which of the following compounds, the size of cation and anion ratio is minimum?
- LiI
 - LiF
 - CsF
 - CsI
58. Which of the following is not an antipyretic drug?
- Paracetamol
 - Aspirin
 - Phenacetin
 - Marijuanae
59. Which of the following represent the heat of formation of $CO_2(g)$?
- $CO(g) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g)$
 - $C_6H_6(l) + 7\frac{1}{2}O_2(g) \longrightarrow 6CO_2(g) + 3H_2O(l)$
 - $C(\text{graphite}) + O_2(g) \longrightarrow CO_2(g)$
 - $CH_4(g) + 2O_2(g) \longrightarrow CO_2(g) + 2H_2O(l)$
60. Which of the following factors is of no significance for roasting sulphide ores to the oxides and not subjecting the sulphide ores to carbon reduction directly?
- Metal sulphides are thermodynamically more stable than CS_2
 - Metal sulphides are less stable than the corresponding oxides
 - CO_2 is more volatile than CS_2
 - CO_2 is thermodynamically more stable than CS_2
61. The reaction of acetaldehyde with excess of ethanol in the presence of H_2SO_4 gives the product
- $CH_3CH(OC_2H_5)_2$
 - $CH_3CH(OH)_2$
 - Ketal
 - $C_2H_5OCH_2CH_2OC_2H_5$
62. When mercuric iodide is added to the aqueous solution of KI, the
- freezing point is raised
 - freezing point does not change
 - freezing point is lowered
 - boiling point does not change
63. $KMnO_4$ oxidises NO_2^- to NO_3^- in basic medium, the number of moles of NO_2^- oxidised by 1 mole of $KMnO_4$ is
- $\frac{3}{2}$
 - 2
 - $\frac{4}{3}$
 - None of these

- I. 2-butene II. 2-methyl-2-butene
III. 2,3-dimethyl-2-butene IV. ethene
- a. III > I > II > IV b. IV > I > II > III
c. IV > II > I > III d. I > III > IV > II

- a. 33.6 JK^{-1} b. -33.6 JK^{-1}
c. 41.2 JK^{-1} d. -41.2 JK^{-1}

- a. copper
b. copper nitride
c. copper nitrite
d. copper oxide

- a. 250 min b. 200 min c. 180 min d. 300 min

- 
- (I) CH_2^+ CH
 (II) CH_2^+ NO_2
 (III) CH_2^+ CH_3
 (IV) CH_2^+ NH_2

- a. $IV > I > III > II$
b. $II > III > I > IV$
c. $IV > III > I > II$
d. $III > IV > I > II$

- a. Kolbe's reaction
b. Wurtz reaction
c. Sandmeyer's reaction
d. Williamson's synthesis

- adsorption of Ca^{2+} and Mg^{2+} ions of hard water replacing Al^{3+} ions
- adsorption of Ca^{2+} and Mg^{2+} ions of hard water replacing Na^{+} ions
- Both (a) and (b) are true
- None of the above is true

- a. 2 b. 3 c. 6 d. 4

- a. -0.732 V b. 0.059 V
c. -0.177 V d. -0.059 V

- a. $\text{Fe}_2[\text{Fe}(\text{CN})_6]$ b. $\text{Na}_4[\text{Fe}(\text{CN})_6]$
c. $\text{Fe}_4[\text{Fe}(\text{CN})_6]_3$ d. $\text{Fe}_3[\text{Fe}(\text{CN})_6]_4$

- a. Na_2ZnO_2 b. $\text{Zn}(\text{OH})_2$
c. ZnO d. None

- a. Allyl amine
b. Propan-2-amine
c. N,N-diethyl butan-1-amine
d. N-methyl ethanamine

- a. square planar
b. tetrahedral
c. trigonal
d. trigonal bipyramidal

- a. 0.321 b. 0.437
c. 0.812 d. 0.617

- Sn-Cassiterite-Gravity separation
- Fe-Haematite-Magnetic separation
- Al-Bauxite-Leaching
- Ag-Cinnabar-Thermal decomposition

- a. $\text{NO}_2^- + \text{CH}_3\text{COOH} \longrightarrow \text{CH}_3\text{COO}^- + \text{HNO}_2$
(in aqueous medium)

- b.** $\text{HCl} + \text{CH}_3\text{COOH} \longrightarrow \text{CH}_3\text{COOH}_2^+ + \text{Cl}^-$
(in non-aqueous medium)

- c. $\text{HNO}_2 + \text{CH}_3\text{COO}^- \longrightarrow \text{CH}_3\text{COOH} + \text{NO}_2^-$
(in aqueous medium)

- d. $\text{HF} + \text{HCl} \longrightarrow \text{H}_2\text{Cl}^+ + \text{F}^-$ (in non-aqueous medium)

- $\text{Ni}(\text{PPh}_3)_2 \text{Cl}_2$ has tetrahedral geometry with magnetic moment 2.82 M
- $[\text{FeCl}_6]^{4-}$ is an inner orbital complex with zero magnetic moment

- c. The magnetic moment of $[\text{Fe}(\text{CN})_6]^{4-}$ is zero
- d. Wavelength of absorption in the visible region for $[\text{Ni}(\text{H}_2\text{O})_6]^{2+}$

Part III

a. English Proficiency

Directions (Q. Nos. 81 to 84) *Spot the error in the following sentences and indicate your answer.*

81. It is a pity (a)/ that even five years old boys (b)/ are engaged in hazardous factories. (c)/ No error (d)
82. I gave him(a)/ two hundred rupees notes (b)/ for depositing. (c)/ No error (d)
83. It is not my business (a)/ to give an advice to those (b)/ who are not sensible enough to deal with their own problems. (c)/ No error (d)
84. I don't think(a)/ it is your house, (b)/ it is someboy's else. (c)/ No error (d)

Directions (Q. Nos. 85 to 88) *Rearrange the jumbled parts of the sentence and then indicate the correct option.*

85. P : it is a pity that
Q : by offering a handsome dowry
R : a number of parents think that
S : they will able to ensure the happiness of their daughters.
The proper sequence should be
a. SQRP b. PRSQ c. PSRQ d. PRQS
86. The common man
P : in nuturing
Q : a more active role
R : communal harmony
S : should play
The proper sequence should be
a. PRSQ b. SQPR c. SQRP d. PRQS
87. The doctor
P : able to find out
Q : what had caused
R : the food poisoning
S : had not been
The proper sequence should be
a. SPRQ b. PRQS c. PRSQ d. SPQR

88. P : was suspended
Q : the officer being corrupt
R : before his dismissal
S : from service
The proper sequence should be
a. QPSR b. QPRS
c. RSQP d. RSQP

Directions (Q. Nos. 89 to 92) *Find the synonym of the word written in capital letters in the given sentences.*

89. EXAGGERATION of facts would always lead to confusion
a. Simplification b. Negation
c. Emancipation d. Amplification
90. After his father's death, he became INSOLVENT.
a. Rich b. Poor
c. Bankrupt d. Nonchalant
91. He INDUCES human beings to want things they don't want.
a. Influences b. Dictates
c. Persuades d. Appreciates
92. His information is not AUTHENTIC.
a. Real b. Reliable c. Believable d. Genuine

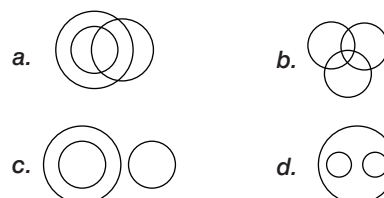
Directions (Q. Nos. 93 to 95) *Find the antonym of the word written in the capital letters.*

93. He finally CONCEDED that he was involved in smuggling
a. Admitted b. Accepted c. Denied d. Concealed
94. The accused emphatically DENIED the charge in the court.
a. Accepted b. Agreed
c. Asserted d. Affirmed
95. The cloth is COARSE of touch.
a. Delicate b. Rough c. Painful d. Harsh

b. Logical Reasoning

96. 'Doctor' is related to 'Patient', in the same way 'Lawyer' is related to
a. Customer
b. Criminal
c. Magistrate
d. Client
97. Find the odd one out.
a. 8-11 b. 1-4 c. 7-10 d. 3-5

98. Which figure will best represent the relationship amongst the three classes doctor, teacher, women ?



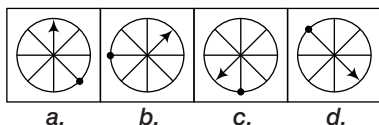
99. Find the missing number.



- a. 26
c. 32

- b. 36
d. 117

100. Find the odd one out.



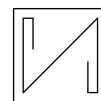
- a. b. c. d.

101. Count the number of squares in the following figure.

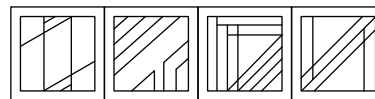


- a. 4 b. 5 c. 6 d. 7

102. In the question below, figure (X) is embedded in one of the following. Then, find out the correct alternative.



(X)



- a. b. c. d.

103. If LOFTY is coded as LPFUY, then DWARF will be written as

- a. DXASF b. DXBSG c. DXATF d. DWBSG

104. If the day before yesterday was Thursday, when will Sunday be ?

- a. Today b. Two days after today
c. Tomorrow d. Day after Tomorrow

105. Find the next term in the series 5, 16, 51, 158.

- a. 1452 b. 483 c. 481 d. 1454

PART IV

Mathematics

106. If the median of 25 observations is 45 and if the observations greater than the median are increased by 4, then the median of the new data is

- a. 49 b. 41 c. 45 d. 40

107. The differential equation of rectangular hyperbolas whose axes are asymptotes of the hyperbola $x^2 - y^2 = a^2$, is

- a. $y \frac{dy}{dx} = x$ b. $x \frac{dy}{dx} = -y$
c. $x \frac{dy}{dx} = y$ d. $x dy + y dx = C$

108. If $f(x) = \max \left\{ \sin x, \cos x, \frac{1}{2} \right\}$, then the area of the region bounded by the curves $y = f(x)$, X-axis, Y-axis and $x = \frac{5\pi}{3}$, is

- a. $\sqrt{2} - \sqrt{3} + \frac{5\pi}{12}$ b. $\sqrt{2} + \frac{\sqrt{3}}{2} + \frac{5\pi}{12}$
c. $\sqrt{2} + \sqrt{3} + \frac{5\pi}{12}$ d. None of these

109. The value of $\int_0^{[x]} \frac{2^x}{2^{[x]}} dx$ is

- a. $[x] \log 2$ b. $\frac{[x]}{\log 2}$
c. $\frac{1}{2} \cdot \frac{[x]}{\log 2}$ d. None of these

110. If $\int \frac{1}{x+x^5} dx = f(x) + C$, then the value of $\int \frac{x^4}{x+x^5} dx$ is

- a. $\log x - f(x) + C$ b. $f(x) + \log x + C$
c. $f(x) - \log x + C$ d. None of these

111. If $f(x) = a \log|x| + bx^2 + x$ has its extremum values at $x = -1$ and $x = 2$, then

- a. $a = 2, b = -1$
b. $a = 2, b = -1/2$
c. $a = -2, b = 1/2$
d. None of the above

112. Let $f(x) = \tan^{-1}\{g(x)\}$, where $g(x)$ is monotonically increasing for $0 < x < \frac{\pi}{2}$. Then, $f(x)$ is

- a. increasing on $(0, \pi/2)$
b. decreasing on $(0, \pi/2)$
c. increasing on $(0, \pi/4)$ and decreasing on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$
d. None of these

113. The equation $\sin x + x \cos x = 0$ has atleast one root in the interval

- a. $(-\pi/2, 0)$
b. $(0, \pi)$
c. $(-\pi/2, \pi/2)$

- d. None of the above
114. If the tangent at $(1, 1)$ on $y^2 = x(2-x)^2$ meets the curve again at P , then P is
 a. $(4, 4)$ b. $(-1, 2)$
 c. $(9/4, 3/8)$ d. None of these
115. If $y = |x - x^2|$, then $\frac{dy}{dx}$ at $x = 1$ is
 a. -1
 b. 1
 c. Does not exist
 d. None of the above
116. If $f(x) = |x - 2|$ and $g(x) = f\{f(x)\}$, then $g'(x)$ for $x > 20$ is
 a. 1 b. 2
 c. -1 d. None these
117. If α and β are the roots of the equation $ax^2 + bx + c = 0$ then $\lim_{x \rightarrow \alpha} (1 + ax^2 + bx + c)^{1/x-\alpha}$ is
 a. $e^{a(\alpha-\beta)}$ b. $e^{\alpha(\beta-\alpha)}$
 c. 1 d. None of these
118. If $f(x) = ax^2 + bx + c$ satisfies the identity $f(x+1) - f(x) = 8x + 3$ for all $x \in \mathbb{R}$. Then, (a, b) equals
 a. $(2, 1)$ b. $(4, -1)$
 c. $(-1, 4)$ d. $(-1, 1)$
119. The number of times a die must be tossed to obtain a 6 atleast once with probability exceeding 0.9 is atleast
 a. 13 b. 19
 c. 25 d. None of these
120. For all $n \in \mathbb{N}$, $3^{3n} - 26^n - 1$ is divisible by
 a. 24 b. 64
 c. 17 d. 676
121. If $x = 5$ and $y = -2$, then $x - 2y = 9$. The contrapositive of this proposition is
 a. if $x - 2y \neq 9$, then $x \neq 5$ or $y \neq -2$
 b. if $x - 2y = 9$, then $x \neq 5$ and $y \neq -2$
 c. $x - 2y = 9$ if and only if $x = 5$ and $y = -2$
 d. None of the above
122. The inverse of the function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = \log_a(x + \sqrt{x^2 + 1})$; ($a > 0, a \neq 1$), is
 a. $\frac{1}{2}(a^x + a^{-x})$ b. $\frac{1}{2}(a^x - a^{-x})$
 c. $\frac{1}{2}\left(\frac{a^x + a^{-x}}{a^x - a^{-x}}\right)$ d. Not defined
123. The relation R defined on the set $A = \{1, 2, 3, 4, 5\}$ by $R = \{(a, b) : |a^2 - b^2| < 16\}$ is given by
 a. $\{(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)\}$
 b. $\{(2, 2), (3, 2), (4, 2), (2, 4)\}$
 c. $\{(3, 3), (4, 3), (5, 4), (3, 4)\}$
 d. None of the above
124. If $A = \left\{ p : p = \frac{(x+2)(2x^5 + 3x^4 + 4x^3 + 5x^2 + 6)}{x^2 + 2x}; x, p \in \mathbb{Z}^+ \right\}$, then the number of elements in the set A , is
 a. 2 b. 3 c. 4 d. 6
125. If $(21.4)^a = (0.00214)^b = 100$, then the value of $\frac{1}{a} - \frac{1}{b}$ is
 a. 0 b. 1 c. 2 d. 4
126. If $\sin \theta + \cos \theta = \frac{1}{5}$ and $0 \leq \theta < \pi$, then $\tan \theta$ equals
 a. $-\frac{4}{3}$ b. $-\frac{3}{4}$ c. $\frac{3}{4}$ d. $\frac{4}{3}$
127. If a, b, c are the sides of $\triangle ABC$, right angled at C , then the minimum value of $\left(\frac{c}{a} + \frac{c}{b}\right)^2$ is
 a. 0 b. 4 c. 6 d. 8
128. The equation $\sin^4 x - 2\cos^2 x + a^2 = 0$ can be solved, if
 a. $-\sqrt{3} \leq a \leq \sqrt{3}$ b. $-\sqrt{2} \leq a \leq \sqrt{2}$
 c. $-1 \leq a \leq 1$ d. None of these
129. Complete solution set of $[\cot^{-1} x] + 2[\tan^{-1} x] = 0$, where $[\]$ denotes the greatest integer function, is equal to
 a. $(0, \cot 1)$ b. $(0, \tan 1)$ c. $(\tan 1, \infty)$ d. $(\cot 1, \tan 1)$
130. In $\triangle ABC$, base BC and area of triangle are fixed. The locus of the centroid of $\triangle ABC$ is a straight line that is
 a. parallel to side BC
 b. right bisector of side BC
 c. right angle of BC
 d. inclined at an angle $\sin^{-1}(\sqrt{\Delta/BC})$ to side BC
131. A triangle is formed by the lines $x + y = 0$, $x - y = 0$ and $lx + my = 1$. If l and m vary subject to the condition $l^2 + m^2 = 1$, then the locus of its circumcentre is
 a. $(x^2 - y^2)^2 = x^2 + y^2$ b. $(x^2 + y^2)^2 = x^2 - y^2$
 c. $x^2 + y^2 = 4x^2y^2$ d. $(x^2 - y^2)^2 = (x^2 + y^2)^2$
132. If the pairs of lines $x^2 + 2xy + ay^2 = 0$ and $ax^2 + 2xy + y^2 = 0$ have exactly one line in common, then the joint equation of the other two lines is given by
 a. $3x^2 + 8xy - 3y^2 = 0$ b. $3x^2 + 10xy + 3y^2 = 0$
 c. $y^2 + 2xy - 3x^2 = 0$ d. $x^2 + 2xy - 3y^2 = 0$
133. If the line $x \cos \theta + y \sin \theta = 2$ is the equation of a transverse common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6\sqrt{3}x - 6y + 20 = 0$, then the value of θ is
 a. $5\pi/6$ b. $2\pi/3$ c. $\pi/3$ d. $\pi/6$

134. If the parabola $y = ax^2 - 6x + b$ passes through (0, 2) and has its tangent at $x = 3/2$ parallel to X -axis, then
 a. $a = 2, b = -2$ b. $a = 2, b = 2$
 c. $a = -2, b = 2$ d. $a = -2, b = -3$
135. An ellipse is sliding along the coordinate axes. If the foci of the ellipse are (1, 1) and (3, 3), then the area of the director circle of the ellipse (in sq units) is
 a. 2π b. 4π c. 6π d. 8π
136. A normal to the hyperbola $\frac{x^2}{4} - \frac{y^2}{1} = 1$ has equal intercepts on the positive X -axis and Y -axis. If this normal touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then $a^2 + b^2$ is equal to
 a. 5 b. 25
 c. 16 d. None of these
137. India plays two matches each with West Indies and Australia. In any match, the probabilities of India getting points 0, 1 and 2 are 0.45, 0.05 and 0.50, respectively. Assuming that the outcomes are independent, the probability of India getting atleast 7 points is
 a. 0.8750 b. 0.0875 c. 0.0625 d. 0.0250
138. The number of 3×3 matrices A whose entries are either 0 or 1 and for which the system $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is
 a. 0 b. $2^9 - 1$
 c. 168 d. 2
139. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. What is the probability that the value of determinant chosen is positive?
 a. $3/16$ b. $3/6$
 c. $13/16$ d. $5/16$
140. The coefficient of t^{24} in $(1+t^2)^{12}(1+t^{12})(1+t^{24})$ is
 a. ${}^{12}C_6 + 3$ b. ${}^{12}C_6 + 1$
 c. ${}^{12}C_6$ d. ${}^{12}C_6 + 2$
141. How many different nine-digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions?
 a. 16 b. 36
 c. 60 d. 180
142. If a, b, c, d are positive real numbers such that $a + b + c + d = 2$, then $M = (a + b)(c + d)$ satisfies the relation
 a. $0 \leq M \leq 1$ b. $1 \leq M \leq 2$
 c. $2 \leq M \leq 3$ d. $3 \leq M \leq 4$
143. In the quadratic equation $ax^2 + bx + c$, if $\Delta = b^2 - 4ac$ and $\alpha + \beta, \alpha^2 + \beta^2, \alpha^3 + \beta^3$ are in GP, where α, β are the roots of $ax^2 + bx + c = 0$, then
 a. $\Delta \neq 0$ b. $b\Delta = 0$
 c. $c\Delta = 0$ d. $\Delta = 0$
144. Let $z = x + iy$ be a complex number, where x and y are integers. Then, the area of the rectangle whose vertices are the roots of the equation $\bar{z}z^3 + z\bar{z}^3 = 350$, is
 a. 48 sq units
 b. 32 sq units
 c. 40 sq units
 d. 80 sq units
145. The largest interval for which $x^{12} - x^9 + x^4 - x + 1 > 0$, is
 a. $-4 < x \leq 0$ b. $0 < x < 1$
 c. $-100 < x < 100$ d. $-\infty < x < \infty$
146. The set of all real numbers x for which $x^2 - |x + 2| + x > 0$, is
 a. $(-\infty, -2)$
 b. $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$
 c. $(-\infty, -1) \cup (1, \infty)$
 d. $(\sqrt{2}, \infty)$
147. Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three vectors of magnitude $\sqrt{3}, 1, 2$ such that $\mathbf{a} \times (\mathbf{a} \times \mathbf{c}) + 3\mathbf{b} = \mathbf{0}$. If θ is the angle between $\mathbf{a}$ and $\mathbf{c}$, then $\cos^2 \theta$ is equal to
 a. $\frac{3}{4}$ b. $\frac{1}{2}$
 c. $\frac{1}{4}$ d. None these
148. If resolved part of vector $\mathbf{a}$ along the vector $\mathbf{b}$ is $\mathbf{a}_1$ and that perpendicular to $\mathbf{b}$ is $\mathbf{a}_2$, then $\mathbf{a}_1 \times \mathbf{a}_2$ is equal to
 a. $\frac{(\mathbf{a} \times \mathbf{b}) \mathbf{b}}{|\mathbf{b}|^2}$ b. $\frac{(\mathbf{a} \times \mathbf{b}) \mathbf{a}}{|\mathbf{a}|^2}$
 c. $\frac{(\mathbf{a} \cdot \mathbf{b})(\mathbf{b} \times \mathbf{a})}{|\mathbf{b}|^2}$ d. $\frac{(\mathbf{a} \cdot \mathbf{b})(\mathbf{b} \times \mathbf{a})}{|\mathbf{b} \times \mathbf{a}|}$
149. The graph of the equation $x^2 + y^2 = 0$ in the three dimensional space, is
 a. X -axis b. Y -axis
 c. Z -axis d. xy -plane
150. If the straight lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ intersect at a point, then integer k is equal to
 a. 2 b. -2
 c. -5 d.

Answers

Physics

1. (b)	2. (c)	3. (b)	4. (b)	5. (b)	6. (d)	7. (b)	8. (a)	9. (b)	10. (d)
11. (c)	12. (c)	13. (c)	14. (c)	15. (c)	16. (c)	17. (c)	18. (b)	19. (d)	20. (c)
21. (a)	22. (d)	23. (a)	24. (b)	25. (d)	26. (b)	27. (d)	28. (b)	29. (d)	30. (d)
31. (c)	32. (a)	33. (b)	34. (c)	35. (a)	36. (a)	37. (c)	38. (b)	39. (c)	40. (a)

Chemistry

41. (b)	42. (c)	43. (a)	44. (c)	45. (c)	46. (d)	47. (b)	48. (a)	49. (d)	50. (b)
51. (c)	52. (b)	53. (b)	54. (c)	55. (b)	56. (b)	57. (a)	58. (d)	59. (c)	60. (a)
61. (a)	62. (c)	63. (d)	64. (b)	65. (b)	66. (d)	67. (b)	68. (c)	69. (a)	70. (b)
71. (d)	72. (c)	73. (c)	74. (a)	75. (c)	76. (a)	77. (b)	78. (d)	79. (a)	80. (c)

English Proficiency

81. (b)	82. (b)	83. (b)	84. (c)	85. (b)	86. (b)	87. (d)	88. (a)	89. (d)	90. (c)
91. (c)	92. (d)	93. (c)	94. (a)	95. (a)					

Logical Reasoning

96. (d)	97. (d)	98. (d)	99. (b)	100. (d)	101. (b)	102. (d)	103. (a)	104. (c)	105. (c)
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Mathematics

106. (c)	107. (b)	108. (b)	109. (b)	110. (a)	111. (b)	112. (a)	113. (b)	114. (c)	115. (c)
116. (a)	117. (a)	118. (b)	119. (a)	120. (d)	121. (a)	122. (b)	123. (d)	124. (c)	125. (c)
126. (a)	127. (d)	128. (b)	129. (d)	130. (a)	131. (a)	132. (b)	133. (d)	134. (b)	135. (d)
136. (d)	137. (b)	138. (a)	139. (a)	140. (d)	141. (c)	142. (a)	143. (c)	144. (a)	145. (d)
146. (b)	147. (a)	148. (c)	149. (c)	150. (c)					

Practice Set 4

BITSAT

Instructions

- There are 150 questions in all. The number of questions in each part is as follows Subject
- All questions are multiple choice questions with four options, only one is correct.
- Each correct answer fetches 3 marks while incorrect answer has a penalty of 1 mark.

No. of Questions

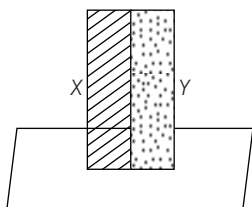
Part I Physics	1-40
Part II Chemistry	41-80
Part III (a) English Proficiency	81-95
(b) Logical Reasoning	96-105
Part IV Mathematics	106-150

PART I

Physics

- The coordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by
 - $\sqrt{\alpha^2 + \beta^2}$
 - $t\sqrt{\alpha^2 + \beta^2}$
 - $t^2\sqrt{\alpha^2 + \beta^2}$
 - $3t^2\sqrt{\alpha^2 + \beta^2}$
- If power (P), surface tension (T) and Planck's constant (h) are arranged so that the dimensions of time in their dimensional formula are in ascending order, then which of the following is correct ?
 - P, T, h
 - P, h, T
 - T, P, h
 - T, h, P
- A girl runs at a speed of 4 m/s to overtake a standing bus. When she is 6m behind the door at $t = 0$, the bus moves forward and continues this with an acceleration of 1.2 ms^{-2} . The girl reaches the door in time t , then
 - $4t = 6 + 0.6 t^2$
 - $1.2 t^2 = 4t$
 - $4t^2 = 1.2t$
 - $6 + 4t = 0.2 t^2$
- A mass m is attached to the end of a rod of length l . The mass goes along a vertical circular path with the other end hinged at its centre. What should be the minimum velocity of the mass at the bottom of the circle, so that the mass completes the circle?
 - $\sqrt{5 gl}$
 - $\sqrt{2 gl}$
 - $\sqrt{3 gl}$
 - $\sqrt{4 gl}$
- The upper half of an inclined plane with inclination ϕ is perfectly smooth, while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom, if the coefficient of friction for the lower half is
 - $2 \cos \phi$
 - $2 \sin \phi$
 - $2 \tan \phi$
 - $\tan \phi$
- A tennis ball of mass m is floating in air by a jet of water emerging out of a nozzle. If the water strikes the tennis ball with a speed v and just after collision water falls dead, the rate of mass flow of water in the nozzle is equal to
 - $\frac{mv}{g}$
 - $\frac{mg}{v}$
 - $\frac{2mg}{v}$
 - $\frac{2mv}{g}$

7. Which of the following is not an example of perfectly inelastic collision?
- A bullet fired into a block, if bullet gets embedded in block
 - Capture of an electron by an atom
 - A man jumping onto a moving boat
 - A ball bearing striking another ball bearing
8. Consider a uniform square plate of side a and mass m . The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is
- $\frac{5}{2} ma^2$
 - $\frac{ma^2}{12}$
 - $\frac{7}{12} ma^2$
 - $\frac{2}{3} ma^2$
9. A bimetallic strip consists of metal X and Y. It is mounted rigidly at the base as shown in figure. Metal X has higher coefficient of expansion as compared to metal Y. When the bimetallic strip is placed in a cold bath



- it will bend towards left
 - it will bend towards right
 - it will neither bend nor shrink
 - it will not bend but shrink
10. The excess pressure inside one soap bubble is 3 times that inside a second bubble, then the ratio of their surface area is
- 9 : 1
 - 1 : 27
 - 1 : 3
 - 1 : 9
11. Two equal drops are falling through air with terminal velocity 5 cm/s. If the drop gets combined, the new terminal velocity will be
- 5 × 2 cm/s
 - 5 × 4 cm/s
 - 5 × (4)^{1/3} cm/s
 - 5 × (4)^{2/3} cm/s
12. A wire of length L and cross-sectional area A is made of a material of Young's modulus Y . If the wire is stretched by the amount x , the work done is
- $\frac{YA}{L} x^2$
 - $\frac{YA}{2L} x^2$
 - $\frac{YAL}{x^2}$
 - $Y \frac{AL}{2x^2}$
13. A lead bullet of initial temperature 27°C and speed v km/h penetrates into a solid object and melts. If 50% of the kinetic energy is used to heat it, the value of v in km/h is (for lead, melting point = 600 K, latent heat of fusion = 2.5×10^4 J/kg and specific heat = 125 J/kg K)
- 1000
 - 1800
 - 3600
 - 1200

14. A Carnot's engine operates with source at 127°C and sink at 27°C. If the source supplies 40 kJ of heat energy, the work done by the engine is
- 1 kJ
 - 4 kJ
 - 10 kJ
 - 30 kJ
15. The mass of 1 mole of air is 29×10^{-3} kg, then the speed of sound in air at standard temperature and pressure is
- 332.5 ms⁻¹
 - 300 ms⁻¹
 - 280 ms⁻¹
 - 250 ms⁻¹
16. The equation of wave on a string of linear mass density 0.04 kg/m³ is given by

$$y = 0.02 \text{ (m)} \sin \left[2\pi \left(\frac{t}{0.04 \text{ (s)}} - \frac{x}{0.50 \text{ (m)}} \right) \right]$$

The tension in the string is

- 6.25 N
 - 4.0 N
 - 12.5 N
 - 0.5 N
17. Two bodies M and N of equal masses are suspended from two separate massless springs of spring constants k_1 and k_2 , respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of vibration of M to that of N is
- $\sqrt{k_1/k_2}$
 - $\sqrt{k_2/k_1}$
 - k_1/k_2
 - k_2/k_1
18. Two identical capacitors, have the same capacitance C . One of them is charged to potential V_1 and the other to V_2 . The negative end of the capacitors are connected together. When the positive ends are also connected, the decrease in the energy of the combined system is
- $\frac{1}{4} C (V_1^2 - V_2^2)$
 - $\frac{1}{4} C (V_1^2 + V_2^2)$
 - $\frac{1}{4} C (V_1 - V_2)^2$
 - $\frac{1}{4} C (V_1 + V_2)^2$
19. A parallel plate capacitor has its capacitance of 100 pF, when the plates of the capacitor are separated by a distance of t . Then, a metallic foil of thickness $t/3$ is introduced between the plates. The capacitance will then become
- 100 pF
 - 150 pF
 - $\frac{200}{3}$ pF
 - $\frac{100}{3}$ pF
20. Two charges, each equal to q , are kept at $x = -a$ and $x = +a$ on X-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement ($y \ll a$) along Y-axis, the net force acting on the particle is proportional to
- y
 - $-y$
 - $\frac{1}{y}$
 - $-\frac{1}{y}$

21. An electric cell of emf E is connected across a copper wire of diameter d and length l . The drift velocity of electrons in the wire is v_d . If the length of the wire is changed to $2l$, the new drift velocity of electrons in the copper wire will be

a. v_d b. $2v_d$
c. $\frac{v_d}{2}$ d. $\frac{v_d}{4}$

22. The resistance of a wire is $5\ \Omega$ at 50°C and $6\ \Omega$ at 100°C , the resistance of the wire at 0°C will be

a. $4\ \Omega$ b. $3\ \Omega$
c. $2\ \Omega$ d. $1\ \Omega$

23. An electric current is passed through a circuit containing two wires of same material connected in parallel. If the lengths and radii of the wires are in the ratio of $\frac{4}{3}$ and $\frac{2}{3}$, then the ratio of the currents passing through the wire will be

a. 3 b. $\frac{1}{3}$
c. $\frac{8}{9}$ d. $\frac{1}{2}$

24. A battery of 10V and internal resistance $0.5\ \Omega$ is connected in parallel with a battery of 12V and internal resistance $0.8\ \Omega$. The terminals are connected by an external resistance of $20\ \Omega$. The current flowing through the $20\ \Omega$ resistance is

a. 0.75 A b. 1.74 A
c. 0.53 A d. 1.21 A

25. A magnetic field of $(4 \times 10^{-3}\ \hat{k})\text{T}$ exerts a force of $(4\hat{i} + 3\hat{j}) \times 10^{-10}\text{N}$ on a particle having charge of $1 \times 10^{-9}\text{C}$ and moving in xy -plane. Find the velocity of the particle.

a. $(75\hat{i} + 100\hat{j})\text{ms}^{-1}$
b. $(75\hat{i} - 100\hat{j})\text{ms}^{-1}$
c. $(-75\hat{i} + 100\hat{j})\text{ms}^{-1}$
d. $(-75\hat{i} - 100\hat{j})\text{ms}^{-1}$

26. A voltmeter has resistance G ohm and range of V volt. The value of resistance used in series to convert into a voltmeter of range nV volt is

a. nG b. $(n-1)G$
c. $\frac{G}{n}$ d. $\frac{G}{n-1}$

27. The earth's magnetic field at a certain place has a horizontal component of 0.3G and total strength 0.5G . Find angle of dip.

a. $\delta = \tan^{-1}\left(\frac{4}{3}\right)$ b. $\delta = \tan^{-1}\left(\frac{3}{4}\right)$
c. $\delta = \tan^{-1}\left(\frac{5}{3}\right)$ d. $\delta = \tan^{-1}\left(\frac{3}{5}\right)$

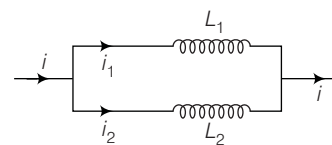
28. A cylindrical magnet has a length of 5 cm and a diameter of 1 cm. It has a uniform magnetisation of $5.30 \times 10^3\text{A/m}$. What is its magnetic dipole moment?

a. $1 \times 10^{-2}\text{J/T}$ b. $2.08 \times 10^{-2}\text{J/T}$
c. $3.08 \times 10^{-2}\text{J/T}$ d. $1.52 \times 10^{-2}\text{J/T}$

29. A coil of wire of certain radius has 600 turns and a self-inductance of 108 mH. The self-inductance of a second similar coil of 500 turns will be

a. 74 mH b. 75 mH c. 76 mH d. 77 mH

30. Two inductors L_1 and L_2 are connected in parallel and a time varying current flows as shown in figure. The ratio of currents $i_1 : i_2$ at any time t is



a. $\frac{L_2}{L_1}$ b. $\frac{L_1}{L_2}$ c. $\frac{L_2^2}{(L_1 + L_2)^2}$ d. $\frac{L_1^2}{(L_1 + L_2)^2}$

31. The intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ = wavelength of light) is I . If I_0 is the maximum intensity, then I/I_0 is equal to

a. $\frac{\sqrt{3}}{2}$ b. $\frac{1}{2}$ c. $\frac{3}{4}$ d. $\frac{1}{\sqrt{2}}$

32. If a thin prism of glass is dipped into water, then minimum deviation (w.r.t. air) of light produced by prism will be $[(^g\mu_a = 3/2) \text{ and } (^a\mu_w = 4/3)]$

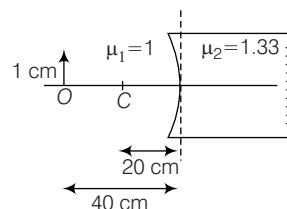
a. $\frac{1}{2}$ b. $\frac{1}{4}$ c. 2 d. $\frac{1}{5}$

33. A ray of light passes from vacuum into a medium of refractive index μ , the angle of incidence is found to be twice the angle of refraction.

Then, angle of incidence is

a. $\cos^{-1}\left(\frac{\mu}{2}\right)$ b. $\sin^{-1}(\mu)$
c. $\sin^{-1}\left(\frac{\mu}{2}\right)$ d. $2 \cos^{-1}\left(\frac{\mu}{2}\right)$

34. For an optical arrangement shown in figure. Find the position of image.



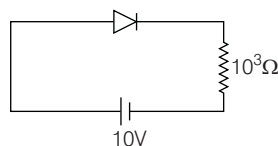
a. + 32 cm b. + 0.6 cm
c. - 6 cm d. 0.5 cm

35. The energy flux of sunlight reaching the surface of the earth is $1.39 \times 10^3 \text{ W m}^{-2}$. If the average wavelength of sunlight is 550 nm, then the number of photons per square metre incident on the earth per second are
 a. 2×10^{21} b. 3×10^{21}
 c. 4×10^{21} d. 4×10^{19}
36. If radius of the $^{27}\text{Al}_{13}$ nucleus is estimated to be 3.6 fermi, then the radius of $^{125}\text{Te}_{52}$ nucleus will be nearly
 a. 5 fermi b. 4 fermi c. 8 fermi d. 6 fermi
37. The binding energy of an electron in the ground state of He is equal to 24.6 eV. The energy required to remove both the electrons is
 a. 49.2 eV b. 24.6 eV
 c. 38.2 eV d. 79.0 eV
38. A radioactive nucleus can decay simultaneously by two different processes which have decay constant λ_1 and λ_2 .

The effective decay constant of the nuclide is λ , where

- a. $\lambda = \lambda_1 + \lambda_2$ b. $\lambda = 2(\lambda_1 + \lambda_2)$
 c. $\frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$ d. $\lambda = \sqrt{\lambda_1 \lambda_2}$

39. A junction diode is connected to a 10 V source and $10^3 \Omega$ rheostat figure. The slope of load line on the characteristics curve of diode will be



- a. 10^{-2} AV^{-1} b. 10^{-3} AV^{-1}
 c. 10^{-4} AV^{-1} d. 10^{-5} AV^{-1}
40. If minimum voltage in an AM wave was found to be 2V and maximum voltage 10V. Find % modulation index.
 a. 80% b. 66.67% c. 64.25% d. 76.25%

PART II

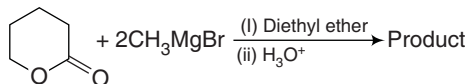
Chemistry

41. For the reaction, $2A(g) + B(g) \rightleftharpoons 3C(g) + D(g)$, 2 moles each of A and B were taken into a flask. The following must always be true when the system attained equilibrium.
 a. $[A] < [B]$ b. $[A] = [B]$ c. $[B] = [C]$ d. $[A] > [B]$
42. The pH at which an acid indicator with $K_a = 1 \times 10^{-5}$ changes colour when the indicator is $1 \times 10^{-3} \text{ M}$, is
 a. 4 b. 3 c. 5 d. 6
43. Which of the following reactions does not involve oxidation-reduction?
 I. $2\text{Cs} + 2\text{H}_2\text{O} \longrightarrow 2\text{CsOH} + \text{H}_2$
 II. $2\text{CuI}_2 \longrightarrow 2\text{CuI} + \text{I}_2$
 III. $\text{NH}_4\text{Br} + \text{KOH} \longrightarrow \text{KBr} + \text{NH}_3 + \text{H}_2\text{O}$
 IV. $4\text{KCN} + \text{Fe}(\text{CN})_2 \longrightarrow \text{K}_4[\text{Fe}(\text{CN})_6]$
 a. I and II b. I, III and IV
 c. I and III d. III and IV
44. 1 L of $N/2$ HCl solution was heated in a beaker. When the volume was reduced to 600 mL, 9.125 g of HCl was lost out. The new normality of solution is approximately
 a. 0.2 b. 0.4 c. 0.8 d. 0.7
45. Which of the following is the strongest Bronsted acid?
 a. b. c. d.
46. The compounds which are functional isomers from the following, are
 a. and b. and
 c. and d. and
47. At which of the following four conditions will the density of nitrogen be the largest?
 a. 273 K and 2 atm b. 546 K and 1 atm
 c. 546 K and 2 atm d. STP
48. The IUPAC name of is
 a. 3-methylbutane-1,4-diamine
 b. 3-(aminomethyl) butanamine
 c. 2-(aminomethyl) butan-4-amine
 d. 2-methylbutane-1, 4-diamine
49. The vapour pressure of pure water at 25°C is 23 torr. Then, the vapour pressure of 100 g of water to which 100 g of $\text{C}_6\text{H}_{12}\text{O}_6$ has been added, is
 a. 15.6 torr
 b. 21.3 torr
 c. 26.7 torr
 d. 24.8 torr

50. The correct order of magnetic moment (in BM) of the following is

- $[\text{Fe}(\text{CN})_6]^{4-} > [\text{MnCl}_4]^{2-} > [\text{CoCl}_4]^{2-}$
- $[\text{MnCl}_4]^{2-} > [\text{Fe}(\text{CN})_6]^{4-} > [\text{CoCl}_4]^{2-}$
- $[\text{MnCl}_4]^{2-} > [\text{CoCl}_4]^{2-} > [\text{Fe}(\text{CN})_6]^{4-}$
- $[\text{Fe}(\text{CN})_6]^{4-} > [\text{CoCl}_4]^{2-} > [\text{MnCl}_4]^{2-}$

51. What is the product of the following reaction?



- $\text{CH}_3\text{OCH}_2\text{CH}_2\text{CH}_2\text{CH}_2\text{CH}(\text{OH})\text{CH}_3$
- $\text{HOCH}_2\text{CH}_2\text{CH}_2\text{CH}_2\text{C}(\text{OH})(\text{CH}_3)_2$
- $\text{HO}-\text{CH}(\text{CH}_3)\text{CH}_2\text{CH}_2\text{CH}_2\text{CH}(\text{CH}_3)-\text{OH}$
- $\text{HOCH}_2\text{CH}_2\text{CH}_2\text{CH}_2\text{C}(\text{OH})(\text{CH}_3)_2$

52. Which of the following alkyl halide undergo rearrangement in $\text{S}_{\text{N}}1$ reaction?

-
-
- $\text{CH}_3-\text{C}(\text{CH}_3)_2-\text{CH}(\text{I})-\text{CH}_3$
- All of these

53. The ionic equivalent conductivity of $\text{C}_2\text{O}_4^{2-}$, K^+ and Na^+ ions are x , y , z $\text{S cm}^2 \text{eq}^{-1}$ respectively. Then, $\Lambda^\circ_{\text{eq}}$ of $(\text{NaOOC}-\text{COOK})$ is

- $x + y + z$
- $x - \frac{y}{2} + \frac{z}{2}$
- $x + \frac{y}{2} + \frac{z}{2}$
- $x + \frac{y}{2} - \frac{z}{2}$

54. The decreasing order of basic character of the transition metal monoxides is

- $\text{VO} > \text{CrO} > \text{TiO} > \text{FeO}$
- $\text{TiO} > \text{FeO} > \text{VO} > \text{CrO}$
- $\text{CrO} > \text{VO} > \text{FeO} > \text{TiO}$
- $\text{TiO} > \text{VO} > \text{CrO} > \text{FeO}$

55. At 25°C in the saturated solution of Ag_2CrO_4 , if concentration of Ag^+ ion is $1.0 \times 10^{-5} \text{ mol/L}$, then the solubility product of Ag_2CrO_4 at 25°C , is

- $5 \times 10^{-3} \text{ mol}^2 \text{L}^{-3}$
- $5 \times 10^{-15} \text{ mol}^2 \text{L}^{-3}$
- $5 \times 10^{-16} \text{ mol}^2 \text{L}^{-3}$
- $5 \times 10^{-14} \text{ mol}^2 \text{L}^{-3}$

56. The gold numbers of protective colloids A, B, C and D are 0.04, 0.002, 10 and 25, respectively. The protective powers of A, B, C and D are in the order

- $D > C > A > B$
- $B > C > A > D$
- $D > C > B > A$
- $B > A > C > D$

57. Ammonia, manufactured by the Haber's process is usually carried out at about 500°C . If a temperature of about 250°C was used instead of 500°C , then

- no ammonia would be formed at all
- the percentage of ammonia in the equilibrium mixture would be too low
- a catalyst would be of no use at all at this temperature
- the rate of formation of ammonia would be too slow

58. The edge length of a body-centred cubic unit cell is 390 pm. If the radius of the cation is 150 pm, then, the radius of the anion is

- 180.3 pm
- 187.7 pm
- 193.4 pm
- 162.4 pm

59. For the reaction $2\text{NO}_2 \longrightarrow 2\text{NO} + \text{O}_2$, activation energy is 110 kJ/mol. At 400°C , the rate constant is $7.8 \text{ mol}^{-1} \text{L s}^{-1}$. The value of rate constant at 430°C is

- $16 \text{ mol}^{-1} \text{L s}^{-1}$
- $18 \text{ mol}^{-1} \text{L s}^{-1}$
- $13 \text{ mol}^{-1} \text{L s}^{-1}$
- $19 \text{ mol}^{-1} \text{L s}^{-1}$

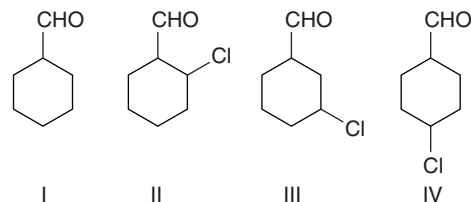
60. Choose the correct statement from the following.

- Both CH_3I and CF_3I reacts with $\text{Na}^+ [\text{Mn}(\text{CO})_5]^-$ and forms $\text{I}^+ [\text{Mn}(\text{CO})_5]^-$
- Both CH_3I and CF_3I reacts with $\text{Na}^+ [\text{Mn}(\text{CO})_5]^-$ and for $[\text{CH}_3]^+ [\text{Mn}(\text{CO})_5]^-$
- CH_3I reacts with $\text{Na}^+ [\text{Mn}(\text{CO})_5]^-$ forms $[\text{CH}_3]^+ [\text{Mn}(\text{CO})_5]^-$ while CF_3I forms $\text{I}^+ [\text{Mn}(\text{CO})_5]^-$
- CH_3I reacts with $\text{Na}^+ [\text{Mn}(\text{CO})_5]^-$ forms $\text{I}^+ [\text{Mn}(\text{CO})_5]^-$ while CF_3I forms $[\text{CF}_3]^+ [\text{Mn}(\text{CO})_5]^-$

61. 6×10^{24} atoms of an element weigh 200 g. If this element form homodiatom gas, then the molar mass of gas is

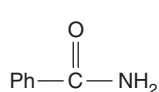
- 30
- 40
- 20
- 45

62. The correct decreasing order of the following towards nucleophilic addition reaction is

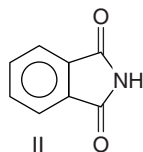


- $\text{IV} > \text{III} > \text{II} > \text{I}$
- $\text{II} > \text{III} > \text{IV} > \text{I}$
- $\text{I} > \text{II} > \text{III} > \text{IV}$
- $\text{I} > \text{IV} > \text{III} > \text{II}$

79. The correct increasing order of acidic character of the following is



I



II



III

- a. III < I < II b. II < I < III c. I < III < II d. II < III < I

80. Which of the following contains greatest number of N-atoms?

- a. 1.0 mole of NH_4Cl
 b. 6.02×10^{23} molecules of NO_2
 c. 22.4 L nitrogen gas at STP
 d. 500 mL of 2.0 M NH_3

PART III

a. English Proficiency

Directions (Q. Nos. 81 to 84) *Read the passage and then choose the most suitable option from the given ones.*

It was a very cold evening and so few people were seen out on the streets. I did not go out myself although it was my habit not to keep indoors after sunset. So, I closed all the doors and windows of my room, took the book which had been lying opened on the table and tried to read it. The cold was getting so severe that I started shivering, so I wrapped myself up with a bigger blanket. But I could not continue reading the book because I was nearly rendered incapable of turning the pages.

81. The author's habit was to
 a. read in the evening b. sleep in the evening
 c. go out in the evening d. play in the evening
82. If it was not so cold the author would have
 a. liked to work
 b. liked to sit and look out to the streets
 c. liked to read
 d. liked to have a stroll
83. The author could not continue reading the book because
 a. he did not like to b. he was feeling tired
 c. he was feeling very cold d. the lights had gone off
84. There were not many people outside because
 a. it was a rainy evening
 b. it was a cold evening
 c. it was a dark evening
 d. there was heavy show outside

Directions (Q. Nos. 85 to 88) *Rearrange the jumbled up parts of the sentence and choose the appropriate option.*

85. With an unsteady hand
 P : on my desk
 Q : from his pocket
 R : he took an envelope
 S : and threw it
 The proper sequence should be
 a. QRPS b. QRSP c. RQSP d. RQSP

86. P : she gave her old coat
 Q : to a beggar
 R : the one with the brown fur on it
 S : shivering with cold

The proper sequence should be

- a. SQRP b. SPRQ
 c. PRQS d. PSQR

87. S1 : I last visited Kandy almost 10 years ago.

S6 : Set around low, forested hills. with the Mahavelli river flowing nearby, the landscape has an instance yet tranquil beauty.

P : The bomb blasts and ugly face of ethnic conflict have not robbed the place of its gracious pace of life

Q : Theartificial lake, which dominates the landscape of the city, was built by its last king in 1806

R : Located in central Sri Lanka, at an elevation of about 1600 ft, Kandy was the last Sinhala outpost of autonomy, resisting both Protuguese and Dutch rule before it succumbed to the British in 1818

S : Most famous for its Temple of the Tooth, a golden pagoda, elegant and imperaturable city

The proper sequence should be

- a. RPQS b. SPRQ c. QSPR d. PSQR

88. S1 : The status of women in our country is, on the whole, far from high.

S6 : Education can lift these moment out of the depths of misery and ignorance into which they have sunk.

P : But the plight of women in village is still miserable

Q : The educated women in cities enjoy equality with the men folk.

R : The moment for the freedom and rights of women has certainly been steadily gaining momentum

S : Their education has been throughly neglected.

The proper sequence should be

- a. RPQS b. RQPS c. SQPR d. SPQR

Directions (Q. Nos. 89-92) *In the sentences given below there is an error in one of the parts. Choose that part as your answer.*

89. She misplaced her spectacle (a)/ is now feeling (b)/ great difficulty in studying. (c)/ No error (d)
90. Arabian Nights are (a)/a collection of (b)/ very interesting episodes of adventure. (c)/No error (d)
91. I hope to visit (a)/my uncle only next year (b)/during summer vacations. (c)/No error (d)
92. Ration has run out (a)/and the District Magistrate (b)/ has been informed. (c)/No error (d)

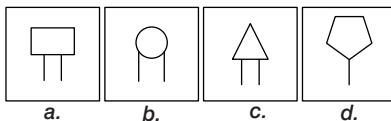
Directions (Q. Nos. 93 to 95) *The underlined part of the sentence may need an improvement. Choose the option that suits the improvement in best way.*

93. Five year ago today, I am sitting in a small Japanese car driving across Poland toward Berlin.
a. was sitting b. sat
c. have been sitting d. No improvement
94. Sumit had told me that he hasn't done it yet.
a. told
b. tells
c. was telling
d. No improvement
95. The record for the biggest tiger hunt has not been met since 1911 when Lord Hardinge, then Viceroy of India, shot a tiger measured 11 feet and 6 inches.
a. improved b. broken
c. bettered d. No improvement

b. Logical Reasoning

96. In a class, boy stand in a single line. One of the boys in nineteenth in order from both the ends. How many boys are there in the class?
a. 27 b. 37 c. 38 d. 39

97. Find out the odd figure

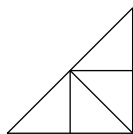


98. Find out next term in the series.

5, 6, 9, 14, 21, ?

- a. 28 b. 30 c. 31 d. 29

99. How many triangles are there in the given figure?



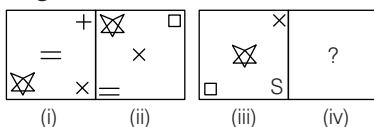
- a. 5 b. 6 c. 7 d. 8

100. If the day before yesterday was Friday, what day will two days after the day after tomorrow be ?

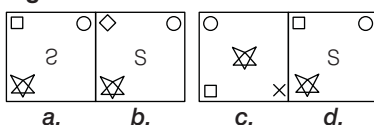
- a. Saturday b. Thursday c. Friday d. Sunday

101. In the problem figure, which figure will replace the question mark (?) from the answer figures ?

Problem Figures

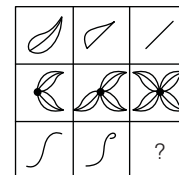


Answer Figures

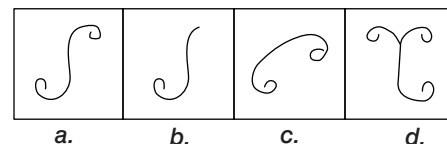


102. Find out which of the answer figures (a), (b), (c) and (d) completes the figures matrix.

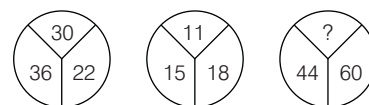
Problem Figure



Answer Figures



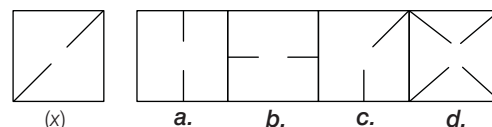
103. The missing in the figure shown below is



- a. 45 b. 54
c. 72 d. 90

104. Figure (X) is embedded in one of the four alternative figures (a), (b), (c) and (d).

Find the alternative which contains figures (X) as its part.



105. Find the water image of the word given below

TOP

- a. 1ob b. .toq
c. tob d. 1op

PART IV

Mathematics

- 106.** The set of all values of x satisfying $x^{\log_x(1-x)^2} = 9$, is
 a. a subset of R containing N
 b. a subset of R containing Z (set of all integers)
 c. a finite set containing atleast two elements
 d. a finite set
- 107.** If $f(x) = \sin^6 x + \cos^6 x$, then range of $f(x)$ is
 a. $\left[\frac{1}{4}, 1\right]$
 b. $\left[\frac{1}{4}, \frac{3}{4}\right]$
 c. $\left[\frac{3}{4}, 1\right]$
 d. None of these
- 108.** If $a \sin x + b \cos(x + \theta) + b \cos(x - \theta) = d$, then the minimum value of $|\cos \theta|$ is
 a. $\frac{1}{2|b|} \sqrt{d^2 - a^2}$
 b. $\frac{1}{2|a|} \sqrt{d^2 - a^2}$
 c. $\frac{1}{2|d|} \sqrt{d^2 - a^2}$
 d. None of the above
- 109.** The set of values of x in $(0, \pi)$ satisfying the equation $1 + \log_2 \sin x + \log_2 \sin 3x \geq 0$, is
 a. $\left(\frac{2\pi}{3}, \frac{3\pi}{4}\right]$
 b. $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$
 c. $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{2\pi}{3}, \pi\right)$
 d. $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$
- 110.** If $\cot^{-1}(\sqrt{\cos \alpha}) - \tan^{-1}(\sqrt{\cos \alpha}) = x$, then $\sin x$ equals
 a. $\tan^2 \frac{\alpha}{2}$
 b. $\cot^2 \frac{\alpha}{2}$
 c. $\tan \alpha$
 d. $\cot \frac{\alpha}{2}$
- 111.** In $\triangle ABC$, if P, Q, R divides sides BC, AC and AB respectively in the ratio $k : 1$ and $\frac{\text{area of } \triangle PQR}{\text{area of } \triangle ABC} = \frac{1}{3}$, then k is equal to
 a. $1/3$
 b. 2
 c. 3
 d. None of these
- 112.** A line is drawn perpendicular to line $y = 5x$, meeting the coordinate axes at A and B . If the area of $\triangle OAB$ is 10 sq units, where O is the origin, then the equation of drawn line is
 a. $3x - y = 9$
 b. $x + 5y = 10$
 c. $x + 4y = 10$
 d. $x - 4y = 10$
- 113.** If the slope of one of the lines represented by $ax^2 + 2hxy + by^2 = 0$ is the square of other, then $\frac{a+b}{h} + \frac{bh^2}{ab}$ equals
 a. 4
 b. 6
 c. 8
 d. None of these
- 114.** If the circle having radii r_1 and r_2 intersect orthogonally, then the length of their common chord is
 a. $\frac{2r_1r_2}{\sqrt{r_1^2 + r_2^2}}$
 b. $\frac{\sqrt{r_1^2 + r_2^2}}{2r_1r_2}$
 c. $\frac{r_1r_2}{\sqrt{r_1^2 + r_2^2}}$
 d. $\frac{\sqrt{r_1^2 + r_2^2}}{r_1r_2}$
- 115.** AB is double ordinate of the parabola $y^2 = 4ax$. Tangents drawn to the parabola at A and B meet Y -axis at A_1 and B_1 , respectively. If the area of trapezium AA_1B_1B is equal to $12a^2$, then the angle subtended by A_1B_1 at the focus of the parabola is equal to
 a. $2 \tan^{-1} 3$
 b. $\tan^{-1} 3$
 c. $2 \tan^{-1} 2$
 d. $\tan^{-1} 2$
- 116.** The line $y = mx - \frac{(a^2 - b^2)m}{\sqrt{a^2 + b^2m^2}}$ is normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ for all values of m belonging to
 a. $(0, 1)$
 b. $(0, \infty)$
 c. R
 d. None
- 117.** Let E^c denotes the complement of an event E . If E, F, G are pairwise independent events with $P(G) > 0$ and $P(E \cap F \cap G) = 0$, then $P(E^c \cap F^c / G)$ equals
 a. $P(E^c) + P(F^c)$
 b. $P(E^c) - P(F^c)$
 c. $P(E^c) - P(F)$
 d. $P(E) - P(F^c)$
- 118.** Let M and N be two 3×3 non-singular and skew-symmetric matrices such that $MN = NM$. If P^T denotes the transpose of P , then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to
 a. M^2
 b. $-N^2$
 c. $-M^2$
 d. MN
- 119.** Consider the set A of all determinants of order 3 with entries 0 or 1 only. Let B be the subset of A consisting of all determinants with values -1 . Then,
 a. C is empty
 b. B has as many elements as C
 c. $A = B \cup C$
 d. B has twice as many elements as C
- 120.** The value of $\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + \binom{30}{2}\binom{30}{12} - \dots + \binom{30}{20}\binom{30}{30}$ where $\binom{n}{r} = {}^nC_r$, is
 a. $\binom{30}{10}$
 b. $\binom{30}{15}$
 c. $\binom{60}{30}$
 d. $\binom{31}{10}$
- 121.** Let $S = \{1, 2, 3, 4\}$, then the total number of unordered pairs of disjoint subsets of S is equal to
 a. 25
 b. 34
 c. 42
 d. 41

- 122.** If $a_1, a_2, \dots, a_n$ are positive real numbers whose product is a fixed number c , then the minimum value of $a_1 + a_2 + \dots + a_{n-1} + 2a_n$ is
 a. $n(2c)^{1/n}$ b. $(n+1)c^{1/n}$
 c. $2nc^{1/n}$ d. $(n+1)(2c)^{1/n}$
- 123.** For a a positive integer n , let $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{(2^n) - 1}$. Then,
 a. $a(200) < 100$ b. $a(200) \geq 100$
 c. $a(200) \leq 100$ d. $a(200) > 100$
- 124.** Let z be a complex number such that the imaginary part of z is non-zero and $a = z^2 + z + 1$ is real. Then, a cannot take the value
 a. -1 b. $1/3$ c. $1/2$ d. $3/4$
- 125.** If α and β are the roots of $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + s = 0$, then the equation of $x^2 - 4qx + 2q^2 - r = 0$ has always
 a. one positive and one negative root
 b. two positive roots
 c. two negative roots
 d. can't say anything
- 126.** If x satisfies $|x - 1| + |x - 2| + |x - 3| \geq 6$, then
 a. $0 \leq x \leq 4$ b. $x \leq -2$ or $x \geq 4$
 c. $x \leq 0$ or $x \geq 4$ d. None of these
- 127.** The standard deviation of some temperature data (in $^\circ\text{C}$) is 5. If the data were converted into $^\circ\text{F}$, then the variance would be
 a. 81 b. 57 c. 36 d. 25
- 128.** The solution of the differential equation $xdy - ydx = \sqrt{x^2 + y^2}dx$, is
 a. $x + \sqrt{x^2 + y^2} = Cx^2$ b. $y - \sqrt{x^2 + y^2} = Cx$
 c. $x - \sqrt{x^2 + y^2} = Cx$ d. $y + \sqrt{x^2 + y^2} = Cx^2$
- 129.** Let the straight line $x = b$ divide the area enclosed by $y = (1-x)^2$, $y = 0$ and $x = 0$ into two parts R_1 ($0 \leq x \leq b$) and R_2 ($b \leq x \leq 1$) such that $R_1 - R_2 = \frac{1}{4}$. Then, b equals
 a. $\frac{3}{4}$ b. $\frac{1}{2}$ c. $\frac{1}{3}$ d. $\frac{1}{4}$
- 130.** The value of $\int_0^{100} [\tan^{-1} x] dx$ is
 a. 100 b. $100 - \tan^{-1} 1$
 c. $100 - \tan 1$ d. None of these
- 131.** $\int \frac{x-1}{(x+1)\sqrt{x^3+x^2+x}} dx$ equals
 a. $\tan^{-1}\left(\frac{x^2+x+1}{x}\right) + C$ b. $2 \tan^{-1}\left(\frac{x^2+x+1}{x}\right) + C$
 c. $3 \tan^{-1}\left(\frac{x^2+x+1}{x}\right) + C$ d. None of these
- 132.** The critical points of $f(x) = \frac{|2-x|}{x^2}$ are
 a. $x = 0, 2$ b. $x = 2, 4$
 c. $x = 2, -4$ d. None of these
- 133.** The length of the longest interval in which the function $3 \sin x - 4 \sin^3 x$ is increasing, is
 a. $\pi/3$ b. $\pi/2$ c. $3\pi/2$ d. π
- 134.** If $27a + 9b + 3c + d = 0$, the equation $4ax^3 + 3bx^2 + 2cx + d = 0$ has atleast one real root lying between
 a. 0 and 1 b. 1 and 3
 c. 0 and 3 d. None
- 135.** If the parabolas $y = x^2 + ax + b$ and $y = x(c - x)$ touch each other at the point (1, 0), then $a + b + c$ equals
 a. 1 b. -1 c. 0 d. None
- 136.** Let f be a differentiable function satisfying $[f(x)]^n = f(nx)$ for all $x \in R$. Then, $f'(x) f(nx)$ equals
 a. $f(x)$ b. 0
 c. $f(x) f'(nx)$ d. None of these
- 137.** If $f(x - y), f(x) f(y)$ and $f(x + y)$ are in A.P. for all $x, y \in R$ and $f(0) = 0$, then
 a. $f'(-2) = f'(2)$ b. $f'(-3) = -f'(3)$
 c. $f'(-2) + f'(2) = 0$ d. None of these
- 138.** If $f(x) = \frac{[x]}{|x|}$, $x \neq 0$, where $[]$ denotes the greatest integer function, then $f'(1)$ equals
 a. -1 b. 1
 c. Does not exist d. None of these
- 139.** The value of $\lim_{n \rightarrow \infty} \{\sqrt[3]{n^2 - n^3} + n\}$, is
 a. $\frac{1}{3}$ b. $-\frac{1}{3}$ c. $\frac{2}{3}$ d. $-\frac{2}{3}$
- 140.** The domain of definition of $f(x) = \sin^{-1}\{\log_2(x^2 + 3x + 4)\}$ is
 a. $[-4, -1/4]$ b. $[-3, -1/3]$
 c. $[-2, -1]$ d. None of these
- 141.** If the range of a random variable X is 0, 1, 2, 3, ... with $P(X = k) = \left(\frac{k+1}{3^k}\right) a$ for $k \geq 0$, then a equals
 a. $2/3$ b. $4/9$ c. $8/27$ d. $16/81$
- 142.** Let $S(k) = 1 + 3 + 5 + \dots + (2k - 1) = 3 + k^2$. Then, which one of the following is true?
 a. Principle of mathematical induction can be used to prove the formula
 b. $S(k) \Rightarrow S(k+1)$
 c. $S(k) \nRightarrow S(k+1)$
 d. $S(1)$ is correct

143. The false statement in the following is
 a. $p \wedge (\sim p)$ is a contradiction
 b. $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a contradiction
 c. $\sim (\sim p) \leftrightarrow p$ is tautology
 d. $p \wedge (\sim p)$ is a tautology
144. Let $f: R \rightarrow R$ be given by $f(x) = [x^2] + [x + 1] - 3$, where $[x]$ denotes the greatest integer less than or equal to x . Then, $f(x)$ is
 a. many-one and onto b. many-one and into
 c. one-one and into d. one-one and onto
145. The relation on the set $A = \{x: |x| < 3, x \in Z\}$ is defined by $R = \{(x, y): y = |x|, x \neq -1\}$. Then, the number of elements in the power set of R is
 a. 32 b. 16 c. 8 d. 64
146. If $A = \{\theta: \cos \theta > -1/2, 0 \leq \theta \leq \pi\}$ and $B = \{\theta: \sin \theta > 1/2, \pi/3 \leq \theta \leq \pi\}$, then
 a. $A \cap B = \{\theta: \pi/3 \leq \theta \leq 2\pi/3\}$
 b. $A \cap B = \{\theta: -\pi/3 \leq \theta \leq 2\pi/3\}$
 c. $A \cup B = \{\theta: -5\pi/6 \leq \theta \leq 5\pi/6\}$
 d. $A \cup B = \{\theta: 0 \leq \theta \leq \pi/6\}$
147. A vector which makes equal angles with the vectors $\frac{1}{3}(\hat{i} - 2\hat{j} + 2\hat{k}), \frac{1}{5}(-4\hat{i} - 3\hat{k}), \hat{j}$, is
 a. $5\hat{i} + \hat{j} + 5\hat{k}$ b. $-5\hat{i} + \hat{j} + 5\hat{k}$
 c. $5\hat{i} - \hat{j} - 5\hat{k}$ d. $5\hat{i} + \hat{j} - 5\hat{k}$
148. The vectors $\mathbf{a} = x\hat{i} + (x+1)\hat{j} + (x+2)\hat{k}$, $\mathbf{b} = (x+3)\hat{i} + (x+4)\hat{j} + (x+5)\hat{k}$ and $\mathbf{c} = (x+6)\hat{i} + (x+7)\hat{j} + (x+8)\hat{k}$ are coplanar for
 a. all values of x b. $x < 0$
 c. $x > 0$ d. None of the above
149. If the direction cosines of a straight line are k, k, k , then
 a. $k > 0$ b. $0 < k < 1$
 c. $k = 1$ d. $k = \frac{1}{\sqrt{3}}$ or $\frac{-1}{\sqrt{3}}$
150. Let the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lie in the plane $x + 3y - \alpha z + \beta = 0$. Then, (α, β) equals
 a. $(6, -17)$ b. $(-6, 7)$
 c. $(5, -15)$ d. $(-5, 5)$

Answers

Physics

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (d) | 2. (a) | 3. (a) | 4. (d) | 5. (c) | 6. (b) | 7. (d) | 8. (d) | 9. (a) | 10. (a) |
| 11. (c) | 12. (b) | 13. (b) | 14. (c) | 15. (a) | 16. (a) | 17. (b) | 18. (c) | 19. (b) | 20. (a) |
| 21. (c) | 22. (a) | 23. (b) | 24. (c) | 25. (c) | 26. (b) | 27. (a) | 28. (b) | 29. (b) | 30. (a) |
| 31. (c) | 32. (b) | 33. (d) | 34. (a) | 35. (c) | 36. (d) | 37. (d) | 38. (a) | 39. (b) | 40. (b) |

Chemistry

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 41. (a) | 42. (c) | 43. (d) | 44. (b) | 45. (d) | 46. (a) | 47. (a) | 48. (d) | 49. (b) | 50. (c) |
| 51. (b) | 52. (d) | 53. (c) | 54. (d) | 55. (c) | 56. (d) | 57. (d) | 58. (b) | 59. (b) | 60. (c) |
| 61. (b) | 62. (b) | 63. (c) | 64. (b) | 65. (d) | 66. (b) | 67. (a) | 68. (b) | 69. (a) | 70. (a) |
| 71. (b) | 72. (b) | 73. (b) | 74. (a) | 75. (a) | 76. (c) | 77. (d) | 78. (c) | 79. (a) | 80. (c) |

English Proficiency

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 81. (c) | 82. (d) | 83. (c) | 84. (b) | 85. (d) | 86. (c) | 87. (d) | 88. (b) | 89. (a) | 90. (a) |
| 91. (c) | 92. (a) | 93. (a) | 94. (b) | 95. (b) | | | | | |

Logical Reasoning

- | | | | | | | | | | |
|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| 96. (b) | 97. (d) | 98. (b) | 99. (c) | 100. (b) | 101. (d) | 102. (a) | 103. (c) | 104. (d) | 105. (a) |
|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|

Mathematics

- | | | | | | | | | | |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 106. (d) | 107. (a) | 108. (a) | 109. (a) | 110. (a) | 111. (b) | 112. (b) | 113. (b) | 114. (a) | 115. (c) |
| 116. (c) | 117. (c) | 118. (c) | 119. (b) | 120. (a) | 121. (d) | 122. (a) | 123. (d) | 124. (d) | 125. (a) |
| 126. (c) | 127. (a) | 128. (d) | 129. (b) | 130. (c) | 131. (b) | 132. (d) | 133. (a) | 134. (c) | 135. (c) |
| 136. (c) | 137. (a) | 138. (c) | 139. (a) | 140. (c) | 141. (b) | 142. (b) | 143. (b) | 144. (b) | 145. (b) |
| 146. (a) | 147. (b) | 148. (a) | 149. (d) | 150. (b) | | | | | |

Practice Set 5

BITSAT

Instructions

- There are 150 questions in all. The number of questions in each part is as follows Subject
- All questions are multiple choice questions with four options, only one is correct.
- Each correct answer fetches 3 marks while incorrect answer has a penalty of 1 mark.

No. of Questions

Part I Physics	1-40
Part II Chemistry	41-80
Part III (a) English Proficiency	81-95
(b) Logical Reasoning	96-105
Part IV Mathematics	106-150

PART I

Physics

- Charges $+q$ and $-q$ are placed at points A and B respectively, which are a distance $2L$ apart. C is the mid-point between the points A and B . The work done in moving a charge $+Q$ along the semi-circle CRD is
 - $\frac{-qQ}{6\pi\epsilon_0 L}$
 - $\frac{-qQ}{4\pi\epsilon_0 L}$
 - $\frac{qQ}{6\pi\epsilon_0 L}$
 - $\frac{-Qq}{2\pi\epsilon_0 L}$
- In the given figures, which one of the diode shown is reverse biased?

a.

b.

c.

d.
- The wavelength of the first line of Lyman series for hydrogen atom is equal to that of the second line of Balmer series of hydrogen like ion. The atomic number Z of hydrogen like atom is
 - 2
 - 3
 - 4
 - 1
- In a Young's double slit experiment, the angular position (θ) of a bright fringe having intensity one forth of the maximum intensity is given by

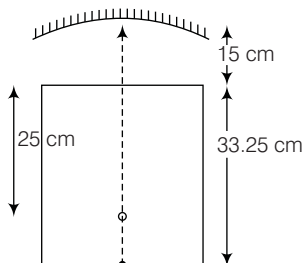
a. $\sin^{-1}\left(\frac{\lambda}{4d}\right)$

c. $\sin^{-1}\left(\frac{\lambda}{2d}\right)$

b. $\sin^{-1}\left(\frac{\lambda}{3d}\right)$

d. $\sin^{-1}\left(\frac{\lambda}{d}\right)$

5. A tank of height 33.25 cm is completely filled with a liquid of index of refraction 1.33. A concave mirror of focal length 15.0 cm is placed above the tank as shown in figure. An object is placed at the bottom of the tank on the axis of concave mirror and its image is formed at a distance 25.0 cm below the surface of the liquid. The focal length of the mirror is

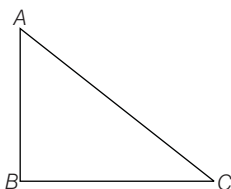


- a. 10 cm b. 18.3 cm c. 20.3 cm d. 25.0 cm

6. In a series resonant LCR circuit, the voltage across R is 100 V and $R = 1\text{ k}\Omega$ with $C = 2\mu\text{F}$. The resonant frequency ω is 200 rad/s. At resonance the voltage across L is
 a. $4 \times 10^{-3}\text{ V}$ b. $2.5 \times 10^{-2}\text{ V}$
 c. 40 V d. 250 V
7. A 2700Ω resistor and a $1.1\mu\text{F}$ capacitor are connected in series across a generator (60 Hz, 120 V). Determine the average power consumed in the circuit.
 a. 3 W b. 9 W c. 10 W d. 7 W
8. Needles N_1 , N_2 and N_3 are made of ferromagnetic, a paramagnetic and a diamagnetic substance, respectively. A magnet when brought close to them will
 a. attract all three of them
 b. attract N_1 and N_2 strongly but repel N_3
 c. attract N_1 strongly, N_2 weakly and repel N_3 weakly
 d. attract N_1 strongly, but repel N_2 and N_3 weakly
9. A long straight wire of radius a carries a steady current I . The current is uniformly distributed across its cross-section, the ratio of the magnetic field at $\frac{a}{2}$ and $2a$ is

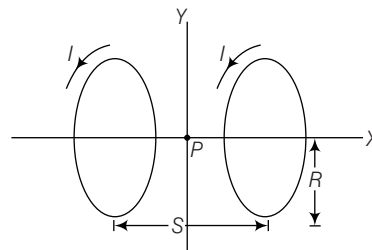
- a. $\frac{1}{4}$ b. 4 c. 1 d. $\frac{1}{2}$

10. A current carrying closed loop in the form of a right angle isosceles $\triangle ABC$ is placed in a uniform magnetic field acting along AB . If the magnetic force on the arm BC is F , then force on arm AC is



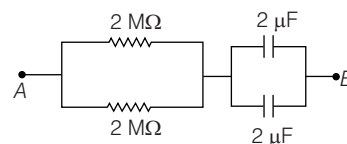
- a. $\sqrt{2} F$ b. $-\sqrt{2} F$ c. $-F$ d. F

11. Figure shows an arrangement known as a Helmholtz coil. It consists of two circular coaxial coils each of 200 turns and radius $R = 25.0\text{ cm}$ separated by a distance $S = R$. The two coils carry equal currents $I = 12.2\text{ mA}$ in the same direction. Find the magnitude of the net magnetic field at P , mid-way between the coils.



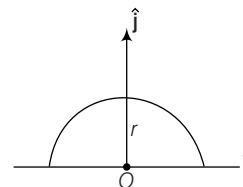
- a. $8.7 \times 10^{-6}\text{ T}$ b. $6.7 \times 10^{-6}\text{ T}$
 c. $5.7 \times 10^{-6}\text{ T}$ d. $10.7 \times 10^{-6}\text{ T}$

12. A material B has twice the specific resistance of another material A . A circular wire made of B has twice the diameter of a wire made of A . Then for the two wires to have the same resistance the ratio $\frac{l_A}{l_B}$ of their respective length must be
 a. 2 b. 1 c. $\frac{1}{2}$ d. $\frac{1}{4}$
13. In a potentiometer experiment, the balancing with a cell is at length 240 cm on shunting the cell with a resistance of 2Ω the balancing length 120 cm. The internal resistance of the cell is
 a. 1Ω b. 0.5Ω c. 4Ω d. 2Ω
14. At time $t = 0$, a battery of 10 V is connected across points A and B shown in figure. If the capacitors have no charge initially at what time does the voltage across becomes 4V?



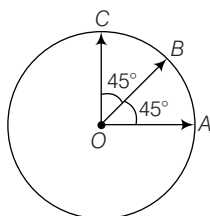
- a. 2s b. 3s
 c. 4s d. 5s

15. A thin semicircular ring of radius r as shown in figure as a positive charge q distributed uniformly over it. The net field E at the centre O is



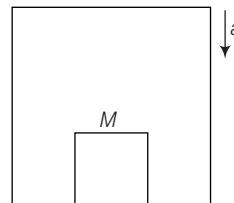
- a. $\frac{q}{2\pi^2\epsilon_0 r} \hat{j}$ b. $\frac{q}{4\pi^2\epsilon_0 r^2} \hat{j}$
 c. $-\frac{q}{4\pi^2\epsilon_0 r^2} \hat{j}$ d. $-\frac{q}{2\pi^2\epsilon_0 r^2} \hat{j}$

16. The decreasing order of wavelength of infrared, microwaves, ultraviolet and gamma rays is
 a. infrared, microwaves, ultraviolet, gamma rays
 b. microwaves, infrared, ultraviolet, gamma rays
 c. gamma rays, ultraviolet, infrared, microwaves
 d. microwaves, gamma rays, infrared, ultraviolet
17. Light with an energy flux of 18 W/cm^2 on a non-reflecting surface at normal incidence. If the surface has an area of 20 cm^2 . Find the average force exerted on the surface during a 30 min time span
 a. $1.2 \times 10^{-6} \text{ N}$ b. $2.4 \times 10^{-6} \text{ N}$
 c. $4.8 \times 10^{-6} \text{ N}$ d. $6.2 \times 10^{-6} \text{ N}$
18. An anatomist is viewing heart muscles cells with a microscope that has two selectable objectives with refracting powers of 100 and 300 D. When she uses the 100 D objective, the image of a cell subtends an angle of $3 \times 10^{-3} \text{ rad}$ with the eye. What angle is subtended when she uses the 300 D objective?
 a. $9 \times 10^{-3} \text{ rad}$ b. $3 \times 10^{-3} \text{ rad}$
 c. $2 \times 10^{-3} \text{ rad}$ d. $10 \times 10^{-3} \text{ rad}$
19. A sheet of silver is illuminate by monochromatic ultraviolet radiations of wavelength $1810 \text{ \AA}$. What is the maximum energy of the emitted electron?
 The threshold wavelength of silver is $2640 \text{ \AA}$.
 a. 9.16 eV b. 2.16 eV
 c. 3.16 eV d. 4.16 eV
20. Two radioactive materials X_1 and X_2 have decay constants 5λ and λ , respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of X_1 to that of X_2 will be $1/e$ after a time
 a. $\frac{e}{\lambda}$ b. λ c. $\frac{1}{2}\lambda$ d. $\frac{1}{4\lambda}$
21. The period of oscillation of simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$ measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1s resolution. What is the accuracy in the determination of g ?
 a. 3 % b. 1 % c. 5 % d. 2 %
22. Find the resultant of three vectors $\vec{OA}$, $\vec{OB}$ and $\vec{OC}$ as shown in figure. Radius of the circle is R .

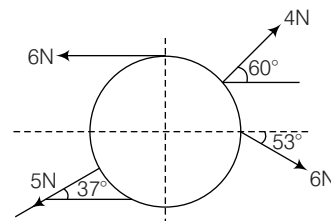


- a. $R(1 + \sqrt{2})$ b. $R(2 + \sqrt{2})$
 c. $R(3 + \sqrt{2})$ d. $R(4 + \sqrt{2})$

23. A ball is thrown from a field with a speed of 12.0 m/s at an angle of 45° with the horizontal. At what distance will it hit the field again? (Take, $g = 10.0 \text{ m/s}^2$)
 a. 10 m b. 14.4 m
 c. 16.4 m d. 18.4 m
24. With what acceleration a should the box of figure descend, so that the block of mass M exerts a force $mg/4$ on the floor of the box?

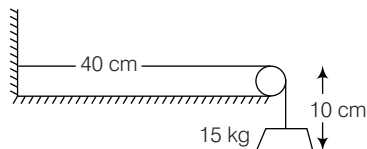


- a. $\frac{2g}{5}$ b. $\frac{3g}{5}$ c. $\frac{g}{2}$ d. $\frac{3g}{4}$
25. A block slides down and inclined of 30° with an acceleration of $g/4$. Find the kinetic friction coefficient.
 a. $\frac{1}{2\sqrt{3}}$ b. $\frac{1}{4\sqrt{3}}$ c. $\frac{1}{5\sqrt{3}}$ d. $\frac{1}{6\sqrt{3}}$
26. A small block of mass 100 g moves with uniform speed in a horizontal circular groove, with vertical side walls of radius 25 cm. If the block takes 2.0 s to complete one round, then find the normal contact force by the side wall of the groove.
 a. 1 N b. 0.25 N
 c. 0.4 N d. 2 N
27. A body dropped from a height H reaches the ground with a speed of $1.2\sqrt{gH}$. Calculate the work done by air friction.
 a. $-0.28 mgh$ b. $0.28 mgh$
 c. $0.56 mgh$ d. $-0.56 mgh$
28. A body of mass 2.5 kg is subjected to the force as shown in figure. Find the acceleration of the centre of mass.

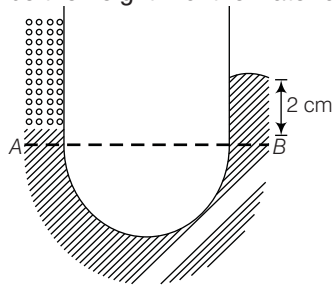


- a. 2 m/s^2 b. 1.6 m/s^2
 c. 3.2 m/s^2 d. None of these
29. A parallel beam of green light of wavelength 546 nm passes through a slit of width 0.40 mm. the transmitted light is collected on a screen 40 cm away. What will be the distance between two first order minima?
 a. 2.6 mm b. 1.8 mm
 c. 1.7 mm d. 1.1 mm

30. A 50 cm long wire of mass 20 g supports a mass of 1.6 kg as shown in figure. Find the fundamental frequency of the position of the string between the wall and the pulley.



- a. 25 Hz b. 50 Hz
c. 75 Hz d. 100 Hz
31. A cylinder is released from rest from the top of an incline of inclination θ and length l . If the cylinder rolls without slipping, then what will be its speed when it reaches the bottom?
- a. $\sqrt{\frac{2}{3}gl \sin \theta}$ b. $\sqrt{\frac{4}{3}gl \sin \theta}$
c. $\sqrt{\frac{6}{7}gl \sin \theta}$ d. None of these
32. The gravitational field in a region is g given by $\mathbf{F} = 10 \text{ N}(\hat{i} + \hat{j}) \text{ kg}^{-1}$. What will be work done by an external agent to slowly shift a particle of mass 2 kg from the point (0,0), to a point (5, 4) ?
- a. -180 J b. +100 J
c. +180 J d. -100 J
33. The liquids shown in figure in the two arms are mercury (specific gravity = 13.6) and water. If the difference of heights of the mercury columns is 2 cm. What will be the height h of the water column?

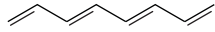
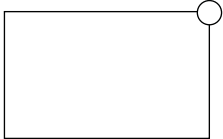
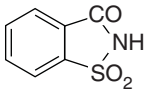


- a. 27 cm b. 30 cm c. 40 cm d. 47 cm
34. A water drop of radius 10^{-2} m is broken into 1000 equal droplets. Calculate the gain in surface energy. Surface tension of water is 0.075 N/m .
- a. $8.5 \times 10^{-4} \text{ J}$ b. $9.5 \times 10^{-4} \text{ J}$
c. $10 \times 10^{-4} \text{ J}$ d. None of these

35. The speed of transverse wave giving on a wire having a length 50 cm and mass 5.0 g is 80 m/s. The area of cross-section of the wire is 1.0 mm^2 and its Young's modulus is $16 \times 10^{11} \text{ N/m}^2$. Find the extension of the wire over its natural length.
- a. 0.04 mm b. 0.02 mm
c. 0.06 mm d. 1 mm
36. A sound wave of wavelength 40 cm travels in air. If the difference between the maximum and minimum pressures at a given point is $1 \times 10^{-3} \text{ N/m}^2$. What will be the amplitude of vibration of the particle of the medium, the bulk modulus of air is $1.4 \times 10^{-10} \text{ N/m}^2$?
- a. $4.4 \times 10^{-10} \text{ m}$
b. $3.3 \times 10^{-10} \text{ m}$
c. $1.1 \times 10^{-10} \text{ m}$
d. $2.2 \times 10^{-10} \text{ m}$
37. A particle of mass 40 g executes a simple harmonic motion of amplitude 2.0 cm. If the time period is 0.20 s, find the total mechanical energy of the system.
- a. $7.9 \times 10^{-3} \text{ J}$
b. $8.9 \times 10^{-3} \text{ J}$
c. $9.9 \times 10^{-3} \text{ J}$
d. $6.9 \times 10^{-3} \text{ J}$
38. The sound level at a point is increased by 30 dB. By what factor is the pressure amplitude increased?
- a. 30 b. 25
c. 32 d. 27
39. White light is mixture of light of wavelength between 400 nm to 700 nm. If this light goes through water ($\mu = 1.33$) what will be the limits of the wavelength there?
- a. (300 to 525) nm
b. (600 to 700) nm
c. (800 to 1000) nm
d. (1200 to 1400) nm
40. What will be the moment of inertia of a solid cylinder of mass M and radius R about a line parallel to the axis of the cylinder and on the surface of the cylinder?
- a. $\frac{7}{2} MR^3$ b. $\frac{5}{2} MR^2$
c. $\frac{7}{5} MR^2$ d. $\frac{2}{3} MR^2$

PART II

Chemistry

41. In which of the following pairs, the two species are not iso-structural?
- CO_3^{2-} and NO_3^-
 - PCl_4^+ and SiCl_4
 - PF_3 and BrF_5
 - AlF_6^{3-} and SF_6
42. Which of the following statements regarding dacron is correct?
- Dacron is an example of polyester.
 - In the manufacturing of dacron zinc acetate antimonytrioxide used as a catalyst.
 - Dacron fibres is increases resistant.
 - It is a polymer of natural rubber.
- Only IV
 - Both IV and I
 - I, II and III
 - None of these
43. Bombardment of aluminium by α -particle leads to its artificial disintegration in two ways as shown. Products X, Y and Z respectively are
- $$\begin{array}{ccc}
 {}_{13}^{27}\text{Al} + {}_2^4\text{He} & \xrightarrow{\text{(ii)}} & {}_{15}^{30}\text{P} + \text{Y} \\
 \downarrow \text{(i)} & & \downarrow \\
 {}_{14}^{30}\text{Si} + \text{X} & & {}_{14}^{30}\text{Si} + \text{Z}
 \end{array}$$
- proton, neutron, positron
 - neutron, proton, positron
 - proton, positron, neutron
 - positron, neutron, proton
44. Predict Kroll process in the following reaction,
- $\text{TiCl}_4(g) + 2\text{Mg} \longrightarrow \text{Ti}(s) + 2\text{MgCl}_2(l)$
 - $\text{Fe}_2\text{O}_3 + 3\text{C} \longrightarrow 3\text{CO} + 2\text{Fe}$
 - $\text{Al}_2\text{O}_3 \cdot x\text{H}_2\text{O}(s) \xrightarrow{1470\text{K}} \text{Al}_2\text{O}_3 + x\text{H}_2\text{O}(g)$
 - $\text{Cr}_2\text{O}_3 + 2\text{Al} \longrightarrow 2\text{Cr} + \text{Al}_2\text{O}_3$
45. The complementary strand of DNA for the given single stranded DNA sequence
 $5'-\text{A}-\text{T}-\text{C}-\text{A}-\text{T}-\text{G}-\text{C}-3'$ is
- $5'-\text{A}-\text{T}-\text{C}-\text{A}-\text{T}-\text{G}-\text{C}-3'$
 - $5'-\text{T}-\text{A}-\text{G}-\text{T}-\text{A}-\text{C}-\text{G}-3'$
 - $5'-\text{G}-\text{C}-\text{A}-\text{T}-\text{G}-\text{A}-\text{T}-3'$
 - $5'-\text{C}-\text{G}-\text{T}-\text{A}-\text{C}-\text{T}-\text{A}-3'$
46. The IUPAC name of the compound  is
- 2, 8-dimethyl-3,6-decadiene
 - 2, 2-dimethylpropane
 - 2, 4, 6, 8 octatetraene
 - 1, 3, 5, 7 octatetraene
47. Which of the following dissolves in water but does not give any oxyacid solution?
- SO_2
 - OF_2
 - SCl_4
 - SO_3
48. Heat capacity (C_p) is an extensive property but specific heat (C) is an intensive property. The relation between C_p and C for 1 mole of water is
- $+5.18 \text{ JK}^{-1}$
 - -5.18 JK^{-1}
 - -75.3 JK^{-1}
 - $+75.3 \text{ JK}^{-1}$
49. Select the correct statement regarding the compound.
- 
- The IUPAC name of this compound is 1, 3-epoxy propane.
 - The functional group of this compound is ether.
 - Root word in the IUPAC name of this compound is prop.
 - This is a saturated open chain structure.
- Only I
 - I, II and III
 - Only IV
 - IV and III
50. Aniline does not undergo Friedel-Crafts reaction due to
- lone pair nitrogen atoms activate the benzene ring
 - salt formation with aluminium chloride
 - meta derivative formation
 - None of the above
51. The edges length of a face-centred cubic cell of an ionic substance is 508 pm. If the radius of the cation is 110 pm, the radius of the anion is
- 618 pm
 - 144 pm
 - 288 pm
 - 398 pm
52. What is the sweeteners value of the structure in comparison to cane sugar?
- 
- 100
 - 500
 - 200
 - 550
53. Interstitial compounds are formed when small atoms are.....
- trapped inside the crystal lattice of metals
 - chemically very reactive
 - decomposes in inner orbitals
 - progressively filled

54. $\text{C}_2\text{H}_5\text{NH}_2$ I $\text{C}_6\text{H}_5\text{NHCH}_3$ II
 $(\text{C}_2\text{H}_5)_2\text{NH}$ and $\text{C}_6\text{H}_5\text{NH}_2$ III

Above the following which forms H-bonding faster in aqueous medium?

- a. Only I
 b. Only III
 c. I, II and III
 d. None of these
55. Identify the reaction upto P and product Q.
-
- a. Sandmeyer reaction, benzoic acid
 b. Fittig reaction, benzoidal chloride
 c. Friedel-Craft's reaction benzaldehyde
 d. Reimer-Tiemann reaction, benzaldehyde
56. Now a days, the pesticide industry has shifted its attention to herbicide such as.....
- a. sodium chlorate
 b. sodium arsenite
 c. Both (a) and (b)
 d. aldrin
57. Ampicillin and amoxycillin are synthetic modification of
- a. amino glycosides
 b. ofloxacin
 c. tetracycline
 d. penicillins
58. A sample of wustite is $\text{Fe}_{0.93}\text{O}_{1.00}$. The percentage of iron present in form of Fe (III) is.....
- a. 15.05%
 b. 17.5%
 c. 19.05%
 d. 8%
59. An alcohol A is treated with "Lucas reagent" to predict whether the alcohol is primary, secondary or tertiary. Which reacts fastest?
- a. Tertiary alcohol
 b. Secondary alcohol
 c. Primary alcohol
 d. None of the above
60. Fullerenes are the only pure form of carbon because
- a. layered structured held by van der Waals' forces
 b. they have smooth structure without having 'dangling' bonds
 c. it has 3-dimensional structure
 d. extremely hard refractory solid of high melting point

61. For a sparingly soluble salt A_pB_q solubility product (L_S) with its solubility (S) is best represented by

a. $L_S = S^{p+q} \cdot p^p \cdot q^q$
 b. $L_S = S^{p+q} \cdot p^q \cdot q^p$
 c. $L_S = S^{pq} \cdot p^p \cdot q^q$
 d. $L_S = S^{pq} \cdot (pq)^{p+q}$

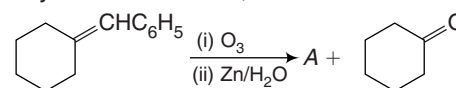
62. Sodium has body-centred cubic structure having nearest neighbour distance 365.9 pm. The density.....

a. 2 g cm^3
 b. 1.01 g cm^3
 c. 5 g cm^3
 d. 3.02 g cm^3

63. RNA contains four bases, the first three bases are same as in DNA but the fourth one is.....

a. adenine
 b. cytosine
 c. thymine
 d. uracil

64. Identify A in the reaction,



a. $\text{C}_6\text{H}_5\text{CHO}$
 b. $\text{C}_6\text{H}_5\text{CH}_2\text{CH}_3$
 c. CH_3COCl
 d. $\text{C}_6\text{H}_5\text{COCH}_3$

65. Dihydrogen is used in the manufacturing of
- a. antiseptic
 b. disinfectant
 c. vanaspati fat
 d. antigens

66. Thyroxine produced in the thyroid gland is an iodinated derivatives of

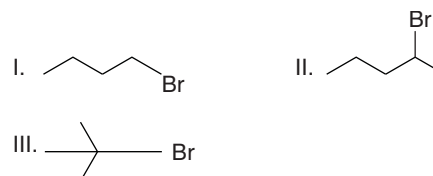
a. acetic acid
 b. lactic acid
 c. amino acid
 d. picric acid

67. $[\text{Fe}(\text{CO})_4(\text{PPh}_3)_2]^{2+}$ I $[\text{Co}(\text{CO})_4(\text{PhO})_2]^{2+}$ II
 $[\text{V}(\text{CO})_4(\text{PF}_3)_2]^{2+}$ III $[\text{Mn}(\text{CO})_4(\text{PCl}_3)_2]^{2+}$ IV

Arrange the following in the decreasing order of C—O bond length.

a. I > II > III > IV
 b. III > II > I > IV
 c. III > IV > II > I
 d. II > I > III > IV

68. Dehydrobromination ($-\text{HBr}$) of the following in increasing order is



a. I < II < III
 b. III < II < I
 c. II = III < I
 d. III < I < II

69. If a thin slice of sugar beet is placed in concentrated solution of NaCl, then
- sugar beet will lose water from its cells
 - sugar beet will absorb water from solution
 - sugar beet will neither absorb nor lose water
 - sugar beet will dissolve in solution

70. 0.02 molar solution of NaCl having degree of dissociation of 90% at 27°C has osmotic pressure equal to

- 0.94 bar
- 9.4 bar
- 0.94 bar
- 9.4×10^{-4} bar

71. Which of the following statements is correct?

- $(n-1)d$ subshell has lower energy than ns subshell
- $(n-1)d$ subshell has higher energy than ns subshell
- $(n+1)d$ subshell has lower energy than nf subshell
- nf subshell has lower energy than $(n+s)s$ subshell

72. If $a = \frac{h}{4\pi^2 me^2}$, then the correct expression for the calculation of the circumference of the first orbit of hydrogen atom is

- $\sqrt{4} \pi h a$
- $2\pi r$
- $\sqrt{4h^2 \pi a}$
- (a) and (c) are correct

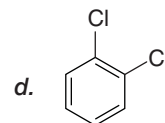
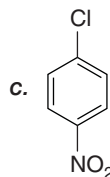
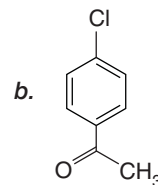
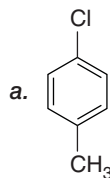
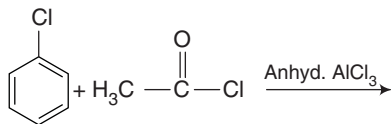
73. By the addition of 0.035 g of starch the coagulation of 10 cm³ of gold sol is completely prevented. The gold number of starch is

- 0.00035
- 0.35
- 55
- 35

74. In chain growth polymerisation addition takes place head-to-tail when monomer is

- $\text{CH}_2=\text{CH}-\text{C}_6\text{H}_5$
- $\text{CH}_2=\text{CH}-\text{CH}=\text{CH}_2$
- $\text{CH}_2=\text{C}(\text{CH}_3)-\text{C}(\text{OCH}_3)=\text{CH}_2$
- $\text{CH}_2=\text{CH}-\text{C}\equiv\text{N}$

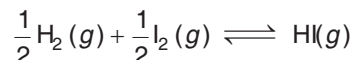
75. The major product(s) of the following reaction is (are)



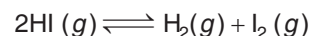
76. ΔU° of combustion of methane is $-x \text{ kJ mol}^{-1}$. The value of ΔH° is

- $= \Delta U^\circ$
- $> \Delta U^\circ$
- $< \Delta U^\circ$
- Equal to zero

77. At 500 K, equilibrium constant K_C for the following reaction is 5.

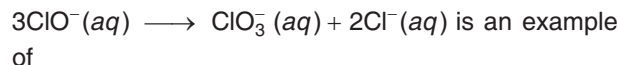


What would be the equilibrium constant K_C for the reaction?



- 0.04
- 0.4
- 25
- 2.5

78. The reaction,



- is an example of
- oxidation reaction
 - reduction reaction
 - disproportionation reaction
 - decomposition reaction

79. Gaseous nitric acid has the structure

- planar
- square planar
- pyramidal
- trigonal bipyramidal

80. *p*-nitrophenol is less volatile due to

- intermolecular hydrogen bonding
- intramolecular hydrogen bonding
- picric acid
- van der Waals' force

PART III

a. English Proficiency

Directions (Q. Nos. 81 to 84) *In the sentences that are given below, a part of it has been underlined. That part may be improved by one of the options. The part with the best improving ability will be your answer. In case, no improvement is needed then "No improvement" is your answer.*

81. If he would have tried, he would have succeeded.
 a. is tried b. was tried
 c. had tried d. No improvement
82. I will not go to school, if it shall rain tomorrow.
 a. it would rain tomorrow b. it will rain tomorrow
 c. it rains tomorrow d. No improvement
83. Why the dinosaurs died out is not known.
 a. it is not known b. the reason is not known
 c. that is not known d. No improvement
84. He is quite well now, except a slight cold.
 a. except with a slight cold b. excepting a slight cold
 c. except for a slight cold d. No improvement

Directions (Q. Nos. 85 to 88) *In the sentences given below there is an error in one of the parts which will be your answer. If there is no error then "No error" is your answer.*

85. The table's wood (a)/ is infested with mite (b)/ and I am likely to dispose it off. (c)/ No error (d)
86. The morale of the army (a)/ was high because the news (b)/ coming from the front are very encouraging. (c)/ No error (d)
87. I can't come to you now (a)/ because a lot of works (b)/ remains to be done. (c)/ No error (d)
88. A farmer was leading oxes (a)/ to his field for ploughing (b)/ early in the morning. (c)/ No error (d)

Directions (Q. Nos. 89 to 95) *Read the passage carefully and then choose the most suitable option from the given ones.*

There are eccentric people who enjoy saving money for no other reason than the pleasure of saving money. It is a passion like drinking and a hobby like collecting of China. Does it usually begin with a money-box ? Imagine a painter drawing the miser 's progress in a number of scenes, with the first scene showing a benevolent grandfather holding out a harmless looking tin money-box to an infant scarcely able to walk .

The gift should always be accompanied by a box of tools. As a young man the infant has grown into a miser. By the age of forty he has substantial bank account. But he persuades himself that he is so poor that he never goes to the theater, never invites a friend to dinner. By sixty, he is a rich man and is convinced that he is all but a pauper.

89. Some people enjoy saving money because
 a. they are able to live a happy life
 b. saving is a passion for them
 c. they are able to enjoy the pleasures of life
 d. they can entertain others
90. If a money-box is given to a child, what should accompany it?
 a. Tips on the advantages of saving money
 b. Guidelines to preserve it
 c. instructions on when and how to open it
 d. A box of tools to open it
91. A miserly man of forty does not go to the theatre because
 a. he has no liking for plays
 b. he does not want to waste his valuable time
 c. he persuades himself that he is very poor
 d. he is frightened by the darkness of the theatre hall

Directions (Q. Nos. 92 and 93) *Find the synonym of the word given in capital letters.*

92. The vehicle slowed down as they moved up the GRADIENT.
 a. Road with sharp bends
 b. Bumpy, uneven path
 c. Slope or incline
 d. Mountainous area
93. It is DEPLORABLE that millions of people in India are below the poverty line.
 a. Hateful b. Regrettable
 c. Undesirable d. Unpleasant

Directions (Q. Nos. 94 and 95) *Rearrange the jumbled part of the sentence and then choose the correct option.*

94. He was a man
 P : even if he had to starve
 Q : who would not beg
 R : borrow or steal
 S : from anyone
 The proper sequence should be
 a. PQRS b. PRQS
 c. QRSP d. QPRS
95. P : in the progress of
 Q : universities play a crucial role
 R : our civilisation
 S : in the present age
 The proper sequence should be
 a. SQPR b. QRSP
 c. QRPS d. SQRP

b. Logical Reasoning

96. If STUDENT is coded as RUTEDOS, which word would be coded as RDGPKBQ ?

- a. SHERBET b. SHINGLE
c. SHACKLE d. SCHOLAR

97. Find the next term in the series.

1, 2, 2, 4, 8, ?

- a. 8 b. 9 c. 32 d. 16

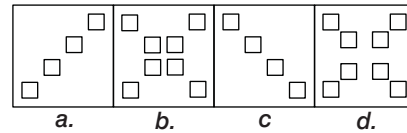
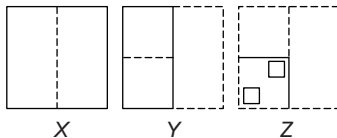
98. Which of the following figures represents village, district, state ?



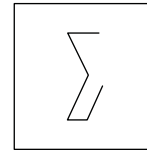
99. Which of the following diagrams represents humans, females and girls ?



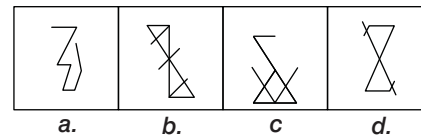
100. From amongst the four alternative figures (a), (b), (c) and (d), select the one showing the unfolded position of Z.



101. Examine the following figure.



Which one of the following figures has the above figure embedded in it ?



102. 'Flower' is related to 'Petal' in the same way as 'Book' is related to

- a. Pages b. Content
c. Author d. Library

103. Find the odd one out.

- a. Cheese b. Wine c. Milk d. Curd

104. Find the odd one out.

- a. Moon b. Planet c. Sky d. Star

105. Find the odd one out.

- a. 42-4 b. 36-6 c. 32-2 d. 15-5

PART IV

Mathematics

106. If $\int f(x) dx = 2\{f(x)\}^3 + C$, then $f(x)$ is

- a. $\frac{x}{2}$ b. x^3
c. $\frac{1}{\sqrt{x}}$ d. $\sqrt{\frac{x}{3}}$

107. The interval in which $2x^3 + 5$ increase less rapidly than $9x^2 - 12x$, is

- a. $(-\infty, 1)$ b. $(1, 2)$
c. $(2, \infty)$ d. None of these

108. The number of real roots of the equation $e^{x-1} + x - 2 = 0$, is

- a. 1 b. 2
c. 3 d. 4

109. The abscissae of the point on the curve $xy = (a + x)^2$, the normal at which cuts off numerically equal intercepts from the axes of coordinates, is

- a. $\frac{a}{\sqrt{2}}$ b. a c. $\sqrt{2}a$ d. $\frac{-a}{2}$

110. If $4x + 3|y| = 5y$, then y as a function of x is

- a. differentiable at $x = 0$ b. continuous at $x = 0$
c. $\frac{dy}{dx} = 2$ for all x d. None of these

111. If $a \cdot \lim_{x \rightarrow 1} x^{\frac{1}{1-x}} + b = e^{-1}$; $a \geq 1, b \geq 0$, then

- a. $a = 1, b = e^{-1}$ b. $a = 2, b = e^{-1}$
c. $a = -1, b = e^{-1}$ d. $a = 1, b = 0$

112. If $e^x + e^{f(x)} = e$, then the domain of definition of $f(x)$ is
 a. $(-\infty, 1)$ b. $(-\infty, 0)$ c. $(1, \infty)$ d. None
113. A coin is tossed n times. The probability that head will turn up an odd number of times, is
 a. $\frac{1}{2}$ b. $\frac{x+1}{2x}$ c. $\frac{x-1}{2x}$ d. $\frac{2^{x-1}-1}{2^x}$
114. For $n \in N$, $3^{2n+2} - 2^3 n - 9$ is divisible by
 a. 3 b. 9 c. 64 d. 81
115. If p, q, r have truth values T, F, T respectively, then which of the following is true?
 a. $(p \rightarrow q) \wedge r$ b. $(p \rightarrow q) \wedge \sim r$
 c. $(p \wedge q) \wedge (p \vee r)$ d. $q \rightarrow (p \wedge r)$
116. Let $A = \{x \in R : x \leq 1\}$ and $f: A \rightarrow A$ be defined as $f(x) = x(2-x)$. Then, $f^{-1}(x)$ is
 a. $1 + \sqrt{1-x}$ b. $1 - \sqrt{1-x}$ c. $\sqrt{1-x}$ d. $1 \pm \sqrt{1-x}$
117. The relation $R = \{(1,3), (3,5)\}$ is defined on the set with minimum number of elements of natural numbers. The minimum number of elements to be included in R , so that R is an equivalence relation, is
 a. 5 b. 6 c. 9 d. 8
118. If S is the set of squares and R is the set of rectangles, then $(S \cup R) - (S \cap R)$ is
 a. S
 b. R
 c. set of squares but not rectangles
 d. set of rectangles but not squares
119. If x_1 and x_2 are the roots of the equation $e^2 \cdot x^{\ln x} = x^3$ with $x_1 > x_2$, then
 a. $x_1 = 2x_2$ b. $x_1 = x_2^2$ c. $2x_1 = x_2^2$ d. $x_1^2 = x_2^3$
120. If $2 \sec^2 A - \sec^4 A - 2 \operatorname{cosec}^2 A + \operatorname{cosec}^4 A = 15/4$, then $\tan A$ is equal to
 a. $\pm \frac{1}{\sqrt{2}}$ b. $\pm \frac{1}{2}$ c. ± 2 d. $\pm \frac{1}{2}\sqrt{2}$
121. The range of R for which the inequality $k \cos^2 x - k \cos x + 1 \geq 0, \forall x \in (-\infty, \infty)$, is
 a. $k < -\frac{1}{2}$ b. $-\frac{1}{2} \leq k \leq 4$ c. $k > 4$ d. $\frac{1}{2} \leq k \leq 5$
122. The number of ordered pairs which satisfy the equation $x^2 + 2x \sin(xy) + 1 = 0, y \in [0, 2\pi]$, are
 a. 1 b. 2 c. 3 d. 0
123. The value of $2 \tan^{-1} \left[\sqrt{\frac{a-b}{a+b}} \tan \frac{\theta}{2} \right]$ is
 a. $\cos^{-1} \left(\frac{a \cos \theta + b}{a + b \cos \theta} \right)$ b. $\cos^{-1} \left(\frac{a + b \cos \theta}{a \cos \theta + b} \right)$
 c. $\cos^{-1} \left(\frac{a \cos \theta}{a + b \cos \theta} \right)$ d. $\cos^{-1} \left(\frac{b \cos \theta}{a \cos \theta + b} \right)$
124. In $\triangle ABC$, medians AD and CE are drawn. If $AD = 5, \angle DAC = \pi/8$ and $\angle ACE = \pi/4$, then the area of $\triangle ABC$ (in sq units) is equal to
 a. $\frac{25}{9}$ b. $\frac{10}{3}$ c. $\frac{25}{18}$ d. $\frac{25}{3}$
125. The locus of the foot of the perpendicular drawn from either focus on a variable tangent to the hyperbola $16y^2 - 9x^2 = 1$, is
 a. $x^2 + y^2 = 9$ b. $x^2 + y^2 = 1/9$
 c. $x^2 + y^2 = 7/144$ d. $x^2 + y^2 = 1/16$
126. The length of the sides of the square which can be made by four perpendicular tangents to the ellipse $\frac{x^2}{7} + \frac{2y^2}{11} = 1$, is
 a. 10 units b. 8 units c. 6 units d. 5 units
127. The set of points on the axis of the parabola $(x-1)^2 = 8(y+2)$ from where three distinct normals can be drawn to the parabola, is the set (h, k) of points satisfying
 a. $h > 2$ b. $h > 1$
 c. $k > 2$ d. None of these
128. If the chords of contact of tangents from three points A, B and C to the circle $x^2 + y^2 = a^2$ are concurrent, then A, B and C will
 a. be concyclic
 b. be collinear
 c. form the vertices of triangle
 d. None of the above
129. The condition that one of the straight lines given by the equation $ax^2 + 2hxy + by^2 = 0$ may coincide with one of those given by the equation $a'x^2 + 2h'xy + b'y^2 = 0$, is
 a. $(ab' - a'b)^2 = 4(ha' - h'a)(bh' - b'h)$
 b. $(ab' - a'b)^2 = (ha' - h'a)(bh' - b'h)$
 c. $(ha' - h'a)^2 = 4(ab' - a'b)(bh' - b'h)$
 d. $(bh' - b'h)^2 = 4(ab' - a'b)(ha' - h'a)$
130. If $P(1+t\sqrt{2}, 2+t\sqrt{2})$ is any point on the line, then the range of the value of t for which the point P lies between the parallel lines $x + 2y = 1$ and $2x + 4y = 15$ is
 a. $-4\sqrt{2}/3 < t < 5\sqrt{2}/3$ b. $0 < t < 5\sqrt{2}/6$
 c. $4\sqrt{2} < t < 0$ d. None of these
131. One Indian and four American men and their wives are to be seated randomly around a circular table. Then, the conditional probability that the Indian man is seated adjacent to his wife, given that each American man is seated adjacent to his wife, is
 a. $1/2$ b. $1/3$
 c. $2/5$ d. $1/5$

132. Let p be an odd prime number and T_p be the following set of 2×2 matrices :

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, \dots, p-1\} \right\}$$

The number of A in T_p such that A is either symmetric or skew-symmetric or both and $\det(A)$ is divisible by p , is

- a. $(p-1)^2$ b. $2(p-1)$ c. $(p-1)^2 + 1$ d. $2p-1$
133. Let a, b, c be the real numbers. Then, the system of equations in $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and $-\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, has
- a. no solution b. unique solution
c. infinitely many solutions d. finitely many solutions
134. If the coefficients of three consecutive terms of $(1+x)^{n+5}$ are in the ratio 5:10:14, then n equals
- a. 3 b. 4 c. 5 d. 6
135. Consider the set of eight vectors $\mathbf{V} = \{a\hat{i} + b\hat{j} + c\hat{k}; a, b, c \in \{-1, 1\}\}$. If three non-coplanar vectors can be chosen from $\mathbf{V}$ in 2^p ways, then p is
- a. 4 b. 5 c. 6 d. 7
136. If $\alpha \in \left(0, \frac{\pi}{2}\right)$, then $\sqrt{x^2 + x} + \frac{\tan^2 \alpha}{\sqrt{x^2 + x}}$ is always greater than or equal to
- a. $2 \tan \alpha$ b. 1 c. 2 d. $\sec^2 \alpha$
137. Let complex numbers α and $\frac{1}{\alpha}$ lie on circles $(x-x_0)^2 + (y-y_0)^2 = r^2$ and $(x-x_0)^2 + (y-y_0)^2 = 4r^2$, respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$, then $|\alpha|$ equals
- a. $1/\sqrt{2}$ b. $1/2$ c. $1/\sqrt{7}$ d. $1/3$
138. Let a, b, c be real numbers and $a \neq 0$. If α is the root of $a^2x^2 + bx + c = 0$ and β is the root of $a^2x^2 - bx - c = 0$; $0 < \alpha < \beta$, then the equation $a^2x^2 + 2bx + 2c = 0$ has a root γ that always satisfies
- a. $\gamma = \frac{\alpha + \beta}{2}$ b. $\gamma = \alpha + \frac{\beta}{2}$ c. $\gamma = \alpha$ d. $\alpha < \gamma < \beta$
139. If $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ lies in the interval
- a. $\left[\frac{1}{2}, 2\right]$ b. $[-1, 2]$ c. $\left[-\frac{1}{2}, 1\right]$ d. $\left[-1, \frac{1}{2}\right]$
140. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non-coplanar vectors, then $\frac{\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})}{(\mathbf{c} \times \mathbf{a}) \cdot \mathbf{b}} + \frac{\mathbf{b} \cdot (\mathbf{a} \times \mathbf{c})}{\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})}$ is equal to
- a. 0 b. 1
c. 2 d. None of these

141. Let $\mathbf{a} = x\hat{i} + x^2\hat{j} + 2\hat{k}$, $\mathbf{b} = -3\hat{i} + \hat{j} + \hat{k}$ and the $\mathbf{c} = (3x+11)\hat{i} + (x-9)\hat{j} - 3\hat{k}$ be three vectors. Then, the angle between $\mathbf{a}$ and $\mathbf{b}$ is acute angle and between $\mathbf{c}$ and $\mathbf{a}$ is obtuse, if x lies in

a. $(-\infty, 1) \cup (2, 3)$ b. $(-\infty, 1]$
c. $[2, 3]$ d. None of these

142. The mean of the numbers $a, b, 8, 5, 10$ is 6 and the variance is 6.80. Then, which one of the following gives possible values of a and b ?

a. $a = 1, b = 6$ b. $a = 3, b = 4$ c. $a = 0, b = 7$ d. $a = 5, b = 2$

143. The degree of the differential equation $x = 1 + \frac{dy}{dx} + \frac{1}{2!} \left(\frac{dy}{dx}\right)^2 + \frac{1}{3!} \left(\frac{dy}{dx}\right)^3 + \dots$, is

a. 3 b. 1
c. Not defined d. None of these

144. The area bounded by $y = xe^{|x|}$ and lines $|x| = 1, y = 0$, is

a. 4 sq units b. 6 sq units c. 1 sq units d. 2 sq units

145. If $\int_{\pi/2}^{\theta} \sin x \, dx = \sin 2\theta$, then the value of θ satisfying $0 < \theta < \pi$, is

a. $3\pi/2$ b. $\pi/6$ c. $5\pi/6$ d. $\pi/2$

146. Let $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$. Then, S_n can take value(s)

a. 1056 b. 1088 c. 1120 d. 1333

147. If the function $f(x) = (4 \sin^2 x - 1)^n (x^2 - x + 1)$, $n \in \mathbb{N}$, has a local maximum at $x = \pi/6$, then n

a. can be any odd natural number
b. can only be an odd prime number
c. can be any even natural number
d. can only be a multiple of 4

148. In three dimensional space, the equation $x^2 - x - 2 = 0$ represents

a. a pair of straight lines
b. a set containing two distinct points
c. a pair of parallel planes
d. None of the above

149. If the sum of the slopes of the normal from a point P to the hyperbola $xy = c^2$ is equal to λ ; $\lambda \in \mathbb{R}^+$, then the locus of point P is

a. $x^2 = \lambda c^2$ b. $y^2 = \lambda c^2$ c. $xy = \lambda c^2$ d. None

150. Consider the lines $L_1: \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$ and

$L_2: \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$. The unit vector perpendicular to both L_1 and L_2 is

a. $\frac{1}{\sqrt{99}}(-\hat{i} + 7\hat{j} + 5\hat{k})$ b. $\frac{1}{5\sqrt{3}}(-\hat{i} - 7\hat{j} + 5\hat{k})$
c. $\frac{1}{5\sqrt{3}}(-\hat{i} + 7\hat{j} + 5\hat{k})$ d. $\frac{1}{\sqrt{99}}(7\hat{i} - 7\hat{j} - \hat{k})$

Answers

Physics

1. (a)	2. (a)	3. (a)	4. (b)	5. (d)	6. (d)	7. (a)	8. (c)	9. (c)	10. (c)
11. (a)	12. (c)	13. (d)	14. (a)	15. (d)	16. (b)	17. (a)	18. (a)	19. (b)	20. (d)
21. (a)	22. (a)	23. (b)	24. (d)	25. (a)	26. (b)	27. (a)	28. (b)	29. (d)	30. (a)
31. (b)	32. (a)	33. (a)	34. (a)	35. (b)	36. (d)	37. (a)	38. (c)	39. (a)	40. (c)

Chemistry

41. (c)	42. (c)	43. (a)	44. (a)	45. (a)	46. (d)	47. (b)	48. (d)	49. (b)	50. (c)
51. (b)	52. (d)	53. (a)	54. (a)	55. (a)	56. (c)	57. (d)	58. (a)	59. (a)	60. (b)
61. (a)	62. (b)	63. (d)	64. (a)	65. (c)	66. (c)	67. (c)	68. (a)	69. (a)	70. (a)
71. (b)	72. (d)	73. (d)	74. (d)	75. (b)	76. (c)	77. (a)	78. (c)	79. (a)	80. (a)

English Proficiency

81. (c)	82. (c)	83. (d)	84. (c)	85. (a)	86. (c)	87. (b)	88. (a)	89. (b)	90. (d)
91. (c)	92. (c)	93. (b)	94. (c)	95. (a)					

Logical Reasoning

96. (d)	97. (d)	98. (b)	99. (d)	100. (b)	101. (c)	102. (a)	103. (b)	104. (c)	105. (a)
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Mathematics

106. (d)	107. (b)	108. (a)	109. (a)	110. (b)	111. (d)	112. (a)	113. (a)	114. (c)	115. (d)
116. (b)	117. (c)	118. (d)	119. (b)	120. (a)	121. (b)	122. (b)	123. (a)	124. (d)	125. (d)
126. (d)	127. (c)	128. (b)	129. (a)	130. (a)	131. (c)	132. (d)	133. (b)	134. (d)	135. (b)
136. (a)	137. (c)	138. (d)	139. (c)	140. (a)	141. (a)	142. (b)	143. (b)	144. (d)	145. (d)
146. (a)	147. (c)	148. (c)	149. (a)	150. (b)					

Solved Paper 2017

BITSAT

Instructions

- There are 150 questions in all. The number of questions in each part is as given below.
Part I Physics
Part II Chemistry
Part III (a) English Proficiency
(b) Logical Reasoning
Part IV Mathematics
- All questions are multiple choice questions with four options, only one is correct.
- Each correct answer fetches 3 marks while incorrect answer has a penalty of 1 mark.

No. of Questions

1-40
41-80
81-95
96-105
106-150

PART I

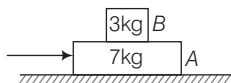
Physics

- If temperature of a black body increases from 300 K to 900 K, then the rate of energy radiation increases by
a. 81 b. 3 c. 9 d. 2
- A whistle of frequency 500 Hz tied to the end of a string of length 1.2 m revolves at 400 rev/min. A listener standing some distance away in the plane of rotation of whistle hears frequencies in the range.
(Speed of sound = 340 m/s)
a. 436 to 574 b. 426 to 586
c. 426 to 574 d. 436 to 586
- The focal length of a thin convex lens for red and blue rays are 100 cm and 96.8 cm respectively. Then, the dispersive power of the material of the lens is
a. 0.968 b. 0.98
c. 0.0325 d. 0.325
- Two metal plates having a potential difference of 800 V are 2 cm apart. It is found that a particle of mass 1.96×10^{-15} kg remains suspended in the region between the plates. The charge on the particle must be (e = elementary charge).
a. $2e$ b. $3e$
c. $6e$ d. $8e$
- At what angle θ to the horizontal should an object is projected so that the maximum height reached is equal to the horizontal range?
a. $\tan^{-1}(2)$ b. $\tan^{-1}(4)$
c. $\tan^{-1}\left(\frac{2}{3}\right)$ d. $\tan^{-1}(3)$
- A body of mass 1 kg is executing simple harmonic motion. Its displacement y (cm) at t seconds is given by $y = 6 \sin\left(100t + \frac{\pi}{4}\right)$.
Its maximum kinetic energy is
a. 6 J b. 18 J
c. 24 J d. 36 J
- A positive charge q is projected in magnetic field of width $\frac{mv}{\sqrt{2}qB}$ with velocity v . Then, the time taken by charged particle to emerge from the magnetic field is
a. $\frac{m}{\sqrt{2}qB}$ b. $\frac{\pi m}{4qB}$
c. $\frac{\pi m}{2qB}$ d. $\frac{\pi m}{\sqrt{2} \cdot qB}$

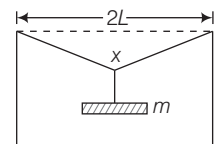
2 SELF STUDY GUIDE BITSAT

8. In Young's double slit experiment, the slits are 2 mm apart and are illuminated by photons of two wavelengths $\lambda_1 = 12000 \text{ \AA}$ and $\lambda_2 = 10000 \text{ \AA}$. At what minimum distance from the common central bright fringe on the screen 2m from the slit will a bright fringe from one interference pattern coincide with a bright fringe from the other?
- a. 8 mm b. 6 mm c. 4 mm d. 3 mm
9. Two blocks *A* and *B* are placed one over the other on a smooth horizontal surface. The maximum horizontal force that can be applied on lower block *B*, so that *A* and *B* move without separation is 49 N.

The coefficient of friction between *A* and *B* is



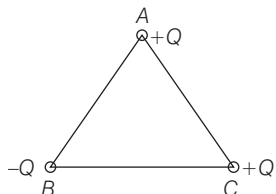
- a. 0.2 b. 0.3
c. 0.5 d. 0.8
10. An aeroplane is flying in a horizontal direction with a velocity u and at a height of 2000 m. When it is vertically below a point *A* on the ground a food packet is released from it. The packet strikes the ground at point *B*. If $AB = 3 \text{ km}$ and $g = 10 \text{ m/s}^2$, then the value of u is
- a. 54 km/h b. 540 km/h
c. 150 km/h d. 300 km/h
11. A conducting circular loop is placed in a uniform magnetic field, $B = 0.025 \text{ T}$ with its plane perpendicular to the loop. The radius of the loop is made to shrink at a constant rate of 1 mm/s. The induced emf when the radius is 2 cm, is
- a. $2 \pi \mu\text{V}$ b. $\pi \mu\text{V}$
c. $\frac{\pi}{2} \mu\text{V}$ d. $2 \mu\text{V}$
12. A mild steel wire of length $2L$ and cross-sectional area A is stretched, well within the elastic limit, horizontally between two pillars as shown in figure. A mass m is suspended from the mid-point of the wire strain in the wire is



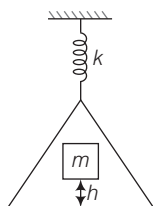
- a. $\frac{x^2}{2L^2}$ b. $\frac{x}{L}$
c. $\frac{x^2}{L}$ d. $\frac{x^2}{2L}$
13. The resistance of a wire at 20°C is 20Ω and 500°C is 60Ω . At which temperature, its resistance will be 25Ω ?
- a. 50°C b. 60°C
c. 70°C d. 80°C

14. The de-Broglie wavelength of a proton (charge $= 1.6 \times 10^{-19} \text{ C}$, $m = 1.6 \times 10^{-27} \text{ kg}$) accelerated through a potential difference of 1 kV is
- a. $600 \text{ \AA}$ b. $0.9 \times 10^{-12} \text{ m}$
c. $7 \text{ \AA}$ d. 0.9 nm
15. An ice-berg of density 900 kgm^{-3} is floating in water of density 1000 kgm^{-3} . The percentage of volume of ice-berg outside the water is
- a. 20% b. 35%
c. 10% d. 11%
16. The total energy of an electron in the first excited state of hydrogen is about -3.4 eV . Its kinetic energy in this state is
- a. -3.4 eV b. -6.8 eV
c. 6.8 eV d. 3.4 eV
17. A common emitter amplifier has a voltage gain of 50, an input impedance of 100Ω and an output impedance of 200Ω . The power gain of the amplifier is
- a. 500 b. 1000
c. 1250 d. 50
18. The horizontal range and maximum height attained by a projectile are R and H respectively. If a constant horizontal acceleration $a = \frac{g}{4}$ is imparted to the projectile due to wind, then its horizontal range and maximum height will be
- a. $(R + H), \frac{H}{2}$
b. $\left(R + \frac{H}{2}\right), 2H$
c. $(R + 2H), H$
d. $(R + H), H$
19. A balloon is filled at 27°C and 1 atm pressure by 500 m^3 He. At -3°C and 0.5 atm pressure, the volume of He will be
- a. 700 m^3 b. 900 m^3
c. 1000 m^3 d. 500 m^3
20. The ratio of intensity at the centre of a bright fringe to the intensity at a point distance one-fourth of the distance between two successive bright fringes will be
- a. 4 b. 3
c. 2 d. 1
21. A rectangular block of mass m and area of cross-section A floats in a liquid of density ρ . If it is given a vertical displacement from equilibrium, it undergoes oscillation with a time period T . Then
- a. $T \propto \sqrt{\rho}$ b. $T \propto \frac{1}{\sqrt{A}}$
c. $T \propto \frac{1}{\sqrt{\rho}}$ d. $T \propto \frac{1}{\sqrt{m}}$

22. Three charges are placed at the three vertices of an equilateral triangle of side 'a' as shown in the figure. The force experienced by the charge placed at the vertex A in a direction normal to BC is



- a. $\frac{Q^2}{4\pi\epsilon_0 a^2}$
 b. $-\frac{Q^2}{4\pi\epsilon_0 a^2}$
 c. zero
 d. $\frac{Q^2}{2\pi\epsilon_0 a^2}$
23. A load of mass m falls from a height h on the scale pan hung from a spring as shown. If the spring constant is k and the mass of the scale pan is zero and the mass m does not bounce relative to the pan, then the amplitude of vibration is



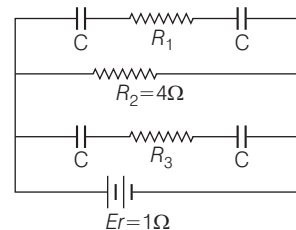
- a. $\frac{mg}{k}$
 b. $\frac{mg}{k} \sqrt{1 + \frac{2hk}{mg}}$
 c. $\frac{mg}{k} + \frac{mg}{k} \sqrt{\frac{1 + 2hk}{mg}}$
 d. None of these
24. The activity of a radioactive sample is measured as N_0 counts per minute at $t = 0$ and $\frac{N_0}{e}$ counts per minute at $t = 5$ min. The time (in minutes) at which the activity reduces to half its value is

- a. $\log_e \frac{2}{5}$
 b. $\frac{5}{\log_e 2}$
 c. $5 \log_{10} 2$
 d. $5 \log_e 2$
25. A plano-convex lens fits exactly into a plano-concave lens. Their plane surfaces are parallel to each other. If lenses are made of different materials of refractive indices μ_1 and μ_2 , R is the radius of curvature of the curved surface of the lenses, then the focal length of combination is

- a. $\frac{R}{2(\mu_1 + \mu_2)}$
 b. $\frac{R}{2(\mu_1 - \mu_2)}$
 c. $\frac{R}{(\mu_1 - \mu_2)}$
 d. $\frac{2R}{\mu_1 - \mu_2}$

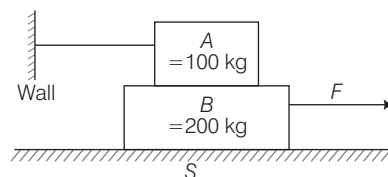
26. In the given circuit diagram,

$$E = 5 \text{ V}, r = 1\Omega, R_2 = 4\Omega, R_1 = R_3 = 1\Omega \text{ and } C = 3\mu\text{F}$$



Then, what will be the numerical value of charge on each plates of the capacitor.

- a. $24\mu\text{C}$
 b. $12\mu\text{C}$
 c. $6\mu\text{C}$
 d. $3\mu\text{C}$
27. A block A of mass 100 kg rests on another block B of mass 200 kg and is tied to a wall as shown in the figure. The coefficient of friction between A and B is 0.2 and that between B and ground is 0.3. The minimum force required to move the block B is ($g = 10\text{ms}^{-2}$)



- a. 900 N
 b. 200 N
 c. 1100 N
 d. 700 N
28. A uniform rod of length l and mass m is free to rotate in a vertical plane about A. The rod initially in horizontal position is released. The initial angular acceleration of the rod is (Moment of inertia of rod about A is $\frac{ml^2}{3}$)



- a. $\frac{3g}{2l}$
 b. $\frac{2l}{3g}$
 c. $\frac{3g}{2l^2}$
 d. $mg \frac{l}{2}$
29. Monochromatic radiation of wavelength λ is incident on a hydrogen sample in ground state. Hydrogen atom absorbs a fraction of light and subsequently emits radiations of six different wavelengths. The wavelength λ is

- a. 97.5 nm
 b. 121.6 nm
 c. 110.3 nm
 d. 45.2 nm

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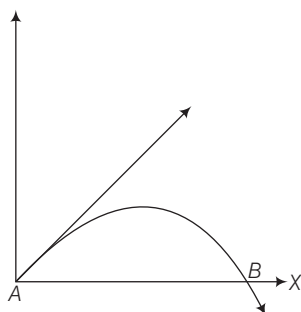
30. A coil in the shape of an equilateral triangle of side l is suspended between the pole pieces of a permanent magnet such that B is in plane of the coil. If due to a current i in the triangle, a torque τ rests on it, the side l of the triangle is

a. $2 \left(\frac{\tau}{\sqrt{3} Bi} \right)^{\frac{1}{2}}$ b. $\frac{2}{\sqrt{3}} \left(\frac{\tau}{Bi} \right)$
 c. $2 \left(\frac{\tau}{Bi} \right)^{\frac{1}{2}}$ d. $\frac{1}{\sqrt{3}} \left(\frac{\tau}{Bi} \right)$

31. Work done in increasing the size of a soap bubble from radius of (3 to 5) cm is nearly (surface tension of soap solution = 0.03 Nm^{-1})

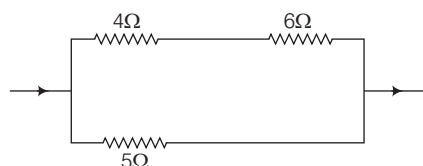
a. $0.2 \pi \text{ mJ}$ b. $2\pi \text{ mJ}$ c. 0.4 mJ d. $4\pi \text{ mJ}$

32. The velocity of a projectile at the initial point A is $(2\hat{i} + 3\hat{j}) \text{ m/s}$. Its velocity (in m/s) at point B is



a. $-2\hat{i} - 3\hat{j}$ b. $-2\hat{i} + 3\hat{j}$
 c. $2\hat{i} - 3\hat{j}$ d. $2\hat{i} + 3\hat{j}$

33. In the circuit shown, the heat produced in 5Ω resistor is 10 cal s^{-1} . The heat produced per sec in 4Ω resistor will be



a. 1 cal b. 2 cal c. 3 cal d. 4 cal

34. An α -particle after passing through potential difference of V volt collides with a nucleus. If the atomic number of the nucleus is Z , then distance of closest approach is

a. $14.4 \cdot \frac{Z}{V}$
 b. $14.4 \frac{Z}{V} \text{ m}$
 c. $14.4 \frac{V}{Z} \text{ m}$
 d. $14.4 \frac{V}{Z}$

35. Two simple pendulums of lengths 5m and 20m respectively are given small displacement in one direction at the same time they will again be in the same sense when the pendulum of shorter length has completed n oscillations. Then, n is

a. 5 b. 1 c. 2 d. 3

36. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between the plates is d . The space between the plates is now filled with two dielectrics constant $K_1 = 3$ and thickness $\frac{d}{3}$ while the other one has dielectric constant $K_2 = 6$ and thickness $\frac{2d}{3}$. Capacitance of the capacitor is now

a. 1.8 pF b. 45 pF
 c. 40.5 pF d. 20.25 pF

37. A particle moving along X -axis has acceleration f , at time t given by $f = f_0 \left(1 - \frac{t}{T} \right)$, where f_0 and T are constants. The particle at $t = 0$ and the instant when $f = 0$, the particle's velocity v_x is

a. $f_0 T$ b. $\frac{1}{2} f_0 T^2$
 c. $f_0 T^2$ d. $\frac{1}{2} f_0 T$

38. A geostationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a sky satellite orbiting a few 100 km above the earth's surface ($R = 64000 \text{ km}$) will approximately be

a. $\frac{1}{2} \text{ h}$ b. 1 h c. 2 h d. 4 h

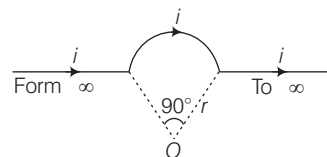
39. A transverse wave propagating on a stretched string of linear density $3 \times 10^{-4} \text{ kgm}^{-1}$ is represented by the equation

$$y = 0.2 \sin(1.5x + 60t)$$

where, x in metres and t is in seconds. The tension in the string (in Newton) is

a. 0.24 b. 0.48
 c. 1.20 d. 1.80

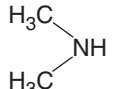
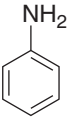
40. What is the magnetic field at the centre of arc in the figure below?



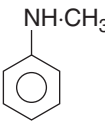
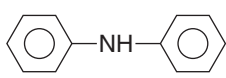
a. $\frac{\mu_0}{4\pi} \cdot \frac{2i}{r} [\sqrt{2} + \pi]$ b. $\frac{\mu_0}{4\pi} \cdot \frac{2i}{r} \left[\sqrt{2} + \frac{\pi}{4} \right]$
 c. $\frac{\mu_0}{4\pi} \cdot \frac{i}{r} [\sqrt{2} + \pi]$ d. $\frac{\mu_0}{4\pi} \cdot \frac{i}{r} \left[\sqrt{2} + \frac{\pi}{4} \right]$

PART II

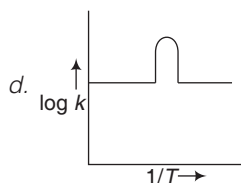
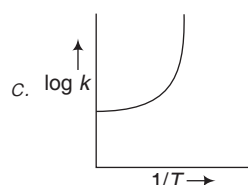
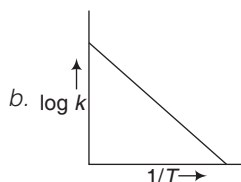
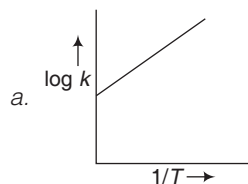
Chemistry

41. 4g of copper was dissolved in conc. HNO_3 . The copper nitrate thus obtained gave 5g of its oxide on strong heating the equivalent weight of copper is
 a. 23 b. 32
 c. 12 d. 20
42. Choose the incorrect statement.
 a. Sodium borohydride reacts very slowly with cold water
 b. Sodium borohydride reacts violently with cold water to give $\text{H}_2(g)$
 c. Solubility of sodium borohydride in water at 25°C is 10.05 g/ml
 d. Melting point of sodium borohydride is 500°C
43. The orbital angular momentum of an electron is 2 S orbital is
 a. $\frac{h}{4\pi}$ b. zero
 c. $\frac{h}{2\pi}$ d. $\sqrt{2} \cdot \frac{h}{2\pi}$
44. Which of the following are isoelectronic species?
 I. C^+H_3 II. NH_2^-
 III. NH_4^+ IV. NH_3
 a. I, II and III b. II, III and IV
 c. I, II and IV d. II and I
45. Two elements A and B have electronegativities 1.2 and 3.0 respectively. The nature of bond between A and B would be
 a. ionic b. covalent
 c. co-ordinate d. metallic
46. In the compound,
 $\text{CH}_2 = \text{CH} - \text{C} \equiv \text{CH}$
 (I) (II) (III)
 The most acidic hydrogen atom is
 a. Only I b. Only II
 c. Only III d. All are equally acidic
47. Reductive ozonolysis of $(\text{CH}_3)_2\text{C} = \text{C}(\text{CH}_3)_2$ followed by hydrolysis gives
 a. only one type of ketone
 b. only one type of aldehyde
 c. two types of ketone
 d. two types of aldehyde
48. Which of the following reactions does not involved absorption energy?
 I. $\text{O}(g) + e^- \longrightarrow \text{O}^-(g)$
 II. $\text{S}(g) + e^- \longrightarrow \text{S}^-(g)$
 III. $\text{O}^-(g) + e^- \longrightarrow \text{O}^{2-}(g)$
 IV. $\text{Cl}(g) + e^- \longrightarrow \text{Cl}^-(g)$
 a. Only II b. I and III
 c. I, II and III d. I, II and IV
49. The most reactive amine towards reactions with dil. HCl is
 a. $\text{CH}_3\text{—NH}_2$ b.  c. $(\text{CH}_3)_3\text{N}$ d. 
50. Blocks of magnesium are fixed to the bottom of a ship to
 a. block hole in the ship
 b. acidity of sea water
 c. make the ship lighter
 d. prevent the action of water and salt
51. On electrolysis of water, a total of 1 mole of gases is evolved. The amount of water decomposed is
 a. 1 mol b. 2 mol c. $\frac{1}{3}$ mol d. $\frac{2}{3}$ mol
52. How many moles of Fe^{2+} ions are formed when excess of iron react with 500 mL O_2 , 4NHCl , under inert atmosphere?
 (assume no change in volume)
 a. 0.4 b. 0.1 c. 0.2 d. 0.8
53. Sodium sulphate is soluble in water but barium sulphate is insoluble because
 a. hydration energy of Na_2SO_4 is more than of its lattice energy
 b. lattice energy of BaSO_4 is more than its hydration energy
 c. Both (a) and (b)
 d. None of the above
54. In the reaction,
 $\text{CH}_3 \cdot \text{CH}_2 \cdot \text{Cl} \xrightarrow{\text{KCN}} (\text{A}) \xrightarrow{\text{LiAlH}_4} (\text{P})$ end product (P) is
 a. $\text{CH}_3\text{CH}_2 \cdot \text{NO}_2$ b. $\text{CH}_3\text{CH}_2\text{CH}_2\text{NO}_2$
 c. $\text{CH}_3\text{CH}_2\text{NH}_2$ d. $\text{CH}_3 \cdot \text{CH}_2 \cdot \text{CH}_2\text{NH}_2$
55. Which of the following reactions is an example of calcination process?

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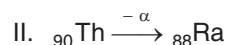
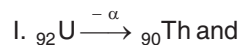
- a. $2\text{Ag} + 2\text{HCl} + [\text{O}] \longrightarrow 2\text{AgCl} + \text{H}_2\text{O}$
 b. $2\text{Zn} + \text{O}_2 \longrightarrow 2\text{ZnO}$
 c. $2\text{ZnS} + 3\text{O}_2 \longrightarrow 2\text{ZnO} + 2\text{SO}_2$
 d. $\text{MgCO}_3 \xrightarrow{\Delta} \text{MgO} + \text{CO}_2$
56. For an endothermic reaction, where ΔH represent the enthalpy of reaction in kJ/mol, the minimum value for energy of activation (for forward reaction) will be
 a. less than ΔH b. zero
 c. more than ΔH d. equal to ΔH
57. Which of the following metal is leached by cyanide process
 a. Ag b. Na c. Al d. Cu
58. Which of the following is a diamagnetic complex
 a. $[\text{Co}(\text{NH}_3)_6]^{3+}$ b. $[\text{NiCl}_4]^{2-}$
 c. $[\text{CuCl}_4]^{2-}$ d. $[\text{Fe}(\text{H}_2\text{O})_6]^{3+}$
59. Neoprene is a
 a. monomer of rubber b. synthetic rubber
 c. a natural rubber d. vulcanised rubber
60. Among the following, which have highest melting point?
 a. Ionic solids b. Pseudo solids
 c. Molecular solids d. Amorphous solids
61. The night-blindness is developed due to deficiency of vitamin
 a. B₆ b. C c. B₁₂ d. A
62. The transfer RNA anticodon for the messenger RNA codon G – C – A is
 a. C – G – U b. C – C – U
 c. U – C – C d. G – U – C
63. 0.765 g of an acid gives 0.535 g of CO_2 and 0.138 g of H_2O . Then, the ratio of percentage of carbon and hydrogen is
 a. 19 : 2 b. 18 : 11
 c. 20 : 17 d. 1 : 7
64. Maximum pK_b value is of
 a.  b. $(\text{CH}_3\text{CH}_2)_2\text{NH}$
 c. $(\text{CH}_3)_2\text{NH}$ d. 
65. Which of the following is an incorrect set of quantum members?
 a. $n = 2, l = 0, m = 0$ b. $n = 1, l = 0, m = 0$
 c. $n = 3, l = 3, m = 0$ d. $n = 2, l = 1, m = 1$
66. The most acidic oxide for nitrogen is
 a. NO_2 b. N_2O
 c. NO d. N_2O_5
67. Which of the following show maximum bond-order?
 a. O_2 b. O_2^-
 c. O_2^+ d. O_2^{2-}
68. Which of the following show an increase in entropy?
 I. Boiling of water
 II. Melting of ice
 III. Freezing of water
 IV. Formation of hydrogen gas from water
 a. (I) and (II) b. Only (III)
 c. (I), (II) and (IV) d. (III) and (IV)
69. BF_3 is an acid, according to
 a. Lewis
 b. Arrhenius
 c. Bronsted and Lowery
 d. All of the above
70. For the reaction
 $\text{N}_2\text{O}_4(g) \longrightarrow 2\text{NO}_2(g)$
 a. $\Delta H > \Delta E$ b. $\Delta H < \Delta E$
 c. $\Delta H = \Delta E$ d. $\Delta H = 0$
71. Which of the following elements mostly form covalent compounds?
 a. Cs b. Rb
 c. K d. Li
72. When aqueous solutions of borax is acidified with HCl, we get
 a. B_2H_6 b. H_3BO_3
 c. B_2O_3 d. All of these
73. Which of the following compound does not follow Huckel's rule?
 a.  b. 
 c.  d. 

74. A graph is plotted between $\log K$ versus $\frac{1}{T}$ for calculation of activation energy (E_a). The correct plot is



75. The hybridisation of Fe is $K_3Fe(CN)_6$ is
 a. sp^3 b. dsp^3 c. sp^3d^2 d. d^2sp^3
76. Which of the following shows maximum magnetic moment?
 a. Mg^{2+} b. Ti^{3+} c. V^{3+} d. Fe^{2+}

77. Consider the following radioactive decays



In which case group of parent and daughter elements remains unchanged.

- a. In (I)
 b. In (II)
 c. Both in (I) and (II)
 d. None of the above

78. Chlorobenzene $\xrightarrow{\text{Reaction-(X)}}$ Phenol
 $\xrightarrow{\text{Reaction-(Y)}}$ Salicylaldehyde.

The reactions(s) 'X' and 'Y' respectively are

- a. Fries rearrangement and Kolbe
 b. Cumene and Reimer-Tiemann
 c. Dow and Reimer-Tiemann
 d. Dow and Sandmeyer.

79. Which of the following has largest number of moles?

- a. 8 g of oxygen atoms
 b. 16 g of oxygen gas
 c. 14 g of nitrogen gas (N_2)
 d. All have same number of moles

80. One moles each of four ideal gases are kept as follows.

- I. 5 L of gas (A) at 2 atm pressure
 II. 2.5 L of gas (B) at 2 atm pressure
 III. 1.25 L of gas (C) at 2 atm pressure
 IV. 2.5 L of gas (D) at 2.5 atm pressure

Which of the above gases is kept at highest temperature.

- a. Gas (A) b. Gas (B)
 c. Gas (C) d. Gas (D)

Part III

a. English Proficiency

Directions (Q. Nos. 81-83) *In the following questions the sentences may or may not be grammatically correct. Find out which part of a sentence has an error and mark that part. If there is no error mark part 'd' as your answer.*

81. Along the northern frontier of India is seen
a *b*
the Himalayas mighty in their splendour. No error
c *d*
82. The father with the son were mysteriously missing
a *b*
from the house. No error
c *d*
83. It is not advisable to take heavy luggages
a *b*
while on journey these days. No error
c *d*

Directions (Q. Nos. 84-85) *Fill in the blanks with suitable preposition from the alternatives given under each sentence.*

84. The problem of communal harmony cannot be glossed by the government.
a. at b. on c. over d. for
85. She could not muster courage to stand against the maltreatment.
a. to b. up c. about d. on

Directions (Q. Nos. 86-88) *The following sentences consist of a word or a phrase which is written in italicised letters. Each sentence is followed by four words or phrases. Select the word or the phrase which is closest to the opposite in meaning of the italicised word or phrase.*

- 86.** Philosophers say that the world is *an illusion*.
a. a fact b. a reality c. an actuality d. a truth
- 87.** She used to *disparage* her neighbours every now and then.
a. please b. praise c. belittle d. denigrate
- 88.** The momentum of the movement *slackened* in course of time.
a. stopped b. quickened c. multiplied d. recovered

Directions (Q. Nos. 89-90) *In the following sentences, a word or a phrase is written in italicised letters. For each italicised part four words/phrases are listed below each sentence. Choose the word nearest in meaning to the italicised word/phrase.*

89. The opposition criticised the ruling party for the *deteriorating* law and order situation in the state.
a. disrupting b. worsening c. crumbling d. eroding
90. The two opposing parties have reached *stalemate*.

- a. dilemma b. deadlock c. exhaustion d. settlement

Directions (Q. Nos. 91-95) *Read the passage given below and answer the questions that follow.*

A pioneering scheme has been started recently in Southampton of England's south coast to educate tourists who have been convicted of drunken driving.

The penalty for drunken driving might be the loss of the driving licence and a heavy fine. But under the new scheme, convicted drivers do not pay the fine. Instead they have to attend eight training sessions; one a week organised by the local authority probation service.

Designed to demonstrate the damage alcohol can do, the scheme was devised by senior probation officer John Cook. He said that about a quarter of the people who came to him had a drinking problem, and had not realised how much they were drinking. One way of getting the message across was to make the drivers pour out their usual ration of alcohol and then measure it. Almost everyone poured out not a single measure, but a double at least, an example of how easy it is to have more than just one drink and to encourage other people to do the same. The instructors on the course are giving clinical evidence of the effects of alcohol on the body and brain. The sober truth is that drinking badly affects driving skills, although the drinker might like to believe otherwise.

91. The Southampton scheme requires convicted drivers
 - a. to pay a heavy fine
 - b. to attend eight driving sessions-one a week
 - c. to undergo a probation service
 - d. to surrender their driving licence
92. John Cook devised the scheme
 - a. as a demonstration technique for driving
 - b. to deny the harmful effects of alcohol
 - c. to show that Southampton was concerned about drivers
 - d. to prove that alcohol does influence driving
93. The problem with a quarter of the people who went to John Cook was that they
 - a. did not want to stop drinking
 - b. were unaware of the fact that they could get drunk
 - c. would not admit that they had a drinking problem
 - d. did not know how much they were drinking
94. Most drivers start off with atleast
 - a. a double measure b. a single measure
 - c. a little less than a single measure
 - d. two doubles
95. The truth is that alcohol
 - a. does not affect the body but only the brain
 - b. affects only the brain
 - c. affects the body and the brain
 - d. has no effect on the body or the brain

b. Logical Reasoning

96. 'Shoes' is related to 'Leather', in the same way as 'Rubber' is related to

- a. Plastic b. Polythene
c. Latex d. Chappal

97. Find the odd one from the following options

- a. 81 : 243 b. 25 : 75
c. 64 : 192 d. 16 : 64

98. Complete the series by replacing '?' mark.

4, 11, 30, 67, 128, ?

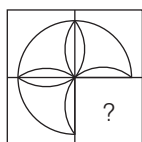
- a. 219 b. 228
c. 237 d. 240

99. Lakshmi is elder than Meenu. Leela is elder than Meenu but younger than Lakshmi. Latha is younger than both Meenu and Hari but Hari is younger than Meenu. Who is the youngest?

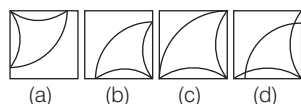
- a. Lakshmi b. Meenu
c. Leela d. Latha

100. In the following question a part of problem figure is missing. Find out from the given answer figures *a, b, c* and *d* that can replace the question mark (?) to complete the figure.

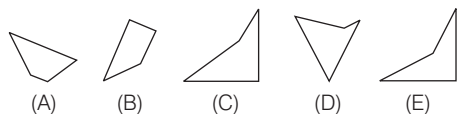
Question Figure



Answer Figure



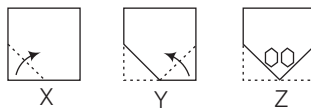
101. In the following question, five figures are given. Out of them find the three figures that can be joined to form a square.



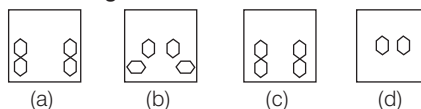
- a. ACD b. BCD
c. BDE d. CDE

102. The three problem figures marked X, Y and Z show the manner in which a piece of paper is folded step by step and then cut. From the answer figures *a, b, c* and *d* select the one showing the unfolded pattern of the paper after the cut.

Question Figures

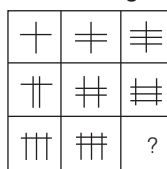


Answer Figures

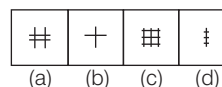


103. Choose the answer figure which completes the problem figure matrix.

Question Figures

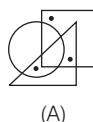


Answer Figures

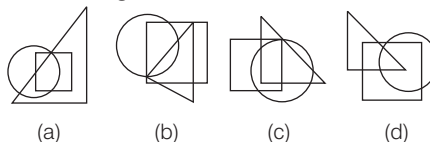


104. In the following question, one or more dots are placed in the figure marked as (A). This figure is followed by four alternatives marked as *a, b, c* and *d*. One out of these four options contains region(s) common to circle, square and triangles, similar to that marked by the dot in figures (A). Find that figure

Question Figure



Answer Figures



105. How many different triangles are there in the figures shown below?



- a. 28 b. 24
c. 20 d. 16

Part IV

Mathematics

- 106.** The coefficient of x^5 in the expansion of $(1+x)^{21} + (1+x)^{22} + \dots + (1+x)^{30}$ is
- a. ${}^{51}C_5$ b. 9C_5
c. ${}^{31}C_6 - {}^{21}C_6$ d. ${}^{30}C_5 + {}^{20}C_5$
- 107.** If $z = a + ib$ satisfies $\arg(z-1) = \arg(z+3i)$, then $(a-1):b =$
- a. $2:1$ b. $1:3$
c. $-1:3$ d. None of these
- 108.** If p and p' denote the lengths of the perpendicular from a focus and the centre of an ellipse with semi-major axis of length a , respectively, on a tangent to the ellipse and r denotes the focal distance of the point, then
- a. $ap = rp'$ b. $rp = ap'$
c. $ap = rp' + 1$ d. $ap' + rp = 1$
- 109.** The value of $\sum_{r=1}^{10} r \cdot \frac{{}^nC_r}{{}^nC_{r-1}}$ is equal to
- a. $5(2n-9)$ b. $10n$
c. $9(n-4)$ d. None of these
- 110.** The numbers $3^{2\sin 2\alpha - 1}$, 14 and $3^{4-2\sin 2\alpha}$ form first three terms of an A.P., its fifth term is
- a. -25 b. -12 c. 40 d. 53
- 111.** For the equation $3x^2 + px + 3 = 0$, $p > 0$, if one of the roots is square of the other, then p is equal to
- a. $\frac{1}{2}$ b. 1 c. 3 d. $\frac{2}{3}$
- 112.** If $a = \log_2 3$, $b = \log_2 5$ and $c = \log_7 2$, then $\log_{140} 63$ in terms of a, b, c is
- a. $\frac{2ac+1}{2c+abc+1}$ b. $\frac{2ac+1}{2a+c+a}$
c. $\frac{2ac+1}{2c+ab+a}$ d. None of these
- 113.** If $\cos(x-y)$, $\cos x$ and $\cos(x+y)$ are in HP, then $\cos x \sec(y/2)$ is equal to
- a. $\pm \sqrt{2}$ b. $\pm 1/\sqrt{2}$
c. ± 2 d. None of these
- 114.** Let $A = \{1, 2, 3, 4, 5\}$ and R be a relation defined by $R = \{(x, y) : x, y \in A, x+y=5\}$. Then, R is
- a. reflexive and symmetric but not transitive
b. an equivalence relation
c. symmetric but neither reflexive nor transitive
d. neither reflexive nor symmetric but transitive
- 115.** The number of times the digit 5 will be written when listing the integers from 1 to 1000, is
- a. 271 b. 272
c. 300 d. None of these
- 116.** Let A and B be two sets such that $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for same set X . Then,
- a. $A = B$ b. $A = X$
c. $B = X$ d. $A \cup B = X$
- 117.** Let $A = [-1, 1]$ and $f: A \rightarrow A$ be defined as $f(x) = x|x|$ for all $x \in A$, then $f(x)$ is
- a. many-one and into function
b. one-one and into function
c. many-one and into function
d. one-one and onto function
- 118.** The general solution of $\sin x - 3 \sin 2x + \sin 3x = \cos x - 3 \cos 2x + \cos 3x$ is
- a. $n\pi + \frac{\pi}{8}$ b. $\frac{n\pi}{2} + \frac{\pi}{8}$
c. $(-1)^n \frac{n\pi}{2} + \frac{\pi}{8}$ d. $2n\pi + \cos^{-1} \frac{3}{2}$
- 119.** Two equal sides of an isosceles triangle are $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point $(1, -10)$. Find the equation of the third side
- a. $x - 3y = -31$ b. $x - 3y = 31$
c. $x + 3y = 31$ d. $x + 3y = -31$
- 120.** If two distinct chords drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ (where $pq \neq 0$) are bisected by the X -axis, then
- a. $p^2 = q^2$ b. $p^2 = 8q^2$
c. $p^2 < 8q^2$ d. $p^2 > 8q^2$
- 121.** The length of perpendicular drawn from the point $(2, 3, 4)$ to line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$, is
- a. $\frac{3}{7} \sqrt{101}$ b. $\frac{2}{7} \sqrt{101}$
c. $\frac{2}{7} \sqrt{103}$ d. $\frac{3}{7} \sqrt{103}$
- 122.** The image of the point $(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is
- a. $(-1, 0, 7)$ b. $(-1, 0, -7)$
c. $(1, 0, 7)$ d. $(2, 0, 7)$

123. The distances of the point $(1, -5, 9)$ from the plane $x + y + z = 5$ measured along a straight line $x = y = z$ is $2\sqrt{3}k$, then the value of k is

a. 5
b. 6
c. $\sqrt{3}$
d. 4

124. $\lim_{n \rightarrow \infty} \sin[\pi\sqrt{n^2 + 1}]$ is equal to

a. ∞
b. 0
c. does not exist
d. None of these

125. If $f(x) = \begin{cases} e^x, & x \leq 0 \\ |1-x|, & x > 0 \end{cases}$, then

a. $f(x)$ is differentiable at $x = 0$
b. $f(x)$ is continuous at $x = 0$, 1
c. $f(x)$ is differentiable at $x = 1$
d. None of the above

126. If a function $f: \mathbb{R} \rightarrow \mathbb{R}$ satisfy the equation $f(x+y) = f(x) + f(y)$, $\forall x, y$ and the function $f(x)$ is continuous at $x = 0$, then

a. $f(x)$ is continuous for all positive real values of x
b. $f(x)$ is continuous for all x
c. $f(x) = 0$ for all x
d. None of the above

127. The value of $f(0)$, so that the function $f(x) = \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$ is continuous at each point in its domain, is

a. $\frac{1}{3}$
b. $-\frac{1}{3}$
c. $\frac{2}{3}$
d. $-\frac{2}{3}$

128. Consider the greatest integer function, defined by $f(x) = [x]$, $0 \leq x < 2$. Then,

a. f is derivable at $x = 1$
b. f is not derivable at $x = 1$
c. f is derivable at $x = 2$
d. None of these

129. Let $f(x) = -2x^3 + 21x^2 - 60x + 41$, then

a. $f(x)$ is decreasing in $(-\infty, 1)$
b. $f(x)$ is decreasing in $(-\infty, 2)$
c. $f(x)$ is increasing in $(-\infty, 1)$
d. $f(x)$ is increasing in $(-\infty, 2)$

130. Rolle's theorem is not applicable for the function $f(x) = |x|$ in the interval $[-1, 1]$ because

a. $f'(1)$ does not exist
b. $f'(-1)$ does not exist
c. $f(x)$ is discontinuous at $x = 0$
d. $f'(0)$ does not exist

131. If the curve $y = a^x$ and $y = b^x$ intersect at angle α , then $\tan \alpha$ is equal to

a. $\frac{a-b}{1+ab}$
b. $\frac{\log a - \log b}{1 + \log a \log b}$
c. $\frac{a+b}{1-ab}$
d. $\frac{\log a + \log b}{1 - \log a \log b}$

132. The expression $\frac{\int_0^n [x] dx}{\int_0^n \{x\} dx}$, where $[x]$ and $\{x\}$ are integral

and fractional part of x and $n \in \mathbb{N}$, is equal to

a. $\frac{1}{n-1}$
b. $\frac{1}{n}$
c. n
d. $n-1$

133. The maximum value of $f(x) = x + \sin 2x$, $x \in [0, 2\pi]$ is

a. $\frac{\pi}{2}$
b. 2π
c. $\frac{3\pi}{4}$
d. $\frac{3\pi}{2}$

134. The area under the curve $y = |\cos x - \sin x|$, $0 \leq x \leq \frac{\pi}{2}$ and above X -axis, is

a. $2\sqrt{2}$
b. $2\sqrt{2} - 2$
c. $2\sqrt{2} + 2$
d. 0

135. The solution of $\frac{dy}{dx} = \cos(x+y) + \sin(x+y)$, is

a. $\log \left[1 + \tan \left(\frac{x+y}{2} \right) \right] + C = 0$
b. $\log \left[1 + \tan \left(\frac{x+y}{2} \right) \right] = x + C$
c. $\log \left[1 - \tan \left(\frac{x+y}{2} \right) \right] = x + C$
d. None of the above

136. The area enclosed by the curves $y = x^3$ and $y = \sqrt{x}$ is

a. $\frac{5}{3}$ sq units
b. $\frac{5}{4}$ sq units
c. $\frac{5}{12}$ sq units
d. $\frac{12}{5}$ sq units

137. If $\lim_{x \rightarrow \infty} \left(an - \frac{1+n^2}{1+n} \right) = b$, where a is finite number, then

a. $a = 2$
b. $a = 0$
c. $b = 1$
d. $b = -1$

138. If the papers of 4 students can be checked by anyone of the 7 teachers, then the probability that all the 4 papers are checked by exactly 2 teachers, is equal to

a. $\frac{12}{49}$
b. $\frac{6}{49}$
c. $\frac{9}{49}$
d. $\frac{15}{49}$

12 SELF STUDY GUIDE **BITSAT**

- 139.** If equation $(10x - 5)^2 + (10y - 4)^2 = \lambda^2 (3x + 4y - 1)^2$ represents a hyperbola, then
 a. $-2 < \lambda < 2$
 b. $\lambda > 2$
 c. $\lambda < -2$ or $\lambda > 2$
 d. $0 < \lambda < 2$

140. Let $\hat{a}$ and $\hat{b}$ be two non-collinear unit vectors. If $u = \hat{a} - (\hat{a} \cdot \hat{b})\hat{b}$ and $v = \hat{a} \times \hat{b}$, then $|v|$ is equal to
 a. $|u|$
 b. $|u| + |v \cdot \hat{a}|$
 c. $2|v|$
 d. $|v| + u \cdot (\hat{a} + \hat{b})$

141. If the variance of the observations $x_1, x_2, \dots, x_n$ is σ^2 , then the variance of $\alpha x_1, \alpha x_2, \dots, \alpha x_n$, $\alpha \neq 0$ is
 a. σ^2
 b. $\alpha \sigma^2$
 c. $\alpha^2 \sigma^2$
 d. $\frac{\sigma^2}{\alpha^2}$

142. Coefficient of variation of two distributions are 50 and 60 and their arithmetic means are 30 and 25, respectively. Difference of their standard deviation is
 a. 0
 b. 1
 c. 1.3
 d. 2.5

143. The maximum value of $z = 9x + 13y$ subject to constraints $2x + 3y \leq 18$, $2x + y \leq 10$, $x \geq 0$, $y \geq 0$ is
 a. 130
 b. 81
 c. 79
 d. 99

144. A coin is tossed 7 times. Each time a man calls head. The probability that he wins the toss atleast 4 occasions is
 a. $\frac{1}{4}$
 b. $\frac{5}{8}$
 c. $\frac{1}{2}$
 d. $\frac{1}{6}$

145. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$ and $f(1) = 2$,
 $f(p + q) = f(p) \cdot f(q)$, $\forall p, q \in R$, then
 $x^{f(1)} + y^{f(2)} + z^{f(3)} - \frac{(x + y + z)}{x^{f(1)} + y^{f(2)} + z^{f(3)}}$ is equal to
 a. 0
 b. 1
 c. 2
 d. 3

146. The value of $\frac{2}{1!} + \frac{2+4}{2!} + \frac{2+4+6}{3!} + \dots$ is
 a. e
 b. $2e$
 c. $3e$
 d. None of these

147. If z_1, z_2 and z_3 represent the vertices of an equilateral triangle such that $|z_1| = |z_2| = |z_3|$, then
 a. $z_1 + z_2 = z_3$
 b. $z_1 + z_2 + z_3 = 0$
 c. $z_1 z_2 = \frac{1}{z_3}$
 d. $z_1 - z_2 = z_3 - z_2$

148. If $\int \frac{(\sqrt{x})^5 dx}{(\sqrt{x})^7 + x^6} = \lambda \log \left(\frac{x^9}{x^9 + 1} \right) + C$, then $a + \lambda$ equal to
 a. 2
 b. > 2
 c. < 2
 d. > 3

149. Line joining the points $(0, 3)$ and $(5, -2)$ is a tangent to the curve $y = \frac{ax}{1+x}$, then
 a. $a = 1 \pm \sqrt{3}$
 b. $a \in \phi$
 c. $a = -1 \pm \sqrt{3}$
 d. $a = -2 \pm 2\sqrt{3}$

150. The shortest distance between the parabolas $y^2 = 4x$ and $y^2 = 2x - 6$ is
 a. 2
 b. $\sqrt{5}$
 c. 3
 d. None of these

Answers

Physics

1. (a)	2. (d)	3. (c)	4. (b)	5. (b)	6. (b)	7. (b)	8. (b)	9. (c)	10. (b)
11. (b)	12. (a)	13. (d)	14. (b)	15. (c)	16. (d)	17. (c)	18. (d)	19. (b)	20. (c)
21. (b)	22. (c)	23. (b)	24. (d)	25. (c)	26. (c)	27. (c)	28. (a)	29. (a)	30. (a)
31. (d)	32. (c)	33. (b)	34. (a)	35. (c)	36. (c)	37. (d)	38. (c)	39. (b)	40. (b)

Chemistry

41. (b)	42. (b)	43. (b)	44. (b)	45. (a)	46. (c)	47. (a)	48. (d)	49. (b)	50. (d)
51. (d)	52. (b)	53. (c)	54. (d)	55. (d)	56. (c)	57. (a)	58. (a)	59. (b)	60. (a)
61. (d)	62. (a)	63. (a)	64. (d)	65. (c)	66. (d)	67. (c)	68. (c)	69. (a)	70. (a)
71. (d)	72. (b)	73. (d)	74. (b)	75. (d)	76. (d)	77. (a)	78. (c)	79. (d)	80. (d)

English Proficiency

81. (b)	82. (b)	83. (b)	84. (c)	85. (b)	86. (b)	87. (b)	88. (b)	89. (b)	90. (b)
91. (b)	92. (d)	93. (d)	94. (a)	95. (c)					

Logical Reasoning

96. (c)	97. (d)	98. (a)	99. (d)	100. (a)	101. (a)	102. (b)	103. (c)	104. (c)	105. (a)
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Mathematics

106. (c)	107. (b)	108. (a)	109. (a)	110. (d)	111. (c)	112. (d)	113. (a)	114. (c)	115. (c)
116. (a)	117. (d)	118. (b)	119. (b)	120. (d)	121. (a)	122. (c)	123. (a)	124. (b)	125. (b)
126. (b)	127. (a)	128. (b)	129. (b)	130. (d)	131. (b)	132. (d)	133. (b)	134. (b)	135. (b)
136. (c)	137. (c)	138. (b)	139. (c)	140. (a)	141. (c)	142. (a)	143. (c)	144. (c)	145. (c)
146. (c)	147. (b)	148. (b)	149. (b)	150. (b)					

SOLUTIONS

Physics

1. (a) According to Stefan's law $\rho = \frac{E}{t} = \sigma A e T^4$

So, we can write as

$$\frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4$$

$$\frac{E_2}{E_1} = \left(\frac{900}{300}\right)^4 \Rightarrow \frac{E_2}{E_1} = (3)^4$$

$$E_2 = 81 E_1 \Rightarrow \frac{E_2}{E_1} = 81$$

2. (d) $v_s = r\omega = 1.2 \times 2\pi f$

$$= 1.2 \times 2 \times 3.14 \times \left(\frac{400}{60}\right) = 50 \text{ m/s}$$

$$v_{\min} = \frac{v}{v + v_s} \cdot v$$

$$= \frac{340}{340 + 50} \times 500$$

$$= 436 \text{ Hz}$$

$$v_{\max} = \frac{v}{v - v_s} \cdot v$$

$$= \frac{340}{340 - 50} \times 500$$

$$= 586 \text{ Hz}$$

3. (c) We have, $f_r - f_b = \omega f_y$

$$\omega = \frac{f_r - f_b}{f_y}$$

Here, $f_r = 100 \text{ cm}$

$$f_b = 96.8 \text{ cm}$$

$$f_y = \sqrt{f_b \times f_r}$$

$$= \sqrt{96.8 \times 100}$$

$$= 98 \text{ cm}$$

$\therefore$ Dispersive power

$$\omega = \frac{f_r - f_b}{f_y}$$

$$= \frac{100 - 96.8}{98}$$

$$= \frac{3.2}{98} = 0.0325$$

4. (b) $V = 800 \text{ V}$

$$E = \frac{V}{d} = \frac{800}{2 \times 10^{-2}} \text{ V/m}$$

$$= 4 \times 10^4 \text{ V/m}$$

$$qE = mg$$

$$\Rightarrow q = \frac{mg}{E}$$

$$= \frac{1.96 \times 10^{-15} \times 9.8}{4 \times 10^4} \text{ coulomb}$$

$$= \frac{1.96 \times 10^{-15} \times 9.8}{4 \times 10^4 \times 1.6 \times 10^{-19}} \text{ elementary charges}$$

$$= 3e$$

5. (b) Given, $H = R$

$$\text{or } \frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{2u^2 \sin \theta \cos \theta}{g}$$

$$\text{or } \tan \theta = 4$$

$$\Rightarrow \theta = \tan^{-1}(4)$$

6. (b) Given, $a = 6 \text{ cm} = 6 \times 10^{-2} \text{ m}$

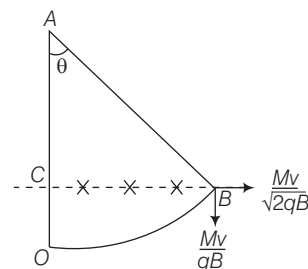
$$\omega = 100 \text{ rad/s}, m = 1 \text{ kg}$$

$$K_{\max} = \frac{1}{2} m \omega^2 a^2$$

$$= \frac{1}{2} \times 1 \times (100)^2 \times (6 \times 10^{-2})^2$$

$$= 18 \text{ J}$$

7. (b) From triangle ABC



$$\sin \theta = \frac{l}{H} = \frac{\frac{mv}{\sqrt{2} qB}}{\frac{mv}{qB}}$$

$$= \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin \theta = \sin 45^\circ$$

$$= \sin \frac{\pi}{4}$$

$$\text{Time period for angle } 2\pi \text{ is } T = \frac{2\pi m}{qB}$$

∴ Time by charged particle to emerge from magnetic field is $\frac{\pi m}{4qB}$.

8. (b) Given, $\lambda_1 = 12000 \text{ \AA}$

and $\lambda_2 = 10000 \text{ \AA}$

$D = 2 \text{ cm}$

$d = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$

We have,

$$\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1} = \frac{12000}{10000} = \frac{6}{5}$$

as $X = \frac{n_1 \lambda_1 D}{d}$

$$= \frac{5 \times 12000 \times 10^{-10} \times 2}{2 \times 10^{-3}}$$

$$= 5 \times 1.2 \times 10^4 \times 10^{-10} \times 10^3 = 6 \text{ mm}$$

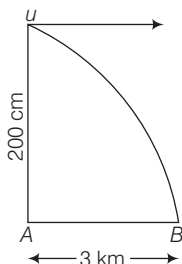
9. (c) Equation of motion of lower block is

$$F = \mu(M_A + M_B) \cdot g$$

$$\mu = \frac{F}{(M_A + M_B)g}$$

$$= \frac{49}{10 \times 9.8} = \frac{1}{2} = 0.5$$

10. (b) Given $R = 3 \text{ km} = 3000 \text{ m}$



Range, $R = u \sqrt{\frac{2h}{g}}$

$$\Rightarrow u = R \sqrt{\frac{g}{2h}} \text{ or } u = 3000 \times \sqrt{\frac{10}{2 \times 2000}}$$

$$= 3000 \times \frac{1}{20}$$

$$= 150 \text{ m/s}$$

$$= 150 \times \frac{18}{5} \text{ km/h}$$

$$= 540 \text{ km/h}$$

11. (b) Here, magnetic field, $B = 0.025 \text{ T}$

∴ Magnetic flux $\phi = B \cdot A = B \cdot \pi r^2$

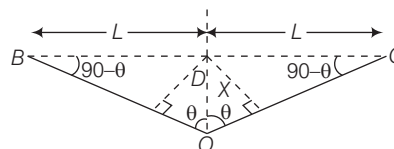
Induced emf $|e| = \frac{d\phi}{dt}$

$$= B \pi 2r \cdot \frac{dr}{dt}$$

$$= 0.025 \times \pi \times 2 \times 2 \times 10^{-2} \times 1 \times 10^{-3}$$

$$= \pi \mu \text{ V}$$

12. (a) Increase in length



$$\Delta L = BO + OC - (BC)$$

$$= 2BO - 2BD$$

$$\Rightarrow \Delta L = 2BO - 2L$$

$$= 2[L^2 + x^2]^{\frac{1}{2}} - 2L$$

or $\Delta L = 2L \left[1 + \frac{x^2}{L^2} \right]^{\frac{1}{2}} - 2L$ [$\therefore$ using Binomial theorem]

$$\Rightarrow \Delta L \approx 2L \left[1 + \frac{1}{2} \frac{x^2}{L^2} - 1 \right]$$

$$= \frac{x^2}{L}$$

$$[\therefore x \ll L]$$

$$\therefore \text{Strain} = \frac{\Delta L}{2L}$$

$$= \frac{x^2}{2L} = \frac{x^2}{2L^2}$$

13. (d) We have, $R_t = R_0(1 + \alpha t)$

Here, $20 = R_0(1 + 20\alpha)$

$$60 = R_0(1 + 500\alpha)$$

$$\Rightarrow \frac{60}{20} = \frac{R_0(1 + 500\alpha)}{R_0(1 + 20\alpha)}$$

$$3 = \frac{1 + 500\alpha}{1 + 20\alpha}$$

$$\Rightarrow 3(1 + 20\alpha) = 1 + 500\alpha$$

$$\Rightarrow 3 + 60\alpha = 1 + 500\alpha$$

$$\Rightarrow 440\alpha = 2$$

$$\Rightarrow \alpha = \frac{2}{440} = \frac{1}{220} / ^\circ \text{C}$$

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Also given $R_t = 25^\circ \text{C}$, $t = ?$

$$\text{Now, } \frac{20}{25} = \frac{R_0 \left(1 + \frac{20}{220}\right)}{R_0 \left(1 + \frac{t}{220}\right)}$$

$$4 \left(1 + \frac{t}{220}\right) = 5 \left(1 + \frac{20}{220}\right)$$

On solving,

$$t = 80^\circ \text{C}$$

14. (b) de-Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$$

$$\Rightarrow \lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19} \times 1000}}$$

$$\Rightarrow \lambda = \frac{6.6 \times 10^{-34}}{7.16 \times 10^{-22}} = 0.9 \times 10^{-12} \text{ m}$$

15. (c) Let the volume of ice-berg is V and its density is ρ . If this ice-berg floats in water with volume V_{in} inside it, then

$$V_{in}\sigma g = V\rho g$$

σ = density of water

$$\Rightarrow V_{in} = \left(\frac{\rho}{\sigma}\right) \cdot V$$

$$\begin{aligned} \Rightarrow V_{out} &= V - V_{in} \\ &= \left[\frac{\sigma - \rho}{\sigma}\right] V = \frac{1000 - 900}{1000} V \\ &= \frac{V}{10} \end{aligned}$$

$$\Rightarrow V_{out} / V = 0.1 = 10\%$$

16. (d) We have, Kinetic energy of electron,

$$K = \frac{Ze^2}{8\pi\epsilon_0 \cdot r}$$

also, Potential energy of electron,

$$U = -\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r}$$

$\therefore$ Total energy

$$\begin{aligned} E &= K + U \\ &= \frac{Ze^2}{8\pi\epsilon_0 \cdot r} - \frac{Ze^2}{4\pi\epsilon_0 \cdot r} \\ E &= -\frac{Ze^2}{8\pi\epsilon_0 r} \text{ or } K = -E = -(-3.4) \\ &= 3.4 \text{ eV} \end{aligned}$$

17. (c) Voltage gain $= \beta \times \text{impedance gain}$

$$\Rightarrow 50 = \beta \times \frac{200}{100}$$

$$\Rightarrow \beta = 25$$

Also power gain $= \beta^2 \times \text{impedance gain}$

$$= (25)^2 \times \frac{200}{100} = 1250$$

$$18. (d) T = \frac{2U_y}{g}$$

$$H = \frac{U_y^2}{2g}$$

and $R = U_x \times T$

When a horizontal acceleration is also given to the projectile U_y , T and H will remain unchanged while range will become

$$\begin{aligned} R' &= U_x \times T + \frac{1}{2} a T^2 \\ &= R + \frac{1}{2} \frac{g}{4} \left(\frac{4U_y^2}{g^2}\right) = R + H \quad \left[\because a = \frac{g}{4}\right] \end{aligned}$$

$$19. (b) \text{ We have, } \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\text{or } V_2 = \frac{P_1 V_1 T_2}{P_2 T_1}$$

$$= \frac{1 \times 500 \times (273 - 3)}{0.5 \times (273 + 27)} = \frac{1 \times 500 \times 270}{0.5 \times 300}$$

$$V_2 = 900 \text{ m}^3$$

20. (c) Intensity at centre of bright fringe,

$$I_0 = I + I + 2\sqrt{II} \cos 0$$

$$I_0 = 2I + 2I$$

$$= 4I$$

$$[\because \cos 0^\circ = 1]$$

Similarly Intensity at point $\frac{P}{4}$

(with phase difference $= \frac{2\pi}{4} = \frac{\pi}{2}$)

$$I' = I + I + 2\sqrt{II} \cos \frac{\pi}{2}$$

$$= 2I + 2\sqrt{II} \times 0$$

$$= 2I$$

$$\therefore \frac{I_0}{I'} = \frac{4I}{2I} = 2$$

21. (b) Let block is displaced through x m, then weight of displaced water or upthrust (upwards)

$$= -Axp g$$

Where, A is area of a cross-section of the block and ρ is density. This must be equal to force $(= ma)$ applied where m is mass of the block and a is acceleration.

$$\therefore ma = -Axp g$$

$$\text{or } a = -\frac{A\rho g}{m} x = -\omega^2 x$$

This is equation of SHM.

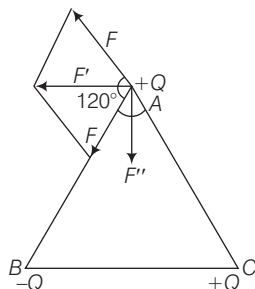
The time period of oscillation,

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{A\rho g}}$$

$$\Rightarrow T \propto \frac{1}{\sqrt{A}}$$

22. (c) Resultant force

$$F' = \sqrt{F^2 + F^2 + 2FF \cos 120^\circ} = F \left[\because \cos 120^\circ = -\frac{1}{2} \right]$$



Now, from figure

$$F = \sqrt{F'^2 + F'^2 + 2F'F' \cos 90^\circ}$$

Now, the force normal to BC at vertex A is

$$F'' = \sqrt{F^2 - F'^2} = 0 \quad (\because F' = F)$$

23. (b) From the conservation principle,

$$mgh = \frac{1}{2} kX_0^2 - mgX_0$$

where, X_0 is maximum elongation in spring.

$$\Rightarrow \frac{1}{2} kX_0^2 - mgX_0 - mgh = 0$$

$$\Rightarrow X_0^2 - \frac{2mg}{k} X_0 - \frac{2mg}{k} h = 0$$

$$X_0 = \frac{\frac{2mg}{k} \pm \sqrt{\left(\frac{2mg}{k}\right)^2 + 4 \times \frac{2mg}{k} h}}{2}$$

Amplitude = elongation in spring for lowest extreme position

– elongation in spring for equilibrium position

$$= X_0 - X_1 = \frac{mg}{k} \sqrt{1 + \frac{2hk}{mg}} \quad \left[\because X_1 = \frac{mg}{k} \right]$$

24. (d) Fraction remains after n half-lives

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{\frac{t}{T}}$$

$$\text{Given, } N = \frac{N_0}{e} \Rightarrow \frac{N_0}{eN_0} = \left(\frac{1}{2}\right)^{\frac{5}{T}}$$

$$\Rightarrow \frac{1}{e} = \left(\frac{1}{2}\right)^{\frac{5}{T}}$$

Taking log on both sides, we get

$$\log 1 - \log e = \frac{5}{T} \log \frac{1}{2}$$

$$\Rightarrow -1 = \frac{5}{T} (-\log 2)$$

$$\Rightarrow T = 5 \log_e 2$$

Now, let t' be the time after which activity reduces to half

$$\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^{t'/5 \log_e 2}$$

$$\Rightarrow t' = 5 \log_e 2$$

25. (c) Focal length of combination

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \quad \dots(i)$$

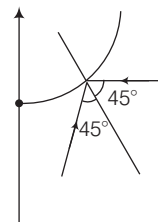
Using lens maker's formula

$$\frac{1}{f_2} = (\mu_2 - 1) \left(\frac{1}{-R} - \frac{1}{\infty} \right) = -\frac{(\mu_2 - 1)}{R}$$

$$\frac{1}{f_1} = (\mu_1 - 1) \left(\frac{1}{\infty} - \frac{1}{-R} \right) = \frac{\mu_1 - 1}{R}$$

$$\text{or } \frac{1}{f_1} = \frac{(\mu_1 - 1)}{R}$$

$$\frac{1}{f_2} = -\frac{(\mu_2 - 1)}{R}$$



Putting these values in Eq (i), we get

$$\Rightarrow \frac{1}{f} = \frac{(\mu_1 - 1)}{R} - \frac{(\mu_2 - 1)}{R}$$

$$\Rightarrow \frac{1}{f} = \frac{(\mu_1 - 1 - \mu_2 + 1)}{R}$$

$$\Rightarrow \frac{1}{f} = \frac{\mu_1 - \mu_2}{R}$$

$$f = \frac{R}{(\mu_1 - \mu_2)}$$

26. (c) Current will flow only through the branch containing resistance R_2

$$\therefore i = \frac{E}{R_2 + r} = \frac{5}{4 + 1} = 1 \text{ A}$$

Potential difference across R_2

$$= 1 \times 4 = 4 \text{ V}$$

If q be the charge on each plate of capacitor, then

$$\frac{q}{C} + \frac{q}{C} = 4$$

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or $\frac{2q}{c} = 4$

or $\frac{2q}{3 \times 10^{-6}} = 4$

$$q = 6 \mu\text{C}$$

27. (c) Friction force between A and B and between block B and surface S will oppose F

$$\begin{aligned} \therefore F &= F_{AB} + F_{BS} = \mu_{AB} M_A g + \mu_{BS} (m_A + m_B) g \\ &= 0.2 \times 100 \times 10 + 0.3(100 + 200) \times 10 \\ &= 200 + 900 = 1100 \text{ N} \end{aligned}$$

28. (a) The moment of inertia of the uniform rod about an axis through one end and perpendicular to its length is

$$= \frac{ml^2}{3}$$

where, m is the mass and l its length.

Torque ($\tau = I \cdot \alpha$) acting on centre of gravity of rod is given by

$$\tau = mg \frac{l}{2}$$

also, $\frac{ml^2}{3} \cdot \alpha = mg \frac{l}{2}$

$$\alpha = \frac{3g}{2l}$$

29. (a) As H-atom emits 6 spectral lines $\frac{n(n-1)}{2} = 6$

$$\therefore n = 4$$

$$\Delta E = E_4 - E_1 = 13.6 - 0.85$$

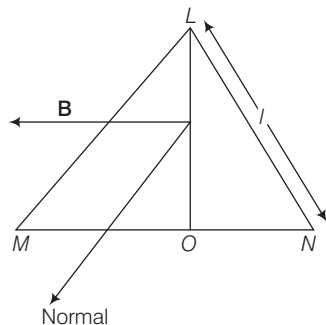
$$= 12.75 \text{ eV}$$

$$\lambda = \frac{1242 \text{ nm}}{12.75}$$

$$= 97.5 \text{ nm}$$

30. (a) Torque acting on equilateral triangle in a magnetic field B is

$$\tau = i AB \sin \theta$$



Area of triangle LMN ,

$$A = \frac{\sqrt{3}}{2} l^2 \text{ and } \theta = 90^\circ$$

Substituting the given values in expression for torque, we get

$$\tau = i \times \frac{\sqrt{3}}{2} l^2 B \sin 90^\circ$$

$$\tau = \frac{\sqrt{3}}{2} i l^2 B \quad (\because \sin 90^\circ = 1)$$

Here, $l = 2 \left(\frac{\tau}{\sqrt{3} Bi} \right)^{\frac{1}{2}}$

31. (d) Work done = Change in surface energy

$$r_2 = 5 \text{ cm}$$

$$r_1 = 3 \text{ cm}$$

$$\begin{aligned} \Rightarrow W &= 2T \times 4\pi(r_2^2 - r_1^2) \\ &= 2 \times 0.03 \times 4\pi [(5)^2 - (3)^2] \times 10^{-4} \\ &= 0.4\pi \text{ mJ} \end{aligned}$$

32. (c) From the figure, the X -component remains unchanged while the Y -component is reverse. Then, the velocity at point B is $(2\hat{i} - 3\hat{j}) \text{ m/s}$.

33. (b) $H = I^2 Rt$

$$\therefore \frac{H_1}{H_2} = \left(\frac{I_1}{I_2} \right)^2 \times \left(\frac{R_1}{R_2} \right)$$

$$\text{or } \frac{10}{H_2} = \left[\frac{1}{\left(\frac{1}{2} \right)} \right] \times \frac{5}{4}$$

$$H_2 = 2 \text{ cal}$$

34. (a) Given KE of α -particle = 2 eV

$$r = \frac{(Ze)(e)}{4\pi\epsilon_0 \cdot (\text{KE})}$$

$$= \frac{2Ze \times 9 \times 10^9}{2V}$$

$$\Rightarrow r = \frac{2 \times Z \times 16 \times 10^{-19} \times 9 \times 10^9}{2V}$$

$$= 14.4 \cdot \frac{Z}{V} \text{ \AA}$$

35. (c) According to question

$$n_1 T_1 = n_2 T_2$$

$$\Rightarrow n \cdot 2\pi \sqrt{\frac{5}{g}} = (n-1) 2\pi \sqrt{\frac{20}{g}}$$

$$\frac{n}{n-1} = 2 \Rightarrow n = 2$$

36. (c) The two capacitors formed are in parallel, hence capacitance of the combination

$$C = \frac{C_1 C_2}{C_1 + C_2} \quad \dots(i)$$

where, $C_1 = \frac{K_1 \epsilon_0 A}{\frac{d}{3}} \quad \left(\because C = \frac{\epsilon_0 A}{d} \right) \dots (ii)$

$$C_2 = \frac{K_2 \epsilon_0 A}{\frac{2d}{3}} \dots (iii)$$

$$C_{eq} = \frac{\frac{3K_1 \epsilon_0 A}{d} \times \frac{K_2 \epsilon_0 A \cdot 3}{2d}}{\frac{3K_1 \epsilon_0 A}{d} + \frac{3K_2 \epsilon_0 A}{2d}}$$

$$= \frac{9K_1 K_2 \epsilon_0^2 A^2}{2d^2} \times \frac{d}{\epsilon_0 A \times 18}$$

It is given

$\frac{2d/3}{K_1}$	$\frac{d/3}{K_2}$
--------------------	-------------------

$$\frac{\epsilon_0 A}{d} = 9 \text{ pF}$$

Using given values

$$C_{eq} = 40.5 \text{ pF}$$

37. (d) Acceleration

$$f = \frac{dv}{dt} = f_0 \left(1 + \frac{t}{T} \right)$$

or $dv = f_0 \left(1 + \frac{t}{T} \right) \cdot dt \dots (i)$

Integrating Eq. (i) on both sides, we get

$$v = f_0 t - \frac{f_0}{T} \cdot \frac{t^2}{2} + C \dots (ii)$$

After applying boundary conditions

$$v = 0 \text{ at } t = 0$$

We get, $C = 0$

$$\Rightarrow v = f_0 t - \frac{f_0}{T} \cdot \frac{t^2}{2} \dots (iii)$$

$$\text{As } f = f_0 \left(1 + \frac{t}{T} \right)$$

When $f_0 = 0$, $t = T$

Substituting $t = T$ in Eq. (iii), then velocity

$$v_x = f_0 T - \frac{f_0}{T} \cdot \frac{T^2}{2} = \frac{1}{2} f_0 T$$

38. (c) By Kepler's law of planetary motion,

$$T^2 \propto r^3, \text{ hence } T_1^2 \propto r_1^3$$

$$\text{Similarly } T_2^2 \propto r_2^3$$

$$\therefore T_1^2 \propto (36000)^3$$

$$T_2^2 \propto (6400 + h)^3$$

Therefore,

$$(T_2)^2 = T_1^2 \left(\frac{6400 + h}{36000} \right)^3$$

$$> T_1^2 \left[\frac{6400}{3600} \right]^3$$

$$> (24)^2 \left[\frac{8}{45} \right]^3$$

$$\therefore T_2 > \frac{24 \times 8 \times \sqrt{8}}{45 \times \sqrt{45}}$$

$$> 1.8 \text{ h}$$

$$\text{So, } T_2 \cong 2 \text{ h}$$

39. (b) Equation of wave,

$$y = 0.2 \sin [1.5x + 60t]$$

Comparing with standard equation, we get

$$y = A \sin [kx + \omega t]$$

$$k = 1.5 = \frac{2\pi}{\lambda}$$

and $\omega = 60 = \frac{2\pi}{T}$

$$\therefore \text{Velocity of wave, } v = \frac{\omega}{k}$$

$$= \frac{60}{1.5} = 40 \text{ m/s}$$

$$\text{Velocity of wave in stretched string, } v = \sqrt{\frac{T}{m}},$$

where, m is the linear density, T is tension in the string.

$$\text{So, } T = v^2 m = (40)^2 \times 3 \times 10^{-4}$$

$$= 0.48 \text{ N}$$

40. (b) Distance of straight path from O

$$= \frac{r}{\sqrt{2}}$$

$$\text{Hence, } B = B_1 + B_2$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2i}{\frac{r}{\sqrt{2}}} + \frac{\mu_0}{4\pi} \cdot \frac{2\pi i}{r} \left(\frac{1}{4} \right)$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2i}{r} \left[\sqrt{2} + \frac{\pi}{4} \right]$$

Chemistry

41. (b) $\because$ 4g of copper gave 5g of its oxide means,
1 g of oxygen combine with 4g of copper.

$\therefore$ Eq. wt of oxygen = 8,

Therefore,

8g of oxygen combine with = 4×8 g of copper = 32 g

Hence,

Equivalent weight of copper = 32

42. (b) Option (b) is the incorrect statement as reaction of NaBH_4 with cold water is very slow. It is not violent. All other are correct statements.

43. (b) $\because$ for s -subshell,

$$l = 0$$

Hence, orbital angular momentum of an electron is $2s$ orbital = zero.

$$\left[\because \text{orbital angular momentum} = \sqrt{l(l+1)} \cdot \frac{h}{2\pi} \right]$$

44. (b) I. Number of electrons in $\text{C}^+ \text{H}_3 = 6 + 3 - 1 = 8$

II. Number of electrons in $\text{NH}_2^- = 7 + 2 + 1 = 10$

III. Number of electrons in $\text{NH}_4^+ = 7 + 4 - 1 = 10$

IV. Number of electrons in $\text{NH}_3 = 7 + 3 = 10$

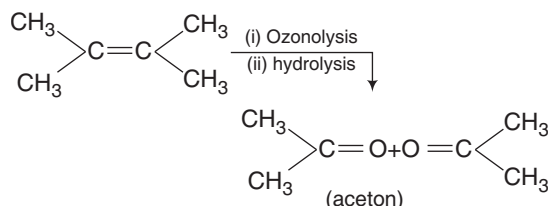
Since, species with same number of electrons are called isoelectronic species, Hence, IIInd, IIIrd and IVth are isoelectronic.

45. (a) $\because$ Difference of electronegativity between (A) and (B) = $3.0 - 1.2 = 1.8$ and, a bond having electronegativity difference greater than 1.65 is of ionic nature.

Hence, bond between (A) and (B) would be of ionic nature.

46. (c) The hydrogen, attached with sp -hybrid C-atom is most acidic because of highest electronegativity of sp hybrid carbon atom, H atom becomes relatively more acidic.

47. (a) Reductive ozonolysis of $(\text{CH}_3)_2 \text{C} = \text{CH}(\text{CH}_3)_2$ followed by hydrolysis gives only one type of ketones because of symmetry across the $\text{C} = \text{C}$ and bonding of two same alkyl groups (CH_3) with both the carbons bonded through double bond.



48. (d) Reaction (I), (II) and (IV) are of exothermic nature, thus do not involve absorption energy.

49. (b) More be the ability of N-atom of $-\text{NH}_2$ group to donate lone pair of electrons to HCl (dil.), more be the reactivity of amine.

In (b) two alkyl groups are attached to the $-\text{NH}_2$ (having electron releasing nature) which increases the electron donating ability of N-atom of $-\text{NH}_2$.

Hence is most reactive.

Note



50. (d) Block of magnesium prevents the action of water and salt of iron (of ship) by sacrificing itself.

This is method of protection of ship (i.e. of iron), known as sacrificial protection,

51. (d) $\because$ Only one mole of total gases are evolved and H_2O decomposes as



i.e. total moles = $1 + \frac{1}{2} = \frac{3}{2}$ moles by one mole of H_2O

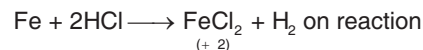
Now,

$\therefore \frac{3}{2}$ moles of gases are evolved by = 1 mole of H_2O

$\therefore$ 1 mole of gases are evolved by = $\frac{1}{3/2} = \frac{2}{3}$

Hence, 1 mole of total gases are evolved by $\frac{2}{3}$ mole of H_2O .

52. (b) $\because$ Reaction between Fe and HCl is as follows



n (moles of Fe) = n (moles of HCl)

$n(\text{Fe}) = \text{Normality}(\text{HCl}) \times \text{Volume of HCl}$

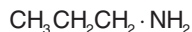
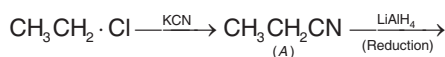
$$n(\text{Fe}) = \frac{0.4 \times 500}{1000} = 0.2 \text{ moles}$$

As each Fe consumers 2 electrons to change to Fe^{2+} ion.

$\therefore$ Number of Fe^{2+} ions produced = $\frac{0.2}{2} = 0.1$ mole

53. (c) For dissolution hydration energy must be more higher (–ve) value than of lattice energy. Thus, (c) is the correct answer.

54. (d) In the given reaction the end product is (d). It is obtained as follows.



55. (d) Conversion of carbonate are into its oxide (by heating in absence of air) is known as calcination.

∴ (d) is the correct answer i.e.

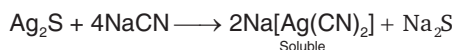


56. (c) We know that, $\Delta H = E_f - E_b$

where, E_b & E_f = activation energy for backward and forward respectively.

Thus, $E_f = \Delta H + E_b$, means $E_f > \Delta H$.

57. (a) Ag is leached by cyanide process by making the complex using NaCN/KCN as follows



58. (a) Among the given option (a) is a diamagnetic complex.

(a) $[\text{Co}(\text{NH}_3)_6]^{3+} \Rightarrow \text{Co} (27) \Rightarrow \text{Co}^{3+} = 3d^6 4s^0$. Thus, has no unpaired electrons and is a diamagnetic complex.

(b) $[\text{NiCl}_4]^{2-} \Rightarrow \text{Ni} (28) \Rightarrow \text{Ni}^{2+} = 3d^8 4s^0$

Thus, has two unpaired electrons is paramagnetic.

(c) $[\text{CuCl}_4]^{2-} \Rightarrow \text{Cu} (29) \Rightarrow \text{Cu}^{2+} = 3d^9 4s^0$

Thus, has one unpaired electrons and is paramagnetic.

(d) $[\text{Fe}(\text{H}_2\text{O})_6]^{3+} \Rightarrow \text{Fe} (26) \Rightarrow \text{Fe}^{3+} = 3d^5 4s^0$

Thus, has five unpaired electrons and is paramagnetic.

59. (b) Neoprene is a synthetic rubber. It is a polymer of chloroprene (2-chlorobuta,1, 3 diene).

60. (a) Ionic solids have highest melting point due to strong electrostatic force of attraction.

61. (d) Night blindness is developed due to deficiency of vitamin-A.

62. (a) The transfer RNA anticodon for messenger RNA codon G – C – A is

C – G – U, because in anticodon of RNA,

G replaced by C

C replaced by G

A replaced by U

63. (a) % for CO_2

∴ 0.765 g of acid gives $\text{CO}_2 = 0.535 \text{ g}$

∴ 100 g of acid gives $\text{CO}_2 = \frac{0.535 \times 100}{0.765}$

% of $\text{CO}_2 = 70.00$

% for carbon

∴ 44 g of CO_2 gives, C = 12 g

∴ 70 g of CO_2 gives, C = $\frac{12 \times 70}{44}$

= 19.09 g

∴ % of carbon = 19.00

Similarly,

(i) for hydrogen

% of H_2O

$$\Rightarrow \frac{0.138 \times 100}{0.765}$$

= 18.03 = 18.00

% of Hydrogen

$$\Rightarrow \frac{2 \times 18}{18} = 2.00$$

Hence ratio of carbon and hydrogen is 19 : 2

64. (d) Least basic has maximum value of pK_b .

Since, in (d), lone pair of electrons over N-atom are delocalised to two benzene rings, thus is least basic.

65. (c) ∴ for (l) It can be upto $(n - 1)$ and

Thus, $m = \pm l$, i.e. + l, 0, – l

for (a) $n = 2$, $l = 0$, $m = 0$ (correct),

for (b) $n = 1$, $l = 0$, $m = 0$ (correct),

(c) $n = 3$, $l = 3$, (not correct),

(d) $n = 2$, $l = 1$, $m = 1$ (correct),

66. (d) More higher be the (+) ve oxidation number of N-atom in its oxide more acidic be the nature of oxide. Thus N_2O_5 is most acidic oxide.

67. (c) ∴ Bond order (BO) = $\frac{N_b - N_a}{2}$

Thus,

$$\text{for } \text{O}_2, \text{BO} = \frac{10 - 6}{2} = 2$$

$$\text{for } \text{O}_2^-, \text{BO} = \frac{10 - 7}{2} + 1.5$$

$$\text{for } \text{O}_2^+, \text{BO} = \frac{10 - 5}{2} = 2.5$$

$$\text{for } \text{O}_2^{2-}, \text{BO} = \frac{10 - 8}{2} = 1.0$$

Hence maximum value for BO is for O_2^+ .

68. (c) Option (c) is the correct option as in (i), (ii) and (iv) the state changes towards more random state, thus entropy increases.

69. (a) BF_3 is a Lewis acid, as it accepts lone pair of electrons.

It is not an Arrhenius acid because it does not furnish H^+ ion in its aqueous solution.

It is also not a Bronsted Lowry acid as it does not give H^+ ions in any solvent.

70. (a) $\therefore \Delta H = \Delta E + \Delta n g R T$
and Δn (for the given reaction) = $2 - 1 = +1$
 $\therefore \Delta H > \Delta E$
71. (d) First member of 1st group i.e. lithium, mostly form covalent compounds due to its small size and comparatively high I.E. (with respect to other members of its group).
72. (b) When aqueous solution of borax ($\text{Na}_2\text{B}_4\text{O}_7$) is acidified with HCl, we get H_3BO_3 and NaCl as follows
- $$\text{Na}_2\text{B}_4\text{O}_7 + 2\text{HCl} + 5\text{H}_2\text{O} \longrightarrow 2\text{NaCl} + 4\text{H}_3\text{BO}_3$$
- BoroxBoric acid
73. (d) An aromatic compound can follow Huckel's rule if it has $(4n + 2)\pi$ -electrons
where, n = integer 0, 1, 2, 3, ... etc.
Thus,
(a) $n = 0$; $4 \times 0 + 2 = 2\pi$
 $\therefore$ follow Huckel's rule
(b) $n = 1$; Number of π -electrons = 6
 $\therefore$ follows Huckels rules.
(c) $n = 1$; Number of π -electrons = 6
 $\therefore$ follows Huckel's rule.
(d) $n = 1$, Number of π -electrons = 8
 $(4n + 2 = 4 \times 1 + 2 = 6)$
but this compound has 8 electrons which distorts the plane and hence does not follows Huckel's rule.
74. (b) According to Arrhenius equation's
$$\log K = \log A - \frac{E_a}{2.303 R T}$$

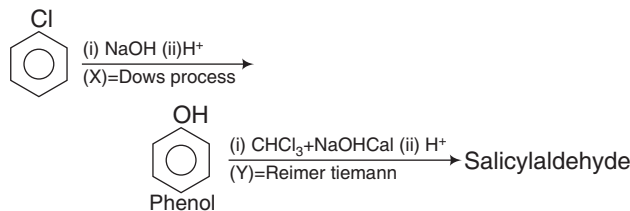
on comparing it with the equation of st. line $y = mx + c$ plot (b) is the correct answer.
75. (d) $\therefore \text{CN}^-$ is a strong ligand and from six Coordinats bonds with Fe-elements. Thus Fe show d^2sp^3 hybriisation in $\text{K}_3\text{Fe}(\text{CN})_6$.
76. (d) More be the number of unpaired electrons $\rightarrow$ more be the magnetic moment.
and
Number of unpaired electrons in $\text{Mg}^{2+} = 0$
Number of unpaired electrons in $\text{Ti}^{3+} = 1$

Number of unpaired electrons in $\text{Fe}^{2+} = 4$

Thus, Fe^{2+} show maximum value of magnetic moment (μ).
also, $\mu = \sqrt{n(n+2)}$. BM (n = Number of unpaired electrons).

- 77.** (a) $\therefore$ Elements with atomic Number 90 to 103 belongs to same group, i.e. III-B 3rd group (i.e. actinoids)

- 78. (c)**



- 79.** (d) $\therefore n = \frac{10}{m}$ (actual mass) (molar mass)

for

$$(a) \ n = \frac{w}{m} = \frac{8}{16} = \frac{1}{2} \text{ mole}$$

$$(b) \ n = \frac{16}{32} = \frac{1}{2} \text{ mole}$$

$$(c) \ n = \frac{14}{28} = \frac{1}{2} \text{ mole}$$

- (d) Thus, all have same number of moles.

- 80. (d)** $\because$ For ideal gas

$$pV = nRT$$

or $T = \frac{pV}{nR}$

$$\therefore n = 1 = \text{constant}$$

$$R = 0.083 = \text{constant}$$

$$\therefore T \propto pV$$

Thus,

The gas that shows highest product of $p \times V$, has highest temperature.

for (A) $p \times V = 1 \times 5 = 5$

(B) $p \times V = 2 \times 2.5 = 5$

(C) $p \times V = 2 \times 1.25 = 2.5$

(D) $p \times V = 2.5 \times 2.5 = 6.25$

Hence gas (D) show highest temperature.

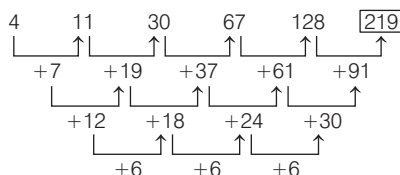
English Proficiency

81. (b) 'Are' should be used in place of 'is'.
82. (b) 'Was' should be used in place of were.
83. (b) Luggage should be used in place of luggages.
84. (c) *Gloss over something* is used.
85. (b) Preposition *up* is used with muster.
86. (b) **Illusion** means a false idea or belief; a deceptive appearance or impression. Its antonym will be **reality**
87. (b) **Disparage** means to criticise someone in a way that shows that you do not respect or value him. Its antonym will be **praise**
88. (b) **Slackened** means reduce or decrease in speed or intensity. Its antonym will be **quickened**.

Logical Reasoning

96. (c) 'Leather' is a raw material used to make 'Shoes'. Same as, 'Rubber' is made using 'Latex' which is a raw material.
97. (d) Here, $81 \times 3 = 243$
 $25 \times 3 = 75$
 $64 \times 3 = 192$
 but $16 \times 4 = 64$
 We can see that in all options second number is three times the first number except 16 : 64.
 Hence, 16 : 64 is odd one.

98. (a) The sequence to given series is as follows



Hence, 219 will come in place of question mark '?'.

99. (d) From given information, we have
 Lakshmi > Meenu ... (i)
 Lakshmi > Leela > Meenu ... (ii)

89. (b) **Deteriorating** means becoming progressively worse worsening.
90. (b) **Stalemate** means a situation in which neither group involved in an argument or an act can win or get an advantage and no action can be taken.
91. (b) The answer is clear from the second paragraph of the passage.
92. (d) The answer is clear from the sentence Designed to demonstrate the damage alcohol can do
93. (d) The answer is clear from the sentence who came to him had not realised how much they were drinking
94. (a) The answer is clear from the sentence 'Almost everyone poured out not a single measure, but a double at least
95. (c) The answer is clear from the first sentence of the last paragraph.

Meenu > Hari > Latha ... (iii)

After arranging the equations,

Lakshmi > Leela > Meenu > Hari > Latha

∴ Latha is the youngest.

100. (a) Here, all the three equal parts have same design. So, we can obtain the answer figure for the missing portion by rotating question figure by 90° clockwise.
 Hence, option(a) is the right answer.
101. (a) Out of five given figures, we can take figure (A), (C) and (D) to make a square.
102. (b) After folding, paper step, by step and then cut as shown in the question, it will show as an option (b).
103. (c) Moving row wise the number of horizontal lines increases by one in each step continuing the process, we will get figure of option (c) which will replace the question mark '?'.
104. (c) Required figure given in option (c).
105. (a) There are 28 triangles in the given figure.

Mathematics

106. (c) $(1+x)^{21} + (1+x)^{22} + \dots + (1+x)^{30}$
 $= (1+x)^{21} [1 + (1+x)^1 + \dots + (1+x)^9]$
 $= (1+x)^{21} \left[\frac{(1+x)^{10} - 1}{(1+x) - 1} \right]$
 $= \frac{1}{x} [(1+x)^{31} - (1+x)^{21}]$

∴ Coefficient of x^5 in the given expression

$$= \text{Coefficient of } x^5 \text{ in } \frac{1}{x} [(1+x)^{31} - (1+x)^{21}]$$

$$= \text{Coefficient of } x^6 \text{ in } [(1+x)^{31} - (1+x)^{21}]$$

$$= {}^{31}C_6 - {}^{21}C_6$$

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107. (b) We have,

$$\begin{aligned}\arg(z-1) &= \arg(z+3i) \\ \Rightarrow \arg((a-1)+ib) &= \arg(a+(b+3)i) \\ \Rightarrow \tan^{-1}\left(\frac{b}{a-1}\right) &= \tan^{-1}\left(\frac{b+3}{a}\right) \\ \Rightarrow \frac{b}{a-1} &= \frac{b+3}{a} \\ \Rightarrow ab &= (a-1)(b+3) \\ \Rightarrow 3(a-1) &= b \\ \Rightarrow (a-1):b &= 1:3\end{aligned}$$

108. (a) Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\text{and let } \frac{x \cos \theta}{a} + \frac{y \sin \theta}{b} = 1 \quad \dots(i)$$

be a tangent to it at point $(a \cos \theta, b \sin \theta)$. Then, p = length of the perpendicular from $S(ae, 0)$ on Eq. (i)

$$\Rightarrow p = \left| \frac{e \cos \theta - 1}{\sqrt{\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}}} \right|$$

p' = length of the perpendicular from $O(0, 0)$ on Eq. (i)

$$\Rightarrow p' = \left| \frac{1}{\sqrt{\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}}} \right|$$

$$\text{and } r = ae \cos \theta - a$$

$$\text{Clearly, } rp' = ap$$

109. (a) We have,

$$\begin{aligned}\sum_{r=1}^{10} r \cdot \frac{{}^nC_r}{{}^nC_{r-1}} &= \sum_{r=1}^{10} (n-r+1) \\ \Rightarrow \sum_{r=1}^{10} r \cdot \frac{{}^nC_r}{{}^nC_{r-1}} &= \sum_{r=1}^{10} \{(n+1)-r\} \\ \Rightarrow \sum_{r=1}^{10} r \cdot \frac{{}^nC_r}{{}^nC_{r-1}} &= 10(n+1) - \sum_{r=1}^{10} r \\ \Rightarrow \sum_{r=1}^{10} r \cdot \frac{{}^nC_r}{{}^nC_{r-1}} &= 10(n+1) - 55 \\ &= 10n - 45 = 5(2n-9)\end{aligned}$$

110. (d) Since, $3^{2 \sin 2\alpha - 1}$, 14 and $3^{4 - 2 \sin 2\alpha}$ are in AP. Therefore,

$$\begin{aligned}2 \times 14 &= 3^{2 \sin 2\alpha - 1} + 3^{4 - 2 \sin 2\alpha} \\ \Rightarrow 28 &= \frac{a}{3} + \frac{3^4}{a}, \text{ where } a = 3^{2 \sin 2\alpha} \\ \Rightarrow a^2 - 84a + 243 &= 0 \\ \Rightarrow (a-81)(a-3) &= 0 \\ \Rightarrow a &= 81, a = 3 \\ \Rightarrow 3^{2 \sin 2\alpha} &= 3^4 \text{ or } 3^{2 \sin 2\alpha} = 3 \\ \Rightarrow 2 \sin 2\alpha &= 1 \quad [\because 2 \sin 2\alpha \neq 4]\end{aligned}$$

$$\Rightarrow \sin 2\alpha = \frac{1}{2} \Rightarrow 2\alpha = 30^\circ \quad [\because \sin 30^\circ = 1/2]$$

Thus, the first three terms of the AP are 1, 14, 27.

$$\begin{aligned}\text{Hence, its fifth term } a_5 &= a_1 + (5-1)d \\ &= 1 + 4 \times 13 = 1 + 52 = 53\end{aligned}$$

111. (c) Let α, α^2 be the roots of $3x^2 + px + 3$

$\therefore$ Product of the roots,

$$\alpha \cdot \alpha^2 = 1 \Rightarrow \alpha^3 = 1$$

$$\Rightarrow \alpha = 1, \omega, \omega^2$$

$$\text{Again, } \alpha + \alpha^2 = -\frac{p}{3}$$

$$\Rightarrow 1 + 1 = -\frac{p}{3} \quad (\text{if } \alpha = 1)$$

$$\Rightarrow p = -6$$

But $p > 0$

$\therefore \alpha = 1$ is not possible.

$$\text{If } \alpha = \omega, \text{ then } \alpha + \alpha^2 = \omega + \omega^2 = -1$$

$$\therefore -1 = -\frac{p}{3} \Rightarrow p = 3$$

Again, if $\alpha = \omega^2$, then

$$\alpha + \alpha^2 = \omega^2 + \omega^4 = \omega^2 + \omega = -1$$

$$\therefore -1 = -\frac{p}{3}$$

$$\Rightarrow p = 3$$

112. (d) We have,

$$\begin{aligned}\log_{140} 63 &= \log_{2^2 \times 5 \times 7} (3 \times 3 \times 7) \\ &= \frac{\log_2 (3 \times 3 \times 7)}{\log_2 (2^2 \times 5 \times 7)} = \frac{\log_2 3 + \log_2 3 + \log_2 7}{2 \log_2 2 + \log_2 5 + \log_2 7} \\ &= \frac{2a + \frac{1}{c}}{2 + b + \frac{1}{c}} = \frac{2ac + 1}{2c + bc + 1}\end{aligned}$$

113. (a) Given that, $\cos(x-y)$, $\cos x$, $\cos(x+y)$ are in HP.

$$\text{Then, } \cos x = \frac{2 \cos(x-y) \cos(x+y)}{\cos(x-y) + \cos(x+y)}$$

$$\Rightarrow \cos x = \frac{2(\cos^2 x - \sin^2 y)}{2 \cos x \cos y}$$

$$\Rightarrow \cos^2 x \cos y = \cos^2 x - \sin^2 y$$

$$\Rightarrow \cos^2 x (\cos y - 1) = -\sin^2 y$$

$$\Rightarrow \cos^2 x (1 - \cos y) = 1 - \cos^2 y$$

$$\Rightarrow \cos^2 x (1 - \cos y) = (1 - \cos y)(1 + \cos y)$$

$$\Rightarrow \cos^2 x = 1 + \cos y \Rightarrow \cos^2 x = 2 \cos^2 y / 2$$

$$\Rightarrow \cos^2 x \sec^2(y/2) = 2$$

$$\Rightarrow \cos x \sec(y/2) = \pm \sqrt{2}$$

114. (c) We first write the elements of the set R .

$$\text{i.e. } R = \{(1, 4), (4, 1), (2, 3), (3, 2)\}$$

Since, $(1, 1) \notin R \Rightarrow R$ is not reflexive.

Now as, $(1, 4) \in R \Rightarrow (4, 1) \in R$ and $(2, 3) \in R$

$\Rightarrow (3, 2) \in R$

So, R is symmetric

Now, $(1, 4) \in R$ and $(4, 1) \in R$

$\nRightarrow (1, 1) \in R$

Thus, R is not transitive.

Hence, R is symmetric but neither reflexive nor transitive.

- 115.** (c) Any number between 1 to 999 is of the form abc when $0 \leq a, b, c \leq 9$. Let us first count the number in which 5 occurs exactly once.

Since, 5 can occur at one place in $1 \times {}^3C_1 \times 9 \times 9 = 243$ ways, next 5 can occur in exactly two places in ${}^3C_2 \times 9 = 27$. Lastly, 5 can occur in all three digits in only one way.

Hence, the number of times 5 occurs

$$= 1 \times 243 + 27 \times 2 + 1 \times 3 \\ = 243 + 54 + 3 = 300$$

- 116.** (a) Given, that, $A \cup X = B \cup X$

$$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$$

$$\Rightarrow (A \cap A) \cup (A \cap X) = (A \cap B) \cup (A \cap X)$$

[Using distributive law]

$$\Rightarrow A \cup \phi = (A \cap B) \cup \phi [\because A \cap X = \phi]$$

$$\Rightarrow A = A \cap B \quad \dots(i)$$

Again, consider $A \cup X = B \cup X$

$$\Rightarrow B \cap (A \cup X) = B \cap (B \cup X)$$

$$\Rightarrow (B \cap A) \cup (B \cap X) = (B \cap B) \cup (B \cap X)$$

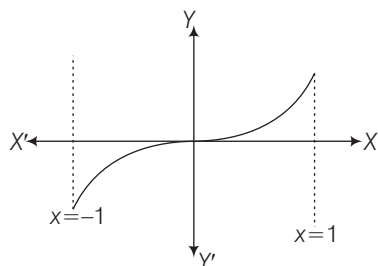
$$\Rightarrow (B \cap A) \cup \phi = B \cup \phi [B \cap X = \phi]$$

$$\Rightarrow A \cap B = B \quad \dots(ii)$$

Thus from Eq. (i) and (ii), we get

$$A = B$$

- 117.** (d) $f(x) = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$



Since, $-1 \leq x \leq 1$, therefore $-1 \leq f(x) \leq 1$

$\therefore$ Function is one-one and onto.

- 118.** (b) We have,

$$\begin{aligned} \sin x - 3 \sin 2x + \sin 3x &= \cos x - 3 \cos 2x + \cos 3x \\ \Rightarrow \sin x + \sin 3x - 3 \sin 2x &= \cos x + \cos 3x - 3 \cos 2x \\ \Rightarrow 2 \sin 2x \cos x - 3 \sin 2x - 2 \cos 2x \cos x + 3 \cos 2x &= 0 \\ \Rightarrow \sin 2x (2 \cos x - 3) - \cos 2x (2 \cos x - 3) &= 0 \end{aligned}$$

$$\Rightarrow (\sin 2x - \cos 2x) (2 \cos x - 3) = 0$$

$$\Rightarrow \sin 2x = \cos 2x \quad [\because \cos x \neq 3/2]$$

$$\Rightarrow 2x = 2x\pi \pm \left(\frac{\pi}{2} - 2x\right)$$

$$\Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{8} \quad [\because \text{neglect -ve sign}]$$

- 119.** (b) The third side is parallel to a bisector of the angle between equal sides.

The bisectors are $7x - y + 3 = \pm 5(x + y - 3)$

$$\Rightarrow 2x - 6y + 18 = 0$$

$$\text{or } 12x + 4y - 12 = 0$$

$$\Rightarrow x - 3y + 9 = 0$$

$$\text{or } 3x + y - 3 = 0$$

Let the third side be $x - 3y = k$ or $3x + y = L$

It passes through $(1, -10)$.

$$k = 31, L = -7$$

Hence, required lines are $x - 3y = 31, 3x + y = -7$

- 120.** (d) Let (t, m) be the other end of the chord drawn from the point (p, q) on the circle

$$x^2 + y^2 = px + qy$$

Their mid-point is $\left(\frac{t+p}{2}, \frac{m+q}{2}\right)$

Since, mid-point lies on X-axis i.e. $y = 0$

$$m + q = 0 \quad \dots(i)$$

Also, (t, m) lies on the circle.

$$\therefore t^2 + m^2 - pt - qm = 0 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get $t^2 - pt + 2q^2 = 0$

Which is quadratic in t such that, Discriminant > 0

$$\Rightarrow p^2 - 8q^2 > 0 \Rightarrow p^2 > 8q^2$$

- 121.** (a) Let P be the foot of the perpendicular drawn from $A(2, 3, 4)$ to the given line l .

$$\text{Then, } \frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3}$$

Now, any point on the line l is given by

$$x = 4 - 2\lambda, y = 6\lambda, z = 1 - 3\lambda$$

The coordinates of P are $(4 - 2\lambda, 6\lambda, 1 - 3\lambda)$

The direction ratios of AP are

$$(4 - 2\lambda - 2, 6\lambda - 3, 1 - 3\lambda - 4)$$

$$\text{i.e. } (2 - 2\lambda, 6\lambda - 3, -3 - 3\lambda)$$

And the direction ratios of l are $-2, 6$ and -3 .

Given, $AP \perp l$

$$\therefore -2(2 - 2\lambda) + 6(6\lambda - 3) - 3(-3 - 3\lambda) = 0$$

$$\Rightarrow \lambda = \frac{13}{49}$$

$$\therefore AP^2 = (4 - 2\lambda - 2)^2 + (6\lambda - 3)^2 + (1 - 3\lambda - 4)^2 \\ = 22 - 26\lambda + 49\lambda^2$$

Put $\lambda = \frac{13}{49}$, we get

$$AP^2 = \frac{909}{49} \Rightarrow AP = \frac{3}{7}\sqrt{101}$$

122. (c) Let $P(1, 6, 3)$ be the given point and L be the foot of perpendicular from P to the given line. The coordinates of a general point on the given line are

$$\frac{x-0}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda \text{ (say)}$$

i.e. $x = \lambda$

$$y = 2\lambda + 1, \quad z = 3\lambda + 2$$

If the coordinates of L are $(\lambda, 2\lambda + 1, 3\lambda + 2)$, then the direction ratios of PL are $(\lambda - 1, 2\lambda - 5, 3\lambda - 1)$.

Since, the direction ratios of given line which is perpendicular to PL , are 1, 2 and 3. Therefore, $(\lambda - 1)1 + (2\lambda - 5)2 + (3\lambda - 1)3 = 0$, which gives $\lambda = 1$.

Hence, coordinates of L are $(1, 3, 5)$.

Let $Q(x_1, y_1, z_1)$ be the image of $P(1, 6, 3)$ in the given line. Then, L is the mid-point of PQ .

$$\text{Therefore, } \frac{x_1 + 1}{2} = 1, \frac{y_1 + 6}{2} = 3, \frac{z_1 + 3}{2} = 5$$

$$\Rightarrow x_1 = 1, y_1 = 0, z_1 = 7$$

Hence, the image of $(1, 6, 3)$ in the given line is $(1, 0, 7)$.

123. (a) Given equation of plane is $x + y + z = 5$.

The distance measured along the line $x = y = z$.

Direction ratios of the given line is $(1, 1, 1)$.

So, the equation of line PQ is

$$\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda$$

$$\Rightarrow x = \lambda + 1, y = \lambda - 5, z = \lambda + 9$$

lies on the plane $x + y + z = 5$

$$\therefore (\lambda + 1) - (-\lambda + 5) + (\lambda + 9) = 5$$

$$\Rightarrow \lambda = -10$$

The coordinate of Q is $(-9, -15, -1)$ and the coordinate of P is $(1, -5, 9)$

$$PQ = \sqrt{(10)^2 + (10)^2 + (10)^2} = 10\sqrt{3}$$

$$\therefore 2\sqrt{3}k = 10\sqrt{3} \Rightarrow k = 5$$

124. (b) $\lim_{n \rightarrow \infty} \sin \left\{ n\pi \left(1 + \frac{1}{n^2} \right)^{1/2} \right\}$
- $$= \lim_{n \rightarrow \infty} \sin \left\{ n\pi \left(1 + \frac{1}{2n^2} - \frac{1}{8n^4} + \dots \right) \right\}$$
- $$= \lim_{n \rightarrow \infty} \sin \left\{ n\pi + \frac{\pi}{2n} - \frac{\pi}{8n^3} + \dots \right\}$$
- $$= \lim_{n \rightarrow \infty} (-1)^n \sin \pi \left(\frac{1}{2n} - \frac{1}{8n^3} + \dots \right) = 0$$

$$125. (b) f(x) = \begin{cases} e^x, & x \leq 0 \\ 1-x, & 0 < x \leq 1 \\ x-1, & x > 1 \end{cases}$$

$$Rf'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \left[\frac{h-1}{h} \right] = -1$$

$$Lf'(0) = \lim_{h \rightarrow 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h \rightarrow 0} \frac{e^{-h} - 1}{-h} = 1$$

So, it is not differentiable at $x = 0$.

Similarly, it is not differentiable at $x = 1$ but it is continuous at $x = 0$ and 1 .

$$126. (b) \because \lim_{x \rightarrow 0} f(x) = f(0)$$

Let a be any point

$$\text{Now, at } x = a, \lim_{x \rightarrow a} f(x) = \lim_{h \rightarrow 0} f(a+h)$$

$$= \lim_{h \rightarrow 0} f(a) + \lim_{h \rightarrow 0} f(h)$$

$$= f(a) + f(0) = f(a+0) = f(a)$$

$\therefore f(x)$ is continuous at $x = a$, where a is any arbitrary point.

Hence, $f(x)$ is continuous for all x .

127. (a) Since, $x, \sin^{-1} x, \tan^{-1} x$ are continuous functions, so the function f is clearly continuous at each point of its domain except possibly at $x = 0$.

So, for f to be continuous at $x = 0$

$$f(0) = \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{2 - \frac{\sin^{-1} x}{x}}{2 + \frac{\tan^{-1} x}{x}} = \frac{1}{3}$$

$$\left[\because \lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} = 1, \lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x} = 1 \right]$$

128. (b) The given function is

$$f(x) = \begin{cases} 0, & 0 \leq x < 1 \\ 1, & 1 \leq x < 2 \end{cases}$$

$$\text{Now, LHD} = \lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1^-} \frac{0 - 1}{x - 1}$$

Which does not exist

$\therefore f$ is not derivable at $x = 1$.

129. (b) Given, $f(x) = -2x^3 + 21x^2 - 60x + 41$... (i)

On differentiating Eq. (i) w.r.t. x , we get

$$f'(x) = -6x^2 + 42x - 60$$

$$= -6(x^2 - 7x + 10) = -6(x-2)(x-5)$$

If $x < 2$, $f'(x) < 0$, i.e., $f(x)$ is decreasing, then

$$f'(x) < 0 \text{ for } x < 2$$

130. (d) Rolle's theorem is not applicable for the function

$$f(x) = |x| \text{ in } [-1, 1]$$

$\therefore f'(0)$ does not exist.

131. (b) Clearly, the point of intersection of curves is (0, 1).

Now, slope of tangent of first curve

$$m_1 = \frac{dy}{dx} = a^x \log a$$

$$\Rightarrow \left(\frac{dy}{dx} \right)_{(0,1)} = m_1 = \log a$$

Slope of tangent of second curve,

$$m_2 = \frac{dy}{dx} = b^x \log b$$

$$\Rightarrow m_2 = \left(\frac{dy}{dx} \right)_{(0,1)} = \log b$$

$$\therefore \tan \alpha = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\log a - \log b}{1 + \log a \log b}$$

132. (d) We have, $\int_0^n [x] dx$

$$\begin{aligned} &= \int_0^1 0 dx + \int_1^2 1 dx + \int_2^3 2 dx + \dots + \int_{n-1}^n (n-1) dx \\ &= 1(2-1) + 2(3-2) + 3(4-3) + \dots + (n-1)\{n-(n-1)\} \\ &= 1 + 2 + 3 + \dots + (n-1) = \frac{n(n-1)}{2} \end{aligned}$$

$$\text{and } \int_0^n \{x\} dx = \int_0^n (x - [x]) dx = \frac{n}{2}$$

$$\therefore \frac{\int_0^n [x] dx}{\int_0^n \{x\} dx} = n - 1$$

133. (b) Given, $f(x) = x + \sin 2x$

$$\Rightarrow f'(x) = 1 + 2 \cos 2x$$

For maximum or minimum value, $f'(x) = 0$

$$\Rightarrow 1 + 2 \cos 2x = 0 \Rightarrow \cos 2x = -\frac{1}{2}$$

$$\Rightarrow 2x = 2n\pi \pm \frac{2\pi}{3}$$

$$\Rightarrow x = n\pi \pm \frac{\pi}{3}$$

$$\Rightarrow x = \frac{4\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}$$

$$\text{Find } f(0), f\left(\frac{2\pi}{3}\right), f\left(\frac{4\pi}{3}\right), f\left(\frac{5\pi}{3}\right), f(2\pi)$$

$$\Rightarrow f(0) = 0, f\left(\frac{2\pi}{3}\right) = \frac{2\pi}{3} - 0.8$$

$$\Rightarrow f\left(\frac{4\pi}{3}\right) = \frac{4\pi}{3} + 0.8, f\left(\frac{5\pi}{3}\right) = \frac{5\pi}{3} - 0.8$$

$$\Rightarrow f(2\pi) = 2\pi + 0 = 2\pi$$

$\therefore$ Maximum value of $f(x) = 2\pi$.

134. (b) Required area = $\int_0^{\pi/2} |\cos x - \sin x| dx$

$$= \int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x) dx$$

$$\begin{aligned} &= [\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2} \\ &= (\sqrt{2} - 1) - (1 - \sqrt{2}) = 2\sqrt{2} - 2 \end{aligned}$$

135. (b) Put $x + y = v$ and $1 + \frac{dy}{dx} = \frac{dv}{dx}$

Therefore, the differential equation reduces to

$$\frac{dv}{dx} = (1 + \cos v) + \sin v = 2 \cos^2 \frac{v}{2} + 2 \sin \frac{v}{2} \cos \frac{v}{2}$$

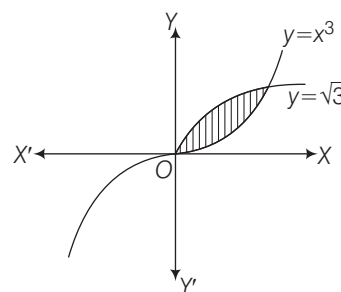
$$= 2 \cos^2 \frac{v}{2} \left(1 + \frac{\tan v}{2} \right)$$

$$\Rightarrow \int \frac{\sec^2 \frac{v}{2}}{2 \left[1 + \tan \left(\frac{v}{2} \right) \right]} dv = \int dx$$

$$\therefore \log \left[1 + \tan \left(\frac{x+y}{2} \right) \right] = x + C$$

136. (c) Since, the intersection of two curves

$y = x^3$ and $y = \sqrt{x}$ are $x = 0$ and $x = 1$.



$$\begin{aligned} \therefore A &= \left| \int_0^1 (x^3 - \sqrt{x}) dx \right| = \left| \left[\frac{x^4}{4} - \frac{2x^{3/2}}{3} \right]_0^1 \right| \\ &= \left| \left[\frac{1}{4} - \frac{2}{3} \right] \right| = \frac{5}{12} \text{ sq units} \end{aligned}$$

137. (c) $\lim_{n \rightarrow \infty} \frac{an(1+n) - (1+n^2)}{1+n}$

$$= \lim_{n \rightarrow \infty} \frac{(a-1)n^2 + an - 1}{n+1} = \infty,$$

If $a - 1 \neq 0$ limit does not exist and if $a - 1 = 0$, then

$$\lim_{n \rightarrow \infty} \frac{an - 1}{n+1} = a = b$$

$$\Rightarrow a = b = 1$$

138. (b) Total ways in which papers can be checked is equal to 7^4 . Now, two teachers who have to check all the papers can be selected in 7C_2 ways and papers can be checked by them is $(2^4 - 2)$ favourable ways.

Thus, required probability

$$= \frac{{}^7C_2 (2^4 - 2)}{7^4} = \frac{6}{49}$$

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139. (c) Given, equation of hyperbola is

$(10x - 5)^2 + (10y - 4)^2 = \lambda^2 (3x + 4y - 1)^2$ can be rewritten as

$$\frac{\sqrt{\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{2}{5}\right)^2}}{\left|\frac{3x + 4y - 1}{5}\right|} = \left|\frac{\lambda}{2}\right|$$

This is of the form of $\frac{PS}{PM} = e$

Where, P is any point on the hyperbola and S is a focus and M is the point of directrix.

Here, $\left|\frac{\lambda}{2}\right| > 1 \Rightarrow |\lambda| > 2 (\because e > 1)$

$$\Rightarrow \lambda < -2 \text{ or } \lambda > 2$$

140. (a) $v = \hat{a} \times \hat{b} = |\hat{a}| |\hat{b}| \sin \theta \hat{n} = \sin \theta \hat{n}$

Where, θ is the angle between a and b . $[\because |a| = 1 = |b|]$

$$\therefore |v| = \sin \theta$$

Now, $u = \hat{a} - (\hat{a} \cdot \hat{b}) \hat{b} = \hat{a} - (\cos \theta) \hat{b}$

$$\begin{aligned} \Rightarrow |u|^2 &= (\hat{a} - (\cos \theta) \hat{b}) \cdot (\hat{a} - (\cos \theta) \hat{b}) \\ &= a^2 + \cos^2 \theta b^2 - 2 \cos \theta ab \quad [|a|^2 = |b|^2 = |a||b| = 1] \\ &= 1 + \cos^2 \theta - 2 \cos^2 \theta \\ &= 1 - \cos^2 \theta = \sin^2 \theta \end{aligned}$$

$$\therefore |u| = \sin \theta$$

Thus $|v| = |u|$

141. (c) Variance = $\frac{1}{n} \sum (x - \bar{x})^2 \sigma^2$

$$\begin{aligned} \text{New variance} &= \frac{1}{n} \sum (\alpha x - \alpha \bar{x})^2 \\ &= \alpha^2 \frac{1}{n} \sum (x - \bar{x})^2 = \alpha^2 \sigma^2 \end{aligned}$$

142. (a) Given, coefficient of variation $C_1 = 50$

and coefficient of variation $C_2 = 60$.

We have, $\bar{x}_1 = 30$ and $\bar{x}_2 = 25$

$$\therefore C = \frac{\sigma}{\bar{x}} \times 100$$

$$50 = \frac{\sigma_1}{30} \times 100$$

$$\Rightarrow \sigma_1 = 15$$

$$\text{and } 60 = \frac{\sigma_2}{25} \times 100$$

$$\Rightarrow \sigma_2 = 15$$

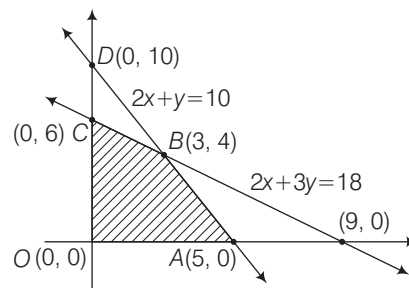
$$\therefore \text{Required difference } \sigma_1 - \sigma_2 = 15 - 15 = 0$$

143. (c) Given, constraints are $2x + 3y \leq 18$, $2x + y \leq 10$,

$x \geq 0$, $y \geq 0$.

The feasible region is $OABCO$.

Also, $z = 9x + 13y$



At $O(0, 0)$, $z = 0$

At $A(5, 0)$, $z = 45$

At $B(3, 4)$, $z = 27 + 52 = 79$

At $C(0, 6)$, $z = 78$

$\therefore$ Maximum value of z is 79.

144. (c) The man has to win atleast 4 times.

$\therefore$ Required probability

$$\begin{aligned} &= {}^7C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^3 + {}^7C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^2 + {}^7C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right) + {}^7C_7 \left(\frac{1}{2}\right)^7 \\ &= ({}^7C_4 + {}^7C_5 + {}^7C_6 + {}^7C_7) \times \frac{1}{2^7} = \frac{64}{2^7} = \frac{1}{2} \end{aligned}$$

145. (c) $\therefore \frac{-\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$, $\frac{-\pi}{2} \leq \sin^{-1} y \leq \frac{\pi}{2}$

$$\text{and } \frac{-\pi}{2} \leq \sin^{-1} z \leq \frac{\pi}{2}$$

Given that,

$$\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$$

Which is possible only when

$$\sin^{-1} x = \sin^{-1} y = \sin^{-1} z = \frac{\pi}{2}$$

$$\Rightarrow x = y = z = 1$$

Put $p = q = 1$

Then, $f(2) = f(1) \quad f(1) = 2 \times 2 = 4$

and put $p = 1, q = 2$

then, $f(3) = f(1) f(2)$

$$= 2 \cdot 2^2 = 8$$

$$\begin{aligned} \therefore x^{f(1)} + y^{f(2)} + z^{f(3)} &= \frac{x + y + z}{x^{f(1)} + y^{f(2)} + z^{f(3)}} \\ &= 1 + 1 + 1 - \frac{3}{1 + 1 + 1} = 3 - 1 = 2 \end{aligned}$$

146. (c) Here,

$$T_n = \frac{n(n+1)}{n!} = \frac{n-1+2}{(n-1)!} = \frac{1}{(n-2)!} + \frac{2}{(n-1)!}$$

$$\begin{aligned} \therefore S &= \sum_{n=1}^{\infty} T_n = \sum_{n=1}^{\infty} \frac{1}{(n-2)!} + 2 \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \\ &= e + 2e = 3e \end{aligned}$$

147. (b) Since, $|z_1| = |z_2| = |z_3|$

$\therefore$ 0 is the circumcentre of an equilateral $\triangle ABC$.

$$\therefore \frac{x_1 + x_2 + x_3}{3} = 0 = \frac{y_1 + y_2 + y_3}{3}$$

where, $z_1 = x_1 + iy_1$

$$\Rightarrow \frac{x_1 + x_2 + x_3}{3} + i \left(\frac{y_1 + y_2 + y_3}{3} \right) = 0$$

$$\Rightarrow (x_1 + iy_1) + (x_2 + iy_2) + (x_3 + iy_3) = 0$$

$$z_1 + z_2 + z_3 = 0$$

148. (b) Let $I = \int \frac{dx}{(\sqrt{x})^2 + (\sqrt{x})^7}$

$$= \int \frac{dx}{(\sqrt{x})^7 \left(\frac{1}{(\sqrt{x})^5} + 1 \right)}$$

Put $1 + \frac{1}{(\sqrt{x})^5} = t$

$$\Rightarrow -\frac{5}{2} \cdot \frac{1}{(\sqrt{x})^7} dx = dt$$

$$\therefore I = -\frac{2}{5} \int \frac{dt}{t} = -\frac{2}{5} \log t + C$$

$$= -\frac{2}{5} \log \left[1 + \frac{1}{(\sqrt{x})^5} \right] + C$$

$$= -\frac{2}{5} \log \left(\frac{x^{5/2} + 1}{x^{5/2}} \right) + C$$

$$\Rightarrow \lambda = \frac{-2}{5} \text{ and } a = \frac{5}{2}$$

$$\therefore a + \lambda = 2.1 > 2$$

149. (b) Equation of line joining the points (0, 3) and (5, -2) is

$$y = 3 - x.$$

If this line is tangent to $y = \frac{ax}{a(x+1)}$, then

$$(3-x)(x-11) = ax \text{ should have equal roots.}$$

$$\text{Thus, } (a-2)^2 + 12 = 0$$

$$\Rightarrow \text{No value of } a \Rightarrow a \in \phi$$

150. (b) Shortest distance between two curves occurred along the common normal.

$\therefore$ Normal to $y^2 = 4x$ at $(m^2, 2m)$ is

$$y + mx - 2m - m^3 = 0$$

Normal to $y^2 = 2(x-3)$ at $\left(\frac{m^2}{2} + 3, m\right)$ is

$$y + m(x-3) - m - \frac{m^3}{2} = 0$$

Both normals are same, if

$$-2m - m^3 = -4m - \frac{1}{2}m^3 \Rightarrow m = 0, 2$$

So, points will be (4, 4) and (5, 2) or (4, -4) and (5, -2).

Hence, shortest distance will be

$$\sqrt{(1+4)} = \sqrt{5}$$

Solved Paper 2018

BITSAT

Instructions

- There are 150 questions in all. The number of questions in each part is as given below.
- All questions are multiple choice questions with four options, only one is correct.
- Each correct answer fetches 3 marks while incorrect answer has a penalty of 1 mark.

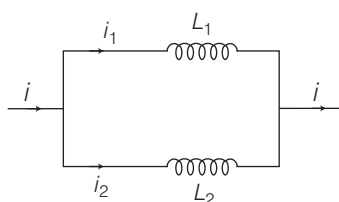
No. of Questions

Part I Physics	1-40
Part II Chemistry	41-80
Part III (a) English Proficiency	81-95
(b) Logical Reasoning	96-105
Part IV Mathematics	106-150

PART I

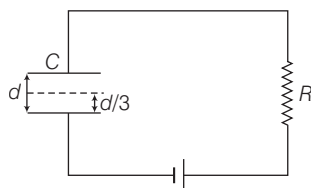
Physics

- Two inductors L_1 and L_2 are connected in parallel and a time varying current flows as shown in figure. The ratio of current $\frac{i_1}{i_2}$ at any time t is



- a. $\frac{L_2}{L_1}$ b. $\frac{L_1}{L_2}$ c. $\frac{L_2^2}{(L_1 + L_2)^2}$ d. $\frac{L_1^2}{(L_1 + L_2)^2}$

- A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant $K = 2$, the level of liquid is $\frac{d}{3}$, initially.



Suppose, the liquid level decreases at a constant speed v , the time constant as a function of time is

- a. $\frac{6\epsilon_0 R}{5d + 3vt}$ b. $\frac{(15d + 9vt)\epsilon_0 R}{2d^3 - 3dvt - gv^2t^2}$
 c. $\frac{6\epsilon_0 R}{5d - 3vt}$ d. $\frac{(15d - 9vt)\epsilon_0 R}{2d^3 + 3dvt - 9v^2t^2}$

- Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent to 0.2 kg at 15°C till the temperature of the calorimeter and its contents rises to 80°C . The mass of steam condensed (in kg) is

- a. 0.130 b. 0.065 c. 0.260 d. 0.135

- Dimension of which base quantity corresponds to that of $\sqrt{\frac{Gh}{c^3}}$ = ?

- a. Time b. Length
c. Mass d. Temperature

- A reservoir is at 827°C and Carnot's engine takes a thousand kilocalories of heat from it and exhausts it to a sink at 27°C . What is the amount of work and the efficiency of the engine?

- a. 2.7×10^5 cal, 70.70% b. 2.72×10^5 cal, 72.72%
c. 2.70×10^5 cal, 80.70% d. 3.70×10^5 cal, 70.70%

- A train moves towards stationary observer with speed 34 m/s. The train blows whistle and its frequency is registered by the observer as f_1 . If the train's speed is reduced to 17 m/s, the frequency registered is f_2 . If the speed of sound is 340 m/s, then the ratio $\frac{f_1}{f_2}$ is

- a. $\frac{19}{18}$ b. $\frac{18}{19}$ c. 2 d. 1/2

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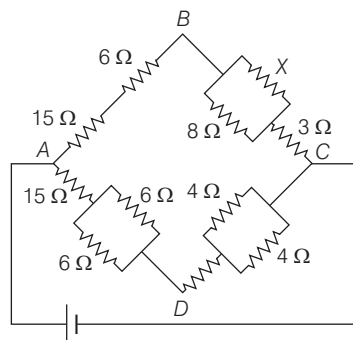
7. An object of mass 5 kg is projected with a velocity 20 ms^{-1} at an angle 60° , to the horizontal. At the highest point of its path, the projectile explodes and breaks up into two fragments of masses 1 kg and 4 kg. The fragments separate horizontally after the explosion, which releases internal energy such that the KE of the system at the highest point is doubled. The separation between the two fragments when they reach the ground is

a. 52.25 m b. 44.25 m
c. 65.32 m d. 78.76 m

8. An automobile moving with a speed of 36 km/h reaches an upward inclined road of angle 30° , its engine becomes switch off. If the coefficient of friction is 0.1, then how much distance will automobile move before coming to rest?

a. 12.53 m b. 21.42 m
c. 15.43 m d. 8.53 m

9. In the circuit given below, the value of resistance X , when the potential difference between the points B and D is zero, will be



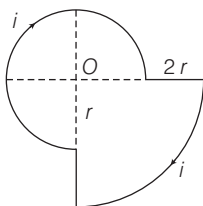
a. 9Ω b. 8Ω c. 4Ω d. 6Ω

10. A block of wood floats in water with $(4/5)$ th of its volume submerged. If the same block just floats in a liquid, the density of liquid in (kg m^{-3}) is

a. 1250 b. 600 c. 400 d. 800

11. As current i flowing through the loop as shown in figure. The magnetic field at the centre O is

a. $\frac{7\mu_0 i}{12r}$ acting downwards
b. $\frac{5\mu_0 i}{12r}$ acting upwards
c. $\frac{7\mu_0 i}{12r}$ acting upwards
d. $\frac{5\mu_0 i}{12r}$ acting downwards



12. The ratio of angular momentum L to the atomic dipole moment μ_l for hydrogen like atoms and ions is

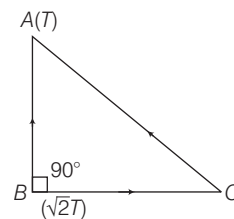
a. always constant and is equal to the ratio of mass to the charge of electron

b. always constant and is equal to twice the ratio of mass to the charge of electron to the principal quantum number n

c. proportional to the principal quantum number n

d. proportional to $\frac{1}{n^2}$

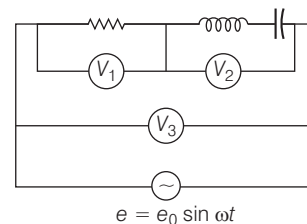
13. Three rods of identical cross-sectional area and made from the same metal, form the sides of an isosceles triangle ABC right angled at B as shown in figure. The point A and B are maintained at temperature T and $\sqrt{2}T$ respectively, in the



steady state. Now, assuming that only heat conduction takes place. The temperature of point C will be

a. $\frac{T}{\sqrt{2}+1}$ b. $\frac{T}{\sqrt{2}-1}$ c. $\frac{3T}{\sqrt{2}+1}$ d. $\frac{\sqrt{3}T}{(\sqrt{2}+1)}$

14. A resistor R , an inductor L , a capacitor C and voltmeters V_1, V_2 and V_3 are connected to an oscillator in the circuit as shown in the adjoining diagram. When the frequency of the



oscillation is increased, then at the resonant frequency, the voltmeter reading is zero in the case of

a. voltmeter V_1 only b. voltmeter V_2 only
c. voltmeter V_3 only d. All the three voltmeters

15. In Young's double slit experiment, intensity at a point is $\left(\frac{1}{4}\right)$ of the maximum intensity. Angular position of this point is

a. $\sin^{-1}\left(\frac{\lambda}{d}\right)$ b. $\sin^{-1}\left(\frac{\lambda}{2d}\right)$
c. $\sin^{-1}\left(\frac{\lambda}{3d}\right)$ d. $\sin^{-1}\left(\frac{\lambda}{4d}\right)$

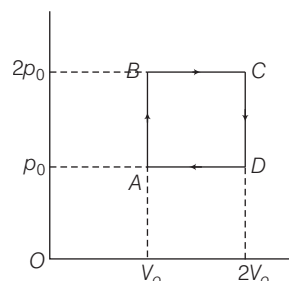
16. The bob of simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob get suddenly unplugged. During observation, till water is coming out, the time period of oscillation would

a. first increase and then decrease to the original value
b. first decrease and then increase to the original value
c. remain unchanged
d. increase towards a saturation value

17. At a certain temperature, the number density of charge carriers in a semiconductor is n . When an electric field is applied to it, the charge carriers drift with an average speed v . If the temperature of the semiconductor is raised.

- a. n will increase but v will decrease
 b. n will decrease but v will increase
 c. Both n and v will increase
 d. Both n and v will decrease
18. Consider the acceleration, velocity and displacement of a tennis ball as it falls to the ground and bounces back. Directions of which of these change in the process?
 a. Velocity only
 b. Displacement and velocity
 c. Acceleration, velocity and displacement
 d. Displacement and acceleration
19. A convex lens shown in the figure is made up of two types of transparent materials. A point sources of light is placed on its principal axis. If reflections from the boundaries between layers are ignored, the lens will form
 a. only one image
 b. two images
 c. infinite images
 d. no image at all
- 
20. If the time period is doubled, then the angular momentum of the body will (provided the moment of inertia of the body is constant)
 a. remain constant
 b. quadruple
 c. become half
 d. double
21. Breaking stress of a steel wire is p and the density of steel is ρ . The greatest length of steel wire that can hang vertically without breaking is
 a. $\frac{p}{\rho g}$
 b. $\frac{p}{2\rho g}$
 c. $\frac{2p}{\rho g}$
 d. None of these
22. A stone is projected with velocity $2\sqrt{gh}$, so that it just clears two walls of equal height h , at distance of $2h$ from each other. The time interval of passing between the two walls is
 a. $\sqrt{\frac{h}{g}}$
 b. $\sqrt{\frac{2h}{g}}$
 c. $2\sqrt{\frac{h}{g}}$
 d. $\frac{2h}{g}$
23. An object takes n times as much time to slide down a 45° rough inclined plane as it takes to slide down a perfectly smooth 45° inclined plane. The coefficient of kinetic friction between the rough plane and the object is
 a. $n^2 - 1$
 b. $1 - \frac{1}{n^2}$
 c. $n^2 + 1$
 d. $1 + \frac{1}{n^2}$
24. Some amount of a radioactive substance (half-life = 10 days) is spread inside a room and consequently the level of radiation becomes 50 times the permissible level for normal occupancy of the room. After how many days will the room be safe for occupation?
 a. 20 days
 b. 34.8 days
 c. 56.4 days
 d. 62.9 days

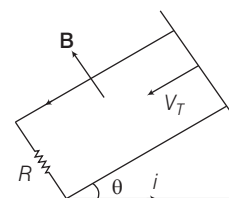
25. Helium gas goes through a cycle $ABCD$ (consisting of two isochoric and two isobaric lines) as shown in figure. The efficiency of this cycle is approximately



- a. 15.4% b. 9.1% c. 10.5% d. 12.5%

26. The frequency and the intensity of incident beam of light falling on the surface of a photoelectric material is increased by a factor of two. This will
 a. increase the maximum kinetic energy of the ejected photoelectrons by a factor of more than two and would increase the photoelectric current by a factor of two.
 b. increase the maximum kinetic energy of the photoelectrons and would increase the photoelectric current both by a factor of two.
 c. increase the maximum kinetic energy of the photoelectrons by a factor of two and will have no effect on the magnitude of the photoelectric current produced.
 d. not produce any effect on the kinetic energy of the emitted photoelectrons but will increase the photoelectric current by a factor of two.

27. A copper rod of mass m slides under gravity on two smooth parallel rails l distance apart and set an angle θ to the horizontal. At the bottom, the rails are joined by a resistance R in figure. There is a uniform magnetic field B perpendicular to the plane of the rails. The terminal velocity of rod is
 a. $\frac{mgR \tan \theta}{B^2 l^2}$
 b. $\frac{mgR \cot \theta}{B^2 l^2}$
 c. $\frac{mgR \sin \theta}{B^2 l^2}$
 d. $\frac{mgR \cos \theta}{B^2 l^2}$



28. An asteroid of mass m is approaching earth, initially at a distance $10 R_e$ with speed v_i . It hits earth with a speed v_f (R_e and M_e are radius and mass of earth), then
 a. $v_f^2 = v_i^2 + \frac{2Gm}{R_e} \left(1 + \frac{1}{10}\right)$
 b. $v_f^2 = v_i^2 + \frac{2GM_e}{R_e} \left(1 + \frac{1}{10}\right)$
 c. $v_f^2 = v_i^2 + \frac{2GM_e}{R_e} \left(1 - \frac{1}{10}\right)$
 d. $v_f^2 = v_i^2 + \frac{2Gm}{R_e} \left(1 - \frac{1}{10}\right)$

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29. When photon of energy 4.0 eV strikes the surface of a metal A , the ejected photoelectrons have maximum kinetic energy T_A eV and de-Broglie wavelength λ_A . The maximum kinetic energy of photoelectrons liberated from another metal B by photon of energy 4.50 eV is $T_B = (T_A - 1.50)$ eV. If the de-Broglie wavelength of these photoelectrons $\lambda_B = 2\lambda_A$, then choose the correct statement(s).

- The work function of A is 1.50 eV
- The work function of B is 4.0 eV
- $T_A = 3.2$ eV
- All of the above

30. A pulley of radius 2 m is rotated about its axis by a force $= (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2 , the number of rotation made by the pulley before its direction of motion is reversed is

- more than 3 but less than 6
- more than 6 but less than 9
- more than 9
- less than 3

31. A planoconvex lens has thickness of 4 cm. When placed on a horizontal table, with the curved surface in contact with it, the apparent depth of the bottom most point of the lens is found to be 3 cm. If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the centre of the plane face is found to be $\frac{25}{8}$ cm. The focal length of the lens is

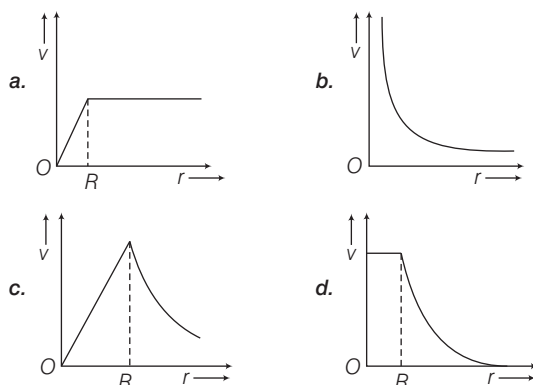
(assume thickness of lens to be negligible)

- 85 cm
- 59 cm
- 75 cm
- 7.5 cm

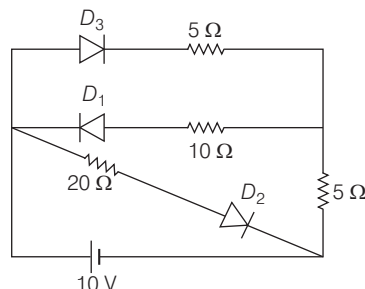
32. A spherically symmetric gravitational system of particles has mass density

$$\rho = \begin{cases} \rho_0 & \text{for } r \leq R \\ 0 & \text{for } r > R \end{cases}$$

where, ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by



33. In the given circuit,



The current through battery is

- 0.5 A
- 1 A
- 1.5 A
- 2.5 A

34. A trolley having mass of 200 kg moves with uniform speed of 36 kmh^{-1} on a frictionless track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 ms^{-1} relative to the trolley in a direction opposite to its motion and ultimately jumps out of the trolley. With how much velocity has the trolley moved from the time the child begins to run?

- 10.36 ms^{-1}
- 11.36 ms^{-1}
- 12.36 ms^{-1}
- 14.40 ms^{-1}

35. A gas has molar heat capacity $C = 37.55 \text{ J mol}^{-1}\text{K}^{-1}$, in this process $pT = \text{constant}$. The number of degree of freedom of the molecule of gas is

- 2
- 3
- 5
- 7

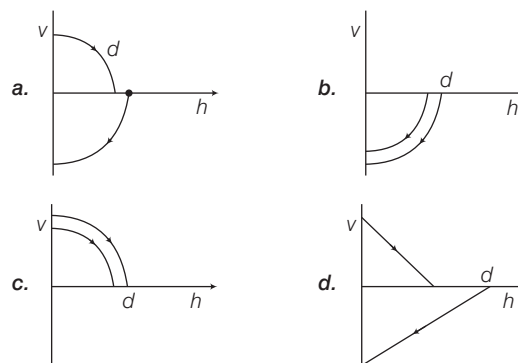
36. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing SHM of time period T . Then, which of the following does not change with time?

- $\frac{aT}{x}$
- $aT + 2\pi v$
- $\frac{aT}{v}$
- $a^2T^2 + 4\pi^2v^2$

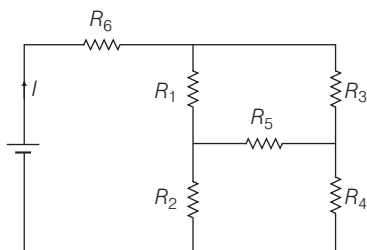
37. To increase the current sensitivity of a moving coil galvanometer by 50%, its resistance is increased, so that new resistance becomes twice its initial resistance, by what factor does its voltage sensitivity change?

- Increases by 15%
- Decreases by 15%
- Increases by 25%
- Decreases by 25%

38. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its velocity v varies with height h above the ground as



39. In the given circuit, it is observe that the current I is independent of the value of resistance R_6 . Then, the resistance value must satisfy



a. $\frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}$

- b. $R_1 R_4 = R_2 R_3$
 c. $R_1 R_2 R_5 = R_3 R_4 R_6$
 d. $R_1 R_3 = R_2 R_4 = R_5 R_6$

40. If a drop of liquid breaks into smaller droplets, it results in lowering of temperature of the droplets. Let a drop of radius R , break into N small droplets each of radius r , then decrease (drop) in temperature Q' (given, specific heat of liquid drop = S and surface tension = T)

a. $\frac{3T}{\rho S} \left[\frac{1}{r} - \frac{1}{R} \right]$ b. $-\frac{2T}{\rho S} \left[\frac{1}{r} - \frac{1}{R} \right]$
 c. $\frac{2R}{\rho S} \left[\frac{1}{R} - \frac{1}{r} \right]$ d. $\frac{3T}{\rho S} \left[\frac{1}{R} - \frac{1}{r} \right]$

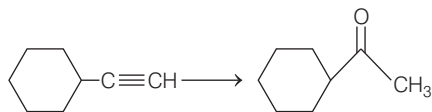
PART II

Chemistry

41. A metallic element has a cubic lattice. Each edge of the unit cell is $2\text{\AA}$ and the density of metal is 2.5 g cm^{-3} . The unit cell in 200 g of metal are
 a. 1×10^{24} b. 1×10^{22}
 c. 1×10^{20} d. 1×10^{25}
42. Four gases P , Q , R and S have almost same values of 'b' but their 'a' values (a and b are van der Waals' constant) are in the order $Q < R < S < P$. At a particular temperature, among the four gases, the most easily liquifiable one is
 a. P b. Q c. R d. S
43. If $\left[\frac{0.51 \times 10^{-10}}{4} \right]$ metre is the radius of smallest electron orbit in hydrogen like atom, then the atom is
 a. hydrogen atom b. He^+
 c. Li^{2+} d. Be^{3+}
44. Which of the following is the correct order for the wavelength if absorption in the visible region.
 a. $[\text{Ni}(\text{NO}_2)_6]^{4-} < [\text{Ni}(\text{NH}_3)_6]^{2+} < [\text{Ni}(\text{H}_2\text{O})_6]^{2+}$
 b. $[\text{Ni}(\text{NO}_2)_6]^{4-} < [\text{Ni}(\text{H}_2\text{O})_6]^{2+} < [\text{Ni}(\text{NH}_3)_6]^{2+}$
 c. $[\text{Ni}(\text{H}_2\text{O})_6]^{2+} < [\text{Ni}(\text{NH}_3)_6]^{2+} < [\text{Ni}(\text{NO}_2)_6]^{4-}$
 d. $[\text{Ni}(\text{NH}_3)_6]^{2+} < [\text{Ni}(\text{H}_2\text{O})_6]^{2+} < [\text{Ni}(\text{NO}_2)_6]^{4-}$
45. The empirical formula and molecular mass of a compound are CH_2O and 180g respectively. The molecular formula of the compound will be
 a. $\text{C}_9\text{H}_{18}\text{O}_9$ b. CH_2O
 c. $\text{C}_6\text{H}_{12}\text{O}_6$ d. $\text{C}_2\text{H}_4\text{O}_2$
46. In the context of the Hall-Heroult process for the extraction of Al, which of the following statement is incorrect.
 a. CO and CO_2 are produced in the process.
 b. Al_2O_3 is mixed with CaF_2 which lowers the melting point of the mixture and brings conductivity.
 c. Al^{3+} is reduced at the cathode to form Al.
 d. Na_3AlF_6 serves as the electrolyte.
47. Which test among the following is not used for the distinction among 1° , 2° and 3° aliphatic amine.
 a. Hinsberg's reagent test b. Carbylamine reaction
 c. Azo dye test d. Action with nitrous acid
48. The incorrect statement about carbonate (CO_3^{2-}) ion is,
 a. It has planar structure
 b. It has one coordinate bond
 c. It has three resonating structure
 d. Hydrolysis of CO_3^{2-} ion gives basic solution
49. Under the same reaction conditions, initial concentration of 1.386 mol dm^{-3} of a substance becomes half in 40 s and 20 s through first order and zero order kinetics, respectively. Ratio $\left(\frac{k_1}{k_0} \right)$ of the rate of constants for first order (k_1) and zero order (k_0) of the reaction is
 a. $0.5\text{ mol}^{-1}\text{dm}^3$ b. 1.0 mol dm^{-3}
 c. 1.5 mol dm^{-3} d. $2.0\text{ mol}^{-1}\text{dm}^3$
50. Which substance has a dipole moment?
 a. CCl_4 b. CH_2Cl_2
 c. C_2Cl_2 d. C_2Cl_4
51. Enthalpy of combustion of methane and ethane are -210 kcal/mol and -368 kcal/mol respectively. The enthalpy of combustion of decane is
 a. -1582 kcal b. -1632 kcal
 c. -1700 kcal d. -1480 kcal

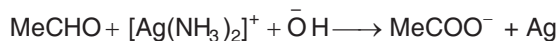
6 SELF STUDY GUIDE BITSAT

52. The correct sequence of reagents for the following conversion will be



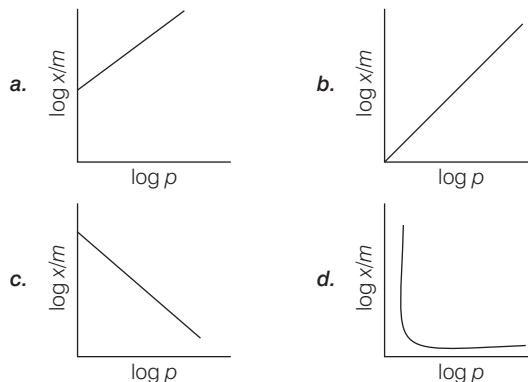
- a. O_3 / Red P, $AlCl_3$, $MeCOOH$
 b. $H_2SO_4 + HgSO_4, H_2O$ / Heat
 c. O_3 / $Zn - AcOH, H_2SO_4 + HgSO_4 / H_2O$ / Heat
 d. $CH_3COOH, H_2O_2 + \bar{O}H / H_2O$
53. In an atom, an electron is moving with a speed of 600 m/s with an accuracy of 0.005%. Certainly with which the position of the electron can be located is ($h = 6.6 \times 10^{-34} \text{ kg m}^2\text{s}^{-1}$, mass of electron, $e_m = 9.1 \times 10^{-31} \text{ kg}$)
 a. $1.52 \times 10^{-4} \text{ m}$ b. $5.10 \times 10^{-3} \text{ m}$
 c. $1.92 \times 10^{-3} \text{ m}$ d. $3.84 \times 10^{-3} \text{ m}$
54. Which of the following diatomic molecules would be stabilised by the removal of an electron?
 a. C_2 b. CN
 c. N_2 d. O_2
55. 0.5F of electricity is through 500 mL of copper sulphate solution. The amount of copper which can be deposited will be
 a. 63.5g b. 31.75g
 c. 15.80g d. unpredictable
56. Consider the following sequence of reactions.

$$Z \xrightarrow{PCl_5} X \xrightarrow{\text{Alc. KOH}} Y \xrightarrow[2H_2O/\Delta]{\text{Conc. } H_2SO_4} Z$$
 'Z' is
 a. $CH_3CH_2CH_2OH$ b. $(CH_3)_2COH$
 c. $CH_3 - \underset{\substack{| \\ OH}}{CH} - CH_3$ d. None of these
57. An equilibrium mixture at 300K contains N_2O_4 and NO_2 at 0.28 and 1.1 atm pressure, respectively. If the volume of the container is doubled, the new equilibrium pressure of these two gases are respectively.
 a. 0.064 atm and 0.095 atm
 b. 0.64 atm and 0.095 atm
 c. 0.095 atm and 0.632 atm
 d. 0.095 atm and 0.64 atm
58. Which among the following actinoids does not have stable electronic configuration
 a. Protactinium b. Nobelium
 c. Americium d. Lawrencium
59. Which of the following statement is incorrect regarding the equation



- a. The equivalent weight of $MeCHO$ is 22.
 b. Three moles of $\bar{O}H$ are required in the reaction.
 c. $MeCHO$ acts as an oxidising agent.
 d. $[Ag(NH_3)_2]^+$ gets reduced.

60. Which of the following graph is correctly represented according to Freundlich isotherm.



61. pH of a saturated solution of $Ba(OH)_2$ is 12. The value of its K_{sp} is
 a. $3.3 \times 10^{-7} \text{ M}$ b. $5.0 \times 10^{-7} \text{ M}$
 c. $4.0 \times 10^{-6} \text{ M}$ d. $5.0 \times 10^{-6} \text{ M}$
62. An unsaturated hydrocarbon 'X' gives white precipitate with Tollen's reagent. If X is gaseous in nature, the molecular formula of X is
 a. C_3H_6 b. C_2H_4
 c. C_2H_2 d. C_4H_8
63. The rate of a reaction triples when temperature changes from 20°C to 50°C . The energy of activation for the reaction is ($R = 8.314 \text{ JK}^{-1} \text{ mol}^{-1}$)
 a. $181.327 \text{ J mol}^{-1}$
 b. $428.141 \text{ J mol}^{-1}$
 c. $32.4321 \text{ kJ mol}^{-1}$
 d. $28.8118 \text{ kJ mol}^{-1}$
64. Which of the following compound will give blood red colour while doing the Lassaigne's test for N.
 a. $(NH_2)C=O$ b. $H_2N(C_6H_4)SO_3H$
 c. $C_6H_5SO_3H$ d. $CHCl_3$
65. For a reaction, $A + B^{2+} \longrightarrow B + A^{2+}$, at 25°C $E^\circ = 0.2955 \text{ V}$. The value of K_{eq} is
 a. 10 b. 10^{10}
 c. -10 d. 10^{-10}
66. Which of the following is the correct order of stability of conformations for $NH_2-CH_2-CH_2-OH$.
 a. gauche > eclipsed > anti
 b. gauche > anti > eclipsed
 c. eclipsed > gauche > anti
 d. anti > eclipsed > gauche

67. When H_2S gas is passed into a mixture of Mn^{2+} , Ni^{2+} , Cu^{2+} and Hg^{2+} ion in an acidified aqueous solution, the precipitates formed are

- CuS and HgS
- MnS and CuS
- MnS and NiS
- NiS and HgS

68. A mixture of bromo trichloride and hydrogen is subjected to silent electric discharge to form x and HCl . x is mixed with NH_3 and heated to 200°C to form y . The formula of y is

- B_2O_3
- $\text{B}_3\text{N}_3\text{H}_6$
- H_3BO_3
- B_2H_6

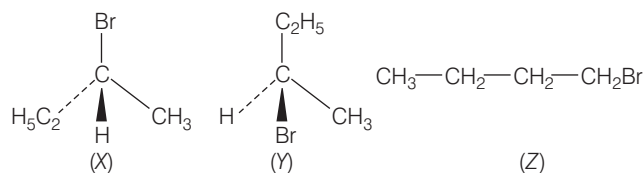
69. Which of the following reactions increases the production of dihydrogen from synthesis gas?

- $\text{CH}_4(\text{g}) + \text{H}_2\text{O}(\text{g}) \xrightarrow[\text{Ni}]{1270\text{K}} \text{CO}(\text{g}) + 3\text{H}_2(\text{g})$
- $\text{C}(\text{s}) + \text{H}_2\text{O}(\text{g}) \xrightarrow{1270\text{K}} \text{CO}(\text{g}) + \text{H}_2(\text{g})$
- $\text{CO}(\text{g}) + \text{H}_2\text{O}(\text{g}) \xrightarrow[\text{Catalyst}]{673\text{K}} \text{CO}_2(\text{g}) + \text{H}_2(\text{g})$
- $\text{C}_2\text{H}_6 + 2\text{H}_2\text{O} \xrightarrow[\text{Ni}]{1270\text{K}} 2\text{CO} + 5\text{H}_2$

70. When the heat of a reaction at constant pressure is -25×10^{-3} cal and entropy change for the reaction is 7.4 cal deg^{-1} , it is predicted that the reaction at 25°C is

- reversible
- spontaneous
- non-spontaneous
- irreversible

71. The addition of HBr to 1-butene gives a mixture of products x , y and z .



The mixture consists of

- x and y as major and z as minor products
- y as major, x and z as minor products
- y as minor, x and z as major products
- x and y as minor and z as major products

72. The correct statement about silicone is

- They are ketones with silyl group (SiH_3) similar to alkyl, $(\text{SiH}_3)_2\text{CO}$.
- They are synthetic polymer containing repeated R_2SiO_2 units.
- They are formed by hydrolysis of R_2SiCl_2 .
- All of the above

73. When dil. sulphuric acid reacts with aqueous solution of potassium chromate, the colour changes from yellow to orange. This shows that

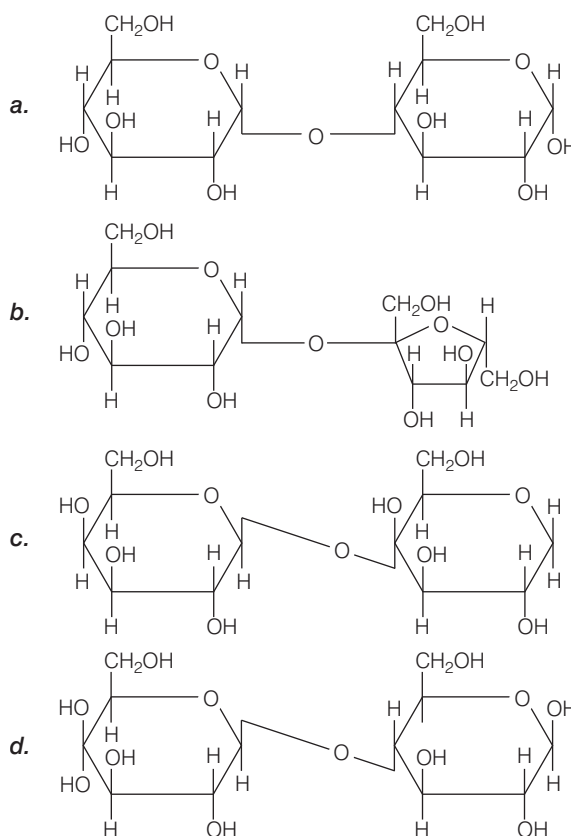
- chromate ions reduced
- chromate ions are oxidised
- monocentric complex is converted into dicentric complex
- oxygen gets removed from chromate ions

74. Valence electrons in the element A are 3 and that in element B are 6. Most probable compound formed from A and B is

- A_2B
- AB_2
- A_6B_3
- A_2B_3

75. In non-reducing disaccharide, the reducing group of monosaccharides i.e. aldehydic or ketonic group are bounded.

Which of the following disaccharide is a non-reducing sugar?



76. Which of the following 0.1 M aqueous solution will have lowest freezing point?

- Potassium sulphate
- Sodium chloride
- Urea
- Glucose

77. A penicillin is a member of a family of drugs that have a

- four membered cyclic amide fused to a five membered thiazolidine ring.
- three membered cyclic amide fused to a five-membered thiazolidine ring
- four-membered cyclic amide fused to have a four membered thiazolidine ring.
- five-membered cyclic amide fused to have a five membered thiazolidine ring

SELF STUDY GUIDE **BITSAT**

- $$5\text{Br}^-(aq) + \text{BrO}_3^-(aq) + 6\text{H}^+(aq) \longrightarrow 3\text{Br}_2(l) + 3\text{H}_2\text{O}(l)$$

a. $\frac{\Delta[\text{Br}^-]}{\Delta t} = \frac{5}{6} \frac{\Delta[\text{H}^+]}{\Delta t}$ b. $\frac{\Delta[\text{Br}^-]}{\Delta t} = \frac{6}{5} \frac{\Delta[\text{H}^+]}{\Delta t}$

c. $\frac{\Delta[\text{Br}^-]}{\Delta t} = \frac{5\Delta[\text{H}^+]}{\Delta t}$ d. $\frac{\Delta[\text{Br}^-]}{\Delta t} = \frac{6\Delta[\text{H}^+]}{\Delta t}$

- | Column I | Column II |
|-------------------|------------------------|
| A. Neoprene | I. Isoprene |
| B. Natural rubber | II. Tetrafluoro ethane |
| C. Teflon | III. Chloroprene |
| D. Acrilan | IV. Acrylnitrile |

	A	B	C	D		A	B	C	D
a.	IV	III	II	I	b.	I	II	III	IV
c.	III	I	II	IV	d.	II	IV	I	II

Part III

a. English Proficiency

Directions (Q. Nos. 81-83) *In the following questions, the sentences may or may not be grammatically correct. Find out which part of a sentence has an error and mark that part. If there is no error mark part 'd' as your answer.*

- 81.** The captain along with his team (a)/are practising very hard (b)/ for the forthcoming match. (c)/ No error (d).
- 82.** I am going (a)/to have this certificate (b)/attest by the director. (c)/No error (d).
- 83.** He is (a)/ having many (b)/friends here.(c)/No error...(d).

Directions (Q. Nos. 84-85) *Fill in the blanks with suitable preposition from the alternatives given under each sentence.*

- 84.** Is not learning superior wealth?
a. than b. from
c. by d. to
- 85.** He could not cope the heavy workload.
a. in with b. up with
c. up d. with

Directions (Q. Nos. 86-88) *Select the word or the phrase which is closest to the opposite in meaning of the italicized word or phrase.*

- 86.** He was in a *dejected* mood.
a. jubilant b. rejected
c. irritable d. romantic
- 87.** The attack on the freedom of the press is a *retrograde* step.
a. progressive b. stubborn
c. punitive d. aggressive
- 88.** We should not *belittle* the value of small things.
a. extal b. praise
c. inflate d. expand

Directions (Q. Nos. 89-90) Choose the word nearest in meaning to the underlined word.

- 89.** Before I could make out anything he had spoken again.
a. find out b. apprehend
c. explain d. reveal
- 90.** He wrote a scathing review of the prize winning novel.
a. biased b. scornful
c. unbalanced d. subjective

Directions (Q. Nos. 91-95) *Read the passage given below and answer the questions that follow.*

The megalomaniac differs from the narcissist by the fact that he wishes to be powerful rather than charming and seeks to be feared rather than loved. To this type belong many lunatics and most of the great men in history. Love of power, like vanity, is a strong element in normal human nature and as such is to be accepted; it becomes deplorable only when it is excessive or associated with an insufficient sense of reality. Where this occurs, it makes a man unhappy or foolish, if not both. The lunatic who thinks he is crowned head may be, in a sense, happy, but his happiness is not of a kind that any sane person would envy. Alexander the Great was psychologically of the same type as the lunatic, though he possessed the talent to achieve the lunatic's dream.

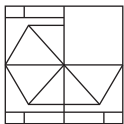
He could not, however, achieve his own dream, which enlarged his scope as his achievement grew. When it became clear that he was the greatest conqueror known to fame, he decided that he was a God. Was he a happy man ? His drunkenness, his furious rages, his indifference to women and his claim to divinity, suggest that he was not. There is no ultimate satisfaction in the cultivation of one element of human nature at the expense of all the others, nor in viewing all the world as raw material for the magnificence of one's own ego.

91. What is the difference between an ordinary megalomaniac and a megalomaniac like Alexander the Great?
- The ordinary megalomaniac does not have excessive desire for power which Alexander the Great had
 - The ordinary megalomaniac does not have the talent to realise his wish which Alexander the Great had
 - The ordinary megalomaniac is a lunatic while Alexander the Great was not a lunatic
 - The ordinary megalomaniac is not great while Alexander the Great was great
92. How does a megalomaniac differ from a narcissist?
- By wishing to be charming and feared
 - By wishing to be loved and not feared
 - By wishing to be powerful and not feared
 - By wishing to be powerful and feared

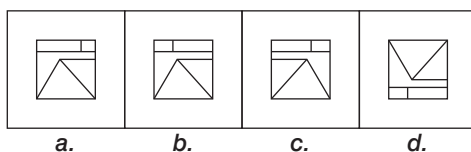
b. Logical Reasoning

96. 'Umpire' is related to 'Match' in the same way as 'Judge' is related to
- Bar council
 - Lawyer
 - Judgement
 - Lawsuit
97. Find the odd one from the following options.
- 9, 49
 - 13, 121
 - 10, 61
 - 7, 25
98. Complete the series by replacing question mark '?'.
10, 9, 16, 45, 176, ?
- 815
 - 222
 - 555
 - 875
99. In a cricket team, Dhoni is taller than Virat but not as tall as Raina, Rohit is shorter than Dhoni but taller than Shikhar. Who among them is the shortest?
- Dhoni
 - Virat
 - Shikhar
 - Cannot be determined
100. Identify the missing part of the question figure and select it from given answer figures.

Question figures



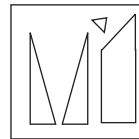
Answer figures



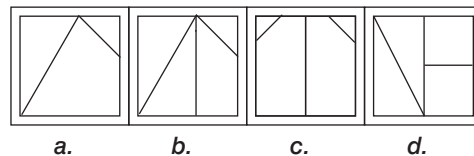
93. In "Where this occurs it makes a man..... if not both," 'this' refers to
- vanity
 - lunacy
 - love of power
 - excessive lover of power
94. Which among the following is the reason for unhappiness?
- Dealing with the raw material of the world
 - Realising one's dream as a megalomaniac
 - The nurturing of only one element in human nature
 - Being indifferent towards women
95. Why has love of power to be accepted?
- Because it can become unreal
 - Because it is an excess in human nature
 - Because it is a part of human nature
 - Because it is vanity in human nature

101. Find out which of the figure (a), (b), (c) and (d) can be formed from the pieces given in question figures.

Questions figure

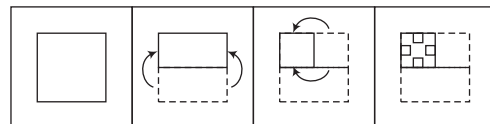


Answer figures

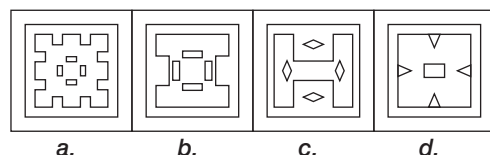


102. In this question, a piece of paper is folded and then cut as shown below. The dotted lines shown are the portion which have been folded.
- The curve arrow shows the directions of folding. And the number of scissors beneath the figure show the number of portions cut. From the given responses, indicate how it will appear when opened. The opening is in the same order as folding.

Question figures

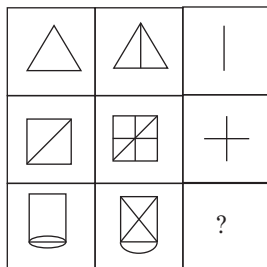


Answer figures

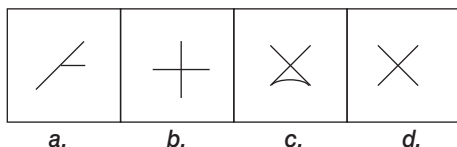


103. Which of the answer figures (a), (b), (c) or (d) completes the figure matrix ?

Questions figures



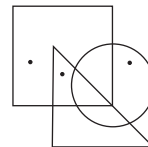
Answer figures



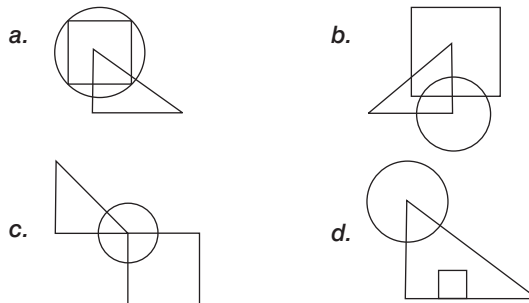
104. In the following question, one or more dots are placed in the question figure.

This figure is followed by four alternatives marked as (a), (b), (c) and (d). One out of these four options contains region(s) common to circle, square and triangles, similar to that marked by the dot in question figure. Find that figure.

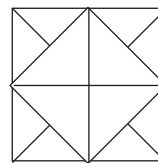
Question figure



Answer figures



105. How many triangles are there in the following figure?



- a. 16 b. 20 c. 12 d. 22

Part IV

Mathematics

106. The coefficient of x^{-n} in $(1+x)^n \left(1 + \frac{1}{x}\right)^n$ is

- a. 0 b. 1 c. 2^n d. $2n$

107. The greatest term in the expansion of $\sqrt{3} \left(1 + \frac{1}{\sqrt{3}}\right)^{20}$ is

- a. $\frac{26840}{9}$ b. $\frac{24840}{9}$
c. $\frac{25840}{9}$ d. None of these

108. The n th roots of unity are in

- a. AP b. GP
c. HP d. None of these

109. If $P = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, $abc = P^T P = I$, then the value of

$$a^3 + b^3 + c^3 \text{ is}$$

- a. 2 b. 1 c. 0 d. 5

110. If $x^2 = \begin{vmatrix} \sin \theta & \cos \theta & 0 \\ -\cos \theta & \sin \theta & 1 \\ \sin \theta & \cos \theta & 2 \end{vmatrix}$, then the value of

$$4x^2 + x \sin \frac{3\pi}{2} + 5 \text{ is}$$

- a. $13 - \sqrt{2}$ b. $13 + \sqrt{2}$
c. $\sqrt{2} - 13$ d. Both (a) and (b)

111. If a, b, c are in GP and $\log a - \log 2b, \log 2b - \log 3c$ and $\log 3c - \log a$ are in AP, then a, b and c are the lengths of the sides of a triangle, which is

- a. equilateral b. right angled
c. acute-angled d. obtuse angled

112. If $\sum_{r=1}^n t_r = \frac{n(n+1)(n+2)(n+3)}{8}$, where t_r denotes the r th

term of a series, then $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{t_r}$ is

- a. $\frac{1}{8}$ b. $\frac{1}{4}$ c. $\frac{1}{2}$ d. 1

113. Which of the following statement is a tautology?
 a. $(p \vee q) \vee (\sim p)$
 b. $(\sim q \wedge p) \vee (p \vee \sim p)$
 c. Both (a) and (b)
 d. None of the above
114. If a parallelogram is cut by two sets of m lines parallel to its sides, then the number of parallelogram thus formed, is
 a. ${}^m C_2 \times {}^m C_2$
 b. $2({}^{m+2} C_2)$
 c. $({}^{m+2} C_2)^2$
 d. None of these
115. The inverse of the function $f(x) = \log_a(x + \sqrt{x^2 + 1})$ (where, $a < 0, a \neq 1$) is
 a. $\frac{1}{2}(a^x - a^{-x})$
 b. not defined for all x
 c. defined for $x > 0$
 d. None of these
116. The value of $S = \sum_{n=1}^{\infty} \tan^{-1} \frac{2n}{n^4 + n^2 + 2}$ is equal to
 a. $\frac{\pi}{2}$
 b. π
 c. $\frac{\pi}{4}$
 d. None of these
117. Equation $\sin x + \cos(t+x) + \cos(t-x) = 2$ has real solution, then $\sin t$ can be
 a. $\frac{1}{2}$
 b. $\frac{3}{5}$
 c. $\frac{3}{4}$
 d. $-\frac{3}{4}$
118. A line makes angles α, β, γ with the coordinate axes. If $\alpha + \beta = \frac{\pi}{2}$, then $(\cos \alpha + \cos \beta + \cos \gamma)^2$ is equal to
 a. $1 + \cos 2\alpha$
 b. $1 - \sin 2\alpha$
 c. $1 + \sin 2\alpha$
 d. None of these
119. Straight lines $3x + 4y = 5$ and $4x - 3y = 15$ intersect at the point A . If point B and C are chosen on these two lines such that $AB = AC$, then the possible equation of the line BC passing through the point $(1, 2)$ is
 a. $x + 7y + 13 = 0$ or $7x + y + 9 = 0$
 b. $x + 7y + 13 = 0$ or $7x + 2y + 7 = 0$
 c. $x - 7y + 13 = 0$ or $7x + y - 9 = 0$
 d. None of the above
120. Normals drawn to $y^2 = 4ax$ at the points where it is intersected by the line $y = mx + c$ intersected at P . Coordinates of foot of the another normal drawn to the parabola from the point ' P ' is
 a. $\left(\frac{a}{m^2}, -\frac{2a}{m}\right)$
 b. $\frac{9}{m}, -\frac{6a}{m}$
 c. $(am^2, -2am)$
 d. $\left(\frac{4a}{m^2}, -\frac{4a}{m}\right)$
121. The area of the triangle formed by joining the origin to the point of intersection of the line $x\sqrt{5} + 2y = 3\sqrt{5}$ and circle $x^2 + y^2 = 10$ is
 a. 3
 b. 4
 c. 5
 d. 6
122. Radius of the largest circle which passes through the focus of the parabola $y^2 = 4x$ and contained in it, is
 a. 8
 b. 4
 c. 2
 d. 5
123. A tangent drawn to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $P\left(\frac{\pi}{6}\right)$ forms a triangle of area $3a^2$ square units, with coordinate axes. If the eccentricity of hyperbola is e , then the value of $e^2 - 9$ is
 a. 9
 b. 10
 c. 11
 d. 8
124. If the sum of squares of distances of a point from the planes $x + y + z = 0$, $x - z = 0$ and $x - 2y + z = 0$ is p^2 , then locus of the point is
 a. $x^2 + z^2 = p^2$
 b. $x^2 + 2xy + y^2 + z^2 = p^2$
 c. $x + y + z = p^2$
 d. $x^2 + y^2 + z^2 = p^2$
125. Line $\frac{(x+1)}{\lambda} = y - 1 = \frac{(z+2)}{-4}$ is perpendicular to $2x + 2y - 8z + 5 = 0$, then λ is
 a. 1
 b. -4
 c. -5
 d. -3
126. $OPQR$ is a square and M, N are the middle points of the sides PQ and QR respectively, then the ratio of the areas of the square and $\triangle OMN$ is
 a. 4 : 1
 b. 2 : 1
 c. 8 : 3
 d. 4 : 3
127. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets its auxiliary circle at the point M . Then, the area of the triangle with vertices at A, M and the origin O is
 a. $\frac{31}{10}$
 b. $\frac{29}{10}$
 c. $\frac{21}{10}$
 d. $\frac{27}{10}$
128. If e_1 and e_2 are the eccentricities of a hyperbola $3x^2 - 3y^2 = 25$ and its conjugate, then
 a. $e_1^2 + e_2^2 = 2$
 b. $e_1^2 + e_2^2 = 4$
 c. $e_1 + e_2 = 4$
 d. $e_1 + e_2 = \sqrt{2}$

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129. Let $f: R \rightarrow R$ be a function satisfying $f(x+y) = f(x) + 2y^2 + kxy$ for all $x, y \in R$. If $f(1) = 2$ and $f(2) = 8$, then $f(x)$ is equal to
- $2x^2$
 - $6x - 4$
 - $x^2 + 3x - 2$
 - $-x^2 + 9x - 6$
130. If the planes $\mathbf{r}(2\hat{i} - \lambda\hat{j} + 3\hat{k}) = 0$ and $\mathbf{r} \cdot (\lambda\hat{i} + 5\hat{j} - \hat{k}) = 5$ are perpendicular to each other, then the value of $\lambda^2 + \lambda$ is
- 0
 - 2
 - 1
 - 3
131. Solution of the differential equation $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$ is equal to
- $\log\left(2 + \sec\frac{x+y}{2}\right) = x + C$
 - $\log(1 + \tan(x+y)) = x + C$
 - $\log\left(1 + \tan\frac{x+y}{2}\right) = y + C$
 - $\log\left(1 + \tan\frac{x+y}{2}\right) = x + C$
132. The value of α , so that $\lim_{x \rightarrow 0} \frac{1}{x^2} (e^{\alpha x} - e^x - x) = \frac{3}{2}$, is
- 1
 - 0
 - 4
 - 2
133. An inverted conical flask is being filled with water at the rate of $3 \text{ cm}^3/\text{sec}$. The height of the flask is 10 cm and the radius of the base is 5 cm. How fast is the water level rising when the level is 4 cm?
- $\frac{4}{3} \pi \text{ cm/sec}$
 - $\frac{3}{4\pi} \text{ cm/sec}$
 - $\frac{3\pi}{4} \text{ cm/sec}$
 - $\frac{4}{3\pi} \text{ cm/sec}$
134. The equation of the curve whose slope at any point is equal to $y + 2x$ and which passes through the origin is
- $y = 2(x - 1)$
 - $y = 2(e^x - x - 1)$
 - $y = 2(e^x - 1)$
 - $y = 2(e^x x - 1)$
135. Let $f(x) = \begin{cases} x^p \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then $f(x)$ is continuous but not differentiable at $x = 0$, if
- $p < 0$
 - $p = 0$
 - $0 < p \leq 1$
 - $p \geq 1$
136. The solution of the differential equation $\frac{d^2y}{dx^2} = \sin 3x + e^x + x^2$ when $y_1(0) = 1$ and $y(0) = 0$, is
- $-\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x - 1$
 - $-\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x$
 - $-\frac{\cos 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x + 1$
 - None of the above
137. For which interval the given function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is decreasing?
- $(-2, \infty)$
 - $(-2, -1)$
 - $(-\infty, -1)$
 - $(-\infty, -2)$ or $(-1, \infty)$
138. If θ is the angle between the vectors $4(i - k)$ and $\hat{i} + \hat{j} + \hat{k}$, then $(\sin \theta + \cos \theta)$ equals to
- 0
 - $\frac{1}{2}$
 - 1
 - 2
139. In a $\triangle ABC$, D, E, F are the mid-points of the sides BC, CA and AB respectively, the vector $\overrightarrow{AD}$ is equal to
- $\overrightarrow{BE} + \overrightarrow{CF}$
 - $\overrightarrow{BE} - \overrightarrow{CF}$
 - $\overrightarrow{CF} - \overrightarrow{BE}$
 - $-\overrightarrow{BE} - \overrightarrow{CF}$
140. The arithmetic mean of a set of observation is $\bar{X}$. If each observation is divided by α and increased by 10, then the mean of the new series is
- $\frac{\bar{X}}{\alpha}$
 - $\frac{\bar{X} + 10}{\alpha}$
 - $\frac{\bar{X} + 10\alpha}{\alpha}$
 - $\alpha\bar{X} + 10$
141. If h is the altitude of a parallelopiped determined by the vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and the base is taken to be the parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$ where $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}$, $\mathbf{b} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\mathbf{c} = \hat{i} + \hat{j} + 3\hat{k}$, then the value of $19h^2$ is
- 19
 - 16
 - 8
 - None of these
142. The mean and variance of a Binomial distribution ($\overrightarrow{BD}$) for 3 trials is 2.7, then the $\overrightarrow{BD}$ is given by
- $(0.2 + 0.8)^5$
 - $(0.3 + 0.7)^5$
 - $(0.4 + 0.6)^5$
 - None of these
143. Let $P(x) = \int \frac{dx}{e^x + 8e^{-x} + 4e^{-3x}}$, $Q(x) = \int \frac{dx}{e^{3x} + 8e^x + 4e^{-x}}$ and $R(x) = P(x) - 2Q(x)$. If $R(x) = \frac{1}{2} A \left(\frac{B + 2e^{-x}}{C} \right) + K$, then the value of (A, B, C) is
- $(\tan^{-1}, 2, e^x)$
 - $(\tan^{-1}, e^x, 2)$
 - $\left(\tan^{-1}, \frac{1}{2}, \frac{1}{e^x} \right)$
 - $\left(\tan^{-1}, \frac{1}{e^x}, \frac{1}{2} \right)$

144. The value of $\int_0^1 \cot^{-1}(1-x+x^2)dx$ is

- a. $\log 2$
- b. $\frac{\pi}{2} - \log 2$
- c. $\frac{\pi}{2} + \log 2$
- d. $-\log 2$

145. The area of the region included between the curves $x^2 + y^2 = a^2$ and $\sqrt{|x|} + \sqrt{|y|} = \sqrt{a}$ ($a > 0$), is

- a. $\left(\pi - \frac{2}{3}\right)a^2$ sq units
- b. $\left(\frac{2}{3} - \pi\right)a^2$ sq units
- c. $\frac{2}{3}\pi a^2$ sq units
- d. $\left(\pi + \frac{2}{3}\right)a^2$ sq units

146. Let A and B are two independent events. If the probability that both A and B occur together is $\frac{1}{6}$ and the probability that neither of them occurs is $\frac{1}{3}$, then the probability of occurrence of A is

- a. 0 or 1
- b. $\frac{1}{2}$ or $\frac{1}{3}$
- c. $\frac{1}{2}$ or $\frac{1}{4}$
- d. $\frac{1}{3}$ or $\frac{1}{4}$

147. In a test an examiner either guesses or copies or knows the answer to a multiple choice question with 4 choices.

The probability that he/she makes a guess is $\frac{1}{3}$. The

probability that he/she copies the answer is $\frac{1}{6}$. If the

probability that the answer is correct, given that he/she copied, it is $\frac{1}{8}$, then the probability that he/she knows

the answer to a question given that he/she correctly answered it, is

- a. $\frac{27}{29}$
- b. $\frac{26}{29}$
- c. $\frac{25}{29}$
- d. $\frac{24}{29}$

148. If $p : 4$ is an even prime number, $q : 6$ is a divisor of 12 and $r : \text{the HCF of 4 and 6 is 2}$, then which of the following is correct?

- a. $(p \wedge q)$
- b. $(p \vee q) \wedge \sim r$
- c. $\sim (q \wedge r) \vee p$
- d. $\sim p \vee (q \wedge r)$

149. The maximum value of $Z = 4x + 2y$ subject to constraints $2x + 3y \leq 18$, $x + y \geq 10$ and $x, y \geq 0$ is

- a. 20
- b. 36
- c. 40
- d. None of these

150. The coordinates of the point at which minimum value of $Z = 7x - 8y$ subject to constraints $x + y - 20 \leq 0$, $y \geq 5$, $x \geq 0$, $y \geq 0$ is attained, is

- a. (20, 0)
- b. (15, 5)
- c. (0, 5)
- d. (0, 20)

Answers

Physics

1. (a)	2. (a)	3. (a)	4. (b)	5. (b)	6. (a)	7. (b)	8. (d)	9. (b)	10. (d)
11. (d)	12. (a)	13. (c)	14. (b)	15. (c)	16. (a)	17. (a)	18. (b)	19. (b)	20. (c)
21. (a)	22. (c)	23. (b)	24. (c)	25. (a)	26. (a)	27. (c)	28. (c)	29. (b)	30. (a)
31. (c)	32. (c)	33. (c)	34. (a)	35. (c)	36. (a)	37. (d)	38. (a)	39. (b)	40. (d)

Chemistry

41. (d)	42. (a)	43. (d)	44. (a)	45. (c)	46. (d)	47. (c)	48. (b)	49. (a)	50. (b)
51. (b)	52. (b)	53. (c)	54. (d)	55. (c)	56. (c)	57. (d)	58. (a)	59. (c)	60. (a)
61. (b)	62. (c)	63. (d)	64. (b)	65. (b)	66. (b)	67. (a)	68. (b)	69. (c)	70. (b)
71. (a)	72. (c)	73. (c)	74. (d)	75. (b)	76. (a)	77. (a)	78. (b)	79. (a)	80. (c)

English Proficiency

81. (a)	82. (c)	83. (b)	84. (d)	85. (d)	86. (a)	87. (a)	88. (a)	89. (b)	90. (b)
91. (b)	92. (d)	93. (d)	94. (c)	95. (c)					

Logical Reasoning

96. (d)	97. (c)	98. (d)	99. (d)	100. (c)	101. (b)	102. (a)	103. (c)	104. (b)	105. (b)
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Mathematics

106. (b)	107. (c)	108. (b)	109. (a)	110. (d)	111. (d)	112. (c)	113. (c)	114. (c)	115. (a)
116. (c)	117. (a)	118. (c)	119. (c)	120. (d)	121. (c)	122. (b)	123. (d)	124. (d)	125. (a)
126. (c)	127. (d)	128. (b)	129. (a)	130. (a)	131. (d)	132. (d)	133. (b)	134. (b)	135. (c)
136. (a)	137. (d)	138. (c)	139. (d)	140. (c)	141. (c)	142. (b)	143. (b)	144. (b)	145. (a)
146. (b)	147. (d)	148. (d)	149. (d)	150. (d)					

SOLUTIONS

Physics

1. (a) As the inductors are in parallel, induced emf across the two inductors is the same, i.e.

$$e_1 = e_2$$

$$L_1 \left(\frac{di_1}{dt} \right) = L_2 \left(\frac{di_2}{dt} \right)$$

On integrating both sides, we get

$$L_1 \int \frac{di_1}{dt} = L_2 \int \frac{di_2}{dt}$$

$$L_1 i_1 = L_2 i_2$$

$$\Rightarrow \frac{i_1}{i_2} = \frac{L_2}{L_1}$$

2. (a) We have, the time constant, $\tau = RC'$... (i)

$$\text{Now, } C' = \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{\left(\frac{A \epsilon_0}{d-x} \right) \left(\frac{KA \epsilon_0}{x} \right)}{\frac{A \epsilon_0}{d-x} + \frac{KA \epsilon_0}{x}}$$

$$\therefore C' = \frac{KA \epsilon_0}{x + K(d-x)}$$

Putting the value of C' in Eq. (i), we get

$$\tau = \frac{RKA \epsilon_0}{\frac{d}{3} - vt + K \left(d - \frac{d}{3} + vt \right)} \left[\because x = \frac{d}{3} - vt \right]$$

Given, $A = 1$ and $K = 2$

$$\therefore \tau = \frac{3 \times 2 \epsilon_0 \times R}{d - 3vt + 6d - 2d + 6vt} = \frac{6R \epsilon_0}{5d + 3vt}$$

3. (a) According to principle of calorimetry,
heat gained = heat lost

Heat is lost by steam in two stages

- (i) Change of state from steam at 100°C to water at 100°C is $m \times 540$.

- (ii) To change water at 100°C to water at 80°C is $m \times 1 \times (100 - 80)$, where m is the mass of the steam condensed.

Total heat lost by steam is

$$m \times 540 + m \times 20 = m(540 + 20) = 560m$$

Heat gained by calorimeter and its contents is

$$(11 + 0.2) \times (80 - 15) = 1.12 \times 65 \text{ cal}$$

$$\Rightarrow 560m = 1.12 \times 65$$

$$\Rightarrow m = \frac{1.12 \times 65}{560} = 0.130 \text{ kg}$$

4. (b) As, gravitational constant,

$$[G] = [M^{-1}L^3T^{-2}]$$

Planck's constant, $[h] = [ML^2T^{-1}]$

$$\text{Hence, } [G][h] = [M^{-1}L^3T^{-2}][ML^2T^{-1}]$$

$$= [M^0L^5T^{-3}]$$

Velocity of light, $[c] = [LT^{-1}]$

Now,

$$\left[\frac{Gh}{c^3} \right]^{1/2} = \frac{[L^5T^{-3}]^{1/2}}{[L^3T^{-3}]^{1/2}}$$

$$= [L^2]^{1/2} = [L]$$

Hence, $[L] = \text{length}$

5. (b) Given, $Q = 10^6 \text{ cal}$

$$T_1 = 827^\circ\text{C} = (827 + 273) = 1100 \text{ K}$$

$$T_2 = 27^\circ\text{C} = (27 + 273) = 300 \text{ K}$$

$$\text{As, } \frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\therefore Q_2 = \frac{T_2}{T_1} Q_1 = \frac{300}{1100} \times 10^6$$

$$= 2.72 \times 10^5 \text{ cal}$$

Efficiency of the engine,

$$\eta = \left(1 - \frac{T_2}{T_1} \right) \times 100$$

$$\eta = \left(1 - \frac{300}{1100} \right) \times 100$$

$$= 72.72\%$$

6. (a) According to Doppler's effect, the approximate frequency heard by the stationary observer,

$$v = \frac{v}{v - v_s} v_0$$

Case (i) $v_s = 34 \text{ m/s}$

where, v = speed of sound in air,

u_s = speed of source

and v_0 = frequency of the source.

$$\therefore v_1 = \frac{340}{340 - 34} v_0$$

$$= \frac{340}{306} v_0 \quad \dots (i)$$

Case (ii) $v_s = 17 \text{ m/s}$

$$\therefore v_2 = \frac{340}{340 - 17} v_0 = \frac{340}{323} v_0 \quad \dots (ii)$$

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From Eqs. (i) and (ii), we get

$$\therefore \frac{v_1}{v_2} = \frac{340/306}{340/323} \Rightarrow \frac{323}{306} \approx \frac{19}{18}$$

7. (b) Given, $m = 5 \text{ kg}$, $v = 20 \text{ ms}^{-1}$, $\theta = 60^\circ$

Vertical component of velocity, $v_y = v \sin 60^\circ$

$$= 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} \text{ ms}^{-1}$$

Time taken to reach the highest point = Time taken to reach the ground from highest point.

$$t = \frac{v \sin \theta}{g} = \frac{v_y}{g} = \frac{10\sqrt{3}}{9.8} = 1.77 \text{ s}$$

If the highest point, m splits up into two parts of masses $m_1 = 1 \text{ kg}$ and $m_2 = 4 \text{ kg}$.

If their velocities v_1 and v_2 respectively, then applying the principle of conservation of linear momentum, we get

$$m_1 v_1 + m_2 v_2 = mv \cos \theta$$

$$v_1 + v_2 = 5 \times 20 \times \frac{1}{2} \quad [\because \theta = 60^\circ]$$

$$v_1 + 4v_2 = 5 \times 10 = 50 \quad \dots (i)$$

$$\text{Initial KE} = \frac{1}{2} m (v \cos \theta)^2$$

$$= \frac{1}{2} \times 5 \times (10)^2 = 250 \text{ J}$$

Final KE = 2 (initial KE) = $2 \times 250 = 500 \text{ J}$

$$\therefore \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = 500$$

$$\text{or} \quad \frac{1}{2} \times 1 \times v_1^2 + \frac{1}{2} \times 4 \times v_2^2 = 500$$

$$\text{or} \quad v_1^2 + 4v_2^2 = 1000 \quad \dots (ii)$$

Solving Eqs. (i) and (ii), we get

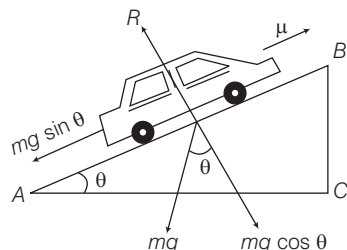
$$v_1 = 30 \text{ m/s}, v_2 = 5 \text{ m/s}$$

Hence, the separation between the two fragments

$$= (v_1 - v_2) \times t = (30 - 5) \times 1.77 \text{ m} = 44.25 \text{ m}$$

8. (d) Given, initial speed, $u = 36 \text{ km/h}$

$$= \frac{36 \times 1000}{60 \times 60} = 10 \text{ ms}^{-1}$$



$$\theta = 30^\circ, \mu = 0.1, s = ?$$

Here, work done in moving up the inclined road

$$= \text{KE of the vehicle}$$

$$(mg \sin \theta + F)s = \frac{1}{2} mu^2$$

$$(mg \sin \theta + \mu R) \times s = \frac{1}{2} mu^2$$

$$(mg \sin \theta + \mu mg \cos \theta) \times s = \frac{1}{2} mu^2$$

$$s = \frac{\frac{1}{2} mu^2}{mg(\sin \theta + \mu \cos \theta)} = \frac{u^2}{2g(\sin \theta + \mu \cos \theta)}$$

$$= \frac{10 \times 10}{2 \times 10 \times (\sin 30^\circ + 0.1 \cos 30^\circ)} = 8.53 \text{ m}$$

$$9. (b) P = 15 + 6 = 21 \Omega, Q = \frac{8X}{8+X} + 3$$

$$R = 15 + \frac{6 \times 6}{6+6} = 18 \Omega, S = 4 + \frac{4 \times 4}{4+4} = 6 \Omega$$

$$\text{As, } \frac{P}{Q} = \frac{R}{S}, \text{ so } Q = \frac{PS}{R} = \frac{21 \times 6}{18} = 7 \Omega$$

$$3 + \frac{8X}{8+X} = 7 \Rightarrow X = 8 \Omega$$

10. (d) Let V be the volume of the block. When block floats in

$$\text{water, then } V \rho_{\text{block}} g = \left(\frac{4}{5} V \right) \rho_{\text{water}} g$$

$$\text{or} \quad \rho_{\text{block}} = \frac{4}{5} \rho_{\text{water}} \quad \dots (i)$$

When block floats in liquid.

$$V \rho_{\text{block}} g = V \rho_{\text{liquid}} g$$

$$\rho_{\text{block}} = \rho_{\text{liquid}}$$

$$\rho_{\text{liquid}} = \frac{4}{5} \rho_{\text{water}} \quad [\text{from Eq. (i)}]$$

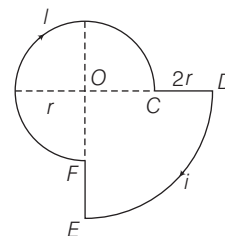
$$= \frac{4}{5} \times 10^3 \text{ kg m}^{-3}$$

$$= 800 \text{ kg m}^{-3}$$

11. (d) The angle, subtended by arc DE at O is $\frac{\pi}{2}$ and FC at

$O = \frac{3\pi}{2}$ the effective magnetic field at O is

$$B = B_{DE} + B_{FC}$$

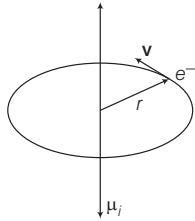


$$= \frac{\mu_0 i}{4\pi 3r} \times \frac{\pi}{2} + \frac{\mu_0 i}{4\pi r} \times \left(\frac{3\pi}{2} \right)$$

$$= \frac{\mu_0 i}{8r} \left[\frac{1}{3} + 3 \right]$$

$$= \frac{5\mu_0 i}{12r} \text{ acting downwards}$$

12. (a) The electron moving with a speed v in the circular Bohr orbit of radius r constitutes a current of magnitude.



$i = \frac{e}{T}$, where T is the orbital period of the electron.

$$T = 2\pi r / v$$

$$i = \frac{ev}{2\pi r}$$

By definition of atomic dipole moment μ_i , its magnitude is given by

$$\mu_i = iA = \frac{ev}{2\pi r} \cdot \pi r^2 = \frac{evr}{2}$$

$\therefore$ Angular momentum, $L = mvr$

$$\therefore \frac{L}{\mu_i} = \frac{2mvr}{evr} = \frac{2m}{e} = \text{constant}$$

13. (c) Let T_0 be the temperature of point C and x be the length of rod AB or BC .

$$\text{Then, } CA = \sqrt{x^2 + x^2} = \sqrt{2} x$$

At steady state, the rate of heat flowing from B to C = rate of heat flowing from C to A .

$$\text{So, } \frac{KA(\sqrt{2}T - T_0)}{x} = \frac{KA(T_0 - T)}{\sqrt{2}x}$$

$$\sqrt{2}(\sqrt{2}T - T_0) = T_0 - T$$

$$\text{By solving, } T_0 = \frac{3T}{(\sqrt{2} + 1)}$$

14. (b) At resonance, $X_L = X_C$ or $\omega L = \frac{1}{\omega C}$

Voltage across the series L - C combination,

$$V_2 = i(X_L - X_C) = 0$$

15. (c) $I = I_{\max} \cos^2\left(\frac{\phi}{2}\right)$

$$\frac{I_{\max}}{4} = I_{\max} \cos^2 \frac{\phi}{2}$$

$$\cos \frac{\phi}{2} = \frac{1}{2}$$

$$\frac{\phi}{2} = \frac{\pi}{3}$$

$$\Rightarrow \phi = \frac{2\pi}{3} = \left(\frac{2\pi}{\lambda}\right) \times \Delta x \quad \dots (i)$$

where, $\Delta x = d \sin \theta$

Putting the value of Δx in Eq. (i), we get

$$\sin \theta = \frac{\lambda}{3d}$$

$$\theta = \sin^{-1}\left(\frac{\lambda}{3d}\right)$$

16. (a) The bob filled completely with water has its centre of mass at its centre. The time period of oscillation is

$$T = 2\pi \sqrt{\frac{l}{g}}$$

As the water starts coming out of the bob, its centre of mass shifts vertically downward as a result effective length of the pendulum increases and hence its period also increases. When the bob is empty, again its centre of mass appears at its centre and as a result, the period of oscillation again reaches to its original value.

17. (a) On raising the temperature of the semiconductor, covalent bonds start breaking up and thus more charge carriers release. This will increase the value of n and as a result the rate of collision of charge carriers will increase. This will decrease the drift speed v .

18. (b) When a tennis ball falls on the ground and bounces back, its velocity and displacement changes in reverse direction while acceleration remains unchanged.

19. (b) Since, the lens is made up of two kinds of transparent material, it has two refractive indices for the incident beam of light. Hence, there will be two focal lengths of the lens and therefore two images will be observed.

20. (c) As we know,

$$L = I\omega$$

$$\Rightarrow L = I \times \frac{2\pi}{T}$$

$$L \propto \frac{1}{T}$$

Given, $T_2 = 2T$

$$\text{Hence, } \frac{L_1}{L_2} = \frac{2T}{T}$$

$$\Rightarrow L_2 = \frac{L_1}{2}$$

21. (a) Maximum stress = $\frac{\text{Maximum weight}}{\text{Cross-sectional area}}$

Now, maximum weight of steel wire

$$= \text{Volume} \times \text{Density} \times g$$

$$= A l \rho g$$

where, l is the maximum length of steel wire that can hang vertically without breaking, ρ is the density of steel and A is the cross-sectional area of steel wire.

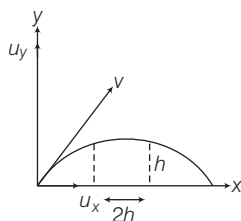
$$\therefore \text{Maximum stress, } p = \frac{A l \rho g}{A} = l \rho g$$

$$\therefore l = \frac{p}{\rho g}$$

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22. (c) $2h = ut$
 $u_x = \frac{2h}{\Delta t}$ $[\because u = u_x, t = \Delta t] \dots (i)$

$$h = u_y t - \frac{1}{2} g t^2$$



$$g t^2 - 2u_y t + 2h = 0$$

$$t_1 = \frac{2u_y + \sqrt{4u_y^2 - 8gh}}{2g}$$

$$t_2 = \frac{2u_y - \sqrt{4u_y^2 - 8gh}}{2g}$$

$$\Delta t = t_1 - t_2 = \frac{\sqrt{4u_y^2 - 8gh}}{g}$$

$$u_y^2 = \frac{g^2(\Delta t)^2}{4} + 2gh$$

$$u_x^2 + u_y^2 = u^2 = (2\sqrt{gh})^2$$

$$\frac{4h^2}{(\Delta t)^2} + \frac{g^2(\Delta t)^2}{4} + 2gh = 4gh$$

$$\frac{g^2}{4} (\Delta t)^4 - 2gh(\Delta t)^2 + 4h^2 = 0$$

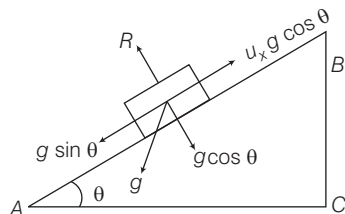
$$(\Delta t)^2 = \frac{2gh \pm \sqrt{4g^2h^2 - 4g^2h^2}}{g^2/2} = \frac{4h}{g}$$

$$\Delta t = 2\sqrt{\frac{h}{g}}$$

23. (b) $s = ut + \frac{1}{2} at^2$, $a = g \sin \theta$, $u = 0$

Then, $s = 0 + \frac{1}{2} at^2$

or $t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{g \sin \theta}}$ [for smooth plane]



For the rough plane, the effective value of acceleration along the incline is

$$a' = g \sin \theta - \mu_k g \cos \theta$$

and $t' = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{g \sin \theta - \mu_k g \cos \theta}}$

Now, $\frac{t'}{t} = n = \sqrt{\frac{g \sin \theta}{g \sin \theta - \mu_k g \cos \theta}}$
 $= \sqrt{\frac{\sin 45^\circ}{\sin 45^\circ - \mu_k \cos 45^\circ}}$
 $= \sqrt{\frac{1}{1 - \mu_k}}$

$$\Rightarrow \frac{1}{n^2} = 1 - \mu_k \Rightarrow \mu_k = 1 - \frac{1}{n^2}$$

24. (c) Since, the initial activity is 50 times the activity for safe occupancy, therefore $R_0 = 50R$, where $R = \lambda N$.

Since, $R \propto N$

$$\frac{R}{R_0} = \frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{t/T}$$

or $\left(\frac{1}{2}\right)^{t/10} = \frac{1}{50}$

$$(2)^{t/10} = 50 \Rightarrow \frac{t}{10} \log_{10} 2 = \log_{10} 50$$

$$\Rightarrow t = \frac{10 \log_{10} 50}{\log_{10} 2} = \frac{10 \times 1.699}{0.301} = 56.4 \text{ days}$$

25. (a) Helium is monoatomic gas, for which

$$C_V = \frac{3}{2} R, C_p = \frac{5}{2} R$$

Work done by the gas in one complete cycle $W = \text{area } ABCDA = p_0 V_0$

From A to B,

Heat given to gas $= nC_V \Delta T$

$$= 1 \times \left(\frac{3}{2} R\right) \times \Delta T = \frac{3}{2} V_0 (\Delta p) = \frac{3}{2} V_0 p_0$$

From B to C, heat given to gas $= nC_p \Delta T$

$$= 1 \times \left(\frac{5}{2} R\right) \times \Delta T = \frac{5}{2} (2p_0) \Delta V = 5p_0 V_0$$

Efficiency of cycle $= \frac{\text{Work done by the gas / cycle}}{\text{Total heat given to gas / cycle}}$

$$= \frac{p_0 V_0}{\frac{3}{2} p_0 V_0 + 5p_0 V_0} = \frac{2}{13}$$

$$\text{Efficiency (\%)} = \frac{2}{13} \times 100 = 15.4\%$$

26. (a) Using Einstein's photoelectric equations,

$$h\nu = \phi + eV_s$$

$$eV_s = h\nu - \phi \dots (i)$$

Thus, when the frequency of incident light radiation (photons) ν increases to 2ν , the stopping potential V_s changes to V'_s .

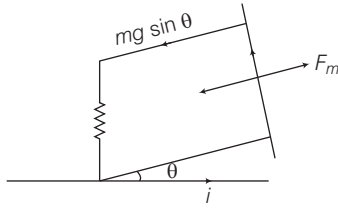
$$eV'_s = h(2\nu) - \phi \dots (ii)$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{V'_s}{V_s} = \frac{2h\nu - \phi}{h\nu - \phi} = \frac{2h\nu - 2\phi + \phi}{h\nu - \phi} = \frac{2(h\nu - \phi) + \phi}{h\nu - \phi} = 2 + \frac{\phi}{h\nu - \phi}$$

$$\therefore \frac{V'_s}{V_s} > 2 \Rightarrow V'_s > 2V_s$$

27. (c) Terminal velocity of the rod is attained when magnetic force on the rod (Bi) balances the component of weight of the rod ($mg \sin \theta$), as in figure.



So, $Bi = mg \sin \theta$

$$B \left(\frac{e}{R} \right) l = mg \sin \theta$$

$$\frac{Ble}{R} = mg \sin \theta$$

$$\frac{Bl(Blv_T)}{R} = mg \sin \theta$$

$$v_T = \frac{mg \sin \theta R}{B^2 l^2}$$

28. (c) Initial energy of the asteroid is

$$E_i = K_i + U_i = \frac{1}{2}mv_i^2 - \frac{GM_e m}{10R_e}$$

Final energy of the asteroid,

$$E_f = \frac{1}{2}mv_f^2 - \frac{GM_e m}{R_e}$$

According to law of conservation of energy, $E_i = E_f$

$$\frac{1}{2}mv_i^2 - \frac{GM_e m}{10R_e} = \frac{1}{2}mv_f^2 - \frac{GM_e m}{R_e}$$

$$v_f^2 - \frac{2GM_e}{R_e} = v_i^2 - \frac{2GM_e}{10R_e}$$

$$\Rightarrow v_f^2 = v_i^2 + \frac{2GM_e}{R_e} \left(1 - \frac{1}{10} \right)$$

29. (b) From Einstein photoelectric equation,

$$E = \phi_0 + KE_{\max}$$

$$\text{For metal A} \quad 4 = \phi_A + T_A \quad \dots (i)$$

$$\text{For metal B} \quad 4.5 = \phi_B + (T_A - 1.5) \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

$$\phi_B - \phi_A = 2$$

Now, according to de-Broglie hypothesis,

$$\lambda_A = \frac{h}{mv} = \frac{h}{\sqrt{2mT_A}}$$

$$\text{Similarly, } \lambda_B = \frac{h}{\sqrt{2mT_B}}$$

$$\therefore \frac{\lambda_A}{\lambda_B} = \sqrt{\frac{T_B}{T_A}} = \sqrt{\frac{T_A - 1.5}{T_A}} = \left(1 - \frac{1.5}{T_A} \right)^{1/2}$$

$$\left(\frac{1}{2} \right)^2 = 1 - \frac{1.5}{T_A}$$

On solving, $T_A = 2.0 \text{ eV}$

$$\text{So, } \phi_A = 4 - T_A = 4 - 2 = 2.0 \text{ eV}$$

$$\phi_B = 6 - T_A = 6 - 2 = 4.0 \text{ eV}$$

30. (a) We have,

$$\text{Torque} = r \times F = I\alpha$$

$$\text{So, } 2(20t - 5t^2) = 10\alpha$$

$$\alpha = 4t - t^2$$

But $\alpha = \frac{d\omega}{dt}$, so

$$\frac{d\omega}{dt} = 4t - t^2$$

$$d\omega = (4t - t^2) dt$$

$$\text{Integrating, } \omega = 2t^2 - \frac{t^3}{3}$$

ω will be zero at $t = 6 \text{ s}$.

$$\text{So, } \omega = \frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3}$$

$$\text{or } d\theta = \left(2t^2 - \frac{t^3}{3} \right) dt$$

Again, integrating both sides

$$\theta = \frac{2t^3}{3} - \frac{t^4}{12}$$

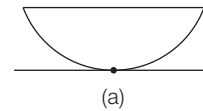
Since, $t = 6 \text{ s}$

$$\text{so, } \theta = \frac{2 \times 6^3}{3} - \frac{6^4}{12} = 36$$

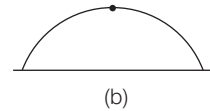
$$\text{Number of turns, } n = \frac{\theta}{2\pi} = \frac{36}{2\pi} = 5.73$$

So, option (a) is right.

31. (c)



(a)



(b)

In first case refraction of the rays, takes place from a plane

surface, so we can use $d_{\text{app}} = \frac{d_{\text{actual}}}{\mu}$

$$3 = \frac{4}{\mu} \Rightarrow \mu = \frac{4}{3}$$

where, d_{app} and d_{actual} = apparent and real depth, respectively.

Now, in second case, as from Fig. (b), refraction takes place from a spherical surface, so

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

or $\frac{1}{(-25/8)} - \frac{4/3}{(-4)} = \frac{1-4/3}{-R}$

or $\frac{1}{3R} = \frac{1}{3} - \frac{8}{25} = \frac{1}{75}$

$$R = 25 \text{ cm}$$

Now, using Lens' Makers formula, to calculate focal length

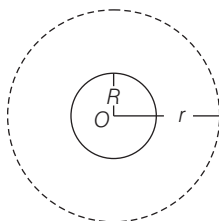
$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f} = \left(\frac{4}{3} - 1 \right) \left(\frac{1}{\infty} - \frac{1}{(-25)} \right) = \frac{1}{75}$$

$$f = 75 \text{ cm}$$

32. (c) When $r \leq R$, then force on the test mass m at the surface of the sphere $= mg$

Force on the test mass at distance r from the centre of sphere is



If $r < R$, then $F = \frac{GMm}{R^2} \cdot \frac{r}{R} = \frac{GMm}{R^3} \cdot r$

$$\therefore \frac{mv^2}{r} = \frac{GMm}{R^3} \cdot r$$

$$\therefore v \propto r$$

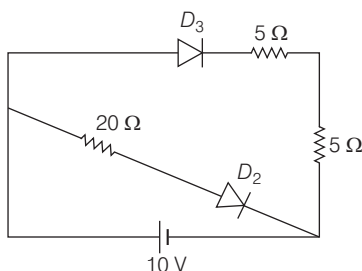
If $r > R$, then $F = \frac{GMm}{r^2}$

$$\therefore \frac{Mv^2}{r} = \frac{GMm}{r^2}$$

$$\therefore v \propto \frac{1}{\sqrt{r}}$$

Hence, option (c) is correct.

33. (c) In the given circuit, diode D_1 is reverse biased, so it will not conduct but D_2, D_3 are forward biased, so they will conduct, hence corresponding equivalent circuit. For the given circuit is



Now, the equivalent resistance of the circuit is

$$R_{eq} = \frac{(5+5) \times 20}{(5+5) + 20} + \frac{20}{3} \Omega$$

Current through battery, $I = \frac{10V}{\frac{20}{3} \Omega} = 1.5 \text{ A}$

34. (a) Since, no external force is acting on the system, we can apply conservation of linear momentum.

$$\text{Speed of 200 kg trolley} = \frac{36 \times 1000}{60 \times 60} = 10 \text{ ms}^{-1}$$

If u be the initial velocity of trolley, v_b be the absolute velocity of the boy after the beginning of journey of the boy, their relative velocity is 4.

$$\text{So, } v' - v_b = 4 \Rightarrow v_b = (v' - 4)$$

Now, applying law of conservation of momentum,

Momentum before the boy begins to run
 = Momentum after the beginning of boy's running

$$\Rightarrow 220 \times 10 = 200 v' + 20(v' - 4)$$

$$2200 = 220 v' - 80 \Rightarrow 220 v' = 2280$$

$$v' = \frac{2280}{220} = 10.36 \text{ ms}^{-1}$$

35. (c) Given, $C = 37.55 \text{ J mol}^{-1} \text{ K}^{-1}$

Also, $pT = \text{constant } (K)$... (i)

According to ideal gas equation, $pV = RT$

$$\Rightarrow p = \frac{RT}{V} \quad \dots \text{ (ii)}$$

Putting the value of p in Eq. (i), we get

$$\frac{RT}{V} \times T = K \Rightarrow V = \frac{RT^2}{K}$$

On differentiating above equation both sides, we get

$$\frac{dV}{dT} = \frac{2RT}{K} \quad \dots \text{ (iii)}$$

But $\frac{T}{K} = \frac{1}{p}$ [from Eq. (i)]

Hence, Eq. (iii) becomes

$$\frac{dV}{dT} = \frac{2R}{p}$$

So, $C = C_V + \frac{pdV}{dT}$

or $C = C_V + \frac{p \times 2R}{p} = C_V + 2R$

or $C_V = C - 2R$... (iv)

As $C_V = \frac{nR}{2}$, where n = number of degrees of freedom.

Putting the value of C_V in Eq. (iv), we get

$$\frac{nR}{2} = C - 2R$$

$$n = \frac{2(C - 2R)}{R} = \frac{2(37.55 - 2 \times 8.3)}{8.3} = 5.048 \approx 5$$

36. (a) We know, instantaneous displacement,

$$x = r \sin \omega t$$

$$\therefore \text{Instantaneous velocity, } v = \frac{dx}{dt} = r\omega \cos \omega t$$

$$\therefore \text{Instantaneous acceleration,}$$

$$a = \frac{dv}{dt} = -r\omega^2 \sin \omega t = -\omega^2 x$$

$$\text{So, } \frac{aT}{x} = \frac{-\omega^2 x \times T}{x} = -\omega^2 T = \frac{-4\pi^2}{T^2} \times T$$

$$= \frac{-4\pi^2}{T} = \text{constant}$$

$$\Rightarrow \frac{aT}{V} = \frac{-\omega^2 r \sin \omega t \times T}{\omega r \cos \omega t} = -\omega T \tan \omega t$$

$$= -\frac{2\pi}{T} \times T \tan \omega t = \text{not constant}$$

Similarly, $aT + 2\pi v$ and $a^2 T^2 + 4\pi^2 v^2$ is also not constant, i.e. both are function of t .

37. (d) New, current sensitivity, $I'_s = I_s + \frac{50}{100} I_s$

$$= \frac{150}{100} I_s = \frac{3}{2} I_s$$

New resistance, $R' = 2R$

Initial voltage sensitivity, $V_s = \frac{I_s}{R}$

Now, new voltage sensitivity,

$$V'_s = \frac{I'_s}{R'} = \frac{\frac{3}{2} I_s}{2R} = \frac{3}{4} V_s$$

% decrease in voltage sensitivity

$$= \frac{V_s - V'_s}{V_s} \times 100$$

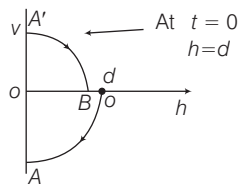
$$= \left(1 - \frac{V'_s}{V_s}\right) \times 100$$

$$= \left(1 - \frac{3}{4}\right) \times 100 = 25\%$$

38. (a) For the uniformly accelerated/decelerated motion,

$$v^2 = u^2 \pm 2gh$$

So, from this equation, we can say v - h graph is parabola.



Initially, velocity is downwards ($-ve$), after collision, it reverses the direction with smaller magnitude and velocity is upwards ($+ve$). So, graph (a) satisfies these conditions.

Also,

When $t = 0$, $h = d$

Velocity increases downwards ($0 \rightarrow A$)

When $t = 1$, velocity reverses its direction ($A' \rightarrow B$)

39. (b) From the given circuit, we can say that I is independent of resistance R_5 , so no current flows through R_5 . This required that the junction of R_1 and R_2 is at the same potential as the junction of R_3 and R_4 ($\therefore$ Wheatstone bridge condition)

So, $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

or $R_1 R_4 = R_3 R_2$

40. (d) Since, volume remains unchanged, during this phenomenon, so

$$\frac{4}{3} \pi R^3 = N \times \frac{4}{3} \pi r^3$$

$$N = \frac{R^3}{r^3}$$

Now, change in surface area $= 4\pi R^2 - N 4\pi r^2$

$$= 4\pi (R^2 - N r^2)$$

Energy released (ΔU) $= T \times \text{change in surface area}$

$$= T \times 4\pi [R^2 - N r^2]$$

Here, all this energy released is at the cost of lowering the temperature and mass of the big drop of liquid $= \frac{4}{3} \pi R^2 \rho$.

Now, change in temperature,

$$\Delta \theta = \frac{\Delta U}{ms}$$

$$= \frac{T \times 4\pi (R^2 - N r^2)}{\left(\frac{4}{3} \pi R^3 \rho\right) S}$$

$$= \frac{3T}{\rho S} \left(\frac{1}{R} - \frac{N r^2}{R^3}\right)$$

$$= \frac{3T}{\rho S} \left(\frac{1}{R} - \frac{R^3 \times r^2}{r^3 \times R^3}\right)$$

$$= \frac{3T}{\rho S} \left(\frac{1}{R} - \frac{1}{r}\right)$$

Chemistry

41. (d) Edge length of the unit cell =
- $2 \text{ \AA} = 2 \times 10^{-8} \text{ cm}$

$$\text{Volume of unit cell} = (2 \times 10^{-8})^3 = 8 \times 10^{-24} \text{ cm}^3$$

$$\text{Mass of one unit cell} = \text{volume} \times \text{density}$$

$$= 8 \times 10^{-24} \times 2.5$$

$$\text{Number of unit cell in 200g of metal}$$

$$= \frac{\text{mass of the metal}}{\text{mass of one unit cell}}$$

$$= \frac{200}{8 \times 10^{-24} \times 2.5} = 1 \times 10^{25}$$

42. (a) Higher the values of 'a' more will be the tendency to get liquefy. Value of
- a
- is highest for gas
- P
- . Thus, it is the most liquefiable gas among the given gases.

43. (d) Radius of hydrogen like atom,

$$r_n = \frac{n^2}{Z} r_0$$

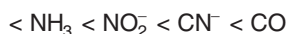
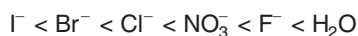
$$\text{Where, } r_0 = 0.51 \times 10^{-10} \text{ m}$$

$$\text{and } r_n = \frac{0.51 \times 10^{-10} \text{ m}}{4}$$

$$\text{At ground state, } n = 1$$

$$\text{Hence, the atom is } \text{Be}^{3+} \quad [\because Z = 4]$$

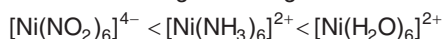
44. (a) When central metal ion is same (here
- Ni^{2+}
-), the absorption of colour depends on the ligand. According to spectrochemical series, various ligands are as follows



Thus, H_2O is the weakest ligand among these, therefore the absorbed energy will be lowest in $[\text{Ni}(\text{H}_2\text{O})_6]^{2+}$, so it will absorb highest wavelength (red light).

$$E \propto \frac{1}{\lambda}$$

$\therefore$ The order of increasing wavelength is



45. (c) Empirical formula mass =
- $\text{CH}_2\text{O} = 12 + 2 + 1 \times 16 = 30$

$$\text{Molecular mass} = 180$$

$$n = \frac{\text{Molecular Mass}}{\text{Empirical formula mass}} = \frac{180}{30} = 6$$

$$\therefore \text{Molecular formula} = n \times \text{empirical formula}$$

$$\therefore \text{Molecular formula} = 6 \times \text{CH}_2\text{O} = \text{C}_6\text{H}_{12}\text{O}_6$$

46. (d) (i) In Hall-Heroult process for extraction of Al, carbon anode is oxidised to CO and
- CO_2
- .

- (ii) When
- Al_2O_3
- is mixed with
- CaF_2
- , it lowers the melting point of the mixture and brings conductivity.

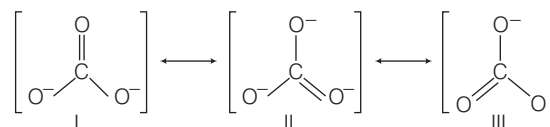
- (iii)
- Al^{3+}
- is reduced at cathode to form Al.

- (iv)
- Al_2O_3
- serves as the electrolyte, undergoing the redox process.
- Na_3AlF_6
- although is an electrolyte but serves as a solvent, not electrolyte.

47. (c) Azo dye test is not used for the distinction in
- 1°
- ,
- 2°
- and
- 3°
- aliphatic amines. The other tests can be easily used to distinguish between
- 1°
- ,
- 2°
- and
- 3°
- amines.

Test	Primary amine	Secondary amine	Tertiary amine
i Hinsberg's reagent test	Forms a sulphonamide soluble in alkali	Forms a sulphonamide insoluble in alkali	No reaction
ii Carbylamine reaction	Forms a carbylamine with unpleasant smell	No reaction	No reaction
iii Reaction with nitrous acid	Forms primary alcohol and evolves nitrogen gas with effervescence	Forms nitrosamine which gives Libermann's nitroso amine reaction	Forms nitrite salt

48. (b) Resonating structure of carbonate
- CO_3^{2-}
- ion are



Thus, there is no coordinate bond present in CO_3^{2-} .

Hence, the second statement is incorrect.

49. (a) For first order reaction,

$$t_{1/2} = \frac{\ln 2}{k_1} = 40 \text{ s} \quad \dots(i)$$

$$\text{For zero order reaction, } t_{1/2} = \frac{[A]_0}{2k_0} = 20 \text{ s} \quad \dots(ii)$$

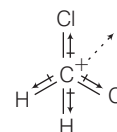
Divide eqn. (i) by (ii), we get

$$\frac{1}{2} = \frac{[A]_0}{2k_0} \times \frac{k_1}{\ln 2}$$

$$\frac{k_1}{k_0} = \frac{\ln 2}{[A]_0} = \frac{0.693}{1.386} = 0.5$$

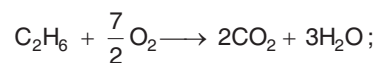
50. (b)
- CH_2Cl_2
- is similar to
- CH_4
- (tetrahedral) but it has three different bond angles.

HCH , HCCl , ClCCl , which are close to but not equal to 109.5° . Since, electronegativity of $\text{Cl} > \text{C} > \text{H}$ the bond dipole moments do not cancel and the molecule has a dipole moment.



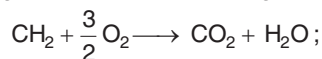
51. (b) Given,
- $\text{CH}_4 + 2\text{O}_2 \longrightarrow \text{CO}_2 + 2\text{H}_2\text{O}$
- ;

$$\Delta H = -210 \text{ kcal/mol} \quad \dots(i)$$



$$\Delta H = -368 \text{ kcal/mol} \quad \dots(ii)$$

On subtracting Eq. (i) from Eq (ii), we get



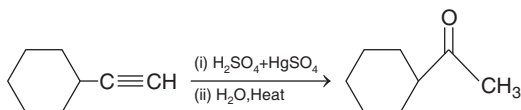
$$\Delta H = -158 \text{ kcal / mol}$$

$\therefore$ Enthalpy of combustion of one CH_2 unit

$$= -158 \text{ kcal / mol}$$

$$\begin{aligned} \Delta H_{\text{comb}}(\text{C}_{10}\text{H}_{22}) &= \Delta H_{\text{comb}}(\text{CH}_4) + 9 \times \Delta H_{\text{comb}}(\text{CH}_2) \\ &= -210 + (9 \times -158) \\ &= -1632 \text{ kcal} \end{aligned}$$

52. (b) $R-\text{C}\equiv\text{CH}$ is converted to ketone by catalytic hydration with reagents.



53. (c) By Heisenberg's uncertainty principle,

$$\Delta m \Delta v = \frac{h}{4\pi}$$

$$\Delta v = 0.005\% \text{ or } 600 \text{ m/s}$$

$$= \frac{600 \times 0.005}{100} = 0.03$$

$$\Delta x \times 9.1 \times 10^{-31} \times 0.03 = \frac{6.6 \times 10^{-34}}{4 \times 3.14}$$

$$\begin{aligned} \text{Hence, } \Delta x &= \frac{6.6 \times 10^{-34}}{4 \times 3.14 \times 0.03 \times 9.1 \times 10^{-31}} \\ &= 192 \times 10^{-3} \text{ m} \end{aligned}$$

54. (d) In O_2 molecule, two electrons are present in antibonding orbitals.

$$\text{O}_2 (8 + 8 = 16) = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2,$$

$$\sigma^2 2pz^2, \pi 2px^2 = \pi 2py^2, \pi^* 2px = \pi^* 2py^1$$

Removal of one electron from the O_2 molecule gives O_2^+ in which the number of antibonding electrons is one less and hence, BO increases. Thus, removal of one electron from O_2 stabilises the molecule.

55. (c) $\text{Cu}^{2+} + 2e^- \longrightarrow \text{Cu} \quad \dots(i)$

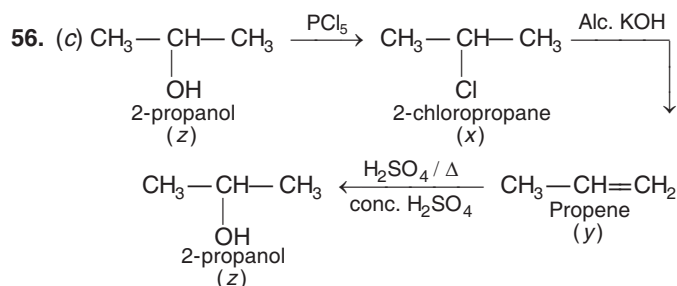
According to Faraday's first law, when an electric current is passed through an electrolyte, amount of substance deposited is directly proportional to the quantity of electric charge passed through the electrolyte.

$\therefore$ 1 mol of 63.5g of Cu is obtained by passing 2F of electricity.

$$\text{Mass of Cu obtained by passing 1F of electricity} = \frac{63.5}{2} \text{ g}$$

$\therefore$ Mass of Cu obtained by passing 0.5F of electricity

$$= \frac{63.5}{2} \times 0.5 = 15.80 \text{ g}$$



57. (d) $\text{N}_2\text{O}_4(g) \rightleftharpoons 2\text{NO}_2(g)$
- 0.28 1.1

Pressure at equilibrium

$$K_p = \frac{p_{\text{NO}_2}^2}{p_{\text{N}_2\text{O}_4}} = \frac{(1.1)^2}{0.28} = 4.32 \text{ atm}$$

If volume of the container is doubled, the pressure will reduced to half

$$\begin{aligned} \text{New pressure, } \left[\frac{0.028}{2} - p \right] & \left[\frac{1.1}{2} + 2p \right] \\ K_p &= \frac{\left[\frac{1.1}{2} + 2p \right]^2}{\left[\frac{0.28}{2} - p \right]} = 4.32 \end{aligned}$$

On solving, we get

$$p = 0.045$$

$$\therefore p_{\text{N}_2\text{O}_4} = 0.14 - 0.045 = 0.095 \text{ atm}$$

$$p_{\text{NO}_2} = 0.55 + (2 \times 0.045) = 0.64 \text{ atm}$$

58. (a) Protoactinium (Pa) is the element that does not have stable electronic configuration.

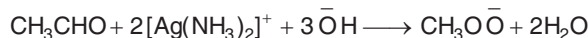
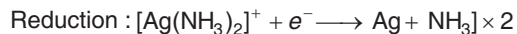
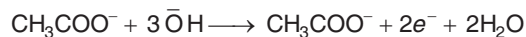
$$\text{Protoactinium (Pa)} = [\text{Rn}] 5f^2 6d^1 7s^2 \text{ (Atomic number} = 91)$$

$$\text{Nobelium (No)} = [\text{Rn}] 5f^{14} 6d^0 7s^2 \text{ (Atomic number} = 102)$$

$$\text{Lawrencium (Lr)} = [\text{Rn}] 5f^{14} 6d^1 7s^2 \text{ (Atomic number} = 103)$$

$$\text{Americium (Am)} = [\text{Rn}] 5f^7 6d^0 7s^2 \text{ (Atomic number} = 95)$$

59. (c) Oxidation :



$$\text{Molecular mass (M) of } \text{CH}_3\text{CO} = 44 \text{ g mol}^{-1}$$

$$\text{Equivalent weight (Ew)} = \frac{M}{n\text{-factor}} = \frac{44}{2} = 22 \text{ g mol}^{-1}$$

60. (a) According to Freundlich adsorption isotherm,

$$x/m = k_p \frac{1}{n}$$

where, x = mass of gas adsorbed on mass ' m ' of the absorbent at pressure p .

k and n = constants.

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Taking logarithm on both sides, we get

$$\log x / m = \log k + \frac{1}{n} \log p$$

This is equation of a straight line with slope of $\frac{1}{n}$ and intercept of $\log k$.

61. (b) Given, pH of $\text{Ba(OH)}_2 = 12$

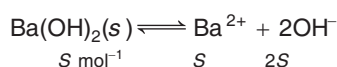
$$\therefore \text{pH} + \text{pOH} = 14$$

$$\therefore \text{pOH} = 14 - \text{pH} = 14 - 12 = 2$$

$$\text{Now, } \text{pOH} = -\log [\text{OH}^-]$$

$$\Rightarrow 2 = -\log [\text{OH}^-]$$

$$[\text{OH}^-] = 10^{-2}$$



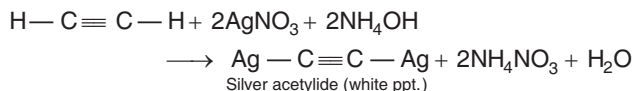
$$[\text{OH}^-] = 2S = 10^{-2}$$

$$\therefore S = 10^{-1}$$

$$K_{sp} = [\text{Ba}^{2+}] [\text{OH}^-]^2$$

$$\Rightarrow [10^{-1}] [10^{-2}]^2 = 0.5 \times 10^{-6} = 5 \times 10^{-7} \text{ M}$$

62. (c) With Tollen's reagent (ammoniacal AgNO_3), a white precipitate of silver salt is obtained



63. (d) Arrhenius equation is given by,

$$\log_{10} \frac{K_2}{K_1} = \frac{E_a}{2.303 \times R} \left[\frac{T_2 - T_1}{T_1 T_2} \right]$$

$$\text{Given, } \frac{K_2}{K_1} = 3; R = -8.314 \text{ JK}^{-1} \text{ mol}^{-1}$$

$$T_1 = 20 + 273 = 293 \text{ K}$$

$$\text{and } T_2 = 50 + 273 = 323 \text{ K}$$

Substituting the given values in Arrhenius equation,

$$\begin{aligned} \log_{10} 3 &= \frac{E_a}{8.314 \times 2.303} \left[\frac{323 - 293}{323 \times 293} \right] \\ E_a &= \frac{2.303 \times 8.314 \times 323 \times 293 \times 0.477}{30} \\ &= 28811.8 \text{ J mol}^{-1} \\ &= 28.8118 \text{ kJ mol}^{-1} \end{aligned}$$

64. (b) Compounds containing both N and S give blood red colour in Lassaigne's test due to the formation of Fe(SCN)_2 . Thus, $\text{H}_2\text{N}(\text{C}_6\text{H}_4)\text{SO}_3\text{H}$ gives blood red colour in Lassaigne's test of nitrogen.

65. (b) Given that,



Using Nernst equation,

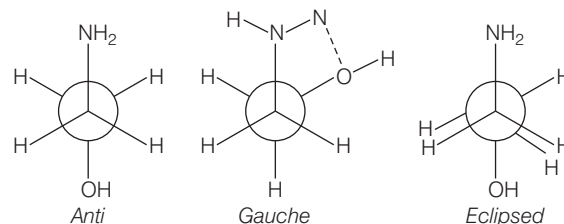
$$E^\circ = \frac{2.303nRT}{nF} \log k_{eq}$$

$$E^\circ = \frac{0.0591}{n} \log k_{eq} \text{ (at } 25^\circ\text{C)}$$

$$\Rightarrow 0.2955 = \frac{0.0591}{2} \log k_{eq}$$

$$k_{eq} = 10^{10}$$

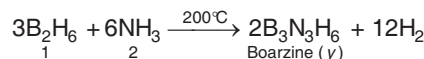
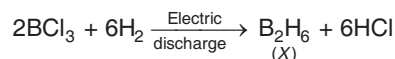
66. (b)



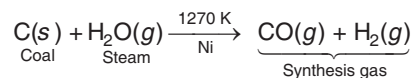
Since, gauche form is stabilised by intermolecular hydrogen bonding, hence it is more stable than anti.

67. (a) In acidic medium, H_2S is very feebly ionised giving very small concentration of sulphide ion for precipitation. Therefore, the most insoluble salts CuS and HgS are precipitated.

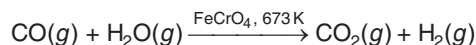
68. (b) When a mixture of BCl_3 and H_2 is subjected to silent electric discharge, diborane and HCl are formed. Diborane reacts with NH_3 at 200°C to give borazine (inorganic benzene).



69. (b) The process of producing *syn* gas or synthesis gas from coal is called 'coal gasification'?



The production of hydrogen can be increased by reaching carbon monoxide of the *syn* gas with steam in the presence of iron chromate as a catalyst at 673 K .



CO_2 is removed by scrubbing with a solution of sodium arsenite.

70. (b) Heat at constant pressure means enthalpy. i.e.

$$\Delta H = 2.5 \times 10^3 \text{ cal}$$

$$\Delta S = 7.4 \text{ cal deg}^{-1}$$

$$T = 298 \text{ K}$$

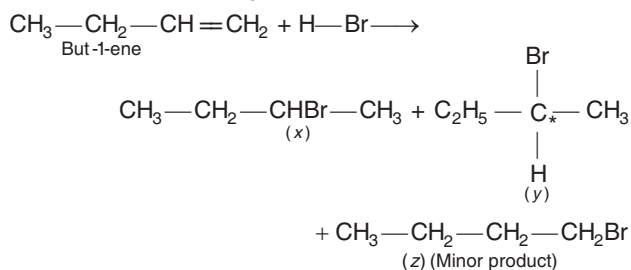
$$\Delta G = \Delta H - T\Delta S$$

$$= 2.5 \times 10^3 - 298 \times 7.4$$

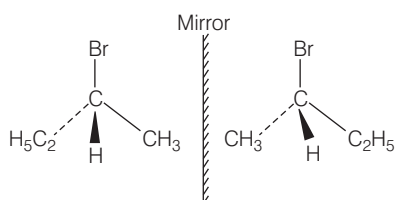
$$= 4705 \text{ cal}$$

Hence, the process is spontaneous.

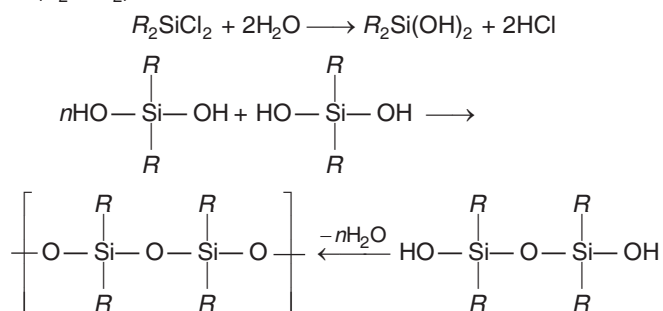
71. (a) The alkene is unsymmetrical, hence will follow Markownikoff's rule to give major product.



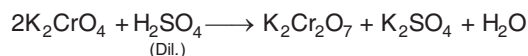
Since, y contains, a chiral carbon, it exists in two enantiomers (x and y) which are mirror images of each other.



72. (c) Silicons are synthetic organo-silicon polymers containing repeated $R_2\text{SiO}$ units. Since the empirical formula is same as that of a ketone ($R_2\text{CO}$), the name silicon has been given to these materials. They can be formed by hydrolysis of dichlorosilanes ($R_2\text{SiCl}_2$)



73. (c) On acidification of potassium chromate solution, yellow colour changes to orange colour due to formation of dichromate which suggests that monocentric complex is converted into dicentric complex.



74. (d) Valence electrons in A = 3

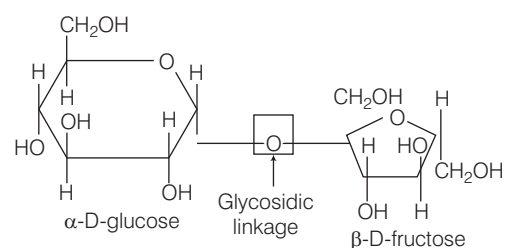
Valence electron in B = 6

Thus, A is electropositive and B is electronegative, A can loose three electrons and B can gain two electrons to attain stable configuration

Hence, A exist as A^{3+} and B as B^{2-}

$\therefore$ Compound formed is A_2B_3 .

75. (b)



This structure represents sucrose in which α -D-glucose and β -D fructose is attached to each other by $C_1 - C_2$ glycosidic linkage. Since reducing groups of glucose and fructose are involved in glycosidic bond formation, this is considered as non-reducing sugar.

76. (a) Potassium sulphate (K_2SO_4), $i = 3$

Sodium chloride (NaCl), $i = 2$

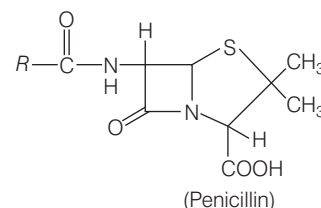
Urea, $i = 1$

Glucose, $i = 1$

Greater the value of i , greater lowering in freezing point and hence, lower will be the freezing temperature.

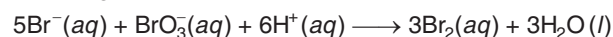
Therefore, K_2SO_4 solution has the lowest freezing point.

77. (a) A penicillin is a member of a family of drugs that have a four membered cyclic amide fused to a five membered thiazolidine ring. It is narrow spectrum antibiotic



78. (b) The lattice energy of the hydroxides of alkaline earth metal decreases more rapidly than their hydration energy leading to more negative value of ΔH_{sol} down the group. More negative is ΔH_{sol} , more is the solubility of compounds. Hence the solubility of hydroxides of alkaline earth metal increases with their atomic number.

79. (a) The given chemical reaction is



Rate law expression for the given reaction can be written as

$$-\frac{1}{5} \frac{\Delta[\text{Br}^-]}{\Delta t} = -\frac{\Delta[\text{BrO}_3^-]}{\Delta t}$$

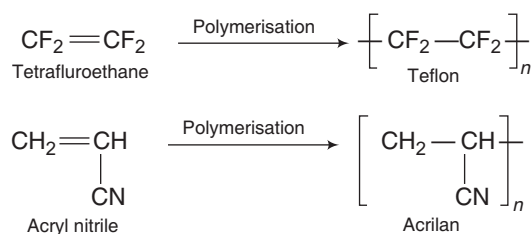
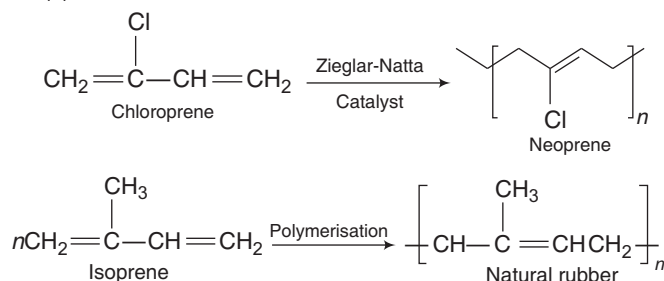
$$-\frac{1}{6} \frac{\Delta[\text{H}^+]}{\Delta t} = +\frac{1}{3} \frac{\Delta[\text{Br}_2]}{\Delta t}$$

$$\Rightarrow -\frac{\Delta[\text{Br}^-]}{\Delta t} = -\frac{5\Delta[\text{BrO}_3^-]}{\Delta t} = -\frac{5}{6} \frac{\Delta[\text{H}^+]}{\Delta t}$$

$$\Rightarrow \frac{\Delta[\text{Br}^-]}{\Delta t} = \frac{5}{6} \frac{\Delta[\text{H}^+]}{\Delta t}$$

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80. (c) A-III, B-I, C-II, D-IV



English Proficiency

81. (a) Use 'is' in place of 'are' as when two subjects are joined 'along with', then helping verb is used according to first subject.
82. (c) Use 'attested' in place of 'attest' as sentence is in passive voice.
83. (b) Use 'making' in place of 'having' as 'have' is not used in progressive sense.
84. (d) 'Superior' is followed by 'to' preposition.
85. (d) 'Cope' always agree with preposition 'with' and use of any other auxiliary term is improper.
91. (b) The ordinary megalomaniac does not have the talent to realise his wish which Alexander the Great had
92. (d) By wishing to be powerful and feared.
93. (d) Excessive lover of power
94. (c) The nurturing of only one element in human nature
95. (c) Because it is a part of human nature

Logical Reasoning

96. (d) 'Umpire' is required to give decision in 'Match'.

Likewise 'Judge' is required to give decision in a 'Lawsuit'.

97. (c) As, $9 - 2 = 7 \rightarrow 7^2 = 49$

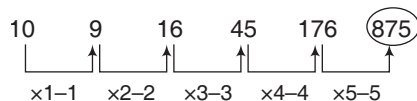
$$13 - 2 = 11 \rightarrow 11^2 = 121$$

$$10 - 2 = 8 \rightarrow 8^2 \neq 61$$

$$7 - 2 = 5 \rightarrow 5^2 = 25$$

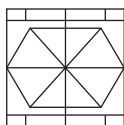
Thus, option (c) is odd.

98. (d) The pattern is as follows



99. (d) The data is inadequate because it is not given that who is taller between Virat and Shikhar. Hence, answer cannot be determined.

100. (c) Option (c) will complete the given pattern as follow.



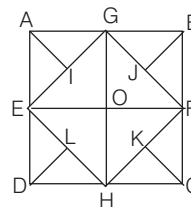
101. (b) Options (b) can be formed from the pieces given in problem figures.

102. (a) After folding and cutting the paper, answer figure (a) will appear.

103. (c) The third figure is each row comprises of parts which are not common in the first two figures.

104. (b) Correct answer figure is (b).

105. (b) The figure in the question may be labelled as shown below



There are nine triangle in the upper half of the figure $ABFE = \triangle AEI, \triangle AIG, \triangle AEG, \triangle GEO, \triangle GBJ, \triangle BFJ, \triangle GBF, \triangle GOF$ and $\triangle GEF$.

Similarly, there are nine triangles in the lower half figure, i.e. $EFCD$.

There are two more triangles $\triangle EGH$ and $\triangle FGH$.

Hence, there are a total of 20 triangles.

Mathematics

$$106. (b) (1+x)^n \left(1 + \frac{1}{x}\right)^n = \left[(2+x) + \frac{1}{x}\right]^n$$

$$= {}^nC_0(2+x)^n \cdot \left(\frac{1}{x}\right) + \dots + {}^nC_n \cdot \frac{1}{x^n}$$

$\therefore$ Coefficient of $x^{-n} = {}^nC_n = 1$

$$107. (c) T_{r+1} = \sqrt{3} \cdot {}^{20}C_r \left(\frac{1}{\sqrt{3}}\right)^r$$

$$\text{and } T_r = \sqrt{3} \cdot {}^{20}C_{r-1} \left(\frac{1}{\sqrt{3}}\right)^{r-1}$$

$$\text{Now, } \frac{T_{r+1}}{T_r} = \frac{20-r+1}{r} \left(\frac{1}{\sqrt{3}}\right)$$

$$\text{Since, } T_{r+1} \geq T_r$$

$$\Rightarrow 20-r+1 \geq \sqrt{3}r$$

$$\Rightarrow r \leq \frac{21}{\sqrt{3}+1} = \frac{21}{2.73}$$

$$\Rightarrow r \leq 7.692 \Rightarrow r = 7$$

$$\therefore \text{ The greatest term is } T_8 = \sqrt{3} \cdot {}^{20}C_7 \left(\frac{1}{\sqrt{3}}\right)^7 = \frac{25840}{9}$$

$$108. (b) \text{ Let } x = 1$$

$$= \cos 0^\circ + i \sin 0^\circ$$

$$= \cos 2r\pi + i \sin 2r\pi = e^{i 2r\pi}$$

$$\Rightarrow x^{1/n} = e^{i(2r\pi)/n}, r = 0, 1, 2, \dots$$

$$\text{Then, the roots are } 1, e^{\frac{2\pi i}{n}}, e^{\frac{4\pi i}{n}}, \dots$$

which are clearly on GP with common ratio $e^{2\pi i/n}$.

$$109. (a) \text{ Given, } P = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$$

$$\therefore P^T = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$$

$$\text{Also given, } P^T P = I$$

$$\therefore |P^T P| = |I| = 1$$

$$\text{or } \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = 1 \Rightarrow \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \pm 1$$

$$\text{or } 3abc - a^3 - b^3 - c^3 = \pm 1$$

$$\text{or } 3 \mp 1 = a^3 + b^3 + c^3$$

$$\therefore a^3 + b^3 + c^3 = 2 \text{ or } 4$$

$\therefore$ Option (a) is correct.

$$110. (d) \text{ Given, } x^2 = \begin{vmatrix} \sin \theta & \cos \theta & 0 \\ -\cos \theta & \sin \theta & 1 \\ \sin \theta & \cos \theta & 2 \end{vmatrix}$$

$$\text{On expanding } \begin{vmatrix} \sin \theta & \cos \theta & 0 \\ -\cos \theta & \sin \theta & 1 \\ \sin \theta & \cos \theta & 2 \end{vmatrix} \text{ along } C_3, \text{ we get}$$

$$x^2 = 0 - 1 \begin{vmatrix} \sin \theta & \cos \theta \\ \sin \theta & \cos \theta \end{vmatrix} + 2 \begin{vmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{vmatrix}$$

$$= -1(\sin \theta \cos \theta - \sin \theta \cos \theta) + 2(\sin^2 \theta + \cos^2 \theta)$$

$$= -1 \times 0 + 2 \times 1$$

$$\Rightarrow x^2 = 2 \Rightarrow x = \pm \sqrt{2}$$

If $x = \sqrt{2}$, then

$$4x^2 + x \sin \frac{3\pi}{2} + 5 = 4 \times (\sqrt{2})^2 - \sqrt{2} + 5$$

$$= 8 - \sqrt{2} + 5 = (13 - \sqrt{2})$$

$$\text{If } x = -\sqrt{2}, \text{ then } 4x^2 + x \sin \frac{3\pi}{2} + 5$$

$$= 4 \times (-\sqrt{2})^2 + \sqrt{2} + 5$$

$$= 8 + \sqrt{2} + 5 = (13 + \sqrt{2})$$

$$111. (d) \text{ Given, } a, b, c \text{ are in GP.}$$

$$\therefore b^2 = ac$$

$$\text{and } 2(\log 2b - \log 3c) = \log a - \log 2b + \log 3c - \log a$$

$$\Rightarrow b^2 = ac \text{ and } 2b = 3c$$

$$\Rightarrow c = \frac{4a}{9} \text{ and } b = \frac{2a}{3}$$

$$\therefore a + b = \frac{5a}{3} > c, b + c = \frac{10a}{9} > a$$

$$\text{and } c + a = \frac{13a}{9} > b.$$

$\therefore a, b, c$ are the sides of a triangle.

Also, a is the greatest side

$$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2bc} = -\frac{29}{48} < 0$$

$\therefore \triangle ABC$ is an obtuse angled triangle.

Hence, option (d) is correct.

$$112. (c) \text{ Given, } \sum_{r=1}^n t_r = \frac{n(n+1)(n+2)(n+3)}{8} = S_n \text{ (say)}$$

$$\therefore \sum_{r=1}^{n-1} t_r = \frac{(n-1)n(n+1)(n+2)}{8} = S_{n-1}$$

$$\text{Now, } t_n = S_n - S_{n-1} = \frac{n(n+1)(n+2)}{2}$$

$$\therefore \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{t_r} = \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{2}{n(n+1)(n+2)}$$

$$\begin{aligned}
&= \lim_{n \rightarrow \infty} \sum_{r=1}^n \left(\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)} \right) \\
&= -\lim_{n \rightarrow \infty} \sum_{r=1}^n \left(\frac{1}{(n+1)(n+2)} - \frac{1}{n(n+1)} \right) \\
&= -\lim_{n \rightarrow \infty} \left(\frac{1}{(n+1)(n+2)} - \frac{1}{2} \right) \\
&= -\left(0 - \frac{1}{2} \right) = \frac{1}{2}
\end{aligned}$$

Hence, option (c) is correct.

113. (c) Truth table

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim q \wedge p$	$p \vee \sim p$	$(p \vee q) \vee (\sim p)$	$(\sim q \wedge p) \vee (p \vee \sim p)$
T	T	F	F	T	F	T	T	T
T	F	F	T	T	T	T	T	T
F	T	T	F	T	F	T	T	T
F	F	T	T	F	F	T	T	T

Hence, option (c) is correct.

114. (c) The two sets of m parallel lines along with two sets of two parallel lines of the given parallelogram will form two sets of $(m+2)$ parallel lines. Each parallelogram is formed by choosing two parallel lines from each of the above

$\therefore$ Total number of parallelograms

$$= {}^{m+2}C_2 \times {}^{m+2}C_2 = ({}^{m+2}C_2)^2$$

Hence, option (c) is correct.

115. (a) Let $f(x) = y$, then

$$\begin{aligned}
a^y &= x + \sqrt{x^2 + 1} \\
\Rightarrow a^{-y} &= \frac{1}{x + \sqrt{x^2 + 1}} \\
\Rightarrow a^{-y} &= \frac{x - \sqrt{x^2 + 1}}{-1} \quad [\text{rationalising}] \\
\therefore a^y - a^{-y} &= 2x \\
\Rightarrow x &= \frac{1}{2}(a^y - a^{-y}) \\
\therefore f^{-1}(y) &= \frac{1}{2}(a^y - a^{-y}) \quad [\because f(x) = y \Rightarrow x = f^{-1}(y)] \\
\Rightarrow f^{-1}(x) &= \frac{1}{2}(a^x - a^{-x}) \quad [\text{replacing } y \text{ by } x]
\end{aligned}$$

Hence, option (a) is correct.

116. (c) Given, $S = \sum_{n=1}^{\infty} \tan^{-1} \frac{2n}{n^4 + n^2 + 2}$... (i)

$$\begin{aligned}
\text{Let } n^4 + n^2 + 1 &= [(n^2)^2 + 1^2 + 2(n^2)(1)] - n^2 \\
&= (n^2 + 1)^2 - n^2 \\
&= (n^2 + n + 1)(n^2 - n + 1) \quad \dots (ii)
\end{aligned}$$

$$\text{Let } u_n = \tan^{-1} \left(\frac{2n}{n^4 + n^2 + 2} \right)$$

$$\begin{aligned}
&= \tan^{-1} \frac{2n}{1 + (n^4 + n^2 + 1)} \\
&= \tan^{-1} \frac{(n^2 + n + 1) - (n^2 - n + 1)}{1 + (n^2 + n + 1)(n^2 - n + 1)} \\
u_n &= \tan^{-1}(n^2 + n + 1) - \tan^{-1}(n^2 - n + 1) \dots (iii)
\end{aligned}$$

On putting $n = 1, 2, 3, \dots$ successively in Eq. (iii), we get

$$u_1 = \tan^{-1} 3 - \tan^{-1} 1$$

$$u_2 = \tan^{-1} 7 - \tan^{-1} 3$$

$$u_3 = \tan^{-1} 13 - \tan^{-1} 7$$

$$\dots \dots \dots$$

$$u_n = \tan^{-1}(n^2 + n + 1) - \tan^{-1}(n^2 - n + 1)$$

On adding vertically, we get

$$\sum_{n=1}^{\infty} u_n = \tan^{-1}(n^2 + n + 1) - \tan^{-1} 1$$

$$S = \lim_{n \rightarrow \infty} \sum_{n=1}^{\infty} u_n \quad [\text{from Eq. (i)}]$$

$$= \lim_{n \rightarrow \infty} \tan^{-1}(n^2 + n + 1) - \tan^{-1} 1$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

117. (a) Given, $\sin x + \cos(t+x) + \cos(t-x) = 2$

$$\Rightarrow \sin x + 2 \cos t \cdot \cos x = 2$$

For real solution

$$\sqrt{1 + 4 \cos^2 t} \geq 2$$

$$\Rightarrow \cos^2 t \geq \frac{3}{4}$$

$$\Rightarrow -\frac{1}{2} \leq \sin t \leq \frac{1}{2}$$

Hence, option (a) is correct.

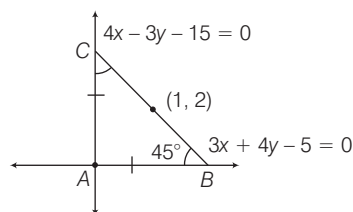
118. (c) We have,

$$\begin{aligned}
&\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \\
\Rightarrow \cos^2 \alpha + \cos^2 \left(\frac{\pi}{2} - \alpha \right) + \cos^2 \gamma &= 1 \quad \left[\text{given, } \alpha + \beta = \frac{\pi}{2} \right] \\
\Rightarrow \cos^2 \alpha + \sin^2 \alpha + \cos^2 \gamma &= 1 \\
\Rightarrow 1 + \cos^2 \gamma &= 1 \\
\Rightarrow \cos^2 \gamma &= 0 \\
\Rightarrow \cos \gamma &= 0 \\
\therefore (\cos \alpha + \cos \beta + \cos \gamma)^2 &= (\cos \alpha + \sin \alpha)^2 \\
&= 1 + 2 \sin \alpha \cdot \cos \alpha = 1 + \sin 2\alpha
\end{aligned}$$

Hence, option (c) is correct.

119. (c) The given straight lines are $3x + 4y = 5$ and $4x - 3y = 15$. Clearly, these straight lines are perpendicular to each other ($m_1 m_2 = -1$) and intersect at A. Now, B and C are points on these lines such that $AB = AC$ and BC passes through (1, 2).

From figure it is clear that $\angle B = \angle C = 45^\circ$



Let slope of BC be m . Then,

$$\tan 45^\circ = \left| \frac{m + \frac{3}{4}}{1 - \frac{3}{4}m} \right|$$

$$\Rightarrow \pm 1 = \frac{4m + 3}{4 - 3m}$$

$$4m + 3 = \pm (4 - 3m)$$

$$4m + 3 = 4 - 3m$$

$$\text{or } 4m + 3 = -4 + 3m$$

$$m = \frac{1}{7}$$

$$\text{or } m = -7$$

Hence, equation of BC is

$$y - 2 = \frac{1}{7}(x - 1)$$

$$\text{or } y - 2 = -7(x - 1)$$

$$\Rightarrow 7y - 14 = x - 1$$

$$\text{or } y - 2 = -7x + 7$$

$$\Rightarrow x - 7y + 13 = 0$$

$$\text{or } 7x + y - 9 = 0$$

Hence, option (c) is correct.

120. (d) Let $y = mx + c$, intersect $y^2 = 4ax$ at $A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$

Then,

$$\frac{2}{t_1 + t_2} = m$$

$$\Rightarrow t_1 + t_2 = \frac{2}{m}$$

Let the foot of another normal be $C(at_3^2, 2at_3)$.

$$\text{Then, } t_1 + t_2 + t_3 = 0$$

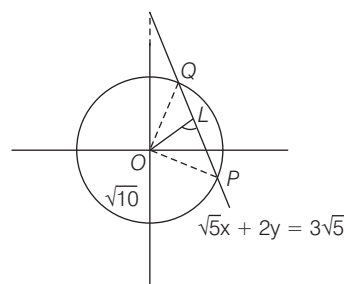
$$t_3 = (t_1 + t_2) = -\frac{2}{m}$$

$$\text{Thus, other foot is } \left(\frac{4a}{m^2}, \frac{-4a}{m} \right).$$

Hence, option (d) is correct.

121. (c) Length of perpendicular from origin to the line $x\sqrt{5} + 2y = 3\sqrt{5}$ is

$$OL = \frac{3\sqrt{5}}{\sqrt{(\sqrt{5})^2 + 2^2}} = \frac{3\sqrt{5}}{\sqrt{9}} = \sqrt{5}$$



Radius of the given circle $= \sqrt{10} = OQ = OP$

$$PQ = 2QL$$

$$= 2\sqrt{OQ^2 - OL^2}$$

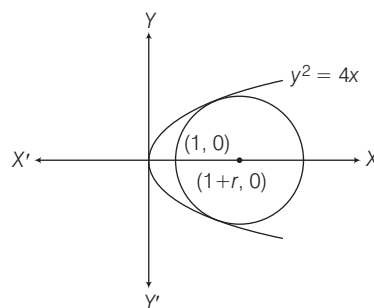
$$= 2\sqrt{10 - 5} = 2\sqrt{5}$$

$$\text{Thus, area of } \triangle OPQ = \frac{1}{2} \times PQ \times OL$$

$$= \frac{1}{2} \times 2\sqrt{5} \times \sqrt{5} = 5 \text{ sq units}$$

Hence, option (c) is correct.

122. (b) Let r be the radius of the largest circle passing through the focus $(1, 0)$ of $y^2 = 4x$



Clearly, centre of the circle will be on X -axis and its coordinates are $(1 + r, 0)$.

The equation of the circle, is $(x - 1 - r)^2 + y^2 = r^2$.

It touches $y^2 = 4x$. Therefore, the equation $(x - r - 1)^2 + 4x = r^2$ must have equal roots

$$\therefore 4(1 - r)^2 - 4(2r + 1) = 0$$

$$\Rightarrow r = 4$$

Hence, option (b) is correct.

123. (d) The point $P\left(\frac{\pi}{6}\right)$ is $\left(a \sec \frac{\pi}{6}, b \tan \frac{\pi}{6}\right)$ or $P\left(\frac{2a}{\sqrt{3}}, \frac{b}{\sqrt{3}}\right)$

$$\therefore \text{Equation of tangent at } P \text{ is } \frac{x}{\frac{\sqrt{3}a}{2}} - \frac{y}{\frac{b}{\sqrt{3}}} = 1$$

$$\therefore \text{Area of the triangle} = \frac{1}{2} \times \frac{\sqrt{3}a}{2} \times \sqrt{3}b = 3a^2$$

$$\therefore \frac{b}{a} = 4$$

$$\therefore e^2 = 1 + \frac{b^2}{a^2} = 17$$

$$\text{Now, } e^2 - 9 = 17 - 9 = 8$$

Hence, option (d) is correct.

124. (d) Let distances of a point $p(x, y, z)$ from the planes $x + y + z = 0$, $x - z = 0$ and $x - 2y + z = 0$ are $\frac{x + y + z}{\sqrt{3}}$, $\frac{x - z}{\sqrt{2}}$ and $\frac{x - 2y + z}{\sqrt{6}}$ respectively, then

the sum of the squares of distances, is as

$$\begin{aligned} \Rightarrow & \left(\frac{x + y + z}{\sqrt{3}} \right)^2 + \left(\frac{x - z}{\sqrt{2}} \right)^2 + \left(\frac{x - 2y + z}{\sqrt{6}} \right)^2 = p^2 \\ \Rightarrow & 2(x + y + z)^2 + 3(x - z)^2 + (x - 2y + z)^2 = 6p^2 \\ \Rightarrow & 2x^2 + 2y^2 + 2z^2 + 4xy + 4yz + 4zx + 3x^2 \\ & + 3z^2 - 6xz + x^2 + 4y^2 + z^2 - 4xy \\ & - 4yz + 2xz = 6p^2 \\ \Rightarrow & 6x^2 + 6y + 6z^2 = 6p^2 \\ \Rightarrow & x^2 + y^2 + z^2 = p^2 \end{aligned}$$

Hence, option (d) is correct.

125. (a) When line

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$$

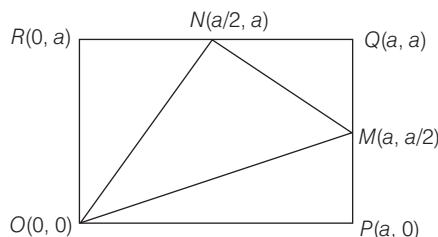
is perpendicular to plane $ax + by + cz + d = 0$, then

$$\begin{aligned} \frac{a_1}{a} &= \frac{-b_1}{b} = \frac{c_1}{c} \\ \therefore \frac{\lambda}{2} &= \frac{1}{2} = \frac{-4}{-8} \\ \Rightarrow \lambda &= 1 \end{aligned}$$

Hence, option (a) is correct.

126. (c) Taking the coordinates of vertices O, P, Q, R as $(0, 0)$, $(a, 0)$, (a, a) , $(0, a)$, respectively.

$\therefore$ The coordinates of M is $\left(a, \frac{a}{2}\right)$ and N is $\left(\frac{a}{2}, a\right)$.



$$\therefore \text{Area of } \triangle OMN = \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ a & a/2 & 1 \\ a/2 & a & 1 \end{vmatrix} = \frac{3a^2}{8}$$

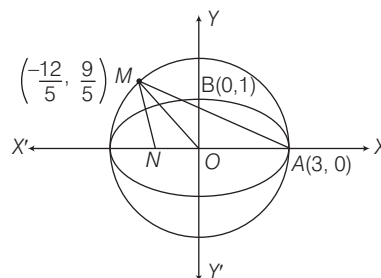
and area of the square $= a^2$

$\therefore$ The required ratio is $8 : 3$.

Hence, option (c) is correct.

127. (d) Equation of auxiliary circle is

$$x^2 + y^2 = 9 \quad \dots(i)$$



$$\therefore \text{Equation of } AM \text{ is } \frac{x}{3} + \frac{y}{1} = 1 \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$M\left(-\frac{12}{5}, \frac{9}{5}\right)$$

$$\text{Now, area of } \triangle AOM = \frac{1}{2} OA \times MN = \frac{27}{10} \text{ sq units}$$

Hence, option (d) is correct.

128. (b) Given equation can be written as

$$x^2 - y^2 = \frac{25}{3}$$

$$\begin{aligned} \therefore e_1 &= \sqrt{1 + \frac{b^2}{a^2}} \\ &= \sqrt{1 + 1} = \sqrt{2} \end{aligned}$$

The equation of conjugate hyperbola is

$$-x^2 + y^2 = \frac{25}{3}$$

$$\therefore e_2 = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + 1} = \sqrt{2}$$

$$\therefore e_1^2 + e_2^2 = (\sqrt{2})^2 + (\sqrt{2})^2 = 4$$

Hence, option (b) is correct.

129. (a) We have,

$$f(x + y) = f(x) + 2y^2 + kxy \text{ for all } x, y \in R$$

$$\Rightarrow \frac{f(x + y) - f(x)}{y} = 2y + kx \text{ for all } x \in R$$

$$\Rightarrow \lim_{y \rightarrow 0} \frac{f(x + y) - f(x)}{y} = \lim_{y \rightarrow 0} (2y + kx)$$

$$\Rightarrow f'(x) = kx \text{ for all } x \in R$$

$$\Rightarrow f(x) = \frac{kx^2}{2} + C \text{ for all } x \in R \quad [\text{by integration}]$$

$$\text{But, } f(1) = 2 \text{ and } f(2) = 8.$$

$$\therefore 2 = \frac{k}{2} + C \text{ and } 8 = 2k + C$$

$$k = 4 \text{ and } C = 0$$

$$\text{Hence, } f(x) = 2x^2 \text{ for all } x \in R$$

So, option (a) is correct.

- Hence, option (a) is correct.

- Hence, option (d) is correct.

- Hence, option (d) is correct.

-

∴ Option (b) is correct.

- Hence, option (b) is correct.

- $$\therefore \lim_{x \rightarrow 0} x^p \sin \frac{1}{x} = 0$$

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This possible only when $p > 0$

$$\begin{aligned} f'(0) &= \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{h^p \sin \frac{1}{h} - 0}{h} \\ &= \lim_{h \rightarrow 0} h^{p-1} \sin \frac{1}{h} \end{aligned}$$

$\Rightarrow f'(0)$ will exist only when $p > 0$

$\therefore f(x)$ will not be differentiable if

$$p \leq 1$$

...(i)

...(ii)

From Eqs. (i) and (ii), for $f(x)$ to be not differentiable but continuous at $x = 0$, possible values of p are given by $0 < p \leq 1$. Hence option (c) is correct.

136. (a) Integrating the given differential equation, we have

$$\frac{dy}{dx} = \frac{-\cos 3x}{3} + e^x + \frac{x^3}{3} + C_1$$

But $y_1(0) = 1$

$$\text{So, } 1 = \left(-\frac{1}{3}\right) + 1 + C_1$$

$$\Rightarrow C_1 = \frac{1}{3}$$

$$\therefore \frac{dy}{dx} = \frac{-\cos 3x}{3} + e^x + \frac{x^3}{3} + \frac{1}{3}$$

Again integrating, we get

$$y = -\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x + C_2$$

But $y(0) = 0$, so $0 = 0 + 1 + C_2$

$$\Rightarrow C_2 = -1$$

$$\text{Thus, } y = -\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x - 1$$

Hence, option (a) correct.

137. (d) Given, $f(x) = -2x^3 - 9x^2 - 12x + 1$

$$\Rightarrow f'(x) = -6x^2 - 18x - 12$$

To be decreasing $f'(x) < 0$

$$\Rightarrow -6x^2 - 18x - 12 < 0$$

$$\Rightarrow x^2 + 3x + 2 > 0$$

$$\Rightarrow (x+2)(x+1) > 0$$

Therefore, either $x < -2$ or $x > -1$

$$\Rightarrow x \in (-1, \infty) \text{ or } (-\infty, -2)$$

Hence, option (d) is correct.

138. (c) Let $\mathbf{a} = 4\hat{i} - 4\hat{k}$ and $\mathbf{b} = \hat{i} + \hat{j} + \hat{k}$

Let θ be the angle between $\mathbf{a}$ and $\mathbf{b}$

$$\begin{aligned} \therefore \cos \theta &= \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} \\ &= \frac{(4\hat{i} - 4\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})}{|\mathbf{a}| |\mathbf{b}|} \end{aligned}$$

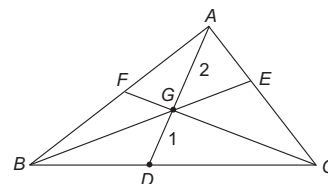
$$= \frac{4 + 0 - 4}{|\mathbf{a}| |\mathbf{b}|} = 0$$

$$\cos \theta = \cos 90^\circ \Rightarrow \theta = 90^\circ$$

$$\therefore \sin \theta + \cos \theta = \cos 90^\circ + \sin 90^\circ = 0 + 1 = 1$$

Hence, option (c) is correct.

139. (d)



$$\begin{aligned} \therefore \vec{AD} &= 3\vec{GD} \\ &= 3 \cdot \frac{1}{2}(\vec{GB} + \vec{GC}) \\ &= \frac{3}{2} \left(\frac{2}{3}\vec{EB} + \frac{2}{3}\vec{FC} \right) \\ &= -\vec{BE} - \vec{CF} \end{aligned}$$

Hence, option (d) is correct.

140. (c) Let $x_1, x_2, \dots, x_n$ be n observations.

$$\text{Then, } \bar{x} = \frac{1}{n} \sum x_i$$

$$\text{Let } y_i = \frac{x_i}{\alpha} + 10$$

$$\text{Then, } \frac{1}{n} \sum_{i=1}^n y_i = \frac{1}{\alpha} \left(\frac{1}{n} \sum x_i \right) + \frac{1}{n} (10n)$$

$$\Rightarrow \bar{x}_{\text{new}} = \frac{1}{\alpha} \bar{x} + 10 = \frac{\bar{x} + 10\alpha}{\alpha}$$

Hence option (c) is correct.

141. (c) Volume of the parallelopiped = $||[\mathbf{a} \mathbf{b} \mathbf{c}]||$

$$\Rightarrow (\text{area of the base parallelogram}) \times h = ||[\mathbf{a} \mathbf{b} \mathbf{c}]||$$

$$\Rightarrow ||\mathbf{a} \times \mathbf{b}|| h = ||[\mathbf{a} \mathbf{b} \mathbf{c}]||$$

$$\Rightarrow |-5\hat{i} + 3\hat{j} + 2\hat{k}| h = \left| \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 1 & 1 & 3 \end{vmatrix} \right|$$

$$\Rightarrow \sqrt{38} h = 4 \Rightarrow h = \frac{4}{\sqrt{38}}$$

$$\Rightarrow 38h^2 = 4^2 \Rightarrow 19h^2 = 8$$

Hence, option (c) is correct.

142. (b) Mean (m) for $BD = np$

and variance (σ^2) for $BD = npq$

Given, $np + npq = 2.7$ and $n = 3$

$$\therefore np(1+q) = 2.7 \Rightarrow p(1+q) = \frac{2.7}{3}$$

$$\Rightarrow p(1+q) = 0.9$$

$$\Rightarrow (1-q)(1+q) = 0.9$$

[$\because$ for BD probability standard deviation (SD), $p + q = 1$]

$$\Rightarrow 1 - q^2 = 0.9$$

$$\begin{aligned} \Rightarrow q^2 &= 0.1 \Rightarrow q = \pm 0.3 \\ \therefore q &= 0.3 \\ [\because -0.3 \text{ can't possible as } 0 \leq q \leq 1] \\ \Rightarrow p &= 0.7 \text{ and } q = 0.3 \\ \therefore \text{B.D.} &= (0.3 + 0.7)^5 \end{aligned}$$

Hence, option (b) is correct.

143. (b) We have,

$$R(x) = \int \frac{dx}{e^x + 8e^{-x} + 4e^{-3x}} - 2 \int \frac{dx}{e^{3x} + 8e^x + 4e^{-x}}$$

$$\Rightarrow R(x) = \int \frac{e^x (e^{2x} - 2)}{e^{4x} + 8e^{2x} + 4} dx$$

On substituting

$$e^x = t \Rightarrow e^x dx = dt,$$

we get

$$\begin{aligned} R(t) &= \int \frac{(t^2 - 2) dt}{t^4 + 8t^2 + 4} = \int \frac{(1 - 2t^{-2}) dt}{(t + 2t^{-1})^2 + 4} \\ &= \frac{1}{2} \tan^{-1} \left(\frac{t + 2t^{-1}}{2} \right) + K \end{aligned}$$

$$\Rightarrow R(x) = \frac{1}{2} \tan^{-1} \left(\frac{e^x + 2e^{-x}}{2} \right) + K$$

Hence, $(A, B, C) = (\tan^{-1}, e^x, 2)$

$\therefore$ Option (b) is correct.

$$\begin{aligned} 144. (b) \cot^{-1}(1-x+x^2) &= \tan^{-1} \left(\frac{1}{1-x+x^2} \right) \\ &= \tan^{-1} \left(\frac{1}{1-x(1-x)} \right) \\ &= \tan^{-1} \left(\frac{x+(1-x)}{1-x(1-x)} \right) \\ \Rightarrow \cot^{-1}(1-x+x^2) &= \tan^{-1} x - \tan^{-1}(1-x) \\ \therefore \int_0^1 \cot^{-1}(1-x+x^2) dx &= \int_0^1 \tan^{-1} x dx - \int_0^1 \tan^{-1}(1-x) dx \\ &= \int_0^1 \tan^{-1} x dx + \int_0^1 \tan^{-1} x dx \\ &\quad \left[\because \int_0^a f(x) dx = - \int_0^a f(a-x) dx \right] \\ &= 2 \int_0^1 \tan^{-1} x dx \end{aligned}$$

On evaluating by integration by parts, we have

$$\begin{aligned} &= 2 \left\{ [\tan^{-1} x \cdot x]_0^1 - \int_0^1 \frac{x}{1+x^2} dx \right\} \\ &= 2 \left\{ \frac{\pi}{4} - \left[\frac{1}{2} \ln(1+x^2) \right]_0^1 \right\} \\ &= 2 \left[\frac{\pi}{4} - \frac{1}{2} \log 2 \right] = \frac{\pi}{2} - \log 2 \end{aligned}$$

$$\text{Hence, } \int_0^1 \cot^{-1}(1-x+x^2) dx = \frac{\pi}{2} - \log 2$$

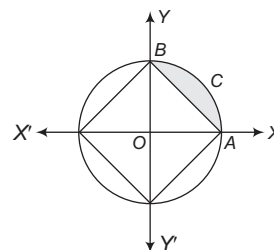
So, option (b) is correct.

145. (a) Given curves are

$$\sqrt{|x|} + \sqrt{|y|} = \sqrt{a} \quad \dots(i)$$

and

$$x^2 + y^2 = a^2 \quad \dots(ii)$$



Now, required areas = 4 [shaded area in the first quadrant]

$$\begin{aligned} &= 4 \left[\frac{\pi a^2}{4} - \int_0^a (\sqrt{a} - \sqrt{x})^2 dx \right] \\ &\quad \text{[from Equ. (i) and (ii)]} \\ &= 4 \left[\frac{\pi a^2}{4} - \int_0^a (a + x - 2\sqrt{a}\sqrt{x}) dx \right] \\ &= 4 \frac{\pi a^2}{4} - 4 \left[ax + \frac{x^2}{2} - \frac{4}{3} \sqrt{a} x^{3/2} \right]_0^a \\ &= \left(\pi - \frac{2}{3} \right) a^2 \text{ sq units} \end{aligned}$$

Hence, option (a) is correct.

$$146. (b) P(A \cap B) = \frac{1}{6} \text{ and } P(A^C \cap B^C) = \frac{1}{3}$$

$$\text{Now, } P(A \cup B)^C = P(A^C \cap B^C) = \frac{1}{3}$$

$$\Rightarrow 1 - P(A \cup B) = \frac{1}{3}$$

$$\Rightarrow P(A \cup B) = \frac{2}{3}$$

$$\text{But } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A) + P(B) = \frac{5}{6} \quad \dots(i)$$

$\therefore A$ and B are independent events

$$\therefore P(A \cap B) = P(A) P(B)$$

$$\Rightarrow P(A) P(B) = \frac{1}{6}$$

$$\begin{aligned} [P(A) - P(B)]^2 &= [P(A) + P(B)]^2 - 4P(A)P(B) \\ &= \frac{25}{36} - \frac{4}{6} = \frac{1}{36} \end{aligned}$$

$$\Rightarrow P(A) - P(B) = \pm \frac{1}{6} \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$P(A) = \frac{1}{2} \text{ or } \frac{1}{3}$$

Hence, option (b) is correct.

147. (d) Let A be the event that the examinee gives the correct answer.

Let G, C, K stand for guessing, copying and knowing, respectively.

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Given, $P(C) = \frac{1}{6}, P(G) = \frac{1}{3}$

and $P\left(\frac{A}{C}\right) = \frac{1}{8}$

$\Rightarrow P(C) + P(G) + P(K) = 1$

$\Rightarrow \frac{1}{6} + \frac{1}{3} + P(K) = 1$

$\therefore P(K) = \frac{1}{2}$

Also, $P\left(\frac{A}{K}\right) = 1$, for if the examinee knows, he/she will correctly answer it and $P\left(\frac{A}{G}\right) = \frac{1}{4}$, since there are four choices.

Now, total probability

$$P(A) = P(G)P\left(\frac{A}{G}\right) + P(C)P\left(\frac{A}{C}\right) + P(K)P\left(\frac{A}{K}\right)$$

$$= \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{6} \cdot \frac{1}{8} + \frac{1}{2} \cdot 1$$

$$= \frac{1}{12} + \frac{1}{48} + \frac{1}{2} = \frac{4 + 1 + 24}{48} = \frac{29}{48}$$

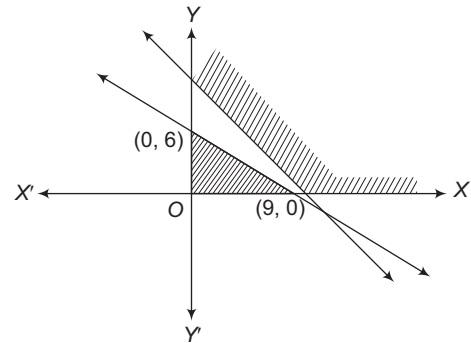
Now,
$$P\left(\frac{K}{A}\right) = \frac{P(K)P\left(\frac{A}{K}\right)}{P(A)}$$

$$= \frac{\frac{1}{2} \cdot 1}{\frac{29}{48}} = \frac{1}{2} \cdot \frac{48}{29} = \frac{24}{29}$$

Hence, option (d) is correct.

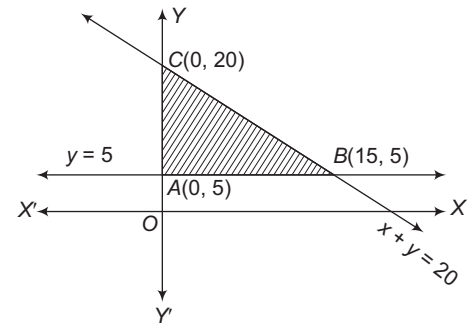
148. (d) Since, $p : 4$ is an even prime number,
 $q : 6$ is a divisor of 12 and $r : \text{the HCF of 4 and 6 is 2}$.
 So, $\sim p \vee (q \wedge r)$ is correct.
 Hence, option (d) is correct.

149. (d) From the figure, it is clear that there is no common area.
 So, we cannot find maximum value of z .



Hence, option (d) is correct.

150. (d) Feasible region is ABCA and $Z = 7x - 8y$



at $A(0, 5), Z = 7 \times 0 - 8 \times 5 = -40$

at $B(15, 5), Z = 7(15) - 8(5) = 65$

at $C(0, 20), Z = 7(0) - 8(20) = -160$

Hence, the minimum value of Z is attained at point $(0, 20)$.
 So, option (d) is correct.

Solved Paper 2019

BITSAT

Instructions

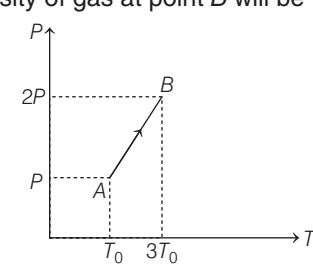
- There are 150 questions in all. The number of questions in each part is as given below.

Part I Physics	1-40
Part II Chemistry	41-80
Part III a. English Proficiency	81-95
b. Logical Reasoning	96-105
Part IV Mathematics	106-150
- All questions are multiple choice questions with four options, only one is correct.
- Each correct answer fetches 3 marks while incorrect answer has a penalty of 1 mark.

No. of Questions

PART I

Physics

- A radioactive sample of half life 23.1 days is disintegrating continuously. The percentage decay of its in 15th to 16th days will be
[Take, $e^{0.03} = 1.03$]
a. 5% b. 1% c. 2.9% d. 3.5%
- Two separate soap bubbles of radii 3×10^{-3} m and 2×10^{-3} m respectively, formed of same liquid (surface tension 6.5×10^{-2} N/m) come together to form a double bubble. The radius of interface of double bubble is
a. 6×10^{-3} m b. 4×10^{-3} m
c. 15×10^{-3} m d. 0.66×10^{-3} m
- An electron revolves in a circular orbit of radius r with angular speed ω . The magnetic field at the centre of electron orbit is
a. $\frac{\mu_0 e \omega}{\pi r}$ b. $\frac{\mu_0 e \omega}{4 \pi r}$
c. $\frac{\mu_0 e^2 \omega}{4 \pi r}$ d. $\frac{\mu_0 e \omega}{2 \pi r}$
- A steel wire of cross-sectional area 4 cm^2 has elastic limit of $2.2 \times 10^8 \text{ N/m}^2$. The maximum upward acceleration that can be given to a 1000 kg elevator supported by this steel wire if the stress is to exceed one-fourth of the elastic limit is [Take, $g = 10 \text{ m/s}^2$]
a. 10 m/s^2 b. 9 m/s^2 c. 15 m/s^2 d. 12 m/s^2
- An ideal monoatomic gas at 300 K expands adiabatically to twice its volume. The final temperature of gas is
a. $300\sqrt{2}$ b. $300\sqrt{3}$ c. $300\left(\frac{1}{2}\right)^{2/3}$ d. $300(2)^{2/3}$
- Two sound producing sources A and B are moving towards and away from a stationary observer with same speed respectively. If frequency of sound produced by both sources are equal as 400 Hz, then speed of sources (approximately) when observer detects 4 beats per second, is
[Given, speed of sound = 340 m/s]
a. 1.7 m/s b. 3.4 m/s c. 2.4 m/s d. 1 m/s
- Pressure versus temperature graph of an ideal gas is shown in the given figure. Density of gas at point A is ρ_0 , then density of gas at point B will be

a. $2\rho_0$ b. $\frac{2}{3}\rho_0$ c. $\frac{3}{2}\rho_0$ d. $3\rho_0$

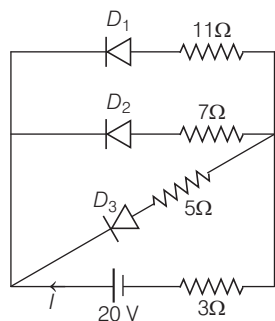
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8. A rubber cord has a cross-sectional area 10^{-6} m^2 and total unstretched length 0.1 m. It is stretched to 0.125 m and then released to project a particle of mass 5.0 g. The velocity of projection is [Given, Young's modulus of rubber, $Y = 5 \times 10^8 \text{ N/m}^2$]
- a. 45 m/s b. 30 m/s c. 25 m/s d. 15 m/s

9. A body is moving unidirectionally under the influence of a sources of constant power. The square of its displacement in time t is proportional to
- a. t^3 b. t^2 c. t d. $\sqrt{t}$

10. Two electrons are moving with speed of $5 \times 10^5 \text{ m/s}$ parallel to each other, then the electrostatic and magnetic force between them is
- a. 5×10^5 b. 2.5×10^4
c. 3.6×10^5 d. 4.4×10^3

11. In the following circuit diagram, the current through battery is



- a. 4 A b. 3 A
c. 1 A d. 2 A

12. A semicircular disc of mass M and radius R is free to rotate about its diameter. The moment of inertia of semicircular disc about a line perpendicular to its plane through centre is

- a. $\frac{3}{4} MR^2$ b. $\frac{MR^2}{2}$
c. $\frac{MR^2}{3}$ d. $\frac{MR^2}{4}$

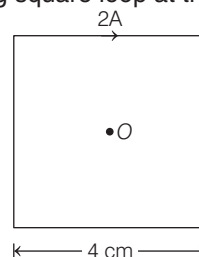
13. 12 eV energy is given to electron in third orbit of H-atom, then final energy of electron when it ionise from third orbit, is

- a. 5.25 eV b. 10.49 eV
c. 12 eV d. 11.15 eV

14. In Young's double slit experiment, the distance between slits and screen is 2 m and distance between slits is 0.25 mm. A light of wavelength 800 nm is used to find fringes on the screen. If screen moves with a speed of 5 m/s, then first maxima will move with a speed of

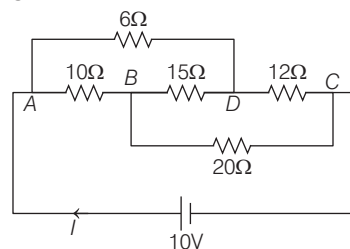
- a. 16 mm/s b. 8 mm/s
c. 20 m/s d. 50 mm/s

15. The magnetic field at the point O in the following current carrying square loop at the centre will be



- a. $12 \times 10^{-5} \text{ T}$ b. $4 \times 10^{-5} \text{ T}$
c. $4\sqrt{2} \times 10^{-5} \text{ T}$ d. $2\sqrt{2} \times 10^{-5} \text{ T}$

16. The value of current I as shown in the given circuit diagram is



- a. 2 A b. 1.5 A c. 0.9 A d. 0.2 A

17. A semicircular wire of radius r rotates in uniform magnetic field B about its diameter with angular velocity ω . If the total resistance of the circuit is R , then the mean power generated per period of rotation is

- a. $\frac{(\pi Br^2 \omega)^2}{8R}$ b. $\frac{\pi Br \omega^2}{8R}$ c. $\frac{(\pi Br \omega)^2}{8R}$ d. 0

18. A convex lens of focal length 25 cm produces images of the same magnification 2, when an object is kept at two positions x_1 and x_2 ($x_1 > x_2$) from the lens. The ratio of x_2 and x_1 is

- a. 2 : 1 b. 1 : 3 c. 1 : 2 d. 3 : 1

19. A particle X of mass m and initial velocity u collide with another particle Y of mass $\frac{3m}{4}$ which is at rest, The

- collision is head on and perfectly elastic. The ratio of de-Broglie wavelengths λ_Y and λ_X after the collision is
- a. 4 : 3 b. 2 : 32 c. 3 : 4 d. 3 : 32

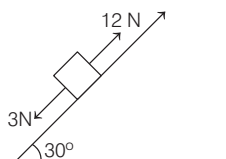
20. The dimensional formula of $\sqrt{\frac{\mu_0}{\epsilon_0}}$ is

- a. $[\text{ML}^2\text{T}^{-3}\text{A}^2]$ b. $[\text{M}^0\text{L}\text{T}^{-1}\text{A}^0]$
c. $[\text{ML}^2\text{T}^{-3}\text{A}^{-2}]$ d. $[\text{M}^{-1}\text{L}^{-2}\text{T}^3\text{A}^2]$

21. A body of mass m is moving in a straight line with momentum p . Starting at time $t = 0$, a force $F = at$ acts in the same direction on the moving particle during time interval of T . So that its momentum changes from p to $2p$. The value of T is

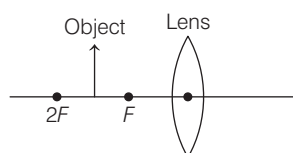
- a. $\sqrt{\frac{2p}{a}}$ b. $\sqrt{\frac{p}{a}}$ c. $2\sqrt{\frac{2p}{a}}$ d. $\frac{2p}{a}$

22. When a maximum force of 3 N is applied on a body kept on rough inclined plane of shown in the figure, then body remains stationary. The maximum external force up the inclined plane that does not move the block is 12 N. The coefficient of static friction between the block and the plane is [Take, $g = 10 \text{ m/s}^2$]

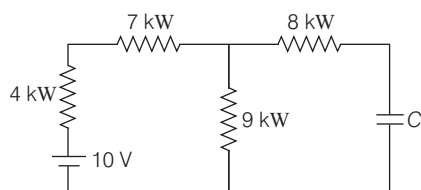


- a. $\frac{\sqrt{3}}{5}$ b. $\frac{1}{\sqrt{3}}$ c. $\frac{3\sqrt{3}}{5}$ d. $\frac{5}{3\sqrt{3}}$

23. In the shown situation, if middle portion of the lens is painted black, then



- a. complete image of the object will form with low intensity
 b. complete image of the object will form with high intensity
 c. incomplete image of the object will form with low intensity
 d. incomplete image of the object will form with high intensity
24. If sound travels in air with the speed of 340 m/s, then number of tones present in an open organ pipe of length 2 m and its maximum frequency 1200 Hz, are
 a. 17 b. 11 c. 9 d. 14
25. A proton is revolving on a circular path of radius 2 mm with frequency 10 Hz. Magnetic dipole moment associated with proton is
 a. $2 \times 10^{-24} \text{ A-m}^2$ b. $4 \times 10^{-24} \text{ A-m}^2$
 c. $3 \times 10^{-20} \text{ A-m}^2$ d. $6 \times 10^{-20} \text{ A-m}^2$
26. When a capacitor is fully charged as shown in the following figure, then current drawn from the cell is



- a. 1 mA b. 0.5 mA c. 2 mA d. 5 mA
27. A charged capacitor of capacitance C is discharging through a resistor of resistance R . At what time the charge on the capacitor is equal to one half of its initial value?

- a. $\frac{RC}{2}$ b. $\frac{RC}{\log_e 2}$ c. $RC \log_e 2$ d. $\frac{1}{RC} \log_e 2$

28. The escape velocity of the earth surface is v_e . A body is projected with velocity $3v_e$. With what constant velocity will it move in the inter planetary space?
 a. v_e b. $3v_e$ c. $\sqrt{2} v_e$ d. $2\sqrt{2} v_e$

29. The time period of a bob performing simple harmonic motion in water is 2s. If density of bob is $\frac{4}{3} \times 10^3 \text{ kg/m}^3$, then time period of bob performing simple harmonic motion in air will be
 a. 3s b. 4s c. 2s d. 1s

30. A uniform solid cylindrical roller of mass m is being pulled on horizontal surface with force F parallel to the surface applied at its centre. If the acceleration of the cylinder is a and it is rolling without slipping, then the value of F is
 a. $2ma$ b. $3ma$ c. $\frac{3ma}{2}$ d. $\frac{5ma}{2}$

31. A projectile is given an initial velocity of $(\hat{i} + \sqrt{3} \hat{j}) \text{ m/s}$, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. Then, the equation of the path of projectile is [Take, $g = 10 \text{ m/s}^2$]

- a. $y = \sqrt{3}x - 5x^2$ b. $y = \sqrt{3}x + 5x^2$
 c. $x = \sqrt{3}y + 5x^2$ d. $x^2 = y^2 + \sqrt{3}$

32. The force on a body of mass 1 kg is $(20\hat{i} + 10\hat{j}) \text{ N}$. If it starts from rest, then the position of the body at time $t = 2 \text{ s}$, is
 a. $-20\hat{i} - 40\hat{j}$ b. $20\hat{i} - 40\hat{j}$
 c. $40\hat{i} - 20\hat{j}$ d. $40\hat{i} + 20\hat{j}$

33. A cylinder of mass 2 kg is released from rest from the top of an inclined plane of inclination 30° and length 1m. If the cylinder rolls without slipping, then its speed when it reaches the bottom, is [Take, $g = 10 \text{ m/s}^2$]
 a. $\frac{20}{3} \text{ m/s}$ b. $\sqrt{\frac{20}{3}} \text{ m/s}$
 c. $\frac{10}{3} \text{ m/s}$ d. $\sqrt{\frac{10}{3}} \text{ m/s}$

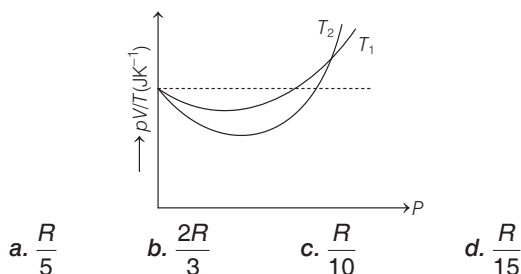
34. Frequency of oscillation of a body is 5 Hz when a force F_1 is applied and 12 Hz when another force F_2 is applied. If both forces F_1 and F_2 are applied together, then frequency of oscillation of the body will be
 a. 13 Hz b. 169 Hz
 c. 62 Hz d. 52 Hz

35. The figure shows the graph of $\frac{pV}{T}$ versus p for

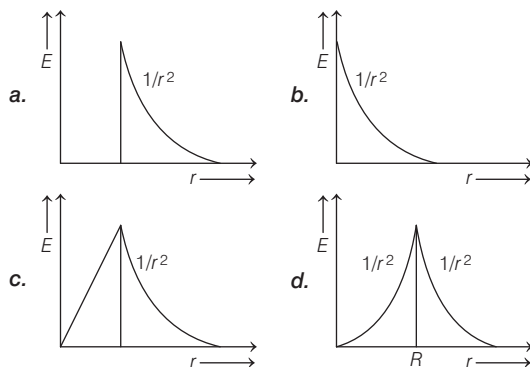
$2 \times 10^{-4} \text{ kg}$ of hydrogen gas at two different temperatures, where p , V and T represents pressure, volume and temperature respectively.

Then, the value of $\frac{pV}{T}$, where the curve meet on the vertical axis, is

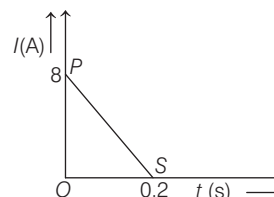
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36. An ammeter of resistance 1.5Ω can measure currents upto 1 A. The value of shunt resistance to measure current upto 4 A is
a. 0.5Ω b. 1Ω c. 1.5Ω d. 0.2Ω
37. In the following figures, which represents the variation of the electric field with distance from the centre of a uniformly charged non-conducting spheres of radius R ?



38. A carbon resistor of $(56 \pm 5.6) \text{ k}\Omega$ is to be marked with rings of different colours for its identification. The colour code sequence will be
a. green, blue, orange, silver
b. blue, yellow, orange, silver
c. green, red, orange, silver
d. red, green, blue, gold
39. Two monochromatic light waves are travelling with same frequency and constant phase different. If both waves interfere, then
a. there is a gain of energy
b. there is a loss of energy
c. the energy is redistributed and the distribution remains constant in time
d. the energy is redistributed and the distribution changes with time
40. In a coil of resistance 50Ω , the induced current developed by changing magnetic flux through it, is shown in figure as a function of time. The magnitude of change in flux through the coil is

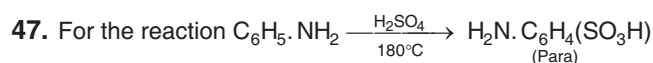


- a. 10 Wb b. 20 Wb
c. 32 Wb d. 40 Wb

PART II

Chemistry

41. In a compound, atoms of elements Y form ccp lattice and those of element X occupy $2/3$ rd of tetrahedral voids. The formula of the compound can be
a. X_4Y_3 b. X_2Y_3 c. X_2Y d. X_3Y_4
42. The density of a certain gas at 30°C and 768 torr is 1.35 kg/m^3 , then density of the gas at STP is
a. 1.48 kg/m^3 b. 1.27 kg/m^3
c. 1.35 kg/m^3 d. 1.00 kg/m^3
43. For the octahedral complex, which of the following d -electron configuration will give maximum value of CFSE
a. high spin, d^6 b. low spin, d^4
c. low spin, d^5 d. high spin, d^7
44. The wavelength corresponding to maximum energy for hydrogen is 91.2 nm. The corresponding wavelength for He^+ ion is
a. 2.28 nm b. 22.8 nm c. 182.4 nm d. 364.8 nm
45. The heat of reaction for
 $\text{C}_{18}\text{H}_8(\text{s}) + 12 \text{ O}_2(\text{g}) \longrightarrow 10 \text{ CO}_2(\text{g}) + 4 \text{ H}_2\text{O}(\text{l})$ at constant volume is -1228.2 kcal at 25°C . The heat of reaction at constant pressure and same temperature is
a. -1228.2 kcal b. -1229.3 kcal
c. -1232.9 kcal d. -1242.6 kcal
46. Given the hypothetical reaction mechanism
 $A \xrightarrow{\text{I}} B \xrightarrow{\text{II}} C \xrightarrow{\text{III}} D \xrightarrow{\text{IV}} E$ and the rate as
- | Species formed | Rate of its formation |
|----------------|---------------------------------------|
| B | 0.002 mol/h per mole of A |
| C | 0.030 mol/h per mole of B |
| D | 0.011 mol/h per mole of C |
| E | 0.420 mol/h per mole of D |
- The rate determining step is
a. step I b. step II c. step III d. step IV



the true statement about the product is

- it does not exist as Zwitter ion
- $-\text{NH}_2$ group displays a powerful basic character
- it does not act as inner salt
- $-\text{SO}_3\text{H}$ diminishes the basic character of $-\text{NH}_2$

48. The hybridisation of phosphorous in PO_4^{3-} is

- sp
- sp^2
- sp^3
- sp^3d

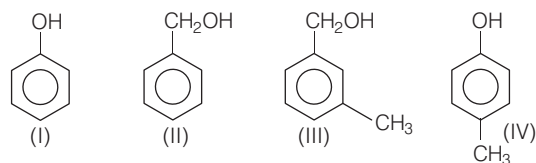
49. Which of the following lanthanoid is diamagnetic?

Atomic no's

Sm = 62, Ce = 58, Eu = 63, Yb = 70

- Sm^{2+}
- Eu^{2+}
- Yb^{2+}
- Ce^{2+}

50. Which of the following is/are aromatic alcohol?

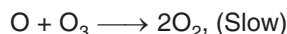
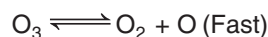
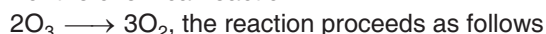


- I, II, III, IV
- II and III
- I and IV
- Only I

51. Among the following substituted silanes, the one which will give rise to cross linked silicone polymer on hydrolysis is?

- R_4Si
- R_2SiCl
- R_3SiCl
- R_3SiCl_2

52. For the chemical reaction



the rate law expression should be given as

- $r = k[\text{O}_3]^2[\text{O}_2]^{-1}$
- $r = k[\text{O}_3]^2$
- $r = k[\text{O}_3][\text{O}_2]$
- unpredictable

53. An aqueous solution freezes at 272.4 K, while pure water freezes at 273 K, given $K_f = 1.86 \text{ K kg mol}^{-1}$, and $K_b = 0.512 - k \text{ kg mol}^{-1}$; the molality of solution and boiling point of solution respectively will be—

- 0.322 and 373.16 K
- 0.222 and 273.15 K
- 0.413 and 400 K
- 0.5 and 300.73 K

54. The order of acidic strength of boron trihalides

- $\text{BF}_3 < \text{BCl}_3 < \text{BBr}_3 < \text{BI}_3$
- $\text{BI}_3 < \text{BBr}_3 < \text{BCl}_3 < \text{BF}_3$
- $\text{BCl}_3 < \text{BBr}_3 < \text{BI}_3 < \text{BF}_3$
- $\text{BBr}_3 < \text{BCl}_3 < \text{BF}_3 < \text{BI}_3$

55. The correct order of mobility of alkali metal ions in aqueous solution is—

- $\text{Li}^+ > \text{Na}^+ > \text{K}^+ > \text{Rb}^+$
- $\text{Na}^+ > \text{K}^+ > \text{Rb}^+ > \text{Li}^+$
- $\text{K}^+ > \text{Rb}^+ > \text{Na}^+ > \text{Li}^+$
- $\text{Rb}^+ > \text{K}^+ > \text{Na}^+ > \text{Li}^+$

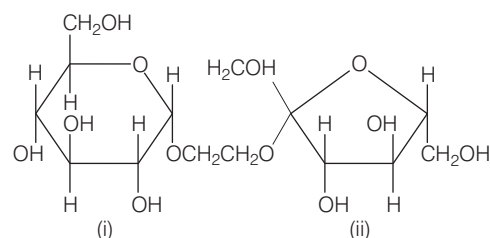
56. Thermosetting polymer bakelite is formed by the reaction of phenol with

- $\text{CH}_3\cdot\text{CH}_2\cdot\text{CHO}$
- $\text{CH}_3\cdot\text{CHO}$
- $\text{H}\cdot\text{CHO}$
- HCOOH

57. The drug that is antipyretic as well as analgesic is

- chloroquine
- penicillin
- paracetamol
- chlorpromazine hydrochloride

58. The correct statement about the following disaccharide is—



- Ring (I) is pyranose with α -glycosidic link
- Ring (I) is furanose with α -glycosidic link
- Ring (II) is furanose with α -glycosidic link
- Ring (II) is pyranose with α -glycosidic link

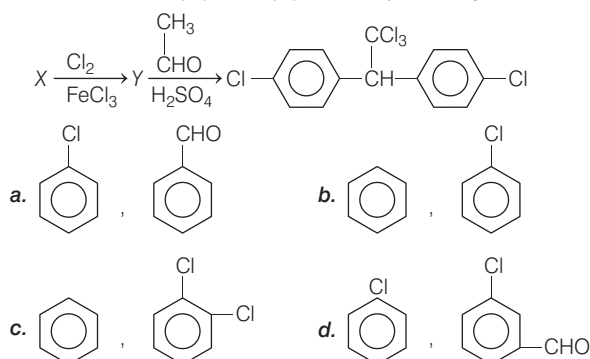
59. If one strand of DNA has sequence ATCGTATG, the sequence in the complementary strand would be

- TAGCTTAC
- TCACATAC
- TAGCATAC
- TACGATAC

60. Which of the following salt would give SO_2 with hot and dil H_2SO_4 and also decolourises Br_2 water?

- Na_2SO_3
- NaHSO_4
- Na_2SO_4
- Na_2S

61. In the reaction (X) and (Y) are respectively.



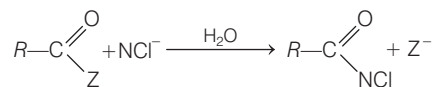
62. Which of the following reactions is an example of use of water gas in the synthesis of other compounds?

- $\text{CH}_4(g) + \text{H}_2\text{O}(g) \xrightarrow[1270 \text{ K}]{\text{Ni}} \text{CO}(g) + \text{H}_2(g)$
- $\text{CO}(g) + \text{H}_2\text{O}(g) \xrightarrow[673 \text{ K}]{\text{Catalyst}} \text{CO}_2(g) + \text{H}_2(g)$
- $\text{C}_n\text{H}_{2n+2} + n \cdot \text{H}_2\text{O}(g) \xrightarrow[1270 \text{ K}]{\text{Ni}} n \cdot \text{CO} + (2n + 1) \text{H}_2$
- $\text{CO}(g) + 2\text{H}_2(g) \xrightarrow[\text{Catalyst}]{\text{Cobalt}} \text{CH}_3\cdot\text{OH}(l)$

6 SELF STUDY GUIDE BITSAT

63. Which of the following compounds, on reaction with NaOH and Na_2O_2 gives yellow colour?
a. $\text{Zn}(\text{OH})_2$ b. $\text{Al}(\text{OH})_3$ c. $\text{Cr}(\text{OH})_3$ d. CaCO_3

64. For the reaction—



rate of reaction is faster, when Z is—

- a. Cl b. NH_2 c. OC_2H_5 d. OCOCH_3
65. E_1 , E_2 and E_3 are the emf of the following three galvanic cells respectively
(i) $\text{Zn}(\text{s}) | \text{Zn}^{2+} (0.1\text{M}) || \text{Cu}^{2+} (1\text{M}) | \text{Cu}(\text{s})$
(ii) $\text{Zn}(\text{s}) | \text{Zn}^{2+} (1\text{M}) || \text{Cu}^{2+} (1\text{M}) | \text{Cu}(\text{s})$
(iii) $\text{Zn}(\text{s}) | \text{Zn}^{2+} (1\text{M}) || \text{Cu}^{2+} (0.1\text{M}) | \text{Cu}(\text{s})$

Which of the following is true?

- a. $E_2 > E_1 > E_3$ b. $E_1 > E_2 > E_3$
c. $E_3 > E_1 > E_2$ d. $E_3 > E_2 > E_1$
66. Sodium nitroprusside, when added to an alkaline solution of sulphide ions, produce a
a. red colouration b. blue colouration
c. purple colouration d. brown colouration
67. For a reverse reaction, $A \rightleftharpoons B$, which one of the following statement is wrong from the given energy. Profile diagram?
a. Activation energy of forward reaction is greater than backward reaction
b. The forward reaction is endothermic
c. The threshold energy is less than that of activation energy
d. The energy of activation of forward reaction is equal to the sum of heat of reaction and the energy of activation of backward reaction
68. Which of the following reactants is used for the preparation of ethyl benzene, where anhyd. AlCl_3 is a catalyst?
a. $\text{CH}_3 \cdot \text{CH}_2 \cdot \text{OH} + \text{C}_6\text{H}_6$ b. $\text{CH}_3-\text{CH}=\text{CH}_2 + \text{C}_6\text{H}_6$
c. $\text{H}_2\text{C}=\text{CH}_2 + \text{C}_6\text{H}_6$ d. $\text{CH}_3-\text{CH}_3 + \text{C}_6\text{H}_6$
69. Given pH of a solution 'A' is '3' and it is mixed with another solution 'B' having pH '2'. If both solutions are mixed, the resultant pH of the solution will be
a. 3.2 b. 1.9 c. 3.4 d. 3.5
70. In Freundlich adsorption isotherm, the value of $1/n$ is
a. between 0 and 1 in all cases
b. between 2 and 4 in all cases
c. always 1 in case of physical adsorption
d. always 1 in use of chemical adsorption

71. Spin only magnetic moment of Mn^{4+} ion is nearly
a. 3 BM b. 6 BM
c. 4 BM d. 5 BM

72. 0.45 g acid of molecular mass 90 was neutralise by 20 ml of 0.5 N KOH. The basicity of the acid is
a. 2 b. 4 c. 1 d. 3

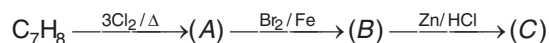
73. 4 moles of A are mixed with 4 moles of B. At equilibrium for the reaction $A + B \rightleftharpoons C + D$, 2 moles of C and D are formed, the equilibrium constant for the reaction will be,
a. 1 b. 1/2
c. 4 d. 1/4

74. What time in sec is required for depositing all the silver present in 125 ml of 1 M AgNO_3 solution by passing a current of 241.25 A (Given $\text{IF} = 96500 \text{ C}$)
a. 10 b. 50
c. 100 d. 1000

75. In a homonuclear molecule, which of the following set of orbitals is degenerated?
a. $\sigma^* 1s$ and $\sigma 2s$ b. $\pi 2p_x$ and $\pi 2p_y$
c. $\pi^* 2p_x$ and $\sigma^* 2p_z$ d. $\sigma 2p_x$ and $\sigma^* 2p_z$

76. If the photon of wavelength 150 pm strikes an atom and one of its inner bond electron is ejected out with a velocity of $1.5 \times 10^7 \text{ m/s}$, what is the energy with which it is bond to the nucleus?
a. $1.2 \times 10^2 \text{ eV}$ b. $2.15 \times 10^3 \text{ eV}$
c. $7.6 \times 10^3 \text{ eV}$ d. $8.12 \times 10^3 \text{ eV}$

77. Identify compound (C) in the following reaction



- a. o-bromotoluene b. m-bromotoluene
c. p-bromotoluene d. 3-bromo, 2, 4, 6 trichlorotoluene
78. In a compound C, H and N are present in 9 : 1 : 3.5 by weight. If molecular weight of compound is 108, the molecular formula of compound is
a. $\text{C}_2\text{H}_6\text{N}_2$ b. $\text{C}_3\text{H}_4\text{N}$
c. $\text{C}_6\text{H}_8\text{N}_2$ d. $\text{C}_9\text{H}_{12}\text{N}_3$
79. Which of the following compounds contain all the carbon atoms in the same hybridisation state
a. $\text{H}-\text{C}\equiv\text{C}-\text{C}\equiv\text{C}-\text{H}$ b. $\text{CH}_3-\text{C}\equiv\text{C}-\text{CH}_3$
c. $\text{CH}_2=\text{C}=\text{CH}_2$ d. $\text{CH}_2=\text{CH}-\text{C}\equiv\text{CH}$
80. The final step for extraction of copper from copper pyrite in bessemer converter involves the reaction
a. $\text{Cu}_2\text{S} + 2\text{Cu}_2\text{O} \longrightarrow 6\text{Cu} + \text{SO}_2$
b. $4\text{Cu}_2\text{O} + \text{FeS} \longrightarrow 8\text{Cu} + \text{FeSO}_4$
c. $2\text{Cu}_2\text{O} + \text{FeS} \longrightarrow 4\text{Cu} + \text{Fe} + \text{SO}_2$
d. $\text{Cu}_2\text{S} + 2\text{FeO} \longrightarrow 2\text{Cu} + \text{Fe} + \text{SO}_2$

Part III

a. English Proficiency

Directions (Q. Nos 81-83) *In the following questions, the sentences may or may not be grammatically correct. Find out which part of a sentence has an error and mark that part. If is no error mark part 'd' as your answer.*

81. Many a man **a.** / have come to India from Bangladesh **b.** / to live here permanently. **c.** / No error **d.**
82. How long it takes **a.** / to travel from Chennai to Trichy **b.** / by train? **c.** / No error **d.**
83. One should respect **a.** / the religions of others **b.** / as much as his own. **c.** / No error **d.**

Directions (Q. Nos. 84 and 85) *Fill in the blanks with suitable preposition from the alternatives given under each sentence.*

84. The master assured her success in the examination.
a. of **b.** in **c.** for **d.** with
85. Why do we always have to submit his authority?
a. under **b.** with **c.** for **d.** to

Directions (Q. Nos. 86-88) *Select the word or the phrase that is closest to the opposite in meaning of the italicised or phrase.*

86. The officer made an *audacious* remark.
a. Mute **b.** Visible **c.** Cultivated **d.** Timid
87. They put forward *cogent* arguments for their membership.
a. Logical **b.** Weighty **c.** Dissuasive **d.** Persuasive
88. His administration was *tainted* by scandal
a. Construct **b.** Clear **c.** Purify **d.** Repair

Directions (Q. Nos. 89 and 90) *Choose the word nearest meaning to the underlined word.*

89. Her mother is vivacious and elegant.
a. Imaginary **b.** Lively **c.** Perceptible **d.** Languid
90. Sporadic fighting broke out in the office.
a. Timely **b.** Scattered **c.** Frequent **d.** Irrelevant

Directions (Q. 91-95) *Read the passage given below and answer the questions that follow.*

Radically changing monsoon patterns, reduction in the winter rice harvest and a quantum increase in respiratory diseases-all part of the environmental doomsday scenario which is reportedly playing out in South Asia. According to a United Nations Environment Programme report, a deadly 3-km deep blanket of pollution comprising a fearsome cocktail of ash, acids, aerosols and other particles has enveloped this region.

For India, already struggling to cope with a drought, the implications of this are devastating and further crop failure will amount to a life and death question for many Indians. The increase in premature deaths will have adverse social and economic consequences and a rise in **morbidities** will place an unbearable burden on our crumbling health system. And there is no one to blame but ourselves.

Both official and corporate India has always been **allergic** to any mention of clean technology. Most mechanical two wheelers roll off the assembly line without proper pollution control system.

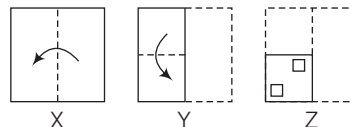
Little effort is made for R & D on simple technologies, which could make a difference to people's lives and the environment. However, while there is no denying that South Asia must clean up its act, skeptics might question the timing of the haze report.

91. Both official and corporate India is allergic to
a. failure of monsoon
b. poverty and inequality
c. slowdown in industrial production
d. mention of clean technology
92. Which, according to the passage, is a life and death question to many Indians?
a. Increase in respiratory diseases
b. Use of clean technology
c. Thick blanket of pollution over the region
d. Failure in crops
93. Choose the word which is similar in meaning to the word 'profligacy' as used in the passage.
a. wastefulness **b.** conservation
c. upliftment **d.** criticalness
94. According to the passage, India cannot tolerate any further
a. crop failure
b. deterioration of healthcare system
c. increase in respiratory diseases
d. proliferation of nuclear devices
95. What could be the reason behind timing of the haze report just before the Johannesburg meet, as indicated in the passage?
a. United Nations is working hand-in-glove with US
b. Organises the forthcoming meet to teach a lesson to the US
c. Drawing attentions of the world towards devastating effects of environment degradation
d. US wants to use it as a handle against the developing countries in the forthcoming meet

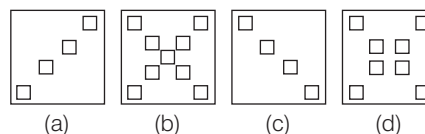
b. Logical Reasoning

96. College' is related to 'Student' in the same way as 'Hospital' is related to
a. Doctor b. Nurse c. Medicine d. Patient
97. Choose the odd one.
a. 27 b. 35
c. 18 d. 9
98. Complete the series by replacing questions mark "?"
1, 2, 2, 4, 3, 8, 7, 10, ?
a. 9 b. 8
c. 11 d. 13
99. Mohan is taller than Shyam but shorter than Ramesh. Ramesh is taller than Rajat but shorter than Gautam. If Shyam is taller than Rajat, then who is the shortest among all?
a. Gautam b. Rajat
c. Shyam d. Ramesh
100. In the following question, complete the missing segment by selecting the appropriate figure from the given alternatives, a., b., c. and d..
- Problem Figure** **Answer Figures**
-
101. In the question given below, find out which of the figures can be formed from the pieces given in the problem figure.
- Problem Figure** **Answer Figures**
-
102. In the following question, a set of three figures (X), (Y) and (Z) have been given, showing a sequence in which paper is folded and finally cut from a particular section. These figures are followed by a set of answer figures marked a., b., c. and d. showing the design which the paper actually acquires when it is unfolded. You need to select the answer figure which is closest to the unfolded piece of paper.

Problem Figures

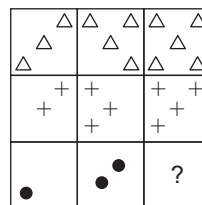


Answer Figures

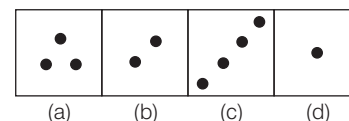


103. Which of the answer figure (a), (b), (c) or (d) completes the figures matrix?

Problem Figure

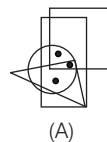


Answer Figures

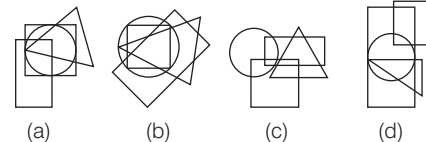


104. In the following question, some dots are placed in the figure marked as (A). The figure is followed by four alternatives marked as (a), (b), (c) and (d). One out of these four options contains region(s) common to the circle, square, triangle, similar to that marked by the dot in figure (A). Choose the correct option.

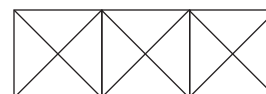
Question Figure



Answer Figures



105. How many triangles are there in the given figure?



- a. 28 b. 24
c. 25 d. 26

Part IV

Mathematics

106. If the length of the major axis of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is three times the length of minor axis, its eccentricity is
 a. $\frac{1}{3}$ b. $\frac{1}{\sqrt{3}}$ c. $\sqrt{\frac{2}{3}}$ d. $\frac{2\sqrt{2}}{3}$
107. $\sum_{m=1}^n \tan^{-1}\left(\frac{2m}{m^4 + m^2 + 2}\right)$ is equal to
 a. $\tan^{-1}\left(\frac{n^2 + n}{n^2 + n + 2}\right)$ b. $\tan^{-1}\left(\frac{n^2 - n}{n^2 - n + 2}\right)$
 c. $\tan^{-1}\left(\frac{n^2 + n + 2}{n^2 + n}\right)$ d. None of these
108. Let $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$, then the value of $\frac{a_8}{a_{12}}$, is
 a. 2 b. 4 c. 8 d. 16
109. Let ABC be an acute-angled triangle and let D be the mid-point of BC . If $AB = AD$, then $\frac{\tan B}{\tan C}$ equal
 a. $\sqrt{2}$ b. $\sqrt{3}$
 c. 2 d. 3
110. Suppose the limit $L = \lim_{n \rightarrow \infty} \sqrt{n} \int_0^1 \frac{1}{(1+x^2)^n} dx$ exists and is larger than $\frac{1}{2}$, then
 a. $\frac{1}{2} < L < 2$ b. $2 < L < 3$ c. $3 < L < 4$ d. $L \geq 4$
111. In the real number system, the equation $\sqrt{x+3} - 4\sqrt{x-1} + \sqrt{x+8} - 6\sqrt{x-1} = 1$ has
 a. No solution
 b. Exactly two distinct solution
 c. Exactly four distinct solution
 d. Infinitely many solution
112. If system of equation $ax + y + z = a$, $x + by + z = b$ and $x + y + cz = c$ is inconsistent, then which of the following is correct?
 a. $abc - a - b - c + 2 = 0$
 b. $abc - a - b - c + 3 = 0, a = 1$
 c. $abc - a - b - c + 3 = 0$
 d. $abc - a - b - c + 2 = 0, a \neq 1, b \neq 1, c \neq 1$
113. A die is thrown 7 times. What is the probability that an even number turns up at least 4 times?
 a. $\frac{1}{2}$ b. $\frac{31}{64}$ c. $\frac{51}{128}$ d. $\frac{35}{128}$
114. The equation of plane containing line $x - y = 1, z = 1$ and parallel to $\frac{x}{2} - \frac{z}{3} = 1, y = 3$ is
 a. $3x + 3y - 2z = 1$ b. $3x - 3y - 2z = 1$
 c. $3x + 3y + 2z = 1$ d. $3x + 3y + 2z = -1$
115. If $(1+x+x^2)^{20} = \sum_{r=0}^{40} a_r \cdot x^r$, then $\sum_{r=0}^{39} (-1)^r \cdot a_r \cdot a_{r+1}$ equal to
 a. 79 b. $2^{39} \cdot {}^{78}C_{39}$ c. $3^{39} \cdot {}^{78}C_{39}$ d. 0
116. The value of $\lim_{x \rightarrow \frac{3\pi}{4}} \frac{4 \sin^2 x \cos x - \cos x + \sin x}{\sin x + \cos x}$ is
 equal to
 a. -1 b. 0 c. 1 d. None of these
117. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non-coplanar vectors such that $\mathbf{b} \times \mathbf{c} = \mathbf{a}$; $\mathbf{c} \times \mathbf{a} = \mathbf{b}$; $\mathbf{a} \times \mathbf{b} = \mathbf{c}$, then which of the following is not TRUE?
 a. $|\mathbf{a}| - |\mathbf{b}| = 0$ b. $|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = 2$
 c. $[\mathbf{a} \ \mathbf{b} \ \mathbf{c}] = 1$ d. $|\mathbf{a}| |\mathbf{b}| |\mathbf{c}| = 1$
118. The value of ' λ ' for which the loci $\arg z = \frac{\pi}{6}$ and $|z - 2\sqrt{3}i| = \lambda$ on the argand plane touch each other is
 a. 3 b. 4 c. 5 d. 6
119. The angle between the lines whose direction cosine satisfy the equations $l + m + n = 0$ and $l^2 = m^2 + n^2$ is
 a. $\frac{\pi}{3}$ b. $\frac{\pi}{4}$
 c. $\frac{\pi}{6}$ d. $\frac{\pi}{2}$
120. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is
 a. $(x^2 - y^2)^2 = 6x^2 + 2y^2$ b. $(x^2 - y^2)^2 = 6x^2 - 2y^2$
 c. $(x^2 + y^2)^2 = 6x^2 + 2y^2$ d. $(x^2 + y^2)^2 = 6x^2 - 2y^2$
121. If f and g are differentiable function in $[0, 1]$ satisfying $f(0) = 2 = g(1)$, $g(0) = 0$ and $f(1) = 6$, then for some $c \in (0, 1)$
 a. $2f'(c) = g(c)$ b. $2f'(c) = 3g'(c)$
 c. $f'(c) = g'(c)$ d. $f'(c) = 2g'(c)$
122. The integral $\int \left(1 + x - \frac{1}{x}\right) e^{x + \frac{1}{x}} dx$ is equal to
 a. $(x-1)e^{x + \frac{1}{x}} + C$ b. $x e^{x + \frac{1}{x}} + C$
 c. $(x+1)e^{x + \frac{1}{x}} + C$ d. $-x e^{x + \frac{1}{x}} + C$

- 123.** If the coefficient of x^3 and x^4 in the expansion of $(1+ax+bx^2)(1-2x)^{18}$ in powers of x are both zero, then (a, b) is equal to
 a. $\left(16, \frac{251}{3}\right)$ b. $\left(14, \frac{251}{3}\right)$ c. $\left(14, \frac{272}{3}\right)$ d. $\left(16, \frac{272}{3}\right)$
- 124.** An equation of plane parallel to plane $x-2y+2z-5=0$ and at a unit distance from the origin is
 a. $x-2y+2z=3$ b. $x-2y+2z+1=0$
 c. $x-2y+2z-1=0$ d. $x-2y+2z+5=0$
- 125.** Three numbers are chosen at random without replacement from $\{1, 2, 3, \dots, 8\}$. The probability that their minimum is 3, given that their maximum is 6, is
 a. $\frac{3}{8}$ b. $\frac{1}{5}$ c. $\frac{1}{4}$ d. $\frac{2}{5}$
- 126.** The number of real numbers λ for which the equality $\frac{\sin(\lambda\alpha)}{\sin\alpha} - \frac{\cos(\lambda\alpha)}{\cos\alpha} = \lambda - 1$, holds for all real α which are not integral multiple of $\pi/2$ is
 a. 1 b. 2
 c. 3 d. Infinite
- 127.** Suppose a parabola $y = ax^2 + bx + c$ has two x intercepts, one positive and one negative, and its vertex is $(2, -2)$, then which of the following is true?
 a. $ab > 0$ b. $bc > 0$
 c. $ac > 0$ d. $a + b + c > 0$
- 128.** The larger of two angles made with the X -axis of a straight line drawn through $(1, 2)$ so that it intersects the line $x + y = 4$ at a point distant $\sqrt{6}/3$ from the point $(1, 2)$ is
 a. 60° b. 75°
 c. 105° d. None of these
- 129.** The point $([P+1], [P])$ (where, $[x]$ is the greatest integer function) lying inside the region bounded by the circle $x^2 + y^2 - 2x - 15 = 0$ and $x^2 + y^2 - 2x - 7 = 0$, then
 a. $P \in [-1, 2) - \{0, 1\}$ b. $P \in [-1, 0) \cup (0, 1) \cup (1, 2]$
 c. $P \in (-1, 2)$ d. None of these
- 130.** Solution of the equation $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$ is
 a. $e^y = e^x - 1 + ce^{-e^x}$ b. $e^{y-x} = -1 + ce^{-e^x}$
 c. $e^x + e^y = ce^{-e^x}$ d. None of these
- 131.** Area bounded by two branches of the curve $(y-x)^2 = x^3$ and $x = 1$ is
 a. $\frac{5}{4}$ sq unit b. $\frac{3}{2}$ sq unit
 c. $\frac{1}{4}$ sq unit d. $\frac{4}{5}$ sq unit
- 132.** The least value of the function $f(x) = \int_0^x (3 \sin x + 4 \cos x) dx$ on the interval $\left[\frac{5\pi}{4}, \frac{4\pi}{3}\right]$ is
 a. $\frac{3}{2} - \frac{\sqrt{3}}{2}$ b. $\frac{5-4\sqrt{3}}{2}$ c. $\frac{7-4\sqrt{3}}{2}$ d. $\frac{9-4\sqrt{3}}{2}$
- 133.** If z_1 and $\bar{z}_1$ represents adjacent vertices of a regular polygon of n sides and if $\frac{I_m(z_1)}{R_e(z_1)} = \sqrt{2} - 1$, then n is equal to
 a. 8 b. 16 c. 18 d. 24
- 134.** In the expansion of $(1+x+x^3+x^4)^{10}$, the coefficient of x^4 is
 a. 235 b. 310 c. 285 d. 325
- 135.** A person writes letter to six friends and addresses the corresponding envelopes. Let x be the numbers of ways so that at least two of the letters are in wrong envelopes and y be the numbers of ways so that all the letters are in wrong envelopes. Then $x - y =$
 a. 719 b. 265 c. 454 d. 720
- 136.** If $x = \log_5 3 + \log_7 5 + \log_9 7$, then
 a. $x > \frac{3}{2}$ b. $x > \frac{1}{\sqrt[3]{2}}$ c. $x > \frac{3}{\sqrt[3]{2}}$ d. $x > \sqrt[3]{2}$
- 137.** Let p and q be roots of the equation $x^2 - 2x + A = 0$ and let r and s be the roots of the equation $x^2 - 18x + B = 0$, If $p < q < r < s$ are in AP, then A and B are
 a. $-3, -77$ b. $3, -77$ c. $-3, 77$ d. $3, 77$
- 138.** Let $a, b, c \in R^+$ and the system of equations $(1-a)x + y + z = 0$, $x + (1-b)y + z = 0$ and $x + y + (1-c)z = 0$ has infinitely many solutions, the minimum value of ' abc ' is
 a. $3\sqrt{3}$ b. 9 c. 27 d. 3
- 139.** If A is an 3×3 non-singular matrix, such that $AA' = A' A$ and $B = A^{-1}A'$, then BB' equals to
 a. $(B^{-1})'$ b. $I + B$ c. I d. B^{-1}
- 140.** If $\sin^{-1} x + \tan^{-1} x = \pi/2$, then $2x^2 + 1 =$
 a. $\sqrt{5}$ b. $\frac{\sqrt{5}-1}{2}$ c. 2 d. $\frac{\sqrt{5}+1}{2}$
- 141.** Number of solution of the equation $|\cos x| = 2[x]$ are (where $|x|, [x]$ are modulus and greatest integer function respectively).
 a. 0 b. 2 c. 1 d. infinitely many
- 142.** Let $\cos(\alpha + \beta) = \frac{4}{5}$ and let $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \leq \alpha, \beta \leq \frac{\pi}{4}$, then $\tan 2\alpha =$
 a. $\frac{20}{7}$ b. $\frac{25}{16}$ c. $\frac{56}{33}$ d. $\frac{19}{2}$

- 143.** Cards are drawn one by one at random from a well shuffled pack of 52 cards until 2 aces are obtained for the first time. The probability that 18 draws are required for this is
 a. $\frac{3}{34}$ b. $\frac{17}{455}$
 c. $\frac{561}{15925}$ d. None of these
- 144.** If $\mathbf{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\mathbf{b} = -\hat{i} + 2\hat{j} + \hat{k}$, $\mathbf{c} = 3\hat{i} + \hat{j}$ and $\mathbf{a} + p\mathbf{b}$ is normal to $\mathbf{c}$, then p is equal to
 a. 0 b. 1
 c. 5 d. 3
- 145.** Let $f(x) = x[x]$, $x \in \mathbb{Z}$, ($[\cdot]$ denotes greatest integer function), then $f'(x)$ is equal to
 a. $2x$ b. $[x]$
 c. $2[x]$ d. 1
- 146.** The difference of maximum and minimum values of $f(x) = x^2 e^{-x}$ is
 a. e b. $1/e$
 c. $1 - \frac{1}{e}$ d. $1 + \frac{1}{e}$
- 147.** If $f'(x) = f(x) + \int_0^1 f(x) dx$, $f(0) = 1$, then $f(x) =$
 a. $\frac{2e^x}{3-e} + \frac{1-e}{3-e}$ b. $\frac{e^x}{3-e} + \frac{1+e}{1-e}$
 c. $\frac{3e^x}{2-e} + \frac{1+e}{1-e}$ d. $\frac{3e^x}{2-e} + \frac{1-e}{3+e}$
- 148.** The distance between the origin and the normal to curve $y = e^{2x} + x^2$ at $x = 0$ is
 a. $\frac{2}{\sqrt{3}}$ b. $\frac{2}{\sqrt{5}}$ c. $\frac{1}{\sqrt{3}}$ d. $\frac{1}{\sqrt{5}}$
- 149.** Reflection of the line $\frac{x-1}{-1} = \frac{y-2}{3} = \frac{z-4}{1}$ in the plane $x + y + z = 7$ is
 a. $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-4}{1}$ b. $\frac{x-1}{-3} = \frac{y-2}{-1} = \frac{z-4}{1}$
 c. $\frac{x-1}{-3} = \frac{y-2}{1} = \frac{z-4}{-1}$ d. $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-4}{1}$
- 150.** If $x, y, z \in \mathbb{R}$, $x + y + z = 5$, $x^2 + y^2 + z^2 = 9$, then length of interval in which x lies is
 a. $8/3$ b. $4/3$ c. $2/3$ d. $1/3$

Answers

Physics

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (c) | 2. (a) | 3. (b) | 4. (d) | 5. (c) | 6. (a) | 7. (b) | 8. (c) | 9. (a) | 10. (c) |
| 11. (d) | 12. (b) | 13. (b) | 14. (a) | 15. (c) | 16. (c) | 17. (a) | 18. (b) | 19. (d) | 20. (c) |
| 21. (a) | 22. (d) | 23. (a) | 24. (d) | 25. (a) | 26. (b) | 27. (c) | 28. (d) | 29. (d) | 30. (c) |
| 31. (a) | 32. (d) | 33. (b) | 34. (a) | 35. (c) | 36. (a) | 37. (c) | 38. (a) | 39. (c) | 40. (d) |

Chemistry

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 41. (a) | 42. (a) | 43. (c) | 44. (b) | 45. (b) | 46. (a) | 47. (d) | 48. (c) | 49. (c) | 50. (b) |
| 51. (b) | 52. (a) | 53. (a) | 54. (a) | 55. (d) | 56. (c) | 57. (c) | 58. (a) | 59. (c) | 60. (a) |
| 61. (b) | 62. (d) | 63. (c) | 64. (a) | 65. (b) | 66. (c) | 67. (c) | 68. (c) | 69. (b) | 70. (a) |
| 71. (c) | 72. (a) | 73. (a) | 74. (b) | 75. (b) | 76. (c) | 77. (b) | 78. (c) | 79. (a) | 80. (a) |

English Proficiency

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 81. (b) | 82. (a) | 83. (c) | 84. (a) | 85. (d) | 86. (d) | 87. (c) | 88. (b) | 89. (b) | 90. (b) |
| 91. (d) | 92. (d) | 93. (a) | 94. (a) | 95. (d) | | | | | |

Logical Reasoning

- | | | | | | | | | | |
|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| 96. (d) | 97. (b) | 98. (c) | 99. (b) | 100. (b) | 101. (b) | 102. (d) | 103. (a) | 104. (b) | 105. (a) |
|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|

Mathematics

- | | | | | | | | | | |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 106. (d) | 107. (a) | 108. (b) | 109. (d) | 110. (a) | 111. (d) | 112. (d) | 113. (a) | 114. (b) | 115. (d) |
| 116. (a) | 117. (b) | 118. (a) | 119. (a) | 120. (c) | 121. (d) | 122. (b) | 123. (d) | 124. (a) | 125. (b) |
| 126. (b) | 127. (b) | 128. (b) | 129. (d) | 130. (a) | 131. (d) | 132. (d) | 133. (a) | 134. (b) | 135. (c) |
| 136. (c) | 137. (c) | 138. (c) | 139. (c) | 140. (a) | 141. (a) | 142. (c) | 143. (c) | 144. (c) | 145. (b) |
| 146. (b) | 147. (a) | 148. (b) | 149. (c) | 150. (b) | | | | | |

Solutions

Physics

1. (c) Given, half life ($T_{1/2}$) = 23.1 days

If N_0 be the initial amount, then active nuclei at $t = 15$ days

$$N_1 = N_0 e^{-15\lambda} \quad \dots (i)$$

Active nuclei at $t = 16$ days

$$N_2 = N_0 e^{-16\lambda} \quad \dots (ii)$$

Disintegration constant,

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{23.1} = 0.03$$

$\therefore$ % decay in 15th to 16th days

$$\begin{aligned} &= \frac{N_1 - N_2}{N_1} \times 100 = \frac{N_0 e^{-15\lambda} - N_0 e^{-16\lambda}}{N_0 e^{-15\lambda}} \times 100 \\ &= (1 - e^{-\lambda}) \times 100 = (1 - e^{-0.03}) \times 100 \\ &= \left(1 - \frac{1}{e^{0.03}}\right) \times 100 = \left(1 - \frac{1}{1.03}\right) \times 100 = 2.9\% \end{aligned}$$

2. (a) Excess pressure inside first bubble of radius r_1 ,

$$p_1 = p_0 + \frac{4T}{r_1}$$

Excess pressure inside second bubble of radius r_2 ,

$$p_2 = p_0 + \frac{4T}{r_2}$$

Excess pressure inside double bubble,

$$\begin{aligned} p &= p_2 - p_1 \\ p &= 4T \left(\frac{r_1 - r_2}{r_1 r_2} \right) \end{aligned}$$

If R be the radius of double bubble, then

$$p = \frac{4T}{R}$$

$$\text{or } 4T \left(\frac{r_1 - r_2}{r_1 r_2} \right) = \frac{4T}{R}$$

$$\Rightarrow R = \frac{r_1 r_2}{r_1 - r_2} = \frac{3 \times 10^{-3} \times 2 \times 10^{-3}}{(3 - 2) \times 10^{-3}} = 6 \times 10^{-3} \text{ m}$$

$$R = 6 \times 10^{-3} \text{ m}$$

3. (b) Magnetic field at the centre of circular loop,

$$B = \frac{\mu_0 I}{2r} \quad \dots (i)$$

Period of revolution of electron, where ω = angular velocity of electron

$$T = \frac{2\pi}{\omega}$$

where, ω = angular velocity of electron

Current produced due to movement of electron,

$$I = \frac{e}{T} = \frac{e}{\frac{2\pi}{\omega}} = \frac{e\omega}{2\pi}$$

On substituting the value of I in Eq. (i), we get

$$B = \frac{\mu_0}{2r} \cdot \frac{e\omega}{2\pi}$$

$$\Rightarrow B = \frac{\mu_0 e \omega}{4\pi r}$$

4. (d) Given, $Y = 2.2 \times 10^8 \text{ m/s}^2$

Area of cross-section of steel wire,

$$A = 4 \text{ cm}^2 = 4 \times 10^{-4} \text{ m}^2$$

Mass of elevator, $m = 1000 \text{ kg}$

The maximum tension in the steel wire that can support elevator is given by

$$\begin{aligned} T &= \frac{1}{4} \times \text{stress} \times \text{area of cross-section} \\ &= \frac{1}{4} \times 2.2 \times 10^8 \times 4 \times 10^{-4} \\ &= 2.2 \times 10^4 \text{ N/m}^2 \end{aligned}$$

If f be the maximum upward acceleration of the elevator, then

$$\begin{aligned} \Rightarrow T &= m(g + f) \\ 2.2 \times 10^4 &= 1000(10 + f) \\ \Rightarrow f &= 12 \text{ m/s}^2 \end{aligned}$$

5. (c) For an ideal monoatomic gas, the heat capacity T_i, T_f = initial and final temperatures of gas

$$\text{ratio, } \gamma = \frac{5}{3}$$

In an adiabatic process,

$$TV^{\gamma-1} = \text{constant}$$

$$\begin{aligned} \text{i.e. } T_f V_f^{\gamma-1} &= T_i V_i^{\gamma-1} \Rightarrow T_f = T_i \left(\frac{V_i}{V_f} \right)^{\gamma-1} = 300 \left(\frac{1}{2} \right)^{\frac{5}{3}-1} \\ &= 300 \left(\frac{1}{2} \right)^{2/3} \end{aligned}$$

6. (a) Given, frequency of sound produced by sources A and B , $f_A = f_B = 400 \text{ Hz}$

Speed of sound in air, $v = 340 \text{ m/s}$

Apparent frequency heard by the observer when source A is moving towards him,

$$f_A' = \frac{v}{v - v_S} \times f_A = \frac{340}{340 - v_S} \times 400$$

Apparent frequency heard by the observer when source B is moving away from him,

$$f_B' = \frac{v}{v + v_S} \times f_B = \frac{340}{340 + v_S} \times 400$$

Given, Beats = $f_A' - f_B' = 4$

$$\begin{aligned} \frac{340}{340 - v_S} \times 400 - \frac{340}{340 + v_S} \times 400 &= 4 \\ \Rightarrow \frac{34000 \times 2v_S}{340^2 - v_S^2} &= 1 \\ \frac{34000 \times 2v_S}{340^2} &= 1 \quad [\because v_S^2 < 340^2] \\ \Rightarrow v_S &= 1.7 \text{ m/s} \end{aligned}$$

7. (b) Given, density of gas at point A , $\rho_A = \rho_0$

General equation for an ideal gas

$$pV = nRT$$

$$\Rightarrow pV = \frac{m}{M} \cdot RT \quad [m \rightarrow \text{mass}, M \rightarrow \text{molecular mass}]$$

$$\Rightarrow p = \frac{m}{V} \cdot \frac{R}{M} \cdot T$$

$$\Rightarrow p = \rho \left(\frac{R}{M} \right) T \quad \left[\text{where, density } \rho = \frac{m}{V} \right] \dots (i)$$

Applying Eq. (i) at point A and B , respectively.

$$p = \rho_A \left(\frac{R}{M} \right) T_0 \quad \dots (ii)$$

$$2p = \rho_B \left(\frac{R}{M} \right) 3T_0 \quad \dots (iii)$$

From Eqs. (ii) and (iii), we get

$$\begin{aligned} \frac{1}{2} &= \frac{\rho_A}{\rho_B} \cdot \frac{1}{3} \\ \Rightarrow \rho_B &= \frac{2}{3} \rho_A = \frac{2}{3} \rho_0 \end{aligned}$$

8. (c) Force constant K of rubber is given by

$$K = \frac{YA}{l} = \frac{5 \times 10^8 \times 10^{-6}}{0.1} = 5 \times 10^3 \text{ N/m}$$

Now, from conservation of energy

elastic potential energy of cord = kinetic energy of particle

$$\text{i.e. } \frac{1}{2} K (\Delta l)^2 = \frac{1}{2} mv^2$$

$$\begin{aligned} \Rightarrow v &= \sqrt{\frac{K}{m}} \cdot \Delta l \\ &= \sqrt{\frac{5 \times 10^3}{5 \times 10^{-3}}} (0.125 - 0.1) \\ &= 10^3 \times 0.025 = 25 \text{ m/s} \end{aligned}$$

9. (a) By work-energy theorem,

$$W = P \times t = \frac{1}{2} mv^2$$

$$\Rightarrow v^2 = 2 \frac{Pt}{m}$$

$$v = \left(\frac{2Pt}{m} \right)^{1/2}$$

$$\text{or } \frac{ds}{dt} = \left(\frac{2Pt}{m} \right)^{1/2}$$

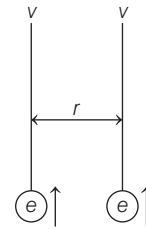
$$\int ds = \int \left(\frac{2Pt}{m} \right)^{1/2} dt$$

$$s = \left(\frac{2P}{m} \right)^{1/2} \cdot \frac{2}{3} t^{3/2}$$

$$\text{i.e. } s \propto t^{3/2}$$

$$s^2 \propto t^3$$

10. (c) Given, speed of electrons, $v = 5 \times 10^5 \text{ m/s}$



Electrostatic force between two electrons,

$$F_e = k \cdot \frac{e^2}{r^2} \quad \dots (i)$$

Magnetic field produced due to moving electron with velocity v ,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{ev}{r^2} \quad \dots (ii)$$

Magnetic force on proton

$$F_m = Bev = \frac{\mu_0}{4\pi} \cdot \frac{ev}{r^2} \cdot ev \quad (\text{from Eq. (ii)})$$

$$F_m = \frac{\mu_0}{4\pi} \cdot \frac{e^2 v^2}{r^2} \quad \dots (iii)$$

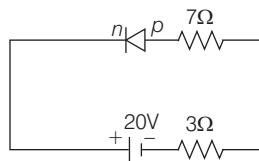
$\therefore$ From Eqs. (i) and (iii), we have

$$\begin{aligned} \frac{F_e}{F_m} &= \frac{ke^2/r^2}{\frac{\mu_0 e^2 v^2}{4\pi r^2}} = \frac{k \cdot 4\pi}{\mu_0 v^2} \\ &= \frac{9 \times 10^9 \times 4 \times 3.14}{4\pi \times 10^{-7} (5 \times 10^5)^2} = 3.6 \times 10^5 \end{aligned}$$

11. (d) From the figure, It is clear that diodes D_1 and D_3 are reverse biased, therefore they will not conduct resistance 11Ω and 5Ω will be not in use but diode D_2 is in forward biased. Therefore, current flows through diode D_2 is same as current through battery, according to the circuit shown after redrawing,

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∴ Current, $I = \frac{20}{7+3} = 2 \text{ A}$



12. (b) The moment of inertia of disc of radius R about its axis is

$$I = \frac{M' R^2}{2}$$

where, $M' = \text{mass of complete disc}$
 $= 2M$

∴ $I = \frac{2MR^2}{2}$
 $I = MR^2 \dots (i)$

If I_1 be the moment of inertia of semicircular disc about its axis, then disc may be assumed as combination of two semi-circular parts,

Thus, $I = I_1 + I_1 \Rightarrow I_1 = \frac{I}{2} = \frac{MR^2}{2}$

13. (b) Given, total energy given to electron in third orbit,

$$E = 12 \text{ eV}$$

Energy of third orbit of H-atom, $E' = \frac{-13.6}{n^2}$

$$= \frac{-13.6}{3^2} = -1.51 \text{ eV}$$

Ionisation energy of electron of third orbit, $E'' = -E'$

$$= -(-1.51 \text{ eV}) \Rightarrow E'' = 1.51 \text{ eV}$$

Hence, final energy of electron when it comes out of H-atom from third orbit

$$E_F = E - E''$$

$$= 12 - 1.51 = 10.49 \text{ eV}$$

14. (a) Given, $D = 2 \text{ m}$, $d = 0.25 \text{ mm} = 2.5 \times 10^{-4} \text{ cm}$

Velocity of screen, $\frac{dD}{dt} = 5 \text{ m/s}$

Speed of first maxima $= \frac{d\beta}{dt} = ?$

In Young's double slit experiment, fringe width $\beta = \frac{D\lambda}{d}$

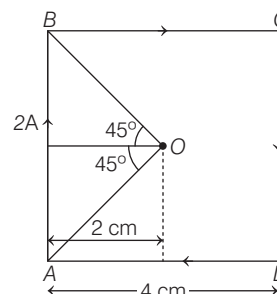
On differentiating with respect to t , we get

$$\frac{d\beta}{dt} = \frac{\lambda}{d} \cdot \frac{dD}{dt}$$

$$= \frac{800 \times 10^{-9}}{2.5 \times 10^{-4}} \cdot 5$$

$$= 0.16 \times 10^{-3} \text{ m/s} = 16 \text{ mm/s}$$

15. (c) Given, current through the square loop is $2A$.



$$I = 2A$$

The given situation is shown in the figure. Magnetic field due to wire AB at the centre O is

$$B_{AB} = \frac{\mu_0}{4\pi} \cdot \frac{I}{r} [\sin 45^\circ + \sin 45^\circ]$$

$$= 10^{-7} \times \frac{2}{2 \times 10^{-2}} \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right]$$

$$= \sqrt{2} \times 10^{-5} \text{ T [Downward to the plane of square loop]}$$

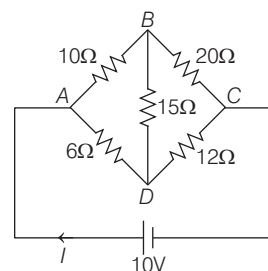
Since, magnetic field due to side AB, BC, CD and DA is same in magnitude and direction. Hence, net magnetic field at O

$$B = 4 \times B_{AB} = 4\sqrt{2} \times 10^{-5} \text{ T}$$

Wheatstone bridge in balanced condition maintains $\frac{P}{Q} = \frac{R}{S}$

i.e. no current flows through the galvanometer (i.e. $i_g = 0$)

16. (c) The given circuit can be redrawn as



Since, $\frac{R_{AB}}{R_{BC}} = \frac{R_{AD}}{R_{DC}} = \frac{1}{2}$, hence, it is a balanced Wheatstone bridge, therefore current flowing in arm $BD = 0$

$$\therefore R_{eq} = (10 + 20) || (6 + 12) = 30 || 18$$

$$= \frac{30 \times 18}{30 + 18} = 11.25 \Omega$$

$$\therefore I = \frac{V}{R_{eq}} = \frac{10}{11.25} = 0.89 \text{ A} \approx 0.9 \text{ A}$$

17. (a) The flux linked with coil (made of wire of radius r) of area A and magnetic field B , is given by

$$\phi = BA \cos \theta = B \left(\frac{\pi r^2}{2} \right) \cos \omega t$$

$$\phi = \frac{\pi B r^2}{2} \cos \omega t \quad \left[\begin{array}{l} \therefore A = \frac{\pi r^2}{2} \\ \text{and } \theta = \omega t \end{array} \right]$$

By Faraday's law of electromagnetic induction, induced emf, $e = -\frac{d\phi}{dt}$

Hence, after differentiating,

$$e = -\frac{d}{dt} \left(\frac{\pi B r^2}{2} \cos \omega t \right) = \frac{\pi B r^2 \omega}{2} \sin \omega t$$

(where, R = resistance in the circuit)

$$\therefore \text{Power, } P = \frac{e^2}{R} = \frac{\pi^2 B^2 r^4 \omega^2 \sin^2 \omega t}{4R}$$

$$\begin{aligned} \therefore \text{Mean power, } \bar{P} &= \frac{\pi^2 B^2 r^4 \omega^2}{4R} \cdot \overline{\sin^2 \omega t} \\ &= \frac{\pi^2 B^2 r^4 \omega^2}{4R} \cdot \frac{1}{2} \quad \left[\because \overline{\sin^2 \omega t} = \frac{1}{2} \right] \\ &= \frac{(\pi B r^2 \omega)^2}{8R} \end{aligned}$$

18. (b) For the given convex lens, focal length, $f = 25$ cm

Magnification of lens in term of u and f is given by

$$m = \frac{f}{f + u} = \frac{25}{25 + u} \quad \dots (i)$$

When the image is real, then $m = -2$ and $u = -x_1$

$$\therefore \text{From Eq. (i), } -2 = \frac{25}{25 - x_1} \Rightarrow x_1 = \frac{75}{2}$$

Again, when the image is virtual, then $m = 2$ and $u = -x_2$

$$\therefore \text{From Eq. (i), } 2 = \frac{25}{25 - x_2} \Rightarrow x_2 = \frac{25}{2}$$

Hence, the ratio of x_2 and x_1 is

$$\therefore \frac{x_2}{x_1} = \frac{25/2}{75/2} = \frac{1}{3}$$

19. (d) For perfectly elastic collision,

total momentum before collision

= total momentum after collision

$$\text{i.e. } m \cdot u + \frac{3m}{4} \times 0 = mv_x + \frac{3m}{4} \cdot v_y$$

$$\Rightarrow 4u = 4v_x + 3v_y \quad \dots (i)$$

Coefficient of restitution,

$$e = \frac{\text{Velocity of separation } (v_y - v_x) \text{ after collision}}{\text{Velocity of approach } (u_x - u_y) \text{ before collision}}$$

$$\Rightarrow 1 = \frac{v_y - v_x}{u - 0} \quad \left[\because u_x = v \text{ and } u_y = v \right]$$

$$u = v_y - v_x \quad \dots (ii)$$

Solving Eqs. (i) and (ii), we have

$$\begin{aligned} v_x &= \frac{u}{7} \text{ and } u_y = \frac{8u}{7} \\ \therefore \frac{\lambda_y}{\lambda_x} &= \frac{h/mv_y}{h/\frac{3m}{4} \cdot v_x} = \frac{3}{4} \cdot \frac{v_x}{v_y} = \frac{3}{4} \cdot \frac{u/7}{8u/7} = \frac{3}{32} \end{aligned}$$

20. (c) We know that, velocity of light c is terms of ϵ_0 and μ_0 is given by

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \Rightarrow c^2 = \frac{1}{\mu_0 \epsilon_0}$$

$$c^2 = \frac{1}{\mu_0^2} \cdot \frac{\mu_0}{\epsilon_0} \Rightarrow c = \frac{1}{\mu_0} \sqrt{\frac{\mu_0}{\epsilon_0}} \Rightarrow \sqrt{\frac{\mu_0}{\epsilon_0}} = c \mu_0 \quad \dots (i)$$

$$\begin{aligned} \text{Dimension of magnetic field, } B &= \frac{\tau}{N/A} = \frac{[ML^2T^{-2}]}{[A][L^2]} \\ &= [MA^{-1}T^{-2}] \end{aligned}$$

Again, $B = \mu_0 n I$

$$\therefore \mu_0 = \frac{B}{nI} = \frac{[MA^{-1}T^{-2}]}{[L^{-1}][A]} = [MLA^{-2}T^{-2}]$$

$$\mu_0 = [MLA^{-2}T^{-2}] \quad \dots (ii)$$

$$\text{and } C = [LT^{-1}] \quad \dots (iii)$$

From Eqs. (i), (ii) and (iii), we get

$$\sqrt{\frac{\mu_0}{\epsilon_0}} = [LT^{-1}] [MLA^{-2}T^{-2}] = [ML^2T^{-3}A^{-2}]$$

21. (a) Given, $F = at$

when, $t = 0$, then linear momentum = p

when, $t = T$, then linear momentum = $2p$

According to Newton's law of motion,

$$\text{applied force, } F = \frac{dp}{dt}$$

$$\text{or, } dp = F dt$$

$$dp = at dt$$

$$\text{or, } \int_p^{2p} dp = a \int_0^T t dt$$

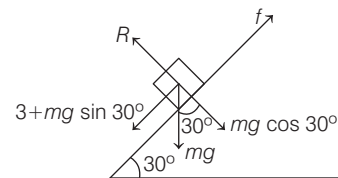
$$(2p - p) = a \cdot \frac{T^2}{2}$$

$$\Rightarrow \frac{2p}{a} = T^2 \Rightarrow T = \sqrt{\frac{2p}{a}}$$

22. (d) Block does not move upto a maximum applied force of 3 N down the inclined plane.

Equation of motion is $3 + mg \sin 30^\circ = F$

$$3 + mg \sin 30^\circ = \mu mg \cos 30^\circ \quad \dots (i)$$



Similarly, block also does not move upto a maximum applied force of 12 N up the plane.

Now, equation of motion is

$$mg \sin 30^\circ + F = 12$$

$$\text{or, } mg \sin 30^\circ + \mu mg \cos 30^\circ = 12 \quad \dots (ii)$$

Solving Eqs. (i) and (ii), we have

$$mg \sin 30^\circ = \frac{9}{2} \text{ and } \mu mg \cos 30^\circ = \frac{15}{2}$$

Hence, dividing

$$\therefore \frac{1}{\mu} \tan 30^\circ = \frac{9/2}{15/2} = \frac{3}{2} \Rightarrow \mu = \frac{5}{3\sqrt{3}}$$

23. (a) If middle portion of lens is painted black, then less number of refracted rays will intersect, therefore intensity of the corresponding image of object will be less. Though, the rays coming from object refracted with small portion of lens, but complete image of object will be formed but of lesser intensity.

24. (d) Given, speed of sound, $v = 340$ m/s

Length of open organ pipe, $l = 2$ m

Frequency, $f = 1200$ Hz

For open organ pipe,

$$\text{Fundamental frequency, } f_0 = \frac{v}{2l} = \frac{340}{2 \times 2} = 85$$

$\therefore$ Number of tones present in the open organ pipe

$$= \frac{f}{f_0} = \frac{1200}{85} = 14.11 \approx 14$$

25. (a) Given, charge on proton, $q = e$

$$= 1.6 \times 10^{-19} \text{ C}$$

Radius, $r = 2 \text{ mm} = 2 \times 10^{-3} \text{ cm}$

Frequency, $f = 10$ Hz

Current associated by proton is given by

$$I = \frac{q}{t} \quad [T \rightarrow \text{time period}]$$

$$= qf \quad \left[\because T = \frac{1}{f} \right]$$

$$= 1.6 \times 10^{-19} \times 10 = 1.6 \times 10^{-18} \text{ A}$$

$\therefore$ Magnetic dipole moment, $M = IA$

$$= I \cdot \pi r^2 = 1.6 \times 10^{-18} \times 3.14 \times (2 \times 10^{-3})^2$$

$$= 2 \times 10^{-24} \text{ A-m}^2$$

26. (b) When capacitor is fully charged, then it acts like a open switch, therefore $8 \text{ k}\Omega$ resistor has no use. So, rest of the resistances $4 \text{ k}\Omega$, $7 \text{ k}\Omega$ and $9 \text{ k}\Omega$ will be in series.

Hence, current drawn from cell

$$I = \frac{10}{(4 + 7 + 9) \times 10^3} = \frac{10}{20 \times 10^3} = 0.5 \text{ mA}$$

27. (c) The charge on the capacitor at time t in a discharging circuit is given by $q = q_0 e^{-t/\tau}$

where, $\tau = RC$ = time constant

$$\therefore q = q_0 e^{-t/RC}$$

$$\text{when, } q = \frac{q_0}{2}$$

$$\text{then } \frac{q_0}{2} = q_0 e^{-t/RC}$$

$$\frac{1}{2} = e^{-t/RC}$$

Taking natural log on both side $\log_e \left(\frac{1}{2} \right) = \log_e e^{-t/RC}$

$$-\log_e 2 = -\frac{t}{RC} \log_e e \Rightarrow t = RC \log_e 2$$

28. (d) If v be the velocity in the inter planetary space and U_i , U_f initial and final PE

Then, by the law of conservation of energy,

$$U_i + K_i = U_f + K_f$$

$$\frac{-GM_e m}{R_e} + \frac{1}{2} m (3v_e)^2 = 0 + \frac{1}{2} m v^2$$

$$\frac{-GM_e}{R_e} + \frac{9}{2} v_e^2 = \frac{1}{2} v^2$$

$$\Rightarrow \frac{-2GM_e}{R_e} + 9v_e^2 = v^2$$

$$\Rightarrow \frac{-2gR_e^2}{R_e} + 9v_e^2 = v^2 \quad [\because GM_e = gR_e^2]$$

$$-2gR_e + 9 \times 2 = gR_e = v^2, \quad [\because v_e^2 = 2gR_e]$$

$$16gR_e = v^2$$

$$8.2gR_e = v^2$$

$$\Rightarrow v = \sqrt{8.2} \cdot \sqrt{2gR_e} = 2\sqrt{2} v_e$$

29. (d) Given, density of bob, $\rho = \frac{4}{3} \times 10^3 \text{ kg/m}^3$

Density of water, $\sigma = 10^3 \text{ kg/m}^3$

If g' be gravitational acceleration in water then,

$$g' = g \left(1 - \frac{\sigma}{\rho} \right) = g \left(1 - \frac{10^3}{\frac{4}{3} \times 10^3} \right) = \frac{g}{4}$$

$$\text{As, } T_{\text{air}} = 2\pi \sqrt{\frac{l}{g}},$$

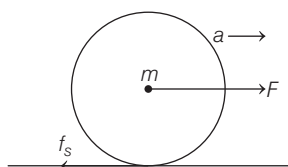
$$\text{Similarly, } T_{\text{water}} = 2\pi \sqrt{\frac{l}{g'}} [T_{\text{water}} = 2s] \left(g' = \frac{g}{4} \right)$$

$$2 = 2\pi \sqrt{\frac{l}{g/4}} = 2\pi \sqrt{\frac{l}{g}} \cdot 2$$

$$2 = 2 \cdot T_{\text{air}} \Rightarrow T_{\text{air}} = 1 \text{ s}$$

30. (c) By Newton's law of motion

$$F - f_s = ma \quad \dots (i)$$


 and torque, $\tau = I\alpha$

 where, α = tangential component of acceleration

 I = moment of inertia

$$\Rightarrow fR = I\alpha$$

$$fR = \frac{MR^2}{2} \cdot \frac{a}{R} \quad \left[\because I = \frac{MR^2}{2} \text{ and } \alpha = \frac{a}{R} \right]$$

$$\Rightarrow f = \frac{ma}{2}$$

$$\therefore F = f + ma = \frac{ma}{2} + ma = \frac{3ma}{2}$$

 31. (a) Velocity of projectile at time t is given by

$$\mathbf{v} = \mathbf{u} - \mathbf{g}t\mathbf{j}$$

$$\mathbf{v} = \hat{i} + \sqrt{3}\hat{j} - 10t\hat{j}$$

$$\therefore \mathbf{r} = \int \mathbf{v} dt$$

$$= \int (\hat{i} + \sqrt{3}\hat{j} - 10t\hat{j}) dt$$

$$= t\hat{i} + \sqrt{3}t\hat{j} - 5t^2\hat{j}$$

$$= t\hat{i} + \sqrt{3}t\hat{j} - 5t^2\hat{j}$$

$$\mathbf{r} = t\hat{i} + (\sqrt{3}t - 5t^2)\hat{j}$$

$$\text{or } x\hat{i} + y\hat{j} = t\hat{i} + (\sqrt{3}t - 5t^2)\hat{j}$$

$$\text{On comparing, } x = t \quad \dots (i)$$

$$\text{and } y = \sqrt{3}t - 5t^2 \quad \dots (ii)$$

From Eqs (i) and (ii), we get

$$y = \sqrt{3}x - 5x^2$$

 32. (d) $\mathbf{F} = 20\hat{i} + 10\hat{j}$

$$\therefore F_x = 20 \text{ N and } F_y = 10 \text{ N}$$

 $\therefore$ Acceleration,

$$a_x = \frac{F_x}{m} = \frac{20}{1}$$

$$a_x = 20 \text{ m/s}^2$$

and

$$a_y = \frac{F_y}{m} = \frac{10}{1} = 10 \text{ m/s}^2$$

 $\therefore$ Displacement of body in x-direction in 2s.

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$= 0 + \frac{1}{2} \times 20 \times 2^2$$

$$= 40 \text{ m}$$

 $\therefore$ Displacement of body in y-direction in 2s.

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$= 0 + \frac{1}{2} \times 10 \times 2^2$$

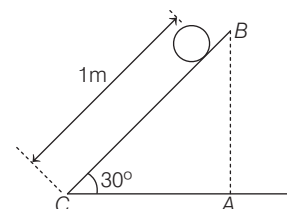
$$= 20 \text{ m}$$

 $\therefore$ Position of the body at $t = 2 \text{ s}$

$$\mathbf{r} = x\hat{i} + y\hat{j}$$

$$\mathbf{r} = 40\hat{i} + 20\hat{j}$$

33. (b) Potential energy of cylinder at top position



$$U = mg(AB)$$

$$= 2 \times 10 [1 \cdot \sin 30^\circ] \quad \left[\because \sin 30^\circ = \frac{AB}{BC} \right]$$

$$= 10 \text{ J}$$

 If v is the linear speed of cylinder when it reaches at bottom, then at bottom, total potential energy of cylinder is converted into kinetic energy,

$$\text{i.e. } K = U$$

$$\frac{1}{2} I\omega^2 + \frac{1}{2} mv^2 = 10$$

$$\frac{1}{2} \cdot \frac{mr^2}{2} \cdot \frac{v^2}{r^2} + \frac{1}{2} mv^2 = 10 \quad \left[\because \omega = \frac{v}{r} \right]$$

$$\Rightarrow \frac{3}{4} mv^2 = 10$$

$$v = \sqrt{\frac{40}{3m}} = \sqrt{\frac{40}{3 \times 2}} = \sqrt{\frac{20}{3}} \text{ m/s}$$

34. (a) According to the question,

$$F_1 = -k_1 x \text{ and } F_2 = -k_2 x$$

 $\therefore$ Frequency of oscillation

$$v_1 = \frac{1}{2\pi} \sqrt{\frac{k_1}{m}}$$

$$5 = \frac{1}{2\pi} \sqrt{\frac{k_1}{m}} \Rightarrow k_1 = 100 \pi^2 m$$

$$\text{Similarly, } 12 = \frac{1}{2\pi} \sqrt{\frac{k_2}{m}}$$

$$\Rightarrow k_2 = 576 \pi^2 m$$

Now, $F = F_1 + F_2 = -(k_1 + k_2) x$

$\therefore$ Frequency of oscillation,

$$v = \frac{1}{2\pi} \sqrt{\frac{k_1 + k_2}{m}} = \frac{1}{2\pi} \sqrt{\frac{100\pi^2 m + 576\pi^2 m}{m}}$$

$$\Rightarrow v = 13 \text{ Hz}$$

35. (c) Number of moles in $2 \times 10^{-4} \text{ kg}$

$$\mu = \frac{\text{Mass of hydrogen in gram}}{\text{Molecules mass}}$$

$$= \frac{2 \times 10^{-4} \times 10^3}{2}$$

$$= 0.1 \text{ mole}$$

$$\therefore pV = \mu RT$$

$$\frac{pV}{T} = \mu R = 0.1 \times R = \frac{R}{10}$$

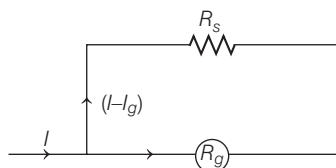
36. (a) Given, for an ammeter, $I_g = 1 \text{ A}$, $R_g = 15 \Omega$

Total current, $I = 4 \text{ A}$

If R_s be the shunt resistance, then

according to the question circuit diagram is given,

($R_s \parallel R_g$, hence have equal potential difference)



where, $I_g R_g = (I - I_g) R_s$

$$R_s = \frac{I_g}{I - I_g} \times R_g = \frac{1}{4 - 1} \times 15 = 5 \Omega$$

37. (c) The electric field intensity at a point lying outside the sphere (non-conducting) is

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

$$\text{i.e. } E \propto \frac{1}{r^2} \quad \dots (i)$$

The electric field intensity at the surface ($r = R$),

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} \Rightarrow E \propto \frac{1}{R^2}$$

where, R being the radius of sphere,

The electric field intensity inside the sphere is

$$E = \frac{qr}{4\pi\epsilon_0 R^3} \Rightarrow E \propto r$$

At the centre of sphere, $r = 0$, $\therefore E = 0$

38. (a) Given, $R = (56 \pm 5.6) \text{ k}\Omega$

$$= 56 \text{ k}\Omega \pm 10\% \text{ of } 56 \text{ k}\Omega$$

$$= 56 \times 10^3 \pm 10\% \Omega$$

As per the colour code for carbon resistors, the colour assigned to numbers

5 $\rightarrow$ Green

6 $\rightarrow$ Blue

3 $\rightarrow$ Orange

For 10% accuracy or tolerance, the colour is silver.

Hence, the bands of colours of carbon resistor in sequence are green, blue, orange and silver.

39. (c) When two waves with same frequency and constant phase difference interfere to each other, then after interference, intensity of resultant wave is distributed such that it is maximum at some points [constructive interference or bright fringes] and minimum at another points. [destructive interference or dark fringes].

Hence, energy is distributed and remains constant [in the form of bright and dark fringes]

40. (d) According to the given graph ($I - t$) induced current (I) is obtained from the slope.

$$\text{Hence, } I = \frac{1}{R} \frac{d\phi}{dt}$$

$$\Rightarrow d\phi = I dt. R = \text{area of triangle } POS \times R = \frac{8 \times 0.2}{2} \times 50 = 40 \text{ Wb}$$

Chemistry

41. (a) Let atoms of Y in ccp structure = 100

Then, number of tetrahedral voids = 2×100

$\therefore$ Number of X atoms = $\frac{2}{3}$ rd of tetrahedral voids

$$\therefore \text{Number of X atoms} = \frac{2}{3} \times 200 = \frac{400}{3}$$

$$\text{Thus, } \frac{X}{Y} = \frac{400}{3 \times 100} = \frac{400}{300} = \frac{4}{3}$$

Hence, formula of compound can be = X_4Y_3 .

42. (a) Given, density (d_1) at 30°C (T_1) = 1.35 kg/m^3

To find density (d_2) at S.T.P, (i.e at 760 torr and 273 K (T_2) temperature), we use formula

$$\frac{d_2}{d_1} = \frac{p_2 \cdot T_1}{p_1 T_2}$$

where, $p_1 = 768 \text{ torr}$ (Pressure at 30°C , i.e. at 303 K)

$p_2 = 760 \text{ torr}$ (Pressure at 273 K)

$$\therefore d_2 = \frac{d_1 \times p_2 \times T_1}{p_1 \times T_2} = \frac{1.35 \times 760 \times 303}{768 \times 273} = 1.48 \text{ kg/m}^3$$

43. (c) $\Delta_0 = (-0.4 \times n_{t_{2g}}) + (0.6 \times n_{e_g})$

where, n = number of electrons in $n_{t_{2g}}$ and n_{e_g} .

CFSE of given d -electrons configuration are as follows

a. For high spin, d^6 ;

$$\Delta_0 (\text{CFSE}) = -0.4 \times 4 + 0.6 \times 2 = -0.4$$

b. For low spin, d^4 ;

$$\Delta_0 (\text{CFSE}) = -0.4 \times 4 + 0.6 \times 0 = 1.6$$

c. For low spin d^5

$$\Delta_0 (\text{CFSE}) = -0.4 \times 5 + 0.6 \times 0 = -2.0$$

d. For high spin, d^7

$$\Delta_0 (\text{CFSE}) = -0.4 \times 5 + 0.6 \times 2 = -0.8$$

Neglecting -ve sign (as it indicates that energy is released).

Hence, maximum Δ_0 (CFSE) value is shown by low spin d^5 configuration.

44. (b) According to Rydberg's equation

$$\bar{\nu} = \frac{1}{\lambda} = RZ^2 \left[\frac{1}{n_L^2} - \frac{1}{n_H^2} \right]$$

where,

$\bar{\nu}$ = wave number

λ = wavelength

R = Rydberg constant ($= 109677 \text{ cm}^{-1}$)

Z = atomic number

n_L = lower energy state

n_H = higher energy state.

i.e. $\frac{1}{\lambda} \propto Z^2$

Thus, $\frac{\lambda(\text{He}^+)}{\lambda(\text{H})} = \frac{Z^2(\text{H})}{Z^2(\text{He}^+)}$

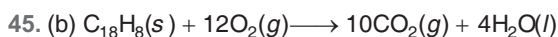
To find $\lambda(\text{He}) = ?$

Given, $\lambda(\text{H}) = 912 \text{ nm}$

$$Z(\text{H}) = 1$$

$$Z(\text{He}) = 2$$

$$\therefore \lambda(\text{He}^+) = \frac{1 \times 912}{4} = 22.8 \text{ nm}$$



We known that $\Delta H = \Delta E + \Delta nRT$

$$\Delta n = 10 - 12 = -2$$

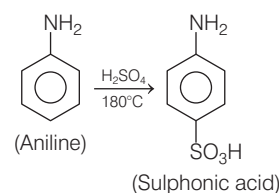
$$\Delta H = -1228.2 \times 10^3 + (-2) \times 2 \times 298$$

$$= -1229393 \text{ Ca}$$

$$= -1229.39 \text{ kcal}$$

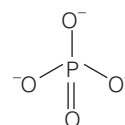
46. (a) Among the formation of B , C , D and E , the slowest step is the rate determining step. As formation of B , i.e. 0.002 mol/h per mole of A is the slowest step. Hence, step-(I) is the rate determining step.

47. (d)



Sulphanilic acid exist as dipolar ion (zwitter ion); act as inner salt and SO_3H group diminishes the basic character of $-\text{NH}_2$.

48. (c) The structure of PO_4^{3-} is as follows



As phosphorous central atom forms four (4) sigma bonds and has no lone pair of electrons. It uses four hybrid orbital and thus, hybridisation of p -atom is sp^3 .

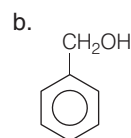
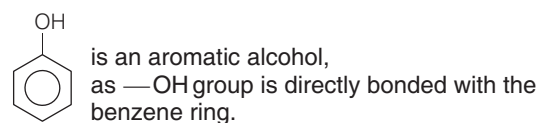
49. (c) Electronic configuration of the given species are

a. Sm^{2+} ($Z = 62$) = $[\text{Xe}] 4f^6$ b. Eu^{2+} ($Z = 63$) = $[\text{Xe}] 4f^7$

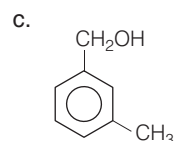
c. Yb^{2+} ($Z = 70$) = $[\text{Xe}] 4f^{14}$ d. Ce^{2+} ($Z = 58$) = $[\text{Xe}] 4f^1 5d^1$

On the basis of above electronic configuration, only Yb^{2+} ($Z = 70$) has no unpaired electrons i.e. has fully-filled orbitals. Thus it is dimagnetic in nature.

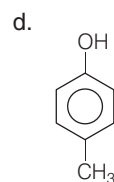
50. (b) Alcohol in which OH group is directly bonded to benzene are aromatic alcohols.



It is not an aromatic alcohol because OH is not bonded directly with the benzene group.

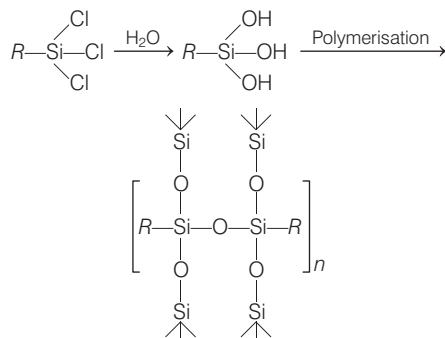


It is also not an aromatic alcohol because $-\text{OH}$ group is not directly bonded with the benzene ring.

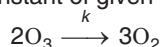


It is an aromatic alcohol because $-\text{OH}$ is directly bonded with the benzene ring.

51. (b) RSiCl_3 gives cross-linked silicon polymer on hydrolysis.



52. (a) Let k be the rate constant of given reaction



From the slowest step $r = k'[\text{O}][\text{O}_3]$... (i)

To eliminate $[\text{O}]$, (From fast step)

$$k_{\text{eq}} = \frac{[\text{O}_2][\text{O}]}{[\text{O}_3]}$$

$$[\text{O}] = \frac{k_{\text{eq}}[\text{O}_3]}{[\text{O}_2]}$$

Now, substituting value of $[\text{O}]$ in Eq (i)

$$r = \frac{k'k_{\text{eq}}[\text{O}_3][\text{O}_3]}{[\text{O}_2]}$$

Let $k'k_{\text{eq}} = k$

Thus, $r = k[\text{O}_3]^2[\text{O}_2]^{-1}$

53. (a) Depression in freezing point is given as $\Delta T_f = K_f$

$$\text{Molality of solution } (m) = \frac{\Delta T_f}{K_f}$$

$$\text{or } m = \frac{0.6}{1.86} = 0.322 \text{ m}$$

Elevation in boiling point of solution is given as

$$\Delta T_b = K_b \times m$$

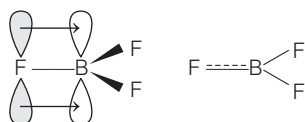
$$\Delta T_b = 0.512 \times 0.322$$

$$\Delta T_b = 0.165$$

$$\therefore \text{Boiling point of solution} = 373 + 0.16 \\ = 373.16 \text{ K}$$

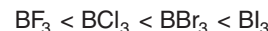
54. (a) The strength of acidic character of boron trihalides depends upon $p\pi - p\pi$ back bonding.

In boron trihalides, $p\pi - p\pi$ back bonding occurs due to empty orbital of boron and filled orbitals of halogens.



Phenomenon of back bonding in BF_3 molecule

The $p\pi - p\pi$ back bonding is shown maximum by BF_3 , as the size of B and F are small and comparatively same. Due to this effect tendency of accepting lone-pair of electrons of boron decreases as size of halogen increases. The order of size of halogens are $\text{F} < \text{Cl} < \text{Br} < \text{I}$. Thus, acidic nature is in order

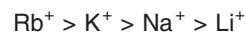


55. (d) **Key idea** Mobility of alkali ions in aqueous solution $\propto$ size of ion

$$\propto \frac{1}{\text{size of hydrated ion}}$$

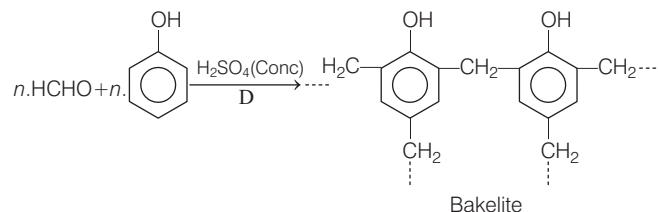
The mobility of alkali ions is inversely proportional to the size of ion in hydrated state, because smaller the ion more it will get hydrated and becomes larger in size, thus its ionic mobility decreases.

Hence, correct order of mobility is



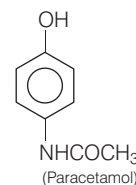
56. (c) Bakelite is a thermosetting plastic which is formed by the reaction of phenol with HCHO in the presence of H_2SO_4 (conc).

The reaction occurs as follows:



A cross-linked condensation polymer (at o - and p -position) bakelite is thus produced.

57. (c) Paracetamol is the drug, that can be used as an antipyretic as well as an analgesic, i.e. to reduce the fever and gives relieve from pain. Its structure is as follows

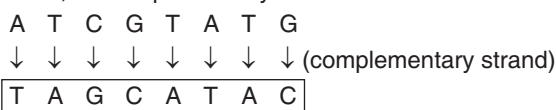


58. (a) In the given structure, ring one (I) has one o -atom and is a 6 membered ring, thus is called pyranose ring, whereas ring (II) has one o -atom and is a five membered ring, thus is called furanose.

The disaccharide is linked by a glycosidic bond between C—1 of glucose (in the α -position, α linkage) and C—2 of fructose (in the β -position, β -linkage).

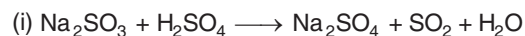
59. (c) Due to size and geometries of bases, the only possible pairing in DNA are between G (guanine) and C (cytosine) through three H-bonds and between A (adenine) and T (thymine) through two H-bonds.

Thus, the complementary strand of DNA will be

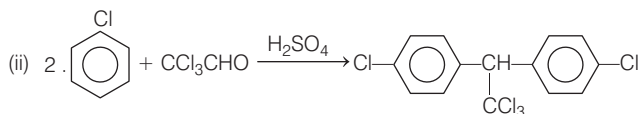
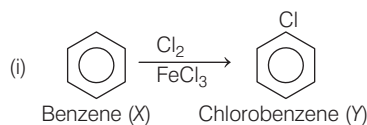


60. (a) When Na_2SO_3 reacts with hot and dil H_2SO_4 , it gives $\text{SO}_2(\text{g})$ which decolourises bromine-water.

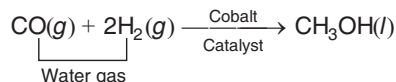
The reaction proceed as follows:



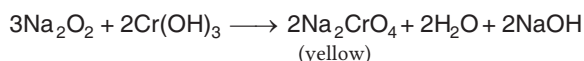
61. (b) The given reaction is for preparation of DDT (dichlorodiphenyl trichloroethane). Complete reaction is given as follows.



62. (d) The mixture of $\text{CO}(\text{g})$ and $\text{H}_2(\text{g})$ is known water gas as it is used in the synthesis of other compounds,



63. (c) Chromium hydroxide, i.e. $\text{Cr}(\text{OH})_3$ when reacts with NaOH and Na_2O_2 , gives yellow colour of sodium chromate. The whole reaction is as follows



64. (a) Cl is the good leaving group, i.e. is the weakest nucleophile among the given options

$[\text{Cl}, \text{NH}_2, \text{OC}_2\text{H}_5 \text{ and } \text{OCOCH}_3]$, the rate of reaction is faster, when 'Z' is 'Cl'.

65. (b) **Key-idea**

Use Nernst equation, i.e.

$$E(\text{emf}) = E_{\text{cell}}^\circ - \frac{0.0591}{2} \log \frac{[\text{Zn}^{2+}]}{[\text{Cu}^{2+}]}$$

For reaction (i)

$$E_1 = E_{\text{cell}}^\circ - \frac{0.0591}{2} \log \frac{0.1}{1}$$

$$E_1 = E_{\text{cell}}^\circ + \frac{0.0591}{2}$$

For reaction (ii)

$$E_2 = E_{\text{cell}}^\circ - \frac{0.0591}{2} \log \frac{1}{1} = 0$$

$$E_2 = E_{\text{cell}}^\circ$$

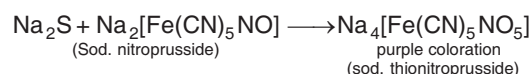
For reaction (iii)

$$E_3 = E_{\text{cell}}^\circ - \frac{0.0591}{2} \log \frac{1}{0.1}$$

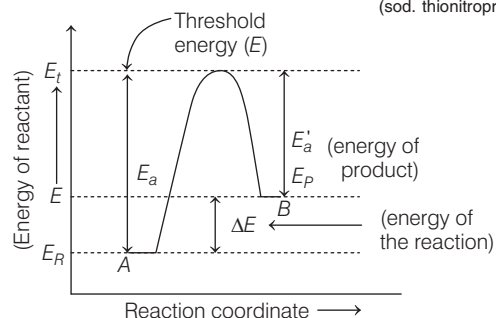
$$E_3 = E_{\text{cell}}^\circ - \frac{0.0591}{2}$$

Hence, $E_1 > E_2 > E_3$.

66. (c) This test is used for the presence of sulphur in any compound. When sodium nitroprusside, react with sulphide ion, (e.g. Na_2S) the following reaction takes place



67. (c)



where, E_a = activation energy of forward reaction

E_a' = activation energy of backward reaction

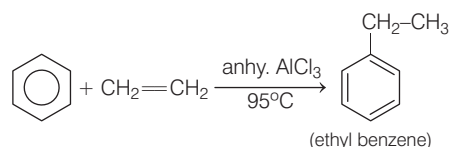
The above energy profile diagram shows that $E_a > E_a'$

The potential energy of the product is greater than that of the reactant, so the reaction is endothermic.

$$E_a = E_a' + \Delta E$$

$$E_t = E_a \text{ or } E_t > E_a'$$

68. (c) The $\text{H}_2\text{C}=\text{CH}_2 + \text{C}_6\text{H}_6$ on reaction produce ethyl benzene as follows



69. (b) Given, pH of solution A = 3

$[\text{H}^+]$ of A = 10^{-3}M and, pH of solution B = 2

$[\text{H}^+]$ of B = 10^{-2}M

Thus, total $[\text{H}^+]$ are $= 10^{-3} + 10^{-2} = 1.1 \times 10^{-2}\text{M}$

$$\begin{aligned} \therefore \text{pH}_{\text{(final)}} &= -\log [\text{H}^+] \\ &= -\log [1.1 \times 10^{-2}] \\ &= 3 - \log 1.1 \\ &= 3 - 1.04 = 1.9 \end{aligned}$$

22 SELF STUDY GUIDE BITSAT

70. (a) Freundlich isotherm is related with the pressure of gas adsorbed, as follows $\frac{x}{m} = k \cdot p^{1/n}$

where, $\frac{x}{m}$ = amount of gas (x) adsorbed over the surface of mass (m).

p = pressure of gas

k = freundlich constant

Thus, value of 'n' is always greater than 1 and therefore value of $1/n$ lies between 0 and 1.

71. (c) Electronic configuration of Mn is ($Z = 25$) = $[Ar]3d^5 4s^2$

$\therefore$ Electronic configuration of Mn^{4+} is $Mn^{4+} = [Ar] 3d^3$

Thus, Mn^{4+} has 3-unpaired electron and its spin only magnetic moment (μ) is, $\mu = \sqrt{n(n+2)} \text{ BM}$

where, n = number of unpaired electrons.

$$\mu = \sqrt{3(3+2)} \cdot \text{BM} \Rightarrow \mu = \sqrt{15} = 3.87 \text{ BM} \approx 4 \text{ BM}$$

72. (a) $\therefore$ Gram equivalents of an acid

$$= \text{Gram equivalents of KOH i.e. } \frac{0.45}{E} = \frac{20 \times 0.5}{1000}$$

$$\text{where, } E = \text{Equivalent mass } E = \frac{0.45 \times 1000}{20 \times 0.5} = 45$$

$$\text{Thus, basicity} = \frac{90}{45} = 2$$

$$\left[\therefore \text{Basicity} = \frac{\text{Molecular mass}}{\text{Equivalent mass}} \right]$$

73. (a) Given, $A + B \rightleftharpoons C + D$

initial conc. 4 4 0 0

at equilibrium (4 - 2) (4 - 2) 2 2

i.e. 2 2 2 2

$$\therefore K_{(eq)} = \frac{2 \times 2}{2 \times 2} = 1$$

74. (b) Given, 125 ml of 1 M $AgNO_3$

which is equal to 13.5 g of Ag

$\therefore$ 1000 ml of $AgNO_3$ = 108 g of Ag

$$\therefore 125 \text{ ml of } AgNO_3 = \frac{108 \times 125}{1000} = 13.5 \text{ g}$$

Also, $\therefore$ 108 g of Ag is deposited by = 96500 C

$$\therefore 13.5 \text{ g of Ag is deposited by} = \frac{96500 \times 13.5}{108} = 12062.5 \text{ C}$$

$$\therefore Q = it$$

$$\therefore t \text{ (time)} = Q / i = \frac{12062.50}{241.25}, t = 50 \text{ sec}$$

75. (b) Orbitals of nearly equal energy are called degenerated orbitals. As, energy of,

$$a. \sigma 2s > \sigma^* 2s \quad b. \pi 2p_x \approx \pi 2p_y$$

$$c. \pi^* 2p_x < \sigma^* 2p_z \quad d. \sigma 2p_x < \sigma^* 2p_z$$

Thus, only $\pi 2p_x$ and $\pi 2p_y$ are degenerate orbitals.

76. (c) Total energy (E) of any photon is given by the relation :

$$E = \frac{hc}{\lambda}$$

where, h = Planck's constant = $6.6 \times 10^{-34} \text{ J-s}$

c = velocity of light = $3 \times 10^{10} \text{ m/s}$

λ = wave-length = $1.5 \times 10^{-10} \text{ m}$, i.e. (150 pm)

$$\text{Thus, } E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.5 \times 10^{-10}}; E = 1.32 \times 10^{-15} \text{ J}$$

and, energy of ejected electron (E')

$$E' = \frac{1}{2} mv^2,$$

where,

m = mass of electron = $9.1 \times 10^{-31} \text{ kg}$

v = velocity of electron = $1.5 \times 10^7 \text{ m/s}$

$$E' = \frac{1}{2} mv^2 = \frac{1}{2} \times 9.1 \times 10^{-31} \times (1.5 \times 10^7)^2$$

$$E' = 1.024 \times 10^{-16}$$

Thus, total energy of photon = binding energy of electron (B) + energy of ejected electron (E')

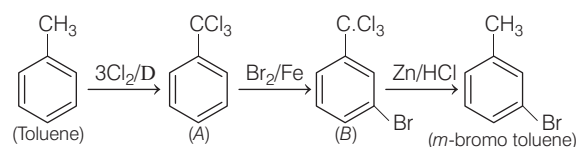
$$\text{Thus, } 1.32 \times 10^{-15} = (B) + (E')$$

$$\therefore (B) = (E) - (E') = [1.32 \times 10^{-15}] - [1.024 \times 10^{-16}]$$

$$= 1.2176 \times 10^{-15} \text{ J}$$

$$= \frac{1.2176 \times 10^{-15}}{1.6 \times 10^{-19}} \text{ eV} = 7.6 \times 10^3 \text{ eV}$$

77. (b) The degree of unsaturation for compound C_7H_8 is three (3), as it reacts with Cl_2/Δ . Hence C_7H_8 is toluene, i.e. $C_6H_5 \cdot CH_3$. The whole reaction occurs as follows:



78. (c)

Element	Ratio (by weight)	Atomic weight	Mol Ratio	Simplest Mol Ratio
C	9	12	$\frac{9}{12} = 0.75$	$\frac{0.75}{0.25} = 3$
H	1	1	$\frac{1}{1} = 1.00$	$\frac{1.00}{0.25} = 4$
N	3.5	14	$\frac{3.5}{14} = 0.25$	$\frac{0.25}{0.25} = 1$

$$\therefore \text{Empirical formula} = C_3H_4N$$

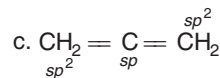
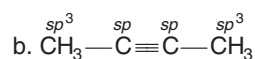
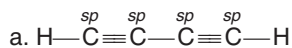
The empirical formula mass = $3 \times 12 + 4 \times 1 + 14 = 54 \text{ g}$

Given, molecular mass = 108 g

Thus,
$$n = \frac{108}{54} = 2$$

Hence, molecular formula = (Empirical formula) $\times$ 2
 $(C_3H_4N) \times 2 = C_6H_8N_2$

79. (a) **Key idea** pi-bond do not participate in hybridisation



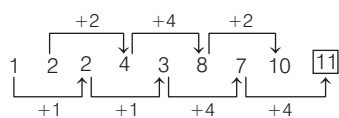
80. (a) In bassemer converter, copper-sulphide is partially oxidised to cuprous oxide, which further reacts with the remaining copper sulphide to give copper and sulphur di oxide, i.e. $Cu_2S + 2Cu_2O \longrightarrow 6Cu + SO_2$

English Proficiency

81. (b) 'Many a' takes singular noun, as well as singular verb. So replace 'have' by 'has'.
 e.g. Many a player is injured lately.
82. (a) The correct syntax for interrogative sentences is 'wh' word + do/does+sub+verb+?
 So, we should replace 'it takes' by 'does it take' e.g. How long does it take to build a new house?
83. (c) Replace 'his' by 'ones' to make the possessive correct.
 e.g. One should respect one's parents.
84. (a) Assured is followed by preposition 'of'.
 e.g. I am assured of my promotion by next month.
85. (d) Preposition 'to' should be used with 'submit' here.
 e.g., When exams are over then candidates are required to submit their papers.
86. (d) Audacious (Adjective) means bold or fearless. So, its correct antonym will be 'timid', which means 'lacking in courage or confidence'.
Antonyms Careful, meek, mild
Synonyms Daring, bold, unafraid.
 e.g. He is an audacious soldier.
87. (c) Cogent (Adjective), which means reasonable and convincing. The correct antonym will be 'dissuasive', which means divert from the measure or purpose.
Antonyms Unconvincing, muddle, vague
Synonyms Convincing, logical, compelling.
 e.g. He put forward some cogent reasons for initiating the project.

Logical Reasoning

96. (d) In the 'College', education is given to 'Students', in the same way, treatment is given to the 'Patient' in 'Hospital'.
97. (b) Except 35, all others are multiples of 9, but 35 is the multiple of 5 and 7.
98. (c) Pattern of the series is as shown below



88. (b) Taint (Noun) means a mark of disgrace or blemish. The correct antonym will be 'clear', which means free of ambiguity or doubt.
Antonyms Blank, esteem, respect.
Synonyms Contamination, corruption, stigma e.g. His reputation was tainted by allegations of illegal activity.
89. (b) Vivacious (Adjective) means full of life and energy or animated.
 So, its correct synonym will be 'Lively'.
Synonyms Spirited, active, jolly
Antonyms Boring, unhappy, dull
 e.g. She is a dark-haired and vivacious lady.
90. (b) Sporadic (Adjective) means rare and scattered in occurrence. Here 'scattered' has the same sense. It is the correct synonym.
Synonyms Desultory, isolated, random.
Antonyms Common, systematic, usual
 e.g. Suddenly, sporadic fighting broke out in the town.
91. (d) Both official and corporate India is allergic to 'mention of clean technology.'
92. (d) 'Failure In crops' is a life and death question to many Indians.
93. (a) The most similar in meaning to the word 'profligacy' is 'wastefulness'.
94. (a) India cannot tolerate any further 'crop failure'.
95. (d) The reason could be 'US wants to use it as a handle against the developing countries in the forthcoming meet'.

This series contains two separate series. In first series, 1 is added in first two steps and 4 is added in third and fourth steps.

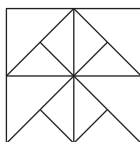
In second series, 2 and 4 is added alternately.

99. (b) From given information,
 Ramesh > Mohan > Shyam ... (i)
 Gautam > Ramesh > Rajat ... (ii)
 Shyam > Rajat ... (iii)
 Combining Eqs. (i), (ii) and (iii), we get,

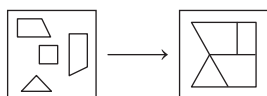
Gautam > Ramesh > Mohan > Shyam > Rajat

Hence, Rajat is the shortest among all.

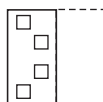
100. (b) If figure shown in option (b), is placed in the place of missing portion of the original figure, then it is completed as shown below



101. (b) Figure given in option b. can be formed by joining the pieces given in question figure, as shown below.

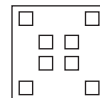


102. (d) In figure (X), the square sheet of paper is being folded along the vertical line of symmetry, so that right half of the sheet overlaps the left half. In figure (Y), the sheet is folded further to a quarter. In figure (Z), two squares are punched in the folded sheet. Clearly, the punched squares will be created in each quarter of the paper and after unfolding the first fold the figure will look like as,

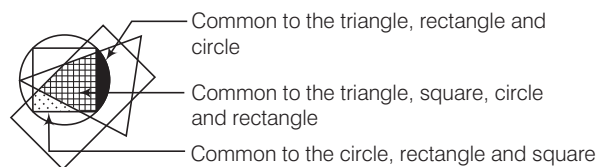


then after unfolding the last fold the transparent sheet will look like as given in option d.

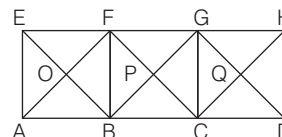
i.e.



103. (a) In each row, the number of objects increases by 1 at each step from left to right. So, in third row, the missing segment contain $2 + 1 = 3$ dots.
104. (b) One of the three dots occupies the region which is common to the circle, rectangle and triangle; another dot occupies the region which is common to the triangle, circle, rectangle and square and the third dot occupies the region which is common to the circle, rectangle and square. These three characteristics as shown by these three dots are found in figure b.. It possesses region which is common to the circle, rectangle and triangle, a region which is common to the triangle circle, rectangle, and square and a region which is common to the circle, rectangle and square



105. (a) Naming the figure,



Clearly, there are 28 triangles in the given figure namely, EOF, AOE, AOB, BOF, ABF, BEF, ABE, AEF, BPF, FPG, CPG, BPC, BFG, BCG, CFG, BCF, GQC, CDQ, DQH, GQH, GDC, GDH, GHC, CDH, AFC, BGD, EBG and FCH.

Mathematics

106. (d) Given, $a = 3b$

$$e = \sqrt{1 - \frac{b^2}{a^2}} \Rightarrow e = \sqrt{1 - \frac{b^2}{9b^2}}$$

$$\Rightarrow e = \sqrt{1 - \frac{1}{9}} = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}$$

107. (a) We have, $\sum_{m=1}^n \tan^{-1} \left(\frac{2m}{m^4 + m^2 + 2} \right)$
- $$= \sum_{m=1}^n \tan^{-1} \left(\frac{(m^2 + m + 1) - (m^2 - m + 1)}{1 + (m^2 + m + 1)(m^2 - m + 1)} \right)$$
- $$= \sum_{m=1}^n [\tan^{-1}(m^2 + m + 1) - \tan^{-1}(m^2 - m + 1)]$$
- $$= (\tan^{-1} 3 - \tan^{-1} 1) + (\tan^{-1} 7 - \tan^{-1} 3)$$
- $$+ (\tan^{-1}(13) - \tan^{-1} 7) + \dots + (\tan^{-1}(n^2 + n + 1) - \tan^{-1}(n^2 - n + 1))$$

$$= \tan^{-1}(n^2 + n + 1) - \tan^{-1} 1$$

$$= \tan^{-1} \left(\frac{n^2 + n + 1 - 1}{1 + (n^2 + n + 1)} \right) = \tan^{-1} \left(\frac{n^2 + n}{n^2 + n + 2} \right)$$

108. (b) Given, $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$

Replacing x by $\frac{2}{x}$, we get

$$\left(\frac{8}{x^2} + \frac{6}{x} + 4 \right)^{10} = \sum_{r=0}^{20} a_r \left(\frac{2}{x} \right)^r$$

$$\Rightarrow 2^{10} (2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r 2^r x^{20-r}$$

$$\Rightarrow 2^{10} \sum_{r=0}^{20} a_r x^r = \sum_{r=0}^{20} a_r 2^r x^{20-r}$$

Comparing coefficient of x^8 both sides, we get

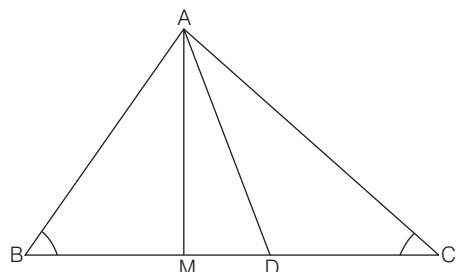
$$2^{10} a_8 = a_{12} 2^{12} \Rightarrow \frac{a_8}{a_{12}} = \frac{2^{12}}{2^{10}} = 4$$

109. (d) In $\triangle ABC$, D be the mid-point of BC and $AB = AD$. $\triangle ABD$ is an isosceles.

$\therefore AM$ is perpendicular bisector of BD

$$BM = MD$$

In $\triangle ABM$, $\tan B = \frac{AM}{BM}$



In $\triangle AMC$, $\tan C = \frac{AM}{CM}$

$$\therefore \frac{\tan B}{\tan C} = \frac{CM}{BM} = \frac{3BM}{BM} = 3 \quad [\because CM = DM + CD]$$

110. (a) We have,

$$L = \lim_{n \rightarrow \infty} \sqrt{n} \int_0^1 \frac{dx}{(1+x^2)^n}$$

We know that, $(1+x^2)^n > 1+nx^2$

$$\Rightarrow \frac{1}{(1+x^2)^n} < \frac{1}{1+nx^2}$$

$$\Rightarrow \int_0^1 \frac{dx}{(1+x^2)^n} < \int_0^1 \frac{1}{1+nx^2} dx$$

$$\Rightarrow \int_0^1 \frac{1}{(1+x^2)^n} < \frac{1}{\sqrt{n}} [\tan^{-1} \sqrt{n} x]_0^1$$

$$= \frac{1}{\sqrt{n}} \tan^{-1} \sqrt{n}$$

$$L = \lim_{n \rightarrow \infty} \sqrt{n} \int_0^1 \frac{1}{(1+x^2)^n} dx < \lim_{n \rightarrow \infty} \sqrt{n} \frac{1}{\sqrt{n}} \tan^{-1} \sqrt{n}$$

$$\therefore L = \frac{\pi}{2}$$

111. (d) We have, $\sqrt{x+3-4\sqrt{x-1}} + \sqrt{x+8-6\sqrt{x-1}} = 1$

$$\sqrt{(\sqrt{x-1})^2 - 2 \times 2\sqrt{x-1} + 4} + \sqrt{(\sqrt{x-1})^2 - 2 \times 3\sqrt{x-1} + 9} = 1$$

$$\Rightarrow \sqrt{(\sqrt{x-1}-2)^2} + \sqrt{(\sqrt{x-1}-3)^2} = 1$$

$$\Rightarrow |\sqrt{x-1}-2| + |\sqrt{x-1}-3| = 1$$

$$\Rightarrow |\sqrt{x-1}-2| + |\sqrt{x-1}-3| = 1$$

$$= (\sqrt{x-1}-2) - (\sqrt{x-1}-3)$$

We know that,

If $|x-a| + |x-b| = (x-a) - (x-b)$

then, $(x-a)(x-b) < 0$

$$\therefore (\sqrt{x-1}-2)(\sqrt{x-1}-3) < 0$$

$$\Rightarrow 2 < \sqrt{x-1} < 3 \Rightarrow 5 < x < 10$$

$\therefore$ Equation have infinite many solutions.

112. (d) We have,

$$ax + y + z = a \quad \dots (i)$$

$$x + by + z = b \quad \dots (ii)$$

$$x + y + cz = c \quad \dots (iii)$$

are inconsistent.

$$\therefore \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0 \text{ and } \begin{vmatrix} a & 1 & 1 \\ b & b & 1 \\ c & 1 & c \end{vmatrix} \neq 0$$

$$\Rightarrow \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$$\Rightarrow a(bc-1) - 1(c-1) + 1(1-b) = 0$$

$$abc - a - c + 1 + 1 - b = 0$$

$$abc - a - b - c + 2 = 0$$

and

$$\begin{vmatrix} a & 1 & 1 \\ b & b & 1 \\ c & 1 & c \end{vmatrix} \neq 0$$

It is possible only when $a \neq 1, b \neq 1$ and $c \neq 1$

113. (a) For atleast 4 times an even number, probability is

$$= {}^7C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^3 + {}^4C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^2 + {}^7C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^1 + {}^7C_7 \left(\frac{1}{2}\right)^7$$

$$= \frac{1}{2}$$

114. (b) Equation of plane containing line $x - y - 1 = 0$ and

$$z - 1 = 0 \text{ is } (x - y - 1) + \lambda(z - 1) = 0$$

$$\Rightarrow x - y + \lambda z - 1 - \lambda = 0 \quad \dots (i)$$

Since, this plane is parallel to line

$$\frac{x}{2} - \frac{z}{3} = 1 \text{ and } y = 3$$

$$\frac{x-1}{2} = \frac{y-3}{0} = \frac{z}{3} \quad \dots (ii)$$

$$\therefore 2(1) + (0)(-1) + 3(\lambda) = 0$$

$$\Rightarrow \lambda = -2/3$$

Putting the value of λ in Eq. (i), we get

$$x - y - \frac{2}{3}z - 1 + \frac{2}{3} = 0; 3x - 3y - 2z = 1$$

115. (d) We have, $(1+x+x^2)^{20} = \sum_{r=0}^{40} a_r x^r$

x is replaced by $-\frac{1}{x}$, we get

$$\left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^{20} = \sum_{r=0}^{40} (-1)^r a_r \left(\frac{1}{x}\right)^r$$

$$(1-x+x^2)^{20} = \sum_{r=0}^{40} (-1)^r a_r x^{40-r}$$

$\sum_{r=0}^{39} (-1)^r a_r \cdot a_{r+1}$ is coefficient of x in product of

$$(1+x+x^2)^{20} \left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^{20} = 0$$

116. (a) Let

$$L = \lim_{x \rightarrow \frac{3\pi}{4}} \frac{4 \sin^2 x \cos x - \cos x + \sin x}{\sin x + \cos x}$$

$$L = \lim_{x \rightarrow \frac{3\pi}{4}} \frac{2 \sin x (\sin^2 x + \cos^2 x + 2 \sin x \cos x - 1) - \cos x + \sin x}{\sin x + \cos x}$$

$$L = \lim_{x \rightarrow \frac{3\pi}{4}} \frac{2 \sin x [(\sin x + \cos x)^2 - 1] - \cos x + \sin x}{\sin x + \cos x}$$

$$L = \lim_{x \rightarrow \frac{3\pi}{4}} \frac{2 \sin x (\sin x + \cos x)^2 - 2 \sin x - \cos x + \sin x}{\sin x + \cos x}$$

$$L = \lim_{x \rightarrow \frac{3\pi}{4}} \frac{(\sin x + \cos x) (2 \sin x (\sin x + \cos x) - 1)}{\sin x + \cos x}$$

$$L = \lim_{x \rightarrow \frac{3\pi}{4}} 2 \sin x (\sin x + \cos x) - 1 = -1$$

117. (b) Given,

$$\mathbf{a} \times \mathbf{b} = \mathbf{c}$$

$$\mathbf{b} \times \mathbf{c} = \mathbf{a}$$

$$\mathbf{c} \times \mathbf{a} = \mathbf{b}$$

$$(\mathbf{b} \times \mathbf{c}) \cdot \mathbf{a} = \mathbf{a} \cdot \mathbf{a}$$

$$(\mathbf{c} \times \mathbf{a}) \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{b}$$

$$\mathbf{a}^2 - \mathbf{b}^2 = (\mathbf{b} \times \mathbf{c}) \cdot \mathbf{a} - (\mathbf{c} \times \mathbf{a}) \cdot \mathbf{b}$$

$$= [\mathbf{b} \mathbf{c} \mathbf{a}] - [\mathbf{c} \mathbf{a} \mathbf{b}] = 0$$

$$\therefore |\mathbf{a}|^2 - |\mathbf{b}|^2 = 0$$

$$|\mathbf{a}| = |\mathbf{b}|$$

We know that,

$$|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 \sin^2 \theta$$

$$|\mathbf{a} \mathbf{b} \mathbf{c}| = |\mathbf{a}| |\mathbf{b}| |\mathbf{c}| \sin \theta$$

$$|\mathbf{a} \mathbf{b} \mathbf{c}| [-|\mathbf{a} \mathbf{b} \mathbf{c}| - 1] = 0$$

$$|\mathbf{a} \mathbf{b} \mathbf{c}| = 1$$

$$|\mathbf{a}| |\mathbf{b}| |\mathbf{c}| = 1$$

$$\therefore |\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| \neq 2$$

118. (a) Let

$$z = x + iy$$

$$\arg z = \tan^{-1} \left(\frac{y}{x} \right)$$

$$\arg(z) = \frac{\pi}{6}$$

$$\therefore \tan^{-1} \frac{y}{x} = \frac{\pi}{6}$$

$$\frac{y}{x} = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow x = \sqrt{3} y$$

Also given,

$$|z - 2\sqrt{3}i| = \lambda$$

$$|x + iy - 2\sqrt{3}i| = \lambda$$

$$x^2 + (y - 2\sqrt{3})^2 = \lambda^2$$

This is the equation of circle whose centre is $(0, 2\sqrt{3})$ and $r = \lambda$. Now, $x - \sqrt{3}y = 0$ touches the circle

$$\therefore \lambda = \left| \frac{0 - \sqrt{3}(2\sqrt{3})}{\sqrt{1+3}} \right|$$

$$\lambda = \left| \frac{6}{2} \right| = 3$$

$$\therefore \lambda = 3$$

119. (a) We have,

$$l + m + n = 0 \quad \dots(i)$$

$$\text{and} \quad l^2 = m^2 + n^2 \quad \dots(ii)$$

Putting the value of l in Eq. (ii), we get

$$(m+n)^2 = m^2 + n^2$$

$$\Rightarrow m^2 + n^2 + 2mn = m^2 + n^2$$

$$\Rightarrow mn = 0$$

$$\Rightarrow m = 0, n = 0$$

$$\text{when} \quad m = 0, l = -n$$

$$\therefore \text{Direction cosines are} \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right).$$

$$\text{When} \quad n = 0, l = -m$$

$$\therefore \text{Direction cosines are} \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right).$$

Angle between lines are

$$\cos \theta = \left(\frac{-1}{\sqrt{2}} \times -\frac{1}{\sqrt{2}} \right) + \left(0 \times \frac{1}{\sqrt{2}} \right) + \left(\frac{1}{\sqrt{2}} \times 0 \right)$$

$$\cos \theta = \frac{1}{2}, \theta = \pi/3$$

120. (c) Let the foot of perpendicular be (h, k) .

Equation of tangent with slope m passing (h, k) is

$$y = mx \pm \sqrt{6m^2 + 2}, \text{ where } m = -h/k$$

$$\Rightarrow \sqrt{\frac{6h^2}{k^2} + 2} = \frac{h^2 + k^2}{k}$$

$$6h^2 + 2k^2 = (h^2 + k^2)^2$$

$$\text{So, required locus is } 6x^2 + 2y^2 = (x^2 + y^2)^2.$$

121. (d) Let

$$h(x) = f(x) - 2g(x)$$

$$h(0) = f(0) - 2g(0)$$

$$h(0) = 2 - 2(0) \quad [\because f(0) = 2, g(0) = 0]$$

$$h(0) = 2$$

$$h(1) = f(1) - 2g(1)$$

$$h(1) = 6 - 2(2) \quad [\because f(1) = 6, g(1) = 2]$$

$$h(1) = 2$$

$$h(0) = h(1)$$

$$h'(x) = f'(x) - 2g'(x)$$

By Rolle's theorem $h'(c) = 0$

$$\therefore f'(c) - 2g'(c) = 0, f'(c) = 2g'(c)$$

122. (b) We have,

$$I = \int \left(1 + x - \frac{1}{x}\right) e^{x + \frac{1}{x}} dx$$

$$I = \int e^{x + \frac{1}{x}} dx + \int \left(x - \frac{1}{x}\right) e^{x + \frac{1}{x}} dx$$

$$I = \int e^{x + \frac{1}{x}} dx + \int x \left(1 - \frac{1}{x^2}\right) e^{x + \frac{1}{x}} dx$$

$$I = \int e^{x + \frac{1}{x}} dx + x e^{x + \frac{1}{x}} - \int e^{x + \frac{1}{x}} dx$$

$$I = x e^{x + \frac{1}{x}} + C$$

123. (d) We have,

$$(1 + ax + bx^2)(1 - 2x)^{18}$$

$$(1 + ax + bx^2)(1 - {}^{18}C_1 2x + {}^{18}C_2 (2x^2) - {}^{18}C_3 (2x)^3 + {}^{18}C_4 (2x)^4 \dots)$$

$$\text{Coefficient of } x^3 \text{ is } -{}^{18}C_3 (2)^3 + a \cdot {}^{18}C_2 (2)^2 - b {}^{18}C_1 (2)$$

$$\text{and coefficient of } x^4 \text{ is } {}^{18}C_4 (2)^4 - {}^{18}C_3 (2)^3 a + {}^{18}C_2 (2)^2 b$$

Coefficient of x^3 and x^4 are zero.

$$\therefore -{}^{18}C_3 (2)^3 + {}^{18}C_2 (2)^2 (a) - {}^{18}C_1 (2) b = 0$$

$$\Rightarrow \frac{4 \times 17 \times 16}{3 \times 2} - 17a + b = 0 \quad \dots (i)$$

$$\text{and } 80 - \frac{32}{3}a + b = 0 \quad \dots (ii)$$

Solving Eqs. (i) and (ii), we get

$$a = 16, b = \frac{272}{3}$$

124. (a) Equation of plane parallel to the plane

$$x - 2y + 2z - 5 = 0 \text{ is } x - 2y + 2z + \lambda = 0.$$

Since, this plane is unit distance from origin

$$\therefore 1 = \left| \frac{\lambda}{\sqrt{1 + 4 + 4}} \right|$$

$$|\lambda| = 3 \Rightarrow \lambda = \pm 3$$

$\therefore$ Equation of plane is

$$x - 2y + 2z = 3$$

125. (b) Let A be the event that maximum is 6.

B be the event that minimum is 3.

$$P(A) = \frac{{}^5C_2}{{}^8C_3} \text{ (number less than 6 is 5)}$$

$$P(B) = \frac{{}^5C_2}{{}^8C_3} \text{ (number greater than 3 is 5)}$$

$$P(A \cap B) = \frac{{}^2C_1}{{}^8C_3}$$

$\therefore$ Required probability

$$= P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{{}^2C_1}{{}^5C_2} = \frac{2}{10} = \frac{1}{5}$$

126. (b) We have, $\frac{\sin(\lambda\alpha)}{\sin \alpha} - \frac{\cos(\lambda\alpha)}{\cos \alpha} = \lambda - 1$

$$\sin(\lambda\alpha) \cos \alpha - \cos(\lambda\alpha) \sin \alpha = (\lambda - 1) \sin \alpha \cos \alpha$$

$$\sin(\lambda - 1)\alpha = (\lambda - 1) \sin \alpha \cos \alpha$$

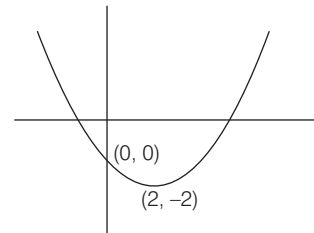
$$\sin(\lambda - 1)\alpha = \frac{\lambda - 1}{2} \sin 2\alpha$$

It is possible, when $\lambda - 1 = 0 \Rightarrow \lambda = 1$

$$\text{and } \frac{\lambda - 1}{2} = 1 \Rightarrow \lambda = 3$$

$\therefore \lambda$ has only two solution.

127. (b) Given, $y = ax^2 + bx + c$ has two x-intercepts one is positive and one is negative and vertex is $(2, -2)$. The graph of $y = ax^2 + bx + c$ is



Clearly from graph

$$c > 0$$

$$-\frac{b}{a} > 0 \Rightarrow -b > 0$$

$$\Rightarrow b < 0$$

$$f(1) = a + b + c < 0$$

$$ab < 0, ac < 0, bc > 0$$

128. (b) Let $A = (1, 2)$ equation of line is

$$\frac{x-1}{\cos \theta} = \frac{y-2}{\sin \theta} = r = \frac{\sqrt{6}}{3}$$

$$\therefore x = \frac{\sqrt{6}}{3} \cos \theta + 1, y = \frac{\sqrt{6}}{3} \sin \theta + 2$$

This point lies on line $x + y = 4$

$$\therefore \frac{\sqrt{6}}{3} \cos \theta + \frac{\sqrt{6}}{3} \sin \theta = 1$$

$$\Rightarrow \sin \theta + \cos \theta = \frac{\sqrt{3}}{\sqrt{2}}$$

$$\Rightarrow \sin \left(\frac{\pi}{4} + \theta \right) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{\pi}{4} + \theta = 60^\circ \text{ or } 120^\circ$$

$$\Rightarrow \theta = 15^\circ \text{ or } 75^\circ$$

$$\therefore \text{Larger angle} = 75^\circ.$$

129. (d) $[P + 1] = [P] + 1$, Let $[P] = n$, then n is integer

$\therefore ([P + 1], [P]) = (n + 1, n)$ lie inside the region of circles

$$S_1 = x^2 + y^2 - 2x - 15 = 0, C_1 = (1, 0), r_1 = 4$$

$$\text{and } S_2 = x^2 + y^2 - 2x - 7 = 0, C_2 = (1, 0), r_2 = 2\sqrt{2}$$

Both circles are concentric.

$$\therefore (n + 1)^2 + n^2 - 2(n + 1) - 7 > 0$$

$$\text{and } (n + 1)^2 + n^2 - 2(n + 1) - 15 < 0 \Rightarrow 4 < n^2 < 8$$

Which is not possible for any integer.

130. (a) We have,

$$\frac{dy}{dx} = e^{x-y}(e^x - e^y)$$

$$\Rightarrow e^y \frac{dy}{dx} + e^y e^x = e^{2x}$$

$$\text{Put } e^y = v$$

$$\therefore e^y \frac{dy}{dx} = \frac{dv}{dx}$$

$$\therefore \frac{dv}{dx} + ve^x = e^{2x}$$

This is linear equation

$$\text{I.F} = e^{\int e^x dx} = e^{e^x}$$

$\therefore$ Required solution is

$$v \cdot e^{e^x} = \int e^{e^x} \cdot e^{2x} dx + C$$

$$v \cdot e^{e^x} = e^{e^x} (e^x - 1) + C$$

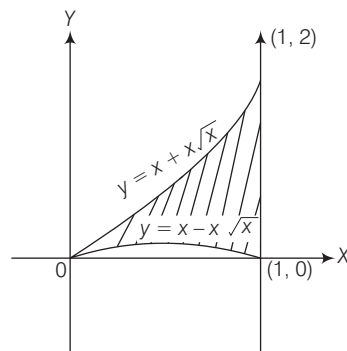
$$\Rightarrow e^y \cdot e^{e^x} = e^{e^x} (e^x - 1) + C$$

$$\Rightarrow e^y = e^x - 1 + ce^{-e^x}$$

131. (d) Given, curve is $(y - x)^2 = x^3$

$$\Rightarrow y - x = \pm x\sqrt{x}$$

$$\Rightarrow y = x \pm x\sqrt{x}$$



$$\therefore \text{Area bounded} = \left| \int_0^1 \{(x + x\sqrt{x}) - (x - x\sqrt{x})\} dx \right|$$

$$= \left| 2 \int_0^1 x^{3/2} dx \right| = \frac{4}{5} \text{ sq unit}$$

132. (d) We have, $f(x) = \int_0^x (3 \sin x + 4 \cos x) dx$

$$\Rightarrow f'(x) = 3 \sin x + 4 \cos x = 5 \sin \left(x + \tan^{-1} \frac{4}{3} \right)$$

$$\frac{5\pi}{4} < x < \frac{4\pi}{3}$$

$$\Rightarrow \pi < x + \tan^{-1} 4/3 < 2\pi$$

$\therefore f'(x) < 0$, i.e. $f(x)$ is decreasing.

$$\therefore \text{least value of } f(x) \text{ is } \left(\frac{5\pi}{4}, \frac{4\pi}{3} \right) = f \left(\frac{4\pi}{3} \right)$$

$$\text{least } f(x) = \int_0^{4\pi/3} (3 \sin x + 4 \cos x) dx$$

$$= [-3 \cos x + 4 \sin x]_0^{4\pi/3}$$

$$= -3 \left(-\frac{1}{2} - 1 \right) + 4 \left(-\frac{\sqrt{3}}{2} \right)$$

$$= \frac{9 - 4\sqrt{3}}{2}$$

133. (a) Let $z_1 = re^{i\theta}$

$$\text{then, } \bar{z}_1 = re^{-i\theta}$$

$$\frac{z_1}{\bar{z}_1} = e^{2i\theta} = e^{2\pi i/n}$$

$$\Rightarrow \theta = \frac{\pi}{n}$$

$$\text{Also, } \frac{I_m(z_1)}{R_e(z_1)} = \tan \theta = \tan \frac{\pi}{n} = \sqrt{2} - 1$$

$$\Rightarrow \tan \frac{\pi}{8} = \sqrt{2} - 1$$

$$\therefore n = 8$$

134. (b) $(1 + x + x^3 + x^4)^{10} = [(1 + x)(1 + x^3)]^{10}$

$$= (1 + x)^{10} (1 + x^3)^{10}$$

$$= (1 + {}^{10}C_1 x + {}^{10}C_2 x^2 + {}^{10}C_3 x^3 + {}^{10}C_4 x^4 \dots)$$

$$\times (1 + {}^{10}C_1 x^3 + {}^{10}C_2 x^6 + \dots)$$

$$\therefore \text{Coefficient of } x^4 = ({}^{10}C_1)({}^{10}C_1) + {}^{10}C_4$$

$$= 100 + 210 = 310$$

135. (c) Since only one letter can not be in wrong envelope so that at least two letters are in wrong envelope means all the letters are not in right envelopes.

$$\therefore x = 6! - 1 = 720 - 1 = 719$$

Y = number of ways, so that all the letters are in wrong envelopes

$$= 6! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} \right)$$

$$= 360 - 120 + 30 - 6 + 1 = 265$$

$$\therefore x - y = 719 - 265 = 454$$

136. (c) We have, $x = \log_5 3 + \log_7 5 + \log_9 7$

AM $\geq$ GM

$$\therefore \frac{\log_5 3 + \log_7 5 + \log_9 7}{3}$$

$$\geq (\log_5 3 \cdot \log_7 5 \cdot \log_9 7)^{1/3}$$

$$\Rightarrow x \geq 3 \left(\frac{\log 3}{\log 5} \times \frac{\log 5}{\log 7} \times \frac{\log 7}{\log 9} \right)^{1/3}$$

$$\Rightarrow x > 3 \left(\frac{1}{2} \right)^{1/3} \Rightarrow x > \frac{3}{\sqrt[3]{2}}$$

137. (c) $x^2 - 2x + A = 0$

$$\Rightarrow x = 1 \pm \sqrt{1 - A}$$

$$\therefore p = 1 - \sqrt{1 - A}, q = 1 + \sqrt{1 - A}$$

$$\text{Similarly, } r = 9 - \sqrt{81 - B}, s = 9 + \sqrt{81 - B}$$

p, q, r, s are in AP.

$$\therefore q - p = s - r$$

$$\Rightarrow 2\sqrt{1 - A} = 2\sqrt{81 - B}$$

$$\Rightarrow B = 80 + A$$

$$\text{and } q - p = r - q$$

$$\Rightarrow 3\sqrt{1 - A} = 8 - \sqrt{81 - B}$$

$$\Rightarrow A = -3, B = 77$$

138. (c) We have,

$$(1 - a)x + y + z = 0$$

$$x + (1 - b)y + z = 0$$

$$x + y + (1 - c)z = 0$$

has many solutions

$$\therefore \begin{vmatrix} 1-a & 1 & 1 \\ 1 & 1-b & 1 \\ 1 & 1 & 1-c \end{vmatrix} = 0$$

$$\Rightarrow ab + ac + bc = abc$$

$$\therefore \frac{ab + bc + ac}{3} \geq (a^2 b^2 c^2)^{1/3} = (abc)^{1/3} \geq 3$$

$$\therefore \text{Minimum of } abc = (3)^3 = 27$$

139. (c) Given,

$$AA' = A' A$$

and

$$B = A^{-1} A'$$

$$BB' = (A^{-1} A')(A^{-1} A')'$$

$$= (A^{-1} A')[(A')'(A^{-1})']$$

$$= (A^{-1} A')[A(A')^{-1}]$$

$$[\because (A')' = A, (A^{-1})' = (A')^{-1}]$$

$$= A^{-1}(A' A)(A')^{-1}$$

$$= A^{-1}(AA')(A')^{-1}$$

$$= (A^{-1} A)(A^{-1} A)'$$

$$= I \cdot I = I$$

140. (a) We have,

$$\sin^{-1} x + \tan^{-1} x = \frac{\pi}{2} \Rightarrow 0 < x < 1$$

$$\therefore \tan^{-1} x = \frac{\pi}{2} - \sin^{-1} x = \cos^{-1} x$$

$$\Rightarrow \tan^{-1} x = \tan^{-1} \left(\frac{\sqrt{1-x^2}}{x} \right)$$

$$\Rightarrow x^2 = \sqrt{1-x^2}$$

$$\Rightarrow x^4 + x^2 - 1 = 0 \Rightarrow x^2 = \frac{-1 \pm \sqrt{5}}{2}$$

$$\Rightarrow x^2 > 0$$

$$\therefore x^2 = \frac{\sqrt{5} - 1}{2}$$

$$\Rightarrow 2x^2 + 1 = \sqrt{5}$$

141. (a) Given, $|\cos x| = 2[x]$

$$0 \leq |\cos x| \leq 1$$

$$\therefore 0 \leq 2[x] \leq 1$$

$$0 \leq [x] \leq \frac{1}{2}$$

$$x = 0$$

$$\therefore |\cos x| = 0$$

$$x : n\pi + \frac{\pi}{2}$$

For any value of $n \in N, [n] \neq 0$

Hence, the equation has no solution.

142. (c) $0 \leq \alpha, \beta \leq \frac{\pi}{4}; \alpha + \beta \leq \frac{\pi}{2}$

$\cos(\alpha + \beta) = 4/5 \Rightarrow \tan(\alpha + \beta) = 3/4$

$\sin(\alpha - \beta) = 5/13 \Rightarrow \alpha - \beta \in \theta_1$

and $\tan(\alpha - \beta) = 5/12$

$$\begin{aligned} \tan 2\alpha &= \tan(\alpha + \beta + \alpha - \beta) \\ &= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \tan(\alpha - \beta)} \\ &= \frac{3/4 + 5/12}{1 - 3/4 \times \frac{5}{12}} = \frac{56}{33} \end{aligned}$$

143. (c) 18 draws are required for 2 aces means in the first 17 draws, there is one ace and 16 non-ace and 18th draw 2nd ace.

$\therefore$ Required probability $= \frac{{}^{48}C_{16} \times {}^4C_1}{{}^{52}C_{17}} \times \frac{3}{35} = \frac{561}{15925}$

144. (c) $(\mathbf{a} + p\mathbf{b})$ is normal to $\mathbf{c}$

$$\begin{aligned} \therefore (\mathbf{a} + p\mathbf{b}) \cdot \mathbf{c} &= 0 \\ ((1-p)\hat{\mathbf{i}} + (2+2p)\hat{\mathbf{j}} + (3+p)\hat{\mathbf{k}}) \cdot (3\hat{\mathbf{i}} + \hat{\mathbf{j}}) &= 0 \\ 3-3p+2+2p &= 0 \Rightarrow p=5 \end{aligned}$$

145. (b) We have,

$$\begin{aligned} f(x) &= x[x] \\ f(x) &= px \quad [\because [x] = p] \\ f'(x) &= p = [x] \\ \therefore f'(x) &= [x] \end{aligned}$$

146. (b) We have,

$$\begin{aligned} f(x) &= x^2 e^{-x^2} \\ f'(x) &= 2xe^{-x^2} - 2x^3 e^{-x^2} \\ f'(x) &= 2xe^{-x^2}(1-x^2) \\ f'(x) &= 0, x=0, -1, 1 \end{aligned}$$

$f'(-h) < 0, f'(h) > 0$, So minimum at $x=0$

$f'(1-h) > 0, f'(1+h) < 0$, So maximum at $x=1$

$f'(-1-h) > 0, f'(-1+h) < 0$, So maximum at $x=-1$

Minimum $f(x) = f(0) = 0$

Maximum $f(x) = e^{-1} = \frac{1}{e}$

Since, $f(x) \geq 0 \forall x$

So, difference between maximum and minimum values $= \frac{1}{e} - 0 = \frac{1}{e}$

147. (a) Given,

$f'(x) = f(x) + \int_0^1 f(x) dx, f(0) = 1 \quad \dots(i)$

$\Rightarrow f''(x) = f'(x) + 0$

$\Rightarrow \frac{f''(x)}{f'(x)} = 1$

$\Rightarrow \int \frac{f''(x)}{f'(x)} dx = \int dx$

$\Rightarrow \log f'(x) = x + C$

$\Rightarrow f'(x) = Ae^x$

$\Rightarrow f(x) = Ae^x + K$

$f(0) = A + K = 1 \quad \dots(ii)$

$\therefore Ae^x = Ae^x + K + \int_0^1 (Ae^x + k) dx$

$\Rightarrow k + [Ae^x + Kx]_0^1 = 0$

$\Rightarrow k + Ae - A + K = 0$

$\Rightarrow A(e-1) + 2k = 0 \quad \dots(iii)$

From Eqs. (ii) and (iii), we get

$A = \frac{2}{3-e}, k = \frac{1-e}{3-e}$

$\therefore f(x) = \frac{2e^x}{3-e} + \frac{1-e}{3-e}$

148. (b) We have,

$y = e^{2x} + x^2$

at $x=0, y=1$

$\frac{dy}{dx} = 2e^{2x} + 2x$

$\left(\frac{dy}{dx}\right)_{x=0} = 2$

$\therefore$ Equation of normal at $(0, 1)$ is

$y-1 = -\frac{1}{2}(x)$

$\Rightarrow x+2y-2=0$

$\therefore$ Distance from $(0, 0)$ from normal $= \frac{2}{\sqrt{5}}$

149. (c) Given line $\frac{x-1}{-1} = \frac{y-2}{3} = \frac{z-4}{1}$ is passes through

$P(1, 2, 4)$.

To find the reflection of line we need one more point on the line clearly $M(0, 5, 5)$ also lie on line.

Let $N(\alpha, \beta, \gamma)$ be the reflection M in the

$x+y+z=7$

$\therefore \frac{\alpha}{2} + \frac{\beta+5}{2} + \frac{\gamma+5}{2} + 7$

$\Rightarrow \alpha + \beta + \gamma = 4$

Also, MN is perpendicular, i.e. parallel to the normal of the plane

$$\therefore \frac{\alpha}{1} = \frac{\beta-5}{1} = \frac{\gamma-5}{1} = \lambda$$

$$\Rightarrow \alpha = \lambda, \beta = \lambda + 5, \gamma = \lambda - 5$$

$$\therefore \lambda + \lambda + 5 + \lambda + 5 = 4$$

$$\Rightarrow \lambda = -2$$

$$\therefore N = (-2, 3, 5)$$

$$\text{Equation } PN \text{ is } \frac{x-1}{-3} = \frac{y-2}{1} = \frac{z-4}{-1}$$

150. (b) We have,

$$x + y + z = 5$$

$$x^2 + y^2 + z^2 = 9$$

$$\Rightarrow x^2 + y^2 + (5 - x - y)^2 = 9$$

$$\Rightarrow y^2 + x^2 - 5x - 5y + xy + 8 = 0$$

$$\Rightarrow y^2 + y(x-5) + (x^2 - 5x + 8) = 0$$

$$\Rightarrow \text{Since } y \in R$$

$$(x-5)^2 - 4(x^2 - 5x + 8) \geq 0$$

$$\Rightarrow x^2 - 10x + 25 - 4x^2 + 20x - 32 \geq 0$$

$$\Rightarrow 3x^2 - 10x + 7 \leq 0$$

$$\Rightarrow 3x^2 - 7x - 3x + 7 \leq 0$$

$$\Rightarrow (3x-7)(x-1) \leq 0$$

$$\Rightarrow x \in \left[1, \frac{7}{3}\right]$$

$$\text{Length of interval} = \left(\frac{7}{3} - 1\right) = \frac{4}{3}$$

Solved Paper 2020

BITSAT

Instructions

- There are 150 questions in all. The number of questions in each part is as given below.

Part I Physics	1-40
Part II Chemistry	41-80
Part III a. English Proficiency	81-95
b. Logical Reasoning	96-105
Part IV Mathematics	106-150
- All questions are multiple choice questions with four options, only one is correct.
- Each correct answer fetches 3 marks while incorrect answer has a penalty of 1 mark.

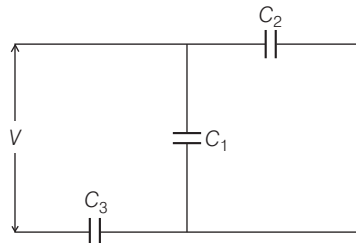
No. of Questions

PART I

Physics

- Three solenoid coils of same dimensions, same number of turns and same number of layers of windings are taken. Coil 1 has inductance L_1 wound by Mn wire of resistance $6\Omega\text{m}^{-1}$, coil 2 with inductance L_2 wound by similar wire but in reverse direction in each layer. Coil 3 with inductance L_3 wound by a superconducting wire. The relation between their self inductances will be

a. $L_1 = L_2, L_3 = 0$	b. $L_1 = L_2 = L_3$
c. $L_1 = L_3, L_2 = 0$	d. $L_1 > L_2 > L_3$
- Three capacitors C_1, C_2 and C_3 are connected as shown in the figure below. If capacitor C_3 breaks down electrically, the change in total charge on the combination of capacitors is



- a. $(C_1 + C_2)V \left[1 - \left(\frac{C_3}{C_1 + C_2 + C_3} \right) \right]$
- b. $(C_3 + C_2) \left[1 - \left(\frac{C_1}{(C_1 + C_2 + C_3)} \right) \right]$

c. $(C_1 + C_2)V \left[1 - \left(\frac{C_2 + C_1}{C_1 + C_3} \right) \right]$

d. $(C_1 + C_2)V \left[1 - \left(\frac{C_2}{(C_1 + C_3)} \right) \right]$

- A black body radiates energy at the rate $E \text{ Wm}^{-2}$ at high temperature $T \text{ K}$. When the temperature is reduced to $\left(\frac{T}{4}\right) \text{ K}$, the new radiant energy is

a. $\frac{E}{256}$	b. $4E$	c. $\frac{E}{4}$	d. $\frac{E}{16}$
--------------------	---------	------------------	-------------------
- The length of the rectangle is $l = 15.2 \text{ cm}$ and breadth is $b = 2.9 \text{ cm}$ and the minimum possible measurement by scale = 0.1 cm . Then, the area of the rectangle is (Taking significant figures into consideration)

a. 44.08 cm^2	b. 24.8 cm^2
c. 44 cm^2	d. 94.008 cm^2
- In an adiabatic process, where pressure is decreased by $\frac{3}{4}\%$, if $\frac{C_p}{C_v} = \frac{4}{3}$, then the volume increases by

a. $\frac{3}{4}\%$	b. $\frac{9}{16}\%$
c. $\frac{16}{9}\%$	d. $\frac{4}{3}\%$

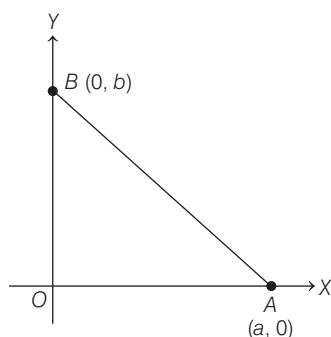
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6. The vibrations of a string of length 60 cm fixed at both ends are represented by the equation

$$y = 4 \sin\left(\frac{\pi x}{15}\right) \cos(96\pi t)$$

where x and y are in cm and t is in second. Calculate the velocity of the particle at $x = 7.5$ cm at $t = 0.25$ s.

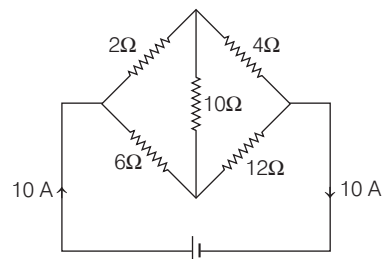
- a. 4 m/s
b. 0
c. 16 m/s
d. 9.8 m/s
7. A charge $+q$ is placed at the origin O of XY -axes as shown in the figure. The work done in taking a charge Q from A to B along the straight line AB is



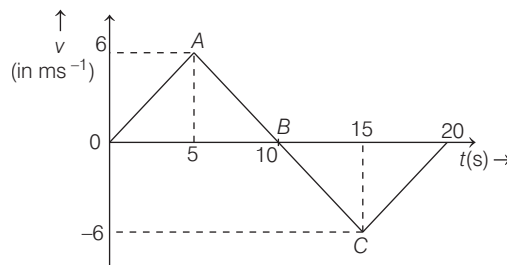
- a. $\frac{qQ}{4\pi\epsilon_0} \left(\frac{a-b}{ab} \right)$
b. $\frac{qQ}{4\pi\epsilon_0} \left(\frac{b-a}{ab} \right)$
c. $\frac{qQ}{4\pi\epsilon_0} \left(\frac{b}{a^2} - \frac{1}{b} \right)$
d. $\frac{qQ}{4\pi\epsilon_0} \left(\frac{a}{b^2} - \frac{1}{b} \right)$
8. The hydrogen-like element that has a spectrum whose lines have wavelength four times shorter than those of atomic hydrogen is
a. Lithium
b. Helium
c. Berilliyum
d. Potassium
9. Two small conducting spheres of equal radius have charges $+20\mu\text{C}$ and $-40\mu\text{C}$ respectively and placed at a distance R from each other experience force F_1 . If they are brought in contact and separated to the same distance, they experience force F_2 . The ratio of F_1 to F_2 is
a. 1 : 4
b. +8 : 1
c. -8 : 1
d. 1 : 8
10. A Carnot engine has the same efficiency between 600 K to 300 K and 1600 K to x K, then the value of x is
a. 1600 K
b. 800 K
c. 819 K
d. 900 K

11. A ball of mass 0.5 kg is thrown up with initial speed 16 ms^{-1} and reaches maximum height of 9 m. How much energy is dissipated by air drag acting on the ball during the ascent?
a. 199 J
b. 19.9 J
c. 20.9 J
d. 9.9 J

12. In the circuit shown, if the 10Ω resistor is replaced by a resistor of 15Ω , then what is the amount of current drawn from the battery?

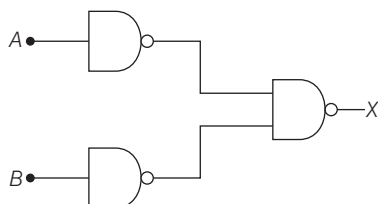


- a. 100 A
b. 10 A
c. 1 A
d. 2.4 A
13. A time dependent force $F = (8t)$ acts on a particle of mass 2 kg. If the particle starts from rest, the work done by the force during the first 1s will be
a. 0.4 J
b. 4 J
c. 19 J
d. 4.5 J
14. If L, R, C and V represent inductance, resistance, capacitance and potential difference respectively, then dimensions of $\frac{L}{RCV}$ are the same as those of
a. current
b. $\frac{1}{\text{current}}$
c. charge
d. $\frac{1}{\text{charge}}$
15. The magnetic field of a beam emerging from a fitter facing a flood light is given by
 $B = 10 \times 10^{-8} \sin(1 \times 10^7 z - 3.6 \times 10^{15} t)$ T. The average intensity of the beam is
a. 1.82 W/m^2
b. 1.19 W/m^2
c. 1.18 W/m^2
d. 1.17 W/m^2
16. From the velocity-time graph of a body moving in a straight line, the distance travelled and the average velocity in the time interval $t = 0$ to $t = 20$ s are, respectively,

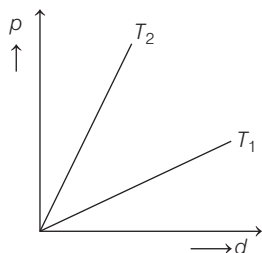


- a. 0, 0
b. 120 m, 60 m
c. 60 m, 0
d. 0, 60 m
17. A thin equi-convex lens is made of glass of refractive index 1.5 and its focal length is 0.2 m. If it acts as a concave lens of focal length 0.5 cm when dipped in a liquid, the refractive index of the liquid is
a. $\frac{17}{8}$
b. $\frac{15}{8}$
c. $\frac{13}{8}$
d. $\frac{9}{8}$

18. A moving coil galvanometer has a resistance of $60\ \Omega$ and it indicates full deflection on passing a current of $4.5\ \text{mA}$. A voltmeter is made using this galvanometer and a $4.5\ \text{k}\Omega$ resistance. The maximum voltage, that can be measured using this voltmeter, will be close to
 a. $21\ \text{V}$ b. $20.5\ \text{V}$ c. $20\ \text{V}$ d. $19.5\ \text{V}$
19. The combination of the gates shown in following figure yields



- a. NAND gate b. OR gate
 c. NOT gate d. XOR gate
20. A vessel contains one mole of O_2 gas (molar mass 32) at a temperature T . The pressure of the gas is p . An identical vessel containing one mole of He gas (molar mass 4) at a temperature $2T$ has a pressure of
 a. $\frac{p}{8}$ b. p c. $2p$ d. $8p$
21. The acceleration of a particle in m/s^2 is given by $a = (3t^2 - 2t + 1)$, where t is in second. If the particle starts with a velocity $v = 1\ \text{m/s}$ at $t = 1\ \text{s}$, then velocity of the particle at the end of $4\ \text{s}$ is
 a. $40\ \text{m/s}$ b. $52\ \text{m/s}$ c. $48\ \text{m/s}$ d. $84\ \text{m/s}$
22. The figure shows graphs of pressure (p) versus density (d) for an ideal gas at two temperatures T_1 and T_2 , then



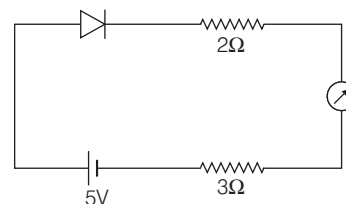
- a. $T_1 > T_2$ b. $T_1 < T_2$
 c. $T_1 = T_2$ d. None of these
23. Two spheres of the same material and same radii r are touching each other. The gravitational force between the spheres is proportional to
 a. $\frac{1}{r^2}$ b. r^2 c. $\frac{1}{r^4}$ d. r^4
24. A spherical lens of power $-4\ \text{D}$ is placed at a distance of $15\ \text{cm}$ from another spherical lens of power $5\ \text{D}$. A beam of parallel light falls on the first spherical lens. The final image formed is
 a. real and at a distance of $40\ \text{cm}$ from the lens of power $5\ \text{D}$

- b. real and at a distance of $10\ \text{cm}$ from the lens of power $-4\ \text{D}$.
 c. virtual and at a distance of $40\ \text{cm}$ from the lens of power $-5\ \text{D}$
 d. None of the above

25. The wheel of a car, accelerated uniformly from rest, rotates through 5 radians during the first second. The angle (in radians) rotated during the next second is
 a. 15 b. 7.5 c. 12.5 d. 20

26. Lights of two different frequencies whose photons have energies $1.5\ \text{eV}$ and $2.5\ \text{eV}$ respectively illuminate a metallic surface whose work function is $0.5\ \text{eV}$ successively. Ratio of maximum speeds of emitted electrons will be
 a. $3 : 2$ b. $2 : 3$
 c. $\sqrt{3} : \sqrt{2}$ d. $\sqrt{2} : \sqrt{3}$

27. When a body is dropped from a height h , then it hits the ground with a momentum p . If the same body is dropped from a height which is three times more than previous height, the percentage change in momentum when it hits the ground is
 a. 25% b. 50% c. 75% d. 100%
28. The reading of the ammeter for a germanium diode in the given circuit is



- a. $0.94\ \text{A}$ b. 0 c. $2.8\ \text{A}$ d. $5\ \text{A}$
29. The decay constants of two radioactive substances X and Y are 4λ and λ respectively. At $t = 0$, a sample has the same number of two nuclei. The time taken for the ratio of number of nuclei to become $\frac{1}{e^3}$ will be
 a. $\frac{1}{3\lambda}$ b. $\frac{1}{2\lambda}$ c. $\frac{2}{3\lambda}$ d. $\frac{3}{2\lambda}$
30. The magnitude of the force vector acting on a unit length of a thin wire carrying a current $I = 10\ \text{A}$ at a point O, if the wire is bent in the form of a semi-circle (shown below) with radius $R = 20\pi\ \text{cm}$, is
 a. $30\ \mu\text{N/m}$ b. $40\ \mu\text{N/m}$
 c. $50\ \mu\text{N/m}$ d. $60\ \mu\text{N/m}$
31. A long cylindrical iron core of cross-sectional area $5\ \text{cm}^2$ is inserted into a long solenoid having 4000 turns/meter and carrying a current $5\ \text{A}$. The magnetic field inside the core is $\pi\ \text{T}$. Find the pole strength developed.
 a. $1000\ \text{A-m}$ b. $1240\ \text{A-m}$
 c. $882\ \text{A-m}$ d. $760\ \text{A-m}$

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32. A plane requires for take off a speed of 72 kmh^{-1} , the run on the ground being 50 m. The mass of the plane is 10000 kg and the coefficient of friction between the plane and the ground is 0.2. Assume that the plane accelerates uniformly during take off. The minimum force required by the engine of the plane for take off is

a. $4.43 \times 10^4 \text{ N}$ b. $5.96 \times 10^4 \text{ N}$
c. $2.25 \times 10^4 \text{ N}$ d. $3.45 \times 10^4 \text{ N}$

33. In a fluorescent lamp choke, 120 V of reverse voltage is produced when the choke current changes uniformly 0.50 A to 0.20 A in a duration of 0.030 ms. The self inductance of the choke (in mH) is estimated to be

a. 12 H b. $12 \times 10^{-3} \text{ mH}$
c. $12 \times 10^{-3} \text{ H}$ d. 0

34. A man grows into a giant such that his linear dimensions increase by a factor of 8. Assuming that his density remains same, the stress in his leg will change by a factor of

a. $\frac{1}{8}$ b. 8 c. 36 d. 64

35. When 2 moles of a monoatomic gas are mixed with 3 moles of a diatomic gas, the value of adiabatic exponent for the mixture is

a. $\frac{15}{16}$ b. $\frac{7}{5}$
c. $\frac{31}{21}$ d. $\frac{38}{59}$

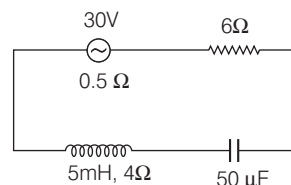
36. A simple pendulum is placed inside a lift, the lift is moving with a uniform acceleration. If the time periods of the pendulum while the lift is moving upwards and downwards are in the ratio of 1 : 3, then the acceleration of the lift is

[Take acceleration due to gravity, $g = 10 \text{ m/s}^2$]
a. 4 m/s^2 b. 6 m/s^2
c. 8 m/s^2 d. 10 m/s^2

37. A satellite is moving around the earth with speed v in a circular orbit of radius r . If the orbit radius is decreased by 2%, its speed will increase by

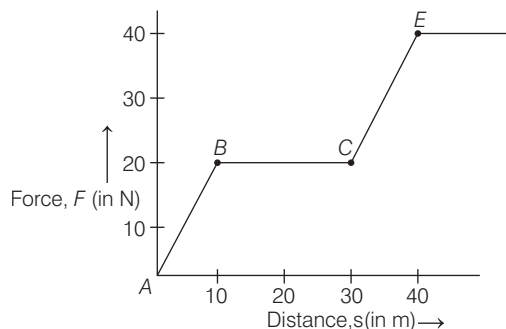
a. 1% b. 2% c. 1.5% d. 1.414%

38. In the circuit shown in figure, the AC source has angular frequency $\omega = 2000 \text{ rad s}^{-1}$. The amplitude of the current will be nearest to



a. 2.85 A b. 3 A
c. 0.5 A d. 3.625 A

39. The work done by a force acting on a body is as shown in the following graph. The total work done in covering an initial distance of 40 m is



a. 500 J b. 600 J c. 400 J d. 800 J

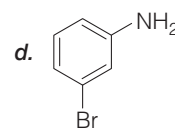
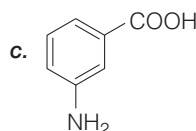
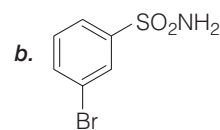
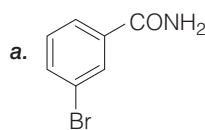
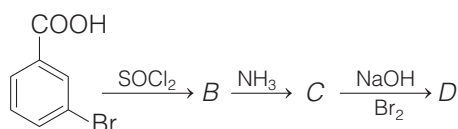
40. A potentiometer wire, 10 m long, has a resistance of 40Ω . It is connected in series with a resistance box and a 4 V storage cell. If the potential gradient along the wire is 0.4 mVcm^{-1} , the resistance unplugged in the box is

a. 220Ω b. 360Ω
c. 760Ω d. 848.3Ω

PART II

Chemistry

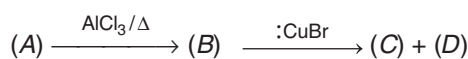
41. In a set of reactions, *m*-bromobenzoic acid gives a product *D*. Identify the product *D*:



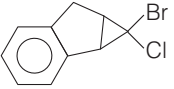
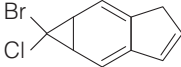
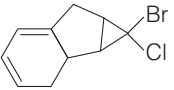
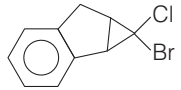
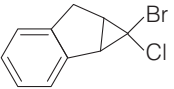
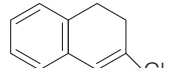
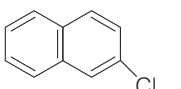
42. A volume of 50.00 mL of a weak acid of unknown concentration is titrated with 0.10 M solution of NaOH. The equivalence point is reached after 39.30 mL of NaOH solution has been added. At the half equivalence point (19.65 mL) the pH is 4.85. Thus, initial concentration of the acid and its pK_a values are

[HA]	pK_a
a. 0.1 M	4.85
b. 0.079 M	4.85
c. 0.1 M	3.70
d. 0.097 M	2.93

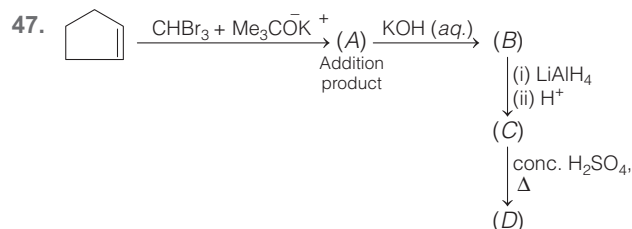
43. 1-phenyl-1, 3-dibromopropane on treatment with alc. KOH gives diastereomeric mixture in which compound (A) is major product. (A) gives the following reaction



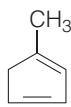
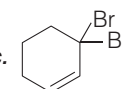
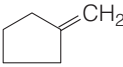
Compound (C) and (D) are

- a.  and 
- b.  and 
- c.  and 
- d.  and 

44. Which of the following statement is incorrect?
- α -D-fructose and β -D-fructose are enantiomers of each other.
 - D-glyceraldehyde and L-glyceraldehyde are enantiomers of each other.
 - The reserve carbohydrate of animals is glycogen.
 - Aldohexoses which react with phenyl hydrazine to give identical osazones are C-2 epimers.
45. Which of the following is not a mixed pair of oxides?
- Mn_3O_4 and Co_3O_4
 - Co_3O_4 and Pb_3O_4
 - Pb_3O_4 and Mn_3O_4
 - Fe_3O_4 and Fe_2O_3
46. A solution of copper sulphate is electrolysed between copper electrodes by a current of 10.0A passing for one hour. Which of the following statements is correct regarding the changes occur at the electrodes and in the solution?
- 11.84 g of copper will deposit on the cathode
 - 11.84 g of copper will deposit on the anode
 - 11.84 g of copper will deposit on the anode as well as on the cathode
 - copper will not deposit on any of the electrode



Identify structure of compound (D).

- a.  b.  c.  d. 

48. Which of the following statement is correct?

- ΔS for $\frac{1}{2} Cl_2(g) \rightarrow Cl(g)$ is positive.
- $\Delta E < 0$ for combustion of $CH_4(g)$ in a sealed container with rigid adiabatic system.
- ΔG is always zero for a reversible process in a closed system.
- ΔG° for an ideal gas reaction is a function of pressure.

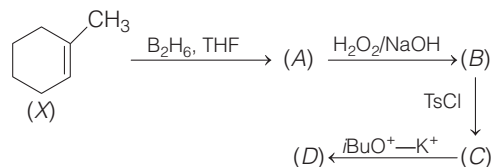
49. Which of the following statements on critical constants of gases are correct?

- Larger the T_c/p_c value of a gas, larger would be the included volume.
- Critical temperature (T_c) of a gas is greater than its Boyle temperature (T_B).
- At the critical point in the van der Waals' gas isotherm, $\left(\frac{\partial p}{\partial V_m}\right)_{T_c} = 0$

Select the correct answer using the codes given below.

- Both I and II
- Both I and III
- Both II and III
- I, II and III

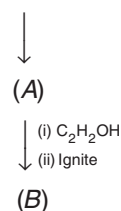
50. In the given reaction,



Product (D) is

- a positional isomer of X
- identical to X
- chain isomer of X
- an oxidation product of X

51. $Na_2B_4O_7 + \text{conc. } H_2SO_4 + H_2O$



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(B) is identified by the characteristic colour of the flame. (A) and (B) are respectively:

- a. H_3BO_3 and $\text{Na}_2\text{B}_4\text{O}_7$ b. $\text{B}(\text{OC}_2\text{H}_5)_3$ and H_3BO_3
c. NaBO_2 and H_3BO_3 d. H_3BO_3 and $\text{B}(\text{OC}_2\text{H}_5)_3$

52. How many mL of perhydrol is required to produce sufficient oxygen which can be used to completely convert 2 L of SO_2 gas?
a. 10 mL b. 5 mL
c. 20 mL d. 30 mL
53. At the top of a mountain the thermometer reads 0°C and the barometer reads 710 mm Hg. At the bottom of the mountain the temperature is 30°C and the pressure is 760 mm Hg. The ratio of the density of air at the top to that of the bottom is
a. 1 : 1.04 b. 0.4 : 1
c. 1.04 : 1 d. 1 : 0.4

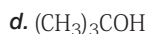
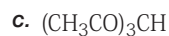
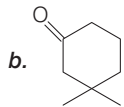
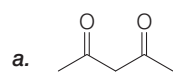
54. Match the following columns :

Column I (Compound)	Column II (Structure)
A. ClF_3	1. Square planar
B. PCl_5	2. Tetrahedral
C. IF_5	3. Trigonal bipyramidal
D. CCl_4	4. Square pyramidal
E. XeF_4	5. T-shaped

Codes

A	B	C	D	E
a. 5	4	3	2	1
b. 5	3	4	2	1
c. 5	3	4	1	2
d. 4	3	5	2	1

55. Most acidic hydrogen is present in



56. The process of 'eutrophication' is due to the
a. increase in concentration of insecticide in H_2O .
b. increase in concentration of fluoride ion in H_2O
c. reduction in concentration of the dissolved oxygen in water due to phosphate pollution in water.
d. attack of younger leaves of a plant by peroxyacetyl nitrate
57. Ionisation energy of H-atom is 13.6 eV. The wavelengths of the spectral line emitted when an electron in Be^{3+} comes from 5th energy level to 2nd energy level is
a. 43.5 nm b. 4350 nm
c. 4.35 nm d. 435 nm

58. The enthalpies of combustion of carbon and carbon monoxide in excess of oxygen at 298 K and constant pressure are -393.5 kJ/mol and -280.0 kJ/mol respectively. The heat of formation of carbon monoxide at constant volume is
a. $+111.7 \text{ kJ/mol}$ b. -1111.7 kJ/mol
c. -111.7 kJ/mol d. -11.7 kJ/mol

59. If the quantum number l could have the value ' n ' also then, $\text{Sc}(21)$ would have electronic configuration as (other rules strictly followed)
a. $1s^2 1p^6 2s^2 2p^6 2d^3 3s^2$
b. $1s^2 1p^6 2s^2 2p^6 3s^2 2d^3$
c. $1s^2 2s^2 2p^6 3s^2 3p^6 3d^1 4s^2$
d. $1s^2 2s^2 2p^6 3s^2 3p^6 3d^3$

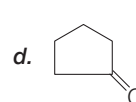
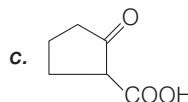
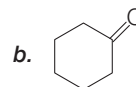
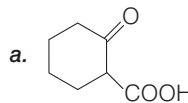
60. 1.1 mole of A mixed with 2.2 moles of B and the mixture is kept in a 1 L flask and the equilibrium, $A + 2B \rightleftharpoons 2C + D$ is reached. If at equilibrium 0.2 mole of C is formed then the value of K_c will be
a. 0.1 b. 0.01 c. 0.001 d. 1

61. Choose the correct chemical reaction among the following:
a. $\text{CaCN}_2 + \text{H}_2\text{O} \longrightarrow \text{Ca}(\text{OH})_2 + \text{C}_2\text{H}_2 + \text{NH}_3$
b. $2\text{NH}_3 + \text{CaSO}_4 + \text{CO}_2 + \text{H}_2\text{O} \longrightarrow \text{CaCN}_2 + (\text{NH}_4)_2\text{SO}_4$
c. $\text{CaCl}_2 + \text{Na}_2\text{SO}_4 \longrightarrow \text{CaSO}_4 + 2\text{NaCl}$
d. $\text{CaC}_2 + \text{H}_2\text{O} \longrightarrow \text{Ca}(\text{OH})_2 + 2\text{NaCl}$

62. Which of the following reactions represents disproportionation?
a. $\text{CrO}_5 \longrightarrow \text{Cr}^{3+} + \text{O}_2$
b. $\text{IO}_3^- + \text{I}^- + \text{H}^+ \longrightarrow \text{I}_2$
c. $\text{CrO}_2\text{Cl}_2 + \text{NaOH} \longrightarrow \text{Na}_2\text{CrO}_4 + \text{NaCl} + \text{H}_2\text{O}$
d. $\text{Na}_2\text{S}_2\text{O}_3 + \text{H}_2\text{SO}_4 \longrightarrow \text{Na}_2\text{SO}_4 + \text{SO}_2 + \text{S}_8 + \text{H}_2\text{O}$

- 63.

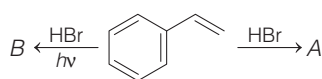
Compound C is

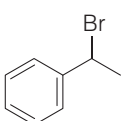
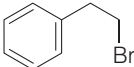
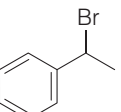
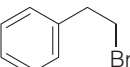
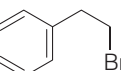
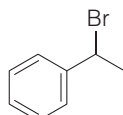


64. On the basis of Ellingham diagram, which of the following is not correct?
a. Entropy change for all metal oxides is roughly same.
b. Below the boiling point, $T\Delta S$ factor is same irrespective of metal.
c. Above $\Delta G = 0$ line, oxide decomposes into metal and oxygen.
d. If randomness increases the slope increases.

65. Which of the following is not explained by adsorption?
- When acetic acid solution is shaken with charcoal the concentration of acid decreases.
 - The white ppt. of $\text{Mg}(\text{OH})_2$ attains blue colour when precipitated in the presence of magneson reagent.
 - An aqueous solution of NaOH attains pink colour with a drop of phenolphthalein.
 - When animal charcoal is shaken with coloured methylene blue solution, the solution turns colourless.

66. Identify 'A' and 'B' in the following reaction

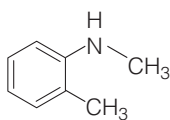
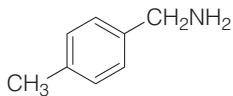
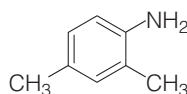
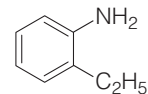


- Both A and B are 
- Both A and B are 
- A is  and B is 
- A is  and B is 

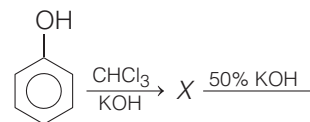
67. An element occurs in two crystalline forms α and β . The α -form has an fcc with $a = 3.68 \text{ \AA}$ and β form has a bcc with $a = 2.92 \text{ \AA}$. Calculate the ratio of their densities.
- 1 : 1
 - 1 : 2
 - 2 : 1
 - 2 : 3

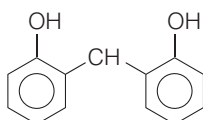
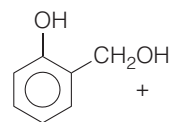
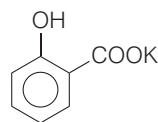
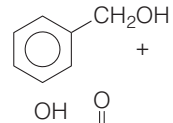
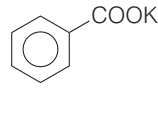
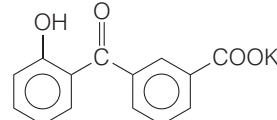
68. The molar heat of vaporisation of water at 100°C is 40.585 kJ/mol . The temperature at which a solution containing 5.60 g of $\text{Al}_2(\text{SO}_4)_3$ per 1000 g of water boil is _____. (Assuming the degree of ionisation of salt to be 1).
- 1000.042°C
 - 10.0042°C
 - 100.042°C
 - 105°C

69. An organic base $\text{C}_8\text{H}_7\text{N}(X)$ reacts with nitrous acid at 0°C to give a clear solution. Heating the solution with KCN and cuprous cyanide followed by continued heating with conc. HCl gives a crystalline solid. Heating this solid with alkaline potassium permanganate gives a compound which dehydrates on heating to an anhydride ($\text{C}_8\text{H}_4\text{O}_3$). Compound X is

- 
- 
- 
- 

70. The final product of the following reaction is/are



- 
-  + 
-  + 
- 

71. The correct order of pseudohalide, polyhalide and interhalogen is

- BrI_2^- , OCN^- , IF_5
- IF_5 , BrI_2^- , OCN^-
- OCN^- , IF_5 , BrI_2^-
- OCN^- , BrI_2^- , IF_5

72. An inorganic halide (A) reacts with water to form two acids (B) and (C). (A) also reacts with NaOH to form two salts (D) and (E) which remain in solution. The solution gives white precipitate with both AgNO_3 and BaCl_2 solutions respectively. (A) is a useful organic reagent. The compound (A) is

- SOCl_2
- SO_2Cl_2
- S_2Cl_2
- SF_4

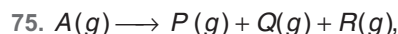
73. Following statements regarding the periodic trends of chemical reactivity to the alkali metals and the halogens are given. Which of these statements gives the correct picture?

- The reactivity decreases in the alkali metals but increases in the halogens with increase in atomic number down the group.
- In both the alkali metals and the halogens the chemical reactivity decreases with increase in atomic number down the group.
- Chemical reactivity increases with increase in atomic number down the group in both the alkali metals and halogens.
- In alkali metals the reactivity increases but in the halogens it decreases with increase in atomic number down the group.

74. According to IUPAC nomenclature sodium nitroprusside is named as

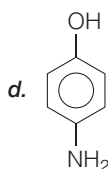
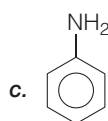
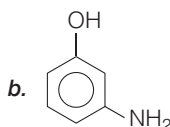
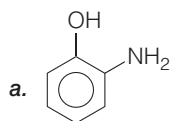
- sodiumpentacyanonitrosylferrate (II)
- sodiumpentacyanonitrosylferrate (III)
- sodiumnitroferrocyanide
- sodiumnitroferrocyanide

8 SELF STUDY GUIDE BITSAT



Follow first order kinetics with a half-life of 69.3 s at 500°C. Starting from the gas 'A' in a container at 500°C and at a pressure of 0.4 atm, the total pressure of the system after 230 s will be
a. 1.15 atm b. 1.32 atm c. 1.22 atm d. 1.12 atm

76. Which of the following gives paracetamol on acetylation?



77. In qualitative analysis when H_2S is passed through an aqueous solution of salt acidified with dilute HCl , a black ppt. is obtained. On boiling the precipitate with dil. HNO_3 , it forms a solution of blue colour. Addition of excess of aqueous solution of NH_3 to this solution gives

- a. deep blue ppt. of $Cu(OH)_2$
b. deep blue solution of $[Cu(NH_3)_4]^{2+}$
c. deep blue solution of $Cu(NO_3)_2$
d. deep blue solution of $Cu(OH)_2 \cdot Cu(NO_3)_2$

78. Titration of 0.1467 g of primary standard $Na_2C_2O_4$ required 28.85 mL of $KMnO_4$ solution. Calculate the molar concentration of $KMnO_4$ solution.

- a. 0.01518 M b. 0.001518 M
c. 0.15180 M d. 1.5180 M

79. Which of the following facts about the complex $[Cr(NH_3)_6]Cl_3$ is wrong?

- a. The complex involves d^2sp^3 hybridisation and its octahedral in shape.
b. The complex is paramagnetic.
c. The complex is an outer orbital complex.
d. The complex gives white precipitate with silver nitrate solution.

80. Sulphuric acid is a dibasic acid. It ionises in two stages and hence has two dissociation constants K_{a1} and K_{a2} . Which of the following is the correct observation regarding K_{a1} and K_{a2} ?

- a. $K_{a1} > K_{a2}$ b. $K_{a1} < K_{a2}$
c. $K_{a1} = K_{a2}$ d. $K_{a1} = 1.2 \times 10^{-2}$, $K_{a2} > 10$

PART III

a. English Proficiency

Directions (Q. Nos. 81-83) *In the following questions, find out which part of a sentence has an error. If there is no error mark part (d) as your answer.*

81. The road a. / to famous monument b. / passes through a forest. c./ No error d.
82. The master did not know a./ who of the servants b./ broke the glass c./ No error d.
83. Had I come a./ to know about his difficulties b./ I would have certainly helped c./ No error d.

Directions (Q. Nos. 84 and 85) *Fill in the blanks with suitable preposition from the alternatives given under each sentence.*

84. Her trekking was met obstacles.
a. with b. from c. by d. of
85. She has not got the shock of losing her father.
a. over b. at c. from d. with

Directions (Q. Nos. 86-88) *Select the word or phrase which is closest to the opposite in meaning of the italicised word or phrase.*

86. Yuvraj Singh is suffering from a *BENIGN* cancer.
a. Unfriendly b. Friendly c. Fatal d. Malignant

87. He is a *NOTED* figure of film industry.

- a. Known b. Unknown
c. Famous d. Infamous

88. *SAGACIOUS* decisions taken at right time in one's career has long effects.

- a. Foolish b. Intelligent c. Thoughtful d. Intuitive

Directions (Q. Nos. 89 and 90) *Choose the word nearest in meaning to the underlined word.*

89. The actor got *PEEVISH* on asking personal questions.

- a. Irritated b. Happy c. Shy d. Satisfied

90. The engineer *ROUGHED OUT* his ideas on a piece of paper while he talked.

- a. Shaped roughly b. Rejected
c. Drew a quick plan d. Describe inaccurately

Directions (Q. Nos. 91-95) *Read the passage given below and answer the questions that follow.*

The Centre and the States must become partners in the planning process to determine national priorities together. The process of planning would undergo a change in view of the changes in domestic economic situation and momentous trends emerging in the world. The development of human resource and the building up of an institutional framework

would have to receive priority attention. The role of the government would also have to be examined so as to fully involve the people in the process of nation-building. The main task would be to ensure that the real initiative, is transferred to the people. The private sector which would register expansion hereafter should keep this objective firmly in view. The need for an effective population policy is an urgent necessity in the country's planning strategy. The family welfare programme should not be treated as the Centre's responsibility alone. The States should evolve a suitable mechanism for closer involvement of the Government agencies, Zilla Parishads and Panchayats for making the family welfare programme a success.

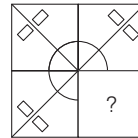
91. Which one of the following statements is correct?
- Effective family welfare programme is Centre's responsibility alone.
 - Population policy and planning process are interlinked.
 - Family welfare programme should be left to the State Governments alone.
 - The State Government should use punitive¹⁰ measures to control population.
92. What should be given priority attention?
- Role of the Government
 - Decentralisation of power
 - Involvement of people in labour welfare
 - Human resource and institutional framework
93. Which one of the following statements is not correct?
- Role of the government in nation-building should be examined.
 - Real initiatives should be transferred to the people.
 - There should be no role for the government as far as planning is concerned.
 - The Centre and the States must become equal partners in the planning process.
94. What would force the planning process to undergo a change?
- Free market forces
 - Domestic economic situation and world trends
 - Domestic compulsions
 - International pressures
95. Which one of the following is implied by the expression 'momentous trends'?
- GDP growth of the country
 - Memorable historical events
 - Important changes in the international scene
 - Improvement of foreign exchange reserves

b. Logical Reasoning

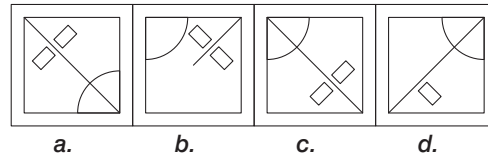
96. Find out the wrong number.
- 2, 6, 12, 72, 865, 62208
- 72
 - 12
 - 62208
 - 865
97. Each of P, Q, T, A and B has different heights. T is taller than P and B but shorter than A and Q. P is not the shortest, who is the tallest?
- A
 - Q
 - A or Q
 - P or B

98. Identify the missing part of the question figure and select it from given answer figures.

Question Figure



Answer Figures

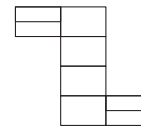


99. Select the related word from the given alternatives.

Mechanic : Spanner :: Carpenter : ?

- Tree
- Wood
- Furniture
- Saw

100. How many rectangles are there in the following figure?



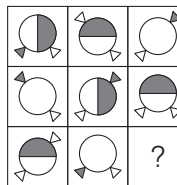
- 8
- 18
- 17
- 20

101. In the following question find the odd letters/group from the given alternatives.

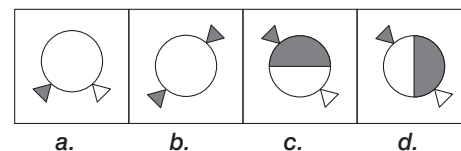
- BADC
- JILK
- NMPO
- VUWX

102. Find out which of the answer figures (a), (b), (c) and (d) completes the figure matrix?

Question Figure

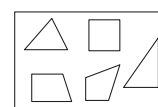


Answer Figures

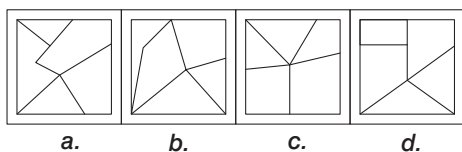


103. Among the four answer figures, which one can be formed from the cut out pieces given below in the question figures?

Question Figure

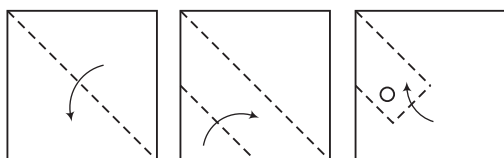


Answer Figures

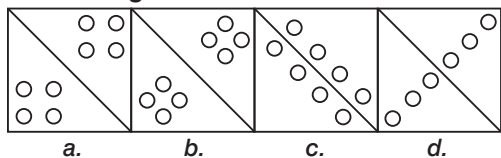


104. A piece of paper is folded and cut as shown below in the question figures. From the given answer figures, indicates how it will appear when opened.

Question Figures

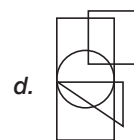
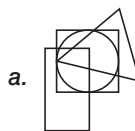
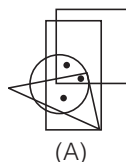


Answer Figures



105. In the following question three dots are placed in the figure marked as (A). The figure is followed by four alternatives marked as (a), (c), (c) and (d). One out of these four options contains region(s) common to the circle, square, triangle, similar to that marked by the dot in figure (A).

Question Figure



PART IV

Mathematics

106. If $A = \{x : x^2 = 1\}$ and $B = \{x : x^4 = 1\}$, then $A \Delta B$ is equal to

a. $\{-i, i\}$ b. $\{-1, i\}$
c. $\{-1, 1, -i, i\}$ d. None of these

107. If $2f(xy) = (f(x))^x + (f(y))^x$ for all $x, y \in R$ and $f(1) = a$ ($a \neq 1$). Then $\sum_{k=1}^n f(k) =$

a. $(a^n - 1) / (a - 1)$
b. $a(a^{n-1} - 1) / (a - 1)$
c. $a(a^n - 1) / (a - 1)$
d. $(a^n - 1) / a + 1$

108. Let $f(x) = x - 3$, $g(x) = 4 - x$. Then the set of values of x for which $|f(x) + g(x)| < |f(x)| + |g(x)|$ is true, is given by :

a. R b. $R - (3, 4)$
c. $R - [3, 4]$ d. None of these

109. If $a_1, a_2, a_3, \dots, a_{20}$ are AM's between 13 and 67, then the maximum value of $a_1, a_2, a_3, \dots, a_{20}$ is equal to

a. $(20)^{20}$ b. $(40)^{20}$
c. $(60)^{20}$ d. $(80)^{20}$

110. If p, q, r are in AP and are positive, the roots of the quadratic equation $px^2 + qx + r = 0$ are all real for

a. $\left| \frac{r}{p} - 7 \right| \geq 4\sqrt{3}$ b. $\left| \frac{p}{r} - 7 \right| < 4\sqrt{3}$
c. All p and r d. No p and r

111. The value of ${}^{47}C_4 + \sum_{r=1}^5 {}^{52-r}C_3$ is equal to

a. ${}^{47}C_6$ b. ${}^{52}C_5$
c. ${}^{52}C_4$ d. None of these

112. The number of numbers divisible by 3 that can be formed by four different even digits is

a. 36 b. 18
c. 0 d. None of these

113. If $n(A) = 1000$, $n(B) = 500$, $n(A \cap B) \geq 1$ and $n(A \cup B) = P$, then

a. $500 \leq P \leq 1000$ b. $1001 \leq P \leq 1498$
c. $1000 \leq P \leq 1498$ d. $1000 \leq P \leq 1499$

114. $\left\{ x \in R : \frac{2x-1}{x^3+4x^2+3x} \in R \right\}$ is equal to

a. $R - \{0\}$ b. $R - \{0, 1, 3\}$
c. $R - \{0, -1, -3\}$ d. $R - \left\{ 0, -1, -3, \frac{1}{2} \right\}$

115. Let $f(x)$ be a polynomial function of second degree. If $f(1) = f(-1)$ and a, b, c are in AP, then $f'(a), f'(b)$ and $f'(c)$ are in.
- AP
 - GP
 - Arithmetic-Geometric progression
 - None of the above
116. The value of $\lim_{x \rightarrow \infty} \frac{1}{n} \left\{ \frac{1}{n+1} + \frac{2}{n+2} + \dots + \frac{3n}{4n} \right\}$ is
- $5 - 2 \log 2$
 - $4 - 2 \log 2$
 - $3 - 2 \log 2$
 - $2 - 2 \log 2$
117. The coefficient of x^8 in the polynomial $(x-1)(x-2)\dots(x-10)$
- 2640
 - 1320
 - 1370
 - 2740
118. If $z = \frac{7+i}{3+4i}$, then z^{14} is
- 2^7
 - $2^7 i$
 - $(-2)^7$
 - $(-2)^7 i$
119. The solution of the equation $\frac{dy}{dx} + \frac{1}{x} \tan y = \frac{1}{x^2} \tan y \sin y$ is
- $2y = \sin y(1 - 2cx^2)$
 - $2x = \cot y(1 + 2cx^2)$
 - $2x = \sin y(1 - 2cx^2)$
 - $2x \sin y = 1 - 2cx^2$
120. The value of the definite integral $\int_0^{\pi/2} \frac{dx}{\tan x + \cot x + \operatorname{cosec} x + \sec x}$
- $1 - \frac{\pi}{4}$
 - $1 + \frac{\pi}{4}$
 - $\pi + \frac{1}{4}$
 - None of these
121. If $\mathbf{a}$ and $\mathbf{b}$ are two vectors such that $|\mathbf{a}| = 1, |\mathbf{b}| = 4$ and $\mathbf{a} \cdot \mathbf{b} = 2$, If $\mathbf{c} = (2\mathbf{a} \times \mathbf{b}) - 3\mathbf{b}$, then angle between $\mathbf{b}$ and $\mathbf{c}$
- $\frac{\pi}{6}$
 - $\frac{\pi}{3}$
 - $\frac{2\pi}{3}$
 - $\frac{5\pi}{6}$
122. Let x_1 and x_2 be the real roots of the equation $x^2 - (k-2)x + (k^2 + 3k + 5) = 0$, then maximum value of $x_1^2 + x_2^2$ is
- 19
 - 22
 - 18
 - 17
123. Circle centered at origin and having radius π units is divided by the curve $y = \sin x$ in two parts. Then area of upper parts equals to
- $\frac{\pi^2}{2}$
 - $\frac{\pi^3}{4}$
 - $\frac{\pi^3}{2}$
 - $\frac{\pi^3}{8}$
124. The root of the equation $2(1+i)x^2 - 4(2-i)x - 5 - 3i = 0$, where $i = \sqrt{-1}$, which has greater modulus, is
- $\frac{3-5i}{2}$
 - $\frac{5-3i}{2}$
 - $\frac{3+i}{2}$
 - $\frac{3i+1}{2}$
125. The equation $(\cos \beta - 1)x^2 + (\cos \beta)x + \sin \beta = 0$ in the variable x has real roots, then β lies in the interval
- $(0, 2\pi)$
 - $(-\pi, 0)$
 - $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
 - $(0, \pi)$
126. An ordered pair (α, β) for which the system of linear $(1+\alpha)x + \beta y + z = 2, \alpha x + (1+\beta)y + z = 3, \alpha x + \beta y + 2z$ has a unique solution.
- $(1, -3)$
 - $(-3, 1)$
 - $(2, 4)$
 - $(-4, 2)$
127. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45° . If flies off horizontally straight way from the point O . After one second, the elevation of the bird from O is reduced to 30° , then the speed (in m/s) of the bird is
- $40(\sqrt{2} - 1)$
 - $40(\sqrt{3} - \sqrt{2})$
 - $20\sqrt{2}$
 - $20(\sqrt{3} - 1)$
128. If one GM, g and two AM's p and q are inserted between two numbers a and b , then $(2p - q)(p - 2q)$ is equal to
- g^2
 - $-g^2$
 - $2g$
 - $3g^2$
129. When x^{100} is divided by $x^2 - 3x + 2$, the remainder is $(2^{k+1} - 1)x - 2(2^k - 1)$, then k is
- 97
 - 99
 - 100
 - 101
130. The mean of five observation is 5 and their variance is 9.20. If three of the given five observation are 1, 3 and 8, then a ratio of other two observations is
- 4 : 9
 - 6 : 7
 - 5 : 8
 - 10 : 3
131. How many three digit number satisfy the property that the middle digit is arithmetic mean of the first and the last digit.
- 41
 - 45
 - 43
 - 44
132. If $z = re^{i\theta}$, then $\arg(e^{iz})$ is
- $-r \sin \theta$
 - $r \cos \theta$
 - $e^{-r \sin \theta}$
 - $-r \cos \theta$
133. If 4 dice are rolled, then the number of ways of getting the sum 10 is
- 56
 - 64
 - 72
 - 80
134. Distance of point $A(1, 2)$ measured parallel to the line $3x - y = 10$ from the line $x + y + 5 = 0$, is
- $2\sqrt{5}$
 - $2\sqrt{10}$
 - $4\sqrt{5}$
 - $4\sqrt{10}$
135. Let $f(x) = a_0 + a_1 x^2 + a_2 x^4 + a_3 x^6 + \dots + a_n x^{2n}$ be a polynomial in a real variable x with $0 < a_1 < a_2 < a_3 < \dots < a_n$, the function $f(x)$ has
- neither a maxima nor a minima

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- b. only one maxima
c. both maxima and minima
d. only one minima

136. If $f(x) = 2x^3 + x^4 + \log x$ and g is the inverse of f , then $g'(3)$ is equal to
a. $\frac{1}{9}$ b. $\frac{1}{7}$ c. $\frac{1}{11}$ d. $\frac{1}{8}$

137. A line passing through $P(3, 7, 1)$ and $R(2, 5, 7)$ meet the plane $3x + 2y + 11z - 9 = 0$ at Q . Then PQ is equal to

- a. $\frac{5\sqrt{41}}{59}$ b. $\frac{\sqrt{41}}{59}$
c. $\frac{50\sqrt{41}}{59}$ d. $\frac{25\sqrt{41}}{59}$

138. $\int \frac{8x^{43} + 13x^{38}}{(x^{13} + x^5 + 1)^4} dx$ equals to

- a. $\frac{x^{39}}{3(x^{13} + x^5 + 1)^3} + C$ b. $\frac{x^{39}}{(x^{13} + x^5 + 1)^3} + C$
c. $\frac{x^{39}}{5(x^{13} + x^5 + 1)^5} + C$ d. None of these

139. If $\mathbf{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\mathbf{b} = 2\hat{i} + \hat{k}$, then find z component of a vector $\mathbf{r}$, which is coplanar with $\mathbf{a}$ and $\mathbf{b}$, $\mathbf{r} \cdot \mathbf{b} = 0$ and $\mathbf{r} \cdot \mathbf{a} = 7$.

- a. 0 b. 3 c. 6 d. 5/2

140. If x, y, z are three consecutive positive integers, $x - z + 2 = 0$, then

$$\frac{1}{2} \log_e x + \frac{1}{2} \log_e z + \frac{1}{2xz+1} + \frac{1}{3} \left(\frac{1}{2xz+1} \right)^3 + \dots \text{ is}$$

equal to

- a. $\log_e x$ b. $\log_e y$
c. $\log_e z$ d. None of these

141. The solution of differential equation

$$(xy^5 + 2y) dx - xdy = 0, \text{ is}$$

- a. $9x^8 + 4x^9y^4 = 9y^4C$ b. $9x^8 - 4x^9y^4 - 9y^4C = 0$
c. $x^8(9 + 4y^4) = 10y^4C$ d. None of these

142. The solution set of $\frac{|x-2|-1}{|x-2|-2} \leq 0$ is

- a. $[0, 1] \cup (3, 4)$ b. $[0, 1] \cup [3, 4]$
c. $[-1, 1] \cup (3, 4]$ d. None of these

143. Let $f(x) = \frac{x}{\sqrt{1+x^2}}$, the $\underbrace{\text{fofofo} \dots \text{fofofo}}_{x \text{ times}}(x)$ is

- a. $\frac{x}{\sqrt{1 + \left(\sum_{r=1}^n r\right)x^2}}$ b. $\frac{x}{\sqrt{1 + \left(\sum_{r=1}^n 1\right)x^2}}$
c. $\left(\frac{x}{\sqrt{1+x^2}}\right)^x$ d. $\frac{nx}{\sqrt{1+nx^2}}$

144. If $\log_5 \frac{(a+b)}{3} = \frac{\log_5 a + \log_5 b}{2}$, then $\frac{a^4 + b^4}{a^2 b^2}$ is equal to
a. 50 b. 47 c. 44 d. 53

145. The number of distinct solutions of the equation $\frac{5}{4} \cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x = 2$ in the interval $[0, 2\pi]$ is
a. 8 b. 10 c. 6 d. 15

146. If the tangent at a point $\left(4 \cos \phi, \frac{16}{\sqrt{11}} \sin \phi\right)$ to the ellipse $16x^2 + 11y^2 = 256$ is also a tangent to $x^2 + y^2 - 2x = 15$, then ϕ equals

- a. $\frac{\pi}{3}$ b. $\frac{\pi}{6}$ c. $-\frac{\pi}{6}$ d. $\frac{\pi}{4}$

147. The distance of point of intersection of the tangents to the parabola $x = 4y - y^2$ drawn at the points where it is meet by Y -axis, from its focus is

- a. $\frac{11}{4}$ b. $\frac{17}{4}$ c. $\frac{13}{4}$ d. 3

148. The value of the sum $\sum_{k=1}^{\infty} \sum_{n=1}^{\infty} \frac{k}{2^{n+k}}$ is

- a. 5 b. 4
c. 3 d. 2

149. A curve passes through $(2, 0)$ and the slope of the tangent at $P(x, y)$ is equal to $\frac{(x+1)^2 + y - 3}{x+1}$ then the

equation of the curve is

- a. $y = x^2 - 2x$ b. $y = x^3 - 8$
c. $y^2 = x^2 + 2x$ d. $y^2 = 5x^2 - 6$

150. Consider matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, if $A^{-1} = \alpha I + \beta A$, where

$\alpha, \beta \in R$, then $(\alpha + \beta)$ is equal to (where A^{-1} denotes the inverse of matrix A)

- a. 1 b. $\frac{4}{3}$ c. $\frac{5}{3}$ d. $\frac{1}{3}$

Answers

Physics

1. (c)	2. (a)	3. (a)	4. (c)	5. (b)	6. (b)	7. (a)	8. (b)	9. (c)	10. (b)
11. (b)	12. (b)	13. (b)	14. (b)	15. (b)	16. (c)	17. (b)	18. (b)	19. (b)	20. (c)
21. (b)	22. (b)	23. (d)	24. (a)	25. (a)	26. (d)	27. (d)	28. (a)	29. (d)	30. (c)
31. (b)	32. (b)	33. (c)	34. (b)	35. (c)	36. (c)	37. (a)	38. (a)	39. (d)	40. (b)

Chemistry

41. (d)	42. (b)	43. (d)	44. (a)	45. (d)	46. (a)	47. (b)	48. (a)	49. (b)	50. (a)
51. (d)	52. (a)	53. (c)	54. (b)	55. (c)	56. (c)	57. (d)	58. (c)	59. (b)	60. (c)
61. (c)	62. (d)	63. (c)	64. (a)	65. (c)	66. (c)	67. (a)	68. (c)	69. (d)	70. (b)
71. (d)	72. (b)	73. (d)	74. (b)	75. (d)	76. (d)	77. (b)	78. (a)	79. (c)	80. (a)

English Proficiency

81. (b)	82. (b)	83. (c)	84. (a)	85. (b)	86. (c)	87. (b)	88. (a)	89. (a)	90. (c)
91. (b)	92. (d)	93. (c)	94. (b)	95. (c)					

Logical Reasoning

96. (d)	97. (c)	98. (c)	99. (d)	100. (b)	101. (d)	102. (d)	103. (c)	104. (b)	105. (b)
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Mathematics

106. (a)	107. (c)	108. (c)	109. (b)	110. (a)	111. (c)	112. (a)	113. (d)	114. (c)	115. (a)
116. (c)	117. (b)	118. (b)	119. (c)	120. (a)	121. (d)	122. (a)	123. (c)	124. (a)	125. (d)
126. (c)	127. (d)	128. (b)	129. (b)	130. (a)	131. (b)	132. (b)	133. (d)	134. (b)	135. (d)
136. (c)	137. (d)	138. (a)	139. (b)	140. (b)	141. (a)	142. (b)	143. (b)	144. (b)	145. (a)
146. (a)	147. (b)	148. (d)	149. (a)	150. (a)					

Solutions

Physics

1. (c) As coil 2 with inductance L_2 has been wound by a similar wire as coil 1 but the direction of windings is reversed in each layer, so flux through L_2 will be zero.

As, $L_2 \propto \phi \Rightarrow L_2 = 0$

Also, since the dimensions, number of turns and number of layers of windings are equal in coil 1 and coil 3.

So, $L_1 = L_3$

2. (a) Since capacitors C_1 and C_2 are parallel, so their equivalent capacitance will be $(C_1 + C_2)$. Now, $(C_1 + C_2)$ and C_3 are in series, so the net equivalent capacitance of circuit will be,

$$\frac{1}{C} = \frac{1}{C_3} + \frac{1}{C_1 + C_2} = \frac{C_1 + C_2 + C_3}{(C_1 + C_2)C_3}$$

or $C = \frac{(C_1 + C_2)C_3}{C_1 + C_2 + C_3}$

Since V is the voltage of the battery, so charge on this system,

$$q = CV \Rightarrow q = \frac{(C_1 + C_2)C_3V}{(C_1 + C_2 + C_3)}$$

If the capacitance C_3 breaks down, then total equivalent capacitance will become

$$C' = C_1 + C_2$$

$\therefore$ New charge stored,

$$q' = C'V$$

$$\Rightarrow q' = (C_1 + C_2)V$$

Now, change in total charge,

$$\begin{aligned} \Delta q &= q' - q \quad [\because q' > q] \\ &= (C_1 + C_2)V - \frac{(C_1 + C_2)C_3V}{C_1 + C_2 + C_3} \end{aligned}$$

or $\Delta q = (C_1 + C_2)V \left[1 - \left(\frac{C_3}{C_1 + C_2 + C_3} \right) \right]$

3. (a) Let E' be the new radiant energy.

According to Stefan's law,

$$E \propto T^4$$

$$\Rightarrow \frac{E_2}{E_1} = \left(\frac{T_2}{T_1} \right)^4$$

$$\Rightarrow \frac{E'}{E} = \left\{ \frac{(T/4)}{T} \right\}^4 = \left(\frac{1}{4} \right)^4$$

or $E' = \frac{E}{256}$

4. (c) Area of rectangle, $A = lb = 15.2 \times 2.9 = 44.08 \text{ cm}^2$

Minimum possible measurement of scale = 0.1 cm

As we know, the product should have significant figures that are present in the measurement with least number of significant figures which is 2 in this case. So, area measured by scale = 44 cm^2 .

5. (b) In adiabatic process,

$$pV^\gamma = K$$

where, K is a constant.

Given that, $\gamma = \frac{4}{3} \Rightarrow pV^{4/3} = K$

Taking logarithm on both sides, we get

$$\log p + \frac{4}{3} \log V = \log K$$

$$\Rightarrow \frac{\Delta p}{p} + \frac{4}{3} \frac{\Delta V}{V} = 0$$

$$\Rightarrow \frac{\Delta V}{V} = -\frac{3}{4} \frac{\Delta p}{p}$$

Now, % change in volume,

$$\frac{\Delta V}{V} \times 100 = -\frac{3}{4} \left(\frac{\Delta p}{p} \times 100 \right)$$

It is given that, pressure decreases by $\frac{3}{4}\%$,

i.e. $\frac{\Delta p}{p} \times 100 = -\frac{3}{4}$

$$\therefore \frac{\Delta V}{V} \times 100 = -\frac{3}{4} \times \left(-\frac{3}{4} \right) = \frac{9}{16} \%$$

Hence, the volume increases by $\frac{9}{16} \%$.

6. (b) The given equation is

$$y = 4 \sin \left(\frac{\pi x}{15} \right) \cos(96\pi t) \quad \dots(i)$$

This equation is similar to the displacement equation of a travelling wave, $y = A \sin(\omega t - kx)$

On comparing these two equations, we get

$$\omega = 2\pi\nu = 96\pi$$

or $\nu = \frac{96\pi}{2\pi} = 48 \text{ Hz}$

Now, the velocity of the string at a point x at time t is obtained by differentiating Eq. (i) w.r.t.

$$\therefore \nu = \frac{dy}{dt} = -(4 \times 96\pi) \times \sin \left(\frac{\pi x}{15} \right) \sin(96\pi t)$$

At $x = 7.5 \text{ cm} = 7.5 \times 10^{-2} \text{ m}$ and $t = 0.25 \text{ s}$,

$$\begin{aligned} v &= -384\pi \sin\left(\pi \times \frac{7.5 \times 10^{-2}}{15}\right) \sin\left(96\pi \times \frac{25}{100}\right) \\ &= -384\pi \sin\left(\frac{\pi}{200}\right) \sin(24\pi) \\ &= 0 \end{aligned} \quad (\because \sin 24\pi = 0)$$

Hence, the velocity of the particle is zero.

7. (a) Potential at point A,

$$V_A = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a}$$

Potential at point B,

$$V_B = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{b}$$

Work done in taking a charge Q from A to B,

$$W = Q(V_B - V_A) = \frac{Qq}{4\pi\epsilon_0} \left[\frac{1}{b} - \frac{1}{a} \right] = \frac{Qq}{4\pi\epsilon_0} \left[\frac{a-b}{ab} \right]$$

8. (b) For hydrogen-like atom,

$$\frac{1}{\lambda'} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad \dots(i)$$

Similarly, for hydrogen atom,

$$\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad \dots(ii)$$

For the element to have hydrogen-like spectrum, the ratio of the wavelengths for transition of electron between any two levels n_1 and n_2 should be constant.

Dividing eq. (ii) by eq. (i), we get

$$\frac{\lambda'}{\lambda} = \frac{R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)}{RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)} \quad \text{or} \quad \frac{\lambda'}{\lambda} = \frac{1}{Z^2}$$

It is given that wavelength of the hydrogen atom is four times the wavelength of the hydrogen-like atom, i.e.

$$\lambda = 4\lambda'$$

Hence,

$$Z = 2$$

Now, the element with atomic number 2 is helium.

9. (c) Given, two small conducting spheres of equal radius (R).

Charge on first sphere, $q_1 = +20\mu\text{C}$

Charge on second sphere, $q_2 = -40\mu\text{C}$

$$\therefore F_1 = \frac{k(+20)(-40)}{R^2} = \frac{-k(800)}{R^2} \quad \dots(i)$$

The spheres have equal radii, so their capacities will be same. Now, if they are brought in contact, then the net charge,

$$q_{\text{net}} = (+20 - 40)\mu\text{C} = -20\mu\text{C}$$

So, each sphere will have charge, $q = \frac{q_{\text{net}}}{2} = -10\mu\text{C}$

$$\text{Now, } F_2 = \frac{k(q)(q)}{R^2} = \frac{k(-10)(-10)}{R^2} = \frac{k(100)}{R^2} \quad \dots(ii)$$

$$\text{Ratio, } \frac{F_1}{F_2} = -\frac{800}{100} = -8:1$$

Thus, ratio of F_1 to $F_2 = -8:1$

10. (b) Efficiency of carnot engine is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

Given, $T_1 = 600 \text{ K}$

$$T_2 = 300 \text{ K}$$

$$T'_1 = 1600 \text{ K and } T'_2 = x \text{ K}$$

Carnot engine has same efficiency between 600 K to 300 K and 1600 K to x K, so

$$\left(1 - \frac{T_2}{T_1} \right) = \left(1 - \frac{T'_2}{T'_1} \right)$$

Substituting all the given values in above equation, we get

$$1 - \frac{300}{600} = 1 - \frac{x}{1600}$$

$$1 - \frac{1}{2} = 1 - \frac{x}{1600}$$

$$\frac{x}{1600} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow x = \frac{1600}{2} = 800 \text{ K}$$

11. (b) Given, initial speed of ball, $v = 16 \text{ ms}^{-1}$

Maximum height, $h = 9 \text{ m}$

Mass of ball, $m = 0.5 \text{ kg}$

Energy supplied to ball while throwing up,

$$E_1 = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.5 \times 16 \times 16 = 64 \text{ J}$$

Energy stored when ball reaches the maximum height,

$$E_2 = mgh = 0.5 \times 9.8 \times 9 = 44.1 \text{ J}$$

Energy dissipated by air drag acting on the ball during the ascent,

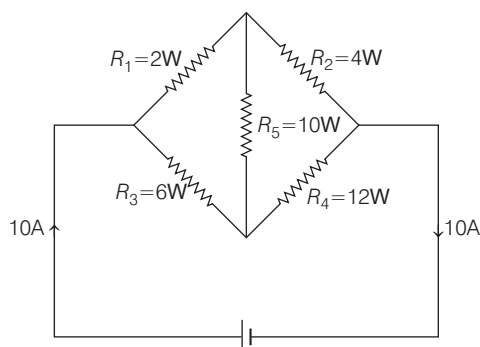
$$E = E_1 - E_2 = 64 - 44.1 = 19.9 \text{ J}$$

12. (b) From the following circuit diagram, $\frac{R_1}{R_3} = \frac{R_2}{R_4}$, so the

given Wheatstone bridge is balanced.

Hence, there is no current in the wire of 10Ω resistance.

As,



Now, if the $10\ \Omega$ wire is replaced with $15\ \Omega$ wire, the bridge still remains balanced. Hence, current drawn from battery will be 10A.

13. (b) Given, force, $F = (8t)\text{ N}$, Mass, $m = 2\text{ kg}$

From Newton's second law,

Force = Rate of change of momentum

$$F = \frac{dp}{dt}$$

i.e. $dp = F dt$

$$\therefore \text{Momentum, } p = \int dp = \int_0^1 F \cdot dt = \int_0^1 8t \cdot dt$$

$$= 8 \left[\frac{t^2}{2} \right]_0^1 = 8 \left(\frac{1}{2} - 0 \right)$$

$$\Rightarrow p = 4\text{ kg ms}^{-1}$$

$$\text{Change in kinetic energy, } \Delta K = \frac{p^2}{2m} = \frac{(4)^2}{2 \times 2} = 4\text{ J}$$

From work-energy theorem,

Work done = Change in kinetic energy

So, work done by the force during the first 1s,

$$W = \Delta K = 4\text{ J}$$

14. (b) As, RC is the time constant, so it has the dimension of time and V has the dimensions of $L \frac{dI}{dt}$.

$$\therefore \text{Dimensions of } \frac{L}{RCV} \cdot \left[\frac{L}{RCV} \right] = \frac{[L]}{[RC][V]}$$

$$\frac{[L]}{[T][L]} \left[\frac{A}{T} \right] = \frac{[L][T]}{[T][L][A]} = \frac{1}{[A]}$$

So, $\left(\frac{L}{RCV} \right)$ has the dimensions $[A^{-1}]$, i.e. same as the dimensions of $\frac{1}{\text{current}}$.

15. (b) Given, magnetic field,

$$B = 10 \times 10^{-8} \sin(1 \times 10^7 z - 3.6 \times 10^{15} t)\text{ T} \quad \dots(i)$$

Standard equation of magnetic field,

$$B = B_0 \sin(kz - \omega t) \quad \dots(ii)$$

On comparing Eqs. (i) and (ii), we get

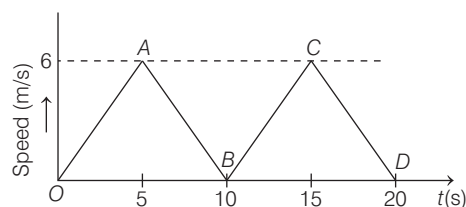
$$B_0 = 10 \times 10^{-8}\text{ T}$$

The average intensity of the beam,

$$I_{av} = \frac{B_0^2 c}{2\mu_0} = \frac{1}{2} \times \frac{(10 \times 10^{-8})^2 \times 3 \times 10^8}{4\pi \times 10^{-7}} [\because c = 3 \times 10^8\text{ ms}^{-1}]$$

$$= 1.19\text{ W/m}^2$$

16. (c) Distance = area under speed~time graph which is shown in the figure below.



$\therefore$ Distance travelled in 20s = area of $\triangle OAB$ + area of $\triangle BCD$

$$= \frac{1}{2} \times 6 \times 10 + \frac{1}{2} \times 6 \times 10 = 60\text{ m}$$

Displacement in 20s = area of $\triangle OAB$ - area of $\triangle BCD$

$$= \frac{1}{2} \times 6 \times 10 - \frac{1}{2} \times 6 \times 10 = 30 - 30 = 0$$

$$\text{Hence, average velocity} = \frac{\text{displacement}}{\text{time}} = \frac{0}{t} = 0$$

17. (b) The focal length of a convex lens of refractive index μ_g in air,

$$\frac{1}{f_{air}} = (\mu_g - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(i)$$

where, R_1, R_2 = radii of curvatures of its first and second surface.

When the lens is immersed in a liquid of refractive index μ_l , then refractive index of material of lens (glass) w.r.t. liquid is,

$${}_l\mu_g = \frac{\mu_g}{\mu_l} \quad \dots(ii)$$

Now, focal length of lens in liquid

$$\frac{1}{f'} = ({}_l\mu_g - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(iii)$$

Dividing Eq. (i) by Eq (iii), we get

$$\frac{f'}{f_{air}} = \frac{\mu_g - 1}{({}_l\mu_g - 1)} = \frac{\mu_g - 1}{\left(\frac{\mu_g}{\mu_l} - 1 \right)}$$

Given, $f' = 0.5\text{ m}$, $f_{air} = 0.2\text{ m}$, $\mu_g = 1.5$

Substituting all these values in above equation, we get

$$-\frac{0.5}{0.2} = \frac{1.5 - 1}{\left(\frac{1.5}{\mu_l} - 1 \right)} = \frac{0.5}{\left(\frac{1.5}{\mu_l} - 1 \right)}$$

$$\Rightarrow \frac{1.5}{\mu_l} - 1 = -0.2 \Rightarrow \frac{1.5}{\mu_l} = 1 - 0.2 = 0.8$$

$$\Rightarrow \mu_l = \frac{15}{8}$$

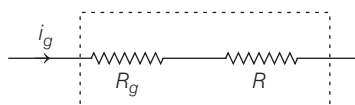
$$\therefore \text{Refractive index of liquid} = \frac{15}{8}$$

18. (b) Given, resistance of galvanometer, $R_g = 60 \Omega$

$$\text{Current, } I_g = 4.5 \text{ mA} = 4.5 \times 10^{-3} \text{ A}$$

Resistance used in converting a galvanometer into voltmeter, $R = 4.5 \text{ k}\Omega = 4.5 \times 10^3 \Omega$

To make the voltmeter, the galvanometer and the high resistance (R) have to be connected in series combination as shown below.

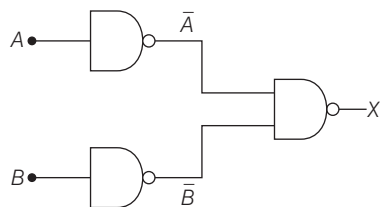


$\therefore$ Maximum current in galvanometer is

$$I_g = \frac{E}{R + R_g}$$

$$\Rightarrow E = I_g(R + R_g) = 4.5 \times 10^{-3} \times (4.5 \times 10^3 + 60) = 20.5 \text{ V}$$

19. (b) The given combination of gates is shown below



The output of this combination can be determined by Boolean algebra.

$$\therefore \text{Output, } X = \overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} \text{ [Using De Morgan's theorem]}$$

$$\Rightarrow X = A + B$$

This is the output of OR gate. Hence, the given combination of gates yield OR gate.

20. (c) From gas equation,

$$pV = nRT \Rightarrow p = \frac{nRT}{V}$$

Pressure of one mole of O_2 gas,

$$(p)_{\text{O}_2} = \frac{1 \times RT}{V} \quad \dots(i)$$

Pressure of one mole of He gas,

$$(p)_{\text{He}} = \frac{1 \times R(2T)}{V} \quad \dots(ii)$$

Dividing Eq. (ii) by Eq. (i), we get

$$\therefore \frac{(p)_{\text{He}}}{(p)_{\text{O}_2}} = 2 \Rightarrow (p)_{\text{He}} = 2(p)_{\text{O}_2} = 2p$$

$$21. (b) \text{ Acceleration, } a = \frac{dv}{dt} \Rightarrow dv = adt$$

$$\text{Integrating both sides, } \int dv = \int adt$$

$$\text{Given, } a = (3t^2 - 2t + 1) \text{ ms}^{-2}$$

Substituting this value in above equation, we get

$$v = \int (3t^2 - 2t + 1) dt$$

$$\Rightarrow v = t^3 - t^2 + t + C \quad \dots(i)$$

$$\text{At } t = 1 \text{ s, } v = 1 \text{ m/s}$$

Substituting these values in Eq. (i), we get

$$\therefore 1 = 1 - 1 + 1 + C \Rightarrow C = 0$$

Substituting this value of C in Eq. (i), we get

$$v = t^3 - t^2 + t$$

$$\text{When } t = 4 \text{ s, } v = (4)^3 - (4)^2 + 4 = 52 \text{ m/s}$$

22. (b) From Ideal gas equation,

$$pV = nRT = \frac{mRT}{M}$$

$$n = \frac{\text{Molar mass (m)}}{\text{Molecular weight (M)}}$$

Here, n = number of moles

R = Gas constant

V = Volume

$$\Rightarrow \frac{pV}{m} = \frac{RT}{M}$$

$$\Rightarrow \frac{p}{(m/V)} = \frac{RT}{M}$$

$$\frac{p}{d} = \frac{RT}{m} \quad \left[\because \frac{m}{V} = d \right]$$

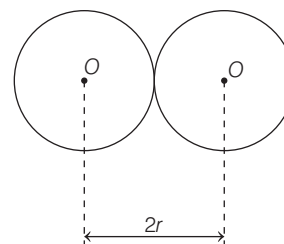
$$\Rightarrow \frac{p}{d} \propto T$$

$\Rightarrow$ Slope of p versus d graph $\propto T$

Since, slope is less for the gas at temperature T_1 .

$$\text{Therefore, } T_1 < T_2$$

23. (d) The two spheres are touching each other as shown below.



Let m be the mass of each sphere.

Gravitational force between the spheres,

$$\begin{aligned}
 F &= G \frac{m \cdot m}{(2r)^2} = G \cdot \frac{m^2}{4r^2} \\
 &= G \cdot \left[\frac{4}{3} \pi r^3 \cdot \rho \right]^2 \quad [\text{Mass} = \text{volume} \times \text{density}] \\
 &= \frac{4G\pi^2 r^6 \rho^2}{3 \times 4r^2} \\
 \Rightarrow F &= \left(\frac{4G\pi^2 \rho^2}{3} \right) r^4 \Rightarrow F \propto r^4
 \end{aligned}$$

24. (a) Power of first spherical lens, $P_1 = -4$ D

The power of this lens is negative, hence it is diverging lens.

Focal length of first spherical lens,

$$f_1 = \frac{1}{P_1} = \frac{1}{(-4D)} = -0.25 \text{ m} = -25 \text{ cm}$$

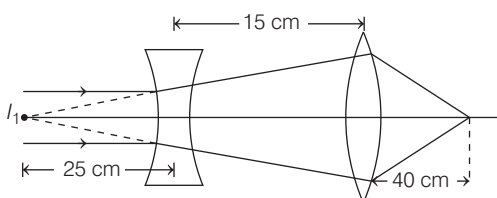
Power of second spherical lens, $P_2 = 5$ D

The power of this lens is positive, hence it is converging lens.

Focal length of second spherical lens,

$$f_2 = \frac{1}{P_2} = \frac{1}{5D} = 0.2 \text{ m} = 20 \text{ cm}$$

The given situation is shown in the following figure.



Now, the image I_1 will be considered as object for converging lens.

For converging lens, distance of object,

$$u = -(25 + 15) = -40 \text{ cm}$$

and $f_2 = 20 \text{ cm}$

$\therefore$ Using lens formula,

$$\frac{1}{f_2} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{20} = \frac{1}{v} - \frac{1}{-40}$$

$$\Rightarrow v = 40 \text{ cm}$$

Since v is positive, so image formed will be real and will be formed at a distance of 40 cm from the lens of power 5D (converging lens).

25. (a) From 2nd equation of rotational motion,

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

Given, $\theta = 5 \text{ rad}$, $t = 1 \text{ s}$, $\omega_0 = 0$

Substituting all these values in above equation, we get

$$5 = 0 + \frac{1}{2} \alpha (1)^2 \Rightarrow \alpha = 10 \text{ rad/s}^2$$

The angle rotated by wheel of car during first two seconds,

$$\theta_2 = 0 + \frac{1}{2} \alpha t^2 = \frac{1}{2} \times 10 \times 2^2 = 20 \text{ radians}$$

Hence, angle rotated during the 2nd second

$$\begin{aligned}
 &= \theta_2 - \theta \\
 &= 20 - 5 = 15 \text{ radians}
 \end{aligned}$$

26. (d) Kinetic energy in photoelectric effect can be given as

$$KE = W - W_0 \quad \begin{cases} W \rightarrow \text{incident energy} \\ W_0 \rightarrow \text{work function} \end{cases}$$

Given, $(KE)_1 = 1.5 - 0.5 = 1 \text{ eV}$

$$(KE)_2 = 2.5 - 0.5 = 2 \text{ eV}$$

$$\therefore \frac{(KE)_1}{(KE)_2} = \frac{1}{2} = \frac{2}{3}$$

$$\Rightarrow \frac{\frac{1}{2} m v_1^2}{\frac{1}{2} m v_2^2} = \frac{2}{3} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{2}{3}}$$

$$\therefore v_1 : v_2 = \sqrt{2} : \sqrt{3}$$

So, the ratio of maximum speeds of emitted electrons is $\sqrt{2} : \sqrt{3}$.

27. (d) Initial momentum, $P_i = mv$

$$P_i = m\sqrt{2gh} \quad [\because v = \sqrt{2gh}]$$

New height of body, $h_1 = h + 3h = 4h$

Now, final momentum,

$$\begin{aligned}
 P_f &= m\sqrt{2g \cdot h_1} \\
 &= m\sqrt{2g(4h)} \\
 &= m \cdot 2\sqrt{2gh} = 2P_i
 \end{aligned}$$

$$\begin{aligned}
 \therefore \% \text{ change in momentum} &= \frac{P_f - P_i}{P_i} \times 100 \\
 &= \frac{2P_i - P_i}{P_i} \times 100 \\
 &= 100\%
 \end{aligned}$$

28. (a) In this case, the diode is forward biased. Potential drop in a germanium diode in forward bias is around 0.3V.

Hence, current through the given circuit,

$$\begin{aligned}
 I &= \frac{V_{\text{net}}}{R_{\text{eq}}} = \frac{5 - 0.3}{2 + 3} = \frac{4.7}{5} \\
 I &= 0.94 \text{ A}
 \end{aligned}$$

29. (d) Number of active nuclei after time t in a sample of radioactive substance is given by

$$N = N_0 e^{-\lambda t}$$

where, N_0 = initial number of nuclei at $t = 0$ and λ = decay constant.

Here, at $t = 0$, number of nuclei in sample X and Y are equal,

$$\text{i.e., } (N_0)_X = (N_0)_Y = N_0$$

$$\text{Also, } \lambda_X = 4\lambda \text{ and } \lambda_Y = \lambda$$

So, after time t , number of active nuclei in X and Y are

$$N_X = N_0 e^{-4\lambda t} \text{ and } N_Y = N_0 e^{-\lambda t}$$

$$\therefore \frac{N_X}{N_Y} = e^{-2\lambda t}$$

$$\frac{1}{e^3} = \frac{1}{e^{2\lambda t}} \quad \left[\text{Given, } \frac{N_X}{N_Y} = \frac{1}{e^3} \right]$$

$$\Rightarrow 2\lambda t = 3 \Rightarrow t = \frac{3}{2\lambda}$$

30. (c) Given, current in the wire, $I = 10\text{ A}$

$$\text{Radius of the wire, } R = 20\pi \text{ cm} = 20\pi \times 10^{-2} \text{ m}$$

Magnetic field produced by semi-circular current carrying thin wire at centre O ,

$$B = \frac{\mu_0 I}{4R}$$

Substituting all the given values in above equation, we get

$$B = \frac{4\pi \times 10^{-7} \times 10}{4 \times 20\pi \times 10^{-2}}$$

$$B = 5 \times 10^{-6} \text{ T}$$

Now, magnitude of force per unit length,

$$\begin{aligned} F &= IB = 10 \times 5 \times 10^{-6} \\ &= 50 \times 10^{-6} \text{ N/m} \\ &= 50 \mu\text{N/m} \end{aligned}$$

31. (b) Given, area, $A = 5 \text{ cm}^2 = 5 \times 10^{-4} \text{ m}^2$

Number of turns per unit length, $n = 4000 \text{ turns/m}$

Magnetic field, $B = \pi T$

Current, $i = 5 \text{ A}$

Now, magnetic intensity, $H = ni = 4000 \times 5 = 2 \times 10^4 \text{ Am}^{-1}$

$$\text{As, } B = \mu_0 (H + I) \quad [I = \text{intensity of magnetisation}]$$

Substituting all the values in above equation, we get

$$\pi = 4\pi \times 10^{-7} ((2 \times 10^4) + I)$$

$$\frac{1}{4} \times 10^7 = (2 \times 10^4) + I$$

$$250 \times 10^4 = 2 \times 10^4 + I$$

$$I = 250 \times 10^4 - 2 \times 10^4$$

$$\Rightarrow I = 248 \times 10^4 \text{ A/m}$$

Now, pole strength, $m = I \times A = 248 \times 10^4 \times 5 \times 10^{-4}$

$$= 1240 \text{ A-m}$$

32. (b) Given, mass of plane, $M = 10000 \text{ kg}$

Coefficient of friction, $\mu = 0.2$

Distance, $s = 50 \text{ m}$

Initial speed, $u = 0$

Final speed, $v = 72 \text{ kmh}^{-1} = 20 \text{ ms}^{-1}$

Let a be the acceleration of the plane.

From third equation of motion,

$$v^2 - u^2 = 2as$$

$$(20)^2 - (0)^2 = 2 \times a \times 50$$

$$\therefore a = \frac{20 \times 20}{100} = 4 \text{ ms}^{-2}$$

Force required to provide necessary acceleration to the plane,

$$F_1 = ma = 10000 \times 4 = 4 \times 10^4 \text{ N}$$

Force required to overcome friction by engine,

$$\begin{aligned} F_2 &= \mu R = \mu mg = 0.2 \times 10000 \times 9.8 \\ &= 1.96 \times 10^4 \text{ N} \end{aligned}$$

Therefore, the minimum force required by the engine,

$$F_1 + F_2 = 4 \times 10^4 + 1.96 \times 10^4 = 5.96 \times 10^4 \text{ N}$$

33. (c) Given, reverse voltage, $E = 120 \text{ V}$

Change in current, $\Delta I = 0.50 - 0.20 = 0.30 \text{ A}$

Time, $\Delta t = 0.030 \text{ ms} = 0.030 \times 10^{-3} \text{ s}$

In an inductor (choke),

$$\text{Emf induced, } E_{\text{induced}} = L \frac{dI}{dt}$$

$$\begin{aligned} \therefore \text{Self inductance of choke, } L &= \frac{E \times \Delta t}{\Delta I} \\ &= \frac{120 \times 0.030 \times 10^{-3}}{0.30} \\ &= 12 \times 10^{-3} \text{ H} \end{aligned}$$

34. (b) Given, linear dimensions increase by a factor of 8.

So, volume will become $(8)^3$ times and area of cross-section will become $(8)^2$ times.

$$\text{Now, stress} = \frac{\text{weight}}{\text{area}}$$

Weight = (volume $\times$ density $\times g$) will also become $(8)^3$ times.

$$\therefore \text{New stress} = \frac{(8)^3 \times w_0}{(8)^2 \times A_0} = 8 \left(\frac{w_0}{A_0} \right)$$

Hence, the stress increases by a factor of 8.

35. (c) Let, γ be the adiabatic exponent for the mixture.

For monoatomic gas,

$$n_1 = 2 \text{ and } \gamma_1 = \frac{5}{3}$$

For diatomic gas,

$$n_2 = 3 \text{ and } \gamma_2 = \frac{7}{5}$$

For the mixture,

$$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

Substituting all the given values in above equation, we get

$$\begin{aligned} \frac{2+3}{\gamma-1} &= \frac{2}{\frac{5}{3}-1} + \frac{3}{\frac{7}{5}-1} \\ \Rightarrow \frac{5}{\gamma-1} &= \frac{21}{2} \Rightarrow \gamma = \frac{31}{21} \end{aligned}$$

36. (c) Let a be the acceleration of the lift.

Given, the ratio of time periods of pendulum while the lift is moving upwards and downwards,

$$T_1 : T_2 = 1 : 3$$

When lift is moving upwards, then total time period,

$$T_1 = 2\pi \sqrt{\frac{l}{g+a}} \quad \dots(i)$$

When lift is moving downwards, then total time period,

$$T_2 = 2\pi \sqrt{\frac{l}{g-a}} \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{T_1}{T_2} = \sqrt{\frac{g-a}{g+a}} \Rightarrow \frac{1}{3} = \sqrt{\frac{g-a}{g+a}}$$

$$\text{or } \frac{g-a}{g+a} = \left(\frac{1}{3}\right)^2 \quad \text{or } \frac{g-a}{g+a} = \frac{1}{9}$$

$$\Rightarrow 9g - 9a = g + a$$

$$\Rightarrow 9g - g = 9a + a$$

$$\Rightarrow 10a = 8g$$

$$a = \frac{8g}{10} = \frac{8 \times 10}{10} = 8 \text{ m/s}^2$$

37. (a) As, orbital velocity of a satellite, $v = \sqrt{\frac{GM}{r}}$

$$\text{Hence, } v \propto \frac{1}{\sqrt{r}}$$

$$\% \text{ increase in speed} = \frac{1}{2} (\% \text{ decrease in radius})$$

$$= \frac{1}{2} (2\%) = 1\%$$

Thus, the speed of satellite will increase by 1%.

38. (a) Given, $\omega = 2000 \text{ rad s}^{-1}$

$$L = 5 \text{ mH} = 5 \times 10^{-3} \text{ H}$$

$$C = 50 \mu\text{F} = 50 \times 10^{-6} \text{ F}$$

$$V = 30 \text{ V}$$

Inductive reactance,

$$X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10 \Omega$$

and capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{50 \times 10^{-6} \times 2000} = \frac{100}{10} = 10 \Omega$$

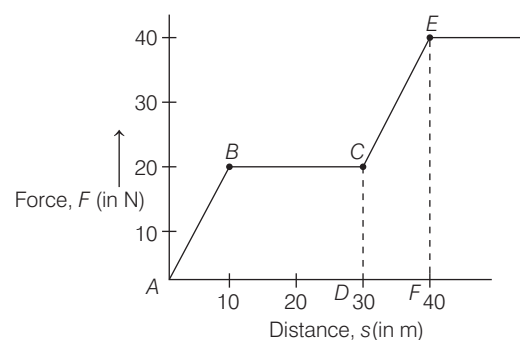
$$\Rightarrow X_L = X_C$$

The given circuit is resonant circuit.

Hence, impedance of circuit, $Z = R = 4 + 6 + 0.5 = 10.5 \Omega$

$$\text{Amplitude of current, } I_0 = \frac{V}{Z} = \frac{30}{10.5} = 2.85 \text{ A}$$

39. (d) Work done, $W = \text{area under } F\text{-}s \text{ graph}$



$\therefore$ Total work done is covering an initial distance of 40 m,

$W = \text{area of trapezium } ABCD + \text{area of trapezium } CEFD$

$$= \frac{1}{2} \times 20(20 + 30) + \frac{1}{2} \times 10 \times (20 + 40) = 800 \text{ J}$$

$$[\because \text{Area of trapezium} = \frac{1}{2} \times \text{height} \times \text{sum of parallel sides}]$$

$$\Rightarrow W = 800 \text{ J}$$

40. (b) Given, potential gradient = 0.4 mV cm^{-1}

$$= 0.4 \times 10^{-3} \text{ V cm}^{-1}$$

Length of wire, $l = 10 \text{ m} = 1000 \text{ cm}$

Electromagnetic force of cell, $E = 4 \text{ V}$

Potential gradient along wire

$$= \frac{\text{potential difference along wire}}{\text{length of wire}}$$

$$\Rightarrow 0.4 \times 10^{-3} = \frac{l \times 40}{1000} \text{ Vcm}^{-1}$$

Let R be the required resistance unplugged in the box.

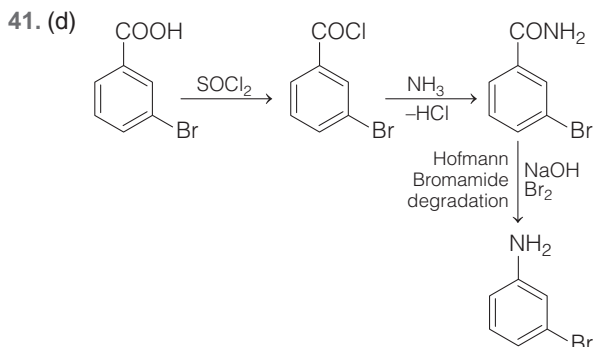
$$\therefore \text{Current in wire, } I = \frac{4}{400} \text{ A} \quad \dots(i)$$

$$\text{Also, } I = \frac{E}{R + R'} = \frac{4}{40 + R} \quad \dots(ii)$$

Equating Eqs. (i) and (ii), we get

$$\frac{4}{40 + R} = \frac{4}{400} \Rightarrow R = 400 - 40 = 360 \Omega$$

Chemistry



42. (b) From molarity equation,

$$M_1 V_1 = M_2 V_2$$

Acid Base

$$M_1 \times 50.0 = 0.10 \times 39.30$$

$$\therefore M_1 = \frac{0.10 \times 39.30}{50} = 0.0786 \text{ M}$$

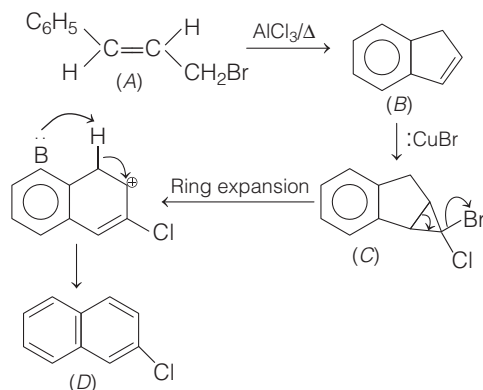
When half equivalence point is reached,

$$[\text{HA}] = [\text{A}^-]$$

$$\text{pH} = \text{p}K_a + \log \frac{[\text{A}^-]}{[\text{HA}]}$$

$$\text{pH} = \text{p}K_a = 4.85$$

43. (d) The complete equation is given as follows:



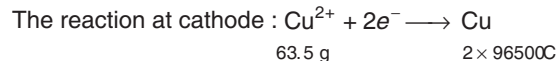
44. (a) Statement (a) is incorrect. Because α -D-fructose and β -D-fructose are anomers, diastereomers and geometrical isomers and not enantiomers.

Rest of the given statements are correct.

45. (d) Option (d) does not contain a pair of mixed oxides. Fe_2O_3 is not mixed oxide.

While, Mn_3O_4 ($2\text{MnO} + \text{MnO}_2$); Co_3O_4 ($\text{CoO} + \text{Co}_2\text{O}_3$); Fe_3O_4 ($\text{Fe}_2\text{O}_3 + \text{FeO}$) and Pb_3O_4 ($2\text{PbO} + \text{PbO}_2$), all are mixed salts.

46. (a) According to Faraday's first law of electrolysis:



The quantity of charge passed

$$= I \times t = (10 \text{ amp}) \times (60 \times 60 \text{ s}) = 36000 \text{ C.}$$

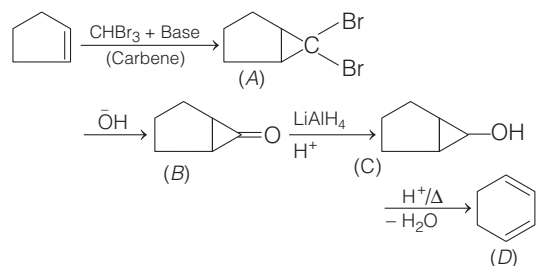
$2 \times 96500 \text{ C}$ of charge deposit copper– 63.5 g

36000 C of charge deposit copper

$$= \frac{(63.5 \text{ g})}{(2 \times 96500 \text{ C})} \times (36000 \text{ C}) = 11.84 \text{ g}$$

Thus, 11.84 g of copper will dissolve from the anode and the same amount from the solution will get deposited on the cathode. The concentration of the solution will remain unchanged.

47. (b) The complete reaction is given as follows:



48. (a) Statement (a) is correct. Rest of the all statements are incorrect. These are explained as follows:

(a) $\text{Cl}_2(g) \longrightarrow 2\text{Cl}(g)$

As randomness is increasing. Therefore, ΔS is positive for this reaction.

$\therefore$ Statement is correct.

(b) In closed container, $\Delta V = 0$, hence work done is zero. There is no heat exchange. Hence, $\Delta E = q + W = 0$.

$\therefore$ Statement is incorrect.

(c) ΔG will be zero only when equilibrium is reached.

$\therefore$ Statement is incorrect.

(d) $\Delta G^\circ = -RT \ln K_{eq}$, not a function of pressure. Thus, this statement is also incorrect.

49. (b) Statement I and III are correct and statement II incorrect.

$$(I) \frac{T_c}{p_c} = \frac{8a}{27Rb} \cdot \frac{27b^2}{a} = \frac{8b}{R} \therefore \frac{T_c}{p_c} \propto b$$

$$(II) T_c = \frac{8a}{27Rb} \text{ and } T_b = \frac{a}{Rb}$$

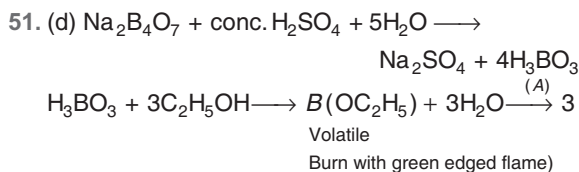
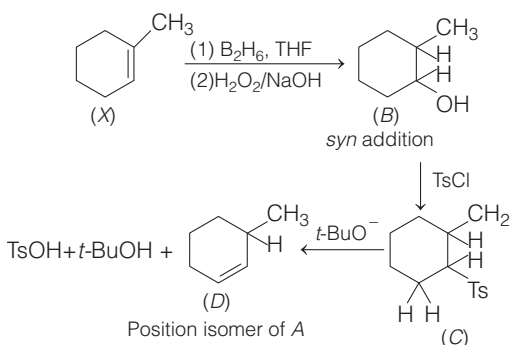
$$\text{Thus, } T_b > T_c$$

(III) When, T_c is attained.

$$\therefore \left(\frac{\partial p}{\partial V} \right) = 0$$

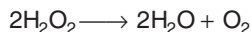
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50. (a) The reaction can be completed as follows:

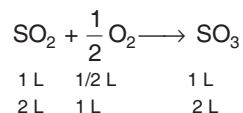


52. (a) Perhydrol means 30% solution of H_2O_2 .

H_2O_2 decomposes as :



Volume strength of 30% H_2O_2 solution is 100 that means 1 mL of this solution on decomposition given 100 mL oxygen.



Since, 100 mL of oxygen is obtained by = 1 mL of H_2O_2

$\therefore$ 1000 mL of oxygen will be obtained by

$$= \frac{1}{100} \times 1000 \text{ mL of H}_2\text{O}_2$$

$$= 10 \text{ mL of H}_2\text{O}_2$$

53. (c) At top of a mountain,

$$T_1 = 0 + 273\text{K}, p_1 = \frac{710}{760} \text{ atm and density} = \rho_1$$

At bottom of mountain,

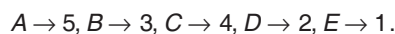
$$T_2 = 303 \text{ K}, p_2 = 1 \text{ atm and density} = \rho_2$$

$$\therefore pV = \frac{w}{M} RT \text{ or } p = \rho \frac{RT}{M}$$

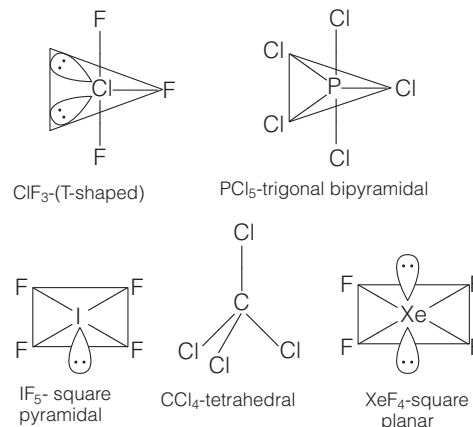
$$\therefore \frac{\rho_1}{\rho_2} = \frac{p_1}{p_2} \times \frac{T_2}{T_1}$$

$$\text{or } \frac{\rho_1}{\rho_2} = \frac{710/760 \times 303}{273 \times 1} = \frac{104}{1}$$

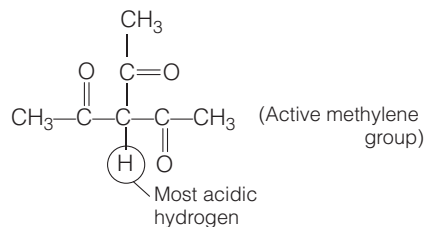
54. (b) The correct option is (b).



The shapes of molecules are given below.



55. (c) In option (c), the hydrogen of $-\text{CH}$ group is surrounded by three $\text{C}-\text{CH}_3$ groups as:



Here, O of CO attracts, the electron density towards itself. Thus, (c) has most acidic hydrogen among given compounds.

56. (c) The process by which the aquatic life gets deprived oxygen results in subsequent loss of biodiversity is known as eutrophication. It is due to the reduction in concentration of the dissolved oxygen in water due to phosphate pollution in water.

57. (d) As we know,

$$v = \frac{1}{\lambda} = \frac{(\text{IE})_H Z^2}{hc} \times \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

Given, $n_1 = 2, n_2 = 5$

$$(\text{IE})_H = 13.6 \text{ eV} = 13.6 \times 1.6 \times 10^{-19} \text{ J}$$

$$v = \frac{1}{\lambda} = \frac{13.6 \times 1.6 \times 10^{-19} \text{ J}}{6.6 \times 10^{-34} \text{ Js} \times 3 \times 10^8 \text{ ms}^{-1}} \left[\frac{1}{2^2} - \frac{1}{5^2} \right]$$

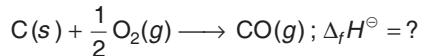
$$= 2.2988 \times 10^6 \text{ m}^{-1}$$

$$\therefore \lambda = \frac{1}{v} \Rightarrow \lambda = 4.35 \times 10^{-7} \text{ m} = 435 \times 10^{-9} \text{ m}$$

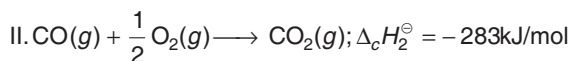
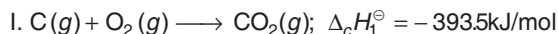
$$= 435 \text{ nm}$$

58. (c) Heat change at constant pressure means enthalpy change ($\Delta_r H = q_p$).

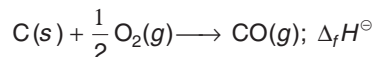
Heat of formation of CO is written as :



Now we have,



Subtracting appropriately (i.e., I-II), we get,



From Hess's law, $\Delta_f H^\ominus = \Delta_c H_1^\ominus - \Delta_c H_2^\ominus$

$$\Rightarrow \Delta_f H^\ominus = -393.5 - (-283) = 110.5 \text{ kJ/mol}$$

Now, calculation of the heat of formation at constant volume means that we have to calculate change in internal energy (i.e. $\Delta_r U$)

$$\begin{aligned} \text{Using, } \Delta_r H &= \Delta_r U + p\Delta V & [\text{For a chemical reaction}] \\ &= \Delta_r U + \Delta n_g RT & [\text{As } pV = nRT] \end{aligned}$$

$$[\Delta n_g = \text{gaseous moles of products} - \text{gaseous moles of reactant}]$$

$$\Rightarrow \Delta_r U = \Delta_r H - \Delta n_g RT$$

$$\text{Now putting the values : } \Delta n_g = 1 - \frac{1}{2} = \frac{1}{2}, \quad T = 298 \text{ K,}$$

$$R = 8.314 \text{ J/K/mol and } \Delta_f H^\ominus = -110.5 \text{ kJ/mol}$$

$$\begin{aligned} \Rightarrow \Delta_r U^\ominus &= -110.5 - \frac{1}{2} \times 8.314 \times 2.98 \times 10^{-3} \\ &= -111.7 \text{ kJ/mol} \end{aligned}$$

59. (b) If $n = n$, then $l = 0, 1, 2 \dots n$

n	l	Sub-orbit	$(n + l)$
1	0	1s	1
	1	1p	2
2	0	2s	2
	1	2p	3
	2	2d	4
3	0	3s	3
	1	3p	4
	2	3d	5
	3	3f	6

Thus, energy order is,

$$1s < 1p < 2s < 2p < 3s < 2d < 3p$$

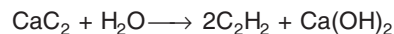
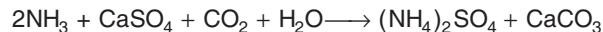
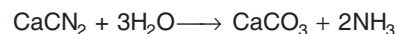
Sc (21) will have electronic configuration as :

$$1s^2 1p^6 2s^2 2p^6 3s^2 2d^3$$

60. (c)	A	+	2B	$\rightleftharpoons$	2C	+	D	
Initial	1.1		2.2		0		0	
At equili.	1.1-x		2.2-2x		2x		x	[2x = 0.2]
	1.1-0.1		2.2-0.2		0.2		0.1	[x = 0.1]
	1		2		0.2		0.1	

$$K_C = \frac{[C]^2 [D]}{[A] [B]^2} = \frac{(0.2)^2 (0.1)}{(1)(2)^2} = 0.001$$

61. (c) Chemical reaction given in option 'c' is correct. Rest of the given chemical reactions are incorrect. These can be corrected as:



62. (d) $Na_2S_2O_3 + H_2SO_4 \longrightarrow Na_2SO_4 + SO_2 + S_8 + H_2O$

Oxidation number of S in $Na_2S_2O_3 = +2$

Oxidation number of S in $H_2SO_4 = +6$

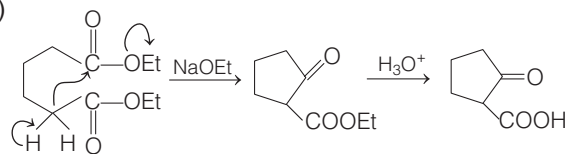
Oxidation number of S in $SO_2 = +4$

Oxidation number of S in $Na_2SO_4 = +6$

Oxidation number of S in $S_8 = 0$

So, sulphur can oxidise from +2 to +4 as well as reduce to 0 (S_8).

63. (c)

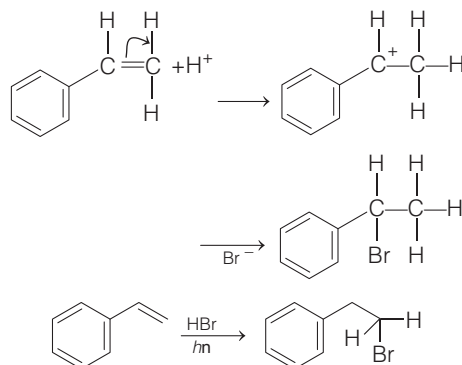


64. (a) $-\Delta S = \frac{\Delta G}{T}$, also $\frac{\Delta G}{T}$ is slope in Ellingham diagram. ΔS is different for different reactions as they have different slopes.

Rest of the statements (b), (c) and (d) are correct about Ellingham diagram.

65. (c) Phenolphthalein indicator changes into pink colour due to change in pH and not due to adsorption.

66. (c) $HBr \longrightarrow H^+ + Br^-$



67. (a) Z_{eff} for fcc = 4

Z_{eff} for bcc = 2

$$\text{Atomic volume of } \alpha\text{-form} = \frac{(3.68 \times 10^{-8})^3}{4} \times N_A$$

$$\text{Atomic volume of } \beta\text{-form} = \frac{(2.92 \times 10^{-8})^3}{2} \times N_A$$

As atomic weight is same, element is same, so the density ratio is

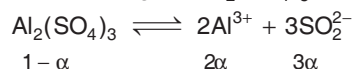
$$\rho_\alpha : \rho_\beta = V_\beta : V_\alpha = \frac{(292)^3}{2} : \frac{(3.68)^2}{4}$$

$$= 12.448 : 12.459 = 1 : 1$$

68. (c) $\therefore K_b = \frac{RT_b^2 M_1}{\Delta H_{vap}} = \frac{8.314 \times (373)^2 \times 0.0180}{40585}$

$$= 0.513 \text{ K kg/mol}$$

Now, molecular weight of $\text{Al}_2(\text{SO}_4)_3 = 342$



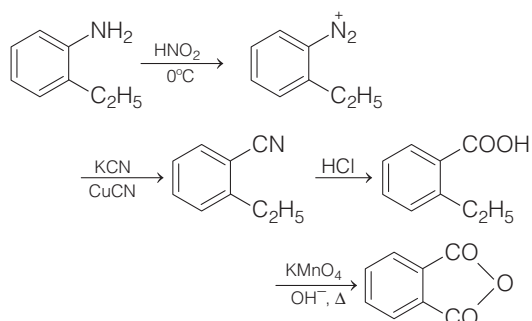
$$i = 1 - \alpha + 2\alpha + 3\alpha = 1 + 4\alpha \Rightarrow i = 5 \quad (\because \alpha = 1)$$

$$\Delta T_b = i \times K_b \cdot m$$

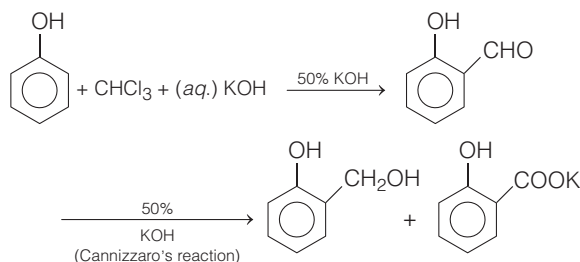
$$= 5 \times 0.513 \times \frac{5.6}{342 \times 1} = 0.042$$

$$\text{Boiling point of solution} = 100 + 0.042 = 100.042^\circ\text{C}$$

69. (d)



70. (b) Phenol on reaction with chloroform and KOH gives salicylaldehyde, which with 50% KOH solution undergoes Cannizzaro's reaction.

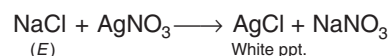
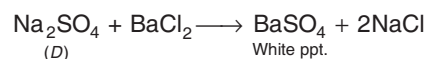
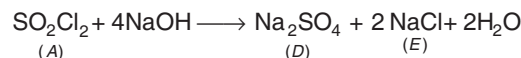
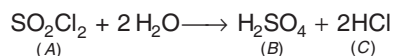


71. (d) Some complex ions of nitrogen show similarities with halide ions. These are called pseudohalide ions e.g., OCN^- , NCO^- , CN^- etc.

Halogens, among themselves, form complex ions which are called polyhalide ions. e.g., I_3^- , BrI_2^- etc.

Similarly, interhalogens are the compounds of halogens in which one halogen is cation and other halogen is anion. e.g., IF_7 , ICl_5 , BrF_5 , IF_5 etc.

72. (b) The inorganic halide (A) is SO_2Cl_2 .



73. (d) In alkali metals, reactivity increases down the group as electropositivity increases but for halogens reactivity decreases down the group as molecular stability of halogens increases.

74. (b) IUPAC name of sodiumnitroprusside $\text{Na}_2[\text{Fe}(\text{CN})_5\text{NO}]$ is sodiumpentacyanonitrosyl ferrate (III) because in it NO is neutral ligand and the oxidation number of Fe is III.

75. (d) $\text{A(g)} \longrightarrow \text{P(g)} + \text{Q(g)} + \text{R(g)}$

$$\text{At } t = 0, \quad 0.4 \text{ atm} \quad 0 \quad 0 \quad 0$$

$$\text{At time } = t, \quad 0.4 - x \quad x \quad x \quad x$$

$$P_t = 0.4 - x + x + x + x = 0.4 + 2x \text{ or } x = \frac{P_t - 0.4}{2}$$

$$P_A = P_0 - x = P_0 - \frac{P_t - 0.4}{2} = \frac{2 \times 0.4 - P_t + 0.4}{2} = \frac{1.2 - P_t}{2}$$

$$\text{From,} \quad k = \frac{2.303}{t} \log \frac{P_0}{P_A}$$

$$\frac{0.693}{t_{1/2}} = \frac{2.303}{230} \log \left(\frac{0.4 \times 2}{1.2 - P_t} \right)$$

$$\frac{0.693}{t_{1/2}} = \frac{2.303}{230} \log \left(\frac{0.8}{1.2 - P_t} \right)$$

$$\log \frac{0.8}{1.2 - P_t} = \frac{0.693 \times 230}{69.3 \times 2.303} = 0.9987$$

$$\frac{0.8}{1.2 - P_t} = 10^{0.9987}$$

$$0.8 = (10)(1.2 - P_t)$$

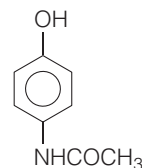
$$0.8 = 12 - 10P_t$$

$$10P_t = 12 - 0.8$$

$$10P_t = 11.2$$

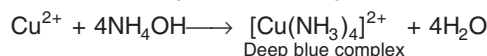
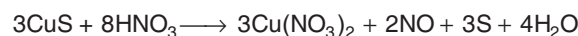
$$P_t = \frac{11.2}{10} = 1.12 \text{ atm}$$

76. (d) The structure of an important antipyretic paracetamol is



So *p*-amino phenol gives paracetamol an acylation.

77. (b) On passing H_2S gas in the aqueous solution of salt in the presence of dilute HCl, black ppt of CuS is formed.





Note In a reaction (redox/neutralisation) number of equivalents of different reactants are same.

Equivalents of $\text{C}_2\text{O}_4^{2-}$ = Equivalents of MnO_4^-

$$\text{Equivalent weight of } \text{C}_2\text{O}_4^{2-} = \frac{\text{Molecular weight}}{2}$$

$$\therefore \text{Molarity} = 2 \times \text{Normality}$$

$$\text{Equivalent weight of } \text{MnO}_4^- = \frac{\text{Molecular weight}}{5}$$

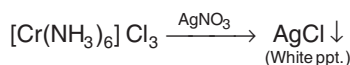
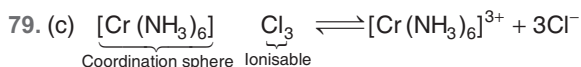
$$\text{Molarity} = 5 \times \text{Normality}$$

$$\text{Number of equivalents of } \text{MnO}_4^- = \frac{28.85 \times 5 M_1}{1000}$$

$$\text{Number of equivalents of } \text{Na}_2\text{C}_2\text{O}_4 = \frac{0.1467}{67}$$

$$\therefore \frac{28.85 \times 5 M_1}{1000} = \frac{0.1467}{67}$$

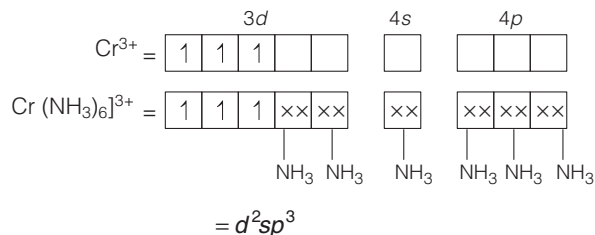
$$M_1 = 0.01518 \text{ M}$$



Config. of Cr (24) = $3d^5 4s^1$

Config. $\text{Cr}^{3+} = 3d^3 4s^0$

Thus option (d) is correct.



(a) d^2sp^3 hybridisation, octahedral. Thus, option (a) is correct.

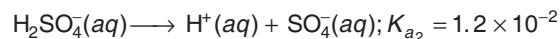
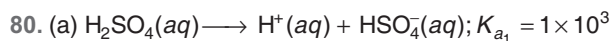
(b) here are three unpaired electrons, hence it is paramagnetic. Thus, option (b) is correct.

So the incorrect/... statement is (c)

(c) d^2sp^3 inner orbital complex. Thus option (c) is as incorrect.

(d) Due to ionisable Cl^- ions, white ppt with AgNO_3 . Thus, option (d) is correct

So, the incorrect statement is (c).



Here, $K_{a_2} < K_{a_1}$ because the negatively charged HSO_4^- ion has much less tendency to donate a proton to H_2O as compared to neutral H_2SO_4 .

English Proficiency

81. (b) The article 'the' should be used before 'famous' as the sentence refers to a particular thing i.e. a famous monument.

82. (b) As the sentence refers to a choice among more than two persons (servants), 'which' will be used in place of 'who'.

83. (c) The sentence is incomplete as it does not answer the question 'helped whom?' So, we add 'him' at the end of the sentence.

84. (a) Use of preposition 'with' is suitable to fill the given blank.

85. (b) Use of preposition 'at' is suitable to fill the given blank.

86. (c) 'Benign' means not likely to cause death. 'Fatal' would be its opposite.

87. (b) 'Noted' is known by many people because of particular qualities. Its opposite would be unknown.

88. (a) 'Sagacious' means having or showing understanding and the ability to make good judgements. So, 'Foolish' would be its correct antonym.

89. (a) Getting peevish means to get irritated.

90. (c) 'Roughed out' is a phrasal verb means to draw a 'rough draft'. So, 'drew a quick plan' is its correct synonym.

91. (b) The need for an effective population policy is an urgent necessity in the country's planning strategy. Hence, option (b) is the correct answer.

92. (d) The development of human resource and the building up of an institutional framework would have to receive priority attention.

93. (c) The Centre and the States government must become partners in the planning process to determine national priorities together. Hence, the statement given in option (c) is not correct.

94. (b) Domestic economic situation and world trends would force the planning process to undergo a change.

95. (c) Important changes in the international scene is implied by the expression 'momentous trends.'

Logical Reasoning

96. (d) The pattern is as follows

$$2 \times 6 = 12$$

$$6 \times 12 = 72$$

$$12 \times 72 = 864 \neq 865$$

$$72 \times 864 = 62208$$

Hence, number 865 is wrong and should be replaced by 864

97. (c) According to the question,

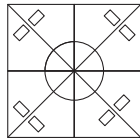
$$A/Q > T > P/B$$

But P is not the shortest

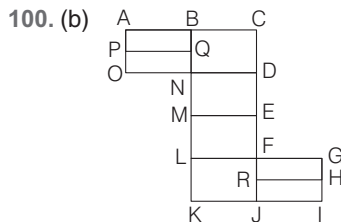
$$A/Q > T > P > B$$

So, tallest is either A or Q.

98. (c) Option figure (c) will complete the given pattern as



99. (d) 'Spanner' is used by mechanic as a tool to loose or tight nut and bolt. In the same way, 'saw' is used by carpenter as a tool to cut wood. Hence, option (d) is correct.



Total 18 rectangles are as follow

ABQP, PQNO, BCDN,
NDEM, MEFL, LFJK,
FGHR, RHIJ, ABNO,
BCEM, NDFL, MEJK,
FGIJ, ACDO, BCFL,
NDJK, LGIK and BCJK

101. (d) As, $B \xrightarrow{-1} A \xrightarrow{+3} D \xrightarrow{-1} C$
 $J \xrightarrow{-1} I \xrightarrow{+3} L \xrightarrow{-1} K$
 $N \xrightarrow{-1} M \xrightarrow{+3} P \xrightarrow{-1} O$

But, $V \xrightarrow{-1} U \xrightarrow{+2} W \xrightarrow{+1} X$

Except option (d), all others follow same pattern
 $(-1, +3, -1)$.

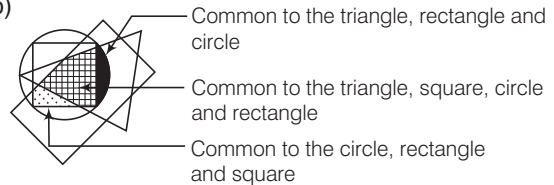
102. (d) Clearly, there are three types of shadings of circles, one is unshaded another has its right half shaded and yet another has its upper half shaded. Also, two of the figures in each row have one triangle shaded.

103. (c) Option figure (c) can be formed from the cut out pieces given in question figure.

104. (b) After unfolding the folded and cut piece of paper it will look like as answer figure (b).

So, Option (c) is correct

105. (b)



Mathematics

106. (a) Given that, $A = \{x : x^2 = 1\} = \{-1, 1\}$ and $B = \{x : x^4 = 1\}$
 $= \{-1, 1, -i, i\}$

Now, $A - B = \emptyset$ and $B - A = \{-i, i\}$

$$\therefore A \Delta B = (A - B) \cup (B - A) = \{-i, i\}$$

107. (c) We have, $2f(xy) = (f(x))^y + (f(y))^x$, $x, y \in R$ and $f(1) = a$.

Putting $x = 1, y = 2$, we get

$$2f(2) = (f(1))^2 + f(2) \Rightarrow f(2) = a^2$$

Putting $x = 1, y = 3$, we get

$$2f(3) = (f(1))^3 + f(3) \Rightarrow f(3) = a^3, \text{ and so on.}$$

$$\therefore f(n) = a^n$$

$$\sum_{i=1}^n f(i) = a + a^2 + \dots + a^n = \frac{a(a^n - 1)}{a - 1}.$$

108. (c) The given inequality is equivalent to

$$1 < |x - 3| + |4 - x|$$

$$\text{i.e., } 1 < |x - 3| + |x - 4| \quad \dots(i)$$

For $x < 3$, we have

$$3 - x + 4 - x = 7 - 2x > 1$$

i.e., $x < 3$, which is true in the domain.

For $3 \leq x < 4$, Eq. (i) gives

$$x - 3 + 4 - x = 1 \not> 1 \quad \dots(ii)$$

For $4 \leq x$, we have

$$2x - 7 > 1 \Rightarrow x > 4 \quad \dots(iii)$$

From Eqs. (i) and (iii), we have the solution of the inequality as $x < 3$ or $x > 4$.

i.e., $x \in]-\infty, 3) \cup (4, \infty[$ which is equal to $R - [3, 4]$.

109. (b) $\because 13, a_1, a_2, \dots, a_{20}, 67$ are in AP.

$$\therefore a_1 + a_2 + a_3 + \dots + a_{20} = 20 \left(\frac{13 + 67}{2} \right) = 800$$

Also, $AM > GM$

$$\Rightarrow \frac{a_1 + a_2 + \dots + a_{20}}{20} \geq (a_1 \cdot a_2 \cdot a_3 \cdot \dots \cdot a_{20})^{\frac{1}{20}}$$

$$\Rightarrow 40 \geq (a_1 \cdot a_2 \cdot a_3 \cdot \dots \cdot a_{20})^{\frac{1}{20}}$$

Hence, maximum value of $a_1 \cdot a_2 \cdot a_3 \cdot \dots \cdot a_{20}$ is $(40)^{20}$.

110. (a) Since, p, q and r are positive and are in AP.

$$\therefore q = \frac{p+r}{2} \quad \dots(i)$$

$\therefore$ The roots of the equation, $px^2 + qx + r = 0$ are real.

$$\Rightarrow q^2 \geq 4pr \Rightarrow \left[\frac{p+r}{2} \right]^2 \geq 4pr \quad [\text{from Eq. (i)}]$$

$$\Rightarrow p^2 + r^2 - 14pr \geq 0$$

$$\Rightarrow \left(\frac{r}{p} \right)^2 - 14 \left(\frac{r}{p} \right) + 1 \geq 0 \quad (\because p > 0)$$

$$\Rightarrow \left(\frac{r}{p} - 7 \right)^2 - 48 \geq 0$$

$$\Rightarrow \left(\frac{r}{p} - 7 \right)^2 - (4\sqrt{3})^2 \geq 0 \Rightarrow \left| \frac{r}{p} - 7 \right| \geq 4\sqrt{3}$$

$$\begin{aligned} 111. (c) \quad {}^{47}C_4 + \sum_{r=1}^5 {}^{52-r}C_3 &= {}^{47}C_4 + {}^{51}C_3 \\ &\quad + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 \\ &= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + ({}^{47}C_3 + {}^{47}C_4) = {}^{52}C_4 \end{aligned}$$

112. (a) Possible even digits are 2, 4, 6, 8, 0.

Case I Number has digits 4, 6, 8, 0.

(Here, sum of digits is 18, divisible by 3)

$$\square \square \square \square$$

Number of arrangements = $3 \times 3!$

[1st place can be filled using 4, 6, 8]

$$= 3 \times 6 = 18$$

Case II Number has digits 2, 4, 6, 0

(Here, sum of digits is 12, divisible by 3)

$$\square \square \square \square$$

1st place cannot be filled by 0.

Number of arrangements = $3 \times 3! = 18$

$\therefore$ Number of numbers = $18 + 18 = 36$

113. (d) We know that,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\therefore P = 1500 - n(A \cap B)$$

$$\Rightarrow n(A \cap B) = 1500 - P$$

Clearly, $1 \leq n(A \cap B) \leq 500$

$$\Rightarrow 1 \leq 1500 - P \leq 500$$

$$\Rightarrow -1499 \leq -P \leq -1000$$

$$\Rightarrow 1000 \leq P \leq 1499$$

114. (c) Here, $\frac{2x-1}{x^3+4x^2+3x} \in R$ only when $x^3 + 4x^2 + 3x \neq 0$

$$\Rightarrow x(x^2 + 4x + 3) = 0$$

$$\Rightarrow x(x+1)(x+3) = 0$$

$$\Rightarrow x = 0, -1, -3$$

$$\therefore \left\{ x \in R : \frac{2x-1}{x^3+4x^2+3x} \in R \right\} \\ = R - \{0, -1, -3\}$$

115. (a) Let $f(x) = Ax^2 + Bx + C$

$$\therefore f(1) = A + B + C$$

$$\text{and } f(-1) = A - B + C$$

$$\therefore f(1) = f(-1)$$

$$\Rightarrow A + B + C = A - B + C$$

$$\Rightarrow 2B = 0 \Rightarrow B = 0$$

$$\therefore f(x) = Ax^2 + C$$

$$\Rightarrow f'(x) = 2Ax$$

$$\Rightarrow f'(a) = 2Aa$$

$$f'(b) = 2Ab \text{ and } f'(c) = 2Ac$$

Also, a, b, c are in APSo, $2Aa, 2Ab$ and $2Ac$ are in AP. $\Rightarrow f'(a), f'(b)$ and $f'(c)$ are also in AP.

$$116. (c) \lim_{n \rightarrow \infty} \frac{1}{n} \left\{ \frac{1}{n+1} + \frac{2}{n+2} + \dots + \frac{3n}{n+3n} \right\}$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^{3n} \frac{1}{n} \left(\frac{r}{n+r} \right) = \int_0^3 \frac{x}{1+x} dx$$

$$= \int_0^3 \left(1 - \frac{1}{1+x} \right) dx$$

$$= [x - \log(1+x)]_0^3$$

$$= 3 - \log 4 = 3 - 2 \log 2$$

117. (b) We have,

$$(x-1)(x-2)(x-3) \dots (x-10)$$

Coefficient of x^8 = Sum of the terms taken two at a time i.e.,

$$\sum_{1 \leq i < j \leq 10} x_i \cdot x_j = \frac{1}{2} \left[\left(\sum_{i=1}^{10} x_i \right)^2 - \left(\sum_{i=1}^{10} x_i^2 \right) \right]$$

$$= \frac{1}{2} [(1+2+3+\dots+10)^2 - (1^2+2^2+3^2+\dots+10^2)]$$

$$= \frac{1}{2} \left[\left(\frac{10 \times 11}{2} \right)^2 - \left(\frac{10 \times 11 \times 21}{6} \right) \right]$$

$$= \frac{1}{2} ((55)^2 - 385) = \frac{1}{2} (3025 - 385) = \frac{2640}{2} = 1320$$

118. (b) We have, $Z = \frac{7+i}{3+4i}$

$$\Rightarrow Z = \left(\frac{7+i}{3+4i} \right) \left(\frac{3-4i}{3-4i} \right) = \frac{21+4-28i+3i}{9+16} = 1-i$$

$$\therefore Z^{14} = (1-i)^{14} = [(1-i)^2]^7 = (1-1-2i)^7$$

$$= (-2)^7 (i)^7 = 2^7 i$$

119. (c) We have, $\frac{dy}{dx} + \frac{\tan y}{x} = \frac{\tan y \sin y}{x^2}$ Now, divide by $\tan y \sin y$ both sides, we get

$$\cot y \operatorname{cosec} y \frac{dy}{dx} + \frac{\operatorname{cosec} y}{x} = \frac{1}{x^2} \quad \dots(i)$$

Put $\operatorname{cosec} y = z$

$$\therefore -\operatorname{cosec} y \cot y \frac{dy}{dx} = \frac{dz}{dx}$$

 $\therefore$ Eq. (i) reduces to

$$\frac{dz}{dx} - \frac{z}{x} = -\frac{1}{x^2}$$

$$\text{IF} = e^{\int -\frac{1}{x} dx} = e^{-\log x} = \frac{1}{x}$$

Hence solution is

$$\frac{z}{x} = \int -\frac{1}{x^3} dx = \frac{1}{2x^2} + c$$

$$\Rightarrow \frac{1}{x \sin y} = \frac{1-2cx^2}{2x^2}$$

$$\Rightarrow 2x = \sin y(1-2cx^2)^2$$

$$\Rightarrow 2x = \sin y(1-2cx^2)$$

120. (a) We have,

$$I = \int_0^{\frac{\pi}{2}} \frac{dx}{\tan x + \cot x + \operatorname{cosec} x + \sec x}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{dx}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} + \frac{1}{\sin x} + \frac{1}{\cos x}}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x dx}{1 + \sin x + \cos x}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{2 \sin \frac{x}{2} \cos \frac{x}{2} \left(\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \right)}{2 \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin \frac{x}{2} \left(\cos \frac{x}{2} - \sin \frac{x}{2} \right) dx$$

$$\Rightarrow I = \frac{1}{2} \int_0^{\frac{\pi}{2}} (\sin x + \cos x - 1) dx$$

$$\Rightarrow I = \frac{1}{2} [-\cos x + \sin x - x]_0^{\pi/2}$$

$$\Rightarrow I = \frac{1}{2} \left[\left(-\cos \frac{\pi}{2} + \sin \frac{\pi}{2} - \frac{\pi}{2} \right) - (-\cos 0 + \sin 0 - 0) \right]$$

$$\Rightarrow I = \frac{1}{2} \left[0 + 1 - \frac{\pi}{2} + 1 \right] = 1 - \frac{\pi}{4}$$

121. (d) We have given,

$$|a| = 1, |b| = 4, a \cdot b = 2$$

Now, $c = (2a \times b) - 3b$ Multiplying by b both sides, we get

$$\begin{aligned}
 b \cdot c &= (2a \times b) \cdot b - 3|b|^2 \\
 \Rightarrow b \cdot c &= 0 - 3(4)^2 = -48 \\
 \text{and } |c|^2 &= |(2a \times b) \cdot 3b|^2 \\
 |c|^2 &= (2a \times b)^2 + 9|b|^2 = 4(a \times b)^2 + 9(4)^2 \\
 \Rightarrow |c|^2 &= 4(|a|^2 \cdot |b|^2 - (a \cdot b)^2) + 144 \\
 &= 4(16 - 4) + 144 = 48 + 144 = 192 \\
 \therefore b \cdot c &= |b||c|\cos\theta \\
 \Rightarrow \cos\theta &= \frac{b \cdot c}{|b||c|} = \frac{-48}{4 \times \sqrt{192}} = -\frac{\sqrt{3}}{2} \\
 \theta &= \pi - \frac{\pi}{6} = \frac{5\pi}{6}
 \end{aligned}$$

122. (a) We have,

$$x^2 - (k-2)x + (k^2 + 3k + 5) = 0$$

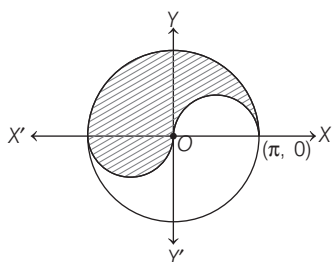
x_1, x_2 are roots of equations

$$\therefore x_1 + x_2 = k - 2, x_1 x_2 = k^2 + 3k + 5$$

$$\begin{aligned}
 x_1^2 + x_2^2 &= (x_1 + x_2)^2 - 2x_1 x_2 \\
 &= (k-2)^2 - 2(k^2 + 3k + 5) \\
 &= k^2 - 4k + 4 - 2k^2 - 6k - 10 \\
 &= -(k^2 + 10k + 6) \\
 &= -[(k+5)^2 - 19] \\
 &= 19 - (k+5)^2 \leq 19
 \end{aligned}$$

$\therefore$ Maximum value of $x_1^2 + x_2^2 = 19$

123. (c) Graph of circle whose centre is origin and radius is π units and $y = \sin x$ is



$$\begin{aligned}
 \text{Area of shaded region} &= \frac{1}{2} \text{ Area of circle} \\
 &= \frac{1}{2} (\pi r^2) = \frac{1}{2} \pi (\pi^2) \quad [\because r = \pi] \\
 &= \frac{\pi^3}{2}
 \end{aligned}$$

124. (a) The given equation is

$$\begin{aligned}
 2(1+i)x^2 - 4(2-i)x - 5 - 3i &= 0 \\
 \Rightarrow x &= \frac{4(2-i) \pm \sqrt{16(2-i)^2 + 8(1+i)(5+3i)}}{4(1+i)}
 \end{aligned}$$

$$= -\frac{i}{1+i} \text{ or } \frac{4-i}{1+i} = \frac{-1-i}{2} \text{ or } \frac{3-5i}{2}$$

$$\text{Now, } \left| \frac{-1-i}{2} \right| = \sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{1}{2}}$$

$$\text{and } \left| \frac{3-5i}{2} \right| = \sqrt{\frac{9}{4} + \frac{25}{4}} = \sqrt{\frac{17}{2}}$$

$$\text{Also, } \sqrt{\frac{17}{2}} > \sqrt{\frac{1}{2}}$$

Hence, required root is $\frac{3-5i}{2}$.

125. (d) For real roots, discriminant,

$$D = b^2 - 4ac \geq 0$$

$$= \cos^2 \beta - 4(\cos \beta - 1) \sin \beta \geq 0$$

$$= \cos^2 \beta + 4(1 - \cos \beta) \sin \beta \geq 0$$

So, $\sin \beta$ should be > 0 . [$\because \cos^2 \beta \geq 0, 1 - \cos \beta \geq 0$]

$$\Rightarrow \beta \in (0, \pi)$$

126. (c) We have,

$$(1+\alpha)x + \beta y + z = 2$$

$$\alpha x + (1+\beta)y + z = 3$$

$$\alpha x + \beta y + 2z = 2$$

$$\text{For unique solution } \begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

Apply $R_1 \rightarrow R_1 - R_2$ and $R_2 \rightarrow R_2 - R_3$, we get

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

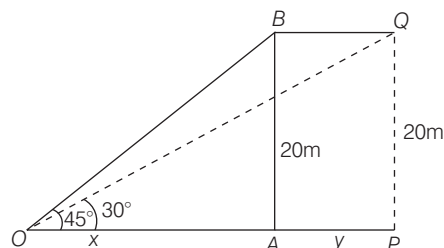
Expanding along R_1 , we get

$$\begin{aligned}
 1(2+\beta) + 1(0+\alpha) &\neq 0 \\
 &= \alpha + \beta + 2 \neq 0
 \end{aligned}$$

Only option (c) satisfied above equation.

127. (d) Let $AB = h =$ height of vertical pole

$OA = x =$ Horizontal distance from O to pole



In $\triangle OAB$

$$\tan 45^\circ = \frac{AB}{OA} = 1 = \frac{20}{x} \Rightarrow x = 20 \text{ m}$$

In $\triangle OPQ$

$$\tan 30^\circ = \frac{PQ}{OP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{20}{x+y}$$

$$\Rightarrow x + y = 20\sqrt{3}$$

$$\Rightarrow y = 20\sqrt{3} - x = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1)$$

$\therefore$ Speed of bird is $20(\sqrt{3} - 1)$ m/s.

128. (b) Since, $g = \sqrt{ab}$. Also, a, p, q and b are in AP.

So, common difference d is $\frac{b-a}{3}$.

$$\therefore p = a + d = a + \frac{b-a}{3} = \frac{2a+b}{3}$$

$$q = b - d = b - \frac{b-a}{3} = \frac{a+2b}{3}$$

Now, $(2p - q)(p - 2q)$

$$= \frac{(4a + 2b - a - 2b)}{3} \cdot \frac{(2a + b - 2a - 4b)}{3}$$

$$= -ab = -g^2$$

129. (b) Let $f(x) = x^{100}$

$$g(x) = x^2 - 3x + 2$$

$q(x)$ = quotient

$r(x)$ = remainder $= (ax + b)$

$$\therefore f(x) = g(x)q(x) + r(x)$$

$$x^{100} = (x^2 - 3x + 2)q(x) + ax + b$$

$$\text{Put } x = 1 \Rightarrow 1 = a + b$$

$$\text{Put } x = 2 \Rightarrow 2^{100} = 2a + b$$

From Eqs. (i) and (ii), we get

$$a = 2^{100} - 1, b = 2 - 2^{100}$$

$$\text{Remainder} = (2^{100} - 1)x + (2 - 2^{100})$$

$$= (2^{99+1} - 1)x - (2^{99} - 1)$$

$$\therefore k = 99$$

130. (a) Let two observations be x and y .

$$\therefore \text{Mean} = \frac{1 + 3 + 8 + x + y}{5} = \frac{12 + x + y}{5}$$

$$\Rightarrow 5 = \frac{x + y + 12}{5} \Rightarrow x + y = 13$$

$$\text{Now, variance} = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$\Rightarrow 9.20 = \frac{1^2 + 3^2 + 8^2 + x^2 + y^2}{5} - 25$$

$$\Rightarrow 46 = 74 + x^2 + y^2 - 125$$

$$\Rightarrow x^2 + y^2 = 97$$

From Eqs. (i) and (ii), we get

$$(x + y)^2 - 2xy = 97$$

$$\Rightarrow (13)^2 - 2xy = 97$$

$$\Rightarrow 2xy = 169 - 97 = 72$$

$$\Rightarrow xy = 36$$

$$\therefore x : y = 4 : 9$$

131. (b) Let the three digit number be abc

Given middle digit is A.M of first and last

$$\therefore b = \frac{a+c}{2} \Rightarrow 2b = a + c$$

$2b$ is an even

$\therefore a$ and c should be both even or both odd

$$\therefore \text{Total number of ways} = {}^5C_1 \times {}^5C_1 + {}^4C_1 \times {}^5C_1 \\ = 5 \times 5 + 4 \times 5 = 25 + 20 = 45$$

132. (b) We have,

$$Z = re^{i\theta}$$

$$Z = r(\cos \theta + i \sin \theta)$$

$$e^{iz} = e^{ir(\cos \theta + i \sin \theta)}$$

$$\Rightarrow e^{iz} = e^{ir \cos \theta + i^2 r \sin \theta}$$

$$\Rightarrow e^{iz} = e^{ir \cos \theta - r \sin \theta}$$

$$\Rightarrow e^{iz} = e^{-r \sin \theta} \cdot e^{ir \cos \theta}$$

$$\therefore \arg(e^{iz}) = r \cos \theta$$

133. (d) Coefficient of x^{10} in $(x + x^2 + \dots + x^6)^4$

$$= \text{Coefficient of } x^6 \text{ in } (1 + x + \dots + x^5)^4$$

...(i)

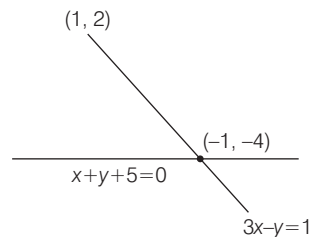
...(ii)

$$= (1 - x^6)^4 (1 - x)^{-4} \text{ in } (1 - 4x^6 + \dots) \left[1 + \binom{4}{1}x + \dots \right]$$

$$\text{Hence, coefficient of } x^6 \text{ is } \binom{9}{6} - 4 = 80.$$

134. (b) A line passing through $(1, 2)$ and parallel to the line $3x - y = 10$ is

$$3(x - 1) - (y - 2) = 0$$



...(i)

...(ii)

$$\Rightarrow 3x - 3 - y + 2 = 0 \Rightarrow 3x - y - 1 = 0$$

The intersection point of line

$$x + y + 5 = 0$$

...(i)

and

$$3x - y - 1 = 0$$

...(ii)

are $(-1, -4)$

$\therefore$ Distance from the line $x + y + 5$ to the point $(1, 2)$ measured parallel to the line $3x - y = 10$ is

$$\sqrt{(-1-1)^2 + (-4-2)^2}$$

$$\sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$$

135. (d) We have,

$$f(x) = a_0 + a_1x^2 + a_2x^4 + a_3x^6 + \dots + a_nx^{2n}$$

$$f'(x) = 2a_1x + 4a_2x^3 + 6a_3x^5 + \dots + 2na_nx^{2n-1}$$

$$f'(x) = 2x(a_1 + 2a_2x^2 + 3a_3x^4 + \dots + na_nx^{2n-2})$$

For maxima or minima

$$\text{Put } f'(x) = 0 \Rightarrow x = 0$$

$$f''(x) = 2(a_1 + 6a_2x^2 + 15a_3x^4 + \dots + n(2n-1)a_nx^{2n-2})$$

$$(f''(x))_{x=0} = 2a_1 > 0$$

$\therefore f(x)$ has only one minima at $x = 0$.

136. (c) We have,

$$f(x) = 2x^3 + x^4 + \log x$$

Put $f(x) = 3$, we get

$$x = 1$$

$$\therefore f(1) = 3 \Rightarrow 1 = f^{-1}(3)$$

Now, given g is the inverse of f

$$\therefore g(x) = f^{-1}(x) \Rightarrow f(g(x)) = x$$

Differentiating with respect to x , we get

$$f'(g(x)) \cdot g'(x) = 1$$

$$g'(x) = \frac{1}{f'(g(x))}$$

$$g'(x) = \frac{1}{(g(x))^2 + 4(g(x))^3 + \frac{1}{g(x)}} \left[\because f'(x) = 6x^2 + 4x^3 + \frac{1}{x} \right]$$

$$g'(3) = \frac{1}{6(g(3))^2 + 4(g(3))^3 + \frac{1}{g(3)}}$$

$$g'(3) = \frac{1}{6+4+1} \quad [\because g(3) = f^{-1}(3) = 1]$$

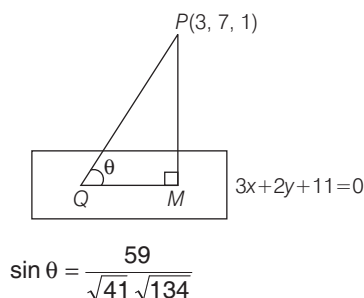
$$g'(3) = \frac{1}{11}$$

137. (d) We have $P(3, 7, 1)$ and $R(2, 5, 7)$

DR's of $PR = (-1, -2, 6)$

Angle between line PQ and plane is

$$\sin \theta = \frac{(-1)(3) + (-2) \times 2 + 6 \times 11}{\sqrt{1+4+36} \sqrt{9+4+121}}$$



$$PM = \left| \frac{3(3) + 2(7) + 11(\times 1) - 9}{\sqrt{9+4+121}} \right|$$

$$= \left| \frac{9+14+11-9}{\sqrt{134}} \right| = \frac{25}{\sqrt{134}}$$

$$\sin \theta = \frac{PM}{PQ}$$

$$\Rightarrow PQ = \frac{PM}{\sin \theta} = \frac{25}{\sqrt{134}} \times \frac{\sqrt{41} \times \sqrt{134}}{59} = \frac{25\sqrt{41}}{59}$$

138. (a) We have,

$$I = \int \frac{8x^{43} + 13x^{38}}{(x^{13} + x^5 + 1)^4} dx$$

$$= \int \frac{8x^{43} + 13x^{38}}{x^{52} \left(1 + \frac{1}{x^8} + \frac{1}{x^{13}} \right)^4} dx$$

$$= \int \frac{8 \left(\frac{1}{x^9} \right) + 13 \left(\frac{1}{x^{14}} \right)}{\left(1 + \frac{1}{x^8} + \frac{1}{x^{13}} \right)^4} dx$$

$$\text{Put, } 1 + \frac{1}{x^8} + \frac{1}{x^{13}} = t$$

$$\Rightarrow \left(-\frac{8}{x^9} - \frac{13}{x^{14}} \right) dx = dt$$

$$\therefore I = - \int \frac{dt}{t^4} = \frac{1}{3t^3} + C$$

$$\Rightarrow I = \frac{1}{3 \left(1 + \frac{1}{x^8} + \frac{1}{x^{13}} \right)^3} + C = \frac{x^{39}}{3(x^{13} + x^5 + 1)^3} + C$$

139. (b) We have,

Vector r is coplanar with vector a and b

$$\therefore r = \lambda(b \times (a \times b))$$

$$\Rightarrow r = \lambda((b \cdot b)a - (b \cdot a)b)$$

$$\Rightarrow r = \lambda(|b|^2 a - (a \cdot b)b)$$

$$\Rightarrow r = \lambda[5a + b] \quad [\because |b| = \sqrt{2^2 + 1^2} = \sqrt{5}]$$

$$\text{and } a \cdot b = (-\hat{i} + \hat{j} + \hat{k}) \cdot (2\hat{i} + \hat{k}) = -1$$

$$\text{Given, } r \cdot a = 7$$

$$\therefore \lambda(5a + b) \cdot a = 7$$

$$\Rightarrow \lambda[5|a|^2 + a \cdot b] = 7$$

$$\Rightarrow \lambda[15 - 1] = 7 \quad [\because |a|^2 = 1 + 1 + 1 = 3]$$

$$\Rightarrow \lambda = \frac{1}{2}$$

$$\therefore r = \frac{1}{2}(5a + b)$$

$$r = \frac{1}{2}(-5\hat{i} + 5\hat{j} + 5\hat{k} + 2\hat{i} + \hat{k})$$

$$\mathbf{r} = \frac{-3}{2} \hat{\mathbf{i}} + \frac{5}{2} \hat{\mathbf{j}} + \frac{6}{2} \hat{\mathbf{k}}$$

$$z \text{ component of } \mathbf{r} = \frac{6}{2} = 3$$

140. (b) Since x, y, z are three consecutive integers

$$\therefore 2y = x + z$$

$$\Rightarrow (4y^2) = (x + z)^2$$

$$= (x - z)^2 + 4xz$$

$$= (-2)^2 + 4xz \quad [\because x - z = -2]$$

$$\Rightarrow 4y^2 = 4 + 4xz$$

$$\Rightarrow y^2 = 1 + xz$$

Now,

$$\frac{1}{2} \log_e x + \frac{1}{2} \log_e z + \frac{1}{2zx+1} + \frac{1}{3(2xz+1)^3} + \dots$$

$$= \frac{1}{2} \left[\log_e (xz) + 2 \left(\frac{1}{2zx+1} + \frac{1}{3} \cdot \frac{1}{(2zx+1)^3} + \dots \right) \right]$$

$$= \frac{1}{2} \left[\log_e (xz) + \log_e \left(\frac{1 + \frac{1}{2zx+1}}{1 - \frac{1}{2zx+1}} \right) \right]$$

$$= \frac{1}{2} \left[\log_e (xz) + \log_e \left(\frac{1+xz}{xz} \right) \right]$$

$$= \frac{1}{2} [\log_e (1+xz)] = \frac{1}{2} [\log_e (y)^2] = \log_e (y)$$

141. (a) We have, $(xy^5 + 2y) dx = x dy$

$$\Rightarrow x \frac{dy}{dx} - 2y = xy^5$$

$$\Rightarrow \frac{dy}{dx} - \frac{2y}{x} = y^5$$

$$\Rightarrow y^{-5} \frac{dy}{dx} - \frac{2y^{-4}}{x} = 1$$

$$\text{Put, } y^{-4} = t$$

$$\Rightarrow -4y^{-5} \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow y^{-5} \frac{dy}{dx} = \frac{-1}{4} \frac{dt}{dx}$$

From Eqs. (i) and (ii), we get

$$-\frac{1}{4} \frac{dt}{dx} - \frac{2t}{x} = 1$$

$$\Rightarrow \frac{dt}{dx} + \frac{8t}{x} = -4$$

$$\text{Now, IF} = e^{\int \frac{8}{x} dx} = e^{8 \log x} = x^8$$

and the solution is

$$t \cdot x^8 = \int (-4)x^8 dx + C$$

$$\Rightarrow \frac{x^8}{y^4} = -\frac{4 \cdot x^9}{9} + C$$

$$\Rightarrow 9x^8 + 4x^9 \cdot y^4 = 9y^4 C$$

$$142. (b) \text{ Given, } \frac{|x-2|-1}{|x-2|-2} \leq 0$$

$$\text{Let } |x-2| = k$$

Then, given equation,

$$\frac{k-1}{k-2} \leq 0 \Rightarrow \frac{(k-1)(k-2)}{(k-2)^2} \leq 0$$

$$\Rightarrow (k-1)(k-2) \leq 0 \Rightarrow 1 \leq k \leq 2$$

$$\Rightarrow 1 \leq |x-2| \leq 2$$

Case I When $1 \leq |x-2|$

$$\Rightarrow |x-2| \geq 1$$

$$\Rightarrow x-2 \geq 1 \text{ or } x-2 \leq -1$$

$$\Rightarrow x \geq 3 \text{ and } x \leq 1$$

...(i)

Case II When $|x-2| \leq 2$

$$\Rightarrow -2 \leq x-2 \leq 2$$

$$\Rightarrow -2+2 \leq x \leq 2+2$$

$$\Rightarrow 0 \leq x \leq 4$$

...(ii)

From Eqs. (i) and (ii), $x \in [0, 1] \cup [3, 4]$

$$143. (b) \text{ We have, } f(x) = \frac{x}{\sqrt{1+x^2}}$$

$$\Rightarrow f(f(x)) = \frac{f(x)}{\sqrt{1+(f(x))^2}} = \frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+\frac{x^2}{1+x^2}}} = \frac{x}{\sqrt{1+2x^2}}$$

$$\text{Similarly, } f(f(f(x))) = \frac{x}{\sqrt{1+3x^2}}$$

$$\underbrace{f \circ f \circ \dots \circ f}_{n \text{ times}}(x) = \frac{x}{\sqrt{1+nx^2}} = \frac{x}{\sqrt{1+\left(\sum_{r=1}^n 1\right)x^2}}$$

144. (b) We have,

$$\log_5 \left(\frac{a+b}{3} \right) = \frac{\log_5 a + \log_5 b}{2}$$

$$\Rightarrow 2 \log_5 \left(\frac{a+b}{3} \right) = \log_5 ab$$

$$\Rightarrow \log_5 \left(\frac{a+b}{3} \right)^2 = \log_5 ab$$

$$\Rightarrow \frac{(a+b)^2}{9} = ab$$

$$\Rightarrow (a+b)^2 = 9ab$$

$$\Rightarrow a^2 + b^2 = 7ab$$

$$\Rightarrow \frac{a}{b} + \frac{b}{a} = 7$$

$$\Rightarrow \left(\frac{a}{b} + \frac{b}{a} \right)^2 = 49$$

$$\Rightarrow \frac{a^2}{b^2} + \frac{b^2}{a^2} + 2 = 49$$

$$\Rightarrow \frac{a^4 + b^4}{a^2 b^2} = 49 - 2 = 47$$

145. (a) We have,

$$\frac{5}{4} \cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$$

$$\Rightarrow \frac{5}{4} \cos^2 2x + (\cos^2 x + \sin^2 x)^2 - 2 \sin^2 x \cos^2 x + (\cos^2 x + \sin^2 x)^3 - 3 \sin^2 x \cos^2 x (\sin^2 x + \cos^2 x) = 2$$

$$\Rightarrow \frac{5}{4} \cos^2 2x + 1 - 2 \sin^2 x \cos^2 x + 1 - 3 \sin^2 x \cos^2 x = 2$$

$$\Rightarrow \frac{5}{4} \cos^2 2x - \frac{1}{2} \sin^2 2x - \frac{3}{4} \sin^2 2x = 0$$

$$\Rightarrow \frac{5}{4} (\cos^2 2x - \sin^2 2x) = 0$$

$$\Rightarrow \cos 4x = 0$$

$$\Rightarrow 4x = 2n\pi \pm \frac{\pi}{2}, n \in \mathbb{Z}$$

$$\Rightarrow x = (4n \pm 1) \frac{\pi}{8}, n \in \mathbb{Z}$$

$$\Rightarrow x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$$

So, number of solution = 8

146. (a) Given equation of ellipse

$$16x^2 + 11y^2 = 256 \Rightarrow \frac{x^2}{16} + \frac{11y^2}{256} = 1$$

Equation of tangent of ellipse at $\left(4 \cos \phi, \frac{16}{\sqrt{11}} \sin \phi \right)$ is

$$\frac{4x \cos \phi}{16} + \frac{11 \times 16y}{256 \sqrt{11}} \sin \phi = 1$$

$$\Rightarrow \frac{x \cos \phi}{4} + \frac{\sqrt{11}}{16} \sin \phi y = 1$$

is also a tangent of circle $x^2 + y^2 - 2x = 15$

$$\Rightarrow (x-1)^2 + y^2 = 16$$

radius of circle = 4 and centre = (1, 0)

Now, $\frac{x \cos \phi}{4} + \frac{\sqrt{11}}{16} \sin \phi y = 1$ is tangent of circle

$$\therefore 4 = \left| \frac{\frac{\cos \phi}{4} - 1}{\sqrt{\frac{\cos^2 \phi}{16} + \frac{11}{256} \sin^2 \phi}} \right| \Rightarrow \phi = \pm \frac{\pi}{3}$$

$$\therefore \phi = \frac{\pi}{3}$$

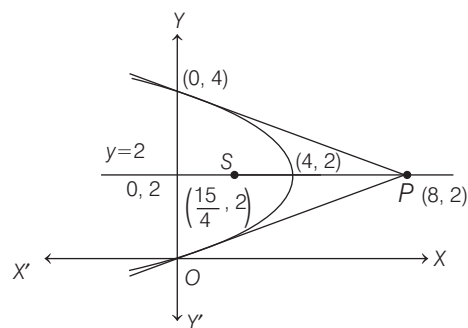
147. (b) Given equation of parabola is

$$x = 4y - y^2$$

$$\Rightarrow y^2 - 4y + 4 = -x + 4$$

$$\Rightarrow (y-2)^2 = -(x-4)$$

$$a = \frac{1}{4}$$



$$\text{Focus} = \left(\frac{15}{4}, 2 \right)$$

Parabola cuts Y-axis of (0, 0) and (0, 4)

Now, equation of tangent at (0, 0) is

$$x = 4y \quad \dots(i)$$

and equation of tangent at (0, 4) is

$$x + 4y = 16 \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we get

$$x = 8, y = 2$$

$\therefore$ Intersection point of tangent is $P(8, 2)$.

Distance from point P to focus S

$$\begin{aligned} \text{Distance} &= \sqrt{\left(8 - \frac{15}{4} \right)^2 + (2 - 2)^2} \\ &= 8 - \frac{15}{4} = \frac{17}{4} \end{aligned}$$

148. (d) We have, $S_{\infty} = \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} \frac{k}{2^{n+k}}$

$$S_{\infty} = \sum_{k=1}^{\infty} \frac{k}{2^k} \cdot \sum_{n=1}^{\infty} \frac{1}{2^n}$$

$$S_{\infty} = \sum_{k=1}^{\infty} \frac{k}{2^k} \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots \right)$$

$$S_{\infty} = \sum_{k=1}^{\infty} \frac{k}{2^k} \left[\frac{1}{2} \left(1 + \frac{1}{2} + \frac{1}{2^2} + \dots \right) \right]$$

$$S_{\infty} = \sum_{k=1}^{\infty} \frac{k}{2^k} \left(\frac{1}{2} \times \frac{1}{1 - \frac{1}{2}} \right) = \sum_{k=1}^{\infty} \frac{k}{2^k}$$

$$S_{\infty} = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \dots \quad \dots(i)$$

$$\frac{1}{2} S_{\infty} = \frac{1}{2^2} + \frac{2}{2^3} + \frac{3}{2^4} + \dots$$

From Eqs. (i) and (ii), we get

$$\frac{1}{2} S_{\infty} = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots$$

$$\frac{1}{2} S_{\infty} = \frac{1}{2} \left(\frac{1}{1 - \frac{1}{2}} \right)$$

$$\frac{1}{2} S_{\infty} = 1$$

$$S_{\infty} = 2$$

149. (a) We have,

Slope of tangent of the curve is $\frac{(x+1)^2 + y - 3}{x+1}$

$$\therefore \frac{dy}{dx} = \frac{(x+1)^2 + y - 3}{x+1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(x+1)^2 - 3}{x+1} + \frac{y}{x+1}$$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x+1} = \frac{(x+1)^2 - 3}{x+1}$$

This is a form of linear differential equations

$$\therefore \text{IF} = e^{\int \frac{-1}{x+1} dx} = e^{\log \frac{1}{1+x}} = \frac{1}{1+x}$$

Solution of given differential equation is

$$\frac{y}{x+1} = \int \frac{(x+1)^2 - 3}{(x+1)^2} dx$$

$$\frac{y}{x+1} = \int \left(1 - \frac{3}{(x+1)^2} \right) dx$$

$$\frac{y}{x+1} = x + \frac{3}{x+1} + c$$

...(ii)

$\therefore$ Curve passes through (2, 0)

$$\therefore 0 = 2 + \frac{3}{3} + c$$

$$\Rightarrow c = -3$$

Hence, equation of curve is

$$\frac{y}{x+1} = x + \frac{3}{x+1} - 3$$

$$y = x^2 + x + 3 - 3x - 3$$

$$y = x^2 - 2x$$

150. (a) We have,

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

We know that, $|A - \lambda I| = 0$

$$\left| \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$= \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)^2 - 1 = 0$$

$$\Rightarrow 4 - 4\lambda + \lambda^2 - 1 = 0$$

$$\Rightarrow \lambda^2 - 4\lambda + 3 = 0$$

$$\therefore A^2 - 4A + 3I = 0$$

Multiply by A^{-1} , we get

$$A - 4I + 3A^{-1} = 0$$

$$\Rightarrow A^{-1} = \frac{-1}{3}A + \frac{4}{3}I$$

$$\therefore \alpha = -\frac{1}{3} \text{ and } \beta = \frac{4}{3}$$

$$\alpha + \beta = -\frac{1}{3} + \frac{4}{3} = \frac{3}{3} = 1$$

Solved Paper 2021 BITSAT

Instructions

- There are 150 questions in all. The number of questions in each part is as given below.

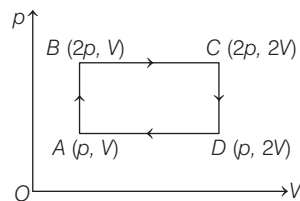
Part I Physics	1-40
Part II Chemistry	41-80
Part III a. English Proficiency	81-95
b. Logical Reasoning	96-105
Part IV Mathematics	106-150
- All questions are multiple choice questions with four options, only one is correct.
- Each correct answer fetches 3 marks while incorrect answer has a penalty of 1 mark.

No. of Questions

PART I

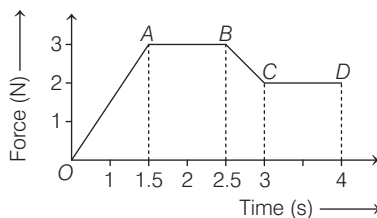
Physics

- An ideal monoatomic gas is taken round the cycle $ABCD$ as shown in the p - V diagram.



The work done during the cycle is

- pV
 - $2pV$
 - $\frac{pV}{2}$
 - zero
- The initial speed of a body of mass 2.0 kg is 5 m/s. A force acts for 4 s in the direction of motion of the body, as shown in force-time graph. The impulse of force is



- 8.50 Ns
- 8 Ns
- 5.5 Ns
- 6 Ns

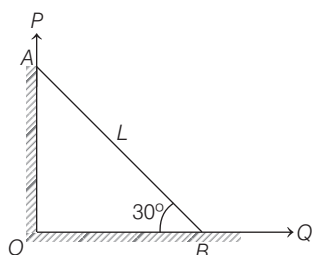
- For a constant hydraulic stress on an object, the fractional change in the object's volume $\left(\frac{\Delta V}{V}\right)$ and its

bulk modulus B are related as

- $\frac{\Delta V}{V} \propto B$
 - $\frac{\Delta V}{V} \propto B^2$
 - $\frac{\Delta V}{V} \propto \frac{1}{B}$
 - $\frac{\Delta V}{V} \propto \frac{1}{B^2}$
- A shunt is connected in parallel with a galvanometer. Why?
 - To prevent galvanometer from strong current
 - To convert galvanometer into ammeter
 - To increase the range of ammeter
 - All of the above
 - The phase difference between the V_{out} and V_{in} of CE amplifier circuit is
 - 90°
 - 180°
 - 0°
 - 270°
 - If n th division of main scale coincides with $(n+1)$ th division of vernier scale. The least count of the vernier is (Given, one main scale division is equal to a units)
 - $\frac{a+1}{n+1}$
 - $\frac{a}{n}$
 - $\frac{a}{n+1}$
 - $\frac{a+1}{n}$

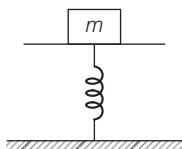
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7. A uniform rod AB of length $L = 1\text{ m}$ is sliding along two mutually perpendicular surfaces OP and OQ as shown in figure.



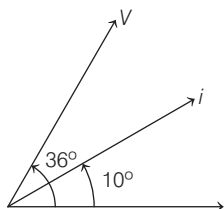
When the rod subtends an angle $\theta = 30^\circ$ with OQ , the end B has a velocity $\sqrt{3}\text{ m/s}$. The velocity of end A at that time is

- a. 1 m/s
b. 0.5 m/s
c. $\sqrt{3}\text{ m/s}$
d. $\frac{1}{\sqrt{3}}\text{ m/s}$
8. A tray of mass (M) 12 kg is supported by a spring as shown in figure.



When the tray is pressed down and released, it executes SHM with a period of 1.5 s . When a block of mass m placed on the tray, the period of SHM changes to 3.0 s . The mass of block is

- a. 36 kg
b. 48 kg
c. 12 kg
d. 24 kg
9. The phasor diagram of a load represents which circuit?



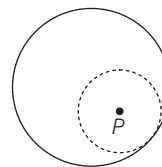
- a. Purely capacitive
b. Purely inductive
c. R - L - C circuit with X_L more than X_C
d. R - L - C circuit with X_L less than X_C
10. The equation of progressive wave is given by $Y = \sin\left[x\left(\frac{t}{5} - \frac{x}{9}\right) + \frac{\pi}{6}\right]\text{ cm}$. Which one of the following is correct?
- a. $v = 5\text{ cm/s}$
b. $\lambda = 18\text{ cm}$
c. $A = 0.04\text{ cm}$
d. $f = 50\text{ Hz}$

11. The property of light used in optical fibre cables is
a. total internal reflection
b. refraction
c. interference
d. polarisation

12. A man walks in a straight line for 5 min with a velocity of 45 m/s . What is the speed with which he has to move in order to comeback to its original position in 1.5 min ?

- a. 90 m/s
b. 150 m/s
c. 135 m/s
d. 115 m/s

13. From a solid sphere of mass M and radius R , a spherical portion of radius $\frac{R}{2}$ is removed as shown in the figure.



Taking gravitational potential $V = 0$ at $r = \infty$, the potential at the centre of the cavity thus formed is

- a. $-\frac{GM}{2R}$
b. $-\frac{GM}{R}$
c. $-\frac{2GM}{3R}$
d. $-\frac{2GM}{R}$

14. Four charges equal to $+Q$ are placed at the four corners of a square and a charge $(-q)$ is at its centre. If the system is in equilibrium, then the value of $-q$ is

- a. $-\frac{Q}{4}(1 + 2\sqrt{2})$
b. $\frac{Q}{2}(2\sqrt{2} + 1)$
c. $\frac{Q}{4}(1 + 2\sqrt{2})$
d. $\frac{Q}{4}(1 + \sqrt{2})$

15. A force $F = a + bx$ acts on a particle in the x -direction, where a and b are constants. The work done by this force during a displacement from $x = 0$ to $x = d$ is

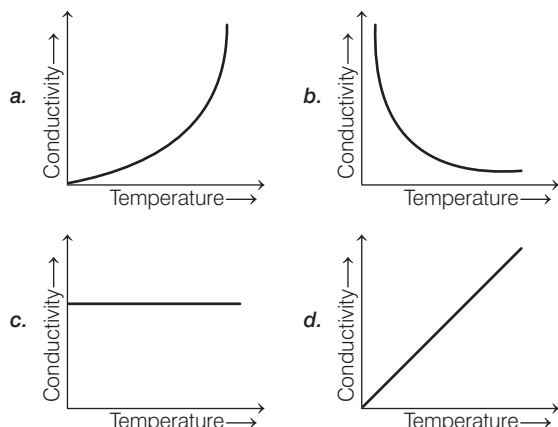
- a. $\left(a + \frac{bd}{2}\right)d$
b. $\left(ad + \frac{bd}{2}\right)$
c. $\left(\frac{ad + bd}{2}\right)$
d. $(ad + b)\frac{d}{2}$

16. A proton has kinetic energy $E = 100\text{ eV}$ which is equal to that of a photon. The wavelength of photon is λ_2 and that of proton is λ_1 . The ratio $\frac{\lambda_2}{\lambda_1}$ is proportional to

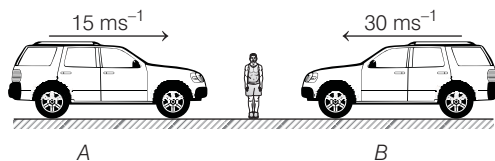
- a. E^2
b. $E^{\frac{1}{2}}$
c. E^{-1}
d. $E^{-\frac{1}{2}}$

17. A block of mass 10 kg rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.1 . The friction force on the block is
a. 4.9 N
b. $49\sqrt{3}\text{ N}$
c. 49 N
d. $0.1 \times 49\sqrt{3}\text{ N}$

18. Which of the following graphs show the correct relation between conductivity and temperature for a metallic conductor?



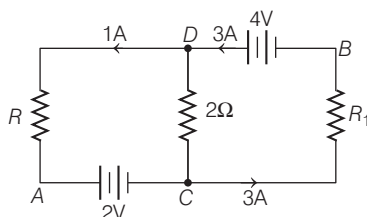
19. The radius of a muonic hydrogen atom is 2.5×10^{-13} m. The total atomic volume (in m^3) of a mole of such hydrogen atoms is (Take, $\pi = 3.14$)
- a. 3.94×10^{-14} b. 3.09×10^{-14}
 c. 4×10^{-14} d. 3.9×10^{-14}
20. The angular momentum of a body placed at origin of mass 1 kg and having position vector $\mathbf{r} = 3t\hat{i} + 4\hat{j}$ is
- a. time dependent b. $3\hat{k}$ J-s
 c. $-12\hat{k}$ J-s d. 0
21. If the earth stops rotating about its axis, then what will be the change in the value of g at a place in the equatorial plane? (Radius of earth = 6400 km)
- a. 3.7 cm/s^2 b. 9.8 m/s^2
 c. 0 d. 3.4 cm/s^2
22. Two cars approach a stationary observer from opposite sides as shown in the figure.



The observer hears no beats. If the frequency of the horn of the car B is 504 Hz, then the frequency of the horn of the car A will be

- a. 529.2 Hz b. 440.5 Hz
 c. 295.2 Hz d. None of these

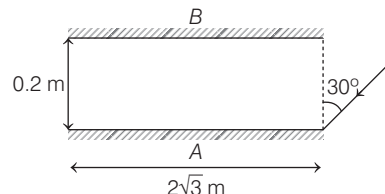
23.



In the following circuit, assuming point A to be at zero potential, then what is the potential at point B?

- a. 1 V b. 2 V
 c. 4 V d. 3 V

24. Two plane mirrors A and B are aligned parallel to each other as shown in the figure.



A ray of light is incident at an angle 30° at a point just inside one end of A. The plane of incidence coincides with the plane of the figure. The maximum number of times, the ray undergoes reflection (excluding the first one) before it emerges out is

- a. 28 b. 34
 c. 30 d. 29

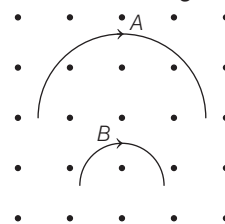
25. A projectile is given an initial velocity of $(\hat{i} + \hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10 \text{ m/s}^2$, then the equation of its trajectory is

- a. $y = x + 5x^2$ b. $y = x - 5x^2$
 c. $y = x^2 + 5x$ d. $y = x^2 - 5x$

26. Two drops of equal radius (R) coalesce to form a bigger drop. What is the ratio of surface energy of bigger drop to smaller one?

- a. $2^{-1/3} : 1$ b. $2^{2/3} : 1$
 c. 1 : 1 d. $2^{1/2} : 1$

27. Two particles A and B of masses m_A and m_B respectively, are having same charge and moving on same plane. A uniform magnetic field exists perpendicular to this plane. The speeds of the particles are v_A and v_B respectively and the trajectories are as shown in figure. Then,



- a. $m_A = m_B$ and $v_A = v_B$ b. $m_A v_A > m_B v_B$
 c. $m_A < m_B$ and $v_A < v_B$ d. $m_A v_A < m_B v_B$

28. The potential difference applied to an X-ray tube is decreased. As a result, in the emitted radiation,

- a. the intensity increases
 b. the intensity decreases
 c. the minimum wavelength increases
 d. the minimum wavelength decreases

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29. The ratio of the specific heats $\frac{C_V}{C_P} = \frac{1}{\gamma}$ in terms of degrees of freedom (n) is given by

a. $\left(1 + \frac{n}{3}\right)$ b. $\left(1 + \frac{2}{n}\right)$
c. $\left(1 + \frac{n}{2}\right)$ d. $\left(1 + \frac{1}{n}\right)$

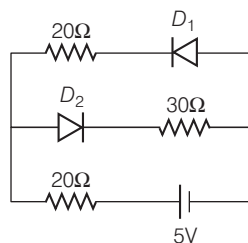
30. A cyclist speeding at 6 m/s in a circle of 18 m radius makes an angle θ with the vertical. The minimum possible value of coefficient of friction between the tyres and the ground is
a. 12.041
b. 0.2041
c. 11.32
d. 10.020

31. Three particles each of mass m are kept at the vertices of an equilateral triangle of side b . Moment of inertia of the system about an axis passing through the centroid and perpendicular to its plane is
a. $3mb^2$ b. mb^2
c. $\frac{mb^2}{3}$ d. $\frac{2}{3}mb^2$

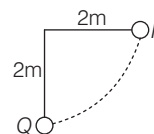
32. An electric dipole is placed at an angle of 30° in a non-uniform electric field. The dipole will experience
a. torque only
b. translational force only in the direction of the field
c. translational force only in a direction normal to the direction of the field
d. torque as well as a translational force

33. Two coils A and B have a mutual inductance 0.001 H. The current changes in the first coil according to the equation $i = i_0 \sin \omega t$, where $i_0 = 10\text{A}$ and $\omega = 10\pi \text{ rads}^{-1}$. The maximum value of emf in the second coil is
a. $0.01\pi \text{ V}$ b. $1\pi \text{ V}$
c. $0.1\pi \text{ V}$ d. $0.05\pi \text{ V}$

34. The current in the circuit will be



- a. 0.125 A b. 0.1 A
c. 0.5 A d. 0.25 A
35. The bob of a pendulum of length 2m lies at P , when it reaches Q , it loses 10% of its total energy due to air resistance.



The velocity of bob at Q is

- a. 6 m/s b. 1 m/s
c. 2 m/s d. 8 m/s

36. A particle executes SHM with a frequency f . The frequency with which its kinetic energy oscillates is
a. $\frac{f}{2}$ b. f
c. $2f$ d. $4f$

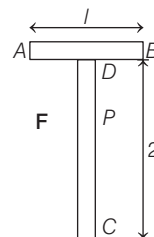
37. A radioactive sample at any instant has its disintegration rate 5000 disintegrations per min. After 5 min, the rate is 1250 disintegrations per min. Then, the disintegration constant (per min) is
a. $0.4 \log_e 2$
b. $0.2 \log_e 2$
c. $0.1 \log_e 2$
d. $0.8 \log_e 2$

38. The separation between two parallel plates of capacitor is 1 mm. What is the electric potential generated between the plates of capacitor, when electric field of 2000 N/C is applied on it?
a. 2 V b. 2000 V
c. 0.2 V d. 200 V

39. Choose the correct statement.

- a. The speed of light in the meta-material is $v = c |n|$.
b. The speed of light in the meta-material is $v = \frac{c}{|n|}$.
c. The speed of light in the meta-material is $v = c$.
d. The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{\text{air}} |n|$.

40. A T-shaped object with dimensions shown in figure, is lying on a smooth floor. A force F is applied at the point P parallel to AB , such that the object has only the translational motion without rotation. Find the location of P with respect to C .



- a. $\frac{2}{3}l$ b. $\frac{3}{4}l$
c. $\frac{3}{2}l$ d. $\frac{4}{3}l$

PART II

Chemistry

41. The most volatile compound among the given option is
 a. *o*-nitrophenol b. *p*-nitrophenol
 c. *m*-nitrophenol d. can't say
42. What is the magnetic moment of Ti^{2+} ?
 (Given : Atomic number = 22)
 a. $\sqrt{8}$ b. $\sqrt{10}$ c. $\sqrt{6}$ d. $\sqrt{9}$
43. Why only Xe can form compounds with fluorine among noble gases?
 a. Large size
 b. Low electronegativity
 c. High ionisation energy
 d. Low electron gain enthalpy
44. Which of the following inert gas is used as cryogenic agent?
 a. He b. Ne c. Ar d. Kr
45. For the following reaction,

$$2A + 3B \longrightarrow 3C + 4D$$
 expression for rate of reaction is
 a. $\frac{d[A]}{dt}$ b. $\frac{1}{2} \frac{d[B]}{dt}$ c. $\frac{1}{3} \frac{d[C]}{dt}$ d. $\frac{1}{4} \frac{d[D]}{dt}$
46. Cyclohexanol on reacting with H_2SO_4 and then heating gives
 a. cyclohexene b. cyclohexanone
 c. cyclohexanol d. None of these
47. The major product obtained on reaction of 3-methylbutene with HCl is
 a. 2-chloro-2-methylbutane
 b. 3-chloro-2-methylbutane
 c. 1-chloro-2-methylbutane
 d. 3-chloro - 3-methylbutane
48. If Rydberg constant is same for all elements, the angular momentum and energy of Li^{2+} of which orbital is equal to angular momentum and energy of 1s-orbital of hydrogen atom?
 a. 3s b. 4s c. 2p d. 3d
49. Sodium iodide reacts with ammonia to gives
 a. $[\text{Na}(\text{NH}_3)_4]\text{I}$ b. $[\text{Na}(\text{NH}_3)_4]\text{I}_3$
 c. $[\text{Na}(\text{NH}_3)_3]\text{I}$ d. $[\text{Na}(\text{NH}_3)_3]\text{I}$
50. In photography, which compound is used as a fixing agent
 a. sodium thiosulphate
 b. ammonium thiosulphate
 c. sodium chloride
 d. Both (a) and (b)
51. The compound formed on reaction of epoxy ethane with NH_3 and H_2O is
 a. mono-ethanol amines b. di-ethanol amines
 c. tri-ethanol amines d. All of these
52. The process of removal of excess electrolyte from colloidal solution is
 a. coagulation b. dialysis
 c. ultra-filtration d. peptisation
53. Maltase converts maltose into
 a. glucose b. sucrose
 c. fructose d. starch
54. Starch is a polymer of
 a. glucose b. fructose
 c. Both a. and (b) d. None of these
55. Methyl alcohol can be distinguished from ethyl alcohol using
 a. Fehling solution
 b. Schiff's reagent
 c. Sodium hydroxide and iodine
 d. Phthalein fusion test
56. Sodium stearate ($\text{C}_{17}\text{H}_{35}\text{COO}^-\text{Na}^+$) is
 a. anionic soap b. cationic detergent
 c. anionic detergent d. Non-ionic soap
57. Which is not an anti-fluorite structure
 a. Rb_2S b. BaF_2 c. K_2O d. Li_2O
58. The structure of ClF_3 is
 a. T-shape b. bent shape
 c. linear d. trigonal planar
59. Which of the following is diamagnetic in nature?
 a. O_2 b. O_2^{2-} c. N_2^{2+} d. B_2
60. The correct order of bond order in SO_2 , SO_3 , SO_4^{2-} , SO_3^{2-} is
 a. $\text{SO}_3^{2-} > \text{SO}_4^{2-} > \text{SO}_3 = \text{SO}_2$
 b. $\text{SO}_4^{2-} > \text{SO}_3 > \text{SO}_2 > \text{SO}_3^{2-}$
 c. $\text{SO}_2 = \text{SO}_3 > \text{SO}_4^{2-} > \text{SO}_3^{2-}$
 d. Can't say
61. Ionisation energy for H^+ ion is proportional to r^n , then value of n is
 a. 1 b. -1 c. 2 d. -2
62. Role of BHA in food industries
 a. prevent oxidation b. prevent reduction
 c. add sweeteners d. None of these

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63. Permanganate ion (MnO_4^-) is dark purple coloured though Mn is in + 7 oxidation state with d^0 configuration. This is due to
- $d-d$ transition
 - charge transfer from metal to ligand
 - charge transfer from ligand to metal
 - All of the above
64. Rutherford model could not explain
- electronic structure of an atom
 - stability of an atom
 - Both (a) and (b)
 - None of the above
65. Which is true in case of $[\text{Ni}(\text{CO})_4]$?
- Hybridisation of Ni is sp^3
 - Hybridisation of Ni is dsp^2
 - Paramagnetic
 - Square planar
66. SI unit of Boltzmann's constant is
- JK^{-1}
 - eVK^{-1}
 - erg K^{-1}
 - JK
67. Acetone does not undergo which type of reaction?
- Substitution reaction
 - Polymerisation reaction
 - Condensation reaction
 - Addition reaction
68. Magnetic moment of in $[\text{Co}(\text{F}_6)]^{2-}$ of unpaired electron
- $\sqrt{35}$
 - $\sqrt{45}$
 - $\sqrt{40}$
 - $\sqrt{30}$
69. Some statements about heavy water are given below
- Heavy water is used as a moderator in nuclear reactors.
 - Heavy water is more associated than ordinary water.
 - Heavy water is more effective solvent than ordinary water.
- Which of the above statements are correct?
- 1 and 2
 - 1, 2 and 3
 - 2 and 3
 - 1 and 3
70. $\text{CH}_3-\text{CHCl}-\text{CH}_2-\text{CH}_3$ has a chiral centre, which of the following represents its R configurations?
- (a) $\begin{array}{c} \text{C}_2\text{H}_5 \\ | \\ \text{H}_3\text{C}-\text{C}-\text{Cl} \\ | \\ \text{H} \end{array}$

(b) $\begin{array}{c} \text{C}_2\text{H}_5 \\ | \\ \text{H}-\text{C}-\text{CH}_3 \\ | \\ \text{Cl} \end{array}$
- (c) $\begin{array}{c} \text{C}_2\text{H}_5 \\ | \\ \text{Cl}-\text{C}-\text{CH}_3 \\ | \\ \text{H} \end{array}$

(d) $\begin{array}{c} \text{CH}_3 \\ | \\ \text{H}-\text{C}-\text{Cl} \\ | \\ \text{C}_2\text{H}_5 \end{array}$
71. In Victor Meyer's method 0.2 g of an organic substance displaced 56 mL of air at STP the molecular weight of the compound
- 56
 - 112
 - 80
 - 28
72. For the complete combustion of ethanol, $\text{C}_2\text{H}_5\text{OH}(l) + 3\text{O}_2(g) \longrightarrow 2\text{CO}_2(g) + 3\text{H}_2\text{O}(l)$, the amount of heat produced as measured in bomb calorimeter is $1364.47 \text{ kJ mol}^{-1}$ at 25°C . Assuming ideality the enthalpy of combustion, ΔH_c , for the reaction will be ($R = 8.314 \text{ JK}^{-1} \text{ mol}^{-1}$)
- $-1366.95 \text{ kJ mol}^{-1}$
 - $-136195 \text{ kJ mol}^{-1}$
 - $-1460.50 \text{ kJ mol}^{-1}$
 - $-1350.50 \text{ kJ mol}^{-1}$
73. The incorrect expression among the following is
- $\frac{\Delta G_{\text{system}}}{\Delta S_{\text{total}}} = -T$
 - In isothermal process, $W_{\text{reversible}} = -nRT \ln \frac{V_f}{V_i}$
 - $\ln K = \frac{\Delta H^\circ - T\Delta S^\circ}{RT}$
 - $K = e^{-\Delta G^\circ/RT}$
74. The solubility of $\text{Pb}(\text{OH})_2$ in water is $6.7 \times 10^{-6} \text{ M}$. Its solubility in a buffer solution of $\text{pH} = 8$ would be
- 1.2×10^{-2}
 - 1.6×10^{-3}
 - 1.6×10^{-2}
 - 1.2×10^{-3}
75. The density (in g mL^{-1}) of a 3.60 M sulphuric acid solution that is 29% H_2SO_4 (molar mass = 98 g mol^{-1}) by mass will be
- 1.64
 - 1.88
 - 1.22
 - 1.45
76. The relative lowering of vapour pressure of a dilute aqueous solution containing non-volatile solute is 0.0125. The molality of the solution is about
- 0.70
 - 0.50
 - 0.90
 - 0.80
77. Equal masses of methane and oxygen are mixed in an empty container at 25°C . The fraction of the total pressure exerted by oxygen is
- $\frac{2}{3}$
 - $\frac{1}{3} \times \frac{273}{298}$
 - $\frac{1}{3}$
 - $\frac{1}{2}$
78. The molar conductivities of KCl , NaCl and KNO_3 are 152, 128 and $111 \text{ S cm}^2 \text{ mol}^{-1}$ respectively. What is the molar conductivity of NaNO_3 ?
- $101 \text{ S cm}^2 \text{ mol}^{-1}$
 - $87 \text{ S cm}^2 \text{ mol}^{-1}$
 - $-101 \text{ S cm}^2 \text{ mol}^{-1}$
 - $-391 \text{ S cm}^2 \text{ mol}^{-1}$
79. The approximate time duration in hours to electroplate 30 g of calcium from molten calcium chloride using a current of 5 A is (Atomic mass of $\text{Ca} = 40$)
- 80
 - 10
 - 16
 - 8
80. Given, the reduction potential of Na^+ , Mg^{2+} , Al^{3+} and Ag^+ as $E^\circ_{\text{Na}^+/\text{Na}} = -2.17 \text{ V}$; $E^\circ_{\text{Mg}^{2+}/\text{Mg}} = -2.37 \text{ V}$; $E^\circ_{\text{Ag}^+/\text{Ag}} = -0.08 \text{ V}$. The least stable oxide is
- Ag_2O
 - Al_2O_3
 - MgO
 - Na_2O

PART III

a. English Proficiency

Directions (Q. Nos. 81-83) *Choose the word which best expresses the meaning of the underlined word in the sentence.*

81. Decay is an immutable factor of human life.
a. important b. unique
c. unchangeable d. awful
82. It was an ignominious defect for the team.
a. shameful b. admirable
c. unaccountable d. worthy
83. His conjecture was the better than mine.
a. guess b. fact c. surprise d. doubt

Directions (Q. Nos. 84-86) *Fill in the blank.*

84. Freedom and equality are the rights of every human.
a. inalienable b. inscrutable
c. incalculable d. institutional
85. Pradeep's face spoke of the happiness he was feeling.
a. elegantly b. tons
c. volumes d. much
86. His speech was disappointing : it all the major issues.
a. projected b. revealed
c. skirted d. analysed

Directions (Q. Nos. 87-89) *Choose the word which is closest to the opposite in meaning of the given italicised word.*

87. Hydra is biologically believed to be *immortal*.
a. undying b. perishable
c. ancient d. eternal
88. The Gupta rulers *patronised* all cultural activities and thus Gupta period was called the golden era in Indian History.
a. criticised b. rejected
c. opposed d. spurned
89. This is a *barbarous* act.
a. bad b. good
c. civilised d. exemplary

Directions (Q. Nos. 90-92) *In each of the following questions, out of the four alternatives, choose the one which can be substituted for the given words/sentence.*

90. A person who does not believe in any religion
a. Philatelist b. Rationalist
c. Atheist d. Pagan

91. A person who believes that pleasure is the chief good
a. Stoic b. Hedonist c. Epicure d. Sensual

92. One who loves mankind
a. Anthropologist b. Philanthropist
c. Seismologist d. Optometrist

Directions (Q. Nos. 93-95) *Choose the order of the sentences marked A, B, C, D and E to form a logical paragraph.*

93. A. Tasty and healthy food can help you bring out their best.
B. One minute they are toddlers and next you see them in their next adventure.
C. Your young ones seem to be growing so fast.
D. Being their loving custodians, you always want to see them doing well.
E. Their eyes sparkle with curiosity and endless questions on their tongues.

Codes

- a. DBCEA b. CADEB c. CBEDA d. ECABD

94. A. It is hoping that overseas friends will bring in big money and lift the morale of the people.
B. But a lot needs to be done to kick start industrial revival.
C. People had big hopes from the new government.
D. So far government has only given an incremental push to existing policies and programmes.
E. Government is to go for big time reforms, which it promised.

Codes

- a. BCDAE b. EADCB c. DABEC d. CDEAB

95. A. However, women hiring is catching up at a slow and steady rate in the recent times.
B. Gender ratio has been inclined more towards male employees.
C. As a result, recent reports have highlighted the rise in demand for women employees.
D. Women constitute a little over half of world's total population.
E. But, their contribution to measured economic activity is far below the potential.

Codes

- a. DEBAC b. CDAEB c. BCDEA d. AEDBC

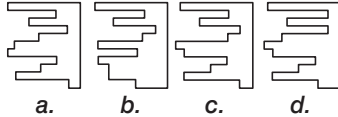
b. Logical Reasoning

96. Choose the correct answer figure which will make a complete square on joining with the problem figure.

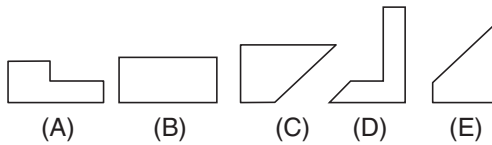
Problem figure



Answer figures



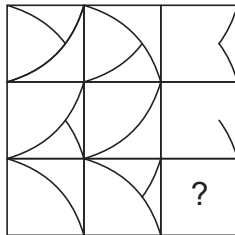
97. In the following question, five figures are given. Out of them, find the three figures that can be joined to form a square.



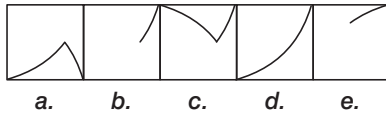
- a. ABD b. BCD
c. ACE d. CDE

98. Choose the answer figure which completes the problem figure matrix.

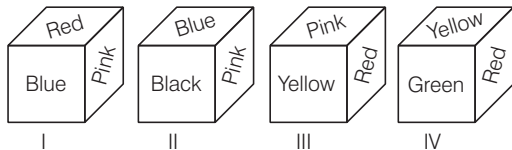
Problem Figure



Answer Figures



99. From the given four positions of a single dice, find the colour at the face opposite to the face having red colour.



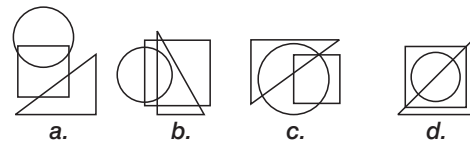
- a. Yellow b. Pink
c. Green d. Black

100. In the following questions, one or more dots are placed in the figure marked as (A). The figure is followed by four alternatives marked as (a), (b), (c) and (d). One out of these four options contains region(s) common to the circle, square, triangle, similar to that marked by the dot in figure (A).

Problem Figure



Answer Figures



101. Complete the series by replacing '?' mark.

G4T, J9R, M20P, P43N, S90L, ?

- a. S90L b. V185J
c. M20P d. P43N

102. Neeraj starts walking towards South. After walking 15 m, he turns towards North. After walking 20 m, he turns towards East and walks 10 m. He then turns towards South and walks 5 m. How far is he from his original position and in which direction?

- a. 10 m, East
b. 10 m, South-East
c. 10 m, West
d. 10 m, North-East

103. The average age of 8 men is increased by 2 yrs. when one of them whose age is 20 yr is replaced by a new man. What is the age of the new man?

- a. 28 yr b. 36 yr c. 34 yr d. 35 yr

104. Shikha is mother-in-law of Ekta who is sister-in-law of Ankit. Pankaj is father of Sanjay, the only brother of Ankit. How is Shikha related to Ankit?

- a. Mother-in-law b. Aunt
c. Wife d. Mother

105. In a row of forty children, P is thirteenth from the left end and Q is ninth from the right end. How many children are there between P and R, if R is fourth to the left of Q?

- a. 12 b. 13
c. 14 d. 15

PART IV

Mathematics

- 106.** If $\operatorname{Re}(z + 2) = |z - 2|$, then the locus of z is
 a. parabola
 b. circle
 c. ellipse
 d. hyperbola
- 107.** If $a \in R, b \in R$, then the equation $x^2 - abx - a^2 = 0$ has
 a. one positive root and one negative root
 b. Both positive roots
 c. Both negative roots
 d. Non-real roots
- 108.** If $a + 2b + 3c = 12, (a, b, c \in R^+)$, then the maximum value of ab^2c^3 is
 a. 2^3
 b. 2^4
 c. 2^6
 d. 2^5

- 109.** Sum of n terms of the infinite series

$1.3^2 + 2.5^2 + 3.7^2 + \dots \infty$ is

- a. $\frac{n}{6}(n+1)(6n^2+14n+7)$
b. $\frac{n}{6}(n+1)(6n^2+14n+5)$
c. $\frac{n}{6}(n+1)(2n+1)(3n+1)$
d. $4n^3+4n^2+n$

- 110.** If $\log_7 5 = a$, $\log_5 3 = b$ and $\log_3 2 = c$, then the logarithm of the number 70 to the base 225 is

- a. $\frac{1-a+abc}{2a(1+b)}$ b. $\frac{1-a-abc}{2a(1+b)}$
- c. $\frac{1+a-abc}{2a(1+b)}$ d. $\frac{1+a+abc}{2a(1+b)}$

- 111.** The maximum number of points of intersection of 10 circles is

- a. 80 b. 90 c. 85 d. 95

- 112.** $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + 4\frac{C_4}{C_3} + \dots 20\frac{C_{20}}{C_{19}} =$

- a. 120 b. 260 c. 210 d. 180

- 113.** If $p \neq q \neq r$ and $\begin{vmatrix} 0 & x-p & x-q \\ x+p & 0 & x-r \\ x+q & x-r & 0 \end{vmatrix} = 0$, then the

value of x which satisfy the equation is

- a. $x = p$ b. $x = q$ c. $x = r$ d. $x = 0$

- 114.** Matrix $A = \begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$, if $xyz = 60$ and

$8x + 4y + 3z = 20$, then $A(\text{adj } A)$ is equal to

a. $\begin{bmatrix} 64 & 0 & 0 \\ 0 & 64 & 0 \\ 0 & 0 & 64 \end{bmatrix}$

c. $\begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$

$$b. \begin{bmatrix} 88 & 0 & 0 \\ 0 & 88 & 0 \\ 0 & 0 & 88 \end{bmatrix}$$

$$d. \begin{bmatrix} 34 & 0 & 0 \\ 0 & 34 & 0 \\ 0 & 0 & 34 \end{bmatrix}$$

- 115.** If $f(x) = 4x - x^2$, $x \in R$, and $f(a + 1) - f(a - 1) = 0$, then a is equal to

- a. 0 b. 2
c. 1 d. 3

- 116.** Which of the following is not an equivalence relation in $\mathbb{Z}$?

- a. $aRb \Leftrightarrow a + b$ is an even integer
 b. $aRb \Leftrightarrow a - b$ is an even integer
 c. $aRb \Leftrightarrow a < b$
 d. $aRb \Leftrightarrow a = b$

- 117.** Which of the following is always true?

- a. $(\sim p \vee \sim q) \equiv (p \wedge q)$
 b. $(p \rightarrow q) \equiv (\sim q \rightarrow \sim p)$
 c. $\sim(p \rightarrow \sim q) \equiv (p \wedge \sim q)$
 d. $\sim(p \leftrightarrow q) \equiv (p \rightarrow q) \rightarrow (q \rightarrow p)$

- 118.** The solution of the inequation $4^{-x+0.5} - 7 \cdot 2^{-x} < 4$, $x \in R$ is

- a. $(-2, \infty)$
b. $(2, \infty)$
c. $\left(2, \frac{7}{2}\right)$
d. None of these

- 119.** If $\cos^3 x \cdot \sin 2x = \sum_{m=1}^n a_m \sin mx$ is identity in x , then

- a. $a_3 = \frac{3}{8}, a_2 = 0$ b. $n = 6, a_1 = \frac{1}{2}$
c. $n = 5, a_1 = \frac{3}{4}$ d. $\sum a_m = \frac{1}{4}$

- 120.** Total number of solutions of

$$|\cot x| = \cot x + \frac{1}{\sin x}, x \in [0, 3\pi] \text{ is equal to}$$

- a. 1 b. 2
c. 3 d. 0

- 121.** The minimum value of $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$ is equal to

- a. $\frac{\pi^3}{32}$ b. $\frac{5\pi^3}{32}$ c. $\frac{9\pi^3}{32}$ d. $\frac{11\pi^3}{32}$

- 122.** The origin is shifted to $(1, 2)$. The equation $y^2 - 8x - 4y + 12 = 0$ changes to $y^2 = 4ax$, then a is equal to
 a. 1 b. 2 c. -2 d. -1
- 123.** The equations of the bisector of the angles between the straight lines $3x + 4y + 7 = 0$ and $12x + 5y - 8 = 0$ are
 a. $7x + 9y + 17 = 0, 99x + 77y + 51 = 0$
 b. $7x - 9y - 17 = 0, 99x + 77y - 51 = 0$
 c. $7x - 9y + 17 = 0, 99x + 77y + 51 = 0$
 d. None of the above
- 124.** Equation of circle which passes through the points $(1, -2)$ and $(3, -4)$ and touch the X -axis is
 a. $x^2 + y^2 + 6x + 2y + 9 = 0$
 b. $x^2 + y^2 + 10x + 20y + 25 = 0$
 c. $x^2 + y^2 + 6x + 4y + 9 = 0$
 d. None of the above
- 125.** If $x = 9$ is the chord of contact of the hyperbola $x^2 - y^2 = 9$, then the equation of the corresponding pair of tangent is
 a. $9x^2 - 8y^2 + 18x - 9 = 0$ b. $9x^2 - 8y^2 - 18x + 9 = 0$
 c. $9x^2 - 8y^2 - 18x - 9 = 0$ d. $9x^2 - 8y^2 + 18x + 9 = 0$
- 126.** The points with position vectors $10\hat{i} + 3\hat{j}$, $12\hat{i} - 15\hat{j}$ and $a\hat{i} + 11\hat{j}$ are collinear, if a is
 a. -8 b. 4 c. 2 d. $\frac{82}{9}$
- 127.** Let a, b, c be vectors of lengths 3, 4, 5 respectively and a be perpendicular to $(b + c)$, b to $(c + a)$ and c to $(a + b)$, then the value of $(a + b + c)$ is
 a. $2\sqrt{5}$ b. $2\sqrt{2}$ c. $10\sqrt{5}$ d. $5\sqrt{2}$
- 128.** For non-zero vectors a, b, c ; $|(a \times b) \cdot c| = |a||b||c|$ holds if and only if
 a. $a \cdot b = 0, b \cdot c = 0$ b. $b \cdot c = 0, c \cdot a = 0$
 c. $c \cdot a = 0, a \cdot b = 0$ d. $a \cdot b = b \cdot c = c \cdot a = 0$
- 129.** Angle between the diagonals of a cube is
 a. $\pi/3$ b. $\pi/2$ c. $\cos^{-1}(1/3)$ d. $\cos^{-1}(1/\sqrt{3})$
- 130.** Consider the two lines
 $L_1: \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$ and $L_2: \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$
 The unit vector perpendicular to both the lines L_1 and L_2 is
 a. $\frac{-\hat{i} + 7\hat{j} + 7\hat{k}}{\sqrt{99}}$ b. $\frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$
 c. $\frac{-\hat{i} + 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$ d. $\frac{7\hat{i} - 7\hat{j} + \hat{k}}{\sqrt{99}}$
- 131.** The distance between the line $r = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ and the plane $a \cdot (\hat{i} + 5\hat{j} + \hat{k}) = 5$ is
 a. $\frac{10}{9}$ b. $\frac{10}{3\sqrt{3}}$
 c. $\frac{10}{3}$ d. None of these
- 132.** Two cards are drawn from a pack of 52 cards. What is the probability that either both are red or both are kings?
 a. $\frac{7}{13}$ b. $\frac{63}{221}$ c. $\frac{55}{221}$ d. $\frac{3}{26}$
- 133.** If A and B are two independent events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{5}$, then which of the following is correct?
 a. $P\left(\frac{A}{B}\right) = \frac{1}{2}$ b. $P\left(\frac{A}{A \cup B}\right) = \frac{5}{6}$
 c. $P\left(\frac{A \cap B}{A' \cup B'}\right) = 0$ d. All of these
- 134.** Box I contains 5 red and 2 blue balls, while box II contains 2 red and 6 blue balls. A fair coin is tossed. If it turns up head, a ball is drawn from box I, else a ball is drawn from box II. The probability ball drawn is from box I, if it is blue, is
 a. $\frac{27}{56}$ b. $\frac{8}{29}$ c. $\frac{21}{29}$ d. $\frac{29}{56}$
- 135.** For a random variable X , $E(X) = 3$ and $E(X^2) = 11$. The variable of X is
 a. 8 b. 5 c. 2 d. 1
- 136.** The sum of 10 items is 12 and the sum of their squares is 18, then the standard deviation will be
 a. $-3/5$ b. $6/5$ c. $4/5$ d. $3/5$
- 137.** The height of the chimney when it is found that on walking towards it 50 m in the horizontal line through its base, the angle of elevation of its top changes from 30° to 60° is
 a. 25 m b. $25\sqrt{2}$ m
 c. $25\sqrt{3}$ m d. None of these
- 138.** The value of $\lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos x^2}}{1 - \cos x}$ is
 a. $\frac{1}{2}$ b. 2
 c. $\sqrt{2}$ d. None of these
- 139.** If $f(x) = \begin{cases} ax^2 + 1, & x \leq 1 \\ x^2 + ax + b, & x > 1 \end{cases}$ is differentiable at $x = 1$, then
 a. $a = 1, b = 0$ b. $a = 1, b = 0$
 c. $a = 2, b = 0$ d. $a = 2, b = 1$

140. The slope of the tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point $t = 2$ is

a. $\frac{7}{6}$ b. $\frac{5}{6}$ c. $\frac{6}{7}$ d. 1

141. $\int \frac{1}{1 - 2 \sin x} dx$ is equal to

a. $\frac{1}{2\sqrt{3}} \log \left| \frac{\tan \frac{x}{2} - 2 - \sqrt{3}}{\tan \frac{x}{2} - 2 + \sqrt{3}} \right| + c$

b. $\frac{\sqrt{3}}{2} \log \left| \frac{\tan \frac{x}{2} - 2 - \sqrt{3}}{\tan \frac{x}{2} - 2 + \sqrt{3}} \right| + c$

c. $\frac{1}{\sqrt{3}} \log \left| \frac{\tan \frac{x}{2} - 2 - \sqrt{3}}{\tan \frac{x}{2} - 2 + \sqrt{3}} \right| + c$

d. None of the above

142. $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$ is equal to

a. $\frac{\pi}{8} \log 2$

b. $\frac{\pi}{8} \log \frac{1}{2}$

c. $\frac{\pi}{4} \log 2$

d. None of these

143. The area of one curvilinear triangle formed by curves $y = \sin x$, $y = \cos x$ and X-axis, is

a. 2 sq units

b. $(2 + \sqrt{2})$ sq units

c. $(2 - \sqrt{2})$ sq units

d. None of these

144. Solution of $\left(\frac{x+y-1}{x+y-2} \right) \frac{dy}{dx} = \left(\frac{x+y+1}{x+y+2} \right)$, given that

$y = 1$ when $x = 1$ is

a. $\ln \left| \frac{(x-y)^2 - 2}{2} \right| = 2(x+y)$

b. $\ln \left| \frac{(x+y)^2 - 2}{2} \right| = 2(x-y)$

c. $\ln \left| \frac{(x-y)^2 + 2}{2} \right| = 2(x+y)$

d. $\ln \left| \frac{(x+y)^2 + 2}{2} \right| = 2(x-y)$

145. If $y = \sin \left(2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right)$, then $\frac{dy}{dx}$ is

a. 1

b. -1

c. $\frac{x}{\sqrt{x^2 - 1}}$

d. $\frac{-x}{\sqrt{1-x^2}}$

146. The maximum value of the function $y = x(x-1)^2$, is

a. 0

b. $\frac{4}{27}$

c. -4

d. None of these

147. The solution of $x^3 \frac{dy}{dx} + 4x^2 \tan y = e^x \sec y$ satisfying

$y(1) = 0$, is

a. $\tan y = (x-2)e^x \log x$

b. $\sin y = e^x (x-1)x^{-4}$

c. $\tan y = (x-1)e^x x^{-3}$

d. $\sin y = e^x (x-1)x^{-3}$

148. The runs of two players for 10 innings each are as follows

A	58	59	60	54	65	66	52	75	69	52
B	94	26	92	65	96	78	14	34	98	13

The more consistent player is

a. player A

b. player B

c. both player A and B

d. None of these

149. The linear programming problem minimise $z = 3x + 2y$ subject to constraints $x + y \geq 8$, $3x + 5y \leq 15$, $x \geq 0$ and $y \geq 0$, has

a. one solution

b. no feasible solution

c. two solutions

d. infinitely many solutions

150. Find the area enclosed by the loop in the curve

$4y^2 = 4x^2 - x^3$.

a. $\frac{128}{15}$

b. $\frac{15}{128}$

c. $\frac{130}{17}$

d. $\frac{17}{130}$

Answers

Physics

1. (a)	2. (a)	3. (c)	4. (d)	5. (b)	6. (c)	7. (a)	8. (a)	9. (c)	10. (b)
11. (a)	12. (b)	13. (b)	14. (c)	15. (a)	16. (d)	17. (c)	18. (b)	19. (a)	20. (c)
21. (d)	22. (a)	23. (b)	24. (d)	25. (b)	26. (a)	27. (b)	28. (c)	29. (b)	30. (b)
31. (b)	32. (d)	33. (c)	34. (b)	35. (a)	36. (c)	37. (a)	38. (a)	39. (b)	40. (d)

Chemistry

41. (a)	42. (a)	43. (a)	44. (a)	45. (c)	46. (a)	47. (a)	48. (a)	49. (a)	50. (d)
51. (d)	52. (b)	53. (a)	54. (a)	55. (c)	56. (a)	57. (b)	58. (a)	59. (b)	60. (c)
61. (b)	62. (a)	63. (c)	64. (c)	65. (a)	66. (a)	67. (a)	68. (a)	69. (a)	70. (c)
71. (c)	72. (a)	73. (c)	74. (d)	75. (c)	76. (a)	77. (c)	78. (b)	79. (d)	80. (a)

English Proficiency

81. (c)	82. (a)	83. (a)	84. (d)	85. (c)	86. (c)	87. (b)	88. (c)	89. (c)	90. (c)
91. (c)	92. (b)	93. (c)	94. (d)	95. (a)					

Logical Reasoning

96. (c)	97. (c)	98. (b)	99. (d)	100. (c)	101. (b)	102. (a)	103. (b)	104. (d)	105. (c)
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Mathematics

106. (a)	107. (a)	108. (c)	109. (a)	110. (d)	111. (b)	112. (c)	113. (c)	114. (c)	115. (b)
116. (c)	117. (b)	118. (a)	119. (a)	120. (b)	121. (a)	122. (b)	123. (d)	124. (b)	125. (b)
126. (d)	127. (d)	128. (d)	129. (c)	130. (b)	131. (b)	132. (c)	133. (d)	134. (b)	135. (c)
136. (d)	137. (c)	138. (c)	139. (c)	140. (c)	141. (c)	142. (a)	143. (c)	144. (b)	145. (d)
146. (b)	147. (b)	148. (a)	149. (b)	150. (a)					

Solutions

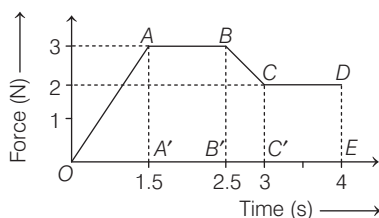
Physics

1. (a) Work done in the cyclic process

$$= \text{Area of the loop } ABCD$$

$$= (2p - p) \times (2V - V) = pV$$

2. (a) Impulse of a force = Area between the force-time graph



Hence, impulse of force = area of $\triangle OAA'$ + area of rectangle $A'BBA'$ + area of trapezium $BB'C'C$ + area of rectangle $CC'ED$

$$= \frac{1}{2} \times 1.5 \times 3 + 1 \times 3 + \frac{1}{2} (3 + 2) (3 - 2.5) + 2 \times 1$$

$$= 2.25 + 3 + 1.25 + 2$$

$$= 8.50 \text{ Ns}$$

3. (c) Bulk modulus = $\frac{\text{Hydraulic stress}}{\text{Volumetric strain}}$

$$\Rightarrow B = \frac{\text{Hydraulic stress}}{\frac{\Delta V}{V}}$$

$$\text{or } \frac{\Delta V}{V} = \text{Hydraulic stress} \times \frac{1}{B}$$

$$\therefore \text{For constant hydraulic stress, } \frac{\Delta V}{V} \propto \frac{1}{B}$$

4. (d) A shunt is a low resistance which is connected in parallel with a galvanometer (or ammeter) to protect it from large current and to increase the range of ammeter.
5. (b) There is a phase difference of 180° between the signal voltage and output voltage in a common emitter amplifier. It is also known as phase reversal.
6. (c) $(n + 1)$ divisions of vernier scale = n division of main scale

$$\therefore 1 \text{ VSD} = \frac{n}{n + 1} \text{ MSD}$$

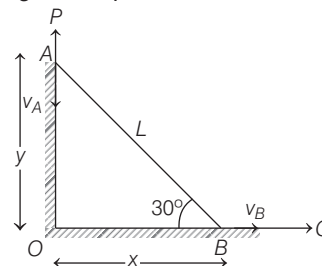
$$\text{Least count} = 1 \text{ MSD} - 1 \text{ VSD}$$

$$= 1 \text{ MSD} - \frac{n}{n + 1} \text{ MSD} = \frac{1}{n + 1} \text{ MSD}$$

$$= \frac{1}{n + 1} \times a \text{ units} = \frac{a}{n + 1} \text{ units}$$

[$\because$ given, $1 \text{ MSD} = a \text{ units}$]

7. (a) According to the question,



Let $OB = x$, $OA = y$ and $x = L \cos \theta$

So, by Pythagoras theorem,

$$\Rightarrow x^2 + y^2 = L^2$$

Differentiating w.r.t. 't', we get

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$2x v_B + 2y v_A = 0 \quad \left[\because \frac{dx}{dt} = v_B \text{ and } \frac{dy}{dt} = v_A \right]$$

$$\Rightarrow v_A = -\frac{x}{y} v_B$$

$$\Rightarrow |v_A| = \frac{x}{y} v_B = v_B \cot \theta \quad \left[\because \frac{x}{y} = \cot \theta \right]$$

$$= \sqrt{3} \times \cot 30^\circ$$

$$= \sqrt{3} \times \frac{1}{\sqrt{3}} = 1 \text{ m/s}$$

8. (a) Let k be the force constant of spring.

$$\text{Time period, } T = 2\pi \sqrt{\frac{M}{k}} \quad \dots(i)$$

When block of mass m is placed in the tray, the time period of oscillations becomes

$$T' = 2\pi \sqrt{\frac{M + m}{k}} \quad \dots(ii)$$

On dividing Eq. (ii) by Eq. (i), we get

$$\Rightarrow \frac{T'}{T} = \frac{\sqrt{M + m}}{\sqrt{M}}$$

$$\Rightarrow \frac{3}{1.5} = \sqrt{\frac{12 + m}{12}} \quad \left[\text{given, } M = 12 \text{ kg, } T = 1.5 \text{ s, } T' = 3 \text{ s} \right]$$

$$\Rightarrow \sqrt{\frac{12 + m}{12}} = 2$$

$$\Rightarrow 12 + m = 48$$

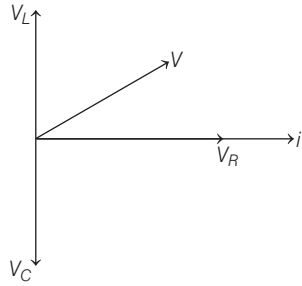
$$\Rightarrow m = 36 \text{ kg}$$

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9. (c) As shown in phasor diagram, voltage leads the current.

As in purely inductive circuit, current lags behind the voltage by an angle of 90° , here angle is not 90° but it is lesser than that.

So, this type of case arise in $R-L-C$ circuit when $R-L-C$ load with inductive reactance is more than the capacitive reactance as shown below.



10. (b) Given,

$$Y = \sin \left[\pi \left(\frac{t}{5} - \frac{x}{9} \right) + \frac{\pi}{6} \right] \quad \dots(i)$$

The general equation of progressive wave is

$$Y = A \sin \left[2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right) + \phi \right] \quad \dots(ii)$$

On comparing Eq. (i) with Eq. (ii), we get

$$A = 1 \text{ cm}, \quad f = 0.1 \text{ Hz}, \quad v = 1.8 \text{ cm/s}$$

$$\text{and} \quad \frac{2\pi}{\lambda} = \frac{\pi}{9}$$

$$\Rightarrow \quad \lambda = 18 \text{ cm}$$

11. (a) In fibre optic communication, signals are transmitted through an optical fibre. The property of light used in transmission through optical fibre cables is total internal reflection.

12. (b) Distance moved by man in 5 min with velocity of 45 m/s is

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$= 45 \text{ m/s} \times (5 \times 60) \text{ s} = 13500 \text{ m}$$

When he move back, it covers the same distance to come back to its original position.

Now, time taken = 15 min

$$= 15 \times 60 \text{ s} = 90 \text{ s}$$

Distance travelled = 13500 m

$$\therefore \text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{13500}{90} = 150 \text{ m/s}$$

13. (b) Potential at point P due to the solid sphere,

$$\begin{aligned} V_1 &= -\frac{GM}{2R^3} \left[3R^2 - \left(\frac{R}{2} \right)^2 \right] \\ &= -\frac{11GM}{8R} \end{aligned}$$

Potential at point P due to the cavity part,

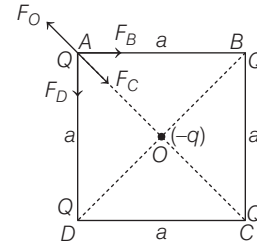
$$V_2 = -\frac{3}{2} \frac{G \left(\frac{M}{8} \right)}{\frac{R}{2}} = -\frac{3GM}{8R}$$

Potential due to the remaining part at point P ,

$$V = V_1 - V_2 = -\frac{11GM}{8R} + \frac{3GM}{8R}$$

$$\Rightarrow \quad V = -\frac{GM}{R}$$

14. (c) Consider the equilibrium of charge Q at corner A



$$F_B = F_D = \frac{1}{4\pi\epsilon_0} \frac{Q^2}{a^2} = \frac{kQ^2}{a^2} \quad \left[\because \frac{1}{4\pi\epsilon_0} = k \right]$$

$$F_C = \frac{kQ^2}{(\sqrt{2}a)^2} \quad [\because AC = \sqrt{2}a]$$

$$= \frac{kQ^2}{2a^2}$$

$$F_O = \frac{k Q q}{\left(\frac{a}{\sqrt{2}} \right)^2} = \frac{k 2Qq}{a^2} \quad \left[\because OC = \frac{a}{\sqrt{2}} \right]$$

For equilibrium of charge Q at corner A , the net force on this charge along AC must be zero.

$$\begin{aligned} \therefore \quad F_O &= F_C + F_B \cos 45^\circ + F_D \cos 45^\circ \\ &= F_C + 2F_B \cos 45^\circ \quad [\because F_B = F_D] \\ &= F_C + 2F_B \times \frac{1}{\sqrt{2}} \quad \left[\because \cos 45^\circ = \frac{1}{\sqrt{2}} \right] \end{aligned}$$

$$\Rightarrow \quad \frac{k 2Qq}{a^2} = \frac{k Q^2}{2a^2} + \frac{2 \times k Q^2}{a^2} \times \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad 2Qq = \frac{Q^2}{2} + \frac{Q^2}{\sqrt{2}} \times 2$$

$$\Rightarrow \quad 2q = \frac{Q}{2} + Q\sqrt{2}$$

$$\Rightarrow \quad q = \frac{Q}{4} (1 + 2\sqrt{2})$$

15. (a) Work done by the particle in X -direction is given by $dW = F dx = (a + bx) dx$

Total work done in displacement from

$x = 0$ to $x = d$ will be

$$W = \int dW = \int_0^d (a + bx) dx$$

$$= \left[ax + \frac{bx^2}{2} \right]_0^d$$

$$\Rightarrow W = \left(a + \frac{bd}{2} \right) d$$

16. (d) Kinetic energy of a proton is given by

$$E = \frac{1}{2} m_p v_p^2$$

$$\Rightarrow E = \frac{p^2}{2m_p} \quad [\because p = mv]$$

$\therefore$ Wavelength of proton is, $\lambda_1 = \frac{h}{p}$

$$\Rightarrow \lambda_1 = \frac{h}{\sqrt{2m_p E}} \quad \dots(i)$$

Now, energy of photon is given by

$$E = h\nu_2 \Rightarrow E = \frac{hc}{\lambda_2}$$

$$\Rightarrow \lambda_2 = \frac{hc}{E} \quad \dots(ii)$$

where, λ_2 is wavelength of photon.

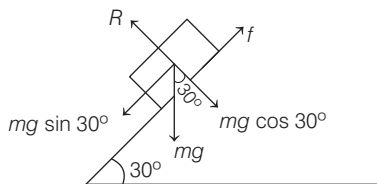
On dividing Eq. (ii) by Eq. (i), we get

$$\frac{\lambda_2}{\lambda_1} = \frac{hc}{E} \times \frac{\sqrt{2m_p E}}{h}$$

$$\Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{c}{\sqrt{E}} \sqrt{2m_p}$$

$$\Rightarrow \frac{\lambda_2}{\lambda_1} \propto E^{-1/2}$$

17. (c) The FBD of block of mass 10 kg is



For the equilibrium of block, the frictional force must be balanced the sine component of mg .

$$\therefore f = mg \sin 30^\circ$$

$$= 10 \times 9.8 \times \frac{1}{2} = 49 \text{ N}$$

18. (b) Conductivity (σ) of a metallic conductor is given by

$$\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$$

where, ρ = resistivity of conductor,

τ = relaxation time,

n = number of free electrons

and m = mass.

For metals, as the temperature increases, the relaxation time τ decreases because collisions become more frequent. Hence, the conductivity of a metallic conductor decreases with the increases of temperature.

19. (a) Given, radius of hydrogen atom, $r = 2.5 \times 10^{-13} \text{ m}$

$$\text{Volume of one atom, } V = \frac{4}{3} \pi r^3$$

$$\text{Number of atoms in 1 mole} = 6.023 \times 10^{23}$$

$$\begin{aligned} \text{Volume of 1 mole of H-atoms} &= N \times \frac{4}{3} \pi r^3 \\ &= 6.023 \times 10^{23} \times \frac{4}{3} \times 3.14 \times (2.5 \times 10^{-13})^3 \\ &= 3.94 \times 10^{-14} \text{ m}^3 \end{aligned}$$

20. (c) Given, $m = 1 \text{ kg}$, $\mathbf{r} = 3t \hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

$$\begin{aligned} \text{Velocity of body, } \mathbf{v} &= \frac{d\mathbf{r}}{dt} \\ &= \frac{d}{dt} (3t\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \Rightarrow \mathbf{v} = 3\hat{\mathbf{i}} \end{aligned}$$

Angular momentum, $\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$

$$= 1 \times [(3t\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \times 3\hat{\mathbf{i}}]$$

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3t & 4 & 0 \\ 3 & 0 & 0 \end{vmatrix}$$

$$= [3t \times 0 - 4 \times 3] \hat{\mathbf{k}}$$

$$\Rightarrow \mathbf{L} = -12\hat{\mathbf{k}} \text{ J-s}$$

21. (d) The value of g at equator is expressed as

$$g_e = g - R\omega^2 \cos \phi = g - R\omega^2 \quad [\because \cos \phi = 1]$$

$\Rightarrow$ Change in value of g is $g - g_e$

$$= R\omega^2 = R \left(\frac{2\pi}{T} \right)^2$$

$$= 6.4 \times 10^6 \times \left[\frac{2\pi}{24 \times 60 \times 60} \right]^2$$

$$= 3.37 \times 10^{-2} \text{ m/s}^2$$

$$\approx 3.4 \text{ cm/s}^2$$

22. (a) Apparent frequency of car A = Apparent frequency of car B

$$\Rightarrow \frac{v}{v - v_s} \times v = \frac{v}{v - v_s'} \times v'$$

$$\Rightarrow \frac{v}{v - 15} = \frac{504}{v - 30}$$

$$\Rightarrow v = \frac{340 - 15}{340 - 30} \times 504 \quad \left[\because \text{velocity of sound in air is } 340 \text{ m/s} \right]$$

$$\Rightarrow v = 529.2 \text{ Hz}$$

23. (b) Using Kirchhoff's loop law, in loop $BDCR_1 B$, we get

$$2 \times 2 + 3R_1 = 4 \quad \text{or} \quad R_1 = 0$$

$\therefore$ Potential drop at B = Potential drop at C

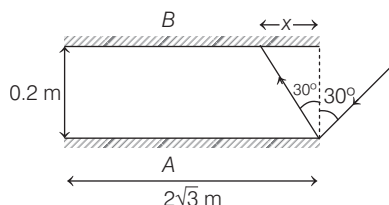
$$\Rightarrow V_B = V_C = 2 + V_A$$

$$= 2 + 0$$

[$\because V_A = 0$, given]

$$\Rightarrow V_B = 2 \text{ V}$$

24. (d) Let N be the maximum number of reflection that ray undergoes.



Suppose it covers x distance in one reflection, then

$$Nx = 2\sqrt{3}$$

$$\Rightarrow N = \frac{2\sqrt{3}}{x} = \frac{2\sqrt{3}}{0.2 \tan 30^\circ} \quad [\because x = 0.2 \tan 30^\circ]$$

$$N = \frac{6}{0.2} = 30$$

Number of reflections ray undergoes before it emerges excluding the first one are $N - 1$

$$= 30 - 1 = 29$$

25. (b) According to equation of motion, in vector form,

$$\mathbf{v} = \mathbf{u} - gt \cdot \hat{\mathbf{j}}$$

$$\Rightarrow \mathbf{v} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 10t \hat{\mathbf{j}}$$

Integrating w.r.t. ' t ' both sides, we get

$$\mathbf{r} = \int \mathbf{v} dt = \int (\hat{\mathbf{i}} + \hat{\mathbf{j}} - 10t \hat{\mathbf{j}}) dt$$

$$\text{or} \quad x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = t\hat{\mathbf{i}} + t\hat{\mathbf{j}} - 5t^2 \hat{\mathbf{j}}$$

$\therefore$ On comparing, we get

$$x = t$$

$$\text{and} \quad y = t - 5t^2$$

$$\Rightarrow y = x - 5x^2$$

This is the required equation of trajectory of particle.

26. (a) Volume of bigger drop = Volume of two smaller drops

$$\Rightarrow \frac{4}{3} \pi R^3 = 2 \times \frac{4}{3} \pi r^3$$

where, R is the radius of bigger drop.

$$\Rightarrow R^3 = 2r^3$$

$$\Rightarrow R = 2^{1/3} r$$

Initial surface energy of two small drops, $U_1 = 8\pi r^2 \sigma$... (i)

Final surface energy of big drop, $U_2 = 4\pi R^2 \sigma$

$$= 4\pi \times 2^{2/3} r^2 \sigma \quad \dots (ii)$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{U_2}{U_1} = \frac{4\pi \times 2^{2/3} r^2 \sigma}{8\pi r^2 \sigma} = \frac{2^{2/3}}{2} = \frac{2^{-1/3}}{1}$$

Ratio of surface energy of bigger drop to smaller one is $2^{-1/3} : 1$.

27. (b) Force on charge particle in magnetic field is balanced by

centripetal force, so $\frac{mv^2}{r} = qvB \sin 90^\circ$

[$\theta = 90^\circ$, therefore magnetic field is perpendicular]

$$\frac{mv^2}{r} = qvB \quad [\sin 90^\circ = 1]$$

$$\Rightarrow r = \frac{mv}{qB}$$

For both particles, q and B are same.

Therefore, $r \propto mv$

$$\text{or} \quad \frac{r_A}{r_B} = \frac{m_A v_A}{m_B v_B}$$

As, $r_A > r_B$ [given]

$$\Rightarrow m_A v_A > m_B v_B$$

28. (c) Minimum wavelength for X-ray is given by

$$\lambda_{\min} = \frac{12375}{V} \text{ \AA}$$

As the accelerating voltage V is decreased, $\lambda_{\min}$ increases.

The intensity of emitted radiation is determined by the number of electrons bombarding the target.

$\therefore$ Accelerating voltage does not change the intensity of X-rays emitted.

29. (b) As we know, $C_V = \frac{n}{2} R$... (i)

$$C_p = \left(\frac{n}{2} + 1 \right) R \quad \dots (ii)$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{C_p}{C_V} = \gamma = \frac{\left(\frac{n}{2} + 1 \right)}{\frac{n}{2}} = 1 + \frac{2}{n}$$

$$\Rightarrow \frac{C_p}{C_V} = 1 + \frac{2}{n}$$

30. (b) Given, $v = 6 \text{ m/s}$, $r = 18 \text{ m}$, $g = 9.8 \text{ m/s}^2$

Now, the velocity of cyclist is given by

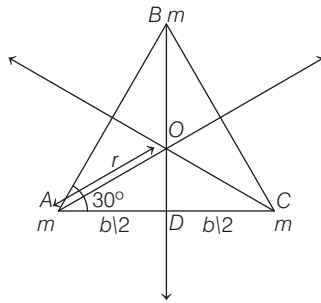
$$v^2 = rg \tan \theta \quad \text{or} \quad \tan \theta = \frac{v^2}{rg} = \frac{6 \times 6}{18 \times 9.8}$$

$$\Rightarrow \tan \theta = 0.2041$$

The minimum value of coefficient of friction, $\mu = \tan \theta$

$$\Rightarrow \mu = 0.2041$$

31. (b) Let us consider an equilateral $\triangle ABC$ of side b .



From figure,

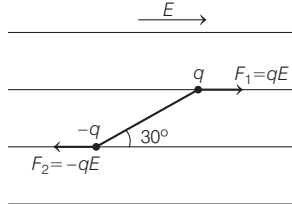
In $\triangle AOD$,

$$\begin{aligned} \cos 30^\circ &= \frac{AD}{AO} \\ \Rightarrow \frac{\sqrt{3}}{2} &= \frac{\frac{b}{2}}{r} \quad \left[\because \cos 30^\circ = \frac{\sqrt{3}}{2} \right] \\ \Rightarrow r &= \frac{b}{2} \times \frac{2}{\sqrt{3}} = \frac{b}{\sqrt{3}} \end{aligned}$$

Moment of inertia about line passing through centroid,

$$\begin{aligned} I &= 3 \times mr^2 \quad [\text{due to 3 masses present in system}] \\ \Rightarrow &= 3 \times m \times \left(\frac{b}{\sqrt{3}} \right)^2 \\ \Rightarrow I &= mb^2 \end{aligned}$$

32. (d) When a dipole is placed in a non-uniform electric field, opposite charges of the dipole experiences a force due to $\mathbf{E}$. It is as shown in the figure below



These forces will act in opposite direction. Now, since the electric field is not uniform, so the force experienced by the charges will be unequal.

Hence, there will be a net force acting on the dipole, as they do not cancel each other.

Also, forces on the charges are not linear as shown above. So, they will experience a non-zero torque. Hence, the dipole will experience both translational force and torque in a non-uniform electric field.

33. (c) Induced emf, $\varepsilon = -M \frac{di}{dt}$

where, i is given as, $i = i_0 \sin \omega t$

$$\begin{aligned} \Rightarrow \varepsilon &= -M \frac{d}{dt} [i_0 \sin \omega t] \\ &= M i_0 \omega \cos \omega t \end{aligned}$$

For maximum value of emf in second coil, $\cos \omega t$ is maximum, i.e. 1.

$$\begin{aligned} E_{\max} &= M i_0 \omega \quad [\because \cos \omega t = 1] \\ \Rightarrow &= 0.001 \times 10 \times 10\pi \\ \Rightarrow E_{\max} &= 0.1 \pi \text{ V} \end{aligned}$$

Hence, maximum emf value induced in second coil is $0.1 \pi \text{ V}$.

34. (b) Diode D_1 does not conduct because it is connected in reversed biased position.

Hence, only D_2 conducts as it is forward biased.

$\therefore$ Current in circuit is given by

$$\begin{aligned} i &= \frac{V}{R} \\ &= \frac{5}{30 + 20} = \frac{5}{50} \text{ A} \Rightarrow i = 0.1 \text{ A} \end{aligned}$$

35. (a) Kinetic energy at $Q = 90\%$ of potential energy at P

$$\begin{aligned} \Rightarrow \frac{1}{2} mv^2 &= \frac{90}{100} \times mgh \\ \Rightarrow v &= \sqrt{1.8gh} \Rightarrow v = \sqrt{1.8 \times 10 \times 2} \\ \Rightarrow v &= 6 \text{ m/s} \end{aligned}$$

36. (c) Let displacement, $x = A \sin \omega t$

$$\text{Velocity, } v = \frac{dx}{dt} = A \omega \cos \omega t$$

Kinetic energy of particle in SHM,

$$\begin{aligned} K &= \frac{1}{2} mv^2 \Rightarrow K = \frac{1}{2} m \omega^2 A^2 \cos^2 \omega t \\ &= \frac{1}{2} m \omega^2 A^2 \left(\frac{1 + \cos 2\omega t}{2} \right) \quad \left[\because \cos^2 \theta = \frac{1 + \cos 2\theta}{2} \right] \\ &= \frac{1}{4} m \omega^2 A^2 (1 + \cos 2\omega t) \end{aligned}$$

$\therefore$ New angular frequency, $\omega_K = 2\omega$

$\therefore$ Frequency of oscillation of kinetic energy = $2f$ $\left[\because f = \frac{\omega}{2\pi} \right]$

37. (a) According to law of radioactivity, the rate of disintegration is given as

$$R = \lambda N = \lambda N_0 e^{-\lambda t}$$

Given, $R = 5000$

$$\Rightarrow 5000 = \lambda N_0 e^{-\lambda t} \quad \dots (i)$$

After 5 min, $R = 1250$

$$\therefore 1250 = \lambda N_0 e^{-\lambda(t+5)} \quad \dots (ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\begin{aligned} \frac{5000}{1250} &= \frac{e^{-\lambda t}}{e^{-\lambda(t+5)}} \\ \Rightarrow 4 &= e^{5\lambda} \end{aligned}$$

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Taking log both sides, we get

$$\log_e 4 = \log_e (e^{5\lambda})$$

$$\Rightarrow \log_e 4 = 5\lambda \Rightarrow \lambda = \frac{1}{5} \log_e 2^2$$

$$\Rightarrow \lambda = 0.4 \log_e 2$$

38. (a) In a uniform electric field, potential difference across two parallel plates of capacitor,

$$V = Ed$$

where, E is electric field and d is separation between plates.

$$\Rightarrow V = 2000 \times (1 \times 10^{-3}) = 2 \text{ V}$$

39. (b) When light travel in meta-material, physical characteristics remain unchanged.

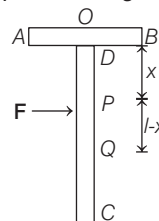
$$\therefore v = \frac{c}{|n|}$$

where, $|n|$ is the relative refractive index of meta-material.

$$\text{As, } n = \frac{c}{v} \Rightarrow |n| = \frac{n_{\text{med}}}{n_{\text{air}}}$$

$$\Rightarrow |n| = \frac{\lambda_{\text{air}}}{\lambda_{\text{med}}} \Rightarrow \lambda_{\text{med}} = \frac{\lambda_{\text{air}}}{|n|}$$

40. (d) Let O be the centre of mass of part AB and Q that of CD . Let M be the mass per unit length of the parts AB and CD .



As, no rotation is set up about point P , so

moment of part AB about P = moment of part CD about P

$$\text{or } (ml)x = (2ml)(l-x) \quad \left[\because m = \frac{M}{l} \right]$$

$$\text{or } x = 2l - 2x$$

$$\text{or } x = 2l/3$$

Distance of P from the end $C = 2l - 2l/3 = 4l/3$

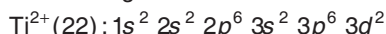
Chemistry

41. (a) *o*-nitrophenol forms intramolecular H-bonding whereas molecules of *p*-nitrophenol get associated through intermolecular H-bond.
 $\therefore$ *o*-nitrophenol is the most volatile compound.

42. (a) The magnetic moment is given by $\mu = \sqrt{n(n+2)}$ BM.

(where, n = number of unpaired electron).

Electronic configuration of



$\therefore$ Number of unpaired electron = 2

$$\therefore \mu = \sqrt{2(2+2)} = \sqrt{8}$$

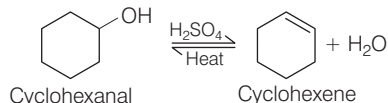
43. (a) Only Xe can form compounds with fluorine among noble gases because Xe is large in size and have high atomic mass. Due to having larger atomic radius the force of attraction between the outer electrons and protons in the nucleus is weaker. Hence, they are easily available to form compound.

44. (a) Helium is used as a cryogenic agent due to its very low melting point.

45. (c) The rate of reaction for the given reaction is

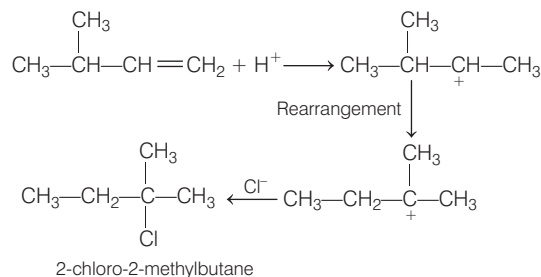
$$r = -\frac{1}{2} \frac{d[A]}{dt} = -\frac{1}{3} \frac{d[B]}{dt} = \frac{1}{3} \frac{d[C]}{dt} = \frac{1}{4} \frac{d[A]}{dt}$$

46. (a) Cyclohexanol on reaction with H_2SO_4 and heating gives cyclohexene as follows



47. (a) Addition of HCl to 3-methylbutene gives 2-chloro-2-methylbutane as major product.

The mechanism reaction is as follows



48. (a) Energy for H-like atom, $(E) \propto \frac{Z^2}{n^2}$

For hydrogen, $Z = 1, n = 1$

For lithium $Z = 3, n = ?$

$\therefore$ For same energy,

$$E_H = E_{Li} \Rightarrow \frac{Z_H^2}{n_H^2} = \frac{Z_{Li}^2}{n_{Li}^2}$$

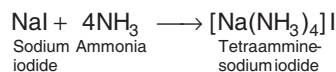
$$\therefore n_{Li}^2 = \frac{Z_{Li}^2 \times n_H^2}{Z_H^2} \Rightarrow n_{Li}^2 = \frac{3^2 \times 1}{1}$$

$$n_{Li} = 3$$

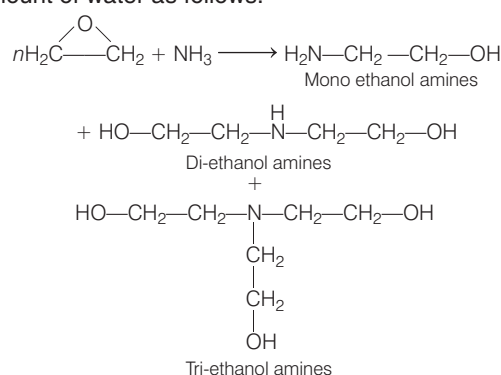
While angular momentum $(nh/2\pi)$ is independent of atomic number.

$\therefore$ For 3s-orbital of lithium atom, energy and angular momentum is equal to that of 1s orbital of hydrogen atom.

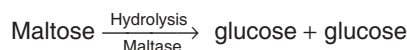
49. (a) Sodium iodide reacts with ammonia to produce tetraamminesodium iodide. The reaction is conducted in liquid ammonia. The reaction is as follows



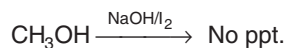
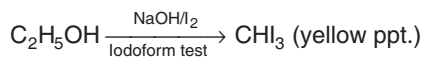
50. (d) Sodium or ammonium thiosulphate is used as a fixing agent in photography. They convert the silver halide into soluble, complex silver salts that dissolve in the fixer. During this process the film loses its original silver halide milky appearance overlaying the image and becomes clear.
51. (d) Epoxy ethane reacts with ammonia forming a mixture of mono, di-, tri-ethanol amines, in the presence of small amount of water as follows.



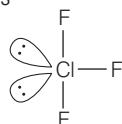
52. (b) Dialysis is a process of removing excess electrolyte from colloidal solution by means of diffusion through a suitable membrane.
53. (a) Maltase converts maltose into glucose by hydrolysis as follows



54. (a) Starch is a polymer of glucose. In sucrose, two monosaccharide (glucose) units are joined by α -1, 2 glycoside bond.
55. (c) Methyl alcohol can be distinguished from ethyl alcohol by iodoform test as follows by using sodium hydroxide and iodine.



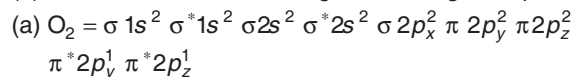
56. (a) Sodium stearate ($\text{C}_{17}\text{H}_{35}\text{COO}^-\text{Na}^+$) is an anionic soap. Anionic part of sodium stearate is involved in cleansing action.
57. (b) BaF_2 is not an anti-fluorite structure. It is of AB_2 type i.e. fluorite (CaF_2) structure. While Rb_2S , K_2O and Li_2O are anti-fluorite structures.
58. (a) The shape of ClF_3 is as follows



T-shape structure

It has two lone pairs at equatorial position.

59. (b) Molecular electronic configuration for given species are

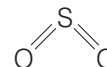


It has two unpaired electrons.

- (b) O_2^{2-} : It has no unpaired electron.
- (c) N_2^{2+} : It has one unpaired electron.
- (d) B_2 : It has two unpaired electrons.
- $\therefore \text{O}_2^{2-}$ is diamagnetic in nature.

60. (c) Bond order for the various compounds are as follows

For SO_2 ,

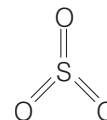


Number of bonds = 4

Number of (SO) groups = 2

$$\therefore \text{Bond order} = \frac{4}{2} = 2$$

For SO_3 ,

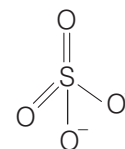


Number of bonds = 6

Number of (SO) groups = 3

$$\therefore \text{Bond order} = \frac{6}{3} = 2$$

For SO_4^{2-} ,

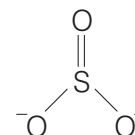


Number of bonds = 6

Number of (SO) groups = 4

$$\therefore \text{Bond order} = \frac{6}{4} = 1.5$$

For SO_3^{2-} ,



Number of bonds = 4

Number of (SO) groups = 3

$$\therefore \text{Bond order} = \frac{4}{3} = 1.33$$

Hence, correct order is $\text{SO}_2 = \text{SO}_3 > \text{SO}_4^{2-} > \text{SO}_3^{2-}$.

61. (b) The energy required for ionisation

$$\text{H-atom is given by, } E = \frac{-kZe^2}{2r}$$

where, Z = atomic number e = charge on proton/electron r = radius of that electron

$$\therefore E \propto \frac{1}{r} \quad \text{or} \quad E \propto r^{-1}$$

$$\therefore n = -1$$

62. (a) BHA is antioxidant (i.e. prevent oxidation). It help in food preservation by retarding the action of oxygen on food. As it is more reactive towards oxygen than the food material.

63. (c) In
- MnO_4^-
- ,

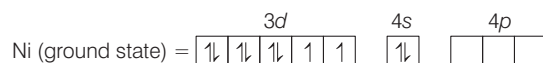
$$x + (-2)4 = -1 \Rightarrow x = +7$$

 ${}_{25}\text{Mn}^{+7} = [\text{Ar}]$, no unpaired electrons.Thus, it will not show $d-d$ transition. It is dark purple coloured due to charge transfer from ligand to metal.

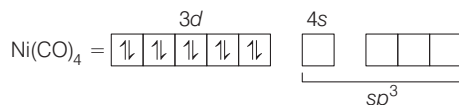
64. (c) Rutherford model could not explain.

- (i) Electronic structure of an atom.
- (ii) Stability of an atom.

65. (a) The oxidation state of Ni in
- $[\text{Ni}(\text{CO})_4]$
- is 0.

The electronic configuration of Ni is $3d^8 4s^2$.

As CO is a strong ligand, pairing of electrons occur.

As, it has sp^3 -hybridisation, so the geometry is tetrahedral. Since, it has no unpaired electrons. So, the complex is diamagnetic.

66. (a) The SI unit of Boltzmann's constant is
- JK^{-1}
- .

67. (a) Acetone does not undergo substitution reaction because they do not contain suitable leaving group.

68. (a) Oxidation number of Co,

$$x + (-1 \times 6) = -2$$

$$\therefore x = 4$$

 $\therefore$ Electronic configuration of Co^{4+} ,

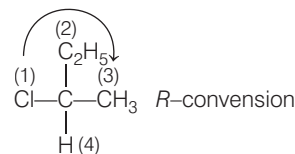
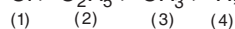
$$\text{Co} : [\text{Ar}] 3d^5$$

 $\therefore$ Number of unpaired electrons = 5

$$\therefore \text{Magnetic moment} = \sqrt{n(n+2)} = \sqrt{5 \times 7} = \sqrt{35} \text{ BM}$$

69. (a) Dielectric constant of
- $\text{H}_2\text{O} > \text{D}_2\text{O}$
- . Therefore,
- H_2O
- is more effective solvent. B. P. of
- $\text{D}_2\text{O} > \text{B.P. of } \text{H}_2\text{O}$
- .

70. (c) According to sequence rule, the priority order are

Cl > C_2H_5 > CH_3 > H, so in R configuration.

71. (c) Molecular mass

$$= \frac{\text{Weight of organic substance taken}}{\text{Air displaced at STP}} \times 22400$$

$$= \frac{0.2}{56} \times 22400 = 80$$

72. (a)
- $\text{C}_2\text{H}_5\text{OH}(l) + 3\text{O}_2(g) \longrightarrow 2\text{CO}_2(g) + 3\text{H}_2\text{O}(l)$

$$\Delta U = -1364.47 \text{ kJ/mol,}$$

$$\Rightarrow \Delta n_g = -1$$

$$T = 25^\circ \text{C} = 298 \text{ K}$$

$$\therefore \Delta H = \Delta U + \Delta n_g RT$$

$$\Delta H = -1364.47 + \frac{-1 \times 8.314 \times 298}{1000}$$

[Here, value of R in unit of J must be converted into kJ]

$$= -1364.47 - 2.4776 = -1366.95 \text{ kJ/mol}$$

73. (c) (a)
- $\Delta G = \Delta H - T \Delta S$

For a system, total entropy change is ΔS_{total} ,

$$\Delta H_{\text{total}} = 0$$

$$\therefore \Delta G_{\text{system}} = -T \Delta S_{\text{total}}$$

$$\therefore \frac{\Delta G_{\text{system}}}{\Delta S_{\text{total}}} = -T$$

Thus, (a) is true,

(b) For isothermal reversible process, $\Delta E = 0$.

By first law of thermodynamics,

$$\Delta E = q + W$$

$$\therefore W_{\text{reversible}} = -q = - \int_{V_i}^{V_f} p dV$$

$$W_{\text{reversible}} = -nRT \ln \frac{V_f}{V_i}$$

Thus, (b) is true.

- (c)
- $\Delta G^\circ = \Delta H^\circ - T \Delta S^\circ$

$$\Delta G^\circ = -nRT \ln K = -RT \ln K \quad (\text{for } n = 1)$$

$$\therefore -RT \ln K = \Delta H^\circ - T \Delta S^\circ$$

$$\therefore \ln K = - \left(\frac{\Delta H^\circ - T \Delta S^\circ}{RT} \right)$$

Thus, (c) is incorrect.

- (d)
- $\Delta G^\circ = -RT \ln K$

$$\therefore \ln K = - \frac{\Delta G^\circ}{RT} \therefore K = e^{-\Delta G^\circ/RT}$$

Thus, (d) is also true.

74. (d)
- $\text{Pb}(\text{OH})_2 \rightleftharpoons \text{Pb}^{2+} + 2\text{OH}^-$

$$K_{\text{sp}} = [\text{Pb}^{2+}][\text{OH}^-]^2 = S \times (2S)^2$$

$$K_{\text{sp}} = 4S^3 = 4 \times (6.7 \times 10^{-6})^3 = 1.20 \times 10^{-15}$$

In a solution with pH = 8

$$[H^+] = 10^{-8} \text{ and } [OH^-] = 10^{-6}$$

$$1.20 \times 10^{-15} = [Pb^{2+}][10^{-6}]^2$$

$$[Pb^{2+}] = \frac{1.2 \times 10^{-15}}{[10^{-6}]^2} = 1.2 \times 10^{-3} \text{ M}$$

75. (c) Molarity = $\frac{10 \times \text{density} \times \text{percentage weight of solute}}{\text{molecular weight of the solute}}$

$$\Rightarrow \text{Density} = \frac{3.60 \times 98}{10 \times 29} = 1.216 = 1.22$$

76. (a) According to Raoult's law, relative lowering of vapour pressure is equal to mole fraction of solute,

$$\therefore \frac{p - p_s}{p} = \frac{n}{n + N} = \frac{w}{m} \frac{w/m}{\frac{w}{m} + \frac{W}{M}} \Rightarrow \frac{w}{m} < < \frac{W}{M}$$

$$\therefore \frac{p^\circ - p_s}{p^\circ} = \frac{w/m}{\frac{W}{M}} = \frac{w}{m} \times \frac{M}{W}$$

$$0.0125 = \frac{w \times 18}{m \times W} \Rightarrow \frac{w}{mW} = \frac{0.0125}{18}$$

$$= \frac{\text{Weight of solute (w)} \times 1000}{\text{Molar mass of solute (m)} \times \text{Weight of H}_2\text{O (W)}}$$

$$\text{Now, molality } m = \frac{0.0125}{18} \times 1000 = 0.69 \approx 0.7$$

77. (c) Suppose the equal mass of methane and oxygen, $w = 1 \text{ g}$

$$\text{Mole fraction of oxygen} = \frac{\frac{w/32}{\frac{w}{32} + \frac{w}{16}}} = \frac{1/32}{\frac{3}{32}} = \frac{1}{3}$$

a. English Proficiency

81. (c) 'Immutable' means 'unchangeable'.

82. (a) 'Ignominious' means 'shameful'.

83. (a) 'Conjecture' means 'making a guess'.

84. (d) Institutional is the correct word to fill the blank. Institutional is the word which means relating to principles esp of law, so legally also every human has rights of freedom and equality.

85. (c) Volumes is the most appropriate option here. Other options do not match here.

86. (c) In the context of the sentence this option 'skirted' is a appropriate word which means to avoid or evade.

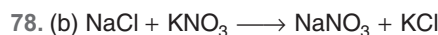
87. (b) Immortal means living forever; never dying or decaying. So, among the given, 'perishable' would be its

Let, the total pressure = p

Pressure exerted by oxygen (partial pressure)

$$= \chi_{O_2} \times p_{\text{total}} = p \times \frac{1}{3}$$

$\therefore$ The fraction of total pressure exerted by oxygen is $\frac{1}{3} p$.



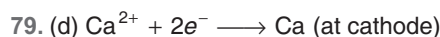
Sum of molar conductivity of reactant
= Sum of molar conductivity of products

$$\Lambda^\circ(\text{NaNO}_3) = \Lambda^\circ_m(\text{NaCl}) + \Lambda^\circ_m(\text{KNO}_3) - \Lambda^\circ_m(\text{KCl})$$

$$\text{KCl} - \text{NaCl} = -\Lambda^\circ_m \text{KCl} - \Lambda^\circ_m(\text{NaCl}) = 24$$

Molar conductivities of $\text{KNO}_3 = 111$

$$\text{Molar conductivities of } \text{NaNO}_3 \\ = 111 - 24 = 87 \text{ S cm}^2 \text{ mol}^{-1}$$



$$1 \text{ mole } \text{CaCl}_2 = 2 \text{ F}$$

$$\text{From } Q = it \Rightarrow n \cdot 2F = it$$

$$\Rightarrow t = \frac{n \cdot 2F}{i} = \frac{30}{40} \times \frac{2 \times 96500}{5}$$

$$= 28950 \text{ s} = \frac{28960}{60 \times 60} = 8.04 = 8 \text{ h}$$

80. (a) As the value of reduction potential of metal ion increases, the tendency of metal oxide to get reduce into metal increases.

Since, reduction potential of only Ag is positive among the given, thus Ag_2O readily gets reduced to Ag metal. In other words, it can be said that Ag_2O is the least stable oxide among the given.

correct opposite meaning word. Perishable means likely to decay easily.

88. (c) 'Opposed' is the correct. Opposite meaning word to the given italicised word 'patronised'.

Patronised means provide favour or support.

Opposed means disagreeing with someone/something.

89. (c) Barbarous means extremely brutal, uncivilised. So, 'civilised' would be its correct opposite meaning word.

90. (c) 'Atheist' is the best alternative.

91. (c) 'Epicure' is the best alternative.

92. (b) 'Philanthropist' is the correct answer, other alternatives are not relevant.

93. (c) CBEDA

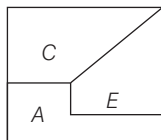
94. (d) CDEAB

95. (d) DEBAC

b. Logical Reasoning

96. (c) Figure shown in option (c) will make a complete square on joining with the problem figure.

97. (c) From figures (A), (C) and (E),



98. (b) The contents of the third figure in each row (and column) are determined by the contents of the first two figures. Lines are carried forward from the first two figures to the third one, except where two lines appear in the same position, in which they are cancelled out.

99. (d) From the given four positions of a single block,
Faces adjacent to face having red colour
= blue, pink, yellow, green
Clearly, black is opposite to red.

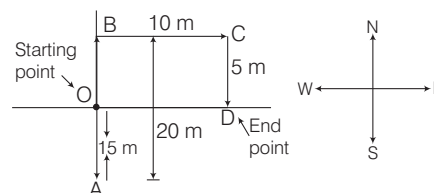
100. (c) In figure (A), the dot is placed in the region which is common to the circle and triangle. Now, we have to search similar common region in the four options. Only in figure (c), we find such a region which is common to the circle and triangle.

101. (b) The pattern is as following

$$\begin{array}{ccccccc} G & \xrightarrow{+3} & J & \xrightarrow{+3} & M & \xrightarrow{+3} & P & \xrightarrow{+3} & S & \xrightarrow{+3} & V \\ 4 & \xrightarrow{\times 2+1} & 9 & \xrightarrow{\times 2+2} & 20 & \xrightarrow{\times 2+3} & 43 & \xrightarrow{\times 2+4} & 90 & \xrightarrow{\times 2+5} & 185 \\ T & \xrightarrow{-2} & R & \xrightarrow{-2} & P & \xrightarrow{-2} & N & \xrightarrow{-2} & L & \xrightarrow{-2} & J \end{array}$$

So, V185J will replace the question mark.

102. (a) According to the given information, the direction of Neeraj is as following



So, it is clearly shown that, Neeraj is 10 m for in East direction from his starting position.

103. (b) Let the average age of 8 men = x yr

Total age of 8 man = $8x$ yr

Now, new average age = $x + 2$ yr

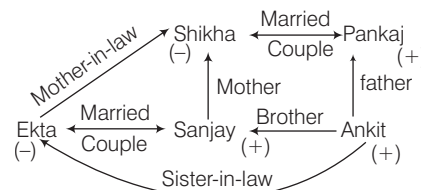
Total age = $8(x + 2)$ yr

Difference of ages = $8(x + 2) - 8x = 8x + 16 - 8x = 16$ yr

$\therefore$ Age of new man = $20 + 16 = 36$ yr

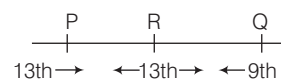
So, the new man is 36 yr older to the man by whom the new man is replaced.

104. (d) The relation is as following



It is clearly shown that, Shikha is the mother of Ankit.

105. (c)



Number of children between P and $R = 40 - (13 + 9 + 4) = 14$

Mathematics

106. (a) Let $z = x + iy$

$$\operatorname{Re}(z + 2) = |z - 2|$$

$$\operatorname{Re}(x + iy + 2) = |x + iy - 2|$$

$$x + 2 = \sqrt{(x - 2)^2 + y^2}$$

Squaring on both sides,

$$(x + 2)^2 = (x - 2)^2 + y^2$$

$$x^2 + 4 + 4x = x^2 + 4 - 4x + y^2 \Rightarrow y^2 = 8x$$

The locus of z is a parabola.

107. (a) Let α, β be the roots of $x^2 - abx - a^2 = 0$

where, $\alpha + \beta = ab$ and $\alpha\beta = -a^2$, which shows product of roots < 0 , i.e. one root must be negative and the other must be positive.

Hence, equation has one positive root and one negative root.

108. (c) Given, $a + 2b + 3c = 12 \forall a, b, c \in \mathbb{R}^+$

As, $AM \geq GM$

$$\frac{a + b + b + c + c + c}{6} \geq \sqrt[6]{ab^2c^3}$$

$$\Rightarrow \frac{12}{6} \geq \sqrt[6]{ab^2c^3} \Rightarrow ab^2c^3 \leq 2^6$$

Hence, the maximum value of ab^2c^3 is 2^6 .

109. (a) a_n of the series = $n(2n + 1)^2 \forall n \in \mathbb{N}$

$$a_n = n[4n^2 + 4n + 1] \Rightarrow a_n = 4n^3 + 4n^2 + n$$

$$S_n = \sum a_n = 4\sum n^3 + 4\sum n^2 + \sum n$$

$$\begin{aligned}
&= 4 \left(\frac{n(n+1)}{2} \right)^2 + \frac{4n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \\
&= \frac{n(n+1)}{2} \left[2n^2 + 2n + \frac{4(2n+1)}{3} + 1 \right] \\
&= \frac{n(n+1)}{2} \left[\frac{6n^2 + 6n + (8n+4) + 3}{3} \right] \\
&= \frac{n(n+1)}{2} \left[\frac{6n^2 + 14n + 7}{3} \right] = \frac{n}{6} (n+1)(6n^2 + 14n + 7)
\end{aligned}$$

110. (d) $\log_{225} 70 = \frac{\log_e 70}{\log_e 225} = \frac{\log_e 7 + \log_e 5 + \log_e 2}{\log_e 25 + \log_e 9}$

$$\begin{aligned}
&= \frac{1 + \frac{\log_e 5}{\log_e 7} + \frac{\log_e 2}{\log_e 7}}{2 \frac{\log_e 5}{\log_e 7} + 2 \frac{\log_e 3}{\log_e 7}} \\
&= \frac{1 + \log_7 5 + \log_3 2 \times \log_5 3 \times \log_7 5}{2[\log_7 5 + \log_5 3 \times \log_7 5]} \\
&= \frac{1 + a + abc}{2[a + ab]} = \frac{1 + a + abc}{2a(1 + b)}
\end{aligned}$$

111. (b) Two circles intersect maximum at two distinct points.

Now, two circles can be selected in ${}^{10}C_2$ ways.

The total number of points of intersection are ${}^{10}C_2 \times 2$

$$= \frac{10 \times 9}{1 \times 2} \times 2 = 90$$

112. (c) $\frac{C_1}{C_0} = n, \frac{C_2}{C_1} = \frac{(n-1)}{2}, \frac{C_3}{C_2} = \frac{n-2}{3}$ and so on.

$$\frac{C_1}{C_0} + 2 \frac{C_2}{C_1} + 3 \frac{C_3}{C_2} + \dots + n \frac{C_n}{C_{n-1}} = \Sigma n = \frac{n(n+1)}{2}$$

On putting $n = 20$,

$$\frac{C_1}{C_0} + 2 \frac{C_2}{C_1} + 3 \frac{C_3}{C_2} + \dots + 20 \frac{C_{20}}{C_{19}} = \frac{20 \times 21}{2} = 210$$

113. (c) Given, $\begin{vmatrix} 0 & x-p & x-q \\ x+p & 0 & x-r \\ x+q & x-r & 0 \end{vmatrix} = 0$

$$-(x-p) \begin{vmatrix} x+p & x-r \\ x+q & 0 \end{vmatrix} + (x-q) \begin{vmatrix} x+p & 0 \\ x+q & x-r \end{vmatrix} = 0$$

$$\Rightarrow (x-p)(x+q)(x-r) + (x-q)(x+p)(x-r) = 0$$

$$\Rightarrow (x-r)[(x-p)(x+q) + (x-q)(x+p)] = 0$$

$$\Rightarrow (x-r)[x^2 - px + qx - pq + x^2 - qx + px - pq] = 0$$

$$\Rightarrow (x-r)[2x^2 - 2pq] = 0 \Rightarrow x-r = 0 \text{ or } x^2 - pq = 0$$

either $x = r$ or pq

114. (c) $A \cdot \text{adj } A = |A|I$

$$|A| = xyz - 8x - 3(z-8) + 2(2-2y)$$

$$|A| = xyz - (8x + 3z + 4y) + 28$$

$$\Rightarrow 60 - 20 + 28 = 68$$

$\Rightarrow (\text{adj } A)^{-1}$ always exists whenever $(A)^{-1}$ exists.

$$\therefore A \cdot \text{adj } A = |A|I$$

$$= 68 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$$

115. (b) Given, $f(x) = 4x - x^2$

$$f(a+1) - f(a-1)$$

$$= [4(a+1) - (a+1)^2] - [4(a-1) - (a-1)^2]$$

$$= [4a + 4 - a^2 - 1 - 2a] - [4a - 4 - a^2 - 1 + 2a]$$

$$\Rightarrow 8 - 4a = 0$$

$$\therefore a = 2$$

116. (c) Let $R = \{(a, b) : a + b \text{ is an even integer } a, b \in \mathbb{Z}\}$

For $a \in \mathbb{Z}$, $a + a = 2a$ is an even integer.

$$\therefore (a, a) \in R \quad \forall a \in \mathbb{Z}$$

$\therefore R$ is reflexive.

Let $(a, b) \in R \Rightarrow a + b$ is an even integer.

$$\Rightarrow (b + a) \text{ is an even integer} \Rightarrow (b, a) \in R$$

$\Rightarrow R$ is symmetric.

$$\text{Let } (a, b), (b, c) \in R$$

$\Rightarrow (a + b)$ and $(b + c)$ are even integers.

$$\Rightarrow (a + b) + (b + c) = (a + c + 2b) \text{ is an even integer.}$$

$$\Rightarrow (a + c + 2b) - 2b = (a + c) \text{ is an even integer.}$$

$$\Rightarrow (a, c) \in R \Rightarrow R \text{ is transitive.}$$

$\therefore R$ is an equivalence relation.

$$\text{Let } R = \{(a, b) : (a - b) \text{ is an even integer, } a, b \in \mathbb{Z}\}.$$

For $a \in \mathbb{Z}$, $a - a = 0$ is an even integer.

$$\therefore (a, a) \in R \quad \forall a \in \mathbb{Z}$$

$\therefore R$ is reflexive.

$$\text{Let } (a, b) \in R \Rightarrow (a - b) \text{ is an even integer.}$$

$$\Rightarrow -(a - b) \text{ is an even integer.}$$

$$\Rightarrow (b - a) \text{ is an even integer} \Rightarrow (b, a) \in R$$

$\Rightarrow R$ is symmetric.

$$\text{Let } (a, b), (b, c) \in R \Rightarrow (a - b), (b - c) \text{ are even integers.}$$

$$\Rightarrow (a - b) + (b - c) \text{ is an even integer.}$$

$$\Rightarrow (a - c) \text{ is an even integer.}$$

$$\Rightarrow (a, c) \in R$$

$\Rightarrow R$ is transitive.

$\therefore R$ is an equivalence relation.

$$\text{Let } R = \{(a, b) : a < b, a, b \in \mathbb{Z}\}$$

Let $a \in \mathbb{Z}$. $a < a$ is false.

$\therefore R$ is not reflexive.

$\therefore R$ is not an equivalence relation.

$$\text{Let } R = \{(a, b) : a = b, a, b \in \mathbb{Z}\}.$$

It is quite easy to check that R is an equivalence relation.

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117. (b) Since, $\sim(p \vee q) \equiv (\sim p \wedge \sim q)$ and $\sim(p \wedge q) \equiv (\sim p \vee \sim q)$

So, options (a) and (d) are not true.

$\sim(p \rightarrow q) \equiv p \wedge \sim q$, so option (c) is not true.

Now, $p \rightarrow q \equiv p \vee q$

$\sim q \rightarrow \sim p \equiv [\sim(\sim q) \vee \sim p]$

$\equiv q \vee \sim p \equiv \sim p \vee q \Rightarrow p \rightarrow q \equiv (\sim q \rightarrow \sim p)$

118. (a) $4^{-x+0.5} - 7 \cdot 2^{-x} < 4$

Let $2^{-x} = t$

The equation becomes

$$2t^2 - 7t - 4 < 0 \Rightarrow (t - 4)(2t + 1) < 0$$

$$\Rightarrow 2(t - 4) \left\{ t - \left(-\frac{1}{2} \right) \right\} < 0 \Rightarrow -\frac{1}{2} < t < 4$$

Since, $t = 2^{-x} > 0 \forall x \in R$

$$\Rightarrow 0 < t < 4 \Rightarrow 0 < 2^{-x} < 2^2$$

As, 2^x is an increasing function,

$$-x < 2 \text{ or } x > -2$$

Solution is $(-2, \infty)$.

119. (a) $\cos^3 x \cdot \sin 2x = \frac{\cos 3x + 3 \cos x}{4} \times \sin 2x$

$$= \frac{1}{8} (\sin 5x - \sin x) + \frac{3}{8} (\sin 3x + \sin x)$$

$$= \frac{1}{4} \sin x + \frac{3}{8} \sin 3x + \frac{1}{8} \sin 5x$$

$$\text{Here, } n = 5, a_1 = \frac{1}{4}, a_2 = 0, a_3 = \frac{3}{8}, a_4 = 0, a_5 = \frac{1}{8}$$

120. (b) $|\cot x| = \cot x + \frac{1}{\sin x}$

$$\text{Let } \cot x > 0 \Rightarrow \cot x = \cot x + \frac{1}{\sin x} = 0$$

$$\Rightarrow \frac{1}{\sin x} = 0 \text{ which is not possible.}$$

$$\text{Let } \cot x \leq 0 \Rightarrow -\cot x = \cot x + \frac{1}{\sin x}$$

$$\Rightarrow -2 \cot x = \frac{1}{\sin x} \Rightarrow \cos x = -\frac{1}{2} \Rightarrow x = \frac{2\pi}{3}, \frac{8\pi}{3}$$

$\therefore$ The number of solutions is 2.

121. (a) Let $I = (\sin^{-1} x)^3 + (\cos^{-1} x)^3$

$$= (\sin^{-1} x + \cos^{-1} x) [(\sin^{-1} x)^2 + (\cos^{-1} x)^2$$

$$- (\sin^{-1} x)(\cos^{-1} x)]$$

$$= \frac{\pi}{2} \left[(\sin^{-1} x + \cos^{-1} x)^2 - 3 \sin^{-1} x \left(\frac{\pi}{2} - \sin^{-1} x \right) \right]$$

$$= \frac{\pi}{2} \left(\frac{\pi^2}{4} - \frac{3\pi}{2} \sin^{-1} x + 3(\sin^{-1} x)^2 \right)$$

$$= \frac{\pi}{2} \left[3 \left((\sin^{-1} x)^2 - \frac{\pi}{2} (\sin^{-1} x) + \frac{\pi^2}{16} - \frac{\pi^2}{16} \right) + \frac{\pi^2}{4} \right]$$

$$= \frac{\pi}{2} \left(\frac{\pi^2}{16} + 3 \left(\sin^{-1} x - \frac{\pi}{4} \right)^2 \right)$$

$$\text{Now, } \frac{\pi}{4} \geq \sin^{-1} x - \frac{\pi}{4} \geq -\frac{3\pi}{4}$$

$$\Rightarrow \frac{9\pi^2}{16} \geq \left(\sin^{-1} x - \frac{\pi}{4} \right)^2 \geq 0 \Rightarrow I \geq \frac{\pi}{2} \cdot \frac{\pi^2}{16} = \frac{\pi^3}{32}$$

122. (b) Let $P(x, y)$ be the original position of the point w.r.t. the original axes. Let us move the origin at new position to (h, k) .

Hence, the position of the same point P in the new system is

$$x' = x - h \Rightarrow y' = y - k$$

Here, $(h, k) = (1, 2)$

$$\therefore x' = (x - 1), y' = (y - 2)$$

According to the question,

$$y^2 - 8x - 4y + 12 = (y - 2)^2 - 4a(x - 1)$$

$$\Rightarrow y^2 - 8x - 4y + 12 = y^2 - 4y + 4 - 4ax + 4a$$

On comparing respective coefficients, we get

$$4a = 8 \Rightarrow a = 2$$

123. (d) Equations of the bisectors of the angles between the given straight lines are given by

$$\frac{3x + 4y + 7}{\sqrt{9 + 16}} = \pm \frac{12x + 5y - 8}{\sqrt{144 + 25}}$$

$$\Rightarrow 13(3x + 4y + 7) = \pm 5(12x + 5y - 8)$$

$$\Rightarrow 39x + 52y + 91 = \pm (60x + 25y - 40)$$

Taking positive signs,

$$39x + 52y + 91 = 60x + 25y - 40$$

$$\Rightarrow -21x + 27y + 131 = 0$$

$$\Rightarrow 21x - 27y - 131 = 0$$

Taking negative signs,

$$(39x + 52y + 91) = -(60x + 25y - 40)$$

$$\Rightarrow 99x + 77y + 51 = 0$$

124. (b) Since, the circle touches X-axis,

$$(x - h)^2 + (y - k)^2 = k^2 \quad \dots (i)$$

Also, it passes through the points $(1, -2)$ and $(3, -4)$.

$$(1 - h)^2 + (-2 - k)^2 = k^2 \quad \dots (ii)$$

$$\text{and } (3 - h)^2 + (-4 - k)^2 = k^2 \quad \dots (iii)$$

Subtracting Eq. (iii) from Eq. (ii), we get

$$h = k + 5$$

On solving these equations, we get

$$k = -10, -2 \text{ and } h = -5, 3$$

By putting the values of $(h, k) = (-5, -10)$ or $(3, -2)$ in Eq. (i), we get

$$x^2 + y^2 + 10x + 20y + 25 = 0 \text{ or } x^2 + y^2 - 6x + 4y + 9 = 0$$

125. (b) $x = 9$ meets the hyperbola at $(9, 6\sqrt{2})$ and $(9, -6\sqrt{2})$.
Then, the equations of tangent at these points are $3x - 2\sqrt{2}y - 3 = 0$ and $3x + 2\sqrt{2}y - 3 = 0$. The combined equation of these two tangents is $9x^2 - 8y^2 - 18x + 9 = 0$.

126. (d) Let the points be $A(10\hat{i} + 3\hat{j})$,

$$B(12\hat{i} - 15\hat{j}) \text{ and } C(a\hat{i} + 11\hat{j}).$$

$$AB = |2\hat{i} - 18\hat{j}| \text{ and } AC = (a - 10)\hat{i} + 8\hat{j}$$

Since, A, B and C are collinear, then

$$\frac{2}{a - 10} = -\frac{18}{8} \Rightarrow a = \frac{82}{9}$$

127. (d) We have, $|a| = 3, |b| = 4$ and $|c| = 5$.

It is given that

$$a \perp (b + c), b \perp (c + a) \text{ and } c \perp (a + b)$$

$$\Rightarrow a \cdot (b + c) = 0, b \cdot (c + a) = 0 \text{ and } c \cdot (a + b) = 0$$

$$\Rightarrow a \cdot b + a \cdot c = b \cdot c + b \cdot a = c \cdot a + c \cdot b = 0$$

$$\text{or } a \cdot b + b \cdot c + c \cdot a = 0$$

(adding all the above equations)

$$\begin{aligned} \text{Now, } |a + b + c|^2 &= |a|^2 + |b|^2 + |c|^2 + 2(a \cdot b + b \cdot c + c \cdot a) \\ &= 3^2 + 4^2 + 5^2 = 50 \end{aligned}$$

$$\therefore |a + b + c| = 5\sqrt{2}$$

128. (d) We have, $|(a \times b) \cdot c| = |a| |b| |c|$

$$\Rightarrow ||a|| |b| |c| |\sin \theta \cos \alpha| = |a| |b| |c|$$

$$\Rightarrow |\sin \theta| |\cos \alpha| = 1 \Rightarrow \theta = \frac{\pi}{2} \text{ and } \alpha = 0$$

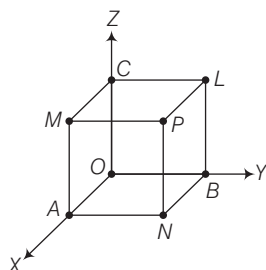
$$\Rightarrow a \perp b \text{ and } c \parallel \hat{n} \Rightarrow a \perp b \text{ and } c \perp \text{both } a \text{ and } b.$$

$$\Rightarrow a, b, c \text{ are mutually perpendicular.}$$

$$\Rightarrow a \cdot b = b \cdot c = c \cdot a = 0$$

129. (c) Let each edge of cube be a , then coordinates of the vertices of cube are

$$O(0, 0, 0), A(a, 0, 0), B(0, a, 0), C(0, 0, a), N(a, a, 0), P(a, a, a), L(0, a, a), M(a, 0, a)$$



Direction ratios of the diagonals OP, AL, BM and CN are $(a, a, a), (-a, a, a), (a, -a, a)$ and $(a, a, -a)$.

Let θ be the acute angle between diagonals OP and AL .

$$\begin{aligned} \therefore \cos \theta &= \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \\ &= \frac{a \times (-a) + a \times a + a \times a}{\sqrt{a^2 + a^2 + a^2} \sqrt{(-a)^2 + a^2 + a^2}} \\ &= \frac{-a^2 + a^2 + a^2}{\sqrt{3a^2} \sqrt{3a^2}} = \frac{a^2}{a\sqrt{3} \times a\sqrt{3}} \end{aligned}$$

$$\Rightarrow \cos \theta = \frac{1}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{3}\right)$$

130. (b) Given lines are

$$L_1: \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2} \text{ and } L_2: \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$$

Now, convert into vector form

$$L_1: (-\hat{i} - 2\hat{j} - \hat{k}) + \lambda(3\hat{i} + \hat{j} + 2\hat{k})$$

$$L_2: (2\hat{i} + 2\hat{j} + 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 3\hat{k})$$

Line L_1 comparing with $a_1 + \lambda b_1$,

and L_2 comparing with $a_2 + \mu b_2$, then we have

$$b_1 = 3\hat{i} + \hat{j} + 2\hat{k} \text{ and } b_2 = \hat{i} + 2\hat{j} + 3\hat{k}$$

Perpendicular to both b_1 and $b_2 = b_1 \times b_2$

$$\begin{aligned} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} \\ &= \hat{i}(3-4) - \hat{j}(9-2) + \hat{k}(6-1) = -\hat{i} - 7\hat{j} + 5\hat{k} \end{aligned}$$

$$\begin{aligned} \therefore \text{Required unit vector} &= \frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{\sqrt{(-1)^2 + (-7)^2 + (5)^2}} \\ &= \frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{\sqrt{75}} = \frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}} \end{aligned}$$

131. (b) The given line is $r = a + tb$, where $a = 2\hat{i} - 2\hat{j} + 3\hat{k}$, $b = \hat{i} - \hat{j} + 4\hat{k}$ and given plane is $r \cdot n = d$, where $n = \hat{i} + 5\hat{j} + \hat{k}, d = 5$.

$$\text{Since, } b \cdot n = 1 - 5 + 4 = 0$$

$\therefore$ Given line is parallel to given plane.

$\therefore$ The distance between the line and the plane is equal to length of the perpendicular from the point $a = 2\hat{i} - 2\hat{j} + 3\hat{k}$ on the line to the given plane.

$$\begin{aligned} \therefore \text{Required distance} &= \left| \frac{(2\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (\hat{i} + 5\hat{j} + \hat{k}) - 5}{\sqrt{1 + 25 + 1}} \right| \\ &= \left| \frac{2 - 10 + 3 - 5}{\sqrt{27}} \right| = \frac{10}{3\sqrt{3}} \end{aligned}$$

132. (c) Let E_1 = event of getting both red cards

$$E_2 = \text{event of getting both kings}$$

and $E_1 \cap E_2$ = event of getting 2 kings of red cards

$$\therefore P(E_1) = \frac{{}^{26}C_2}{{}^{52}C_2} = \frac{325}{1326}, P(E_2) = \frac{{}^4C_2}{{}^{52}C_2} = \frac{6}{1326}$$

and $P(E_1 \cap E_2) = \frac{{}^2C_2}{{}^{52}C_2} = \frac{1}{1326}$

$$\begin{aligned} \therefore P(\text{both red or both kings}) &= P(E_1 \cup E_2) \\ &= P(E_1) + P(E_2) - P(E_1 \cap E_2) \\ &= \frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326} = \frac{330}{1326} = \frac{55}{221} \end{aligned}$$

133. (d) Since, A and B are independent events.

$$\therefore P(A/B) = P(A) = \frac{1}{2} \Rightarrow P\left(\frac{A}{A \cup B}\right) = \frac{P[A \cap (A \cup B)]}{P(A \cup B)}$$

$$\begin{aligned} \Rightarrow P\left[\frac{A}{A \cup B}\right] &= \frac{P(A)}{P(A \cup B)} = \frac{1/2}{\frac{1}{2} + \frac{1}{5} - \frac{1}{10}} \\ &= \frac{1/2}{\frac{1}{2} + \frac{1}{5} - \frac{1}{10}} = \frac{1/2}{\frac{6}{10}} = \frac{5}{6} \end{aligned}$$

$$\text{Similarly, } P\left(\frac{A \cap B}{A' \cup B'}\right) = P\left(\frac{A \cap B}{(A \cap B)'}\right) = 0$$

134. (b) Let E_1 = coin shows head, E_2 = coin shows tail,

A = drawn ball is blue

$$P(E_1) = \frac{1}{2} = P(E_2)$$

$P(A/E_1)$ = Probability of drawing a blue ball from bag I = $2/7$

$$P\left(\frac{A}{E_2}\right) = \text{Probability of drawing a blue ball from bag II} = \frac{6}{8}$$

By Baye's theorem, we have

$$\begin{aligned} P\left(\frac{E_1}{A}\right) &= \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)} \\ &= \frac{\frac{1}{2} \times \frac{2}{7}}{\frac{1}{2} \times \frac{2}{7} + \frac{1}{2} \times \frac{6}{8}} = \frac{\frac{2}{7}}{\frac{2}{7} + \frac{6}{8}} = \frac{2}{7} \times \frac{56}{58} = \frac{8}{29} \end{aligned}$$

135. (c) Given, $E(X) = 3$ and $E(X^2) = 11$

$$\text{Variable of } X = E(X^2) - [E(X)]^2 = 11 - 3^2 = 11 - 9 = 2$$

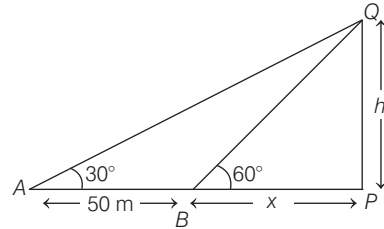
136. (d) Given, $x_1 + x_2 + x_3 + \dots + x_{10} = 12$

$$\text{and } x_1^2 + x_2^2 + \dots + x_{10}^2 = 18$$

$$\begin{aligned} \sigma^2 &= \frac{1}{n} \sum x^2 - \left(\frac{1}{n} \sum x\right)^2 \\ &= \frac{1}{10} \times 18 - \left(\frac{1}{10} \times 12\right)^2 = \frac{9}{5} - \frac{36}{25} = \frac{9}{25} \end{aligned}$$

$\therefore$ Standard deviation = $3/5$

137. (c) Let PQ be the chimney whose height is h metres.



$$\text{In } \triangle BPQ, \tan 60^\circ = \frac{PQ}{BP} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x \quad \dots(i)$$

$$\text{and in } \triangle APQ, \tan 30^\circ = \frac{PQ}{AP}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{50+x} \Rightarrow 50+x = h\sqrt{3}$$

$$\Rightarrow 50+x = 3x \Rightarrow x = 25 \quad [\text{using Eq. (i)}]$$

$\therefore$ Height of the chimney = $25\sqrt{3}$ m

138. (c) The required limit

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{\sqrt{(1-\cos x^2)(1+\cos x^2)}}{(1-\cos x)\sqrt{1+\cos x^2}} \\ &= \lim_{x \rightarrow 0} \left(\frac{\frac{\sin x^2}{x^2}}{2\left(\frac{\sin^2 x/2}{x^2}\right)} \right) \cdot \frac{1}{\sqrt{1+\cos x^2}} = \frac{1}{2 \times \frac{1}{4} \times \sqrt{2}} = \sqrt{2} \end{aligned}$$

$$139. (c) f'(1-0) = \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{a(1-h)^2 + 1 - (1+a)}{-h} = \lim_{h \rightarrow 0} \frac{a(h^2 - 2h)}{-h} = 2a$$

$$f'(1+0) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(1+h)^2 + a(1+h) + b - (1+a)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h^2 + 2h + ah + b}{h} = 2 + a, \text{ if } b = 0$$

$$\text{Thus, } 2a = 2 + a, b = 0 \Rightarrow a = 2, b = 0$$

140. (c) We have, $\frac{dx}{dt} = 2t + 3$ and $\frac{dy}{dt} = 4t - 2$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4t-2}{2t+3}$$

Thus, slope of the tangent to the curve at the point $t = 2$ is

$$\left[\frac{dy}{dx}\right]_{t=2} = \frac{4(2)-2}{2(2)+3} = \frac{6}{7}$$

$$141. (c) \text{ Let } I = \int \frac{1}{1-2\sin x} dx$$

$$\text{On putting } \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \text{ we get}$$

$$\begin{aligned}
 I &= \int \frac{1}{1 - \frac{4 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}} dx = \int \frac{1 + \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2} - 4 \tan \frac{x}{2}} dx \\
 &= \int \frac{\sec^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2} - 4 \tan \frac{x}{2}} dx \\
 &= 2 \int \frac{1}{\left(\tan \frac{x}{2} - 2\right)^2 - (\sqrt{3})^2} d\left(\tan \frac{x}{2}\right) \\
 &= 2 \int \frac{dt}{(t-2)^2 - (\sqrt{3})^2}, \text{ where } t = \tan \frac{x}{2} \\
 &= 2 \cdot \frac{1}{2\sqrt{3}} \log \left| \frac{t-2-\sqrt{3}}{t-2+\sqrt{3}} \right| + c = \frac{1}{\sqrt{3}} \log \left| \frac{\tan \frac{x}{2} - 2 - \sqrt{3}}{\tan \frac{x}{2} - 2 + \sqrt{3}} \right| + c
 \end{aligned}$$

142. (a) Let $I = \int_0^1 \frac{\log(1+x)}{1+x^2} dx = \int_0^{\pi/4} \frac{\log(1+\tan\theta)}{(1+\tan^2\theta)} \sec^2\theta d\theta$

Let $x = \tan\theta \Rightarrow dx = \sec^2\theta d\theta$

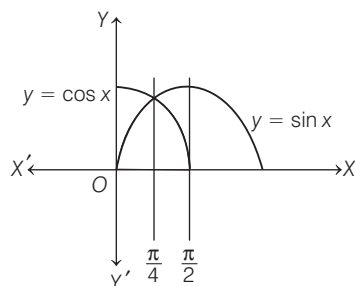
$$= \int_0^{\pi/4} \log(1+\tan\theta) d\theta = \int_0^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right] d\theta$$

$$= \int_0^{\pi/4} \log\left(1+\frac{1-\tan\theta}{1+\tan\theta}\right) d\theta$$

$$= \int_0^{\pi/4} \log\left(\frac{2}{1+\tan\theta}\right) d\theta = \log 2 \int_0^{\pi/4} 1 d\theta - I$$

$$\Rightarrow 2I = \frac{\pi}{4} \log 2 \Rightarrow I = \frac{\pi}{8} \log 2$$

143. (c) Required area



$$= \int_0^{\pi/4} \sin x dx + \int_{\pi/4}^{\pi/2} \cos x dx$$

$$= -[\cos]_0^{\pi/4} + [\sin x]_{\pi/4}^{\pi/2} = 2\left(1 - \frac{1}{\sqrt{2}}\right) = (2 - \sqrt{2}) \text{ sq units}$$

144. (b) $\left(\frac{x+y-1}{x+y-2}\right) \frac{dy}{dx} = \left(\frac{x+y+1}{x+y+2}\right)$

Put $x+y=v$

$$\therefore 1 + \frac{dy}{dx} = \frac{dv}{dx} \text{ or } \left(\frac{v-1}{v-2}\right) \left(\frac{dv}{dx} - 1\right) = \left(\frac{v+1}{v+2}\right)$$

$$\Rightarrow \frac{dv}{dx} - 1 = \frac{(v+1)(v-2)}{(v-1)(v+2)} = \frac{v^2 - v - 2}{v^2 + v - 2} \text{ or } \frac{dv}{dx} = \frac{2v^2 - 4}{(v^2 + v - 2)}$$

$$\Rightarrow \frac{(v^2 + v - 2)}{(v^2 - 2)} dv = 2 dx \Rightarrow \left(1 + \frac{v}{v^2 - 2}\right) dv = 2 dx$$

On integrating, we get

$$v + \frac{1}{2} \log |v^2 - 2| = 2x + c$$

or $(y-x) + \frac{1}{2} \log |(x+y)^2 - 2| = c$

Given, $y = 1$ when $x = 1$

$$\therefore 0 + \frac{1}{2} \log 2 = c \text{ or } (y-x) + \frac{1}{2} \log \left| \frac{(x+y)^2 - 2}{2} \right| = 0$$

or $\log \left| \frac{(x+y)^2 - 2}{2} \right| = 2(x-y)$

145. (d) Given, $y = \sin\left(2 \tan^{-1} \sqrt{\frac{1-x}{1+x}}\right)$

$$\text{Let } x = \cos 2\theta \Rightarrow y = \sin\left(2 \tan^{-1} \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\right)$$

$$\Rightarrow y = \sin\left(2 \tan^{-1} \sqrt{\frac{2 \sin^2 \theta}{2 \cos^2 \theta}}\right) \Rightarrow y = \sin[2 \tan^{-1}(\tan \theta)]$$

$$\Rightarrow y = \sin 2\theta \Rightarrow y = \sqrt{1 - \cos^2 2\theta} \Rightarrow y = \sqrt{1 - x^2}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}} (-2x) = -\frac{x}{\sqrt{1-x^2}}$$

146. (b) We have, $y = x(x-1)^2$

$$\therefore \frac{dy}{dx} = (x-1)^2 \cdot 1 + 2x(x-1) = 3x^2 - 4x + 1 = (x-1)(3x-1)$$

For maximum or minimum, $\frac{dy}{dx} = 0$

$$\Rightarrow (x-1)(3x-1) = 0 \Rightarrow x = \frac{1}{3}, 1$$

Now, $\frac{d^2y}{dx^2} = 6x - 4 \Rightarrow \left[\frac{d^2y}{dx^2}\right]_{x=\frac{1}{3}} = -2 < 0$

$\therefore y$ is maximum when $x = \frac{1}{3}$ and maximum value is

$$[y]_{x=\frac{1}{3}} = \frac{4}{27}$$

Also,

$$\left[\frac{d^2y}{dx^2}\right]_{x=1} = 2 > 0$$

$\therefore y$ is minimum when $x = 1$

147. (b) We have, $\cos y \frac{dy}{dx} + \frac{4}{x} \sin y = \frac{e^x}{x^3}$

Let $\sin y = t$

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$$\Rightarrow \cos y \frac{dy}{dx} = \frac{dt}{dx} \therefore \frac{dt}{dx} + \frac{4}{x}t = \frac{e^x}{x^3}$$

$$\text{IF} = e^{\int \frac{4}{x} dx} = e^{4 \log x} = x^4$$

The solution is

$$\therefore tx^4 = \int x^4 \cdot \frac{e^x}{x^3} dx = xe^x - e^x + c \therefore \sin y x^4 = xe^x - e^x + c$$

$$\therefore x = 1, y = 0$$

$$[\because c = 0]$$

$$\therefore \sin y = e^x(x-1)x^{-4}$$

148. (a) Let us make the table from the given data

Player A			Player B		
x_i	$x_i - 61$	$(x_i - 61)^2$	y_i	$y_i - 61$	$(y_i - 61)^2$
58	-3	9	94	33	1089
59	-2	4	26	-35	1225
60	-1	1	92	31	961
54	-7	49	65	4	16
65	4	16	96	35	1225
66	5	25	78	17	289
52	-9	81	14	-47	2209
75	14	196	34	-27	729
69	8	64	98	37	1369
52	-9	81	13	-48	2304
$\sum x_i = 610$		$\sum (x_i - 61)^2 = 526$	$\sum y_i = 610$		$\sum (y_i - 61)^2 = 11416$

$$\text{For player A, Mean} = \frac{\sum x_i}{n} = \frac{610}{10} = 61$$

$$\begin{aligned} \text{SD} &= \sqrt{\frac{\sum (x_i - 61)^2}{N}} \\ &= \sqrt{\frac{526}{10}} = 7.25 \end{aligned}$$

$$\text{For player B, Mean} = \frac{\sum y_i}{n} = \frac{610}{10} = 61$$

$$\begin{aligned} \text{SD} &= \sqrt{\frac{\sum (y_i - 61)^2}{N}} \\ &= \sqrt{\frac{11416}{10}} = 33.79 \end{aligned}$$

Since, SD for player A is 7.25 < SD for player B is 33.79.

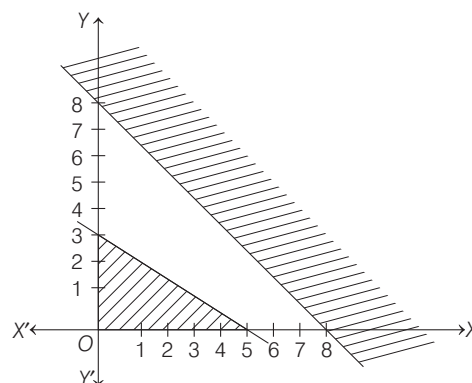
Hence, player A is more consistent player.

149. (b) Table for equation $x + y = 8$ is

x	0	8
$y = 8 - x$	8	0

Table for equation $3x + 5y = 15$ is

x	0	5
$y = \frac{15 - 3x}{5}$	3	0



It can be concluded from the graph, that there is no point which can satisfy all the constraints simultaneously. Therefore, the problem has no feasible solution.

$$\begin{aligned} 150. (a) \text{ Substitute 0 for } y \text{ in the equation } 4y^2 &= 4x^2 - x^3, \\ 0 &= x^2(4 - x) \Rightarrow x = 0, x = 4 \end{aligned}$$

It means curve makes the loop symmetric about X-axis between 0 and 4.

$$\text{Area} = 2 \int_0^4 y dx = \frac{2}{2} \int_0^4 \sqrt{4x^2 - x^3} dx = \int_0^4 x \sqrt{4 - x} dx$$

$$\text{Let } 4 - x = t \Rightarrow -dx = dt$$

$$\text{Area} = - \int_4^0 (4 - t) \sqrt{t} dt$$

$$= \int_0^4 (4\sqrt{t} - t\sqrt{t}) dt = \left[4 \times \frac{2}{3} t^{3/2} - \frac{2}{5} t^{5/2} \right]_0^4$$

$$= \frac{8}{3} \times 8 - \frac{2}{5} \times 32 = \frac{64}{3} - \frac{64}{5} = 64 \times \frac{2}{15} = \frac{128}{15}$$